Analysis of shear strength of rock joints with PFC2D

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Abstract

Joints are the main features encountered in rock and sliding of rock blocks on joints is classified as the principal source of instability in underground excavations. In this regard, joints’ peak shear strength is the controlling parameter. However, given the difficulty in estimating it, shear tests are often performed. These are often quite expensive and also time consuming and, therefore, it would be valuable if shear tests could be artificially performed using numerical models. The objective of this study is to prove the possibility to perform virtual numerical shear tests in a PCF2D environment that resemble the laboratory ones.

A numerical model of a granite rock joint has been created by means of a calibration process. Both the intact rock microparameters and the smooth joint scale have been calibrated against macroparameters derived from shear tests performed in laboratory. A new parameter, the length ratio, is introduced which takes into account the effective length of the smooth joint compared to the theoretical one. The normal and shear stiffnesses, the cohesion and the tensile force ought to be scaled against the length ratio.

Four simple regular joint profiles have been tested in the PFC2D environment. The analysis shows good results both from a qualitative and from a quantitative point of view. The difference in peak shear strength with respect to the one computed with Patton’s formula is in the order of 1% which indicates a good accuracy of the model.

In addition, four profiles of one real rough mated joint have been tested. From the scanned surface data, a two-dimensional profile has been extracted with four different resolutions. In this case, however, interlocking of particles along the smooth joint occurs, giving rise to an unrealistic distribution of normal and shear forces. A possible explanation to the problem is discussed based on recent developments in the study of numerical shear tests with PFC2D.

Key words: PFC2D, rock joints, peak shear strength, smooth joint contact model, numerical shear test.
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1 Introduction
Rock engineering could be considered as the most fascinating field of engineering, and for sure is one of the most dated. Stones and the soil itself have always been used as construction materials since the earliest ages. Nevertheless, rock remains a complex material to deal with, due to its nature of natural material and the variability of its parameters.

What differentiate rock from other construction materials are its heterogeneity, discontinuity and anisotropy; rock mass, in fact, is defined as the sum of two components: intact rock characterized by its composition and texture, and fractures characterized by their orientation, geometry and properties. These comprise bedding planes, folds, faults, shear zones, dykes and joints. All these features added to the geological uncertainties and limited numbers of investigation data that can be acquired for a specific site, make the work of rock engineers very challenging and fascinating. Structural engineers deal with manmade materials which have predetermined (or almost) characteristics, whereas rock engineers refers to in situ material whose parameters determination requires careful analysis.

Joints are the main features encountered in rock and are defined as ruptures of geological origin along which no relative displacement is visible (Zhang 2005). Since sliding of rock blocks on joints is classified as the principal source of instability in underground excavations, one parameter which has attracted lots of attention for its major importance in this context is the peak shear strength of joints. The shear strength of these fractures is in fact the controlling parameter which governs the sliding behaviour of rock blocks on such types of discontinuities. It is thus not surprising why so much effort has been put in the determination of a criterion which could satisfactorily determine the mechanical behaviour of the joint given certain input parameters.

Given the difficulty in estimating the shear strength of rock joints, shear tests are often performed. However, these tests are often quite expensive and also time consuming. Especially if in situ tests need to be performed. Therefore, it would be valuable if shear tests could be artificially performed using numerical models.

1.1 Objective
The main focus of the present work is the creation of a numerical model of a granite rock joint and of its shear test using PFC2D (Particle Flow Code 2D). The objective of this thesis is to prove the possibility to perform virtual numerical shear tests in a PCF2D environment that resemble the laboratory ones.

1.2 Disposition of the Thesis
The thesis starts with a general literature study about the peak shear strength: the most important parameters affecting it are described and criteria developed from the 1960’s to the present days are reviewed in Chapter 2. Also, a brief introduction and theoretical background regarding PFC2D is given. In Chapter 3, the overall procedure aimed at the performing of the shear tests in PFC2D is explained. Chapter 4 deals with the calibration of the PFC2D model based on these data. Results are then presented in Chapter 5 and discussed in Chapter 6. Conclusions of the work and suggestion for future research are presented in Chapter 7.

1.3 Limitations
The subject that this thesis is dealing with is broad and complex. There exist many different types of rock all around the world and every joint is unique. This work focus on rough unfilled joints in granite which is an intrusive igneous rock.
The study is focused on the peak behaviour of the rock joints. Due to time and computational requirements, it has not been possible to investigate in detail the post peak behaviour.
2 Theoretical Background

2.1 Introduction

Rock engineering deals with many different applications ranging from dam abutments, mining (open-pit or underground), nuclear power station foundations, to newer applications including geothermal energy, radioactive waste disposal, domestic refuse treatment and high-energy particle accelerators (Hudson and Harrison 1997). Due to the large scale of such type of projects, it is of primary importance to design them in such a way that time and costs are optimized. This can be achieved by an extensive knowledge of the rock mechanics principles and applications and experience in the field.

Rock is not a typical engineering material which is manufactured and tested to specifications. Properties of rock are not decided a priori, but rock engineers must deal with the rock material found in situ which is normally highly variable. Rock mass is a unique material. It has been subjected to millions of years of chemical, thermal and mechanical action; during this time, discontinuities were induced in different times, scales and as result of different stress states. Their geometrical and mechanical features govern the overall stability of near surface structures and it is thus vital to understand their behaviour.

A fracture can be formed in three ways: one by pulling apart and two by shearing. The first mechanism leads to the creation of joints, which have been simply opened without any relative displacement between them (Hudson and Harrison 1997). They usually occur in parallel and regularly spaced sets. There are usually several sets oriented in different directions forming a joint system; the main join set is referred to as major joints and is formed by more or less planar and parallel joints. This is usually the set of greater importance. Cross joints are usually of less importance, but it is worthy to underline how they contribute in dividing the rock mass in a blocky structure providing sliding planes, but also hydraulic paths for fluids (Jaeger et al. 2007). Joints spacing can be highly variable, from centimetres to decimetres and they can be either open or filled with different minerals as quartz, calcite, dolomite or clay.

Given the dimensions of the rock joints, which can span from millimetres to several meters, researchers have to face big challenges in order to correctly represent the in situ conditions while testing in laboratory. This becomes even more complicated under the consideration that rock and rock joints are highly scale dependant.

Rock joints present negligible tensile strength, while they have a shear strength that is smaller than the surrounding rock (Zhang 2005). Indeed, shear failure along discontinuities is in many cases the controlling mechanism of rock stability.

In this chapter, some of the past proposals for shear strength of joints are reviewed and the main parameters affecting it are described.

2.2 Fundamental Frictional Behaviour

The nature of contact between rough surfaces and of their deformation has been extensively investigated both from a theoretically and from a practical point of view. Most of the studies have dealt with materials with very low RMS (root mean square) roughness, i.e. metals. Rock surfaces show much higher roughness properties (Reeves 1985).
The two basic laws describing the basic frictional behaviour date back to the fifteenth century but named after Amontous (1699) and known as Amontous’s laws:

- The shear resistance is proportional to the applied normal load;
- The shear resistance is independent of the apparent area of contact.

The adhesion theory formulated by Terzaghi (1925), is the generally accepted theory for friction processes; it states that all surface, even the smoothest ones are rough at microscopic level. This means that two surfaces in contact produce discrete contact points; once a normal force is applied, the stress at these contact points is so high that quickly reaches the yield strength. Yielding of the contact points gives rise to adhesive bonds which provide for a shear resistance \( T \):

\[
T = \sum_{i} s_{i} A_{c,i} = s A_{c}
\]

Eq. 2.1

With:

\[
A_{c} = \frac{N}{q_{u}}
\]

Where \( s_{i} \) is the strength of the \( i\text{th} \) adhesive bond, \( A_{c,i} \) is the area of the \( i\text{th} \) contact point, \( N \) is the normal force applied and \( q_{u} \) is the yield strength of the contact points. \( A_{c} \) is named true contact area and is the sum of the finite contact areas \( A_{c,i} \).

Thus:

\[
T = N \frac{s}{q_{u}} = N \mu = N \tan \phi
\]

Eq. 2.2

Assuming \( s \) and \( q_{u} \) constant, the shear force will be proportional to the normal force applied.

It is clear how in the equation above, the two laws of friction are implicitly included.

2.3 Parameters governing the shear strength of unfilled rough joints

In the present work unfilled rough rock joints will be taken into consideration. Considering the shear strength, the factors affecting it are multiple and their measure and assessment is not straightforward. Here, different parameters affecting the shear strength of joints are briefly described.

2.3.1 Effective normal stress

The effective normal stress is the difference between the normal stress and the pore pressure, if present, and represents the actual stress acting on the joint surface. It varies, in general, as a function of depth, but it is also influenced by the topography, the geologic history of the rock and the tectonic forces (Jaeger et al. 2007).

The effective normal stress is linearly related to the shear stress acting on the joint surface by means of the coefficient of friction \( \mu \), according to the well-known relation:

\[
\tau = \mu \sigma_{n}
\]

Eq. 2.3
2.3.2 Uniaxial compressive strength of the joint surface

The uniaxial compressive strength (UCS) is the most basic parameter for the description of the intact rock strength. It is the ability of the material to resist a uniaxial compressive load. It is generally determined in laboratory through a destructive testing during which a sample of intact rock undergoes a uniaxial compressive load until failure. However, indirect tests are often conducted to estimate the unconfined compressive strength such as the point load, the hammer test and the sound velocity test (Zhang 2005).

The uniaxial compressive strength of the joint surface is usually expressed by the JCS (Joint Compressive Strength) parameter, introduced by Barton (1973). It is equivalent to the uniaxial compressive strength of the intact rock if the discontinuity is unweathered. In case the fracture is weathered or has suffered from saturation, the strength should be decreased.

It has been found that the ratio of applied normal stress and uniaxial compressive strength of the joint affects the shearing mechanism. For low values of the ratio ($\sigma_n/\sigma_{UCS} \approx 10^{-4}$), the effect of surface roughness on the shear strength is more evident and shearing occurs by overriding the asperities. At higher values ($\sigma_n/\sigma_{UCS} \approx 10^{-2}$), the asperities begin to be sheared off. At relatively high normal stress ($\sigma_n/\sigma_{UCS} = 0.15 - 0.2$), the dilation is completely replaced by shearing (Grasselli 2006).

2.3.3 Roughness

The roughness is a parameter affecting the mechanical properties of the joint. In particular, it plays an important role in the determination of the shear strength, and in the characterization of various mechanical and hydraulic properties of joints. It is defined as the vertical deviation of the joint surface from its mean plane. It is a measure of the unevenness and waviness of the surface texture. The term roughness is usually associated with the small-scale irregularity of the surface, while at larger scale it is more appropriate to use the terms undulations or waviness.


According to Hudson and Harrison (1997), from an engineering point of view, the Joint Roughness Coefficient chart developed by Barton and Choubey (1977) and Barton (1973) is the only technique with any degree of universality. It consists in the comparison of the discontinuity profile with a chart listing 10 standard profiles and thus reducing the roughness characteristic of a joint to a number (0-20) named Joint Roughness Coefficient (JRC). “Despite the obvious limitations of reducing all roughness information to a single scalar value, the possibly subjective nature of the assessment and its wholly empirical nature, the JRC profiles have been proved to be of significant value in rock engineering” (Hudson and Harrison 1997). Hsiung et al. (1993) states furthermore that this approach may introduce large errors in the evaluation of joint's strength under low normal stress condition.

In order to overcome the subjectivity of JRC profile assessment, other methods than the visual comparison have been proposed. Barton and Choubey (1977) suggested the back calculation of JRC by performing laboratory tilt tests. Furthermore, different approaches to describe the roughness have been proposed, from statistics (Tse and Cruden 1979) to fractal analysis (Mandelbrot 1967; Lee et al. 1990; Huang et al. 1992). The former makes an attempt to relate the JRC number to $Z_2$ which is the root mean square of the tangents of
the asperities angles on the joint surface. Fractal analysis will be discussed in more detail in the following section in relation to the scale properties of roughness.

Despite the efforts in trying to find a suitable method for the estimation of JRC, Grasselli (2003, 2006) points out that there is actually not a reliable method able to compute the JRC value needed to calculate the same value of shear strength that would be obtained during the shear test. Moreover, roughness anisotropy, which implies that different responses are expected according to different shear directions, has not been clearly treated in literature.

2.3.4 Matedness

Joints undergo various geological processes which cause weathering, alteration and deformation of the joint surfaces. These processes affect the matedness (or matching) of the joint surfaces. As a result, a joint will present different joint surfaces on each side and therefore a certain degree of matching.

Zhao (1997) introduced a new parameter, the Joint Matching Coefficient (JMC), to describe the matedness of a joint. The JMC value spans from 0 to 1 and is based on the percentage of the area in contact between the joint walls with respect to the total joint surface area. A JMC value close to 0 indicates a totally mismatched joint, while a JMC value equal to 1 indicates a perfectly matching joint with the asperities perfectly interlocked.

The degree of matching affects highly the peak shear strength. For a mismatched rough joint, the asperities are not interlocked and relatively low shear stress is required to overcome the dilation angle and/or to shear off the asperities. On the other hand, for a perfectly matched rough joint, the degree of interlocking of the asperities will lead to much higher shear strength.

2.3.5 Scale

It has been recognised that the shear strength is strongly scale dependant (Bandis et al. 1981). More precisely, various parameters affecting the shear strength have been discovered being scale dependant. These include the JRC and JCS values (Barton 1973).

It has been showed that the scale effect on JRC and JCS is qualitatively related and proportional to the joint length up to some critical length (Barton and Choubey 1977). Subsequently, Bandis et al. (1981) proposed these relations linking the JRC values of the laboratory size sample and the natural block:

\[
\frac{\text{JRC}_o}{\text{JRC}_{\text{lab}}} = \frac{\bar{a}_o}{\bar{a}_{\text{lab}}} \quad \text{Eq. 2.4}
\]

Where \(\bar{a}\) is the mean inclination angle of the asperities and the notations o and lab refer to natural block and laboratory sample size respectively.

Barton and Bandis (1980) and Bandis et al. (1981) showed as both JRC and JCS reduces significantly as the joint length increases. The magnitude of this effect is depending upon the roughness, where higher roughness implies higher scale effect. Based on this this, Barton and Bandis (1982) proposed the following relations to account for scale for both JRC and JCS parameters:

\[
\text{JRC}_n = \text{JRC}_o \left(\frac{L_n}{L_o}\right)^{-0.02\text{JRC}_o} \quad \text{Eq. 2.5}
\]
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\[ JCS_n = JCS_0 \left( \frac{L_n}{L_0} \right)^{-0.03} \]  

Eq. 2.6

Where \( L \) is the length of the sample taken and the notations \( o \) and \( n \) refer respectively to 100mm laboratory scale sample and to natural block size.

Generally, the sample tested in laboratory is much smaller than the natural one, and this is due to both technical and economic reasons. However, small samples testing often produces unrepresentative data and, in general, higher peak shear strength (Barton and Bandis 1980). The scale effect was explained by Barton and Bandis as: “Smaller blocks have greater freedom to follow and ‘feel’ the smaller scale and steeper asperities of the component joints hence the higher JRC values” (Barton and Bandis 1980). Bandis et al. (1981) suggested thus that the natural block-size may represent a scale effect size limit. They also proposed the tilt or pull tests on single jointed block, with length equal to the average joint spacing, as the best inexpensive methods to obtain scale free JRC values.

Given the fundamental role played by the roughness in the determination of the shear peak strength, it arises the need for a description of the roughness using a scale-invariant parameter (Fardin 2003). Several researches (Lee et al. 1990, Huang et al. 1992) have recognised the fractals as suitable for this task.

Mandelbrot (1967) introduced the concept of fractal geometry which allows for the description of irregular forms typical of geological features such as coastlines, landscapes, clouds and joint surfaces (Huang et al. 1992). These features are characterized by a fractal dimension \( D \) which “describes the degree of variation that a curve, a surface or a volume has from its topological ideal” (Lee et al. 1990). Mandelbrot (1977) stated that the length of such types of curves is a function of the measuring unit length and this dependence can be measured by the \( D \) parameter.

Consider an irregular curve and a compass of aperture \( r \), walking the compass along the curve the length \( L(r) \) is obtained which is clearly function of the compass aperture (see Figure 2.1: Interaction of an irregular curve and the measuring “ruler length” (Lee et al. 1990).1). This procedure is called step divider method. The function \( L(r) \) tends to infinity as \( r \) tends to zero; “As the dividers’ opening become smaller and smaller, (...) you (...) expect \( L(r) \) to settle rapidly to a well defined value called the true length. (...) The observed \( L(r) \) tends to increase without limit. The concept of a true length for the surface profile does not apply when fractal theory is considered” (Lee et al. 1990). \( L(r) \) and \( r \) are related by the fractal dimension \( D \) in this way (Mandelbrot 1967):

\[ L(r) = r^{1-D} \]  

Eq. 2.7
There are two different types of fractal surfaces: self-similar and self-affine. The former indicates those profiles independent of scale: a small portion of the profile displays, from a statistical point of view, the same roughness as the whole profile. For self-affine profiles, two perpendicular directions have to be scaled by different amounts to obtain statistical equivalency (Huang et al. 1992). Rock surfaces can be described by self-affine fractals (Mandelbrot 1967, Lee et al. 1990, Kulatilake and Um 1997).

Two parameters are required to define a self-affine profile: the fractal dimension $D$ and an amplitude parameter $A$, which describes the variance slope at a reference scale (Fardin 2003). Defining the Hurst component $H = E - D$ where $E$ is the topological (Euclidean) dimension of the profile (for a profile it is 2 since it can be contained in a plane), it represents the variation of roughness with scale. The relation between the asperities height and the compass aperture is then given by:

$$S(r) = A r^H$$

Eq. 2.8

Where $S(r)$ is the standard deviation of the asperity height (Malinverno 1990).

Thus the scale relation between the asperities height and length can be expressed by:

$$h_{asp} = a l_{asp}^H$$

Eq. 2.9

Where $a$ is an amplitude constant, $h_{asp}$ is the asperity height and $l_{asp}$ is the asperity base length. Usually, the Hurst component $H$ is lower than 1, implying that the increase in asperity length is lower than the increase in asperity height. This leads to an exponential decrease of the dilation angle $i$ (inclination of the asperities) when the base length increases (Johansson 2009).
Even if measurements have given unusual low fractal values as observed by Huang et al. (1992), “The value of the fractal dimension is directly proportional to surface roughness. (...) The fractal dimension offers a unique approach to the quantitative description of surface roughness, which has otherwise been qualitatively described.” (Lee et al. 1990).

2.4 Shear Strength: Past Proposals

It is generally known that shear resistance between rock surfaces arises from two components: a frictional component resulting from two rock surfaces sliding on top of each other and which is linked to the angle of friction $\phi$ depending on the rock material properties only, and a sliding resistance due to geometric irregularities of the surfaces. The latter component is linked to the dilation angle $i$ (Tse et al. 1979).

Before the sixties, it was however customary describing the shear strength of joints according to the well-known Mohr-Coulomb failure criterion:

$$\tau_f = c + \sigma_n' \tan \phi$$  \hspace{1cm} \text{Eq. 2.10}

Where $\tau_f$ is the shear strength of the joint, $c$ is the cohesion of the joint, $\phi$ is the friction angle of the joint and $\sigma_n'$ is the effective normal stress acting on the joint surface.

The basic friction angle is dependent on the strength of the rock, thus on its mineral composition (Tse et al. 1979). It is generally determined by tilt tests or direct shear test.

Neglecting the cohesion $c$, it leads to the reduced Mohr-Coulomb criterion:

$$\tau_f = \sigma_n' \tan \phi$$  \hspace{1cm} \text{Eq. 2.11}

This is a conservative assumption since joints possess some apparent cohesion. It is however generally assumed that the shear strength is a function of the friction angle rather than of the cohesion (Jaeger et al. 2007).

In the 1960’s, Patton (1966) developed a bilinear failure criterion more suitable for rough joints. He investigated the different behaviour of rough joints according to different stress levels. He considered an idealized surface whose roughness is approximated by a series of triangles with constant inclination $i$. At low normal stress, the shear stress causes the asperities to slide over each other resulting in dilation as shear displacement occurs, giving an effective friction of $\phi_b + i$. At high level of normal stress, the fracture possess a cohesion $c$ and asperities are sheared off with the two walls of the joint riding on top of each other (Jaeger et al. 2007). The bilinear model he derived is described by the following equations:

$$\tau_f = \sigma_n' \tan (\phi_b + i)$$  \hspace{1cm} \text{Eq. 2.12} \text{ for } \sigma_n' < \sigma_T'$$

$$\tau_f = c + \sigma_n' \tan \phi_r$$  \hspace{1cm} \text{Eq. 2.13} \text{ for } \sigma_n' > \sigma_T'$$

Where $\tau_f$ is the shear strength of the joint, $c$ is the cohesion derived from the asperities, $\sigma_n'$ is the effective normal stress acting on the joint surface, $\phi_b$ is the internal basic friction angle for an macroscopic smooth surface, $\phi_r$ is the residual friction angle of the material forming the asperities($\phi_r < \phi_b + i$) and $\sigma_T'$ is the threshold value of the effective normal stress.

Jaeger (1971) proposed a shear strength model to provide for a smoother transition than the bilinear model:
\[ \tau_f = (1 - e^{-d\sigma_n})c + \sigma_n' \tan \phi_r \]  
\text{Eq. 2.14}

Where \( d \) is determined experimentally.

Ladanyi and Archambault (1970) proposed a new criterion in which they considered the shear strength force \( S \) composed by several distinct contributions:

\[ S = (S_1 + S_2 + S_3)(1 - a_s) + a_s S_4 \]  
\text{Eq. 2.15}

Where \( S_1 \) is due to external work done in dilatation, \( S_2 \) is the internal work in friction, \( S_3 \) is the friction component for a flat, rough surface, \( S_4 \) is due to shearing of asperities and \( a_s \) is the ratio of the area of the asperities sheared off and the total shear area.

The shear strength then resulted as:

\[ \tau_f = \frac{\sigma_n' (1 - a_s) (\dot{\omega} + \tan \phi_u) + a_s (\sigma_n' \tan \phi_i + \eta c_i)}{1 - (1 - a_s) \dot{\omega} \tan \phi_f} \]  
\text{Eq. 2.16}

Where \( \dot{\omega} \) is the rate of dilation at failure, \( \phi_u \) is the frictional resistance along the contact areas of the teeth, \( \phi_i \) is the intact material friction angle, \( \eta \) is the degree of locking, \( c_i \) is the intact rock material cohesion and \( \phi_f \) is mean friction angle during sliding. The authors did not consider any scale effects.

Barton (1973) developed an empirical shear strength criterion with a curved failure envelope:

\[ \tau_f = \sigma_n' \tan (JRC \log_{10} \left( \frac{JCS}{\sigma_n'} \right) + \phi_b) \]  
\text{Eq. 2.17}

Where \( \tau_f \) is the shear strength of the joint, \( \sigma'_n \) is the effective normal stress acting on the joint surface, \( \phi_b \) is the basic friction angle measured from a saw-tooth sample, \( JRC \) is the joint roughness coefficient and \( JCS \) is the joint wall compressive strength.

Barton’s model applies for joints whose walls present contact points all over the length of the joint. If the joint is filled with material as clay, its behaviour under shearing will be influenced by the properties of the filling material and by its thickness (Zhang 2005). If the thickness is more than 25 to 50% of the asperities amplitude, there will be such a low rock contact that the shear strength will be controlled by the shear strength properties of the filling material (Goodman 1970).

Grasselli (2003) proposed a constitutive criterion based on experimental shear tests conducted in laboratory both on concrete replicas and joint samples. Before the shear tests, the three-dimensional joint surfaces were digitized using a particular optical measurement system (ATS scanner, Advanced Topometric System). The surfaces were measured and reconstructed by triangulation before and after shearing. It was observed that asperities have failed mainly by means of tensile failure, leading to the conclusion that tensile strength may play a more important role than compressive strength in quantifying the shear strength of joints.

The criterion he eventually proposed was:

\[ \tau_f = \sigma_n' \tan \phi_r^* (1 + g) \]  
\text{Eq. 2.18}
Where \( g \) quantifies the contribution due to the surface related parameters:

\[
g = e^{-\left(\theta_{\text{max}}^* / C \sigma'_t / \sigma'_n\right)}
\]

Eq. 2.19

Where \( \theta_{\text{max}}^* \) is the maximum apparent dip angle of the surface with respect to the shear direction, \( C \) is the roughness parameter, \( \sigma'_t \) is the tensile strength of the intact material obtained from a Brasilian test and \( A_0 \) is the maximum potential contact area for the specified shear direction.

While \( \phi_r^* \) is the residual friction angle, measured after a displacement of 5mm, where the shear strength is supposed to have reached its residual strength.

Laboratory analysis led to the conclusion that \( \phi_r^* \) is function of the basic angle of friction of the material and of the roughness of the joint surface in the way:

\[
\phi_r^* = \phi_b + \beta_r
\]

Eq. 2.20

Where \( \beta_r \) accounts for the roughness of the joint surface such that:

\[
\beta_r = \left(\theta_{\text{max}}^*/C\right)^{1.18 \cos \alpha}
\]

Eq. 2.21

Where \( \alpha \) accounts for the presence of schistosity planes.

The criterion proposed by Grasselli (2003) shows well agreement with laboratory results. The author himself however, points out the need for further studies regarding applicability of the model to in situ conditions. Moreover, the author states that no attempt was made to take into account for scale effect on the shear strength and on the parameters which determine it.

Johansson (2009) proposed a conceptual model for the shear strength aimed to overcome the uncertainties for full sized joints.

Summarizing the focus points of his proposal:

- For a rough surface in hard rock, the peak friction angle is the sum of two contributions: the basic friction angle \( \phi_b \) and the dilation angle \( i \);
- \( \phi_b \) is explained by the adhesion theory and is the contribution from a smooth but microscopic rough surface;
- \( i \) originates from a macroscopic rough surface and it is related to failure by sliding over the asperities;
- Change in the dilation angle \( i \) for different joint sizes are due to changes in the size of contact points;
- For perfectly mated joint, the number of contact points increases proportionally to the area of the sample;
- For maximally unmated joint, the number of contact points is constant at an increased scale.

Considering the points above, an expression for the dilation angle was proposed with respect to roughness, strength of the surface, normal stress and increased scale.

\[
i_n = \theta_{\text{max}}^* \left(1 - 10 \frac{\log \sigma'_b - \log A_0}{C} \right) \left(\frac{L_{\text{in}}}{L_g}\right)^{kH-k}
\]

Eq. 2.22
The figure below shows the dilation angle against scale and at different matedness.

![Diagram showing dilation angle against scale and matedness](image)

**Figure 2.2: Conceptual behaviour of the dilation angle at different scales and matedness (Johansson 2009)**

A proposal by Ghazvinian et al. (2012) aims to investigate the shear behaviour of discontinuities having distinct joint wall compressive strength (i.e. with two different rock wall types with different compressive strength). They found that the shear behaviour of such type of joints is the same as joints having the identical type of rock on both side, but with the lower strength. They also observed that failure of the asperities can occurs either in pure tension, in pure shear or in a combination of the two. It was also observed that the values predicted by Barton’s criterion were greater than the experimental results. Therefore, they proposed a modified Barton’s strength criterion substituting the compressive strength with the tensile one:

\[
\tau_f = \sigma_n' \tan \left( \phi_b + a \frac{\sigma_t'}{\sigma_n'} \frac{\sigma_t'}{\sigma_c'} \right)
\]

**Eq. 2.23**

With

\[
a = 19.4 + 3.74 \frac{\sigma_t'}{\sigma_n'} + \text{JRC}
\]

**Eq. 2.24**

Where JRC is the roughness coefficient, \(\phi_b\) is the basic friction angle, \(\sigma_t'\) is the intact tensile strength of the weaker of the joint walls and \(\sigma_n'\) is the effective normal stress acting on the surface.

This criterion was shown to have validity in the range \(\frac{\sigma_n'}{\sigma_c'} = 0.015 - 0.25\) and \(\frac{\sigma_t'}{\sigma_c'} = 5 - 15\) for the samples tested as well for natural joints.
2.5 PFC2D

2.5.1 Introduction
The chapter is mainly based on the PFC2D Online Manual (Theory and Background cap. 1) as well as on notes provided by Diego Mas Ivars and Abel Sánchez Juncal during the course on PFC held at KTH in February 2013.

2.5.2 PFC Fundamentals
PFC is a discrete element program implementing the Bonded Particle Modeling (BPM). It simulates the dynamic response of particle-based systems using the Distinct-Element Method (DEM). It describes the behaviour of solid materials, like rock, as well as of granular materials.

In PFC2D, the fundamental entities are particles (or balls), clumps and walls. The particles are circular bodies (or spherical in 3D) with a finite mass that occupy a finite amount of space. They are rigid, and interact with other particles and walls only at soft contacts which are characterized by normal and shear stiffness. Particles are allowed to overlap each other by a small amount in relation to their size. To mimic more complex behaviours, like the presence of cement among grains or interlocking between particles, bond can exist at contacts and they can carry loads and may break. The clump logic allows the creation of arbitrarily shaped particles. A clump consists of a set of overlapping balls that acts as a rigid body with a deformable boundary.

PFC is a discrete element method, i.e. it allows finite displacements and rotations of discrete bodies, including complete detachment, and recognizes new contacts automatically during the calculation. Newton’s law is solved via explicit finite-difference procedure at each time step, which makes PFC a distinct element method. The full equation of motion is solved at each step and time step adjusts automatically according to local conditions. Damping formulation allows for dissipation of kinetic energy, thus reaching equilibrium more quickly.

2.5.3 BPM Bonded Particle Model
Rock can be represented as a heterogeneous material composed of cemented grains. In sedimentary rock, a real cement is present, while in crystalline rock, as granite, the granular interlock can be considered as notional cement.

In such a system, many different items that influence the mechanical behaviour may evolve under load application, such as locked-in stresses produced during material genesis, deformability and strength of the grain and the cement, grain size, grain shape, grain packing and degree of cementation. Thus, rock behaves like a cemented granular material of complex-shaped grains where both, grains and cement, are deformable and breakable. The Bonded-Particle Modelling mimics this system through a dense packing of non-uniform-sized circular or spherical particles bonded together at contact points by means of parallel bonds. Their mechanical behaviour is simulated by the distinct-element method implemented in PFC2D (Potyondy and Cundall 2004).

BPM is a direct modelling approach which means that it does not impose theoretical assumptions and limitations on material behaviour. Particles, contacts and bonds are assigned microproperties and, through Newton’s laws of motion, the microforces and micromoments acting on them cause microstructural rearrangements. These involve bond breakages, sliding, and contacts forming thus resulting in macroscopic effects that can be investigated and compared with real cases.
The BPM is generally characterized by the grain density, grain shape, grain size distribution, grain packing and grain-cement microproperties: all these items affect the behaviour of the model. Grain and cement microproperties are listed below and they will be discussed later on:

\[ \{E_c, (k_n/k_s), \mu \}, \text{ grain microproperties} \]

\[ \{\lambda, \overline{E_c}, (\overline{k_n}/\overline{k_s}), \overline{\sigma_c}, \overline{\tau_c} \}, \text{ cement microproperties}. \]

### 2.5.4 PFC: a Distinct Element Code

In PFC2D, the interaction of particles is treated as a dynamic process. The contact forces and displacements of a stressed assembly of particles are determined by tracing the movements of the single particles which result from the propagation through the system, of disturbances due to boundary conditions and body forces. The dynamic behaviour is represented numerically by a timestepping algorithm in which it is assumed that velocities and accelerations are constant within each timestep. The solution scheme is the explicit finite-difference method as for continuum analysis. The DEM is based upon the idea that the timestep chosen should be so small that, during a single timestep, disturbances cannot propagate further from any particle than its closest particles. Thus the forces acting on any particle at any time, are determined exclusively by its interaction with the particles which it is in contact with.

The calculations performed in the DEM alternate between the application of Newton’s second law to the particles and a force-displacement law at the contacts. Newton’s second law is used to trace the motion of each particle arising from its contact and body forces, while the force-displacement law is used to update the contact forces arising from the relative motion at each contact. The calculation cycle in PFC2D is a timestepping algorithm that consists of the repeated application of the law of motion to each particle, a force displacement law to each contact and a constant updating of wall positions.

![Diagram](image)

**Figure 2.3: Calculation cycle in PFC2D (From notes provided by Itasca)**

The PFC2D environment is two dimensional in nature. The out-of-plane force component and the two in-plane moment components are not considered in any way in the equations of motion or in the force-displacement laws.
2.5.5 How PFC2D is organized
PFC provides a distinct-element model wrapped by a graphical user interface.

FISH is an embedded language in PFC.

The PFC Fishtank is a consistent set of FISH functions that extends the range of modelling that can be done with PFC. In particular, Fishtank functions support bonded-particle modelling by means of material-genesis procedure and measurement of properties. It contains function for the performing of Brazilian test, direct tensile test, uniaxial and triaxial compression test and fracture toughness test.

In the present work, the material-genesis procedure, as well as the uniaxial compressive stress functions provided by the Fishtank have been used to create the sample and perform the calibration.

2.5.6 Previous work with PFC2D
PFC is a relatively new software and the literature regarding shear tests on rock joints is limited. Park and Song (2009) pointed out that simulation of direct shear using PFC2D have been mainly done on granular material and few ones have been carried out on rock joints.

Cundall (2000) reported the capability of PFC2D to perform a simulated direct shear test on virtual rough joints. He observed that “The representation of a rough joint in rock by a micromechanical model leads to simulated behaviour that is similar to that observed in real joints”. It was the first time that the bonded particle model was used to simulate the shear test of rough rock joints. In this simulation, the joint profile was identified as a path of unbounded particles whose bond strength is set to zero.

Park and Song (2009) performed a numerical simulation of a direct shear test on a rock joint using PFC3D. They observed that using an explicit finite difference scheme, PFC allows the observations of forces transmission through the contacts, and has the ability to track the bond breakage at each stage. The pointed out, however, some limitations of this simulation due mainly to the relationship between the micro-parameters and macro-response of a rock joint that has to be calibrated, and the enhanced computing capacity needed to represent the actual situation with a satisfactory number of particles.

Recently, Asadi and Rasouli (2010, 2011) presented an application of the contact bond model for rock joint with symmetric triangular profiles and with irregular rough profiles. They showed that the numerical simulation with PFC2D is able to simulate progressive shear behaviour of rough fractures. However, using the contact bond model, particular attention has to be directed towards a good calibration of the particles friction coefficient. Moreover, they highlighted the advantages of using PFC for simulations of fracture shearing since “it can easily simulate the complex failure process in pre and post-peak behaviour of intact rocks with an ability to visualize the entire process of crack initiation, growth, coalescence, localization and complete breakdown process without requiring continuous system re-configuration”.

The smooth joint contact model has been however applied only to simulate a shear test on a smooth plain joint profile (Asadi and Rasouli (2010)).

Bahaaddini et al. (2013) modelled a shear test on Hawkesbury sandstone using the bonded particle model (referred to in the paper as the bond removal method) and the smooth joint model. Issues with both methods are found and discussed and a new shear box genesis approach is proposed.
Chapter 3 Methodology

The modelling of the shear test requires the creation of a consistent model for the specimen that is going to be tested. Laboratory tests, whom to compare the results with, must be available. The numerical assembly has to resemble the actual piece of rock used in the laboratory test, with same geometrical and mechanical properties. Also, the simulated test itself must resemble the laboratory procedure in order to get comparable results. The steps needed to the creation and testing of the model can be summarized as follows:

- Calibration on PFC2D of the intact rock (Chapter 4)
- Calibration on PFC2D of the smooth joint (Chapter 4)
- Performing of the shear test in the PFC2D shear test environment (Chapter 5)
- Comparison of the numerical results with the real laboratory test ones (Chapter 6)

3.1 Material-genesis procedure

Mechanical data on the intact rock have been collected from shear tests on samples of fractures in granite performed at SP, the Swedish National Testing and Research Institute, and some data have been assumed. Based on these parameters, the calibration of the intact rock has been performed by means of the material-genesis support functions provided by the PFC2D Fishtank.

The material genesis procedure produces a system of a dense packing of particles joined through parallel bonds in a way that locked in forces are low with respect to material strength.

The files needed for the creation of the synthetic material and for its testing, are found in ...

The result is a compacted assembly of particles whose radius distribution is uniform between a minimum and a maximum radius input by the user, with locked-in forces small in comparison to the strength of the material. The microproperties for grains and cement (balls and parallel bonds) are also input as well as the vessel geometry.
3.2 Smooth Joint Contact Model

The smooth joint contact model, abbreviated as sj contact model, is a type of contact available for all users of PFC2D. It “simulates the behaviour of a joint regardless of contact orientation along the joint” (Mas Ivars 2008). It assigns the smooth-joint model to all contacts between particles that lie upon opposite sides of the joint and the behaviour which results is a smooth scroll of the two sides of the joint with respect to each other: the particle pairs joined through a sj contact can overlap and slide past each other, instead of being forced to move around one another. In this way, the bumpiness of the DEM model is overcome and perfectly plane joints can be modelled; the mechanical response is analogous to flattened particles.

Each smooth-joint contact resolves relative displacement increment between the two particle surfaces into a joint coordinate system. During each time step, these components are multiplied by the relative stiffness producing increments of joint forces. The force-displacement law provides either Coulomb sliding with dilation or bonded behaviour.
3.2.1 Formulation

The joint geometry consists of two planar surfaces parallel to each other (surface 1 and 2). The plane orientation is defined by the unit-normal vector $\hat{n}_j$ which, according to its orientation, defines the surfaces (surface 2 is the one $\hat{n}_j$ is directed towards). The unit normal is defined by the dip angle, assuming clockwise angles as positive with respect to the global x-axis.

When the smooth joint is created, the previous contact model and parallel bond are deleted and replaced by the new contact with no parallel bond and the force, displacement and gap are set to 0.

The properties of the contact can either be inherited from the previous contact, or be assigned manually through input data. They are:

$\bar{k}_n$: normal stiffness of the sj

$\bar{k}_s$: shear stiffness of the sj

$\mu = \tan(\phi_b)$: friction coefficient of the sj

$\psi$: dilation angle of the sj

$M = \begin{cases} 
0 & \text{not bonded and never failed} \\
1 & \text{not bonded and failed in tension} \\
2 & \text{not bonded and failed in shear} \\
3 & \text{bonded} 
\end{cases}$: bond mode

$\sigma_c$: tensile strength of the sj

$c_b$: cohesion of the sj

$\phi_b$: friction angle of the sj
Using the data derived by the analysis of the three plane joint tests, the calibration of the parameters of the smooth joint model has been carried out. The model thus resulting resembles the real plain rock joint with same uniaxial compressive strength and same behaviour at peak during the shear test.

So far, the PFC2D shear test environment used has been provided by Itasca and the work done up to now is a calibration of both the rock (the particle assembly) and the smooth joint against the laboratory test results.

3.3 Shear test environment

After the virtual rock specimen has been created with the calibrated mechanical properties and the smooth joint geometry has been inserted and its relative properties assigned, the sample is ready for the shear test.

The shear test environment is composed by two .dvr files (_st1.dvr and _st2.dvr) which contains the commands and two .fis files (st.fis and st_post_proc) which contains the FISH functions for the test itself and for the post processing. A list of the most important functions is here given and briefly commented.

The file _st1.dvr initializes the specimen preparing it to the shear test and initializes the variables to be monitored during the test:

- mv_remove: removes the specimen from the material vessel, then allow it to relax.
- st_carve: removes the excess of particles leaving two lower wings in order to avoid boundary effects where the load is applied
- st_walls: adds the walls
- st_grips: assigns the grip
- st_seat_shear: seats the sample
- et2_servo_yon = 1: controls the confinement walls such that the confinement stress remains constant
- sj_seat_disp_corr: registers the displacements occurred while sitting the sample in order to subtract them from the displacements later calculated from the shear test.

![Figure 3.4: Joint, sj segments and sj contacts](image-url)
The file _st2.dvr performs the test; it accelerates the grip particles gradually and once the final velocity is reached, it cycles thousands of times solving at each cycle the equation of motion for each particle and the force-displacement law for each contact.

The variables monitored during the test are:

- \textit{sj}_{\text{Ux\_tot}}: x-displacement in the global x direction (total) [m]
- \textit{sj}_{\text{Uy\_tot}}: y-displacement in the global y direction (total) [m]
- \textit{sj}_{\text{xUs\_tot}}: projection of the shear displacement relative to the joint plane on the global x-axis (total) [m]
- \textit{sj}_{\text{Un\_tot}}: normal displacement relative to the joint plane (total) [m]
- \textit{sj}_{\text{Ss\_Th\_tot}}: theoretical shear stress from the smooth joint force (taking into account for the length ratio) [Pa]
- \textit{sj}_{\text{Ss\_Re\_tot}}: real shear stress from the smooth joint force (without accounting for the length ratio) [Pa]
- \textit{sj}_{\text{Conf\_Th\_tot}}: theoretical confinement stress (taking into account for the length ratio) [Pa]
- \textit{sj}_{\text{Conf\_Re\_tot}}: real confinement stress (without accounting for the length ratio) [Pa]
- \textit{sj}_{\text{Fx\_tot}}: x-force in the global x direction (total) [N]
- \textit{sj}_{\text{Fy\_tot}}: y-force in the global y direction (total) [N]
- \textit{sj}_{\text{xFs\_tot}}: projection of the shear force relative to the joint plane on the global x-axis (total) [N]
- \textit{sj}_{\text{Fn\_tot}}: normal force relative to the joint plane (total) [N]
- \textit{sj}_{\text{Ss\_Lab\_tot}}: shear force as calculated in laboratory test from the smooth joint force [Pa]
- \textit{wall\_ypos}: y-displacement in the global y direction of the top wall (id=2) [m]
- \textit{wall\_yvel}: y-velocity in the global y direction of the top wall [m/s]
- \textit{wall\_xvel}: x-velocity in the global x direction of the top wall [m/s]
- \textit{wall\_tcs}: applied stress on the top wall [Pa]
- \textit{wall\_fob}: out-of-balance force of the top wall [N]
- \textit{wall\_low\_right\_xforce}: out-of-balance force of the right wall (id=4) [N]
- \textit{st\_Fx\_grip}: grip force [N]
- \textit{st\_Ss\_lab\_grip}: shear force as calculated in laboratory test from the grip force [Pa]
- \textit{st\_Ss\_Th\_grip}: theoretical shear force from the grip force [Pa]

In particular, the shear stresses of interest are defined into \textit{st\_post\_proc.fis} as follow:

- \textit{sj\_Ss\_Th\_tot} = \textit{sj\_Fx\_tot} / \textit{Th\_SJ\_Length\_tot}
Analysis of shear strength of rock joints with PFC2D

- \( sj_{Ss\_Re\_tot} = sj_{Fx\_tot} / sj_{instant\_Re\_L\_tot} \)
- \( sj_{Ss\_Lab\_tot} = sj_{Fx\_tot} / ((xl\_R-xu\_L) \times 1.0) \)
- \( st_{Ss\_lab\_grip} = sj_{Fx\_grip} / ((xl\_R-xu\_L) \times 1.0) \)
- \( st_{Ss\_Th\_grip} = sj_{Fx\_grip} / Th_{SJ\_Length\_tot} \)

Where:
- \( sj_{Fx\_tot} = \sum_{sj\_contacts} Fx_i \)
- \( sj_{Fx\_grip} = \sum_{grip\ balls} Fx_i \)
- \( Th_{SJ\_Length\_tot} = \sum_{sj\_segments} L_i \)
- \( sj_{instant\_Re\_L\_tot} = \sum_{sj\_contacts} L_i \)
- \( ((xl\_R-xu\_L) \times 1.0) = 60mm \)

The grip force has been introduced to resemble the laboratory procedure where the shear stress is computed from the shear force registered at the grip. This force is then divided by the sample length (60mm) giving rise to \( st_{Ss\_lab\_grip} \).

The \( sj_{Ss\_Lab\_tot} \) differs from the latter since it takes into account the forces at the smooth joint contacts. Theoretically, the two quantities should be identical, since the forces they are considering should be equal due to equilibrium. In reality, given that PFC does not work with a continuum, but with an assembly of particles, it seems reasonable that some differences in the two quantities exist.

The same reasoning applies for what concern \( sj_{Ss\_Th\_tot} \) and \( st_{Ss\_Th\_grip} \), while \( sj_{Ss\_Re\_tot} \) differs inasmuch it does not consider the length ratio, as described in Chapter 5.

### 3.4 Shear tests
First, four different simple joint profiles have been tested in order to verify the correctness of the code and to get acquainted with the shear test environment. The four profiles are depicted in Figure 3.5 and are:

- Triangular asperities with 10° angle
- Triangular asperities with 20° angle
- Triangular right angle with 10° angle
- Triangular right angle with 20° angle
Given the regular pattern of these profiles, it has been considered a minimum radius of the particles of 0.375mm to be enough to represent correctly the behaviour of the joint.

Moving to the numerical testing of real rough joints, it has been chosen to test one single profile of one mated joint, Test 1. From the scanned surface data, the two-dimensional profiles have been extracted with different resolutions using Matlab and inserted in the PFC2D code and the numerical shear test has been performed (Figure 3.6 to 3.10). The profile has been extracted from Matlab and inserted in PFC2D with four different resolutions, i.e. each joint profile has been described by four different numbers of segments: 60, 40, 20 and 6, which means that the profile has been sampled every 1.0, 1.5, 3 and 10mm respectively. The higher the number of segments describing the profile, the more accurate its description is. However, numerical limitations, in terms of time required to run the tests, put a limit to the degree of refinement the joint profile can reach: increasing the number of segments describing the joint requires to lower the particles radii in order to assure that the behaviour of the joint would not become too stiff due to the high stiffness of single particles. Lowering the radius even of few ratios of millimetre has found to be an extremely time consuming option. It has thus been decided to keep the minimum step to sample the joint profile with equal to 1.0mm which leads to a minimum particle radius of 0.1mm. Changing the particles radii, it has been necessary to perform a new calibration both for the intact rock and for the smooth joint properties. The study on different joint profiles has been carried out with the intention to analyse the effect of the contact points on the peak shear strength. The higher the resolution, the higher is the number of contact points. As a consequence, the size of the contact points will be smaller, leading to higher inclinations of the asperities in contact and, consequently, higher shear strength.
Analysis of shear strength of rock joints with PFC2D

Figure 3.6: Scanned surface of the sample Test1. Shear direction from left to right

Figure 3.7: Joint profile with sampling step 1.0mm

Figure 3.8: Joint profile with sampling step 1.5mm
Analysis of shear strength of rock joints with PFC2D

Figure 3.9: Joint profile with sampling step 3.0mm

Figure 3.10: Joint profile with sampling step 10.0mm
4 Calibration of the PFC Model

4.1 Introduction
Contrarily to indirect modelling (continuum models), where the input properties are directly derived from laboratory tests, for the BPM, the input properties of the components are not usually known. “The relation between model parameters and commonly measured material properties (...) is found by means of a calibration process in which a particular instance of a BPM (...) is used to simulate a set of material tests, and the BPM parameters are then chosen to reproduce the relevant material properties measured in such tests.” (Potyondy and Cundall 2004).

In the following subsections it is explained more in detail the creation and calibration in PFC2D of a synthetic particle assembly. As earlier explained, all the processes have been performed more than once given the different particle assemblies created according to the different joint profiles needed to be tested. Here, it will be presented in detail only one of the calibrations of the parameters relative to the regular profile, while the results relative to the calibration of the real rough profiles will be summarized at the end of every section in forms of tables.

4.2 Creation of the sample for the uniaxial compressive test
As previously stated, the creation of the sample has been done with the support of the material-genesis procedure offered by the PFC Fishtank. The first specimen to be generated has been a square of dimension 70*70mm which has been subsequently tested in a simulated biaxial cell according to the EN 1926:

\[
\begin{align*}
\text{mv}_H &= 70.0 \times 10^{-3} \text{ m} \\
\text{mv}_W &= 70.0 \times 10^{-3} \text{ m}
\end{align*}
\]

The particle size has been chosen to give a resolution 15; the resolution is defined as the number of particles along the shortest edge of the sample. Resolution higher than 12 is enough to assure heterogeneity in the particle distribution (Potyondy and Cundall 2004); alternatively, a lower resolution could have been used, but an increasing number of tests would have been required in order to get the same heterogeneity. The distribution of particles is uniform between the minimum radius and the maximum:

\[
\begin{align*}
\text{mg}_R_{\text{min}} &= 1.75 \times 10^{-3} \text{ m} \\
\text{mg}_R_{\text{rat}} &= 1.66 \\
\end{align*}
\]

Such that:

\[
R_{\text{max}} = \text{mg}_R_{\text{rat}} \times \text{mg}_R_{\text{min}} = 1.66 \times 1.75 = 2.91 \text{ mm}
\]

After the creation of the sample, the particle assembly is supposed to be in equilibrium with an overall isotropic stress, resembling the interlock stresses present in natural rock which is small compared to the strength of the material:

\[
\text{mg}_{\text{ts0}} = -0.1 \times 10^6
\]

The number of particles is chosen automatically by PFC2D in order to reach a defined density:

\[
\text{ba}_\text{rho}(1) = 2660.0 \text{ [kg/m}^3]\]
In the present case, the mechanical parameters regard both the grain and the cement, modelled by means of balls and parallel bonds. These parameters enter the code in terms of microproperties which have to be calibrated against the elastic properties of the intact rock, the Young modulus $E$ and the Poisson’s ratio $\nu$, and the uniaxial compressive strength (UCS) derived from the uniaxial compressive test.

Unfortunately, the only tested parameter available for the rock considered is the UCS; for the elastic constants, assumptions have been made considering typical values for granite:

$$UCS = 197 \text{ MPa}$$

$$E = 84 \text{ GPa}$$

$$\nu = 0.26$$

The three parameters above will be referred to as macroproperties of the intact rock, since they characterize the overall behaviour of the rock sample.

The following list shows the microparameters needed as an input for the material-genesis support functions in PFC2D and which have to be calibrated. They are collected in the `sW_ml_st-param.dat` file:

- **Ball properties** ($ba_\text{____}$):
  - $ba_Ec(1)$: ball’s Young’s modulus.
  - $ba_krat(1)$: ratio between normal and shear stiffness of the ball.

- **Parallel Bond properties** ($pb_\text{____}$):
  - $pb_Ec(1)$: parallel bond’s Young modulus.
  - $pb_krat(1)$: ratio between normal and shear stiffness of the parallel bond.
  - $pb_{ss\_mean}(1)$: average value of shear strength of the parallel bond.
  - $pb_{sn\_mean}(1)$: average value of normal strength of the parallel bond.
  - $pb_{ss\_sdev}(1)$: standard deviation of the shear strength of the parallel bond.
  - $pb_{sn\_sdev}(1)$: standard deviation of the normal strength of the parallel bond.

In order to create the particle assembly to test the real rough joint profile, assumptions on the minimum resolution needed have been done. The profile will be extracted from the scanned data using different resolutions. The highest resolution intended to be used is 1.0mm per joint segment, which implies that a 60mm long profile base length will be described by 60 segments. It has been supposed that a minimum of 3 particles per segment was necessary to assure that the behaviour of the joint would not result in too stiff behaviour due to the very high stiffness of the single particle. Thus, considering 3 particles per 1.0mm segment with an average radius of $mg_{Rmin} \ast 2.66 / 2$ and 0.80% of space filled with particles, the minimum radius to use as input is: $mg_{Rmin} = \frac{1.0 \ast 0.80}{3 \ast 2} = \frac{2}{2.66} = 0.1 \text{ mm}$. 
4.3 Calibration of the intact rock

The ball and parallel bond moduli and stiffness ratios are set equal to one another in order to reduce the number of free parameters. The same has been done for what concern shear and normal strength (and standard deviation) of the parallel bond. This allows for both tensile and shear microfailures. The standard deviation has been taken equal to 0.2 times the strength in order to get heterogeneity of the strengths and the grain friction coefficient has been set equal to 2.50 which implies a friction angle at particle level of around 75°. This last parameter is not clear what should be calibrated against, however it seems to affect only the post-peak response (Potyondy and Cundall 2004).

\[ \text{ba_fric(1)} = 2.50 \]

The first parameters to be matched are the Young modulus \( E \) and the Poisson’s ratio \( \nu \). \( E \) depends on the ball’s and pbond’s Young’s modulus \( E_c \) and \( E\overline{c} \) or \( \text{ba_Ec(1)} \) and \( \text{pb_Ec(1)} \). Thus, setting a high value of the parallel bond strength (in order to have a long elastic behaviour before failure), the micromoduli are varied to match the macromodulus \( E \). The Poisson’s ratio \( \nu \) depends on the ratio of the normal to shear stiffnesses of both the ball and the pbond. Few iterations have been necessary to match both values and they result in:

\[ \begin{align*}
\text{ba_Ec(1)} & = 63.0e9 [\text{Pa}] \\
\text{ba_krat(1)} & = 2.2 \\
\text{pb_Ec(1)} & = 63.0e9 [\text{Pa}] \\
\text{pb_krat(1)} & = 2.2
\end{align*} \]

Once the elastic constants have been calibrated, the UCS has been taken into account. It is proportional to the pbond normal and shear strengths. The result of the calibration is:

\[ \begin{align*}
\text{pb_sn_mean(1)} & = 160e6 [\text{Pa}] \\
\text{pb_sn_sdev(1)} & = 32e6 [\text{Pa}] \\
\text{pb_ss_mean(1)} & = 160e6 [\text{Pa}] \\
\text{pb_ss_sdev(1)} & = 32e6 [\text{Pa}]
\end{align*} \]

The calibration of the intact rock has been performed on samples 70x70 mm with different resolutions as well as with samples with 125*250 mm dimensions. The results are presented in Table 4.1.
4.4 Calibration of the smooth joint scale

Once the intact rock has been calibrated against the laboratory-derived macroparameters, a new specimen is created with different dimensions in order to be tested in the shear test environment created in PFC2D. The resolution is set again 15:

\[ mv_H = 30.0 \times 10^{-3} \text{ [m]} \]
\[ mv_W = 60.0 \times 10^{-3} \text{ [m]} \]
\[ mg_{Rmin} = 0.375 \times 10^{-3} \text{ [m]} \]

As described in Chapter 2, the smooth joint input parameters are eight and are collected in st_sj_propparam.dat file. The parameters are here reported with both the PFC2D variable name and the mathematical symbol:

\[ sj_{kn}, \bar{k}_n: \text{normal stiffness} \]
\[ sj_{ks}, \bar{k}_s: \text{shear stiffness} \]
\[ sj_{fric}, \mu: \text{friction coefficient} \]
\[ sj_{da}, \psi: \text{dilation angle} \]
\[ sj_{bmode}, M: \text{bond mode} \]
\[ sj_{bns}, \sigma_c: \text{tensile strength} \]
\[ sj_{bcoh}, c_b: \text{cohesion} \]
\[ sj_{bfa}, \phi_b: \text{friction angle} \]

The joints this work is dealing with, are open joints, which means that the tensile strength and the bond mode are set to 0 (joint not bonded). The cohesion is as well set to 0 since it is disregarded when dealing with rock.

For what concern the two stiffnesses, their value must be scaled according to a parameter that can be called length ratio \( L_{rat} \) (or area ratio since a similar concept can be developed in PFC3D). The concept of the length ratio is here explained. Considering a plane joint which runs throughout the sample, its theoretical
length \( (L_{\text{th}}) \) is of 60mm. The way it is modelled however, gives rise to an effective (real) length \( (L_{\text{real}}) \) higher than 60mm. This real length is the sum of the sj contacts lengths all along the smooth joint.

\[ L_{\text{th}} = 60\text{mm} \quad \text{(length of the sample)} \]

\[ L_{\text{real}} = \sum_{n_{\text{sj contacts}}} l_{\text{sj contacts}} \]

\[ L_{\text{rat}} = \frac{L_{\text{real}}}{L_{\text{th}}} \quad \text{Eq. 4.1} \]

\[ L_{\text{rat}} = 1.59 \quad \text{for resolution 15} \]
Where:

\( l_{sj,i} \) is the length of the i-th smooth-joint contact.

In the code, these quantities are named as follows:

\( L_{th} \rightarrow \text{Th}_\text{SJ}_\text{Length}_\text{tot} \)

\( L_{real} \rightarrow \text{Re}_\text{SJ}_\text{Length}_\text{tot} \)

\( L_{rat} \rightarrow \text{Ratio}_\text{SJ}_\text{Length}_\text{tot} \)

The scaling relationship is described by the following table:

<table>
<thead>
<tr>
<th>sj_properties (microproperties)</th>
<th>Lab_properties (macroproperties)</th>
<th>PFC2D Macroscopic Behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>sj_kn</td>
<td>Kn</td>
<td>( L_{rat} \ast sj_kn )</td>
</tr>
<tr>
<td>sj_ks</td>
<td>Ks</td>
<td>( L_{rat} \ast sj_ks )</td>
</tr>
<tr>
<td>sj_\varphi</td>
<td>( \varphi )</td>
<td>sj_\varphi</td>
</tr>
<tr>
<td>sj_\mu</td>
<td>( \mu )</td>
<td>sj_\mu</td>
</tr>
<tr>
<td>sj_\sigma_n</td>
<td>( \sigma_n )</td>
<td>( L_{rat} \ast sj_\sigma_n )</td>
</tr>
<tr>
<td>sj_\tau_n</td>
<td>( \tau_n )</td>
<td>( L_{rat} \ast sj_\tau_n )</td>
</tr>
</tbody>
</table>

In the second column of Table 4.2, the parameters derived from laboratory testing are collected. In the first column are presented the corresponding microparameters. The outermost column of the table shows how the microparameters are reflected in the macroscopic behaviour by means of the length ratio. As it can be seen, the scaling relation should be taken into account in the input of the sj normal and shear stiffnesses in order to obtain the correct theoretical stiffnesses values.

Thus, if the goal is to match the macroparameter \( K_s \), since

\[
L_{rat} \ast sj\_ks = K_s \tag{4.2}
\]

The sj microparameter for the shear stiffness will be given by:

\[
sj\_ks = K_s/L_{rat} \tag{4.3}
\]

The scaling relation does not affect dilation and friction angles.

To calibrate the smooth joint scale, the results from shear test performed on 3 planar joints (A, B and C) have been used. The mean values for values of normal and shear stiffness and shear strength have been derived by analysis of shear test data.
Table 4.3: Results from the laboratory shear tests

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\sigma_n$ [MPa]</th>
<th>$\tau_p$ [MPa]</th>
<th>$\phi_{b,p}$ [°]</th>
<th>$i_{avg}$ [°]</th>
<th>$K_n$ [GPa/m]</th>
<th>$K_s$ [GPa/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>0.56</td>
<td>30</td>
<td>-1</td>
<td>26</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>1.00</td>
<td>0.64</td>
<td>33</td>
<td>0</td>
<td>26</td>
<td>22</td>
</tr>
<tr>
<td>C</td>
<td>1.00</td>
<td>0.62</td>
<td>31</td>
<td>0</td>
<td>38</td>
<td>5</td>
</tr>
<tr>
<td>average</td>
<td>0.61</td>
<td>31</td>
<td>0</td>
<td>30</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

Where:

$\sigma_n$: normal stress (confinement)

$\tau_p$: peak shear stress

$\phi_{b,p}$: basic friction angle at peak

$i_{avg}$: average dilation angle

$K_n$: normal stiffness

$K_s$: shear stiffness

$$i = \arctan\left(\frac{dn}{ds}\right)$$  \hspace{1cm} \text{Eq.4.4}

$$\phi = \arctan\left(\frac{\sigma}{\tau}\right)$$  \hspace{1cm} \text{Eq. 4.5}

$$\phi_b = \phi - i$$  \hspace{1cm} \text{Eq. 4.6}

$ds$: chosen increment of shear displacement

$dn$: increment of normal displacement relative to $ds$

The increment of shear displacement $ds$ has been chosen to be 0.1 mm.

The graph showing the stress-displacement diagram are shown in Graph 4.1 for Test A. The diagrams for the other tests are collected in Appendix B.
Analysis of shear strength of rock joints with PFC2D

Graph 4.1: Shear stress-displacement curve of Test A

Given the laboratory measured stiffnesses:

\[ K_n = 30 \, \text{GPa/m} \]
\[ K_s = 11 \, \text{GPa/m} \]

And the length ratio:

\[ L_{rat} = 1.59 \]

This leads to:

\[ sj_{kn} = 19 \, \text{[GPa/m]} \]
\[ sj_{ks} = 7 \, \text{[GPa/m]} \]

The set of sj properties has then been defined and here summarized:

\[ sj_{kn} = 19 \, \text{[GPa/m]} \]
\[ sj_{ks} = 7 \, \text{[GPa/m]} \]
\[ sj_{fric} = 0.61 \]
\[ sj_{da} = 0 \]
\[ sj_{bmode} = 0 \]
\[ sj_{bns} = 0 \]
\[ sj_{bcoh} = 0 \]
\[ sj_{bfa} = 0 \]
Table 4.4 summarizes the calculation for all the resolutions previously considered. The length ratio is observed to be increasing with increasing resolution, i.e. decreasing of particles radii; however its value seems to converge towards 1.8.

Table 4.4: Summary of the calibration of the smooth joint scale

<table>
<thead>
<tr>
<th>res</th>
<th>L_ratio</th>
<th>Kn</th>
<th>Ks</th>
<th>sj_kn</th>
<th>sj_ks</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.59</td>
<td>19</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>1.62</td>
<td>19</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>1.77</td>
<td>17</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>1.78</td>
<td>17</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5 Results from PFC Shear Tests

5.1 Regular Profiles
As earlier stated, the PFC shear test environment records a high number of variable histories, spanning from displacements, to forces and stresses. The main focus throughout this work has been the peak shear strength and here are the reported shear stress-shear displacement curves for 4 cases considered.

In Figure 5.1 to 5.4, the red line is \( st_{Ss\_lab\_grip} \); the black line is \( sj_{Ss\_Lab\_tot} \). Stresses are expressed in [Pa], while displacements are expressed in [m]. A summary table is provided which shows the magnitude of the peak shear stress for both curves and their difference in percentage calculated as

\[
\text{diff} \% = \frac{sj_{Ss\_Lab\_tot} - st_{Ss\_lab\_grip}}{st_{Ss\_lab\_grip}}.
\]

Figure 5.1: Shear stress-displacement diagram for isosceles 10°

Figure 5.2: Shear stress-displacement diagram for isosceles 20°
Analysis of shear strength of rock joints with PFC2D

Figure 5.3: Shear stress-displacement diagram for right angle 10°

Figure 5.4: Shear stress-displacement diagram for right angle 20°
Table 5.1: Summary of results in terms of peak shear strength for the regular profiles

<table>
<thead>
<tr>
<th>slope</th>
<th>Lab tot</th>
<th>Lab grip</th>
<th>diff [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[°]</td>
<td>[MPa]</td>
<td>[MPa]</td>
<td></td>
</tr>
<tr>
<td><strong>isosceles</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0,90</td>
<td>0,89</td>
<td>1,1</td>
</tr>
<tr>
<td>20</td>
<td>1,28</td>
<td>1,25</td>
<td>2,4</td>
</tr>
<tr>
<td><strong>right angle</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0,89</td>
<td>0,89</td>
<td>0,0</td>
</tr>
<tr>
<td>20</td>
<td>1,32</td>
<td>1,27</td>
<td>3,9</td>
</tr>
</tbody>
</table>

Figures 5.5 to 5.6 show the distribution of shear forces (in black) at balls ‘contacts after the peak.

Figure 5.5: Shear force distribution after peak for isosceles 10° and 20°

Figure 5.6: Shear force distribution after peak for right angle 10° and 20°
5.2 Real Profiles

The results from the shear stress-shear displacement curves for the three cases considered, i.e. with step sampling of the joint profile of 1.0, 1.5, 3.0 and 10mm are presented in Figure 5.7 to 5.10. The legend for the line is the same as for the regular profiles.

Figure 5.7: Shear stress-displacement diagram for real profile sampled every 1.0mm

Figure 5.8: Shear stress-displacement diagram for real profile sampled every 1.5mm
Figure 5.9: Shear stress-displacement diagram for real profile sampled every 3.0mm

Figure 5.10: Shear stress-displacement diagram for real profile sampled every 10mm
Analysis of shear strength of rock joints with PFC2D

Figure 5.11: Distribution of shear forces after the peak for real profile sampled every 1.0mm

Figure 5.12: Distribution of shear forces after the peak for real profile sampled every 1.5mm

Figure 5.13: Distribution of shear forces after the peak for real profile sampled every 3.0mm
Figure 5.14: Distribution of shear forces after the peak for real profile sampled every 10mm
6 Discussion of Results

6.1 Regular Profiles
From a qualitative point of view, the shear stress-shear displacement graphs (Figures 5.1 to 5.4) show a clear elastic pre-peak behaviour and a defined peak stress followed by a perfectly-plastic behaviour. Considering Figures 5.5 and 5.6 it can be noticed how the shear forces at contacts at peak, and thus the shear stresses, result concentrated along the contact areas facing the shear direction. In both the types of profile, increasing the faces inclination results in an increasing of forces (i.e. an increasing of peak shear strength), while the distribution of contact forces remains uniform among the three faces facing the shear direction.

From a quantitative point of view, the peak shear stresses obtained are comparable with Patton’s peak shear strength formula $\tau_f = \sigma'_n \tan(\phi_b + i)$ whose results are summarized in Table 6.1. The outermost column is the percentage difference between Patton’s formula result and lab_grip result calculated as $diff[\%] = \frac{st_{SS_{lab\_grip}} - st_{SS_{Patton}}}{st_{SS_{Patton}}}$. The high stiffness of the joint is due to the perfect matedness of the joint; real joints are never perfectly mated in reality. Thus, applying a shear force make them rearrange a little creating some displacement. The joint model created in PFC2D is perfectly mated (due to the nature of the joint description in the code) and once the load is applied to the grip, the shear mechanisms start right away resulting in a very high stiffness (i.e. a small displacement at peak).

The behaviour of the model is thus very good for these simplified profiles. It has to be pointed out once again how PFC2D does not impose any theoretical assumption or limitation on material behaviour. The behaviour observed in the model is the result of rearrangement of microforces according to the microparameters assigned to the intact rock and smooth joint.

The difference between the two curves plotted in Figures 5.1 to 5.4 is very little and their mismatch could be due to the nature of the model; PFC deals with a particle assembly, not a continuum; thus, small errors in the transmission of forces from particle to particle should be expected.

6.2 Real rough profile
Table 6.2 shows the results in terms of peak shear strength from PFC2D for the different resolutions adopted. Figure 6.1 shows the shear stress-displacement curve of Test1 obtained in laboratory. The outermost column presents the result collected from the laboratory shear test. The shear strength values obtained in PFC2D fall into the range of shear strengths calculated with Patton’s formula considering on one side the maximum dilation angle of the profile, and on the other its average value. This is valid for large enough resolutions to
allow for a minimum number of particles per joint segment of 5 units. The basic friction angle \( \varphi \) is taken as the average value of the basic friction angle computed from Tests A-C, 31.4\(^{\circ} \). However, looking at the plots, it is obvious that something is wrong. First, the two curves, \( st_{\text{SS} \_\text{lab}_\text{grip}} \) and \( sj_{\text{SS} \_\text{Lab}_\text{tot}} \) (which for sake of simplicity will be called from now on grip shear stress and sj shear stress curves) do not match, and their difference cannot be explained by the not-continuum nature of the model. The sample with the profile consisting of 6 segments is the only one showing a matching of the two curves.

Table 6.2: peak shear strength from PFC2D for the different resolutions adopted

<table>
<thead>
<tr>
<th>step</th>
<th>( n^\circ _\text{seg} )</th>
<th>( n^\circ _\text{particles/seg} )</th>
<th>( i_{\text{max}} )</th>
<th>( i_{\text{avg}} )</th>
<th>( \varphi + i_{\text{max}} )</th>
<th>( \varphi + i_{\text{avg}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>120</td>
<td>2</td>
<td>37.1</td>
<td>11.9</td>
<td>2.54</td>
<td>0.94</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
<td>3</td>
<td>32.8</td>
<td>10.3</td>
<td>2.07</td>
<td>0.89</td>
</tr>
<tr>
<td>1.5</td>
<td>40</td>
<td>5</td>
<td>24.2</td>
<td>9.8</td>
<td>1.46</td>
<td>0.87</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>9</td>
<td>12.6</td>
<td>7.4</td>
<td>0.97</td>
<td>0.80</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>30</td>
<td>8.5</td>
<td>6.6</td>
<td>0.84</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Where:

\[
i = \arctan \left( \frac{dy}{dx} \right)
\]

Test 1

Figure 6.1: Shear stress-displacement diagram

Considering the plots of the contacts shear forces, it would have been expected a behaviour similar to the regular profiles, where the shear forces are transmitted throughout the whole length of the joint. However, only the sample with 6 joint segments shows a similar distribution of shear forces. Regarding the other three samples (Figures 5.11 to 5.13), it is possible to see in all of them only one point by which the shear force is transmitted to the lower part of the sample. This has been found to be related to the distribution of normal and shear forces right after the sitting of the sample, i.e. before the grip is accelerated. Considering the normal forces distribution, the sample is expected to display vertical lines of force patterns connecting the upper wall to the lower wall where the confinement pressure of 1MPa is applied. Indeed, looking at the plots of the normal forces distribution for the regular profiles (Figure 6.2 left), this is the case. Also, the shear
force distribution is uniform along the joint plane (Figure 6.2 right). Here it is shown only the plots relative to the right triangles profile with $10^\circ$, but similar plots result from the other regular cases.

![Figure 6.2: Distribution of normal (left) and shear (right) forces after the sitting of the sample for right triangles with $10^\circ$](image)

However, in the real profiles, right after the sitting of the sample, the normal (and shear) force distribution does not follow the expected pattern, but is concentrated in one point only. Exception is for the profiles with a step resolution of 1.5mm and, as expected, 10mm. In these two cases, the distribution is homogeneous along the joint profile. Considering the shear stress-displacement diagrams for the three cases (Figures 5.7 to 5.9), it can be noticed how in the case of 1.5mm step resolution, the grip and sj shear stress curves are matching in the first part of the shear test, while in the other two the two curves are completely divided from the beginning. This seems to suggest that perhaps there is some numerical instability issue in the code relative to the sitting portion and to the shear test itself that fail to represent the actual behaviour of the sample. What also strikes the attention is the difference in behaviour of the four samples given that their only difference is in the joint profile resolution.

It is clear that the real profile sampled every 10mm, i.e. made of 6 joint segments, would resemble more a triangular regular profile, than the actual profile (see Figure 6.3 left). Indeed, the normal force distribution right after the sitting and the shear force distribution at peak show an expected behaviour resembling the one of the regular profiles with shear forces crossing the join throughout the slopes facing the shear direction. However, if the result from the laboratory test is considered, it can be noticed the difference in peak shear strength: 0.81 MPa from PFC2D and 2.16 MPa from the laboratory test. This big difference is due to the fact that the 6 segments profile completely fails to represent the joint surface because of its low resolution. Thus, the result in term of peak shear strength tends to be closer to the results from the regular triangular profiles.

![Figure 6.3: Geometry of the joint profile samples every 10mm (left). Distribution of normal forces after sitting (right)](image)

Recently, a study performed by Bahaaddini et al. (2013) faced the same issue. Results show that the smooth joint model suffers from interlocking whenever a particle, assigned with a smooth joint contact, moves to the other side of the joint segment during the sitting of the sample or the shearing. In this case, the smooth joint contact is removed (since the two balls previously connected by the sj contact now lie on the same side) and
interlocking occurs. This leads to a stress concentration around the single particle, producing an unrealistic distribution of forces.

The problem arose in the present work appears to be strongly related to the one faced by Bahaaddini and his colleagues. The confinement applied during the sitting of the sample could be the cause of particle displacement to the opposite side of the joint segment and, thus, could explain the stress concentrations around few points along the joint profile showed by Figure 5.11 and 5.13. The same reasoning applies to the sample with 1.5mm step profile, only this time the phenomenon happens during the shearing.

In attempt to overcome this problem, in their paper Bahaaddini et al. (2013) propose a new shear box genesis approach. It consists of the generation of the upper and lower part of the sample separately. The smooth joint contact is applied only at contacts between the particles of the two blocks. By doing this, the particles belonging to either of the two parts of the sample can be detected easily in each phase of the test.
7 Conclusions and Further Research
Starting from laboratory data about shear tests of joints in granite rock, a particle assembly and a smooth joint model have been calibrated by means of microparameters in PFC2D. Itasca provided a shear test environment to which few modifications have been made. Four simple and regular profiles have been tested and subsequently four real profiles, taken from the same sample’s scanned data, with four different resolutions have been tested.

The results from the regular profiles are good both from a qualitative point of view and from a quantitative one. The difference in shear strength with respect to the one computed with Patton’s formula is in the order of 1% which, considering the particle nature of the model, is a very good result.

However, moving to the testing of more complex and less regular profiles, something changes in the numerical computation, leading to results that are apparently wrong. Moreover, where the curves relative to the grip and the sj shear stress do not match, the distribution of normal and shear forces is unrealistic; where the two curves match the results in terms of force distribution seem reliable. Recent developments in the study of numerical shear tests with PFC2D by Bahaaddini et al. (2013) may offer an explanation to the problem and an effective solution. However, well-performing models need a significant improvement in terms of refinement of the number of segments describing the joint in order to obtain reasonable results in terms of peak shear strength.

This work is intended to be the start of a larger project aimed to the development and creation of a shear environment fully working for all type of profiles and able to represent correctly the behaviour of real joint profiles. This thesis shows the state of the art of this work. Time and computational requirements put a limit to the work that was possible to do. Improvement in the code shall be done and with more time and computational power also post peak behaviour could be investigated.
8 References


Appendix A

Matlab code for the analysis of the scanned data.

```matlab
% Test_1--> Analysis of scannad data
% Vectors with x,y,z measured data
data= b_pre_1_e; % read the imported .txt file with the scanned data

% Changing coordinates according to the shear direction (in order to have % the sample displayed like in the test):
% y'=x
% x'=-y
% being x' and y' the new reference system and x and y the reference
% system of the scanned data
length=size(data,1);
X= - data(1:length,2);
Y=data(1:length,1);
Z=data(1:length,3);

% Net for interpolation
step=0.5;
tx=-32.2:step:27.8;
ty=-37.1:step:22.3;

% Create the surface
[XI,YI]=meshgrid(tx,ty);
ZI=griddata(X,Y,Z,XI,YI); % it interpolates X Y Z in the points defined by XI and YI

% Extract the data for 2D profile--> 5 profiles (u:upper, l:lower; wrt half)
np = 5; % number of profiles
pt_mid= round(np/2); % i-th corresponding to the mid profile (odd only)
d_ty = size(ty,2) * step / (np+1); % distance between profiles [mm]
p_ty = size(ty,2) / (np+1); % pointer distance

for i=1:np
    pt(i)= round(i*p_ty); % pointer vector to the equally-spaced profiles
    profiles(:,i)=ZI(pt(i),:); % each column a profile
    profiles(:,i)= profiles(:,i)-profiles(1,i); % setting 0 the first coordinate
    table(:,1,i)= 490; % create the i-th 490 table
    table(:,2,i)= tx_pfc'/1000; % [mm]-->[m]
    table(:,3,i)=(profiles(:,i)/1000)' % [mm]-->[m]
    p_table(:,1,i)= 491;
    p_table(:,2,i)= cc';
    p_table(:,3,i)= 1; % number of the property
end
```
end

% Tables for the mid profile
table_mid = table(:,:,pt_mid);
p_table_mid = p_table(:,:,pt_mid);

% Plot surface
figure(1)
surf(XI,YI,ZI), hold;
axis equal
view(2)
light
shading interp

% figure (2)
for i= 1: np
    subplot(np,1,i), plot (tx,profiles(:,i))
end

% plot (tx, profiles(:,3))
xlabel('Joint x coordinate [mm]');
ylabel('Asperities height [mm]');
axis tight;
grid on;
10 Appendix B

**Figure B1: Shear stress-displacement diagram**

Test B

**Figure B2: Shear stress-displacement diagram**

Test C