Master of Science Thesis

Astrophysical Constraints on Secret Neutrino Interactions

Björn Ahlgren

Theoretical Particle Physics, Department of Theoretical Physics,
School of Engineering Sciences
KTH Royal Institute of Technology, SE-106 91 Stockholm, Sweden

Stockholm, Sweden 2013

Typeset in LaTeX

Examensarbete för avläggande av masterexamen inom ämnesområdet teoretisk fysik, inom utbildningsprogrammet Teknisk fysik.

Graduation thesis for the degree Master of Science in Engineering Physics.

TRITA-FYS 2013:59
ISSN 0280-316X
ISRN KTH/FYS/--13:59--SE

Printed in Sweden by Universitetsservice US AB, Stockholm October 2013

Abstract

This thesis aims to use astrophysics to derive constraints on a certain type of models of Beyond Standard Model (BSM) physics. Specifically a model proposed to account for small scale problems of Λ-CDM cosmology, by the introduction of a new massive vector boson, is examined. There is a surprising lack of analysis of models with secret neutrino interactions with ~ 1 MeV vector bosons. We calculate the Extra Degrees of Freedom for neutrinos induced by the vector boson in the framework of Big Bang Nucleosynthesis (BBN) and consider the existing experimental constraints, in order to derive constraints on the parameter space of the model. This is performed by solving the non-integrated Boltzmann equation for the interactions of the boson in the early universe around the time of BBN. Another constraint is also developed, based on terrestrial experiments. The introduction of a new particle may alter the total decay width of some particles, and this leads to a straight-forward constraint on the parameter space of the model. Of these two constraints, the BBN constraint is the harder to evade by tweaking this type of model, since it is invariant under sterile neutrinos and inert longitudinal polarization of the boson.

Key words: Neutrino interactions, Massive vector boson, Big Bang nucleosynthesis, Boltzmann equation, Decay width.

Preface

This thesis sums up my work from January to October 2013, in the Particle Physics group at the Department of Theoretical Physics at KTH Royal Institute of Technology, for my degree Master of Science in Engineering Physics. The work concerns astrophysical and other possible constraints on a specific type of BSM physics.

Overview

The thesis is divided into four chapters. Chapter 1 contains a general introduction and a more specific introduction to the basic physics of the Standard Model of Particle physics. It ends with a small peek at what might lie beyond the Standard Model. Chapter 2 is about Big Bang Nucleosynthesis and the more specific framework of the early Universe that has been used in this thesis. This includes the key concepts of the thesis, such as *Extra Degrees of Freedom*. In Chapter 3 the concrete motivation for this work is shown along with the aim of the work. Firstly an approximative constraint is worked out, to get a feeling for the situation. Then the bulk of the work, in the form of the Boltzmann equation and its numerical solutions, is presented in a rather slim fashion. At the end of the third chapter we present another type of constraint namely that from particle decays, and its results. Chapter 4, Summary and Conclusions, shortly sums up the ideas and results of the thesis. Lastly there is an appendix containing a comment to Ref. [1] which the work on the thesis led to.

Notation and Conventions

Throughout the thesis natural units are employed *i.e.* $\hbar = c = k_b = 1$. Furthermore, if nothing else is stated, the metric used is the Minkowski metric and the Einstein summation convention is used; meaning that repeated indices are summed over in the framework of the metric.

Acknowledgements

Firstly I would like to thank my supervisor Prof. Tommy Ohlsson for the opportunity to do my diploma thesis in the Theoretical Particle Physics group. Many thanks to my supervisor Dr. Shun Zhou for guiding and inspiring me with great patience through my work. I would also like to take the opportunity to thank the other two people with whom I shared office and had many interesting discussions with; Stella Riad and Dr. Johannes Bergström. Additionally, thanks go to Dr. Mattias Blennow for helpful discussions. Lastly I would like to thank my friends, especially Martin, Emil and Axel, and my girlfriend, Desirée, for their help and support with everything.

Contents

Abstract . . . iii

Preface **v**

Contents **vii**

1 Introduction **1**
1.1 The Standard Model of Particle Physics . . . 3
1.1.1 Particle Content . . . 3
1.1.2 Particle Interaction . . . 6
1.2 Beyond the Standard Model . . . 7
1.2.1 Neutrino Oscillations . . . 7
1.2.2 Dark Matter . . . 8

2 Big Bang Nucleosynthesis **11**
2.1 Standard Cosmology . . . 11
2.1.1 The Epochs of the Early Universe . . . 13
2.2 Relativistic Degrees of Freedom . . . 14
2.3 Big Bang Nucleosynthesis . . . 15
2.4 Effective Degrees of Freedom . . . 17
2.5 Approximative Constraints From V Boson Decay . . . 17

3 Secret Neutrino Interactions **21**
3.1 A New Vector Boson . . . 21
3.1.1 The Decay Rate of V . . . 22
3.2 The Boltzmann Equation . . . 25
3.2.1 The Integrated Boltzmann Equation . . . 26
3.2.2 The Non-Integrated Boltzmann Equation . . . 28
3.2.3 Numerical Treatment . . . 29
3.3 Other Constraints . . . 30
3.3.1 Particle Decays . . . 30

4 Summary and Conclusions **41**

A Comment on Ref. [1] **43**

Bibliography **45**

Chapter 1

Introduction

Questioning led to science through a long and ever refined process. As it progressed, science branched in many directions, each striving for answers to its own questions. Each branch of science has led to new knowledge and to yet new parts of science, not imagined by previous eras. The branch of science known as physics originates from philosophy and was referred to as *natural philosophy* until the late 1800. This particular science was born from the observation of natural phenomena and the will to describe and explain these by other means than resorting to the supernatural. Several fields of physics were slowly united by the development of geometry as a mathematical tool to describe them. These were fields such as astronomy or mechanics; things easily available to the thinkers of that time.

At first physics was rather straight-forward in both its concept and the way to carry through with the science of it. After the introduction of the scientific way of confirming theories by experiments, the general scheme was, simplified, something like the following. You looked at Nature where you observed some graspable phenomenon (like a falling apple, as in the famous anecdote with Newton). You recreated said phenomenon in a more controlled environment to perform a series of measurements of physical quantities, such as time or weight. These quantities were then used to construct a physical model of how the phenomenon works. Of course it was not always so easy and often it was not just to add numbers to find a simple relation, say between the movement of different stellar objects and satellites in the sky. In order to find said relation there was a need for increasingly complicated mathematics. Where the early physicists needed new geometry to describe what they saw in the night sky, Newton needed to invent infinitesimal calculus to be able to describe the motion of bodies and to introduce his famous laws [2]. Thus, mathematics and physics has evolved and pushed each other to new heights throughout history and are still very much entwined.

Backtracking to the early physics, the approach with which one performed the experiments and compared it with theory, and vice versa, was rather simple. The set up was: observation, experiment and theory, but not always in that order.

However, as we could make more and more detailed observations, the world seemed to become increasingly complicated, forcing the experiments to be likewise. Even if the idea remained the same, the experiments became harder to perform and the results harder to relate to as the theories became more abstract. When we approach the century shift into 1900, the experiments revealed a structure of matter consisting of atoms of a nearly incomprehensibly small size. The word atom refers to something indivisible, which was somewhat of a premature name, as Joseph J. Thomson in 1897 discovered the electron [3]. As it turned out, the electron was indeed a subatomic particle, possible to separate from the atom. The notion of *fundamental particles*, i.e. a particle with no (known) substructure, dates back to ancient Greece but was getting an increasingly important role in physics during the early 1900s. This field of physics came to be a central part of modern physics, known as *particle physics*.

We also had the development of quantum mechanics and the theory of relativity that revolutionized the physics of the time. These theories were revolutionizing in many ways. Firstly they were highly unexpected when they were discovered, especially quantum mechanics can be profoundly counter-intuitive, and they also show that the reality we know from our every-day lives is not valid in some extreme limits, as one would have naïvely assumed. Secondly they are examples of theories sprung from the mind and from mathematics, loosely based on previous theories, but not derived from them. There was no theory, nor any experiment, from which Einstein derived his theory of relativity. There was only an idea. We had entered a field of physics where theories were formed not from experiments but from other theories, ideas and mathematics alone.

Today we make a distinction between experimental and theoretical physics as these areas are separated by quite different work schemes. An experimental physicist tries to design an experimental set up with which he or she may measure some desired quantity, and later on treat the normally large amount of data acquired. Experimental physics challenges the frontier of technology by demanding more and more complex experimental set-ups. Today one of the biggest experiments is CERN, the international particle accelerator in Switzerland, delving deeper and deeper into the fabric of the Universe.

The theorist may rather try to interpret or explain some experimental results in the frame of some theory or develop a new theory, either to account for some experimental result or to come closer to finding the ultimate theory of everything. i.e. a theory that treats all physical processes in the Universe we know of, or at least that covers everything in a certain field. Regardless of whether or not such a theory exists, theoretical physicists are not seldom driven by the will to find a better, more beautiful and more all-inclusive theory. Our best approximation of such a theory lies in the field of particle physics, and has for a long time been the Standard Model (SM) of particle physics.

It is the theory of all the fundamental particles in the Universe and their corresponding interactions. The theory has a long history that dates back to the discovery of fundamental particles, with the electron being the first. After Joseph

Thompson had discovered the electron in 1897 the structure of the atom was revealed by the famous "Gold Foil Experiment", devised by Ernest Rutherford [4, 5]. Rutherford also discovered and named the proton and predicted the neutron, which was subsequently found by James Chadwick 1932 [6, 7]. For a while these three, the electron, proton and the neutron, were considered to be the only subatomic particles. But as more particles were found, aided by new technology in terms of particle accelerators, as well as some inexplicable phenomena, more theories were proposed. In 1930 Wolfgang Pauli proposed the neutrino as a way to fix the seemingly violation of the energy-momentum conservation in α and β decays. In 1964 Murray Gell-Mann and George Zweig proposed the quark model as a solution to the many particles found in the accelerators [8, 9]. In this model hadrons were consisting of yet smaller particles, called quarks, and their antiparticles. The different quarks were introduced as to deal with certain issues, such as the top and bottom quarks that could explain CP-violation, that had been observed. Recently the last remaining particle predicted by the standard model was discovered, when, on 4 July 2012, it was announced that a particle that appeared to be the Higgs boson had been detected at CERN [10, 11]. That it was indeed the sought after Higgs boson was later confirmed on 14 March 2013.

1.1 The Standard Model of Particle Physics

The SM classifies the fundamental particles in different categories depending on what attributes we are considering and deals with three of the four known fundamental forces in the Universe. There are the electromagnetic force, the weak force and the strong force. The SM does not include gravity, which is the weakest force by far in comparison. The forces of the SM all work on relatively short range, whereas gravity is a long-range force and thus not very interesting on small scales where the strong force for instance, is 10^{38} times stronger.

Mathematically the SM describes the fundamental particles and their forces as a non-Abelian gauge theory of the gauge group $SU(3)_{\mathrm{c}} \otimes SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{Y}}$. The subscripts denote the group's associated quantum number, except for the "L", which signifies that only left-handed particles carry the associated quantum number. At high energy scales ($\gtrsim 246$ GeV), before the spontaneous breaking of the electroweak symmetry, the electromagnetic and the weak force constitute the electroweak interactions of the gauge group $SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{Y}}$. The strong force is described by the theory of Quantum Chromo Dynamics with the gauge group $SU(3)_{\mathrm{c}}$.

1.1.1 Particle Content

The SM chooses to separate the particles in the SM primarily by their spins. We thus have spin-1/2 particles known as *fermions*, which build the normal matter; everything that we see and feel in everyday life, and basically almost everything else too. There are also particles of spin-1, so-called *gauge bosons*. They mediate the interactions and act as force carriers. Lastly there is also the newly confirmed *Higgs boson*, which appears to be a spin-0 particle.

Fermions

Fermions are divided into leptons and quarks, based on which gauge bosons they interact with, *i.e.* by which forces they interact. Those particles not interacting with the strong force are called leptons, while those that do interact with the strong force are called quarks. Fermions are also divided into three so-called generations for both leptons and quarks, with the particles of each generation coupling identically to the gauge bosons. To each fermion there is also a corresponding antiparticle. Depending on whether the particle is of Dirac or Majorana nature, its antiparticle will be either another particle or it will be its own antiparticle, respectively. The six leptons, arranged in generations, are

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}. \tag{1.1}$$

The reason that there are exactly three generations is not theoretical but experimental; we have simply not observed any particles from a fourth generation. The electron (e^-) has, just like the muon (μ^-) and the tau (τ^-) a charge of $Q = -|e|$, the elementary charge of $e = 1.602 \cdot 10^{-19}$ C. Each of these particles has in its respective generation an uncharged neutrino (ν_e, ν_μ and ν_τ). Then there are the quarks which are logically defined as the fermions that do interact with the strong force. We may use a similar grouping for the quarks and present them in their respective generations as

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}. \tag{1.2}$$

They are called: up (u), down (d), charm (c), strange (s), top (t) and bottom (b). As for the leptons there is a logic to how they have been arranged, with the upper quark of each generation having a charge of $Q = +|\frac{2}{3}e|$ and the lower quark having $Q = -|\frac{1}{3}e|$. When the quarks bind together they always do so to form a particle of integer charge; we have still not observed a particle with something else then integer charge. The quarks are,along with gluons, the building blocks of nuclei, and are relatively heavy, whereas leptons in general are lighter. For both types of fermions it holds though, that they get heavier the further up in generation you go. There are several quantum numbers and properties associated with each particle and they can be classified thereafter. A property of fermions is chirality. It is basically the same chirality one would speak of in e.g. chemistry. A fermion may be either right- or left-handed, distinguishing between the two even if the particles in all other ways are identical. It is only the weak interaction that makes this distinction as the gauge bosons of the $SU(2)_L$ does not interact with right-handed particles. To describe this mathematically we have projections operators (P_R and P_L) to project a particle to a certain chirality. If we let a particle be represented by the fermionic field Ψ we then have its right- and left-handed components as $\Psi_R = P_R\Psi$ and $\Psi_L = P_L\Psi$, respectively. These projection operators are

$$P_R = \frac{1+\gamma^5}{2},$$
$$P_L = \frac{1-\gamma^5}{2},$$

and are also known as chirality operators. Here we have used the Dirac gamma matrices; γ^μ, with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. It then makes sense to have the left-handed fermions in the SM to be $SU(2)_L$-doublets and the right-handed ones as $SU(2)_L$-singlets. Since the $W^\pm$ boson have a coupling that is a vector minus axial vector, $(1-\gamma^5)$, *i.e.* left-handed, it only interacts with other left-handed particles, or their right-handed antiparticles. Now one can summarize one fermion generation with this language as its left-handed doublets and its right-handed doublet. For the first generation this could be written as

$$L_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad e_R^-, \quad u_R, \quad d_R. \tag{1.3}$$

An important detail at this point is that all neutrinos in the SM are right-handed and should thus be massless. This is not true according to experiments and is an area for Beyond Standard Model (BSM) research.

Gauge bosons

In the standard model all forces have force carriers, *i.e.* particles that mediate the force. Hence a particle exerting force on another particle is the exchange of a third particle and these particles are called gauge bosons. With the categorization above, each group of particles has one or several gauge bosons associated with it. After electroweak symmetry breaking we end with the $W^\pm$ and Z bosons that mediate the weak force. They are massive with a mass of 80.385 GeV and 91.187 GeV respectively [12]. In the same group we also have the massless photon, the mediator of the electromagnetic force. Corresponding to the last gauge group, the $SU(3)_c$, there are eight spin-1 gluons that handle the strong force. Only particles with a color charge (the quarks) are effected by this force.

The Higgs Boson

This is the newest confirmed particle in the SM and was until recently the only remaining missing particle in the SM. This is a boson that does not mediate a certain force but rather is a result of the electroweak spontaneous symmetry breaking [13, 14, 15], giving mass to the other particles. Though this is a rather simplified picture, a more detailed excursion will not be given here, see instead Ref. [16]. On 8 October 2013, François Englert and Peter Higgs were awarded the Nobel Prize in physics, for their work on the theory behind the Higgs mechanism.

1.1.2 Particle Interaction

A way to derive the interactions of the SM particles is to require gauge invariance of the Lagrangian under group transformations. If we start by considering a free field Lagrangian of a Dirac particle (a fermion) we have

$$\mathcal{L}_{\text{free}} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi. \tag{1.4}$$

If we want to make this invariant under the possible group transformations of the SM gauge group, it needs to be invariant under e.g. the $U(1)$-transformation

$$\psi(x) \to e^{i\alpha(x)}\psi(x). \tag{1.5}$$

Now this is obviously not invariant under the Lagrangian in Eq. (1.4). But with a modification of the derivative, to the *covariant derivative*

$$\partial_\mu \to D_\mu = \partial_\mu + igA_\mu, \tag{1.6}$$

Eq. (1.4) becomes invariant under the transformation in Eq. (1.5). Here g is the coupling constant and A_μ transforms as

$$A_\mu \to A_\mu - \frac{1}{g}\partial_\mu\alpha(x), \tag{1.7}$$

and defines the field strength tensor as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{1.8}$$

Of course, we would want the Lagrangian to be invariant under all group transformations for the entire gauge group of the standard model. If we take a general non-Abelian gauge field A_μ^a we can construct a covariant derivative

$$D_\mu = \partial_\mu - igA_\mu^a T^a, \tag{1.9}$$

with g being the coupling constant and T^a the group generators. To relate this to the field strength tensor we introduce a structure constant f^{abc} and define the following relations

$$[T^a, T^b] = if^{abc}T^c, \tag{1.10}$$
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c. \tag{1.11}$$

When this is applied to the SM gauge group, we end up with the covariant derivative

$$D_\mu = \partial_\mu - ig_1 W_\mu^a T^a - ig_2 B_\mu Y - ig_3 G_\mu^b T^b, \tag{1.12}$$

where $b = 1, \ldots, 8$ corresponds to the eight gluons and $a = 1, 2, 3$ and Y is the hypercharge. The quantities B_μ and W_μ^a are vector fields, which mix at the electroweak symmetry breaking to form the Z and $W^\pm$ bosons and the photon. In the case of the $SU(2)_L \otimes U(1)_Y$ group, i.e. the electroweak force, $T^a = i\sigma^a/2$, with σ^a being the Pauli matrices. For the strong force, i.e. the $SU(3)_c$ group, $T^b = i\lambda^b/2$ where λ^b are the Gell-Mann matrices.

1.2 Beyond the Standard Model

The SM has in its current formulation, with uncanny precision, predicted and described new particles over the last 40 years [17]. So how do we know there is something beyond the SM? There are several issues with the SM as it is today. Firstly, it is not compatible with the general theory of relativity, which in turn is extremely good at describing gravity. Secondly there is the matter of neutrino oscillations. The SM predicts that neutrinos have zero mass, but experiments tell of neutrino oscillations, which are only possible if neutrinos indeed have non-zero mass. Thirdly there is the problem of Dark Matter (DM), or rather the lack of observation thereof. DM is the proposed explanation for the observed discrepancy between the determined mass of large objects in space by measurements of their gravitational effects and calculations of the amount of luminous matter in them. There are also additional problems of varying degree, such as the matter-antimatter asymmetry, that might also be resolved by BSM physics.

1.2.1 Neutrino Oscillations

There was an issue called *the solar neutrino problem*. The problem was that roughly only a third of the expected neutrino flux coming from charged-current interactions in the Sun, was observed. This problem was resolved by inferring that the neutrinos can spontaneously change flavour, by so-called *flavour oscillations*. Thus about two thirds of the electron neutrinos produced in the Sun could change to muon and tau neutrinos on their way to Earth. This was actually confirmed in an experiment 2001 [18]. There has been several other experiments confirming the massive nature of the neutrinos since, although no measurement of their actual mass, only upper bounds [19, 20, 21, 22, 23]. The oscillations can be parametrized in vacuum by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [24], given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{23}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1.13}$$

where $s_{ij} = \sin(\theta_{ij}), c_{ij} = \cos(\theta_{ij})$ for the mixing angles θ_{ij}, with $i, j = 1, 2, 3$ and where δ is the CP violating phase. One of the big questions to be answered in neutrino physics is whether neutrinos are Dirac or Majorana particles. If the neutrinos are Majorana particles, the matrix U must also be multiplied by the diagonal matrix

$$\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.14}$$

where α_1 and α_2 are Majorana phases. We consider the case of three neutrino flavours, letting $\alpha = e, \mu, \tau$. If we denote $U_{\alpha i}$ as an element of the PMNS matrix, we have that the probability of detecting a neutrino, originally of flavour α, as a neutrino of flavour β, is

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_i U^*_{\alpha i} U_{\beta i} e^{-i \frac{m_i^2}{2E} t} \right|^2 , \tag{1.15}$$

where t is the time. It is worth noticing that this is only valid for neutrino propagation in vacuum and that the case of propagation through matter is far more complicated. We are interested in neutrino oscillations mainly because the phenomenon provides the best proof of non-zero neutrino masses.

1.2.2 Dark Matter

Dark matter (DM) was first introduced in the 1930's to resolve the issue of the galaxy rotation curves [25], and it was at the same time the name was coined. More specifically the Coma cluster lacked the majority of the mass needed for it to bound smaller high-velocity galaxies to it by gravitational effects. Additionally, it was required a much larger mass in the galaxies than observed, in order to account for the high velocities of some stars in the perimeter of galaxies. Hence DM was introduced to yield the extra mass, and the name is simply a hint that it is matter we have yet to detect, or that it does not interact with the particles in the SM.

There are several theories to explain what DM is and what its constituents are, see Ref. [26] for more details. The most common theory is that DM consists of some particles that does not interact with the SM particles or that they interact only very weakly. They would have to be electrically neutral and be stable or have a very long lifetime, since there is such an abundance of DM in the Universe today. If one considers the DM particles to be non-relativistic, the model is a cold DM model, as compared to a relativistic, hot DM model. Each model has different candidates on DM particles, where neutrinos are the only suitable SM particle. The standard neutrinos are however disqualified since they are light and not nearly sufficiently abundant. Another popular model is that of supersymmetry, that predicts heavy BSM particles as DM candidates.

Problems with the models

There are issues of all current DM models that hinder them from fully explaining DM. We will specifically look at the some of the problems of Λ-CDM cosmology [27]. One of the concerns is the *missing satellites problem*. Simulations predict that there should be numerous dwarf-sized subhalos contained as satellites to hosting galaxies such as the Milky Way. However, as the name of the problem suggests, these satellites have not been observed in abundance anywhere near what is predicted by simulations. Additionally, simulations tells us that the most massive subhalos in the Milky Way should be too dense to form and host the observed satellites, which is in contradiction to the other predictions of simulations. This particular part of this problem is referred to as the *too big to fail* problem.

There is also a problem of the structure of the observed galaxies, with simulations of Λ-CDM cosmology implying that there should be a CM cusp rather than the observed cored profiles of some observed galaxies. *i.e.* the density distribution of galaxies has been observed to be nearly constant at the cores, instead of having a density distribution that varies highly with the radius.

There is also an issue that *structure formation may not be fast enough.* In Λ-CDM cosmology we have a prediction of bulk flow velocities of ~ 200 km/s. There have however been observations of galaxy bulk flows of over 3000 km/h, which is too high a velocity for Λ-CDM to account for.

Chapter 2

Big Bang Nucleosynthesis

This chapter presents the general ideas about standard cosmology and the early universe and its thermodynamics. For more details, see Ref. [28].

2.1 Standard Cosmology

The Friedmann-Robertson-Walker (FRW) cosmological model is the most commonly used theoretical framework for the early universe. It is also known as the *hot Big Bang model*. This theory lets us probe the earliest times after the big bang, as well as the subsequent evolution until today. We can reach back in time with the help of particle accelerators, yielding high energies resembling the early Universe, when all the energy was once focused in a singularity.

This model is founded upon three basic astrophysical observations; the Hubble law, the black body spectrum of the background photon radiation, and the homogeneity and isotropy of the Universe on large scales. The homogeneity and isotropy of the Universe suggests that the metric used to describe the Universe should itself be homogeneous and isotropic. This yields the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which is an exact solution to Einstein's field equations of general relativity. In comoving spherical coordinates this becomes

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2)\right], \tag{2.1}$$

with $a(t)$ being the cosmic scale-factor and $k = -1, 0, 1$ depending on if the space is elliptic, Euclidean or hyperbolic, respectively. If one uses this metric with the Einstein field equations and consider the equation of state of the fluid filling the Universe $P = P(\rho)$, meaning that the pressure is a function of the energy density, one obtains a simple equation governing the evolution of the energy density in the Universe,

$$\frac{\mathrm{d}(\rho a^3)}{\mathrm{d}a} = -3Pa^2. \tag{2.2}$$

Solving this yields

$$\rho_M \propto a^{-3}, \tag{2.3}$$
$$\rho_R \propto a^{-4}, \tag{2.4}$$
$$\rho_\Lambda \propto \text{const}, \tag{2.5}$$

for matter, radiation and the cosmological constant, respectively. It is convenient to think of the content of the early Universe as fluids of matter and radiation. These can in turn be thought of as excitations of the corresponding particles' quantum fields and can be treated with phase-space distributions for each particle specie. At high enough temperatures the interactions will be sufficiently rapid to guarantee thermodynamic equilibrium, yielding that a specific particle specie has its distribution function given by the equilibrium distribution

$$f_i(|\mathbf{p}|, T) = \left[\exp\left(\frac{E_i(|\mathbf{p}|) - \mu_i}{T}\right) \pm 1\right]^{-1}. \tag{2.6}$$

Here, $E_i(|\mathbf{p}|) = \sqrt{|\mathbf{p}|^2 + m_i^2}$, μ_i is the chemical potential and $\pm$ holds for the Fermi-Dirac and the Bose-Einstein distribution, respectively. The chemical potential is zero for particles such as photons or Z bosons, that can be emitted and absorbed in *ad infinitum*, due to their lack of conserved quantum number. However, since empirical studies show that the net electrical charge in the Universe is zero and the baryon number density contra the photon number density is negligible, we can set $\mu = 0$ for most purposes.

With the notion of the phase-space distribution of particles, we can, in the comoving frame, utilize the following expressions for the number density, energy density and pressure of the given particle specie at a given temperature,

$$n_i(T) = g_i \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} f_i(|\mathbf{p}|, T), \tag{2.7}$$
$$\rho_i(T) = g_i \int \frac{\mathrm{d}^3|\mathbf{p}|}{(2\pi)^3} E_i(|\mathbf{p}|) f_i(|\mathbf{p}|, T), \tag{2.8}$$
$$P_i(T) = g_i \int \frac{\mathrm{d}^3|\mathbf{p}|}{(2\pi)^3} \frac{|\mathbf{p}|^2}{3E_i(|\mathbf{p}|)} f_i(|\mathbf{p}|, T), \tag{2.9}$$

where g_i is the number of internal degrees of freedom. These quantities are the thermodynamical observables of this model. It is worth noticing some special cases of Eqs. (2.7) and (2.8) and to introduce two new variables, $x = m/T$, and $y = E/T$ yielding

$$n_i(T) = \frac{g_i}{2\pi^2} T^3 I_i^{11}(\mp), \tag{2.10}$$

$$\rho_i(T) = \frac{g_i}{2\pi^2} T^4 I_i^{21}(\mp), \tag{2.11}$$

with

$$I_i^{jk}(\mp) = \int_{x_i}^{\infty} y^j (y^2 - x_i^2)^{k/2} \frac{1}{e^y \mp 1} \mathrm{d}y. \tag{2.12}$$

Then, for relativistic (R) bosons ($x \ll 1$) and fermions we have

$$n_{i,\mathrm{R}}^b(T) = g_i \frac{\zeta(3)}{\pi^2} T^3, \tag{2.13}$$

$$\rho_{i,\mathrm{R}}^b(T) = g_i \frac{\pi^2}{30} T^4, \tag{2.14}$$

and

$$n_{i,\mathrm{R}}^f(T) = g_i \frac{\zeta(3)}{4\pi^2} T^3, \tag{2.15}$$

$$\rho_{i,\mathrm{R}}^f(T) = g_i \frac{7\pi^2}{240} T^4, \tag{2.16}$$

respectively. Note that ζ is the Riemann zeta function. For non-relativistic (NR) particles ($x \gg 1$), irrespective of the bosonic or fermionic nature of the particle, this becomes

$$n_{i,\mathrm{NR}}^{\mathrm{eq}}(T) = g_i \frac{x^{3/2} e^{-x}}{(2\pi)^{3/2}} T^3, \tag{2.17}$$

$$\rho_{i,\mathrm{NR}}^{\mathrm{eq}}(T) = g_i \frac{x^{5/2} e^{-x}}{(2\pi)^{3/2}} T^4, \tag{2.18}$$

which is just the Boltzmann distribution. It is worth noticing that

$$\frac{\rho_{\mathrm{R}}^b}{\rho_{\mathrm{R}}^f} = \frac{8}{7}, \tag{2.19}$$

a common factor when distinguishing between bosons and fermions.

2.1.1 The Epochs of the Early Universe

There are several models and ways to classify the chronology of the Universe. One model of the chronology of the Universe begins with the so-called *the Planck epoch*, which lasts until about 10^{-43} seconds (10^{18} GeV) after the Big Bang. Here the temperature is so high that all the four forces are unified in one fundamental force. There are several hypothesis of this force such as superstrings or quantum gravitation.

However, at about 10^{-36} seconds (10^{15} GeV), this force splits into gravitation and the gauge forces. The theory is that the non-gravitational forces in this epoch would be described by some *Grand Unification Theory* (GUT).

Then comes the electroweak epoch where the strong force and the electroweak force part at 10^{-12} seconds (10^3 GeV), followed by the electroweak symmetry breaking at 10^{-10} seconds (10^2 GeV). At this stage the four fundamental forces exist as we know them today. It takes until 10^{-4} seconds (150 MeV) before the Universe has cooled so much that the quarks can form hadrons and baryons in the Hadron epoch. One second after the Big Bang the temperature is low enough for the majority of the hadrons to annihilate with anti-hadrons, *i.e.* they leave thermal equilibrium with the thermal bath that is the photons. This process reheats the Universe. At this time leptons start to form and the mass of the Universe is dominated by leptons and anti-leptons. Again, as the temperature continues to fall, the larger parts of leptons and anti-leptons annihilate in a process that again reheats the Universe. Next enters the radiation dominated era, eventually followed, at a time of about 70000 years, by the mass dominated era.

2.2 Relativistic Degrees of Freedom

If all relativistic particles are in thermal equilibrium during a time in the radiation dominated era, we can parametrize the energy density of every particle species, in terms of the photon energy density [29]. This is intuitively sound since the photons make out the thermal bath and indeed define the temperature. We start off by stating the relation

$$\rho_i^{\text{eq}} = \left(\frac{g_i}{2}\right)\rho_\gamma. \tag{2.20}$$

We then obtain the total energy density as

$$\rho_R = \left(\frac{g_R}{2}\right)\rho_\gamma, \tag{2.21}$$

for

$$g_R = \sum_{\text{bosons}} g_i + \frac{7}{8}\sum_{\text{fermions}} g_i, \tag{2.22}$$

which is known as the *relativistc degrees of freedom.* The quantity g_R is a function of temperature that varies over large temperature intervals, depending on what particles are relativistic and in thermal equilibrium with the thermal bath.

We can also make a more general parametrization where we parametrize the energy density of all relativistic particles, regardless of whether they are in thermal equilibrium or not. We can write

$$\rho_R(T) = \left(\frac{g_*}{2}\right)\rho_\gamma. \tag{2.23}$$

Hence we have that

$$g_* = \sum_{j\neq i} g_{*_j}(T) + g_{*_i}(T_i)\left(\frac{T_i}{T}\right)^4, \tag{2.24}$$

where i and j represent the particle species and T denotes the photon temperature. When all particles are relativistic, we can instead sum over particles as fermions and bosons, and obtain

$$g_* \approx \sum_{\text{bosons}} g_i(T_i)\left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\text{fermions}} g_i(T_i)\left(\frac{T_i}{T}\right)^4. \tag{2.25}$$

At the time of BBN, the SM has $g_* = g_R = \frac{43}{4}$.

2.3 Big Bang Nucleosynthesis

During the radiation dominated epoch, at $\mathcal{O}(1)$ MeV we infer that we at this time have a plasma consisting of photons, electrons, positrons, neutrinos, protons and neutrons as well as heavier nuclei. Due to the high temperature and thus the fast interaction rates, they are in thermal equilibrium with each other. The Universe is expanding with an expansion rate, H, which is temperature dependent. When the Universe expands it demands a relatively higher interaction rate for the particles to continue to be in thermal equilibrium. As the temperature falls the interaction rates grows slower. For neutrinos, which only interact via the weak force and have no way of interacting directly with the thermal bath, this happens rather early. The neutrinos will eventually have an interaction rate which does not allow them to retain equilibrium with the rest of the Universe and hence they *freeze out*. This freeze out occurs at $2-3$ MeV. At $T \approx 0.7$ MeV, there is another freeze out when the neutron-proton charged-current interactions no longer keep them in chemical equilibrium. This leaves a proton/neutron fraction of $\sim 1/7$ that only changes by neutron decays into protons. When a particle specie freezes out it is called that it *decouples*. A rough criterion for when a particle is coupled or decoupled is by comparing the expansion rate, H, of the Universe, with the interaction rate, Γ, of the particle specie

$$\Gamma \gtrsim H \quad \text{coupled}, \tag{2.26}$$

$$\Gamma \lesssim H \quad \text{decoupled}. \tag{2.27}$$

Obviously, the decoupling of a particle is not something that happens momentarily, but rather is a rather complex process over some time interval. It is however a fairly good, and very useful, approximation to say that the decoupling is *instantaneous*. This is an approximation we will use throughout this thesis.

In addition to the process of freeze out, there may be the case that the particle specie starts out of equilibrium with the thermal bath, due to a very weak coupling to the ordinary particles. If, in addition, its interaction rate is proportional to T^i for some $i < 2$, the particle will actually reach equilibrium later on. We illustrate with two examples. First let us consider the neutrinos. Their interaction rates come mainly from their interactions with electrons and positrons, through various processes such as neutral and charged current elastic scattering processes. This gives them an interaction rate of $\Gamma = n\langle\sigma v\rangle$ [29], where $\langle\sigma v\rangle \approx G_\mathrm{F}^2 T^2$ is the thermally averaged cross section and n is the number density, which is proportional to T^3 for a relativistic particle. We have that

$$H = 1.66\sqrt{g_*}\frac{T^2}{M_\mathrm{P}} \approx \frac{T^2}{M_\mathrm{P}}, \tag{2.28}$$

where $M_\mathrm{p} = 1.22 \cdot 10^{19}$ GeV, being the Planck mass. This yields the decoupling condition

$$G_\mathrm{F}^2 T^5 < \frac{T^2}{M_\mathrm{P}}, \tag{2.29}$$

which in turn yields the decoupling temperature $T_D(\nu) \approx 1$ MeV. More complex calculations yields a decoupling temperature of $2 - 3$ MeV.

Next, let us have a look at another case. If we have a particle that interacts with SM particles only through its decay into some particle and its corresponding antiparticle, and the corresponding inverse decay, we can have a decay rate

$$\Gamma \propto \frac{f^2 M^2}{T}, \tag{2.30}$$

for some coupling constant f and particle mass M. This yields the equilibrium condition

$$\frac{f^2 M^2}{T} > \frac{T^2}{M_\mathrm{P}}. \tag{2.31}$$

Rearranging this gives us that the particle specie is coupled for

$$T < (f^2 M^2 M_\mathrm{P})^{1/3}. \tag{2.32}$$

This means that the particle begins out of equilibrium and couples to the thermal bath at $T^3 \sim f^2 M^2 M_\mathrm{P}$ and then stays in equilibrium. This process is also available to us, and is called *freeze in*.

2.4 Effective Degrees of Freedom

The energy density of the early Universe can also be parametrized in terms of the neutrino energy density, and not just in terms of the photon energy density. This is particularly helpful when we try to introduce a new particle. A new particle can alter the energy density of the early Universe and thus gives rise to these *extra degrees of freedom* for the neutrinos. *i.e.* the parametrization yields a result as if we had additional neutrino flavours, and this is something that can be measured and constrained today.

In the standard BBN picture the total radiation density, ρ_R, of the early Universe consists of photons and neutrinos, since we neglect the contribution of non-relativistic particles. Hence we obtain

$$\rho_R = \rho_\gamma + \rho_\nu. \tag{2.33}$$

If we consider some additional particle in the early Universe that could contribute to the total energy density, say a vector boson, V, we would instead have

$$\rho'_R = \rho_\gamma + \rho_\nu + \rho_V. \tag{2.34}$$

We could then choose to define *effective degrees of neutrinos*, $N_{\text{eff}} = 3 + \Delta N_\nu$, such that Eq. (2.34) becomes

$$\rho'_R = \rho_R + \Delta N_\nu \rho_\nu, \tag{2.35}$$

$$\frac{\rho'_R}{\rho_\nu} = \frac{\rho_R}{\rho_\nu} + \Delta N_\nu, \tag{2.36}$$

$$\Delta N_\nu = \frac{\rho'_R}{\rho_\nu} - \frac{\rho_R}{\rho_\nu} = \frac{\rho_V}{\rho_\nu}. \tag{2.37}$$

2.5 Approximative Constraints From V Boson Decay

By using the condition of decoupling in Eq. (2.27), along with the decay rate of the V boson and the experimental constraints on the extra degrees of freedom for neutrinos, one can find constraints on the parameter space of a massive vector boson model.

We call the coupling constant of the particle's relevant interaction g_V. This will be clarified later on. For certain values of the coupling constant g_V, the vector boson V will be in thermal equilibrium with the photons in the early Universe. We thus know that its density of states in this case is described by Eq. (2.6). Using Eq. (2.8) and assuming the chemical potential to be zero, we find

$$\rho_V = \frac{\tilde{g}_V}{(2\pi)^3} \int_0^\infty \frac{E}{e^{\frac{E}{T}} - 1} \mathrm{d}^3\mathbf{p}, \tag{2.38}$$

with $\tilde{g}_V = 3$, the number of degrees of freedom for the boson, V. As already noted, this is not analytically solvable, so we examine it in its two extreme cases. For relativistic particles we find, by the usage of Eq. (2.14) ($x \ll 1$)

$$\rho_{V,\mathrm{R}} = \frac{T^4\pi^2}{10}, \tag{2.39}$$

while Eq. (2.18) ($x \gg 1$) yields

$$\rho_{V,\mathrm{NR}} = 3m^{5/2}\left(\frac{T}{2\pi}\right)^{3/2} e^{-\frac{m}{T}} = 3x^{5/2}\frac{T^3}{(2\pi)^{3/2}}e^{-x}. \tag{2.40}$$

We have the well-known photon density

$$\rho_\gamma = \frac{\pi^2 T^4}{15}. \tag{2.41}$$

We can express the neutrino energy density in terms of the photon energy density as

$$\rho_\nu = \frac{7}{8}\rho_\gamma = \frac{7}{8}\frac{\pi^2 T^4}{15}, \tag{2.42}$$

where the factor $\frac{7}{8}$ accounts for the fermionic nature of the neutrinos. Using these expressions we obtain the following analytical form for the extreme cases of a new boson specie in thermal equilibrium at the time of BBN

$$\Delta N_{\mathrm{eff}} = \frac{\rho_V}{\rho_\nu} = \begin{cases} \dfrac{90\cdot\sqrt{2}e^{-m/T}}{7\cdot\pi^{7/2}}\left(\dfrac{m}{T}\right)^{5/2} \approx 0.33x^{5/2}e^{-x}, & x \gg 1, \\ \dfrac{8}{7}\cdot\dfrac{3}{2} \approx 1.71, \quad x \ll 1. \end{cases} \tag{2.43}$$

With the constraint $\Delta N_\nu < 1$, at 95% C.L. [30], we obtain a constraint on particles that are in thermal equilibrium with the thermal bath at the time of BBN. By observing the function $\Delta N_\nu(x)$, as plotted in Fig. 2.1, we note that we can exclude all particles that are in thermal equilibrium at the time of BBN, with a mass $m_V < 2.3$ MeV. This since we have used the approximation that decoupling is instantaneous and that BBN takes place at $T = 1$ MeV, yielding $x = m_V$ at the time of BBN. We can then numerically extract that $\Delta N > 1$ when $x \lesssim 2.3$.

Hence one would need to know for what values of the theory's free parameters the boson would be in thermal equilibrium at the time of BBN. In order to know this we either must solve the Boltzmann equation for the early Universe or use the approximative condition in Eq. (2.27). We will do both in the subsequent chapters.

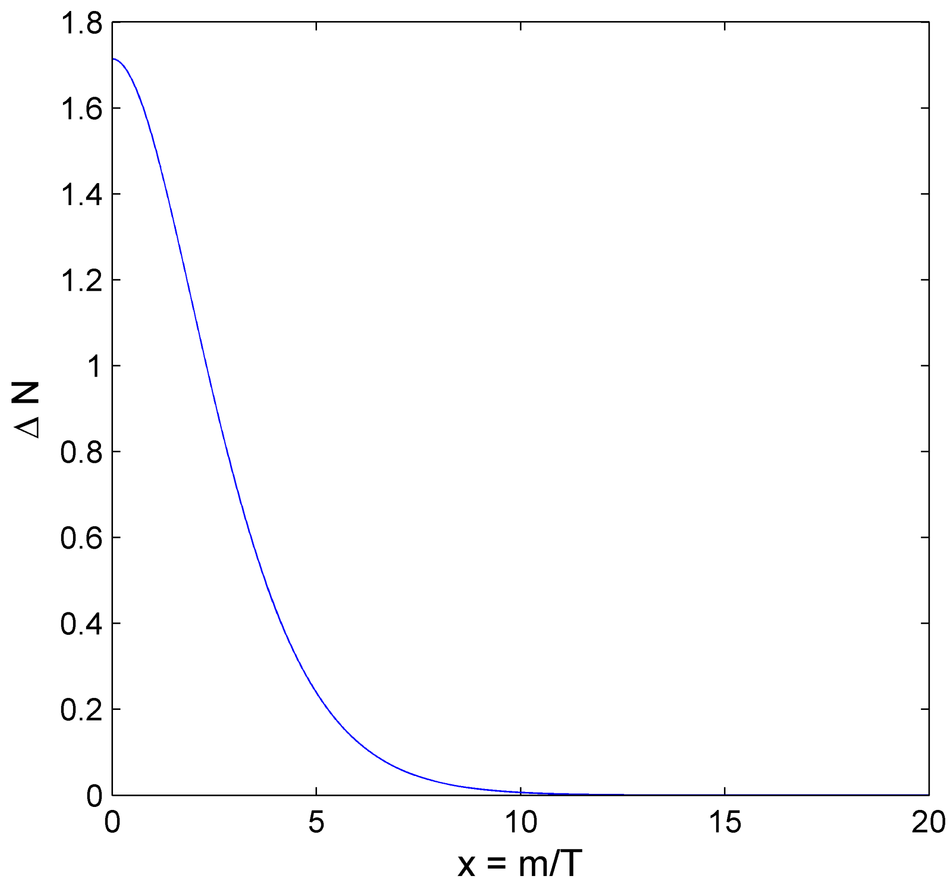

Figure 2.1. The extra degrees of freedom, ΔN_ν, plotted as a function of $x = m_V/T$. A more careful, numerical analysis reveals that $\Delta N_\nu(x) > 1$ for $x \lesssim 2.3$. At the time of BBN, this corresponds to that a vector boson of mass $m_V < 2.3$ MeV causes more than one extra degree of freedom, which, according to Ref. [30], is excluded.

Chapter 3

Secret Neutrino Interactions

There are many scenarios where one can use secret neutrino interactions, and several ways to constrain them. Some will be mentioned here. Earlier discussions about these interactions include Refs. [31] and [32], which contemplate secret neutrino interactions with new particles. Ref. [32] considers BBN in combination with a supernova observation as a way to constrain Majoron-emitting double β decay. Ref. [31] develops constraints on the parameter space of a model with secret neutrino interactions and a new vector boson by using the cosmic background and observations from the supernova 1987A [33]. Another paper considers vector bosons in the setting of Big Bang Nucleosynthesis and constrain the parameter space using methods similar to what we will use in this chapter [34]. However, their scope is rather narrow and they look at the cases of a very heavy (order of SM gauge bosons) or a very light (order KeV or eV) vector boson. Our analysis will be more thorough and apply a wider range of constraints and use more robust versions of these methods.

3.1 A New Vector Boson

As a toy model we use the model proposed in Ref. [1], where we have a secret coupling of the new vector boson, V, to neutrinos by the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -g_V \bar{\nu} \not{V} \nu. \tag{3.1}$$

This is similar to the toy model used in Ref. [34], which puts large constraints on the model. However, their constraints are only valid for a rather heavy ($m_V \gg 200$ MeV) or a very light ($m_V \ll 1$ MeV) vector boson. Our analysis will be more careful and considers a massive vector boson of ~ 1 MeV.

For a first analysis we looked at the condition in Eq. (2.26). In order to apply this we need to calculate the decay rate of the particle. The decay rate will also be useful further on when solving the Boltzmann equation.

We will use a non-SM interactions of the vector boson and the neutrinos since this will be simpler to work with. If one would choose to have an A-V coupling between the neutrinos and the vector boson instead, as in the SM, one would obtain a decay rate which is half the rate of what we obtain here. This factor of one half could simply be absorbed into the coupling constant g_V, making a very small change in the final result. The Feynman diagram for the V boson decay can be found in Fig. 3.1.

3.1.1 The Decay Rate of V

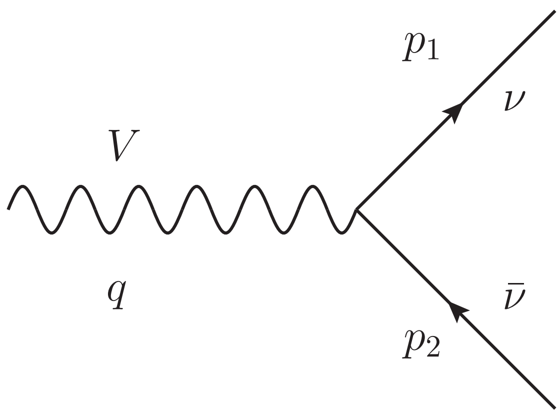

Figure 3.1. Feynman diagram for the V-boson decay into a neutrino anti-neutrino pair. We denote the outgoing momenta as p_1 and p_2 and the ingoing momentum as q.

From the Feynman diagram we find the amplitude for the decay $V \to \nu\bar{\nu}$ as

$$M = \bar{u}^s(p_1) g_V \gamma^\mu \varepsilon_\mu(q) v^{s'}(p_2). \tag{3.2}$$

From this we want to find the decay rate of the particle, meaning that we will have to perform some algebra. It will be presented rather explicitly, since this is a process vital to the work. We multiply Eq. (3.2) with its complex conjugate to obtain

$$\begin{aligned} |M|^2 &= g_V[\bar{u}^s(p_1)\gamma^\mu\varepsilon_\mu(q)v^{s'}(p_2)]g_V^*[\bar{v}^{s'}(p_2)\gamma^\nu\varepsilon_\nu^*(q)u^s(p_1)] \\ &= g_V^2\bar{u}(p_1)\gamma^\mu v(p_2)\bar{v}(p_2)\gamma^\nu u(p_1)\varepsilon_\mu(q)\varepsilon_\nu^*(q). \end{aligned} \tag{3.3}$$

Next we want to sum over ingoing spin and average over polarization in order to consider an unpolarized interaction. We want the unpolarized case since we are not considering any particular polarization. Using the following relations

$$\sum_{\mathrm{s}} u^s(p)\bar{u}^s(p) = \not{p} + m, \tag{3.4}$$

$$\sum_{\mathrm{s}} v^s(p)\bar{v}^s(p) = \not{p} - m, \tag{3.5}$$

$$\sum_{\text{polarization}} \varepsilon_\mu(q)\varepsilon^*_\nu(q) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_V^2}, \tag{3.6}$$

we can simplify our amplitude

$$\begin{aligned}
\overline{|M|^2} &= \frac{1}{3}\sum_{\text{spin}} |M|^2 = \frac{g_V^2}{3}\left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_V^2}\right] \mathrm{Tr}\left[(\not{p}_1 + m_\nu)\gamma^\mu(\not{p}_2 - m_\nu)\gamma^\nu\right] \\
&= \frac{g_V^2}{3}\left[8p_2\cdot p_1 + 16m_\nu^2 + \frac{4}{m_V^2}\left[(q\cdot p_2)(q\cdot p_1)\right.\right. \\
&\qquad \left.\left. +(q\cdot p_1)(q\cdot p_2) - q^2(p_1\cdot p_2 + m_\nu^2)\right]\right] \\
&\approx \frac{g_V^2}{3}\left[8p_2\cdot p_1 + \frac{4}{m_V^2}\left[2(q\cdot p_1)(q\cdot p_2) - q^2(p_1\cdot p_2)\right]\right],
\end{aligned} \tag{3.7}$$

where we have used the approximation that the neutrino mass is zero. We define the following kinematics, working in the center-of-mass frame (CM frame)

$$p_1 = (E, -E\hat{z}), \tag{3.8}$$

$$p_2 = (E, E\hat{z}), \tag{3.9}$$

$$q = (m_V, 0), \tag{3.10}$$

$$p_1\cdot p_2 = 2E^2, \tag{3.11}$$

$$q^2 = (p_1 + p_2)^2 = 4E^2, \tag{3.12}$$

$$q\cdot p_1 = q\cdot p_2 = \frac{m_V^2}{2}. \tag{3.13}$$

Applying these yields

$$\left[(q\cdot p_2)(q\cdot p_1) + (q\cdot p_1)(q\cdot p_2) - q^2(p_1\cdot p_2)\right] = 0, \tag{3.14}$$

$$\overline{|M|^2} = \frac{16}{3}g_V^2 E^2. \tag{3.15}$$

Here we stop and observe that the squared matrix amplitude is independent of the longitudinal polarization, *i.e.* the term $(q_\mu q_\nu)/m_V^2$ in the averaged matrix square amplitude. Hence our constraint will be invariant under things such as an inert longitudinal polarization. This is true for $m_\nu \approx 0$. We then have the well-known expression for the decay of a particle in rest frame in vacuum [16]

$$\begin{aligned} d\Gamma &= \frac{1}{2m_A}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right) \\ &\times \quad |M(m_A \to \{p_f\})|^2 (2\pi)^4\delta^{(4)}(p_A - \sum p_f). \end{aligned} \tag{3.16}$$

The index f denotes the final state particles whereas the index A denotes the incoming particle. In the special case of a two-body decay, such as the one considered here, this can be simplified to

$$d\Gamma = \frac{1}{2m_A}\left(\frac{1}{4\pi}\frac{|\mathbf{p}_1|}{E_{\mathrm{CM}}}\right)|M(m_A \to \{p_f\})|^2. \tag{3.17}$$

Calculating this for our case, using the approximation that $m_V \gg m_\nu$, we have

$$\Gamma = \frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_V^2}\frac{16}{3}g_V^2 E_1^2 \approx \frac{g_V^2}{3\pi m_V}\frac{m_V^2}{4}. \tag{3.18}$$

We have in the above equation used that $E_1 = m_V/2$ in the rest frame of the V boson. This is obviously the case since the two final state particles will obtain an equal share of the initial state energy if one applies a CM frame. Hence we find

$$\Gamma = \frac{g_V^2 m_V}{12\pi}. \tag{3.19}$$

However, we want the decay rate of a moving particle in the comoving frame so we want to multiply this by m_V/E, basically setting the $1/2m_{\mathrm{A}}$ from Eq. (3.16) to $1/2E_{\mathrm{A}}$. We see that in the case of a non-moving particle, we would regain our previous constant of $1/m_{\mathrm{A}}$. The energy of the particle is, in average, about the temperature of the bosons. We show this more carefully by taking the thermal average of the energy

$$\langle E\rangle = \frac{g_\nu \int E\frac{d^3\mathbf{p}}{(2\pi)^3}f_\nu(|\mathbf{p}|,T)}{g_\nu \int \frac{d^3\mathbf{p}}{(2\pi)^3}f_\nu(|\mathbf{p}|,T)} = \frac{\rho_\nu}{n_\nu}. \tag{3.20}$$

This is a simple and useful expression that we can easily evaluate in a relativistic or non-relativistic limit to obtain an analytical expression for $\langle E\rangle$ in the respective limit. This yields

$$\langle E\rangle = \begin{cases} \dfrac{\rho_V}{\rho_V/m} = m, \qquad \text{NR particles,} \\ \\ \left(\frac{3T^4\cdot\pi^4}{2\pi^2\cdot 15}\right)\left(\frac{3T^3\cdot 2\xi(3)}{2\pi^2}\right)^{-1} \approx 2.66T, \qquad \text{R particles.} \end{cases} \tag{3.21}$$

For a not completely relativistic expression, but still not a completely non-relativistic case, we will take $\langle E\rangle = 2T$. This corresponds fairly well to the case of our vector boson around the time of BBN. Hence we have the following expression for the decay rate

$$\Gamma = \frac{g_V^2 m_V^2}{24T\pi}. \tag{3.22}$$

With this, the constraint on the parameters of the toy model would be

$$g_V > 10^{-10} \tag{3.23}$$
$$m_V < 2.3 \quad \text{MeV}. \tag{3.24}$$

These constraints are merely a first approximation and will be refined by looking at the Boltzmann equation.

3.2 The Boltzmann Equation

The Boltzmann equation is often used in non-equilibrium statistical mechanics to describe the overall behaviour of a "large" thermodynamic system; where "large" in this context means that it contains so many particles that it can be treated to good accuracy with statistical methods. A common example of when to make use of the Boltzmann equation is when describing the heat flow from an out-of-equilibrium gas or fluid to its surroundings. Not entirely unlike how we would view certain particle species in the early Universe. The Boltzmann equation can, in Hamiltonain mechanics, be written as

$$\hat{L}[f] = C[f], \tag{3.25}$$

with $\hat{L}$ being the Liouville operator and C the collision term. For the standard cosmology model this is [28]

$$E_V \frac{\partial f_V}{\partial t} - H|\mathbf{q}|^2 \frac{\partial f_V}{\partial E_V} = -\int d\Pi_\nu d\Pi_{\bar{\nu}} (2\pi)^4 \delta^{(4)}(q - p_1 - p_2)$$
$$\times \left[\overline{|M|^2}_{V\to\nu\bar{\nu}} f_V \cdot (1 - f_\nu)(1 - f_{\bar{\nu}}) - \overline{|M|^2}_{\nu\bar{\nu}\to V} f_\nu f_{\bar{\nu}} \cdot (f_V + 1)\right], \tag{3.26}$$

where

$$d\Pi_a = g_a \frac{1}{(2\pi)^3} \frac{d^3 p_a}{2E_a}, \tag{3.27}$$
$$d\Pi_\nu = \frac{1}{4\pi^3} \frac{d^3 p_1}{2E_1}, \tag{3.28}$$
$$d\Pi_{\bar{\nu}} = \frac{1}{4\pi^3} \frac{d^3 p_2}{2E_2}, \tag{3.29}$$

and $\overline{|M|^2}$ is the matrix amplitude squared, averaged over initial spins and summed over final spins, of the specific process. In the scenario to which we shall apply this equation it is well motivated to use the approximation of CP, or T, conservation, meaning that $\overline{|M|^2}_{V\to\nu\bar{\nu}} = \overline{|M|^2}_{\nu\bar{\nu}\to V} = \overline{|M|^2}$. We also postulate that all particle species except the one of interest, f_V, have their respective equilibrium distributions and that these distributions may be approximated by Maxwell-Boltzmann statistics instead of Fermi-Dirac or Bose-Einstein statistics. This is a good approximation in the radiation dominated epoch, since the particles have very high interaction rates and can be considered to be in equilibrium in this stage. Lastly we make the approximation that $(1 \pm f_i) \approx 1$. We thus obtain

$$f_\nu = \exp(-E_1/T), \tag{3.30}$$
$$f_{\bar{\nu}} = \exp(-E_2/T). \tag{3.31}$$

We want to rewrite the left-hand side of Eq. (3.26) to something more convenient, so we note that

$$\frac{\partial f_V}{\partial E_V} = \frac{\partial f_V}{\partial q}\frac{\partial q}{\partial E_V} = \frac{\partial f_V}{\partial q}\frac{E_V}{q}. \tag{3.32}$$

Putting all this together we obtain the Boltzmann equation as

$$\begin{aligned}&\frac{\partial f_V}{\partial t} - H|\mathbf{q}|\frac{\partial f_V}{\partial q}\\ &= -\frac{1}{E_V}\int \frac{2}{(2\pi)^3}\frac{\mathrm{d}^3 p_1}{2E_1}\frac{2}{(2\pi)^3}\frac{\mathrm{d}^3 p_2}{2E_2}(2\pi)^4\\ &\times\delta^{(4)}(q-p_1-p_2)\overline{|M|^2}(f_V - f_\nu f_{\bar{\nu}}).\end{aligned} \tag{3.33}$$

Note that we have H as a parameter in the Boltzmann equation. We will use Eq. (2.28) when we need to express H in our variables.

3.2.1 The Integrated Boltzmann Equation

It is possible to rewrite Eq. (3.33) in terms of the number density n. This can be very helpful in some situations, even though we will not use it for more than illustrative purposes of the dynamics of the equation, and as a comparison to the non-integrated Boltzmann equation. It is common that one studies the Boltzmann equation for the early universe to find various number densities and it is then very convenient to directly obtain the number density when solving the equation. In addition, the equation takes on a simpler form when stated in terms of the number density, as we will see. We begin by noting that the expression for the number density,

$$n_V(t) = \frac{\tilde{g}_V}{(2\pi)^3}\int \mathrm{d}^3 q f(E,t), \tag{3.34}$$

is contained in Eq. (3.33). This allows us, after some algebra and tricks, to express Eq. (3.33) in terms of $n_V(t)$ as

$$\dot{n}_V(t) + 3Hn_V = \int \frac{3}{(2\pi)^3}\frac{C[f_V]}{E_V}\mathrm{d}^3 q, \tag{3.35}$$

with the collision term on the right-hand side as

$$\begin{aligned}\frac{3}{(2\pi)^3}\int\frac{C[f_V]}{E_V}\mathrm{d}^3q &= -\int\frac{3}{(2\pi)^3}\frac{\mathrm{d}^3q}{E_V}\frac{2}{(2\pi)^3}\frac{\mathrm{d}^3p_1}{2E_1}\frac{2}{(2\pi)^3}\frac{\mathrm{d}^3p_2}{2E_2}(2\pi)^4\\ &\times\ \delta^{(4)}(q-p_1-p_2)\overline{|M|^2}(f_V-f_\nu f_{\bar\nu}).\end{aligned} \tag{3.36}$$

If we ponder upon the delta-term and the energy conservation it dictates, we see that

$$(f_V - f_\nu f_{\bar\nu}) = (f_V - f_V^{EQ}). \tag{3.37}$$

This is a very useful form, since we have an easy expression for f_V^{EQ}, using Boltzmann statistics. As we will see, this term survives throughout our manipulation of the equation, giving us part of a term proportional to how far off from equilibrium the distribution function or the number density is. The next observation one can make is that the right-hand side of Eq. (3.36) can be identified as

$$-\int\frac{3}{(2\pi)^3}\mathrm{d}^3q\cdot 8\Gamma\cdot[f_V - f_V^{EQ}], \tag{3.38}$$

with Γ being the decay rate of the V boson. Another convenient relation to implement is the thermal average of the decay rate Γ is

$$\langle\Gamma\rangle = \frac{\int\frac{3}{(2\pi)^3}\mathrm{d}^3q\Gamma\cdot f_V^{EQ}}{\int\frac{3}{(2\pi)^3}\mathrm{d}^3q f_V^{EQ}}. \tag{3.39}$$

Using this in the integrated Boltzmann equation yields, after some calculations and rearrangements

$$\dot n_V(t) + 3Hn_V = -\langle\Gamma\rangle[n_V - n_V^{EQ}], \tag{3.40}$$

with

$$\langle\Gamma\rangle = 8\cdot\Gamma\frac{K_1(x)}{K_2(x)}. \tag{3.41}$$

Here $K_n(x)$ is the modified Bessel function of the second kind and of order n. We now have a very elegant form for the Boltzmann equation. Solving this directly yields the number density as a function of time. However, it is clearly preferable to obtain the number density as a function of some other, dimensionless, variable, say $x = m/T$. So finally we observe that this can be put on a dimensionless form in a comoving frame, by the variable substitution

$$Y = n_V/s, \tag{3.42}$$

$$s \approx \frac{2\pi}{45}g_{*s}n_\gamma, \tag{3.43}$$

where s is the entropy, and g_{*s} is the relativistic degrees of freedom scaled with the entropy. Applying this variable substitution finally leads to the dimensionless, integrated Boltzmann equation

$$\frac{\mathrm{d}Y}{\mathrm{d}x} = -\frac{0.602 M_{pl} x}{m_V^2 \sqrt{g_{*s}}} \cdot 8 \cdot \Gamma \frac{K_1(x)}{K_2(x)} [Y - Y^{EQ}]. \tag{3.44}$$

3.2.2 The Non-Integrated Boltzmann Equation

Equation (3.44) is a very convenient form for many purposes, but not so much for finding the energy density. In order to calculate the energy density function it is more beneficial to go back to Eq. (3.33), the so-called non-integrated Boltzmann equation. This may be rewritten to dimensionless derivatives with the variable substitutions $x = m/T$ and $y = q/T$. Using these variables we obtain the following relations

$$\begin{aligned} t &= 0.301 g_*^{-1/2} \frac{M_{pl}}{T^2} = 0.301 g_*^{-1/2} \frac{M_{pl}}{m_V^2} x^2 = \frac{1}{2H}, & (3.45) \\ \frac{\mathrm{d}T}{\mathrm{d}t} &= -HT, & (3.46) \\ \frac{\partial x}{\partial t} &= Hx, & (3.47) \\ \frac{\partial y}{\partial t} &= Hy, & (3.48) \\ \frac{\partial x}{\partial q} &= 0, & (3.49) \\ \frac{\partial y}{\partial q} &= \frac{1}{T}. & (3.50) \end{aligned}$$

Then

$$\frac{\partial f_V}{\partial t} - qH \frac{\partial f_V}{\partial q} = \frac{\partial f_V}{\partial x} Hx. \tag{3.51}$$

Additionally, just as in the case of the integrated Boltzmann equation, we can use conservation of energy to see that

$$\begin{aligned} &f_\nu f_{\bar{\nu}} = \exp(-(E_1 + E_2)/T) = \exp(-(E_V)/T) = f_V^{EQ}, \\ &(f_V - f_\nu f_{\bar{\nu}}) = (f_V - f_V^{EQ}), & (3.52) \end{aligned}$$

since we have assumed Boltzmann statistics for all particles in thermal equilibrium. Much like in the procedure for the integrated Boltzmann equation we may express the right-hand side of Eq. (3.33) in terms of the decay rate. Since

$$\begin{aligned}\Gamma &= \frac{1}{2E_V}\int \frac{1}{(2\pi)^3}\frac{\mathrm{d}^3p_1}{2E_1}\frac{1}{(2\pi)^3}\frac{\mathrm{d}^3p_2}{2E_2} \\ &\times (2\pi)^4\delta^{(4)}(q-p_1-p_2)\overline{|M|^2}, \qquad (3.53)\\ \frac{1}{E_V}C[f_V] &= -8\Gamma. \qquad (3.54)\end{aligned}$$

From here, though, we cannot use the trick of the thermally averaged decay rate, nor do we end up with any modified Bessel functions. Finally, after some simplification, we have the following form of our non-integrated Boltzmann equation

$$\frac{\partial f_V}{\partial x} = -\frac{2g_V^2 M_{pl}}{1.66\sqrt{g_*}\cdot 3\pi}\frac{x}{E_V}\left(f_V - e^{-E_V}\right). \qquad (3.55)$$

3.2.3 Numerical Treatment

When solving Eq. (3.55) numerically we first normalize it with the following variable substitution

$$Y_V = \frac{f_V}{f_V^{\mathrm{EQ}}} \quad \Rightarrow \quad \frac{\partial f_V}{\partial x} = f_V^{\mathrm{EQ}}\frac{\partial Y_V}{\partial x}. \qquad (3.56)$$

This is essential in order to obtain numerical stability over large intervals, since the equation otherwise solves for values over many orders of magnitude. Hence the equation we solve is given by

$$\frac{\partial Y_V}{\partial x} = -\frac{2g_V^2 M_{pl}}{1.66\sqrt{g_*}\cdot 3\pi}\frac{x}{E_V}\left(Y_V - 1\right). \qquad (3.57)$$

We solve this in Matlab, using the integrated solver *ode23s*. To simplify the equation, without changing the result too much, we set $g_* = \text{constant} = 10.75$, *i.e.* the SM value for g_* at the time of BBN. In reality g_* varies but is constant on large intervals. Since we are really only interested in the time just around the BBN, it does not matter if this would change the behaviour of the differential equation far from $T = 1$ MeV.

The program solved Y as a function of $x = m_V/T$ over an interval $0 < x \leq 100$ MeV. The interval was split into parts with different incrementing, speeding up the numerics and making the solution more precise. This was done for a constant mass, alas meaning that x simply takes the role of the inverse temperature scaled, and a constant energy E_V. The quantity Y was subsequently transformed back into f_V, which was then to be integrated in Eq. (2.8). However, instead of integrating over momentum it is more convenient at this stage to change to an integration over energy. Hence the energy density integral becomes

$$\frac{3}{2\pi^2}\int_{m_V}^{\infty}\frac{E^2\sqrt{E^2-m_V^2}}{e^{\frac{E}{T}}-1}\mathrm{d}E. \tag{3.58}$$

In order to perform said integration over a sufficiently large and fine interval, the process of solving the differential equation in Eq. (3.57) was done for 400 different energies, also with an adapted incrementing. The result was then a vector containing $\rho_{V,i}(T = m_{V,i}/x)$ for a set of parameter values g_i and $m_{V,i}$. An example of such a simulation with a fixed value of m_V and two different values of g_V, namely for $m_V = 1$ and $g_V = \{10^{-8}, 10^{-5}\}$, is shown in Fig. 3.2, where we see three distinct lines. The straight, dashed line corresponds to the neutrino energy density, and is seen to vary as $\rho_\nu \propto T^{-4}$. Then there is the solid line, starting parallel to the neutrinos'; this is ρ_V for $g_V = 10^{-5}$. This overlaps completely with the equilibrium distribution for the energy density of the plotted interval. We may note that at about $x = 2.3$, the equilibrium energy density of the V boson, falls below ρ_ν, which will yield a $\Delta N < 1$ when $x > 2.3$. This is due to that the V boson will leave the relation $\rho \propto T^{-4}$ as it becomes less and less relativistic. This is the same thing we found when doing a simpler analysis in Chapter 2. This should not be surprising since this is the same physics but in a slightly different framework, but it is nonetheless reassuring that we obtain the same result. Lastly there is the dotted line, corresponding to $g_V = 10^{-8}$, which begins out of equilibrium, but reaches the equilibrium distribution at about $x = 10^{-1}$ or $T = 10$ MeV. This is just as we foresaw when looking at the equilibrium condition $\Gamma > H$ in Chapter 2. Hence our particle might *freeze in* before or after BBN.

Finally, this process was done for the entire parameter space of interest. To speed up the process the code was parallelized using Matlab's built in function *parfor*. Then ΔN was calculated as $\Delta N = \frac{\rho_V}{\rho_\nu}$, as in previous chapters. The result is shown in Fig. 3.3.

3.3 Other Constraints

Models such as the one in Ref. [1] can also be constrained by terrestrial experiments such as the observation of decay rates. The total decay width of some particles has been measured with great accuracy, giving a robust limit on changes that can be made on the partial decay widths for these particles.

3.3.1 Particle Decays

The introduction of a new particle which couples to neutrinos will alter the interaction and decay rates involving neutrinos. Especially the decay rates can be heavily affected by this since a regular two-body decay can, by a subsequent decay of an energetic neutrino into the new particle and something else, instead turn into

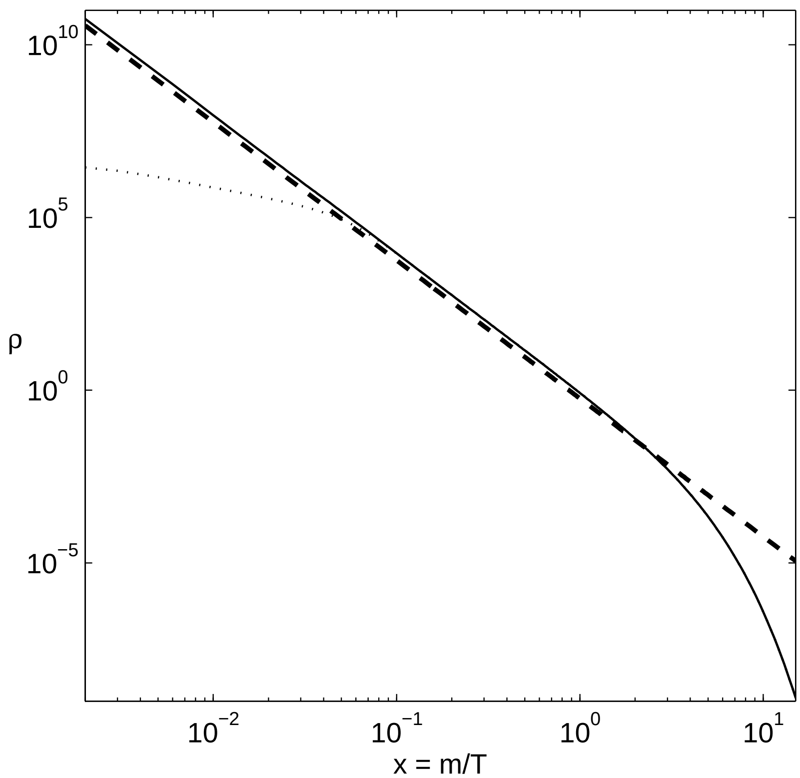

Figure 3.2. The energy density of the V boson for the parameters $m_V = 1$ MeV, $g_V = \{10^{-8}, 10^{-5}\}$ and its equilibrium distribution, ρ_V^{EQ}. $\rho_V(g_V = 10^{-5})$ and ρ_V^{eq} overlap completely in the plotted region and are represented by the solid line. $\rho_V(g_V = 10^{-8})$ is the thin, dotted line. This line follows the equilibrium distribution once they come together in the plot. For reference the energy density of the neutrinos is also plotted, which corresponds to the thick, dashed line.

a three-body decay for instance. This can have a significant impact on the total decay rate [35]. We will in this section present an example of such a constraint.

W Boson Decay

The difference from a normal W boson decay will be that the antineutrino now can decay into our V boson and a neutrino, giving us an extra decay channel. We begin with finding the invariant matrix element of the W boson decay $W \to \bar{\nu} l^- V$, by calculating it from the Feynman diagram, which may be found in Fig. 3.4. We obtain

$$M = g_V \frac{\mathrm{i} g_2}{\sqrt{2}} \bar{u}(p_1) \gamma^\mu \frac{1}{2} \left(1 - \gamma^5\right) \frac{-\mathrm{i}(\not{p}_2 + \not{p}_3)}{(p_2 + p_3)^2} \gamma^\nu v(p_3) \varepsilon_\nu^*(p_2) \varepsilon_\mu(q). \tag{3.59}$$

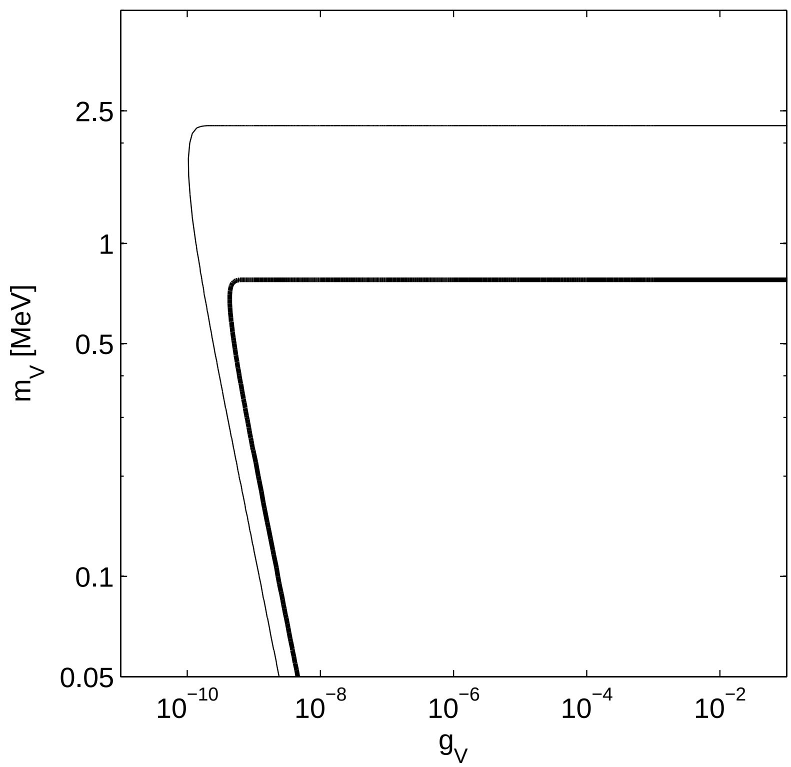

Figure 3.3. The constraint of the parameter space for the two conditions $\Delta N < 1$ (thin line) and $\Delta N < 1.5$ (thick line). The area confined under the two lines represents the excluded parameter space of the model, under the corresponding constraint on the extra degrees of freedom.

We proceed in the same manner we did when we calculated the decay rate of the V boson. Squaring the matrix amplitude we obtain

$$\begin{aligned} |M|^2 &= \frac{g_V^2 g_2^2}{4} \frac{1}{(p_2+p_3)^4} \varepsilon_\nu^*(p_2)\varepsilon_\mu(q)\varepsilon_\alpha(p_2)\varepsilon_\beta^*(q)\bar{u}(p_1)\gamma^\mu\left(1-\gamma^5\right) \\ &\times \quad (\not{p}_2+\not{p}_3)\gamma^\nu v(p_3)\bar{v}(p_3)\gamma^\alpha(\not{p}_2+\not{p}_3)\gamma^\beta u(p_1). \end{aligned} \tag{3.60}$$

Then we average over incoming spin and sum over outgoing spin, to obtain

$$\begin{aligned} \Sigma_{\text{spin}}|M|^2 &= \frac{1}{3}\frac{g_V^2 g_2^2}{4(m_V^2+2p_2\cdot p_3)^2}\left[-g_{\mu\beta}+\frac{q_\mu q_\beta}{m_Z^2}\right]\left[-g_{\nu\alpha}+\frac{p_{2,\nu}p_{2,\alpha}}{m_V^2}\right] \\ &\times \quad \text{Tr}\left[\not{p}_1\gamma^\mu\left(1-\gamma^5\right)\left(\not{p}_2+\not{p}_3\right)\gamma^\nu\not{p}_3\gamma^\alpha\left(\not{p}_2+\not{p}_3\right)\gamma^\beta\right]. \end{aligned} \tag{3.61}$$

Calculating this is not as straight-forward as for the V boson, since it is a three body problem. One may use the Dalitz variables $m_{12}^2=(p_1+p_2)^2$ and $m_{23}^2=(p_2+p_3)^2$ to express the result [12]. These are quite convenient and Dalitz plots are common

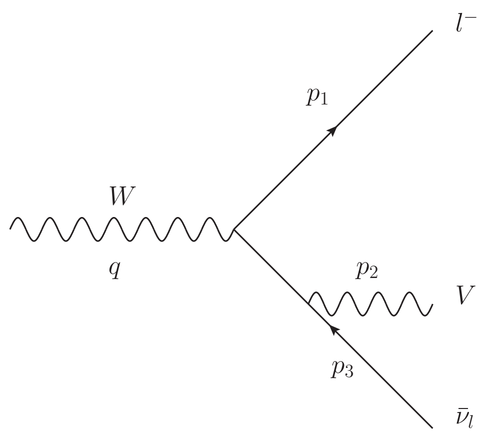

Figure 3.4. The Feynman diagram for the decay $W \to \bar{\nu} l^- V$.

when studying many-body decays. However, we choose a different method where we use conservation of 4-momentum in order to define the scaling variables

$$x_\alpha = 2E_\alpha/m_W = 2q \cdot p_\alpha/m_W^2, \quad \alpha = 1, 2, 3 \tag{3.62}$$

such that in the rest frame of the W boson we have $x_1 + x_2 + x_3 = 2$. If we consider the neutrino masses to be negligible this yields the scalar products

$$p_1 \cdot p_3 = \frac{m_W^2}{2}\left(1 - x_2 + \frac{m_V^2}{m_W^2} - \frac{m_l^2}{m_W^2}\right), \tag{3.63}$$

$$p_1 \cdot p_2 = \frac{m_W^2}{2}\left(1 - x_3 - \frac{m_V^2}{m_W^2} - \frac{m_l^2}{m_W^2}\right), \tag{3.64}$$

$$p_2 \cdot p_3 = \frac{m_W^2}{2}\left(1 - x_1 - \frac{m_V^2}{m_W^2} + \frac{m_l^2}{m_W^2}\right). \tag{3.65}$$

This problem is not too hard to do by hand, but as there were harder problems up the road, this was implemented in Mathematica, with the help of the package *Feynpar*. Using these variables, and Mathematica to calculate, we obtain

$$\begin{aligned} \Sigma_{\text{spin}}|M|^2 &= -\frac{g_V^2 g_2^2}{6m_V^2 m_W^4 (1-x_1)^2}\left[E_1(8E_2 m_V^2(m_V^2 + m_W^2(x_1 - 1))\right. \\ &+ 4E_3(2m_V^4 - m_W^4(x_1-1)^2)) + m_W^2(m_W^4(x_1-1)^2(x_2-1) \\ &+ \left. 2m_V^4 - m_V^2 m_W^2(x_1-1)(x_1+2x_3-3))\right]. \end{aligned} \tag{3.66}$$

From here it can be useful to express the partial decay rate as a function of the scaling variables, but we will go back to an expression in terms of the energies, since they are easy to work with in our case. We then want to integrate this according to Eq. (3.16), here again stated as

$$\mathrm{d}\Gamma = \frac{1}{2m_A}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)|M(m_A \to \{p_f\})|^2 (2\pi)^4\delta^{(4)}(p_A - \sum p_f). \quad (3.67)$$

We said that we wanted an expression in terms of the energy, meaning that we would also like to have the integration in terms of the energy. In the case of an unpolarized three-body decay we can integrate out the angles and use the conservation of energy and momentum in order to transform Eq. (3.67) into [12]

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^3}\frac{1}{8m_A}\overline{|M|^2}\mathrm{d}E_1\mathrm{d}E_2. \quad (3.68)$$

When performing the integration the following parameter values were used

$$\begin{aligned} g_2 &= 0.66, && (3.69)\\ m_W &= 80.38 \quad (\mathrm{GeV}), && (3.70)\\ m_l &= 0.511, && (3.71)\\ m_V &= 10^i, && (3.72) \end{aligned}$$

for $i = \{-3, -2, -1, 0, 1\}$ in order to find Γ as a function of m_V in the studied parameter space. For us to perform the integration one more thing is needed, namely the integration limits. We find these limits by using energy conservation once more. Using the expression for one of the energies, say E_3, chosen since the corresponding mass will always be negligible, we have

$$\begin{aligned} m_W &= E_1 + E_2 + E_3, && (3.73)\\ E_3 &= \sqrt{|\bar{p}_1|^2 + |\bar{p}_2|^2 + 2|\bar{p}_1||\bar{p}_2|^2\cos\theta_{12} + m_3^2}. && (3.74) \end{aligned}$$

We see that the corresponding minimum and maximum value of E_3 is acquired at $\cos\theta_{12} = -1$ and $+1$ respectively, *i.e.* E_3 takes on its minimum value when the other two final state particles have opposite momentum and it reaches its maximum value for the case of parallel momenta for the other two final state particles. This leads to the relation

$$\sqrt{(|\bar{p}_1| - |\bar{p}_2|)^2 + m_3^2} \le m_W - E_1 - E_2 \le \sqrt{(|\bar{p}_1| + |\bar{p}_2|)^2 + m_3^2}, \quad (3.75)$$

for $|\bar{p}_1| = \sqrt{E_i^2 - m_i^2}$. If we insert the expressions for the momenta in Eq. (3.75) and use the approximation that we may neglect outgoing particle masses, we obtain the much simpler relation

$$|E_1 - E_2| \le m_W - E_1 - E_2 \le E_1 + E_2. \quad (3.76)$$

Sorting out the two cases of $E_1 > E_2$ and $E_2 > E_1$ we obtain the integration limits

$$0 \leq \quad E_1 \quad \leq \frac{m_W}{2}, \tag{3.77}$$
$$0 \leq \quad E_2 \quad \leq \frac{m_W}{2} - E_1. \tag{3.78}$$

Using these integration limits we obtain our sought after partial decay width, with the mass of m_V expressed in MeV,

$$\Gamma(W \to \bar{\nu} l^- V) = 2.57 \cdot 10^9 \frac{1}{m_V^2} \quad \text{(MeV)}. \tag{3.79}$$

Now, to turn this into a constraint on the parameter space of the model, we compare this with the observed decay rate of the W boson. The total decay width for W is $\Gamma_{\text{tot}} = 2.085 \pm 0.042$ GeV. Having just calculated the decay rate of a decay channel not included in the SM, this decay channel would, if existing, still contribute to the total decay width of the W boson. Obviously the observed decay width will not change due to our calculations and thus this new, calculated decay rate must lie within the uncertainty of the measurement of the total decay width. Hence, using a 95 % confidence level (C.L.), yielding an extra factor of 1.28, we obtain the constraint

$$\Gamma(W \to \bar{\nu} l^- V) < 1.28 \cdot 42 \quad \text{MeV}, \tag{3.80}$$

which leads to the following constraint of the parameters space

$$g_V < 14 \cdot 10^{-5} \left(\frac{m_V}{\text{MeV}}\right). \tag{3.81}$$

The constraint is plotted in Fig. 3.5 together with the favoured region of the model suggested by Ref. [1].

Z Boson Decay

The same thing was done for the Z boson, with the hope that it would yield a stronger constraint, since the uncertainty of the measurement is more than a magnitude smaller for this decay. The Z boson has a total decay rate of 2.4952 ± 0.0023 GeV [12]. This yields a constraint on the massive vector boson such that

$$\Gamma(Z \to \nu\bar{\nu}V) < 0.0023 \cdot 1.28, \tag{3.82}$$

again for a 95 % C.L. In this case, it is slightly more complicated than for the W boson to find the decay rate, since there are two contributing diagrams, as shown in Fig. 3.6. These yield the invariant matrix element

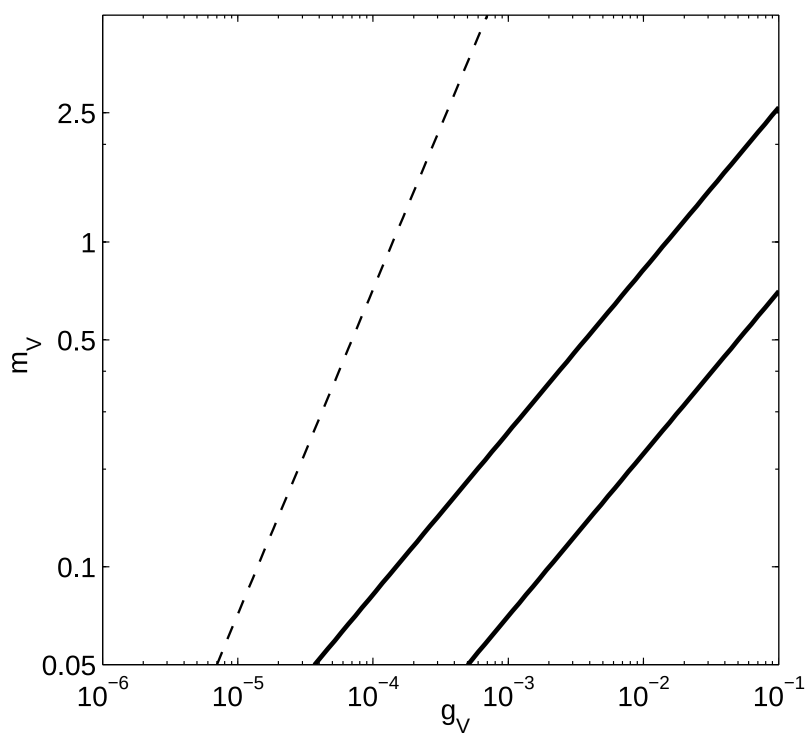

Figure 3.5. The constraint from the decay rate of the W boson is marked by the dashed line. Everything to the right of this line is excluded parameter space. For reference, the favoured region of the model proposed in Ref. [1], is marked as the region between the two thick lines.

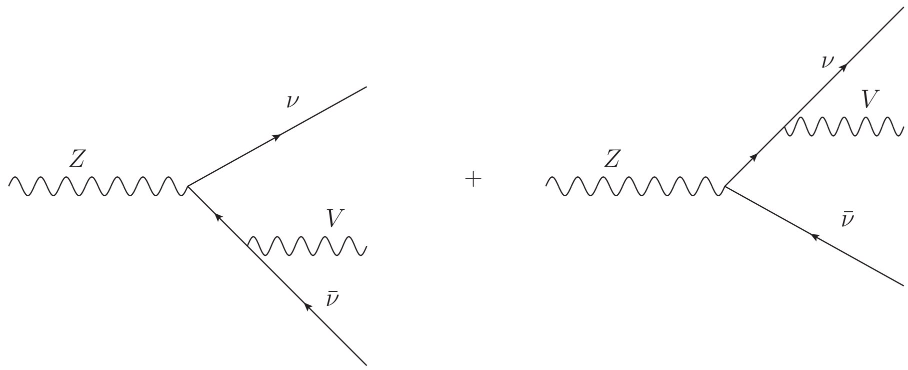

Figure 3.6. Feynman diagram for the Z boson decay into a neutrino-antineutrino pair and a V boson.

$$
\begin{aligned}
M &= g_V \frac{g_2}{\sqrt{2}\cos\theta} \bar{u}(p_1)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right) \frac{\mathrm{i}(\not{p}_2+\not{p}_3)}{(p_2+p_3)^2} \gamma^\nu v(p_3)\varepsilon^*_\nu(p_2)\varepsilon_\mu(q) \\
&+ g_V \frac{g_2}{\sqrt{2}\cos\theta} \bar{u}(p_1)\gamma^\nu \frac{\mathrm{i}(\not{p}_1+\not{p}_2)}{(p_1+p_2)^2} \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right) v(p_3)\varepsilon^*_\nu(p_2)\varepsilon_\mu(q). \quad (3.83)
\end{aligned}
$$

Using the notation

$$K_{11} = g_V^2 \frac{g_2^2}{8\cos\theta} \frac{1}{(p_2+p_3)^4}, \quad (3.84)$$

$$K_{22} = g_V^2 \frac{g_2^2}{8\cos\theta} \frac{1}{(p_1+p_2)^4}, \quad (3.85)$$

$$K_{12} = K_{21} = g_V^2 \frac{g_2^2}{8\cos\theta} \frac{1}{(p_2+p_3)^2(p_1+p_2)^2}, \quad (3.86)$$

we can write this spin averaged matrix element as the following expression:

$$\begin{aligned}
\Sigma_{\rm spin}|M|^2 &= K_{11}\Big\{\mathrm{Tr}\Big[\not{p}_1\gamma^\mu(1-\gamma^5)\left(\not{p}_2+\not{p}_3\right)\gamma^\nu\not{p}_3\gamma^\alpha\left(\not{p}_2+\not{p}_3\right)\gamma^\beta\Big] \\
&\times\quad [-g_{\mu\beta}+\frac{q_\mu q_\beta}{m_Z^2}][-g_{\nu\alpha}+\frac{p_{2\nu}p_{2\alpha}}{m_V^2}]\Big\} \\
&+\quad K_{22}\Big\{\mathrm{Tr}\Big[\not{p}_1\gamma^\nu(1-\gamma^5)\left(\not{p}_1+\not{p}_2\right)\gamma^\mu\not{p}_3\gamma^\beta\left(\not{p}_1+\not{p}_2\right)\gamma^\alpha\Big] \\
&\times\quad [-g_{\mu\beta}+\frac{q_\mu q_\beta}{m_Z^2}][-g_{\nu\alpha}+\frac{p_{2\nu}p_{2\alpha}}{m_V^2}]\Big\} \\
&-\quad K_{12}\Big\{\mathrm{Tr}\Big[\not{p}_1\gamma^\mu(1-\gamma^5)\left(\not{p}_2+\not{p}_3\right)\gamma^\nu\not{p}_3\gamma^\beta\left(\not{p}_1+\not{p}_2\right)\gamma^\alpha\Big] \\
&\times\quad [-g_{\mu\beta}+\frac{q_\mu q_\beta}{m_Z^2}][-g_{\nu\alpha}+\frac{p_{2\nu}p_{2\alpha}}{m_V^2}]\Big\} \\
&-\quad K_{21}\Big\{\mathrm{Tr}\Big[\not{p}_1\gamma^\nu(1-\gamma^5)\left(\not{p}_1+\not{p}_2\right)\gamma^\mu\not{p}_3\gamma^\alpha\left(\not{p}_2+\not{p}_3\right)\gamma^\beta\Big] \\
&\times\quad [-g_{\mu\beta}+\frac{q_\mu q_\beta}{m_Z^2}][-g_{\nu\alpha}+\frac{p_{2\nu}p_{2\alpha}}{m_V^2}]\Big\}. \qquad (3.87)
\end{aligned}$$

Note the two minus signs before K_{12} and K_{21}. They arise due to the opposite direction of the momentum versus the particle number flow, of the two diagrams. This indicates that there is destructive interference between the diagrams. We use the same method and basically the same scaling variables as for the W boson decay

$$x_\alpha = 2E_\alpha/m_Z = 2q\cdot p_\alpha/m_Z^2, \quad \alpha = 1,2,3 \qquad (3.88)$$

again such that we in the rest frame of the Z boson have $x_1+x_2+x_3 = 2$. Still considering the neutrino masses to be negligible this yields the scalar products

$$\begin{aligned}
p_1\cdot p_3 &= \frac{m_Z^2}{2}\left(1-x_2+\frac{m_V^2}{m_Z^2}\right), \qquad (3.89)\\
p_1\cdot p_2 &= \frac{m_Z^2}{2}\left(1-x_3-\frac{m_V^2}{m_Z^2}\right), \qquad (3.90)\\
p_2\cdot p_3 &= \frac{m_Z^2}{2}\left(1-x_1-\frac{m_V^2}{m_Z^2}\right). \qquad (3.91)
\end{aligned}$$

This is not so easy to do by hand and was thus implemented in Mathematica, like the W boson decay. The partial decay rate was calculated and then conveniently expressed in terms of the energies E_1 and E_2 along with the masses m_Z and m_V. The integration limits are derived in an almost identical fashion to what was done for the W boson and become, ignoring decay product masses,

$$\begin{aligned}
E_1 &= [0, m_Z/2], \qquad (3.92)\\
E_2 &= [0, m_Z/2 - E_1]. \qquad (3.93)
\end{aligned}$$

We then insert it into Eq. (3.68) and integrate numerically to obtain the decay width $\Gamma(Z \to \nu\bar{\nu}V)$. The SM value for the constants was used in the integration

$$g_2 = 0.66, \tag{3.94}$$

$$m_Z = 91.18 \quad (\text{GeV}), \tag{3.95}$$

$$\sin^2\theta = \frac{1}{4}. \tag{3.96}$$

When performing said integration there were large problems in getting it to converge properly. The problems were numerical and thus no definite number will be given here as none has been found at the time of writing. However, all calculations pointed towards that the constraint will be significantly smaller than that of the W boson. This is confirmed in Ref. [35].

The complete summary of the constraints found, compared with the favoured region in Ref. [1], can be found in the Appendix.

Chapter 4

Summary and Conclusions

In this thesis we have looked at what constraints one can have on a specific model involving the introduction of a ~ 1 MeV, vector boson, in addition to the SM particles. Specifically the model considered in this thesis can be found in Ref. [1]. The relevant part of the model was the interaction Lagrangian

$$\mathcal{L}_{\text{Int}} = -g_V \nu \not{V} \bar{\nu}. \tag{4.1}$$

There is a surprising lack of discussions in the literature on the topic of secret neutrino interactions with ~ 1 MeV vector bosons. Most consider only heavy or very light vector bosons and few utilize BBN to constrain these theories. We showed in this thesis that these constraints may be very restrictive and should most definitely be included in such a model.

The main constraint considered was the extra degrees of freedom during BBN. According to Ref. [30], there is a limit of $\Delta N < 1$ at 95% C.L. In order to find the extra degrees of freedom generated by the model, we solved the Boltzmann equation for the models' parameter space and plotted the result. We found that there were indeed heavy constraints on the suggested model, for $m_V < 2.3$. However, for masses above that there were no constraint. Almost the entire region favoured by the model in question, was excluded by this method. It is possible to extend the model to be valid in a lower mass-region by implementing scattering processes in the Boltzmann equation.

The other constraint treated in this thesis was that from particle decays. Introducing a new particle that couples to some SM particle can alter the decay rates of some particles, by introducing new branches of the decay. If we calculate the new, partial decay width of a particle, this cannot be larger than what fits within the uncertainty of the experimentally observed total decay width. We looked at the decay of the W boson and the Z boson and numerically found their decay rates, which gave us the following constraint for the W decay

$$g_V < 14 \cdot 10^{-5} \left(\frac{m_V}{\text{MeV}}\right). \tag{4.2}$$

For the Z boson the constraints were considerably smaller due to destructive interference between the two contributing Feynman diagrams. For a graphical summary of the constraints found, along with the favoured region of the model from Ref. [1] and the constraints in Ref. [35], of which one was also derived in this thesis, see the Appendix.

Appendix A

Comment on Ref. [1]

Comment on “Is Dark Matter with Long-Range Interactions a Solution to All Small-Scale Problems of ΛCDM Cosmology?”

Björn Ahlgren, Tommy Ohlsson, and Shun Zhou
Department of Theoretical Physics, KTH Royal Institute of Technology, 106 91 Stockholm, Sweden

In a recent Letter [1], van den Aarssen *et al.* suggested a general scenario for dark matter, where both the dark-matter particle χ and ordinary neutrinos ν interact with an MeV-mass vector boson V via $\mathcal{L} \supset g_\chi \overline{\chi}\gamma^\mu \chi V_\mu + g_\nu \overline{\nu}\gamma^\mu \nu V_\mu$ with g_χ and g_ν being the corresponding coupling constants. Given a vector-boson mass $0.05~\mathrm{MeV} \lesssim m_V \lesssim 1~\mathrm{MeV}$, one needs $10^{-5} \lesssim g_\nu \lesssim 0.1$ to solve all the small-scale structure problems in the scenario of cold dark matter [1].

Recently, Laha *et al.* [2] found that the ν-V interaction might lead to too large decay rates of $K^- \to \mu^- + \overline{\nu}_\mu + V$ and $W^- \to l^- + \overline{\nu}_l + V$, indicating that the scenario proposed in Ref. [1] is severely constrained. However, such experimental bounds can be evaded if the longitudinal polarization state of V is sterile or if V is coupled to sterile neutrinos rather than ordinary ones [1, 2].

Now, we show that the constraints on g_ν and m_V from Big Bang Nucleosynthesis (BBN) are very restrictive. In the early Universe, V can be thermalized via the inverse decay $\nu + \overline{\nu} \to V$ and pair annihilation $\nu + \overline{\nu} \to V + V$ and contribute to the energy density. If the inverse-decay rate exceeds the expansion rate H around the temperature $T = 1$ MeV, we obtain $g_\nu > 1.5 \times 10^{-10}~\mathrm{MeV}/m_V$ for $m_V \lesssim 1$ MeV. For $m_V \ll 1$ MeV, pair annihilation is more efficient than inverse decay to thermalize V. Requiring the annihilation rate $\Gamma_{\rm pair} \propto g_\nu^4 T > H$, one obtains $g_\nu > 3.4 \times 10^{-5}$. For $m_V > 1$ MeV, however, even if V is in thermal equilibrium, its number density will be suppressed by a Boltzmann factor. To derive the BBN bound, we first solve the non-integrated Boltzmann equation for the distribution function of V for given g_ν and m_V, where only the decay and inverse-decay processes are included in the computations [3–7]. Then, we calculate its energy density, and require the extra number of neutrino species $\Delta N_\nu < 1$ at $T = 1$ MeV [8]. Thus, we can exclude a large region of the parameter space, as shown in Fig. 1. The contribution from V in thermal equilibrium reaches its maximum $\Delta N_\nu \approx 1.71$ in the relativistic limit.

Note that we have assumed ΔN_ν to be constant, but for $m_V > 1$ MeV it actually decreases during the BBN era, so our constraint should be somehow relaxed in the large-mass region. Since only the transverse polarizations of V are involved in inverse decay in the limit of zero neutrino masses, the BBN constraint does not depend on whether the longitudinal polarization is thermalized or not. If neutrinos are Dirac particles, the right-handed neutrinos $\nu_{\rm R}$ can be in thermal equilibrium as well. Both V and $\nu_{\rm R}$ contribute to the energy density, so one or more species of $\nu_{\rm R}$ is obviously ruled out. In addition, if V is coupled to sterile neutrinos, which are supposed to be thermalized, the BBN bound on g_ν and m_V becomes more stringent.

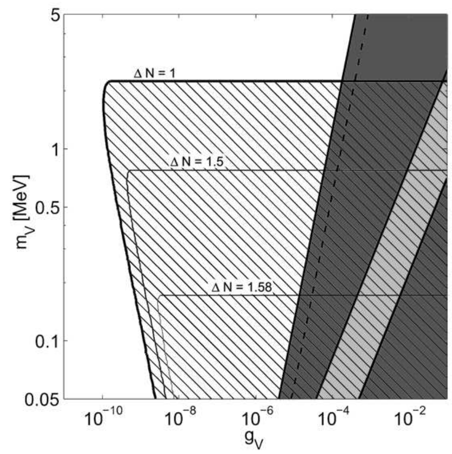

FIG. 1: Constraints on g_ν and m_V from K decays (gray, solid line), W decays (gray, dashed line), and BBN. The two former are reproduced from Ref. [2], and the sample region in Fig. 3 of Ref. [1] is also presented (light gray). The hashed region bounded by the thick solid curve is excluded by $\Delta N_\nu < 1$ at 95% C.L. [8]. The excluded region will shrink for $\Delta N_\nu < 1.5$ (see, e.g., Ref. [9]), and even further for the “maximally conservative” limit $\Delta N_\nu \lesssim 1.58$ for $m_V > 0.05$ MeV.

We are grateful to Torsten Bringmann for valuable communication. Financial support from the Swedish Research Council and the Göran Gustafsson Foundation is acknowledged.

[1] L.G. van den Aarssen, T. Bringmann and C. Pfrommer, Phys. Rev. Lett. **109**, 231301 (2012).
[2] R. Laha, B. Dasgupta and J.F. Beacom, arXiv:1304.3460.
[3] E.W. Kolb and M.S. Turner, *The Early Universe*, Front. Phys. **69**, 1 (1990).
[4] M. Kawasaki, G. Steigman and H.-S. Kang, Nucl. Phys. B **403**, 671 (1993).
[5] A. Basbøll and S. Hannestad, JCAP **0701**, 003 (2007).
[6] J. Garayoa, S. Pastor, T. Pinto, N. Rius and O. Vives, JCAP **0909**, 035 (2009).
[7] F. Hahn-Woernle, M. Plümacher and Y.Y.Y. Wong, JCAP **0908**, 028 (2009).

[8] G. Mangano and P.D. Serpico, Phys. Lett. B **701**, 296 (2011).
[9] J. Hamann, S. Hannestad, G.G. Raffelt and Y.Y.Y. Wong, JCAP **1109**, 034 (2011).

Bibliography

[1] L. G. van den Aarssen, T. Bringmann and C. Pfrommer, *Is dark matter with long-range interactions a solution to all small-scale problems of Λ CDM cosmology?*, Phys. Rev. Lett. **109**, 231301 (2012), 1205.5809.

[2] I. Newton, Philosophiæ naturalis principia mathematica, 1687.

[3] J. Thomson, *Cathode rays*, Phil. Mag. **44**, 293 (1897).

[4] E. Rutherford, *The Scattering of α and β Particles by Matter and the Structure of the Atom*, Philosophical Magazine Series 6 **21**, 669 (1911).

[5] H. Geiger and E. Marsden, *On a Diffuse Reflection of the α-Particles*, Proceedings of the Royal Society, Series A **82**, 495 (1909).

[6] E. Rutherford, *Collisions of alpha particles with light atoms. IV. An anomalous effect in nitrogen*, Phil. Mag. **37**, 581 (1920).

[7] J. Chadwick, *Possible Existence of a Neutron*, Nature **129**, 312 (1932).

[8] M. Gell-Mann, *A Schematic Model of Baryons and Mesons*, Phys. Lett. **8**, 214 (1964).

[9] G. Zweig, *An SU(3) Model for Strong Interaction Symmetry and its Breaking. PT. 2*, (1964).

[10] ATLAS Collaboration, G. Aad *et al.*, *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, Phys. Lett. **B716**, 1 (2012), 1207.7214.

[11] CMS Collaboration, S. Chatrchyan *et al.*, *A New Boson with a Mass of 125 GeV Observed with the CMS Experiment at the Large Hadron Collider*, Science **338**, 1569 (2012).

[12] Particle Data Group, J. Beringer *et al.*, *Review of Particle Physics (RPP)*, Phys. Rev. **D86**, 010001 (2012).

[13] P. W. Higgs, *Broken Symmetries and the Masses of Gauge Bosons*, Phys. Rev. Lett. **13**, 508 (1964).

[14] F. Englert and R. Brout, *Broken Symmetry and the Mass of Gauge Vector Mesons*, Phys. Rev. Lett. **13**, 321 (1964).

[15] G. Guralnik, C. Hagen and T. Kibble, *Global Conservation Laws and Massless Particles*, Phys. Rev. Lett. **13**, 585 (1964).

[16] M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory*, (1995).

[17] R. Oerter, *The theory of almost everything: The standard model, the unsung triumph of modern physics*, (2006).

[18] SNO Collaboration, Q. Ahmad *et al.*, *Measurement of the rate of interactions produced by B-8 solar neutrinos at the Sudbury Neutrino Observatory*, Phys. Rev. Lett. **87**, 071301 (2001), `nucl-ex/0106015`.

[19] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, *Evidence for oscillation of atmospheric neutrinos*, Phys. Rev. Lett. **81**, 1562 (1998), `hep-ex/9807003`.

[20] SNO Collaboration, Q. Ahmad *et al.*, *Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory*, Phys. Rev. Lett. **89**, 011301 (2002), `nucl-ex/0204008`.

[21] KamLAND Collaboration, K. Eguchi *et al.*, *First results from KamLAND: Evidence for reactor anti-neutrino disappearance*, Phys. Rev. Lett. **90**, 021802 (2003), `hep-ex/0212021`.

[22] RENO collaboration, J. Ahn *et al.*, *Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment*, Phys. Rev. Lett. **108**, 191802 (2012), `1204.0626`.

[23] DAYA-BAY Collaboration, F. An *et al.*, *Observation of electron-antineutrino disappearance at Daya Bay*, Phys. Rev. Lett. **108**, 171803 (2012), `1203.1669`.

[24] Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, Prog. Theor. Phys. **28**, 870 (1962).

[25] F. Zwicky, *Spectral displacement of extra galactic nebulae*, Helv. Phys. Acta **6**, 110 (1933).

[26] G. Bertone, D. Hooper and J. Silk, *Particle dark matter: Evidence, candidates and constraints*, Phys. Rept. **405**, 279 (2005), `hep-ph/0404175`.

[27] B. Famaey and S. McGaugh, *Challenges for Lambda-CDM and MOND*, J. Phys. Conf. Ser. **437**, 012001 (2013), `1301.0623`.

[28] E. W. Kolb and M. S. Turner, *The Early Universe*, Front.Phys. **69**, 1 (1990).

[29] S. Sarkar, *Big bang nucleosynthesis and physics beyond the standard model*, Rept. Prog. Phys. **59**, 1493 (1996), `hep-ph/9602260`.

[30] G. Mangano and P. D. Serpico, *A robust upper limit on $N_{\rm eff}$ from BBN, circa 2011*, Phys. Lett. **B701**, 296 (2011), `1103.1261`.

[31] E. W. Kolb and M. S. Turner, *Supernova SN 1987a and the Secret Interactions of Neutrinos*, Phys. Rev. **D36**, 2895 (1987).

[32] S. Chang and K. Choi, *Constraints from nucleosynthesis and SN1987A on majoron emitting double beta decay*, Phys. Rev. **D49**, 12 (1994), `hep-ph/9303243`.

[33] S. Woosley and T. Weaver, *The Great Supernova of 1987*, Sci. Am. **261**, 24 (1989).

[34] E. Masso and R. Toldra, *Constraints on neutrino-neutrino interactions from primordial nucleosynthesis*, Phys. Lett. **B333**, 132 (1994), `hep-ph/9404339`.

[35] R. Laha, B. Dasgupta and J. F. Beacom, *Strong constraint on dark matter - neutrino interactions via light vector bosons*, (2013), `1304.3460`.