Influence of Frequency on Compaction of Sand in Small-Scale Tests

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Stockholm, 2013
Preface

This project was carried out between February 2011 and November 2013 at the Division of Soil and Rock Mechanics, Department of Civil and Architectural Engineering, Royal Institute of Technology (KTH) in Stockholm, Sweden. Supervisor was Prof. Stefan Larsson at KTH and assistant supervisor was Dr. Nils Rydén at the Faculty of Engineering, Lund University (LTH) and PEAB AB.

The project was funded by the Development Fund of the Swedish Construction Industry (SBUF), Dynapac Compaction Equipment AB, PEAB AB and KTH. Sincere thanks go to the funders, making this project possible.

I would like to express my gratitude to the supervisor Prof. Stefan Larsson and to the assistant supervisor Dr. Nils Rydén for their support, enthusiasm and guidance throughout the project.

Special thanks to Dr. Kent Lindgren at the Marcus Wallenberg Laboratory for Sound and Vibration Research, KTH, for manufacturing and lending test equipment, calibrating measurement systems and always being available for assistance in laboratory work. I would also like to thank Ingmar Nordfelt and his colleagues at Dynapac for fruitful discussions and assistance in planning tests and evaluating data. Dr. Anders Bodare and Dr. Rainer Massarsch at Geo Risk and Vibration Scandinavia AB have provided valuable comments throughout the work. Their contributions are highly appreciated.

Furthermore, I would like to express my sincere gratitude to all my colleagues at the Division of Soil and Rock Mechanics for valuable discussions and making my time at the department such an enjoyable experience.

Finally, I would like to thank my family and my friends. Without their never-ending support, this work would not at all have been possible.

Stockholm, November 2013

Carl Wersäll
Abstract

Vibratory rollers are commonly used for compaction of embankments and landfills. In a majority of large construction projects, this activity constitutes a significant part of the project cost and causes considerable emissions. Thus, by improving the compaction efficiency, the construction industry would reduce costs and environmental impact. In recent years, rollers have been significantly improved in regard to engine efficiency, control systems, safety and driver comfort. However, very little progress has been made in compaction effectiveness. While the compaction procedure (e.g. layer thickness and number of passes) has been optimized over the years, the process in which the machine compacts the underlying soil is essentially identical to the situation in the 1970s.

This research project investigates the influence of one crucial parameter, namely vibration frequency of the drum, which normally is a fixed roller parameter. Frequency is essential in all dynamic systems but its influence on the compaction efficiency has not been studied since the early days of soil compaction. Since laboratory and field equipment, measurement systems and analysis techniques at the time were not as developed as they are today, no explicit conclusion was drawn. Frequency-variable oscillators, digital sensors and computer-based analysis now provide possibilities to accurately study this concept in detail.

In order to examine the influence of vibration frequency on the compaction of granular soil, small-scale tests were conducted under varying conditions. A vertically oscillating plate was placed on a sand bed contained in a test box. The experiments were carried out in laboratory conditions to maximize controllability. The first test setup utilized an electro-dynamic oscillator where dynamic quantities, such as frequency and particle velocity amplitude, could be varied in real-time. The second test setup included two counter-rotating eccentric mass oscillators, where tests were conducted at discrete frequencies. This type of oscillator has a force amplitude that is governed by frequency.

The main objectives of the tests were to determine the optimal compaction frequency and whether resonance can be utilized to improve compaction efficiency. Results showed that resonance had a major influence in the electro-dynamic oscillator tests, where the applied force amplitude is low, and the optimal compaction frequency is the resonant frequency under these circumstances. In the rotating mass oscillator tests, where a high force was applied to the plate, resonant amplification was present but not as pronounced. Since force increase with frequency, the optimal frequency to obtain the highest degree of compaction is very large. In a practical regard, however, frequency should be kept as low as possible to minimize machine wear and emissions while still achieving a sufficient compaction of the soil. Considering the practical issues, it is proposed that surface compactors should operate slightly above the resonant frequency. However, the applicability to vibratory rollers must be confirmed in full-scale tests.

The thesis also presents an iterative method to calculate the frequency response of a vibrating plate, incorporating strain-dependent soil properties. Calculated dynamic quantities are compared to measured values, confirming that the method accurately predicts the response.
Sammanfattning


Detta forskningsprojekt undersöker inflytandet av en grundläggande parameter, nämligen valsens vibrationsfrekvens, vilken vanligtvis är en icke-variabel vältparameter. Frekvensen är av avgörande betydelse i alla dynamiska system men dess inflytande på packningseffektiviteten har inte undersöpts sedan jordpackningens barndom. Eftersom dåtidens laboratorie- och fältutrustning, mätsystem och analysförfarande inte var så utvecklade som de är idag uppnäddes inga konkreta slutsatser. Frekvensvariabla vibratorer, digitala mätsystem och datorbaserad utvärdering tillhandahåller nu nya möjligheter för att studera detta concept i detalj.


Avhandlingen presenterar också en iterativ metod för att beräkna frekvensresponsen av en viberande platta, som tar hänsyn till töningsberoende jordegenskaper. Beräknade dynamiska kvantiteter jämförs med uppmätta värden och bekräftar att metoden framgångsrikt kan förutsäga responsen.
List of Publications

The following papers are appended to the thesis:

Paper I


Wersäll conducted the experiments, performed the analysis and wrote the paper. Larsson supervised the work and assisted in interpreting the results and writing the paper.

Paper II


Wersäll conducted the experiments, performed the analysis and wrote the paper. Larsson supervised the work and assisted in writing the paper. Rydén and Nordfelt assisted in interpreting the results and provided valuable comments on planning the tests and writing the paper.
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<tr>
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<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>Dimensionless stiffness coefficient</td>
</tr>
<tr>
<td>$B_z$</td>
<td>Mass ratio</td>
</tr>
<tr>
<td>$F$</td>
<td>Force</td>
</tr>
<tr>
<td>$F_0$</td>
<td>Force amplitude</td>
</tr>
<tr>
<td>$F_c$</td>
<td>Force in damper element</td>
</tr>
<tr>
<td>$F_k$</td>
<td>Force in spring element</td>
</tr>
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<td>$F_m$</td>
<td>Force in mass element</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus</td>
</tr>
<tr>
<td>$G_{\text{max}}$</td>
<td>Small-strain shear modulus</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Length of strained element</td>
</tr>
<tr>
<td>$M$</td>
<td>Dynamic magnification factor for constant force</td>
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<tr>
<td>$M'$</td>
<td>Dynamic magnification factor for rotating mass oscillators</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of loading cycles</td>
</tr>
<tr>
<td>OCR</td>
<td>Overconsolidation ratio</td>
</tr>
<tr>
<td>$P$</td>
<td>Power</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Atmospheric pressure</td>
</tr>
<tr>
<td>$PI$</td>
<td>Plasticity index</td>
</tr>
<tr>
<td>$S$</td>
<td>Settlement of the plate</td>
</tr>
<tr>
<td>$W_c$</td>
<td>Energy consumed in one cycle</td>
</tr>
<tr>
<td>$a$</td>
<td>Acceleration</td>
</tr>
<tr>
<td>$a_0$</td>
<td>Acceleration amplitude</td>
</tr>
<tr>
<td>$c$</td>
<td>Damping coefficient</td>
</tr>
<tr>
<td>$c_{\text{cr}}$</td>
<td>Critical damping coefficient</td>
</tr>
<tr>
<td>$c_R$</td>
<td>Rayleigh wave speed</td>
</tr>
<tr>
<td>$c_S$</td>
<td>Shear wave speed</td>
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<td>Eccentricity</td>
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<td>Diameter for 10% passing</td>
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<tr>
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<td>Exponent depending on $\text{PI}$</td>
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<tr>
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<tr>
<td>$m_{e\varepsilon}$</td>
<td>Eccentric moment</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Apparent mass</td>
</tr>
</tbody>
</table>
\( n \) Exponent depending on stress state
\( r_0 \) Footing radius
\( t \) Time
\( u \) Displacement
\( u_0 \) Displacement amplitude
\( u_A \) Nominal displacement amplitude
\( u_{LVDT} \) Settlement of the plate
\( v \) Velocity
\( v_0 \) Velocity amplitude
\( v_{LVDT} \) Displacement velocity of the plate
\( w \) Water content
\( \Delta V_c \) Compacted volume
\( \Delta V_d \) Displaced volume
\( \alpha \) Empirical factor depending on PI
\( \beta \) Dimensionless frequency
\( \beta \) Empirical exponent depending on PI
\( \gamma \) Shear strain
\( \gamma_r \) Reference shear strain
\( \varepsilon \) Compressive strain
\( \zeta \) Damping ratio
\( \zeta_{max} \) Maximum damping ratio
\( \nu \) Poisson's ratio
\( \rho \) Mass density
\( \sigma_0^n \) Effective isotropic confining pressure
\( \tau \) Shear stress
\( \omega \) Circular frequency
\( \omega_h \) Circular natural frequency
1 INTRODUCTION

1.1 Background

Soil compaction is the most common ground improvement method and is often necessary to reduce settlement, increase stability and stiffness of the subgrade, control swelling and creep, lower the risk of liquefaction and decrease the permeability. It implies densification of the soil by reducing its pore volume. In granular soil, this is normally achieved by vibration or impact, producing stress-waves that rearrange the soil particles into a denser state. In construction of embankments and landfills, soil is placed in layers and compacted using vibratory roller (Figure 1). This process is time-consuming and normally constitutes a significant part of the project cost as well as giving rise to considerable emissions. It is thus in the interest of the industry to improve the compaction efficiency and reduce the time for this activity.

As vibratory rollers became popular around the 1950s, the optimal compaction procedure became a topic of research. One fundamental property that was investigated was the compaction frequency. All rollers operate with rotating eccentric mass oscillators that produce increasing force amplitude with frequency. However, all dynamic systems have a resonant frequency where vibrations are amplified. For roller compaction, this is within the operating frequency of the roller and taking advantage of this amplification might therefore be feasible. Several studies were conducted in the early years of this research field (especially in the 1950s and 1960s), with varying results. However, the available compaction equipment, measurement systems and evaluation techniques at the time were far from what they are today. Frequency was normally varied by adjusting the speed of the engine, which is a crude method for frequency variation. Since digital sensors or computers were not

Figure 1. Vibratory roller (courtesy of Dynapac Compaction Equipment AB).
available, results were difficult to interpret. Furthermore, there are many aspects that affect the results, not all of which were known at the time. First of all, a dynamic system behaves very differently below, close to or above resonance. Hence, it is important to be aware of the compaction frequency in relation to the resonant frequency. The acceleration amplitude is also of great importance. Several authors have found that compaction should be performed at accelerations above 1 g to be effective (D’Appolonia et al. 1969; Dobry & Whitman 1973). There are several other aspects, such as dynamic-to-static load ratio, shape of the contact surface and soil properties. Due to the complexity of the problem, the early studies had varying conclusions.

The first to propose a compactor, utilizing frequency to obtain the maximum degree of compaction was Hertwig (1936). Tschebotarioff & McAlpin (1947) concluded that the subsidence of a piston, vibrating on the soil was independent of frequency as long as the total number of cycles was constant. However, the frequency in those tests was very low, less than 20 Hz. Bernhard (1952) conducted laboratory tests with variable frequency and constant force, obtaining a more efficient compaction at the resonant frequency. Converse (1953) conducted field compaction tests of sand and also concluded that resonance could be utilized. Forssblad (1965) highlighted that in the tests by Bernhard and Converse the dynamic load was only in the same order of magnitude as the static weight and argued that the results could not be compared to roller compaction. Several other authors found a correlation between resonance and increased compaction efficiency (Johnson & Sallberg 1960; Lorenz 1960). Forssblad (1965) argued that the increase in force amplitude with frequency would be too significant for the resonant amplification to influence the compaction effect and that the technical difficulties for utilizing resonance would exceed the practical advantages. Thus, there was no agreement among researchers on whether resonant amplification could be used to improve roller compaction. There was, however, one conclusion on with the community agreed, namely that effective compaction must be performed above the resonant frequency.

There have been many attempts to model the roller behavior by mathematical or numerical methods. Yoo & Selig (1979) presented a lumped-parameter model that formed the basis for most subsequent models of roller behavior. These studies have mainly been conducted for the purpose of continuous compaction control and intelligent compaction (e.g., Forssblad 1980; Thurner & Sandström 1980; Adam 1996; Anderegg & Kaufmann 2004; Mooney & Rinehart 2009; Facas et al. 2011). Modeling the dynamic behavior of compaction equipment is complicated by the fact that soil shows very nonlinear stress-strain behavior. Most models do not take this into account. However, Susante & Mooney (2008) developed a model that includes nonlinear soil stress-strain behavior, calculating the response in time domain.

In the 1970s, computer programs using an equivalent linear approach to determine the nonlinear seismic response during earthquakes, such as SHAKE (Schnabel et al. 1972), became popular. These programs apply an iterative procedure to determine the nonlinear response of a transient time history. As finite element and other numerical methods were introduced, these became dominating in calculating the nonlinear response. However, numerical methods are time-consuming and require skilled operators to be reliable. Thus, equivalent linear methods are still useful but there has hardly been any development of these concepts in recent years. No one has previously used this approach
for calculating the nonlinear response of an oscillating foundation (such as surface compaction equipment) on soil with strain-dependent properties.

1.2 Objectives

This research project aims at determining the optimal compaction frequency of vibratory rollers and to investigate whether resonance in the roller-soil system can be utilized for increasing the compaction efficiency. As a first step, the fundamental dynamic behavior during frequency-variable compaction is studied. The main objective of this thesis is to investigate the influence of frequency on the compaction of sand in small-scale tests. These are conducted under varying conditions to quantify the effect of, not only frequency, but also type of oscillator, dynamic load and soil water content. The small-scale tests form a basis for full-scale tests using vibratory roller.

Another objective is to develop an equivalent linear calculation procedure that can be performed in frequency domain. These calculations are compared to results of the small-scale tests.

1.3 Outline of Thesis

This thesis consists of an introductory part and two appended paper, one published in a peer-reviewed journal and the other submitted to the same journal. The introductory part is intended as an introduction and a complement to the appended papers. It contains background information, summary of the main findings and further development of some concepts that are included in the papers.

Chapter 2 describes the fundamentals of dynamic single degree of freedom systems and vertically oscillating foundations. The linear equivalent calculation procedure developed in Paper II is described in detail. All necessary background information for understanding of this procedure is provided.

Chapter 3 is a description of the small-scale tests. Since the tests are described thoroughly in the papers, this chapter summarizes briefly the test setups and provides additional photographs of the equipment. The test results of the two papers are summarized and discussed in relation to each other.

Chapter 4 contains a summary of the appended papers.

Chapter 5 provides the main conclusions of the thesis and papers and suggests further research.
2 OSCILLATING FOUNDATIONS ON SOFTENING SOIL

Studies on vibrating foundations began with the objective to analyze ground vibrations from rotating machinery founded on the ground surface. This has become the basis for dynamic soil-structure interaction analysis, including many more applications than just rotating machinery, such as wind turbines and bridge abutments subjected to traffic load. This chapter describes how basic equations for vibrating foundations can be combined with empirical knowledge for nonlinear stress-strain behavior of soil to predict the dynamic response of vibrating foundations on softening soil. Since this thesis deals with vertical oscillations on granular soil, other oscillatory motions or plastic soils are not treated herein. For other vibration modes, such as rocking or horizontal oscillation, reference is made to Richart et al. (1970) and Gazetas (1983).

2.1 Single Degree of Freedom Systems

The dynamic behavior of a vertically oscillating foundation can be estimated by analyzing a single degree of freedom (SDOF) system consisting of a mass, a dashpot and a spring, where the force in these three components are proportional to acceleration, velocity and displacement, respectively. The forces in each element \( F_m, F_c \) and \( F_k \) are determined by Equations 1 to 3.

\[
F_m = ma \\
F_c = cv \\
F_k = ku
\]  

where \( m \) is mass, \( a \) is acceleration, \( c \) is damping coefficient, \( v \) is vibration velocity, \( k \) is spring stiffness and \( u \) is displacement. Since the velocity is given by \( v = \frac{du}{dt} \) and the acceleration is given by \( a = \frac{d^2u}{dt^2} \), and since all forces need to be in equilibrium, a SDOF system can be described by the second order differential equation presented in Equation 4.

\[
m \frac{d^2u}{dt^2} + c \frac{du}{dt} + ku = F(t)
\]  

where \( F(t) \) is the externally applied force. The system may be either under-damped, critically damped or over-damped depending on if the damping coefficient is less than, equal to or larger than the critical damping coefficient \( c_{cr} \). The ratio between these is denoted damping ratio, \( \zeta \), and is shown in Equation 5.

\[
\zeta = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}}
\]

All dynamic systems have one or several natural frequencies. The circular natural frequency, \( \omega_n \), of a SDOF system is calculated by Equation 6.
\[ \omega_n = \sqrt{\frac{k}{m}} \] (6)

It is convenient to express frequency normalized by the natural frequency, the so-called dimensionless frequency, \( \beta \), as shown in Equation 7.

\[ \beta = \frac{\omega}{\omega_n} \] (7)

where \( \omega \) is the circular frequency. For a harmonic external load, the solution to Equation 4 may then be expressed by Equation 8.

\[ u_0 = \frac{F_0}{k} \frac{1}{\sqrt{(1 - \beta^2)^2 + (2\zeta\beta)^2}} \] (8)

where \( u_0 \) is displacement amplitude and \( F_0 \) is force amplitude. The dynamic displacement in relation to the displacement that would be obtained from static loading by the same force is called dynamic magnification factor. It is calculated by Equation 9 and shown in Figure 2 for different values of the damping ratio.

\[ M = \frac{u_0}{F_0/k} = \frac{1}{\sqrt{(1 - \beta^2)^2 + (2\zeta\beta)^2}} \] (9)

When the frequency approaches zero, the magnification factor approaches unity. As the excitation frequency approaches the natural frequency, the dynamic response is significantly magnified. The

![Figure 2. Dynamic magnification factor for constant force and different damping ratios.](image-url)
frequency where maximum magnification occurs is the resonant frequency, which is equal to the natural frequency when damping is zero and slightly lower as the damping ratio becomes larger. When the frequency is increased above resonance, the magnification factor (and thus the displacement amplitude) decreases and approaches zero for large frequencies. If the damping ratio is zero, the resonant amplification is infinite. This is an unrealistic case as there are no real systems without damping. However, for a damping ratio of 10 %, which represents quite high damping, the resonant amplification is still as high as 5 times the static value. The damping ratios of 20-30 % shown in the figure are uncommon but can occur for example during large strain in soil, as will be discussed below.

The magnification factor shown in Figure 2 is for the case where the applied force amplitude is constant with frequency. If the load would be produced by rotating mass oscillators, force amplitude would increase rapidly with frequency according to Equation 10.

\[ F_0 = m_e e \omega^2 \]  

(10)

where \( m_e \) is the eccentric mass and \( e \) is the eccentricity. The nominal displacement amplitude of a rotating mass oscillator, \( u_h \), is given by Equation 11.

\[ u_A = \frac{m_e e}{m} \]  

(11)

Details regarding the properties of rotating mass oscillators can be found in, for example, Forssblad (1981). By applying Equations 6 and 10 and 11, Equation 8 may be rewritten as Equation 12 with a corresponding dynamic magnification factor for rotating mass oscillators, \( M' \), given by Equation 13.

\[ u_0 = u_A \frac{\beta^2}{\sqrt{(1 - \beta^2)^2 + (2\zeta \beta)^2}} \]  

(12)

\[ M' = \frac{u_0}{u_A} = \frac{\beta^2}{\sqrt{(1 - \beta^2)^2 + (2\zeta \beta)^2}} \]  

(13)

The magnification factor is shown in Figure 3. Since the dynamic force is generated by the rotating masses, there is no or very little dynamic displacement when the frequency is zero or close to zero. Thus the magnification factor approaches zero when the frequency goes toward zero. The behavior around resonance is similar to the case with constant force, except from the resonant frequency being slightly larger than the natural frequency. As the frequency is increased, the magnification factor converges toward unity for all damping ratios, i.e. the displacement amplitude approaches the nominal amplitude. The curves in Figure 2 and Figure 3 are called frequency response functions since they describe the response of a system to frequency-dependent dynamic input variable. Instead of magnification factor, they can be displayed for any other dynamic amplitude quantity, such as dynamic displacement, velocity, acceleration or force but are then not, by definition, frequency response functions. The displayed dynamic output is then herein simply denoted frequency response or response diagram.
2.2 Vertically Oscillating Foundations

The previous section described fundamental dynamic properties of SDOF systems. This section focuses on calculation of the dynamic response of a vertically oscillating foundation on an elastic half-space, as described by Lysmer & Richart (1966). For horizontal or rocking motion, see Hall (1967), and for torsion, see Richart et al. (1970). Gazetas (1983) presented equations for foundations on layered soil. Model tests have been conducted to experimentally determine the response of oscillating foundations under various conditions (e.g. Novak 1970; Baidya & Murali Krishna 2001; Mandal et al. 2012).

Lysmer & Richart (1966) showed how the behavior a vertically oscillating foundation on an elastic half-space can be simulated by a SDOF model, where spring stiffness and damping ratio are given by Equations 14 and 15.

\[ k = \frac{4G\rho_0}{1 - \nu} \]  
\[ \zeta = \frac{0.425}{\sqrt{B_z}} \]  

where \( G \) is soil shear modulus, \( \rho_0 \) is the footing radius, \( \nu \) is Poisson’s ratio of the soil and \( B_z \) is the mass ratio obtained by Equation 16.

\[ B_z = \frac{1 - \nu}{4 \rho r_0^3} \]  

Figure 3. Dynamic magnification factor for rotating mass oscillators and different damping ratios.
where $m$ is the total mass and $\rho$ is the mass density of the soil. The total mass consists of two components. One is the mass of the foundation, $m_0$, including any external static load on it. The other part is called apparent mass, $m_s$, which corrects for the fact that stiffness decreases with frequency (Gazetas 1983). Different equations exist for calculating the apparent mass. In this study it becomes very small and is thus neglected. The total mass is given by Equation 17 and one expression for the apparent mass is given by Equation 18.

$$m = m_0 + m_s$$  \hspace{1cm} (17)

$$m_s = \frac{1.08}{1 - \nu} \rho r_0^3$$ \hspace{1cm} (18)

By applying the above equations to the SDOF model presented in the previous section, the dynamic behavior of a vertically oscillating foundation can be estimated. The main limitation with this and many other studies on the subject is the assumption that the subgrade is elastic. Since the stress-strain behavior of soil (especially non-plastic soil) is highly nonlinear, this simplification can lead to very large discrepancies between calculated and real dynamic responses. Soils with high plasticity, however, behave more elastic and the implications of treating the soil as perfectly linear are thus less severe. Nonlinear stress-strain behavior of soil, and a method to take these properties into account, is explained below.

### 2.3 Soil Nonlinearity

Deformation behavior of granular soil is often modeled by a hyperbolic stress-strain formulation (Kondner 1963a; Kondner 1963b; Hardin & Drnevíc 1972a; Hardin & Drnevíc 1972b). The hyperbolic shear stress $\tau$ is given by Equation 19.

$$\tau = \frac{G_{\text{max}} \gamma}{1 + \left| \frac{\gamma}{\gamma_r} \right|}$$ \hspace{1cm} (19)

where $G_{\text{max}}$ is the small-strain shear modulus, $\gamma$ is the shear strain and $\gamma_r$ is a reference strain. At very low strains the shear modulus is at its maximum, hence the denotation $G_{\text{max}}$. Equation 19 describes the so-called backbone curve (also called virgin curve or skeleton curve), which applies to virgin loading. When soil is subjected to cyclic loading, the stress-strain relationship forms a hysteresis loop often modeled by Masing Rule (Masing 1926), which implies magnifying the backbone curve by a factor of two during unloading and reloading. The backbone curve and hysteresis loop are shown in Figure 4.
The small strain shear modulus can be estimated by Equation 20 according to Hardin (1978).

\[ G_{\text{max}} = \frac{A \cdot OCR^k}{0.3 + 0.7\varepsilon^2} P_a (1-n) \sigma_0^m \]  

(20)

where \( A \) and \( n \) are dimensionless parameters, \( OCR \) is the overconsolidation ratio, \( k \) is a parameter depending on plasticity index (PI), \( \varepsilon \) is the void ratio, \( P_a \) is the atmospheric pressure (100 kPa) and \( \sigma_0^m \) is the effective isotropic confining pressure. Equation 20 is often seen with fixed values of \( A \) and \( n \). However, these parameters vary with soil type and applying the equation with fixed values can thus be misleading. Studies investigating the values of the above parameters (e.g. Stokoe et al. 1999) have found \( A \) to vary in wide interval and \( n \) to vary slightly.

As can be seen in Figure 4, stiffness decreases with strain. Many authors have studied the strain-softening effect on the shear modulus (Seed et al. 1986; Vucetic & Dobry 1991; Rollins et al. 1998; Stokoe et al. 1999; Assimaki et al. 2000; Kausel & Assimaki 2002; Tatsuoka et al. 2003; Massarsch 2004; Zhang et al. 2005, among others). In the hyperbolic formulation described above, the shear modulus \( G \) decreases according to Equation 21.

\[ \frac{G}{G_{\text{max}}} = \frac{1}{1 + \left( \frac{\gamma}{\gamma_r} \right)} \]  

(21)
The reference strain is a curve-fitting parameter that depends on soil properties. It represents the strain at which the shear modulus has half the value of the small strain shear modulus. As shear strain increases, it affects not only the shear modulus, but also the damping ratio increases significantly. Hardin & Drnevic (1972b) proposed the formulation for damping ratio presented in Equation 22.

\[
\frac{\zeta}{\zeta_{\text{max}}} = 1 - \frac{G}{G_{\text{max}}}
\]  

(22)

where \(\zeta_{\text{max}}\) is the maximum damping ratio, which depends on the soil type and number of loading cycles \(N\). Equation 23 shows the maximum damping ratio (in percent) for clean dry sand and Equation 24 presents the same parameter for saturated sand.

\[
\zeta_{\text{max}} = 33 - 1.5\log(N)
\]  

(23)

\[
\zeta_{\text{max}} = 28 - 1.5\log(N)
\]  

(24)

Rollins et al. proposed a model for shear modulus, Equation 25, and damping ratio, Equation 26, based on tests conducted on gravel.

\[
\frac{G}{G_{\text{max}}} = \frac{1}{1.2 + 16|\gamma|(1 + 10^{-20}|\gamma|)}
\]  

(25)

\[
\zeta = 0.8 + 18(1 + 0.15|\gamma|^{-0.9})^{-0.75}
\]  

(26)

There is an apparent uncertainty in Rollins’ equations. The shear modulus reduction ratio does not approach one for small strains due to the factor 1.2 in the denominator. As the ratio has to become unity for zero strain, the most obvious assumption is that this is a misprint in the paper.

A further formulation for shear modulus was proposed by Massarsch (2004), as presented in Equation 27.

\[
\frac{G}{G_{\text{max}}} = \frac{1}{1 + \alpha|\gamma|(1 + 10^{-\beta|\gamma|})}
\]  

(27)

where \(\alpha\) and \(\beta\) are empirical factors depending on PI. The variation of \(\alpha\) and \(\beta\) are shown in Figure 5. The study was conducted with focus on fine-grained soils and thus no values are available for PI less than 10%. However, Stokoe et al. (1999) found that strain-softening relationships of natural non-plastic soils and soils with low plasticity are very similar. The behavior of granular soil (non-plastic) can thus be estimated by applying values for PI = 10%.
The shear modulus reduction ratios according to Equations 21, 25 and 27 are shown in Figure 6. A reference strain of 0.05 % was chosen, which is a typical value for sand (Stokoe et al. 1999). A modified version of Rollins’ equation is also shown, where the term 1.2 has been replaced by 1.0. The reduction according to Massarsch is very similar to Hardin & Drnevic at low strains but implies slightly higher values of the shear modulus at large strains. The original expression by Rollins et al. is obviously not correct at small strains. The modified equation shows smaller shear modulus than the other expression at small and moderate strains, while larger at quite high strain level and similar to the other curves at very high strains.

Figure 7 shows the damping ratio calculated by Equation 26 (Rollins et al.), Equations 22 and 23 (Hardin & Drnevic, dry sand, first loading cycle) and Equations 22 and 24 (Hardin & Drnevic, saturated sand, first loading cycle). Seed et al. (1986) compiled results from many laboratory and field studies for strain-dependent damping ratio of sand. Equation 22 (using the reference strain 0.05 %, as above) is fitted to that data and shown in the same figure. The data from Seed et al. show a damping ratio close to that of clean saturated sand according to Hardin & Drnevic. For clean dry sand, the damping ratio is higher. The curve from Rollins et al. has a significantly lower damping ratio at high strains. All curves based on hyperbolic strain have one major disadvantage, namely that they approach zero for small strains. Since the damping ratio always is greater than zero, these models are unreliable at small strains.
2.4 Calculation of Foundation Response

The expressions given in Sections 2.1 and 2.2 can be used to calculate the frequency response for a vertically oscillating foundation on an elastic half-space. However, this is usually not sufficient for capturing the dynamic behavior of foundations on softening soil unless the strains are very small or the soil is highly plastic, as discussed above. During vibratory compaction, the case is normally the
opposite, i.e. very large strains and non-plastic soil. This section describes a simple method to incorporate strain-softening behavior into the calculation of frequency response, proposed by Wersäll et al. (2013). The procedure is explained in relation to the small-scale tests conducted with rotating mass oscillators described in Chapter 3 but is equally applicable to oscillations of other types of foundations under different dynamic load.

The first step is to determine the uncorrected displacement amplitude frequency response. This is done by first estimating the shear wave speed, $c_s$, and the mass density of the soil. The small-strain shear modulus can then be calculated by Equation 28.

$$G_{\text{max}} = \rho c_s^2$$  \hspace{1cm} (28)

Note that the shear wave speed in Equation 28 represents that at small strain and that it will decrease at larger strains. The small-strain shear wave speed can be measured by, for example, seismic tests. Alternatively, the small-strain shear wave speed can be estimated directly by Equation 20. Determining Poisson’s ratio of the soil and knowing the radius and mass of the foundation, the spring stiffness and damping ratio can be calculated by Equations 14-16. Depending on the relative size of the calculated apparent mass, it may be neglected and the mass of the foundation can be adopted as the total mass. Since the stiffness varies with frequency, each point on the curve will have a different natural frequency. The natural and dimensionless frequencies are calculated by Equations 6 and 7. Normally, the eccentric moment of the oscillator, $m_e e$, is known and the force amplitude can thus be calculated by Equation 10 for the frequency range of interest. The uncorrected frequency response for displacement amplitude is then obtained by Equation 8.

The next step is to calculate the shear strain in the soil. Each point in the response diagram represents a value of vertical displacement amplitude. This must first be converted to compressive strain and then to shear strain. Since a single value of strain is necessary for each frequency, strain must be assumed to be evenly distributed down to a certain depth. However, strain is not constant over depth but can rather be assumed to follow the Boussinesq distribution shown in Figure 8 (assuming the soil moduli are constant over depth). Figure 8 shows conceptually how the Boussinesq strain distribution can be approximated as triangular and then further simplified to a rectangular distribution.
The depth to which the simplified rectangular strain distribution extends must thus be assumed, which gives the length of the strained element, \( L_e \). The compressive strain, \( \varepsilon \), can then be calculated for each frequency by Equation 29 by using the displacement given by the response diagram.

\[
\varepsilon = \frac{u_0}{L_e}
\]  

(29)

By assuming axisymmetric conditions, the shear strain is calculated by Equation 30 (Atkinson & Bransby 1978).

\[
\gamma = \frac{2}{3} \varepsilon (1 + \nu)
\]

(30)

After the shear strain has been obtained, new strain-dependent values of the shear modulus and damping ratio can be calculated for every frequency by a suitable formulation. In this study, Equation 27 was applied for shear modulus and Equation 22 for damping ratio. The maximum damping ratio was assumed to be 33 %, based on Equation 23.

The new shear modulus and damping ratio are then used, applying the same procedure, to calculate the frequency response for displacement, compressive strain and shear strain. The new shear strain again yields new values of the shear modulus and damping ratio and the process is repeated. This is iterated until the response diagrams converge with sufficiently small variations between iterations, for each value of frequency. The final displacement function then gives the frequency response for velocity amplitude \( v_0 \), acceleration amplitude \( a_0 \) and force amplitude by Equations 31 to 33.
\[ v_0 = \omega u_0 \]  
\[ a_0 = \omega^2 u_0 \]  
\[ F_0 = ku_0 \]
3 DESCRIPTION OF SMALL-SCALE TESTS

Small-scale compaction tests were conducted in laboratory environment. The tests were divided over two main setups and several test series. An electro-dynamic oscillator was used in the first setup, creating high controllability. The second setup utilized rotating mass oscillators with less controllability but higher resemblance to field conditions. Both setups were purpose-built for the tests. Since the tests are described in detail in the appended papers, this chapter provides only a summary of the test setups and results.

Sand was placed in a box having inner measurements 1100 mm x 700 mm x 370 mm (width x length x height). The boundaries were coated with 30 mm of expanded polystyrene to reduce vibration reflections and the bottom of the box consisted of the concrete floor below the box. The filling method is crucial to obtain similar test conditions as it has a strong influence on the initial density of the sand (Rad & Tumay 1987). Due to the large number of tests and the large sand volume, the material was filled by pouring. Since there was no target density but rather a similar density in all tests that was important, this method was considered sufficient. Other more precise methods, such as raining, would be unrealistically time-consuming. The pouring was performed in the same way by the same person to minimize any differences in initial density.

3.1 Tests with Electro-Dynamic Oscillator

This type of oscillator consists of a static mass and a significantly smaller oscillating mass on top. In the first setup, shown in Figure 9, the static mass was connected to a steel rod with a circular steel plate, 84 mm in diameter, at the other end. The rod was running through two low-friction polytetrafluoroethylene (Teflon) rings, allowing the rod to move only in the vertical direction. The plate was placed directly on the sand surface. The advantage with an electro-dynamic oscillator is that dynamic quantities can be adjusted in real-time, thus making the tests very controllable. The system can be illustrated as a coupled mass-spring-dashpot model, shown in Figure 10. The dynamic force $F(t)$ is generated in the spring between the oscillating and static masses. Since the oscillating mass is much smaller than the static mass, the soil response does not influence its vibrations, which means that measurements on the oscillating mass are independent of soil-compactor resonances. This provides the opportunity to conduct tests under constant dynamic load. The total mass of the vertically moving system was 37.4 kg.

One accelerometer was placed on the oscillating mass and one on the static mass. A force transducer was placed between the plate and the rod measuring the reaction force. Furthermore, the rod was connected to a linear variable differential transformer (LVDT), measuring the vertical settlement of the plate. Acceleration signals from the accelerometers were integrated in the amplifiers so that particle velocity was recorded. An external amplifier controlled the amplitude of the oscillator and the frequency was adjusted by a function generator. Geophones were placed in the sand, on the box perimeter and on the concrete floor. A vertical accelerometer was buried in the sand, 20 cm below the plate.
Figure 9. Tests with electro-dynamic oscillator. (a) Preparation of test box. (b) Preloading. (c) The complete test setup. (d) Settlement and heave after compaction. (e) Measurement with geophones inside and outside of the test box. (f) Test for investigation of soil displacement.
Figure 10. Representation of tests with the electro-dynamic oscillator as a coupled mass-spring-dashpot system.

Each test was conducted with frequency sweep and a constant particle velocity on the moving mass. The frequency was controlled by the function generator and the velocity amplitude was adjusted manually on the oscillator amplifier. The measured acceleration signal was integrated and plotted on a computer screen in real-time for adjusting the amplitude. The tests are described thoroughly in Wersäll and Larsson (2013).

### 3.2 Tests with Rotating Mass Oscillators

To obtain conditions that are more similar to those during roller compaction, a new compactor was manufactured using two rotating mass oscillators together giving rise only to a vertical component. Except for the new type of oscillators, the equipment, shown in Figure 11, was very similar as in the previous small-scale tests. A mass-spring-dashpot representation would here only include one mass where the force is directly applied. An accelerometer was mounted on the bottom plate and a force transducer was placed between the plate and the rod. In the same manner as the previous tests, the vertical settlement was measured by an LVDT. In some tests, geophones were placed in the sand and on the box perimeter or outside the box. The mass of the vertically moving system was 28.8 kg.

The tests, described in Wersäll et al. (2013), were conducted at discrete frequencies, i.e. not using frequency sweep as in the previous tests. The sand was replaced between each test. When using rotating mass oscillators, the eccentric moment is constant and the applied force increases with the square of frequency. It was thus not possible to control particle velocity or any other dynamic quantity. This is true also for compaction with vibratory roller. In each test, the sand was compacted for 30 seconds and the settlement was recorded.
Figure 11. Tests with rotating mass oscillators. (a) Test setup. (b) Oscillators with protective caps removed. (c) Preloading by vibrating a wooden plate. (d) After completion of a test on dry sand. (e) After completion of a test on wet sand. (f) Imprint in wet sand after test and removal of the plate.
3.3 Results of Small-Scale Tests

In the tests using the electro-dynamic oscillator, compaction was significantly enhanced close to the resonant frequency with hardly any compaction sufficiently below or above this frequency. The tests with rotating mass oscillators, on the other hand, showed a more complex relationship between frequency and compaction. The applied force increased with frequency, producing a very high degree of compaction at the higher frequencies and hardly any compaction at the low frequencies. In the mid-range, however, there was a resonant amplification, which was quite modest compared to the amplification in the previous tests. The main differences between the two test setups were the following:

- The dynamic load in comparison with the static weight.
- The variation of input load with frequency – constant particle velocity in the first test setup and force increasing with the square of frequency in the second setup.
- Higher dynamic loads in the second setup.

The difference in resonant amplification originates from the differences between tests, as listed above. Since the dynamic-to-static load ratios are significantly lower than one and far above one, respectively, the fundamental dynamic behavior is essentially different. Furthermore, since force is increasing drastically with frequency during operation of the rotating mass oscillators, resonant amplification becomes less pronounced. The most influential aspect, however, is most likely the high dynamic load, giving rise to large strains, which produces a significant reduction in the soil stiffness while the damping ratio increases, as has been explained in Section 2.3. This causes the curve to flatten out (Wersäll et al. 2013). The effect of increased damping ratio can be understood by observing Figure 3. In spite of the modest amplification at resonance, it is probable that this effect can be utilized in compaction by vibratory roller, as has been discussed in Paper II.

Frequency response of dynamic quantities was calculated by the equivalent linear calculation procedure described in Chapter 2. When parameters of the rotating mass oscillator tests were applied to the equations, the results matched measured data well. The conclusion is thus drawn that this method accurately can predict the dynamic behavior of oscillating foundations during compaction or other applications where large strains are involved.
4 SUMMARY OF APPENDED PAPERS

4.1 Paper I

Small-Scale Testing of Frequency-Dependent Compaction of Sand Using a Vertically Vibrating Plate

Carl Wersäll and Stefan Larsson

Published in ASTM Geotechnical Testing Journal 2013:36(3)

The paper presents results from 85 small-scale tests that were conducted using a vertical electrodynamic oscillator, connected to a plate and placed on a sand bed. Frequency was adjusted continuously to assess its influence on compaction of the underlying sand. The results showed that the rate of compaction with this type of compactor is significantly magnified at, and close to, the resonant frequency. The results indicated that velocity amplitude is a crucial quantity in obtaining sufficient compaction in for the test setup used. While a large velocity amplitude gave rise to a large degree of compaction, it also caused significant soil displacement and heave. Tests showed that compaction is closely related to strain-softening since the strain above which moduli start to decrease coincides with the strain required for compaction of the soil.

4.2 Paper II

Frequency Variable Surface Compaction of Sand Using Rotating Mass Oscillators

Carl Wersäll, Stefan Larsson, Nils Rydén and Ingmar Nordfelt

Submitted to ASTM Geotechnical Testing Journal in November 2013

The objective of this paper is to study the influence of frequency in compaction tests using rotating mass oscillators. Results from 105 small-scale tests, conducted using a vertically oscillating plate, are presented. The soil underlying the plate was dry sand, or sand close to the optimum water content. The results showed that there is a resonant amplification, providing slightly higher degree of compaction. Most effective compaction is obtained at very high frequencies. The paper discusses the implications for roller compaction and suggests that a slightly lower frequency may prove more efficient. An iterative method for calculating dynamic response of the plate, incorporating strain-dependent properties of the soil, is also presented. The calculated frequency response agrees well with measured quantities.
5 CONCLUSIONS AND FURTHER RESEARCH

This thesis presents results from small-scale tests using a vertically oscillating plate. Two test setups were manufactured, the first using an electro-dynamic oscillator and the second utilizing two rotating mass oscillators. The frequency response was calculated by combining theory for vibrating foundations on elastic half-space with an iterative procedure for estimating strain-dependent properties of soil. The main conclusions of all studies incorporated in this thesis are listed below:

- Soil compaction by a vibratory plate is frequency-dependent, providing an amplified degree of compaction close to the resonant frequency.
- The resonant amplification is more modest during compaction using a high dynamic force, mainly due to large strain causing high damping and strain-softening.
- The small-scale tests using an electro-dynamic oscillator gave quite a large amount of soil displacement and heave while these were small in the rotating mass oscillator tests.
- No dynamic quantity is solely governing for the degree of compaction.
- The water content of the soil has no apparent effect on the dynamic behavior of the vibrating plate or on the frequency dependence of compaction. It does, however, have positive effect on the degree of compaction.
- Compaction is closely related to strain-softening. The strain level, above which the stiffness of the soil starts to decrease, coincides with the strain required to obtain rearrangement of soil particles.
- The proposed method to calculate frequency response captures well the measured dynamic behavior the second small-scale test.

The settlement velocities in all tests using the electro-dynamic oscillator are shown in Figure 12. A higher velocity implies a higher rate of compaction. The total settlement in the tests using the rotating mass oscillators is shown in Figure 13. The figures illustrate the frequency dependence of compaction using the different equipment.

The results above are valuable for a fundamental understanding of soil compaction and also for the dynamics of oscillating foundations. To develop this further and to make it practically applicable to compaction using vibratory roller, the following proposals for further research are suggested:

- Investigate the influence of changing the eccentricity, i.e. the dynamic force, in rotating mass oscillator tests.
- Investigate the influence of sand layer thickness and stiffness of the subsoil.
- Compare calculated frequency response to small-scale tests with electro-dynamic oscillator and to other practical applications.
- Do extensive full-scale tests with vibratory roller, compacting granular soil at discrete frequencies, but in a wide frequency span.
- Simulate the small-scale tests and roller compaction by distinct element modeling.
Figure 12. Displacement velocity in tests using electro-dynamic oscillator. Modified after Wersäll and Larsson (2013).

Figure 13. Total settlement in tests using rotating mass oscillators. From Wersäll et al. (2013).
REFERENCES


