Simple flexible wheelset model for low frequency instability simulations

C. Casanueva¹, A. Alonso², I. Eziolaza³ and G. Gimenez⁴

¹Rail Vehicle Division, Royal Institute of Technology (KTH), Stockholm, Sweden

²CEIT and TECNUN (University of Navarra), Spain

³Construcciones y Auxiliar de Ferrocarriles, S.A., R&D Department, Spain

⁴Construcciones y Auxiliar de Ferrocarriles, S.A., R&D Department and TECNUN (University of Navarra), Spain

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Abstract:

As a general rule, multibody simulation models used by railway vehicle designers use totally rigid wheelsets, thus leading to possible errors when calculating the critical speed of the vehicle under study. This article suggests a wheelset model that takes into account wheelset flexibility for the study of dynamic stability. The model is simple to implement and easily parametrised, which can be applied to both conventional and variable gauge wheelsets. The parameters corresponding to wheelset flexibility that most influence critical speed of high speed and variable gauge vehicles are also analysed.

Keywords: flexible wheelsets, variable gauge, low frequency instability, critical speed

1. Introduction

The critical speed of a train is an interesting parameter for designers of railway vehicles, and becomes increasingly more relevant in the design as the speed at which the vehicle has to travel increases. Classic models do not contemplate the influence of the flexibility associated with the wheelset. However there are two cases in which it is advisable to include this flexibility: in High-Speed and in variable gauge vehicles. In the first case, the vehicle's maximum operating speed is very close to its critical speed, meaning that the precision required when making calculations is much greater. Furthermore, the stiffness of the primary suspension on high-speed vehicles is much greater than that of conventional vehicles. In the second case, the lateral distance between the wheels can adopt two positions that are assured by mechanisms that remain blocked during operation. The complex construction of these wheelsets leads to increased flexibility. This situation becomes particularly critical when variable gauge vehicles are used at high speed.

1.1. Study of flexibility associated with the wheelset

Bibliography contains several references to the structural flexibility of solid wheelsets. These references can be divided into various categories:

Depending on the precision of the models:

- those simplifying the flexibility components to be analysed (mainly bending and/or torsion) with flexible beam models coupled with wheel and brake disc masses [1–13] .
- those considering the wheelset as a model based on finite three-dimensional elements [14–22].

Alternatively, according to the effects to be studied:

- the influence of the flexibility of the wheelset on the dynamic stability of the vehicle [1–3], [21].

calculations on the interaction between wheel and track, principally in order to predict rail corrugation and wheel polygonalisation [4], [6–13], [15–22].

As can be seen, the majority of the articles study the influence of wheelset flexibility on contact forces and the development of wear patterns on the wheels and tracks.

Their influence on the stability of the vehicle at high speeds has been studied relatively little, with three articles dating from before 1990 [1–3] and one written in 2006 [21] which makes use of a model created to analyse corrugation and irregular wear and uses it to calculate their influence on the critical speed of the vehicle. Despite the limited number of references, all of them do conclude that wheelset flexibility has an effect on the low-frequency stability of the vehicle: torsional flexibility increases critical speed by less than 4% in the studied cases, whereas bending decreases it by around 10% [21].

The models used to carry out the analysis include a very large number of vibration modes in order to achieve a suitable representation of real systems. All these modes have very high natural frequencies, meaning that the numerical integration step needs to be reduced, which leads to high computational costs.

1.2. Objectives

The objectives of the study are as follows:

- develop a model for flexible wheelsets which represents the contribution of the flexibility to the dynamic stability of railway vehicles while reducing the model's computational costs without affecting its precision;
- analyse the effect of the different modes of wheelset deformation on the dynamic stability of the vehicle and
- apply the model which has been developed to a high-speed and variable gauge vehicle.

The objective of the study is not to describe the different techniques for calculating the critical speed of the vehicle, as this has already been analysed in depth in previous publications [23], [24]. For the studied cases, the linear critical speed has been calculated by solving the eigenvalue problem.

2. Modelling wheelset flexibility for low frequency instability analyses

Critical speed is defined as the maximum speed at which a vehicle can travel without any instability occurring in any part of the system. In theory, instability occurs when one of the vibration modes reaches a negative damping value. Simple linear models of vehicles with wheelsets represented by double cones on specific tracks or circular profiles [25] determine that the most unstable modes of vibration are vehicle hunting modes, whether on the carbody, bogie or wheelset. Running under unstable conditions leads to increased lateral displacement of the wheelsets which causes impact between the wheel flanges and tracks.

When including flexibility of the wheelsets in the dynamic simulation model, it must represent adequately the displacement and internal rotation of the wheel affecting the creepages in the wheel/rail contact, which are:

- longitudinal and lateral displacement
- Roll, torsion and yaw

For this reason, the flexible wheelset model is simplified but trying to model accurately these displacements and rotations.

2.1. Mathematical model

The position of any point L on the wheelset can be expressed as in equation 2.1.

$$\mathbf{x}(t) = \mathbf{x}_0(t) + \mathbf{T}\mathbf{x}_L + \mathbf{x}_f(\mathbf{x}_L, t) \quad (2.1)$$

Figure 1 – Displacement of point L on the wheelset in a set of coordinates $\vec{X}$.

$\vec{x}_0$ is the displacement of the centre of gravity of the wheelset as a rigid body, $\mathbf{T} = \mathbf{T}(\vec{\varphi})$ is the transformation matrix due to the rotation as a rigid body $\vec{\varphi} = \{\varphi(t), \chi(t), \psi(t)\}$, $\vec{x}_L$ is the vector of local coordinates of a generic point on the wheelset and $\vec{x}_f(\vec{x}_L, t)$ the displacement of point $\vec{x}_L$ due to the flexibility of the system. In conventional models, the flexibility of the wheelset is not taken into account, thus $\vec{x}_f(\vec{x}_L, t) = 0$. However, when taking the modal flexibility of the system into account, the term $\vec{x}_f$ may be expressed according to the different flexible modes of the wheelset (equation 2.2).

$$\mathbf{x}_f(\mathbf{x}_L, t) = \mathbf{T} \sum \boldsymbol{\phi}_k(\mathbf{x}_L) \boldsymbol{\eta}_k(t) \quad (2.2)$$

$\phi_k(x_L)$ is the mode shape at position x_L and $\eta_k(t)$ is the amplitude of the mode of vibration k at instant t . In this way, when introducing the flexibility of the wheelset, k new equations are included in the system for each wheelset. Additionally, the flexibility will not be represented adequately unless a large number of modal degrees of freedom are included. A more efficient way of representing this is with equation 2.3.

$$x(t) = x_0(t) + T x_L + T [\sum_{k=1}^n \phi_k(x_L) \eta_k(t) + \delta_R] \quad (2.3)$$

where the flexibility of the system is represented by the sum of the flexibility of a limited number of modes up to mode n plus the residual deformation associated with the modes above n , shown as δ_R . This approach allows us to represent very precisely the dynamic response of the wheelset for the range of frequencies covered by the modes that have been taken into account, adding only n new equations to the system.

In this particular case, the first natural bending frequency of the axle of the wheelset is greater than 300 Hz for the vast majority of railway wheelsets, whereas the yaw instability phenomenon occurs at frequencies below 15 Hz

Figure 2 – Transfer function for the first two bending modes (H_{mod01} and H_{mod02}), residual flexibility of modes above 2 (F_{res}) and the sum of them (F_{sum}), compared to the static flexibility (F_{st}).). Thus, they are dynamically separate issues and can be disregarded with respect to the modal inertia and damping contributions associated with flexible modes. The contribution of each of these modes is reduced to their static flexibility (F_{est}).

Figure 2 – Transfer function for the first two bending modes (H_{modo1} and H_{modo2}), residual flexibility of modes above 2 (F_{res}) and the sum of them (F_{sum}), compared to the static flexibility (F_{st}).

According to the modal analysis theory and based on the hypothesis of proportional damping, the transfer function matrix $\mathbf{H}(\omega)$ of a system of k degrees of freedom can be expressed as a function of the k modes of the same (equation 2.4).

$$\mathbf{H}(\omega) = \sum_k \frac{\frac{1}{m_k} \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T}{\omega_k^2 - \omega^2 + 2i\xi_k \omega \omega_k} \quad (2.4)$$

where for the mode of vibration k , m_k represents the modal mass, ω_k its natural frequency, ξ_k its equivalent damping and $\boldsymbol{\phi}_k$ its modal vector. This implies that the static flexibility ($\omega = 0$) of the system is the sum of the static flexibility of each of the vibration modes (equation 2.5).

$$\mathbf{F}_{st} = \sum_k \frac{1}{m_k \omega_k^2} \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T \quad (2.5)$$

This means that, if wheelset flexibility is considered when performing limit cycles or critical speed simulations, the flexible wheelset model could be simplified to account only for static flexibility. From a vehicle design engineering point of view this is very useful as the designer can forget about complex flexible element modelling and still be able to represent accurately its influence. However, the model must still be capable of representing all the deformations that occur in the wheelset, and thus the following simplified model is proposed.

2.2. Simplified model

A flexible wheelset model has been developed which is valid for both conventional and variable gauge wheelsets. The main difference between a variable gauge wheelset and a conventional wheelset is the existence of mechanisms between the wheel, axle and primary suspension that lead to non-linear behaviour on those joints, as well as reduced stiffness values.

As well as the static flexibility of the axle body, the elements located between the axle and wheels may also add a certain additional flexibility, meaning that the displacements and rotations of the wheel will be different than those of the section of axle on which they are mounted (Figure 3).

Figure 3 – Displacement due to elements providing flexibility in a model with a flexible axle.

In Figure 3, different angles are considered for the rotation corresponding to the ends of the axle (ψ_{1a}, ψ_{2a}) and the rotation corresponding to each wheel (ψ_{1w}, ψ_{2w}). Furthermore, in variable gauge wheelsets, the wheels are fixed to the axle using locking elements with a certain amount of flexibility (including some small clearance). For this reason it is necessary to include the lateral displacements of the wheels (y_1, y_2) as additional degrees of freedom in the model.

In order to take into account the displacements in Figure 3, the model given in Figure 4 is used, known as the Three-Body Model. A two-dimensional model is depicted in order to simplify the equations. The model is made up of three rigid bodies (two wheels and the axle) which are connected using angular (K_A, K_B) and longitudinal (K_y) stiffness. The degrees of freedom between wheels and axle are as follows:

- lateral displacement (y);
- wheel rolling (φ), torsion (χ) and yaw (ψ) angles.

- Figure 4 – Three-body model for a variable gauge wheelset. The rotational stiffness elements K_A and K_B have 3 degrees of freedom (φ, χ, ψ).

It is possible to identify three uncoupled behaviours in the wheelset: bending, lateral and torsional displacements. Lateral stiffness is represented by a spring between the wheel and the axle, K_y . Torsional stiffness between the wheels, K_T , is calculated by taking into account the static stiffness of the three-body model (equation 2.6).

$$K_T = K_{\chi B} + K_{\chi A}/2 \quad (2.6)$$

For a conventional axle there is an infinite number of combinations of $K_{\chi A}$ and $K_{\chi B}$ for the same value of K_T . This is not the case for a variable gauge wheelset where both wheels are connected in torsion through a hollow shaft and the connection in bending is carried out by the axle.

In the case of conventional axles, the easiest form of modelling torsional stiffness is to adopt a value of zero for either $K_{\chi A}$ or $K_{\chi B}$. If $K_{\chi A}$ has a value of zero, the axle is disconnected from the wheels for the degree of freedom of torsion, and it is necessary to join it to one of the wheels in order to avoid problems when calculating the model. This would create an asymmetric model that would not necessarily represent the real behaviour of the wheelset correctly. This problem would not occur when making $K_{\chi B}$ zero (equation 2.7), as both wheels are connected to the axle through $K_{\chi A}$.

$$\begin{cases} K_{\chi A} = 2K_T \\ K_{\chi B} = 0 \end{cases} \quad (2.7)$$

For the conventional wheelset K_T is obtained as the torsional stiffness of a beam element with the geometry of the axle. For the variable gauge wheelset, the value of K_T is obtained experimentally.

Obtaining the parameters for bending stiffness is more complicated. Based on the bending flexibility matrix between the sections of the axle supporting both wheels $[F]$ and the rotational flexibility between the wheel and the axle F_w , the stiffness matrix of the system can be obtained (equations 2.8 to 2.10).

$$\begin{cases} \psi_1 = \psi_{1a} + \psi_{1w} = M_1 \cdot F_{11} + M_2 \cdot F_{12} + M_1 \cdot F_w \\ \psi_2 = \psi_{2a} + \psi_{2w} = M_1 \cdot F_{21} + M_2 \cdot F_{22} + M_2 \cdot F_w \end{cases} \quad (2.8)$$

$$\boldsymbol{\psi} = \mathbf{F} \cdot \mathbf{M} \rightarrow \mathbf{M} = \mathbf{K} \cdot \boldsymbol{\psi} \quad (2.9)$$

Being $\boldsymbol{\psi}$ and $\mathbf{M}$ the angle and moment vectors, $\mathbf{F}$ the flexibility matrix and $\mathbf{K}$ the stiffness matrix (equation 2.10)

$$\mathbf{K} = \mathbf{F}^{-1} \rightarrow \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} F_{11} + F_w & F_{12} \\ F_{21} & F_{22} + F_w \end{bmatrix}^{-1} \quad (2.10)$$

The values for the flexibility matrix of the axle body ($F_{11}, F_{12}, F_{21}, F_{22}$) are calculated using the inverse of its static stiffness matrix. Contribution from rigid body modes is removed in advance in order to be able to calculate the inverse matrix.

Comparing the resulting stiffness matrix with the corresponding stiffness matrix derived from the three-body model,

$$\begin{cases} M_1 = K_{11}(\psi_1 - \psi) + K_{12}(\psi_2 - \psi) = K_A(\psi_1 - \psi) + K_B(\psi_1 - \psi_2) \\ M_2 = K_{21}(\psi_1 - \psi) + K_{22}(\psi_2 - \psi) = K_A(\psi_2 - \psi) - K_B(\psi_1 - \psi_2) \end{cases} \quad (2.11)$$

Taking into account the symmetry of the flexibility matrix, $K_{22} = K_{11}$ and $K_{21} = K_{12}$

$$\begin{cases} K_A = K_{11} + K_{12} \\ K_B = -K_{12} \end{cases} \quad (2.12)$$

The value of K_B is negative due to the fact that the stiffness for symmetric deformation is less than the stiffness for anti-symmetric deformation.

A variable gauge rail wheelset may not have revolution symmetry, meaning it is necessary to calculate K_A and K_B for both the plane XY ($K_{\psi A}, K_{\psi B}$) and the plane YZ ($K_{\phi A}, K_{\phi B}$). Furthermore, the rotational flexibility $K_{\phi A}$ means that the initial pre-load of the vehicle causes the wheel to have a roll angle with respect to the longitudinal axis. For this reason, an initial pre-load is required in $K_{\phi A}$ elements.

For the conventional wheelset F_{11} , F_{12} , and F_{22} are calculated as the flexural stiffness of a beam element with the geometry of the axle. For the variable gauge wheelset, these values are obtained through the Finite Element Method simulation (FEM) and are validated with experimental measurements. As an example, the values for the stiffness of different elements in the model for a conventional wheelset in which $F_w = 0$ are given in Table 1.

Table 1 – Stiffness values for a conventional, standard gauge wheelset model, 170 mm diameter.

Standard gauge	
$K_{\phi A}, K_{\psi A}$	$24.73 \cdot 10^6 \text{ Nm/rad}$
$K_{\phi B}, K_{\psi B}$	$-7.13 \cdot 10^6 \text{ Nm/rad}$
$K_{\phi B}$	$3 \cdot 10^6 \text{ Nm/rad}$
K_y	∞

In the case of variable gauge wheelsets, the values for the difference in stiffness cannot easily be calculated theoretically (except for the stiffness of the axle). For this reason, the flexibility characteristics for the gauge change mechanism have been calculated using a finite element model and have been experimentally validated.

2.3. Experimental characterisation of the variable gauge wheelset

A variable gauge wheelset is mounted onto a test bench with the purpose of measuring the distance between the inside faces of the wheels. The test bench allows both vertical preloading of the wheelset between the primary suspension and the wheel-rail contact, and the application of lateral forces on the wheel close to the wheel-rail contact (Figure 5). The primary suspension used is that of a Spanish RENFE 120 Series vehicle and the wheels are

supported on real rails. Lateral forces at the wheel/rail contact are applied symmetrically both inward and outward on the wheelset.

Figure 5 – Mounting of the variable gauge wheelset on the test bench.

The following parameters have been measured for all the different load cases; of particular interest are the relative lateral displacement and rotation with respect to the longitudinal axis (φ) between the axle body and the sliding support in the wheel area.

The experimental measurements have been contrasted with the finite element model of half a wheelset. For a better understanding of the friction forces and clearances of the system, a sketch of the gauge change mechanism is depicted in

Figure 6.

There are four main pieces in the variable gauge wheelset: i) axle; ii) sliding bushing; iii) wheel; and iv) fake axle box (which covers the whole system but does not hold the bearings). The wheel is the only element that rotates, and is coupled to the sliding bushing by means of two

tapered roller bearings. The sliding bushing is placed around the axle and locked to the fake axle box, which is the cover of the whole system and is coupled to the primary suspension.

The gauge change is performed by unlocking both axle and bushing, pushing laterally the wheel and locking the whole device back in the new position.

Vertical forces are transmitted to the primary suspension through a polymeric tile placed between the fake axle box and the bushing. There is also vertical load transmission between bushing and axle. Lateral forces are transmitted through two paths: i) through the axle, by means of friction through the sliding bushing ($\mu=0.1$) and the axle locking device; and ii) directly to the fake axle box through friction of the tile ($\mu=0.3$) and bushing locking devices.

Figure 6 – Sketch of the lateral behaviour of the variable gauge wheelset.

The results are given in

Figure 7 and

Figure 8.

Figure 7 – Experimental validation of the lateral displacement between wheel and axle vs. wheel-rail contact force. $\mu=0.1$ marks the coulomb force for the bushing, d_b the clearance of the bushing locking device; $\mu=0.3$ marks the coulomb force for the tile, d_a the clearance of the axle locking device;

Figure 8 – Experimental validation of the rotation with respect to longitudinal axis (φ) between wheel and axle vs. moment introduced by lateral wheel-rail contact force.

The force/lateral displacement characteristic for the finite element model is validated by the

experimental results (

Figure 7). This characteristic is not linear due to the blocking mechanisms, which have a certain clearance (d_a and d_b), as well as the contact surfaces of the components connected by those mechanisms that have frictional characteristics.

The moment/angle characteristic with regards to the longitudinal axis calculated by the finite element model has a certain non-linearity that is not observed during experimental testing

Figure 8). However the correlation between theoretical and experimental results is satisfactory, hence these flexibility values have been used in the construction of the model.

With the available experimental facility it was not possible to obtain the values of rotational stiffness with respect to the vertical axis of the gauge variation mechanism. Therefore it is

assumed that the finite element model predicts that stiffness accurately

Figure 9) given that the other flexibilities have already been correctly validated using the same model.

Figure 9 – Rotational deformation with respect to the vertical (ψ) axis of the gauge variation mechanism vs. moment introduced by lateral wheel-rail contact force.

3. Results

In order to study in depth the influence of the flexibility of the wheelset on dynamic instability, the influence which each of the components has on the critical velocity has been analysed. Given that the model for the conventional flexible wheelset is a particular case of the model for the variable gauge wheelset, the latter is used in the following analyses.

The results of this study are presented in two steps:

- In the first one the flexibility associated with each degree of freedom of the wheelset model is introduced separately and their influence on dynamic stability is assessed.
- In the second step a more detailed analysis is depicted for the parameters resulting as the most relevant in the first step.

3.1. Influence of the different flexibilities in the model

In order to determine which displacement or rotation has the greatest effect on the critical speed, the influence of the different flexibility parameters is checked separately for the entire range of equivalent conicity values.

Figure 10 depicts the critical speed for models including different sources of flexibility. The flexibility values used have been determined for an extremely flexible variable gauge wheelset. The influence of the flexibilities associated with different degrees of freedom have been analysed independently, as well as a stiff wheelset and a wheelset including the flexibilities associated with all degrees of freedom considered in the model.

Figure 10 – Comparison of the linear critical speed of a reference vehicle with a flexible wheelset taking into account different flexibilities separately: stiff wheelset, lateral flexibility, torsional flexibility, rolling flexibility, yaw flexibility, all flexibilities.

It can be observed that the rolling flexibility (φ) and lateral flexibility (y) between the wheel and axle body have hardly any influence. There is a very small variation in the figure, so the lines corresponding to *Stiff*, *Flex. K_y* and *Flex. K_φ* overlap. The one with the greatest influence is the yaw flexibility (ψ), represented by $K_{\psi A}$ and $K_{\psi B}$. It should be noted that the torsional flexibility (ϑ) of the model reduces the critical velocity for the entire range of cone angles, contrary to the behaviour observed in the existing bibliography.

For the flexible wheelsets including yaw flexibility (*Flex. K_ψ* and *Flexible* in

Figure 10) there is a noteworthy behaviour: for low conicity values the critical speed is very low. While the equivalent conicity increases, the critical speed also increases until it reaches a maximum value between $\delta=0.1$ and $\delta=0.15$. Then it starts decreasing again, following the trend of the more rigid wheelsets. This is caused by two different behaviours of the wheelset: the influence of the conicity and the influence of the yaw flexibility on the instability. The influence of equivalent conicity is clearly seen when considering rigid wheelsets: for higher values, lower critical speed. When the yaw flexibility of the wheelset is included, both sources of instability act at the same time, and the effect of this flexibility cannot be isolated from the effect of the equivalent conicity. For low equivalent conicity values, the flexibility has more influence than the conicity. Moreover, this low conicity favours the instability as the yaw deformations of the wheelsets are more difficult to damp. As the conicity increases, the gravitational stiffness acts as a damper of the anti-symmetric deformation of the wheelset, thus increasing the critical speed until the effect of increasing the conicity is higher than the one damping the yaw deformations. In accordance with these results, the three-body model has been simplified so as it does not take into account lateral displacement between the wheel and axle ($K_y = \infty$) and the rolling between the wheel and axle body ($K_{\varphi A} = \infty$; $K_{\varphi B} = \infty$). The model is therefore reduced to 3 rigid bodies with a shared roll angle (φ) and displacement (x, y, z). The 3 rigid bodies that have independent torsional and yaw angles are joined by springs which represent the torsional flexibility between the wheels (K_ϑ) and the yaw stiffness between the wheel and axle body ($K_{\psi A}, K_{\psi B}$). This way, the model developed only introduces four additional degrees of freedom in each wheelset with respect to a fixed solid wheelset model.

These theoretical results have not been experimentally validated. However, the validity of the proposed model is assured based on the fact that it is a combination of different static stiffness values. In section 2.1 it has been demonstrated that this simplification is very accurate as the modes of the wheelset have much higher frequency than the hunting modes.

3.2. Influence of the different variables and equivalent conicity

In this step, the parameters with the biggest influence on the dynamic stability in the previous step are also considered jointly with others of which the influence is commonly accepted.

Yaw flexibility of the axle has been studied by both parameters $K_{\psi A}$ and $K_{\psi B}$.

$K_{\psi A}$ represents the equivalent stiffness between each wheel and the axle (taking into account that both are considered in the model as rigid bodies). For a variable gauge wheelset this value is dependent on the flexibility of the axle and the flexibility between the axle and wheel.

Figure 11 shows how dependent the critical speed of the train is on $K_{\psi A}$ for the full range of equivalent conicity values. Three ranges of values can be seen: for stiffness values lower than $8 \cdot 10^6$ Nm/rad, the critical speed is reduced drastically, irrespective of the equivalent conicity. For values of between $8 \cdot 10^6$ Nm/rad and $15 \cdot 10^6$ Nm/rad, there is a range of influences for $K_{\psi A}$. For $K_{\psi A}$ values greater than $15 \cdot 10^6$ Nm/rad, improvements in the critical speed cease to be significant.

Figure 11 – Variation in critical speed against stiffness $K_{\psi A}$ for the complete range of equivalent conicity values. International gauge.

In the case of a conventional wheelset, the value $K_{\psi A}$ is close to $30 \cdot 10^6 \text{ Nm/rad}$, where the influence is no longer significant. In the case of wheelsets with mechanisms between the wheel and axle body, the stiffness can be reduced down to $10 \cdot 10^6 \text{ Nm/rad}$, whereby the variation in the critical speed is significant. This means that the flexibility of the components on the wheelset must be correctly characterised in order to analyse its influence on the stability of the vehicle with good precision.

$K_{\psi B}$ represents the stiffness connecting yaw rotations of both wheels on the three-body model. Its value is also dependent on the flexibility of the axle and the flexibility between the

Figure 12, we can observe that the critical speed remains constant for the range of variation of $K_{\psi B}$ for the studied wheelsets, meaning that the angular stiffness between both wheels in the three-body model has no influence on the dynamic stability.

Figure 12 – Variation in critical speed against stiffness $K_{\psi B}$ for the complete range of equivalent conicities. International gauge.

This stiffness, and it should be emphasized that it is negative, is responsible for offering increased flexibility in symmetrical deformation of the wheels rather than parallel deformation

Figure 13). In linear models this result means that symmetrical deformations do not influence the critical speed of the vehicle. Only deformations that rotate both wheels in the same direction affects the dynamic stability, as this movement favours the occurrence of yaw behaviour on the wheelset.

Figure 13 – Symmetric and anti-symmetric deformation of wheels on the flexible three-body wheelset model, and their respective displacement patterns along the rails.

For this reason, stiffness $K_{\psi B}$ does not have any influence on the critical speed of the vehicle. When studying the dynamic stability of vehicles with flexible wheelsets, the precision with which this parameter is calculated is not decisive when calculating the critical speed of the train. For this reason, the model is simplified by assigning a single, common Degree of Freedom (DoF) to both wheels, which reduces to three the number of additional DoF introduced by each flexible wheelset model.

K_{θ} represents the torsional flexibility between both wheels and has a value of $K_{\theta} = 3 \cdot 10^6 \text{ Nm/rad}$ for a conventional wheelset and $K_{\theta} = 5.1 \cdot 10^6 \text{ Nm/rad}$ for a variable gauge wheelset. Beforehand we observed that there is a certain reduction in the critical speed

depending on the torsional flexibility between the wheels; this is confirmed in

Figure 14: the lower the resistance to torsion, the lower the critical speed of the vehicle. We should remember that according to the bibliography, including torsional flexibility in the wheelset model improves its dynamic stability, which is contrary to what we observe with this model.

Figure 14 – Variation in critical speed against torsional stiffness between the wheels (K_θ) for the complete range of equivalent conicities. International gauge.

In order to confirm this behaviour, extra simulations have been carried out on an underground train model. Similar results have been obtained (Table 2). The critical speed is reduced by approximately 2% by making the wheelset flexible for the studied equivalent conicity values.

Table 2 – Influence of torsional stiffness of the wheelset on the critical speed of an underground train for two different equivalent conicity values.

Torsional Stiffness	$\delta = 0.05$	$\delta = 0.4$
$K_T = 3 \cdot 10^6 \text{ Nm/rad}$	327km/h	130km/h
$K_T = 5 \cdot 10^6 \text{ Nm/rad}$	333km/h	132km/h

Assuming rolling contact between wheel and rail, a relative torsional angle between both wheels will cause a difference on the longitudinal rolling distance between them, which will consequently generate a specific yaw angle. So a torsional deformation of the wheelset causes a yaw angle, thus increasing the instability of the vehicle. In previous publications there cannot be found a discussion on the causes of critical speed increase, so no reason is given for the behaviour improvement when adding torsional stiffness. Thus, a direct comparison of the results would be incomplete.

In variable gauge wheelsets, the lateral flexibility present in the gauge change mechanism may also have an effect on the primary suspension, reducing slightly the stiffness value. The influence on the lateral stiffness of the primary suspension on vehicle stability is shown in

Figure 16 where we can observe that a reduction in the value of the lateral stiffness on the primary suspension has a positive effect on the dynamics of the vehicle. At a lower stiffness, the critical speed increases for any conicity value, although the improvement cannot be considered significant.

Figure 15 – Variation in critical speed in relation to the lateral stiffness of the primary suspension (K_{Y1}) for the complete range of equivalent conicity values. International gauge.

It has been observed that the yaw angle of the wheels represents the greatest influence on the critical speed of the vehicle. In some conventional and high speed vehicles the yaw centre of

the primary suspension is shifted longitudinally S_y from the axle (i.e. for arm axlebox types). In these cases lateral forces are transmitted between axle and bogie at a distance S_y from the axle and creates a yaw moment on the axle box

Figure 16). Thus, it is necessary to study the positioning of the yaw centre.

Figure 16 – Outline of the geometry of the primary suspension.

The results depicted in

Figure 17 reveal that $S_Y > 0$ reduces the dynamic stability, whereas a primary suspension centred with regards to the axle ($S_Y = 0$) (which is also a common arrangement) improves the critical speed significantly for the entire range of conicity values.

Figure 17 – Variation in the critical speed depending on the longitudinal positioning of the lateral load on the primary suspension (S_Y) for the complete range of equivalent conicities. International gauge.

The unsprung mass of the vehicle has a considerable influence on the critical speed due to the fact that wheelset yaw is one of the causes of instability at high speeds. The three-body model differentiates the mass of the wheelset and that of the wheels, meaning that the influence of each element can be analysed separately. The moments of inertia of each body have been modified in accordance with their mass.

Figure 18 – Variation in critical speed according to the mass of the axle body (m_{axle}) and the wheels (m_w) for the complete range of equivalent conicity values. International gauge.

Figure 18 depicts the influence of the unsprung mass on the dynamic stability of the vehicle with flexible wheelsets. The influence is equivalent to that of the models with rigid body axles: the greater the mass, the lower the critical speed. Both components, wheel and axle body, have a similar influence, so the influence of the unsprung mass can be regarded as a whole.

Table 3 shows the different parameters according to their influence on the critical speed of the vehicle.

Table 3 – Influence of the different parameters on the flexible axle affecting the critical speed.

Parameter	Parameter	Effect
Anti-symmetric flexibility of wheelset	K_A	High
Longitudinal position of the primary suspension yaw centre	S_Y	High
Unsuspended mass	m_a, m_w	Medium
Lateral stiffness between wheel and axle	K_y	Negligible
Symmetrical flexibility of wheelset	K_B	Negligible

4. Conclusions

This paper proposes a wheelset model which adequately represents the influence of its flexibility on the dynamic stability of a high-speed and/or variable gauge vehicle.

For the dynamic analysis of a rail vehicle, a flexible axle based on a modal representation was proposed as the point of departure, to which the residual flexibility of the mode shapes not included in the modal representation was added.

Due to the fact that problems in the dynamic stability of rail vehicles occur at frequencies lower than 15 Hz and that natural frequencies of the axle body are much higher than that, a model of the wheelset based exclusively on its static flexibility is accurate enough.

The structural flexibility of the wheelset influences considerably its dynamic stability:

- the influence of the symmetric bending mode shapes is negligible, it is the anti-symmetric bending mode shapes that affect the dynamic stability;
- this influence needs to be taken into account when modelling HST;

- it is even more important in variable gauge vehicles.

The influence of different parameters on the dynamic stability of the variable gauge bogie has been evaluated and quantified based on the model developed. The most influential factors are the anti-symmetric flexibility of the axle, particularly at low conicity values, and the longitudinal position of the primary suspension yaw centre (S_y).

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The authors declare that there is no conflict of interests.

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LIST OF NOTATION

x_0	displacement of the rigid body of the wheelset
T	transformation matrix due to the rotation of the rigid body
φ	rotation of the rigid body
x_L	location of a point on the wheelset
x_f	displacement of point $\vec{x}_L$ due to the flexibility of the system
ϕ_k	shape of the mode of vibration k
η_k	amplitude of the mode of vibration k
δ_R	residual deformation
H	transfer function matrix
m_k	modal mass of mode k
ω_k	natural frequency of mode k
ξ_k	equivalent damping of mode k
F_{st}	static flexibility
φ	roll angle
χ	torsion angle
ψ	yaw angle
y_i	lateral displacement of wheel i
ψ_{ia}	rotation corresponding to the end i of the axle
ψ_{iw}	rotation corresponding to wheel i
K_A	wheel-axle angular stiffness
K_B	wheel-wheel angular stiffness
K_y	wheel-axle lateral stiffness
K_T	wheelset torsional stiffness between wheels
M_i	moment corresponding to wheel i
F_w	complementary flexibility between wheel and axle
F	flexibility matrix
M	moment vector
S_Y	Longitudinal positioning of the lateral load on the primary suspension