Acoustic properties of perforates under high level multi-tone excitation

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This paper discusses the effect of high level multi-tone acoustic excitation on the acoustic properties of perforates. It is based on a large experimental study of the nonlinear properties of these types of samples without mean grazing or bias flow. Compared to previously published results the present investigation concentrates on the effect of multiple harmonics. It is known from previous studies that high level acoustic excitation at one frequency will change the acoustic impedance of perforates at other frequencies, thereby changing the boundary condition seen by the acoustic waves. This effect could be used to change the impedance boundary conditions and for instance increase the absorption. It could obviously also pose a problem for the correct modelling of sound transmission through ducts lined with such impedance surfaces. Experimental results are compared to a quasi-stationary model. The effect of the combination of frequency components and phase in the excitation signal is studied.

I. Introduction

There is a large number of papers on the effect of high level acoustic excitation on the acoustic properties of perforates and orifice plates¹⁻¹⁸. Orifice plates and perforates appear in many technical applications where they are exposed to a combination of high acoustic excitation levels and either grazing or bias flow or a combination. Examples are automotive mufflers and aircraft engine liners. Non-linear losses are associated with vortex shedding at the outlet side of the orifice or perforate openings. In the linear case the impedance is independent of the sound field but when the sound pressure level is high the perforate impedance will be dependent on the acoustic particle velocity in the holes. For pure tone excitation the impedance will be controlled by the acoustic particle velocity at that frequency. If the acoustic excitation is random or periodic with multiple harmonics the impedance at a certain frequency will depend on the particle velocity at other frequencies¹⁷⁻¹⁸. In¹⁸ the effect of mainly two-tone excitation on the acoustic properties of perforates was discussed and the present paper discusses the effect of high level multi-tone acoustic excitation. It is based on an experimental study of the nonlinear properties of these types of samples without mean grazing or bias flow. It is known from previous studies that high level acoustic excitation at one frequency will change the acoustic impedance of perforates at other frequencies, thereby changing the boundary condition seen by the acoustic waves. This effect could be used to change the impedance boundary conditions and for instance increase the absorption, as in the so-called zero mass-flow liner¹⁹. This effect can also pose a problem for the correct modelling of sound transmission through ducts lined with such impedance surfaces. The present paper will study the effect of harmonic or non-harmonic excitation as well as the influence of combination of signal phase.

II. Perforate impedance model

The model used here takes the starting point in the model presented by Cummings⁷. This model can excluding bias flow effects be written

$$l(t) \frac{dV(t)}{dt} + \frac{1}{2} \left(\frac{V(t)}{C_D} \right) \cdot \left| \frac{V(t)}{C_D} \right| = \frac{\Delta p(t)}{\rho_0}, \quad (1)$$

where $l(t)$ is an effective orifice thickness including end corrections which can be time varying, $V(t)$ is the fluctuating acoustic velocity in the orifice, C_D is a discharge coefficient to consider the vena contracta effect and $\Delta p(t)$ is the fluctuating pressure difference over the orifice. In the model according to Eq. (1) linear viscous losses are neglected. Such a term can be added either from theory¹⁶ or from experimental data giving

$$l(t) \frac{dV(t)}{dt} + \frac{1}{2} \left(\frac{V(t)}{C_D} \right) \cdot \left| \frac{V(t)}{C_D} \right| + R_L V(t) = \frac{\Delta p(t)}{\rho_0}. \quad (2)$$

In the article by Cummings⁷ an empirical expression for the time varying effective orifice length was presented

$$l(t) = l_0 + \frac{l_w + l_0}{1 + \frac{(L_v(t)/d)^{1.585}}{3}}, \quad (3)$$

where l_0 is the end correction on one side of the orifice, l_w is the orifice length, $L_v(t)$ is a time varying jet length caused by the high level acoustic excitation and d is the orifice diameter. Cummings suggested that the jet length should be estimated from

$$L_v(t) = \int_0^{\tau} |V(t)| dt, \quad (4)$$

where τ is the time from the beginning of the previous acoustic half cycle after $V(t)$ has changed sign. Here the result of using Eq. (4) has been compared to the assumption that L_v does not vary with time but only with the level of acoustic excitation such that

$$L_v = \int_0^{\tau/2} |V(t)| dt. \quad (5)$$

It was found by comparison with experimental data that a modified version of Eq. (3), shown in Eq. (6), gave a better agreement with experimental results for some perforates. This has, together with jet length estimation using Eq. (5) been used for some of the simulation studies presented in section III.

$$l(t) = l_0 + \frac{l_w + l_0}{1 + \frac{(L_v(t)/d)^{1.2}}{24}}, \quad (6)$$

II. Experimental setup

The experimental data presented were obtained using an impedance tube with the perforate sample mounted at the end and an open termination behind the perforate sample. The sample was mounted using a holder causing a slight area reduction. The impedance change caused by the holder was measured separately and deducted from the perforate sample results. Three quarter inch condenser microphones were used with microphone separations 0.05 m and 0.3 m. The distance between the sample and the nearest microphone was 0.15 m.

Tonal excitation was used either with single tone excitation or with a combination of tones with different frequencies. Measurements in the impedance tube were made for four different circular hole perforate samples, as specified in Table 1. The perforate samples studied are described in Table 1 in terms of their hole diameter (d), plate thickness (l_w) and porosity (σ). Results obtained for sample P2 with porosity $\sigma=0.032$ and sample P4 with $\sigma=0.032$ will be presented in this paper.

Table 1. Specification of circular hole perforate samples tested in the impedance tube.

Sample	d [mm]	t [mm]	σ
P1	3	1	0.28
P2	2	1.5	0.032
P3	2	2	0.086
P4	1	2	0.020

III. Results and discussion

A. Single tone excitation

Figure 1 shows the normalized perforate impedance of sample P2 with porosity 0.032, obtained using single tone excitation at 110 Hz, plotted against peak particle velocity in the holes. A comparison is made between experimental results and simulated results obtained using Cummings equation (2) with the equivalent orifice thickness calculated using Eq. (4) and Eq. (6). Figure 2 shows the corresponding results for sample P4 with porosity 0.02. The agreement between simulated and experimental results is fairly good.

Figure 1. Normalized impedance at 110 Hz for perforate sample with 3.2% porosity, for single tone excitation at 110 Hz, plotted against particle velocity in the holes: black – measurement, red – simulation; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$; a) Real part, b) Imaginary part.

Figure 2. Normalized impedance at 110 Hz for perforate sample with 2% porosity, for single tone excitation at 110 Hz, plotted against particle velocity in the holes: black – measurement, red – simulation; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_W$ and $l = 2l_0 + l_W$; a) Real part, b) Imaginary part.

B. Multi tone excitation

In this section experimental and simulation results for multi tone excitation are presented, starting with two tone excitation shown in Fig. 3 and 4. It can be seen that the simulation still gives a reasonably fair prediction of a large part of the resistance results but with some deviation while the reactance results agree less well. The reason for this scattering in the experimental results will be investigated in the remainder of this section.

Figure 3. Normalized impedance at 110 Hz for perforate sample with 3.2% porosity, for two tone excitation, plotted against summed (total) particle velocity in the holes. The excitation at 110 Hz is kept constant while the excitation at other frequencies is varied: black – measurement, red – simulation; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_W$ and $l = 2l_0 + l_W$; a) Real part, b) Imaginary part.

Figure 4. Normalized impedance at 110 Hz for perforate sample with 2% porosity, for two tone excitation, plotted against summed (total) particle velocity in the holes. The excitation at 110 Hz is kept constant while the excitation at other frequencies is varied: black – measurement, red – simulation; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_W$ and $l = 2l_0 + l_W$; a) Real part, b) Imaginary part.

The results shown in Fig. 3 and Fig. 4 are plotted against the total particle velocity in the holes summed over both frequencies of excitation. This may obviously not be the best quantity to plot against to get a collapse of the data. In Figures 5-8 it is investigated if plotting against the particle velocity at the frequency where the result is evaluated (110 Hz in this case) would be a better candidate. Results are presented for: two tone, three tone, four tone and five tone excitation. It can be seen that even though the voltage supplied to the loudspeaker at 110 Hz is kept constant the actual level of excitation in terms particle velocity is decreased when the level of excitation at another frequency is increased. This is because the perforate impedance at 110 Hz increases causing a decrease in particle velocity and an increase in pressure. If the pressure wave amplitude (p_+) incident on the sample is evaluated it is found that this quantity is unchanged. The results show that plotting against particle velocity at 110 Hz instead of total summed particle velocity gives a better collapse of the resistance results, while the scatter in the reactance results remain more or less the same.

Figure 5. Normalized impedance at 110 Hz for perforate sample with 3.2% porosity, for two tone excitation with the level of excitation at 110 Hz kept constant and excitation at the other frequency varied; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_W$ and $l = 2l_0 + l_W$; a) Real part plotted against total particle velocity, b) Imaginary part plotted against total particle velocity, c) Real part plotted against particle velocity at 110 Hz, d) Imaginary part plotted against particle velocity at 110 Hz.

Figure 6. Normalized impedance at 110 Hz for perforate sample with 3.2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at the other frequency varied; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$; a) Real part plotted against total particle velocity, b) Imaginary part plotted against total particle velocity, c) Real part plotted against particle velocity at 110 Hz, d) Imaginary part plotted against particle velocity at 110 Hz.

Figure 7. Normalized impedance at 110 Hz for perforate sample with 3.2% porosity, for four tone excitation with the level of excitation at 110 Hz kept constant and excitation at the other frequency varied; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_W$ and $l = 2l_0 + l_W$; a) Real part plotted against total particle velocity, b) Imaginary part plotted against total particle velocity, c) Real part plotted against particle velocity at 110 Hz, d) Imaginary part plotted against particle velocity at 110 Hz.

Figure 8. Normalized impedance at 110 Hz for perforate sample with 3.2% porosity, for five tone excitation with the level of excitation at 110 Hz kept constant and excitation at the other frequency varied; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$; a) Real part plotted against total particle velocity, b) Imaginary part plotted against total particle velocity, c) Real part plotted against particle velocity at 110 Hz, d) Imaginary part plotted against particle velocity at 110 Hz.

C. Multi tone excitation – effect of combination of frequencies

It is also of interest to see if the choice of frequency components has an influence on the result. The frequency components used in the present study are: 60 Hz, 120 Hz, 220 Hz, 440 Hz and 880 Hz. For these frequencies the level of excitation has been varied while the excitation, i.e. the loudspeaker voltage and amplitude of incident pressure wave (p_+), at 110 Hz remained unchanged. Figures 9 and 10 shows a subset of the two tone excitation results at 110 Hz where the frequency component varied is indicated. It can be seen that excitation at 120 Hz which is close to 110 Hz causes a larger scatter in the results compared to other combinations of frequency components. Even though there is scatter at higher excitation levels also for other combinations of frequencies especially for sample P4 for with smaller porosity. In Figure 11 and 12 results for three tone excitation is shown and again the cases where the level of the 120 Hz component gives a larger scatter in the result at 110 Hz compared to other combinations.

Figures 13 to 20 gives corresponding results for three tone excitation for the impedance at: 60 Hz, 120 Hz, 220 Hz and 440 Hz. In these figures the red symbols are used for cases where the level of the frequency component for which the impedance is presented is varied while the levels of excitation at other frequencies are kept constant. The

black symbols are used for cases where the level of excitation at the frequency studied is instead kept constant while the level at one of the other components is varied. It can be seen that the impedance results when plotted against the particle velocity at the same frequency as where the result is evaluated collapses into two distinct branches, one for the case when the level at the frequency studied is varied and the other for the case that the level at one of the other frequency components is varied. The impedance at 120 Hz, which is close to 110 Hz, again behaves in a different way and can have negative resistance values and very large reactance values.

Figure 9. Normalized impedance at 110 Hz for perforate sample with 3.2% porosity, for two tone excitation with the level of excitation at 110 Hz kept constant and excitation at the other frequency varied: black – measurement, red – simulation; stars – 60 Hz, plus – 120 Hz, squares – 220 Hz, diamonds – 440 Hz, triangles – 880 Hz; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$; a) Real part plotted against summed particle velocity, b) Imaginary part plotted against summed particle velocity, c) Real part plotted against particle velocity at 110 Hz, d) Imaginary part plotted against particle velocity at 110 Hz.

Figure 10. Normalized impedance at 110 Hz for perforate sample with 2% porosity, for two tone excitation with the level of excitation at 110 Hz kept constant and excitation at the other frequency varied: black – measurement, red – simulation; stars – 60 Hz, plus – 120 Hz, squares – 220 Hz, diamonds – 440 Hz, triangles – 880 Hz; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$; a) Real part plotted against summed particle velocity, b) Imaginary part plotted against summed particle velocity, c) Real part plotted against particle velocity at 110 Hz, d) Imaginary part plotted against particle velocity at 110 Hz.

Figure 11. Normalized impedance at 110 Hz for perforate sample with 3.2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies:
black stars – variation at 120 Hz, 60 Hz kept constant, red stars – variation at 60 Hz, 120 Hz kept constant,
black plus – variation at 220 Hz, 60 Hz kept constant, red plus - variation at 60 Hz, 220 Hz kept constant,
black squares – variation at 440 Hz, 60 Hz kept constant, red squares - variation at 60 Hz, 440 Hz kept constant,
black diamonds – variation at 220 Hz, 120 Hz kept constant, red diamonds - variation at 120 Hz, 220 Hz kept constant,
black triangles – variation at 440 Hz, 120 Hz kept constant, red triangles – variation at 120 Hz, 440 Hz kept constant,
black rings – variation at 440 Hz, 220 Hz kept constant ,red rings - variation at 220 Hz, 440 Hz kept constant
; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$;
a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity,
c) Real part plotted against velocity at 110 Hz, d) Imaginary part plotted against velocity at 110 Hz .

Figure 12. Normalized impedance at 110 Hz for perforate sample with 2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies: black stars – variation at 120 Hz, 60 Hz kept constant, red stars – variation at 60 Hz, 120 Hz kept constant, black plus – variation at 220 Hz, 60 Hz kept constant, red plus - variation at 60 Hz, 220 Hz kept constant, black squares – variation at 440 Hz, 60 Hz kept constant, red squares - variation at 60 Hz, 440 Hz kept constant, black diamonds – variation at 220 Hz, 120 Hz kept constant, red diamonds - variation at 120 Hz, 220 Hz kept constant, black triangles – variation at 440 Hz, 120 Hz kept constant, red triangles – variation at 120 Hz, 440 Hz kept constant, black rings – variation at 440 Hz, 220 Hz kept constant ,red rings - variation at 220 Hz, 440 Hz kept constant ; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$;
a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity, c) Real part plotted against velocity at 110 Hz, d) Imaginary part plotted against velocity at 110 Hz .

Figure 13. Normalized impedance at 60 Hz for perforate sample with 3.2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies: black stars – variation at 120 Hz, 60 Hz kept constant, red stars – variation at 60 Hz, 120 Hz kept constant, black plus – variation at 220 Hz, 60 Hz kept constant, red plus - variation at 60 Hz, 220 Hz kept constant, black squares – variation at 440 Hz, 60 Hz kept constant, red squares - variation at 60 Hz, 440 Hz kept constant; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$;
a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity, c) Real part plotted against velocity at 60 Hz, d) Imaginary part plotted against velocity at 60 Hz .

Figure 14. Normalized impedance at 60 Hz for perforate sample with 2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies: black stars – variation at 120 Hz, 60 Hz kept constant, red stars – variation at 60 Hz, 120 Hz kept constant, black plus – variation at 220 Hz, 60 Hz kept constant, red plus - variation at 60 Hz, 220 Hz kept constant, black squares – variation at 440 Hz, 60 Hz kept constant, red squares - variation at 60 Hz, 440 Hz kept constant;

solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$;

a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity,

c) Real part plotted against velocity at 60 Hz, d) Imaginary part plotted against velocity at 60 Hz .

Figure 15. Normalized impedance at 120 Hz for perforate sample with 3.2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies:

black stars – variation at 60 Hz, 120 Hz kept constant, red stars – variation at 120 Hz, 60 Hz kept constant, black plus – variation at 220 Hz, 120 Hz kept constant, red plus - variation at 120 Hz, 220 Hz kept constant, black squares – variation at 440 Hz, 120 Hz kept constant, red squares - variation at 120 Hz, 440 Hz kept constant;

solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_W$ and $l = 2l_0 + l_W$;

a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity, c) Real part plotted against velocity at 120 Hz, d) Imaginary part plotted against velocity at 120 Hz .

Figure 16. Normalized impedance at 120 Hz for perforate sample with 2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies: black stars – variation at 60 Hz, 120 Hz kept constant, red stars – variation at 120 Hz, 60 Hz kept constant, black plus – variation at 220 Hz, 120 Hz kept constant, red plus - variation at 120 Hz, 220 Hz kept constant, black squares – variation at 440 Hz, 120 Hz kept constant, red squares - variation at 120 Hz, 440 Hz kept constant;

solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$;

a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity,

c) Real part plotted against velocity at 120 Hz, d) Imaginary part plotted against velocity at 120 Hz .

Figure 17. Normalized impedance at 220 Hz for perforate sample with 3.2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies:

black stars – variation at 60 Hz, 220 Hz kept constant, red stars – variation at 220 Hz, 60 Hz kept constant, black plus – variation at 120 Hz, 220 Hz kept constant, red plus - variation at 220 Hz, 120 Hz kept constant, black squares – variation at 440 Hz, 220 Hz kept constant, red squares - variation at 220 Hz, 440 Hz kept constant;

solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$;

a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity, c) Real part plotted against velocity at 220 Hz, d) Imaginary part plotted against velocity at 220 Hz .

Figure 18. Normalized impedance at 220 Hz for perforate sample with 2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies: black stars – variation at 60 Hz, 220 Hz kept constant, red stars – variation at 220 Hz, 60 Hz kept constant, black plus – variation at 120 Hz, 220 Hz kept constant, red plus - variation at 220 Hz, 120 Hz kept constant, black squares – variation at 440 Hz, 220 Hz kept constant, red squares - variation at 220 Hz, 440 Hz kept constant; solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$;

a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity, c) Real part plotted against velocity at 220 Hz, d) Imaginary part plotted against velocity at 220 Hz .

Figure 19. Normalized impedance at 440 Hz for perforate sample with 3.2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies:

black stars – variation at 60 Hz, 440 Hz kept constant, red stars – variation at 440 Hz, 60 Hz kept constant, black plus – variation at 120 Hz, 440 Hz kept constant, red plus - variation at 440 Hz, 120 Hz kept constant, black squares – variation at 220 Hz, 440 Hz kept constant, red squares - variation at 440 Hz, 220 Hz kept constant;

solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$;

a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity, c) Real part plotted against velocity at 440 Hz, d) Imaginary part plotted against velocity at 440 Hz .

Figure 20. Normalized impedance at 440 Hz for perforate sample with 2% porosity, for three tone excitation with the level of excitation at 110 Hz kept constant and excitation at other frequencies varied, frequencies: black stars – variation at 60 Hz, 440 Hz kept constant, red stars – variation at 440 Hz, 60 Hz kept constant, black plus – variation at 120 Hz, 440 Hz kept constant, red plus - variation at 440 Hz, 120 Hz kept constant, black squares – variation at 220 Hz, 440 Hz kept constant, red squares - variation at 440 Hz, 220 Hz kept constant;

solid lines – imaginary parts obtained assuming: $l = l_0$, $l = l_0 + l_w$ and $l = 2l_0 + l_w$;

a) Real part plotted against particle total velocity, b) Imaginary part plotted against total velocity, c) Real part plotted against velocity at 440 Hz, d) Imaginary part plotted against velocity at 440 Hz .

D. Multi tone excitation – effect of phase

In order to test if changing the phase of the higher harmonics has an influence on the result a standard impedance tube tests was made with excitation at 110 Hz and 330 Hz in¹⁸. The level of excitation was varied either by changing the excitation level at 110 Hz or 330 Hz. At each excitation level the phase at 330 Hz was shifted 180 degrees. Figures 21 and 22 show comparisons of experimental results and simulation results. In Fig. 21 the normalized impedance at 330 Hz obtained when the level of excitation at 110 Hz was varied is show for cases where the phase at 330 Hz is varied. In the measurements the phase was shifted by 180 degrees while a few additional results are

shown in the simulation results. Figure 22 shows the corresponding results at 110 Hz. When comparing the experimental and simulated results it should be noticed that the reference case shown as solid lines do not have exactly the same phase in the experiments and simulations. What should be compared is the magnitude of the difference caused by a phase shift. It can be seen that this variation is predicted well by the simulation. The phase relation between the tones is of importance at least at higher ratios between the velocities.

Figure 21. Real and imaginary parts of normalised impedance at 330 Hz for a perforate sample with 2% porosity plotted against velocity ratio for excitation at 110 Hz varied and excitation at 330 Hz kept constant, from¹⁸: black – real part, red – imaginary part, solid line – phase at 330 Hz not shifted, dashed – phase at 330 Hz shifted 180 degrees, dashed-dotted – phase at 330 Hz shifted 60 degrees, dotted – phase at 330 Hz shifted 90 degrees. a) Measured, b) simulation.

Figure 22. Real and imaginary parts of normalised impedance at 110 Hz for a perforate sample with 2% porosity plotted against velocity ratio for excitation at 110 Hz varied and excitation at 330 Hz kept constant, from¹⁸: black – real part, red – imaginary part, solid line – phase at 330 Hz not shifted, dashed – phase at 330 Hz shifted 180 degrees, dashed-dotted – phase at 330 Hz shifted 60 degrees, dotted – phase at 330 Hz shifted 90 degrees. a) Measured, b) simulation.

Figures 23 and 24 shows simulated results for the impedance of samples P2 and P4 with 2% and 3.2 % porosity obtained using two tone excitation. Here the impedance at 110 Hz is presented while the level of excitation at other frequencies (60, 120, 220 and 440 Hz) is varied. The phase has been varied from the experimentally obtained particle velocities by either complex conjugating the velocity or adding a phase shift of: $\pi/2$, $\pi/3$ or $\pi/4$. It can be seen that the phase shift produces a scatter in the reactance results and also in the resistance results at higher levels

of excitation. It should be noticed that the level of excitation at the other frequencies is high compared to the level of excitation at 110 Hz for the higher values of summed (total) particle velocity.

Figure 23. Simulated results for normalised impedance at 110 Hz for a perforate sample with 3.2% porosity plotted against summed particle velocity, excitation at 110 Hz kept constant and excitation at other frequencies varied: black stars – phase at 110 Hz according to experiments, red stars – phase at 110 Hz shifted $\pi/2$, black plus - phase at 110 Hz shifted $\pi/3$, red plus – phase at 110 Hz shifted $\pi/4$, black diamonds - phase at 110 Hz complex conjugated. a) Real part, b) imaginary part.

Figure 24. Simulated results for normalised impedance at 110 Hz for a perforate sample with 2% porosity plotted against summed particle velocity, excitation at 110 Hz kept constant and excitation at other frequencies varied: black stars – phase at 110 Hz according to experiments, red stars – phase at 110 Hz shifted $\pi/2$, black plus - phase at 110 Hz shifted $\pi/3$, red plus – phase at 110 Hz shifted $\pi/4$, black diamonds - phase at 110 Hz complex conjugated. a) Real part, b) imaginary part.

IV. Conclusions

The result of high level acoustic multi-tone excitation on the acoustic properties of perforates has been investigated experimentally and using a model. It was shown that a modified version of the Cummings⁷ model gives sufficiently good results, for single tone excitation, when compared to experimental data to use the model for parameter variation studies. For two and multi-tone excitation the model gives reasonable results for the resistance while it is less good for the reactance part of the perforate impedance. It was concluded that the combination of frequencies is of importance for multi tone excitation. In¹⁸ it was shown that harmonically related tones have a stronger influence on the nonlinear interaction results compared to other combinations of frequencies. Here it was also demonstrated that if the tones used for excitation are nearby in the frequency plane, such as the 110 Hz and 120 Hz components used in the present study, the effect on the apparent impedance at these two frequencies will be affected substantially. It was shown just as in¹⁸ that the phase of the tones used also makes a difference for the result

especially at higher levels of excitation. In¹⁸ it was concluded that no single parameter which controls the obtained impedance results for an arbitrary combination of tones could be found. Here it was shown that plotting the impedance result against the particle velocity at the frequency where the impedance is evaluated rather than against the summed particle velocity for all frequencies gives a better collapse of the measured data especially for the resistance

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