Airborne Sound Insulation of Floating Floors

by

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Stockholm 2000
AIRBORNE SOUND INSULATION OF FLOATING FLOORS

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PREFACE

This thesis is the result of the research project “Ljudisoleringsegenskaper hos flytande golv” (Sound Insulation Properties of Floating Floors). The work was carried out at the Division of Building Sciences, Kungl Tekniska Högskolan, Stockholm. The research project was founded by the Swedish Council for Building Research (BFR), which is gratefully acknowledged.

I started my research by comparing and studying different theories for predicting the airborne sound insulation of single plates. This resulted in paper II, entitled “Airborne sound insulation of single walls – a critical review”. The paper considers theories for ‘thin’ plates, i.e., models which are based on the Kirchhoff plate equation. The paper closely follows the theory earlier published by the co-author Prof. Sten Ljunggren in 1991. In this paper the amount of work for Sten was 40%.

After this, I developed an analytical model for predicting the airborne sound insulation of floating floors. This work resulted in paper I, entitled “Airborne sound insulation of floating floors”. This paper is submitted to Acta Acustica.

I wish to thank Professor Sten Ljunggren for the support and guidance during my work with the papers. I also wish to thank my colleague Olivier Fégeant for helping me in my troubles with various softwares for technical computing.
NOMENCLATURE

A  Area of plate (Sewell\(^1\), Leppingon\(^2\)) \([\text{m}^2]\)
B  Bending stiffness \([\text{Nm}]\)
C\(_g\)  Group velocity \([\text{m/s}]\)
E  Young’s modulus, complex \([\text{N/m}^2]\)
E\(_s\)  Young’s modulus, real \([\text{N/m}^2]\)
L  Plate dimension \([\text{m}]\)
L\(_m\)  Mean free path \([\text{m}]\)
L\(_r\)  Rise length \([\text{m}]\)
P\(_i\)  Incident power \([\text{W}]\)
P\(_r\)  Radiated power \([\text{W}]\)
R  Sound reduction index \([\text{dB re. } 2 \times 10^5 \text{ Pa}]\)
S\(_i\)  Area of plate 1 \([\text{m}^2]\)
S\(_3\)  Area of plate 3 \([\text{m}^2]\)
S\(_R\)  Area of plate facing receiving room (Paper II) \([\text{m}^2]\)
S\(_S\)  Area of plate facing sending room (Paper II) \([\text{m}^2]\)
S\(_S\)  Area of excited / radiating part of plate (Paper I) \([\text{m}^2]\)
U\(_S\)  Circumference of the excited / radiating part of plate (Paper I) \([\text{m}]\)
U  Form factor (Sewell\(^1\))
Y  Plate admittance \([\text{m}^3/\text{Ns}]\)
a  Plate dimension \([\text{m}]\)
b  Plate dimension \([\text{m}]\)
c  Velocity of sound in air \([\text{m/s}]\)
c\(_b\)  Velocity of bending waves \([\text{m/s}]\)
f  Frequency \([\text{Hz}]\)
f\(_c\)  Coincidence frequency \([\text{Hz}]\)
j  \(\sqrt{-1}\)
k  Exciting wave number \((k = k_s \sin \theta)\) \([\text{rad/m}]\)
k\(_s\)  Wave number \([\text{rad/m}]\)
\( k_B \quad \text{Bending wave number (complex) [rad/m]} \)
\( k_{B,r} \quad \text{Bending wave number (real) [rad/m]} \)
\( k_r, k_x \quad \text{Transform wave number [rad/m]} \)
\( m \quad \text{Surface density [kg/m}^2\)]
\( n' \quad \text{Number of mounts per unit area} \)
\( p_1 \quad \text{Pressure exciting plate 1 [Pa]} \)
\( p_3 \quad \text{Pressure exciting plate 3 [Pa]} \)
\( s \quad \text{Spring constant [kg/s}^2\)]
\( v \quad \text{velocity [m/s]} \)
\( x \quad \text{direction in coordinate system} \)

**Additional index**

1 \( \quad \) index referring to the floating slab
3 \( \quad \) index referring to the load-bearing slab
r \( \quad \) resonant
f \( \quad \) forced

**Greek letters**

\( \phi \quad \text{Angle between the direction of the velocity of the forced wave and the normal of the boundary [rad]} \)
\( \Lambda \quad \text{Length of boundary [m]}
\quad \text{(in Sewell\(^1\); relation height/width of testing wall)} \)
\( \psi \quad \text{Angle between the direction of the velocity of the free wave and the normal of the boundary [rad]} \)
\( \alpha \quad \text{Absorption factor} \)
\( \varepsilon \quad \text{Loss factor (Josse and Lamure\(^3\))} \)
\( \theta \quad \text{Angle of incidence [rad]} \)
\( \eta \quad \text{Loss factor} \)
\( \eta_{eq} \quad \text{Equivalent loss factor} \)
\( \lambda \quad \text{Wave length [m]} \)
\( \nu \quad \text{Poisson's ratio} \)
\( \pi \quad 3.14159\ldots \)
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INTRODUCTION

Floating floors has been used for a long time as a method of increasing the sound insulation of a floor construction. The result has varied, much because the lack of a usable prediction model. The use of a floating floor is not only beneficial for the airborne sound insulation, which is treated in this thesis, but also for the impact noise insulation. This means that even hard floor coverings, such as ceramic tiles, may by used with a satisfying result.

The sound reduction of a building partition will depend on both the direct sound transmission as well as the flanging transmission. The papers in this thesis only treats the direct sound transmission.

In the first article, an analytical model for predicting the sound reduction index of finite size floating floors is presented. The model consists of two stiff plates with a resilient layer in between, where the bottom plate is the load-bearing slab and the top plate and the resilient layer compose the floating floor.

The problem is treated somewhat differently for different frequency regions. The low frequency range is defined as the range below the critical frequency, $f_c$, of both plates. The mid-frequency range is between the critical frequencies of the plates. The high frequency range is above $f_c$ for both plates.

For low frequencies, the sound reduction index is derived using the same concept of resonant and forced velocity fields of a plate as that used by Ljunggren\textsuperscript{4}. The bottom plate, plate 3, is assumed to be excited by an incident propagating plane sound wave with the trace wave number $k$. It is assumed that the influence of the intermediate layer can be described by the simple spring model used by e.g. Cremer et al.\textsuperscript{4} It is also assumed that the intermediate layer is fairly resilient so that the force acting on the floating slab, plate 1, can be taken as the displacement of plate 3 times the stiffness of the spring.

For midrange frequencies, it is thought that the main coupling between the plates will occur at the coincidence angle of the load-bearing plate. Above the critical frequency of both plates, transmission will occur at the angle of coincidence of each plate.

In the case of high frequencies, it can be assumed that plate 3 will couple to plate 1 at the angle of of each plate, $\theta_{c3}$ and $\theta_{c1}$. For the coupling that occurs at coincidence angle of plate 3, the conditions are the same as for midrange frequencies. The sound transmission factor for the whole construction can now be written as the sum of the transmission at $\theta_{c3}$ and the transmission at $\theta_{c1}$.

In the paper, it is shown that the plates will interact, so that the sound insulation improvement to some extent will depend on the properties of the load-bearing slab. Further, it is shown that the sound insulation improvement of the floating floor will increase with increasing area of the floating slab, but decrease when increasing the
excited and radiating area. It is seen that the floating slab is preferably as heavy and highly damped as possible. Note though the assumption that the bottom plate is much heavier than the top plate. At low and mid-frequencies the sound reduction index will increase with 20lg(m) for increasing mass of plate 1, and somewhat more at high frequencies.

It is found that the expected sound insulation improvement due to the floating floor is significantly lower, especially in the high frequency region, than that presented by Cremer et. al.5.

The second paper contains a critical review of analytical models for the airborne sound insulation of 'thin' plates, that is, models based on the Kirchoff plate equation. It is found that several works agree on the general expression for the sound insulation at frequencies well below the critical frequency except for the radiation factors, which are, of course, very important factors in this context. In the case of the radiation of the forced plate field the disagreement is profound and values of the radiation factor from less than unity to infinity has been found. In the case of reverberant plate fields, the disagreement is much smaller. No serious disagreement in the radiation factor is found for frequencies above the critical frequency. Comparing the transmission factor for frequencies above the coincidence frequency, it is found that in the prediction formula presented by Cremer et. al.5, the radiation factor and the relation in size between the radiating area and the total area was not taken into account. In contrast to Ljunggren,6 neither Cremer et. al.5 nor Josse and Lamure3 have regarded the forced transmission above the coincidence frequency.

References


LIST OF PAPERS

1. Airborne Sound Insulation of Floating Floors
   Ulrica Kernen
   Submitted to Acta Acustica

2. Airborne Sound Insulation of Single Walls – a critical review
   Ulrica Kernen and Sten Ljunggren
Paper I
Airborne Sound Insulation of Floating Floors

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Abstract
An analytical model is presented for the sound reduction index of finite size floating floors. The model consists of two elastic plates with a resilient layer in between. The bottom plate, the load-bearing slab, is assumed to be excited with a diffuse airborne sound field. The top plate and the resilient layer compose the floating floor. The problem is solved for frequencies below, between and above the critical frequencies of the plates. Above the critical frequency of the load-bearing plate, but below that of the floating slab, the main coupling between the plates will occur at the coincidence angle of the load-bearing plate. Above the critical frequency of both plates, transmission will occur at the angle of coincidence of each plate. As the plates will interact, the sound insulation improvement will to some extent depend on the properties of the load-bearing slab. In the article it is shown how the sound reduction index depends on the physical parameters and the geometry of the plates.

1. Introduction

1.1 Historical review
The probably most famous work in this field was published by Cremer\(^1\) in 1952, when he presented his well-known equation

\[ \Delta L = 10 \lg \left( \frac{f}{f_0} \right) \]

where \( f \) is the frequency and \( f_0 \) is the mass-spring-mass resonance frequency. Cremer’s model treats two homogeneous infinite plates with a intermediate resilient layer that acts as a spring. In many cases however this equation was found to overestimate the sound insulation improvement. In 1954 Cremer\(^2\) explained this as caused by sound bridges, which has ever since been a popular explanation.

In 1956, Gösele\(^3\) point out that equation (1) shows good agreement with measurement results when the resilient layer has a relatively high flow resistance. To explain the overestimated sound insulation improvement observed in certain cases, he suggested that wave propagation in the resilient layer could have a deteriorating effect.
Shortly after, Foti\textsuperscript{4} published a new theory where the plates no longer are infinite. Both plates are considered to be simply supported and their vibrations are described by the sum of eigenfunctions. Still, his theory leads to the same frequency dependence as Cremer’s equation, i.e. an increase by 40 dB/octave.

In 1967 Cremer et al.\textsuperscript{5} examined the case where the structural slab is primarily excited. They found that the sound reduction improvement could be written as

$$\Delta L \approx 40 \log \left( \frac{f}{f_0} \right) + 10 \log \left( \frac{f_{c1}}{f_{c1}} \right)^2,$$

where \( f_{c1} \) and \( f_{c3} \) are the coincidence frequencies of the plates. If \( f_{c1} \gg f_{c3} \) this expression turns into equation (1).

A different type of resonantly reacting floating floors was examined by Vér\textsuperscript{6,7,8}. In his model the floating slab is not resting on a continuous resilient layer but on load-bearing resilient unit-mounts. The space between these mounts is filled with flow-resistive material in order to prevent horizontal propagation of sound waves. A number of assumptions are made, such as that there is no correlation of the slab motion at the different mounts. The unit mounts are modelled as pure springs, and all sound transmission from plate 1 to plate 2 is assumed to be transmitted through these mounts. By use of SEA (Statistical Energy Analysis) Vér derived the sound reduction improvement of floating floors as

$$\Delta L = 10 \log \left( \frac{2.3 c_{11} t_1 \eta_1 n' f^3}{2 \eta_0 f_0^4} \right),$$

where \( n' \) is the number of resilient mounts per unit area, \( \eta_1 \) and \( t_1 \) are the loss factor and the thickness of plate 1 and \( c_{11} \) is the longitudinal wave speed in plate 1.

In 1973 Nilsson\textsuperscript{9} presented a model for finite size floating floors. The load-bearing slab, plate 3, was assumed to be simply supported and excited by a bending moment along one of the boundaries. The floating slab has free ends and rests on a continuous elastic interlayer. The coupling between the plates is still considered to be local.

If the floating slab is highly damped Nilsson’s expression agrees exactly with equation (2) by Cremer. If plate 1 was lightly damped the sound insulation improvement was derived as

$$\Delta L = 10 \log \left( \frac{f_0^5 \eta_2 2 \pi \left( f_{c2}^2 - f_{c1}^2 \right)^2 ab}{c f_0^3 f_{c1}^2 f_{c3}^2 (a + b)} \right) + 10 \log \left( \frac{\sigma_3}{\sigma_1} \right),$$

in the case of \( k_{b1} \propto k_{b3} \), and
\[ \Delta L = 10 \log \left( \frac{f^{3/2} \eta_z 2\pi \left( f_{c3}^2 - f_{c1}^2 \right)^2 \sigma_1}{c f_0^3 f_{c3}^3 f_{c1}^3 (a + b)} \right) + 10 \log \left( \frac{\sigma_1}{\sigma_3} \right), \]  

(5)

if \( k_{m1} \gg k_{m3} \). The first part of the equations represents the velocity level difference, by addition of the second term the full expression for the sound insulation improvement is obtained. Note here that \( \eta_3 \) is a loss factor assigned to the intermediate layer, \( a \) and \( b \) are the dimensions of the plates and \( \sigma_1 \) and \( \sigma_3 \) the radiation factors of the plates. In this case \( \Delta L \) will increase with 25 dB/octave, assuming that \( \eta_3, \sigma_1 \) and \( \sigma_3 \) are independent of the frequency.

In 1979 Ljunggren\(^{10}\) published measurements that showed how different parameters influence the sound reduction improvement of floating floors. For frequencies above the critical frequency of the floating slab, plate1, the loss factor of plate1 is shown to increase the sound insulation by approximately 100\( \log(\eta) \) and increased thickness of plate1 will increase the insulation by 30\( \log(t) \).

In 1982 Widén\(^{11}\) presented measurements where he compared concrete tiles with a continuous concrete plate. By this subdivision of the plate, each tile will behave as a lumped mass on a spring, and the sound insulation of the floating floor was increased by approximately 6 – 10 dB.

Gudmundsson\(^{12}\) solved the problem using spatial Fourier Transforms, starting with Heckl’s solution for single infinite plates (see Cremer et.al.\(^{5}\)). In the case of a point force excitation, assuming that the same force is acting on the floating slab as on the structural slab, the insulation improvement for locally reacting floating floors is presented as

\[ \Delta L = 10 \log \left( \frac{\int \left[ N_3 + N_0 \right]^2 \sqrt{1 - \xi^2}^3 \xi d\xi}{\int \left[ N_1 + N_3 + N_2 N_1 / N_2 \right]^2 \sqrt{1 - \xi^2}^3 \xi d\xi} \right), \]  

(6)

where

\[ N_0 = \frac{j \omega c}{\sqrt{1 - \xi^2}}, \]  

(7)

\[ N_1 = \left[ B_1 (k\xi)^4 - \omega^3 m_1 \right] + j \left[ \frac{\omega c}{\sqrt{1 - \xi^2}} + B_1 \eta (k\xi)^4 \right], \]  

(8)

\[ N_2 = s (1 + j \eta_2). \]  

(9)
Here \( \rho \) is the density of the air, \( \omega \) is the angular frequency, \( m \) is the mass per square meter and \( B \) is the bending stiffness. \( N_i \) is defined analogously to \( N_1 \) and \( \xi = k_e/k \), where \( k \) is the exciting wave number and \( k_e \) is a transform wave number.

Gudmundsson\textsuperscript{12} solved this equation numerically and when compared, it agreed well with equation (1) in the case of a heavy structural slab and with equation (2) if the structural slab was light. In the case of airborne excitation, he deduced an equation similar to equation (6), with the square roots \( \sqrt{1 - \xi^2} \) replaced with \( (1 - \xi^2) \).

In the case of resonantly reacting floating floors, Gudmundsson pointed out that the sound isolation improvement should be corrected with a term

\[
\Delta L_{res} = -10 \log \left( 1 + \frac{\sigma_{1,\text{res}}}{\sigma_{\text{forced}}} \right),
\]  

(10)

where \( \sigma_{1,\text{res}} \) and \( \sigma_{\text{forced}} \) are the radiation factors of the resonant and the forced field of the floating slab. For \( f < f_c \), \( \sigma_{1,\text{res}} \) is approximated to 0, for \( f > f_c \) \( \sigma_{1,\text{res}} \approx 1 \) and \( \sigma_{\text{forced}} \approx 1 \).

In the case of a free floating slab and a simply supported load-bearing slab the ratio factor, defined as \( Y = \langle v_{1,\text{res}}^2 \rangle / \langle v_{1,\text{forced}}^2 \rangle \), where \( v_{1,\text{res}} \) and \( v_{1,\text{forced}} \) the velocity of the resonant and forced waves, is presented as

\[
Y_{GR} = \frac{U^2}{3 \pi \eta_1 S} \left( \frac{k_{b3}}{k_{b1}} \right)^2.
\]  

(11)

If the load-bearing slab instead is clamped, Gudmundsson gives the ratio factor as

\[
Y_{CR} = \frac{I_c U^2}{4 \pi \eta_1 S} \left( \frac{k_{b3}}{k_{b1}} \right),
\]  

(12)

where \( I_c \) is the squared equivalent width of the coupling strip (\( w_c \)), averaged over the angles of incidence, \( S \) is the plate area and \( U \) is the length of the plate boundary. This expression has to be evaluated numerically.

1.2 An outline of the present work

In the present article, an analytical model for predicting the sound reduction index for airborne sound of finite floating floors is presented. In the following, the upper floating slab is named plate 1, the load bearing plate is named plate 3. Both the plates have a finite area. In between the plates is a resilient layer, layer 2. Plate 3 is excited by a diffuse airborne sound field, which is composed of plane waves propagating at an angle \( \theta \), see figure (1).

The problem is treated somewhat differently for different frequency regions. The low frequency range is defined as the range below the critical frequency, \( f_c \), of both
plates. The mid-frequency range is between the critical frequencies of the plates. The high frequency range is above $f_c$ for both plates.

For low frequencies, the sound reduction index is derived using the same concept of resonant and forced velocity fields of a plate as that used by Ljunggren\textsuperscript{13}. Plate 3 is assumed to be excited by an incident propagating plane sound wave with the trace wave number $k$. It is assumed that the influence of the intermediate layer can be described by the simple spring model used by e.g. Cremer et.al.\textsuperscript{5}. It is also assumed that the intermediate layer is fairly resilient so that the force acting on plate 1 can be taken as the displacement of plate 3 times the stiffness of the spring. Knowing the excitation of plate 1, the sound transmission of the total construction can be derived.

Both plates exhibit resonant and forced fields. The sound transmission factors for all four combinations are derived separately, and the transmission factors are summed up in the end.

For midrange frequencies, i.e. $f_{c3} < f < f_{c1}$, the coupling of the plates behaves differently. In order to describe the coupling the concept of “plate admittance” is now introduced. It is defined as $Y = \frac{v}{p}$, where $v$ is the forced velocity response of the plate and $p$ is the pressure acting on the plate. This admittance is a complex quantity and a function of the frequency as well as the forcing wave number. The main behaviour of the normalised admittance of the two plates are illustrated in figure (2) as a function of the angle of incidence. $Y_o$ is here defined as $Y_o = \frac{v_o}{p_o}$ where $v_o$ is $10^9$ m/s and $p_o$ is $2 \times 10^5$ Pa.

If a diffuse airborne sound field excites plate 3, figure (2) shows that a predominant part of the response of this plate is due to the coupling at the coincidence angle $\theta_{c3}$. As the response of plate 1 does not vary very much with the angle of incidence and as the sound reduction index of plate 3 is determined by the coupling with the airborne sound field at this angle, it is thought that all important coupling between the plates occurs at $\theta_{c3}$.

The examples have been calculated assuming a density of 2400 kg/m$^3$, a Young’s modulus of 26 GPa and a loss factor of 2%.
In the case of high frequencies, it can be assumed that plate 3 will couple to plate 1 at two angles, $\theta_{c3}$ and $\theta_{c1}$. The normalised admittance with respect to the forced response are illustrated in figure (3). For the coupling that occurs at $\theta_{c3}$ the conditions are the same as for midrange frequencies. The sound transmission factor for the whole construction can now be written as the sum of the transmission at $\theta_{c3}$ and the transmission at $\theta_{c1}$.

The scope of this article is to show the influence of the area, the critical frequency and the damping of the floating slab on the sound insulation.

1.3 Main limitations and assumptions of the model

The plates are assumed to be of uniform thickness and made from a homogeneous, isotropic and linear elastic material. It is also assumed that plate 3 is much heavier than plate 1. No regard is taken of the boundary conditions at the edges of the plates.

As mentioned earlier, it is assumed that the intermediate layer can be described by the simple spring model, used by e.g. Cremer et al., and that the frequency is high enough so that the force acting on plate 1 can be taken as the displacement of plate 3 times the spring constant $s$.

Further, it is assumed that the area of the load-bearing slab, $S_L$, and the floating slab, $S_f$, is larger than the excited part of the area, $S_e$. It is also assumed that the excited area of the load-bearing slab and the radiating area of the floating slab has the same size and are situated directly above each other (see figure (4)).
2. The analytical model

For the present analysis, the plates are assumed to be of uniform thickness and made from a homogeneous, isotropic and linear elastic material. It is also assumed that plate 3 is much heavier than plate 1. The whole construction is supposed to be surrounded by air but the influence of the radiation load on the plates is neglected throughout. The model can therefore be used only for sufficiently heavy plates. Plate 3 is assumed to be excited by an incident propagating plane sound wave. The incident and the reflected wave act on the plate with a pressure \( p(x) \) where

\[
p(x) = p_m e^{-j \omega x - j k_x x}.
\]

(13)

Here \( x \) is the coordinate according to figure (4), \( \omega \) is the angular frequency, \( t \) is the time and \( k \) is the trace wave number of the exciting pressure along the plate surface,

\[
k = k_a \sin \theta,
\]

(14)

where \( \theta \) is the angle of incidence according to figure (1) and \( k_a \) is the wave number in air. The area of plate 1 and 3 are denoted \( S_1 \) and \( S_3 \), respectively, and \( S_s \) is the area of the excited part of plate 3.

2.1 Forced and resonant fields

As the concept of forced an resonant response are defined differently in literature, an elucidation may be appropriate here.

Consider first an infinite plate, excited over part of the area, see figure (5), by an airborne sound field in a room below. A bending wave field will then be excited in the plate. In the part of the plate which is inside the boundaries of the room, the bending wave field will be strongly correlated to the forcing airborne field. This bending wave field is called forced.

Due to the excitation, a bending wave field will also be generated in the plate outside the boundaries of the room. This field consists of waves propagating at the phase speed of the free bending wave. For high frequencies, the waves will mainly

Figure 4. Definition of the areas \( S_1, S_3 \) and \( S_s \)
emerge from the boundaries towards the unexcited part of the plate, but for lower frequencies account must be taken also of the waves propagating in the opposite direction. In reality, no plates are infinite, and there will be reflections at the boundaries of the plate. If the plate is not too large and the damping is not to high, the reflections will give rise to a field where the amplitude does not vary very much over the surface, a resonant field.

![Figure 5. Illustration of the generated bending field outside the excited area.](image)

**2.2 The low frequency range where \( f < f_{C3}, f < f_{C1} \)**

In this first part, excitation with wave numbers smaller than the bending wave number of both plate 3 and plate 1 is considered.

**2.2.1 Transmission due to forced/forced fields**

In this first case forced fields in both plates are considered. The exciting wave number \( k_a \) and hence also the trace wave number \( k \) is much smaller than both the bending wave number of plate 3 and that of plate 1. The transverse velocity of the excited part of plate 3 can be taken as (Ljunggren\(^\text{15}\))

\[
v_3(x) = \frac{P_{in}}{j \omega m_3} \left[ e^{-jkr} - \frac{1}{2} e^{-jkr} \cos(k_{p}x) - \frac{1}{2} e^{+jkr} \cosh(k_{p}x) \right],
\]

where \( m \) is the surface weight, \( P_{in} \) is the amplitude of the exciting pressure. The time factor \( e^{j \omega t} \) is omitted here and in all similar cases in this paper. The first part of the expression represents the forced wave. The other two terms represent the free waves, of which one is the propagating wave and the other an exponential nearfield. Since the amplitude of the free waves is smaller than that of the forced wave and the radiation factor usually smaller, the free waves are neglected. Hence, expression (15) is simplified to

\[
v_{3,f}(x) = \frac{P_{in}}{j \omega m_3} e^{-jkr}.
\]
The first index of the velocity specify the plate number, the second index is \( f \) as in "forced". The motion of plate 3 will cause a pressure on plate 1,

\[
p_i(x) = p_i e^{-j\omega x} = \frac{s v_{1,f}}{j\omega} e^{-j\omega x},
\]

(17)

where \( s \) is the spring stiffness of the elastic intermediate layer. In the same manner the velocity of the forced vibrations of plate 1 can be taken as

\[
v_{1,f}(x) = \frac{P_1}{j\omega m_i} e^{-j\omega x} = \frac{s P_m}{-j\omega^2 m_i m_3} e^{-j\omega x}.
\]

(18)

The Fourier transform of the velocity can be written as

\[
v_{1,f}(k_x) = \int_L v_{1,f}(x)e^{j\omega x}dx = \frac{s P_m}{-j\omega^2 m_i m_3} \frac{2\sin[(k_x-k)L]}{(k_x-k)}.
\]

(19)

where \( L \) is the length according to figure (4). The radiated power can be calculated according to Cremer et al.\(^5\) as

\[
P_{\text{rad}} = \left( \frac{\rho c k_a}{4\pi} \right) \int_{-k_x}^{k_x} v(k_x)^2 \frac{1}{\sqrt{k_a^2-k_x^2}} dk_x =
\]

\[
= \frac{4\rho s^2 P_m^2}{4\pi\omega^2 m_i^2 m_3} \int_{-k_x}^{k_x} \frac{\sin^2[(k_x-k)L]}{(k_x-k)^2} \frac{1}{\sqrt{k_a^2-k_x^2}} dk_x,
\]

(20)

where \( \rho \) is the density of air and \( c \) is the velocity of sound in air. The radiation efficiency \( \sigma \) is introduced as (see also Ljunggren\(^13\))

\[
\sigma = \left( \frac{k_a}{\pi L} \right) \int_{-k_x}^{k_x} \frac{\sin^2[(k_x-k)L]}{(k_x-k)^2} \frac{1}{\sqrt{k_a^2-k_x^2}} dk_x.
\]

(21)

As plate 3 is considered heavy, the power incident on the plate can for this two-dimensional case, be calculated as (Ljunggren\(^13\))

\[
P_m = \frac{P_m^2 2L \cos \theta}{8\rho c}.
\]

(22)

The transmission factor is taken as the ratio between the power radiated from the structure and the incident power, which gives
\[ \tau(\theta) = \frac{P_{\text{rad}}}{P_{\text{m}}} = \left( \frac{\rho cs}{\omega^3 m_1 m_2} \right)^2 \frac{4\sigma}{\cos \theta}. \] (23)

If the transmission factor is integrated over the angle of incidence according to Paris' formula,

\[ \bar{\tau} = 2 \int_0^{\pi} \tau(\theta) \cos \theta \sin \theta \, d\theta, \] (24)

the averaged transmission factor for the three-dimensional case becomes

\[ \bar{\tau}_{f,r} = \left( \frac{2 \rho cs}{\omega^3 m_1 m_2} \right)^2 2\sigma_d. \] (25)

The first index \( f \) stands for forced transmission in plate 3, second index \( r \) for forced transmission in plate 1, and as in resonant transmission will also be used later on. The radiation factor \( \sigma_d \) is defined as

\[ \sigma_d = \int_0^{\pi} \sigma \sin \theta \, d\theta. \] (26)

\( \sigma_d \) is further discussed in the conclusions.

### 2.2.2 Transmission due to forced/resonant fields

The possibility of forced wave in plate 3 giving rise to reverberant field in plate 1 will now be considered. The power radiated from the upper surface of plate 1 can be written as

\[ P_{\text{rad}} = \langle v_{1r}^2 \rangle \rho cs \sigma, \] (27)

or, with the help of the relation between the velocity of the reverberant and forced vibrations derived in the appendix

\[ \frac{\langle v_{1r}^2 \rangle}{\langle v_{1f}^2 \rangle} = \frac{\pi S_s f_c}{4 \eta_1 S_i f} \sigma, \] (28)

and equation (18) as

\[ P_{\text{rad}} = \frac{s^2 p_{m}^2}{\omega^3 m_1^2 m_2^3} \frac{\pi S_s f_c}{8 \eta_1 S_i f} \rho cs \sigma^2. \] (29)
Here \( \eta_1 \) is the loss factor of plate 1 and \( U_s \) is the length of the boundary of the excited part of the plates and \( \sigma_1 \) is the radiation factor with respect to low frequencies, as

\[
\sigma_1 = \frac{1}{2\pi^2} \frac{U_s \lambda_c}{S_s} \frac{\sqrt{f}}{f_{c1}}. \tag{30}
\]

The incident power must in this three-dimensional case be taken as

\[
P_{in} = \frac{p_0^2 S_s \cos \theta}{8 \rho c}, \tag{31}
\]

which gives the transmission factor as

\[
\tau_{f,s} (\theta) = \frac{P_{rad}}{P_{in}} = \left( \frac{2 \rho c s}{\omega^3 m_1 m_3} \right)^2 \frac{\pi}{4 \eta_1} \frac{S_s}{S_1} \frac{f_{c1}}{f} \sigma_1^2 \frac{1}{\cos \theta}. \tag{32}
\]

The average transmission factor, integrated according to equation (24), becomes

\[
\bar{\tau}_{f,s} = \left( \frac{2 \rho c s}{\omega^3 m_1 m_3} \right)^2 \frac{\pi}{2 \eta_1} \frac{S_s}{S_1} \frac{f_{c1}}{f} \sigma_1^2. \tag{33}
\]

### 2.2.3 Transmission due to resonant/forced fields

In this case, the possibility of resonant waves in plate 3 giving rise to a forced field in plate 1 is considered. The radiated power can in this case be written as

\[
P_{rad} = \langle v_{1,f}^2 \rangle \rho c S_s \sigma_3. \tag{34}
\]

Since the vibrational field in plate 1 is forced the radiation factor of plate 1 is equal to that of plate 3, i.e. \( \sigma_3 \) is calculated in the same manner as equation (30) (using \( \lambda_{c3}, f_{c3} \) instead of \( \lambda_{c1} \) and \( f_{c1} \)). \( \langle v_{1,f}^2 \rangle \) is calculated according to the first part of equation (18) where the squared pressure exciting plate 1 in this case will be

\[
p_{1}^2 = \frac{s^2 \langle v_{3,f}^2 \rangle}{\omega^2}. \tag{35}
\]

To calculate the velocity of the resonant field in plate 3, the relation between the velocity of the reverberant and the forced field may be used (see appendix I)
\[
\frac{\langle v_{3,r}^2 \rangle}{\langle v_{3,f}^2 \rangle} = \frac{\pi}{4\eta_3} \frac{S_S f c_3}{S_3 f} \frac{S_f}{S_3} \frac{s^2}{\omega^2 m_1^2} \rho c S_3 \sigma_3.
\]  
\[\text{(36)}\]

and the radiated power becomes
\[
P_{\text{rad}} = \langle v_{3,f}^2 \rangle \frac{\pi}{4\eta_3} \frac{S_S f c_3}{S_3 f} \frac{S_f}{S_3} \frac{s^2}{\omega^2 m_1^2} \rho c S_3 \sigma_3.
\]
\[\text{(37)}\]

With \(P_\text{in}\) according to equation (31) and the velocity of plate 3 according to equation (16), the transmission factor becomes
\[
\tau(\theta) = \left(\frac{2\rho c s}{\omega^2 m_1 m_3}\right)^2 \frac{\pi}{4\eta_3} \frac{S_S f c_3}{S_3 f} \frac{s^2}{\omega^2 m_1^2} \frac{1}{\cos \theta}.
\]
\[\text{(38)}\]

The averaged transmission factor integrated according to equation (24) is
\[
\tau_{av} = \left(\frac{2\rho c s}{\omega^2 m_1 m_3}\right)^2 \frac{\pi}{2\eta_3} \frac{S_S f c_3}{S_3 f} \frac{s^2}{\omega^2 m_1^2} \sigma_3.
\]
\[\text{(39)}\]

2.2.4 Transmission due to resonant/resonant fields

In the forth and last case for the low frequency region, it is assumed that the resonant waves in plate 3 cause a resonant field in plate 1. The radiated power is taken as
\[
P_{\text{rad}} = \langle v_{1,r}^2 \rangle \rho c S_3 \sigma_1.
\]
\[\text{(40)}\]

With the velocity of the resonant field in plate 3 according to equation (36) and (16), the pressure exciting plate 1 can be calculated according to equation (35). The velocity of the reverberant field in plate 1 may now be calculated with help of equation (28) and (18), and the radiated power becomes
\[
P_{\text{rad}} = \frac{\rho c s^2 p_{\text{in}}}{\omega^2 m_1^2 m_3^2} \pi^2 \frac{S_S^2 f c_3}{S_3 S_f} \sigma_1^2 \sigma_3.
\]
\[\text{(41)}\]

With \(P_{\text{in}}\) according to equation (31) the transmission factor becomes
\[
\tau(\theta) = \frac{P_{\text{rad}}}{P_{\text{in}}} = \left(\frac{2\rho c s}{\omega^2 m_1 m_3}\right)^2 \frac{\pi^2}{2\eta_3 \eta_1} \frac{S_S^2 f c_3}{S_3 S_f} \sigma_1^2 \sigma_3 \frac{1}{\cos \theta}
\]
\[\text{(42)}\]

and
\[
\bar{\tau}_{\text{rer}} = \left( \frac{2 \rho \sigma s}{\omega^3 m_s m_3} \right)^2 \left( \frac{\pi}{\eta_1 \eta_3} \right) S_s^2 \frac{f_{\text{Cl}} f_{\text{C3}}}{f^2} \sigma_1^2 \sigma_3^2 . \tag{43}
\]

Four different transmission factors for the low frequency region are now derived. These are all summed up to a total transmission factor,
\[
\bar{\tau}_{\text{tot}} = \bar{\tau}_{f \rightarrow f} + \bar{\tau}_{f \rightarrow r} + \bar{\tau}_{r \rightarrow f} + \bar{\tau}_{r \rightarrow r} , \tag{44}
\]
\[
\bar{\tau}_{\text{tot}} = \left( \frac{2 \rho \sigma s}{\omega^3 m_s m_3} \right)^2 \left( 2 \sigma_4 + \frac{\pi}{2 \eta_1} \frac{S_s}{S_1} \frac{f_{\text{Cl}}}{f} \sigma_1^2 \sigma_3^2 \right)
+ \frac{\pi}{2 \eta_1} \frac{S_s}{S_3} \frac{f_{\text{C3}}}{f} \sigma_3^2 + \frac{\pi^2}{4 \eta_1 \eta_3} \frac{S_s^2}{S_1 S_3} \frac{f_{\text{Cl}} f_{\text{C3}}}{f^2} \sigma_1^2 \sigma_3^2 . \tag{45}
\]

2.3 The mid-frequency range

The mid-frequency range well above the critical frequency of plate 3 but below that of plate 1 will now be considered. As explained in the introduction, all important coupling between the plates will here occur at the coincidence angle \( \theta_{\text{C3}} \). This implies that plate 3 will excite plate 1 with the wave number \( k_{n3} \).

If plate 3 is to be considered large in the sense that \( \eta k_{n} L / 4 \gg 1 \), the plate would be expected to be dominated by forced vibrations. However, since this model is mainly aimed for floating floors in dwellings, these plates will be considered small and not fulfill the criteria above. In Ljunggren\(^{13} \) it is shown that for plates of more modest dimensions, the response will be dominated by the free vibrations and if the plate is not too small, by the propagating waves alone.

Since plate 1 still is excited with small wave numbers though, both resonant and forced transmission will be of importance. First, the forced field is considered. The power radiated from plate 1 is taken as
\[
P_{\text{rd}} = \langle v_{1,f}^2 \rangle \rho c S_s \sigma_3 , \tag{46}
\]

where \( \langle v_{1,f}^2 \rangle \) is calculated according to equation (18) and (35). The velocity of the reverberant field in a plate of finite dimensions can be taken as
\[
\langle v_{3,r}^2 \rangle = \frac{P_r}{\omega m_3 \eta_3 S_s} , \tag{47}
\]

where \( P_r \) is the power travelling away from the excited area,
\[ P_k = 2c \beta m_3 v_{3,R}^2 \Lambda. \]  

(48)

\( \Lambda \) is the length of the boundary at \( x = L \) and \( \langle v_{3,R}^2 \rangle \) is the mean square value of the velocity of the free propagating bending wave \( v_{3,R}(x) \). Using the two-dimensional model of figure (4), Ljunggren\(^{13} \) has shown that \( v_{3,R} \) can be written as

\[ v_{3,R}(x) = \frac{p m}{2 \alpha m_3} \frac{k_{R2} \sin[(k - k_{R2})L]}{(k - k_{R2})} e^{-j_{R2}x}. \]  

(49)

The velocity of the reverberant field becomes

\[ \langle v_{3,R}^2 \rangle = \frac{p_m^2}{4\omega^2 m_2^2} \frac{k_{R2} \Lambda \sin^2[(k - k_{R2})L]}{(k - k_{R2})^2 \eta_3 S_3}. \]  

(50)

and the radiated power from plate 1

\[ P_{rad} = \frac{s^2}{2 \omega^2 m_1^2} \frac{p_m^2}{4 \omega^2 m_2^2} \frac{k_{R2} \Lambda \sin^2[(k - k_{R2})L]}{(k - k_{R2})^2 \eta_3 S_3} \rho c S_3 \sigma_3. \]  

(51)

With \( P_m \) according to equation (22) the transmission factor becomes

\[ \tau(\theta) = \left( \frac{2 \rho c s}{\omega^2 m_1 m_2} \right)^2 \frac{S_3}{S_2} \frac{k_{R2}}{2 \eta_3 L} \sigma_3 \sin^2\left[(k - k_{R2})L\right] \frac{1}{(k - k_{R2})^2} \frac{1}{\cos \theta}. \]  

(52)

where \( \sigma_3 \) is the radiation factor of plate 3. For frequencies above the coincidence frequency, the radiation factor can be evaluated in the same way as in Ljunggren's paper\(^{13} \) to

\[ \sigma_3 = \frac{1}{\sqrt{1 - \frac{f_{C1}}{f}}}. \]  

(53)

The average transmission factor integrated according to Paris' formula becomes

\[ \overline{\tau}_{av} = \left( \frac{2 \rho c s}{\omega^2 m_1 m_2} \right)^2 \frac{\pi}{2 \eta_3} \frac{S_3}{S_2} \frac{f_{C3}}{f} \sigma_3^2. \]  

(54)

Second, the case of resonant transmission in plate 1 is considered. The radiated power must in this case be taken as

\[ P_{rad} = \langle v_{1,R}^2 \rangle \rho c S_3 \sigma_1. \]  

(55)
Once again, the relation between the resonant and forced field according to equation (28) may be used and the radiated power becomes

\[ P_{rad} = \frac{s^2 \rho c^3}{8 \omega_0^2 m_1^2 m_2^2} \frac{S_2^2}{S_1 S_3 \eta_1 \eta_3} \frac{\pi k_{B1} f_{CL}}{f} \frac{\sin^2 \left[ \frac{(k - k_{B1})L}{(k - k_{B1})^2} \right]}{\sigma_1^2} \cdot \]  

(56)

The corresponding transmission factor is

\[ \tau(\theta) = \left( \frac{s \rho c}{\omega_0^2 m_1 m_2} \right)^2 \frac{S_2^2}{S_1 S_3 \eta_1 \eta_3} \frac{\pi k_{B1} f_{CL}}{L} \frac{\sin^2 \left[ \frac{(k - k_{B1})L}{(k - k_{B1})^2} \right]}{\cos \theta} \]  

(57)

and the averaged transmission factor for the case of resonant transmission in both plates becomes

\[ \bar{\tau}_{res} = \left( \frac{2 \rho c s}{\omega_0^2 m_1 m_2} \right)^2 \frac{\pi^2 S_2^2}{4 \eta_1 \eta_3} \frac{f_{CL} f_{C3}}{f^2} \sigma_1^2 \sigma_3. \]  

(58)

The total transmission factor for the mid-frequency region is given as

\[ \bar{\tau} = \bar{\tau}_{res} + \bar{\tau}_{rer} = \left( \frac{2 \rho c s}{\omega_0^2 m_1 m_2} \right)^2 \frac{\pi}{2 \eta_3} \frac{S_2}{S_3} \frac{f_{C3}}{f} \left( \sigma_3^2 + \frac{\pi}{2 \eta_1} \frac{S_2}{S_1} \frac{f_{CL}}{f} \sigma_1^2 \sigma_3 \right). \]  

(59)

2.4 The high frequency range

The high frequency range where \( f > f_{C3} \) and \( f > f_{CL} \), will now be considered. As in the mid-frequency range, all important coupling between the plates will occur at the coincidence angle, but in this case at both \( \theta_{C1} \) and \( \theta_{C3} \). This implies that plate3 will excite plate 1 with the wave numbers \( k = k_{B1} \) and \( k = k_{B3} \). The mean transmission factor for high frequencies is calculated as the sum of the contribution from the transmission at \( \theta_{C1} \) and \( \theta_{C3} \).

2.4.1 Coupling at \( \theta_{C3} \)

In the case of coupling at \( \theta_{C3} \) plate 3 is assumed to be excited with \( k = k_{B3} \) and so the resonant waves will be predominant. Plate 1 on the other hand is still excited with wave numbers smaller than \( k_{B1} \), which means that both forced and resonant transmission will be of importance. This implies that the plates will behave in the exact same manner as in the mid-frequency range and the transmission factors are given by equations (54) and (58).

2.4.2 Coupling at \( \theta_{C1} \)

The radiated power is in the case of coupling at \( \theta_{C1} \) taken as
\[ P_{\text{rad}} = \langle v_{i,k}^2 \rangle \rho c S \sigma_1. \] (60)

As plate 3 is excited with a wave number larger than \( k_{b3} \), the velocity of the excited part of the plate will according to Ljunggren\(^\text{13}\) be

\[ v_3(x) = \frac{\text{j} \omega \mu \rho}{B k^4} e^{-j k x} + \frac{p_{w} k_{b}}{2 \omega m k} \left[ e^{-j k_{b} x} \sin(kL - k_{b} x) + e^{-j k_{b} x} \sinh(k_{b} x - jkL) \right], \] (61)

where the first part of the equation represents the forced wave. The second and third terms are the free waves; the propagating bending wave and the exponential nearfield. Since the amplitude of the free waves is smaller than that of the forced wave, these free waves are disregarded. With \( k=k_{b1} \), the squared pressure on plate 1 calculated according to equation (17) becomes

\[ p_1^2 = \frac{s^2 \langle v_{3,f}^2 \rangle}{-\omega^2} = \frac{p_{w}^2 s^2 f_{c3}^4}{2 \omega^4 m_3^2 f_{c1}^4}. \] (62)

Since plate 1 is excited at its angle of incidence, the resonant waves will dominate. Using the same technique as in the preceding case, the radiated power can be derived as

\[ P_{\text{rad}} = \left( \frac{p_{w} k_{b1}}{2 \omega m_1} \right)^2 \frac{S_{b} \rho c S \sigma_1 \Lambda}{k_{b1} \eta_{1} S_{i}} \frac{\sin^2[(k-k_{b1})L]}{(k-k_{b1})^2}, \] (63)

where \( \sigma_1 \) is the radiation efficiency at high frequencies according to equation (53) and \( \Lambda \) is the length of the boundary at \( x = L \). The transmission factor becomes

\[ \tau(\theta) = \left( \frac{2 \rho c s}{\omega^2 m_1 m_2} \right)^2 \frac{k_{b1} S_{b} f_{c1}^4 \sigma_1}{4 \eta_{1} S_{i} L f_{c1}^4} \frac{\sin^2[(k-k_{b1})L]}{(k-k_{b1})^2} \frac{1}{\cos \theta} \] (64)

and

\[ \overline{\tau}_{rr} = \left( \frac{2 \rho c s}{\omega^2 m_1 m_2} \right)^2 \frac{\pi}{2 \eta_{1} S_{i} f_{c1}^4} \frac{S_{b}}{f_{c3}^4} f_{c1}^2 \sigma_1^2. \] (65)

The total transmission factor for high frequencies is now calculated as the sum of the contributions at \( \theta_{C1} \) and \( \theta_{C3} \) as
\[
\bar{\tau} = \bar{\tau}_{ref} + \bar{\tau}_{res} + \bar{\tau}_{f \tau} = \left( \frac{2 \rho c S}{\omega_0 m_1 m_2} \right)^2 \left( \pi \frac{S_S}{2 \eta_3} \frac{f_{CL} \sigma_3^2}{f} + \frac{\pi^2}{4 \eta_3 \eta_1} \frac{S_S^2}{S_1 S_2} \frac{f_{CL} f_{CL}^*}{f^2} \sigma_1^2 \sigma_3 \sigma_1^2 + \frac{\pi}{2 \eta_1} \frac{S_S}{S_1} \frac{f_{CL} f_{CL}^*}{f} \sigma_3 \sigma_1^2 \right).
\] (66)

Inspection shows that this expression is valid not only above \( f_{CL} \) but also at \( f_{CL} \), providing that an appropriate radiation factor is used.
3. Discussion

3.1 The characteristics of the present model

The sound reduction indices given in section 3 show that the airborne sound insulation is a function of the areas of the both plates. In dwellings, the area of the load-bearing slab, \( S_s \), is often much larger than both \( S_1 \) and \( S_3 \). While the excited part of the plate is thought to represent the area of a room, the load-bearing plate can extend over the entire flat. This is the case if the walls within the flat are lightweight constructions, e.g., gypsum panels and steel studs. The floating slab area can represent one or more rooms. In most lab measurements, though, the areas \( S_1 \), \( S_3 \), and \( S_s \) are the same and the area relation is little investigated.

In Figure (6) and (7), \( S_s \) and \( S_1 \) are varied from 10 to 200 m², while the other areas are kept fixed. Increasing the excited and radiating area \( S_s \) will decrease the sound insulation for all frequencies, but to a varying extent. Increasing the area \( S_1 \) will increase the sound insulation in all frequency regions although, for low and mid-frequencies, the influence of \( S_1 \) is less than for high frequencies (see also the expressions for the transmission factors presented in section 2).

![Figure 6](image1.jpg)  
*Figure 6. The sound reduction index, \( R \) for varying \( S_s \). \( S_1=200 \text{ m}^2 \), \( S_3=200 \text{ m}^2 \), \( S_s=10 \text{ m}^2 \) (---), \( 40 \text{ m}^2 \) (---), \( 120 \text{ m}^2 \) (---), \( 200 \text{ m}^2 \) (---).*

![Figure 7](image2.jpg)  
*Figure 7. R for varying \( S_1 \), \( S_s=10 \text{ m}^2 \), \( S_3=200 \text{ m}^2 \), \( S_1=10 \text{ m}^2 \) (---), \( 40 \text{ m}^2 \) (---), \( 120 \text{ m}^2 \) (---), 200 m² (---).*

In these and the following calculated examples it is assumed, where nothing else is mentioned, that the plates consist of concrete with \( \rho=2400 \text{ kg/m}^3 \), \( E=3\times10^7 \text{ N/m}^2 \), the loss factor \( \eta \) of plate 1 and 3 is 0.5% and 4%, respectively, \( S_1=20 \text{ m}^2 \), \( S_3=20 \text{ m}^2 \), \( S_s=100 \text{ m}^2 \), \( t_1=0.050 \text{ m} \), \( t_2=0.200 \text{ m} \), \( s=2\times10^6 \text{ kg/m}^3\text{s}^2 \).
From the expressions of the transmission factors in section 2 it is seen that the floating slab is preferably as heavy and highly damped as possible. Note though the initial assumption that \( m_3 \gg m_1 \). At low and mid-frequencies the sound reduction index will increase with \( 20 \log(m) \) for increasing mass of plate 1, and somewhat more at high frequencies.

From the sound transmission factors presented in section 2 it is evident that the sound insulation improvement of the floating floor construction will to some extent depend on the properties of plate 3. Figure (8) shows the variation of \( \Delta R \) with the critical frequency of plate 3. \( \Delta R \) is here defined as

\[
\Delta R = 10 \log \left( \frac{1}{\tau} \right) - 10 \log \left( \frac{1}{\tau_3} \right),
\]

where \( \tau \) is the total sound transmission factor according to section 2 and \( \tau_3 \) is the sound transmission factor of plate 3 if there were no floating floor present according to Ljunggren.\(^\text{15}\) In the transmission factor of plate 3 for low frequencies (equation (45)) the term which represents the forced transmission in plate 3 will be dominating. If the resonant part of the transmission is disregarded \( \Delta R \) for the low frequency region will be

\[
\Delta R = 20 \log \left( \frac{\omega^2 m_3}{s} \right) - 10 \log \left( 1 + \frac{\pi \frac{S_3}{2 \eta_1} \frac{S_{cl}}{S_1} \frac{\sigma_i^2}{2 \sigma_d} \right).
\]

In the mid-frequency region \( \Delta R \) becomes

\[
\Delta R = 20 \log \left( \frac{\omega^2 m_3}{s} \right) - 10 \log \left( 1 + \frac{\pi \frac{S_3}{2 \eta_1} \frac{S_{cl}}{S_1} \frac{\sigma_i^2}{\sigma_d} \right).
\]

When comparing the sound transmission at the low- and mid-frequency regions, the sound insulation improvement of the floating floor at a certain frequency will be somewhat higher if the critical frequency of plate 3 is above the frequency studied, than beneath. Since the critical frequency of plate 3 will decide where the low frequency region ends and the mid-frequency region begins, the properties of plate 3 will affect the sound insulation improvement of the floating floor for these regions.

Further, other parameters of plate 3 such as the area and the loss factor will to some extent have a certain influence on the sound insulation improvement of the floating floor.
3.2 Comparison with previous result

If $\Delta R$, defined as above, is compared to the $\Delta L$ presented by Cremer\(^1\) in equation (2), it is seen (figure (9)) that the sound insulation mainly differs in the high frequency region.

The expression for the transmission factor for high frequencies according to equation (66) shows that the sound reduction improvement will increase with $10\ln(\eta)$ for increasing loss factor. This is the same relation found in measurements by Ljunggren\(^10\) and in theory by Vét\(^{5,7,8}\) (see equation (3)). It is also seen that in the second term, which represents the resonant transmission in both plates and is the most dominating of the three terms in equation (66), the sound insulation will increase with $30\ln(t_i)$ for increasing thickness of plate 1 which again agrees with both Ljunggren's results\(^10\) and the theory by Vét\(^{5,7,8}\).
4. Conclusions

4.1 Prediction formulae

4.1.1 Low frequencies

The sound reduction index for low frequencies becomes

\[
R = 20 \log \left( \frac{\omega^3 m_1 m_3}{2 \rho cs} \right) - 10 \log \left( 2 \sigma_4 + \frac{\pi}{2 \eta_3} S_3 f \sigma_3^2 + \frac{\pi}{2 \eta_3} S_3 f \sigma_3^2 + \frac{\frac{\pi^2}{4 \eta_1 \eta_3} S_3^2 f^2 c_1 c_3}{2 \eta_3} \sigma_3^2 \sigma_5 \right) \]  

(70)

It is thought that the expression for \( \sigma_4 \) presented by Ljunggren\(^{13}\) is appropriate here. The radiation efficiency at resonant transmission, i.e. \( \sigma_1 \) and \( \sigma_3 \), is given by equation (30) for frequencies below \( f_c \). However, it should be noted that a value twice as high must be used if the construction is surrounded by orthogonal walls.

4.1.2 Mid-frequency region

For the mid frequency range well above the critical frequency of plate 3 but below that of plate 1 the sound reduction index becomes

\[
R = 20 \log \left( \frac{\omega^3 m_1 m_3}{2 \rho cs} \right) - 10 \log \left( \frac{\pi}{2 \eta_3} S_3 f \sigma_3^2 + \frac{\pi^2}{4 \eta_1 \eta_3} S_3^2 f^2 c_1 c_3 \sigma_3^2 \sigma_5 \right) \]  

(71)

The radiation efficiency \( \sigma_3 \) can here be calculated according to equation (53) but with \( c_1 \) exchanged for \( c_3 \). The radiation efficiency for plate 1 on the other hand is still calculated according to (30) as the plate is excited below \( f c_1 \).

4.1.3 High-frequency region

In the high frequency region, well above the critical frequencies of both plates, the sound reduction index will be

\[
R = 20 \log \left( \frac{\omega^3 m_1 m_3}{2 \rho cs} \right) - 10 \log \left( \frac{\pi}{2 \eta_3} S_3 f \sigma_3^2 + \frac{\pi^2}{4 \eta_1 \eta_3} S_3^2 f^2 c_1 c_3 \sigma_3^2 \sigma_5 + \frac{\pi}{2 \eta_1} S_1 f \sigma_1^4 \right) \]  

(72)
For frequencies above the critical frequency of each plate the radiation efficiencies used is the same as in the mid-frequency case. In the case where plate 1 is excited at the critical frequency a radiation factor valid at the critical frequency of plate 1 must be used for $\sigma_1$. 
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6. References


11. G. Widén, "Monterbart ljudisolerande flytande golv" (Mountable sound-insulating floating floor), Report H-9279-A


Appendix

The velocity relation of a plate at low frequencies

According to equation (18) the transverse velocity of the forced wave in the excited part of a plate can be taken as

$$v_f(x) = \frac{P}{j\omega m} e^{-j\kappa x}. \quad (73)$$

Outside the excited area, for $x > L$, waves will propagate away from the boundary. This response can be approximated to (Ljunggren\(^{13}\))

$$v_{x\infty}(x) = \frac{P}{4\omega m} e^{-j2\kappa x}. \quad (74)$$

The exponential nearfield is here disregarded with the same motivation as for the excited part of the plate. The propagating waves will cause a reverberant field, see figure (5), with the mean square velocity as

$$\left\langle v^2 \right\rangle = \frac{P_r}{\omega m \eta S}, \quad (75)$$

where S is the total area of the plate and $P_r$ is the power travelling away from the excited area. In the case of excitation with plane airborne waves, the power emerging from one side of the boundary can be taken as (compare Cremer et. al.\(^{5}\), p. 109)

$$P_r = C_g m v^2 \Lambda \cos \psi, \quad (76)$$

where $C_g$ is the group velocity, $v^2$ is the rms value of the velocity of the propagating wave emerging from the boundary, and $\Lambda$ is the length of the boundary at $x = L$. The angle $\Psi$ is the angle between the direction of the free wave and the boundary normal. Thus, the mean power $P_r$ can be written as

$$P_r = 2 \left[ \left( \frac{P}{4\omega m} \right)^2 / 2 \right] 2c_g m U_s \left\langle \cos \psi \right\rangle. \quad (77)$$

There will be free waves emerging from both sides of the boundary which gives the first factor 2 in the expression above. According to Ljunggren\(^{13}\) the mean value of
\( \cos^2 \Psi \) can be taken as 1 for low frequencies. The mean square velocity of the resonant field will be

\[
\langle v_r^2 \rangle = \left( \frac{p}{4 \omega m} \right)^2 \frac{2c_p U_s}{\omega \eta S},
\]

(78)

where \( \eta \) is the loss factor of plate 1 and \( U_s \) is the length of the boundary of the excited part of the plates. The squared velocity ratio between resonant and forced vibrations will be

\[
\frac{\langle v_r^2 \rangle}{\langle v_f^2 \rangle} = \frac{U_s}{4 \eta S k_b}.
\]

(79)

The relation between the resonant and the forced field in a plate can be rewritten as a function of the radiation factor as

\[
\frac{\langle v_r^2 \rangle}{\langle v_f^2 \rangle} = \frac{\pi S_s f_c}{4 \eta S f} \sigma,
\]

(80)

where \( \sigma \) is the radiation factor with respect to low frequencies as

\[
\sigma = \frac{1}{2 \pi^2} \frac{U_s \lambda_c}{S_s} \frac{\sqrt{f}}{f_c}.
\]

(81)
Paper II
Airborne Sound Insulation of Single Walls

– a critical review

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Abstract

This report contains a critical review of analytical models for the airborne sound insulation of 'thin' plates, i.e., models based on the Kirchoff plate equation. It is found that several works agree on the general expression for the sound insulation at frequencies well below the critical frequency except for the radiation factors, which are, of course, very important factors in this context. In the case of the radiation of the forced plate field the disagreement is profound and values of the radiation factor from less than unity to very high values has been found. In the case of reverberant plate fields, the disagreement is much smaller. However, it is very important that the correct boundary and mounting conditions are taken into account. No serious disagreement is found for frequencies above the critical frequency.

1. Introduction

In a companion paper, theoretical models are derived for the airborne sound transmission through floating floors, i.e., a construction which consists of two plates with an intermediate resilient layer. That work is mainly founded on an existing theoretical model for the airborne sound transmission through a single walls (Ljunggren'). The purpose of the present paper is to scrutinise that previous work and to compare the results with those from other theoretical models. For that reason, the present presentation closely follows that of the previous paper.
2. Sound insulation at frequencies below the critical frequency

2.1 Excitation by a plane sound wave

2.2.1 General expression for plate response

First the plate response in a two-dimensional case according to figure (1), is studied. The plate is presumed to be surrounded by air and exited over the length \(x = -L\) to \(x = L\) by an incoming plane wave. As the wave is reflected against the surface, the incoming and the reflected wave will cause the pressure \(p(x)\) on the plate, which can be described as

\[
p(x) = p e^{(-j k_0 x + j \omega t)}
\]  

(1)

where \(\omega\) is the angular frequency and \(k\) is the wave number of the exiting pressure on the plate surface. \(k = k_0 \sin \theta\), where \(k_0 = \omega/c_s\) is the wave number of airborne sound wave and \(\theta\) is the angle of incidence. When \(\theta = 0\) the sound wave impinges perpendicularly to the surface.

![Figure 1. A two dimensional model of a plate, exited by a plane wave with the angle of incidence \(\theta\).](image)

The plate is assumed to have a constant thickness, and to be made of a homogeneous and isotropic linear elastic material. The motion of the plate is described by Euler-Bernoullis solution for thin plates, so that the plate can be characterised by its surface weight \(m\) and bending stiffness per unit width. The plate response is derived with Greens formula for bending waves. If the plate is exited by a unit force at the position \(\xi\) and the velocity is observed in position \(x\), the Greens’s function for a plate in vacuum can for this two-dimensional case be written as

\[
G(x, \xi; \omega) = \left( \frac{k_B}{4 \pi \omega} \right) \left( e^{-k_0|x-\xi|} - j e^{-k_0|x+\xi|} \right)
\]  

(2)
where \( k_B = \sqrt{\omega^2 m / B} \) is the bending wavelength for free bending waves.

It should be observed that the longitudinal waves in the plate are not taken under consideration, as the displacement due to the longitudinal waves is negligible compared to that due to free bending waves (Ljunggren\(^b\)). Low frequent Lamb modes are also neglected. These are however of importance close to the boundaries but the amplitude can be shown to be negligibly small at a distance away from the boundaries, which is of the same order of magnitude as the plate thickness. This implies that the Lamb modes can be disregarded in the present context.

The transversal velocity of the plate \( v(x) \) due to the pressure \( p(x) \) can now be written as

\[
v(x) = p \left( \frac{k_B}{4m\omega} \right) \int_{-L}^{L} e^{-j\xi z} \left( e^{-kjx} - je^{-jk|x|-\xi} \right) d\xi.
\]

The time factor \( e^{j\omega t} \) is omitted here and in all similar cases in the remainder of this paper. The solution for the exited part of the plate, region 1: \(-L < x < L\), can be written as

\[
v_1(x) = \left( \frac{pk_B}{4m\omega} \right) \left[ \frac{j4k_B^2}{k^2 - k_B^2} e^{-jkx} \right]
  \left[ -e^{-j\left(k - k_B\right)x} + \frac{je^{-jkx} + \frac{jk - jk_B}{j\omega}}{j\omega - jk + jk_B} + \frac{jk - jk_B}{j\omega - jk - jk_B} - e^{-j\left(k - \omega\right)x} \right].
\]

The first part of the expression represents the forced wave. This could be simplified as

\[
v_f(x) = p \left( \frac{j\omega}{B(k^2 - k_B^2)} \right) e^{-jkx},
\]

which is the same expression as for infinite plates. Outside the exited area the solution consists of a free propagating wave together with an exponential nearfield.

For region 2, \( x > L \), the velocity is given by

\[
v_2(x) = \left( \frac{pk_B}{2m\omega} \right) \frac{\sin\left((k - k_B)L\right) e^{-jk_Bx}}{k - k_B} - \frac{j\sin\left((k + jk_B)L\right) e^{-jk_Bx}}{k + jk_B},
\]

and for region 3, \( x < L \) in the same manner.
\[ v_3(x) = \left( \frac{p k_B}{2m \omega} \right) \left[ \frac{\sin((k + k_B)L)e^{-jk_Bx}}{k + k_B} - \frac{j \sin((k - jk_B)L)e^{jk_Bx}}{k - jk_B} \right]. \] (7)

The solutions for regions 2 and 3 can be interpreted as one propagating wave and one exponential near-field which emanate from each of the two boundaries and interfere with each other.

### 2.2 Approximate solution for frequencies below the critical frequency – forced transmission in the two-dimensional case

If the frequency is low enough, the wave number for airborne sound \( k_a \), and hence also the trace wave number \( k \), will be much smaller than the wave number for a free bending wave \( k_B \). The velocity for different parts of the plate can therefore be approximated as

\[ v_1(x) = \left( \frac{p}{j \omega m} \right) \left[ e^{-j k a x} - \frac{e^{-j k a x} \cos(k_B x) - e^{j k a x} \cosh(k_B x)}{2} \right], \] (8)

\[ v_2(x) = \left( \frac{p}{2 \omega m} \right) \left[ \sin(k_B L)e^{-j k a x} - j \sinh(k_B L)e^{j k a x} \right], \] (9)

\[ v_3(x) = \left( \frac{p}{2 \omega m} \right) \left[ \sin(k_B L)e^{j k a x} - j \sinh(k_B L)e^{j k a x} \right]. \] (10)

It is now assumed that the exited part of the plate is surrounded by an infinite baffle and that its upper face radiates sound into a semi-infinite sphere. The radiated sound power can then be calculated for this two-dimensional case as (Cremer et al.\(^2\) p 528)

\[ P = \left( \frac{\rho c k_a}{4 \pi} \right) \int_{k_a}^{k_s} \frac{|v(k_s)|^2}{\sqrt{(k_s^2 - k_a^2)}} dk_s, \] (11)

where \( \rho \) is the density of the surrounding air, \( c \) is the velocity of sound in air and \( v(k_s) \) is the Fourier transform of the plate velocity of region 1,

\[ v(k_s) = \int_{-L}^{L} v_1(x)e^{jk_s x} dx. \] (12)

If the plate is not too small, the vibrational velocity in the exiting part of the plate is expected to dominate the forced part of the response, i.e. the first part of \( v_1(x) \). The Fourier transform of this part can be written as
\[ v_f(k_x) = \left( \frac{2p}{j\omega m} \right) \frac{\sin((k_x - k)L)}{(k_x - k)} \]  \hspace{1cm} (13)

and the corresponding radiated sound power

\[ P_{rad} = \left( \frac{p^2 \rho}{\pi \omega m^2} \right) \int_{k_a}^{k_x} \frac{\sin^2((k_x - k)L)}{(k_x - k)^2 \sqrt{(k_a^2 - k_x^2)}} dk_x. \]  \hspace{1cm} (14)

For large plates, i.e. when \( k_a L \gg 1 \), this can be approximated as

\[ P_{rad} = \left( \frac{p^2 \rho}{\pi \omega m^2} \right) \frac{1}{\sqrt{(k_a^2 - k_x^2)}} \int_{k_a}^{k_x} \frac{\sin^2((k_x - k)L)}{(k_x - k)^2} dk_x = \frac{p^2 \rho cL}{\omega^2 m^2 \cos \phi}. \]  \hspace{1cm} (15)

As the plate is assumed to be heavy, the incident sound power can be calculated as

\[ P_{in} = \frac{p^2 2L \cos \theta}{8 \rho c}, \]  \hspace{1cm} (16)

which gives the transmission factor as

\[ \tau(\theta) = \frac{P}{P_{in}} = \left( \frac{2 \rho c}{\omega m \cos \theta} \right)^2 \]  \hspace{1cm} (17)

The radiated power from free bending waves could be calculated from the second and third term in the expression of the vibrational velocity of the plate \( v_x(x) \), according to equation (4) in the same manner as for the forced response. If the plate is not too small, i.e. \( k_a L \gg 1 \) is fulfilled, this will be approximated as

\[ P_{free} = \frac{P_{rad} k_a \cos \theta}{16 k_a^2 L}, \]  \hspace{1cm} (18)

where \( P_{rad} \) is the forced part of the radiated sound power. For small plates, where \( k_a L \gg 1 \) is not fulfilled, a radiation factor is introduced as

\[ \sigma = \frac{P_{rad}}{2L \rho c p^2 / 2 \omega m^2} = \frac{k_a}{\pi L} \int_{k_a}^{k_x} \frac{\sin^2((k_x - k)L)}{(k_x - k)^2 \sqrt{k_a^2 - k_x^2}} dk_x, \]  \hspace{1cm} (19)

where \( P_{rad} \) is the radiated power according to equation (14). The transmission factor will then look like
\[
\tau(\theta) = \left( \frac{2\rho c}{\omega m} \right)^2 \frac{\sigma}{\cos \theta},
\]  

(20)

which is the same as the expression for large plates if the radiation factor \(1/\cos \theta\) is used.

2.3 Forced transmission at low frequencies - the three-dimensional case

A comparison between the two expressions for the transmission factor at low frequencies according to equation (17) and equation (20), shows that the expression for small plates must be expected to give a better assumption even in the case of a relatively large plate. Thus, the mean transmission factor should be derived from the expression for a finite plate and not from the expression for an infinite plate.

The mean transmission factor is integrated over the angle of incidence according to Paris’ formula as

\[
\bar{\tau} = 2 \int_0^\pi \tau(\theta) \cos \theta \sin \theta d\theta.
\]  

(21)

Insertion of the transmission factor according to equation (20) then gives

\[
\bar{\tau} = 2 \int_0^\pi \left( \frac{2\rho c}{\omega m} \right)^2 \sigma \sin \theta d\theta = 2 \left( \frac{2\rho c}{\omega m} \right)^2 \sigma, 
\]  

(22)

where

\[
\sigma_d = \int_0^\pi \sigma \sin \theta d\theta.
\]  

(23)

With the definition of the sound reduction index as

\[
R = 10 \log \frac{1}{\tau},
\]  

(24)

the sound reduction index for a forced transmission at low frequencies will according to Ljunggren\(^7\) be

\[
R_f = 20 \log \left( \frac{\omega m}{2\rho c} \right) - 3 - 10 \log \sigma_d.
\]  

(25)
2.4 Resonant transmission

The radiated power from free bending waves could be calculated from the second and third term in the expression of the vibrational velocity of the plate \( v_i(x) \), according to equation (4) in the same manner as for the forced response.

If the plate is now considered finite, the bending waves radiated from the plate boundaries will cause a reverberant field in the sending room. This reverberant field will have a mean square velocity of

\[
\langle v^2 \rangle = \frac{P_R}{\omega \; m \eta \; S_{tot}},
\]  

(26)

where \( S_{tot} \) is the total plate area and \( P_R \) is the power leaving the exited part of the plate. In the case of an excitation with a plane wave, the power travelling in a direction away from the boundary \( x = L \) could be written as (see also Cremer et al.\(^5\) p 109)

\[
P = C_n m v^2 \Lambda \cos \psi ,
\]  

(27)

where \( C_n \) is the group velocity \( = 2c_n \), \( v^2 \) is the mean square value of the velocity for a wave that rises from the boundary, and \( \Lambda \) is the length of the boundary \( x = L \). The angle \( \psi \) is the angle between the direction of the velocity of the free wave and the normal to the boundary.

As the magnitude of the velocity of the wave leaving the boundary is \( p/4 \omega m \), which is a mean value over \( \phi \) and \( \sin(k_B L) \) with the specific angle of incidence \( \theta \), the mean value of the power leaving the boundary will be

\[
P_R = 2 \left( \frac{p}{4 \omega m} \right)^2 \frac{L}{2} C_n m U \langle \cos \psi \rangle ,
\]  

(28)

where \( U \) is the length of the boundary of the excited area and \( \phi \) is the angle between the direction of the velocity of the forced wave and the normal to the boundary. The mean value of \( \cos \psi \) is calculated as

\[
\langle \cos \psi \rangle = \frac{2}{\pi} \int_0^{\pi/2} \sqrt{1 - \left( \frac{k \sin \phi}{k_B} \right)^2} d\phi = \frac{2E}{\pi} \frac{k^2}{k_B^2} .
\]  

(29)

thus for low frequencies \( <\cos \Psi> \) can, according to Ljunggren\(^5\) be set to 1. The radiated power from a part of the plate with the area \( S_B \) is calculated as

\[
P_{rot} = \langle v^2 \rangle S_B \rho c \sigma_k ,
\]  

(30)
where $\sigma_R$ is the radiation factor of the plate as seen from the receiving room. The incident power on the plate is calculated as (compare with equation (16) for two dimensions)

$$P_m = \frac{p^2 S_s \cos \theta}{8 \rho c},$$  \hspace{1cm} (31)

where $S_s$ is the area of excitation, the source rooms area. The transmission factor, defined as $\tau = \frac{P_{rel}}{P_m}$, becomes

$$\tau(\theta) = \frac{(\rho c)^2 U S_R \sigma_R}{\omega^2 m^2 k_B \eta S_m S_s \cos \theta}.$$  \hspace{1cm} (32)

The mean transmission factor, integrated over the angle of incidence according to equation (21), becomes

$$\bar{\tau} = \frac{2(\rho c)^2 U S_R \sigma_R}{\omega^2 m^2 k_B \eta S_m S_s}.$$  \hspace{1cm} (33)

This expression could be compare with an expression used in SEA, statistical energy analysis, by Ver and Holmer

$$\bar{\tau} = \left( \frac{\rho c}{\omega^2 m^2} \right)^2 \frac{\pi}{2 \eta} f_c \sigma^2.$$  \hspace{1cm} (34)

The radiation factor $\sigma$ is in this case equal for sending and receiving room. Further, no difference is made between excited, radiating and the total area of the plate. If equation (33) is rewritten for this case, it will look like

$$\bar{\tau} = \frac{2(\rho c)^2 U \sigma}{\omega^2 m^2 k_B \eta S}$$  \hspace{1cm} (35)

and the expressions becomes equal if the radiation factor

$$\sigma = \frac{1}{2 \pi^2} \frac{U \xi}{S} \sqrt{\frac{f}{f_c}}$$  \hspace{1cm} (36)

is used. In the same manner as for forced transmission, the transmission factor for resonant transmission can be rewritten as a sound reduction index as

$$R_r = 20 \lg \left( \frac{\omega m}{2 \rho c} \right) + 10 \lg \left( \frac{2 \eta}{\pi} \right) + 10 \lg \left( \frac{f}{f_c} \right) - 20 \lg (\sigma_{rel}) - 10 \lg \left( \frac{S_R}{S_{tot}} \right).$$  \hspace{1cm} (37)
The loss factor is calculated as

$$\eta = \eta_i + \eta_p + \eta_r + \left( \frac{2}{\pi k_p S_{\text{sw}}} \right) \sum_j L_j \alpha_j, \quad (38)$$

where \( \eta_i \) is the loss factor due to internal losses, according to Cremer et al.\(^2\) estimated to 0.4\%, \( \eta_p \) is the loss factor due to energy transmission from one floor to another through the structure, and \( \eta_r \) is the loss factor due to sound radiation to the surrounding air. The sum \( \eta_i + \eta_p + \eta_r \) can according to Ljunggren and Ottosson\(^10\) be taken as 0.55\%.

Further, \( k_p \) is the wave number for bending waves, \( \alpha_j \) is the absorption factor at the boundaries and \( L_j \) the length of the boundary.

It has been pointed out (Ljunggren\(^9\), Leppington\(^6\)) that the radiation factor must be doubled if the plate is surrounded by orthogonal partitions and the edge radiators are dominating. For frequencies below \( f_c \) the radiation factor \( \sigma_{\text{rad}} \) is in such cases calculated as

$$\sigma_{\text{rad}} = 2 \left( \frac{1}{2\pi^2} \right) \left( \frac{U \lambda_c}{S} \right) \sqrt{\frac{f}{f_c}}, \quad (39)$$

under the condition that

$$\sigma_{\text{rad}} \leq \sqrt{\frac{a}{\lambda_c}} + \sqrt{\frac{b}{\lambda_c}}, \quad (40)$$

where \( a \) and \( b \) refers to the plate dimensions, \( U \) its circumference and \( \lambda_c \) is the wavelength at the coincidence frequency.

The sound reduction index for resonant and forced transmission are now added as

$$R_{\text{total}} = -10 \log \left( 10^{-10} + 10^{-10} \right). \quad (41)$$

2.5 Validity of the assumptions

It is seen from the preceding presentation that the derivation for the forced transmission is valid only if the influence of the waves excited at the boundaries of the excited area can be disregarded. However, if the plate stiffness is very high and the frequency is low, equation (4) shows that the waves from the boundaries must be expected to be important. On the other hand, it is not difficult to obtain an approximate solution for the case \( k_p L \ll 1, k_p L \ll 1 \).

Thus, for small values of the Helmholtz numbers, equation (3) immediately gives
\[ v = v_0 = p \frac{k_B}{4 \omega m} 2L(1 - j), \]  \hspace{1cm} (42)

\[ v(k_x) = 2v_0 \frac{\sin k_x L}{k_x}. \]  \hspace{1cm} (43)

The radiated power can again be obtained from equation (11) which here gives

\[ P_{\text{rad}} = \frac{\rho c k_a}{4\pi} \int_{k_x} k_x \frac{4v_0^2 \sin^2(k_x L)}{\sqrt{k_x^2 - k_z^2}} dk_x = \rho c k_a L^2 v_0^2. \]  \hspace{1cm} (44)

With the help of the expression for the incident power for this two-dimensional case, equation (16), the transmission factor becomes

\[ \tau(\theta) = \left( \frac{2\rho c}{\omega m} \right)^2 \frac{k_a L^3 k_b^2}{2 \cos \theta}, \]  \hspace{1cm} (45)

which gives the mean transmission factor as

\[ \bar{\tau} = \left( \frac{2\rho c}{\omega m} \right)^2 \frac{1}{(k_a L_m)^3} \frac{f_C}{f}. \]  \hspace{1cm} (46)

Results from this expression are compared with those from equation (41) in figure (2). This result cannot be very accurate for a field situation as the derivation is made for a two-dimensional case only. In the three-dimensional case a still smaller radiation from the excited area must be presumed. However, the results still show that the transmission factor cannot be expected to be very small in the case of small areas. Thus, equation (41) should be used with caution in such cases.

**Figure 2.** The transmission factor according to
- equation (46),
- equation (41).
The example above is calculated for a 180 mm thick concrete plate with an area of 10 m². The Young's modulus is $3 \times 10^{10}$ N/m², Poisson's ratio $\nu = 0.3$, the density of concrete 2400 kg/m³ and the loss factor $\eta = 0.4 \%$. The coincidence frequency of this plate is approximately 96 Hz.

2.6 Comparisons with other results

2.6.1 Cremer

The transmission factor according to Cremer et al.¹ can be written as

$$\tau(\theta) = \frac{1}{1 + \frac{j \omega m' \cos \theta}{2 \rho c}}.$$  \hspace{1cm} (47)

and the corresponding mean transmission factor as

$$\bar{\tau} \approx \left(\frac{2 \rho c}{\omega m}\right)^2 \ln\left(\frac{\omega m}{2 \rho c}\right).$$ \hspace{1cm} (48)

2.6.2 Josse and Lamure

The transmission factor for a plate which obeys the same conditions as above, i.e. no flanking transmission is present, is according to Josse and Lamure⁴ calculated as

$$\tau = \frac{4 \rho^2 c^2}{\omega^2 m^2} \left[2 + \frac{4c^2}{\pi^2 \varepsilon f_c \sqrt{f_c}} \cdot \frac{a^2 + b^2}{a^2 b^2} \left(1 + \frac{2f}{f_c} + \frac{3f^2}{f_c^2}\right)\right].$$  \hspace{1cm} (49)

where $\varepsilon$ corresponds to the loss factor of internal and radiational losses and $a$ and $b$ referring to the plate dimensions. The sound reduction index will be

$$R = 20 \lg \frac{\omega m}{2 \rho c} - 10 \lg \left[2 + \frac{4c^2}{\pi^2 \varepsilon f_c \sqrt{f_c}} \cdot \frac{a^2 + b^2}{a^2 b^2} \left(1 + \frac{2f}{f_c} + \frac{3f^2}{f_c^2}\right)\right].$$ \hspace{1cm} (50)

2.6.3 Heckl

For frequencies well below the critical frequency, Heckl⁵ derived the following expression by means of a reciprocity argument

$$R = 20 \lg \left(\frac{\omega m}{2 \rho c}\right) - 10 \lg \left(4 + \frac{\pi f_c \sigma \sigma'}{2 \rho \eta}\right).$$ \hspace{1cm} (51)
It should be noted though, that Heckl\(^3\) did not make a proper evaluation of the radiation factor for the forced transmission but just wrote that it is approximately 2, which explains the factor 4 within the brackets.

### 2.6.4 Sewell and Leppington

The transmission factor for forced transmission for frequencies below the coincidence frequency could according to Sewell\(^13\) be calculated as

\[
\tau = \Omega^{-2} \left( \ln(k\sqrt{A}) + 0.160 - U(\Lambda) + \frac{1}{4\pi k^2 A} \right),
\]

(52)

where \(A\) is the area of the test object, which is assumed to be a plate simply supported in an infinite baffle, \(A = l_1 l_2\), and where \(k\) is the wave number, \(k = \omega/c\). \(U(\Lambda)\) is a form factor which could be calculated as

\[
U(\Lambda) = -0.804 - \left( \frac{1}{2} + \frac{\Lambda}{\pi} \right) \ln \Lambda + \frac{5\Lambda}{2\pi} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \Lambda^{2n+1}}{2\pi n(n+1)(2n+1)^3},
\]

(53)

for \(\Lambda \leq 1\), where \(\Lambda\) is the relation between the height and width of the testing area, \(\Lambda = l_2/l_1\). \(\Omega\) is calculated as

\[
\Omega = \frac{m\omega}{2\rho c} \left( 1 - \frac{\omega^2}{\omega^2_c} \right) = \frac{m\omega}{2\rho c} \left( 1 - \frac{f^2}{f^2_c} \right).
\]

(54)

If this is rewritten in terms of sound reduction, the sound reduction index for the forced transmission for frequencies well below \(f_c\) could according to Sewell\(^13\) be calculated as

\[
R = 20 \log \frac{m\omega}{2\rho c} + 20 \log \left( 1 - \frac{f^2}{f^2_c} \right) - 10 \log \left( \ln(k\sqrt{A}) + 0.160 - U(\Lambda) + \frac{1}{4\pi k^2 A} \right).
\]

(55)

Sewell’s expression for forced transmission is valid under the condition that \(m > 10\) kg/m\(^2\). In the case of a concrete plate, this is fulfilled at a plate thickness of 5 mm.

Leppington et al.\(^6\) published in 1987 a slightly modified version of Sewell’s expression for the sound reduction index. The expression for the forced transmission factor valid for frequencies well below the coincidence frequency was presented as
\[
\tau = \left( \frac{\rho_0 c_0}{\pi f m (1 - f^2 / f_c^2)} \right)^2 \left( \ln \left( \frac{2\pi f \sqrt{S}}{c_0} \right) + 0.160 + U(l_x, l_y) + \right.
\]
\[+ \frac{1}{4 \mu^2} \left[ (2\mu^2 - 1)(\mu^2 + 1)\ln(\mu^2 - 1) + (2\mu^2 + 1)(\mu^2 - 1)\ln(\mu^2 + 1) - 4\mu^2 - 8\mu^4 \ln \mu \right] \left( 56 \right)
\]

where \( U(l_x, l_y) \) is the form factor and \( \mu = \left( \frac{f_c}{f} \right)^{1/2} \). This could be excluded for normal building partitions, i.e. when \( 0.1 < l_x/l_y < 10 \).

### 2.6.5 Novak

Novak\(^{11}\) presented in 1995 a radiation factor for plates subjected to a diffuse sound field. An approximation of the radiation factor can be calculated as (Novak\(^{12}\))

\[
10 \log \sigma_d = 3 - \frac{7.13}{\sqrt{1.42 k_a l_m - 1}}, \tag{57}
\]

where \( l_m \) is the mean free path, \( l_m = \pi S_R / U_R \) according to Kosten\(^5\) and \( U_R \) is the length of the boundary of the radiating area.

### 2.6.6 A comparison of the radiation factors for forced transmission

It is interesting to note that the expressing by Sewell\(^{13}\) and Leppington\(^6\) contain a term \( 20 \log(1 - f^2 / f_c^2) \), which does not appear in the expressions by the other authors. With regard to many field measurements results, this term does not seem to be valid for frequencies close to the critical frequency; it can certainly be said that the sound insulation is comparatively poor at the critical frequency but it does not equal - \( \infty \)!

In the case of forced transmission, it is seen that the main difference between the different expressions for the sound transmission concerns the radiation factors. On the other hand, the values given by different authors differ widely.

Some of the expressions are compared in figure (3). In the case of Sewell\(^{13}\) and Leppington\(^6\), the radiation factor has been obtained from a comparison of the result in equation (25) with those of equation (55) and (56), respectively.

The radiation factor for an infinite wall can for the forced plate field be written as

\[
\sigma_d = 2 \int_0^\pi \frac{1}{\cos^2 \theta} \sin \theta \cos \theta \, d\theta. \tag{58}
\]
This integral does not converge; the radiation factor is $\infty$. The reason why Cremer's result for the sound reduction index is limited is merely due to the radiation load on the plate. This load causes a reduction of the transverse velocity of the plate which explains the finite value.

Figure 3. Comparison of the radiation factor for forced transmission, for frequencies below the coincidence frequency.
(1) Heckl\textsuperscript{3}, eq. (51)
(2) Josse and Lamure\textsuperscript{4}, eq. (50)
(3) Sewell\textsuperscript{13}, eq. (55), or Leppington\textsuperscript{6}, eq. (56)
(4) Ljunggren\textsuperscript{9}, eq. (23)
(5) Novak\textsuperscript{11,12}, eq. (57)

While the radiation factor presented by Cremer\textsuperscript{2}, Heckl\textsuperscript{3} and Josse and Lamure\textsuperscript{4} are constants, that by Ljunggren\textsuperscript{9}, Sewell\textsuperscript{13} and Leppington\textsuperscript{6} are strongly frequency dependent. In between is the radiation factor presented by Novak\textsuperscript{11,12}, which is strongly frequency dependent for low frequencies but will reach a limit of $10\log \sigma = 3$.

2.6.7 Radiation factors for resonant plate fields

The expression for the sound reduction index given by Josse and Lamure\textsuperscript{4} contains two parts, one obviously related to the forced transmission, the first term within the brackets of equation (50), and the other related to the resonant transmission. The expression for the resonant transmission agrees with that given by equation (37) provided that the radiation factor is taken as

$$
\sigma_{res} = \frac{16c^2}{S_R \pi^3 f_C^2} \frac{\sqrt{f} \left( 1 + \frac{2f}{f_C} + \frac{3f^2}{f_C^2} \right)}{\sqrt{f_C}}
$$

(59)

and that $a = b$ (square panel). This expression is illustrated in figure (4) together with that from equation (39), for walls with orthogonal partitions.
The example above is calculated for a 180 mm thick concrete plate with an area of 10 m², i.e. the same material data as used for figure (2).
3. Response at coincidence

When calculating the plate response the loss factor must be taken into consideration. This is introduced in a usual way with a complex Young's modulus

\[ E = E_r(1 + j\eta) \tag{60} \]

where \( E_r \) is the usual real Young's modulus. Hence the wave number for free bending waves will also be complex and could for low frequencies be written as

\[ k_b = k_{b0}(1 - j\eta/4) \tag{61} \]

where \( k_{b0} \) is the real wave number. Since \( k = k_{b0} \) in this case, only the first two parts of the solution is needed, according to equation (4), to describe the response of the exited plate, since these are the only ones that include the term \( 1/(k-k_b) \). The velocity \( v_1(x) \) can now be calculated as

\[ v_1(x) = \frac{p}{B(k^4 - k_b^4)} e^{-jk_b x} - \frac{j k}{4\omega m (k - k_b)} e^{j\omega(k - k_b)} \tag{62} \]

This expression for \( v_1(x) \) can be modified as

\[
\begin{align*}
v_1(x) & = \frac{j\omega}{B(k^4 - k_b^4)} e^{-jk_b x} - \frac{j k}{4\omega m (k - k_b)} e^{j\omega(k - k_b)} \\
& \approx \frac{j\omega}{4k_b^3(k - k_b)} e^{-jk_b x} - \frac{j k}{4\omega m (k - k_b)} e^{j\omega\eta/4 k_b x} \\
& = \frac{j k}{4\omega m (k - k_b)} \left( e^{-jk_b x} - e^{-jk_b x + j\omega(L+x)/4} \right) \\
& = \frac{p k}{\omega m \eta} e^{-jk_b x} \left( 1 - e^{-j\omega\eta(L+x)/4} \right). \tag{63}
\end{align*}
\]

This solution is also obtained by Lindblad's, but in a slightly different way.

If \( \eta k_b(L+x)/4 << 1 \) it is allowable to use

\[ v_1(x) = \frac{p(L+x)}{4mc_b} e^{-jk_b x} \tag{64} \]

and for \( \eta k_b(L+x)/4 >> 1 \)
\[ v_1(x) = \frac{P}{4m\eta} e^{-\beta z}. \]  \hspace{1cm} (65) \]

According to equation (63) a certain length of the excitation area is required before the final value is attained and the initial response, in terms of the transversal velocity, displacement etc., is proportional to the length of excitation. This agrees with the well-known analogy between coincidence and response (see Cremer). Hence, it is possible to define a “rise length” \( L_r \) according to figure 2.5. This definition gives

\[ L_r = 9.21/k_{\eta}. \]  \hspace{1cm} (66) \]

As the loss factor is rather small for most of the materials used in building acoustics, the “rise length” will be relatively large in many cases. For example, for a 200 mm thick concrete plate the \( k_{\eta} \) will be approximately 4 at 300Hz, and the loss factor approximately 0.4% (regarding internal losses, according to Cremer et al.\(^2\) p. 242). \( L_r \) will in this case be 576m. This example shows that the area of excitation in most cases could be considered as small, and that equation (64) is more general and useful than the expression given in (65).

![Figure 5. The magnitude of the velocity at the coincidence frequency. The velocity is normalised with respect to \( p/(\omega m \eta) \). The rise length is defined as the length from \(-L\) to the point where the response has reach 90% of its final value. In this case 90% equals \( \eta k_{\eta} (L - x) / 4 = 2.3 \)](image)

The response of the plate in region 2 can for small values of \( \eta k_{\eta} (L + x) / 4 \) be approximated to

\[ v_2(x) = \frac{pk_{\eta}}{2\omega m} \left( e^{-\beta z} - \frac{\sin(k_{\eta} L(1 + j)) e^{-k_{\eta} z}}{k_{\eta}(1 - j)} \right) \]  \hspace{1cm} (67) \]

and in the same manner the velocity in region 3

\[ v_3(x) = \frac{pk_{\eta}}{2\omega m} \left( \sin(2k_{\eta} L)e^{\beta z/2k_{\eta}} + \frac{\sin(k_{\eta} L(1 - j)) e^{-k_{\eta} z}}{k_{\eta}(1 + j)} \right). \]  \hspace{1cm} (68) \]
Obviously, the transversal velocity of the propagating wave in region 2 is proportional to the exciting length 2L, and is in practice much larger than the propagating wave in region 3.

![Plate response graph]

Figure 6. The magnitude of the velocity at coincidence for small values of x according to Ljunggren. The velocity is normalised with regards to \( p/\omega m \). The plate size 2L=4m is used. Otherwise, the same material values as in figure 2.2.

---

The exact solution based on equations (4), (6) and (7).

---

The approximated solution based on equations (64), (67) and (68).

If the exited part of the plate is very large, i.e. \( \eta k_b L/2 \gg 1 \), the response of the plate can be described according to equation (65). The corresponding velocity transform will be

\[
v(k_x) = \frac{2p}{\omega m \eta} \frac{\sin((k_x - k_b)L)}{(k_x - k_b)}
\]  (69)

and the radiated power could be calculated by combining this equation with equation (11). The integrand in equation (11) will have two peaks, one at \( k_x = k_b \) and one at \( k_x = k_s \). These peaks will coincide if the \( k_s = k_b \), i.e. if coincidence occurs at grazing incidence. If the peaks are well separated, it is possible to express the power from one side of the plate as

\[
P = \frac{p^2 \rho c L}{\omega^2 m^2 \eta^2 \cos \theta}.
\]  (70)

The transmission factor is now obtained in the same way as before

\[
\tau(\theta) = \frac{P}{P_{in}} = \left( \frac{2 \rho c}{\omega m \eta \cos \theta} \right)^2.
\]  (71)
This result can also be obtained from Cremer et. al.\textsuperscript{2} by using \( \varepsilon = 0 \) in his equation (9.4).

In the case of small plates, i.e. \( \eta k_aL/2 \ll 1 \) equation (64) should be used instead to express the velocity. The corresponding transform could then be written as

\[
v(k_z) = v_a(k_z) - jv_b(k_z),
\]

\[
v_a(k_z) = \frac{2pL \sin ((k_z - k_b)L)}{4mc_B (k_z - k_b)},
\]

\[
v_b(k_z) = \frac{p}{4mc_B} \left( \frac{2L \cos ((k_z - k_b)L)}{(k_z - k_b)} - \frac{2 \sin ((k_z - k_b)L)}{2} \right) \left( k_z - k_b \right).
\]

In figure (2.7) the real part \( v_a(k_z) \) and the imaginary part \( v_b(k_z) \) of \( v(k_z) \) are shown.

The figure shows that \( v_a(k_z) \) will have a broad peak at \( k_z = k_b \). On the other hand, the curve \( v_b(k_z) \) will show two peaks, one slightly below \( k_a \) and one slightly above. This indicates that the largest part of the radiation due to \( v_b(k_z) \) occurs at two angles. One slightly smaller and one slightly larger than the angle of coincidence. The figure also shows that the contribution from \( v_b(k_z) \) is not negligible compared to the contribution from \( v_a(k_z) \).

\[\textbf{Figur 7. The velocity transform near around the coincidence frequency.}\]

The phase difference between \( v_a(k_z) \) and \( v_b(k_z) \) makes it possible to consider them one at a time. Hence it follows that the radiation from the bottom of the plate due to \( v_a(k_z) \) is given by

\[
P_a = \frac{p^2 \rho c L^2}{16m^2 c^2_B \cos \theta}
\]

and the corresponding radiation from \( v_b(k_z) \) is given by
\[ P_b = \left( \frac{p^2 \rho c L^2}{16 \pi m^2 c_b^2 \cos \theta} \right)^x I, \]  

(76)

where

\[ I = \int_{k_a}^{\infty} \left( \frac{\cos((k_a - k_b)L) - \sin((k_a - k_b)L)}{(k_a - k_b)^2 L^2} \right)^2 dk_a. \]  

(77)

Here it is assumed that the angle \( \theta \) corresponds to the radiation from the two peaks in \( v_b(k_a) \).

The integral \( I \) has been solved numerically for \( k_b \) in an interval between 2 and 6, for \( L \) from 2 to 10, and for two different frequencies 500 Hz and 1000 Hz. In this case was \( I = L \), with a deviation of no more than 5%. This result is thought to be generally relevant for all cases studied in this report. Assumed that \( I = L \), the relation will be

\[ P_b = P_a / \pi \]  

(78)

and the transmission becomes

\[ \tau(\theta) = \left(1 + \frac{1}{\pi} \right)^2 \left( \frac{\rho c k_b L}{2 \omega m \cos \theta} \right)^2. \]  

(79)

This expression can be written in the same manner as the transmission factor for large plates (41), if an equivalent loss factor \( \eta_{eq} \) defined as

\[ \eta_{eq} = \frac{4}{(1 + 1/\pi) k_b L} \approx \frac{3}{k_b L}, \]  

(80)

is introduced. It should be observed that \( \eta_{eq} \) is not a real loss factor as it does not represent any power dissipation. It is instead extracted from the power leaving the area of excitation. Observe also that the response in the case with a large excitation area is inversely proportional to the loss factor at coincidence. As the corresponding response in the case of small plates is proportional to the area of excitation, the equivalent loss factor must be inversely proportional to the length of the excited area.

From equation (79) we get the sound reduction index at the coincidence frequency according to Ljunggren as

\[ R = 20 \lg \frac{2 \omega m}{\rho c} + 20 \lg \frac{\cos \theta}{k_b L} + 10 \lg \left(1 + \frac{1}{\pi} \right). \]  

(81)
Josse and Lamure\(^4\) has for the same frequency expressed the transmission factor and the corresponding sound reduction index as

\[
\tau = \frac{4 \rho^2 \epsilon^2}{\omega^2 m^2} \frac{\pi}{2 \epsilon} \left( \frac{f}{\Delta f} \right) \tag{82}
\]

and

\[
R = 20 \log_{10} \frac{\omega m}{2 \rho c} + 10 \log_{10} \frac{2 \epsilon}{\pi} + 10 \log_{10} \frac{\Delta f}{f}, \tag{83}
\]

where \(\Delta f\) is the frequency bandwidth and \(\epsilon\) is the loss factor.
4. Frequencies above the coincidence frequency

When exiting plates with frequencies above the coincidence frequency, and the angle of incidence is not small, the response of the plate can be written as

\[ v_1(x) = \frac{j\omega P}{Bk^4} e^{-jkx} + \frac{pk_B}{2\omega mk} e^{-jks} \sin(kL - k_Bx) + e^{-jks} \sinh(k_Bx - jkL). \quad (84) \]

It should be noted that the angle of incidence is here assumed to be much larger than the coincidence angle. The first part of the solution of \( v_1(x) \) corresponds to the forced part of the solution, the other part corresponds to the response of a free propagating wave. Comparing the magnitudes of these two, it is seen that for frequencies above coincidence the response of a free propagating wave will have a bigger influence on the plate velocity than the forced part of the response

\[ \left| \frac{j\omega P}{Bk^4} \right| > \left| \frac{pk_B}{2\omega mk} \right| = 2 \left( \frac{f_c}{f} \right)^{1.5}. \quad (85) \]

This expression is not valid for extremely large plates, since the free waves will then be attenuated as they propagate away from the boundaries.

In this case the response will be approximated as

\[ v_1(x) = \frac{j\omega P}{Bk^4} e^{-jkx} \]

and the corresponding Fourier transform of the velocity

\[ v_1(k_x) = \frac{j\omega 2p}{Bk^4} \sin((k_x - k)L)/(k_x - k). \quad (87) \]

This expression corresponds to equation (13) for low frequencies and in the same way as before, the radiated power becomes

\[ P = \frac{p^2 \omega^3 \rho c L}{B^2 k^8 \cos \theta}. \quad (88) \]

With \( \tau = P/P_{in} \), the transmission factor will be

\[ \tau(\theta) = \left( \frac{2\omega pc}{Bk^4 \cos \theta} \right)^2. \quad (89) \]
If the plate is of more normal dimensions, the plate is dominated by the free propagating wave, and the plate response can be approximated to

\[
v_1(x) = \frac{-jk_B}{2\omega mk} e^{-jk_L} \sin(kL - k_Bx)
\]  

(90)

The transform of the velocity will in this case consists of two parts, one part for the wave propagating in positive x-direction, and one part for the wave propagating in negative x-direction

\[
v_1(k_x) = -\frac{jpk_B}{2\omega mk} e^{-j(k-k_B)x} \frac{\sin((k_x - k_B)L)}{(k_x - k_B)} + \frac{jpk_B}{2\omega mk} e^{-j(k+k_B)x} \frac{\sin((k_x + k_B)L)}{(k_x + k_B)}
\]  

(91)

If the plate is large, i.e. \(k_BL >> \pi\), the radiated power will be

\[
P = 2\frac{p^2k_B^2\rho c L}{16\omega^2m^2k^2\cos\theta}
\]  

(92)

which will give the transmission factor as

\[
\tau(\theta)_\text{free} = \frac{1}{2} \left( \frac{k_B\rho c}{\omega m k \cos \theta} \right)^2
\]  

(93)

4.1 Forced transmission around the coincidence angle, calculation in three dimensions

First, the case of large plates are studied. A plate is considered large if \(\eta k_B L/4 >> 1\). Form equation (17) an expression for the response in region 1, when exiting with a plain sound wave with the wave number \(k=k_B\), is derived as

\[
v_1(x) = \frac{-jk_B}{4\omega m \Delta k} e^{-j\kappa x}
\]  

(94)

where

\[
\Delta k = k - k_B
\]  

(95)

As the plate is considered large, a plane wave will be radiated from the plate. The amplitude of this wave can be obtained with help of the radiation factor 1/cos\(\theta\), from which the transmission factor can be calculated. Alternatively, the transmission factor could be calculated by using Fourier transforms. This method is used here, even though it is somewhat more circumstantial, since it is also valid for small
plates, which will be treated later. Thus, insertion of the Fourier transform on the right side of equation (94) in equation (11) gives

$$P_{rad} = \frac{\rho c}{4\pi \cos \theta} \int_{-\Delta k}^{\Delta k} \left( \frac{pk_B}{2\omega m|\Delta k|} \right)^2 \sin^2 \left( \frac{(k_x - k)\ell}{2|\Delta k|} \right) \frac{\sin^2 \left( \frac{(k_x - k)L}{2|\Delta k|} \right)}{(k_x - k)^2} \, dk_x. \quad (96)$$

It should be observed here that $\Delta k$ is a constant during this integration. Equation (96) is solved in the same manner as preceding integrals, which in this case will give

$$P_{rad} = \frac{\rho^2 k_B^2 \rho c L}{16\omega^2 m^2 \cos \theta |\Delta k|^2}. \quad (97)$$

The transmission factor with respect to a plane wave, incident at an angle $\theta$, can now be obtained from the equation above, together with equation (16) for incident sound power, as

$$\tau(\theta)_f = \left( \frac{\rho c k_B}{16\omega m \cos \theta |\Delta k|^2} \right)^2. \quad (98)$$

The corresponding mean transmission factor can now be calculated in the same way as by Cremer\(^1\) (see also equation (21)). If we use the following expression for $|\Delta k|^2$

$$|\Delta k|^2 = \left( k_x - k_B \right)^2 + \left( \frac{\eta k_B}{4} \right)^2, \quad (99)$$

the solution of the integral will be

$$\bar{\tau}_f = \left( \frac{2\rho c}{\omega m} \right)^2 \left( \frac{k_B}{k_a} \right)^2 \frac{\pi}{2\eta} \left( 1 - \frac{f_c}{f} \right)^{-1}, \quad (100)$$

which agrees with Cremers\(^1\) result for heavy walls.

However, if instead plates of a more reasonable size are considered, the transmission factor according to Ljunggren\(^9\) is obtained in the same way as for large plates, but with a new definition of $|\Delta k|^2$, where $\eta_{eq}$ is used instead of $\eta$, as

$$|\Delta k|^2 = \left( k_x - k_B \right)^2 + \left( \frac{3}{4L} \right)^2, \quad (101)$$

where the equivalent loss factor $\eta_{eq}$ is calculated as
\[ \eta_{eq} = \frac{6}{k_b L_m}. \]  

Hence, the transmission factor for a plain wave is proportional to \( L^2 \). For the corresponding case, calculated in three dimensions, the length \( 2L \) must vary with the angle \( \phi \), i.e. the mean value of \( (2L)^2 \) must be used. Equation (101) could then be written as

\[ |\Delta k|^2 = (k_y \sin \theta - k_x)^2 + \left( \frac{3}{2L_m} \right)^2, \]  

(103)

where \( L_m \) is defined as \( \langle (2L)^2 \rangle \). This expression is solved by Ljunggren \(^9\), and the mean projected trace length \( L_m \) is calculated as

\[ L_m = \sqrt{\frac{2ab}{\pi}} = \sqrt{\frac{2S_s}{\pi}}. \]  

(104)

For frequencies above \( f_r \) the sound reduction index for resonant transmission is calculated from equation (82) as

\[ R_f = 20 \lg \left( \frac{\omega m}{2 \rho c} \right) + 10 \lg \left( \frac{2 \eta_{eq}}{\pi} \right) + 10 \lg \left( \frac{f_r}{f} \right) + 10 \lg \left( 1 - \frac{f_r}{f} \right). \]  

(105)

4.2 Resonant transmission, calculation in three dimensions

Again, the solution for the response in the case of plane wave excitation is taken as a starting point. The important part of the solution is in this case the first part of the right side of the equation (7), that is

\[ v_2(x) = \frac{pk_b}{2\omega m} e^{-\beta_x x} \sin \left( \frac{(k - k_B)L}{k - k_b} \right). \]  

(106)

The power propagating away from the area of excitation can, for this model which is still two-dimensional, be written as

\[ P_r = c_b m \left( \frac{pk_b}{2\omega m} \right)^2 \Lambda \sin^2 \left( \frac{(k - k_B)L}{k - k_b} \right) \left( \frac{\Lambda}{k - k_b} \right)^2, \]  

(107)

where \( \Lambda \) again is the length of the boundaries at \( x = L \). A transmission factor with regard to the power propagating away from the area of excitation, can now according to Ljunggren \(^7\) be obtained with the help of equation (31) as
\[ \tau(\theta) = \left( \frac{4 \rho cc_g mk^2 \sin^2((k - k_b)L)}{(k - k_b)4\omega^2 m^2 L \cos \theta} \right)^2. \] (108)

It is evident from equation (108) that the main part of the transmission occurs at angels close to coincidence. The transmission factor is independent of \( \Lambda \) and hence also by \( \Delta x \). Further, the transmission factor is, after integration over the angle \( \theta \), also independent of the length \( L \) and \( L(x) \). This means that equation (108) could be used as the integrand in equation (21) when calculating the mean transmission factor

\[ \bar{\tau} = \left( \frac{8 \rho cc_g mk^2}{4\omega^2 m^2 L} \right)^2 \int_0^{\pi} \sin^2(\frac{(k - k_b)L}{(k - k_b)^2}) \sin \theta d\theta = \frac{2\pi \rho cc_g^2}{\omega m k^2 \sqrt{1 - k^2/k^2_c}}. \] (109)

If the power radiated to the receiving room is calculated as

\[ P_{\text{rad}} = \frac{P_n S_h \rho c}{\omega mk^2 \sqrt{1 - f_c/f}}, \] (110)

the mean transmission factor, regarding the radiated power, will be given by

\[ \bar{\tau} = \left( \frac{2 \rho c}{\omega m} \right)^2 \left( \frac{f_c}{f} \right) \frac{\pi}{2\eta} \left( 1 - \frac{f_c}{f} \right)^{-1} \left( \frac{S_R}{S_{\text{tot}}} \right). \] (111)

Hence, the sound reduction index for resonant transmission for frequencies above the coincidence frequency \( f_c \) is given by

\[ R_r = 20 \log \left( \frac{\omega m}{2 \rho c} \right) + 10 \log \left( \frac{2\eta}{\pi} \right) + 10 \log \left( \frac{f}{f_c} \right) - 20 \log(\sigma_{\text{res}}) - 10 \log \left( \frac{S_R}{S_{\text{tot}}} \right), \] (112)

where the radiation factor \( \sigma_{\text{res}} \) is calculated according to

\[ \sigma_{\text{res}} = \frac{1}{\sqrt{1 - f_c/f}}. \] (113)

In the same manner as for frequencies below the critical frequency, the sound reduction index for resonant and forced transmission are added according to

\[ R_{\text{total}} = -10 \log \left( 10^{R_r/10} + 10^{R_{\gamma}^f/10} \right). \] (114)
4.2.1 A note on the equivalent loss factor

It should be noted that the equivalent loss factor defined by equation (102), is valid for airborne sound transmission only. It has been shown (Ljunggren) that a slightly different value should be used for impact sound insulation. This also implies that the wellknown formula for the relation between the sound reduction index and the normalised impact sound becomes slightly different for the forced fields above the critical frequency than for the resonant fields. The reason for this is that the forced plate fields are different in the two cases; the reciprocity principle does still hold of course.

4.2.2 Comparison with other expressions

If $R_e$ is compared to the sound reduction index for frequencies above $f_c$ according to Cremer et. al. (equation VI.103)

$$R = 20 \log \left( \frac{\omega m}{2 \rho c} \right) + 10 \log \left( \frac{2 \eta}{\pi} \right) + 10 \log \left( \frac{f}{f_c} \right).$$  \hspace{1cm} (115)

it is seen that the first three terms is common with the resonant part of the sound reduction index according to equation (112), but in equation (115) the radiation from the plate or the size of this is not taken under consideration.

Josse and Lamure has reached one step further and added a radiation factor to the sound reduction index, which is the same as the one used in equation (112). The sound reduction index for frequencies above the coincidence frequency according to Josse and Lamure is given by

$$R = 20 \log \frac{\omega m}{2 \rho c} + 10 \log \frac{2 \epsilon}{\pi} + 10 \log \frac{f}{f_c} + 10 \log \left( 1 - \frac{f_c}{f} \right).$$  \hspace{1cm} (116)

However, it should be noted that the term $10 \log (1 - f_c / f)$ is included in Cremer's paper from 1942. Neither Cremer et. al. nor Josse and Lamure have regarded the forced transmission above the coincidence frequency.
5. References


