The effect of high level multi-tone excitation on the acoustic properties of perforates and liner samples

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This paper discusses the effect of high level multi-tone acoustic excitation on the acoustic properties of perforates and liner samples. It is based on a large experimental study of the nonlinear properties of these types of samples without mean grazing or bias flow. It is known from previous studies that high level acoustic excitation at one frequency will change the acoustic impedance of perforates at other frequencies, thereby changing the boundary condition seen by the acoustic waves. This effect could be used to change the impedance boundary conditions and for instance increase the absorption. It could obviously also pose a problem for the correct modelling of sound transmission through ducts lined with such impedance surfaces. First a quasi-stationary model for the acoustic properties of the perforate is discussed and the results are compared to experimental data. The effect of the combination of frequency components in the excitation signal is studied to find out if it matters if we are using tones which are harmonically related or not. The effect the phase of the frequency components is studied using both the model and experimental data. It is also discussed if a parameter controlling the impedance can be found for an arbitrary combination of tones with different frequencies.

I. Introduction

There is a large number of papers on the effect of high level acoustic excitation on the acoustic properties of perforates and orifice plates¹⁻¹⁷. Orifice plates and perforates appear in many technical applications where they are exposed to a combination of high acoustic excitation levels and either grazing or bias flow or a combination. Examples are automotive mufflers and aircraft engine liners. It is well known from the literature that perforates can become non-linear at fairly low acoustic excitation levels. The non-linear losses are associated with vortex shedding at the outlet side of the orifice or perforate openings. In the linear case the impedance is independent of the sound field but when the sound pressure level is high the perforate impedance will be dependent on the acoustic particle velocity in the holes. For pure tone excitation the impedance will be controlled by the acoustic particle velocity at that frequency. If the acoustic excitation is random or periodic with multiple harmonics the impedance at a certain frequency will depend on the particle velocity at other frequencies¹⁷. The present paper discusses the effect of high level multi-tone acoustic excitation on the acoustic properties of perforates and liner samples. It is based on a large experimental study of the nonlinear properties of these types of samples without mean grazing or bias flow. It is known from previous studies that high level acoustic excitation at one frequency will change the acoustic impedance of perforates at other frequencies, thereby changing the boundary condition seen by the acoustic waves. This effect could be used to change the impedance boundary conditions and for instance increase the absorption, as in the so-called zero mass-flow liner¹⁸. This effect can also pose a problem for the correct modelling of sound transmission through ducts lined with such impedance surfaces. The present paper will attempt to answer some questions which are important for the understanding these effects such as: Does the combination of frequencies matter? It could for instance make a difference if we are using tones which are harmonically related or not. Does the phase of the frequency components make a difference? This could be yet another factor to use for changing the impedance. Can a parameter controlling the impedance be found which controls the impedance for an arbitrary combination of tones with different frequencies? One possible choice could be the total particle velocity at the sample surface.

II. Perforate impedance model

The model used here takes the starting point in the model presented by Cummings⁷. This model can excluding bias flow effects be written

$$l(t) \frac{dV(t)}{dt} + \frac{1}{2} \left(\frac{V(t)}{C_D} \right) \cdot \left| \frac{V(t)}{C_D} \right| = \frac{\Delta p(t)}{\rho_0}, \quad (1)$$

where $l(t)$ is an effective orifice thickness including end corrections which can be time varying, $V(t)$ is the fluctuating acoustic velocity in the orifice, C_D is a discharge coefficient to consider the vena contracta effect and $\Delta p(t)$ is the fluctuating pressure difference over the orifice. In the model according to Eq. (1) linear viscous losses are neglected. Such a term can be added either from theory¹⁶ or from experimental data giving

$$l(t) \frac{dV(t)}{dt} + \frac{1}{2} \left(\frac{V(t)}{C_D} \right) \cdot \left| \frac{V(t)}{C_D} \right| + R_L V(t) = \frac{\Delta p(t)}{\rho_0}. \quad (2)$$

In the article by Cummings⁷ an empirical expression for the time varying effective orifice length was presented

$$l(t) = l_0 + \frac{l_w + l_0}{1 + \frac{(L_v(t)/d)^{1.585}}{3}}, \quad (3)$$

where l_0 is the end correction on one side of the orifice, l_w is the orifice length, $L_v(t)$ is a time varying jet length caused by the high level acoustic excitation and d is the orifice diameter. Cummings suggested that the jet length should be estimated from

$$L_v(t) = \int_0^{\tau} |V(t)| dt, \quad (4)$$

where τ is the time from the beginning of the previous acoustic half cycle after $V(t)$ has changed sign. Here the result of using Eq. (4) has been compared to the assumption that L_v does not vary with time but only with the level of acoustic excitation such that

$$L_v = \int_0^{\tau/2} |V(t)| dt. \quad (5)$$

It was found by comparison with experimental data that a modified version of Eq. (3), shown in Eq. (6), gave a better agreement with experimental results for some perforates. This has, together with jet length estimation using Eq. (5) been used for some of the simulation studies presented in section III.

$$l(t) = l_0 + \frac{l_w + l_0}{1 + \frac{(L_v(t)/d)^{1.2}}{24}}, \quad (6)$$

II. Experimental setup

Most of the experimental data presented were obtained using an impedance tube with the perforate sample mounted at the end and an open termination behind the perforate sample. The sample was mounted using a holder causing a

slight area reduction. The impedance change caused by the holder was measured separately and deducted from the perforate sample results. Three quarter inch condenser microphones were used with microphone separations 0.05 m and 0.3 m. The distance between the sample and the nearest microphone was 0.15 m. Some data was also used from a test setup according to Fig. 1 where a single orifice was mounted in a duct with two microphones on each side. The test object was an orifice plate with 3 mm thickness and 6 mm hole diameter. The orifice plate was mounted in a rigid tube with a diameter of 40 mm. On the left hand side, a high quality loudspeaker was mounted as the excitation source. The microphone separation was in this case 0.18 m. This test rig was used to study the combination of high acoustic excitation levels and bias flow²⁰ but only no flow results will be presented here.

Figure 1. Test rig for measurement of orifice impedance.

Tonal excitation was used either with single tone excitation or with a combination of tones with different frequencies. Measurements in the impedance tube were made for four different circular hole perforate samples, as specified in Table 1. The perforate samples studied are described in Table 1 in terms of their hole diameter (d), plate thickness (t) and porosity (σ). Results obtained for sample P4 with the smallest porosity will mainly be presented in this paper. Measurements were in the impedance tube also made for a couple of realistic aircraft engine liner samples.

Table 1. Specification of circular hole perforate samples tested in the impedance tube.

Sample	d [mm]	t [mm]	σ
P1	3	1	0.28
P2	2	1.5	0.032
P3	2	2	0.086
P4	1	2	0.020

III. Results and discussion

A. Single tone excitation

Figure 4 shows an example of a typical result obtained using single tone excitation. The real and imaginary parts of the normalised impedance have been plotted against the inverse Strouhal number based on the peak particle velocity in the holes and the hole diameter ($1/St = |\hat{V}|/\omega d$). It can be seen that at an inverse Strouhal number close to one the acoustic properties start to become nonlinear and change with the level of excitation.

Figure 2. Normalized impedance for perforate sample P4 with 2% porosity, plotted against inverse Strouhal number: black stars – real part, red – plus imaginary part.

Equation (2) was solved in the time domain using the assumptions regarding the jet length according to Eq. (4) and Eq. (5) for different levels of single tone excitation. The pressure difference was assumed to be sinusoidal and the equation was solved to obtain $V(t)$. The impedance at the frequency of excitation was then obtained by taking the FFT of Δp and V and then dividing the two. These results were then compared to measured impedances. Figure 3 shows results at 200 Hz for the single orifice with 2.25% porosity.

Figure 3. Normalized impedance for single orifice with 2.25 % porosity, plotted against inverse Strouhal number: black stars – measured real part, red stars – measured imaginary part, black solid line – simulated real part, red dashed line – simulated imaginary part, red thick lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$. a) Jet length according to Eq. (5), b) jet length according to Eq. (4).

It can be seen that the best agreement is obtained using the jet length according to Eq. (5) while the assumption according to Eq. (4) gives an under prediction of the drop in effective orifice length caused by the high level acoustic excitation. Figure 4 show the corresponding results for perforate sample P4 with 2 % porosity where now Eq. (6) has been used together with Eq. (5).

a)

b)

Figure 4. Normalized impedance for perforate with 2 % porosity, plotted against inverse Strouhal number: black stars – measured real part, red stars – measured imaginary part, black solid line – simulated real part, red dashed line – simulated imaginary part, red thick lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$. a) Jet length according to Eq. (5), b) jet length according to Eq. (4).

It can be seen that for this case the assumption of Eq. (5) together with Eq. (6) gives a better agreement with experimental data. The model can give reasonable results when compared to experimental data and will therefore be used to complement experimental results in the subsequent sections. The time varying jet length estimation according to Eq. (4) gives small jumps in the results. This problem gets worse when the excitation is multi tone rather than single tone. For the perforate sample P4 with 2% porosity which will be studied the model with time varying effective length according to Eq. (5) and (6) will be used. The time variation of the effective length during one cycle for the highest level of excitation is shown in Fig. (5) and the variation of effective length with level of excitation obtained when the result using Eq. (5) is inserted in Eq. (3) is shown in Fig 6.

Figure 5. Time variation of jet length and effective orifice length according to Eq. (4) and Eq. (3) for a perforate with 2 % porosity and for the highest level of acoustic excitation according to Fig. (4): black solid line – jet length, red dashed line – effective orifice length, red thick lines – imaginary parts obtained assuming: $l = l_0$ and $l = 2l_0 + l_w$.

Figure 6. Variation of jet length and effective orifice length according to Eq. (5) and Eq. (3) for a perforate with 2 % porosity for different levels of acoustic excitation: black solid line – jet length, red dashed line – effective orifice length, red thick lines – imaginary parts obtained assuming: $l = l_0$ and $l = 2l_0 + l_w$.

B. Multi tone excitation

In this section experimental and simulation results for multi tone excitation are presented. Figure 7 shows the variation of measured impedance, for a 2% porosity perforate, at 330 Hz and 110 Hz when the excitation at the other frequency is varied. For comparison the impedance obtained using single tone excitation is also included. The resistance increases as the level of secondary tone excitation increases while the reactance decreases. For very high secondary tone excitation levels the impedance will go to minus the impedance seen from the sample back towards the impedance tube²¹, so the resistance will have a maximum and then drop down and become negative.

Figure 7. Measured normalized impedance for a perforate with 2 % porosity, plotted against the ratio between velocities at 110 Hz and 330 Hz: a) impedance at 330 Hz: black solid line – real part excitation only at 330 Hz, red solid line – imaginary part excitation only at 330 Hz, black dashed line – real part excitation at 110 Hz and 330 Hz, red dashed line – imaginary part excitation at 110 Hz and 330 Hz, b) impedance at 110 Hz: black solid line – real part excitation only at 110 Hz, red solid line – imaginary part excitation only at 110 Hz, black dashed line – real part excitation at 330 Hz and 110 Hz, red dashed line – imaginary part excitation at 330 Hz and 110 Hz.

Figures 8 and 9 shows comparisons between experimental results and simulations using Eqs. (2), (5) and (6). In Fig. (8) the variation of impedance at 330 Hz when the level of excitation at 110 Hz is varied while the level of excitation at 330 Hz is kept constant is shown. Curves for different levels of excitation at 330 Hz are shown for the simulation results. Figure 9 shows the corresponding results at 110 Hz when the level of excitation at 330 Hz is varied. It can be seen that the simulation model gives reasonable agreement with the experimental results which means that it can be used to study trends caused by other types of variation in the excitation.

Figure 8. Normalized impedance for perforate with 2 % porosity at 330 Hz, for varying level of excitation at 110 Hz while the level of excitation at 330 Hz is kept constant and with impedance for excitation at 330 Hz only removed, plotted against ratio between velocity at 110 Hz and 330 Hz. Excitation levels at 330 Hz: black solid line – simulated 0.06 m/s, blue dashed-dotted line – simulated 0.2 m/s, red dashed line – simulated 0.6 m/s, green dotted line – simulated 2 m/s, stars – measured 0.6 m/s. a) Real part, b) imaginary part.

Figure 9. Normalized impedance for perforate with 2 % porosity at 110 Hz, for varying level of excitation at 330 Hz while the level of excitation at 110 Hz is kept constant and with impedance for excitation at 110 Hz only removed, plotted against ratio between velocity at 330 Hz and 110 Hz. Excitation levels at 110 Hz: black solid line – simulated 0.07 m/s, blue dashed-dotted line – simulated 0.2 m/s, red dashed line – simulated 0.6 m/s, green dotted line – measured 2.1 m/s. a) Real part, b) imaginary part.

C. Multi tone excitation – effect of combination of frequencies

Figure 10 shows normalized impedance results obtained using multi-tone excitation on sample P4 with 2 % porosity, plotted against total peak particle velocity in the holes. Measurements were made with single tone excitation, two-tone excitation, three-tone excitation, four tone excitation and five-tone excitation. The impedance results are also compared to two different semi-empirical models described in^{15,17}. It can be seen that there is a large

scatter in the results showing that the total, summed, peak particle velocity is not the only parameter controlling the results.

Figure 10. Normalized perforate impedance for sample P4 at 110 Hz as a function of peak particle velocity in the holes; left resistance, right reactance, measured using different number of tones; stars – single tone, + two tones, diamonds – three tones, squares – four tones, < - five tones, straight line- theory according to¹⁵, dashed line - theory described in¹⁷.

Measurements have also been made on an aircraft engine liner sample using multi-tone techniques. Figure 11 shows normalised impedance results at 990 Hz obtained with different combinations of excitation frequencies. It can be seen that the effect of varying the level at 330 Hz has the largest effect consistent with the result that nonlinear energy transfer mainly occurs to higher order odd harmonics. The effect should therefore be largest at three times the excitation frequency. In the curve where the level of excitation at 330 Hz is kept constant while the level of excitation at 990 Hz is varied we can see that with low level of excitation at 990 Hz we get a negative resistance because the nonlinear energy transfer from 330 Hz dominates the result. As the level of excitation at 990 Hz is increased we approach the resistance obtained for single tone excitation at 990 Hz, without changing the total particle velocity much, since it is dominated by the excitation at 330 Hz. These results show that it is difficult to find a single parameter controlling the obtained impedance results.

Figure 11. Real part of normalized perforate impedance for liner sample at 990 Hz as a function of peak particle velocity incident on the sample, measured using different combinations of tones; black dots – single tone excitation at 990 Hz, black stars – excitation at 990 Hz constant and excitation at 110 Hz varied, blue boxes – excitation at 990 Hz constant and excitation at 330 Hz varied, red plus – excitation at 110 Hz constant and excitation at 990 Hz varied, green diamonds – excitation at 330 Hz constant and excitation at 990 Hz varied.

Figure 12 presents the results of another experiment where it was studied if it makes a difference for the results obtained using two-tone excitation if the frequencies used are exact harmonics or not. It can be seen that the largest variation is obtained when excitation at 110 Hz interacts with the result at 330 Hz. This again shows that the combination of frequencies does make a difference and that excitation at harmonics have a larger effect compared to excitation at other frequencies.

Figure 12. Normalized impedance for perforate with 2 % porosity at 330 Hz plotted against velocity ratio: Black – real part, red – imaginary part, solid line – $V_I = V_{100}$, dashed line – $V_I = V_{110}$, dashed-dotted line – $V_I = V_{120}$. a) Excitation at 330 Hz varied and excitation at the other frequencies kept constant, b) excitation at 100 Hz, 110 Hz and 120 Hz varied and excitation at 330 Hz kept constant.

D. Multi tone excitation – effect of phase

In order to test if changing the phase of the higher harmonics has an influence on the result a standard impedance tube tests have been made with excitation at 110 Hz and 330 Hz. The level of excitation was varied either by changing the excitation level at 110 Hz or 330 Hz. At each excitation level the phase at 330 Hz was shifted 180 degrees. Figures 13 and 14 shows comparisons of experimental results and simulation results. In Fig. 13 the normalized impedance at 330 Hz obtained when the level of excitation at 110 Hz was varied is shown for cases where the phase at 330 Hz is varied. In the measurements the phase was shifted by 180 degrees while a few additional results are shown in the simulation results. Figure 14 shows the corresponding results at 110 Hz. When comparing the experimental and simulated results it should be noticed that the reference case shown as solid lines do not have exactly the same phase in the experiments and simulations. What should be compared is the magnitude of the difference caused by a phase shift. It can be seen that this variation is predicted well by the simulation. The phase relation between the tones is of importance at least at higher ratios between the velocities.

Figure 13. Real and imaginary parts of normalised impedance at 330 Hz for a perforate sample with 2% porosity plotted against velocity ratio for excitation at 110 Hz varied and excitation at 330 Hz kept constant: black – real part, red – imaginary part, solid line – phase at 330 Hz not shifted , dashed – phase at 330 Hz shifted 180 degrees, dashed-dotted – phase at 330 Hz shifted 60 degrees, dotted – phase at 330 Hz shifted 90 degrees. a) Measured, b) simulation.

Figure 14. Real and imaginary parts of normalised impedance at 110 Hz for a perforate sample with 2% porosity plotted against velocity ratio for excitation at 110 Hz varied and excitation at 330 Hz kept constant: black – real part, red – imaginary part, solid line – phase at 330 Hz not shifted , dashed – phase at 330 Hz shifted 180 degrees, dashed-dotted – phase at 330 Hz shifted 60 degrees, dotted – phase at 330 Hz shifted 90 degrees. a) Measured, b) simulation

IV. Conclusions

The result of high level acoustic multi-tone excitation on the acoustic properties of perforates and liner samples has been investigated experimentally and using a model. It was shown that a modified version of the Cummings⁷ model gives sufficiently good results when compared to experimental data to use the model for parameter variation studies. It was concluded that the combination of frequencies is of importance for multi tone excitation and that harmonically related tones have a stronger influence on the nonlinear interaction results compared to other combinations of frequencies. The phase of the tones used also makes a difference for the result. It was concluded that no single parameter which controls the obtained impedance results for an arbitrary combination of tones could be found.

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