This is the accepted version of a paper published in *Annals of Nuclear Energy*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Suvdantsetseg, E., Wallenius, j. (2014)
An assessment of prompt neutron reproduction time in a reflector dominated fast critical system: ELECTRA.
*Annals of Nuclear Energy*, 71: 159-165
http://dx.doi.org/10.1016/j.anucene.2014.04.001

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:
http://urn.kb.se/resolve?urn=urn:nbn:se:kth:diva-146988
An assessment of prompt neutron reproduction time in a reflector dominated fast critical system: ELECTRA

E. Suvdantsetseg*, J. Wallenius

*KTH, Division of Reactor Physics, Albanova University Centre, 10691 Stockholm, Sweden

Abstract

In this paper, an accurate method to evaluate the prompt neutron reproduction time for a reflector dominated fast critical reactor, ELECTRA, is discussed. To adequately handle the problem, explicit time dependent Monte Carlo calculations with MCNP, applying repeated time cut-off technique, is used and compared against the $\sigma \sim 1/v$ time dependent absorber method, applying artificial cross section data in the Monte Carlo code SERPENT. The results show that when a reflector plays a major role in criticality for fast neutron reactor, the two methods predict different physical parameters ($\Lambda = 69 \pm 2$ ns and $\Lambda = 83 \pm 1$ ns for time cut-off and the $1/v$ method respectively). The reason is explained by applying Avery-Cohn’s two-region prompt neutron model.

Keywords: prompt neutron reproduction time, reflector dominated critical system

*E-mail: esuv@kth.se
1. Introduction

In the design of fast neutron Generation IV reactor systems, breeding blankets are often replaced by inert reflectors, in order to enhance proliferation resistance. The importance of the reflector design for static and dynamic behavior of such systems is larger than for traditional breeder reactor concepts. In particular, for small, reflector dominated systems, the methodology for modeling of kinetic parameters has to be carefully assessed. In this paper, we focus on evaluating the prompt neutron reproduction time (Lewins, 1960, 2006) for the ELECTRA design.

The European Lead Cooled Training Reactor (ELECTRA) concept, was proposed by Wallenius et al. (Wallenius et al., 2012). The use of inert matrix ($\text{Pu}_{0.4} \cdot \text{Zr}_{0.6}$)N fuel results in a very small core with negative reactivity coefficients and a design power of 0.5 MW$_{th}$, which may be removed by natural convection of the coolant. The core consists of a single fuel assembly with an active height of 30 cm and a flat-to-flat distance of 28.2 cm. A detailed geometrical scheme is provided in Figure 1.

Due to the very small size of the active core and large neutron leakage, the reactor belongs to the class of reflector-dominant fast systems, meaning that the neutron lifetime in the fast core region can be orders of magnitude less than the neutron lifetime in the reflector region. In such reactors, it is known that the kinetic parameter $\Lambda$ becomes extremely sensitive to the geometry and composition of the reflector region. Therefore, a reliable evaluation of $\Lambda$ in these systems is difficult to accomplish (Spriggs et al., 1997; Dulin et al., 1997). In ELECTRA, the presence of rotating absorbers in the reflector adds complexity to the geometry and introduces streaming effects, which are challenging to treat with deterministic neutron transport tools. Hence, we investigate the application of Monte Carlo methods for the calculation of the prompt neutron reproduction time.

Unfortunately, current Monte Carlo codes do not provide a direct estimate of the effective neutron reproduction time (Leppänen, 2012; MCNP X-5 Monte Carlo Team, 2003), but rather predicts the unweighted neutron reproduction time, based on estimates of the unweighted neutron lifetime. The latter represents the average life expectancy of all neutrons within the model system boundaries. However, reactor dynamic analysis require an effective reproduction time which is a value weighted
by each neutron’s probability of inducing a fission chain. Note that the discrepancy between the weighted and unweighted reproduction time can be orders of magnitude for the case of reflector dominated system (Suvdantsetseg et al., 2013). Thus any rigorous calculation of the reactor kinetic parameters must involve weighting over the adjoint flux.

Alternatively, the application of deterministic codes is challenging both in terms of modeling unusual geometrical configurations and with respect to the validity of the homogenized assembly approximation, which can result in significant differences in the case of a core consisting of a single assembly. Furthermore, convergence of the neutron flux in the lead reflector region may be difficult to achieve for a small core like the one of ELECTRA (Bortot and Pelloni, 2013).

Returning to Monte Carlo, in order to weight the neutrons, either continuous energy adjoint Monte Carlo methods, which require extensive preprocessing of cross section data (Hoogenboom, 2003) or multi-group adjoint calculations may be implemented (Lux and Koblinger, 1991; Fegghi et al., 2007). The mentioned methods however require either a modification in existing codes or extensive amount of calculations to process.

In addition, the method to calculate the neutron importance-weight factor is still an open-question, in particular in the reflector dominated case, and has been investigated by various authors. Nevertheless, the parameter may possibly be evaluated by repeated Monte Carlo simulations using the artificial $\sigma \sim 1/v$ poisoning method (Verboomin et al., 2006), which is relatively simple and reliable. Furthermore, the reproduction time can also be indirectly evaluated from the $\alpha$-eigenvalue.

In this work, we have chosen the two latter methods, since they are straight-forward to implement and fairly commonly used. However, it is known in the literature that in some reflector dominated cases that simple point kinetics is not satisfactory for dynamic modeling (Vollaire, 2004) meaning that applying a single average value for $\Lambda$ is not sufficient. More detailed dynamic models may need to be considered, including for instance, a two-region prompt neutron reproduction time (Spriggs et al., 1997; Avery et al., 1958; Cohn, 1961, 1962). Thus we should investigate whether the time behavior of ELECTRA can be represented by a single reproduction time (evaluated by the above methods), or if multiple region reproduction times should be defined.

In the next section, we discuss the theory of the implemented methods and demonstrate their applicability on the benchmark Godiva and Flattop-Pu experiments. Finally the results obtained for ELECTRA are compared to each other and the physical explanation of the discrepancy as well as a discussion of a two-region prompt neutron approximation is given.

2. Theoretical approach and methodology

The prompt neutron reproduction time is the average time for a neutron to reproduce itself by fission. In a one-group diffusion-theory approximation, the reproduction time $\Lambda$ for a bare homogeneous reactor may be defined as the inverse of the neutron production rate:

$$\Lambda = \frac{1}{v\nu\Sigma_f}, \quad (1)$$

where $v$ is the average neutron velocity, $\nu$ is the number of prompt neutrons released per fission and $\Sigma_f$ is the macroscopic fission cross section. Similarly, the neutron lifetime $l_p$ is defined by the inverse of the neutron destruction rate and is linked to the reproduction time as follows:

$$l_p = \frac{1}{v(\Sigma_a + DB^2)} \quad (2)$$

$$\Lambda = \frac{l_p}{k_{eff}} \quad (3)$$

where $\Sigma_a$ is the macroscopic absorption cross section, $D$ the diffusion coefficient, $B$ the geometrical buckling, and $k_{eff}$ the steady-state effective multiplication factor. In a critical reactor, the production rate is equal to the destruction rate, thus the two parameters are equal.

Using the stationary homogenous transport equation (Ott and Neuhold, 1985; Stacey, 2007) the reproduction time has in the literature dubiously been evaluated as (Leppänen, 2012; Verboomin et al., 2006):

$$\Lambda = \frac{\langle \frac{1}{v}\Phi_0 \rangle}{\langle F_0 \Phi_0 \rangle} \quad (4)$$

where $\langle \rangle$ denotes integration over space and energy, $\Phi_0$ is the fundamental critical flux and $F_0$ the total fission operator of the configuration. However, not all the neutrons contribute equally to the fission chain. In particular the neutrons born at the
the first order perturbation as:

Thus, in the limit of zero concentration \( c \) of the poison

\[
\Lambda_{eff} = \frac{\langle \Phi^+_0, \frac{1}{2} \Phi_0 \rangle}{\langle \Phi^+_0, \Phi_0 \rangle}
\] (5)

where, \( \Phi^+_0 \) is the critical adjoint flux. Note that the difference between the weighted and the non-weighted parameters can be huge when the system is dominated by a reflector.

2.1. 1/\( v \) poisoning method

In terms of an exact perturbation, a reactivity increment may be written as:

\[
d\rho = k_0^{-1} - k_1^{-1} = \frac{\langle \Phi^+_0, k_1^{-1} \delta F - \delta K | \Phi_1 \rangle}{\langle \Phi^+_0, \Phi_0 \Phi_1 \rangle} = \delta \rho = \frac{1}{\nu} \langle \Phi^+_0, \frac{1}{2} \Phi_0 \rangle
\] (6)

where index 0 is the fundamental stationary state and the index 1 is the perturbed state of the configuration, \( \rho \) the reactivity, \( k \) the multiplication factor, \( F \) the total fission operator and, \( K \) the total destruction operator.

If a uniform\( \Sigma_a = c/\nu \) poison over the system is considered, then the perturbation operators become:

\[
\delta K = \frac{c}{\nu}
\]

\[
\delta F = 0
\] (8)

Thus, in the limit of zero concentration \( c \) of the poison

\[
\left( \frac{\partial \rho}{\partial c} \right)_{c=0} = \lim_{c \to 0} \frac{\rho_c - \rho_0}{c} = -\lim_{c \to 0} \frac{1}{c} \frac{\langle \Phi^+_0, \frac{1}{2} \Phi_0 \rangle}{\langle \Phi^+_0, \Phi_0 \Phi_1 \rangle} = -\Lambda_{eff}
\] (9)

Therefore, when the perturbation introduced by the poison is small, \( \Lambda_{eff} \) can be found according to the first order perturbation as:

\[
\Lambda_{eff} = -\frac{\Delta \rho}{c}
\] (10)

The stated formula (Eq. 10) is indeed a numerical approximation of Eq. 9. Hence, the correct evaluation of it would require a calculation of suitable and sufficiently large series of \( c \) values in the vicinity of 0 in order to extrapolate the limiting value.

2.1.1. Implementation

Typically the method is applied in Monte Carlo codes by uniformly adding the\( ^{10}\text{B} \) isotope in the whole system for the case of thermal reactors. However, the absorption cross section of\( ^{10}\text{B} \) does not behave as \( 1/\nu \) in the fast region, hence, it is not suitable for estimates of \( \Lambda_{eff} \) in fast systems. Thus we have created an artificial \( 1/\nu \) cross section file in ACE format for use with the Serpent code.

2.2. Direct time cut-off method

Time eigenvalues of the one-group diffusion equation are (Stacey, 2007):

\[
\lambda_n = -\frac{k_n - 1}{l_n}
\] (11)

where

\[
k_n = \frac{\nu \Sigma_f}{\Sigma_a + DB^2_n}
\] (12)

\[
l_n = \frac{1}{\nu(\Sigma_a + DB^2_n)}
\] (13)

giving

\[
\lambda_n = \nu(DB^2_n + \Sigma_a - \nu \Sigma_f)
\] (14)

In the fundamental mode solution, one has that \( B^2_n = B^2_1 \), which is the geometrical buckling and \( k_1 = k_p \), \( \lambda_1 = -\alpha \) and \( l_1 = l_{p,eff} \) (see Eq. 2), where, \( k_p \) is the multiplication factor for prompt neutrons only, \( \alpha \) the prompt neutron decay constant or the \( \alpha \)-eigenvalue, and \( l_{p,eff} \) the effective prompt neutron lifetime. Thus if the \( \alpha \)-eigenvalue is known, the effective prompt neutron reproduction time can be estimated as follows:

\[
l_{p,eff} = \frac{k_p - 1}{\alpha} = \frac{k_{eff}(1 - \beta_{eff}) - 1}{\alpha}
\] (15)

\[
\Lambda_{eff} = \frac{l_{p,eff}}{k_{eff}}
\] (16)

where \( \beta_{eff} \) is the effective delayed neutron fraction. Furthermore, by definition \( \alpha \) can be estimated by the tallies of the asymptotic change in neutron population over time as (Parsons et al., 1997):

\[
\alpha = \frac{\ln N_1 - \ln N_2}{t_1 - t_2}
\] (17)

where \( N_i \) is the population of neutrons in the system at time \( t_i \).
2.2.1. Implementation

Accurate $\alpha$-eigenvalue calculations are not an explicit option in Monte Carlo codes. The Serpent code provides an $\alpha$-eigenvalue parameter in the output. However, it is based on non-weighted methodology, hence does not provide accurate values, especially for reflected systems. Nevertheless, alpha-like calculation may be performed without a modification of the code or cross sections by using the time cut-off technique of the MCNP code (Parsons et al., 1997).

MCNP has implemented the possibility to perform Monte-Carlo simulations with time as an explicit variable. Hence, the growth or decrease in neutron population can be tallied, including estimates of statistical uncertainties. In the case of positive reactivity, a time cut-off is introduced to avoid divergence of the neutron population. We have used this cut-off (the "cut:n" card of MCNP) to estimate the alpha-eigenvalue. Figure 2 shows how the method is implemented to evaluate the $\alpha$-eigenvalue of the Godiva benchmark with ENDF/B-VII library.

![Figure 2: $\alpha$-eigenvalue evaluation of Godiva using the time cut-off method with the ENDF/B-VII library.](image)

2.3. Avery-Cohn’s two-region prompt neutron model

In some cases a single average prompt neutron reproduction time might not be satisfactory to treat the dynamic behavior of the system. Avery-Cohn (Avery et al., 1958; Cohn, 1961, 1962) proposed the method of arbitrarily subdividing the multiplying system where each region considered has a multiplication factor contribution $k_i$ and an effective neutron lifetime $l_i$ and the idea was further developed by Spriggs et al. (Spriggs et al., 1997). If we divide the reactor into two regions: core and the reflector, then the simplified inhour equation would become, according to Avery-Cohn theory as shown by Spriggs et al.:

$$\rho = \omega \Lambda_c + \frac{\omega f \Lambda_r}{l_r \omega + 1} + \sum \frac{\beta_i \omega}{\omega + \lambda_i}$$

where $\omega$ is the inverse period, and $\Lambda_c = l_c/k_c$, $\Lambda_r = l_r/k_r$ are the effective neutron reproduction time in the core region and the reflector region respectively, $f = 1 - k_c/k_{eff}$ is the reflector return fraction, $l_r/l_c$ is the effective neutron lifetime in the reflector/core region, and $k_c$ the bare core multiplication factor.

When the reflector lifetime is sufficiently small, such that $l_r \omega << 1$, the dynamic model of the system can be expressed by equivalent one-group point kinetics and the equivalent reproduction time then would be:

$$\Lambda_m = \Lambda_c + f \Lambda_r$$

where $\Lambda_m$ is the equivalent dynamic reproduction time of the system. This equivalent model is valid for reactivities in the vicinity of prompt criticality or $\rho < 18$. When the reactivity is $\rho >> \beta + f/k_c$, which is highly unlikely, the dynamic response of the system would solely depend on $\Lambda_c$, i.e. not on $\Lambda_m$.

Apart from the dynamic reproduction time $\Lambda_m$, another reproduction time appears in the solution of the Avery-Cohn equations (by Spriggs), the so called static reproduction time, $\Lambda_s$. This parameter can be written as:

$$\Lambda_s = \Lambda_c + f_{cr} \Lambda_r$$

where $f_{cr}$ is the probability of core neutrons to leak into the reflector. The static reproduction time expresses the reproduction time in an equilibrium system which corresponds to the average time of fission source neutrons weighted by the total neutron population. On the other hand, the dynamic reproduction time expresses the average time of the fission source neutrons weighted by the neutrons which can potentially propagate fission only.

2.3.1. Implementation

For the purpose of evaluating the Avery-Cohn model parameters, the geometrical model of ELECTRTRA has been subdivided into two regions: the
core, which includes only the fuel assembly part and the reflector, which includes everything else outside the core. The reflector return fraction $f_r$ and the core to reflector leakage fraction $f_{cr}$ are evaluated by the Serpent code.

The parameter $f$ is evaluated according to the previous section via the bare core multiplication factor $k_c$ which is determined by simulating the core region with a black condition on its outer boundary. Moreover, the $f_{cr}$ parameter is found using the fraction of system neutrons absorbed in the core as Spriggs et al. described (Spriggs et al., 1997).  

Furthermore, $\Lambda_c$ and $\Lambda_r$ are estimated by the same technique as described in section 2.1 by partially poisoning the sub regions of the complete model of ELECTRA:

$$\Lambda_c = \frac{-\Delta \rho_{core}}{c}$$

$$\Lambda_r = \frac{1}{f_{cr}} \frac{-\Delta \rho_{ref}}{c}$$

Where, the suffix "core" refers to the core and "ref" to the reflector region. However, it should be noted that neutrons entering from non poisoned subregion to the poisoned subregion may result in a slight over-prediction of the $\Lambda_c$ and $\Lambda_r$.

3. Results

Firstly, before applying our approach on ELECTRA, we have simulated well known critical benchmark experiments to demonstrate the applicability of the 1/v poisoning and the time cut-off methods. For this purpose, Godiva has been selected among other experiments available due to its simple geometry and high quality experimental data. Moreover, similar to ELECTRA, a reflected plutonium benchmark with dedicated experimental data: Flattop-Pu has also been investigated. The benchmark specifications are taken from OECD (OECD Agency, 2010) and the measured values are taken from LANL (Mosteller and Kiedrowski, 2011). The measured alpha-values are performed by Rossi-alpha method.

3.1. Godiva and Flattop-Pu

The results are shown in Table I. Different cross section libraries are used to observe the bias introduced from the choice of cross section library.

Note that on the basis of the 1/v method the effective reproduction time is evaluated first and the $\alpha$-eigenvalue is consequently determined by means of Eq. 15. Vice-versa $\alpha$ is determined first when using the direct time cut-off method. We may expect that the computational uncertainty is higher for the second parameter being determined indirectly.

As we can see in Table I, discrepancies are seen not only between the methodologies but also between the chosen cross section libraries. For the case of Godiva, when the JEFF3.1.1 library is used the measured effective multiplication factor is underestimated by a few hundred pcm, leading to more negative $\alpha$-eigenvalues for both simulation methodologies. Furthermore, the fact that the multiplication factor is close to one introduces a higher bias than the statistical uncertainty on effective multiplication factor and delayed neutron fraction, consequently also on the prompt multiplication factor $k_p$ through the use of Eq. 15. It consequently introduces larger uncertainty in the indirectly evaluated parameters: $\alpha$-eigenvalue or the prompt neutron reproduction time. Therefore, the large discrepancy between simulated and measured $\alpha$-eigenvalues can likely be attributed to nuclear data and also to the inherent methodologies used in the two Monte Carlo codes. Nevertheless, the evaluated parameters shows a consistent and reasonably good agreement between the methods, in respect to reproduction time, thus supporting the applicability of both 1/v poisoning and the direct time cut-off methodology.

For Flattop-Pu, the reflector dominated plutonium system the findings are similar in terms of nuclear data performance with once again MCNP reproducing the experimental value when used in conjunction with the ENDF/B-VII library. When the direct time cut-off method is used in conjunction with the JEFF-3.1.1 library, $k_p$ is almost one. Hence, $\alpha$ is nearly zero implying that the analytical value of the prompt reproduction time based on $\alpha$ may be quite unreliable. Nevertheless, the reproduction time was possible to evaluate by the time cut-off method using ENDF/B-VII library and appears to be slightly lower than the 1/v poisoning method. The reason is explained later in section 3.2.
Finally, the results indicate that ENDF/B-VII library seems to be preferable in this case as it agrees better with the experimental data. Thus we use only ENDF/B-VII library for further simulations of ELECTRA.

3.2. ELECTRA assembly only model in a reflector

An experimental benchmark, such as Flattop-Pu would ideally suit for further investigation of the dynamic parameters for the application of Avery-Cohn’s model. Unfortunately, the depleted uranium reflector of the Flattop-Pu benchmark contributes with about 15% to the total fission rate. Since Avery-Cohn’s model requires a discrete subdivision into core and reflector zones, this experimental benchmark may not be suitable for our purpose. Instead, the authors have created a strongly reflector dominated model case. It consists of the fuel assembly of ELECTRA placed in a large lead pool. A radial cross section view of the assembly in pool model is shown on Figure 3. There is about 50 cm lead above and below the assembly and the outer boundary of the pool from the center is 100 cm. Absence of absorber drums would increase not only the neutron lifetime in the reflector but also reflector returning fraction of the system. Hence reflector influence would be stronger than the ELECTRA reference case. The results of evaluating Λ_{eff} as well as Avery-Cohn’s Λ_{m} and Λ_{s} are shown on Table II and III respectively.

As can be seen from the tables, the effective prompt neutron reproduction times estimated by 1/v method is 2.6 times larger than the value obtained from the time cut-off method. On one hand, the static reproduction time evaluated by Avery-Cohn’s model matches with the 1/v method within the statistical accuracy, whereas on the other, the dynamic reproduction time appears closer to the one estimated by time cut-off method, though out of the statistical error range. It should be mentioned again that the method for evaluating the core and the reflector reproduction time is not perfect.

Spriggs et al. (Spriggs et al., 1997) pointed out that the static reproduction time is predicted by the perturbation method whereas the dynamic reproduction time could correspond to the Rossi-alpha experimental result. According to Avery-Cohn’s model, the point kinetics equivalent reproduction time would be the dynamic reproduction time. Thus, we believe that the methods predict different physical parameters: the 1/v method, which is a perturbation method, leads to the static reproduction time and the time cut-off method seem to be leading to the dynamic reproduction time respectively.

In the following section, we continue with the reference case model of the ELECTRA.

3.3. ELECTRA

Note that when applying Eq. 13 that when \( k_p \approx 1 \), the numerical estimation of the effective prompt neutron lifetime becomes unreliable, as the absolute uncertainties introduced by the code or the cross section library are large comparing to the numerator of Eq. 13. Hence, we have chosen to evaluate the prompt neutron reproduction time of ELECTRA far from criticality; in a supercritical state, where all the absorbers drums are rotated to the maximal distance from the core, in order to avoid such uncertainties.

The estimated values of the prompt neutron reproduction time obtained by the two different methods are shown in Table IV. As it can be seen there is a slight but statistically significant difference in the kinetic parameters where the 1/v method results in larger values for \( \Lambda_{eff} \) and, smaller values for \( \alpha \).

3.4. Avery-Cohn model of ELECTRA

The parameters \( k_c \), \( f \), \( f_{cr} \) for of Avery-Cohn’s two-region prompt neutron model are evaluated and shown in Table V. Note the large probability of leaking into the reflector.
Moreover, subzone reproduction parameters evaluated by partial poisoning, as explained in section 2.3, are found to be \( \Lambda_c = 51.3 \pm 1.3 \) ns and \( \Lambda_r = 89.6 \pm 1.1 \) ns respectively. Using these parameters, the system’s dynamic and static reproduction times are found to be:

\[
\Lambda_{mn} = \Lambda_c + f_c \Lambda_r = 74.4 \pm 1.4 \text{ ns} \tag{23}
\]
\[
\Lambda_s = \Lambda_c + f_c \Lambda_r = 104.9 \pm 1.5 \text{ ns.} \tag{24}
\]

Neither static nor dynamic reproduction times are exactly coinciding with values for \( \Lambda \) obtained with the 1/\( v \) poisoning or the time cut-off method. Nevertheless, considering a slight over-prediction that might have occurred due to non perfect methodology to evaluate \( \Lambda_c \) and \( \Lambda_r \), and relying on the previous sections conclusions, 1/\( v \) poisoning method seems to be predicting the static reproduction time of the system, whereas, the time cut-off method predicts the dynamic reproduction time of the system.

4. Conclusions

Based on our simulations we conclude that methods based on time-dependent absorption may possibly result in a discrepancy between calculated and measured values in a reflector dominated system. The time cut-off method might be the most reliable method to determine \( \Lambda_{eff} \) for use in dynamic modeling of the system.

Being the reflector return fraction of the reference ELECTRA case \( \sim 30\% \) indicates an clear impact of reflector on the effective reproduction time of ELECTRA. Though, due to the very small value of \( \Lambda_{eff} \), the actual value is shown not to play an important role for the reactor overpower transient performance (Suvdantsetseg et al., 2013). However, for a system combining a large value of \( \Lambda_{eff} \) and a large reflector importance, this effect can not be neglected.

Furthermore, dynamic simulations of the studied system may need to consider the non-constant behavior of the reproduction time. For instance, the non-linear relationship between the inverse period and the reactivity when the reactivity is \( > 1 \) $\$ or multiple decay modes during die-away experiments.

In addition, noting that the reproduction time is a parameter which is sensitive to the drum absorber location as well as the fine design details of the reflector, the parameter may change if any further design update is made. Thus following each modification of the reflector design, it must be re-calculated.

Acknowledgement

Dr S. Bortot’s early contribution is appreciated. This work is financially supported from SKB.

References

Bortot, S., Pellicone, S., 2013. Private communication. PSI.


Ott, K. O., Neuhold, R. J., 1985. Introductory Nuclear Reactor Dynamics. ANS.

Table I: Godiva and Flattop-Pu benchmark result

<table>
<thead>
<tr>
<th>Benchmark name</th>
<th>$k_{eff}$, −</th>
<th>$\beta_{eff}, \text{pcm}$</th>
<th>$k_p$, −</th>
<th>$\Lambda^* / \Lambda_{eff}$, ns</th>
<th>$\alpha$, $10^4$ s$^{-1}$</th>
<th>(C-E)/E$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Godiva</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured</td>
<td>1.0000±0.00010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Serpent 1/v method</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JEFF3.1.1 library</td>
<td>0.99638±0.000001</td>
<td>673±0.1</td>
<td>0.98967±0.000001</td>
<td>6.26±0.01/4.34±0.05</td>
<td>-238±2.8</td>
<td>1.14</td>
</tr>
<tr>
<td>ENDF/B-VII library</td>
<td>0.99989±0.000001</td>
<td>675±0.1</td>
<td>0.99314±0.000001</td>
<td>6.25±0.01/4.26±0.05</td>
<td>-161±2.1</td>
<td>0.45</td>
</tr>
<tr>
<td>MCNP time cut-off method</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JEFF3.1.1 library</td>
<td>0.99723±0.000006</td>
<td>650±8.5</td>
<td>0.99075±0.000006</td>
<td>6.25±0.02/5.04±0.09</td>
<td>-184±3.1</td>
<td>0.66</td>
</tr>
<tr>
<td>ENDF/B-VII library</td>
<td>1.00008±0.000006</td>
<td>647±8.5</td>
<td>0.99421±0.000006</td>
<td>6.25±0.02/4.88±0.08</td>
<td>-119±1.5</td>
<td>0.07</td>
</tr>
<tr>
<td><strong>Flattop-Pu</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured</td>
<td>1.0000±0.00030</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Serpent 1/v method</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JEFF3.1.1 library</td>
<td>1.00365±0.000001</td>
<td>282±0.1</td>
<td>1.00082±0.000001</td>
<td>75.1±0.1/10.1±0.2</td>
<td>8.1±0.2</td>
<td>-1.38</td>
</tr>
<tr>
<td>ENDF/B-VII library</td>
<td>1.00148±0.000003</td>
<td>270±0.2</td>
<td>0.99878±0.000003</td>
<td>77.3±0.1/10.1±0.3</td>
<td>-12.1±0.4</td>
<td>-0.44</td>
</tr>
<tr>
<td>MCNP time cut-off method</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JEFF3.1.1 library</td>
<td>1.00269±0.000007</td>
<td>271±7</td>
<td>0.99997±0.000003</td>
<td>75.6±0.1/0.4±1.2</td>
<td>-7.2±2.7</td>
<td>0.66</td>
</tr>
<tr>
<td>ENDF/B-VII library</td>
<td>1.00105±0.000007</td>
<td>273±10</td>
<td>0.99832±0.000007</td>
<td>77.0±0.5/7.82±0.94</td>
<td>-21.4±2.4</td>
<td>0.0</td>
</tr>
</tbody>
</table>

$^a$A is direct output of the code
$^b$C stands for computational value and E stands for experimental value
$^c$(Mosteller and Kiedrowski, 2011; OECD Agency, 2010)
$^d$(Mosteller and Kiedrowski, 2011; OECD Agency, 2010)

Table II: Prompt neutron reproduction time $\Lambda_{eff}$ [ns] for ELECTRA assembly only in pool model

<table>
<thead>
<tr>
<th>Method</th>
<th>$k_{eff}$, −</th>
<th>$\beta_{eff}$, pcm</th>
<th>$\Lambda_{eff}$, ns</th>
<th>$\alpha$, $10^4$ s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serpent 1/v method</td>
<td>1.10725±0.00004</td>
<td>286±1</td>
<td>260.0±1.6</td>
<td>40.1±0.3</td>
</tr>
<tr>
<td>MCNP time cut-off method</td>
<td>1.10252±0.00010</td>
<td>289±13</td>
<td>104.1±3.5</td>
<td>86.6±2.8</td>
</tr>
</tbody>
</table>

Table III: Avery-Cohn parameters for ELECTRA assembly only in pool model

<table>
<thead>
<tr>
<th>Parameter name, unit</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication factor, -</td>
<td>$k_{eff}$</td>
<td>1.10725 ± 0.00004</td>
</tr>
<tr>
<td>Reflector return fraction, -</td>
<td>$f = 1 - k_c/k_{eff}$</td>
<td>0.3017 ± 0.0001</td>
</tr>
<tr>
<td>Probability of leaking into reflector, -</td>
<td>$f_{cr}$</td>
<td>0.7098 ± 0.0001</td>
</tr>
<tr>
<td>Core reproduction time, ns</td>
<td>$\Lambda_c$</td>
<td>46.4±1.6</td>
</tr>
<tr>
<td>Reflector reproduction time, ns</td>
<td>$\Lambda_r$</td>
<td>301.5 ± 1.6</td>
</tr>
<tr>
<td>Dynamic reproduction time, ns</td>
<td>$\Lambda_m$</td>
<td>137.4 ± 1.6</td>
</tr>
<tr>
<td>Static reproduction time, ns</td>
<td>$\Lambda_s$</td>
<td>260.4 ± 1.9</td>
</tr>
</tbody>
</table>

Table IV: Prompt neutron reproduction time $\Lambda_{eff}$ [ns] for ELECTRA model

<table>
<thead>
<tr>
<th>Method</th>
<th>$k_{eff}$, −</th>
<th>$\beta_{eff}$, pcm</th>
<th>$\Lambda_{eff}$, ns</th>
<th>$\alpha$, $10^4$ s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serpent 1/v method</td>
<td>1.04166±0.00003</td>
<td>278±1</td>
<td>83.1±1.1</td>
<td>46.6±0.7</td>
</tr>
<tr>
<td>MCNP time cut-off method</td>
<td>1.03758±0.00007</td>
<td>278±10</td>
<td>68.9±2.1</td>
<td>48.5±1.5</td>
</tr>
</tbody>
</table>

Table V: Avery-Cohn parameters for ELECTRA

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{eff}$</td>
<td>1.04166 ± 0.00003</td>
</tr>
<tr>
<td>$k_c$</td>
<td>0.77316 ± 0.00004</td>
</tr>
<tr>
<td>$f = 1 - k_c/k_{eff}$</td>
<td>0.2578 ± 0.0001</td>
</tr>
<tr>
<td>$f_{cr}$</td>
<td>0.5975 ± 0.0001</td>
</tr>
</tbody>
</table>