Comparison of Idealized 1D and Forecast 2D Wave Spectra in Ship Response Predictions

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Abstract

Commonly, when calculating ship responses one uses idealized wave spectra to represent the sea. In the idealized model, the sea is frequently assumed to consist of swell and wind-waves, which are usually represented by idealized 1D wave spectra, and the directionality of wind-waves is accounted for by multiplication with a standard spreading function. In operational response predictions these idealized spectra are typically generated by extracted parameters from real directional 2D wave spectra obtained from a weather forecast, i.e. spectra that reflects the sea state conditions for the particular place and time. It is generally not known in a statistical sense how large the errors become when idealized wave spectra are used to represent 2D wave spectra, especially not regarding the directionality. The objective with the study is hence to assess the errors that arise when adopting this simplification.

The analysis compares three ship types that cover different combinations of hull form, load condition and operational conditions: a 153m RORO ship, a 219 m PCTC and a 240m bulk carrier. Chosen response parameters are roll motion, vertical acceleration and wave added resistance, which were calculated in 12240 sea states, for 10 speeds and 36 courses for each ship. The sea states are forecast 2D spectra from the North Atlantic 25th of September 2012. Transfer functions were generated from the hull geometry and realistic load conditions at speeds 2-20 knots. For each sea state-speed-course combination, responses were calculated for 2D wave spectra and corresponding generalized spectra. The error is taken as the difference in response between results obtained with 2D and idealized spectra, using 2D-results as reference. Several statistical measures were used to represent the errors for one sea state with only one number, and among them the root-mean-square error (RMSE) and the worst possible error (WPE) are regarded most relevant.

The results show that the relative error decreases with increasing share of wind waves and decreasing share of swell. Multi-directionality of wind waves causes large errors only for small waves, and it is concluded that for higher sea states (for which the wind waves are predominant) the Bretschneider representation with spreading function leads to small relative errors. Absolute errors are considered the only relevant for investigating the effect of the error on seakeeping calculations. In general, the RMS acceleration levels are in the order of percentages of one $g$ for all ships. For the bulker, WPE and RMSE for wave added resistance was found to be 8.3% and 3.8% of the total calm-water hull resistance in general, and almost 50% in worst case. The roll angle bias could reach up to 15$^\circ$. Also, the effect of ship speed was investigated, and it shows that the error increases in general with higher speed. It is concluded that it is necessary to use 2D spectra in order to avoid large errors, and to keep performance predictions correct on average.
Acknowledgements

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1 Introduction

Seaware makes on-board software for vessel routing, ship performance and seakeeping responses. Global ocean weather forecasts are utilized in predicting responses, optimal route etc, and typically come in the form of a real directional 2D wave spectrum. These spectra use very large data sizes and have to be simplified into 1D idealized wave spectra parameters before they can be sent to the vessel, where the calculations are done in the on-board software. Seaware participates in a project where one aim is to move calculations to the shore-side and only send the results to the ship. This opens up for performing the calculations with more detail, i.e. utilizing all of the information in the 2D wave spectrum given from the forecast, in the calculations. A possible benefit from doing this is that results could be gained with higher accuracy.

Variation of ship responses due to wave spectrum directionality is not a very explored area, and only two relevant articles (in english) has been found. Mynett et al [1] perform model experiments in a wave-basin and compare ship motion response (heave and pitch) for uni-directional and directional seas produced by a wave generator. One conclusion is that the effect of wave directionality is most pronounced for sea states having sea and swell contributions with different main directions and spreading characteristics. Some results also indicate that if sea and swell are given incorrect directions, the variation in motion response can easily be a factor of two, while if they are correctly accounted for the same variation can be kept within 10-15%. Graham & Juszko [2] develop a 10-parameter spectrum to account for both bi-modal seas and directionality. Comparison is made between motion responses yielded from this 10-parameter spectrum, a hindcast spectrum (which is regarded true) and a 2-parameter Bretschneider spectrum with standard cosine-squared spreading function. The analysis is made for a destroyer at 20 knots in 144 spectra for heave, pitch and roll degrees of freedom. The conclusion is, in essence, that there can be a significant difference in response for bi-modal seas.

The purpose in [2] is obviously to assess the developed 10-parameter spectrum, why the comparison between Bretschneider and hindcast spectra probably has been secondary. However, the consequence is a ‘standard’ representation of the sea state consisting of only a Bretschneider spectrum, and hence does not account for bi-modal sea states, which is a common feature of the sea. Both [1] and [2] are limited with respect to variation of ship, speed and sea state, and only motions are considered. Neither of them discuss the influence of wave height, nor do they draw conclusions on the quantitative aspects the errors might imply in practical ship response predictions.

In today’s model, operational at Seaware, the idealized sea is represented by an Ochi-Hubble spectrum for swell and a Bretschneider spectrum with spreading function for wind-waves, thus accounting for many common bi-modal sea states. The global ocean weather forecast provides a two-dimensional (2D) wave spectrum, that reflects the actual sea state conditions for the particular place and time. Significant wave height, mean period and direction are extracted from the 2D spectrum in order to produce the idealized spectra. The objective in this study is to assess the errors generated by representing real 2D wave spectra with idealized wave spectra. The analysis compares the response errors for roll, vertical acceleration and wave added resistance utilizing 3 ship types, 10 speeds and 12240 sea states. The outcome is intended to form the basis for evaluating the potential in changing to shore-based calculations using real directional 2D wave spectra.

The report is structured as follows. The theoretical background of waves, spectra and linear responses is described under section 2. A reader already familiar with the theory can go directly to section 3, where input data is presented, and the calculation scheme and post-processing of output is described. In section 4, responses and error statistics are scrutinized in order to compare the difference in ship response of the methods with respect to trends and impact on practical ship calculations.
2 Theory

There are numerous texts about wave modeling for ship dynamics applications. Molland [3] gives a brief overview of basic wave modeling and spectral representation, while Michel [4] surveys common spectral formulations currently in use. Lewis et al [5] and St. Denis & Pierson [6] provide comprehensive information on ocean waves in general, but also derivations for directional spectra. Especially Lewis et al [5] is great reading and a very good introduction to sea spectra and ship dynamics in general, yet being thorough and covering many important aspects. Another resource is Lewandowski [7], which summarizes general theory in use.

2.1 Basic concepts

The sea is usually considered having two types of waves relevant to seakeeping and performance predictions; swell and wind-waves. Swell are waves that originates from remote areas like distant storms. Due to their large wavelength they travel fast and can cover several thousands of sea miles before they dissipate completely. Since they originate from distant areas, swell are mainly traveling in the same direction and are therefore often regarded as uni-directional or long-crested, i.e. as if all waves had the same directions, which is almost true for most of the time, see further [5]. Wind-waves, or actually wind-generated waves, are generated by the wind at the sea surface, and present a different pattern. Wind-waves tend to have a larger directional spread and are frequently regarded short-crested, i.e. they travel in different directions, and a seaway consisting to a large extent of these waves is called a wind sea and regarded directional. Generally the sea is assumed to consist of both wind-waves and swell, or either one of them. When the wind starts to generate waves in one direction, in an area where swell traveling in another direction are present, the sea is typically also termed bi-modal, which implies that the spectrum exhibits two distinct peaks in either the frequency plane or the directional plane, or both. This is even more pronounced for a seaway consisting of two groups of swell traveling in different directions. However, it should also be said that the sea is in general directional in the sense that there is always a small spread over directions, even for the most long-crested wave systems.

2.2 Wave modeling and spectrum formulation in 1D

The simplest way to model the sea is to describe the surface by regular sinusoidal waves that have constant amplitudes $A_i$ (half wave heights) and frequencies $\omega_i$ (corresponding to wavelengths according to $\lambda_{wl} = \frac{2\pi g}{\omega^2}$). An irregular sea is made up of a large number of such waves but having different frequencies, heights and (importantly) random phase $\epsilon_i$, each wave component $\zeta_i$ referred to as a partial wave:

$$\zeta_i = A_i \cdot \cos(\omega_i t + \epsilon_i) \quad (2.1)$$

Using sufficiently many frequency components and adding them according to the linear superposition principle will generate a signal that reproduces a wave record at a certain geographical location, where only the vertical rise and fall of the wave surface is sensed. It is therefore often referred to as a point spectrum, and it is called the linear wave model. Here it is understood that we consider the waves having only one direction. In order to relate the wave model to the ship responses it is necessary to look at the energy content of the waves and to adopt a statistical approach. It is assumed that the sea is a steady-state (i.e. statistically stationary) Gaussian random process with zero mean, which is typically true for a 15-20 min period, and it turns out that the energy content is proportional to the square of the wave amplitude. Further, it can be shown that the variance of the wave surface, which according to the model is the sum of a very large (i.e. infinite) number of different wave components, approaches the sum of the variances of each partial wave, see especially [5] for a complete introduction to sea spectra. This reasoning eventually leads to a statistical spectral formulation $S(\omega)$ of the seaway in the frequency domain, see Figure 1. The irregular sea is seen as the time signal in the figure, which is the sum of the
partial waves. For simplicity only eight frequency components were used but as can be seen the
time signal is already starting to resemble a wave record. Since the spectral density \( S(\omega) \) in this
case is a function of only one variable, the wave frequency, it is called a 1D spectrum.

There are several quantities that can be calculated from the frequency spectrum. The so-
called spectral moment is defined as

\[
m_n = \int_0^\infty S(\omega) \cdot \omega^n \cdot d\omega
\]  

The by far most important measure is the zero spectral moment \( m_0 \) which is obtained when
\( n = 0 \). The entity \( m_0 \) is by definition equal to the variance of the wave surface. From the
moments the significant wave height \( H_s \), the average period of component waves \( T_{-1} \) and average
period between zero upcrossings \( T_z \) are calculated as

\[
H_s = 2\pi \sqrt{m_0} \\
T_{-1} = 2\pi m_{-1}/m_0 \\
T_z = 2\pi \sqrt{m_0/m_2}
\]  

2.3 Directional spectra

The sea has until this point been described as made up by waves traveling in the same direction.
However, it is well-known that the sea presents a much more complicated pattern already for
a moderate breeze, generating waves that are traveling in different directions, as mentioned
previously. The usual way to cope with this fact is to multiply the point spectrum \( S(\omega) \) with a
spreading function \( M(\mu) \), as will be described in more detail in Subsection 2.5.1 below. Figure
2 displays different views of such a directional spectrum \( S(\omega, \mu) \). The top-left plot shows the
3D view, the top-right shows the spectrum in the frequency domain (same as the frequency
spectrum in Figure 1), the bottom-left plot shows the directional spread in the range \( \pm 90^\circ \). The
bottom-right graph is a contour plot i.e. the spectrum is seen from above, but represented by
level curves. This is a complete description of the spectrum for qualitative purposes, that is, we
can judge the complete energy distribution but not the magnitude of spectral density. It should
also be said that the directional wave spectrum still represents the energy distribution in one
single geographical point.

Figure 1: Time-frequency domain representation.
The zero spectral moment for the directional spectrum is the same as in the 1D case, but with the addition of integration over directions

\[ m_0 = \int_0^\infty \int_{-\pi}^{\pi} S(\omega, \mu) \cdot d\omega d\mu \]  \hspace{1cm} (2.6)

### 2.4 Relative heading

Before we continue to idealized spectra, the concept of relative heading is introduced. As mentioned in the introduction mean directions for swell and wind waves are extracted parameters from the 2D wave spectra, in order to create idealized spectra. These are given according to the meteorologic convention, which means waves that have a direction of 0° propagates towards south, 90° towards east etc. On the contrary, the directions that define the 2D spectra follow the oceanographic convention i.e. in the opposite direction relative the extracted swell and wind-wave parameters. To relate to the ship’s course the waves given by meteorologic convention are converted to oceanographic convention. For each particular ship course, the relative direction between the ship and the waves — may it be directional or not — has been defined as

\[ \mu_{rel} = \mu_{wave} - \mu_{ship} \]  \hspace{1cm} (2.7)

where \( \mu_{wave} \) is the propagation direction of the wave(s) and \( \mu_{ship} \) is the ship’s course, both according to oceanographic convention, see Figure 3. Transfer functions are defined relative the ship according to standard naval architecture notation i.e. 0° for waves encountering from astern, 90° or 270° for beam seas and 180° for head seas, defined positive clockwise. Thus, the relation between relative wave directions \( \mu_{rel} \) and encountering wave directions \( \mu_{enc} \) can be expressed

\[ \mu_{enc} = \mu_{rel} \mod 360^\circ \]  \hspace{1cm} (2.8)

where \( \mod \) is the modulo operation. Note that \( \mu_{enc} \) is also called relative heading, which is not the same as \( \mu_{rel} \) according to the definition in equation 2.7.
2.5 Spectral representation of the sea state

The European Centre for Medium-Range Weather Forecasts (ECMWF) provides global ocean wave forecasts as part of their weather service products. The forecast is based on the global Wave Model (WAM), which incorporates numerically solving the energy balance equation. This is the governing equation which deals with the interaction between the ocean and the atmosphere, or to be more specific, wave growth from wind input on the sea surface. See for instance [8] and [9] for more information. The primary output from the operational model is a 2D wave spectrum, for a certain time and grid point. From this model ECMWF extracts wave parameters such as significant wave height, mean period, principal directions etc for both wind waves and swell. These parameters are utilized for producing idealized spectra, in today’s method.

2.5.1 Idealized representation of the sea state

In today’s model, operational at Seaware, a Bretschneider spectrum is used to represent wind waves. The ISSC version is used, which is sometimes also called a modified Pierson-Moskowits spectrum or Bretschneider ITTC78 spectrum. Here the name Bretschneider is preferred. It is given as

\[
S_{\text{bret}}(\omega) = \frac{A e^{-B/\omega^4}}{\omega^5}
\]  

(2.9)

with

\[
A = 123.95 \frac{H_s^2}{T_z^4}
\]

\[
B = 495.8 \frac{T_z^4}{T_z^4}
\]

where \(H_s\) is the significant wave height for wind waves, \(T_z\) is the mean zero crossing period and \(\omega\) is the wave frequency. A standard cosine-squared spreading function is applied to distribute...
the energy over directions according to

$$M(\mu) = \frac{2}{\pi} \cos^2(\mu) \quad (2.10)$$

for

$$-90^\circ \leq \mu \leq 90^\circ \quad (2.11)$$

where $\mu$ is the relative heading centered about the principal wave direction, $M(\mu) = 0$ elsewhere. The directional wind spectrum is then obtained as the product

$$S_{\text{wind}}(\omega, \mu) = M(\mu) \cdot S_{\text{bret}}(\omega) \quad (2.12)$$

Figure 2 is a representation according to equations 2.9-2.12

For swell, a 3-parameter Ochi-Hubble spectrum is used. This originates from the more frequently used so-called 6-parameter Ochi-Hubble spectrum which utilizes 3 parameters twice to obtain two peaks in the sea spectrum corresponding to both swell and wind-waves, and hence constitute 6 parameters, see further [10]. The 3-parameter spectrum is expressed:

$$S_{\text{swell}}(\omega) = \frac{1}{4} \left( \frac{4\lambda+1}{\Gamma(\lambda)} \right) \omega^{\lambda} H_2^2 \exp\left(-\frac{4\lambda+1}{4} \left( \frac{\omega_m}{\omega} \right)^4 \right) \quad (2.13)$$

where $\Gamma$ denotes the gamma function, $\omega_m$ is the modal frequency and $\lambda$ is a shape parameter which controls the sharpness of the spectrum. When $\lambda = 1$ equation (2.13) reduces to the Bretschneider spectrum. An example of $S_{\text{swell}}(\omega)$ is shown in Figure 4.

![Figure 4: The sea represented by an Ochi-Hubble 3-parameter spectrum.](image)

### 2.5.2 Representation using 2D spectra

In this report, 2D spectra is used to refer specifically to the 2-dimensional wave spectra generated in the global wave model at ECMWF, whereas directional spectra is used in a broader sense to denote a wave spectrum spread over more than one direction. The 2D spectra are obtained from the forecast model as described above. In particular, each spectrum is obtained for a discrete point i.e. a longitude-latitude position, and is given as spectral ordinates discretized over 36 frequencies and 36 directions. Figure 5 shows an example of a 2D spectrum. It has several peaks and the energy is spread over many directions. This is obviously very different from the representation in Figure 2.
2.6 Linear response

The theory of linear response implies, in effect, that the motions of the ship can be regarded as linear transformations of the wave surface field. This wave surface field, as described above, is a stationary Gaussian random process. Theory of probability says that a linear transformation of such a process generates another stationary Gaussian random process. It relates the input signal $\zeta$ (the waves) to the ship’s motions $\eta$ by assuming a linear system, wherein a response amplitude operator, RAO, (the ship) transforms an input signal linearly. This means that the output signal (the motions) are linear transformations of the input signal. That is, a sinusoidal wave component is transformed to another sinusoidal component with proportional amplitude, shifted phase and same frequency. The RAOs, often called transfer functions and denoted $Y$, are calculated for the ship’s geometry and load condition for a particular speed and relative heading by solving the equations of motion for all D.O.F. except surge. Transfer functions are generally expressed as the ratio of response amplitude per regular wave amplitude, and can be given on amplitude-phase form (equation 2.1) or as a complex number. Generally the response is obtained by multiplying transfer functions with the wave spectrum according to

$$S_\eta(\omega, \mu) = Y(\omega, \mu)^2 \cdot S_\zeta(\omega, \mu)$$  (2.14)

where $S_\eta$ is the response spectrum, $S_\zeta$ is the wave spectrum and $Y$ is the absolute value of the complex transfer functions. Owing to the properties of linear responses to random processes, we can calculate spectral moments of the response spectrum completely analogous to the definition in equation 2.6. The root-mean-square (RMS) of a zero-mean random process is the same as the standard deviation $\sigma$, which is by definition the square root of the variance:

$$\sigma = RMS = \sqrt{m_0}$$  (2.15)

2.7 Wave added resistance

Since wave added resistance is also included in the study, it will be explained in brief here. There are a number of methods to estimate the wave added resistance. The one that has been used in the analysis was developed within the SPA project [11]. Most methods available for calculating added resistance due to waves are not valid for following seas i.e. not valid for $0^\circ \leq \mu_{enc} \leq 90^\circ$ or $270^\circ \leq \mu_{enc} \leq 360^\circ$. The method in [11] however was developed with the objective of being valid for all relative headings. It is based on regression analysis of model tests and has been verified with towing tank trials.
The added resistance in waves is considered the extra resistance due to a seaway, i.e. the additional resistance from waves compared to still-water conditions. The mean value of a linear force is zero, and hence the wave added resistance is calculated as the time mean value of the second order force. The wave added resistance in irregular seas is according to [5] usually expressed:

\[ R_{AW} = 2 \int Y(\omega_e) \cdot S_\zeta(\omega_e) \cdot d\omega_e \]  

(2.16)

where \( R_{AW} \) is the average response in [N] and not as an RMS value, i.e. the time signal for a period of stationary conditions is statistically averaged.
3 Calculations

This section describes the pre-processing and input, calculation procedure and post-processing of the output. Figure 6 shows the outlines for the process. Step 1 is done once for each ship and load condition, steps 2-6 are repeated for each sea state and steps 3-6 for each course of the ship.

Figure 6: Schematic calculation procedure

3.1 Choice of parameters

The following responses are investigated in this study

- Roll motion
- Vertical acceleration
- Wave added resistance

These are chosen because they are common aspects in seakeeping analyses. Roll is oftentimes regarded one of the most critical degrees of freedom for the ship’s motion. Vertical acceleration is typically interesting in many aspects, for instance to determine Motion Sickness Incidence index and predict loads on cargo. Wave added resistance is crucial to calculations of fuel consumption, which has gained increasing attention recent years. All these responses also affect the choice of optimal route in weather routing calculations.

3.2 Ships

The analysis compares three ship types that cover different combinations of hull form, load condition and operational conditions according to Table 1. Finnbirch is a RORO ship that was
lost 2006 in a storm in the Baltic Sea. The PCTC is managed by the shipping company Wallenius and is among the world’s largest car carriers. The bulk carrier is a panamax-size ship that was designed to transport bulk wheat from Houston to Yokohama i.e. it does not exist but has a realistic load case and geometry, and is designed in accordance with structural classification rules and fulfills standard dynamic and intact stability regulations. Main particulars and load condition for the ships can be found in Appendix A.

<table>
<thead>
<tr>
<th>Type</th>
<th>$L_{oa}$ [m]</th>
<th>$B_{oa}$ [m]</th>
<th>Service speed [knots]</th>
<th>Analysis speed [knots]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RORO</td>
<td>156</td>
<td>22.73</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>PCTC</td>
<td>227.8</td>
<td>32.26</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>BULKER</td>
<td>238.06</td>
<td>32.30</td>
<td>13</td>
<td>12</td>
</tr>
</tbody>
</table>

3.3 Transfer functions

According to Figure 6 the first step in the calculation scheme is to generate transfer functions, which are generated with Seaware’s software according to the linear strip theory method described in [12]. These are calculated for heave, roll and pitch for port side and then mirrored for starboard side. For roll they are also conjugated in order to preserve the sign on the phase. In addition, all this is done for different ship speeds, see Table 2. The ship’s course is also defined in the table.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Min.</th>
<th>Incr.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfer function, speed</td>
<td>[kts]</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>Transfer function, rel. heading</td>
<td>[°]</td>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>Transfer function, frequency</td>
<td>[rad/s]</td>
<td>0.05</td>
<td>2.5</td>
</tr>
<tr>
<td>Ship’s course</td>
<td>[°]</td>
<td>0</td>
<td>350</td>
</tr>
</tbody>
</table>

3.4 2D wave spectra

The spectra utilized in the analysis cover a large part of the North Atlantic Ocean the 25th of September 2012, as shown in Table 3. There are 8 time steps, each containing 1530 sea spectra, which sums up to a total of 12240, see further Appendix B. The area is the rectangle (on a projected map), that has its southeast corner off Morocco’s coast, its east bound at Ireland’s coast, its western limit touching St. John’s at Newfoundland and its northern limit at Greenland’s southernmost tip. Figure 7 shows how the wave heights are distributed over this area at 21.00hrs. A low is apparent in the top-right corner of the figure, just outside Ireland, with wave heights up to 7.3 m. The southeast corner is actually on the African continent.

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Increment</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>N 23°00”</td>
<td>N 59°00”</td>
</tr>
<tr>
<td>Longitude</td>
<td>E -52°00”</td>
<td>E -11°00”</td>
</tr>
<tr>
<td>Time</td>
<td>00.00</td>
<td>21.00</td>
</tr>
</tbody>
</table>
More precisely there were 24 spectra on land per time step - they were all detected and are not included in the analysis.

![Wave height distribution](image.png)

Figure 7: Wave height distribution [m].

3.4.1 Interpolation and scaling

The frequencies given for the 2D spectra start at 0.0345 Hz and are given as

\[ f_{n+1} = 1.1 f_n \]  

(3.1)

giving that the highest frequency is about 6 rad. The 2D spectra are subsequently interpolated on the same frequencies as the transfer functions in Table 2. For each spectrum, the zero spectral moment \( m_0 \) is calculated using the trapezoidal rule according to

\[ m_0 = \int_{\omega=\omega_1}^{\omega=\omega_2} \int_{\mu=5}^{\mu=365} S(\omega, \mu) \cdot d\omega d\mu \]  

(3.2)

It was found that this value did not agree completely with the moment (computed from the significant wave height) extracted from the wave model, given by ECMWF. The reason to this is unknown, but may be due to different interpolation methods, round-off errors in the GRIB files or discretization of the wave model. The mean of the relative error in significant wave height for all spectra was 0.12-0.14% and the standard deviation 0.44-0.48%. This is not actually important for the sake of comparison and the analysis. What is crucial is that the same wave
height is used in order to preserve the energy content when creating idealized wave spectra. Hence, a scaling factor is introduced as

$$\alpha = \frac{m_{0, ECMWF}}{m_0}$$  \hspace{1cm} (3.3)

which is multiplied with all the spectral ordinates in the 2D spectrum.

### 3.5 Idealized representation of the sea state

The idealized sea is represented as described in Section 2. A Bretschneider spectrum with spreading function is used to generate a wind sea according to equations 2.9-2.12. Since ECMWF gives the average period of component waves $T_{-1}$, the zero crossing period can be calculated as

$$T_z \approx \frac{T_{-1, wind}}{1.19}$$  \hspace{1cm} (3.4)

i.e. is used as exactly 1.19 in the analysis. For swell, a 3-parameter Ochi-Hubble spectrum is used according to equation 2.13. Since one has the average period of component waves $T_{-1}$ rather than the modal frequency, a numeric method is required. A MATLAB built-in optimizer was used to find the modal frequency that gives the same $T_{-1}$.

### 3.6 Response calculations

The calculations for each ship are done for all time steps i.e. for the 12240 spectra at the positions given in Table 3. For each combination of the ship’s course, a certain sea state and a certain ship speed, the responses are computed with both methods i.e. using idealized and 2D spectrum according to the procedure in Figure 6. That is, for every sea state the ship has the 36 courses given in Table 2. Below, the motions and vertical accelerations are described. Wave added resistance was treated in section 2.7.

#### 3.6.1 Motion

Since relative heading for swell and wind-waves typically do not coincide with calculated transfer functions, interpolation is necessary to obtain values corresponding to $\mu_{enc}$. It was chosen to use linear interpolation, since higher-order methods generated artifacts in some cases. The transfer functions are obtained with an increment of 5°, which is quite dense and the linear methodology will not likely affect the results more than other discretizations. The response spectrum is obtained as

$$S_\eta(\omega, \mu_{enc}) = Y(\omega, \mu_{enc}) \cdot S_\zeta(\omega, \mu_{enc})$$  \hspace{1cm} (3.5)

where $Y$ is the absolute value of the transfer functions and $S_\zeta$ is the relevant wave spectrum i.e. swell, wind-wave or 2D spectrum, sorted with respect to relative heading $\mu_{enc}$. According to Section 2 the response moments can be calculated by

$$m_{0, wind} = \int_{\omega_1}^{\omega_2} \int_{\mu_{enc, wind}}^{\mu_{enc, wind}} S_\eta(\omega, \mu_{enc, wind}) \cdot d\omega d\mu_{enc, wind}$$

$$m_{0, swell} = \int_{\omega_1}^{\omega_2} S_\eta(\omega, \mu_{enc, swell}) \cdot d\omega$$  \hspace{1cm} (3.6)

$$m_{0, 2D} = \int_{\omega_1}^{\omega_2} \int_{\mu_{enc, 2D}}^{\mu_{enc, 2D}} S_\eta(\omega, \mu_{enc, 2D}) \cdot d\omega d\mu_{enc, 2D}$$

Owing to the fact that the modeled wave-ship system is linear, the total response is the sum of the partial responses. Hence, adding the spectral moments for swell and wind-waves gives the total response for the idealized sea state

$$m_{0, ideal} = m_{0, swell} + m_{0, wind}$$  \hspace{1cm} (3.7)
where \( m_{0,\text{ideal}} \) denotes the combined response of the idealized sea state. Finally, the RMS response is calculated by taking the square root of the spectral moment,

\[
RMS = \sqrt{m_0}
\]  

(3.8)

### 3.6.2 Acceleration

Vertical acceleration is assessed at the point in the ship that coincides with the fore perpendicular on the x-axis, i.e. at \( y = 0 \). The distance in x-direction is taken as the difference of the perpendicular length and the longitudinal center of gravity. The transfer function for the vertical acceleration is

\[
Y_{\text{acc}} = \left| -\omega_e^2 (HE + RO \cdot y - PI \cdot x) \right|
\]  

(3.9)

with

\[
\omega_e = \omega - \omega^2 U \cos \mu_{\text{enc}}
\]  

(3.10)

where \( HE, RO \) and \( PI \) denote transfer functions for heave, roll and pitch D.O.F, and \( \omega_e \) is the encounter frequency. Further, \( U \) is the ship speed and \( g \) the gravitational acceleration. The response spectrum is subsequently obtained as in equation 3.5, and the response moments are calculated according to 3.6 and 3.7.

### 3.7 Post-processing

The output from the calculations are RMS values in the case of roll and acceleration, and averages in case of wave added resistance.

#### 3.7.1 Formulation of error

Since the primary objective is to quantify the errors in statistical measures, it is crucial how the errors are defined. As discussed in next section, the error is regarded as the difference between responses obtained with 2D and idealized spectra. The relative error for a specific course, in a given sea state-speed combination, was first adopted, and defined as

\[
e_{\text{rel},i} = \frac{RMS_{2D,i} - RMS_{\text{ideal},i}}{RMS_{2D,i}}
\]  

(3.11)

i.e. as the normalized error of the root-mean-square response. Subscripts \( 2D, i \) and \( \text{ideal}, i \) refer to the \( i \)th response of methods using 2D spectra and idealized spectra respectively. Inspection of the results however suggested that the relative error in some cases was not representative, since there could be a large relative deviation even though the difference was small. Hence, the absolute error was introduced as an additional measure:

\[
e_{\text{abs},i} = RMS_{2D,i} - RMS_{\text{ideal},i}
\]  

(3.12)

#### 3.7.2 Statistical measures

For every sea state, responses for all 36 ship courses are calculated. Hence, with the definitions in equations 3.11-3.12, there will be 36 error estimates per sea state, error definition and speed. To utilize a more compact notation in the following, all statistical measures apply to both relative and absolute errors. The error estimate for one course is consistently denoted \( e \) and a measure representing a whole sea state is capitalized \( E \). The mean value of the error for one sea state is hence

\[
E = \frac{1}{N} \cdot \sum_{i=1}^{N} e_i
\]  

(3.13)
where \( N = 36 \) and \( e_i \) is the error for course \( i \). However, this measure only says whether the idealized method is over- or under-estimating the response on an average for one sea state. Hence, the positive mean value is introduced as

\[
E_{\text{pos}} = \frac{1}{N} \cdot \sum_{i=1}^{N} |e_i|
\]  

(3.14)

which measures how much bias there is in total for a sea state, since it does not account for the sign on the error. The standard deviation in general is defined about the mean of a population. Since one is more interested in the deviation about the line for which the error is zero, the root-mean-square error (RMSE) is used here. This is sometimes also called the quadratic mean and is defined as

\[
\sigma_0 \equiv E_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} e_i^2}
\]  

(3.15)

Corresponding minimum and maximum values for one sea state have also been utilized and are denoted \( \min[e] \) and \( \max[e] \), i.e. the largest over- and under-predicting error that occur. The worst possible error that can arise in a sea state is the positive maximum of these values:

\[
E_{\text{wp}} = \max \{ |\min[e]|, |\max[e]| \}
\]  

(3.16)

Another measure that is useful for analyzing the correlation between the methods can be expressed

\[
E_{\text{cor}} = \frac{\sum_{i=1}^{N} |e_{\text{abs},i}|}{\sum_{i=1}^{N} RMS_{2D,i}}
\]  

(3.17)

which is obviously a relative measure only, since given as a percentage.
4 Results

In this section, the output from the calculations are analyzed. The reader is first introduced to some representative sample plots of responses in one sea state. To draw further conclusions the discussion is then taken to scatter plots and histograms of statistical measures of all the sea states as function of wave height. A compilation of the discussed errors and some example calculations for the bulker closes this section.

4.1 Prerequisites

The idea is to make a comparative study of predictions for 1D and 2D spectra, assuming results obtained from the predictions with 2D spectra are true. This is not exactly right, but for the sake of analysis it doesn’t matter whether the directional spectra are exactly true or not, what is interesting is how the response is affected using the different methods. Also, it is more important that the spectra are accurate in a statistical sense for this kind of analysis, as pointed out in [2]. However, there are good reasons to believe that the directional spectra from the global wave model is a good model (in fact the only physical model for a certain time and place of interest) and much more accurate than using multiple idealized spectra. Assessment of the correctness of the forecasted 2D spectra is a complex matter. Partly because it is difficult to measure a sea state accurately, and partly due to the complexity of the wave model. ECMWF continuously verifies and validates their own models. The Root Mean Square Error (RMSE) for significant wave height in 2008 was reported to be in the order of 0.3-0.4m compared to wave buoys and 0.25m compared to altimeter data, see [13].

4.1.1 Validation

Currently, no method exists for measuring the errors arising from the simplified sea state; comparing measured ship responses with predictions would not show whether the errors are due to the sea state representation or the response calculations. Not only would this require an error analysis of using linear strip method, but in addition measuring responses accurately aboard ships is difficult and impaired by errors itself.

4.2 Sample responses

There are basically two things that can go wrong when representing a 2D wave spectrum with idealized spectra; either (1) the directional spread of a swell peak is represented with only one direction (by using an Ochi-Hubble spectrum), or (2) the direction of the peaks is erroneously given as a weighted mean-value due to multi-modality in the 2D spectrum. Of course, a combination of the both can occur at the same time and to different degrees. Section 4.2.2 deals with a sea state of type (1) and section 4.2.3 with a sea state of type (2).

4.2.1 Good agreement

To start with, a sea state that results in good correlation between the methods is investigated. Figure 8 shows acceleration response and related errors for the RORO ship as function of course. The responses are obtained in a mixed sea-swell sea state, noted (A), with a total significant wave height of $H_s = 6.3$ m. The dashed line marks the response obtained with the idealized method described in Section 3.5 and the solid line is the response utilizing the 2D spectrum as described in Section 3.6. The other two plots are the relative and absolute error for the same sea state, obtained with equations 3.11 and 3.12. According to the error definitions done in the previous a negative error means the idealized method is overestimating the response and a positive error that it is underestimating the response. Figure 9 shows the corresponding 2D spectrum as level curves. The plot title shows the time step and position, as well as ECMWF
extracted parameters. The notation ‘wind’ refers to wind-waves, ‘H’ is significant wave height, ‘T’ average period of component waves and ‘MU’ is wave direction according to oceanographic convention. It can be seen that the swell direction and wind-sea direction given on top of the plot are approximately within the peak of the energy distribution, and that the energy is distributed over a range of about 100°.

Figure 9: 2D spectrum for the sea state (A) in Figure 8.

4.2.2 Error due to spread

Figure 10 shows roll RMS response for a swell-dominated sea state (B) with significant wave height $H_s = 5.1$ m. The corresponding 2D wave spectrum is shown in Figure 11. The extracted wave parameters show that the significant height of wind-waves is $H_{wind} = 0.47$ m, while the significant height of swell is $H_{swell} = H_s = 5.1$ m. The idealized method in Figure 10 is seen to have large errors corresponding to the peaks and hollows in the response. The response peaks occur at the right places since the main direction for swell $\mu_{swell} = 192^\circ$ coincides with the peak in the energy spectrum, i.e. the directionality is correctly accounted for. There is no idealized response at courses corresponding to bow seas ($\mu_{enc} = 180^\circ$ at $\mu_{ship} = 12^\circ$) which agrees well with linear strip theory. Apparently, these swell have a spread of around 150°, which is more than the mixed sea state (A) in Figure 9, and it is clear that representing such 2D spectrum with only one direction gives a bias in terms of sharp peaks that over- and under-estimates the responses.
Figure 10: Sea state (B). Roll RMS response for the RORO ship at 18 knots speed.

Figure 11: 2D spectrum for the sea state (B) in Figure 10.

4.2.3 Error due to directionality

Figure 12, sea state (C), is an example of roll response where the agreement between the two methods is particularly bad. The left plot shows how the RMS response varies as function of course in a sea state of $H_s = 1.7$ m. Apparently, the error varies a lot depending on which course the ship has i.e. depending on the relative heading to waves. To be able to explain the large deviations in response in this case one has to look at the corresponding sea state. Figure 13 is the contour plot of the 2D wave spectrum for the response in (C). The first thing that can be noticed is that it is double-peaked, and that the peaks occur at both different frequencies and different directions. It is also completely swell-dominated ($H_{swell} = H_{total} = 1.7$ m and $H_{wind} \approx 0$). The extracted direction for swell $\mu_{swell}$ is apparently $286^\circ$, which does not coincide with neither of the peaks, but is a weighted mean-value with respect to spectral density and propagation direction. The resulting idealized representation of the sea state is an Ochi-Hubble swell spectrum with all its energy concentrated in only one direction, which is in addition between the original peaks. This spectrum is shown in Figure 14. Since it is in 2D, one could think of it as if all the energy was compressed into a discrete slice in the 3-dimensional representation of the 2D spectrum, located at $\mu_{swell}$. Figure 15 shows the same roll response as Figure 12, but with the addition of relative heading on the top x-axis. Looking closely at the location of the peaks, one realizes that they do not occur at a relative heading equal to $90^\circ$ or $270^\circ$, as would have been expected for the maximum roll response, but rather at $\mu_{enc} = 56^\circ$ and $306^\circ$. This has to do with the ship’s natural roll frequency $\omega_n$ which is very pronounced for many conventional merchant ships. The natural frequency can usually be approximated by the un-damped natural frequency $\omega_0$. 

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Figure 12: Sea state (C). Roll RMS response for the RORO ship at 18 knots speed.

Figure 13: 2D spectrum for sea state (C).

Figure 14: Ochi-Hubble representation of (C).

Figure 16 shows the encounter frequency $\omega_e$ for swell normalized with the undamped natural frequency $\omega_0$. The crosses mark the points for the evaluation, which is at discrete steps of 10° of the ship’s course according to Table 2. The encounter frequency has been calculated with equation 3.10, with $U = 18$ knots, $\mu_{enc} = 282°$ and $g = 9.81 \text{ m/s}^2$. Apparently $\omega_e$ coincides with $\omega_0$ at $\mu_{ship} \approx 230°$ and $\mu_{ship} \approx 340°$ which corresponds well to the peaks in the idealized response. To sum up the discussion here it can be said that, in the case of swell-dominated spectra, the incorrect magnitude of the response peaks in roll is caused by the combination of two things: coincidence between the ship’s natural roll frequency and the encounter wave frequency, as well as the energy in a 2D spectrum being concentrated to only one direction. The incorrect directional location of the response peaks, however, are simply due to the erroneously given direction for swell.
4.3 Interpretation of statistical measures

Table 4 shows the statistical properties corresponding to the responses in Figures 8, 10 and 12. The relative error in Figure 12 is seen to be very large, the worst possible being $E_{wp} = 580\%$. However, looking at the absolute error for this measure, it only corresponds to a value of $E_{wp} = 1.42^\circ$. Also, the values in Figures 10 and 12 have maximum relative errors of $\max [e] = 98\%$ and $99\%$, but absolute errors of $\max [e] = 2.05^\circ$ and $1.04^\circ$, i.e. the absolute error differs a factor of two for the same relative error. Since these are RMS values, the expected maximum response to be encountered within one hour can be up to almost 4 times higher, which would then be an absolute error of $\sim 8^\circ$ and $\sim 4^\circ$ respectively. Hence, the error made for the sea state in Figure 12 has much less practical influence than the one in Figure 10; the relative error is only interesting when analyzing trends or correlations while the absolute error is only relevant for appreciating the impact on practical ship calculations. Another important conclusion can be drawn by looking at the difference between the RMS error $\sigma_0$ and the arithmetic mean $E$ for the whole sea state. For the relative error in Figure 12, the mean value is seen to be merely $E = 12\%$ while $\sigma_0 = 145\%$. For the absolute error the same measures are 26% and 65% respectively. This means that the average error $E$, as defined in this study, can be small even if the responses in a given sea state is literally never the same for the two methods (which is clear from Figure 12 or 15). The quadratic mean $\sigma_0$, on the other hand, better captures the variability of the error as function of course. The parameter $E_{cor}$ is seen to be in the same order of magnitude as $\sigma_0$, indicating the degree of correlation for the methods. The positive mean value $E_{pos}$ is also seen

<table>
<thead>
<tr>
<th>Sea state</th>
<th>Figure 8 (A)</th>
<th>Figure 10 (B)</th>
<th>Figure 12 (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>Rel Abs</td>
<td>Rel Abs</td>
<td>Rel Abs</td>
</tr>
<tr>
<td>Unit</td>
<td>[-] [m/s²]</td>
<td>[-] [°]</td>
<td>[-] [°]</td>
</tr>
<tr>
<td>$E$</td>
<td>-0.04 -0.03</td>
<td>0.42 0.57</td>
<td>0.12 0.26</td>
</tr>
<tr>
<td>$E_{pos}$</td>
<td>0.07 0.06</td>
<td>0.62 1.10</td>
<td>1.05 0.57</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>0.09 0.08</td>
<td>0.67 1.25</td>
<td>1.45 0.65</td>
</tr>
<tr>
<td>$\min [e]$</td>
<td>-0.19 -0.18</td>
<td>-0.86 -2.35</td>
<td>-5.80 -1.42</td>
</tr>
<tr>
<td>$\max [e]$</td>
<td>0.14 0.08</td>
<td>0.98 2.05</td>
<td>0.99 1.04</td>
</tr>
<tr>
<td>$E_{wp}$</td>
<td>0.19 0.18</td>
<td>0.98 2.35</td>
<td>5.80 1.42</td>
</tr>
<tr>
<td>$E_{cor}$</td>
<td>0.06 -</td>
<td>0.79 -</td>
<td>1.58 -</td>
</tr>
</tbody>
</table>
to differ a lot from the normal mean. Looking at the absolute error for Figure 10, this shows that it can vary with a factor of two, and the values for Figure 12 shows that the relative error can change dramatically (12% versus 105%). The positive mean error and RMS-error is seen to be literally the same, which makes sense considering that $\sigma_0$ is actually a quadratic mean. It is also worth noting that the maximum value for the relative error is $\max[e] \approx 100\%$ for both Figure 10 and 12. It is sufficient that the idealized method is equal to zero while the 2D method is not for this to happen. All this can be summarized as follows:

- Relative error only relevant for analyzing trends
- Absolute error only relevant for assessing the actual effect on ships
- $\sigma_0$ and $E_{cor}$ best measures of correlation between the methods
- The normal mean $E$ does not necessarily say anything about the agreement between the methods for a certain sea state, but tells whether there is an over- or under-prediction in average
- The positive mean $E_{pos}$ does not account for over and under prediction, but indicates the average magnitude of the total errors
- The $\min[e]$, $\max[e]$ and $E_{wp}$ values are the only measures that captures the worst errors that can occur for one sea state

### 4.4 Influence of wave height on relative error

Figure 17 shows $\sigma_0$ and $E_{cor}$ of the relative error for roll as function of total significant wave height. The error is larger for smaller wave heights and tends to approach the vicinities of zero for higher waves. This corresponds well with intuition, that the larger the waves, the more long-crested and concordant with the idealized representation. Sea states of smaller waves are characterized by multiple wave patterns. The first difference that can be noticed is that $\sigma_0$ seems to find the sea states related to high bias roughly in the interval 1-3 m wave height, while $E_{cor}$ defines the largest error within 0.5-1 m. The marked points have the largest error and correspond to the spectra shown in Figure 32 and 33, Appendix C. Let’s start with the spectra corresponding to the left plot in Figure 17. Apparently, all of the spectra are bi-modal, and paying careful attention to the appended data on top of each subplot it can be seen that the given direction for swell never actually coincides with any of the spectrum peaks. Further, in
about half of the plots the wind-waves are negligible, so one must draw the conclusion that the two peaks in every spectrum are both swell peaks (which sometimes also coincide with wind-wave peaks). In this case it is thus not strange at all that the idealized method fails to represent the 2D spectrum. Looking now at the spectra for the right plot in Figure 17 shows a different pattern. These have a huge spread and present more than three peaks in most of them. Thus, it is clear what causes the large biases for roll response at low wave heights.

The extreme values for the relative error can be seen in Figure 18. The trend here is still that the errors diminish for larger wave heights. The reason for the slight peculiar look of the maximum values was described above; it has to do with the fact that, due to the error definition, as soon as the idealized method is zero and the 2D method is non-zero, it will reach 100%. Therefore, it is not interesting to analyze the spectra that generate high max-values of the relative error. However, the 9 spectra for the minimum values on the other hand, proves to be almost identical to the ones in Figure 32. These can be seen in Figure 34, in Appendix C. Figure 19 shows the mean value $E$ of the relative error, which is predominantly positive.

Figure 18: $\min [e]$ and $\max [e]$ Relative error of RMS roll response as function of significant wave height.

Figure 19: Arithmetic $E$ and positive $E_{pos}$ mean value of relative error of RMS roll response.

Figure 20 shows the relative occurrences of significant wave heights for swell, wind waves and total waves. It can be inferred from the figure that 57% of the wind waves have a height less than 1 m, and 80% a height less then 2 m. Also, the distribution is exponentially decreasing. For swell, 93% has a wave height between 1 and 3 m, while for the total wave height the corresponding figure is 76%.
Figure 20: Occurences of significant wave height for total waves, swell and wind waves.

Figure 21 shows the error as function of significant height of wind waves and swell as fractions of total significant wave height, i.e. as $H_{\text{wind}}/H$ and $H_{\text{swell}}/H$, respectively. Histograms of corresponding occurrences are displayed below each scatter plot. The entity on the x-axis thus tells whether the sea state is dominated by swell or wind-waves. Hence, it is clear that the error decreases with increasing share of wind waves, and increases with increasing share of swell. Median and mean values have also been plotted, and are seen to be quite similar. The other measures ($E, E_{wp}, E_{\text{min}}$ etc) were found to have similar behavior, also for accelerations and wave added resistance.

Figure 21: The upper plots show the root-mean-square error $\sigma_0$ versus significant height of wind waves and swell as fractions of total significant wave height. The lower plots show corresponding occurrences.
4.5 Influence of speed on relative errors

To investigate the influence of the ship’s speed, the scatter plots are condensed into one single number, the median, which captures the centroid of the data. Figure 22 and 23 shows the median values for $\sigma_0$ and $E_{wp}$ for the different ships and response types, plotted versus speed. It can be gathered from the figures that the error increases with ship speed. The only exception is that of the RORO ship for wave added resistance, which seems to decrease slightly for speeds above 14 knots, and that for the acceleration of the bulker, which decreases up to 8 knots. It can also be noted that the worst possible error is very similar for all ships regarding wave added resistance.

Figure 22: Median of root-mean-square error $\sigma_0$ for RMS roll, RMS acceleration and wave added resistance, as function of ship speed.

Figure 23: Median of root-mean-square error $\sigma_0$ for RMS roll, RMS acceleration and wave added resistance, as function of ship speed.

4.6 Compilation of absolute errors

As mentioned above, the absolute error is the only interesting measure for determining the effect on the responses of real ships. In order to quantify the results, one has to boil down the information in the scatter plots into manageable numbers. Figure 24 shows box plot characteristics of the different error measures for the RORO ship, the corresponding plots for the PCTC and BULKER have very similar trends and can be seen in Appendix D. Each box represents data corresponding to the errors for all sea states. The central mark is the median, and the edges of the box are the 25th and 75th percentiles, i.e. the cumulated values for 25% and 75% of the data. The corresponding value of the standard deviation is denoted $q_1$ and $q_3$, corresponding to the 25th and 75th percentiles, respectively. That is, the distance between the box edges is $q_3 - q_1$. The ends of the whiskers represent a distance in standard deviations from the box’s lower and upper edges corresponding to $q_1 - 1.5(q_3 - q_1)$ for the lower and $q_3 + 1.5(q_3 - q_1)$ for the upper one. This corresponds to a 99.3% coverage and $\approx \pm 2.7$ standard deviations for
a normally distributed population. Outliers are outside the whiskers and marked with a plus if they are less than $2 \cdot 1.5(q_3 - q_1)$ and with a circle if they are larger. The median is also shown as the corresponding number above each box for clarity.

The average error $E$ is seen to have the smallest spread and to be located nearest zero. However, according to the earlier discussion of the interpretation of measures, the mean practically does not say anything about the magnitude of the error. If one is interested in some mean magnitude, $E_{\text{pos}}$ can be used, while $\sigma_0$ describes better the oscillatory behavior of the error. The negative extreme values $E_{\text{min}}$ are consistently larger than the positive extremes $E_{\text{max}}$, which means there is a systematic over-prediction regarding extrema.

Whether a measure of the error is significant or not depends on the type of response and of application for the seakeeping or performance calculations. That is, if the intention is to estimate if the roll or acceleration response might exceed a certain limit, it is interesting to know whether the idealized method over or under-predicts the response. If calculations are part of ship routing as a means to optimizing fuel consumption, incorrect wave added resistance will have a negative influence regardlessly. Also, it is sometimes more interesting to know what the maximal error might be rather than how much the response differs in average. If several thousands of predictions are made on a daily basis, this is adding up the probability of making such large errors. Therefore the worst possible error $E_{\text{wp}}$ and the root-mean-square error $\sigma_0$ are regarded the most interesting, and can be seen in Table 5. The table shows the median error of the measures for all sea states. The median corresponds to the most common error. Alternatively put, 50% of the sea states generates errors that exceed this limit. The expected value that is encountered with a 95% confidence level to occur during one hour can be up to approximately 4 times the RMS value, but varies depending on the encountered spectrum. This applies only to roll and acceleration since wave added resistance is a mean and not an RMS value. For the bulk carrier, this would lead to an error in roll response of between 2-3° and 4-5°, for $E_{\text{rms}}$ and $E_{\text{wp}}$, respectively. The total calm water resistance for the bulk carrier is $R_T = 612$ kN, which means that $E_{\text{wp}}$ is almost 10% of the hull resistance. Next section carries out some calculations on the bulker in order to get a feel for the results in the table.
### 4.7 Influence of speed on absolute errors

As with the relative errors, the influence of speed is judged by looking at the median of the error scatter. Figure 25 shows the root-mean-square error $\sigma_0$ for all ships and response parameters. The bulker is apparently most affected of the error in roll, while it has the smallest error for acceleration. Regarding roll and acceleration, the RORO and PCTC ships seem to be most coherent in general. The wave added resistance is seen to be somewhat higher for the PCTC and the bulker, which makes sense considering they are much larger than the RORO ship. Figure 26 shows the same plots for the worst possible error $E_{wp}$. It presents the same behavior as for $\sigma_0$, except for the magnitude of the error. Also, there seems to be a minimum in acceleration error at 12 knots for the bulker, which is the speed for which the analysis was performed and is only 1 knot from the design speed.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Roll [°] $\sigma_0$</th>
<th>Acc. [m/s$^2$] $E_{wp}$</th>
<th>W.a.r [kN] $\sigma_0$</th>
<th>$E_{wp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RORO</td>
<td>0.41</td>
<td>0.100</td>
<td>24</td>
<td>50</td>
</tr>
<tr>
<td>PCTC</td>
<td>0.28</td>
<td>0.098</td>
<td>30</td>
<td>67</td>
</tr>
<tr>
<td>BULKER</td>
<td>0.57</td>
<td>0.050</td>
<td>23</td>
<td>51</td>
</tr>
</tbody>
</table>

![Figure 25: Median of root-mean-square error $\sigma_0$ as function of ship speed.](image)

![Figure 26: Median of worst possible error $E_{wp}$ as function of ship speed.](image)

### 4.8 Calculation example

To investigate the effect of the errors on real ships an example is taken of the bulker, for which detailed data exist. As discussed above, both the most common error and the more extreme errors can be of interest. The ship is hence assessed at 12 knots in two sea states; one with a
very large error according to the sea state (B) in section 4.2.2, and one which is among the most common (D). The NORDFORSK seakeeping criteria [14] stipulate limits for roll and acceleration given as RMS values. For this ship’s length, the roll limit is 6° and limit for vertical accelerations in the bow is 1.1747 m/s².

Figure 27 shows responses for sea state (D), and Figure 28 displays the corresponding 2D spectrum. This sea state was picked due to its commonness, both regarding its roll RMS error of 0.57° (as given in Table 5) and its waveheight of 2.3 m. It is a single-peaked, swell-dominated spectrum. As treated in the previous, swell spectra with a spread typically generates roll response curves that are too peaked, as can be seen in the figure. The modal values of the idealized model implies a largest roll error of about -1.2°, and the squared average of 0.57° could be translated as a maximum expected value of roughly 2.2°. The root-mean-square error of the acceleration and wave added resistance also have typical values. For acceleration σ₀ = 0.0519 m/s² and for resistance σ₀ = 21kN.

Figure 27: Responses for the bulker in sea state (D)

Figure 28: Spectrum for sea state (D)

Figure 29 shows responses for sea state (B). The idealized model overestimates the response twice, exceeding the roll criterion. For a course of about 200° the roll is underestimated with about 3.5°. This is typically also a disadvantage in ship routing, where the algorithm has to find its way around an area where a threshold level for roll or acceleration is predicted to be exceeded. The expected maximum value of the maximum roll angle for the idealized method
could hence be summing up to about 30°, while the corresponding number for the 2D method is around 18°. The accelerations, however, are not more than ± 0.2 m/s² off, and thus not even near the limit. Probably due to the low speed for this vessel. The min \([e]\) and max \([e]\) error in wave added resistance is 246 and -300 kN respectively, which should be put in relation to the total hull resistance of \(R_T = 612\) kN, i.e. the error is almost half of the calm water resistance.

The overall impression is that the most common error varies considerably regarding the shape (peaks and hollows), but do not imply very large errors, while the most erroneous sea state proves that the correlation can be really poor.

Figure 29: Responses for the bulker in sea state (B)
5 Conclusions and discussion

Responses for roll, vertical acceleration and wave added resistance has been calculated with two different methods; idealized method utilizing a Bretschneider spectrum with cosine-square spreading function for seas and a Ochi-Hubble spectrum for swell, and a method using directional 2D forecast wave spectra operational at ECMWF. The idealized spectra were fed with extracted wave parameters from the 2D spectra such as wave height, mean period and propagation direction for sea and swell respectively. The responses were calculated for three ship types, at 10 different speeds, for 36 courses and 12240 sea states. The ships are one RORO vessel, a PCTC and a bulk carrier.

The analysis was divided into two parts. Relative errors were used to analyze trends and compare the methods qualitatively, while absolute errors were used to estimate the effect on seakeeping calculations. The error is seen to vary largely as function of ship course. Oftentimes, the idealized method has much more pronounced peaks in the response than the 2D spectrum method, sometimes also at incorrect locations. Several statistical measures were used in order to characterize the error for one sea state with one number. Scatter plots of these measures versus wave height were utilized, for relative and absolute errors. The results show that the relative error decreases with increasing height of wind waves and decreasing height of swell, and approaches zero (or reasonably so) for the highest waves. The arithmetic mean value shows that there is an under-estimation in average for the roll and vertical acceleration response, while for the wave added resistance there is an over-estimation. In general, multi-directionality of wind waves causes large errors only for small waves, and it is concluded that for higher sea states (for which the wind waves are predominant) the Bretschneider representation with spreading function leads to small errors. It was also found that the roll response of the idealized method is particularly prone to generate erroneous peaks in the response, since the encounter frequency sometimes coincides with the roll natural frequency.

To evaluate the absolute errors, a more quantitative approach was adopted. The statistical measures were compiled into box plots, showing the median value for all sea states, and it was seen that the trends were alike between the ships. The box plots are essentially the results themselves since what measure to use depends on the purpose of the calculations. However, it is believed that generally, one either wants to be sure of not making large errors, or wants to know the error related to the probability of the sea state. It was decided that the root-mean-square error (RMSE) and the worst possible error (WPE) best represent these cases. The median values show that the roll RMS errors for the bulker are large, RMSE = 0.57° and WPE = 1.20°. Errors of RMS accelerations for the RORO and PCTC are in the order of twice as large as for the bulker, which is probably due to the difference in speed (18 vs 12 knots). In general, however, the median RMS acceleration levels are in the order of percent of one g and hence do not have any practical impact. Wave added resistance has a median error of 23-67 kN for all ships.

The influence of ship speed was investigated for relative and absolute errors. The median value of the scattered errors for $\sigma_0$ and $E_{wp}$ was plotted for all ships and response types. In general, the error increases with higher speed for both relative and absolute errors. An exception is the absolute errors of the accelerations of the bulker, which has a parabolic shape with a minimum at 12 knots. By looking at the absolute errors for roll, it is concluded that the bulker is to a larger extent affected by the idealized representation, while for accelerations, it is less affected.

A calculation example was carried out on the bulker for two sea states. The most common sea state, with a median error of the squared average of RMS roll of 0.57°, shows biased peaks in the response (especially for roll) but the error measures are rather low. The other sea state is one of the worst, with respect to absolute error. The differences between the methods were in the order of $\sim 15°$ for roll, assumed that the maximum expected value to be encountered within one hour can reach up to about a factor four of the RMS value. For accelerations, the errors
are small yet, even though the look of the response graphs differs. The error in the prediction of resistance due to waves is up to about half of the ship’s total hull resistance. It is concluded that there is no agreement at all between the methods for sea states like this. Thus, to be sure of not making large errors, one should use the 2D wave spectra. However, the idealized method generates small errors when considering the probability of the sea state to occur.

Just like the extracted wave directions, wave periods extracted from multi-modal spectra are also erroneous. For future work, it is suggested to analyze the effect of applying this in the idealized model, as it should also have an effect on the error. It is further suggested that the peakedness and spreading of the spectra is characterized to more clearly see what effect this has on the error. For certain types of analyses, one could also redefine the error extreme values like min \([e]\) and max \([e]\) (and as a consequence \(E_{wp}\)) to account for the largest and smallest response of the idealized and 2D method for the whole sea state rather than comparing the error for each particular course.
References


A Ships

This appendix presents the ships included in the study.

A.1 RORO

Finnbirch is a Roll-On Roll-Off (RORO) ship that was lost in a storm in the Baltic Sea, see [15] for more information. The main particulars can be seen in Table 6, and the sections plan is displayed in Figure 30. The lines have been digitized from [15].

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, over all</td>
<td>156 m</td>
</tr>
<tr>
<td>Beam (b), moulded</td>
<td>22.73 m</td>
</tr>
<tr>
<td>Draft, design/max</td>
<td>6.85/7.30 m</td>
</tr>
<tr>
<td>Deadweight at max draft</td>
<td>13882 dwt</td>
</tr>
<tr>
<td>Block coefficient</td>
<td>0.57 -</td>
</tr>
<tr>
<td>Service speed</td>
<td>18 kts</td>
</tr>
<tr>
<td>Vertical Center of Gravity</td>
<td>10.44 m</td>
</tr>
<tr>
<td>Transverse Center of Gravity</td>
<td>0 m</td>
</tr>
<tr>
<td>Longitudinal Center of Gravity</td>
<td>74.64 m</td>
</tr>
<tr>
<td>Trim</td>
<td>0.375 m</td>
</tr>
<tr>
<td>$GM_0$</td>
<td>1.36 m</td>
</tr>
<tr>
<td>Roll damping</td>
<td>10 %</td>
</tr>
<tr>
<td>Radius of gyration, $r_x$</td>
<td>0.35b m</td>
</tr>
<tr>
<td>Radius of gyration, $r_y$</td>
<td>0.25b m</td>
</tr>
<tr>
<td>Radius of gyration, $r_z$</td>
<td>0.25b m</td>
</tr>
</tbody>
</table>

Figure 30: Sections plan for the RORO ship Finnbirch.
A.2 PCTC

The ship is a large Pure Car and Truck Carrier (PCTC) managed by Wallenius. Most information about her is therefore classified, but the main particulars can be seen in Table 7.

<table>
<thead>
<tr>
<th>Table 7: Main particulars for PCTC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, over all</td>
</tr>
<tr>
<td>Beam, moulded</td>
</tr>
<tr>
<td>Height to upper deck</td>
</tr>
<tr>
<td>Draft, design/max</td>
</tr>
<tr>
<td>Deadweight at max draft</td>
</tr>
<tr>
<td>Number of car decks</td>
</tr>
</tbody>
</table>

A.3 BULKER

The ship is a dry bulk carrier of panamax size, with a displacement of just above 82'000 tonnes, and a length over all of 238 m. It was designed to transport 1 million tonnes of wheat per year from Houston to Yokohama, i.e. it does not exist but has been analyzed rather thoroughly. It is designed according to the DNV classification rules of structural design and fulfills IMO standard intact stability criteria. Moreover, it has been assessed in an operability analysis for the NORDFORSK criteria [14] in the North Atlantic. The operability was evaluated for roll, vertical and lateral accelerations and green water on deck, and was found to be 94%. Top wing tanks and hopper tanks are used for ballast water in all four corners of the midship section, as well as the forepeak. Figure 31 shows the section’s plan. Main particulars are shown in Table 8.

<table>
<thead>
<tr>
<th>Table 8: Main particulars and load condition for bulker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, over all</td>
</tr>
<tr>
<td>Beam (b), moulded</td>
</tr>
<tr>
<td>Draft, design/max</td>
</tr>
<tr>
<td>Deadweight at max draft</td>
</tr>
<tr>
<td>Block coefficient</td>
</tr>
<tr>
<td>Service speed</td>
</tr>
<tr>
<td>Vertical Center of Gravity</td>
</tr>
<tr>
<td>Transverse Center of Gravity</td>
</tr>
<tr>
<td>Longitudinal Center of Gravity</td>
</tr>
<tr>
<td>Trim</td>
</tr>
<tr>
<td>$GM_0$</td>
</tr>
<tr>
<td>Roll damping</td>
</tr>
<tr>
<td>Radius of gyration, $r_x$</td>
</tr>
<tr>
<td>Radius of gyration, $r_y$</td>
</tr>
<tr>
<td>Radius of gyration, $r_z$</td>
</tr>
</tbody>
</table>
Figure 31: Sections plan for the bulk carrier.
B Wave heights for 2D wave spectra

The significant wave heights are distributed according to the colorbars. Each plot shows one time step, which contains 1554 spectra, whereof 1530 are used in the analysis (24 are on land).
The figures show spectra that are marked with circles in the scatter plots, corresponding to the largest errors.

Figure 32: Spectra corresponding to the marked data of $\sigma_0$ in Figure 17
Spectra corresponding to maximum errors
Ship: RORO. Speed: 18 kts. Timesteps: All
Measure: $E_{cor}$. Error type: Relative. Response: RMS Roll

Figure 33: Spectra corresponding to the marked data of $E_{cor}$ in Figure 17
Spectra corresponding to maximum errors
Ship: RORO. Speed: 18 kts. Timesteps: All
Measure: min[e]. Error type: Relative. Response: RMS Roll

Figure 34: Spectra corresponding to the marked data of min\(e\) in Figure 18
Figure 35: Spectra corresponding to the marked data of $E$ in Figure 19
D  Box plots

The figures shows box plot characteristics of the different error measures. Each box represents data corresponding to the errors previously plotted as scatter. The central mark is the median, and the edges of the box are the 25th and 75th percentiles. The median is also shown as the corresponding number above each box for clarity. The coverage between the ends of the whiskers is at least 95%, i.e. the outliers constitute at most 5%.

Figure 36: Box plot for the PCTC summarizing the error scatter statistics.

Figure 37: Box plot for the BULKER summarizing the error scatter statistics.