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The effect of combined high level acoustic excitation and bias flow on the acoustic properties of an in-duct orifice

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This paper investigates the acoustic properties of an orifice with bias flow under medium and high sound level excitation. Orifices with two different edge configurations were tested experimentally. The study includes a wide range of bias flow velocities, various acoustic excitation levels and different frequencies. The so-called Cummings equation was the starting point for the theoretical modelling. It was modified and a novel orifice acoustic discharge coefficient model was developed both for cases with and without bias flow. The model was experimentally validated. With this model the acoustic resistance is obtained by the harmonic balance method, and the results agree fairly well with the experimental results for low frequencies. Experimental results also show that bias flow makes the acoustic properties much more complex compared to the no bias flow case, especially when the velocity ratio between acoustic particle velocity and mean flow velocity is near unity.

Nomenclature

C_C	= Orifice time varying discharge coefficient
C_{CA}	= Acoustic flow discharge coefficient
C_{CA0}	= Acoustic flow discharge coefficient under low acoustic excitation
C_{CAmin}	= Minimum acoustic flow discharge coefficient under high acoustic excitation
C_{CAi}	= Acoustic flow discharge coefficient for different regions, $i=1,2,3,4$
C_{CM}	= Mean flow discharge coefficient
$\bar{l}$	= Effective orifice thickness
$\bar{l}$	= Average effective orifice thickness
l_0	= End correction on one side of orifices $\approx (\pi/4) r$
l_W	= Orifice thickness
L_J	= Orifice jet length
U	= Mean flow velocity
V	= Fluctuating acoustic velocity
$\hat{V}$	= Peak velocity for acoustic flow
V_{rms}	= Root mean square velocity value for acoustic flow
r	= Orifice radius
Re_{rms}	= Reynolds number for acoustic flow
Sr_{rms}	= Strouhal number for acoustic flow
T	= Acoustic period time, angular frequency $= 2\pi/T$, and frequency $= 1/T$
ρ_0	= Air density
σ	= Porosity of orifice plate
Δp	= Fluctuating pressure difference
ΔP	= Steady pressure drop
ΔP_R ΔP_I	= Real and imaginary part of acoustic pressure difference
ν	= Air kinetic viscosity (1.55×10^{-5} Pa s)

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I. Introduction

Orifice plates and perforates appear in many technical applications where they are exposed to high acoustic excitation levels and either grazing or bias flow or a combination. Examples are automotive mufflers and aircraft engine liners. Taken one by one the effect of high acoustic excitation levels, bias flow and grazing flow are reasonably well understood. The nonlinear effect of high level acoustic excitation has for instance been studied in¹⁻¹¹. It is well known from this literature that perforates can become non-linear at fairly low acoustic excitation levels. The non-linear losses are associated with vortex shedding at the outlet side of the orifice or perforate openings⁹⁻¹⁰. The effect of bias flow has for instance been studied in¹²⁻¹⁷. Losses are significantly increased in the presence of bias flow, since it sweeps away the shed vortices and transforms the kinetic energy into heat, without further interaction with the acoustic field. Grazing flow has also received a lot of attention, for instance¹⁸⁻²³. The combination of bias flow and high level acoustic excitation has been discussed and studied in²⁴ and some experimental investigations have been made in²⁵. Luong²⁴ derived a simple Rayleigh conductivity model for cases when bias flow dominates and no flow reversal occurs.

The purpose of the present paper is to make a detailed study of the transition between the case when high level nonlinear acoustic excitation is the factor determining the acoustic properties to the case when bias flow is most important. As discussed in²⁴, it can from a theoretical perspective be expected that this is related to if high level acoustic excitation causes flow reversal in the orifice or if the bias flow maintains the flow direction. Three regions are identified according to different combinations of mean flow velocity and acoustic velocity. For theoretical modelling, Cummings equation is modified and a novel acoustic discharge coefficient model is developed. The harmonic balance method is used to get an analytic acoustic resistance model for different situations. For low frequencies there is a fairly good agreement with experimental results. Experimental acoustic properties, such as impedance are discussed and compared for two orifice plates with two different edge configurations.

II. Modified Cummings equation

For orifices with bias flow, one of the most important models to study the acoustic properties is Cummings⁶ empirical equation. It is based on Bernoulli equation for unsteady flow, which in²⁴ is written as

$$l(t) \frac{dV(t)}{dt} + \frac{1}{2} \left(\frac{U + V(t)}{C_c(t)} \right) \cdot \left| \frac{U + V(t)}{C_c(t)} \right| = \frac{\Delta p(t) + \Delta P}{\rho_0}, \quad (1)$$

where $l(t)$ is an effective orifice thickness including end corrections which can be time varying, $V(t)$ is the fluctuating acoustic velocity in the orifice, U is the mean flow velocity, $\Delta p(t)$ is the fluctuating pressure difference over the orifice, and ΔP is the steady pressure drop over the orifice, $C_c(t)$ is a discharge coefficient to consider the vena contracta effect which should be also time varying.

According to Cummings⁶, the value 0.75 for discharge coefficient is consistent with experimental results for high levels of acoustic excitation. However, as mentioned in⁶ it is possible that the discharge coefficient will vary with time if the level of acoustic excitation is high. In the presence of mean flow, the value of the discharge coefficient should vary according to the acoustic flow as well as mean flow velocity. Therefore, the discharge coefficient can be split into two parts: C_{CM} for mean flow and C_{CA} for acoustic flow. The discharge coefficient for mean flow can be easily found either from theory or experimental result. The acoustic flow discharge coefficient, however, need to be further modelled.

As shown in Fig.1, the acoustic flow caused by single frequency excitation combines with the mean flow, giving three regions: no mean flow (Region 1), mean flow and acoustic excitation of equal importance (Region 2) and mean flow dominates (Region 3). Region 2 is more complicated and could be divided into two subregions according to the period (τ) of flow reversal. Region 2.1 is the part of the cycle where there is flow reversal ($U+V(t)<0$) and Region 2.2 is the rest part of the cycle without flow reversal ($U+V(t)>0$). According to the previous discussion of discharge coefficient it is suggested the Cummings equation (Eq. (1)) can be modified as follows.

Region 1 ($U=0$) and Region 2.1 ($U < -V(t)$ & $U < \hat{V}$):

In these regions, the acoustic flow dominates the behaviour. It happens when there is no mean flow ($U=0$), or when the flow reversal in the present of mean flow ($U < \hat{V}$). Here it is suggested to use acoustic discharge coefficient and to modify the Cummings equation as:

Figure 1. Flow directions through the orifice for different regions

Figure 2. Acoustic flow vena contracta during two half-cycles

$$l(t) \frac{dV(t)}{dt} + \frac{1}{2} \left(\frac{U + V(t)}{C_{CA}} \right) \cdot \left| \frac{U + V(t)}{C_{CA}} \right| = \frac{\Delta p(t) + \Delta P}{\rho_0}. \quad (2)$$

Region 2.2 ($U > -V(t)$ & $U < \hat{V}$) and Region 3 ($U > \hat{V}$):

In these regions, the influence of the acoustic excitation becomes weaker. The nonlinear part of Cummings equation should be determined by mean flow and acoustic flow together. And the Cummings equation can be modified as:

$$l(t) \frac{dV(t)}{dt} + \frac{1}{2} \left(\frac{U}{C_{CM}} + \frac{V(t)}{C_{CA}} \right) \cdot \left| \frac{U}{C_{CM}} + \frac{V(t)}{C_{CA}} \right| = \frac{\Delta p(t) + \Delta P}{\rho_0}. \quad (3)$$

A. Acoustic discharge coefficient C_{CA}

The discharge coefficient for steady mean flow has been studied extensively both theoretically and experimentally. However there is fairly few publications on acoustic flow discharge coefficients²⁶⁻²⁷. As shown in Fig. 2, acoustical flow discharge coefficient of an orifice can be equivalently determined as the average volume flow

rate entering or exiting the orifice during a half-cycle. Hersh²⁶ experimentally studied the acoustic flow discharge coefficient of the orifice for a Helmholtz resonator and observed that it tended to be unity at low acoustic excitation level and decreased according to acoustic excitation. It also followed by the numerical investigation of Zhang²⁷, which, in addition, showed that the discharge coefficient increased with frequency at the same acoustic excitation. However, both studies were limited to no mean flow cases where acoustic flow determines the acoustic properties as in Region 1. In the presence of mean flow as in Region 2, one can expect that the acoustic flow discharge coefficient should be in-between the mean flow discharge coefficient and the minimum acoustic flow discharge coefficient for fully developed turbulent acoustic flow.

Following the discussion above, a model for the acoustic discharge coefficient were developed as

$$C_{CA} = (C_{CA0} - (C_{CA0} - C_{CAmin}) \cdot \text{erf}(\frac{\text{Re}_{ms}^{\alpha_1} \sqrt{\pi}}{\text{Sr}_{ms}^{\alpha_2} \text{Re}_{ref} C_{CAmin}})), \quad (4)$$

where $\text{erf}(x) = (2/\sqrt{\pi}) \int_0^x e^{-\xi^2} d\xi$ is the error function; C_{CA0} is the acoustic discharge coefficient for low acoustic excitation level (=1 in Region 1 and 2.1, = C_{CM} in Region 2.2 and 3); C_{CAmin} is the minimum acoustic flow discharge coefficient for high level acoustic excitation; Re_{ref} is the typical Reynolds number (4000) for full developed turbulence; Re_{ms} , Sr_{ms} are the Reynolds number and Strouhal number for acoustic flow, which are

$$\text{Re}_{ms} = \frac{V_{rms} \cdot r}{\nu}, \quad \text{Sr}_{ms} = \frac{\omega \cdot r}{V_{rms}},$$

V_{rms} is the root mean square velocity value for acoustic flow; α_1, α_2 are power indexes which should be slightly dependant on acoustic frequency, since even with the same ratio of Reynolds number to Strouhal number the acoustic flow details in orifices should be different. However, we assume here they are the same and take unity values, $\alpha_1, \alpha_2=1$.

In summary, the following acoustic discharge coefficient models for different cases will be used,

$$C_{CA} = \begin{cases} (1 - (1 - C_{CAmin}) \cdot \text{erf}(\frac{\sqrt{\pi}}{4000 C_{CAmin}} \frac{V_{ms}^2}{\omega \nu})), & V_{ms}^2 = \frac{1}{\tau} \int_{\tau} (U + V(t))^2 dt \quad (U = 0 \text{ or } U < -V(t)) \\ (C_{CM} - (C_{CM} - C_{CAmin}) \cdot \text{erf}(\frac{\sqrt{\pi}}{4000 C_{CAmin}} \frac{V_{ms}^2}{\omega \nu})), & V_{ms}^2 = \frac{1}{T - \tau} \int_{T-\tau} (U + V(t))^2 dt \quad (U > -V(t) \text{ and } U < \hat{V}) \\ (C_{CM} - (C_{CM} - C_{CAmin}) \cdot \text{erf}(\frac{\sqrt{\pi}}{4000 C_{CAmin}} \frac{V_{ms}^2}{\omega \nu})), & V_{ms}^2 = \frac{1}{T} \int_T V^2(t) dt \quad (U > \hat{V}) \end{cases}, \quad (5)$$

where τ is the flow reversal period in the opposite direction of mean flow, where $T=2\pi/\omega$ is the acoustic period.

Figure 3. Acoustic discharge coefficient as a function of mean acoustic velocity and frequency: (a) no mean flow, (b) with mean flow ($C_{CM}=0.61$, $C_{CAmin}=0.75$)

B. Effective orifice length $l(t)$

The effective orifice length describes the acoustic inertia of the irrotational flow around orifices. Supposing the rotational flow mainly starts within and downstream of the orifices, in the article by Cummings⁶ an empirical expression for the time varying effective orifice length was presented

$$l(L_j) = l_0 + \frac{l_w + l_0}{1 + \frac{(L_j/2r)^{1.585}}{3}}, \quad (6)$$

where $l_0 \approx (\pi/4) r$ is the end correction on one side of the orifice, l_w is the orifice thickness, $L_j(t)$ is a time varying jet length caused by the high level acoustic excitation. Cummings suggested that the jet length should be estimated from

$$L_j(t) = \int_{\tau_j} |U + V(t)| dt, \quad (7)$$

where τ_j is the ‘jet age’ from the beginning of the acoustic half cycle to $V(t)$ changes sign, which means τ_j equals half a period of acoustic flow in absence of mean flow. Following the discussion in²⁴, the jet length should be much more complicated, especially when flow reversal occurs. The average effective length ($\bar{l}$) should tend to have a maximum value ($l_w + 2l_0$) under low acoustic excitation without mean flow, and have a minimum value (l_0) either for high acoustic excitation or with high mean flow.

C. Solution and normalized acoustic impedance

In order to get the acoustic impedance, there are different methods to solve the nonlinear modified Cummings equation (Eq. (2)-(3)). One way is to take a harmonic acoustic pressure loading $\Delta p \cos(\omega t)$ to solve the solution for the acoustic velocity in the orifice, and get the impedance at the end, which was used by Ingard⁸ and Cummings²⁸ for cases without mean flow. It is quite consistent with the experimental situations, since pure tone acoustic excitation is widely used for investigation of acoustic properties. But in presence of mean flow, the solution process is much more complicated and it makes analytical solution impossible. Instead, one can impose the acoustic flow velocity $\hat{V} \cos(\omega t)$ and using the harmonic balance method to analyse the basic harmonic acoustic pressure difference. The imaginary part of acoustic pressure difference ΔP_I is then

$$\frac{\Delta P_I}{\rho_0} = -\frac{2}{T} \int_T l(t) \cdot \hat{V} \sin^2(\omega t) d(\omega t). \quad (8)$$

The real part of acoustic pressure difference ΔP_R is

$$\frac{\Delta P_R}{\rho_0} = \frac{2}{T} \int_T \frac{1}{2} \left(\frac{U + \hat{V} \cos(\omega t)}{C_{CA}} \right) \cdot \left| \frac{U + \hat{V} \cos(\omega t)}{C_{CA}} \right| \cdot \cos(\omega t) dt, \quad (9)$$

or

$$\frac{\Delta P_R}{\rho_0} = \frac{2}{T} \int_T \frac{1}{2} \left(\frac{U}{C_{CM}} + \frac{\hat{V} \cos(\omega t)}{C_{CA}} \right) \cdot \left| \frac{U}{C_{CM}} + \frac{\hat{V} \cos(\omega t)}{C_{CA}} \right| \cdot \cos(\omega t) dt. \quad (10)$$

The normalized impedance can be written as

$$Z(\omega) = \frac{\Delta P_R - i \cdot \Delta P_I}{\hat{V} \cdot \rho_0 c_0}. \quad (11)$$

With the acoustic discharge coefficient according to Eq.(5) and Cummings effective length model according to Eq.(6) the final normalized impedance model can be developed for different regions.

Region 1($U=0$):

$$Z(\omega) = \frac{4}{3\pi} \frac{\hat{V}}{c_0 C_{CA1}^2} + i \cdot \frac{\omega}{c_0} l(L_{j1}), \quad (12)$$

where

$$C_{CA1} = 1 - (1 - C_{CAmin}) \cdot \text{erf}\left(\frac{\sqrt{\pi}}{4000 C_{CAmin}} \frac{V_{ms1}^2}{\omega \nu}\right), \quad V_{ms1}^2 = \frac{\hat{V}^2}{2},$$

$$l(L_{J1}) = l_0 + \frac{l_w + l_0}{1 + \frac{(L_{J1}/d)^{1.585}}{3}}, \quad L_{J1} = \frac{2\hat{V}}{\omega}.$$

Region 2 ($U < \hat{V}$):

$$Z(\omega) = \frac{1}{2\pi c_0} \left[\frac{(\sin 3\theta + 9 \sin \theta) \hat{V}^2}{6C_{CA2}^2} + \frac{(\sin 3\theta + 9 \sin \theta) \hat{V}^2}{6C_{CA3}^2} - \frac{(2\theta + \sin 2\theta) \cdot U \hat{V}}{C_{CA2} C_{CM}} + \frac{(2\pi - 2\theta - \sin 2\theta) \cdot U \hat{V}}{C_{CA3} C_{CM}} + \frac{4 \sin \theta \cdot U^2}{C_{CM}^2} \right] + i \cdot \frac{\omega}{c_0} \bar{l} \quad (13)$$

where

$$C_{CA2} = 1 - (1 - C_{CAmin}) \cdot \text{erf}\left(\frac{\sqrt{\pi}}{4000 C_{CAmin}} \frac{V_{ms2}^2}{\omega \nu}\right), \quad V_{ms2}^2 = \frac{\hat{V}^2}{2} + U^2 + \frac{1}{\theta} \left[\frac{\sin 2\theta \cdot \hat{V}^2}{4} - 2U\hat{V} \sin \theta \right],$$

$$C_{CA3} = C_{CM} - (C_{CM} - C_{CAmin}) \cdot \text{erf}\left(\frac{\sqrt{\pi}}{4000 C_{CAmin}} \frac{V_{ms3}^2}{\omega \nu}\right), \quad V_{ms3}^2 = \frac{\hat{V}^2}{2} + U^2 - \frac{1}{\pi - \theta} \left[\frac{\sin 2\theta \cdot \hat{V}^2}{4} - 2U\hat{V} \sin \theta \right],$$

$$\theta = \arccos(U/\hat{V}).$$

Region 3 ($U > \hat{V}$):

$$Z(\omega) = \frac{U}{c_0 C_{CM} \cdot C_{CA4}} + i \cdot \frac{\omega}{c} \bar{l}, \quad (14)$$

where

$$C_{CA4} = C_{CM} - (C_{CM} - C_{CAmin}) \cdot \text{erf}\left(\frac{\sqrt{\pi}}{4000 C_{CAmin}} \frac{V_{ms4}^2}{\omega \nu}\right), \quad V_{ms4}^2 = \frac{\hat{V}^2}{2}.$$

One thing which should be mentioned is that comparing our nonlinear resistance model (Eq. (12)) with the model (Eq.(24)) in Cummings' paper²⁸, the slope coefficient is slightly different. It is $4/3\pi \approx 0.4246$ for our model and $1.11/2.464 \approx 0.4505$ for Cummings. The slight difference, as stated before, is from the difference whether we take the acoustic velocity as harmonic input or the pressure difference. In the real situation non of these idealized assumptions apply. However it means the value 0.728 for minimum acoustic flow discharge coefficient for our model is equivalent to the value of 0.75 in²⁸.

III. Experimental setup

The experimental setup is illustrated in Fig. 4. The test object is a orifice plate mounted in a duct with a diameter of 40 mm. Six microphones were divided into two groups and symmetrically installed on both sides of the test sample so that we could use the two-microphone method to identify the sound wave components on each side. Two different transducer separations (24mm and 180mm) gave a frequency arrange from 80Hz up to 5000Hz. On the left hand side, a high quality loudspeaker was mounted as the excitation source. Pure tone acoustic excitation was used and we made sure that nonlinear harmonics were sufficiently small when performing high pressure measurement. In order to measure the mean flow velocity, on the upstream side, a laminar flow meter was employed during the experiment. A sound attenuation system, including a tunable Helmholtz resonator and a muffler, were well designed to attenuate the sound to less than 126 dB in the position of laminar flow meter, to reduce the measurement error caused by the fluctuating flow. During the experiment the steady pressure drop over the orifice was also monitored by two pressure sensors installed further away from the test sample than the microphones. The mean flow discharge coefficient could be calculated as

$$C_{CM} = \frac{U}{\sqrt{2\Delta P / \rho_0}}. \quad (15)$$

In the study, a wide range of mean flow (0-19m/s in the orifice), sound levels (100-155dB) and frequencies (100-1000Hz) were considered. Two orifice plates were tested, which have the same thickness and hole diameter, but

Figure 4. Schematic of the experimental setup, dimensions in millimetres

Figure 5. Orifice geometry, Orifice1: chamfer-edged, Orifice 2: thick sharp-edged

Figure 6. Forward and backward travelling wave components

different edges, as shown in Fig.5. Orifice 1 does not have a perfect sharp edge on upstream side. Instead it has an equivalent thickness about 0.6mm for the hole with diameter of 6mm.

Fig. 6 shows the forward and backward travelling waves on both sides of the orifice, which were determined by two microphone wave decomposition method as follows.

$$\begin{bmatrix} e^{-ik_+d_i} & e^{ik_+d_i} \\ e^{-ik_+d_j} & e^{ik_+d_j} \end{bmatrix} \begin{bmatrix} P_+ \\ P_- \end{bmatrix} = \begin{bmatrix} P_i \\ P_j \end{bmatrix}, \quad (16)$$

where $k_{\pm}$ are the wavenumbers for forward and backward planar waves. Following a model proposed by Dokumaci²⁹, the effect of visco-thermal damping in pipe was included as

$$k_{\pm} = \frac{\omega}{c_0} \frac{K_0}{1 \pm K_0 M}, \quad (17)$$

where

$$K_0 = 1 + \left(\frac{1-i}{s\sqrt{2}} \right) \left(1 + \frac{\gamma-1}{\sqrt{\text{Pr}}} \right), \quad (18)$$

Figure 7. Pressure difference level plotted against inverse Strouhal number ($\hat{V} / \omega r$), frequency range: 100-1000Hz, (a): Orifice 1, (b): Orifice 2

where $s = r\sqrt{\omega/\nu}$ is the shear wavenumber, R is the duct radius; γ is the ratio of specific heats and Pr is the Prandtl number; which is 0.712 for the experimental temperature (about 22 °C); and M is mean flow Mach number in pipe .

With the planar wave components (P_{u+} , P_{u-} , P_{d+} , P_{d-}) on both sides, the oscillating velocity V in the orifice and acoustic properties, such as the normalized impedance can be given as

$$V = \frac{P_{u+} - P_{u-}}{\rho_0 c_0 \sigma} , \quad (19)$$

$$Z = \frac{(P_{u+} + P_{u-}) - (P_{d+} + P_{d-})}{\rho_0 c_0 V} . \quad (20)$$

IV. Results and discussion

A. Acoustic impedance without bias flow

A wide range of frequencies and acoustic excitation have been studied in the test campaign. The range of frequency is from 100Hz to 1000Hz ($k r \approx 0.006-0.05$) with a step of 100Hz. As show in Fig.7, the pressure difference is from below 120dB up to about 155dB plotted as a function of acoustic inverse Strouhal number. Fig.8 shows the normalized impedance divided by the Helmholtz number, which makes the curves for different frequencies collapse. There is a fairly good agreement between experimental resistance and the analytical results even when the discharge coefficients are kept as a constant minimum value, which is 0.728 for Orifice 1 and 0.7 for Orifice 2. This difference indicates that the acoustic discharge coefficient, like the mean flow discharge coefficient, could vary according to different orifice geometries. For medium or low acoustic excitation, the errors are larger. One reason could be the viscosity in the orifice; another reason is that the acoustic discharge coefficient should not take the minimum level at low excitation levels. However the last factor should be much more important for low frequencies. With the acoustic discharge coefficient model the results is much better for low and medium acoustic levels, as shown in Fig. 9.

For acoustic reactance with Cummings effective orifice length model, the analytical results have a qualitative consistence with our experimental results as shown in Fig. 8. The experimental results show that the reactance have a constant value with $l = l_w + 2l_0$ at low acoustic levels; decrease with higher acoustic excitation levels; and tend to a constant level with a small value at high excitation levels. This minimum reactance value seems to vary with different orifice geometries. Compared with the thick orifice (Orifice 2) the reactance for the thin orifice (Orifice 1) is much more sensitive to acoustic excitation.

Figure 8. Normalized acoustic impedance divided by Helmholtz number ($\omega r / c_0$) plotted against inverse Strouhal number ($\hat{V} / \omega r$), frequency range: 100-1000Hz, (a): Orifice 1, (b): Orifice 2

Figure 9. Normalized acoustic resistance comparison for acoustic discharge coefficient model Eq.(5), frequency 100Hz, (a): Orifice 1, (b): Orifice 2

B. Acoustic impedance with bias flow

In the presence of bias flow the acoustic properties becomes quite complicated, since it is not only a function of acoustic excitation level and frequency but also influenced by mean flow velocity. In view of the flow pattern, both bias flow and acoustic flow can be laminar or turbulent depending on their Reynolds numbers. Table 1 provides parameters for the bias flow in two orifices used in the experiments. The mean flow discharge coefficient is calculated according to Eq. (15). In most cases the values are between 0.6 and 0.7 which are typical for turbulent flow in orifices. The exception is the case with low Reynolds number for Orifice 2 where the discharge coefficient is around 0.8.

Table 1. Measured bias flow velocity and mean flow discharge coefficient

Orifice 1			Orifice 2		
Bias flow velocity U (m/s)	Reynolds number $(2Ur/\nu)$	Discharge coefficient C_{cM}	Bias flow velocity U (m/s)	Reynolds number $(2Ur/\nu)$	Discharge coefficient C_{cM}
2.8	1084	0.663	3.9	1510	0.799
7.8	3019	0.676	7.4	2865	0.610
11.5	4452	0.697	11.7	4529	0.687
			14.5	5613	0.645
			18.6	7200	0.684

Figure 10. Normalized acoustic impedance for different bias flow velocities, Orifice 1, frequency: 200Hz **Figure 11. Normalized acoustic impedance for different bias flow velocities, Orifice 2, frequency: 200Hz**

Fig.10-11 compares acoustic impedance results for the two orifices with different bias flow velocities and low to high acoustic excitation levels, which is from Region 3 ($U > \hat{V}$) to Region1 ($U = 0$ or $U \ll \hat{V}$). The results show that the acoustic resistance firstly decreases with an increase in acoustic excitation level, and then tend to increase and approach the result without bias flow. The minimum is obtained when the acoustic velocity is similar in magnitude to the bias flow velocity. The reason could be related to the difference in values of mean flow discharge and acoustic discharge coefficient, since normally the acoustic discharge coefficient is a bit larger than the mean flow discharge coefficient, as validated from the fairly good consistence with analytical results for both orifices. The agreement is

Figure 12. Normalized acoustic impedance for different frequencies, Orifice 1, $U=11.5$ m/s

Figure 13. Normalized acoustic impedance for different frequencies, Orifice 2, $U=11.7$ m/s

much better for the thin orifice (Orifice 1) because of the absence of the frequency dependant influence by orifice thickness. The reactance, which is plotted divided by the Helmholtz number, has varying values for low acoustic excitation depending on mean flow velocity and orifice geometries. The values are even smaller than the one-sided end correction for relative high bias flow levels. Compared with the no bias flow case, even a very small bias flow can decrease the reactance substantially for low acoustic excitation levels. With increase of acoustic excitation, the acoustic reactance starts to increase to a maximum value. Then it behaves similar to that in the no bias flow cases. This transfer point for acoustic flow velocity depends on the bias flow velocity. The higher the bias flow velocity, the higher acoustic excitation it needs.

There is no doubt that acoustic impedance is also frequency dependent. Fig. 12-13 show the values of acoustic impedance for different frequencies with the same bias flow velocity for both orifice. For Region 3 ($U > \hat{V}$), low frequencies and low acoustic excitation, the value for resistance is quite close to the analytical result, which is dependent on bias flow velocity and mean flow discharge coefficient. In this case, the flow jet kinetic energy changes slowly. So the flow discharge coefficient should be quite stable and close to the value in the absence of acoustic excitation, which was measured and used for the analytical model. For higher frequencies the dimension of unsteady vorticity out of the incompressible jet should be in the order of magnitude $\sim U/\omega$, which means the scale of turbulence decreases with frequencies. Therefore additional irrotational flow is developed and the flow discharge

coefficient increase with the vena contracta area expansion. As stated in²⁴, this irrotational response in exterior fluid must become essentially similar to that in the absence of the jet (bias flow). So higher frequencies decrease the acoustic resistance and increase the acoustic reactance, as shown in Fig. 12. However, since the turbulence caused by mean flow somehow is still present, the mean flow discharge coefficient should still be less than the acoustic flow discharge coefficient. A minimum value for resistance is obtained with an increase in the acoustic flow velocity, and the reactance increases to a constant value. Comparing the thick orifice (Orifice 2) to the thin (Orifice 1), the resistance for some high frequencies even decreased to a negative value and the reactance sharply increased at low acoustic excitation. The reason is that these frequencies (800-1000Hz) fall into the range of flow instability, where the Strouhal number based on orifice thickness and bias flow ($f t_w / U$) equals 0.2-0.35³⁰. Even though increasing acoustic excitation increases the resistance to positive values. This means the high acoustic level somehow could decrease the flow instability, which conclusion would need further validation.

V. Conclusions

In this paper, the nonlinear acoustic properties of orifices under high acoustic excitation and with bias flow have been studied for different frequencies. The Cummings equation⁶ has been modified and a novel orifice acoustic discharge coefficient model was developed both for no bias flow cases and bias flow cases. An analytical acoustic impedance model has been developed by using the harmonic balance method. Comparisons have been made with the model for two orifices with different edge configurations. It was seen that without bias flow the acoustic impedance only depends on the inverse acoustic Strouhal number and there is a reasonably good agreement between analytical model results and measurement for acoustic resistance. The reactance model based on Cummings effective length model catches the initial decrease with increasing excitation but has larger errors for high excitation levels. For the case with bias flow, when acoustic excitation is low, the resistance decreases with frequency, while the reactance increases accordingly. Orifice thickness influences the flow stability and the resistance tends to be negative while the reactance increases sharply with a relative small increase of acoustic excitation level. For medium acoustic excitation levels, both resistance and reactance increase with the acoustic excitation. A minimum frequency dependent value exists for resistance when the acoustic flow velocity is of the same magnitude or slightly smaller than the bias flow velocity. For high acoustic excitation the acoustic impedance is similar to the no bias flow cases. There is fairly good agreement with the analytical model for resistance either for low or high acoustic excitation for low frequency.

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