ON SMALL-SIGNAL ANALYSIS AND CONTROL OF THE SINGLE-
AND THE DUAL-ACTIVE BRIDGE TOPOLOGIES

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Dedicated

To

Eva, Philippos, and Alexandros
ABSTRACT

High-frequency dc-dc converters are nowadays widely used in a diversity of power electronic applications. High operating frequencies entail a reduction in size of the passive components, such as inductors, capacitors and power transformers. By operating the converter at higher frequencies with conventional hard-switching topologies, the transistor switching losses increase at both turn-on and turn-off. High-voltage converters in the power range of 1-10MW will therefore have excessive switching losses if the switching frequency is higher than 4 kHz. In order to achieve a high-frequency operation with moderate switching losses a number of soft-switched topologies have been studied in [Dem1]. The favourable DC-DC converter was found to be the Dual-Active Bridge when a bi-directional power flow is demanded. Additionally, the Single-Active Bridge (SAB) topology was introduced for the first time.

In this thesis the two topologies are thoroughly studied. The dynamic small-signal models are presented and the dynamic behaviour of the converters is discussed in deep. Different control strategies are presented concerning the two converters and the advantages and the disadvantages of the different control strategies are stated. Critical issues as efficiency and stability are presented separately for the two converters.

Keywords:
- DC-DC converters
- Soft-switching
- High-frequency
- High-power
- Bi-directional
- Single-Active Bridge
- Dual-Active Bridge
- State-space averaging
- Small-signal modelling
- Control
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SYMBOLS

\( A \) state coefficient matrix

\( A_c \) cross-sectional area of the core \([ m^2 ]\)

\( B \) source coefficient matrix

\( \vec{B} \) magnetic flux density \([ T ]\)

\( C \) filter capacitance \([ F ]\)

\( C_r \) series capacitance \([ F ]\)

\( C_s \) snubber capacitance \([ F ]\)

\( C_e \) effective resonance capacitance \([ F ]\)

\( C_w \) winding capacitance \([ F ]\)

\( d \) duty ratio

\( \dot{d} \) duty ratio-small signal perturbation

\( d_n \) duty factor of the \( n \text{th} \) cycle

\( d_{D, on} \) on duty ratio-diode

\( d_{dsc} \) duty ratio for DCM

\( d_{dscESR} \) duty ratio for DCM with the losses and the ESR included
\( d_{R_{d+}} \) on duty ratio-controllable switch

\( d_1 \) duty ratio

\( d_2 \) duty ratio

\( d_3 \) duty ratio

\( D \) dc term of the duty ratio

\( D_{A+} \) upper diode-primary side

\( D_{A-} \) lower diode-primary side

\( D_{s1,s2,s3,s4} \) secondary placed diodes

\( \rightarrow D \) dielectric displacement \[ C \: m^{-2} \]

\( E \) control matrix

\( \rightarrow E \) electric field \[ V \: m^{-1} \]

\( e(t) \) applied step voltage \[ V \]

\( F^T \) coefficient matrix

\( f_c \) cross-over frequency \[ Hz \]

\( f_0 \) resonance frequency \[ Hz \]

\( f_s \) switching frequency \[ Hz \]

\( f_{s0} \) switching frequency-steady state value \[ Hz \]
\( \tilde{f}_s \)  
switching frequency-small-signal perturbation  
[ Hz ]

\( f_{\text{start-up}} \)  
switching frequency start-up  
[ Hz ]

\( G_{sd} \)  
small-signal control-to-state transfer function

\( G_{sda} \)  
small-signal control-to-state transfer function when \( \alpha \) is the control parameter

\( G_{vd} \)  
small-signal control-to-output transfer function

\( G_{vda} \)  
small-signal control-to-output transfer function when \( \alpha \) is the control parameter

\( G_{sg} \)  
small-signal source-to-state transfer function

\( G_{sg} \)  
small-signal source-to-output transfer function

\( \vec{H} \)  
magnetic field  
\([ \text{ Am}^{-1} ]\)

\( I \)  
unity matrix

\( I_{1,2} \)  
current through the windings  
[ A ]

\( I_{S_1} \)  
current through \( S_1 \)  
[ A ]

\( \hat{I}_{S_1} \)  
peak current of \( S_1 \)  
[ A ]

\( I_{S_2} \)  
current through \( S_2 \)  
[ A ]

\( \hat{I}_{S_2} \)  
peak current of \( S_2 \)  
[ A ]
\[ \langle I_{S_1} \rangle \] average current through \( S_1 \) \[ A \]

\[ \langle I_{S_2} \rangle \] average current through \( S_2 \) \[ A \]

\( I_T \) current through the transistor \[ A \]

\( I_{\text{off}} \) turn-off current \[ A \]

\( i_c \) current through the capacitor \( C \) \[ A \]

\( i_{L_{\sigma}} \) inductor current \[ A \]

\[ |i_{L_{\sigma}}| \] rectified inductor current \[ A \]

\[ \langle |i_{L_{\sigma}}| \rangle \] average value of the rectified inductor current \[ A \]

\[ \langle i_{L_{\sigma}} \rangle \] average inductor current \[ A \]

\[ i_{L_{\sigma}}^\wedge \] peak inductor current \[ A \]

\( i_{L_{m}} \) magnetising current \[ A \]

\( i_{r_{	ext{resi}}} \) resonance current \[ A \]

\( i_{r_{	ext{min}}} \) minimum inductor current \[ A \]

\( i_R \) current through the load resistance \( R \) \[ A \]

\( i_{\text{scr}} \) short-circuit current \[ A \]

\( i_0 \) load current \[ A \]
\[ J \] current density \[ A \text{m}^{-2} \]

\[ K_{pi} \] proportional gain for the current PI

\[ K_{pv} \] proportional gain for the voltage PI

\[ l_m \] mean magnetic path length \[ m \]

\[ L_r \] resonance inductance \[ H \]

\[ L_{or} \] leakage inductance \[ H \]

\[ L_m \] magnetising inductance \[ H \]

\[ M \] conversion ratio

\[ N_G \] numerator

\[ N_{1,2} \] winding turns

\[ n_{TF} \] turns ratio of the transformer

\[ P_0 \] output power \[ W \]

\[ P_{0\text{max}} \] maximum output power \[ W \]

\[ Q^T \] coefficient matrix

\[ Q \] loaded quality factor

\[ Q_0 \] unloaded quality factor

\[ q_0 \] area enclosed by \( I_{S_i} \)
\[ q_2 \] area enclosed by \( I_{S_2} \)

\[ R \] load resistance \([\text{[ } \Omega \text{ ]}]\)

\[ R_{\text{core}} \] resistance representing the core losses of the inductor \([\text{[ } \Omega \text{ ]}]\)

\[ R_L \] resistance representing the resistive losses of the inductor \([\text{[ } \Omega \text{ ]}]\)

\[ R_{\text{losses}} \] resistance representing the core losses \([\text{[ } \Omega \text{ ]}]\)

\[ R_{\text{no}} \] no-load resistance \([\text{[ } \Omega \text{ ]}]\)

\[ R \] core reluctance

\[ r_c \] ESR of the output capacitor \([\text{[ } \Omega \text{ ]}]\)

\[ r_L \] series resistance representing converter losses \([\text{[ } \Omega \text{ ]}]\)

\[ S^T \] coefficient matrix

\[ T_{A^+} \] upper controllable switch-primary side

\[ T_{A^-} \] lower controllable switch-primary side

\[ T_{s_{1,2,3,4}} \] secondary placed controllable switches

\[ T_c \] resonance period \([\text{[ } s \text{ ]}]\)

\[ T_r \] resonance period \([\text{[ } s \text{ ]}]\)

\[ T_s \] switching period \([\text{[ } s \text{ ]}]\)

\[ T_{iv} \] integration time for the voltage PI \([\text{[ } s \text{ ]}]\)
\( T_{ii} \) integration time for the current PI [s]

\( t_{d_{dc}} \) discontinuous-time interval [s]

\( t_{D_{A+}} \) conduction time-diode \( D_{A+} \) [s]

\( t_{o_{ff}} \) turn-off time [s]

\( t_{T_{A+}} \) conduction time-controllable switch \( T_{A+} \) [s]

\( u \) source vector

\( \tilde{u} \) source vector-ac term

\( u_0 \) source vector-dc term

\( V_d \) pole-to-pole voltage [V]

\( V_{dc} \) input dc-voltage [V]

\( V_{dc0} \) input dc-voltage-dc term [V]

\( \tilde{V}_{dc} \) input dc-voltage-ac term [V]

\( V_0 \) output voltage [V]

\( V_0' \) output voltage reflected to the primary side of the transformer [V]

\( V_{L_{\sigma}} \) voltage across \( L_{\sigma} \) [V]

\( \langle V_{t_{\sigma}} \rangle \) average inductor voltage [V]

\( V_r \) voltage across the switch [V]
\( v_C \) voltage across the output capacitor \([V]\)

\( v_R \) voltage across the load resistance \([V]\)

\( W_e \) electric energy \([J]\)

\( W_m \) magnetic energy \([J]\)

\( x \) state vector

\( x_{1pk} \) state variable-peak inductor current \([A]\)

\( x_{1pk0} \) state variable-dc component \([A]\)

\( x_{1pk}^- \) state variable-small-signal perturbation \([A]\)

\( x_1 \) state variable-inductor current \([A]\)

\( x_{10} \) state variable-dc component \([A]\)

\( x_1^- \) state variable-small-signal perturbation \([A]\)

\( |x_1| \) rectified inductor current \([A]\)

\( \langle |x_1| \rangle \) average value of the rectified inductor current \([A]\)

\( x_2 \) state variable-voltage across the filter capacitor \([V]\)

\( x_{20} \) state variable-dc component \([V]\)

\( x_2^- \) state variable-small-signal perturbation \([V]\)

\( Z_r \) resonance impedance \([\Omega]\)
$Z_o$  characteristic impedance of the resonant tank  [Ω]
Greek letters

\( \alpha \) phase-shift angle [ rad ]

\( \alpha_0 \) phase-shift angle-dc term [ rad ]

\( \tilde{\alpha} \) phase-shift angle-small-signal perturbations [ rad ]

\( \beta \) the ratio of the series and the magnetising inductance

\( \gamma \) resistive term [ \( \Omega \) ]

\( \Lambda \) determinant

\( \delta_{\text{r, max}} \) required dead time [ s ]

\( \delta_{\text{r}} \) dead time [ s ]

\( \varepsilon \) permittivity [ \( F \, m^{-1} \) ]

\( \varepsilon_r \) permittivity of the material [ \( F \, m^{-1} \) ]

\( \varepsilon_0 \) permittivity of the vacuum [ \( F \, m^{-1} \) ]

\( \zeta \) damping ratio

\( \eta \) transformer ratio

\( \kappa \) resistive term [ \( \Omega \) ]

\( \lambda \) resistive term [ \( \Omega \) ]
\[ \Lambda \quad \text{constant} \]

\[ \mu \quad \text{permeability} \quad \left[ H \, m^{-1} \right] \]

\[ \mu_r \quad \text{permeability of the material} \quad \left[ H \, m^{-1} \right] \]

\[ \mu_0 \quad \text{permeability of the vacuum} \quad \left[ H \, m^{-1} \right] \]

\[ \xi \quad \text{ratio of the length of a TLO wave period versus a length of a non-TLO wave period.} \]

\[ \rho_r \quad \text{relative resistivity of the material} \quad \left[ \Omega \, m \right] \]

\[ \phi \quad \text{phase-shift angle (Dual Active Bridge)} \quad \left[ \text{rad} \right] \]

\[ \phi_0 \quad \text{phase-shift angle-dc term} \quad \left[ \text{rad} \right] \]

\[ \phi \quad \text{phase-shift angle-small-signal perturbations} \quad \left[ \text{rad} \right] \]

\[ \phi_c \quad \text{phase-angle at crossover frequency} \quad \left[ \text{deg} \right] \]

\[ \phi_i \quad \text{phase-shift angle corresponding to } M_i \text{ and } P_{0_{\text{max}}} \quad \left[ \text{deg} \right] \]

\[ \Phi \quad \text{flux density} \quad \left[ \text{Wb} \right] \]

\[ \sigma \quad \text{real part of a pole} \]

\[ \tau \quad \text{time constant} \quad \left[ \text{rad} \right] \]

\[ \omega_0 \quad \text{resonant angular frequency} \quad \left[ \text{rad} \, s^{-1} \right] \]
\[ \omega \quad \text{operating angular frequency} \quad \left[ \text{rad s}^{-1} \right] \]

\[ \omega_c \quad \text{crossover angular frequency} \quad \left[ \text{rad s}^{-1} \right] \]

\[ \omega_n \quad \text{undamped natural frequency} \quad \left[ \text{rad s}^{-1} \right] \]
<table>
<thead>
<tr>
<th>Abbreviation</th>
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<tbody>
<tr>
<td>CCM</td>
<td>Continuous-Conduction Mode</td>
</tr>
<tr>
<td>DAB</td>
<td>Dual-Active Bridge</td>
</tr>
<tr>
<td>DCM</td>
<td>Discontinuous-Conduction Mode</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processor</td>
</tr>
<tr>
<td>EMC</td>
<td>Electromagnetic Compatibility</td>
</tr>
<tr>
<td>EMF</td>
<td>Electromotive force</td>
</tr>
<tr>
<td>EMI</td>
<td>Electromagnetic Interference</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Elements Method</td>
</tr>
<tr>
<td>FM</td>
<td>Frequency Modulation</td>
</tr>
<tr>
<td>HF</td>
<td>High Frequency</td>
</tr>
<tr>
<td>HS-PWM</td>
<td>Hard-Switched PWM</td>
</tr>
<tr>
<td>MMF</td>
<td>Magnetomotive force</td>
</tr>
<tr>
<td>PCB</td>
<td>Printed Circuit Board</td>
</tr>
<tr>
<td>PI</td>
<td>Proportional-Integral</td>
</tr>
<tr>
<td>PSH</td>
<td>Phase Shift</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>RI</td>
<td>Radio Interference</td>
</tr>
<tr>
<td>SAB</td>
<td>Single-Active Bridge</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>SLR</td>
<td>Series-Loaded Resonant</td>
</tr>
<tr>
<td>SOA</td>
<td>Safe-Operating Area</td>
</tr>
<tr>
<td>TLO</td>
<td>Transformer-induced Low-frequency Oscillations</td>
</tr>
<tr>
<td>ZCS</td>
<td>Zero-Current-Switching</td>
</tr>
<tr>
<td>ZVS</td>
<td>Zero-Voltage-Switching</td>
</tr>
</tbody>
</table>
1 INTRODUCTION TO THE THESIS

1.1 Introduction

High-frequency dc-dc converters are nowadays widely used in a diversity of power electronic applications. High operating frequencies entail a reduction in size of the passive components, such as inductors, capacitors and power transformers. By operating the converter at higher frequencies with conventional hard-switching (square-current waveforms) topologies, the transistor switching losses increase at both turn-on and turn-off. High dissipation spikes occur more often and as a result, the average transistor losses are increased. In some cases, R-C snubber circuits can be used but in most of these cases, the losses are just transferred from the switches to the snubber circuit. Circuit parasitics such as semiconductor junction capacitance, transformer leakage inductance and winding capacitance, are major factors hindering high frequency operation of the conventional hard-switched Pulse-Width-Modulated (PWM) converters.

Lower switching losses can be achieved by introducing a resonating L-C tank. Current and/or voltage waveforms become more or less sinusoidal and with a proper control strategy, the switching losses can be dramatically reduced. In this report the above topologies are referred to as soft-switched topologies due to their ability to switch the devices on or off at zero-current or zero-voltage and in some special cases a combination of the two. In [Dem 1] a number of soft-switched topologies have been studied and compared. The most promising topologies were the

- Single-active bridge and,
- Dual-active bridge topologies

The specifications concerning the converter design are as follows:

- Variable input voltage and constant output voltage
- Galvanic isolation
- Bi-directional power flow
Half-bridge configuration on the high-voltage side and full-bridge configuration on the low-voltage side.

This thesis is devoted to high-power applications. High-voltage input of 30-80 kV is considered on the primary side while high current of some hundreds of ampere is considered on the secondary side. Due to the high-voltage input, only half-bridge configurations were considered. The operating frequency is in the range of some kHz and the power rating of the order of some MW. The major objective of this thesis is to prove feasibility of the topologies proposed in [Dem1]. Converter dynamics are of great interest and the dynamic models are derived by using the extended state-space averaging technique. The extended state-space averaging method can be applied in circuits with half-cycle symmetry. Additionally, the controller of all the topologies has been designed based on the small-signal transfer functions as derived in the thesis. Different modulation strategies are proposed and tested and compare with each other. A prototype has been built and the stress parameters of the converter are presented.

1.2 Outline of the thesis

In Chapter 2 a very general description of the basic terminology is given. Load regulation, temperature regulation as well as, line regulation is briefly discussed. Some control theory and regulation terms are discussed as well.

The state-space averaging modelling method is generally derived in Chapter 3. State-space averaging allows a switched system to be approximated as a continuous non-linear system, and linearization allows the resulting non-linear system to be approximated as a linear system.

In Chapter 4 the behaviour of the Single-Active Bridge (SAB) topology is studied both in steady-state and transient mode of operation. The small-signal model for the topology has been derived both in the Continuous-Conduction Mode (CCM) and the Discontinuous-Conduction Mode (DCM). Different modulation strategies are discussed. A small-signal based control scheme is presented and the behaviour of the controller is studied. Oscillations occurring during the DCM are thoroughly discussed.

Experimental measurements done with SAB topology are presented and discussed in Chapter 5. Two modulation strategies are concerned, the turn-off time control operating at DCM, and the turn-off time control operating at intermittent mode. The steady-state and the
dynamics characteristics of the converter are studied and presented. Additionally, the efficiency of the converter is given for a wide load range.

The Dual-Active Bridge (DAB) topology employing a conventional phase-shift control scheme is studied in Chapter 6. The steady-state and the dynamic behaviour of the converter is studied and presented. A new small-signal model for the DAB topology has been derived and a small-signal based control scheme is presented. A high-frequency oscillation occurs during the commutation of the secondary placed controllable switches and is briefly discussed in this chapter.

In Chapter 7 experimental results obtained with the DAB converter are presented. Both the steady-state and the dynamic behaviour of the converter are studied. The controller performance during transient is presented. Additionally, the measured efficiency of the converter in a wide load range is given. Measurements are presented in both step-up and step-down operation mode.

In Chapter 8 a new control strategy concerning the DAB converter is presented and discussed. Relatively lower kvar rating of the filter capacitor has been reported but a power transfer reduction is the trade-off. Both steady-state and dynamic modelling is presented. A small-signal based control configuration is discussed and presented.

In Chapter 9 the behaviour of the DAB topology employing the duty-cycle control scheme has been studied in both steady-state and transient mode of operation. Measurements at step-up and step-down mode are presented and discussed. The performance of the controller is presented under dynamic operating conditions. Additionally, the measured efficiency of the converter is given.

A basic transformer modelling technique is presented in Chapter 10. The major transformer parasitics are thoroughly discussed. A method based on the step response of the transformer for deriving the major parasitics is presented. Experimental verification of the theoretical model is included in this chapter.

In Chapter 11 presents a summary of the most important results and conclusions of the thesis. Suggestions for possible future work in the area of the dc-dc converters for high-power applications are also given.
1.3 Contribution of the thesis

- The half-bridge SAB topology

It is evident to the author that the half-bridge Single-Active Bridge topology has been firstly presented in [Dem1]. The SAB topology has never been published in scientific papers due to the fact that the conventional Pulse-Width Modulation (PWM) cannot be employed.

- Control and modulation

In this thesis two control strategies are presented. The constant switching-frequency turn-off time control, and the constant switching-frequency turn-off time control operating at intermittent mode. Both control strategies have been firstly presented in the present thesis.

- Small-signal modelling

In the literature, a plethora of small-signal models using the conventional state-space averaging technique have been presented for the basic dc-dc converters such as the buck, the boost, and the Cuk converter. The conventional state-space averaging method can however not be used to describe the SAB and the DAB. Instead, a new extended state-space averaging method based on half-cycle symmetry of the circuit was derived. Consequently, a novel small-signal modelling technique is presented in the present thesis.

- Oscillations

The present thesis contributes to the understanding of the transformer-induced low-frequency oscillations of the SAB and DAB. Furthermore, suggestions for how to reduce these oscillations during transients are presented.

- Duty-cycle control strategy

A new Swedish patent was the direct contribution of the thesis as presented in [Dem5]. The novel control strategy can be used to achieve better characteristics of the DAB topology. By the insertion of the freewheeling stages the effective duty-cycle is modulated in order to deliver the appropriate output as ruled by the controller.
• New converter topology

A new dc-dc converter employing the duty-cycle control strategy has been proposed but not studied in the present thesis. The new dc-dc converter employs less semiconductor devices and exhibits less losses and higher efficiency than the conventional DAB topology. Additionally, the complexity of the converter can be reduced considerably. The characteristics of the converter are identical to those of the DAB topology employing the duty-cycle control scheme and has both uni-directional and bi-directional power transfer capability. Furthermore, the converter can operate in both step-up and step-down mode.

The following scientific publications are related to the present thesis:

8. M. D. Manolarou, S. N. Manias, A. Kandianis, G. D. Demetriades, “Transformless driving scheme for synchronous rectifiers”, IEE MED POWER 2002, Athens, Greece, The paper has been accepted for publication in the IEE Proceedings
9. G. D. Demetriades, Hans-Peter Nee, “Characterisation of the soft-switched single-active bridge with novel control scheme for high-power dc-dc applications”, Accepted for publication in IEEE Power Electronics Specialist Conference, PESC 2005, Recife, Brazil, 2005

2 REGULATION AND CONTROL ASPECTS

2.1 Introduction

The definition of power electronics emphasises control of energy flow. Without control, a converter is constrained by its environment [Kre1]. Many loads require very precise regulation of their supply voltage. Control must adjust the switching functions, firstly introduced by Wood [Woo1], in order to maintain precise operation, and adjustment must be performed continuously whenever the converter operates.

Feedback control is considered essential in high-performance power electronics. The converter output and other state parameters as voltages and currents are measured and are used in order to adjust the operation to obtain desired result. Feedforward control uses information about the input waveform or the system behaviour to help determine the correct system operation. Combinations of feedback and feedforward control are used to minimise error between the actual and desired behaviour.

Good regulation and known dynamic behaviour is the purpose of the control.

2.2 Regulation

The term regulation describes the ability of a power converter to compensate for external disturbances as, load variations, input fluctuations or even temperature which can alter the performance. A power supply that behaves as an ideal source would be said to have perfect regulation.

Line regulation refers to the ability of the converter to maintain the output even when the input fluctuates and is given by

\[
\text{Line reg} \approx \frac{\partial V_{\text{out}}}{\partial V_{\text{in}}}
\]  

(2.1)
where $V_{out}$ is the output and $V_{in}$ is the input.

Load regulation can be defined as the ability of a power converter to maintain the output when the output power fluctuates. In power systems and battery-based systems, load regulation is usually referenced to the open-circuit output voltage. Load regulation can be expressed as the partial derivative of the output voltage with respect to the output current as shown in Eq. (2.2)

$$\text{Load reg} \approx \frac{\partial V_{out}}{\partial I_{out}}$$

(2.2)

Other important regulation issue includes temperature regulation and is expressed by the partial derivative of the output voltage as given by Eq.(2.3).

$$\text{Temp reg} \approx \frac{\partial V_{out}}{\partial T}$$

(2.3)

As already mentioned good performance means good regulation. Regulation and insensitivity must be addressed in two very different contexts:

- Static regulation is concerned with effects of changes on the steady-state operation of the converter and is expressed by Eqs. (2.1), (2.2) and (2.3)

- Dynamic regulation, or tracking, is the ability of a system to maintain the desired operation even during rapid changes. The term also refers to the ability to follow time-varying outputs.
2.3 Open-loop and Closed-loop control

Open-loop control is the simplest way to operate a system. The control inputs are independent variables and are not adjusted. By using an open-loop control concept no corrective action can take place during performance diversities. Additionally, open-loop control does not cancel the effects of changes. Although open-loop control is simple, and indeed often preferred if it can meet the needs of a user, it is not sufficient in general for power converter regulation or for many other everyday systems. If an automobile reaches a curve on the road, open-loop steering control will cause some fatal problems, [Kre1]. Similarly if a drifting spacecraft begins to spin, open-loop control will not keep its radio antenna pointed to earth.

![Figure 2.1: Closed-loop system based on feedback control](image)

Closed-loop or feedback control, as illustrated graphically in Figure 2.1, makes use of the output or some other variables of the system. The measured variable can be compared to the desired action and an error signal is developed. Adjustments on control action will be produced by the controller to account for this error. Converter regulation is an excellent example. A proper control parameter must be defined in order to serve as the command input for a system. In the buck dc-dc regulator, as shown in Figure 2.2 (a), the duty ratio can be used as control parameter.
Generally, a control parameter is a value that provides a way to alter the output or other desired variable. In power electronics, the conventional control parameters include phase and, as already mentioned, the duty ratio. Sometimes control parameters appear as functions. In a PWM inverter, the modulating function can be treated as the control parameter.
2.4 State-space design

The idea of state space comes from the state-variable method of describing differential equations as thoroughly described in [Fran1]. In this method the differential equations describing a dynamic system are organised as a set of first-order differential equations in the vector-valued state of the system, and the solution is visualised as a trajectory of this state vector in space. State-space control design is the technique in which the control engineer designs a compensation by working directly with the state-variable description of the system. The ordinary differential equations of physical dynamic systems can be manipulated into state-variable form. The state-variable form is often known as the normal form for the equation. There are several good reasons for studying equations in this form and are listed here:

- To study more general systems. By studying the equations themselves very general models can be developed. By using the equations in their state-variable form give us a compact, standard form for study.

- To introduce the ideas of geometry into differential equations. In physics the plane of position versus velocity of a particle or rigid body is called the phase plane, and the trajectory of the motion can be plotted as a curve in this plane. The state is a generalisation of that idea to include more than two dimensions. The concepts of distance, of orthogonal and parallel lines, and other concepts from geometry can be useful in visualising the solution of a ordinary differential equation as a path in the state space.

- To connect internal and external descriptions. The state of a dynamic system often directly describes the distribution of the internal energy in the system. For example, it is common to select the following state variables: position (potential energy), velocity (kinetic energy), capacitor voltage (electrical energy), inductor current (magnetic energy). The internal energy can always be computed from the state variables. By a system analysis the state of the system inputs can be related with the system state outputs, thus, connecting the internal variables to the external inputs and to the sensed outputs. In contrast, the transfer function relates only the input to the output and does not show the internal behaviour.

The use of the state-space approach has often been referred to as modern control design.
Advantages of the state-space design are especially apparent when the system to be controlled has more than one control input or more than one sensed output. A full state feedback control can be used and the concept of an observer can be introduced, and construct estimates of the state based on the sensed output.
3 STATE-SPACE AVERAGING: APPROXIMATION FOR CONTINUITY

3.1 Introduction
State-space averaging and linearization are analytical approximation techniques that allow switching regulators to be represented as linear systems [Mit1]. More specifically, state-space averaging allows a switched system to be approximated as a continuous non-linear system, and linearization allows the resulting non-linear system to be approximated as a linear system. The criterion for the continuity approximation is that the circuit natural frequencies and all modulation, or signal frequencies of interest must be sufficiently low with respect to the switching frequency. The criterion for the linearity approximation is that the time variations, or ac excursions, of the pertinent variables about the respective dc operating points must be sufficiently small.
Generally speaking, all of us are already familiar with the concept of state-space averaging [Mit1], whether we realise it or not. In our everyday life we are constantly averaging out discrete phenomena, either in space or time, to achieve continuity. When we watch television, we are not usually aware of the raster nor of the colour dots, unless there is a convergence problem. Since utilities went to higher frequencies, i.e. 50 Hz, we are not bothered by flicker in incandescent lamps.
The key ingredient for space continuity is that the size of interest must be large with respect to the size of the discrete part. Similarly, the key ingredients for time continuity are that the rate of change of the desired information must be slow with respect to the rate of change of the discrete events, and there must be some means of storing information from one discrete event to the next.
In switch-mode converters, the inductors and the capacitors serve to store information, or energy, as dictated by the control signals. Even though any set of linearly independent variables can be chosen as the state variables, it is customary and convenient in electrical networks to adopt the inductor currents and capacitor voltages. The total number of storage elements thus determines the order of the system. Since the inductor current and the capacitor
voltage contributes to the amount of the magnetic and electric energy stored respectively they are ideal state variables.

Middlebrook and Cúk [Mid1] have developed a state-space averaging technique that results in a linear model of the power stage including the output filter for small ac signals, linearized around a steady-state dc operating point. Similarly, the controller can be linearized around a steady state operating point.

The state equation can be written in a matrix form as given by

\[ \dot{x} = Ax + Bu \quad (3.1) \]

All state variables and their derivatives must be continuous in order for the system to be continuous. Switch-mode converters do not fulfil this criterion since the voltage across the inductor, and hence, the derivative of the inductor current, is discontinuous. Thus, Eq. (3.1) cannot be applied for any time interval that is long enough to include discontinuity. However, equation Eq. (3.1), can be applied to any switch-mode converter for any given switch condition and in fact the coefficient matrices \( A \) and \( B \) are constant. As a result, the system consists a set of alternating linear systems, each of which describes the system at a particular switch state. In the continuous-conduction mode typically two systems are sufficient to describe the operation. One system describes the operation when the switch is in the on-state and the other system describes the off-state. In the discontinuous-conduction mode additional switch state exists, thus, when all the switches are turned-off and a freewheeling state is present. This additional state must be added to the two states as described in the continuous-conduction mode.

Assuming that \( A_1 \) and \( B_1 \) are the coefficient matrices for the switch condition \( T_{A^+} \) on and \( T_{A^-} \) off, and that \( A_2 \) and \( B_2 \) are the coefficient matrices for the switch condition \( T_{A^+} \) on/ \( T_{A^-} \) off, the system under discussion can be described [Mit1] by,

\[ \dot{x}(t) = A_1 x(t) + B_1 u(t) \quad \frac{n}{f_s} \leq t \leq \frac{n + d_u}{f_s} \quad (3.2) \]

\[ \dot{x}(t) = A_2 x(t) + B_2 u(t) \quad \frac{n + d_u}{f_s} \leq t \leq \frac{n + 1}{f_s} \quad (3.3) \]

where the switching times correspond to the normalised PWM waveform as shown in Figure 3.1.
If the switching frequency is high enough relative to the natural frequencies of the circuit and the signal frequencies, then the switching period is short enough to approximate the derivatives of \( x \) at \( t = \frac{n}{f_s} \) and \( t = \frac{n+D}{f_s} \) as follows

\[
\dot{x}\left(\frac{n}{f_s}\right) = d_n \left( \frac{n}{f_s} \right)
\]  
\[ (3.4) \]

and

\[
\dot{x}\left(\frac{n+d_n}{f_s}\right) = \left[1 - \frac{1-d_n}{f_s}\right] \left( \frac{n+d_n}{f_s} \right)
\]  
\[ (3.5) \]
Combining Eqs. (3.2), (3.3), (3.4) and (3.5) to eliminate $x\left(\frac{n + d_n}{f_s}\right)$ and $x\left(\frac{n}{f_s}\right)$ results in

$$x\left(\frac{n + d_n}{f_s}\right) = x\left(\frac{n}{f_s}\right) + \frac{d_n}{f_s} A_1 x\left(\frac{n}{f_s}\right) + B_1 u\left(\frac{n}{f_s}\right)$$

(3.6)

and

$$x\left(\frac{n + 1}{f_s}\right) = x\left(\frac{n + d_n}{f_s}\right) + \frac{1 - d_n}{f_s} A_2 x\left(\frac{n + d_n}{f_s}\right) + B_2 u\left(\frac{n + d_n}{f_s}\right)$$

(3.7)

Substituting $x\left(\frac{n + d_n}{f_s}\right)$ in Eq. (3.7) with the right-hand side of Eq. (3.6) yields

$$x\left(\frac{n + 1}{f_s}\right) = x\left(\frac{n}{f_s}\right) + \frac{d_n}{f_s} A_1 x\left(\frac{n}{f_s}\right) + B_1 u\left(\frac{n}{f_s}\right)$$

$$+ \frac{1 - d_n}{f_s} A_2 x\left(\frac{n}{f_s}\right) + \frac{d_n}{f_s} A_1 x\left(\frac{n}{f_s}\right) + B_1 u\left(\frac{n}{f_s}\right)$$

$$+ \frac{1 - d_n}{f_s} B_2 u\left(\frac{n + d_n}{f_s}\right)$$

(3.8)

As for the state vector, the time derivative of the input vector $u$ can be defined as

$$\dot{u}\left(\frac{n}{f_s}\right) = \frac{u\left(\frac{n + d_n}{f_s}\right) - u\left(\frac{n}{f_s}\right)}{\frac{d_n}{f_s}}$$

(3.9)

In order to eliminate $u\left(\frac{n + d_n}{f_s}\right)$ from Eq.(3.8), $u\left(\frac{n + d_n}{f_s}\right)$ is solved for in Eq.(3.9) and the result is inserted in Eq.(3.8).

Accordingly,
\[
\frac{x\left(\frac{n+1}{f_s}\right) - x\left(\frac{n}{f_s}\right)}{\frac{1}{f_s}} = \left[d_n A_1 + (1 - d_n) A_2\right] x\left(\frac{n}{f_s}\right) \\
+ \left[d_n B_1 + (1 - d_n) B_2\right] u\left(\frac{n}{f_s}\right) \\
+ \frac{1}{f_s}\left[d_n (1 - d_n)\right]\left[A_2\left[A_1 x\left(\frac{n}{f_s}\right) + B_1 u\left(\frac{n}{f_s}\right)\right] + B_2 \left(\frac{n}{f_s}\right)\right] 
\]  
(3.10)

For high switching frequencies the term \(\frac{1}{f_s}\) can be neglected, and Eq. (3.10) can be rearranged as

\[
\dot{x}\left(\frac{n}{f_s}\right) = \left[d_n A_1 + (1 - d_n) A_2\right] x\left(\frac{n}{f_s}\right) + \left[d_n B_1 + (1 - d_n) B_2\right] u\left(\frac{n}{f_s}\right) 
\]  
(3.11)

where the left-hand side of Eq. (3.10) was identified to be the time-derivative of \(x\).

Where

\[
\dot{x} = \left[d A_1 + (1 - d) A_2\right] x + \left[d B_1 + (1 - d) B_2\right] u 
\]  
(3.12)

From the definition of the state-space formulation as given by Eq.(3.1) and identifying the terms of Eq.(3.11) yields that

\[
A = \left[d A_1 + (1 - d) A_2\right] 
\]  
(3.13)

and

\[
B = \left[d B_1 + (1 - d) B_2\right] 
\]  
(3.14)

The state-space averaging method replaces the state-equations by a single state-space description which represents approximately the behaviour of the circuit across the whole period. This approximation, as shown graphically in Figure 3.2, describes the evolution of the
initial $x(0)$ state to the final $x(T_s)$ state through a single, equivalent system as illustrated by the dotted line [Bar1]. The average step is achieved by taking the average of both dynamic and static equations for the two switched intervals. A different approach has been presented by Al-hosini [Al-h1] and [Al-h2] but will not be used in the present thesis. Al-hosini proposed that the linear positive and negative slopes of the variable $x$ can be replaced, with little loss of information, by a single slope passing through the middle of the two lines, dashed line as shown in Figure 3.2.

**Figure 3.2 : Graphical representation of the state-space averaging**

This new line represents the average value of the variable during the switching period $T_s$. Therefore, if the initial average $x_n$ and the final $x_{n+1}$ at the beginning and end of a single period, respectively, are known the derivative of the state variable $\dot{x}$ is given by

$$\dot{x} = \frac{x_{n+1} - x_n}{T_s} \approx \left( x_{n+1} - x_n \right) f(t) \quad (3.15)$$

For the ideal dead-beat controller, which was the case in [Al-h1] and [Al-h2], the response is achieved in one period of the switching frequency. The averaged value $x_{n+1}$ represents the
demanded value $x_r$ of the state variable, and $x_n$ represents the actual value of the state variable $x$. Therefore, the linearised state-space average model, which is referred to as the governing equation in [Al-h1] and [Al-h2] can be written as

$$f(t)(x_r - x) \approx Ax + Bu$$  \hspace{1cm} (3.16)$$

Equation (3.16) is used in order to calculate the on-line required change in the duty ratio to achieve the demanded output quantity.

### 3.2 Small-signal approximation for linearity

In order to linearise the system the state vector $x$, the input vector $u$, and the duty ratio $d$ are assumed to be subdivided into dc-component and a small ac perturbation. Accordingly,

$$x = x_0 + \tilde{x}$$  \hspace{1cm} (3.17)$$

$$u = u_0 + \tilde{u}$$  \hspace{1cm} (3.18)$$

$$d = D + \tilde{d}$$  \hspace{1cm} (3.19)$$

Where $x_0$, $u_0$ and $D$ are the dc components and $\tilde{x}, \tilde{u}$ and $\tilde{d}$ are the signal frequency ac components.

Substituting Eqs. (3.17), (3.18), and (3.19) into Eq. (3.12) results in

$$\begin{bmatrix} \dot{x}_0 + \tilde{x} \end{bmatrix} = \begin{bmatrix} (D + \tilde{d}) & A_1 & 1 - (D + \tilde{d}) \\ \end{bmatrix} \begin{bmatrix} x_0 + \tilde{x} \\ \\ \end{bmatrix} + \\ + \begin{bmatrix} (D + \tilde{d}) & B_1 & 1 - (D + \tilde{d}) \\ \end{bmatrix} \begin{bmatrix} u_0 + \tilde{u} \\ \\ \end{bmatrix}$$  \hspace{1cm} (3.20)$$

Assuming that the small-signal ac products $\tilde{x}u \approx \tilde{x}d \approx \tilde{d}u \approx 0$, the equation can by simplified. Subdividing the equation into dc and ac components yields
\[ 0 = \left[ A_1 D + A_2 (1 - D) \right] x_0 + \left[ B_1 D + B_2 (1 - D) \right] u_0 \]  

(3.21)

\[ \dot{x} = \left[ A_1 D + A_2 (1 - D) \right] \bar{x} + \left[ B_1 D + B_2 (1 - D) \right] \bar{u} \]  

(3.22)

\[ + \left[ (A_1 - A_2) x_0 + (B_1 - B_2) u_0 \right] \bar{d} \]

Defining

\[ A_0 = A_1 D + A_2 (1 - D) \]  

(3.23)

\[ B_0 = B_1 D + B_2 (1 - D) \]  

(3.24)

\[ E = (A_1 - A_2) x_0 + (B_1 - B_2) u_0 \]  

(3.25)

the dc equation can be expressed as

\[ 0 = A_0 x_0 + B_0 u_0 \]  

(3.26)

and the ac equation as

\[ \dot{x} = A_0 x + B_0 u + E \dot{d} \]  

(3.27)

We now have in our disposal an overall strategy for approximating a switch-mode converter as a linear system. This strategy is depicted in the flow graph [Mit1] as shown in Figure 3.3.
3.3 Control-law considerations

Since Eq. (3.27) is linear the Laplace transformation can be applied resulting in

\[
\tilde{X}(s) = (sI - A_0)^{-1}B_0 \tilde{U}(s) + (sI - A_0)^{-1}E \tilde{d}(s)
\]  

(3.28)

A set of small-signal transfer functions, thus, the control-to-states and the source-to-states can be derived. The transfer function matrices are given by

\[
G_{vd}(s) = \left. \frac{\tilde{X}(s)}{\tilde{d}(s)} \right|_{\tilde{U}(s)=0} = (sI - A_0)^{-1}E
\]  

(3.29)
\[ G_{eg}(s) = \frac{X(s)}{U(s)} \left|_{\tilde{u}(s) = 0} \right. = \left(sI - A_0\right)^{-1} B_0 \] (3.30)

Nevertheless, in most of the cases \( d \) is a function of \( x \), feedback control, and \( u \), feed-forward control. Therefore, the control law can be defined as the equation that describes the dependency of \( \tilde{d}(s) \) on \( \tilde{X}(s) \) and \( \tilde{U}(s) \).

The control law is not necessarily linear and in some cases the duty factor is by design or a consequence of a feedback control, inversely proportional to the input voltage. Consequently, the control-law is nonlinear and it must be linearised using small-signal approximations [Mit1]. The nonlinear term \( \frac{1}{u_1} \) is first written as

\[
\frac{1}{u_1} = \frac{1}{u_{10} + \tilde{u}_1} = \frac{1}{u_{10} \left(1 + \frac{\tilde{u}_1}{u_{10}}\right)}
\]

(3.31)

In order to eliminate the small-signal term from the numerator, the term that includes a small-signal state must be multiplied and divided with its conjugate. Thus,

\[
\frac{1}{u_{10} \left(1 + \frac{\tilde{u}_1}{u_{10}}\right)} \approx \frac{1}{u_{10} \left(1 - \frac{\tilde{u}_1}{u_{10}}\right)}
\]

(3.32)

where terms with products of \( \tilde{u}_1 \) have been neglected.

In some cases the control is frequency dependent, as in cases of the resonant and the quasi-resonant topologies, resulting to a more complicated expression. Consequently, the linearised general control law can be expressed as

\[
\tilde{d}(s) = F^T \tilde{X}(s) + Q^T \tilde{U}(s) + S^T \tilde{F}(s)
\]

(3.33)
The matrix $F^T$ expresses the dependency of the control law on the states of the system. Similarly, $Q^T$ and $S^T$ couples the source vector and the frequency vector respectively with the control law.

Combining Eq. (3.33) with the Laplace transform of Eq. (3.27) results in

$$\ddot{X}(s) = \left[ s I - A_0 - E F^T \right]^{-1} \left[ E S^T \right] \ddot{F}(s)$$  \hspace{1cm} (3.34)

The control-to-state transfer function matrix, when the switching frequency is the control parameter, can be expressed as

$$G_{vd}(s) = \left. \frac{\ddot{X}(s)}{\ddot{F}(s)} \right|_{U(s)=0} = \left[ s I - A_0 - E F^T \right]^{-1} \left[ E S^T \right]$$  \hspace{1cm} (3.35)

Similarly, the general transfer function matrix relating the state variables with source variables can be expressed as

$$\ddot{X}(s) = \left[ s I - A_0 - E F^T \right]^{-1} \left[ B_0 + E Q^T \right] \ddot{U}(s)$$  \hspace{1cm} (3.36)

The source-to-state transfer function matrix is given by

$$G_{vg}(s) = \left. \frac{\ddot{X}(s)}{\ddot{U}(s)} \right|_{\ddot{F}(s)=0} = \left[ s I - A_0 - E F^T \right]^{-1} \left[ B_0 + E Q^T \right]$$  \hspace{1cm} (3.37)
4 SINGLE-ACTIVE BRIDGE TOPOLOGY

4.1 Introduction

In the present Chapter the dynamic behaviour of the Single-Active Bridge (SAB) is thoroughly studied. Key expressions relating the critical parameters as the average current of the inductor and duty ratios for both controllable switches and diodes are derived from the steady-state analysis of the topology operating in the Continuous-Conduction Mode (CCM). Small-signal models for both CCM and Discontinuous-Conduction Mode (DCM) are derived by using the half-period symmetry assumption. The half-period symmetry can be applied since the power delivered by the source to the load during the positive half-cycle is equal to the power delivered during the negative half-cycle. Firstly, the small-signal model of the SAB topology is derived analytically and secondly the losses are added to the circuit resulting in a new small-signal model. Since the SAB topology operates at DCM, the small-signal models for the ideal SAB and the non-ideal converter are derived. Furthermore, the small-signal transfer functions are used in order to study the dynamic behaviour of the system, converter and the control configuration, due to small-signal perturbations.

The present chapter is organised as follows. First in Section 4.2 the steady state analysis of the converter is presented. Key equations and key issues are presented. In Section 4.3 the state-space averaged of the ideal converter is derived and the equivalent circuits and coefficient matrices are derived in Sections 4.4 and 4.5. The dynamic small-signal model of the ideal converter is derived in Section 4.6 and key transfer functions are presented. The steady-state dc transfer function is derived in Section 4.7. Additionally, in Section 4.8 the state-spaced averaged and the dynamic small-signal model of the converter is derived including the losses of the converter. Since the converter is operating at DCM the state-space averaging model and the dynamic small-signal model for both the ideal and the non ideal converter are presented in Section 4.9. Furthermore, the oscillations occurred during the discontinuous-time interval is thoroughly discussed and examined in Section 4.11. Finally, in Section 4.12 different control strategies are proposed and thoroughly discussed. The controller of the converter is presented as well and the dynamic behaviour of the system is examined.
4.2 Introduction to the single-active bridge topology

The single-active bridge was first introduced in [Dem1]. A half-bridge is employed on the primary side and a full-bridge diode rectifier on the secondary side. Thus, conventional control strategies such as Pulse-Width Modulation (PWM) and Phase-Shift (PSH) control cannot be applied when only two controllable switches are used. Phase-shifting the control signals of the two switches would only result in a short-circuit of the dc-link. In order to control the power flow from the primary to the secondary, frequency modulation or duty-cycle modulation can instead be employed. Various control strategies are proposed and thoroughly discussed in the present chapter.

The inductor current is a quasi triangular waveform and the voltage applied on the primary side of the power transformer has a square waveform. The transformer parasitics will influence the operation of the converters, especially the winding capacitance and the magnetising inductance. As is discussed in this chapter significant oscillations are taking place during the discontinuous-time interval. The winding capacitance and the magnetising inductance are the major contributors of the resonant circuit. On the other hand, the leakage inductance is used as the main energy storage element. A high value of the leakage inductance will have an impact on the dynamic behaviour of the converter. An increase in $L_\alpha$ will call for a decrease in duty cycle $D$.

The SAB topology can operate in DCM. At light loads the SAB operates at DCM but at nominal load and at heavy loads operates at CCM.

Since SAB is a buck-derived topology only step-down operation is possible.

The commutation sequence starts by assuming that the inductor current is flowing in the negative direction i.e. through the devices $D_{4+}$, $D_{3s}$ and $D_{5s2}$ as shown in Figure 4.2(a).

Note that the voltage across the inductor is positive, which means that the derivative of the current is positive as shown in Figure 4.4. While the current is flowing through diode $D_{4+}$, the voltage across the switch $T_{4+}$ is zero. The transistor $T_{4+}$ is switched on at zero voltage and eventually the current reverses and starts flowing through the transistor and through the diodes $D_{3s}$ and $D_{5s2}$ as illustrated in Figure 4.2(b) and Figure 4.2(c). At a certain instant the transistor $T_{4+}$ is turned off and the energy stored in the inductance is transferred to the snubber capacitors by resonance. The snubber capacitor, which is placed across $T_{4+}$, will take over the current, and the transistor turn-off occurs under ZVS conditions. Simultaneously, the snubber capacitor, which is connected across the transistor $T_{4-}$, will be discharged and will eventually force the diode $D_{4-}$ to be forward-biased.
Therefore, the inductor current starts flowing through the diode $D_{A-}$ and diodes $D_{s1}$ and $D_{s4}$, as shown in Figure 4.2(a).

Similarly, the transistor $T_{A-}$ is turned on at ZVS as shown in Figure 4.2(b) and Figure 4.2(c), and the commutation sequence will be repeated as above.

Figure 4.1: Modes of operation for the SAB converter when $V_{dc}$ is positive
Figure 4.2: Modes of operation for the SAB topology when $V_{dc}$ is negative

4.2.1 Steady-state analysis

The steady-state analysis of the SAB is devoted only for the CCM operation of the converter. Key expressions relating the average inductor current and the duty ratio for both the controllable switches and the diodes are derived. Since the average current is expressed in terms of the duty ratios, which are inversely proportional to the switching period, steady-state analysis of the topology in the DCM is unnecessary. The results from the steady-state analysis of the converter are used in order to derive the small-signal model of the topology. When the SAB topology operates in the continuous-conduction mode two operating modes are of interest due to symmetry conditions as shown in Figure 4.4.
The two modes of operation can be defined as

- Mode 1: The two voltage sources have different polarity
- Mode 2: The two voltage sources have the same polarity

During Mode 1 the inductor current can be expressed as

**Mode 1**: \(0 \leq \omega t \leq \omega t_1\)

\[i_{L_{\omega}}(\omega t) = \frac{V_{dc} + V_0}{\omega L_{\sigma}} (\omega t_1) + i_{L_{\omega}}(0)\]  \hspace{1cm} (4.1)

**Mode 2**: \(\omega t_1 \leq \omega t \leq \omega t_2\)

\[i_{L_{\omega}}(\omega t) = \frac{V_{dc} - V_0}{\omega L_{\sigma}} (\omega t_2 - \omega t_1) + i_{L_{\omega}}(\omega t_1)\]  \hspace{1cm} (4.2)

Due to symmetry conditions, at the end of the half cycle, Figure 4.4,

\[i_{L_{\omega}}(0) = -i_{L_{\omega}}(\omega t_2)\]  \hspace{1cm} (4.3)

Hence, the complete current waveform can be obtained by solving Eqs. (4.1), (4.2), and (4.3)

\[i_{L_{\omega}}(0) = -\frac{V_{dc}}{2 \omega L_{\sigma}} \left[ \left( 1 - M \right) \omega t_2 + \left( 1 + M \right) \omega t_1 \right]\]  \hspace{1cm} (4.4)

Where

\[M = \frac{V_0}{V_{dc}}\]  \hspace{1cm} (4.5)

is the conversion ratio.

During Mode 1, the diode \(D_{A+}\) is in the on state and during Mode 2 the inductor current will commute to \(T_{A+}\). Therefore, Eq. (4.1) and (4.2) can be expressed as
\[ i_{L_a}(\omega t) = \frac{V_{dc} + V_0}{\omega L_{\sigma}} (\omega t_D) + i_{L_a}(0) \]  
\( (4.6) \)

\[ i_{L_a}(\omega t) = \frac{V_{dc} - V_0}{\omega L_{\sigma}} (\omega t_{T_a}) + i_{L_a}(\omega t_D) \]  
\( (4.7) \)

Where \( t_{D_{a}} \) and \( t_{T_{a}} \) are the conduction times for the diode and the controllable switch respectively.

As a result, the Eq. (4.4) can be expressed in terms of the duty ratios of the two devices.

Thus,

\[ i_{L_a}(0) = -\frac{V_{dc} T_{a}}{2 L_{\sigma}} \left[ \left( 1 - M \right) d_{T_{a}} + \left( 1 + M \right) d_{D_{a}} \right] \]  
\( (4.8) \)

The value of \( i_{L_a}(0) \) can also be found from Eq. (4.6). Since \( i_{L_a} = 0 \) for \( t = t_D \), Eq. (4.6) yields

\[ i_{L_a}(0) = -\frac{V_{dc} + V_0}{\omega L_{\sigma}} (\omega t_D) \]  
\( (4.9) \)

By equating the right hand sides of the Eq. (4.8) and (4.9) the duty ratio of the diode can be expressed in terms of the duty ratio of the transistor as stated in Eq. (4.10).

\[ d_{D_{a}} = \frac{(1 - M)}{(1 + M)} d_{T_{a}} \]  
\( (4.10) \)

Equation (4.10) clarifies the volt-seconds balance of the inductor and is a limiting factor for the duty ratio. Thus, the inductors volt-seconds balance is fulfilled in a half period.

4.2.2 Soft-switching boundaries

In order to achieve soft-switching in a wide operating area the soft-switching conditions have to be fulfilled, see Appendix B. By assuming lossless circuit elements and from energy balance considerations the following expressions must be true for a complete resonant transition.
\[ \frac{1}{2} L_\sigma \left(i_{L_\sigma}(0)\right)^2 \geq \frac{1}{2} \left(2 C_s \right) \left(2 V_{dc}\right)^2 \]  

(4.11)

Thus

\[ \left|i_{L_\sigma}(0)\right| \geq \frac{2 V_{dc}}{Z_0} \]  

(4.12)

Where

\[ Z_0 = \sqrt{\frac{L_\sigma}{2 C_s}} \]  

(4.13)

Combining Eq. (4.9) and (4.12) the duty ratio constraint of the diode can be expressed as

\[ d_{D_{+}} \geq \frac{2 L_\sigma}{Z_0 T_s \left(1 + M\right)} \]  

(4.14)

From Figure 4.3 and by the symmetry considerations the mean value of the absolute value of the current over a half period can be expressed as

\[ \langle i_{L_\sigma} \rangle = \frac{i_{L_\sigma}}{2} \left(d_{D_{+}} + d_{T_{+}}\right) \]  

(4.15)

This quantity will be called average inductor current below.

The peak value of the inductor current when \( T_{A+} \) is turned off can be calculated from Eq.(4.7) as

\[ \hat{i}_{L_\sigma} = \frac{V_{dc}}{L_\sigma} \frac{T_s}{L_\sigma} \left(1 - M\right) d_{T_{+}} \]  

(4.16)

Substituting Eq. (4.16) into Eq. (4.15) results in a new expression for the average inductor current

\[ \langle i_{L_\sigma} \rangle = \frac{V_{dc}}{2 L_\sigma} \frac{T_s}{L_\sigma} \frac{d_{T_{+}} \left(1 - M\right) d_{D_{+}} + d_{T_{+}}}{\left(d_{D_{+}} + d_{T_{+}}\right)} \]  

(4.17)
The duty ratio of the diode can be redefined using Eq. (4.17). Thus, the duty ratio of the diode is defined in terms of the transistor duty ratio as

\[
d_{D_{dc}} = \frac{2 L D}{V_{dc} T_s d_{T_{dc}} (1 - M)} - d_{T_{dc}}
\]  

(4.18)

Eq. (4.18) will be used later on in order to derive the small-signal control-to-output, and control-to-state transfer functions of the SAB converter.
4.3 State-space averaging and linearization

For any given switch condition the SAB topology can be described by a state coefficient matrix $A_N$ and a source coefficient matrix $B_N$, where, $N$ is the state index. In Figure 4.3 the operational behaviour of the single active bridge is illustrated and some information about the current and voltage waveforms is given. The primary voltage is a square waveform and the inductor current has a quasi triangular waveform. At heavy loads the inductor current is increased, compared with the light loads, causing additional conduction losses. Additionally, the ripple current through the filter capacitor is comparably high implying that a filter capacitor with high kvar rating has to be considered.

In the continuous-conduction mode four modes of operation are of interest [Dem1] and are illustrated in Figure 4.3. The conducting sequence of the switches, as well as the current and input voltage sign are given in Table 4.1.

![Figure 4.3: (a) The half-bridge single active bridge topology and (b) Modes of operation](image-url)
For a half-bridge SAB topology, as shown in Figure 4.3, two equivalent circuits are of interest. The switch consists of a transistor with an antiparallel diode. Consequently, the parallel connection of the transistor and diode is treated as a bi-directional switch. In this first approximation the high-frequency transformer is treated as ideal with a turns ratio equal to $n_{TF}$.

As mentioned in [Dem1], capacitive snubbers are required in order to achieve Zero-Voltage-Switching (ZVS) conditions. In Table 4.1, as presented in [Dem1], the voltage across the inductor is expressed by means of the output voltage $V_0$ and the input voltage, $V_{dc}$.

<table>
<thead>
<tr>
<th>Conducting device</th>
<th>Inductor current</th>
<th>Input voltage</th>
<th>$V_{Lσ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{A+}$</td>
<td>$-i_{Lσ}$</td>
<td>$+V_{dc}$</td>
<td>$V_{Lσ} = V_{dc} + V_0$</td>
</tr>
<tr>
<td>$T_{A+}$</td>
<td>$+i_{Lσ}$</td>
<td>$+V_{dc}$</td>
<td>$V_{Lσ} = V_{dc} - V_0$</td>
</tr>
<tr>
<td>$D_{A-}$</td>
<td>$+i_{Lσ}$</td>
<td>$-V_{dc}$</td>
<td>$V_{Lσ} = -V_{dc} - V_0$</td>
</tr>
<tr>
<td>$T_{A-}$</td>
<td>$-i_{Lσ}$</td>
<td>$-V_{dc}$</td>
<td>$V_{Lσ} = -V_{dc} + V_0$</td>
</tr>
</tbody>
</table>

*Table 4.1: The inductor voltage for the SAB topology*

A plethora of state-space averaged models have been derived in the past for various DC-DC converters. Most of the models are examining the behaviour of conventional DC-DC converters, i.e. the buck converter, the boost converter, and the buck-boost converter. The analysis of the conventional converters is quite simple due to the fact that they employ only two switches, a controllable switch and a diode. Additionally, the current through the inductor is always DC with an AC component. Consequently, when the converter is operating in the discontinuous-conduction mode the maximum number of modes are three, and is thoroughly studied in the literature [Mit1], [Mid1], [Bar1] and [Cůk1]. Cůk in [Cůk1] derives a state-space averaged model which is extended to converters with multi-structural topological modes. Nevertheless, it is evident to the author that multi-structural state-space averaged model presented in [Cůk1] is applicable only when the inductor current is DC with an AC component.
In order to derive the state-space model for the half-bridge SAB a step-by-step analysis must be used. Figure 4.4 illustrates the inductor current and the input voltage when the converter operates at CCM.

In order to derive a complete state-space averaged model for the SAB topology the duty ratios for all devices must be derived. This is due to duty-ratio constraints. Thus, when the controllable switch is turned-on the current is still flowing through the antiparallel diode. The inductor current will start flowing through the transistor only when the diode current reaches zero. Consequently, the duty ratio of the controllable switches can neither be directly controlled nor chosen. The duty ratios are dependent on the input and output voltage and the load.

As a result the duty ratios for every active and passive switch must be derived. When $T_{a+}$ is in the on-state, the on-duty ratio can be expressed as

$$d_{T_{a+}} = \frac{t_2 - t_1}{T_s}$$

(4.19)

Similarly when $D_{a+}$ is forward biased the on-duty ratio is given by

$$d_{D_{a+}} = \frac{0 - t_1}{T_s}$$

(4.20)

![Figure 4.4: The input voltage and the inductor current at CCM.](image-url)
In Appendix A, Table A.1, the switch conduction times are derived and as a result the duty ratios can be expressed as

\[
d_{D_{A+}} = \frac{1}{T_s} \left( \frac{L_\sigma \hat{I}_{S_1}}{V_{dc} + x_2} \right)
\]  
(4.21)

and

\[
d_{T_{A+}} = \frac{1}{T_s} \left( \frac{L_\sigma \hat{I}_{S_1}}{V_{dc} - x_2} \right)
\]  
(4.22)

The duty ratios for all devices are summarised in Table 4.2.

<table>
<thead>
<tr>
<th>Conducting device</th>
<th>Duty ratio</th>
<th>Coefficient Matrices</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_{A+})</td>
<td>(d_{D_{A+}} = \frac{1}{T_s} \left( \frac{L_\sigma \hat{I}<em>{S_1}}{V</em>{dc} + x_2} \right))</td>
<td>(A_{II}) and (B_{II})</td>
</tr>
<tr>
<td>(T_{A+})</td>
<td>(d_{T_{A+}} = \frac{1}{T_s} \left( \frac{L_\sigma \hat{I}<em>{S_1}}{V</em>{dc} - x_2} \right))</td>
<td>(A_I) and (B_I)</td>
</tr>
<tr>
<td>(D_{A-})</td>
<td>(d_{D_{A-}} = \frac{1}{T_s} \left( \frac{L_\sigma \hat{I}<em>{S_1}}{V</em>{dc} + x_2} \right))</td>
<td>(A_{IV}) and (B_{IV})</td>
</tr>
<tr>
<td>(T_{A-})</td>
<td>(d_{T_{A-}} = \frac{1}{T_s} \left( \frac{L_\sigma \hat{I}<em>{S_1}}{V</em>{dc} - x_2} \right))</td>
<td>(A_{III}) and (B_{III})</td>
</tr>
</tbody>
</table>

*Table 4.2: Coefficient matrices*

As shown in Table 4.1 two states are of interest due to the symmetry of the circuit [Mit2]. Thus, if the circuit operates in the Continuous-Conduction Mode (CCM), the equivalent circuits when \(T_{A+}\) and \(D_{A-}\) are conducting, respectively, are needed in order to completely describe the dynamic behaviour of SAB.
4.4 State equations when $T_{A+}$ is on and $T_{A-}$ is off

When the upper switch, $T_{A+}$ is turned on the equivalent circuit shown in Figure 4.5 is obtained.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{equivalent_circuit.png}
\caption{The equivalent circuit for the SAB when $T_{A+}$ is forward biased.}
\end{figure}

From Kirchoff's voltage law it is found that

$$V_{dc} = L_{\sigma} \frac{d}{dt} i_{L_{\sigma}} + v_C$$

(4.23)

By applying Kirchoff's Current Law the inductor current is derived in terms of the capacitor current and the load current. Thus,

$$i_{L_{\sigma}} = i_C + i_R$$

(4.24)

Knowing that

$$i_C = C \frac{d}{dt} v_C$$

(4.25)

and

$$i_R = \frac{v_C}{R}$$

(4.26)

Equation 4.24 can be rewritten as
The inductor current and the capacitor voltage are chosen as state-parameters. Accordingly,

\[
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} = \begin{bmatrix}
  i_{L_\sigma} \\
  v_C
\end{bmatrix}
\]  

(4.28)

Equation (4.29) is the state-space equation describing the SAB topology when the switch \( T_{A_+} \) is conducting the inductor current and the controllable switch \( T_{A_-} \) is turned off.

\[
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2
\end{bmatrix} = \begin{bmatrix}
  0 & -\frac{1}{L_\sigma} \\
  \frac{1}{C} & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} + \begin{bmatrix}
  \frac{1}{L_\sigma} \\
  0
\end{bmatrix} V_{dc}
\]  

(4.29)

The state-coefficient matrix \( A \), and the source-coefficient matrix \( B \) are identified as

\[
A_t = \begin{bmatrix}
  0 & -\frac{1}{L_\sigma} \\
  \frac{1}{C} & -\frac{1}{RC}
\end{bmatrix} \quad B_t = \begin{bmatrix}
  \frac{1}{L_\sigma} \\
  0
\end{bmatrix}
\]  

(4.30)

4.5 State equations when all controllable switches are turned off and \( D_{A_-} \) is forward biased

When \( D_{A_-} \) is forward biased and \( T_{A_+} \), and \( T_{A_-} \) are turned off the equivalent circuit presented in Figure 4.6 is obtained. Obtain that the polarity of the input voltage \( V_{dc} \) has been changed from \(+V_{dc}\) to \(-V_{dc}\). The inductor current \( i_{L_\sigma} \) however is still positive.
Figure 4.6: The equivalent circuit for the SAB when $T_a^-$ is forward biased.

The state-equations describing the circuit in Figure 4.6 can be defined as

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & -1/L \sigma \\
1/C & -1/RC
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
-1/L \sigma \\
0
\end{bmatrix} V_{dc}
$$

(4.31)

Where the state-coefficient matrix $A_2$ and the source-coefficient matrix $B_2$ are identified as

$$
A_2 = \begin{bmatrix}
0 & -1/L \sigma \\
1/C & -1/RC
\end{bmatrix} \quad B_2 = \begin{bmatrix}
-1/L \sigma \\
0
\end{bmatrix}
$$

(4.32)
4.6 State-space averaging and small-signal analysis

The continuous approximation of the two switched linear systems can be described, see Section 3.1, by the continuous-time state-space-averaged equation,

\[ \dot{x} = [A_1 \, d + A_2 \, (1 - d)] \, x + [B_1 \, d + B_2 \, (1 - d)] \, u \]  \quad (4.33)

As explained in Chapter 3 and from Eq. (3.13), and Eq. (3.14), the state-coefficient matrix \( A \), and the source-coefficient matrix \( B \) can be defined as

\[ A = A_1 \, d + A_2 \, (1 - d) \]  \quad (4.34)

\[ B = B_1 \, d + B_2 \, (1 - d) \]  \quad (4.35)

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{1}{L_\sigma} \\
\frac{1}{C} & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix} + \begin{bmatrix}
\frac{2 \, d - 1}{L_\sigma} \\
0
\end{bmatrix} V_{dc} \]  \quad (4.36)

A continuous but nonlinear system is described by Eq. (4.36). It is nonlinear because \( d \) can be a function of any combination of \( x_1, x_2 \) and \( u \).

Any continuous nonlinear system can be approximated as a linear system within the vicinity of its dc operating point. The switching frequency terms are effectively separated in the state-space-averaging model as explained in Chapter 3. As a result, in order to linearize Eq. (4.36) the dc terms must be separated from the signal frequency ac terms. Accordingly,

\[ x = x_0 + \tilde{x} \]  \quad (4.37)

\[ V_{dc} = V_{dc0} + \tilde{V}_{dc} \]  \quad (4.38)

\[ d = D + \tilde{d} \]  \quad (4.39)
Where \( x_0 \), \( V_{dc0} \), and \( D \) are the dc terms and \( \tilde{x} \), \( \tilde{V}_{dc} \) and \( \tilde{d} \) are the signal-frequency ac terms.

In order to define the small-signal model for the SAB topology the matrices \( A_0 \), \( B_0 \), and \( E \), must be defined as described in Chapter 3.

\[
A_0 = A_1 D + A_2 (1 - D) \tag{4.40}
\]

\[
B_0 = B_1 D + B_2 (1 - D) \tag{4.41}
\]

\[
E = (A_1 - A_2)x_0 + (B_1 - B_2)V_{dc0} \tag{4.42}
\]

The linear state-equation is derived from Eq. (3.27) and is given by

\[
\ddot{x} = A_0 x + B_0 \tilde{V}_{dc} + E \tilde{d} \tag{4.43}
\]

As mentioned in Section 3.3 and from Eq. (3.33) the duty ratio can be a function of any combination of the state variables. The duty ratio of the controllable switch is strongly dependent on the duty ratio of the diode. Thus, the duty ratio can be expressed as

\[
d = 1 - d_{D_s} \tag{4.44}
\]

The duty ratio is calculated by the insertion the duty ratio of the diode \( d_{D_s} \) as given by Table 4.2 into Eq. (4.44) and is given by

\[
d = 1 - \frac{L_{ir} x_{1pk} f_s}{\left( V_{dc} + x_2 \right)} \tag{4.45}
\]

By insertion of the Equations (4.37), (4.38), and (4.39) into Eq. (4.45), the linearized expression of the duty ratio is given by
\[ d = \frac{L_\sigma x_{1pk0} f_{s0}}{(V_{dc0} + x_{20})^2} x_2 - \frac{L_\sigma f_{s0}}{(V_{dc0} + x_{20})} x_1 + \frac{L_\sigma x_{1pk0} f_{s0}}{(V_{dc0} + x_{20})^2} V_{dc} \]

\[ - \frac{L_\sigma x_{1pk0}}{(V_{dc0} + x_{20})} f_s \]

The state-, source- and control-coefficient matrices as defined by Eq. (3.33) are given by

\[ F^T = \begin{bmatrix} - \frac{L_\sigma f_{s0}}{(V_{dc0} + x_{20})} & \frac{L_\sigma x_{1pk0} f_{s0}}{(V_{dc0} + x_{20})^2} \end{bmatrix} \]

\[ Q^T = \begin{bmatrix} + \frac{L_\sigma x_{1pk0} f_{s0}}{(V_{dc0} + x_{20})^2} \end{bmatrix} \]

\[ S^T = \begin{bmatrix} - \frac{L_\sigma x_{1pk0}}{(V_{dc0} + x_{20})} & 0 \end{bmatrix} \]

Combining equations (4.40), (4.41), (4.42) and (4.43) the linear state-space-averaged model for the SAB can be written as

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L_\sigma} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{2D - 1}{L_\sigma} \\ 0 \end{bmatrix} V_{dc} + \begin{bmatrix} \frac{2V_{dc0}}{L_\sigma} \\ 0 \end{bmatrix} d \]

Where the coefficient matrices \( A_0 \), \( B_0 \), and \( E \) are given by

\[ A_0 = \begin{bmatrix} 0 & -\frac{1}{L_\sigma} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, B_0 = \begin{bmatrix} \frac{2D - 1}{L_\sigma} \\ 0 \end{bmatrix}, E = \begin{bmatrix} \frac{2V_{dc0}}{L_\sigma} \\ 0 \end{bmatrix} \]

The ac and the dc-equations of the system can be derived and by substituting Equations (4.37), (4.38) and (4.39) into Eq. (4.43) yields
\begin{align*}
\dot{x}_1 &= -\frac{1}{L_\sigma} x_2 + \frac{2 D - 1}{L_\sigma} V_{dc} + \frac{2 V_{dc0}}{L_\sigma} d \\
\dot{x}_2 &= \frac{1}{C} x_1 - \frac{1}{RC} x_2 
\end{align*}
\tag{4.52}

and

\begin{align*}
\dot{x}_1 &= -\frac{1}{L_\sigma} x_2 + \frac{2 D - 1}{L_\sigma} V_{dc0} \\
\dot{x}_2 &= \frac{1}{C} x_1 - \frac{1}{RC} x_2 
\end{align*}
\tag{4.53}

Similarly, and knowing that the derivative of a dc quantity is zero, i.e. \( x_{10} = x_{20} = 0 \), the dc equations are given by

\begin{align*}
0 &= -\frac{1}{L_\sigma} x_{20} + \frac{2 D - 1}{L_\sigma} V_{dc0} \\
0 &= \frac{1}{C} x_{10} - \frac{1}{RC} x_{20}
\end{align*}
\tag{4.54, 4.55}

Solving Eq. (4.54) the dc term of the state-variable \( x_{20} \) is given by

\[ x_{20} = (2 D - 1) V_{dc0} \]
\tag{4.56}

Similarly, the dc term of \( x_1 \) can be expressed in terms of \( x_{20} \). Solving Eq. (4.55) yields

\[ x_{10} = \frac{x_{20}}{R} \]
\tag{4.57}

From Eqs. (4.56) and (4.57), \( x_{10} \) is expressed in terms of the input voltage as given by Eq. (4.58).

\[ x_{10} = \frac{(2 D - 1)}{R} V_{dc0} \]
\tag{4.58}
4.6.1 The small-signal control-to-output transfer function

In order to describe the dynamics of the SAB topology, i.e. by means of poles and zeroes, the control-to-output transfer function must be derived. By examining Eq. (4.46) it is obvious that the duty ratio depends on the input voltage, the peak inductor current, the output voltage and the switching frequency. Consequently, the duty ratio introduces a state-feedback and feedforward to the system.

The output of interest is the capacitor voltage, which is given by the state variable $x_2$. Equations (4.47), (4.48), and (4.49) do not contain any information concerning the dependence on small variations of the duty ratio around its steady state value. In order to retain this information Eq.(4.36) is used in combination with the state-space averaged model. Similar methodology has been used by Sun et al [Sun1] concerning dc-dc converters as Buck and Boost converters operating at DCM.

As thoroughly explained in Chapter 3, the state-space averaging model of the SAB can be redefine as

$$A_{alt} = d_{T_s} A_1 + d_{D_s} A_2$$

and

$$B_{alt} = d_{T_s} B_1 + d_{D_s} B_2$$

The state-coefficient matrices $A_1$ and $A_2$, and the source-coefficient matrices $B_1$ and $B_2$ are given by Eq. (4.30) and Eq. (4.32). As presented in section 4.2.1 the duty ratio of the diode can be expressed in terms of the input voltage the inductor current and the transistor duty ratio. The duty ratio of the diode $D_{A-}$ is given by Eq. (4.18). By insertion of the duty ratio of the diode as given by Eq. (4.18) into Eq. (4.59) and, from Eq. (4.30) and Eq. (4.32) the averaged state-coefficient matrix $A_{alt}$, and the averaged source-coefficient matrix $B_{alt}$ are given by

$$A_{alt} = \begin{bmatrix}
0 & -\frac{2 x_1}{V_{dc} - x_2} d_{T_s} T_s \\
\frac{2 L_{dc} x_1}{C (V_{dc} - x_2) d_{T_s} T_s} & -\frac{2 L_{dc} x_1}{R C (V_{dc} - x_2) d_{T_s} T_s}
\end{bmatrix}$$

(4.60)
Linearization of the system can be achieved by inserting Eq. (4.37), Eq. (4.38) and Eq. (4.39) into Eq. (4.60) and Eq. (4.61). From Appendix C yields,

\[
B_{alt} = \begin{bmatrix}
\frac{2 d_T}{L} - \frac{2 x_1}{(V_{dc} - x_2) d_T, T_s}
\end{bmatrix}
\]

(4.61)

\[
A_{alt0} = \begin{bmatrix}
-2 x_1 \left( V_{dc0} + x_2 \right) D_{Tc}, T_s (V_{dc0} - x_2) & \vdots & -4 V_{dc0} x_1 \left( V_{dc0} - x_2 \right)^2 D_{Tc}, T_s \\
\vdots & \cdots & \vdots
\end{bmatrix}
\]

\[
-2 L_x x_1 \left( V_{dc0} - x_2 \right) D_{Tc}, T_s & -2 L_x x_1 \left( V_{dc0} - x_2 \right) D_{Tc}, T_s \left( \frac{1}{R} \right)
\]

(4.62)

\[
B_{alt0} = \begin{bmatrix}
\frac{2 d_T}{L} - \frac{4 x_1 x_2}{(V_{dc0} - x_2)^2 D_{Tc}, T_s}
\end{bmatrix}
\]

(4.62)

\[
E_{alt0} = \begin{bmatrix}
\frac{2 V_{dc0}}{L} + \frac{2 x_1 \left( V_{dc0} + x_2 \right)}{(V_{dc0} - x_2)^2 D_{Tc}, T_s}
\end{bmatrix}
\]

In Equation (4.62) the small-signal state-, source- and control-coefficient matrices are given. Consequently, and as explained in Section 3.3, the control-to-output transfer function can be expressed as
The determinant $\Delta_{alt}$ of the system contributes with poles which are determining the dynamic behaviour of the system, i.e. if the system is stable or unstable.

\[
\Delta_{alt} = s^2 + s \left( \frac{2}{D_{T_{dc}}} \frac{V_{dc0} + x_{20}}{T_s (V_{dc0} - x_{20})} \right) + \frac{2L_\sigma x_{10}}{C (V_{dc0} - x_{20}) D_{T_{dc}} T_s} \left( \frac{1}{R} \right) + \\
+ \frac{4 x_{10} L_\sigma (V_{dc0} + x_{20})}{D_{T_{dc}}^2 T_s^2 (V_{dc0} - x_{20})^2 C} \left( \frac{1}{R} \right) + \frac{8 L_\sigma V_{dc0} x_{10}^2}{C (V_{dc0} - x_{20})^2 D_{T_{dc}}^2 T_s^2} 
\]

(4.64)

The control-to-output transfer function can be written as

\[
G_{cy}(s) = \frac{x_2(s)}{d(s)} = \frac{N_{G_{cy}}(s)}{\Delta_{alt}(s)} 
\]

(4.65)

Where

\[
N_{G_{cy}} = \left( \frac{2L_\sigma x_{10}}{C (V_{dc0} - x_{20}) D_{T_{dc}} T_s} \right) \left( \frac{2 x_{10} (V_{dc0} + x_{20})}{D_{T_{dc}}^2 T_s (V_{dc0} - x_{20})} + \frac{2 V_{dc0}}{L_\sigma} \right) 
\]

(4.66)
In Figure 4.7 the Bode diagram for the SAB is presented. By examining Figure 4.7 a crossover-frequency of $\omega_c = 1.43 \times 10^5 \left( \frac{rad}{s} \right)$ is obtained. The crossover-frequency of a system is the frequency at which the open-loop gain is 0dB. On the other hand the gain margin of a system is the inverse of the open loop-magnitude at the frequency where the phase angle is $180^\circ$. The gain margin of the SAB is approximately equal to -70dB. The control-to-output transfer function of the SAB behaves as a first-order system.

In order to determine the phase-margin of the system the Nichols chart is plotted as shown in Figure 4.8. The phase margin of the system is the difference angle $180^\circ - \phi_c$, where $\phi_c$ is the phase of the transfer function at the crossover frequency. A $60^\circ$ phase margin can be obtained by examining Figure 4.8, which implies that the system is stable.

The poles of the system are examined and their corresponding natural responses are shown in Figure 4.9. Both poles are placed on the Left Half Plane (LHP) implying that the system is stable.
In general, poles further to the left in the s-plane are associated with natural signals that decay faster than those associated with poles closer to the imaginary axis. Figure 4.9 clarifies the presence of a dominant pole, i.e. the pole dominates the early part of a step response. The pole that is placed closer to the imaginary axis is the primary contributor later on.

![Nichols Chart](image1)

**Figure 4.8:** The Nichols chart for the SAB topology.

![Pole-Zero Map](image2)

**Figure 4.9:** The zeroes-pole map for the SAB topology.

The poles of the SAB converter have no imaginary part. This implies that the damping of the system is high. In cases where the poles of the system have imaginary part the step response
of the system is oscillatory. The impact of the real part and the imaginary part of the poles is explained.

Complex poles can be defined in terms of their real and imaginary parts traditionally referred to

\[ s = -\sigma \pm j \omega_d \quad (4.67) \]

Equation (4.67) states that the pole has a negative real part. Additionally, complex poles always appear in complex conjugate pairs. The denominator corresponding to a complex pair can be expressed as

\[ \alpha(s) = (s + \sigma - j \omega_d) (s + \sigma + j \omega_d) = (s + \sigma)^2 + \omega_d^2 \quad (4.68) \]

The transfer function of a system can be expressed as

\[ H(s) = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2} \quad (4.69) \]

In order to find the correspondence between the parameters of the denominator in Eq. (4.69) and the parameters in Eq. (4.67) the coefficients of the same order are compared with each other resulting in

\[ \sigma = \zeta \omega_n \text{ and } \omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (4.70) \]

where \( \zeta \) is the damping ratio and \( \omega_n \) is the undamped natural frequency. The poles of the system examined are located at a radius \( \omega_n \) in the s-plane and at angle \( \theta = \arcsin(\zeta) \) as shown in Figure 4.10.
Therefore, the damping ratio reflects the level of damping as a function of the critical damping value where the poles become real. When damping ratio is zero there is no damping and the damped natural frequency equals the undamped natural frequency. For very low damping ratios the impulse response of the system is oscillatory, while for large damping, $\xi = 1$, the response shows no oscillation. As a result, the change in pole location will influence the time response of the system.

Notice that the negative real part of the pole, $-\sigma$, determines the decay rate of an exponential envelope that multiplies the sinusoid as shown in Figure 4.11. If $\sigma < 0$ and the pole is located in the RHP, the natural response of the system will grow with time so, as defined earlier the system is said to be unstable. Note that if $\sigma = 0$ the natural response neither grows nor decays, so stability is open debate.
The step response of the control-to-output transfer function is a well-damped stable response and is illustrated in Figure 4.12. As predicted by the Bode diagram in Figure 4.7 and from Figure 4.9 the step response will increase monotonically without oscillations.
Similarly, the impulse response of the control-to-output transfer function, as shown in Figure 4.13, is decaying with the pole which is placed on the far end of the zeroes-pole diagram to dominate the early part of the time history.

![Impulse Response](image)

**Figure 4.13:** The impulse response of the control-to-output transfer function.

### 4.6.2 The small-signal control-to-state transfer function

In order to derive the control-to-state transfer function Eq. (4.71) is used resulting to the expression that describes the inductor current sensitivity to small variations of the duty ratio around its steady-state. Consequently, the control-to-state transfer function is derived as shown in Eq. (4.71).

Thus,

$$G_{ss} (s) = \left. \frac{\dot{x}_i (s)}{d (s)} \right|_{V_{dc} = 0} = \frac{N_{G_m} (s)}{\Delta_{alt} (s)}$$

(4.71)

Where,

$$N_{G_m} = \left( s + \frac{2 L_{\sigma} x_{10}}{C (V_{dc0} - x_{20}) D_{f_{ss}} T_s} \left( \frac{1}{R} \right) \right) \left( \frac{2 x_{10} (V_{dc0} + x_{20})}{D_{f_{ss}} T_s (V_{dc0} - x_{20})} + \frac{2 V_{dc0}}{L_{\sigma}} \right)$$

(4.72)
A Bode diagram of Equation (4.71) is plotted in Figure 4.14 (a). At low frequencies the inductor current is amplified. The amplification ratio is increased at frequencies between $100 \leq \omega \leq 10^5$.

Figure 4.14: The control-to-state transfer function, (a) Bode diagram and (b) Step response
For perturbations with a frequency higher than the cross over frequency the inductor current is
damped and the impact of the perturbations on the state is negligible.

The poles and zeroes map of the control-to-state transfer function is illustrated in Figure 4.15.

\[ \text{Figure 4.15: The poles and zeroes map for the control-to-state transfer function.} \]

### 4.6.3 The small-signal source-to-output transfer function

In a power converter, the input source might experience a disturbance, such as line frequency
harmonics or external noise. The source-to-output transfer function should be low at the
disturbance frequencies to reject source noise. This transfer function is often called the audio
susceptibility since it provides a measure of the effect of ac noise at the output. Audio
susceptibility provides a transient measure corresponding to line regulation.

From Equation (3.37) and setting \( f_x = 0 \) the small-signal source-to-output transfer function
is derived as
The small-signal source-to-output transfer function for the examined topology is given by

\[
G_{\text{eq}}(s) = \left. \frac{x_2(s)}{V_{dc}(s)} \right|_{f_s(s)} = \left[ sI - A_0 - EF^T \right]^{-1} \left[ B_0 + EQ^T \right] \tag{4.73}
\]

The small-signal source-to-output transfer function for the examined topology is given by

\[
\frac{\ddot{x}_2(s)}{V_{dc}(s)} = \frac{1}{C} \left( \frac{2V_{dc0} x_{1p0} f_{s0}}{(V_{dc0} - x_{20})^2} + \frac{(2D - 1)}{L_\sigma} \right) \frac{\Delta(s)}{s} \tag{4.74}
\]

where \( \Delta \) is defined by Equation (4.75). Thus,

\[
\Delta(s) = s^2 + s \left( \frac{1}{RC} + \frac{2V_{dc0} f_{s0}}{(V_{dc0} + x_{20})} \right) - \frac{2V_{dc0} x_{1p0} f_{s0}}{C(V_{dc0} + x_{20})^2}
\]

\[+ \frac{2V_{dc0} f_{s0}}{RC(V_{dc0} + x_{20})} + \frac{1}{L_\sigma C} \tag{4.75}\]

The matrix \( sI - A_0 - EF^T \) is given by

\[
\left[ sI - A_0 - EF^T \right]^{-1} = \frac{1}{\Delta(s)} \begin{bmatrix}
\frac{1}{RC} & -\frac{1}{L_\sigma} - \frac{2V_{dc0} x_{1p0} f_{s0}}{V_{dc0} + x_{20}} \\
\frac{1}{C} & s + \frac{2V_{dc0} f_{s0}}{V_{dc0} + x_{20}}
\end{bmatrix} \tag{4.76}
\]

Additionally the matrices \( EQ^T \) and \( B_0 + EQ^T \) can be written as
\[
\begin{bmatrix}
\frac{2 V_{dc0} x_{1pk0} f_s0}{(V_{dc0} - x_{20})^2} \\
0
\end{bmatrix}
\]

The matrix containing both the source-to-output and the source-to-state transfer functions is given by

\[
\left[ B_0 + E Q^T \right] = \begin{bmatrix}
\frac{2 V_{dc0} x_{1pk0} f_s0}{(V_{dc0} - x_{20})^2} + \frac{(2 D - 1)}{L_\sigma} \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{-x_1(s)}{V_{dc}(s)} \\
\frac{-x_2(s)}{V_{dc}(s)}
\end{bmatrix} = \frac{1}{\Delta(s)} \begin{bmatrix}
\left( s + \frac{1}{RC} \right) \left( \frac{2 V_{dc0} x_{1pk0} f_s0}{(V_{dc0} + x_{20})^2} + \frac{2 D - 1}{L_\sigma} \right) \\
\frac{1}{C} \left( \frac{2 V_{dc0} x_{1pk0} f_s0}{(V_{dc0} + x_{20})^2} + \frac{2 D - 1}{L_\sigma} \right)
\end{bmatrix}
\]

\[ (4.79) \]
Figure 4.16: The small-signal source-to-output transfer function, (a) The Bode diagram, (b) Step response and (c) Poles and zeroes map
The Bode diagram of the magnitude and the phase of the source-to-output transfer function is shown in Figure 4.16. At low frequencies, the disturbances in the input source are slightly damped and for frequencies higher than $10^3 \left[ rad \right]$, the disturbances are heavily attenuated.

Due to the fact that the examined topology is a buck-derived topology, the transformer has a negligible effect on the source-to-output transfer function, other than introduction of the turns ratio. In case when the turns ratio of the transformer is not considered to be one, the small-signal source-to-output transfer function should be multiplied by the transformer turns ratio.

### 4.6.4 The small-signal source-to-state transfer function

The small-signal source-to-state transfer function can be derived by Equation (4.80) and is given by

$$\frac{- x_1(s)}{V_{dc}(s)} = \frac{1}{C} \left( s + \frac{1}{R C} \right) \left( \frac{2 V_{dc0} x_{1p0} f_{s0}}{V_{dc0} + x_{20}} \right)^2 + \frac{2 D - 1}{L_{\sigma}} \right) \Delta (s) \right)$$

The source-to-state transfer function describes the dynamic behaviour and/or the dynamic impact of the small-signal line perturbations on the inductor current. In the case of the buck converter small-signal line perturbations can cause severe oscillations resulting in a system which is not stable. Small-signal line perturbations occur frequently and in cases where a six-pulse rectifier is used to achieve a dc input voltage a 300 Hz ripple is present. Additionally, the start-up of different equipments connected to the grid can cause voltage transients which can affect the dynamic behaviour of the converter. Disregarding the dynamic behaviour of the converter due to line perturbations can result in a system which is stable only in steady state.

By examining Figure 4.17(c) the source-to-state transfer function contains a LHP zero as in the case of the control-to-state transfer function. Distortions and/or noise are rejected in the whole frequency range resulting in a well-damped and stable dynamic behaviour.
Figure 4.17: The small-signal input-to-state transfer function, (a) Bode diagram, (b) Step response, and (c) Poles and zeroes map.
4.6.5 The small-signal switching frequency-to-state transfer functions

The small-signal switching frequency-to-state transfer functions can be obtained from Eq. (3.35). By setting the input voltage variations equal to zero the switching frequency-to-output transfer function can be defined as stated in Eq. (4.81).

\[
G_{sf}(s) = \frac{-x_2(s)}{f(s)} = -\left[ sI - A_0 - EF^T \right]^{-1} ES \tag{4.81}
\]

The determinant \( \Delta \) of the system is given by Eq.(4.75). Consequently, the switching frequency-to-output transfer function is defined as

\[
-\frac{x_2(s)}{f(s)} = -2V_{dc0}x_{pl0} \left( \frac{1}{C(V_{dc0} + x_{20})} \right) \frac{\Delta(s)}{\Delta(s)} \tag{4.82}
\]

Similarly, the matrix \( [ES^T] \) is given by

\[
[ES^T] = \begin{bmatrix}
-2V_{dc0}x_{pl0} & 0 \\
\left( V_{dc0} + x_{20} \right) & 0 \\
0 & 0
\end{bmatrix} \tag{4.83}
\]

Additionally the switching frequency-to-state transfer function is derived as

\[
-\frac{x_1(s)}{f(s)} = -2V_{dc0}x_{pl0} \left( \frac{1}{(V_{dc0} + x_{20})} \right) \left( s + \frac{1}{RC} \right) \frac{\Delta(s)}{\Delta(s)} \tag{4.84}
\]

The small-signal switching frequency-to-output and switching frequency-to-state transfer functions are plotted in Figure 4.18 and Figure 4.19 respectively. As indicated by the minus sign in both transfer functions, an increase in the switching frequency results in reduction of
the output voltage and to inductor current. Similarly, when the switching frequency is decreased both of the states will increase in magnitude.

![Bode Diagram](image)

![Step Response](image)

**Figure 4.18:** Bode diagram, and, step response for the switching frequency-to-output transfer function

The dynamic behaviour of the SAB converter in switching frequency perturbations has been examined in order to verify the robustness of the converter. Namely, and as indicated from Figure 4.19 and Figure 4.18, both voltage and current decreases monotonically when the switching frequency rapidly increases. This property is favourable and is used during the start-up of the converter. Specifically, when a start-up command is given to the converter the filter capacitor is not charged. Normally, in three-phase controllable rectifiers the charge-up of the capacitor is achieved through the antiparallel diodes of the rectifier.
Figure 4.19: Bode diagram, and, step response for the switching frequency-to-state transfer function.

In the case of the SAB converter this is not feasible. In order to avoid the inrush current, which can be ten times higher than the nominal peak current during start-up, the SAB converter starts with a switching frequency which is twice the nominal switching frequency. The inrush current is not eliminated but is reduced avoiding transistor and/or diodes failures. When the output voltage reaches the threshold level the switching frequency is rapidly changed to the nominal frequency. This step in switching frequency could cause oscillations or instabilities in the system with fatal for the converter results. However, and by examining the small-signal transfer functions, switching frequency-to-output and, switching frequency-to-state, the system behaves stably.
4.7 Steady-state dc transfer function

The steady-state equation can be obtained from Eq. (4.50) by setting all the perturbation terms and their time derivatives to zero. Therefore, the steady-state equation can be expressed as

\[ AX + BV_{dc} = 0 \]  
(4.85)

The output can be described in terms of the state variables as

\[
\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix} = C \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} + D_{\text{matrix}} \begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
\]  
(4.86)

In case where the output is the load voltage \( V_0 \), Equations (4.87) and (4.88) express \( V_0 \) in terms of the state variables during \( d T_s \) and \((1 - d)T_s \) respectively. Parameter \( T_s \) is defined as the switching period expressed in seconds.

\[ V_0 = C_1 \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} \]  
(4.87)

and

\[ V_0 = C_2 \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} \]  
(4.88)

The averaged expression for the output of the converter is described

\[ C = \left[ C_1 D + C_2 (1 - D) \right] \]  
(4.89)

Using Eq. (4.85) and (4.89), the steady-state dc voltage transfer function can be derived as

\[ \frac{V_0}{V_{dc}} = - C A^{-1} B \]  
(4.90)
The output is given by

\[ C = C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \]  

(4.91)

Combining Equations (4.36) (4.90) and (4.91), the steady-state dc transfer function for the half-bridge SAB is given by,

\[ \frac{V_0}{V_{dc}} = (2D - 1) \]  

(4.92)
4.8 The influence of the ESR of the capacitor, and the semiconductor losses.

The impact of the Equivalent Series Resistance (ESR) of the capacitor on the dynamic behaviour of SAB topology is examined. Additionally, the resistive and the semiconductor losses are included in the model.

4.8.1 State equations when $T_{A^+}$ is on and $T_{A^-}$ is off

The equivalent circuit for the SAB when $T_{A^+}$ is on is shown in Figure 4.20.

![Figure 4.20: The equivalent circuit for the SAB when $T_{A^+}$ is forward biased including ESR and semiconductor losses](image)

Equation (4.93) is the state-equation describing the SAB topology when the switch $T_{A^+}$ is on and $T_{A^-}$ is off.

$$\begin{bmatrix}
\cdot \\
\cdot
\end{bmatrix}
= 
\begin{bmatrix}
- \frac{1}{L_{\sigma}} \left( r_L + \frac{R r_C}{R + r_C} \right) & - \frac{1}{L_{\sigma}} \left( \frac{R}{R + r_C} \right) \\
\frac{R}{C (R + r_C)} & - \frac{1}{C (R + r_C)}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ 
\begin{bmatrix}
\frac{1}{L_{\sigma}} \\
0
\end{bmatrix}
V_{dc}
$$ (4.93)

The state-coefficient matrix $A$, and the source-coefficient matrix $B$ are defined as
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\[
A_1 = \begin{bmatrix}
-\frac{1}{L_\sigma} \left( \frac{r_L + \frac{R r_C}{R + r_C}}{R + r_C} \right) & -\frac{1}{L_\sigma} \left( \frac{R}{R + r_C} \right) \\
\frac{R}{C(R + r_C)} & -\frac{1}{C(R + r_C)}
\end{bmatrix}
\]

\[
B_1 = \begin{bmatrix}
\frac{1}{L_\sigma} \\
0
\end{bmatrix}
\]  

(4.94)

4.8.2 State equations when \( D_{A-} \) is on and \( T_{d+} \) is off

When \( D_{A-} \) is forward biased and \( T_{d+} \) is turned off the equivalent circuit is as shown in Figure 4.21.

![Figure 4.21: The equivalent circuit for the SAB when \( D_{A-} \) is forward biased including ESR and semiconductor losses.](image)

The state-equations describing the circuit in Figure 4.21 can be defined as

\[
\begin{bmatrix}
\cdot \\
\cdot 
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{L_\sigma} \left( \frac{r_L + \frac{R r_C}{R + r_C}}{R + r_C} \right) & -\frac{1}{L_\sigma} \left( \frac{R}{R + r_C} \right) \\
\frac{R}{C(R + r_C)} & -\frac{1}{C(R + r_C)}
\end{bmatrix} \begin{bmatrix}
\cdot \\
\cdot 
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L_\sigma} \\
0
\end{bmatrix} V_{dc}
\]  

(4.95)

The coefficient matrices are as stated in Eq. (4.96)

\[
A_2 = \begin{bmatrix}
-\frac{1}{L_\sigma} \left( \frac{r_L + \frac{R r_C}{R + r_C}}{R + r_C} \right) & -\frac{1}{L_\sigma} \left( \frac{R}{R + r_C} \right) \\
\frac{R}{C(R + r_C)} & -\frac{1}{C(R + r_C)}
\end{bmatrix}
\]

\[B_2 = \begin{bmatrix}
\frac{1}{L_\sigma} \\
0
\end{bmatrix}
\]  

(4.96)
4.8.3 State-space averaging and small-signal analysis

The continuous approximation of the two switched linear systems can be described by the continuous-time state-space-averaged equation, and is generally given by Eq. (3.11). From Equations (3.12), (3.13), (3.14) and from Eqs. (4.94) and (4.96) the averaged state-space model for the SAB is given by Eq. (4.97).

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 
\end{bmatrix} = 
\begin{bmatrix}
-\frac{1}{L_\sigma} \left( \frac{r_L + R r_C}{R + r_C} \right) & -\frac{1}{L_\sigma} \left( \frac{R}{R + r_C} \right) \\
\frac{R}{C \left( R + r_C \right)} & -\frac{1}{C \left( R + r_C \right)}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + 
\begin{bmatrix}
\frac{2 d - 1}{L_\sigma} \\
0
\end{bmatrix} V_{dc}
\] (4.97)

Applying Eqs.(3.22), (3.23), (3.24), (3.25) and from the coefficient matrices as stated in Eqs.(4.94) and (4.96) the small-signal model is expressed as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 
\end{bmatrix} = 
\begin{bmatrix}
-\frac{1}{L_\sigma} \left( \frac{r_L + R r_C}{R + r_C} \right) & -\frac{1}{L_\sigma} \left( \frac{R}{R + r_C} \right) \\
\frac{R}{C \left( R + r_C \right)} & -\frac{1}{C \left( R + r_C \right)}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + 
\begin{bmatrix}
\frac{2 D - 1}{L_\sigma} \\
0
\end{bmatrix} V_{dc}
\] (4.98)

\[
\begin{bmatrix}
2 V_{dc0} \\
\frac{L_\sigma}{-d}
\end{bmatrix}
\]

In order to simplify the mathematical analysis of the problem the terms including the ESR and the semiconductor losses are replaced with constants and are defined according to
\[
\gamma = r_L + \frac{R r_c}{R + r_c}
\]
\[
\kappa = \frac{R}{R + r_c}
\]
\[
\lambda = \frac{1}{R + r_c}
\]  

(4.99)

In order to define the small-signal transfer functions for the SAB topology, the impact of the ESR and the semiconductor losses has to be taken under consideration and the duty ratio of both controllable and passive switches are redefined. Consequently, the duty ratio for \( D_{A_{+}} \) can be expressed as

\[
d_{D_{A_{+}}} = \frac{x_{1, pk} L_{\sigma} f_s}{x_{1, pk} \gamma + x_2 k + V_{dc}}
\]  

(4.100)

Additionally, and by insertion of Eq. (4.100) into Eq. (4.44), the duty ratio of \( T_{A_{+}} \) yields

\[
d = 1 - \frac{x_{1, pk} L_{\sigma} f_s}{x_{1, pk} \gamma + x_2 k + V_{dc}}
\]  

(4.101)

The small-signal expression for the duty ratio can be defined as

\[
\bar{d} = \frac{L_{\sigma} f_{s0}}{x_{1, pk0} \gamma + V_{dc0} x_{20} k} \left( \frac{x_{1, pk0}}{x_{1, pk0} \gamma + V_{dc0} x_{20} k} - 1 \right) x_1 \\
+ \frac{L_{\sigma} x_{1, pk0} f_{s0} \kappa}{\left( x_{1, pk0} \gamma + V_{dc0} + x_{20} \kappa \right)^2} x_2 + \frac{L_{\sigma} f_{s0} x_{1, pk0}}{\left( x_{1, pk0} \gamma + V_{dc0} + x_{20} \kappa \right)^2} V_{dc} \\
- \frac{L_{\sigma} x_{1, pk0} f_{s0} \kappa}{\left( x_{1, pk0} \gamma + V_{dc0} + x_{20} \kappa \right)^2} f_s
\]  

(4.102)

The state, source and control-coefficient matrices as defined by Eq. (3.33) are given by

\[
F^T(s) = \begin{bmatrix} F_1 & F_2 \end{bmatrix}
\]  

(4.103)
Where

\[
F_1 = \frac{L_\sigma f_s \gamma}{(x_{1p0} \gamma + V_{dc0} + x_{20} \kappa)} \left(\frac{x_{1p0}}{x_{1p0} \gamma + V_{dc0} + x_{20} \kappa} - 1\right) \tag{4.104}
\]

\[
F_2 = \frac{L_\sigma x_{1p0} f_s \gamma}{(x_{1p0} \gamma + V_{dc0} + x_{20} \kappa)^2} \tag{4.105}
\]

\[
Q^T = \left[\frac{L_\sigma f_s x_{1p0}}{(x_{1p0} \gamma + V_{dc0} + x_{20} \kappa)^2}\right] \tag{4.106}
\]

\[
S^T = \left[\frac{L_\sigma x_{1p0}}{(x_{1p0} \gamma + V_{dc0} + x_{20} \kappa)}\right] \tag{4.107}
\]

The output of the converter can be expressed in terms of the state variables as indicated in Eq. (4.86). Therefore, the coefficient matrix can be defined as

\[
C_{ESR} = C_{ESR1} = C_{ESR2} = \begin{bmatrix} 1 & 0 \\ r_c & \kappa \end{bmatrix} \tag{4.108}
\]

4.8.4 The small-signal source-to-output and source-to-state transfer functions

Adding the converter losses and the ESR of the output capacitor the source-to-output and the source-to-state transfer functions can be derived as stated in Chapter 3 and Eq. (3.37). Thus,

\[
G_{yEESR} (s) = \left. \frac{-x_2(s)}{V_{dc}(s)} \right|_{f_s=0} = \frac{1}{\Delta_{ESR}(s)} \left[ N_{yEESR} (s) \right] \tag{4.109}
\]

Where

\[
N_{yEESR} (s) = \left( r_c \kappa \left( s + \frac{\lambda}{C} \right) + \frac{\kappa^2}{C} \right) \left( \frac{2D-1}{L_\sigma} + \frac{2 V_{dc0} x_{1p0} f_s \gamma}{(x_{1p0} \gamma + V_{dc0} + x_{20} \kappa)^2} \right) \tag{4.110}
\]
The determinant of the system is given by

\[
\Delta_{\text{ESR}}(s) = s^2 + s \left( \frac{\lambda + \gamma}{C} - \frac{2 V_{dc0} f_{s0}}{V_{dc0} + \kappa x_{20} + \gamma x_{1pk0}} \right) \left( \frac{x_{1p0} \gamma}{V_{dc0} + \kappa x_{20} + \gamma x_{1pk0}} - 1 \right)
\]

\[+ \frac{k^2}{L \sigma C} - \frac{2 V_{dc0} x_{1pk0} f_{s0} \kappa^2}{C \left( V_{dc0} + \kappa x_{20} + \gamma x_{1pk0} \right)^2} \tag{4.111}\]

In Figure 4.22 the source-to-output transfer function has been plotted. The Bode diagram, the step response and the poles and zeroes map are illustrated. Comparing Figure 4.22 with the case where the impact of the ESR and the losses was negligible, it is clear that a LHP zero has been introduced resulting in a deviation in the phase angle and a reduction of the settling time. Additionally, perturbations around the steady-state of the input voltage are damped with higher damping coefficient.
Figure 4.22: Small-signal source-to-output transfer function, (a) Bode diagram, (b) Step response, and (c) Poles and zeroes map
4.8.5 The small-signal source-to-state transfer function

Similarly, the small-signal source-to-state transfer function can be derived from Eq. (3.37) resulting to

\[
G_{\text{vgs,ESR}}(s) = \left. \frac{x_1(s)}{V_{dc}(s)} \right|_{j \omega \epsilon_0} = \frac{1}{\Delta_{\text{ESR}}(s)} \left[ N_{G_{\text{vgs}}}(s) \right] \tag{4.112}
\]

The term \( N_{G_{\text{vgs}}}(s) \) is defined by Eq. (4.113) as

\[
N_{G_{\text{vgs}}}(s) = \left( s + \frac{\lambda}{C} \right) \left( \frac{2D-1}{L} + \frac{2V_{dc0} x_{1pk0} f_{x0}}{(x_{1pk0} + V_{dc0})^2 + (x_{20} \kappa)^2} \right) \tag{4.113}
\]

Figure 4.23 illustrates the source-to-state transfer function taking into consideration the ESR and losses. By including ESR and losses, an increase in the phase magnitude compared with the ideal case is observed and as in the case of the source-to-output transfer function, which means that higher damping ratio is obtained. On the contrary, the magnitude is unchanged. Thus, the ESR and the losses of the circuit will only affect the phase and not the magnitude of the small-signal control-to-output transfer function.
Figure 4.23: The small-signal source-to-state transfer function, (a) Bode diagram, (b) Step response and (c) Poles and zeroes map.
4.8.6 Control-to-output transfer function including ESR and converter losses

Both ESR and converter losses will definitely have an impact on the control-to-output transfer function, namely, both in the Bode diagram and in the step and impulse response. The same procedure as explained in Section 4.6.1 will be followed in order to obtain both control-to-output and control-to-state transfer functions. Equation (4.18) is redefined as

\[
d_{D,s} = \frac{2 x_1 f_s L_\sigma}{d_{T,s} (V_{dc} - \kappa x_2 - \lambda x_1)} - d_{r,s}
\]  

(4.114)

The state-space averaged model of the SAB converter corresponds to

\[
x = A_{altESR} x + B_{altESR} V_{dc}
\]  

(4.115)

Consequently, the coefficient matrices can be defined as

\[
A_{altESR} = \begin{bmatrix} \frac{2 f_s x_1 \gamma}{d_{T,s} (V_{dc} - \kappa x_2 - \lambda x_1)} & \frac{2 f_s x_1 \kappa}{d_{T,s} (V_{dc} - \kappa x_2 - \lambda x_1)} \\ \frac{2 f_s x_1 L_\sigma \kappa}{d_{T,s} C (V_{dc} - \kappa x_2 - \lambda x_1)} & \frac{2 f_s x_1 L_\sigma \lambda}{d_{T,s} C (V_{dc} - \kappa x_2 - \lambda x_1)} \end{bmatrix}
\]  

(4.116)

\[
B_{altESR} = \begin{bmatrix} \frac{2 d_{T,s}}{L_\sigma} - \frac{2 x_1 f_s}{d_{T,s} (V_{dc} - \kappa x_2 - \gamma x_1)} \\ 0 \end{bmatrix}
\]  

(4.117)

The small-signal model of the converter can be derived similarly as in the case of the ideal model. Thus, the small-signal state, source and control matrices are expressed as
$$A_{ahlESRO} = \Lambda \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$B_{ahlESRO} = \begin{bmatrix} \frac{4 x_{10} f_{s0} (\kappa x_{20} + \gamma x_{10})}{D(V_{dc0} - \kappa x_{20} - \gamma x_{10})^2} + \frac{2 D}{L_\sigma} \\ -\frac{2 x_{10} L_\sigma f_{s0} (\kappa x_{10} - \lambda x_{20})}{D \left(C V_{dc0} - \kappa x_{20} - \gamma x_{10}\right)^2} \end{bmatrix}$$

$$E_{ahlESRO} = \begin{bmatrix} \frac{2 x_{10} f_{s0} (V_{dc0} + \gamma x_{10} + \kappa x_{20})}{D^2 \left(V_{dc0} - \kappa x_{20} - \gamma x_{10}\right)} + \frac{2 V_{dc0}}{L_\sigma} \\ -\frac{2 x_{10} L_\sigma f_{s0} (\kappa x_{10} - \lambda x_{20})}{D^2 \left(C V_{dc0} - \kappa x_{20} - \gamma x_{10}\right)} \end{bmatrix}$$

Where

$$A_{11} = -\left(\gamma x_{10} + \left(V_{dc0} + \kappa x_{20} + \gamma x_{10}\right) \left(\frac{\gamma x_{10}}{V_{dc0} - \kappa x_{20} - \gamma x_{10}} + 1\right)\right)$$

$$A_{12} = -\frac{2 x_{10} \kappa V_{dc0}}{V_{dc0} - \kappa x_{20} - \gamma x_{10}}$$

$$A_{21} = \frac{L_\sigma}{C} \left(\kappa x_{10} - \lambda x_{20}\right) \left(1 + \frac{\gamma x_{10}}{V_{dc0} - \kappa x_{20} - \gamma x_{10}}\right) + \kappa x_{10}$$

$$A_{22} = \frac{x_{10} \kappa}{C} \left(\frac{\kappa (\kappa x_{10} - \lambda x_{20})}{V_{dc0} - \kappa x_{20} - \gamma x_{10}} - \lambda\right)$$

$$\Lambda = \frac{2 f_s}{D \left(V_{dc0} - \kappa x_{20} - \gamma x_{10}\right)}$$

From Equation (4.63) the control-to-state transfer function corresponds to
The determinant of the system can be derived as

$$\Delta_{\text{altESR}}(s) = s^2 - s \Lambda \left( A_{11} + A_{22} \right) + \Lambda^2 A_{11} A_{22} - \Lambda^2 A_{12} A_{21}$$

(4.122)

Similarly,

$$N_{\text{vdESR}}(s) = r_e \kappa \left( \left( s - \Lambda A_{22} \right) E_{11} + \Lambda A_{12} E_{21} \right) + \kappa \left( \Lambda A_{21} E_{11} + E_{21} \left( s - \Lambda A_{11} \right) \right)$$

(4.123)

The terms $E_{11}$ and $E_{21}$ are the coefficients of the first and the second row of the matrix $E_{\text{altESR}}$. Examining Equation (4.123), a zero term is present compared with the control-to-output transfer function for the ideal converter. This phenomenon is well known, and a plethora of publications are treating the presence of the zero term in the control-to-output transfer function. In the case of the buck converter, the introduction of the ESR will result in a RHP zero which has to be taken under consideration in the design of the controller.

Figure 4.24: Control-to-output transfer function. Bode diagram including ESR and losses.
Figure 4.25: Zeroes-poles map of the control-to-output transfer function. ESR and losses are included.

### 4.8.7 Control-to-state transfer function including ESR and converter losses

The control-to-state transfer function of the non-ideal SAB converter can be defined as

\[
G_{adESR} (s) = \frac{\frac{\frac{\Delta}{s}}{\Delta}}{\frac{\Delta}{s}} = \frac{N_{adESR} (s)}{\Delta_{adESR} (s)}
\]

The numerator of the control-to-state transfer function corresponds to

\[
N_{adESR} (s) = (s - \Lambda A_{11}) E_{11} + \Lambda A_{12} E_{12}
\]
As shown in Figure 4.26 the impact of the ESR and the converter losses is minor. This is obvious if Figure 4.26 is compared with Figure 4.14 which corresponds to the control-to-state transfer function for the ideal SAB converter.

Figure 4.26: Control-to-state transfer function. Bode diagram and zeroes-pole map. Both converter losses and the ESR of the capacitor are included.
4.8.8 Steady-state dc transfer function

When the converter losses and the ESR of the capacitor are negligible, the steady-state dc transfer function of SAB is derived in Section 4.7. The same procedure is followed in order to derive the steady-state dc transfer function when the converter losses and the ESR of the capacitor are included. As a result, and from Equations (4.90), (4.97) and (4.108), the steady state dc transfer function is given by

$$\frac{V_0}{V_{dc}} = \frac{R}{r_L + R} \left( 2D - 1 \right)$$

(4.126)

In Equation (4.126), the term containing the load resistance and the converter losses is referred to as the correction factor [Moh1]. It is worth noticing that the ESR of the output capacitor has no impact on the steady-state dc transfer function of the converter. Since the average current flowing through the capacitor is zero at steady-state the product $\langle i_c \rangle r_c$ should be $\langle i_c \rangle r_c = 0$. On the contrary, the load resistance and the converter losses will affect the conversion ratio of the converter. As expected, the higher the losses are, the lower the conversion ratio is.
4.9 Discontinuous-conduction mode

Modelling the dynamic behaviour of switch mode power converters demands a detailed derivation of the converters behaviour in both the continuous-conduction mode and in the discontinuous-conduction mode. In the case of the SAB converter the current will always return to zero resulting to characteristics similar to that of the discontinuous-conduction mode of operation. This behaviour is often referred to as boundary continuous operation mode as defined by Chen et al in [Che1].

In the discontinuous-conduction mode an additional mode of operation is present. Thus, none of the switching devices conduct and as a result the inductor current is zero. On the other hand, the snubber capacitors are still charged and connected in series with the inductor forming a resonant tank. At the beginning of the discontinuous-time interval the snubber capacitors will start to oscillate with the inductor and the current is flowing through the load. When the diode rectifier becomes reversed biased, a new resonance mode begins. Consequently, the current starts to flow through the inductor and will charge and/or discharge the winding capacitance of the high-frequency transformer. The two resonance modes are not considered in the small-signal model of the converter due to their high frequency nature but they will be treated separately. Therefore, the equivalent circuit for the discontinuous-time interval is illustrated in Figure 4.27.

![Figure 4.27: The equivalent circuit of the SAB topology during the discontinuous-time interval.](image)

The coefficient matrices for the discontinuous-time interval can be defined as
Similarly, the discontinuous-time interval can be expressed as the remaining time in a half-period and is given by

\[ t_{\text{dsc}} = T_s - \left( t_{T_{\text{c}}} + t_{D_{\text{c}}} \right) \quad (4.128) \]

By substituting the device duty ratios into Eq. (4.128), and from Table 4.2 it is found that

\[ d_{\text{dsc}} = 1 - \left( 2 \frac{V_{dc}}{L_{\sigma}} \frac{x_{1pk}}{f_s} \right) \left( \frac{V_{dc}^2 - x_2^2}{V_{dc}^2 - x_2^2} \right) \quad (4.129) \]

In the case of the discontinuous-conduction mode the coefficient matrices \( A_1 \) and \( B_1 \) apply a fraction \( d \) of a half cycle as in the case of the continuous conduction mode, but the duty ratio of \( A_2 \) and \( B_2 \) is \( d_{\text{dsc}} - d \). Similarly, the matrices \( A_3 \) and \( B_3 \) apply \( (1 - d_{\text{dsc}}) \) of the time, [Mit1]. The averaged state-space system for the SAB topology operating in the discontinuous-conduction mode is therefore given by

\[
A_{\text{dsc0}} = dA_1 + (d_{\text{dsc}} - d)A_2 + (1 - d_{\text{dsc}})A_3 \quad (4.130)
\]

\[
B_{\text{dsc0}} = dB_1 + (d_{\text{dsc}} - d)B_2 + (1 - d_{\text{dsc}})B_3 \quad (4.131)
\]

However, the ESR of the capacitor and the converter losses are of importance, and as a result, the coefficient matrices for the discontinuous-time interval, and the duty ratio corresponds to

\[
A_{\text{ESR}} = \begin{bmatrix} 0 & 0 \\ 0 & - \frac{\lambda}{C} \end{bmatrix} \quad (4.132)
\]

\[
d_{\text{dscESR}} = 1 - \left( \frac{2V_{dc}}{L_{\sigma}} \frac{x_{1pk}}{f_s} \right) \left( \frac{2V_{dc}L_{\sigma}x_{1pk}f_s}{\left( V_{dc}^2 - x_2^2 \right) \left( \gamma \right) - x_{1pk}x_2} \right) \quad (4.133)
\]

The state-space averaged model for the discontinuous-conduction mode can be written as
\[
\begin{bmatrix}
\cdot \\
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
\frac{\gamma}{L_\sigma} d_{\text{discESR}} - \frac{\kappa}{L_\sigma} d_{\text{discESR}} \\
\frac{\kappa}{C} d_{\text{discESR}} & -\frac{\lambda}{C}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
2 d - d_{\text{discESR}} \\
0
\end{bmatrix} V_{dc}
\]

(4.134)

However, the derivation of the small-signal model of Eq.(4.134) is a very tedious mathematical procedure, i.e. Eq. (4.37), Eq. (4.38), and Eq.(4.39) have to be inserted in Eqs. (4.133), and (4.134) resulting to a complicated mathematical model. Additionally, the resulted small-signal model does not contain any information concerning the dependence on small-signal perturbations around the steady-state value of the duty ratio. As a result the procedure described in Section 4.6.1 should be followed.

The coefficient matrix \( A_i \) and the source matrix \( B_i \), as given by Eq. (4.94), apply a fraction \( d_{T_{\text{e}}} \) of the half cycle as in the case of the continuous-conduction mode. Similarly, the duty ratio of \( A_2 \) and \( B_2 \), as shown in Eq. (4.96), is \( d_{D_{\text{e}}} \). The duty ratio of the discontinuous-time interval can be expressed as \( 1 - d_{T_{\text{e}}} - d_{D_{\text{e}}} \) and corresponds to the matrices \( A_3 \) and \( B_3 \) as defined in Eq. (4.127). By substituting the corresponding duty ratios into Eq. (4.130) and Eq. (4.131), the state-space averaged model for the discontinuous-conduction mode yields

\[
\begin{bmatrix}
\cdot \\
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
\frac{\gamma}{L_\sigma} \left( d_{T_{\text{e}}} + d_{D_{\text{e}}} \right) - \frac{\kappa}{L_\sigma} \left( d_{T_{\text{e}}} + d_{D_{\text{e}}} \right) \\
\frac{\kappa}{C} \left( d_{T_{\text{e}}} + d_{D_{\text{e}}} \right) & -\frac{\lambda}{C}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
\frac{d_{T_{\text{e}}} - d_{D_{\text{e}}}}{L_\sigma} \\
0
\end{bmatrix} V_{dc}
\]

(4.135)

The expression for the duty ratio of the diode is given by Eq. (4.114). Therefore, the small-signal coefficient, source, and control matrices are defined as
\[ A_{\text{dscESR0}} = \begin{bmatrix} \Lambda A_{\text{dsc11}} & \Lambda A_{\text{dsc12}} \\ \Lambda A_{\text{dsc21}} & A_{\text{dsc22}} \end{bmatrix} \]

\[ B_{\text{dscESR0}} = \begin{bmatrix} B_{\text{dsc11}} \\ B_{\text{dsc21}} \end{bmatrix} \quad (4.136) \]

\[ E_{\text{dscESR0}} = \begin{bmatrix} E_{\text{dsc11}} \\ E_{\text{dsc21}} \end{bmatrix} \]

where \( \Lambda \) is given by Eq. (4.120).

The elements of the state matrix are defined as

\[ A_{\text{dsc11}} = -\left( \gamma x_{10} + \left( V_{dc0} + \kappa x_{20} + \gamma x_{10} \right) \left( \frac{\gamma x_{10}}{V_{dc0} - \kappa x_{20} - \gamma x_{10}} + 1 \right) \right) \]

\[ A_{\text{dsc12}} = -\frac{2 x_{10} \kappa V_{dco}}{V_{dc0} - \kappa x_{20} - \gamma x_{10}} \]

\[ A_{\text{dsc21}} = \frac{x_{10} L_{\sigma} \kappa}{C} \left( 2 + \frac{\kappa \gamma x_{10}}{V_{dc0} - \kappa x_{20} - \gamma x_{10}} \right) \]

\[ A_{\text{dsc22}} = \Lambda \frac{x_{10}^2 L_{\sigma} \kappa^2}{C \left( V_{dc0} - \kappa x_{20} - \gamma x_{10} \right)} - \frac{\lambda}{C} \]

Similarly, the elements of the source matrix are given by

\[ B_{\text{dsc11}} = \frac{4 x_{10} f_{s0} \left( \kappa x_{20} + \gamma x_{10} \right)}{D \left( V_{dc0} - \kappa x_{20} - \gamma x_{10} \right)^2} + \frac{2 D}{L_{\sigma}} \]

\[ B_{\text{dsc21}} = -\frac{2 f_{s0} x_{10}^2 L_{\sigma} \kappa}{D C \left( V_{dc0} - \kappa x_{20} - \gamma x_{10} \right)^2} \]
Additionally, the elements of the control matrix are given by

\[
E_{dc11} = \frac{2 x_{10} f_{s0}}{D^2} \left( V_{dc0} + \gamma x_{10} + \kappa x_{20} \right) + \frac{2 V_{dc0}}{L_{\sigma}}
\]

\[
E_{dc21} = -\frac{2 f_{s0} x_{10}^2 L_{\sigma} \kappa}{D^2 C \left( V_{dc0} - \kappa x_{20} - \gamma x_{10} \right)}
\]

(4.139)
4.9.1 Control-to-output transfer function operating in the discontinuous conduction mode including ESR and the converter losses

The control-to-output transfer function for the discontinuous-conduction mode corresponds to

\[ G_{vdDSCesr}(s) = \frac{N_{vdDSCesr}(s)}{\Delta_{DSCesr}(s)} (4.140) \]

Where,

\[ N_{vdDSRe,sr}(s) = r_e \kappa \Lambda A_{dc21} E_{dc11} + \kappa E_{dc21} (s - \Lambda A_{dc11}) (4.141) \]

\[ \Delta_{DSCesr}(s) = s^2 - s (A_{dc22} + \Lambda A_{dc11}) + \Lambda (A_{dc11} A_{dc22} + \Lambda A_{dc12} A_{dc21}) (4.142) \]

In Figure 4.29 the control-to-output transfer function, and the zeroes-poles map of the SAB converter operating in the discontinuous conduction mode is shown. The ESR of the capacitor and the converter losses are included in the model.

Figure 4.28: Control-to-output transfer function for the discontinuous conduction mode
In order to obtain the control-to-output transfer function for the ideal converter configuration operating in the discontinuous-conduction mode the ESR of the capacitor and the converter losses are set to zero. By comparing Figure 4.28 and Figure 4.29 with Figure 4.30, a large deviation in the phase angle is achieved by the addition of the ESR and the converter losses. Additionally, the ideal converter exhibits a RHP zero. The RHP zero is sometimes referred to as a nonminimum-phase zero [Fra1]. A step in the reference value of the nonminimum-phase system will cause rapid deviations in the phase angle compared with the minimum-phase system. A minimum-phase system, defines a system with all the poles and zeroes placed in the LHP. The right half-plane zero has a tendency to destabilise wide-bandwidth feedback loops [Roh1]. Specifically, the presence of the RHP-zero during transient, the output of interest will rapidly change in the wrong direction and an increase in the hysteresis time. In this case the hysteresis time is defined as the time needed for the output of interest to reach the equilibrium value. With single-loop feedback systems with wide bandwidth, it is difficult to obtain adequate phase margin when a right half-plane zero is present.

Nevertheless, with the addition of the losses and the ESR, the RHP disappears and a LHP zero takes its place which is quite favourable from the converter control, and, regulation point of view.
Similarly, the control-to-state transfer function is defined as
\[ G_{\text{vsDSC}eN} (s) = \frac{x_1(s)}{d(s)} \bigg|_{\gamma_{\text{dc}} = 0} = \frac{N_{\text{vsDSC}eN} (s)}{\Lambda_{\text{DSC}eN} (s)} \]  \hspace{1cm} (4.143)

Where,

\[ N_{\text{vsDSC}eN} (s) = E_{\text{dc}1} (s - A_{\text{dc}22}) + \Lambda A_{\text{dc}12} E_{\text{dc}21} \]  \hspace{1cm} (4.144)

Figure 4.31: Control-to-state transfer function for the DCM including losses and ESR.
Figure 4.31 illustrates the control-to-state transfer function. When the ESR of the capacitor and the converter losses are negligible the control-to-state transfer function for the DCM can be obtained. Both the Bode diagram and the step response of the system are quite similar with the plots describing the non-ideal SAB converter. Consequently, the ESR of the capacitor and the converter losses has not significant impact on the dynamic behaviour of the control-to-state transfer function.

4.9.3 Source-to-output transfer function in the discontinuous-conduction mode including ESR and the converter losses

The source-to-output transfer function of the SAB converter operating at DCM can be derived as given by Eq. (4.145). The losses of the converter as well as the ESR of the capacitor are included in the model.

\[
G_{v_2SG_{esr}}(s) = \left. \frac{x_2(s)}{V_{dc}(s)} \right|_{d=0} = \frac{N_{v_2SG_{esr}}(s)}{\Lambda_{SG_{esr}}(s)} \tag{4.145}
\]

The numerator of the source-to-output transfer function corresponds to

\[
N_{v_2SG_{esr}}(s) = r_c \kappa \Lambda A_{SG_{esr}} B_{SG_{esr}} + \kappa B_{SG_{esr}} \left( s - \Lambda A_{SG_{esr}} \right) \tag{4.146}
\]

In Figure 4.32 the Bode diagram and the zeroes-pole diagram are shown. As shown in the figure small-signal variations in the input are expected to be damped at the output. At low frequencies the damping ratio is almost constant but in frequency range of \( f \geq 100 \frac{rad}{sec} \) the small signal variations will be damped with a linearly varying damping ratio.
Figure 4.32: Source-to-output transfer function. Bode diagram and the zeroes-poles diagram corresponding to the DCM. The ESR and the converter losses included.
4.9.4 Source-to-state transfer function in the discontinuous-conduction mode including ESR and the converter losses

The source-to-state transfer function describes the dynamic impact of the small-signal line perturbations on the state. Line perturbations can cause instabilities to the system and special attention should be given early in the design stage of the converter.

The impact of the converter losses and the ESR of the capacitor for the DCM are included in the small-signal model as given by Eq. (4.147).

\[
G_{sg\text{DC}esr} (s) = \frac{-x_1(s)}{V_{dc}(s)} \left|_{d=0} \right. = \frac{N_{sg\text{DC}esr}(s)}{A_{\text{DC}esr}(s)}
\]

(4.147)

Where, the numerator can be defined as

\[
N_{sg\text{DC}esr}(s) = B_{dcl11} (s - A_{dcl22}) + \Lambda A_{dcl2}
\]

(4.148)

Both the Bode diagram and the zeroes-poles map of the source-to-state transfer function is shown in Figure 4.33. Low frequency perturbations are damped but between the interval

\[
15 \left[ \frac{\text{rad}}{s} \right] \leq \omega \leq 300 \left[ \frac{\text{rad}}{\text{sec}} \right]
\]

the perturbations are amplified. At frequencies

\[
\omega \geq 2 \cdot 10^4 \left[ \frac{\text{rad}}{\text{sec}} \right]
\]

the perturbations around the steady-state of inductor current are damped heavily.

The system is a minimum-phase system with all the poles located in the LHP with zero imaginary parts. This implies that the system is neither unstable nor oscillatory.
4.10 Verification of the small-signal model

Simulations have been done in order to verify the small-signal model for the SAB topology. A perturbation with a certain frequency is added to the duty ratio and the perturbations at the input voltage are set to zero. The amplitude of the perturbations do not exceed 10% of the steady-state value. For a certain frequency the output voltage and the inductor current are

---

Figure 4.33: Source-to-state transfer function for the DCM.
damped or amplified as predicted by the small-signal transfer functions. Simulations are presented in Appendix E.

4.11 Oscillations in the discontinuous-conduction mode

As mentioned in Section 4.9, the major drawback of the discontinuous-conduction mode is the presence of oscillations taking place under the discontinuous-time interval. The snubber capacitors and the series inductor are connected in series creating a resonance tank. This resonant tank is used in order to assure the soft-commutation of the controllable switches at turn-off. However, during the discontinuous-time interval the snubber capacitors are fully charged to the dc-link voltage and the current through the inductor is zero. Consequently, the snubber capacitors will start to oscillate with the series inductor. Two resonance modes have been identified

- **Resonance mode 1:**

  Resonance mode 1 starts when the current through the antiparallel diode is zero and no other device of the half-bridge is triggered into the on-state. The snubber capacitor is fully charged and the inductor current is zero. Consequently, the capacitor and the inductor will start to oscillate. As a result, the current starts to flow through the snubber capacitor, forcing the voltage to drop in a certain level, and will charge the other snubber capacitor to the corresponding level, as ruled by the charge balance constraint. The current will flow through the inductor, and the load and the oscillation is damped. The equivalent circuit which describes mode 1 is presented in Figure 4.34(a)

- **Resonance mode 2:**

  When the resonance current reaches the zero level a second resonance mode is triggered. The diode rectifier is reversed biased disconnecting the load from the resonance circuit. The current starts to flow through the snubber capacitors, recharging and discharging them, through the inductor, and through the winding capacitance of the transformer. Due to the magnitude of the magnetising inductance, much bigger than the series inductance, the impact of the magnetising inductance to the resonance circuit appears in terms of a certain offset which follows the repetition rate of the magnetising current. The influence of the magnetising inductance is examined in a later stage. However, the load is not connected, the core losses, as
represented by a resistor connected in parallel with the inductor, damps the oscillations. The equivalent circuit of mode 2 corresponds to Figure 4.34(b).

![Equivalent Circuit](image)

*Figure 4.34: Oscillations during the discontinuous-time interval. (a) Mode 1, (b) Mode 2.*

Simulations verify the nature of the two resonance modes and are shown in Figure 4.35 and Figure 4.36. In Figure 4.35 the inductor current, the snubber voltage and, the snubber current are illustrated. For clarity, the oscillations are enlarged on the right hand side of the figure. As shown in the figure during the discontinuous-time interval the inductor is oscillating with the snubber capacitors but the oscillations are damped by the load. The peak value of the oscillating current is only approximately 0.5% of the peak value of the rated inductor current.
In both Figures the transformer is assumed to be ideal, thus having an infinite magnetizing inductance and a negligible winding capacitance. As illustrated in Figure 4.35 and Figure 4.36, the resonance mode 2 is absent which means that it depends on transformer parasitics and specifically on the winding capacitance of the transformer. Another interesting feature that is clarified from both figures is that the oscillating current does not involve any device from the primary half-bridge. However, it does involve the rectifier bridge introducing some additional losses.
Figure 4.36: Simulation results. The rectified current during the resonance mode assuming ideal transformer

Figure 4.37 corresponds to the configuration which includes the transformer parasitics. By taking under consideration the winding capacitance, and the magnetizing inductance of the transformer, the resonance mode 2 is present. Additionally the capacitance between the primary winding and the secondary winding is added to the model. The capacitance between the two windings can cause some severe EMI emissions but it can be minimised by adding electrostatic shields between the primary and the secondary winding. Electrostatic shields are not used in the transformer used in the prototype, and consequently the impact of the capacitance on the converter behaviour has been added.

By examining Figure 4.37 (a) and (b), two resonance modes are present. As in the case with the ideal transformer the devices placed on the primary half-bridge do not conduct any resonance current. Thus, the resonance current will not contribute to any additional losses in the primary bridge. On the other hand, the resonance current is flowing through the rectifier on the secondary side during mode 1 resulting in additional switching and conduction losses. Figure 4.37 (b) illustrates the current through the diode placed on the primary half-bridge. As expected from the analysis above no resonance current flows through the primary diodes. When the conversion ratio $M \geq 0.9$ the diodes on the primary side are forced to commutate the resonance current resulting to additional losses.
Figure 4.37: Oscillations during the discontinuous-time interval. The transformer parasitics are included.
Nevertheless, the diodes placed on the half-bridge will be part of the resonance circuit if the winding capacitance is sufficiently large to force the snubber voltage to drop to zero, forcing the diode to commutate the resonance current. In that case, excessive switching and, conduction losses will be added reducing even more the efficiency of the converter. The oscillations can be quite severe and can cause extensive EMI problems. Additional RC snubber circuits will not solve the problem, on the contrary, the turn-off behaviour of the transistors is degraded and the losses are shifted from the diode to the resistor. As a result, the efficiency of the converter will remain at the same level as before. Another approach to the problem is to treat the problem as a loaded-resonance circuit. Thus, the resistor which represents the core losses of the inductor can be assumed to be the load. If the core losses of the inductor are low the resistor in parallel with the inductor is quite high and vice versa. Evidently, the resonance circuit exhibits the characteristics of a parallel-loaded resonance configuration. The resonance impedance of a parallel-loaded resonance circuit can be derived as

\[ Z_0 = \sqrt{\frac{L_o}{C_r}} \]  

(4.149)

\( C_r \) is defined as the effective resonance capacitance, thus, the sum of all capacitances included in the resonance tank. Additionally, the loaded quality factor of the parallel-loaded resonance circuit corresponds to

\[ Q_0 = \frac{R_{\text{core}}}{Z_0} \]  

(4.150)

Equation (4.150) implies that the higher the core losses are the higher the quality factor is, thus, the oscillations are undamped. If the parallel resistor is infinitely large the inductance and the capacitance forms a resonance circuit which is damped by the resistive losses of the circuit. Consequently, if a resistor is connected in parallel with the inductor in the SAB converter the oscillations during the discontinuous-time interval should be damped to a certain level. However, the parallel resistance must be comparable in size with the impedance of the inductor at the switching frequency. This implies that a considerable amount of current will flow through the resistor and will result to additional losses. The losses demanded to damp the oscillations are huge compared with the power capability of the converter.
In Figure 4.38 the simulated resistor current is presented. A considerable amount of power will be dissipated in the resistor which corresponds to 0.12 pu. Thus the efficiency will decrease with almost 12% which is not acceptable. The introduction of the parallel resistor will damp the oscillations but the efficiency of the converter will be drastically reduced. It is worth noticing that the oscillations are not completely damped. Summarising, any attempt to eliminate the oscillations will degrade the operation, and, the efficiency of the converter.

4.11.1 Transformer-induced Low-frequency Oscillations (TLO)

In [Kles1], see Appendix F, and [Kin1] the different modes of the Transformer-induced Low-frequency Oscillations (TLO) are presented and thoroughly examined. The parasitics of the transformer may interfere with the power circuit and may lead to low-frequency oscillations. The TLO are generated when the secondary side is disconnected, i.e. when the secondary diode bridge is reversed biased. This configuration is obtained at the moment when the inductor current $i_{L_L}$ equals the magnetising current $i_{L_m}$ provided that the absolute value of the primary voltage is lower than the output voltage. The equivalent circuit for the TLO is shown in Figure 4.39.
As reported in [Kles1] the generation of the TLO is dependent on the conversion ratio $M$ and the ratio of the series and the magnetising inductance $\beta = \frac{L_\sigma}{L_m}$. The ratio of the length of TLO wave period versus that of a non-TLO wave as reported in [Kles1] is given by

$$\xi = \sqrt{\frac{L_\sigma + L_m}{L_\sigma}}$$  \hspace{1cm} (4.151)

Since the SAB topology is operating at DCM the power circuit is interfering with the transformer parasitics as discussed in this chapter. Simulations have been done in order to examine the existence of TLO. The inductor and the magnetising current are shown in Figure 4.40.

![Inductor and magnetising current](image-url)

Figure 4.40: Simulated inductor current and magnetising current
As shown in Figure 4.40 the inductor current is equal to the magnetising current at a certain time interval. As a result, the secondary placed diode bridge is reverse biased and the secondary current is discontinuous. In Figure 4.41 the inductor and the secondary current as well as the primary voltage are shown. The oscillations are enlarged on the right hand side of the figure.

![Simulated waveforms](image)

*Figure 4.41: Simulated waveforms, (a) the inductor current, (b) secondary current, and (c) primary voltage*

When the inductor current equals the magnetising current, the secondary voltage is decreased as shown in Figure 4.41. This decrease depends on the conversion ratio $M$ and the inductance ratio $\beta$.

When the duty ratio of the controllable switches is varying the peak value of the inductor current should vary as well. This implies that a dc component is added to the magnetising current. The presence of a dc component can be observed in the primary voltage as voltage dips with amplitude that varies from period to period as shown in Figure 4.42. This means that, if dc component is zero the amplitude of the dips are constant.
A dc component added to the magnetising current can cause dc saturation of the transformer. DC saturation can be avoided if the duty ratio of the controllable switch during the positive half period equals to the duty ratio of the controllable switch during the negative half period. At steady state the controller must ensure that the on duty ratio of the controllable switches, $T_{A+}$ and $T_{A-}$, is $d_{T_{A+}} = d_{T_{A-}}$ in order to avoid dc saturation of the transformer. If the duty ratio constraint, i.e. $d_{T_{A+}} = d_{T_{A-}}$, is ensured by the controller during transients the dc saturation of the transformer can be avoided as well.

In Figure 4.43 the inductor current and the magnetising current are shown. The duty ratio of the switches $T_{A+}$ and $T_{A-}$ is always $d_{T_{A+}} = d_{T_{A-}}$ in a period. As a result, when the duty ratio of the switches varies the added dc component to the magnetising current is zero within two periods.
Figure 4.43: Simulated inductor current and magnetising current during a transient keeping the duty ratios of the controllable switches equal during a period.

In Figure 4.44 (a) the inductor current and the magnetising current are shown when the duty ratio of $T_{A-}$ varies resulting in a dc component which is added to the magnetising current. Observe that the mismatch in duty ratios, i.e. $d_{T_{A-}} \neq d_{T_{A+}}$, occurs only once. This mismatch can cause dc saturation of the transformer resulting in additional losses. In Figure 4.44 (b) the magnetising current is shown. When $d_{T_{A-}} \neq d_{T_{A+}}$ a dc component is inevitably added to the magnetising current. This dc component decays and reaches zero after 17 periods. Therefore, in order to avoid dc saturation of the transformer and additional losses due to duty ratio deviations the controller must ensure always that $d_{T_{A-}} = d_{T_{A+}}$ in a period.
The voltage dips occur when the inductor current $i_L$ equals the magnetising current $i_m$.

When the magnetising current intersects the inductor current and when the absolute value of the primary voltage is lower than the output voltage, the secondary placed diode rectifier is
reverse biased. As shown in Figure 4.45 the voltage dips can occur more than once during a half period. This implies that, the diode rectifier is forward and/or reverse biased each time \( i_{Lm} = i_{Lp} \) if the absolute value of the primary voltage is lower than the output voltage. In Figure 4.45, the diode rectifier is forward and reverse biased a number of times during a half period. This implies that the switching losses of the diode rectifier are increased compared with the ideal configuration and when the converter operates at CCM. The increase in switching losses in the diode bridge not only calls for increased cooling but also decreases the efficiency of the converter.

In Figure 4.45 the inductance ratio \( \beta \) equals to \( \beta = 7.5 \times 10^{-3} \) and the conversion ratio \( M = 0.74 \).

---

**Figure 4.45: Simulated waveforms, (a) Inductor and magnetising current, (b) Rectified current, and (c) Primary voltage**
4.12 Control and regulation of the SAB topology

In order to control power flow from the primary side to the secondary side of the half-bridge configuration of the SAB topology three control algorithms have been identified. Namely,

- Variable-frequency control
- Turn-off time control operating at DCM
- Turn-off time control operating at intermittent mode

The constant switching frequency duty ratio control method cannot be directly used. Specifically, due to the duty ratio limitations as expressed by Eqs. (4.18) and (4.44), the duty ratio of the transistor cannot be freely chosen. This applies when the converter operates in the continuous-conduction mode. The duty ratio of the transistor is strongly dependent on the duty ratio of the diode. Thus, when the current flows through the diode, the transistor cannot commutate the current even though a turn-on command is applied. Consequently, the current through the diode has to reach the zero level resulting in a current commutation from the diode to the transistor. However, by controlling the turn-off time of the controllable switch the duty ratio is indirectly controlled. Thus, the conduction time of the transistor can be chosen by controlling the turn-off time of the transistor.

4.12.1 Variable-switching frequency control

The variable-switching frequency control is a common control method for loaded resonant converters as thoroughly studied in [Dem1]. By varying the switching frequency, the impedance of the resonant tank varies, resulting in power control. Concerning the SAB topology, high switching frequency will increase the impedance of the inductor and the voltage drop over the inductor is increased. Thus, the inductor current is decreased which results in a lower output voltage. On the other hand comparably low switching frequencies yield a lower inductor impedance and, thus, high output power. This means that the transformer has to deliver maximum power at low switching frequencies and that the core has to be dimensioned for comparably low frequencies. Consequently, the transformer becomes bulky.

Additionally, all passive components in the circuit, including the transformer, inductors and capacitors, cannot be optimised for a certain frequency since the frequency is used as the
control variable. As a result, filter design is not optimised. Additionally, in applications where a number of converters are connected together, frequency modulation can be a very significant problem. Variable-frequency converters cannot be synchronised, due to the fact that their frequencies depend on the load. This can generate low-beat frequencies. Concluding, variable-switching frequency modulation corresponds to more drawbacks than benefits, and as a result is not considered in the present thesis.

4.12.2 Turn-off time control operating at DCM

The turn-off time control method is a novel control method firstly presented in the present thesis. At heavy loads and at the nominal load the converter operates in the continuous-conduction mode. The turn-off time of the transistor is determined by the inductor current as ruled by Eq. (4.45) and Table 4.2. In Table 4.2 the duty ratios of each device are expressed in terms of the inductor peak current. A certain output voltage corresponds to a specific peak inductor current. When the peak value of the inductor current reaches the threshold level the controllable switch is turned-off. At light load the converter will shift mode of operation from the continuous-conduction mode to the discontinuous-conduction mode. Therefore, discontinuous-time intervals are introduced in every half cycle. The introduction of the discontinuous-time interval is the major drawback of the method as it introduces an interruption of the power flow. On the other hand, the presented control method provides a peak-current protection and limitation.

4.12.3 Turn-off time control operating at intermittent mode

The turn-off time control operating in the intermittent mode of operation is quite similar to the control method presented in Section 4.12.2. Both control methods are easy to implement and the same control configuration can be used. The intermittent mode of operation is built on the quantum principle. Thus, a pulse train is sent to the controllable switches. When the controlled variable reaches the reference value the controller introduces a discontinuous-time interval. During this discontinuous-time interval, the output voltage which is the control variable of interest, is decaying with a certain load dependent rate. The next pulse-train is send by the controller when the output of interest reaches a certain threshold limit. Actually the implementation of the intermittent control method can be compared with the hysteresis control method but operating with a constant switching frequency. The repetition rate of the discontinuous-time interval is dependent on the width of the hysteresis band. Thus, the higher the hysteresis band is, the longer the discontinuous-time interval is. On the other hand, if the hysteresis band is made sufficiently narrow the repetition rate of the discontinuous-time interval is...
interval becomes so high that the converter operates at DCM. For very low loads the converter will also operate at DCM, since only one single cycle is enough to change the output voltage to the upper hysteresis level.

When a discontinuous-time interval is introduced oscillations are taking place as in the case of DCM. In the intermittent mode of operation the oscillations are not present in every half-cycle but depend on the repetition rate as governed by the controller. Simulation results are shown in Figure 4.46.

![Simulated waveforms. Output voltage, inductor current, and snubber voltage waveforms.](image)

As shown in Figure 4.46 a low frequency ripple corresponding to the rate of the discontinuities is introduced in the output voltage. The simulated results are obtained assuming ideal transformer, i.e. having a negligible winding capacitance and an infinite magnetising inductance. By adding the transformer parasitics to the model oscillations will appear during the discontinuous-time interval as shown in Figure 4.48. The oscillations are enlarged on the right hand side of the figure. Observe that resonance mode 1 is present and is shown in Figure 4.47.
Figure 4.47: Oscillations during the discontinuous-time interval

Figure 4.48: Simulated waveforms presenting the inductor current, the snubber voltage, and the inductor voltage. Transformer parasitics are included.
4.12.4 Controller design

The controller design is based on the results obtained by the small-signal analysis of the single-active bridge. By obtaining the small-signal transfer functions of the converter the dynamics are known and appropriate actions can be taken. From the small-signal analysis, the dynamics of the system are well-defined and conventional Proportional-Integral (PI) regulators can be used to implement the controller. The block diagram of the regulator system is shown in Figure 4.49. Four transfer functions are included in the model: the control-to-output, the control-to-state, the source-to-output, and the source-to-state transfer functions. Evidently, the line regulation, concerning disturbances around the equilibrium of the source, is taken under consideration. Especially, in applications where a six-pulse rectifier is used the presence of a 300 Hz ripple is dominant. The ripple can be reduced by increasing the dc-capacitor bank or damped by the controller if enough bandwidth is available in the feedback loop. Additionally, the feedforward of the input voltage reduces the influence of line regulation.

Additionally, duty ratio regulation is included in the model. Small-signal variations around the steady state value of the duty ratio can cause oscillations and instabilities to the system, as in the case of the buck converter. Duty ratio oscillations can be caused by switching-frequency variations caused by delays in the system. The block diagram of Figure 4.49 completely describes the dynamic behaviour of the SAB topology.

![Figure 4.49: Small-signal representation of the regulated SAB.](image-url)
The small-signal transfer functions as shown in Figure 4.49, represent the power stage of the converter. In order to study the behaviour of the topology both in continuous and discontinuous-conduction mode of operation, the transfer functions corresponding to each of the operating modes have to be added in the model. Consequently, in the continuous-conduction mode the switching frequency-to-output, and the switching frequency-to-state transfer functions have to be included in the model. Thus, the converter operates in the continuous-conduction mode when the frequency transition occurs.

Simulations have been done in order to examine the behaviour of the system during steady-state and during transient operation.

The PI regulators were tuned by using the SISO (Single Input Single Output) tool in Matlab. The following steps were carried out. The inner current loop is infinitely fast compared with the outer voltage loop. Consequently, the inner current loop can be tuned without closing the voltage loop. The step response of the loop is examined and proper regulator parameters can be chosen. When the correct parameters have been chosen the voltage loop is closed and the step response of the complete system is studied. The values for the proportional gains and the integration times are summarised in Table 4.3. The same parameters were used in the laboratory.

<table>
<thead>
<tr>
<th>Voltage regulator</th>
<th>Current regulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proprtional gain $K_{pv} = 1$</td>
<td>Proportional gain $K_{pi} = 0.1$</td>
</tr>
<tr>
<td>Integration time $T_{iv} = 0.01$ s</td>
<td>Integration time $T_{ii} = 0.0005$ s</td>
</tr>
</tbody>
</table>

*Table 4.3: Controller parameters*

In Figure 4.50 the impact of the variations in the reference voltage are examined. As illustrated, the output voltage follows smoothly the reference voltage. The system is neither unstable nor oscillatory. Additionally, the inductor current increases and decreases in magnitude as governed by the controller. Thus, when a step in the reference voltage is given, the current will rapidly increase to deal with the step in the output voltage. When the output voltage is stabilised around the reference value the inductor current is decreased in order to keep the output voltage constant. Similarly, when the reference voltage drops to the steady state value the inductor current is rapidly decreased and then returns smoothly to its steady state value.
Furthermore, Figure 4.51 illustrates the performance of the system in the presence of small-signal line variations. In this case a 300Hz ripple has been chosen. By examining the output voltage waveform, it is obvious that the amplitude of the 300 Hz ripple is suppressed by the controller. This is a favourable property and it implies that the dc link capacitors can be minimised in order to achieve a cost reduction.
On the other hand, the 300 Hz ripple is present on the inductor current waveform. As in the case of the output voltage the magnitude of the ripple is damped but not as damped as the output voltage.

Concluding, small-signal perturbations of the line voltage will neither cause instabilities nor oscillations.

The impact of the small-signal perturbations around the steady state of the duty ratio is presented in Figure 4.52. Duty ratio variations will cause some minor ripple on the output voltage waveform, resulting in variations in the inductor current waveform. The dynamic behaviour of the system is smooth with the proposed control scheme. Consequently, the controller has satisfactory line regulation properties and can successfully deal with duty ratio variations without causing instabilities and/or oscillations of the system. Load regulation is examined experimentally in Chapter 5.

![Figure 4.52: Controller performance. Small-signal duty ratio variations.](image-url)
4.13 Summary and Conclusions

In the present chapter the single-active bridge topology has been thoroughly studied. The steady state behaviour and the dynamic properties are investigated using improved state-space averaging. Due to the symmetry conditions the improved state-space averaging method can be used. The small-signal transfer functions, for both CCM and DCM are presented, and are the basis for the controller design. It has been shown that the dynamics of the converter are well-defined and as a result conventional PI regulators can be used.

Small-signal functions are useful tools in order to linearise nonlinear systems, and conventional control tools as the Laplace transformation can be used to study and verify the dynamics of the system. At the beginning of the chapter a step-by-step procedure has been followed in order to introduce the reader into the linearizing procedure.
5  SINGLE-ACTIVE BRIDGE: EXPERIMENTAL VERIFICATION

5.1  Introduction

In order to benchmark the single-active bridge topology a prototype has been built and tested in the laboratory. The SAB topology can operate only in the step-down mode and the power can be delivered from the source to the load. From the switching losses point of view the use of the SAB topology instead of DAB should be the optimal solution. Thus, the SAB topology employs less semiconducting devices than the DAB topology. Additionally, the turn-off current of the switches is comparable to the current, turn-off current, of the DAB topology, resulting to comparable switching losses. On the other hand, the rms current flowing through the switches is comparably lower in the case of the SAB topology implying to considerably lower conduction losses compared to DAB topology. Consequently, in applications where a uni-directional power flow is demanded the optimum solution, from the losses point of view, is the SAB topology. On the contrary, the DAB topology should be used in cases where a step-up operation is demanded and a bi-directional power flow.

Number of measurements was performed under different load conditions in order to evaluate the performance and the behaviour of the system both at steady-state and during dynamic conditions. The purpose of the testing is to:

- evaluate the limitations of the system, topology and controller
- evaluate and compare different control strategies
- analyse the steady-state and dynamic behaviour of the system
- verify and evaluate the theoretical results
• compare simulations and measurements

The prototype was built as a dual-active bridge, but the secondary switches were turned-off during the experiments with the single-active bridge. Thus, the antiparallel diodes were conducting the load current. The whole idea was to use the single-active bridge in unidirectional power flow demands and the dual active bridge in cases where a step-up operation is demanded and a bi-directional power flow.

In order to meet the design criteria for both topologies certain trade-offs had to be made. Thus, an external inductance was connected in series with the leakage inductance of the transformer. As discussed in Chapter 10, the leakage inductance of the transformer is very low and does not fulfil the energy storage demands of the converter as discussed in Appendix B. The converters specifications are summarised in Table 5.1.

<table>
<thead>
<tr>
<th></th>
<th>Input voltage (pole-to-pole)</th>
<th>$V_{d}$</th>
<th>540 volt</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Output Power</td>
<td>$P_0$</td>
<td>8 kW</td>
</tr>
<tr>
<td>3</td>
<td>Switching frequency start-up</td>
<td>$f_{\text{start-up}}$</td>
<td>20 kHz</td>
</tr>
<tr>
<td>4</td>
<td>Switching frequency normal operation</td>
<td>$f_s$</td>
<td>10 kHz</td>
</tr>
<tr>
<td>5</td>
<td>Total Inductance</td>
<td>$L_\sigma$</td>
<td>150 $\mu$ H</td>
</tr>
<tr>
<td>6</td>
<td>Snubber capacitor per position</td>
<td>$C_s$</td>
<td>10 nF</td>
</tr>
<tr>
<td>7</td>
<td>Transformer turns ratio</td>
<td>$n$</td>
<td>1</td>
</tr>
</tbody>
</table>

*Table 5.1: Converter specifications*
Two control strategies were investigated. Namely, the turn-off time control at DCM and the turn-off time control at intermittent mode. The same control structure was used in both cases, but the proportional and integral gains were adjusted between the experiments.

The measuring equipment is summarised in Appendix G.

5.2 Measurements employing a turn-off time control at DCM

As explained in Chapter 4 the use of the turn-off time control will result in a discontinuous inductor current. The converter operates in CCM at nominal power and at heavy loads, that is, high currents and low voltages. Thus, at light loads, low and moderate currents, the converter operates in DCM. The boundary between the DCM and CCM depends on the conversion ratio.

A conversion ratio of $M = 0.74$ has been chosen as the optimum operating point from the losses point of view. Thus, if the converter operates close to a unity conversion ratio the converter will enter DCM at a considerably lighter load. Similarly, if a conversion ratio of $M \leq 0.6$ is chosen the converter should operate at DCM and will enter CCM at very heavy loads implying to high conduction currents. Consequently, in order to study the converter behaviour and additionally, the control algorithms characteristics a conversion ratio of $M = 0.74$ has been chosen.

5.2.1 Measurements at nominal power

Knowing the power capability of the converter as given in Table 5.1 and the conversion ratio the load current $i_0$ can be calculated by

$$i_0 = \frac{P_0}{M V_d}$$  \hspace{1cm} (5.1)

Since the load of the converter is a resistor in parallel with a large filter capacitor, the load resistance $R$ can be expressed as

$$R = \frac{M V_d}{i_0}$$  \hspace{1cm} (5.2)

From Eqs. (5.1) and (5.2)
At nominal power the converter is operating at continuous-conduction mode. Measurements have been done both at steady-state and during transient operating conditions. In Figure 5.1, the inductor current and voltage as well as the snubber capacitor voltage are illustrated. As foreseen by theory and simulations, the controllable switches are turned on under zero-voltage and zero current conditions and are turned off under and zero-voltage conditions at nominal power. As a result, the major loss contribution should be the conduction losses. Additional losses occur during the snubber capacitor charging and discharging processes.

As shown in Figure 5.1, oscillations are present in both the inductor voltage and snubber capacitor voltage. The semiconducting modules used in the prototype are not ideal. A certain amount of stray inductance due to the loops resulted by the connections of the chips is present. Additionally, the snubber capacitors are placed on a Printed-Circuit-Board (PCB) and are connected in parallel with the switches by means of a buss bar with a certain stray inductance and stray capacitance. Consequently, during the charging and the discharging of the snubber capacitors, the stray components of the module and the snubber circuit are oscillating with the snubber capacitors causing additional losses. Due to the high oscillating frequency the EMI behaviour of the converter can be degraded.

\[
R = \left( \frac{M V_d}{P_0} \right)^2
\]
Figure 5.1: (a) Measured inductor current and inductor voltage and, (b) Measured inductor current and snubber voltage.
The primary voltage and the primary current are illustrated in Figure 5.2. As shown in Figure 5.2, the primary voltage fluctuates at every switch transition. That is, the primary voltage decreases by means of resonance while the current through the switch linearly increases. At point (a) as indicated in Figure 5.2, the inductor current is zero meanwhile the current through the output dc capacitor has reach its negative maximum value. During the interval (a)-(b) the inductor current increases linearly and the capacitor current decreases forcing the voltage to decrease. Due to the presence of the inductor the voltage through the capacitor will decrease by means of an oscillation. When the voltage reaches point (b) the capacitor current is zero implying that the capacitor voltage is constant. During the interval (b)-(c) the capacitor current is increasing and is charging the capacitor. At (c) the current reaches its maximum value.

Furthermore, when the current is commutate from the transistor to the diode a rapid change, step (c), in the magnitude of the primary voltage is taking place. This is due to the fact that the derivative of the current at the point of commutation (c) is changing sign resulting to a step in the waveform of the primary voltage.

*Figure 5.2: Measured primary voltage and current.*
From Figure 5.2 the angle between the fundamental components of the voltage and current is 19.4 degrees resulting to a relatively low reactive power. Thus, the active power of the converter at the fundamental frequency can be expressed as

\[ P_0 = V_0 I_0 \cos \phi \]  

(5.4)

A phase-angle of 19.4 degrees corresponds to

\[ \cos \phi = \cos (19.4^\circ) = 0.943 \]  

(5.5)

As a result, it can be stated that 0.943 p.u of the power delivered by the source will be transferred to the load. Thus, the transformer can be optimized resulting in a straightforward transformer design. In cases where the reactive power is significant, the peak value of the inductor current is comparably high resulting to additional conduction losses. In cases where resonance converters are used the peak value of the resonance current is considerably high as well as the associated conduction losses [Dem1]. Additionally, the transformer has to be designed to handle the reactive power. Therefore, the kvar rating of the transformer in a resonance converter is often considerably higher, [Dem1] and [Dem2], than the nominal kW of the converter.
5.2.2 Measurements at light loads

At light loads, nominal voltage and low output current, the converter will shift the operating mode, from the CCM to the DCM. As mentioned in Chapter 4, and when the converter operates in DCM oscillations are present during the discontinuous-time interval.

Figure 5.4 and Figure 5.3 illustrate the properties of the converter at DCM. During the discontinuous-time interval (as thoroughly discussed in Chapter 4) the inductor and the snubber capacitors are oscillating when the current is clamped to zero. The oscillations will contribute with some additional losses due to the fact that the rectifier bridge will commutate the resonance current. Additionally, the high frequency oscillations will contribute to excessive EMI and radio interference problems. Similarly, during the discontinuous-time interval the snubber capacitors will be discharged and charged. It is true that the switches are turned-on at a finite voltage but the current is nevertheless, clamped to zero. Therefore, the switches are turned on under zero-current switching conditions and are turned off under ZVS.

The primary voltage will also drop during the discontinuous-time interval with a slower rate than the rate of the voltage applied to the terminals of the transformer, i.e. slower voltage derivative. This will not excite the internal resonance of the transformer and the only contribution to the converter is a slight decrease in efficiency.

![Figure 5.3: Measured primary current and voltage](image-url)
Figure 5.4: (a) Measured inductor current and voltage and, (b) Measured inductor current and snubber capacitor voltage
When the discontinuous-time interval is increased, i.e. at low currents, the oscillations become more severe. The current through the rectifier is oscillating and additional EMC and radio interference is generated. Due to the increase of the amplitude of the oscillations the losses are increased.

The voltage spikes/dips during the discontinuous-time interval as shown in Figure 5.3 have not the same amplitude. This is due to the dc-component introduced by the inductor current. The dc-component introduced by the inductor current however will introduce a dc-component on the magnetising current resulting to the unequal voltage spikes/dips as shown in Figure 5.3 and is briefly discussed in Section 4.11.1. DC-component can be introduced if the on-duty ratio of the controllable switches differs. Different duty ratios will result to different peak inductor current and therefore, a dc-component will be added to the inductor current.

In Figure 5.5 the inductor current and voltage, and, the resonance modes are shown. Two resonance modes are present as discussed in Chapter 4. During resonance mode 1, as discussed in Chapter 4, the current will flow through the rectifier bridge and during resonance mode 2 the current will charge and discharge the winding capacitance of the transformer. This is shown in Figure 5.6. Excessive EMC and radio interference may call for effective filters which can affect the converters dynamic behaviour and certainly the costs of the system.

![Figure 5.5: Measured inductor current and voltage and the resonance modes.](image-url)
Figure 5.6: (a) Measured inductor current and snubber voltage and, (b) Measured primary current and voltage
Figure 5.6 (b) can be compared with Figure 4.45. As explained in Section 4.11.1 the diode rectifier is forward and reverse biased a number of times during a half period. This implies that the switching losses of the diode rectifier are increased compared with the ideal configuration and when the converter operates at CCM.

The efficiency of the converter has been measured and is shown in Figure 5.7. As expected the efficiency of the converter is decreased when the load is getting lighter. This is due to the presence of the oscillating current which will force the rectifier bridge to commutate the current more than once in a half cycle. At nominal power the efficiency of the converter is almost 96 % and at light load the efficiency of the converter is 91.3%. On the other hand, for converters in the MW range the efficiency is expected to increase. IGBT:s optimized for soft-switching operation usually have higher switching losses and lower conduction losses. This is relevant when the switching losses are negligible and the dominant losses are the conduction losses. With IGBT:s having lower conduction losses the efficiency of the converter will be increased. In all cases presented in this chapter the controllable switches are soft-switched. Nevertheless, the snubber capacitors have an optimum operating area and by moving to lighter loads the inductor peak current will comparably decreased and the ZVS will be lost. On the other hand, the current is very low and the switching losses are therefore low as well.

![Output power versus efficiency](image_url)

*Figure 5.7: Measured output power versus efficiency*
5.3 Converter dynamics

In this section the dynamic behaviour of the converter has been studied. When the converter operates at steady state the load is rapidly increased or decreased. Thus, additional resistive load is connected to the converter. The voltage loop of the controller is disconnected and only the current loop is connected. Both the inductor current and the output voltage are measured.

As shown in Figure 5.8, the converter dynamics are predictable i.e. stable and damped response, and are as predicted by the small-signal models presented in Chapter 4. A 10% load step will force the voltage to decrease and the inductor current to increase in a controllable rate. Neither oscillations nor instabilities are taking place.

![Figure 5.8: Dynamic behaviour of the converter with open voltage loop.](image)
Figure 5.9 shows the response of the system during a 10% load step with both voltage and current loops closed. As shown in the figure the inductor current is slightly decreased but the output voltage is clamped to its steady state value. Actually the output voltage is slightly increased but is forced back to its steady state value by the controller. This is clarified in Figure 5.10 where the load is increased by 20%.

![Figure 5.9: Load step response.](image)

As discussed in Chapter 4 the converter is started up with a switching frequency corresponding to twice the switching frequency at steady state in order to limit the inrush current. A step in the switching frequency can cause oscillations and instabilities to the system. In the case of the SAB converter and as predicted by the small-signal model the frequency step response is neither oscillatory nor unstable avoiding unpleasant surprises. The frequency transition is shown in Figure 5.11 (a) for light loads and, in Figure 5.11(b) at nominal load. As shown in the figures the response of the converter due to a step in the switching frequency is quite similar for both light loads and nominal load. When the switching frequency is reduced to the half of its initial value the voltage increases. The current increases rapidly in order to deal with the increase in voltage and after a certain time it returns to steady-state.
Figure 5.10: Load step response.

Figure 5.11: Frequency transition. (a) Light load and, (b) Nominal load.
5.4 Turn-off time control at intermittent mode

The converter can operate in intermittent mode of operation. By slightly adjusting the proportional gain and the integral gain of the controller the converter shifts mode and enters the intermittent mode of operation. The oscillations are present but with a lower repetition frequency as discussed in Chapter 4.

Measurements were performed with low output current. As in the case of the turn-off time control at DCM the converter operates at CCM at heavy loads and at nominal load. For output currents below 5% of the nominal current the converter will eventually enter DCM.

In Figure 5.12 the inductor current and the output voltage are presented. As indicated in the figure the output ripple is comparably high which calls for a higher capacitance on the DC side of the converter. In high power applications DC-cables are used in order to transfer the power over a long distance. High-voltage dc-cables will exhibit high ac-losses if the ripple of the dc-voltage exceeds 1%. Consequently, if SAB converter operates in the intermittent mode and is used in high-power applications where dc-cables are used the ripple should not exceed 1%.

![Figure 5.12: Measured inductor current and output voltage.](image-url)
Figure 5.13: (a) Measured snubber voltage and inductor current and, (b) Measured primary current and voltage.
In Figure 5.13 and Figure 5.14 the characteristic waveforms for the converter operating in the intermittent mode are presented. It is clear that the oscillations are present but with a lower repetition rate comparing to DCM. The major disadvantages of the intermittent mode are the high ripple at the output and that the repetition rate of the discontinuous-time interval is in the audible frequency range. Consequently, for high frequency applications special precautions, insulation and thicker walls must be chosen in order to avoid the acoustical noise from the converter. Nevertheless, the efficiency of the converter is slightly higher, in the intermittent mode than at DCM.

Another demerit of the intermittent mode of operation is the need of adaptive control scheme. Thus, by varying the load the controller parameters have to be adjusted in order to achieve acceptable performance. By using the same parameters the converter enters the unstable region, as shown in Figure 5.15, which can cause converter failures. This implies that, further studies should be performed in order to optimise the implementation of the turn-off time control at intermittent mode.

The major challenge is to use a Digital Signal Processor (DSP) for the hysteresis-control algorithm implementation, which would normally require an analog circuit design. Digital implementation calls for high sampling frequency.
5.5 Remarks and Conclusions

The single-active bridge topology has been built and tested in the laboratory. Two control algorithms have been tested. It is evident to the author that the turn-off time control at DCM fulfills the needs and the specifications of the converter. The output voltage ripple is lower than 1% and the efficiency of the converter is at acceptable levels. By using IGBT:s optimized for soft-switching operation the efficiency of the converter will certainly be increased. On the other hand the oscillations are the major disadvantage of the discontinuous-conduction mode. Additional, EMC and radio interference filters must be used affecting the dynamic behaviour of the converter and a certain amount of losses will be added.

The intermittent mode of operation is not the optimal solution at this stage, and should be a subject for future activities. Oscillations are present and by moving to lighter loads the converter will eventually enter the discontinuous-conduction mode.

In both control strategies the controllable switches are turned off under ZVS and are turned on under zero current and/or zero voltage conditions.
6 THE DUAL-ACTIVE BRIDGE TOPOLOGY

6.1 Introduction

The Dual-Active Bridge (DAB) topology which was firstly presented by Kheraluwala et.al in [Khe1, Khe2, Khe3] consists of two switch-mode active bridges, one operating in the inversion mode and the other in the rectification mode. The ac-terminals of the bridges are interconnected by means of a high-frequency transformer, enabling power flow in both directions as shown in Figure 6.2. Each bridge can be controlled to generate a high-frequency square-wave voltage at its transformer terminals ($\pm V_{dc}, \pm V_0$) in a similar manner as described in Chapter 4 for the SAB topology.

By incorporating a controlled amount of leakage inductance into the transformer the two square waves can be appropriately phase-shifted to control the power flow from one dc-source to the other. A bi-directional power transfer can be achieved. Power is delivered from the bridge generating the leading square wave. Maximum power transfer is achieved at a phase shift of 90 degrees. In a wide load range all the devices are operated at ZVS and therefore high efficiency is obtained. The DAB can operate with bi-directional power flow in both step-up and step-down operation.

Furthermore, the DAB topology can be represented as an inductor driven by two controlled square-wave voltage sources. As already explained the two voltage sources are phase-shifted from each other by a certain angle $\phi$. If the square-wave sources are replaced by their fundamental components [Khe2], a model similar to the synchronous machine equivalent circuit is obtained as shown in Figure 6.1.

![Figure 6.1: The fundamental model](image-url)
6.2 Steady-state analysis

The equivalent circuits, during one half cycle, for each different switch state, are shown in Figure 6.2. As in the case of the SAB the commutation sequence starts by assuming that the current is flowing through the devices $D_{A+}$, $D_{s3}$ and $D_{s2}$ as shown in Figure 6.2(a). While the current is flowing through diodes $D_{A+}$, $D_{s3}$ and $D_{s2}$ the voltage across the switches $T_{A s}$ and $T_{s2}$ and $T_{s3}$ is zero. The transistor $T_{A+}$ is switched on at zero voltage and after a certain time instant the switches $T_{s2}$ and $T_{s3}$ are turned on and eventually the current reverses and starts flowing through them as illustrated in Figure 6.2(b) and Figure 6.2(c). The transistors $T_{s2}$ and $T_{s3}$ are turned-off, forcing the diodes $D_{s1}$ and $D_{s4}$ to commutate and conduct the load current as shown in Figure 6.2(d). When the transistor $T_{A+}$ is turned off the energy stored in the inductance is transferred to the snubber capacitors by resonance as explained in Section B. The snubber capacitor, which is placed across $T_{A+}$, will take over the current, and the transistor turn-off occurs under ZVS conditions and when the device is carrying a certain minimum current. Additionally, the snubber capacitor, which is connected across the transistor $T_{A-}$, will be discharged and will eventually force the diode $D_{A-}$ to be forward-biased. Therefore, the inductor current starts flowing through the diode $D_{A-}$ and diodes $D_{s1}$ and $D_{s4}$, as shown in Figure 6.2(e). Similarly, the transistor $T_{A-}$ is turned on at zero voltage and the commutation sequence will be repeated as above.

Due to the symmetrical operation of the circuit, the equivalent circuits concerning the transistor $T_{A-}$ and the secondary placed active switches $T_{s4}$ and $T_{s5}$ can be easily obtained by following the same procedure.
Figure 6.2: The equivalent circuits for each mode of operation
The voltage across the inductor, $V_{L_p}$, can be expressed in terms of the input voltage and the output voltage as shown in Table 6.1.

<table>
<thead>
<tr>
<th>Equivalent circuit</th>
<th>Conducting device</th>
<th>$V_{L_p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 6.2 (a)</td>
<td>$D_{4+}$ and $D_{53}$ &amp; $D_{52}$</td>
<td>$V_{L_p} = V_{dc} + V_0$</td>
</tr>
<tr>
<td>Figure 6.2 (d)</td>
<td>$T_{A+}$ and $D_{41}$ &amp; $D_{44}$</td>
<td>$V_{L_p} = V_{dc} - V_0$</td>
</tr>
<tr>
<td>Figure 6.2 (e)</td>
<td>$D_{A-}$ and $D_{41}$ &amp; $D_{44}$</td>
<td>$V_{L_p} = -V_{dc} - V_0$</td>
</tr>
</tbody>
</table>

Table 6.1: The inductor voltage for the DAB topology

In Figure 6.3 the input voltage, the output voltage and the inductor current are shown.

![Figure 6.3: Voltage and current for the DAB](image-url)
In Figure 6.2, two modes of operation are identified. The two modes of operation can be defined as

- Mode 1: The two voltage sources have different polarity
- Mode 2: The two voltage sources have the same polarity

During mode 1 the inductor current can be expressed as

\[ i_{L_1}(\omega t) = \frac{V_d}{\omega L_1} \omega t + i_{L_1}(0) \]  \hspace{1cm} (6.1)

Similarly, in mode 2 the inductor current corresponds to

\[ i_{L_2}(\omega t) = \frac{V_d}{\omega L_2} (\omega t - \omega t_0) + i_{L_2}(\omega t_0) \]  \hspace{1cm} (6.2)

Due to symmetry conditions, i.e. the turn-off current of the transistor \( T_{A-} \) at \( \omega t = 0 \) is equal to the turn-off current of the transistor \( T_{A+} \) at \( \omega t = \omega t_1 \), at the end of the half cycle yields

\[ i_{L_1}(0) = -i_{L_2}(\omega t_1) \]  \hspace{1cm} (6.3)

The initial current \( i_{L_1}(0) \) can now be calculated in steps. From Eq. (6.1) \( i_{L_1}(\omega t_0) \) is found by insertion of \( \omega t = \omega t_0 \). From Eq. (6.2) \( i_{L_2}(\omega t_1) \) is found by setting \( \omega t = \omega t_1 \) and inserting \( i_{L_1}(\omega t_0) \) from Eq. (6.1). Rearranging the resulting equations and introducing \( M = \frac{V_d}{V_{dc}} \) and from Eq. (6.3) yields

\[ i_{L_1}(0) = -\frac{V_d}{2 \omega L} \left[ (1-M)(\omega t_1 - \omega t_0) + (1+M)\omega t_0 \right] \]  \hspace{1cm} (6.4)
Similarly, \( i_{L,\sigma}(\omega t_0) \) can now be obtained from Eq. (6.1) by inserting \( i_{L,\sigma}(0) \) from Eq. (6.4) and by again introducing \( M = \frac{V_0}{V_{dc}} \). Accordingly,

\[
i_{L,\sigma}(\omega t_0) = \frac{V_{dc}}{2} \omega L_\sigma \left[ \left( 1 + M \right) \omega t_0 - \left( 1 - M \right) \left( \omega t_1 - \omega t_0 \right) \right]
\]

(6.5)

From Figure 6.3 it is found that \( \omega t_1 - \omega t_0 = \pi - \phi \) and that \( \omega t_0 = \phi \). Eqs. (6.4) and (6.5) can now rewritten as

\[
i_{L,\sigma}(0) = -\frac{V_{dc}}{2} \omega L_\sigma \left[ \left( 1 - M \right) \left( \pi - \phi \right) + \left( 1 + M \right) \phi \right]
\]

(6.6)

\[
i_{L,\sigma}(\phi) = \frac{V_{dc}}{2} \omega L_\sigma \left[ \left( 1 + M \right) \phi - \left( 1 - M \right) \left( \pi - \phi \right) \right]
\]

(6.7)

As stated in [Khe1] the output power of the converter corresponds to

\[
P_0 = \frac{2}{\omega} V_{dc}^2 M \phi \left( 1 - \frac{\phi}{\pi} \right)
\]

(6.8)

Equation (6.8) is plotted in Figure 6.4 for different values of the phase-shift angle \( \phi \). The maximum power is delivered by the source to the load at \( \phi = \frac{\pi}{2} \). For positive values of the phase-shift angle the power flow is always delivered from the primary bridge to the secondary bridge. Consequently, for negative values of the phase-shift angle the power flow is reversed, i.e. from the secondary bridge to the primary bridge.
6.2.1 Boundaries for zero-voltage switching

In order to achieve zero-voltage switching, the device must be conducting a certain current in order to charge and discharge the snubber capacitors at a controllable rate. As shown in Figure 6.3 the controllable switch $T_{A-}$ is turned off at $\omega t = 0$ which corresponds to zero degrees and the current through the switch is negative. Similarly, the controllable switch $T_{A+}$ is turning off a positive current at $\omega t = \omega t_0$ which corresponds to the phase-shift angle $\phi$.

Thus, the above statement corresponds to

$$i_{Lp} (0) \leq 0 \quad (6.9)$$
$$i_{Lp} (\phi) \geq 0 \quad (6.10)$$

Combining Eqs (6.6) and (6.9) and Eq.s (6.7) and (6.10) the soft switching boundaries can be defined as stated in Eqs. (6.11) and (6.12) corresponding to the input and output bridge respectively.

$$M \leq \frac{\pi}{\pi - 2 \phi} \quad (6.11)$$
\[ M \geq 1 - \frac{2\phi}{\pi} \] (6.12)

\textbf{Figure 6.5: Soft-switching boundaries}

In Figure 6.5 Eqs. (6.11) and (6.12) are plotted for different power levels. The areas defined by the boundaries and phase-shift axis are the hard-switching areas. As shown in the figure, when \( M = 1 \), the soft-switching conditions are fulfilled along the operating area. When the conversion ratio decreases, the soft-switching area decreases as well.

\section*{6.3 Converter dynamics}

Later in this chapter a controller for the DAB is designed. As basis for the controller design a dynamic small-signal model is derived. Having a small-signal model, the controller design is quite straightforward, and stability problems can easily be foreseen with pole plots. The small-signal model also provides an understanding of the dynamic properties of the converter.

Due to symmetry conditions, the DAB topology can be averaged over one half cycle in the same way as the SAB. State-space averaging is used and the same procedure as described in Chapter 3 and Chapter 4 is followed. Consequently the phase-shift angle \( \phi \) must be
expressed in terms of the conversion ratio $M$. Thus, the duty ratio defined by
\[ \omega t_0 \leq \omega t \leq \omega t_0 \] corresponds to
\[ d_1 = \frac{2 \phi + (M - 1) \pi}{4 \pi (1 + M)} \] (6.13)

Similarly, the duty ratio for the time interval \( \omega t_0 \leq \omega t \leq \omega t_1 \) can be expressed as
\[ d_2 = \frac{1}{2} \left( 1 - \frac{\phi}{\pi} \right) \] (6.14)

The duty ratio for the controllable switch can be defined similarly as in the case of the SAB and corresponds to
\[ d = 1 - d_{D_s} \] (6.15)

Where, the duty ratio of the diode is given in the Table 4.2.

The state-space averaged model for the DAB topology corresponds to
\[
\begin{bmatrix}
\cdot \\
\cdot 
\end{bmatrix}
= 
\begin{bmatrix}
0 & \frac{2 d_1 - 1}{L_{s}} \\
\frac{1}{C} & -\frac{1}{RC}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ 
\begin{bmatrix}
\frac{2 d - 1}{L_{s}} \\
0
\end{bmatrix}
V_{dc}
\] (6.17)

Comparing Eq. (6.17) with the state-space averaged model corresponding to SAB as derived in Chapter 4 the only term which is not identical is the expression \( \frac{2 d_1 - 1}{L_{s}} \), i.e. the mean value of the inductor voltage. This deviation is thoroughly explained in the Appendix in Appendix D.

The state variables are the same as in the case of the SAB topology, thus the inductor current and the output capacitor voltage. Linearization of the state-space averaging model will result to the small-signal model as defined by Eq. (6.18).
\[
\begin{bmatrix}
\cdot \\
- x_1 \\
\cdot \\
- x_2
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
1 & -1
\end{bmatrix} \begin{bmatrix}
- x_1 \\
- x_2
\end{bmatrix} + \begin{bmatrix}
B_{11} \\
0
\end{bmatrix} V_{dc} + \begin{bmatrix}
E_{11}
\end{bmatrix} \phi
\]

(6.18)

Where

\[A_{11} = - \frac{2 V_{dc0}}{T_s \left( V_{dc0} + x_{20} \right)}\]

\[A_{12} = \frac{1}{2} \left( \frac{V_{dc0} \left( \left( \phi_0 - 3 \pi \right) V_{dc0} - 2 x_{20} \pi \right) + 4 \pi L_{\sigma} x_{10} V_{dc0} - \pi T_s x_{20} \pi^2}{\pi T_s L_{\sigma} \left( V_{dc0} + x_{20} \right)^2} \right)\]

(6.19)

\[B_{11} = \frac{x_{20} \left( T_s \phi_0 x_{20} - 2 \pi L_{\sigma} x_{10} \right) + \pi T_s V_{dc0} \left( V_{dc0} + 2 x_{20} \right)}{\pi T_s L_{\sigma} \left( V_{dc0} + x_{20} \right)^2}\]

(6.20)

\[E_{11} = \frac{x_{20}}{\pi L_{\sigma} \left( M + 1 \right)}\]

(6.21)

### 6.3.1 Small-signal control-to-output transfer function

The control-to-output transfer function of the DAB topology can be defined as stated in Chapter 4, thus,

\[G_{\text{ref}}(s) = \frac{\frac{x_2(s)}{\phi(s)}}{\left[ s I - A_0 \right]^{-1} \left[ E \right]} = \left[ s I - A_0 \right]^{-1} \left[ E \right] \]

(6.22)

The control parameter is the phase-shift angle \( \phi \).
\[ G_{vd}(s) = \frac{N_{G_{vd}}(s)}{\Delta(s)} \] (6.23)

The numerator of the control-to-output transfer function corresponds to

\[ N_{G_{vd}}(s) = \frac{E_{11}}{C} \] (6.24)

Similarly, the determinant of the system is defined as

\[ \Delta(s) = s^2 + s \left( \frac{1}{RC} - A_{11} \right) - \frac{A_{11}}{RC} \frac{A_{12}}{RC} \] (6.25)

In Figure 6.6 the Bode diagram of the control-to-output transfer function is shown.

![Bode Diagram](image)

**Figure 6.6: Bode diagram for the control-to-output transfer function**

As shown in Figure 6.6 the control-to-output transfer function for the DAB topology is stable and is not oscillatory either. Since the DAB topology is derived from the SAB topology similar dynamic characteristics should be expected. This implies that the dynamics of the DAB topology are predictable, i.e. neither oscillatory nor unstable behaviour, and as a result
the same control configuration as used in the SAB topology should be adequate. The phase-
margin of the system is 35 degrees and a cross-over frequency of \( f_c = 4.8 \, k \, Hz \).

The step response of the DAB topology is presented in Figure 6.7. As predicted by the Bode
diagram of the transfer function the response of the system to a step is well-damped stable
response and increases with a controllable manner. Neither oscillations nor instabilities are
taking place.

![Step Response](image)

**Figure 6.7: The step response for the control-to-output transfer function**

In Figure 6.8 the poles and zeroes map of the control-to-output transfer function is presented.
Both poles are located on the LHP and are placed along the real axis. This implies that the
system is stable and not oscillatory. The significance of the poles and zeroes is briefly
discussed in Chapter 4.
6.3.2 Small-signal control-to-state transfer function

The small-signal control-to-state transfer function for the DAB converter is defined in Eq. (6.26).

\[
G_{sd}(s) = \left. \frac{x_j(s)}{\phi(s)} \right|_{U(s)\to 0} = \left[ s I - A_0 \right]^{-1} \begin{bmatrix} E \end{bmatrix} \tag{6.26}
\]

\[
G_{sd}(s) = \frac{N_{G,sd}(s)}{\Delta(s)} \tag{6.27}
\]

Where

\[
N_{G,sd}(s) = \frac{x_{20}}{\pi L_\sigma} \frac{1}{M + 1} \left( s + \frac{1}{RC} \right) \tag{6.28}
\]
Figure 6.9: Bode diagram for the control-to-state transfer function

Figure 6.10: Control-to-state transfer function. Step response
By examining Figure 6.9, Figure 6.10, and Figure 6.11, it is obvious that the DAB topology behaves in a very similar way as the SAB topology. Therefore, similar current control loop can be used without degrading the performance of the system.

6.3.3 Small-signal source-to-output transfer function

The audio-susceptibility of the converter is of great interest. Line variations can cause instabilities and/or oscillations to the system with fatal consequences. Additionally, in case where a six-pulse rectifier is used the 300 Hz ripple can be of significance.

The source-to-output transfer function of the DAB topology can be expressed as

\[ G_{sg}(s) = \frac{x_v(s)}{U(s)} \bigg|_{s=0} = [s I - A_0]^{-1} \left[ B_0 \right] \]  

(6.29)

Thus,

\[ G_{sg}(s) = \frac{N_{G_e}(s)}{\Delta(s)} \]  

(6.30)
The numerator of the small-signal transfer function corresponds to

\[ N_{G_{so}}(s) = \frac{B_{11}}{C} \]  

(6.31)

Figure 6.12: Small-signal source-to-output transfer function. Bode diagram.

Figure 6.13: Step response
As shown in Figure 6.12, the phase margin of the source-to-output transfer function is adequate, 145 degrees, implying that the stability criteria are fulfilled. On the other hand, and at low frequencies, small variations around the steady-state value of the input voltage are slightly amplified in the output of the converter. In the SAB topology the small-signal variations where damped resulting in lower filter capacitance. As predicted by Figure 6.12 and Figure 6.13, the 300 Hz ripple should be slightly amplified.

6.3.4 Small-signal source-to-state transfer function

The susceptibility of the state to small-signal variations of the source is defined in Eq. 6.32.

\[
G_{ss}(s) = \frac{\Delta x(s)}{\Delta U(s)} = \left[ sI - A_0 \right]^{-1} B_0
\]

Eq. 6.32 results to

\[
G_{ss}(s) = \frac{N_{gs}}{\Delta(s)}
\]
Where,

\[ N_{Gr0}(s) = B_{11} \left( s + \frac{1}{RC} \right) \]  \hspace{1cm} (6.34)

Figure 6.15: Small-signal source-to-state transfer function. Bode diagram.

Figure 6.16: Step response
The susceptibility of the state to small-signal variations of the source is fully described by Figure 6.15, Figure 6.16 and, Figure 6.17. Small-signal variations of the source are attenuated which is a favourable peculiarity. Comparing, the source-to-state transfer function with the source-to-output transfer function, a LHP zero is present which is forcing the phase angle to overshoot the frequency axis and increase the magnitude of the source-to-state transfer function. Similar characteristics where observed in the SAB topology.

6.4 Remarks and discussion on small-signal modelling

In Section 6.3 the dynamic behaviour of the ideal DAB converter was presented. The dynamic properties of the DAB topology are similar to those of the SAB topology which is expected since the DAB topology is a SAB derived topology. The system is neither unstable nor oscillatory. A significant difference between the two topologies, however, is that the DAB topology is susceptible to source variations. Small-signal variations of the source are slightly amplified in the output voltage. This implies, that the kvar rating of the filter capacitor must be higher, or that the kvar rating of the input filter capacitor must be increased in order to deal with the small-signal variations of the source.

The small-signal model as presented in this thesis is quite different from the small-signal model as presented by Kheraluwala in [Khe4] and [Dem4]. Both models are identical and are
based on the switch averaging technique as presented in the Appendix A. The state variables of the two models are the output voltage and the rectified inductor current. Thus, the large signal model of the DAB topology can be represented as shown in Figure A.5. Consequently, the topology appears to be a first order system which simplifies the controller design. The first order characteristics of the model implies that the system can operate stably with a simple proportional controller in a close loop configuration [Khe4].

In [Li1] the small-signal model of a modified DAB topology has been presented. The method described in the paper cannot be applied to the conventional DAB topology because the inductor current is expressed in terms of the input current, flowing through a dc-link inductor, and the dc-link capacitor. The average inductor current is then a sum of two dc values which makes the analysis quite easy.

Additionally, if a dc filter inductor is used in the output of the converter, the method presented in [Sab1] and [Vla1] can be used. The method is based on the duty ratio loss which is a consequence of the presence of the leakage inductance. In the ideal case the leakage inductance is assumed to be zero. The state variables of the system is the dc filter inductor current and the dc filter capacitor voltage and as a result the conventional state-space averaged methodology can be applied.

In the present thesis the inductor current is a state variable. The inductor current can neither be expressed as a sum of two other variables nor it is dc quantity. As a result, the half-cycle symmetry has been used as in the case of the SAB topology. This will result in some deviations from the models presented in [Khe4] and [Li1] and are expected. The model presented in this thesis completely describes the dynamic behaviour of the DAB topology.

6.5 Oscillations during commutations

As for the SAB topology the transformer parasitics generate oscillations during the commutation of the controllable switches. The oscillations are of a high-frequency nature and cannot be predicted by the small-signal model of the system.

The high-frequency oscillations encountered in the DAB topology are quite different compared to those of the SAB topology. In DAB topology the duty ratio of the controllable switches is constant resulting in a continuous inductor current. As shown in Figure 6.18 no
oscillations occur when the transformer is assumed to be ideal. When the transformer parasitics are introduced into the topology, oscillations are present.

In Figure 6.19 simulated waveforms are presented for the case when the transformer parasitics are introduced into the topology. As illustrated by the figure the oscillations are more severe on the secondary side, indicating that the oscillations are generated in the transformer.

![Simulated waveforms of the inductor current, secondary current, and inductor voltage for the DAB topology employing an ideal transformer.](image)

_Figure 6.18: Simulated waveforms of the inductor current, secondary current, and inductor voltage for the DAB topology employing an ideal transformer._
It is the opinion of the author that the oscillations are excited when the charging and the discharging processes of the snubber capacitors are completed. The primary placed snubbers start to oscillate with the secondary placed snubbers involving the transformer parasitics with the load connected in series. Consequently, the oscillations are more severe at heavy loads. At light loads the oscillations are heavily damped by the load. When the oscillations are damped the winding capacitance is still oscillating with the leakage inductance and probably with the magnetising inductance.
When the secondary placed controllable switches are turned off the current will not commutate to the diode. The current is clamped at zero. This due to the reversal of the voltage across the winding capacitance, thus, the winding capacitance is charged and/or discharged. This is clearly shown in Figure 6.20. Thus, oscillations are present only when the secondary placed controllable switches are turned off.

![Dual-Active Bridge secondary current](image)

*Figure 6.20: Simulated waveform illustrating the secondary current*

Due to the nature of the oscillations and especially when the load is involved, the secondary placed devices, especially the diodes, are conducting the oscillating current. This causes additional losses. Additionally, the high-frequency oscillations can be modelled as a radio interference source which will degrade the EMC and EMI behaviour of the converter. Filters can be however be inserted with the result of a more complex solution with a considerably higher cost.

The oscillations can be avoided by eliminating the winding capacitance of the transformer or possibly by increasing the natural frequency of the transformer, avoiding natural frequencies excitation.

### 6.6 Control and regulation

A constant-frequency constant-duty-ratio control strategy is employed. Thus, in order to increase or decrease the power deliver to the load a phase-shift angle is introduced. As discussed in this Chapter the maximum power can be achieved for a phase-shift angle equal to
90 degrees. Minimum power can be achieved at phase-shift angle which equals zero. Minimum power or zero output power does not imply zero current. The current is still circulating through the devices causing losses. Consequently, zero active power does not correspond to zero reactive power. If the duty ratio is kept constant, it is not possible to achieve zero reactive power if the output power is zero.

In order to study and design a controller for the DAB topology the small-signal transfer functions are employed. The control configuration is identical to the case of the SAB converter. Thus, two PI regulators are connected in a cascade scheme with two loops, a voltage loop and an inner current loop as shown in Figure 6.21. The states of interest are the inductor current and the output voltage.

![Diagram of the small-signal controller for the DAB topology](image)

*Figure 6.21: The small-signal controller for the DAB topology*

In order to avoid the inrush current into the filter capacitor during start-up same started-up sequence for the SAB-case issued. Thus, the secondary placed switches are turned-off, and the switching frequency is twice the nominal switching frequency. At a threshold voltage the switching frequency rapidly changes to the nominal frequency and a phase-shift angle is
introduced. The secondary bridge is the master unit and the primary bridge is the slave unit, switching with a constant duty ratio and a constant switching frequency.

The procedure described in Section 4.12.4 was followed in order to tune the PI regulators. In Table 6.2 the controller parameters are summarised.

<table>
<thead>
<tr>
<th>Voltage regulator</th>
<th>Current regulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional gain $K_{pi} = 0.1$</td>
<td>Proportional gain $K_{pi} = 1$</td>
</tr>
<tr>
<td>Integration time $T_{iv} = 0.01, s$</td>
<td>Integration time $T_{ii} = 0.001, s$</td>
</tr>
</tbody>
</table>

*Table 6.2: Controller parameters*

Simulations have been performed in order to study the controller performance at steady-state, and with small-signal perturbations. In Figure 6.22 small-signal perturbations are added to the reference voltage. The peak value of the perturbation corresponded to 10\% of the steady-state value of the dc-link voltage. As shown in the figure the controller response is well-damped stable response and neither instabilities nor oscillations are present. When the small-signal pulse is added to the reference voltage, the controller adjusts the phase shift angle in order to cope with this variation.

*Figure 6.22: Control performance. Small-signal variations around the reference voltage.*
The output voltage increases with a certain slope while the current rapidly increases and then is settled to its new steady-state value. Both signals are following their new reference values as governed by the controller until the next variation occurs.

In Figure 6.23 a 300 Hz small-signal ripple is added to the dc-link voltage. As predicted by the small-signal source-to-output transfer functions, small-signal perturbations added to the line voltage are amplified and are added to the output voltage of the converter. Despite the inherent property of the converter to amplify the perturbations, the controller should attenuate the small-signal variations to acceptable levels. As shown in the figure the output voltage is oscillating with 300 Hz but the ripple is only 1% of the nominal voltage. The controller is stable and copes in an excellent manner with the small-signal variations.

![Figure 6.23: Controller performance. Small-signal line variations](image)

Similarly, a small-signal step is added to the nominal value of the phase-shift angle as illustrated in Figure 6.24. As shown in the figure the step in the phase-shift angle will force the output voltage to rise to a new value (since the load resistance is constant) but after a certain time the integral part of the voltage controller is forcing it to follow the reference value. The current instantly increases in order to cope with the phase-shift variation but returns back to its steady-state value with a certain derivative. Phase-shift angle variations can
neither cause instabilities nor oscillations to the system assuring that the system is robust and well-defined.

![Figure 6.24: Small-signal step in phase-shift angle](image)

**6.7 Conclusions and discussion**

In the present chapter the phase-shift controlled DAB topology has been studied. A new dynamic model has been presented by using state-space averaging under the constraint that half-cycle symmetry can be assumed. The small-signal model for the power stage appears to deviate from the previous models as presented by [Khe4], [Dem4], and, [Li1]. These deviations are expected and are discussed in this chapter.

Oscillations due to the presence of the transformer parasitics have been reported. The nature of the high frequency oscillations is quite different from those observed in the SAB topology. Oscillations are present during the turn-off procedure of the secondary placed controllable switches. This due to the fact that, the current is firstly clamped to zero and then is commutated to the antiparallel diodes. During this discontinuous-time interval the voltage across the winding capacitance commutates from $\pm V_{dc}$ to $\mp V_{dc}$ and vice versa. High-frequency oscillations will degrade the EMI and EMC behaviour and they will have an impact on the converters cost.
The controller design has been based on the small-signal transfer functions presented in this chapter and the response and the performance of the controller has been studied. Load and line regulation is achieved and the system is well-defined and stable. The controller can successfully cope with phase-shift steps, reference variations, and line voltage variations.
7 EXPERIMENTAL VERIFICATION OF THE PHASE-SHIFT CONTROLLED DUAL-ACTIVE BRIDGE

7.1 Introduction

The DAB is a SAB derived topology and can be used as a step-down and/or as a step-up configuration. Additionally the DAB topology has the ability to deliver the power in both directions, i.e. from the source to the load and, from the load to the source. This bi-directional ability has however not been exploited in this thesis due to hardware limitations.

The load of the converter consists of a capacitor bank and a variable resistive load. Number of measurements was performed under different load conditions in order to evaluate the performance and the behaviour of the system both at steady-state and during dynamic conditions. The purpose of the testing is to:

- evaluate the limitations of the system, topology and controller
- analyse the steady-state and dynamic behaviour of the system
- verify and evaluate the theoretical results
- compare simulations and measurements

The converter specifications are presented in Table 7.1. As discussed in Chapter 6 the converter starts-up as a SAB converter and at a certain threshold voltage the secondary placed controllable switches are turned on in order to limit the inrush current.
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Input voltage (pole-to-pole)</td>
<td>$V_d$</td>
</tr>
<tr>
<td>2</td>
<td>Output Power</td>
<td>$P_0$</td>
</tr>
<tr>
<td>3</td>
<td>Switching frequency start-up</td>
<td>$f_{\text{start-up}}$</td>
</tr>
<tr>
<td>4</td>
<td>Switching frequency normal operation</td>
<td>$f_s$</td>
</tr>
<tr>
<td>5</td>
<td>Total Inductance</td>
<td>$L_{\sigma}$</td>
</tr>
<tr>
<td>6</td>
<td>Snubber capacitor per position</td>
<td>$C_s$</td>
</tr>
<tr>
<td>7</td>
<td>Transformer turns ratio</td>
<td>$n$</td>
</tr>
</tbody>
</table>

Table 7.1: Converter specifications

The measuring equipment is summarised in Appendix G.

7.2 Measurements at nominal power

At nominal power the phase-shift angle should be 90 degrees. Due to load power handling limitations the phase-shift angle was limited to values less than 90 degrees during measurements, and the nominal power delivered to the load was 10 kW. Consequently, the converter can deliver considerably higher power than 10 kW at a phase-shift angle of 90 degrees. Additionally, the converter was operated as a step-down converter and at certain loads as a step-up converter for demonstration reasons.

In Figure 7.1 the inductor current and the secondary snubber capacitor voltage are shown. Thus, at $i_{L,\sigma}(\phi)$ the secondary placed controllable switches are turned-off and the snubber capacitors are charging up, forcing the antiparallel diode to commutate the current.
In order to fulfil the soft-switching constraints, the controllable switches must turn off a finite current as discussed in Chapter 6. Additionally, the phase-shift angle must be limited within a certain range as given in Chapter 6. By examining Figure 7.1 it is obvious that the secondary
placed controllable switches are turned-off under ZVS conditions resulting to almost negligible switching losses. At turn-off, and when the capacitors are fully charged oscillations are generated. Part of the oscillations are caused by the stray inductance in the circuit, but the major part of the oscillations are caused during the reversal of the voltage of the transformer winding capacitance.

The inductor current and voltage at a conversion ratio 0.86 are shown in Figure 7.3. As discussed in Chapter 6 oscillations are present in the inductor voltage but not in the inductor current. Observe that the inductor current presented in the figure is the primary current. Oscillations are presented in the same figure. As shown in the figure the oscillating current is flowing through the snubber capacitors implying that are part of the resonance circuit. Nevertheless, the different frequencies appears in the current, and as a result two different resonance modes are present as discussed in Chapter 6.

![Figure 7.3: Measured inductor current, snubber current, and inductor voltage](image)

In addition very high frequency oscillations are taking place when the charging current is reaching the peak value. These oscillations are result of the stray inductance of the snubber...
circuit and the stray inductance of the IGBT module. Oscillations of this nature are well known and have been thoroughly examined.

Measurements have also been performed at a higher conversion ratio as shown in Figure 7.4. As shown in the figure the oscillations are not affected by the conversion ratio but on the contrary are expected to be damped under light load conditions.

Figure 7.4: Measured inductor current, snubber current, and inductor voltage at $M=0.96$

As discussed in Section 6.5, when the secondary placed controllable switches are turned off the current will not commutate to the diode. The current is clamped at zero. This due to the reversal of the voltage across the winding capacitance, thus, the winding capacitance is charged and/or discharged. This is clearly shown in Figure 7.5. Thus, oscillations are present only when the secondary placed controllable switches are turned off. Figure 7.5 can be compared with Figure 6.20.

In Figure 7.6 the primary and the secondary currents are shown. The difference between the primary and the secondary current is shown in the last plot. As shown in the figure the
oscillations occur during the commutation of the secondary placed controllable switches. Assuming that the primary and the secondary current are in phase the difference of the two currents should represent the current flowing through the magnetising inductance and the winding capacitance of the transformer.

Figure 7.5: Measured secondary current at nominal load

Figure 7.6: Measured primary and secondary current at nominal load.
7.3 Measurements under light load conditions

As discussed above the oscillations are expected to be damped under light load conditions. Soft-switching should be conserved despite the decrease in turn-off current. At very light loads the soft-switching of the controllable switches are expected to be lost resulting in switching losses and a decrease in the efficiency of the converter.

In Figure 7.7 the inductor current and voltage for $R = 25 \, \Omega$ are presented. The oscillations are heavily damped but on the contrary the impact of the magnetising inductance is comparably high. The impact of the magnetising inductance is represented by the finite slope of the inductor voltage during the conducting time of the controllable switch.

![Figure 7.7: Inductor current and voltage](image)

The lighter the load, the higher the impact of the magnetising inductance. This is illustrated in Figure 7.8 where the load is lighter than in Figure 7.7.
In Figure 7.9 the inductor current and the snubber voltage for the lightest load examined are shown. As shown in the figure the snubber capacitor voltage is still fully charged in a controllable manner assuring soft-switched turn-off of the controllable switches. It is the opinion of the author that for lighter loads than those presented in the figure the soft-switching condition is lost. Additionally, if the conversion ratio requested by the controller is lower than this used in the measurement, the phase-shift angle should be comparably low and thereby violating the soft-switching condition as discussed in Chapter 6. As a result, the controllable switches are expected to be turned-off under hard-switching conditions.
7.4 Measurements at step-up mode of operation

The DAB converter has the ability to operate in the step-up mode. In this mode of operation the conduction time of the primary placed diodes corresponds to the conduction time of the primary placed controllable switches when the converter operates in the step-down mode. Thus, the conduction losses of the primary bridge are expected to be lower in the step-up mode than in the step-down mode. On the contrary, the conduction time of the secondary placed controllable switches is considerably increased in the step-up mode. Specifically, the conduction time of the secondary placed controllable switches corresponds to the conduction time of the secondary placed diodes when the converter operates in the step-down mode. This implies that the conduction losses of the secondary bridge are higher in the step-up mode than in the step-down mode.

For clarification reasons, when the converter operates in the step-down mode the conduction time of the primary placed controllable switches is considerably higher than the conduction time of the diodes. For the secondary bridge, the conduction time of the diodes is considerably higher than the conduction time of the controllable switches at step-down. The measurements in the step-up mode were carried out just to demonstrate the ability of the DAB
EXPERIMENTAL VERIFICATION OF THE PHASE-SHIFT CONTROLLED DUAL-ACTIVE BRIDGE

converter to boost the output voltage within reasonable limits with maintained high efficiency.

In both Figure 7.10 and Figure 7.11 the inductor current and voltage and, the charging/discharging snubber capacitor current are shown. The converter has the ability to boost the output voltage corresponding to 1.4 times the input dc link voltage. As in the step-down operation the oscillations are present in the step-up mode as well. The oscillations are expected to be damped at light loads and amplified at heavy loads as already discussed.

Figure 7.10: Step-up operation. Inductor current and voltage
7.5 Efficiency measurements

The efficiency of the converter has been measured at different power levels and at different conversion ratios. Comparing with the SAB topology the efficiency is expected to degrade due to the switching action of the secondary placed switches. The secondary placed controllable switches are contributing to an increase of the overall conduction losses of the converter, assuming that the switching losses are negligible or very low. For very light loads where the soft-switching is lost switching losses from the secondary bridge should be added to the total losses.

The efficiency of the converter at different conversion ratios, both in the step-down and step-up mode is shown in Figure 7.12. As shown in the figure the efficiency of the converter, operating in the step-down mode, is varying between 93% and 95%. This can be a result of the contribution of the conduction losses to the overall efficiency of the converter. Thus, for $M=0.96$ the inductor current is lower compared with the current at $M=1$. As a result the conduction losses are lower, resulting in a higher efficiency. This is also true for low power levels where the soft-switching conditions are fulfilled. For high powers the peak current is considerably higher implying increased conduction losses. Additionally, at heavy loads the
oscillations are more severe than at light loads and are contributing to a decrease in the efficiency of the converter.

![Graph showing output power versus efficiency with different conversion ratios]

*Figure 7.12: The efficiency of the DAB topology at different conversion ratios*

When the converter is operating as a step-up converter the peak value of the inductor current is considerably higher, Figure 7.10, than in the step-down mode as shown in Figure 7.7. At the specified load the peak inductor current is 1.8 times higher in the step-up mode than in the step-down mode. Consequently, the averaged switch and diode current is 1.8 higher than in the step-down operation, and therefore, the conduction losses are considerably higher than in the step-down mode.

### 7.6 Converter dynamics

The converter dynamics have been studied, i.e. the response of the converter with respect to reference steps. In Figure 7.13 the response of the system to step in the reference voltage at heavy load is shown. In Figure 7.13(a) the reference voltage is rapidly increased with 10% of the nominal value and in Figure 7.13(b) the reference voltage is rapidly decreased by 10%. In both cases the system is neither unstable nor oscillatory. The voltage is increased and/or decreased monotonically as governed by the controller, in order to reach the new reference value.
Similarly, in Figure 7.14 the dynamic behaviour of the converter operating at light load is presented. The converter behaves in an identical way as in the heavy load case.
Figure 7.14: Dynamic response at light load for -10% step

Figure 7.15: Controller performance during reference voltage step -10%
Figure 7.16: Controller performance during reference voltage step +10%

In Figure 7.15 and in Figure 7.16 the performance of the controller during voltage reference steps is presented. In both cases the controller is neither unstable nor oscillatory. Both output voltage and inductor current are increasing and decreasing monotonically without causing any instability of the system. The measurements presented in both figures are the actual values sampled by the control unit. The signals presented in the figures are:

- Upper window in the figures: input, output and, reference voltage.
- Second window in the figures: Measured and reference current
- Third window in the figures: Measured and reference value for the phase-shift angle
- Last window in the figures: Alive signal, indicating that both bridges are synchronised and are operating (operating mode).
7.7 Summary and conclusions

The Dual-Active Bridge employing a conventional phase-shift control strategy has been analysed and studied in this Chapter. Both the steady-state and the dynamic behaviour of the converter has been analysed and presented. Converter dynamics have been presented by using a state-space averaging technique based on the symmetry assumption. It has been shown that both the power plants and the whole system including the controller are stable with favourable characteristics. The dynamic behaviour of the system is quite similar to those presented for the solution employing a SAB topology.

Measurements have shown that the oscillations are present even in the DAB topology. However these oscillations are not caused by a discontinuous inductor current as in the SAB, but are a result of a resonance between the inductor and the winding capacitance of the transformer. The measured efficiency of the converter is varying between 93% and 95% and is lower, at nominal power, than the efficiency of the SAB converter at nominal power. This is due to the switching action of the secondary bridge.

The performance of the system due to reference voltage steps was studied in the laboratory and the results are presented in this chapter. The system was proven to be stable and that it can deal successfully with steps in the reference voltage without causing any instabilities.
8 NEW CONTROL STRATEGY FOR THE DAB CONVERTER

8.1 Introduction

A control strategy introducing an additional phase-shift angle has been proposed in order to improve the characteristics of the DAB topology in [Dem5]. This strategy is referred to as duty-cycle modulation, i.e. a variation of the effective duty cycle of the converter. As the additional phase-shift angle is introduced a freewheeling stage which reduces the effective duty cycle is created. During the freewheeling stage the secondary voltage of the transformer is clamped to zero and the current is circulating in the transformer windings. When the converter operates in the step-up mode the primary current is a quasi-sine waveform with a low harmonic content. In [Khe3] identical results have been presented by using a three-phase concept. Even though the idea of the duty-cycle modulation was first presented in [Dem5], the first experimental results of the control method were presented in [Zha1]. In [Zha1] the superiority of the new control strategy was claimed with an efficiency improvement of up to 2%. The efficiency measurements were, however, performed at a unity conversion ratio, i.e. at the optimal operating point with respect to efficiency. At unity conversion ratio the soft-switching constraints are fulfilled. In order to obtain a complete understanding of the behaviour of the converter, measurements have to be performed also at other values of the conversion ratio.

Another benefit of the control strategy is that the kvar rating of the filter capacitor is expected to be decreased [Zha1] compared with the conventional DAB topology.

In [Cha1] another control method has been presented. It is evident to the author, however, that the control method proposed in [Cha1] will increase the circulating power in the converter. Due to associated additional conduction losses this control method has therefore not been considered in the present thesis instead all attention is paid to the original idea as presented in [Dem5].
First in Section 8.2 the steady state analysis of the converter is presented. Key equations and key issues as soft-switching boundaries are presented. In Section 8.3 the state-space averaged and the dynamic small-signal model of the converter is derived and key transfer functions are presented. Furthermore, the oscillations as discussed in Chapter 4 and 6 are briefly discussed. Finally, in Section 8.5 the controller of the converter is presented.

8.2 Steady-state analysis

The equivalent circuits, during one half-cycle, for each mode of operation, are shown in Figure 8.2. As in the case of the conventional DAB the commutation sequence starts by assuming that the current is flowing through the devices $D_{A+}$, $D_{D3}$ and $D_{e2}$ as shown in Figure 8.2(a). Additionally, the transistors $T_{s3}$ and $T_{s2}$ are on. While the current is flowing through diodes $D_{A+}$, $D_{D3}$ and $D_{e2}$ the voltage across the switches $T_{d+}$ and $T_{s2}$ and $T_{s3}$ is zero. The transistor $T_{d+}$ is switched at ZVS and at a certain time $\omega t_0$ corresponding to the phase-shift angle $\alpha$ the switch $T_{s2}$ is turned off and the energy stored in the inductance is transferred to the snubber capacitors by resonance.

\[ \frac{dV_s}{dt} = 0 \]

\[ V_s(t) = \begin{cases} 
  V_{s0} & \text{for } 0 \leq t \leq \alpha \omega t_0 \\
  -V_{s0} & \text{for } \alpha \omega t_0 < t < \alpha \omega t_0 + \phi \\
  V_{s0} & \text{for } \alpha \omega t_0 + \phi \leq t < \omega t_1 \\
  -V_{s0} & \text{for } \omega t_1 \leq t < \omega t_2 \\
  V_{s0} & \text{for } \omega t_2 \leq t < \omega t_3 \\
  -V_{s0} & \text{for } \omega t_3 \leq t < \omega t_4 \\
  \cdots 
\end{cases} \]

Figure 8.1: Steady-state operation waveforms
The snubber capacitor, which is placed across $T_{s2}$, will take over the current, and the transistor turn-off occurs under ZVS conditions provided that the device is carrying a certain minimum current. Additionally, the snubber capacitor, which is connected across the transistor $T_{s1}$, will be discharged and will eventually force the diode $D_{s1}$ to be forward-biased.

The load current is flowing through the diode $D_{s1}$ and the transistor $T_{s3}$ as shown in Figure 8.2(c). The secondary voltage is zero and the power is circulating at the secondary side of the transformer forcing the inductor voltage to be equal to the dc-link voltage. Consequently, the effective duty cycle of the converter is reduced due to the finite slope of the inductor current as reported in [Vla1] and [Sab1]. At a phase-shift angle $\phi$ the switch $T_{s3}$ is turned off at ZVS due to the capacitive snubber action forcing the diode $D_{s4}$ to commutate the load current at ZCS. As a result, the current is flowing through the diodes $D_{s1}$ and $D_{s4}$ as shown in Figure 8.2(d).

When the transistor $T_{a+}$ is turned off the energy stored in the inductance is transferred to the snubber capacitors by resonance. The snubber capacitor, which is placed across $T_{a+}$, will take over the current, and the transistor turn-off occurs under ZVS conditions provided that the device is carrying a certain minimum current. Simultaneously, the snubber capacitor, which is connected across the transistor $T_{a-}$, will be discharged and will eventually force the diode $D_{a-}$ to be forward-biased. Therefore, the inductor current starts flowing through the diode $D_{a-}$ and diodes $D_{s1}$ and $D_{s4}$, as shown in Figure 8.2(e). Similarly, the transistor $T_{a-}$ is turned on at ZVS and the commutation sequence will be repeated as above.

<table>
<thead>
<tr>
<th>Equivalent circuit</th>
<th>Conducting device</th>
<th>$V_{La}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 8.2(a)</td>
<td>$D_{a+}$ and $D_{s1}$ &amp; $D_{s2}$</td>
<td>$V_{La} = V_{dc} + V_0$</td>
</tr>
<tr>
<td>Figure 8.2(c)</td>
<td>$T_{a+}$ and $T_{s3}$ &amp; $D_{s1}$</td>
<td>$V_{L_{a+}} = +V_{dc}$</td>
</tr>
<tr>
<td>Figure 8.2(d)</td>
<td>$T_{a+}$ and $D_{s1}$ &amp; $D_{s4}$</td>
<td>$V_{L_{a+}} = V_{dc} - V_0$</td>
</tr>
</tbody>
</table>

*Table 8.1: The inductor voltage for the DAB topology employing the new control strategy.*
Due to the symmetrical operation of the circuit, the equivalent circuits concerning the transistor $T_A$ and the secondary placed active switches can be easily obtained by following the same procedure.

The voltage across the inductor, $V_{L_v}$, can be expressed as shown in Table 8.1

A quite worth noticing phenomenon taking place during the conduction of the secondary placed controllable switches which are placed on the same diagonal. Assume that $T_{s_3}$ is turned on and after a certain time the switch $T_{s_2}$ is turned on as shown in Figure 8.2. The load current is expected to commutate from the diode $D_{s_2}$ and $D_{s_3}$ to the switches $T_{s_2}$ and $T_{s_3}$ respectively.

Obtain that the current should commutate from the diodes to the switches at zero-crossing. The current is commutating to the switch $T_{s_3}$ but for certain $\theta = \alpha$ the current will never commutate from the diode $D_{s_2}$ to $T_{s_2}$. On the contrary, the current is freewheeling through the diode $D_{s_1}$ and the switch $T_{s_3}$. During the second half-cycle yields the above statement, thus, for certain $\theta = \alpha$ the switch $T_{s_1}$ will never commutate the load current. Consequently, the two controllable switches, $T_{s_1}$ and $T_{s_2}$, can be removed. Thus, the converter will employ one bridge with two controllable switches, $T_{s_3}$ and $T_{s_4}$, and one bridge leg with only diodes, $D_{s_1}$ and $D_{s_2}$.

The complexity of the proposed converter will be comparably decreased and the efficiency is expected to comparably increase. This configuration is not considered in this thesis but it can be studied in the future. A similar topology has been studied in [Mor1] but with an LC filter at the output. The additional inductance at the output will affect the dynamic and the steady-state behaviour of the converter. The converter proposed in this thesis however, should employ a low-pass RC filter.

However, the same work has been done and some results have been presented by Zhang et al. in [Zha2]. It is evident to the author that the converter proposed in this thesis reflects the ideas of the author and the results obtained are due to the research activities conducted (performed) by the author. Clearly, the two research activities, the authors and Zhang et al., were performed in parallel.
Figure 8.2: Equivalent circuits for the different modes of operation
Based on the equivalent circuits and, the circuits behaviour as described in this paragraph, three modes of operation are of interest, i.e.

- Mode 1: The two voltage sources have different polarity
- Mode 2: Freewheeling stage
- Mode 3: The two voltage sources have the same polarity

**Mode 1**: \(0 \leq \omega t \leq \omega t_0\)

In this mode of operation the inductor current corresponds to

\[
i_{L,\sigma}(\omega t) = \frac{V_{dc} + V_0}{\omega L_{\sigma} \omega} (\omega t) + i_{L,\sigma}(0) \quad (8.1)
\]

**Mode 2**: \(\omega t_0 \leq \omega t \leq \omega t_0\)

\[
i_{L,\sigma}(\omega t) = \frac{V_{dc}}{\omega L_{\sigma} \omega} (\omega t - \omega t_0) + i_{L,\sigma}(\omega t_0') \quad (8.2)
\]

Similarly, the inductor current for the converter operating in Mode 3 can be expressed as given by Eq. (8.3).

**Mode 3**: \(\omega t_0 \leq \omega t \leq \omega t_1\)

\[
i_{L,\sigma}(\omega t) = \frac{V_{dc}}{\omega L_{\sigma} \omega} (\omega t - \omega t_0) + i_{L,\sigma}(\omega t_0) \quad (8.3)
\]

From symmetrical operation

\[
i_{L,\sigma}(0) = -i_{L,\sigma}(\omega t_1) \quad (8.4)
\]

\[
i_{L,\sigma}(0) = -i_{L,\sigma}(\pi)
\]

The initial current \(i_{L,\sigma}(0)\) can now be calculated in steps. From Eq. (8.1) \(i_{L,\sigma}(\omega t_0')\) is found by insertion of \(\omega t = \omega t_0'.\) From Eq. (8.2) \(i_{L,\sigma}(\omega t_0)\) is found by setting \(\omega t = \omega t_0.\)
and inserting $i_{L,\sigma}(\omega t_0')$ from Eq. (8.1). Similarly, from Eq. (8.3) $i_{L,\sigma}(\omega t_1)$ is found by setting $\omega t = \omega t_1$ and inserting $i_{L,\sigma}(\omega t_0)$ from Eq. (8.2). From Figure 8.1 it is found that $\omega t_0' - 0 = \alpha$, $\omega t_0 - 0 = \phi$, $\omega t_0 - \omega t_0' = \phi - \alpha$, and $\omega t_1 - \omega t_0 = \pi - \phi$.

Rearranging the resulting equations and introducing $M = \frac{V_0}{V_{dc}}$ and from Eq. (8.4) yields

$$i_{L,\sigma}(0) = -\frac{V_{dc}}{2 \omega L_{\sigma}}(1 - M)\pi + M \phi + M \alpha$$

(8.5)

$$i_{L,\sigma}(\phi) = \frac{V_{dc}}{\omega L_{\sigma}}(\phi - \alpha)$$

(8.6)

Knowing that the inductor current is zero at $\theta = \alpha$, and form Eq. (8.1) the $i_{L,\sigma}(\omega t_0')$ is found by insertion of $\omega t = \omega t_0'$. By equating $i_{L,\sigma}(\omega t_0') = 0$, Eq. (8.1) is solved in terms of $i_{L,\sigma}(0)$. By rearranging Eq. (8.1) and Eq. (8.5), the additional angle $\theta = \alpha$ can be expressed in terms of the conversion ratio and the phase-shift angle $\phi$ as given by

$$\alpha = \frac{1}{2 + M}(1 - M)\pi + M \phi$$

(8.7)

Equation (8.7) was implemented into the control algorithm. The angle $\theta = \alpha$ is calculated online by the controller.

8.2.1 Boundaries for zero-voltage switching

In order to achieve soft-switching, i.e. zero-voltage switching, the device must be conducting a certain current in order to charge and discharge the snubber capacitors at a controllable rate. As shown in Figure 8.1 and at $\omega t = 0$, $\theta = 0$, the controllable switch $T_{L,-}$ is turned off and the charging and the discharging process of the snubber capacitors is initiated. When the snubber capacitor which is placed in parallel with $T_{L,-}$ is fully charged
the diode $D_{A^+}$ is forward biased and is conducting a negative current. Similarly, at $\theta = \phi$ the current is positive when $T_{A^+}$ is turned off.

The above statement corresponds to

\[
i_{L,\sigma}(0) \leq 0 \tag{8.8}
\]

\[
i_{L,\sigma}(\phi) \geq 0 \tag{8.9}
\]

Additionally, from Eq. (8.1) \(i_{L,\sigma}(\omega t_0')\) is found by insertion of $\omega t = \omega t_0'$. Knowing that $i_{L,\sigma}(\omega t_0') = 0$, and $\omega t_0' - 0 = \alpha$, and from Eq. (8.8) yields

\[
\alpha \geq 0 \tag{8.10}
\]

Similarly and by rearranging Eq. (8.6) and (8.9) results to

\[
\alpha \leq \phi \tag{8.11}
\]

From Eq. (8.10), (8.11), and (8.7) the soft switching boundaries can be defined as

\[
\phi \geq \frac{1-M}{2} \pi \tag{8.12}
\]

\[
\phi \geq \frac{M-1}{M} \pi \tag{8.13}
\]

Equation (8.12) is true only for conversion ratios $M < 1$. Similarly, Eq.(8.13) is true only for $M > 1$.

In Figure 8.3 the soft-switching boundaries are shown. When the conversion ratio is $M \geq 1$ the minimum phase-shift angle $\phi$ has to take certain values, which are quite high, in order to achieve ZVS for the controllable switches of the input bridge. Similarly, when $M \leq 1$, ZVS is maintained for the output bridge in the area where the $\phi$ is higher than the minimum phase-shift angle as given by Eq. (8.12). As shown in Figure 8.3 the minimum phase-shift angle for the lowest conversion ratio, $M = 0.4$ is higher than the minimum phase-shift angle for
$M = 0.8$. On the contrary, the phase-shift angle for $M = 0.4$ is comparable with the minimum phase-shift angles corresponding to $M = 1.2$ and $M = 1.4$.

![Figure 8.3: Soft-switching boundaries](image)

### 8.2.2 Output Power capability of the converter

The output power capability of the converter can be derived assuming that the input power is fully delivered to the output and by neglecting the losses. From Eq. (8.5), (8.6), and Eq. (8.7), the mean value of the output current can be expressed as

$$I_0 = \frac{V_{dc}}{2 \phi \omega L_\sigma} \Phi_m \quad (8.14)$$

Where

$$\Phi_m = 4 \phi \left(1 + M + M^2\right) - \frac{2 \phi^2}{\pi} \left(2 + 2M + M^2\right) + \pi \left(1 + M - 2M^2\right) \quad (8.15)$$

The output power of the converter corresponds to
Equation (8.16) clarifies that the power capability of the converter is not depended on \( \theta = \alpha \) but on the contrary the output power is dependent on the components and rating of the converter as, inductance, input and output voltage, switching frequency and, finally on the phase-shift angle \( \phi \).

In order to define the phase-shift angle corresponding to the maximum power, the partial derivative of the output power is defined and set to zero as given by Eq. (8.17).

\[
\frac{\partial P_0}{\partial \phi} = 0
\]  

Solving Eq. (8.17) in terms of the phase-shift angle the maximum power transfer occurs when

\[
\phi = \frac{M^2 + M + 1}{M^2 + 2M + 2} \pi
\]  

The result presented in Eq. (8.18) differs for the results presented in the case of the phase-shift controlled DAB converter. As discussed in Chapter 6, the maximum power transfer for the DAB topology occurs at a phase-shift angle \( \phi = 90^\circ \). The maximum power versus the phase-shift angle is presented in Figure 8.4.

Another interesting remark which can be observed in Figure 8.4 is that the maximum power capability has been considerably reduced compared with the conventional DAB converter. Thus, for identical converters, employing different control strategies, the power transfer delivered to the load is reduced with 0.2 p.u. This could be a disadvantage if the power level cannot be kept by other means, for instance by changing the inductance.

Nevertheless, a quite important feature of the proposed control method is the ability of the converter to fully deliver the input power to the load. By examining Figure 8.1, the polarity of the inductor current is the same as the polarity of the primary voltage. As a result, the average power during a half-cycle has the same polarity as the inductor current and the primary voltage indicating that the input power is fully delivered to the load. If the polarity of the power changes during a half-cycle, as in the case of the phase-shifted DAB, the power will
pulsate back and forth between the source and the load. This can probably explain the 0.2 p.u difference in power capability of the two converters.

Another interesting feature of the converter is the diversity of phase-shift angles needed to deliver the maximum output power at different conversion ratios. In the phase-shifted DAB topology the maximum power is delivered from the source to the load at $\phi = 90^\circ$ for every conversion ratio. When the duty-cycle modulation is used the maximum power is delivered to the load at $\theta = \phi_{M_i}$ where $\phi_{M_i}$ is the phase-shift angle corresponding to a conversion ratio $M_i$. The above conclusion is illustrated in Figure 8.4 and by Eq. (8.18).

![Figure 8.4: Output power versus the phase-shift angle $\phi$.](image)
8.3 Converter dynamics

As in the case of the SAB and the phase-shift controlled DAB converters as basis for the controller design the dynamic small-signal model is derived. Deriving a small-signal model, the controller design is quite straightforward, and critical issues as stability problems can be easily foreseen with pole plots. Understanding of the dynamic properties of the converter is provided as well.

The dynamic behaviour of the converter is expected to be identical to the dynamic behaviour of the phase-shifted DAB converter. Nevertheless, the introduction of the freewheeling interval is affecting the behaviour of the converter both in steady state and transient. Consequently, the phase-shift angle $\alpha$ can be considered as a control parameter. Thus, the control-to-output and the control-to-state transfer functions are no longer dependent only on $\phi$ but on the contrary, $\alpha$ will have a considerable impact on the transfer functions.

The state variables of the system are the same as in the cases of the SAB and the conventional controlled DAB converters. Thus, the inductor current, and the voltage of the capacitor, are the states of interest. The state-space averaging model based on half-cycle symmetry is used resulting in the state-space averaging model of the converter as given by Eq. (8.19).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1-d_1}{L_{\sigma}} \\ \frac{1}{C} & \frac{1}{R C} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{2d-1}{L_{\sigma}} \\ 0 \end{bmatrix} V_{dc}$$

(8.19)

The duty ratio of the primary placed controllable switch corresponds to $d$ and $d_1$ is defined in Eq. (8.20).

$$d_1 = \frac{\phi - \alpha}{2 \pi}$$

(8.20)

Comparing Eq. (8.19) with the state-space averaged model corresponding to SAB as derived in Chapter 4 the only term which is not identical is the expression $-\frac{1-d_1}{L_{\sigma}}$, i.e. the mean value of the inductor voltage. This deviation is thoroughly explained in the Appendix in Appendix D.
By following the small-signal linearization procedure, as applied for the SAB and DAB converters, and, from Eq. (8.19) and (8.20), the small-signal model for the duty-cycle controlled DAB topology is defined as

\[
\begin{bmatrix}
-x_1 \\
-x_2
\end{bmatrix}
= \begin{bmatrix}
A_{11} & A_{12} \\
\frac{1}{C} & -\frac{1}{R C}
\end{bmatrix}
\begin{bmatrix}
-x_1 \\
-x_2
\end{bmatrix}
+ \begin{bmatrix}
B_{11} \\
0
\end{bmatrix} V_{dc} + \begin{bmatrix}
E_{11} & E_{12}
\end{bmatrix}
\begin{bmatrix}
\phi \\
\alpha
\end{bmatrix}
\]

(8.21)

Where

\[
A_{11} = -\frac{2 V_{dc0}}{T_s (V_{dc0} + x_{20})}
\]

(8.22)

\[
A_{12} = -\frac{1}{L_\sigma} \left(1 - \frac{\phi_0 - \alpha_0}{2 \pi}\right) + \frac{2 x_{20} V_{dc0}}{T_s (V_{dc0} + x_{20})^2}
\]

(8.23)

\[
B_{11} = \frac{1}{L_\sigma} - \frac{2 x_{10} x_{20}}{T_s (V_{dc0} + x_{20})^2}
\]

(8.24)

\[
E_{11} = \frac{x_{20}}{2 \pi L_\sigma}
\]

(8.25)

\[
E_{12} = -\frac{x_{20}}{2 \pi L_\sigma}
\]

(8.26)

**8.3.1 Small-signal control-to-output transfer function**

The small-signal control-to-output transfer function can be derived as

\[
G_{sd}(s) = \frac{-x_2(s)}{\phi(s)} \bigg|_{u(s)=0} = \left[ s I - A_0 \right]^{-1} \begin{bmatrix}
E
\end{bmatrix}
\]

(8.27)

The control parameter is the phase-shift angle $\phi$. 

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The numerator of the control-to-output transfer function corresponds to

\[ N_{G_{vd}}(s) = \frac{E_{i1}}{C} \]  

Similarly, the determinant of the system is defined as

\[ \Delta(s) = s^2 + s \left( \frac{1}{RC} - A_{11} \right) - \frac{A_{11}}{RC} \frac{A_{12}}{C} \]  

The control-to-output transfer function is presented in Figure 8.5. As shown in the figure the system is stable with a crossover frequency of 31.6 rad/sec and a phase margin of 10°. The phase margin is not adequate and it should be increased. In the literature, [Kre1], and [Eri1], the minimum phase margin is defined to a value of between 45° and 60°.

![Bode Diagram](image)

*Figure 8.5: Small-signal control-to-output transfer function*
In Figure 8.6 and Figure 8.7 the step response and the poles and zeroes map are presented. The step response of the system is a monotonically increasing function without overshoot or oscillations. This is due to the fact that both poles are located in the LHP and their imaginary part is zero. If the poles had an imaginary part, the system would oscillate.

**Figure 8.6: Step response**

**Figure 8.7: Poles and zeroes map**
8.3.2 The small-signal control-to-state transfer function

The small-signal control-to-state transfer function is defined in Eq. (8.31).

\[
G_{sd}(s) = \left. \frac{\Delta x}{\phi} \right|_{U(s)=0} = \left[ sI - A_0 \right]^{-1} E \tag{8.31}
\]

\[
G_{sd}(s) = \frac{N_{G_{sd}}(s)}{\Delta(s)} \tag{8.32}
\]

Where

\[
N_{G_{sd}}(s) = E_{11} \left( s + \frac{1}{RC} \right) \tag{8.33}
\]

The control-to-state transfer function of the system is shown in Figure 8.8 and stable characteristics are presented.

![Figure 8.8: Control-to-state transfer function](image-url)
In Figure 8.9 the step response of the system is illustrated. As shown in the figure the current has a considerable overshoot and then returns to the steady-state operating point without oscillations. The overshoot is a result of the LHP zero of the transfer function.

![Step Response](image)

*Figure 8.9: Control-to-state transfer function. Step response*

The poles and zeroes map is shown in Figure 8.10. Both poles and the zero are placed in the LHP ensuring the stability of the system. Additionally, as in the case of the control-to-output transfer function, the imaginary parts of the poles are zero, which implies that the system will not exhibit any oscillatory behaviour. This ensures that control and regulation of the system is quite relaxed.
8.3.3 Small-signal source-to-output transfer function

The source-to-output transfer function of the DAB topology can be expressed as

$$G_{sg}(s) = \frac{x_2(s)}{U(s)} = \left[ sI - A_0 \right]^{-1} B_0$$ (8.34)

Thus,

$$G_{sg}(s) = \frac{N_{G_{sg}}(s)}{\Delta(s)}$$ (8.35)

The numerator of the small-signal transfer function corresponds to

$$N_{G_{sg}}(s) = \frac{B_{11}}{C}$$ (8.36)

As shown in Figure 8.11, small-signal variations around the steady-state value of the input voltage are amplified and added to the output voltage. This is not favourable, but on the other hand, small-signal perturbations are not causing instabilities or oscillations to the system.

Figure 8.10: Control-to-state transfer function. Poles and zeroes map.
Furthermore, low-frequency perturbations in the input voltage can be handled by the control. As will be presented later a feedforward of the input voltage effectively eliminates this problem.

In Figure 8.12 and Figure 8.13 the step response and the poles and zeroes map of the transfer function are shown. The transfer function is stable and no oscillations are present.

*Figure 8.11: Small-signal source-to-output transfer function.*
Figure 8.12: Small-signal source-to-output transfer function. Step response.

Figure 8.13: Small-signal source-to-output transfer function. Poles and zeroes map.
8.3.4 Small-signal source-to-state transfer function

The susceptibility of the state to small-signal variations of the source is defined in Eq. (8.37).

\[
G_{ss}(s) = \frac{\bar{x}_1(s)}{U(s)} \bigg|_{s=0} = \left[ sI - A_0 \right]^{-1} B_0 \tag{8.37}
\]

Eq. (8.37) results to

\[
G_{ss}(s) = \frac{N_{G_{ss}}(s)}{\Delta(s)} \tag{8.38}
\]

Where,

\[
N_{G_{ss}}(s) = B_{11} \left( s + \frac{1}{RC} \right) \tag{8.39}
\]

The susceptibility of the state to small-signal variations of the source is fully described by Figure 8.14, Figure 8.15 and, Figure 8.16. Small-signal variations of the source are damped which is a favourable property.

Figure 8.14: Small-signal source-to-state transfer function.
Comparing, the source-to-state transfer function with the source-to-output transfer function, a LHP zero is present which is forcing the phase angle to overshoot the frequency axis and increase the magnitude of the source-to-state transfer function. Similar characteristics where observed in the SAB topology.

**Figure 8.15: Small-signal source-to-state transfer function. Step response.**

**Figure 8.16: Small-signal source-to-state transfer function. Poles and zeroes map.**
The impact of the phase-shift angle $\alpha$ to the transfer functions has been examined as well. Both the control-to-output and control-to-state small-signal transfer functions are shown in Figure 8.17. In Figure 8.17 (a) the Bode diagram corresponding to the control-to-output transfer function is shown and in Figure 8.17 (b) the step response of the control-to-output transfer function is plotted. Additionally, the bode diagram and the step response of the control-to-state small-signal transfer function is shown in Figure 8.17 (c) and Figure 8.17 (d).

Figure 8.17: Small-signal transfer functions corresponding to the phase-shift angle $\alpha$. (a) Control-to-output, (b) Step response, (c) Control-to-state, (d) Step response.
It appears that the transfer functions are identical to those corresponding to the phase-shift angle \( \phi \), but, with a minus sign on the denominator coefficient. This is obvious when studying Eqs. (8.21), (8.25) and (8.26), since \( E_{12} = -E_{11} \). As a result, when a step in the phase-shift angle \( \alpha \) is applied both the output and the state are taking negative values. Despite this behaviour no oscillations are taking place.

8.4 Oscillations during commutations

As in the case of the phase-shift controlled DAB converter, significant oscillations are taking place during the commutation of the secondary placed active devices. The nature of the oscillations is identical as in the DAB converter and is briefly discussed in Chapter 6.

8.5 Control and regulation

The control and regulation strategy used in this chapter is quite different than the one used in the phase-shift controlled DAB topology. In this case, the impact of \( \alpha \) is added to the control scheme. The phase-shift angle \( \alpha \) is calculated on-line as derived by Eq. (8.7). At start-up the converter operates as a SAB topology and at a threshold voltage level the phase-shift angle \( \phi \) is added by the controller. Additionally, the phase-shift angle \( \alpha \) is introduced. Consequently, the inrush current is avoided and the stability of the circuit is assured.

The controller based on the small-signal analysis is presented in Figure 8.18. The cascade configuration is kept from the DAB converter. Thus, two PI regulators are connected in a cascade scheme with two loops, a voltage loop and an inner current loop. Additionally, the phase-shift angle \( \alpha \) is calculated on-line as indicated in Figure 8.18. Consequently, \( \alpha \) will have an impact on the states of interest, and is represented as two additional small-signal transfer functions, \( G_{sda} \) and \( G_{sda} \). The states of interest are the inductor current and the output voltage.
Figure 8.18: Small-signal controller for DAB converter employing new control strategy.

The procedure described in Section 4.12.4 was followed in order to tune the PI regulators. In Table 8.2 the controller parameters are summarised.

<table>
<thead>
<tr>
<th>Voltage regulator</th>
<th>Current regulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional gain $K_{pv} = 0.1$</td>
<td>Proportional gain $K_{pi} = 1$</td>
</tr>
<tr>
<td>Integration time $T_{pv} = 0.01$</td>
<td>Integration time $T_{pi} = 0.001$</td>
</tr>
</tbody>
</table>

Table 8.2: Controller parameters.

In Figure 8.19 the dynamic behaviour of the converter due to step response is examined. In the upper graph the output voltage is plotted. The inductor current is plotted in the second graph in the figure, and the step in phase-shift angle is plotted in the last graph in the figure.
A step has been applied around the steady-state value of the phase-shift angle $\phi$. As shown in Figure 8.19 the controller will force the phase-shift angle to return to its steady-state value in order to keep the output voltage constant. Minor oscillations are present in both the current and the voltage waveform. The phase-shift angle is oscillating as well around its steady-state value since it is derived from the current and voltage errors.

![Figure 8.19: Controller performance, simulated waveforms. Phase-shift angle step.](image)

In Figure 8.20 the output voltage is plotted in the upper graph of the figure with the corresponding oscillations as plotted to the upper graph placed at the right hand side of the figure. The inductor current is plotted in the second graph and the step in the reference voltage is plotted in the last graph of the figure.

In Figure 8.20 a step is applied to the reference voltage. The output voltage increases linearly since the current has reached the current limit. When the output voltage reaches the new reference voltage minor oscillations are observed. The peak value of the oscillation is approximately 0.5% of the output voltage and is damped after ten periods. Identical oscillations are also found in the current and consequently also in the phase-shift angle. The oscillations are damped after ten periods but are clearly visible in the current waveforms.
It is the opinion of the author that these oscillations can be reduced by an advanced controller design. This is however left for future work.

![Graph of small-signal variations around the reference voltage](image1)

![Graph of oscillations](image2)

**Figure 8.20**: Controller performance, simulated waveforms. Reference-voltage step

In Figure 8.21 the output voltage is plotted in the upper graph of the figure and the inductor current is represented by the second graph. The 300 Hz small–signal perturbations on the dc-link voltage are plotted in the last graph.

In order to check the line regulation ability of the converter small-signal perturbations are added around the steady-state value of the dc-link voltage. The frequency of the variations is 300 Hz which is a reasonable scenario in cases where a six-pulse diode rectifier is used. As shown in Figure 8.21 the variations are present in both waveforms but are heavily damped by the controller. This implies that small-signal variations of the source will not cause any stability problems. Additionally, higher ripple can be allowed in the input, thus, the kvar rating of the dc-link capacitor can be considerably reduced. This responds to a comparably small capacitor bank resulting to a considerably cost reduction.
8.6 Discussion and Remarks

In this chapter a new control strategy has been presented. The new control strategy is described in [Dem5] and results have been published in [Zha1]. Improvement of the characteristics of the converter is expected but on the other the power transfer capability of the converter is reduced with 0.2 p.u compared with the phase-shift controlled DAB converter. This is the major disadvantage of the proposed control strategy. On the other hand significant cost reduction is expected.

The small-signal model of the converter is presented and favourable characteristics have been obtained. Additionally, the controller has been designed based on the small-signal transfer functions of the plant as presented in this chapter. The ability of the converter concerning line regulation and load regulation has been studied as well as the dynamic behaviour of the converter due to step in both reference voltage and phase-shift angle has been presented.

Furthermore, oscillations during the commutation of the secondary placed switches are expected as in the SAB converter and the conventional controlled DAB converter. These oscillations are strongly dependent on the parasitics of the transformer and especially on the magnitude of the winding capacitance.
9 EXPERIMENTAL VERIFICATION OF THE DUTY-CYCLE CONTROLLED DAB CONVERTER

9.1 Introduction

In this chapter the duty-cycle control method for the DAB converter is verified experimentally. The specification (see Table 7.1) of the converter is the same as the phase-shift controlled DAB. Similarly to the phase-shift controlled DAB converter the bi-directional power transfer ability of the converter has not been considered in this thesis. The load of the converter consists of a low-pass filter, i.e. a resistor in parallel connection with a capacitor. Additionally, a six-pulse rectifier has been used on the primary side in order to feed the converter with a dc voltage. This prohibits the power to flow from the load to the source.

The purpose of the experimental set-up and testing is to:

- evaluate the limitations of the system, topology and controller
- evaluate the duty-cycle control strategy
- analyse the steady-state and dynamic behaviour of the system
- verify and evaluate the theoretical results
- compare with the results obtained with the phase-shift controlled DAB

The measuring equipment is summarised in Appendix G.
9.2 Measurements at heavy-load conditions

The behaviour of the converter both in steady-state and under transient conditions has been examined in a wide load range. In Figure 9.1 the inductor current and voltage, and the snubber current are illustrated. As in the case of the phase-shift controlled DAB converter, oscillations take place when the secondary placed controllable switches are turning off. The same mechanism as discussed in Chapter 6 is ruling the nature of the oscillations. Thus, when a secondary placed controllable switch is turning off the winding capacitance of the transformer has to be charged and/or discharged. The secondary current will flow through the snubber capacitors and the winding capacitance, and through the load. As a result, at light loads the oscillations are damped.

As shown in the figure the oscillations are not present at the current waveform measured on the primary side of the transformer.

Figure 9.1: Inductor current and voltage, and snubber current.
In Figure 9.2 the inductor and the snubber current as well as the snubber voltage are shown. Additionally the oscillations are enlarged in the right hand side of the figure. As shown in the figure the switches are turned off under ZVS conditions resulting in negligible switching losses.

![Figure 9.2: Measured inductor current, snubber current and voltage.](image)

In Figure 9.3 the primary current and voltage are presented. During the time interval $\phi - \alpha$ the secondary side of the transformer is short circuited and the current is circulating in the transformer windings as shown in the figure. Since the power transfer is interrupted the duty-cycle controlled DAB delivers less power to the load compared with the phase-shifted DAB converter. Additionally, the transformer has to be designed to handle the freewheeling current flowing in the windings.
Figure 9.3: Measured primary current and voltage.
9.3 Measurements at light-loads

Experimental measurements have been done under light load conditions. As discussed in this chapter the oscillations at light load conditions are expected to be damped by the load. This is illustrated in Figure 9.4.

Figure 9.4: Inductor current and voltage and, snubber current.
The primary current and voltage of the converter are illustrated in Figure 9.5. At light loads the freewheeling stage of the converter is less than in heavy loads. Thus, the difference between the phase-shift angle $\phi$ and the phase-shift angle $\alpha$ is reduced compared to the nominal load case. As a result, circulating current in the transformer windings is reduced. The oscillations obtained in the nominal load case at every switch transition are damped at light load as shown in Figure 9.5.

*Figure 9.5: Measured primary current and voltage.*
In Figure 9.6 the inductor current and the snubber voltage are shown. As shown in the figure the snubber capacitors are charged and discharged in a controllable rate ensuring the soft-switching turn-off of the controllable switches. The very high-frequency oscillations shown in the figure are due to the presence of the stray inductance of the snubber module and, and the stray inductance of the semiconductor module.

![Figure 9.6: Inductor current and snubber voltage](image)

In Figure 9.7 the inductor current and voltage and the snubber current are shown. By examining the figure it is clear that the oscillations are damped by the load. On the other hand, the impact of the magnetising inductance increases considerably. The impact of the magnetising inductance [Kles1] is clearly represented by the finite slope of the inductor voltage, following the magnetising current.

![Figure 9.7: Inductor current and snubber voltage](image)
In [Kles1] the impact of the magnetising inductance has been reported concerning resonance converters operating at DCM. At light loads, as reported in [Kles1] transformer-induced low-frequency oscillations (TLO) are presented resulting in loss of the soft-switching conditions of the controllable switches at turn-on. The different TLO modes are summarised in the Appendix F.

In the measurements presented in this thesis the low-frequency oscillations are not influencing the behaviour of the converter. Nevertheless, when the converter is not operating at DCM the impact of the magnetising inductance is reduced to a minimum. Thus, the transformer-induced low-frequency oscillations (TLO) are not that severe as in the case of the resonance converter.

Figure 9.7: Inductor current and voltage and, snubber current
9.4 Measurements during step-up operation

The converter has the ability to operate both at step-down and at step-up mode. Measurements have been done to illustrate the ability of the converter to operate in both modes of operation and to prove the feasibility of the proposed control scheme as explained in [Dem5].

In Figure 9.8 the inductor current and voltage are presented. The inductor voltage is a three-level waveform reducing the harmonic content of the current. Nevertheless, the controller has to be further developed in order to achieve less harmonic content in the current waveform. Thus, the sampling frequency of the controller has to be increased drastically. The used sampling frequency is very low and it corresponds to the switching frequency of the converter. This was due to software limitations.

Figure 9.8: Measured inductor current and voltage. Step-up operation
In Figure 9.9 the inductor current and the snubber voltage are presented. At turn-off the snubber voltage increases at a controllable rate ensuring the soft-switched transition of the controllable switch. Additionally, at every switch transition oscillations occur as seen in the snubber voltage waveform.

Figure 9.9: Measured inductor current and snubber voltage. Step-up operation
In Figure 9.10 the primary current and voltage are shown. The freewheeling stage is considerably longer compared to the step-down operation. This implies that the circulating current flowing through the transformer windings is comparably high, and as a result, the transformer has to be designed to deal with it.

![Graph showing measured primary current and voltage at M=1.4 and R=25](image)

*Figure 9.10: Measured inductor current and primary voltage. Step-up operation.*

As discussed in this chapter the controller was not optimized for step-up operation using the control scheme as proposed in [Dem5]. With some minor adjustments in the limits of the phase-shift angles preliminary measurements were performed at the same load as in the case shown in Figure 9.10.

In Figure 9.11 the inductor current and the snubber voltage are shown. The snubber voltage increases by resonance means and overshoots the steady-state value. A severe oscillation occurs, resulting in excessive EMI. The oscillation occurs at every transition of the controllable switches. Nevertheless, the switches are turned-off under ZVS conditions. Despite the oscillations, the current is of a quasi-sinus waveform ensuring that the harmonic
content is quite low. Consequently, the winding losses of the transformer are expected to be reduced compared to the step-down case.

![Figure 9.11: Measured inductor current and snubber voltage. Step-up operation](image1)

![Figure 9.12: Inductor current and voltage. Step-up operation](image2)
In Figure 9.12 the inductor current and voltage are shown. The inductor voltage is of a three-level type forcing the current waveform to follow a quasi-sinus waveform. As shown in the figure the oscillations are quite severe which can cause additional losses.

![Measured Primary Current and Voltage at M=1.6 and R=25](image)

*Figure 9.13: Primary current and voltage. Step-up operation*

In Figure 9.13 the primary current and the primary voltage of the converter is presented. From the figure it is quite obvious that a considerable amount of reactive power is delivered to the transformer complicating the transformer design. Thus, the transformer has to be designed to deal with excessive reactive power delivered by the inverter.

The reactive power delivered to the transformer at the fundamental frequency is 60% of the input power. Reactive power causes excessive losses, thus, increased inductor current, and therefore, increased conduction losses for all the devices.

The current corresponding to the phase-shift controlled DAB converter and the duty-cycle controlled DAB topology operating at the step-up mode are compared in respect to their harmonic contents and are shown in Figure 9.14.
Figure 9.14: Inductor current and current spectrum

The upper waveform represents the inductor current corresponding to the conventional controlled DAB topology. Similarly, the waveform in the middle and the waveform at the bottom of the figure correspond to the inductor current using duty-cycle controlled DAB converter. As shown in Figure 9.14 the lowest harmonic content in the current corresponds to the DAB converter employing the duty-cycle control strategy as described in [Dem5]. Consequently, losses related with current harmonics are expected to be reduced. On the other hand the lowest waveform demands a considerable amount of reactive power from the inverter. Consequently, a trade-off between harmonic content and reactive power has to be made. In other words transformer losses have to be compared with inverter losses.
9.5 Measured efficiency

The efficiency of the converter has been measured at different conversion ratios and is shown in Figure 9.15. In the measurements presented in the figure the six-pulse rectifier is included. As shown in the figure the efficiency of the converter is approximately 95%. The higher efficiency is obtained at $M = 0.96$ and is close to 96%. Nevertheless, at heavy loads the efficiency of the converter is reduced. The main part of the losses are conduction losses assuming that the soft-switching conditions are fulfilled at the measured operating points. Consequently, by moving to heavy loads the efficiency of the converter is reduced due to the increase of the inductor current. Another contribution to the loss increase at heavy loads can be the high-frequency oscillations which are more severe at heavy loads.

![Output Power versus efficiency graph](image)

*Figure 9.15: Measured efficiency for different conversion ratios*

When the converter is operating in the step-up mode the efficiency drops dramatically compared to the step-down. This is due to an increase of the inductor current resulting in excessive conduction losses. Additionally, the oscillation becomes more severe implying a decrease of the efficiency of the converter.

For high-power applications IGBT:s optimised for soft-switching applications can be used. The soft-switched IGBT is designed in such a way to allow higher switching losses and lower
conduction losses. Since the DAB topology operates under soft-switching conditions in wide load range, the efficiency of the converter can be considerably increased in a wide operating area.

Nevertheless, the efficiency of the DAB converter using duty-cycle control is comparable with the phase-shift controlled DAB converter. In [Zha1] an improvement of the efficiency with 2% has been reported. This improvement in the efficiency, as high as 2%, has not been reached in the present thesis. The maximum improvement achieved is 1%. On the other hand the power transfer capability of the converter is decreased with 0.2 p.u compared with the phase-shift controlled DAB converter.

9.6 Converter dynamics

Experimental measurements have been done in order to verify the dynamic behaviour of the converter as presented in Chapter 8. As in the case of the phase-shifted DAB converter the conversion ratio of the converter is rapidly increased or decreased with 10% of the nominal value. Thus, a small-signal step is introduced to the conversion ratio.

In Figure 9.16 (a) and Figure 9.16 (b) the response of the converter due to a step in the reference voltage is shown. The step in the reference voltage is 10% of the nominal value and is applied by the controller. When the reference step is applied a data log is stored in the memory and it can be plotted in Matlab. As shown in the figures the response of the converter is a monotonically increasing/decreasing function and neither oscillations nor instabilities are presented. In both cases the converter operates at the same load, i.e. at nominal load conditions. Both the inductor current and the output voltage decrease and/or increase monotonically and are adjusted to the new reference values as governed by the controller.
Figure 9.16: Step response. (a) + 10% and (b) -10%
In Figure 9.17 (a) and in Figure 9.17 (b) the performance of the controller during voltage reference steps is presented. In both cases the controller is neither unstable nor oscillatory. Both output voltage and inductor current are increasing and decreasing monotonically without
causing any instability to the system. The measurements presented in both figures are the actual values sampled by the control unit. The signals presented in the figures are:

- Upper window in the figures: input, output and, reference voltage.
- Second window in the figures: Measured and reference current
- Third window in the figures: Measured and reference value for the phase-shift angle
- Last window in the figures: Alive signal (operating mode), indicating that both bridges are synchronised and are operating.

Some interesting features have been obtained at light load conditions, i.e. $R = 25\ \text{ohm}$, and when a step has been applied to the reference voltage.

When a positive step is applied to the reference voltage, as shown in Figure 9.18 (a), both current and voltage are oscillating with a low resonance frequency. This peculiar behaviour has been obtained only at the certain load and only at a positive step value and it has not been predicted by neither small-signal models nor by simulations. On the contrary, and as shown in Figure 9.18 (b) where the step is negative, the step response of the converter is as obtained by the small-signal models and by simulations. Both current and voltage are increasing and/or decreasing as derived by the small-signal analysis of the converter.
Identical results are obtained from the data log of the control system as shown in Figure 9.19 (a) and Figure 9.19 (b). Thus, at a positive step the output voltage is oscillating at a low resonance frequency. The reference current which is given by the PI is a result of the
subtraction of the measured voltage and the reference voltage and, therefore, is oscillating as well. Since the reference current is oscillating and the measured current is following the reference value, the inductor current will start oscillate as governed by the controller. Thus, oscillations at the output voltage are causing the current oscillations.

*Figure 9.19: Controller performance at light loads, (a) +10% and (b) -10%*
By closely examining Figure 9.19 (a) an interesting event takes place when the step in reference is applied. The input voltage drops at the same time instance. Consequently, this small variation around the steady-state value of the dc-link in addition to the small-signal perturbation at the reference voltage could have caused the oscillations. This behaviour cannot be predicted by the small-signal model because perturbations are added only at one parameter with all the others set to zero. On the other hand the same behaviour has been repeatedly obtained.

9.7 Conclusions and discussions

In this chapter measurements have been presented concerning the duty-cycle control method of the DAB converter. Measurements concerning steady-state operation and small-signal behaviour of the converter have been considered. During steady-state operation the converter behaves identically as with the phase-shift controlled DAB converter. Thus, at every transition of the secondary placed controllable switch severe oscillations are taking place. Oscillations of this nature can cause additional losses and degrade the EMI behaviour of the converter. Consequently, in order to follow the EMI directives additional filters have to be added to the converter. Filters will have an impact on the complexity of the converter and will certainly affect the dynamic behaviour of the converter. Therefore, the costs of the converter will be comparably high.

The efficiency of the converter has been found to be slightly higher than the efficiency of the phase-shift controlled DAB converter. At step-up operation the efficiency drops dramatically due to the excessive conduction losses caused by the high inductor current. This implies that the main loss mechanism is the conduction losses in the converter.

Additional work is required in order to obtain a control which can operate satisfactory in the step-up mode.

Dynamics have been studied. The converter behaves as derived by the small-signal models. Nevertheless, at light loads and at a positive step applied at the reference voltage the converter exhibits oscillatory behaviour. This behaviour has been obtained only at a certain specific load.
10 BASIC TRANSFORMER MODELLING

10.1 Introduction

Magnetic devices are integral parts of every switching power converter. Often, the design of the magnetic devices cannot be isolated from the converter design. The conversion process in power electronics requires the use of magnetic devices, components that are frequently the heaviest and the bulkiest items in the conversion circuits. Magnetic devices and especially transformers have a significant effect upon the overall performance and efficiency of the system. Accordingly, their design has an important influence on the overall system weight, power conversion efficiency, and cost.

10.2 Transformers

The power transformer consists of two magnetically coupled electric circuits. Although the static transformer is not an energy conversion device, it is an indispensable component in many conversion systems. As one of the principal reasons for the widespread use of ac-power systems, it makes possible electric generation at the most economical generator voltage, power transfer at the most economical transmission voltage, and power utilisation at the most suitable voltage for the particular utilisation device. Nevertheless, the transformer can be used to match the impedances of the source and its load for maximum power transfer, insulating one circuit from another or isolating direct current while maintaining ac-continuity between two circuits. Moreover, the transformer is one of the simpler devices comprising two or more electric circuits coupled by a common magnetic circuit.
10.2.1 Transformer modelling

Considering the two winding transformer configuration as shown in Figure 10.1, with a core of a cross-sectional area $A_c$, a mean magnetic path length $l_m$, and permeability $\mu$, the core reluctance can be expressed as

$$\mathcal{R} = \frac{l_m}{\mu A_c} \quad (10.1)$$

Due to symmetry the Amperes law can be apply, thus,

$$mmf = \int_c \vec{H} \cdot d\vec{l} = N_1 I_1 \quad (10.2)$$

![Figure 10.1: Two-winding transformer](image)

At this stage it is necessary to point out the duality between magnetic and electric circuits, and is shown in Table 10.1.

Since there are two windings it is necessary to determine the polarities of the mmf generators. The mmf generators are additive, because the currents $I_1$ and $I_2$ pass in the same direction through the core window.

In the ideal transformer the core reluctance is assumed to be zero. Applying Faradays law,

$$V_1 = N_1 \frac{d\Phi}{dt} \quad (10.3)$$

$$V_2 = N_2 \frac{d\Phi}{dt} \quad (10.4)$$
Dividing Eq.(10.3) with Eq.(10.4) results to

\[
\frac{V_1}{V_2} = \frac{N_1}{N_2}
\]  

(10.5)

<table>
<thead>
<tr>
<th>A/A</th>
<th>Electric circuit</th>
<th>Magnetic circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Current $I$</td>
<td>Magnetic flux $\Phi$</td>
</tr>
<tr>
<td>2</td>
<td>Current density $J = \frac{I}{S}$</td>
<td>Magnetic flux density $B = \frac{\Phi}{A_c}$</td>
</tr>
<tr>
<td>3</td>
<td>Electric field $E$</td>
<td>Magnetic field $H$</td>
</tr>
<tr>
<td>4</td>
<td>Electromotive force $emf$</td>
<td>Magnetomotive force $mmf = NI$</td>
</tr>
<tr>
<td>5</td>
<td>Resistance $R = \frac{l}{\rho S}$</td>
<td>Reluctance $\mathcal{R} = \frac{l_m}{\mu A_c}$</td>
</tr>
</tbody>
</table>

*Table 10.1: Duality between magnetic and electric circuits*
10.3 Transformer parasitics

10.3.1 The magnetising inductance

The magnetising inductance models the magnetisation of the core material and exhibits saturation and hysteresis. All physical transformers must contain a magnetising inductance. The magnetising current causes the current ratio of the winding currents to differ from the turns ratio. When the transformer saturates the magnetising current becomes comparably high and the magnetising inductance becomes low. As a result, the transformer windings are short circuited.

The magnetising inductance and the magnetising current as referred to the primary winding are defined as

\[
\begin{align*}
L_m &= \frac{N_1^2}{2} \\
i_m &= I_1 + \frac{N_2}{N_1} I_2
\end{align*}
\]  

In most power conversion applications the magnetising inductance is assumed to be infinitely large resulting in a negligible magnetising current. This can lead to modelling errors. Klesser et al. [Kles1] and King et al. [Kin1] showed that the magnetising inductance can actually interfere with the power electronic circuit and may lead to the generation of low frequency oscillations which ultimately can influence the power transfer capability of the circuit.

10.3.2 Leakage inductance

Ideally the flux must link both windings through the magnetic core. In practice there is a certain amount of flux which links one winding but not the other. Instead of following the magnetic core, this flux leaks through other paths which do not link the other winding. This flux is referred to as leakage flux.

In order to represent the leakage energy storage the term leakage inductance is widely used. If two windings were placed in the same physical position the flux linkage of the two windings has a common magnetic path resulting in zero leakage energy storage. However, this is not feasible due to the 3D geometrical effects, e.g. edge effect [Sar1] and [Dai1], resulting to a certain amount of leakage flux. Consequently, some leakage energy storage will always appear. It is evident that the leakage inductance is a geometrically derived parameter [Fer1].
The higher the separation of the windings, the higher the leakage energy, resulting in higher leakage inductance. By ensuring good proximity between the different winding layers the leakage energy storage can be minimised. On the other hand, in high-voltage multi-winding transformers, the separation of the different winding layers is strongly dependent on the electrical properties of the material used for insulation, i.e. oil. The leakage inductances will appear in series with the resistance of the windings, which represents the winding losses.

### 10.3.3 M.M.F diagrams

As discussed by Dowell [Dow1], the leakage flux crossing the winding space via this layer will induce eddy currents in the conductors of the layer, thereby producing an increase in the impedance of the layer. The leakage flux in the layer will also result in the storing of magnetic energy. This stored magnetic energy is associated with the leakage inductance of the transformer. Thus the leakage flux in the layer determines the ac winding resistance and the leakage inductance which are associated with that layer. When considering the leakage impedance due to a particular layer it is only necessary to consider the other layers of the winding insofar as they affect the flux in the layer being considered. Furthermore, the leakage-flux distribution across any layer depends only on the current in that layer and the total current between the layer and an adjacent position of zero m.m.f. It is clear that, when considering the leakage impedance due to particular layer, the only other layers which need to be considered are those, which lie between that layer and an adjacent position of zero m.m.f. Hence, for the purpose of calculating the leakage impedance, it is permissible to consider the winding space to consist of a number of separate parts, winding portions, each part containing one position of zero m.m.f. The leakage impedances due to those portions can be summed to give the total leakage impedance.

M.M.F diagrams can be used to determine the frequency-independent leakage impedance due to the intersection gaps. Intersection gap is defined as the gap between the primary winding and the secondary winding. A number of circular conductors can be replaced by a single rectangular conductor with the same cross-sectional area as the winding portion under investigation. This is done in order to make the mathematics more manageable. Additionally, the mathematical treatment neglects the curvature of the conductors by considering a mean turn length. For frequencies in the kHz range the errors introduced by these assumptions are relatively low. By increasing the operating frequency of the transformer the current distribution will alter across the conductor, but the total net current flow in each conductor will remain unaltered.
Hence, increasing the frequency will alter the m.m.f acting on the conductors, but will leave the m.m.f acting on the intersection gap unaltered. Thus the leakage inductance due to the intersection gaps and interlayer gaps will be frequency independent up to a frequency where the eddy currents caused by the leakage flux counteract the flux in the intersection gaps. The latter effect is however disregarded. Interlayer gap is defined as the gap between the different winding layers.

From Figure 10.2 and by considering the stored energy in the intersection gaps and the interlayer gaps, the leakage inductance can be calculated as proposed in [Dem3] and [Ase1]. Knowing that the field density in every separation layer equals

\[
H_n = \frac{1}{4\pi} \iiint_{\text{Vol}_n} \frac{\vec{J}_n \times \vec{r}}{r^3} \ d\text{Vol}_n \tag{10.8}
\]

Where \( n = 1,2,3...m-1,m \) and with \( m \) to represent the number of the winding layers used and \( \mu = \mu_0 \mu_r \), \( \vec{J}_n \) and \( d\text{Vol}_n \) are defined as the magnetic permeability, the current density in every winding layer and the volume element respectively. The term \( \vec{r}_n \) is the unity vector and \( r \) is the distance. Evidently, \( \mu_r \) is the permeability of the material and \( \mu_0 \) is representing the permeability of the vacuum.

In Figure 10.2 three different winding configurations are presented. The highest amount of the magnetic energy stored in the winding window corresponds to configuration (a). Furthermore, the magnetic energy stored in the window is proportional to the square of the magnetic field intensity, [Dow1] and, [Van1]. The magnetic energy storage will result in losses due to eddy currents and proximity effect [Sar1]. Additionally, the leakage inductance of configuration (a) is comparably high. In order to reduce the magnetic energy stored in the winding window, interleaving of the windings can be applied as shown in Figure 10.2(b). This measure yields a reduction of the magnetic energy storage by four. Similarly, a considerable reduction of the eddy current losses and the leakage inductance is achieved. Despite the reduction of the leakage inductance the windings are in closer proximity when interleaving is applied resulting in a higher winding capacitance.

The winding configuration shown in Figure 10.2 (c) is known as the non-optimum winding as called by Dowell in [Dow1]. This winding configuration results in the lowest leakage inductance and eddy current losses but, on the contrary the highest winding capacitance of all three winding solutions. However, to minimise leakage inductance and eddy current losses it
is necessary to arrange all the positive and negative peaks of the m.m.f. diagram to be of equal height. This is deduced from the stored-energy considerations as reported in [Dow1] and [Van1].

![Diagram](image)

Figure 10.2: The dc m.m.f diagram.

The magnetic energy stored in a certain oil barrier can be defined as

\[ W_m = \frac{1}{2} L_\sigma i_w^2 \]  

(10.9)
where $L_\sigma$ is the leakage inductance corresponding to the stored magnetic energy in the barrier and $i_w$ is the net current corresponding to the barrier between two winding layers.

The magnetic energy can be expressed in terms of the magnetic field density as

$$W_m = \frac{1}{2} \iiint_{Vol} \mu \vec{H}^2 \ dVol$$

(10.10)

By combining equations (10.8)-(10.10) the total leakage inductance can be obtained.
10.3.4 Step response and leakage inductance.

The leakage inductance of a transformer can be experimentally verified by applying a step-function in the primary side. Consider the two-winding transformer as shown in Figure 10.1 with the secondary winding short-circuited. When the secondary side is short-circuited the magnetising inductance and the winding capacitance of the transformer are short-circuited. The only parasitic components which are part of the circuit are, the winding resistance, and the leakage inductance of the transformer as shown in Figure 10.3.

![Figure 10.3: The equivalent circuit of the transformer with short-circuited secondary winding.](image)

By applying the Kirchhoff’s voltage law results to

\[
e(t) - L_\sigma \frac{d}{dt} i_{scr}(t) - i_{scr}(t) R_L = 0
\]  
(10.11)

Solving Eq. (10.11) the current flowing through the circuit can be expressed as

\[
i_{scr}(t) = \frac{e(t)}{R_L} \left(1 - e^{-\frac{t}{\tau}}\right)
\]  
(10.12)

Where

\[
\tau = \frac{L_\sigma}{R_L}
\]  
(10.13)

Rearranging Eq. (10.11) yields
\[
\frac{d}{dt} i_{\text{scr}}(t) = \frac{e(t) - i_{\text{scr}}(t) R_i}{L_\sigma}
\]  
(10.14)

From Eq. (10.14) and when the current through the primary winding is zero, the maximum \( \frac{d}{dt} i_{\text{scr}}(t) \) can be expressed as

\[
\left[ \frac{d}{dt} i_{\text{scr}}(t) \right]_{\text{max}} = \frac{e(t)}{L_\sigma}
\]  
(10.15)

The leakage inductance referred to the primary side can be expressed as

\[
L_\sigma = \frac{e(t)}{\left[ \frac{d}{dt} i_{\text{scr}}(t) \right]_{\text{max}}}
\]  
(10.16)

The current through the leakage inductance, as given by Eq. (10.12), when a step is applied at the primary winding with the secondary winding short-circuited is shown in Figure 10.4.

![Step response with short-circuited secondary winding](image)

*Figure 10.4: The current flowing through the primary winding. Simulated waveform*
10.3.5 The winding capacitance

The winding capacitance of the transformer is distributed in the whole volume of the device as shown in Figure 10.5.

As in the case of the leakage inductance, the winding capacitance is a geometrically derived parameter. A closer proximity of the conductor layers causes higher capacitance values. However, the electrical parameters of the material used for insulation will drastically influence the capacitance value. Insulators with high permittivity, $\varepsilon_r$, result in a high value of the capacitance. Additionally, the thickness of the insulation material used will strongly influence the capacitance value. The thicker the insulation material, the lower the electric energy storage, and as result, the capacitance is lower. In order to calculate the value of the winding capacitance the electrical energy stored in the insulation layers must be calculated. The electric energy stored in the insulation layer can be obtained by

$$W_c = \frac{1}{2} \iiint_{Vol} \vec{D} \cdot \vec{E} dVol$$  \hspace{1cm} (10.17)

Where
\[ E = -\nabla V \]  

(10.18)

and

\[ D = \varepsilon E \]  

(10.19)

The parameter \( \varepsilon \) is defined as

\[ \varepsilon = \varepsilon_r \varepsilon_0 \]  

(10.20)

where \( \varepsilon_r \) is the relative permittivity of the insulator used and \( \varepsilon_0 \) is the permittivity of vacuum.

Knowing that the energy stored in a capacitor is given by

\[ W_e = \frac{1}{2} CV^2 \]  

(10.21)

Using equations (10.17)-(10.21) the winding capacitance can be defined for every layer.

The capacitive model is a quite complicated model due to the fact that the voltage is varying in space and in time.
10.3.6 Verification of the winding capacitance using FEM simulations tool

A high-voltage high-frequency planar transformer configuration was modelled using FEM simulations tool. The secondary winding of the transformer was interleaved and was placed on both sides of the primary side. As given by Eq. (10.18) the electric field is voltage dependent with a direct impact in the capacitance. Figure 10.6 illustrates the equipotential lines as simulated by a FEM-software. High density indicates that the electric energy storage in the insulator used between the windings is high as well as the electric field density. Consequently, the associated capacitance is high. Knowing that the insulator has a relative resistivity $\rho_r$, and for considerably high values of $\rho_r$, the dielectric losses of the insulator are expected to be increased. As a result, materials with low values of $\rho_r$ are favourable. Dielectric losses can cause hot-spots and life-time reduction of the transformer. As mentioned in this chapter, the thickness of the insulator is affecting the capacitance value. In cases where the insulation material is thick, the equipotential line density is low, and consequently, the electric energy storage is low. Therefore, the capacitance value is low. On the other hand, by increasing the thickness of the insulator material the leakage inductance of the transformer increases. As shown in Figure 10.6 the insulator between the primary winding and the secondary windings is highly stressed. Additionally, the equipotential line density in the bobin is considerably high resulting in high electric energy storage.

![Figure 10.6: Equipotential lines](image-url)
The geometry of the transformer is shown in Figure 10.7 and the different materials are specified in Table 10.2.

**Figure 10.7: Transformer geometry**

<table>
<thead>
<tr>
<th>Area</th>
<th>Resistivity $\rho_r$</th>
<th>Permittivity $\epsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purple Bobin</td>
<td>$2 \times 10^{14}$</td>
<td>2.2</td>
</tr>
<tr>
<td>Red Screening board</td>
<td>$2 \times 10^{14}$</td>
<td>2.2</td>
</tr>
<tr>
<td>Yellow Secondary Winding Board</td>
<td>$2 \times 10^{14}$</td>
<td>2.2</td>
</tr>
<tr>
<td>Cyan Electrostatic Screening Board</td>
<td>$2 \times 10^{14}$</td>
<td>2.2</td>
</tr>
<tr>
<td>Blue Primary Winding Board</td>
<td>$2 \times 10^{14}$</td>
<td>2.2</td>
</tr>
<tr>
<td>Black Oil</td>
<td>$3 \times 10^{12}$</td>
<td>4.5</td>
</tr>
</tbody>
</table>

**Table 10.2: Material and specifications**
10.3.7 Experimental verification of the winding capacitance and the magnetising inductance

The winding capacitance as well as the magnetising inductance of the transformer can easily be calculated from the no-load response characteristic. Additionally, from the no-load response characteristic the magnetising current can be calculated. Consider the lumped parameter equivalent transformer circuit as shown in Figure 10.8.

\[ e(t) = L_{\sigma} \frac{d i_{L_{\sigma}}(t)}{dt} + i_{L_{\sigma}}(t) R_L + v_C(t) \]  

(10.22)

As a result, the derivative of the current through the leakage inductance can be expressed as

\[ \frac{d i_{L_{\sigma}}(t)}{dt} = \frac{e(t)}{L_{\sigma}} - \frac{i_{L_{\sigma}}(t) R_L}{L_{\sigma}} - \frac{v_C(t)}{L_{\sigma}} \]  

(10.23)

Knowing that

\[ v_C(t) = L_m \frac{d i_{L_m}(t)}{dt} \]  

(10.24)

the derivative of the magnetising current can be expressed in terms of the winding capacitor voltage as

\[ \frac{d i_{L_m}(t)}{dt} \]
\[
\frac{d i_{t_n}(t)}{d t} = \frac{v_c(t)}{L_m} \quad (10.25)
\]

By applying the Kirchoff's current law it is found that
\[
I_{L_c}(t) = C_w \frac{d v_c(t)}{d t} + i_{t_w}(t) + \frac{v_c(t)}{R_{no}} \quad (10.26)
\]

Where
\[
i_c(t) = C_w \frac{d v_c(t)}{d t} \quad (10.27)
\]

and
\[
i_{R_{no}}(t) = \frac{v_c(t)}{R_{no}} \quad (10.28)
\]

From Eq. (10.25)
\[
\frac{d v_c(t)}{d t} = \frac{i_{t_n}(t)}{C_w} - \frac{i_{t_w}(t)}{C_w} - \frac{v_c(t)}{R_{no} C_w} \quad (10.29)
\]

From Eqs. (10.23), (10.25) and (10.29) results
\[
\begin{bmatrix}
\frac{d i_{t_n}(t)}{d t} \\
\frac{d v_c(t)}{d t} \\
\frac{d i_{t_w}(t)}{d t}
\end{bmatrix} =
\begin{bmatrix}
\frac{-R_{t_n}}{L_{t_n}} & \frac{-1}{L_{t_n}} & 0 \\
\frac{1}{C_w} & \frac{-1}{R_{no} C_w} & \frac{-1}{C_w} \\
0 & \frac{1}{L_m} & 0
\end{bmatrix}
\begin{bmatrix}
i_{t_n}(t) \\
v_c(t) \\
i_{t_w}(t)
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{L_{t_n}} \\
0 \\
0
\end{bmatrix} e(t) \quad (10.30)
\]

Equation (10.30) is the state-space representation of the transformer and describes the no-load behaviour of the transformer.
The current through the leakage inductance and the winding capacitance as well as the magnetising current can be found by solving Eq. (10.30).

Equation (10.30) can expressed in the s-domain as

\[ sX(s) = AX(s) + Be(s) \] \hspace{1cm} (10.31)

From Eq.(10.31)

\[ X(s) = \left[sI - A\right]^{-1} Be(s) \] \hspace{1cm} (10.32)

where

\[ A = \begin{bmatrix} \frac{R_L}{L_{\sigma}} & \frac{1}{L_{\sigma}} & 0 \\ \frac{1}{C_w} & \frac{1}{R_{no} C_w} & \frac{1}{C_w} \\ 0 & \frac{1}{L_m} & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{1}{L_{\sigma}} \\ 0 \\ 0 \end{bmatrix} \] \hspace{1cm} (10.33)

The transformers state variables can be expressed in the s-domain according to

\[ \begin{bmatrix} i_{L_w}(s) \\ v_c(s) \\ i_{L_m}(s) \end{bmatrix} = \begin{bmatrix} e(s) \left(s^2 \frac{R_{no}}{L_m C_w} + s L_m + R_{no}\right) \\ s \frac{R_{no}}{L_m} e(s) \\ \frac{R_{no} e(s)}{\Delta(s)} \end{bmatrix} \] \hspace{1cm} (10.34)

Where
\[ \Delta(s) = s^3 R_{no} L_\sigma L_m C_w + s^2 L_m \left( L_\sigma + R_L C_w \right) + s \left( L R_{no} - R_L L_m + R_{no} L_m \right) - R_L R_{no} \] (10.35)

The primary current is plotted in the s-domain in Figure 10.9. Two resonance peaks are shown in Figure 10.9 corresponding to two resonance modes. The low frequency resonance peak describes a resonance tank including the magnetising inductance and the winding capacitance. Similarly, the high frequency resonance peak describes an oscillation between the leakage inductance and the winding capacitance. Equations (10.36) and (10.37) define the two resonance frequencies in terms of the dominant parasitics of the transformer.

\[ T_r = 2 \pi \sqrt{L_\sigma C_w} \] (10.36)

\[ T_c = 2 \pi \sqrt{L_m C_w} \] (10.37)

Figure 10.9: State-to-input transfer function

In order to express both primary and magnetising current and the winding capacitance voltage in the time domain, the inverse Laplace transformation has to be used. From Eq. (10.34) and (10.35) the state variables of the transformer are expressed in the time domain as derived according to
In Figure 10.10 the primary current, as given by Eq. (10.38), is shown at no-load. When a step is applied at the primary side the current oscillates with a resonant frequency which corresponds to the resonant tank containing the leakage inductance, the winding capacitance and, the magnetising inductance of the transformer.

\[
\begin{bmatrix}
    i_{L_a}(t) \\
    v_c(t) \\
    i_{L_m}(t)
\end{bmatrix} = \text{Laplace}^{-1}
\begin{bmatrix}
    e(s) \left( \frac{s^2 R_{no} L_m C_w + s L_m + R_n}{\Delta(s)} \right) \\
    \frac{s R_{no} L_m e(s)}{\Delta(s)} \\
    \frac{R_{no} e(s)}{\Delta(s)}
\end{bmatrix}
\]

(10.38)

Figure 10.10: No-load response. Simulated waveform

As explained in paragraph 10.3.4 the leakage inductance can be calculated by using Figure 10.4 and from Eq. (10.16). Knowing the leakage inductance, and from Eq.(10.36) and (10.37) the winding capacitance and the magnetising inductance of the transformer can be obtained. Similarly, from Figure 10.10 and from Eq. (10.36) the winding capacitance can be easily obtained.
10.4 Experimental verification and transformer parasitics

The transformer configuration as shown in Figure 10.6 has been tested in the laboratory and the step response of the transformer under no load conditions and short circuit conditions have been studied. As already mentioned the leakage inductance can be calculated by the short circuit response and the winding capacitance and the magnetising inductance can be predicted by the no load response. The method proposed in this chapter is applicable only when the natural frequencies of the transformer are lower than the rise time of the applied step at the transformer terminals. The applied step should excite the natural frequencies of the transformer and will appear in the current waveform as oscillations.

In Figure 10.11 the no load response of the transformer is presented. The step pulse is applied at the primary side of the transformer and the secondary side is open. As a result, the calculated winding capacitance and magnetising inductance are as referred to the primary side.

![Figure 10.11: Step response at no load](image)

Similarly, the step response of the transformer with a short-circuited secondary side is shown in Figure 10.12. The leakage inductance as referred to the primary side can be calculated as explained in this Chapter.
The applied voltage step at the primary terminals of the transformer is $E = 20$ volts. Knowing the input voltage the leakage inductance of the transformer referred to the primary side can be calculated by using Figure 10.12 and Eq. (10.16). As a result, the leakage inductance of the transformer is $L_{\sigma} = 4.2 \, \mu H$.

Similarly, from Figure 10.11 and from Eq. (10.36) and Eq. (10.37), the magnetizing inductance of the transformer and the winding capacitance corresponds to $L_m = 323 \, \mu H$ and $C_w = 1.18 \, \mu F$.

The parasitics for the transformer used in the prototype are given in Table 10.3.

<table>
<thead>
<tr>
<th>Parasitic</th>
<th>$L_\sigma \rightarrow [\mu H]$</th>
<th>$C_w \rightarrow [n , F]$</th>
<th>$L_m \rightarrow [m , H]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>3.5</td>
<td>1.2</td>
<td>2.75</td>
</tr>
</tbody>
</table>

Table 10.3: Transformer parasitics
10.5 Conclusions and discussion

In this chapter the subject corresponding to the basic transformer modelling has been discussed. Transformer parasitics are briefly discussed and their dependency on the transformer geometry has been discussed. The lumped parameter model has been used in order to derive the theoretical model of the transformer based on the state-space analysis. As discussed in this chapter the transformer parasitics can be derived experimentally by studying the step response of the transformer both at no-load and with one of the windings short-circuited. This method is valid only when the rise time of the step applied at the terminals of the transformer corresponds to a frequency which is higher than the natural frequencies of the transformer. Experimental results are presented and the parasitics of the transformer used in the prototype have been determined.
11 CONCLUSIONS

11.1 Summary of the main results

In the present thesis two converter topologies have been studied and modelled with respect to their steady-state and dynamic behaviour. The two topologies are the Single-Active Bridge and the Dual-Active Bridge. Different control and modulation strategies have been studied and presented. The main results are summarised as follows:

- The Single-Active Bridge topology was first presented in [Dem1] and is the simplest topology employing the lowest number of active and passive components. On the other hand conventional PWM control strategies cannot be used nor can the phase-shift control strategy.

- The small-signal model of the SAB topology has been derived and presented. The conventional state-space averaging method cannot be used because the inductor current is an ac quantity. The extended state-space averaging technique has been presented and is based on the half-cycle symmetry of the circuit. Circuits operating in a symmetrical manner can be easily modelled by using the extended state-space averaging method.

- Three different control strategies are presented and briefly discussed, namely,
  1. Variable-frequency control
  2. Turn-off time control operating at DCM
  3. Turn-off time control operating at intermittent mode

- Only two control strategies have been considered in the thesis, i.e. the turn-off time control operating at DCM and the turn-off time control operating at intermittent
The converter operates at CCM at nominal and at heavy loads. On the contrary, the converter operates at DCM or in the intermittent mode at light loads depending on which of the two control strategies is used. In both cases some severe oscillations are present degrading the efficiency and the EMI behaviour of the converter.

- Oscillations are briefly discussed and it is shown that they are strongly related to the parasitics of the transformer. Any attempt to damp the oscillations will result in higher losses in the damping circuit.

- A control design has been presented based on the small-signal transfer function of the converter.

- Measurements have been done and the behaviour of the converter has been studied by using both control strategies. The measured efficiency of the converter has been presented.

- The dual-active bridge has been studied and a new small-signal model for the DAB converter has been presented. Compared with the previous small-signal models concerning the DAB converter [Khe4], deviations concerning the behaviour of the topology are reported and are briefly discussed in Chapter 6. It has been proven that the topology employing the phase-shift control strategy is stable and no oscillations are present as predicted by the small-signal models.

- Oscillations are reported and as in the case of the SAB topology they are ruled by the transformer parasitics.

- Measurements on the DAB are presented in both step-down operation and step-up operation. The efficiency of the converter has been measured and it was found to be slightly lower than the efficiency of the SAB converter at nominal and at heavy loads. On the other hand the efficiency is higher at light loads. The converter dynamics have been verified in the laboratory.

- A new control strategy is presented [Dem5] but some results have been published by Zhang et al. in [Zha1]. Small-signal models are presented in the present thesis.
• The behaviour of the converter employing the duty-cycle control strategy has been studied in the laboratory both in steady-state and under transient operating conditions.

• The converter has been operated in both step-down and step-up mode. Nevertheless, at step-up operating mode some additional control development has to be done.

• The transformer parasitics can be defined experimentally by deriving the step response of the transformer. Key equations are presented and experimental results obtaining the winding capacitance, the leakage, and the magnetising inductance of the transformer are presented.

Generally, the SAB converter can be used when uni-directional power transfer is requested. Nevertheless, at light loads the converter will operate at DCM and the oscillations will degrade the efficiency of the converter. It is evident to the author that the efficiency of the converter can be improved by using IGBT:s devoted for soft-switching operation. Additionally, oscillations of the nature presented in the thesis can be avoided by optimising the transformer design. Since the series inductance is the main energy storage element, a transformer with very high inductance and negligible winding capacitance can be utilised.

The single-active bridge can operate only in the step-down mode.

The dual-active bridge topology can transfer the power in both directions, i.e. from the source to the load and from the load to the source. Additionally, it can operate as a step-down and as a step-up converter. The efficiency of the converter is slightly lower than the SAB converter at nominal and at heavy loads. This is due to the switching action of the secondary placed controllable switches. Furthermore, with the duty-cycle control strategy the DAB efficiency is increased with 1% implying that a solution with the duty-cycle controlled DAB converter could be appropriate for high-power applications. The current ripple flowing through the filter capacitor has been reported to be considerably lower than the phase-shift controlled DAB converter. This has not been verified in the laboratory.

A proposed solution for high power applications should be a DAB converter employing the new control strategy operating at light-loads. At nominal and at heavy loads the converter can
be operated as a SAB converter improving the efficiency of the system. This of course is possible only when a uni-directional power flow is demanded and operating in the step-down mode. Furthermore, at light loads the oscillations in the DAB converter are damped by the load, improving the EMI behaviour of the converter. When a bi-directional power flow is demanded the duty-cycle controlled DAB converter is preferable.

11.2 Future work

Since all the topologies are linearised as presented in the thesis observers can be used in order to achieve possible control benefits.

The major disadvantage of the turn-off time control operating at intermittent mode, SAB topology, is the high voltage ripple at the output of the converter. A possible solution is the use of the voltage ripple as a stabilising ramp for the inductor current. Consequently, the output ripple should be cancelled even though a low-frequency ripple current might flow through the output capacitor. Further studies should be done in order to verify the feasibility of the proposed control method and its impact on the topology.

As discussed in this thesis when the duty-cycle controlled DAB topology is operating at the step-up mode the behaviour of the controller is not the appropriate. This implies that further control development should be done in order to get the desirable characteristics.

As mentioned in Chapter 8 the duty-cycle control method [Dem5] can operate in the same way by just using a half-bridge with active switches and a half-bridge with only diodes placed on the secondary side as shown in Figure 11.1. On the primary side the same configuration can be used. This implies that a cost reduction can be achieved and the complexity of the converter can be considerably reduced. Additionally, the efficiency of the converter is expected to be improved. The proposed topology should be a subject for future activities.

The DC-DC converter solutions that are presented in this thesis can be used as DC-transformers. In high-power applications and when a DC-transmission network is used tapping is a serious issue. In order to get a certain amount of power from the dc-transmission network step-down and/or step-up transformers should be utilised. DC conventional transformers do not exist and as a result a DC-DC converter can be used. The interactions of the two systems both in steady-state and under transient operating conditions should be
examined. Possible solutions concerning connection of the two systems or with multiple systems should be examined.

Figure 11.1: The proposed dc-dc converter
12 REFERENCES


[Mit2] D. M. Mitchel, Private discussions


[Sar1] E. Sarris, “Electromagnetic theory”, in Greek


A. Appendix A: Averaged switch modelling

By averaging the waveforms as suggested by the circuit averaging technique, [Eri1], the switch network is modified but the remainder of the converter circuit is unchanged. This suggests that, to obtain a small-signal ac converter model, the switch network can be replaced with its averaged model. This procedure is called the averaged switch modelling [Eri1]. The circuit averaged procedure is followed to average the switch terminal waveforms. Consequently, the resulting dc and small-signal ac averaged switch model is then inserted into the converter circuit.

A.1. The averaged switch model of the SAB

As in the case of the state-space averaging modelling the parallel connection of the transistor and diode is treated as a bi-directional switch as shown in Figure A.1.

![Figure A.1: Single active bridge, switch network.](image-url)
The current waveforms for the two bi-directional switches are shown in Figure A.2. In order to model the two switches the conduction times must be defined. The average current through $S_1$ during one switching period can be defined as the sum of the areas $q_1$ and $q_2$ divided by the switching period $T_s$ as given by

$$\langle I_{S_1} \rangle = \frac{q_1 + q_2}{T_s} \quad \text{(A.1)}$$

where $\langle I_{S_1} \rangle$ is the averaged switch current and $q_1$ and $q_2$ are defined as shown in Figure A.2, where,

$$q_1 = -\frac{1}{2} t_1 \dot{I}_{S_1} \quad \text{(A.2)}$$

and

$$q_2 = \frac{1}{2} t_2 \dot{I}_{S_1} \quad \text{(A.3)}$$

![Figure A.2: The switch waveforms.](image)
Knowing that the voltage across the inductance during the two time intervals, \( t_1 \) and \( t_2 \) can be defined in terms of the input voltage \( V_{dc} \), the output voltage \( x_2 \), and the peak switch current, the voltage across the inductor \( L_\sigma \) can be written as

\[
V_{L_\sigma} = L_\sigma \frac{d i_{L_\sigma}}{d t}
\]  
(A.4)

From Table 4.1, the voltage across the inductance when \( D_{t_+} \) is forward biased is given by

\[
V_{L_\sigma} = V_{dc} + x_2
\]  
(A.5)

From Eq. (A.4) and Eq. (A.5) it is found that

\[
t_1 = \frac{L_\sigma \hat{i}_{S_1}}{V_{dc} + x_2}
\]  
(A.6)

The area \( q_1 \) is defined by combining Eq. (A.2) and Eq. (A.6) and is given by

\[
q_1 = -\frac{1}{2} \left( \frac{L_\sigma \hat{i}_{S_1}^2}{V_{dc} + x_2} \right)
\]  
(A.7)

When the switch \( T_{t_+} \) is conducting the inductor voltage is as given in Table 4.1, thus,

\[
V_{L_\sigma} = V_{dc} - x_2
\]  
(A.8)

From Eq. (A.4) and Eq. (A.8), \( t_2 \) can be defined as

\[
t_2 = \frac{L_\sigma \hat{i}_{S_1}}{V_{dc} - x_2}
\]  
(A.9)

Combining Eq. (A.9) and Eq. (A.3) yields

\[
q_2 = \frac{1}{2} \left( \frac{L_\sigma \hat{i}_{S_1}^2}{V_{dc} - x_2} \right)
\]  
(A.10)
The averaged switch current can be obtained by using Eq. (A.1), Eq. (A.7), and Eq. (A.10), and is found that

\[
\langle I_{S_1} \rangle = L_\sigma \hat{I}_{S_1}^2 f_s \left( \frac{x_2}{V_{dc}^2 - x_2^2} \right) \quad (A.11)
\]

Similarly, the averaged switch current for \( S_2 \) yields

\[
\langle I_{S_2} \rangle = -L_\sigma \hat{I}_{S_2}^2 f_s \left( \frac{x_2}{V_{dc}^2 - x_2^2} \right) \quad (A.12)
\]

The time intervals, the areas and the averaged switch currents are summarised in Table A.1.

<table>
<thead>
<tr>
<th>Switch ( S_1 )</th>
<th>Switch ( S_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_1 = \frac{L_\sigma \hat{I}<em>{S_1}}{V</em>{dc} + x_2} )</td>
<td>( t_3 = \frac{L_\sigma \hat{I}<em>{S_2}}{V</em>{dc} + x_2} )</td>
</tr>
<tr>
<td>( t_2 = \frac{L_\sigma \hat{I}<em>{S_1}}{V</em>{dc} - x_2} )</td>
<td>( t_4 = \frac{-L_\sigma \hat{I}<em>{S_2}}{-V</em>{dc} + x_2} )</td>
</tr>
<tr>
<td>( q_1 = -\frac{1}{2} \left( \frac{L_\sigma \hat{I}<em>{S_1}^2}{V</em>{dc} + x_2} \right) )</td>
<td>( q_3 = \frac{1}{2} \left( \frac{L_\sigma \hat{I}<em>{S_2}^2}{V</em>{dc} + x_2} \right) )</td>
</tr>
<tr>
<td>( q_2 = \frac{1}{2} \left( \frac{L_\sigma \hat{I}<em>{S_1}^2}{V</em>{dc} - x_2} \right) )</td>
<td>( q_4 = \frac{1}{2} \left( \frac{L_\sigma \hat{I}<em>{S_2}^2}{-V</em>{dc} + x_2} \right) )</td>
</tr>
<tr>
<td>( \langle I_{S_1} \rangle = L_\sigma \hat{I}<em>{S_1}^2 f_s \left( \frac{x_2}{V</em>{dc}^2 - x_2^2} \right) )</td>
<td>( \langle I_{S_2} \rangle = -L_\sigma \hat{I}<em>{S_2}^2 f_s \left( \frac{x_2}{V</em>{dc}^2 - x_2^2} \right) )</td>
</tr>
</tbody>
</table>

**Table A.1: Switch parameters.**

In the DC-DC converters the output rectifier must be included in the averaged switch model. Thus, switch \( S_1 \) and \( S_2 \) contains a controllable switch with the antiparallel diode and the two
diodes which are forward biased when the voltage $V_{dc}$ is positive or negative as shown in Figure A.3.

*Figure A.3: The averaged switch model including the corresponding diode rectifier*

The rectified inductor current is shown in Figure A.4.

*Figure A.4: The rectified inductor current and the DC-link voltage.*
Calculating $t_1$:

The voltage across the inductor can be expressed in terms of the output voltage $x_2$ and the DC-link voltage $V_{dc}$ as given by

$$V_{L_1} = V_{dc} - x_2$$  \hfill (A.13)

Substituting Eq. (A.13) in to Eq. (A.4) results to

$$t_1 = \frac{L_\sigma \hat{x}_1}{V_{dc} - x_2}$$  \hfill (A.14)

Calculating $t_2$:

The voltage across the inductor can be expressed in terms of the output voltage $x_2$ and the voltage DC-link $V_{dc}$ as given by

$$V_{L_2} = -V_{dc} - x_2$$  \hfill (A.15)

Substituting Eq. (A.15) in to Eq. (A.4) results to

$$t_2 = -\frac{L_\sigma \hat{x}_1}{-V_{dc} - x_2}$$  \hfill (A.16)

The average value of the rectified inductor current can be expressed as the sum of the areas $q_1$ and $q_2$ multiplied by $2 f_s$, where $f_s$ is the switching frequency.

The area $q_1$ can be expressed as

$$q_1 = \frac{L_\sigma \hat{x}_1^2}{2 (V_{dc} - x_2)}$$  \hfill (A.17)

Similarly, $q_2$ is given by

$$q_2 = -\frac{L_\sigma \hat{x}_1^2}{2 (-V_{dc} - x_2)}$$  \hfill (A.18)

The averaged value of the rectified inductor current can be expressed as
From Eq. (A.17), Eq. (A.18) and Eq. (A.19)

\[
\langle \dot{x}_1 \rangle = 2 f_s L\alpha \dot{x}_1 \left( \frac{V_{dc}}{x_1^2 - x_2^2} \right) \quad (A.20)
\]

The simplified equivalent circuit of the single-active bridge is illustrated in Figure A.5.

![Figure A.5: The simplified equivalent circuit for the single-active bridge.](image)

Let's keep the input voltage constant and let \( \tilde{f}_s \) be a small disturbance around the operating point \( f_{s0} \) which causes a small disturbances \( \langle \tilde{x}_1 \rangle \) about \( \langle x_1 \rangle \) and, hence, a disturbance \( \tilde{x}_2 \) about \( x_2 \).

From Eq. (A.20)

\[
\langle \dot{x}_1 \rangle + \langle \tilde{x}_1 \rangle = 2 (f_{s0} + f_s) \frac{V_{dc}}{V_{dc}^2 - x_2^2} \quad (A.21)
\]

Hence, the small-signal perturbation of the output voltage can be written as

\[
\tilde{x}_2(s) = \frac{\langle \tilde{x}_1 \rangle}{1 + s \frac{R}{1 + s RC}} \quad (A.22)
\]

By substituting Eq. (A.21) to Eq. (A.22) the control-to-output transfer function can be expressed as
\[ \frac{x_2(s)}{f_s(s)} = \frac{2 V_{dc} L \dot{x}_1^2}{V_{dc}^2 - x_2^2} \frac{R}{1 + s R C} \]  \hspace{1cm} (A.23)

The Bode diagram is illustrated in Figure A.6, and the poles and zeroes map in Figure A.7.

**Figure A.6:** Control-to-output transfer function.

**Figure A.7:** Control-to-output transfer function, poles and zeroes map.
As shown in the figures the system exhibits characteristics corresponding to a first-order dynamic system. This due to the fact that the actual state of the system is no longer the inductor current but the rectified inductor current. This of course is the main reason why the small-signal model presented in Chapter 4 exhibits different characteristics.
Both models can be used depending on the control strategy employed.
B. Appendix B: Soft-switching principles

Most commercially available converters at low and medium power, up to 200kVA, and medium voltage, up to 800 V dc, are operating at hard-switched mode. During hard-switching the power devices experience simultaneously high-voltages and high currents both at turn-on and turn-off. As a result, the devices are subject to high switching stress and losses. A large square Safe-Operating Area (SOA) is required by the power devices which have to compromise between switching speed and forward saturation voltage [Div1]. Additionally, the power devices and in some cases the converter load have to withstand very high current and voltage derivatives which are characteristic to all hard-switching converters.

At high-switching frequencies or high power levels the switching losses may be very high. Snubber circuits can be used in order to shape the Volt-Ampere switching trajectory of the power device. Passive snubber circuits provide means to reduce the device stress but they do not reduce the system overall efficiency. On the contrary, the energy stored in the snubber capacitor is dissipated in the snubber resistor. Thus, the losses are shifted from the power devices to the snubber circuits. In most cases both a turn-on and a turn-off snubber circuit are required for every phase-leg. This implies that the complexity of the phase-leg is increased and as a result the cost is substantially increased.

Regenerative snubbers have been evaluated for several years but their complexity, low reliability and poor efficiency make them unattractive alternatives [Div1].

To overcome some deficiencies of resonant converters, primarily the increased current stresses and the conduction losses, a number of techniques that enable PWM converters to operate with ZVS have been introduced in the last few decades [Sch1], [Tab1], [Liu1], [Jov1]. In these soft-switching PWM converters [Hua1], [Jov1], [Liu1], lossless turn-on of the switches is achieved without a significant increase of the conduction losses. Due to the minimised conduction losses and negligible amount of circulating energy required to achieve ZVS, these converters have potential to achieve high efficiency at high frequencies.

In recent years quite a lot of soft-switching topologies [Jov1], [Khe1], [Ste1], [Ste2], [Sab1] have been presented and analysed. However, there has been little discussion of where the various topologies could be applied.
Similarly, comparisons of various topologies for a given application are rarely discussed. Soft-switching can be achieved in two ways, zero-current switching, and zero-voltage switching. The two basic configurations are shown in Figure B.1.

In the case of ZCS an inductive element is connected in series with the switch. During turn-on, the voltage across the switch falls to approximately zero before the current rises. Consequently, the turn-on losses are very low. At turn-off in ZCS circuits, the voltage across the switch is reduced to zero. Additionally, the current is reversed and flows in the antiparallel diode, by external circuit means, such as the reversal of current in the resonant circuit. During the current reversal, the device is turned off so that when the voltage is reapplied the device is in the off state.

Similarly, in the case of ZVS, the voltage across the device is reduced to zero through external circuits as the current through the device reverses and eventually flows through the antiparallel diode. The device is gated on during the diode conduction time. Thus, a reduction in turn-on losses is achieved. With ZVS the capacitive energy is not lost but is returned to the circuit through resonant action. ZVS topologies are favourable for higher switching frequencies because the energy of the parasitic capacitance of the switch is not dissipated during turn-on. However, in applications where IGBT:s with ‘extended’ tail currents are used, ZCS is recommended.

At ZVS the device must carry a certain minimum current during turn-off. When the switch $T$ is turned off, the remaining load current $i_c$ charges-up the purely capacitive snubber, thus
limiting the rate of rise of the voltage across the switching device $T$. A purely capacitive snubber requires an inductive load, which in fact interacts with the snubber during turn-off and the voltage is built-up by resonance. This is commonly known as the resonant pole concept [Div1].

**B.1. The resonant pole concept**

The single-phase representation of the resonant pole inverter is shown in Figure B.2.

![Figure B.2: Single-phase resonant pole inverter](image)

The single-phase resonant pole inverter consists of two controllable switches with two antiparallel diodes. Across the switches the snubber capacitors (also referred to as resonant capacitors $C_r$) are connected. A resonant inductor $L_r$, it can be part of the topology or the leakage inductance of the transformer, is used to reset the energy stored in the capacitors. The resonant pole as shown in Figure B.2 possesses only turn-off snubbers in parallel with the switches. As a result, in soft-switched topologies where turn-on snubbers are required, additional turn-on snubbers must be provided. In the topologies examined in the present thesis turn-on snubbers are not a necessity.

The resonant pole concept is the most commonly used concept in nearly all ZVS and resonant converters and the operation principles are discussed below. Assume that the current is flowing through the device $T_{A+}$ as shown in Figure B.3 (a). When $T_{A+}$ is turned off the load current is transferred to the resonant capacitors and the transition from the positive pole voltage to the negative pole voltage is achieved by charging and discharging the resonant capacitors. Thus, the capacitor which is connected across $T_{A+}$ is charged and the capacitor which is connected across $T_{A-}$ is discharged as shown in Figure B.3 (b). When the charging and discharging procedure is completed the diode $D_{A-}$ is forward biased thereby clamping
the resonant pole output voltage as shown in Figure B.3 (c). The controllable switch can be turned-on under ZVS as shown in Figure B.3 (d).

![Figure B.3: Soft-switching modes of the resonant pole](image)

In order to achieve ZVS at turn-off the energy stored in the inductance must be greater or equal to the energy stored in the resonance capacitors as stated in Chapter 4. As a result the minimum inductor current can be expressed as

\[
i_{r,\text{min}} = \sqrt{\frac{V_0^2 - (V_{dc} - V_0)^2}{Z_r}}
\]  \hspace{1cm} (B.1)

Where

\[
Z_r = \sqrt{\frac{L_r}{2C_r}}
\]  \hspace{1cm} (B.2)

The voltage \( V_0 \) is assumed to be constant during the commutation.
Figure B.4: Voltage across the capacitor and the turn-off current when, (a) The energy stored in the inductor is equal to the energy required, (b) The energy stored in the inductor is larger than required and, (c) The energy stored in the inductor is less than required
In order to assure that $T_{A-}$ will turn on at zero voltage, a dead time is introduced between the
turn-off of $T_{A+}$ and turn-on of $T_{A-}$ to ensure that $D_{A-}$ conducts prior to turn on of $T_{A-}$. The
dead time required to ensure the maximum possible load range with ZVS can be determined
from the resonant oscillation. The resonance between $L_r$ and $C_r$ provides a sinusoidal voltage
across the capacitances that reaches a maximum at one forth of the resonant period [Sab1],

$$
\delta \tau_{\text{max}} = \frac{T}{4} = \frac{\pi}{2} \sqrt{\frac{L_r}{2 C_r}}
$$

(B.3)

In Figure B.4 three different cases are presented. When the energy stored in the inductor is
equal to the required energy to charge and discharge the resonance capacitors is shown in
Figure B.4 (a). In Figure B.4 (b) the energy is larger than the required energy and the
resonance capacitors are charged and discharged in less than $\delta \tau_{\text{max}}$, and the voltage is
clamped to the input voltage. When the energy is not sufficient to charge and discharge the
resonance capacitors, as shown in Figure B.4 (c), the ZVS turn-on is lost. The resonance
capacitors are marginally charged and discharged at $\delta \tau_{\text{max}}$ when $T_{A-}$ is turned-on under hard
turn-on conditions. The voltage across a resonance capacitor increases sharply and is finally
clamped to the dc-link voltage.
C. Appendix C: Matrix simplifications

The small-signal coefficient matrices can be achieved by linearizing the state-space averaging model of the topology as given by Eq. (4.60) and Eq. (4.61). Linearization as explained in Chapter 3 yields,

\[
A_{alt00} = \begin{bmatrix}
A_{alt011} & A_{alt012} \\
A_{alt021} & A_{alt022}
\end{bmatrix}
\] (C.1)

\[
B_{alt0} = \left[ \begin{array}{c}
\frac{2 D_{T_{dc}}}{L_s} - \frac{4 x_{t0} x_{20}}{(V_{dc0} - x_{20})^2 D_{T_{dc}} T_s} \\
- \frac{2 x_{t0}}{C (V_{dc0} - x_{20}) D_{T_{dc}} T_s} \left( x_{t0} - \frac{x_{20}}{R} \right)
\end{array} \right]
\] (C.2)

\[
E_{alt0} = \left[ \begin{array}{c}
\frac{2 V_{dc0}}{L_s} + \frac{2 x_{t0} (V_{dc0} + x_{20})}{(V_{dc0} - x_{20})^2 D_{T_{dc}} T_s} \\
- \frac{2 x_{t0} L_s}{C (V_{dc0} - x_{20}) D_{T_{dc}} T_s} \left( x_{t0} - \frac{x_{20}}{R} \right)
\end{array} \right]
\] (C.3)

Where,

\[
A_{alt011} = -\frac{2 x_{t0} (V_{dc0} + x_{20})}{D_{T_{dc}} T_s (V_{dc0} - x_{20})}
\] (C.4)

\[
A_{alt012} = -\frac{4 V_{dc0} x_{t0}}{(V_{dc0} - x_{20})^2 D_{T_{dc}} T_s}
\] (C.5)
From the steady-state analysis of the circuit, Eq. (4.57) yields,

\[
x_{10} = \frac{x_{20}}{R}
\]  \hspace{1cm} (C.8)

The terms

\[
\left( x_{10} - \frac{x_{20}}{R} \right) = 0
\]  \hspace{1cm} (C.9)

and

\[
\left( 2 \frac{x_{10}}{R} \right) = x_{10}
\]  \hspace{1cm} (C.10)

Substituting Eqs. (C.9) and (C.10) into Eq. (C.1), Eq. (C.2) and Eq. (C.3) the small-signal coefficient, source and control matrices can be expressed as

\[
A_{sth0} = \begin{bmatrix}
-\frac{2 L_{\sigma} \left( 2 x_{10} - \frac{x_{20}}{R} \right)}{C \left( V_{dc0} - x_{20} \right) D_{r_s} T_s} & \cdots & -\frac{4 V_{dc0} x_{10}}{V_{dc0} - x_{20}} \frac{x_{10}}{x_{20}} & \frac{1}{D_{r_s} T_s} \\
\vdots & \ddots & \vdots & \vdots \\
-\frac{2 L_{\sigma} x_{10}}{C \left( V_{dc0} - x_{20} \right) D_{r_s} T_s} & \cdots & -\frac{2 L_{\sigma} x_{10}}{C \left( V_{dc0} - x_{20} \right) D_{r_s} T_s} & \frac{1}{R}
\end{bmatrix}
\]  \hspace{1cm} (C.11)
\[ B_{alt0} = \begin{bmatrix} \frac{2 D_{T_{dc}}}{L_{\sigma}} - \frac{4 x_{10} x_{20}}{(V_{dc0} - x_{20})^2 D_{T_{dc}}, T_s} \\ 0 \end{bmatrix} \]  

(C.12)

\[ E_{alt0} = \begin{bmatrix} \frac{2 V_{dc0} + 2 x_{10} (V_{dc0} + x_{20})}{L_{\sigma} (V_{dc0} - x_{20}) D_{T_{dc}}, T_s} \\ 0 \end{bmatrix} \]  

(C.13)
D. Appendix D: State-space averaging model deviations

In Chapter 6 and in Chapter 8 the state-space averaging model for both the phase-shifted DAB and the duty-cycle controlled DAB topology are presented in Eq. (6.17) and Eq. (8.17). Comparing the two models with the state-space averaging model for the SAB topology some minor deviations occur concerning the coefficient representing the mean inductor voltage. In Figure D.1 the inductor current and the inductor voltage corresponding to the phase-shift controlled DAB topology is shown. The mean value of the inductor voltage can be easily obtained and expressed in terms of the duty ratios for the different time intervals.

The on-duty ratio for the switch \( T_{A+} \) can be expressed as

\[
d = d_1 + d_2 \quad \text{(D.1)}
\]

Figure D.1: Inductor voltage and inductor current corresponding to the Phase-shift controlled DAB
The duty ratio \( d_3 \) of the diode \( D_{\text{d}} \) can be derived with respect to \( d \) as

\[
\begin{align*}
  d_3 &= 1 - d \\
  &\quad \text{(D.2)}
\end{align*}
\]

Additionally, and from Figure D.1 the mean value of the inductor voltage during a half-cycle can be derived as

\[
\langle V_{L_a} \rangle = d_1 (V_{dc} + x_2) + d_2 (V_{dc} - x_2) - d_3 (V_{dc} + x_2)
\]

\[
\text{(D.3)}
\]

The mean value of the inductor voltage can now be expressed in terms of \( d_1 \) and \( d_2 \). From Eqs. (D.1) and (D.2) \( \langle V_{L_a} \rangle \) is found by insertion of \( d_2 = d - d_1 \) in Eq. (D.3). Rearranging the terms yields

\[
\langle V_{L_a} \rangle = d_1 (V_{dc} + x_2) - d_1 (V_{dc} - x_2) + d (V_{dc} - x_2) + d (V_{dc} + x_2)
\]

\[
\text{\quad} - (V_{dc} + x_2)
\]

\[
\text{(D.4)}
\]

Following a basic algebraic procedure the mean value of the inductor voltage can be expressed as

\[
\langle V_{L_a} \rangle = (2d_1 - 1)x_2 + (2d - 1)V_{dc}
\]

\[
\text{(D.5)}
\]

The result obtained in Eq. (D.5) can be compared with the state-space averaging model of the DAB converter as derived in Chapter 6. As obtained from Eq. (D.5) the mean value of the inductor voltage in the case of the phase-shifted DAB converter not only depends on \( (2d - 1)V_{dc} \) and \( x_2 \) as the SAB. Due to the fact that the secondary bridge contains controllable switches, the output voltage is multiplied by the factor \( (2d_1 - 1) \).

The deviations obtained between the state-space averaging model concerning the phase-shift controlled DAB and the duty-cycle controlled DAB converter can be explained by following a similar procedure as above. In Figure D.2 the inductor current and the inductor voltage of the duty-cycle controlled DAB converter are shown. The average value of the inductor voltage can be expressed as
Rearranging the terms in Eq. (D.6) yields

\[
\langle V_{L_i} \rangle = d_1 V_{dc} - d_1 \left( V_{dc} - x_2 \right) + d \left( V_{dc} - x_2 \right) + d \left( V_{dc} + x_2 \right) - \left( V_{dc} + x_2 \right)
\]  

\( (D.6) \)

The result obtained in Eq. (D.7) can be compared with the state-space averaging model of the DAB converter as derived in Chapter 8. When comparing Eq. (D.7) with the case of the phase-shift control, i.e. Eq. (D.5), it is found that the factor describing the dependence on the output voltage has changed from \((2d_1 - 1)\) to \((d_1 - 1)\).

\[
\langle V_{L_i} \rangle = (d_1 - 1)x_2 + (2d - 1)V_{dc}
\]  

\( (D.7) \)

**Figure D.2: Inductor voltage and inductor current corresponding to the duty-cycle controlled DAB**
Since these factors are always negative in the step-down mode, the mean value of the inductor voltage is increased by the same amount. As a result the peak value of the inductor current is increased accordingly.

In Figure D.3 the inductor voltage and the inductor current corresponding to SAB converter is shown. In this case the average value of the inductor voltage can be expressed as

$$\left\langle V_{L_a} \right\rangle = d \left( V_{dc} - x_2 \right) + (1 - d) \left( -V_{dc} - x_2 \right)$$  \hspace{1cm} (D.8)

From Eq. (D.8) the mean value of the inductor voltage corresponding to the SAB converter can be derived as

$$\left\langle V_{L_a} \right\rangle = (2d - 1)V_{dc} - x_2$$  \hspace{1cm} (D.9)
Equation (D.9) can be compared with the results obtained by the state-space analysis as presented in Chapter 4.

The deviations obtained concerning the DAB converters compared with the SAB topology are summarised in Table D.1

<table>
<thead>
<tr>
<th>SAB</th>
<th>Phase-shifted DAB</th>
<th>Duty-cycle controlled DAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$(2d_1 - 1)$</td>
<td>$(d_1 - 1)$</td>
</tr>
</tbody>
</table>

*Table D.1: The Factor describing the dependence on the output voltage for each of the three investigated topologies*

From Table D.1 an interesting conclusion can be drawn. As already investigated previously the factors in Table D.1 are always negative in the step-down mode. This means that the phase-shift controlled DAB has the highest mean value of the inductor voltage and the SAB has the lowest. Since the mean value of the inductor voltage is proportional to the peak value of the inductor current it is obvious that the phase-shift controlled DAB has the highest peak-value of the inductor current. This also indicates that the phase-shift controlled DAB has the potential to deliver the highest output power followed by the duty-cycle controlled DAB and the SAB consecutively.
E. Appendix E: Verification of the small-signal model

As mentioned in Chapter 4 simulations have been done in order to verify the small-signal model as presented in the present thesis. The response of the converter due to duty ratio variations has been examined and compared with the results presented in Chapter 4.

![Simulation waveforms. Small-signal duty ratio variations.](image)

*Figure E.1: Simulated waveforms. Small-signal duty ratio variations.*

The simulated waveforms are shown in Figure E.1. Observe that the frequency of the variations is 50 Hz. As shown in the figure both the output voltage and current are amplified when the duty ratio is increased and are decreasing when the duty ratio is decreased. The increase of the output voltage corresponds to an amplification of 51 dB as predicted by the small-signal model. In Figure 4.7 a 51dB amplification corresponds to a frequency magnitude of 50 Hz. This implies that the small-signal model fully describes the dynamic behaviour of the SAB converter.

Similar results have been obtained for all the transfer functions as presented in the present thesis.
F. Appendix F: Transformer-induced Low-frequency Oscillations (TLO)

In [Kles1] and [Kin1] the different modes of the Transformer-induced Low-frequency Oscillations (TLO) are presented and thoroughly examined. In this chapter the TLO modes are summarised.

Consider a series-loaded resonant converter operating at DCM as shown in Figure F.1 (a).

![Series-loaded resonant converter operating at DCM](image)

(a) SLR half-bridge configuration

![Resonance current, transistor (T₆₆) and diode (D₄₅) current](image)

(b) Resonance current, transistor (T₆₆) and diode (D₄₅) current

*Figure F.1: The series-loaded resonant converter operating at DCM.*
In Figure F.1 (b) the resonance current and the current through the transistor $T_{A-}$ and the diode $D_{A+}$ are shown. The magnetising inductance of the transformer is considered to be infinitely large.

The theoretical maximum value of the average output voltage $V_0$ with respect to the input voltage $V_d$ is limited to $\frac{V_0}{V_d} = 1$. To overcome this limitation a transformer has to be introduced to the circuit. The parasitics of the transformer may interference with the resonant circuit and may lead to the generation of transformer-induced low-frequency oscillations. These oscillations can influence the power transfer capacity.

The TLO are not present under heavy load conditions. At light load conditions however, the TLO can influence the operation of the converter. The different TLO modes are presented in Figure F.2, Figure F.3 and in Figure F.4.

In Figure F.2 and Figure F.3 the ratio $\frac{V_0}{V_d} = 0.74$. The TLO as shown in Figure F.3 are much more severe than the oscillations shown in Figure F.2. Both the diode $D_{A+}$ and the transistor $T_{A-}$ are forced to commutate the current twice in a half-cycle. As a result switching losses may appear and the conduction losses may increase.

In Figure F.4 the converter operates at heavy loads and the TLO are almost negligible.

Summarising the TLO is present at light loads, which means that at high values of the ratio $\frac{V_0}{V_d}$ and/or at low switching frequencies. A detail analysis of the the TLO is presented in [Kles1] and [Kin1].
(a) The resonance current and the magnetising current (bold line)

(b) The resonance, the magnetising current and, the current through $T_{A-}$ and $D_{A+}$

Figure F.2: The TLO mode at $\frac{V_s}{V_d} = 0.74$ and at $f_s = \frac{f_0}{2}$
(a) The resonance current and the magnetising current (bold line)

(b) The resonance, the magnetising current and, the current through $T_{A-}$ and $D_{A+}$

Figure F.3: The TLO mode at $\frac{V_s}{V_d} = 0.74$ and at $f_s = \frac{f_0}{10}$
(a) The resonance current and the magnetising current (bold line)

(b) The resonance, the magnetising current and, the current through $T_{A-}$ and $D_{A+}$

Figure F.4: The TLO mode at $\frac{V_o}{V_d} = 0.37$ and at $f_s = \frac{f_0}{2}$
G. Appendix G: Measuring Equipment

The measuring equipment used during the experimental measurements are summarised in Table G.1.

<table>
<thead>
<tr>
<th>Instrumentation</th>
<th>Manufacturer</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscilloscope</td>
<td>LG564A 1GHz</td>
<td>LeCroy</td>
</tr>
<tr>
<td>Inductor Current Transformer</td>
<td>Pearson Electronics INC</td>
<td>≤1%</td>
</tr>
<tr>
<td>Efficiency measurements</td>
<td>Wide Band Power Analyzer D6100</td>
<td>NORMA</td>
</tr>
<tr>
<td>Voltages</td>
<td>×100 voltage probes PPE1.2KV</td>
<td>LeCroy</td>
</tr>
</tbody>
</table>

Table G.1: Measuring equipment.