<http://www.diva-portal.org>

Postprint

This is the accepted version of a paper published in *Composite structures*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Ekh, J., Schön, J., Zenkert, D. (2013)

Simple and efficient prediction of bearing failure in single shear, composite lap joints.

Composite structures, 105: 35-44

<http://dx.doi.org/10.1016/j.compstruct.2013.04.038>

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:kth:diva-126034>

Simple and Efficient Prediction of Bearing Failure in Single Shear, Composite Lap Joints

Johan Ekh^{a,*}, Joakim Schön^b, Dan Zenkert^a

*^aDivision of Lightweight Structures,
Department of Aeronautical and Vehicle Engineering,
The Royal Institute of Technology,
SE-100 44 Stockholm, Sweden*

*^bSwedish Defence Research Agency, FOI
SE-172 90 Stockholm, Sweden*

Abstract

A straightforward procedure to predict bearing strength in bolted composite structures has been developed. The method is based on a finite element analysis, using structural elements, followed by a post-processing procedure. Bolt-hole clearance, friction between member plates, fastener clamp-up and fastener deformation is accounted for. Forces calculated in the FE-analysis are converted into a local stress field which is used in an existing criterion to predict fiber micro buckling in the most critically loaded lamina. Predictions were compared with experiments which validated the method. The small computational cost required by the procedure suggests that the method is applicable on large scale structures and suitable to use in conjunction with iterative schemes such as optimization and statistical investigations.

Keywords: Bolted composite joint, Bearing failure, Fiber micro buckling

*Corresponding author

Phone: +46 70 610 2055

Email: ekh@kth.se

1. Introduction

The present paper addresses the issue of designing effective multi-fastener, single shear composite joints. Composite materials are used increasingly in aircraft structures and bolted joints are weak spots that limit the overall efficiency of the structure. Successful design of such joints requires predictive capability regarding failure. Failure in composite joints is governed by the local stress field and by the way the material reacts to these stresses. Thus, prediction of failure implies that the local multi-axial stress field must be determined first and then used in a suitable failure criterion.

Determining the local stress field in a multi-fastener, composite joint is a complex task that has been attempted by several workers [1, 2, 3, 4, 5, 6] using different techniques. Usually, a relatively coarse analysis is conducted to calculate how the load is distributed among the fasteners, followed by a more detailed analysis of critical fasteners. Some workers have used detailed 3D FE-analysis to conduct both the load distribution analysis and the failure analysis. However, the coarse analysis techniques generally fail to take all important parameters into account, e.g. bolt-hole clearance, friction and fastener clamp-up, whereas the detailed methods are too computationally expensive to be used in analysis of large scale structures or in optimization. A method to calculate the load distribution for a general structure, and accounting for the above parameters, was developed by Ekh and Schön [7]. The method did not include calculation of local stresses.

Assuming that the stresses are known, predicting failure requires detailed knowledge of how the material responds to the stresses. The most important macroscopic joint failure modes are bearing and net-section as illustrated in Fig. 1.

Bearing failure is developed by the compressive forces acting between the fastener and the hole surface. It is a crushing mode which involves several micro mechanical failure modes usually starting with matrix cracking followed by buckling of destabilized fibers. As loading continues to increase, delamination and formation of kink bands start to occur. Kink bands eventually lead to shear cracks progressing from the center of the laminate towards the surface at a 45° angle. Net-section failure is governed by the tensile forces in the laminate and by the stress concentration created by the fastener hole. A crack is initiated at the hole edge which is usually leading to a sudden catastrophic failure of the joint.

Bearing failure is ductile and may be initiated at significantly lower load

Figure 1: Schematic of macroscopic failure modes of composite joints.

levels than the brittle net-section mode. The ductile characteristic of the bearing failure mode makes it the “preferred” failure mode over the net-section mode and focus in the present work is on predicting bearing failure.

Several workers [8, 9, 10, 11, 12, 13] have attempted to predict bearing failure in composite joints using various criteria related to either the Yamada-Sun [14] criterion or the Tsai-Wu criterion [15]. All approaches have in common that they assume the existence of a failure surface in the stress space and that they do not distinguish between different micro mechanical failure modes.

As described above, compressive failure in composite materials is governed by several micro mechanical failure mechanisms and formulation of a failure criterion that includes all possible mechanisms is a rigorous task. A more convenient strategy is to identify the dominating mechanism and formulate a criterion, for that mechanism, in terms of the local stress field.

Several experimental investigations on bearing failure in typical aircraft material systems have been conducted in order to determine the initial failure mechanism as well as the mechanisms governing the progression of damage in the material. The initial mechanism has been found to be either fiber micro buckling in 0° -plies [16, 17] or delamination [18, 19], most likely between 0° - and 90° -plies, close to the laminate surface. In cases where delamination was the initial mechanism, it was found that fiber micro buckling followed shortly after delamination and that fiber micro buckling, rather than delamination, caused the first irregularities in the load displacement curve. The

effect of fracture toughness on failure initiation was investigated by Xiao and Ishikawa [20] and it was found that in material systems with low fracture toughness, the initial failure mode may be shifted from fiber micro buckling to delamination. A similar investigation was conducted by Ye [21] where two different epoxy resins were used with the same graphite fiber to make laminates. Delamination onset was significantly delayed in the laminates based on the epoxy resin with larger tensile strength and rupture elongation.

Based on the above, it was assumed in the present work that accurate prediction of fiber micro buckling would be an important tool in the design process of aircraft structures. If used with a suitable safety margin it would yield a conservative prediction of the amount of load that could be submitted to the structure without introducing any damages, regardless of whether the initial failure mode is delamination or micro buckling for the material system at hand.

Several workers have suggested criteria for fiber micro buckling. A 2D-model of fibers embedded in a matrix was developed by Rosen [22], where the fibers are assumed to buckle in the elastic foundation at the critical compressive stress, σ_c , according to

$$\sigma_c = \frac{G_m}{1 - v_f} = G \quad (1)$$

where v_f the fiber volume fraction, G_m is the shear modulus of the matrix and G is the shear modulus of the lamina. Rosen found that the model overestimates the compressive strength for many materials. Two important assumptions in the model are that the fibers are perfectly aligned in the loading direction and that the matrix material is linear elastic. In reality, a large number of fibers are misaligned and strains in the matrix material exceeds the yield limit which implies that the plastic response is important.

Argon [23] developed an expression for compressive strength according to,

$$\sigma_c = \frac{\tau_y}{\phi_0} \quad (2)$$

where τ_y is the plastic shear strength of the composite and ϕ_0 is the original angle of misalignment of the fibers. It was recognized that when the compressive stress exceeded σ_c , a local instability in the form of a shear collapse band, or kink band, developed. This expression considers plastic micro

Figure 2: Definitions of angles and stresses.

buckling of the fibers as it involves the plastic shear strength instead of the elastic shear modulus of the matrix.

Budiansky [24] generalized Eq. 2 in order to take into account the angle, β , at which the kink band is inclined with the fiber direction, see Fig. 2, according to

$$\sigma_c = \frac{\tau_y/G}{\phi_0 + \tau_y/G} (G + E_{zz} \tan^2 \beta) \quad (3)$$

where E_{zz} is the transverse modulus of elasticity.

Further generalization was provided by Slaughter et. al. [25] such that a multi-axial stress state was accounted for according to

$$\sigma_c = \frac{\alpha \tau_y - \tau_{xz} - \sigma_{zz} \tan \beta}{\phi_0 + \phi} \quad (4)$$

$$\alpha = \sqrt{1 + \left(\frac{\sigma_{zz_y}}{\tau_y} \right)^2 \tan^2 \beta} \quad (5)$$

where σ_{zz_y} is the transverse yield strength and ϕ is the additional rotation of fibers within the kink band.

In the present paper, a general methodology to predict fiber micro buckling in composite joints is presented. The method accounts for all important joint parameters, is computationally effective and can be used for large scale structures, involving a large number of fasteners, and for optimization. This is achieved by extending the structural modeling technique developed by Ekh and Schön [7], such that the local stress field can be estimated, and applying the fiber micro buckling criterion developed by Slaughter et. al. [25].

The paper is outlined as follows: First, the structural modeling technique is briefly reviewed and slightly modified. A post-processing procedure of the output from the FE-model is then developed in order to estimate the local stresses in the vicinity of the fastener holes. An experimental programme is presented which includes measurements of the kink band inclination angle and failure loads for several multi-fastener joints. This is followed by an assessment of the developed methodology including comparison with experiments and a discussion regarding some of the simplifying assumptions used.

2. Calculation of stresses

2.1. Review of the structural modeling technique

A brief description of the modeling technique is given in this section. More details can be found in [7]. The technique was developed for prediction of load transfer in complex, multi-fastener joints, taking into account bolt-hole clearance, bolt clamp-up and friction between member plates. The structure is represented by structural finite elements (beams and shells) along the centroids of the parts and by special purpose connector elements as illustrated in Fig. 3(a). Connector elements are used for two purposes; to represent the contact, including friction, between member plates and to implement the bolt-hole clearance. The original version of the technique presented in [7] utilized four connector elements to represent a single fastener, one at each connection between the fastener and a member plate, and two to maintain the separation between the member plates. In the present study, the latter two has been replaced by a single connector, according to Fig. 3(b), in order to simplify the model.

Bolt-hole clearance is implemented through a stopping mechanism featured by the connector elements. The nodes belonging to the plate and the fastener, respectively, are allowed to separate a certain distance without resistance. When the separation reaches this distance, the two nodes can no longer move relative to each other. This is illustrated in Figure 3(b). The same technique is used to maintain the separation between the two plates. A connector element between points B and E in Figure 3(a) prevents the plates from approaching each other. This generates a compressive force in the connector which is used to represent the frictional force between the plates.

This modeling technique facilitates analysis of complex geometries where bolt-hole clearance and fastener clamp-up can be represented individually

Figure 3: Schematic description of FE-model of a single shear joint based on structural and connector finite elements.

for each fastener. Thus, the effect of manufacturing tolerances in the holes can be investigated as well as the effect of changing the fastener pattern or the properties of individual fasteners. Good agreement with experiments and detailed 3D continuum FE-models in terms of fastener load distribution was obtained with this technique [7]. The model is used to calculate fastener loads, N , fastener moments, M , as defined in Fig. 4(b), and fastener clamping forces, f_{cl} , to be used for estimation of the local stress field in the vicinity of each fastener.

2.2. Estimation of the local stress field

The micro buckling criterion in Eq. 4 requires the stresses σ_{xx} , σ_{zz} and τ_{xz} as input. This implies that the forces and moments calculated with the FE-model must be converted into a multi-axial stress field at the contact interface between the fastener and the hole surface. The following assumptions were made in this process:

1. Radial strain in the laminate, at the contact interface, increases linearly from the top surface to the faying surface of the plate. The fastener is in contact with the plate over the entire thickness of the plate, i.e. the strain is zero or larger at the top surface and reaches a maximum value, ϵ_r^{max} , at the faying surface, as illustrated in Fig. 4(a).
2. Radial strain in the laminate follows a cosine distribution [26] in the tangential direction of the hole. Contact occurs for angles $-\pi/2 \leq \theta \leq \pi/2$ throughout the thickness of the plate, as illustrated in Fig. 4(c).
3. The contact is frictionless in the circumferential direction, i.e. no shear stress, $\tau_{r\theta}$, is generated by the contact between the fastener and the

hole surface since this stress component is not needed in the buckling criterion. However, the shear stress in the z-direction, τ_{rz} , is needed in the criterion and thus the contact is not frictionless in that direction.

4. Individual plies are orthotropic and in a state of plane stress.

(a) Strain variation in the thickness direction. (b) Structural FE-model. (c) Strain variation in the tangential direction.

Figure 4: Assumed strain distribution.

Thus, the radial strain is depending on the two coordinates z and θ and can be expressed as

$$\epsilon_r = \epsilon_r^{max} f(z)g(\theta) = \epsilon_r^{max} \left(1 - \frac{z + h/2}{z_0 + h/2}\right) \cos \theta \quad (6)$$

where z_0 and ϵ_r^{max} are defined in Fig. 4(a). Maximum strain, ϵ_r^{max} , occurs at the faying surface in the bearing plane, i.e. for $z = -h/2$ and $\theta = 0$, and the coordinate z_0 on the z -axis determines the slope of the strain variation in the z -direction. Determining the strain distribution becomes a matter of determining the parameters z_0 and ϵ_r^{max} . This is done based on the moment and force equilibrium in the x -direction. The x - and y -components of the strain are

$$\epsilon_x = \epsilon_r \cos \theta \quad (7)$$

$$\epsilon_y = \epsilon_r \sin \theta \quad (8)$$

and the longitudinal stress, σ_{xx} , is expressed according to Hook's law for an orthotropic material under plane stress as

$$\sigma_{xx} = \frac{E_x \epsilon_x}{1 - \nu_{xy} \nu_{yx}} + \frac{\nu_{yx} E_x \epsilon_y}{1 - \nu_{xy} \nu_{yx}} \quad (9)$$

where E_x is the modulus of elasticity in the x-direction and (ν_{xy}, ν_{yx}) are the Poisson's ratio in the x- and y-directions, associated with the individual lamina.

Off-axis material properties can be expressed in their corresponding on-axis properties [27] according to

$$\frac{1}{E_x} = \frac{\cos^4 \theta}{E_L} + \frac{\sin^4 \theta}{E_T} + \frac{1}{4} \left(\frac{1}{G_{LT}} - \frac{2\nu_{LT}}{E_L} \right) \sin^2 2\theta \quad (10)$$

$$\frac{1}{E_y} = \frac{\sin^4 \theta}{E_L} + \frac{\cos^4 \theta}{E_T} + \frac{1}{4} \left(\frac{1}{G_{LT}} - \frac{2\nu_{LT}}{E_L} \right) \sin^2 2\theta \quad (11)$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{LT}}{E_L} - \frac{1}{4} \left(\frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}} \right) \sin^2 2\theta \quad (12)$$

$$\frac{\nu_{yx}}{E_y} = \frac{\nu_{TL}}{E_T} - \frac{1}{4} \left(\frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}} \right) \sin^2 2\theta \quad (13)$$

where subscripts L and T denotes the longitudinal and transverse material direction, respectively.

Thus, the above modulus of elasticity and Poisson's ratios for each lamina in a symmetric quasi-isotropic laminate with stacking sequence $[\pm 45/0/90]_{4s}$ are obtained from using Eqs. 10-13 in

$$\theta = \pi/2 \quad \pm 4it \leq |z| \leq \pm(4i+1)t \quad (14)$$

$$\theta = 0 \quad \pm(4i+1)t \leq |z| \leq \pm(4i+2)t \quad (15)$$

$$\theta = -\pi/4 \quad \pm(4i+2)t \leq |z| \leq \pm(4i+3)t \quad (16)$$

$$\theta = \pi/4 \quad \pm(4i+3)t \leq |z| \leq \pm 4(i+1)t \quad (17)$$

where t is the lamina thickness and $i = 1 \dots 4$.

Eqs. 6-17 determine the variation of the longitudinal component σ_{xx} of the bolt-hole contact stress in the thickness and tangential directions. Equilibrium with respect to force and moment acting on the fastener as illustrated in Fig. 4(b) yields

$$N = \int_{-\pi/2}^{\pi/2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xx}(z, \theta) \frac{d}{2} dz d\theta \quad (18)$$

$$M = \int_{-\pi/2}^{\pi/2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xx}(z, \theta) \frac{d}{2} z dz d\theta \quad (19)$$

where N is the bolt load, M is the moment generated by the uneven contact stress, h is the thickness of the laminate and d is the diameter of the fastener. The force, N , and moment, M , are calculated with the structural FE-model and are thus regarded as known quantities which implies that parameters z_0 and ϵ_r^{max} can be calculated from Eqs. 18-19.

Using Eqs. 6-9 in Eq. 18 and 19 yields the following expressions for the equilibrium,

$$N = \frac{\epsilon_r^{max} d}{2(1 - \nu_{xy}\nu_{yx})} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[E_x \left(1 - \frac{z + h/2}{z_0 + h/2} \right) (\cos^2 \theta + \nu_{yx} \cos \theta \sin \theta) \right] dz d\theta \quad (20)$$

$$M = \frac{\epsilon_r^{max} d}{2(1 - \nu_{xy}\nu_{yx})} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[E_x \left(1 - \frac{z + h/2}{z_0 + h/2} \right) (\cos^2 \theta + \nu_{yx} \cos \theta \sin \theta) \right] z dz d\theta \quad (21)$$

where E_x is given in Eqs. 14 to 17 on the preceding page. The integrals that need to be evaluated in (20) and (21) are on the form

$$I_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 \theta d\theta = c_1 = \frac{\pi}{2} \quad (22)$$

$$I_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z) dz = c_2 \quad (23)$$

$$I_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z) z dz = c_3 \quad (24)$$

$$I_4 = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z) z^2 dz = c_4 \quad (25)$$

$$I_5 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin \theta \cos \theta d\theta = 0 \quad (26)$$

and are evaluated by recognizing that $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$ and by using Eq. 14 to 17 on page 9. This gives the two equations

$$N = \frac{\pi \epsilon_r^{max} d}{2(1 - \nu_{xy} \nu_{yx})} \left\{ \frac{c_2}{2} - \frac{c_3}{2(z_0 + h/2)} - \frac{hc_2}{4(z_0 + h/2)} \right\} \quad (27)$$

$$M = \frac{\pi \epsilon_r^{max} d}{2(1 - \nu_{xy} \nu_{yx})} \left\{ \frac{c_3}{2} - \frac{c_4}{2(z_0 + h/2)} - \frac{hc_3}{4(z_0 + h/2)} \right\} \quad (28)$$

from which z_0 and ϵ_r^{max} can be extracted according to

$$z_0 = \frac{Nc_4 - Mc_3}{Nc_3 - Mc_2} \quad (29)$$

$$\epsilon_r^{max} = \frac{4N(1 - \nu_{xy} \nu_{yx})}{d\pi \left(\frac{c_2 z_0 - c_3}{z_0 + h/2} \right)}. \quad (30)$$

When z_0 and ϵ_r^{max} are known, the contact stress distribution is determined and it is possible to calculate the longitudinal force reacted by each individual lamina. Of particular interest is the load taken by the first 0-layer, counting from the faying surface of the joint, which is given by

$$N_0 = \frac{E_0 \epsilon_r^{max} d}{2(1 - \nu_{xy} \nu_{yx})} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{h}{2}+2t}^{-\frac{h}{2}+3t} \left(1 - \frac{z + h/2}{z_0 + h/2} \right) \cos^2 \theta dz d\theta \quad (31)$$

where the term $\nu_{yx} \cos \theta \sin \theta$ in Eq. 20 has been disregarded since it evaluates to zero according to Eq. 26. This lamina is the most severely loaded in terms of bearing load and it is assumed that failure initiation will occur in the bearing plane, i.e. for $\theta = 0$, in this lamina. The radial contact stress, σ_r , is assumed to follow the same cosine distribution as the strains (see Eq. 6), thus

$$\sigma_r = \sigma_{xx}^{max} \cos \theta \quad (32)$$

where σ_{xx}^{max} is the stress in the bearing plane. Further, it is expected that the variation of the stress through the thickness of the lamina is small and by neglecting this variation, σ_r denotes the average stress over the lamina thickness. Taking the longitudinal component, σ_{xx} , of σ_r according to $\sigma_{xx} = \sigma_r \cos \theta$ and expressing the longitudinal force equilibrium for the lamina yields

$$N_0 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sigma_r \cos(\theta) t \frac{d}{2} d\theta = \frac{\sigma_{xx}^{max} t d}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 \theta d\theta = \frac{\sigma_{xx}^{max} \pi t d}{4} \quad (33)$$

where t is the lamina thickness. Recalling that N_0 is a known quantity given by Eq. 31 implies that the bearing stress, σ_{xx}^{max} , can be expressed as

$$\sigma_{xx}^{max} = \frac{4N_0}{\pi t d}. \quad (34)$$

The transverse shear stress, τ_{xz} , can be estimated by making the conservative assumption that the frictional force between the fastener and the hole surface is fully developed. This yields

$$\tau_{xz} = \mu \sigma_{xx}^{max} \quad (35)$$

where μ is the coefficient of friction between the fastener and the plate. A simple model is adopted to obtain an approximative value of the transverse stresses caused by the pressure between the washer and the plate. At each fastener, the compressive force carried by the connector element that maintains the separation of the plates is divided by the area of the washer, i.e.

$$\sigma_{zz} = \frac{f_{cl}}{a_w} \quad (36)$$

where f_{cl} and a_w are the clamping force and the washer area, respectively.

The above relations have been implemented in the numerical environment Octave [28] such that the stresses σ_{xx}^{max} , σ_{zz} and τ_{xz} can be calculated based on the results from the structural FE-model.

3. Experimental programme

The experimental programme was conducted in order to measure the kink band inclination angle, β , to be used as input to the fiber micro buckling criterion, and to find the load at which failure was initiated in various joint configurations. Testing involved single fastener double lap specimens and various multi-fastener, single lap configurations.

3.1. Material system

All specimens utilized composite member plates of the carbon-epoxy prepreg material system AS4/8552 with stacking sequence $[\pm 45/0/90]_{4s}$ and aluminium plates of quality AA7475-T76. The fastener system consisted of titanium bolts, alloy steel nuts and stainless steel washers. All elastic properties are tabulated in Table 1. The coefficient of friction between the

Table 1: Elastic properties of materials and their constituents.

	E_{11} [GPa]	E_{22} [GPa]	E_{33} [GPa]	G_{12} [GPa]	G_{13} [GPa]	G_{23} [GPa]	ν_{12} -	ν_{13} -	ν_{32} -
Fiber	238	-	-	22	-	-	0.2	-	-
Matrix	3.3	-	-	1.2	-	-	0.35	-	-
Lamina	140	10	10	5.2	5.2	3.9	0.3	0.3	0.5
Laminate	54.25	54.25	10	20.72	4.55	4.55	0.309	0.332	0.332
Aluminium	71	-	-	-	-	-	0.31	-	-
Titanium	110	-	-	-	-	-	0.29	-	-
Steel	210	-	-	-	-	-	0.30	-	-

composite and aluminium plates used in the present work was measured by Schön [29] to be 0.235.

In addition to the elastic properties in Table 1 and the kink band inclination angle, β , the failure criterion requires the transverse yield stress, σ_{T_y} , the plastic shear strength, τ_y , and the fiber misalignment angle of the material. Transverse yield stress and plastic shear strength for the present material system are specified by the manufacturer to be 81 MPa and 114 MPa, respectively. Fiber misalignment is divided into an initial angle, ϕ_0 , caused by an imperfect manufacturing procedure, and an additional angle, ϕ , generated by the applied loading. However, the two angles can be treated as a single critical misalignment angle, ϕ_c , at which failure occurs. Uniaxial, longitudinal compressive strength, σ_0 , is specified by the manufacturer to be 1531 MPa which can be used in Eq 2 to estimate ϕ_c , provided that ϕ_0 is replaced by ϕ_c . The resulting critical fiber misalignment, ϕ_c , was approximately 4.3° which was used in subsequent analyses.

3.2. Kink band inclination angle

Kink band inclination angle is considered to be a material property and has previously [30] been found to be 10–30° for many materials. According to Eq. 4, compressive strength is reduced for smaller values of β , provided that σ_{zz} is compressive, and thus it is important to estimate a lower bound for β for the material system at hand. Three double lap (Alu.-comp.-Alu.), single fastener specimens were tested in quasi static tension, to different load levels, and then sectioned along the bearing plane and studied in a microscope. Bearing damages for the three specimens are presented in Fig. 5.

(a) Joint loaded to 23 kN. (b) Joint loaded to 26 kN. (c) Joint loaded to 29 kN.

Figure 5: Bearing damages in double lap specimens subjected to different levels of tension loading.

It can be seen that severe damage has developed already at 23 kN. Shear cracks through single laminae occur at several locations and delamination between the outermost 0° - and 90° -plies has started. At 26 kN, the lamina shear cracks have merged into larger cracks that propagate from the center of the laminate towards the surfaces in approximately a 45° angle. Since the laminate surfaces are clamped by the Aluminium plates (single fastener torqued with 6 Nm), the laminate is still capable of carrying an increasing load which generates a second set of shear cracks that progresses from the center to the surfaces of the laminate as shown in Fig. 5(c). This stage-by-stage process is characteristic for clamped laminates [16].

Kink band inclination angles are studied most conveniently in the specimen subjected to 23 kN tension load. A single, 0° -ply kink band is illustrated in Fig. 6.

Figure 6: Close-up of kink band in a 0° -ply.

Several inclination angles of kink bands developed in 0° -plies appeared to be between 10° and 15° and the smallest angle found was approximately 8° . This value was selected to represent a lower bound to be used in the micro buckling criterion.

3.3. Joint failure loads

Several multi-fastener, single lap joint configurations were tested quasi statically in tension in order to generate results for comparison with failure predictions. The baseline joint, referred to as joint number 1, is illustrated in Fig. 7 and the complete test matrix is presented in Table 2.

Figure 7: Baseline joint geometry.

Table 2: Varied parameters in the experimental programme.

<i>Joint</i> [nr]	<i>Torque</i> [Nm]	<i>Row Sp.</i> [mm]	<i>Length</i> [mm]	<i>Al.-thi.</i> [mm]	<i>Bolt</i> <i>Type</i>	<i>Diam.</i> [mm]	<i>Nr of</i> <i>Specs</i>
1	6	32	376	8	P.H.	4×6	4
2	0	32	376	8	P.H.	4×6	3
3	6	32	376	4	P.H.	4×6	3
4	6, 14	32	376	8	P.H.	3×6, 1×8	3
5	6, 14	32	376	8	P.H.	2×6, 2×8	3
6	6, 14	32	376	8	P.H.	1×6, 3×8	3
7	14	32	376	8	P.H.	4×8	3
total							22

Configurations 2 and 3 are generated by removing the fastener torque or reducing the thickness of the aluminium plate, respectively. In configurations 4-7, the 6 mm fasteners were replaced by an increasing number of 8 mm fasteners, starting with bolt number 1 in configuration 4, bolts 1 and 2 in configuration 5 and bolts 1-3 in configuration 6. Configuration 7 used 8 mm

fasteners exclusively. The 6 mm and 8 mm fasteners were torqued with 6 Nm and 14 Nm, respectively. Bolt numbers are taken from Fig. 7. The thickness of the composite plate is 4.16 mm in all specimens.

Deviations from nominal values regarding hole diameters and positions are bound to exist. Some of the joints used in the present study were examined in detail [31] in this respect by means of a coordinate measurement machine. It was found that hole sizes were different in the composite compared to the aluminium and that the holes were not perfectly aligned. Average hole sizes for the two materials and the average fastener diameters are shown in Table 3.

Table 3: Mean values of measured diameters

Nominal [mm]	Alu. [mm]	Comp. [mm]	Bolt [mm]
6	6.042	6.006	5.985
8	8.044	8.001	7.985

It was found that variations of hole diameters in both materials were small. This was also the case for the fasteners which implies that at each 6 mm fastener there exist an effective clearance between $[0 < \delta < 58\mu m]$. The same range in clearance was found in the joints with 8 mm fasteners.

Based on the discussion in Section 1 and on the results presented in Fig. 5 it is assumed that the initial failure events are fiber kinking of single laminae, possibly preceded by matrix cracking and micro buckling of a small number of fibers. The failure criterion, Eq. 4, is applied to the average stress field acting over the thickness of the 0° -ply closest to the faying surface of the joint, which is the most critically loaded lamina in a single shear lap joint. Thus, the prediction of failure corresponds to the development of a kink band in the bearing plane of this lamina. However, the criterion is two dimensional, as it deals only with the stresses occurring in the bearing plane, and little can be said about the extension of the kink band in the width direction of the lamina. The contact stresses between the fastener and the hole surface decreases with the tangential distance from the bearing plane according to Eq. 32. Thus, the amount of damage caused by the developed kink band may be rather limited and is likely to generate a small, sudden, stiffness drop in the load displacement curve which is quickly regained due to redistribution of the contact stresses. The ability to detect such small, temporary stiffness drops depends on the cross bar speed of the testing machine

and the sampling frequency of the data acquisition system. In the present work, cross bar speed and sampling frequency were set to 0.1 mm/min and 50 Hz, respectively, which corresponds to 3×10^4 samples/mm. This enabled a careful examination of the load displacement curves, in which it was possible to detect the stiffness drops, as illustrated in Fig. 8.

Figure 8: Close-up of sudden stiffness drop assumed to be caused by fiber micro buckling.

The first stiffness drop occurring in each curve is interpreted as the initial failure event, i.e. fiber micro buckling in the 0° -ply closest to the faying surface.

4. Results and discussion

The results section is divided in two parts. First, a comparison between predictions and measurements regarding failure load is made. After this follows a discussion about some of the simplifying assumptions that are made in the model together with some suggestions for improvements.

4.1. Comparison with experiments

In order to make meaningful comparisons between predicted and experimental results, it is necessary to account for friction between the fastener and hole surface as it generates transverse shear stresses, τ_{xz} , which affect the failure load according to Eq. 4. The coefficient of friction in this contact interface has not been determined experimentally, but can be expected to be

small as the fasteners are coated with a solid film lubricant. However, since the pressure between fastener and hole surface is large, large shear stresses may develop even for a small coefficient of friction. In particular, with the conservative assumption that the shear stresses are given by Eq. 35, the coefficient of friction may have a strong impact on the predicted failure load. In order to specify a coefficient of friction that generates relevant shear stresses in Eq. 35, results from a previous investigation [7] on joint configuration 1, regarding bolt hole clearances, were used. Detailed measurements of hole sizes and hole locations revealed that the initial clearance at the two outer fasteners (bolts 1 and 4 in Fig. 7(a)) was 22μ and that the clearance at the two inner fasteners (bolts 2 and 3 in Fig. 7(a)) was close to zero. The smallest failure load found in the present investigation for this joint configuration is 39.5 kN. Using this value, in conjunction with the above clearances, enables the calculation of a coefficient of friction that causes agreement between experimental and predicted failure loads. This value was found to be $\mu = 5.61 \times 10^{-3}$ and should not be regarded as a true coefficient of friction but instead as a value that generates relevant shear stresses when used in Eq. 35. As the smallest found failure load is used in the calculation, this value can be considered to be conservative and it is used in all subsequent simulations.

Experimental and predicted failure loads for all joints are presented in Fig. 9. The experimental values corresponds to the first irregularity in the load displacement curve according to the discussion in Section 3.3. Since several specimens of each joint configuration were tested, failure loads are presented in terms of average, minimum and maximum loads. It can be seen that variations among specimens within a specific joint configuration can be large which implies that the number of tested specimens may be too small to ensure that the average value is representative for the joint. Nevertheless, some trends were identified and the experiments still provide important information for the validation of the model.

Predicted loads are presented in terms of nominal, minimum and maximum values. Nominal values correspond to failure loads calculated with zero bolt hole clearance, δ , while minimum and maximum values correspond to combinations of clearances $[0 < \delta_i < 58\mu m]$ resulting in the smallest and largest failure load, respectively. The subscript i refers to the fastener number according to Fig. 7(a). In all cases, the minimum value was generated by assuming maximum clearance at fasteners 1-3 and zero at fastener 4. The opposite situation, i.e. maximum clearance at fastener 4 and zero clearance

Figure 9: Experimental and predicted failure loads. Experiments are presented in terms of average, minimum and maximum values. Corresponding predictions refer to nominal (no bolt-hole clearance), minimum and maximum values.

at the other fasteners resulted in the maximum load in most cases. In some cases, this combination of clearances implied that failure occurred at fastener 1 instead of 4, and thus the maximum load was achieved by introducing a small clearance at fastener 1 in addition to the maximum clearance at fastener 4. Below follows comparisons for each joint configuration. Predicted failure load for joint 1 is conservative as expected due to that the coefficient of friction between fasteners and hole surfaces in the model is calculated based on the smallest experimental failure load. It can be seen that the difference between maximum and minimum failure load is similar in the experiments and the predictions. Joint 2 has finger tightened fasteners and thus only a small amount of load is transferred by friction between the plates. This is reflected by a reduction of failure load, similar in magnitude, in both the experiments and the predictions. The reduced thickness of the aluminium plate in joint 3 increases the predicted failure load but appears to reduce the experimental values. The increase in predicted load can be explained by the increased out-of-plane deformation, referred to as secondary bending, which

follows from the reduced stiffness of the joint. This makes the fastener more aligned with the hole surface and thus increases the contact area between the fastener and the hole surface [32]. This mechanism is represented in the present model by the parameter z_0 in Fig. 4(a). A larger z_0 is predicted for joint 3 compared to joint 1, which indicates a more uniform distribution of the contact stress and hence a larger failure load. The slight reduction in average experimental failure load is not logical and is probably due to testing too few specimens to obtain an accurate mean value for the load.

Regarding joints 4-7, some discrepancy between the experimental and numerical results was found. The numerical results appears to be more logical and will be discussed first. It was found that the failure load increases when the 6 mm fasteners are replaced by highly torqued 8 mm fasteners. Several mechanisms contribute to generate this behavior. The increased clamping load causes more load to be transferred through friction between the plates, which relieves the fasteners of some load. Also, using larger fasteners in holes 1-3 shifts some load from the critical fastener 4 to the other fasteners due to their increased stiffness. However, increasing the load carried by a specific fastener mainly reduces the amount of load carried by adjacent fasteners. Fasteners further away are left virtually unaffected. This explains why the introduction of 8 mm fasteners in position 1 (joint 4) and in positions 1-2 (joint 5) only have a small effect on the joint failure load, which is determined by the conditions at bolt 4. In joint 6, an 8 mm fastener is used also in position 3, leading to an increase in the load carried by this fastener, which is reflected by a decreased load carried by the adjacent fastener 4 and thus an increased failure load. Replacing also the critical fastener 4, and thus utilize only 8 mm fasteners, increases the failure load significantly. This is due to the increased contact area between the critical fastener and the hole surface and to the increased clamping of the material.

The experimental results clearly indicates that too few specimens were tested as the results fluctuated in a non physical manner. From Fig. 9 it can be seen that the variation in experimental failure load, within a specific joint configuration, increases as the applied torque on the fastener increases, i.e. variations are small for the finger tightened configuration (joint 2), larger for the torqued joints (joint 1) and even larger for joints 4-7 where highly torqued 8 mm fasteners were used. This could partly be explained by the fact that only a small portion of the applied torque actually contributes to the clamping force in the fastener. Most of the torque is absorbed in the process of overcoming the friction between the nut and the washer and

between the nut and the threads on the fastener. This implies that small variations in conditions on these surfaces could have a large impact on the resulting clamping force in the fastener, which in turn have a large influence on the fiber micro buckling failure mode.

From the comparisons between the experimental and the predicted failure loads it can be concluded that all predictions appeared to be logical and physically sound. The conservative calibration of the model in terms of a fictitious coefficient of friction between the fasteners and the hole surfaces, implied that the nominal predictions (no clearance) resulted in failure loads close to or below the minimum experimental values. However, it was found that too few specimens were tested in order to determine the distribution of the failure load for a specific joint configuration. This was especially true for joints with large torque applied to the fasteners as the experimental scatter was increased with increased torque.

4.2. Simplifying assumptions

A number of simplifying assumptions regarding stresses, strains and material properties were made in order to establish the failure prediction methodology presented in this paper. Below follows a brief discussion about some of these assumptions and their validity.

The assumptions that strains in the composite material vary linearly in the thickness direction, and follows a cosine distribution in the tangential direction, at the contact interface between the fastener and the hole surface are essential to the procedure of calculating the local stress field. Variation of strain through the laminate thickness in single lap joints has been investigated previously. The three-dimensional continuum finite-element method was used by Ireman [1] to calculate the radial strains at the contact interface for single fastener, perfect fit joints. It was found that the strains increased approximately linearly with the distance from the top surface of the laminate. At a distance from the top surface corresponding to approximately 75% of the laminate thickness, the strain distribution started to deviate from the linear behavior and increase at a greater rate. Thus, the strain variation through the thickness could be approximated by a bi-linear function or possibly by a second order polynomial. Increasing the fastener diameter generated a more uniform strain distribution but still with the same bi-linear or second order characteristics. This suggests that the strain distribution is depending on the fastener compliance such that the nonlinearity becomes stronger when

the fastener is deformed, i.e. the strain distribution becomes more nonuniform when the applied load increases. Similar analyses and results regarding the strain variation through the thickness were presented by McCarthy and McCarthy [3].

Several authors have addressed the issue of the contact strain distribution in the tangential direction. The conforming contact problem between the fastener and the slightly oversized hole is highly nonlinear as the contact area increases with increasing load. This problem can be solved with analytical and numerical methods [33, 34, 3]. A simpler approach suggested by Waszak and Cruse [26] is to assume a cosine distribution of the strains as indicated in Fig. 4(c). This assumed strain distribution has been used by other workers [35, 36] with good results. Obviously, if larger clearances occurs the assumption is no longer valid as the strains will be increased due to the reduced contact area. This was clearly demonstrated by several authors [33, 34, 3].

However, it is believed that the assumptions made regarding the strain distribution, maintain a fair balance between accuracy on one hand, and complexity and computational cost on the other hand. It must be recognized though that increased clearances and fastener deformations are likely to decrease the accuracy of the model. Future improvements may include more sophisticated procedures to obtain the strain distribution which implies that $f(z)$ and $g(\phi)$ in Eq. 6 should be changed accordingly.

Further, it is assumed in the present model that the contact between the fastener and the hole surface is frictionless in the tangential direction. This is a conservative assumption as the tangential stress would have a longitudinal component which would have reduced the longitudinal component of the normal stress.

A severe limitation in the present model is the inability to accurately calculate the shear stresses, τ_{xz} , in the contact interface between the fastener and the hole surface. Relative movement between the two contacting surfaces in the transverse direction is determined by the transverse displacements of the fastener and the hole surface and by the frictional forces in the interface. The contact may be in a sticking or a slipping state which implies that the shear stress at the surface can be expressed as

$$\tau_{xz} = f\mu\sigma_{xx} \quad (37)$$

where $0 \leq f \leq 1$. The slipping state corresponds to $f = 1$ which was used in the present model. However, the contact may be in a sticking state which

would imply that the shear stresses are overestimated in Eq. 37. In the above simulations this problem was circumvented by using a nonrealistic coefficient of friction which made the predictions agree with the experiments in a specific well controlled case. Using a more realistic coefficient of friction of $\mu = 0,05$ in joint configuration 1 and varying f from zero to one, demonstrates the importance of predicting τ_{xz} accurately as a large impact on the predicted failure load was found according to Fig. 10(a).

Figure 10: Influence of f and clamping.

Similarly, the calculation of the compressive stress, σ_{zz} , in the transverse direction due to torquing the fastener is simple and not accurate. It is assumed that the stress is uniform under the washer in both the tangential and the thickness direction which is not the case. The variation of σ_{zz} may be complicated in both directions, especially at fasteners 1 and 4 due to the severe secondary bending. However, the influence of clamping force on the predicted failure load appears to be realistic. It is illustrated in Fig. 10(b) where it can be seen that the predicted failure load increases linearly with increasing clamping force. The compressive stress σ_{zz} is built up from a solution dependent component and the applied torque. The solution dependent component is determined by the geometry of the joint, i.e. by the eccentric load path and the amount of secondary bending, and may be large. Thus, increasing the torque level may only have a limited impact on the predicted failure load. However, it is likely that a complete removal of the compressive stress, i.e. assuming a pin-loaded plate, would significantly reduce the

predicted failure load.

More accurate prediction of τ_{xz} and σ_{zz} are required to improve the accuracy of the model, but this is beyond the scope of the present work.

The effect of fiber misalignment, at the initiation of failure, is illustrated in Fig. 11(a). Fiber misalignment appears to be of great importance and the simple procedure adopted in this work to estimate it may be insufficient. An experimental method to measure the distribution of the fiber misalignment was developed by Yurgartis [37]. The method is based on sectioning the lamina at a small angle off the fiber direction and measure the major axis of each elliptical fiber cross section. The magnitude of the major axis of a particular fiber cross section can be related to the orientation of the fiber relative to the sectioning plane and thereby also to the nominal fiber direction. It is suggested [37] that the procedure should be supported by an image analysis software to automatically measure all elliptical cross sections and calculate the distribution of the fiber misalignment angle. Thus, it seems possible to accurately determine the initial fiber misalignment for any composite material which can improve the accuracy of the strength predictions.

Regarding the kink band angle, β , it appears to be less important compared to the misalignment angle according to Fig. 11(b). The relation between β and predicted failure load is approximately linear with a moderate slope.

Figure 11: Influence of material angles.

5. Summary & Conclusions

A simple, effective and scalable method to predict the bearing strength of composite joints has been suggested. The method accounts for all important factors such as bolt-hole clearance, friction, fastener clamp-up and fastener deformation. A straight forward structural FE-analysis is providing input to a post-processing procedure in which the failure analysis is conducted. This general, computationally effective approach opens the possibility to analyze large scale structures and to incorporate iterative schemes such as optimization and statistical investigations regarding bolt-hole clearances and effects of the complete failure of certain fasteners.

The presented method agrees well with experiments and provides a basis for further development. In particular, more accurate estimation of the transverse shear stress at the contact interface between the fastener and the hole surface is required in order to improve the accuracy.

Appendix A. Acknowledgments

“BOJCAS – Bolted Joints in Composite Aircraft Structures is a RTD project partially funded by the European Union under the European Commission GROWTH programme, Key Action: New Perspectives in Aeronautics, Contract No. G4RD-CT99-00036”. ABB AB, Corporate Research is acknowledged for financial support and for providing computer resources.

References

- [1] T. Ireman. Three-dimensional stress analysis of bolted single-lap composite joints. *Composite Structures*, 43:195–216, 1998.
- [2] M. A. McCarthy, C. T. McCarthy, V. P. Lawlor, and W. F. Stanley. Three-dimensional finite element analysis of single-bolt, single-lap composite bolted joints: Part I - Model development and validation. *Composite Structures*, 71:140–158, 2005.
- [3] M. A. McCarthy and C. T. McCarthy. Three-dimensional finite element analysis of single-bolt, single-lap composite bolted joints: Part II - Effects of bolt-hole clearance. *Composite Structures*, 71:159–175, 2005.

- [4] P. Perugini, A. Riccio, and F. Scaramuzzino. Three-dimensional progressive damage analysis of composite joints. In B. H. V. Topping, editor, *Proceedings of the Eighth International Conference on Civil and Structural Engineering Computing*, 2001.
- [5] A. Riccio and F. Scaramuzzino. Influence of damage onset and propagation on the tensile structural behavior of protruding composite joints. In *4th GRACM Congress on Computational Mechanics*, Patras, June 2002. GRACM.
- [6] K. I. Tserpes, G. Labeas, P. Papanikos, and Th. Kermanidis. Strength prediction of bolted joints in graphite/epoxy composite laminates. *Composites: Part B*, 33:521–529, 2002.
- [7] J. Ekh and J. Schön. Finite element modeling and optimization of load transfer in multi-fastener joints using structural elements. *Composite Structures*, 82(2):245 – 256, 2008.
- [8] F.-K. Chang, R. A. Scott, and G. S. Springer. Strength of mechanically fastened composite joints. *Journal of Composite Materials*, 16:470–494, November 1982.
- [9] F.-K. Chang, R. A. Scott, and G. S. Springer. Failure of composite laminates containing pin loaded holes - Method of solution. *Journal of Composite Materials*, 18:255–278, May 1984.
- [10] F.-K. Chang, R. A. Scott, and G. S. Springer. Failure strength of nonlinearly elastic composite laminates containing a pin loaded hole. *Journal of Composite Materials*, 18:464–477, September 1984.
- [11] H. Hamada, Z.-I. Maekawa, and K. Haruna. Strength prediction of mechanically fastened quasi-isotropic carbon/epoxy joints. *Journal of Composite Materials*, 30(14):1596–1612, 1996.
- [12] A. J. Sawicki and P. J. Minguet. Failure mechanisms in compression-loaded composite laminates containing open and filled holes. *Journal of Reinforced Plastics and Composites*, 18(18):1708–1728, 1999.
- [13] A. J. Sawicki and P. J. Minguet. The influence of fastener clearance upon the failure of compression-loaded composite bolted joints. *Composite Structures*, pages 293–308, 2000.

- [14] C. T. Sun and S. E. Yamada. Strength distribution of a unidirectional fiber composite. *Journal of Composite Materials*, 12, 1978.
- [15] S. W. Tsai and E. M. Wu. A general theory of strength for anisotropic materials. *Journal of Composite Materials*, 5, 1971.
- [16] H.-S. Wang, C.-L. Hung, and F.-K. Chang. Bearing failure of bolted composite joints. Part 1: Experimental characterization. *Journal of Composite Materials*, 30(12), 1996.
- [17] P. S. Wu and C. T. Sun. Modeling bearing failure initiation in pin-contact of composite laminates. *Mechanics of Materials*, 29:325–335, 1998.
- [18] T. Ireman, T. Ranvik, and I. Eriksson. On damage development in mechanically fastened composite laminates. *Composite Structures*, 49:151–171, 2000.
- [19] P. P. Camanho, S. Bowron, and F. L. Matthews. Failure mechanisms in bolted CFRP. *Journal of Reinforced Plastics and Composites*, 17(3):205–233, 1998.
- [20] Y. Xiao and T. Ishikawa. Bearing strength and failure behavior of bolted composite joints (part I: Experimental investigation). *Composites Science and Technology*, 65:1022–1031, 2005.
- [21] L. Ye. The role of matrix resin in delamination onset and growth in composites. *Composite Science and Technology*, 33:257–277, 1988.
- [22] B. W. Rosen. *Fiber Composite Materials*, chapter "Mechanics of composite strengthening", pages 37–75. Americal Society of Metals, 1965.
- [23] A. S. Argon. *Treatise on Material Science and Technology*, volume 1. Academic Press, New York, 1972.
- [24] B. Budiansky. Micromechanics. *Computers & Structures*, 16(1-4):4–12, 1983.
- [25] W. S. Slaughter, N. A. Fleck, and B. Budiansky. Compressive failure of fiber composites: The role of multiaxial loading and creep. *Journal of Engineering Materials and Technology*, 115:308–313, 1993.

- [26] J. P. Waszak and T. A. Cruse. Failure mode and strength predictions of anisotropic bolt bearing specimens. *Journal of Composite Materials*, 5:421–425, 1971.
- [27] B. D. Agarwal and L. J. Broutman. *Analysis and Performance of Fiber Composites*. John Wiley & Sons, Inc., 2 edition, 1990.
- [28] John W. Eaton. *GNU Octave Manual*. Network Theory Limited, 2002.
- [29] J. Schön. Coefficient of friction for Aluminium in contact with a carbon fiber epoxy composite. *Tribology International*, 37(5):395–404, May 2004.
- [30] B. Budiansky and N. A. Fleck. Compressive failure of fiber composites. *Journal of the Mechanics and Physics of Solids*, 41:183–211, 1993.
- [31] J. Ekh and J. Schön. Load transfer in multi row, single shear, composite-to-aluminium lap joints. *Composites Science and Technology*, 66:875–885, 2006.
- [32] J. Ekh and J. Schön. Effect of secondary bending on strength prediction of composite, single shear lap joints. *Composites Science and Technology*, 65:953–965, 2005.
- [33] M. W. Hyer and E. C. Klang. Contact stresses in pin-loaded orthotropic plates. *International Journal of Solids and Structures*, 21:957–975, 1985.
- [34] R. A. Naik and J. H. Crews Jr. Stress analysis method for a clearance-fit bolt under bearing loads. *AIAA Journal*, 24(8), 1986.
- [35] T. A. Collings. On the bearing strengths of CFRP laminates. *Composites*, 13(3):241–252, 1982.
- [36] T. A. Collings and M. J. Beauchamp. Bearing deflection behavior of a loaded hole in CFRP. *Composites*, 15(1):33–38, 1984.
- [37] S. W. Yurgartis. Measurement of small angle fiber misalignments in continuous fiber composites. *Composites Science and Technology*, 30:279–293, 1987.