Input Signal Generation for Constrained Multiple-Input Multple-Output Systems ⋆

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Abstract: In this paper we extend a recent method for generating an input signal with a desired auto-correlation function while satisfying both input and output constraints for the system it is to be applied to. This is an important problem in system identification, firstly, the properties of the identified model is highly dependent on the used excitation signal during the experiment and secondly, on real processes, due to actuator saturation and safety considerations, it is important to constraint the input and output from the process. The proposed method corresponds to a nonlinear model predictive control problem and here we extend it to consider general input horizons and also to the multiple-input multiple-output case. In general this corresponds to solving a multivariate polynomial of order four in each time step. Here two different methods for solving this problem are considered: one based on convex relaxation and one based on a cyclic algorithm. The performance of the algorithm is successfully verified by simulations and the implications of different lengths of the input horizon is discussed.

1. INTRODUCTION

The problem of generating signals with specific second order (auto-correlation) properties arises in many fields. In system identification, it is well known that the quality of the identified model depends on the auto-correlation function of the applied input signal. Therefore, an essential part of an identification experiment is the choice and the realization of the excitation signal.

Input design is available in a variety of flavors but the common central idea is that the statistical properties can be influenced by the choice of input signal through the covariance. Initially, input design was formulated as optimization of some measure of the covariance matrix directly. This is still often the case in practice. The focus has since shifted to consider model quality in terms of the intended use of the model, *c.f., identification for control* (Gevers and Ljung, 1986; Gevers, 1991; Hjalmarsson et al., 1996), *least costly identification* (Bombois et al., 2006) and *applications oriented input design* (Hjalmarsson, 2009). In these later methods, the input is designed in terms of the power spectrum. The actual time signal then has to be realized using a suitable approach. Including time domain constraints on signals in the design is therefore difficult.

In practical applications, it is often vital to limit input and output amplitudes as well as rate of change of variables. This is due to actuator saturations, strain of equipment and the desire to keep the system in a operating region. Related to this, *plant-friendly identification* techniques (Rivera et al., 2003) have been developed where signals variances are kept low and actuator strain kept at a minimum.

Many techniques to handle time domain constraints on the signals in input design have been proposed. One idea is to limit the design to the class of binary signals where it is very easy to control the input amplitudes. (Hannan, 1970; Rojas et al., 2007; He et al., 2012) Another possibility is to consider sums of sinusoids (Schoukens et al., 1991; Pintelon and Schoukens, 2001) where the amplitude is controlled using the phase of the sinusoidal components. Further, one can choose to exclude the time domain constraints in the input design and enforce them when the time domain signal is generated. For example, white noise can be filtered to have the correct correlation properties and then "clipped" to the right amplitude. This approach is simple but changes the spectrum of the signal which can lead to suboptimal results, see Hannan (1970).

This contribution presents a signal generation idea applicable to the latter case. We consider time domain realization of an amplitude constrained signal such that the signal has a prescribed auto-correlation. The algorithm tries to match the auto-correlation while maintaining input and output constraints. The idea was introduced for single-input single-output (SISO) systems in Larsson et al. (2013). In Hägg et al. (2013) extensions of the algorithm to the case of uncertain system knowledge were discussed and tested. Here we extend the algorithm to work for multiple-input multiple-output (MIMO) systems and more general constraints.

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The structure of the paper is as follows. In Section 2 the receding horizon signal generation problem is set up and extended to a more general setting. Section 3 and Section 4 presents two numerical approaches to solve the receding horizon optimization problem and the differences between the two methods are discussed in Section 5. The viablity of the proposed method is evaluated through simulations in Section 6 while Section 7 and Section 8 concludes the paper.

2. RECEDING HORIZON SIGNAL GENERATION

In this section the method in Larsson et al. (2013) is extended to the MIMO (multiple-input multiple-output) case. The objective is to generate N samples of the p signals

$$u_t = [u_t(1), \ldots, u_t(p)]^T, \quad t = 1, \ldots N$$

with a prescribed auto-correlation

$$R^d(\tau) = E\left\{u_t u_{t-\tau}^T\right\}, \quad \text{for } \tau = 0, \ldots, n.$$

Hence we try to match the first $n+1$ lags of the desired auto-correlation function. Furthermore, when the input signals are applied to the p input and m output MIMO system,

$$\begin{aligned} x_{t+1} &= Fx_t + Gu_t \\ y_t &= Hx_t \end{aligned} \tag{1}$$

the input and the output constraints

$$\begin{aligned} &u_t \in \mathcal{U}, \\ &y_t \in \mathcal{Y}, \quad t = 1, \ldots, N \end{aligned} \tag{2}$$

should be satisfied. Here we will only consider amplitude constraint *i.e.,* $|u_t| \leq u_{max}$ and $|y_t| \leq y_{max}$ but both state constraints and other convex constraints are possible. Note that the output constraints can be rewritten as time varying constraints on u_t since y_t is linear in u_t.

Defining the (biased) sample auto-correlation of the signals $u(t)$ for $t = 1, \ldots, N$ as

$$R_N(\tau) = \frac{1}{N} \sum_{i=\tau+1}^{N} u_i u_{i-\tau}^T. \tag{3}$$

it is natural to formulate the constrained signal generation problem as the following optimization problem

$$\begin{aligned} \underset{\{u_t\}_{t=1}^N}{\text{minimize}} \quad & \sum_{\tau=0}^{n} \|R_N(\tau) - R^d(\tau)\|_F^2, \\ \text{subject to} \quad & (1) \text{ and } (2) \end{aligned}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. However, as noted by Larsson et al. (2013), even for the SISO case, this optimization problem is non-convex. To simplify the problem by reducing the number of optimization varaiables a receding horizon solution to the problem was proposed in Larsson et al. (2013). A natural extension of this to the MIMO case is solve the following optimization problem at sample t

$$\begin{aligned} \underset{\{u_k\}_{k=t}^{t+N_u-1}}{\text{minimize}} \quad & \sum_{\tau=0}^{n} \|R_{t+N_u-1}(\tau) - R^d(\tau)\|_F^2, \\ \text{subject to} \quad & \hat{x}_{k+1} = F\hat{x}_k + Gu_k, \\ & \hat{y}_k = H\hat{x}_k, \\ & \hat{x}_1 = x_t, \\ & \begin{matrix} u_k \in \mathcal{U} \\ y_k \in \mathcal{Y}, \end{matrix} \quad k = 1, \ldots, N_y \end{aligned} \tag{4}$$

where N_y is the output horizon, *i.e.,* how far in the future we consider that the output should satisfy the constraints and N_u is the input horizon. The optimization is performed over the whole input horizon but only the first sample, $u_1^\star$, is implemented, *i.e.,* $u_t = u_1^\star$, and the optimization is performed iteratively in receding horizon fashion. If the output horizon is longer than the input horizon, *i.e.,* $N_y > N_u$, we set $u_{k+N_u} = \cdots = u_{k+N_y} = 0$, that is the input is considered zero over the remaining part of the output horizon. This approach can be seen as an MPC-controller where we try to follow a correlation reference while satisfying input and output constraints of the considered system.

Since the sample auto-correlation $R_{t+N_u-1}(\tau)$ is quadratic in u the optimization problem (4) corresponds to minimizing a constrained multivariate polynomial of order 4. In Larsson et al. (2013) it was shown that for the SISO case with input horizon $N_u = 1$ the optimization problem (4) can be solved analytically. However in the general case there is no analytic solution and it is still non convex. In fact, in the extreme case of $N_u = N$ it is as hard to solve as the original problem. However, for a short input horizon N_u the problem is small and can be solved numerically. In this paper we will consider two different numerical approaches to solve the optimization problem (4).

3. SDP RELAXATION

Solving the problem in (4) means finding the global minimizer of a real-valued polynomial of degree four over an admissible set. This problem is in general non convex and difficult to solve. However, recent developments (Lasserre, 2000) on optimization of polynomials over a set defined by polynomial inequalities offer a route to a numerical solution to this problem. The theory is based on the theory of moments and representation of polynomials that are strictly positive on a compact, semi algebraic set. It is shown that a family of convex LMI relaxations has an associated increasing sequence of lower bounds converging to the global optimal value. Here, we briefly present the idea; for a complete treatment of the theory and related references we refer to Lasserre (2000).

Consider the polynomial optimization problem

$$g^\star = \left\{ \min_{x \in \mathbf{R}^n} g_0(x) : g_i(x) \geq 0, i = 1, \ldots, m \right\},$$

where g_i are multivariate polynomials. First note that $g_0(x) - g^\star$ is a positive polynomial on the constraint set and that the problem of finding a sum of squares polynomial

$$p(x) = g_0(x) - g^\star = q_0(x) + \sum_{i=1}^{m} g_i(x) q(x)$$

can be represented as an LMI if the degrees of the polynomials $q_i(x)$ are fixed. The primal formulation is constructed as a minimization over moments with support on the constraint set. Then a condition by Putinar (1993) on the moment and localizing matrices of the polynomials can be used to construct a family of LMI relaxations with the increasing sequence of lower bounds $p_k^\star$. The dual LMI corresponds to the condition that the polynomial $p(x)$ is a sum of squares polynomial, denote the optimum $d_k^\star$. It is proven in Lasserre (2000) that if the constraint set is compact, under mild conditions, $p_k^\star = d_k^\star \leq g^\star$ and that

$$\lim_{k\to\infty} p_k^\star = g^\star.$$

The price to be paid is that the size of the LMI relaxations quickly grows very large. In fact, for a fixed number of polynomial variables n, the number of primal and dual variables grow polynomially in $\mathcal{O}(\delta^n)$ and $\mathcal{O}(m\delta^n)$, respectively. Here δ is half the polynomial degree. However, it has been noted that in practice the convergence is often very fast and therefore the theory has proven to be usefull also for numerical implementation.

4. CYCLIC ALGORITHM

Cyclic or alternating algorithms have been successfully applied to many related signal generation problems. See for example He et al. (2012) for applications to radar beampattern generation or Jansson and Medvedev (2013) for stimulus design for eye-tracking identification. Common for these applications is that a complete sequence of input signals is generated offline and only considers input constraints. Here we will use the cyclic algorithm in a new setting, to solve the receding horizon signal generation with both input and output constraints.

To be able to fit our problem into the cyclic algorithm algorithm suggested in Stoica et al. (2008) we will not directly solve the optimization problem (4) but instead solve a related problem. Consider the cost function

$$\min_{\{u_k\}_{k=t}^{t+N_u}} \quad \|\tilde{R}_{t+N_u-1} - \tilde{R}^d\|_F^2. \tag{5}$$

where $\tilde{R}_{t+N_u-1}$ is a block Toeplitz matrix with first (block) row equal to

$$[R_{t+N_u-1}(0) \ R_{t+N_u-1}(1) \ \cdots \ R_{t+N_u-1}(n)]$$

and $\tilde{R}^d$ is defined analogous.

It is easy to see that this cost functions satisfies

$$\sum_{\tau=0}^{n} \|R_{t+N_u-1}(\tau) - R^d(\tau)\|_F^2 \leq \|\tilde{R}_{t+N_u-1} - \tilde{R}^d\|_F^2.$$

Hence if we with a cyclic algorithm can make the cost of the right hand side small then the cost function of the original problem will also be small. This can be seen as a reweighting of the original optimization problem such that it is more important to match the correlation matrices corresponding to small lags (small τ) than to match them for large lags.

Using the definition of the correlation matrices in (3), the Toeplitz matrix containing the correlation matrices can be written on recursive form as

$$\tilde{R}_{t+N_u-1} = \frac{1}{t+N_u-1}\Big((t-1)\tilde{R}_{t-1} + \Phi_u^T\Phi_u - Q\Big)$$

where

$$\Phi_U = \begin{bmatrix} u_{t+N_u-1}^T & & 0 \\ \vdots & \ddots & \\ u_{t-n}^T & & u_{t+N_u-1}^T \\ & \ddots & \vdots \\ 0 & & u_{t-n}^T \end{bmatrix}$$

and Q is a block Toeplitz matrix with first (block) row

$$\left[\sum_{p=-n}^{-1} u_{t+p}u_{t+p}^T, \ldots, \sum_{p=-1}^{-1} u_{t+p}u_{t+p-n+1}^T, 0\right].$$

The optimization problem (5) can now be written on recursive form as

$$\min_{\{u_k\}_{k=t}^{t+N_u}} \left\| \frac{1}{t+N_u-1}\Big((t-1)\tilde{R}_{t-1} + \Phi_u^T\Phi_u - Q\Big) - \tilde{R}^d \right\|_F^2. \tag{6}$$

Finally, rescaling this problem the optimization to be solved at each time is on the form

$$\begin{aligned} \underset{\{u_k\}_{k=t}^{t+N_u}}{\text{minimize}} \quad & \|\Phi_U^T\Phi_U - Z_t\|_F^2, \\ \text{subject to} \quad & u_k \in \mathcal{U}_{t+k} \\ & k = 0, \ldots, N_u - 1 \end{aligned} \tag{7}$$

where we have rewritten the output constraints as time variable input constraints to shorten the notation. Note that $Z_t = (t-1)\tilde{R}_{t-1} - Q - (t+N_u-1)\tilde{R}^d$ only depends on the previous inputs up to time t, *i.e.*, $\{u_k\}_{k=1}^{t-1}$.

In Stoica et al. (2008) a cyclic algorithm for solving optimization problems on the form (7) was proposed. The idea is the following. If Z_t is a positive semidefinite matrix then the class of signals $\{u_k\}_{k=t}^{t+N_u}$ that satisfies $\Phi_U^T\Phi_U = Z_t$ is given by

$$\Phi_U^T = Z_t^{1/2}U^T,$$

where U is an arbitrary unitary matrix ($U^TU = I$) and $Z_t^{1/2}$ is a Hermitian square root of Z_t. The optimization problem (7) can then be reformulated as

$$\begin{aligned} \underset{\{u_k\}_{k=t}^{t+N_u}, U}{\text{minimize}} \quad & \|\Phi_U - UZ_t^{1/2}\|_F^2, \\ \text{subject to} \quad & u_k \in \mathcal{U}_k, \\ & k = 0, \ldots, N_u - 1. \end{aligned} \tag{8}$$

This problem is still non-convex. However, as we will see, for a fixed U it is straightforward to find the optimal $\{u_k\}_{k=t}^{t+N_u}$ and vice versa for a fixed $\{u_k\}_{k=t}^{t+N_u}$. The main idea is thus to alternate between solving for $\{u_k\}_{k=t}^{t+N_u}$ and U while keeping the other variable fixed. This is repeated until convergence. One can show that if the cost function for this optimization problem is small then the cost function for the original problem will also be small. For more details and properties of the cyclic algorithm we refer to Tropp et al. (2005).

The steps in the cyclic algorithm are

Step 0: Initialize U to an arbitrary matrix.

Step 1: Find the vector $\{u_k\}_{k=t}^{t+N_u}$ that minimizes (8) for U fixed. Since Φ_U is an affine function of $\{u_k\}_{k=t}^{t+N_u}$ the problem becomes a quadratic optimization problem with convex constraints. This problem can be solved efficiently and accurately using numerical optimization (Boyd and Vandenberghe, 2004).

Step 2: For $\{u_k\}_{k=t}^{t+N_u}$ fixed, find a unitary U ($U^TU = I$) that minimizes (8) disregarding the constraints. Defining the singular value decomposition (SVD) of $Z_t^{1/2}\Phi_u^T = \bar{U}\Sigma\tilde{U}^T$, the optimal solution to this problem is given by $U_{opt} = \tilde{U}\bar{U}^T$, see Stoica et al. (2008) for details.

Step 3: Check if the solution satisfy the stop criterion, if not, goto Step 1.

The algorithm alternates between solving the two simpler optimization problems in step 1 and 2 until convergence. Since this only involves solving a quadratic optimization problem and an SVD it is possible to solve relatively large problems on a standard computer.

4.1 Non positive semidefinite Z_t

In the algorithm outlined above the matrix Z_t is required to be positive semidefinite. In our case we cannot guarantee that this is the case. Therefore, we first project Z_t on to the space of positive semidefinite matrices, *i.e.,* we find

$$Z_t^+ = \underset{X \in S_n^+}{\operatorname{argmin}} \|X - Z_t\|.$$

where S_n^+ is the set of positive semidefinite $n \times n$ matrices. This projection is easily done using spectral decomposition (Henrion and Malick, 2012).

Consider

$$Z_t = V \operatorname{Diag}(\lambda_1, \ldots, \lambda_n) V^T$$

where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of Z_t and V is the corresponding matrix with eigenvectors. The projection can then be written

$$Z_t^+ = V \operatorname{Diag}(\max(0, \lambda_1), \ldots, \max(0, \lambda_n)) V^T,$$

and Z_t^+ replaces Z_t in the relevant expressions in the algorithm.

4.2 Termination

We choose to stop the cyclic algorithm when the relative error of the cost function between two consecutive iterations are smaller than a threshold ϵ.

5. COMPARISON

The two methods outlined in this paper have different advantages and disadvantages. For the SDP-relaxation method it is possible to certify numerically and find the global optimum to the constrained polynomial optimization problem in many cases. However, the number of variables in the relaxed problem grows exponentially with the length of the input horizon N_u and the number of inputs, p. Due to memory limitations it is therefore only feasible to solve problems where $N_u p$ is in the order of 10 on a standard desktop computer.

With the cyclic algorithm on the other hand, larger problems can be solved. In each iteration one singular value decomposition of a real matrix of size $(n+1)p \times N_u + 2n$ needs to be calculated and a constrained quadratic optimization problem with $N_u p$ variables needs to be solved. Thus the number of variables $N_u p$ can be in the order of 1000 for this method. However, this method does not solve the original problem on but instead solves a related problem and not much can be guaranteed in terms of the optimality of the solution to the original problem. But if the cost function of the solved problem is small then the original cost function will also be small. Nevertheless, this method have been shown to give good performance in many applications, see for example He et al. (2012) and the references therin.

6. EXAMPLES

In this section we present three simulation examples. The first example illustrates the effect of the length of the input horizon on the quality of the generated input signal when no output constraints are considered. In the second example we look at an example with output constraints where the choice of input horizon is important. The third and final example we apply the algorithm to generate an input signal to the quadruple tank process, a MIMO system with input and output constraints.

Throughout the examples we set the relative error stopping criteria for the cyclic method to $\epsilon = 10^{-5}$. The SDP-relaxation solution to the multivariate polynomial of degree four is calculated using *GlotiPoly* (Henrion et al., 2009), a polynomial global optimization tool for Matlab that uses the results presented in Section 3. In the cyclic algorithm the constrained quadratic optimization problem is solved with the Matlab command *quadprog*.

6.1 Pseudo random white noise

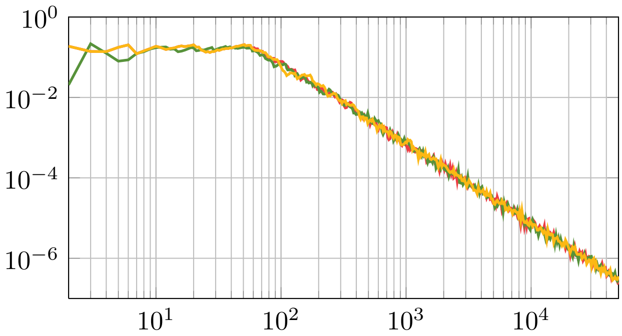
In this first example we will look at the effect of longer input horizons on the signal generation performance of the two proposed methods. The results will also be compared to the optimal solution with input horizon $N_u = 1$ presented in Larsson et al. (2013) where it is possible to find the global optimum analytically. We will try to generate a white noise sequence with unit variance, and we will try to match $n = 50$ correlation lags. The input constraints are $|u_t| \leq 1.5$. Here we are only interested in the signal generation and thus we do not consider any output constraints. Three different input horizons will be considered, $N_u = 1, 2, 5$ for both methods.

We will generate $N = 50000$ samples and look at the convergence rate of the two different solution methods for the different settings.

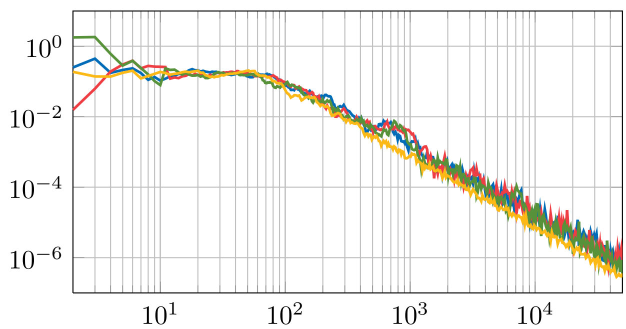
In Figure 1 the cost $\sum_{\tau=0}^{n} \|R_N(\tau) - R^d(\tau)\|_F^2$ as a function of N is plotted. The SDP-relaxation method is able to find the global optimum for all cases and for all time instances except at the first sample. Hence it is expected that the analytically optimal solution for $N_u = 1$ should be close to the one we get with the SDP-relaxation. Indeed, looking at Figure 1, this is the case. One could also note that the performance for all input horizons are very close to each other. For the cyclic method we loose a factor of about two in performance compared to the analytically optimal solution for the case $N_u = 1$ due to the suboptimality of the solutions. However, the convergence rate seems to be the same. Again, with this method, the length of the input horizon does not seem to have any significant effect on the performance.

6.2 AR(1) process

In this example the we will generate a signal corresponding to an AR(1)-process. The generated signal is to be applied to the SISO system



(a) SDP-relaxation method.



(b) Cyclic algorithm.

Fig. 1. Convergence rate for the proposed method for different input horizons, $N_u = 1$ (——), $N_u = 2$ (——) and $N_u = 5$ (——). The analytic solution with $N_u = 1$ is shown as (——).

$$G(z) = \frac{z + 1.25}{z^2 + 0.4z + 0.6}$$

with a non minimum phase zero at $z_{nmp} = -1.25$. Here the goal is to identify the location of this zero, and the input signal that minimizes the variance of this estimate has the following auto-correlation function, see Mårtensson et al. (2005),

$$r^d(\tau) = \frac{a^{|\tau|}}{1 - a^2}$$

where $a = z_{nmp}^{-1} = -0.8$. The generated signal should also satisfy input and output constraints $|u(t)| \leq 1.5$ and $|y(t)| \leq 1.2$ and the output horizon will be $N_y = 5$ in this case. For each of the two methods we will study two different cases; when $N_u = 1$ and when $N_u = N_y = 5$. We set $N = 1000$ and the number of auto-correlation lags to match to $n = 50$.

In the two considered cases the output constraints are never violated. However, looking at the cost function as a function of the number of generated samples N in Figure 2, firstly it is seen that by considering longer input horizon $N_u = 5$ instead of $N_u = 1$ much is gained in terms of performance. This is due to the added flexibility when considering the longer input horizon. Instead of having $u_{t+1} = \cdots = u_{t+4} = 0$ we can now use these variables to control the future predicted output of the system, making it possible to generate a sequence with properties closer to the desired ones. Hence, in certain applications it is important to be able to consider longer input horizons.

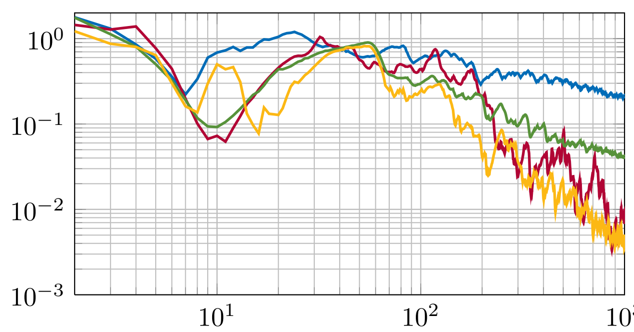


Fig. 2. Convergence for generated input signals for the SDP-relaxation method, $N_u = 1$ (——), $N_u = 5$ (——) and for the cyclic method, $N_u = 1$ (——), $N_u = 5$ (——).

Table 1.

Parameter	Value	Description
a_1	$0.148\,\mathrm{cm}^2$	outlet area tank 1
a_2	$0.167\,\mathrm{cm}^2$	outlet area tank 2
a_3	$0.0673\,\mathrm{cm}^2$	outlet area tank 3
a_4	$0.0619\,\mathrm{cm}^2$	outlet area tank 4
γ_1	0.620	fraction into lower tank
γ_2	0.621	fraction into lower tank
k_1	$2.51\,\mathrm{cm}^3/\mathrm{sV}$	flow constant pump 1
k_2	$2.25\,\mathrm{cm}^3/\mathrm{sV}$	flow constant pump 2
A	$15.15\,\mathrm{cm}^2$	cross-section area of tanks
g	$10^3\,\mathrm{cm}/\mathrm{s}^2$	gravitational constant

6.3 The quadruple tank system

In this example we will consider a quadruple tank system. Its main components are two lower tanks, two upper tanks and two electrical pumps. Each pump delivers water to one of the upper tank and one of the lower tank according. Water then flows from the upper tanks into the lower tanks and from the lower tanks into the water basin through small orifices located in the bottom of each tank. See Johansson et al. (1999) for details about the considered process.

The two input signals to the system are the applied voltages to the pumps and the outputs are the water level in each of the four tanks.

From a nonlinear model of the plant from Johansson et al. (1999), a linearized continuous time model around a working point x^0, u^0 of the process is given by

$$\dot{x}_t = \begin{bmatrix} \tau_1 & 0 & -\tau_3 & 0 \\ 0 & \tau_2 & 0 & -\tau_4 \\ 0 & 0 & \tau_3 & 0 \\ 0 & 0 & 0 & \tau_4 \end{bmatrix} x_t + \frac{1}{A} \begin{bmatrix} k_1\gamma_1 & 0 \\ 0 & k_2\gamma_2 \\ 0 & k_2(1-\gamma_2) \\ k_1(1-\gamma_1) & 0 \end{bmatrix} u_t$$

$$y_t = I x_t$$

where $\tau_i = -\frac{a_i}{A}\sqrt{\frac{g}{2x_i^0}}$. The parameter values are taken from Larsson (2011) and are summarized in Table 1. This model is discretized using zero order hold at a sampling rate of 1 Hz, see Larsson (2011) for details.

We want to excite the system with an input signal with a FIR-spectrum with $n = 50$ lags shown in Figure 3. To keep the water levels close to the equilibrium point we constrain the four outputs to satisfy $|y_t| \leq 0.65$ cm. Furthermore

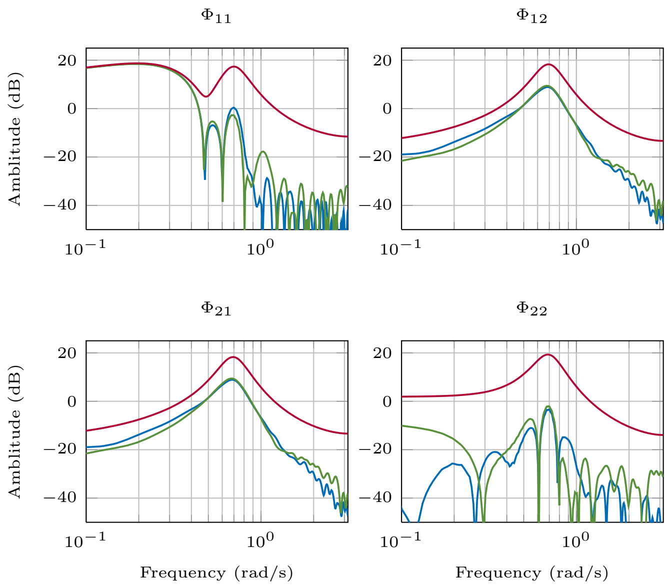


Fig. 3. Error between the desired spectra and spectra of generated for SDP-method (——) and cyclic method (——). Desired input spectra is shown as (——). Φ_{ij} is the cross spectrum between $u(i)$ and $u(j)$.

we require that the input signal amplitude should be less than 2 V. $N = 1000$ samples are generated using the SDP relaxation method and the cyclic method.

The cost function,

$$\sum_{\tau=0}^{n} \|R_N(\tau) - R^d(\tau)\|_F^2,$$

for the generated sequences are 0.16 and 0.25, respectively compared to the norm of the desired auto-correlation sequence

$$\sum_{\tau=0}^{n} \|R^d(\tau)\|_F^2 \approx 3.7.$$

Again the SDP-relaxation method performs slightly better. For comparison, with a pseudo random white Gaussian noise sequence filtered through a spectral factor of the FIR-spectrum the cost 0.69 is obtained. Hence the properties of the signals generated with the two proposed methods are better than just filtered white noise. Moreover the output constraints are never violated for the two proposed methods while it is violated in 186 instances for the filtered white noise.

In Figure 3 the error between the estimated spectrum of the generated signals and the desired spectrum is also shown. The spectrums for the generated input signals are calculated using a Hann window of width 50 (Stoica and Moses, 2005). We see that both methods can successfully generate an input signal with the desired properties.

7. ROBUSTNESS ISSUES

The proposed method in this paper requires perfect knowledge of the true underlying system to be able to predict the effect of the input on the output. This is not possible in real applications. For example the model of the system often have some uncertainties due to undermodeling and noise.

To overcome this a robust and adaptive implementation of the method for the SISO case with input horizon one was proposed by Hägg et al. (2013). The first step is to make the output predictions robust to model uncertainties by using tools from robust MPC (Maciejowski, 2002). The input signal should then satisfy the output constraints, not just for the initial model, but for all possible models in some uncertainty region. But if the uncertainties are large, the generated input could be quite conservative to satisfy the robust output constraints. Furthermore the propagation of the uncertainty as time goes will eventually force the input to be zero.

The sample by sample calculation of the input signal in the proposed algorithm does however allow for realtime implementation. One sample of the generated input is applied to the system and the resulting output is measured. A state observer, such as a Kalman filter, could then be used on the data to estimate the current state of the system. With this new information, a new input sample is generated and so on. This would reduce the propagation of uncertainty. In Hägg et al. (2013) the model was also reidentifed recursively to reduce the uncertainties in the model.

The results from Hägg et al. (2013) should be directly applicable to the method proposed in this paper, making the method practically viable.

8. CONCLUSIONS

In this paper we extended the signal generation method proposed in Larsson et al. (2013) to the MIMO case and to general input horizon length. This is formulated as an receding horizon algorithm where the objective is to generate a input signals over a finite horizon such that the signal has a specified sample auto-covariance while satisfying input and output constraints of the system the signal is to be applied to. We showed that the corresponding optimization problem is a constrained minimization of a polynomial of degree four, which in general is non convex. To this end, two methods from the literature for solving this polynomial optimization problem were investigated; a convex relaxation method and a cyclic algorithm.

With the convex relaxation method it is in many cases possible to find the global optimum. However due to the exponential growth of the complexity the method is only feasible for small problems. The method based on the cyclic algorithm can on the other hand solve larger problems but not much can be said about the convergence of the method. Still, it has shown good performance in many applications.

Three simulations examples were performed to verify the feasibility of the method. In the first example the convergence rate were investigated when the length of the input horizon is varied. It was noted that quality of the generated signal is relatively unaffected by the length of the input horizon. This was also noted for the binary input case in Rojas et al. (2007). However, as noted in the second simulation example, the longer input horizon could improve the performance if output constraints are considered as more flexibility is added to the control of the system. In the third and final example the MIMO capability of the methods were successfully shown.

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