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ACOUSTIC PROPERTIES OF AN IN-DUCT ORIFICE SUBJECTED TO BIAS FLOW AND HIGH LEVEL ACOUSTIC EXCITATION

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ABSTRACT
This paper experimentally investigates the acoustic properties of an orifice with bias flow under medium and high sound level excitation. The test included no bias flow and two bias speeds for three different frequencies. Experimental results are compared and discussed with theory. It is shown that bias flow makes the acoustic properties much more complex compared to theory and with the no bias flow case, especially when velocity ratio between acoustic particle velocity and mean flow velocity is near unity.

NOMENCLATURE

\( c \) Speed of sound  
\( C_c \) Discharge coefficient  
\( d \) Orifice diameter  
\( D \) Diameter of pipe  
\( J \) Bessel function  
\( l \) Variable effective length  
\( l_o \) Orifice thickness  
\( l_0 \) End correction on one side of orifice  
\( L \) Jet length  
\( M_b \) Bias flow Mach number  
\( M_g \) Grazing flow Mach number  
\( k \) Wave number  
\( K_R \) Rayleigh conductivity  
\( U \) Mean flow velocity in the orifice  
\( V \) Acoustic particle velocity in the hole  
\( Z_R \) Acoustic resistance  
\( Z_I \) Acoustic reactance  
\( Z \) Acoustic impedance  
\( \beta = d/D \)  
\( \mu \) Adiabatic dynamic viscosity  
\( \nu \) Kinematic viscosity  
\( \omega \) Radian frequency  
\( \rho_0 \) Air density  
\( \sigma \) Porosity  
\( \tau \) Time for ejection of air flow orifice

Superscripts
\(^*\) Denotes a peak value

Subscripts
\( u \) On source side of orifice  
\( d \) On downside of orifice  
\( + \) Mean flow direction  
\( - \) Opposite direction of mean flow  
\( 1,2 \) Microphones on source side  
\( 3,4 \) Microphones on the downstream side

INTRODUCTION
Orifice plates and perforates appear in many technical applications where they are exposed to a combination of high acoustic excitation levels and either grazing or bias flow or a combination. Examples are automotive mufflers and aircraft engine liners. Taken one by one the effect of high acoustic excitation levels, bias flow and grazing flow are reasonably well understood. The nonlinear effect of high level acoustic excitation has for instance
been studied in [1–11]. It is well known from this literature that perforates can become non-linear at fairly low acoustic excitation levels. The non-linear losses are associated with vortex shedding at the outlet side of the orifice or perforate openings [9, 10]. The effect of bias flow has for instance been studied in [12–17]. Losses are significantly increased in the presence of bias flow, since it sweeps away the shed vortices and transforms the kinetic energy into heat, without further interaction with the acoustic field. Grazing flow has also received a lot of attention see for instance [18–24]. The combination of bias flow and high level acoustic excitation has been discussed and studied in [25] and some experimental investigations have been made in [26]. Luong [25] derived a simple Rayleigh conductivity model for cases when bias flow dominates and no flow reversal occurs.

The purpose of the present paper is to make a detailed experimental study of the transition between the case when high level nonlinear acoustic excitation is the factor determining the acoustic properties to the case when bias flow is most important. As discussed in [25] it can from a theoretical perspective be expected that this is related to if high level acoustic excitation causes flow reversal in the orifice or if the bias flow maintains the flow direction. Acoustic properties, such as impedance, Rayleigh conductivity and absorption coefficient are discussed.

Semi-empirical impedance model

Starting for instance from [4] a number of semi-empirical models have been developed to include the effect of high level acoustic excitation, grazing flow and bias flow have been suggested. One example is the model presented in [8] where the normalised impedance of a perforate is expressed as

\[ Z_R = \text{Re} \left( \frac{ik}{\sigma C_c} \right) \left[ \frac{l_w}{\mathcal{F}(\mu')} \left( \frac{\delta_{re}}{\mathcal{F}(\mu)} \right) f_{int} \right] + \frac{1}{\sigma} \left[ 1 - \frac{2J_1(kd)}{kd} \right] \]

\[ + \left( \frac{1 - \sigma^2}{\sigma^2 C_c^2} \right) \frac{1}{2c} \dot{V} \left( \frac{0.5 \sigma M_g + 1.15}{\sigma C_c M_b} \right) \]  

(1)

\[ Z_I = \text{Im} \left( \frac{ik}{\sigma C_c} \right) \left[ \frac{l_w}{\mathcal{F}(\mu')} \left( \frac{0.5d}{\mathcal{F}(\mu)} \right) f_{int} \right] - \left( \frac{1 - \sigma^2}{\sigma^2 C_c^2} \right) \frac{1}{2c} \dot{V} \left( \frac{3}{\sigma M_g} \right) \]

where \( Z_R \) is the normalized resistance and \( Z_I \) is the normalized reactance, \( k \) is the wave number, \( \sigma \) is the porosity (percentage open area), \( C_c \) is the discharge coefficient, \( l_w \) is the plate thickness, \( \mu \) is the adiabatic dynamic viscosity, \( \mu' = 2.179 \mu \) is the dynamic viscosity close to a conducting wall, \( v = \mu / \rho_0 \) is the kinematic viscosity, \( J \) is the Bessel function, \( d \) is the hole diameter, \( c \) is the speed of sound, \( M_g \) is the mean flow Mach number grazing to the liner surface, \( M_b \) is the bias flow Mach number inside the holes of the perforate and \( \dot{V} \) is the peak value of the acoustic particle velocity in the hole. The rest of the parameters are defined as

\[ K = \sqrt{-\frac{i\omega}{\nu}} \]  

(3)

\[ F(\mu) = 1 - \frac{4J_1(Kd/2)}{Kd \cdot J_0(Kd/2)} \]  

(4)

\[ \delta_{re} = 0.2d + 200d^2 + 16000d^3 \]  

(5)

\[ f_{int} = 1 - 1.47\sqrt{\sigma} + 0.47\sqrt{\sigma^3} \]  

(6)

Using Eqn. (1) and Eqn. (2) the magnitude of the terms related to high level nonlinear effects and bias flow, as well as grazing flow, can be compared. It should be noted that these terms are based on studies of the effect of nonlinearity and flow separately and not simultaneously.

The Cummings Equation

Consider orifice with bias flow, one of the most important models to study the acoustic properties is Cummings [6] empirical equation. It is based on Bernoulli equation for unsteady flow, which in [25] is written as

\[ \bar{\tilde{\tau}}(t) \frac{dV}{dt} + \frac{1}{2C_c^2} (U + V)|U + V| = \frac{P_0 + dP \cos \omega t}{\rho_0} \]  

(7)

where \( P_0 \) is the steady pressure drop; \( dP \cos \omega t \) is the sound pressure difference over the orifice; \( \bar{\tilde{\tau}}(t) \) is variable effective length of the fluid plug in the hole; \( V \) is oscillating velocity averaged over the plane of the orifice; \( U \) is the mean bias flow velocity in the orifice. For irrotational flow, the length \( \bar{\tilde{\tau}}(t) \) is \( 2l_0 + l_w \), where \( l_0 \approx (\pi/8)d \) is the end-correction on one side of the orifice plane. When a jet is formed, it according to Cummings [6] becomes a function of the effective length \( L \):

\[ L(\tau) = \int_0^\tau |U + V| dt \]  

(8)
where the time $\tau$ is measured from the previous flow “changeover” during the sign of $U+V(t)$ is constant. The empirical coefficient $\varepsilon = l(t)/L_0$, where $L_0 = 0.425d + l_w$ in [6] is the maximum possible lost end correction, were expressed as

$$\varepsilon = 1 - [1 + (L/d)^{1.585}/3]^{-1}$$  \hspace{1cm} (9)

This function is a little different in [25] when considering cases with bias flow. In the paper the effective length is divided into two parts: where the end correction on the inflow side remains constant as $l_0$ but on the outflow side decreases according to the jet length, as follows

$$l(t) = l_0 + \frac{l_0 + l_w}{1 + (L/d)^{1.585}/3}$$  \hspace{1cm} (10)

Thus, in the cases of $U \gg \tilde{V}$, Cummingss equation can be expressed as

$$\tilde{l}(t) \frac{dV}{dt} + \frac{V}{C_c^2} (U + \frac{V}{2}) = \frac{d\hat{P}e^{iat}}{\rho_0}$$  \hspace{1cm} (11)

here $\tilde{l}(t) \rightarrow l_0$ for the jet length is large enough. At high frequency($\omega R/U > 1$),$l(t)$ revert to the value $2l_0 + l_w$, for unsteady volume flux through the aperture causes pulsations in the jet cross-sectional area in or just downstream of the aperture. Using an effective hole thickness($l$) which can vary between $l_0$ and $2l_0 + l_w$ we get a normalized impedance

$$Z = \frac{d\hat{P}}{\rho_0 c \tilde{V}} = ikl + \frac{U}{cC_c^2} + \frac{\tilde{V}}{2cC_c^2}$$  \hspace{1cm} (12)

This gives a Rayleigh conductivity

$$K_R = \frac{ik\pi R^2}{Z} = \frac{\pi R^2}{l} \frac{\omega l/\rho}{(\omega l/\rho) + i/C_c^2(1 + \tilde{V}/2U)}$$  \hspace{1cm} (13)

If this is linearized a Rayleigh conductivity model was in [25] developed as

$$K = \frac{K_0(\omega l/U)}{(\omega l/U) + i/C_c^2}$$  \hspace{1cm} (14)

where $K_0 = \pi R^2/l$, $l = 2l_0 + l_w$

**EXPERIMENTAL SETUP**

The experimental configuration is illustrated in Fig. (1). The test object was an orifice plate with 3 mm thickness and 6 mm hole diameter. The orifice plate was mounted in a rigid tube with a diameter of 40 mm. On the left hand side, a high quality loudspeaker was mounted as the excitation source. Pure tone excitation was used and it was checked make sure that nonlinear harmonics generated at the loudspeaker were sufficiently small. Two microphones (1-4) were mounted on each side of the sample so that we could use two-microphone wave decomposition to identify the sound wave components on each side.

In order to get the mean flow velocity passing through the orifice, two steady pressure sensors 1⃝2⃝ were mounted to measure the steady pressure drop $\Delta P$. The calculation is according to ISO5167-1:2003 [27], as follows

$$U = C_c \sqrt{\frac{2\Delta P}{\rho_0(1-\beta^4)}}$$  \hspace{1cm} (15)

where $\beta = d/D$. According to [27] the discharge coefficient $C_c$ should be a function of $\beta$, Reynolds number and etc.. Here we suppose it is 0.75 recommended by Cummins [6]. In the study we consider two mean flow cases: one is with $\Delta P = 14\text{Pa}$ and the mean flow velocity in the orifice is 3.65 m/s; the other is $\Delta P = 40\text{Pa}$ and the mean flow velocity is 6.17 m/s.
Under the plane wave assumption, sound wave components on both sides the orifice can be expressed as

\[ \begin{bmatrix} P_{u+} \\ P_{u-} \end{bmatrix} = \begin{bmatrix} e^{ik_c d_1} e^{-ik_c d_1} \\ e^{ik_c d_2} e^{-ik_c d_2} \end{bmatrix}^{-1} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \] (16)

\[ \begin{bmatrix} P_{d+} \\ P_{d-} \end{bmatrix} = \begin{bmatrix} e^{ik_c d_3} e^{-ik_c d_3} \\ e^{ik_c d_4} e^{-ik_c d_4} \end{bmatrix}^{-1} \begin{bmatrix} P_3 \\ P_4 \end{bmatrix} \] (17)

where \( k = \omega / c, M = U / c, k_+ = k/(1 + M), k_- = k/(1 - M) \). With acoustic waves amplitudes \( P_{u+}, P_{u-}, P_{d+}, P_{d-} \), the oscillating velocity in the orifice \( \dot{V} \) and acoustic properties, such as normalized impedance \( Z \), Rayleigh conductivity \( K_R \) can be given as

\[ \dot{V} = \frac{P_{u+} - P_{u-}}{\rho_0 c \sigma} \] (18)

\[ Z = \frac{P_{u+} + P_{u-} - P_{d+} - P_{d-}}{\rho_0 c \dot{V}} \] (19)

\[ K_R = 2R \cdot \frac{i k \cdot \pi R / 2}{Z} \] (20)

RESULTS AND DISCUSSION

Effects of in incident pressure level

Fig. (2) shows the acoustic particle velocity for different levels of pressure difference over the orifice. The nonlinear behavior at higher levels of excitation can be clearly seen. The corresponding normalized impedance is shown in Fig. (3), and we can see the non-linearity at fairly low acoustic excitation levels. Instead plotting the Rayleigh conductivity makes the curves for different frequencies collapse, as can be seen in Fig. (4). The real part approaches the value \( K_0 / 2R = 0.61 \) at low levels. The real part decrease when the inverse Strouhal number is near 1 and goes to a very low value at high inverse Strouhal numbers; while the imaginary part first increases and reaches a maximum at an inverse Strouhal number around 1.5.
FIGURE 5: Normalised hole impedance divided by the Helmholtz number plotted against inverse Strouhal number \( \hat{V}/\omega d \)

It can be seen from Fig. (5) that dividing the normalized impedance by the Helmholtz number makes the curves for different frequencies collapse which is consistent with the result for the Rayleigh conductivity. There is also a fairly good agreement between experimental resistance and the results from both the Elnady and Cummings models. These two models for the resistance only differs on two points: The Cummings model does not include any linear resistance term and the slope of the nonlinear term is reduced by a factor \( 1 - \sigma^2 \) in the Elnady model compared to the Cummings model. The slope can in both models be changed by choosing another value for the discharge coefficient. Here the discharge coefficient has been set to 0.75 for both models. For the reactance term there is a reasonable agreement between experimental results and the Elnady model at lower inverse Strohal numbers. Higher up the model does not catch the fact that the reactance does not continue to decrease with the increase in particle velocity. The Cummings model, as it is presented in Eqn. (12) does not include any nonlinear effect on the reactance term. In the article by Cummings [6] it is however discussed that the reactance may vary with time and with the effective jet length caused by the high level acoustic excitation. The effective thickness would at low acoustic levels take the value \( l = l_w + 2l_0 \) and would then decrease at higher acoustic excitation levels. It can be seen that this agrees fairly well with the results in Fig. (5), the experimental reactance results start close to the curve for effective thickness \( l = l_w + 2l_0 \) and approaches the curve for effective thickness \( l = l_w \) at high acoustic excitation levels.

FIGURE 6: Measured normalized impedance in the orifice as a function of ratio between a acoustic particle velocity and mean flow velocity

When there is a flow through the orifice the acoustic properties become more complex. Fig. (6) shows the normalized impedance as a function of the ratio of oscillating velocity to flow velocity. We divide the results into three parts according to the value of the velocity ratio: much less than unity (I), near unity (II) and much larger than unity (III). The resistance reduces as the velocity ratio increases in region I, has a minimum in region II and then increases in region II where the acoustic particle velocity dominates the behavior. The reactance has a more complex behavior and can either initially increase or decrease with increasing velocity ratio in region I. It then has a minimum in region II and the increases in region III to finally approach a constant value at high velocity ratios.

Fig. (7) compares the measured result with Elnady and Cummings models calculated by Eqn. (12). Neither resistance nor reactance is consistent with experimental data, especially near the ratio of unity. It seems the mechanism is much more complex and further works should focus on the interaction of frequency, acoustic particle velocities and mean flow velocity.

In the conclusions of [25] it was mentioned that the Rayleigh conductivity for the case without flow reversal (\( \hat{V} \ll U \)) should approach the result in Eqn. (14). In order to check this experimental results for the Rayleigh
FIGURE 7: Normalized impedance in the orifice as a function of ratio between a acoustic particle velocity and mean flow velocity ($U=6.17$ m/s)

FIGURE 8: Rayleigh conductivity plotted against flow Strouhal number $\omega R/U$

conductivity has been compared to the model results. Fig. (8) shows the Rayleigh conductivity plotted against flow Strouhal number ($\omega R/U$). This means that at each Strouhal number there are a number of experimental data points representing different acoustic particle velocity levels. It can be seen that the Rayleigh conductivity does not exhibit a linear behavior since the results vary with acoustic excitation level at each flow Strouhal number point. The agreement with the model result is also not very good. It can be seen that by varying the effective hole thickness results of the right order of magnitude can be obtained but it seems that the Rayleigh conductivity has a more complicated dependence on both mean flow velocity and acoustic excitation level than indicated by Eqn. (14).

CONCLUSIONS

An experimentally study of the acoustic properties for an orifice plate under high acoustic excitation levels and bias flow conditions has been made. Comparisons have been made with a semi-empiric impedance model [7, 8] and the Cummings model as described in [25]. It was seen that without bias flow there is a reasonably good agreement between model results and measurements for the resistance. For the reactance the model according to [7, 8] catches the initial decrease with increasing excitation level but not the subsequent behavior at high excitation levels. The Cummings [6] model as described in [25] discusses the possibility of an end correction which varies with both bias flow and high level acoustic excitation. It can be seen that the measured reactance is within the range predicted by the suggested variations in end corrections. For the case with bias flow three regions were identified in terms of the ratio between acoustic particle velocity and mean flow velocity being: (I) smaller than unity, (II) around unity and (III) larger than unity. For region I there was a decrease in resistance and a variation in reactance with velocity ratio. In region II both parts of the impedance had a minimum. In region III resistance increases while the reactance first has an increase and the approaches a constant value. Compared with experimental data, it seems neither the Elnady nor the Cummings model gives a good prediction result since the nonlinear acoustic mechanism with bias flow is much more complex than that without. In [25] it was predicted that the Rayleigh conductivity would go to the linearized value according to Eqn. (14) for cases when the acoustic particle velocity is smaller than the mean flow velocity in the orifice so that no flow reversal occurs. Comparisons with experimental results shows that this is not the case there is still a nonlinear variation in Rayleigh conductivity even when the velocity ratio is small.

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