Connectivity and embeddability of buildings and manifolds

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Abstract

The results presented in this thesis concern combinatorial and topological properties of objects closely related to geometry, but regarded in combinatorial terms. Papers A and C have in common that they are intended to study properties of buildings, whereas Papers A and B both are concerned with the connectivity of graphs of simplicial complexes.

In Paper A it is shown that graphs of thick, locally finite and 2-spherical buildings have the highest possible connectivity given their regularity and maximal degree. Lower bounds on the connectivity are given also for graphs of order complexes of geometric lattices.

In Paper B an interpolation between two classical results on the connectivity of graphs of combinatorial manifolds is developed. The classical results are by Barnette for general combinatorial manifolds and by Athanasiadis for flag combinatorial manifolds. An invariant $b_\Delta$ of a combinatorial manifold $\Delta$ is introduced and it is shown that the graph of is $(2d - b_\Delta)$-connected.

The concept of banner triangulations of manifolds is defined. This is a generalization of flag triangulations, preserving Athanasiadis’ connectivity bound.

In Paper C we study non-embeddability for order complexes of thick geometric lattices and some classes of finite buildings, all of which are $d$-dimensional order complexes of certain posets. They are shown to be hard to embed, which means that they cannot be embedded in Euclidean space of lower dimension than $2d+1$, which is sufficient for all $d$-dimensional simplicial complexes. The notion of weakly independent atom configurations in general posets is introduced. Using properties of the van Kampen obstruction, it is shown that the existence of such a configuration makes the order complex of a poset hard to embed.