Investigation of
Acoustic Source Characterisation
and Installation Effects
for Small Axial Fans

by

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ABSTRACT

Fans are often used in equipment such as home appliances and electronic equipment where the margin of profit is small but customers demands on a low noise level are high. Therefore, methods for predicting the noise emitted by an application including one or several fans are desirable in order to improve, accelerate and reduce the cost of low-noise design. The Noise Shaping Technology (NST) has been developed within the EC-project NABUCCO in order to fulfil the above requirements on a prediction method. According to NST, the noise source (not necessary a fan) is described by one or several noise descriptors, CSSs, and the corresponding transmission paths through the structure described by one or several transfer functions, ACFs. In this thesis, the applicability of NST is evaluated on a cabinet for electronic equipment where small axial cooling fans constitute the primary sources of the airborne sound.

As an axial fan is a complex source of sound, simplifications are necessary when modelling its acoustic properties. Therefore, the sound radiation of an axial fan in free space was examined by expanding the generated sound pressure field into spherical harmonics. The conclusion on a source model for the cabinet example, where the fans are more or less In-duct mounted, is a modified single axial dipole. The model is expected to be valid in the entire frequency range of interest except in the mid-frequency range where the modal density is low. In order to improve the source model in this frequency range, a future model based on a rotating dipole is proposed.

The sound power of a small axial fan is measured in an ISO 10302 test-rig. In order to take account of flow conditions, acoustically transparent ducts have been developed. These shall be attached to the test-rig when measuring the sound power of the fan. A simple but practical method of how to correct the sound power for the baffling effect of the test-rig has also been developed. Finally, the sound power can be converted into dipole force, which is the airborne CSS corresponding to the single axial dipole model.

The corresponding airborne transfer function (ACF), i.e., from dipole force at the source point to sound pressure at the receiver point, is measured reciprocally by taking use of Lyamshevs reciprocity relation.

From multiplication of the CSS and the ACF, the sound pressure can be predicted. The prediction shows quite good agreement with the measured values.

Keywords: axial fan, airborne sound, source characterisation, transmission path analysis, In-duct, spherical harmonics, rotating dipole, installation effects, ISO 10302, flow conditions, baffling effect, acoustically transparent ducts, Lyamshevs reciprocity relation, reciprocity, CSS, ACF, GSM, NST.
ACKNOWLEDGEMENTS

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1. INTRODUCTION

This licentiate thesis describes work done in the recently terminated EC-project NABUCCO. The Nabucco project aimed through a component approach at improving, accelerating and reducing the cost of low-noise design of equipment such as home appliances, heating, ventilating and air conditioning, refrigerating units, washing machines, electronic equipment, the noise of which although not very high and harmful causes discomfort and stress to users and consumers.

The first fundamental concept of Nabucco is that of a component – noise source – which generates noise through a complex interaction with its structural assembly. Here, a source is taken into account in a realistic rather than an abstract way, the latter being usually the case when the prediction is carried out by computation only. The result will be one or several noise descriptors, the Component Source Strengths (CSS), which together provide full noise information about the source – air-borne (AB), structure-borne (SB), fluid-borne (FB) etc. The second fundamental concept is that of a Generic Structural Model (GSM). The role of a GSM is to provide noise transfer functions, the Assembly Conductivity Functions (ACF), from the component(s) via the connections and the structure to the listener’s ear. The generic term of this technique is Noise Shaping Technology (NST), since if e.g., the ACFs have been established, CSSs corresponding to different components, of the same type though, can be chosen from by the designer in order to shape the noise of the future product in a sensitive way.

Methods from traditional subject fields in technical acoustics like Source Characterisation and Transmission Path Analysis will apply very well when it comes to establishment of suitable CSSs and ACFs. Therefore, in the following text, the traditional notation has been used instead of the NST abbreviations.

The application, on which the applicability of NST is about to be evaluated, is a cabinet for electronic equipment where fans provide the necessary cooling. Consequently, the main noise sources are the fans. As a fan is a complex source of sound, simplifications are necessary in order to model its acoustic properties. The sound generation of a fan is also very dependent on certain installation effects like flow conditions, acoustic influence from the structure etc. Therefore, the main part of this work has been concentrated on acoustic source characterisation and installation effects for small axial fans.
2. THE APPLICATION

In NABUCCO the applicability of NST was to be evaluated by trying to apply the technique to several different products. One of the products was a cabinet, which contains electronics that has to be cooled. As fans provide the cooling, the cabinet emits noise. Concerning cooling flow, the interior of the cabinet consists of two chains. A radial fan drives the internal flow, which provides cooling of the electronics. Two axial fans drive the external flow, which is in contact with the internal flow only through two heat exchangers. This means that there is no direct exchange of air between the two chains and that only the external chain has openings to the exterior of the cabinet. The cabinet itself is made of equally thick steel plates except for the interior and backside of the door, which is made of aluminium plates.

2.1. PRELIMINARY MEASUREMENTS

In order to find the main noise source(s) and the paths of sound transmission through the cabinet, some preliminary measurements was carried out.

In order to investigate which surfaces of the cabinet that emits most noise, the total exterior surface was divided into ten sub-surfaces.

All surfaces are closed except surface S2 and S5 where there are openings to the interior of the cabinet door. The opening in S5 is the intake for the external flow and the opening in S2 its corresponding outlet.
The radiated Sound Power Level from each surface can be calculated from

\[ \langle L_{w_i} \rangle = \langle L_{i_i} \rangle + 10 \log S_i, \]

where \( S_i \) is the area \([m^2]\) of the sub-surface. The spatial averaged Sound Intensity Level, \( \langle L_{ii} \rangle \), was measured using the scanning technique with a distance of about 0.2 m to the surface in order to be out of the near field. Using a spacing of 12 mm between the two microphones of the intensity probe, the measurable frequency range is from 100 Hz to 5 kHz, which by a wide margin includes the frequencies of interest for the cabinet.

The measurement plan was as follows:

<table>
<thead>
<tr>
<th>Measurement no</th>
<th>Internal flow fan running</th>
<th>External flow fans running</th>
<th>Flow guide</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>--</td>
<td>X</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td>X</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

In order to get sufficient cooling capacity the fans have to run at full speed and therefore all measurements were performed with either the internal fan or the external fans, or all fans, running at full speed.

These are the results from measurement number 1:

![Graph showing sound power level vs. frequency](image)

<table>
<thead>
<tr>
<th>( L_{W, A} ) [dB(A)]</th>
<th>Lw [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>51.4</td>
<td>57.4</td>
</tr>
<tr>
<td>57.4</td>
<td>52.3</td>
</tr>
<tr>
<td>52.9</td>
<td>54.2</td>
</tr>
<tr>
<td>48.2</td>
<td>51.7</td>
</tr>
<tr>
<td>53.2</td>
<td>53.9</td>
</tr>
<tr>
<td>55.8</td>
<td>63.7</td>
</tr>
<tr>
<td>62.6</td>
<td>59.3</td>
</tr>
<tr>
<td>56.2</td>
<td>60.3</td>
</tr>
<tr>
<td>62.7</td>
<td>60.7</td>
</tr>
<tr>
<td>61.5</td>
<td>61.5</td>
</tr>
<tr>
<td>70.2</td>
<td>70.2</td>
</tr>
</tbody>
</table>

Figure 2.4: Results from measurement no 1 (internal fan only).
The graphs show the linear third-octave sound power spectrum of each surface and the figures below the graphs show the corresponding total sound power level, linear and A-weighted.

The spectrum of the radial fan itself is rather flat. Therefore, if direct airborne sound from the interior of the cabinet reaches the exterior at any surface, it would show up as an increased level at high frequencies in its corresponding graph. However, as all graphs decrease more or less monotonically at high frequencies, no direct airborne sound from the interior of the cabinet reaches the exterior and hence all sound radiation from the surfaces is due to vibration of the surfaces. As modern fans are well balanced, the fan itself is probably not the primary source to these vibrations. Instead, they are most probably induced by the sound pressure generated by the fan inside the cabinet. Vibrations induced by flow are negligible, as the flow velocity is low inside the cabinet.

Next are the results from measurement number 2:

![Graph showing sound power level vs. frequency](image)

<table>
<thead>
<tr>
<th>Surface</th>
<th>LWA [dB(A)]</th>
<th>LW [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>58.2</td>
<td>64.3</td>
</tr>
<tr>
<td>S2</td>
<td>65.7</td>
<td>71.8</td>
</tr>
<tr>
<td>S3</td>
<td>60.3</td>
<td>66.6</td>
</tr>
<tr>
<td>S4</td>
<td>60.1</td>
<td>66.9</td>
</tr>
<tr>
<td>S5</td>
<td>65.1</td>
<td>71.7</td>
</tr>
<tr>
<td>S6</td>
<td>55</td>
<td>61.8</td>
</tr>
<tr>
<td>S7</td>
<td>56.1</td>
<td>63.6</td>
</tr>
<tr>
<td>S8</td>
<td>50.9</td>
<td>59.4</td>
</tr>
<tr>
<td>S9</td>
<td>54.6</td>
<td>61.9</td>
</tr>
<tr>
<td>S10</td>
<td>56.5</td>
<td>63.6</td>
</tr>
<tr>
<td>S Total</td>
<td>70.5</td>
<td>77.1</td>
</tr>
</tbody>
</table>

Figure 2.5: Results from measurement no 2 (external fans only).

Here a significant increase in level and high frequency contents can be observed at the surfaces with openings, S2 and S5, compared to the other surfaces. This can be expected since S2 and S5 correspond to the inlet and outlet of the external flow ducts and therefore sound emitted by the axial fans reaches these openings directly.
Next are the results from measurement number 3:

![Graph showing sound power levels for different frequencies.](image)

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
<th>S10</th>
<th>S Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>125</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>160</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

LwA [dB(A)] 58.5 65.4 60 60.2 64.6 55.1 56.9 55.5 57.4 59 70.6
Lw [dB] 64.7 71.8 66.5 67.3 71.6 62.2 64.8 63.3 64.7 65.8 77.5

A comparison of these results to the ones obtained in measurement no 2 shows almost no differences except at S8, S9 and S10 where the levels are about equal to the sum of the levels obtained in measurement 1 and 2. The reason to why there is no difference at S6 and S7 is probably partly due to diffraction of sound from the openings in S2 and S5 disturbing the measurements. From the comparison it can be established that the contribution from the internal fan is roughly the same as that from the external fans concerning S8, S9 and S10. If any other surface is concerned, the contribution from the internal fan can be neglected. However, the contribution from S8, S9 and S10 is not important to the total sound power level, as the figures in Figure 2.6 show. Furthermore, S10 is the top surface and therefore the radiation from it is not affecting people standing on the floor. Hence, the conclusion is that the contribution from the internal fan can be completely neglected and therefore only the contribution from the external fans has to be considered.

From the results of measurement no 3, the sound power of all closed surfaces are added into one graph and the sound power of the open surfaces are added into another graph, in Figure 2.7.
As Figure 2.7 show is the sound power radiated from all closed surfaces about equal to the sound power radiated from the openings. Hence, vibration of the plates is important to the total sound power emitted from the cabinet.

In order to check the hypothesis of the sound pressure inside the cabinet being the cause to the vibrations, the sound pressure level inside the cabinet was measured together with the spatial averaged acceleration of S8, which is the surface with largest area. The spatial average was taken as a meanvalue over 8 measurement points randomly distributed over the plate. The measured acceleration was then converted into velocity in order to get a quantity directly proportional to power.
The character of the spectra in Figure 2.8 is very much the same, especially at low frequencies where most of the energy is concentrated. This indicates that it is the sound pressure inside the cabinet that induces the vibration of S8, and probably the vibration of the other surfaces as well, and not direct vibrations from the fans themselves.

Finally, the influence of a flow guide was investigated in measurement no 4. The purpose of the flow guide was to separate the flows entering each axial fan. In the cabinet door, the axial fans are closely mounted, therefore maybe disturbing each others inflow if no flow guide is present. Previous research [1] has shown that the broadband noise of an axial fan decreases if a turbulent inflow can be avoided. So, if the flow guide is able to reduce the turbulence, the measured sound power values shall decrease when it’s mounted.
A comparison with the results in measurement 3, obtained without the flow guide, shows that there is no significant reduction of sound power when the flow guide is present. This could be due to bad design of the flow guide or on the contrary, that the degree of flow disturbance still is very low even when two axial fans are mounted side by side without any flow guide.

2.2. CONCLUSIONS CONCERNING NST MODELLING

As the total level of 70.5 dB(A) in measurement no 2 is almost identical to the total level of 70.6 dB(A) in measurement no 3, and the total level of 63.7 dB(A) in measurement 1 is much lower than the total level in measurement 2, it means that, in terms of total emitted sound power from the cabinet, only the external flow chain have to be considered.

The external flow is driven by two axial fans, which are more or less mounted side by side. As mentioned earlier, the “no effect” of the flow guide indicates that the degree of flow disturbance still is very low even when the fans are mounted like this, a mounting that should constitute a sort of worst case from flow point of view. Hence, there seems to be no flow interaction between the fans and therefore certain sub-structuring should be allowed even within the external flow chain itself. Whether this sub-structuring do concern the acoustics of the fans remains to be investigated, but it seem to be applicable to the inflow conditions.
As measurement no 3 show is the sound power radiated from all closed surfaces about equal to the sound power radiated from the openings. Hence, not only the direct airborne transmission path must be concerned for but also radiation from the plates. This radiation is due to vibration of the plates induced by the sound pressure inside the cabinet. Therefore, it can be considered as part of the airborne transmission path. The vibration of the fans themselves, constituting primary sources of structure borne sound, can be neglected.

Considering the conclusions above, the NST model of the cabinet is drawn in Figure 2.11. Abbreviations according to Chapter 1 have been used.

![Figure 2.11: NST model of the cabinet.](image-url)
3. ACOUSTIC MODELLING OF AN AXIAL FAN

An axial fan is a complex source of sound. Complex in that sense that the physics behind the aerodynamically generated noise require a three-dimensional wave equation including both steady state and turbulence flow models together with appropriate boundary conditions. Hence, calculation of the three-dimensional acoustic field generated by an axial fan is, if not impossible, at least very time consuming and requires a lot of computational effort. Therefore, a need for simplified models occurs where the concept of equivalent acoustic sources is suitable since any acoustic field can be re-created by a superposition of the acoustic fields generated by a certain combination of equivalent sources. Simplified models allow treatment of the fan in a more deterministic way.

3.1. NOISE GENERATING MECHANISMS

Considering only the aerodynamically generated noise of an axial fan, the nature of it is indicated by its spectrum. A typical spectrum of an axial fan consists of discrete frequency components and a broadband component. According to the work of Lighthill, Curle, Ffowes, Williams and Hawkins, the physical origin to the noise components can be found from dimensional analysis of the terms in the multipole expansion of the wave equation source term. Neise [1] has made a summary of the physical origin to the aerodynamically generated noise, according to such an approach, and it’s shown in Figure 3.1.

![Diagram of fan noise generation mechanisms](image)

Figure 3.1: Summary of aeroacoustic fan noise generation mechanisms [1].
Regarding the low to medium speed fans considered here, neither monopole or quadrupole radiation becomes important [1, 2]. Dipole radiation though, is of great importance.

Dipole radiation is generated either by steady rotating forces or by unsteady rotating forces exerted by the fan blades on the fluid. Steady rotating forces are of the same type as those exerted by a propeller on the fluid. The forces experienced by the blades operating in a uniform stationary flow field are steady. An observer in a fixed frame of reference, however, will sense periodic pressure fluctuations at the blade passing frequency and its harmonics. This type of noise is referred to as “Gutin-noise” and in the case of low to medium speed fans, it’s negligible compared to the noise due to unsteady blade forces.

When the blades operate in a stationary but non-uniform flow field, each blade will experience unsteady forces, since both magnitude and angle of attack of the oncoming flow change with angular position. The resulting spectrum will be discrete with spectral lines at the blade passing frequency and its harmonics, and this because of the periodic nature of the unsteady forces. Non-uniform stationary flow fields are for example produced by; obstructions, like rods, present in the oncoming flow, by duct bends or corners, or by asymmetric position of the fan intake with respect to adjacent duct walls.

When the flow distortions on top of being non-uniform also are unsteady, the spectrum changes from discrete to continuous because of the random nature of the unsteady forces. For example, if there is a low-frequency variation in the oncoming flow the well-known haystack-effect appear, which is a band-spreading of the spectrum about the blade passing frequency. With increasing initially turbulent flow close to the fan, the level of the broadband component of the spectrum also increases.

According to Figure 3.1, there are several other causes to unsteady forces. However, in a normal application when the fan is running close to its optimum point of operation and the installation is reasonable, the above mentioned causes are the most significant.

Unsteady forces acting normal to the fan blade curvature gives rise to bending waves in the fan blades. If the direction of force excitation is different from the normal direction, other types of structural waves will be induced in the fan blades. However, as these types of waves have much lower radiation efficiency compared to the bending wave, they can be neglected. The acoustic radiation from an unbaffled finite surface (plate) suffering from bending waves can be modelled by a distribution of dipole sources over the surface as long as the wavelength of the frequency of interest exceed the dimensions of the surface [3]. Because the rotation and curvature of the fan blade surfaces, a three-dimensional acoustic field is created from superposition of the acoustic fields generated by the actual distribution of dipole
sources. Because the axially symmetric nature of an axial fan, the time averaged three-dimensional acoustic field can also be assumed to be axially symmetric about the fan axle.

3.2. IN-DUCT MODEL

In Chapter 3.1 it was established that both the discrete frequencies and the broadband part of the noise could be modelled by a distribution of dipole sources over the blade surfaces. In conjunction with limiting the model of an In-duct fan to the plane wave region of the duct system, this permits the use of a single dipole source to represent the distribution of dipole sources in the impeller plane. Modelling the In-duct axial fan in Figure 3.2, the single dipole source shall therefore be located at the impeller plane, which equals $z = 0$, and oriented along the $z$-axis. As the $z$-axis usually coincide with the symmetry axis of the fan, because of the direction of flow, the direction of the dipole is said to be in the axial direction of the fan. This single axial dipole model of an In-duct axial fan was first proposed by Cremer [4] and further verified by Baade [5], Åbom and Bodén [6].

3.3. RADIATION FROM AN AXIAL FAN IN FREE SPACE CONDITIONS

The limitation to the plane wave range of the duct system permitted the use of a single dipole to represent the distribution of dipole sources in the impeller plane. An In-duct axial fan, in conjunction with no limitation of the frequency to a certain range, therefore implies that the distribution of dipole sources in the impeller plane might not be replaced by a single dipole.

In order to develop a model of an axial fan which is based on a minimum amount of equivalent sources but still accurate enough in the entire frequency range, the radiation of a fan in free space is examined using a spherical harmonics expansion. Hence, if it turns out that the free space radiation of the fan solely is due to contribution from the first order spherical harmonic (dipole radiation), the single axial dipole model will represent the fan at all frequencies under any mounting condition.
3.3.1. **Spherical Harmonics Expansion**

The sound pressure field in free field conditions caused by a vibrating sphere, centred at the origin, is described by the wave equation in spherical coordinates $(r, \phi, \theta)$

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial p}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 p}{\partial \phi^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}.
\]

Assuming axial symmetry, i.e., no dependence on $\phi$, and it reduces to

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial p}{\partial \theta} \right) = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2},
\]

where $c$ is the speed of sound in the fluid.

The harmonic solution to this equation can be found in ordinary textbooks, e.g., Morse & Ingard [7]. Omitting the time dependence $e^{i\omega t}$, and the n'th solution read

\[
\hat{p}_n = \sqrt{\frac{\pi}{2kr}} \hat{E}_n H^{(2)}_{n+1/2}(kr) P_n(\cos \theta), \quad n = 0, 1, 2, \ldots, \tag{3.1}
\]

where $\hat{E}_n$ is the harmonic complex amplitude of order $n$. The Hankel function $H^{(2)}_{n+1/2}$, of the order $n + 1/2$, is dependent on the wave number $k = \omega/c$ and the radius $r$. Finally, the n’th order Legendre polynomial $P_n$, is dependent only on the angle $\theta$.

As different orders are orthogonal to each other, the complete sound pressure at a point in free space is given by

\[
\hat{p}(k, r, \theta) = \sum_{n=0}^{\infty} \hat{p}_n. \tag{3.2}
\]

The advantage using Spherical Harmonics is that the sound pressure pattern of each order corresponds to the pressure pattern generated by different types of equivalent acoustic sources centred at the origin. The pattern of the zero order, $n = 0$, correspond to the pattern generated by a monopole at the centre. Continuing with the first order, its pattern correspond to that of a dipole at the centre, and the second order pattern to that of a quadrupole and so on. Equation 3.2 also tells that the different types of equivalent sources (monopole, dipole etc.), giving rise to the respective sound pressures, are mutually coherent. From measuring the sound pressure field of a real source, like a fan, the expansion into spherical harmonics therefore provides much help in the evaluation of a model based on mutually coherent equivalent sources.

In spherical coordinates the sound power of each order of an axially symmetric field, is given by

\[
W_n = 2\pi r^2 \int_0^{\pi} \sin \theta \, d\theta, \tag{3.3}
\]
where $\bar{I}_{r,n}$ is the time averaged sound intensity of the $n$’th order in radial direction. Assuming a far field approximation and it’s given by

$$\bar{I}_{r,n} = \frac{1}{2} \text{Re} \left( \frac{|\hat{p}_r|^2}{\rho_0 c} \right),$$

Equation 3.4

where $\rho_0$ is the static fluid density. The error in sound intensity caused by this approximation is shown in Figure 3.3.

![Figure 3.3: Sound intensity error caused by the far field approximation.](image)

With $r = 2$ m, $f = 100$ Hz and $c = 343$ m/s the far field approximation will overestimate the sound intensity by $\approx 1$ dB for the 2’nd order and less for the lower orders.

From Equation 3.1, the new harmonic complex amplitude is introduced

$$\hat{A}_n = \frac{\pi}{2kr} \hat{E}_n H_n^{(2)}(kr).$$

Equation 3.5

Combining Equation 3.1, 3.3, 3.4 and 3.5 gives the final expression for the sound power of each order

$$\bar{W}_n = \frac{2 \pi r^4 |\hat{A}_n|^2}{\rho_0 c (2n+1)}, \quad n = 0, 1, 2, \ldots$$

Equation 3.6

Because the orthogonal property of the harmonics described in Equation 3.2, the total sound power is given by
In order to determine the sound power of each order of a real source, the complex amplitudes, $\hat{A}_n$, have to be determined from measurements. Some reformulation of Equation 3.2 is therefore suitable. Use Equation 3.1 and 3.5 in Equation 3.2 and the complete sound pressure at a point $m$, defined by $(r_m, \theta_m)$, is given by

$$\hat{p}_m = \sum_{n=0}^{\infty} \hat{A}_n(k,r_m)P_n(\cos \theta_m).$$

Truncating the series to $N+1$ terms and measuring the sound pressure at the points $m = 1, 2, 3, ..., M$, and a linear system of equations can be formulated. The solution for the complex amplitudes is then generally given by

$$\hat{A} = \mathbf{P}^{-1} \hat{\mathbf{p}},$$

where $\mathbf{P}^{-1}$ is the inverse of the Legendre polynomial matrix

$$\mathbf{P} = \begin{bmatrix}
P_0(\cos \theta_1) & P_1(\cos \theta_1) & \cdots & P_N(\cos \theta_1) \\
P_0(\cos \theta_2) & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
P_0(\cos \theta_M) & \cdots & \cdots & P_N(\cos \theta_M)
\end{bmatrix},$$

and $\hat{\mathbf{p}}$ the measured complex harmonic sound pressure amplitude vector

$$\hat{\mathbf{p}} = \begin{bmatrix}
\hat{p}_1 \\
\hat{p}_2 \\
\vdots \\
\hat{p}_M
\end{bmatrix}.$$

Measuring the complex sound pressure at a point $m$ in the sound field generated by the source at the centre, require a reference signal. As the source is an acoustic source, the only reference signal possible to measure is the sound pressure at a point in the sound field where only sound from the source under investigation is present. The expected main direction of the dipole radiation from the fan then imply that its generated sound pressure level on a certain radius from the fan has its peak at a point on the symmetry axis of the fan. The microphone measuring the reference signal shall consequently be positioned somewhere along that axis. Because the assumption of
the field being axially symmetric, it’s of no importance how the measurement plane is oriented relative to the fan casing as long as it coincides with the symmetry axis of the fan.

In order to get a solution to the equation system not that sensitive to possible errors in the angle when positioning the moving microphone at the respective points, it’s better to use more points than necessary in order to get an overdetermined system which then can be solved in a least square sense. According to expertise in numerical analysis the degree of overdetermination should be at least three (3), which means that with \( N = 2 \) (zero, first and second orders) at least \( M = 3(N+1) = \{N = 2\} = 9 \) measurement points shall be used. According to the same expertise the points shall be chosen to be equidistant in angle.

When performing the measurements, two quantities need to be measured: Autospectrum of the sound pressure at the reference point \( G_{\text{ref,ref}} \) and the transfer function between sound pressure at the reference point to sound pressure at the movable point \( m \). The transfer function is defined by: \( H_{\text{ref,m}} = \hat{p}_m / \hat{p}_\text{ref} \). From signal analysis point of view, the measured “system” can be considered as a Single input/Single output-system with noise present at the output. In this case, “noise” is mainly identified as contribution from other possible non-coherent sources within the source region. The input to the “system” is the reference signal and according to previous argumentation it can be considered as noise-free because of the location of the reference point along the symmetry axis. If also locating the reference point at the inlet side of the fan, there will be no extraneous noise because of flow affecting the microphone. With noise present only at the output, the best estimate of the transfer function is the \( H_1 \) estimate. From the two measured quantities above, the complex sound pressure at point \( m \) is given by

\[
\hat{p}_m = \sqrt{G_{\text{ref,ref}}} H_{\text{ref,m}}.
\]

The measurements must finally be performed in an anechoic chamber in order to fulfil the boundary conditions of free space.

The measurements now to be reported were carried out on the two different types of axial fans showed in Figure 3.4 and 3.5.

![Figure 3.4: Pure axial fan.](image)

![Figure 3.5: Axial fan of mixed flow type.](image)
The difference between the two types is the direction of flow at the outlet. Because of the strong radial component of the flow, the acoustic behaviour of a mixed flow fan can be expected to be quite different from that of a pure axial fan.

The performance of a fan is given by its fan-curve. The principle look of the fan-curve of an axial fan running at a certain constant speed is showed in Figure 3.6.

![Figure 3.6: Typical fan-curve of an axial fan.](image)

When the volume flow \( \dot{V} \) increases, the static pressure difference \( \Delta P \) between the inlet and outlet side of the fan decreases. The optimum point of operation, both from operational and acoustic point of view, is close to the saddle point (approaching from the right side). At the saddle point the sound experience a dramatic change of character because the significantly higher level of the broadband component.

Because the difficulty to choke the volume flow when the fan operate in free space, the measurements were conducted at maximum volume flow, thereby the point of operation is not the optimum one. However, with the assumption that the ratios between the acoustic power of each order remain along the entire fan-curve, the measurements can be performed at this somewhat unusual operating point.

![Figure 3.7: The measurement arrangement with the fan on a stand and centred at the origin. The left microphone is fixed at the reference point and the right microphone, which is the movable microphone, is positioned at the first point. The measurement radius is \( r = 2 \) m and the points located at \( \Theta = 10, 30, 50, 70, 90, 110, 130, 150, 170 \) degrees.](image)
The first measured fan is a Papst DV6248, which is a mixed flow fan. Measurements were performed at two different rotation speeds (determined by applying either 35 or 48 V to the motor). As only monopole, dipole and quadrupole radiation is possible from an axial fan according to Figure 3.1, only the sound power levels of the zero, first and second orders have been calculated.

![Figure 3.8: The Sound Power Levels of different orders for the mixed flow fan running at 3187 rpm (35V).](image)

Figure 3.8 shows that the broadband component of the sound power predominantly is of the first order. At the discrete frequencies though, the result is not that easy to interpret. At some discrete frequencies, the sound power of the zero order is higher than that of the first order. At other frequencies, the relation is the opposite. At no frequency though, is the sound power of the second order the highest.

Assuming the sound power of a certain order being higher than the others is a unique property of each fan-type that is related to its geometry, the same relationships between the powers should be present at a different rotation speed.
As Figure 3.9 show, very much the same relationships present at the lower rotation speed also seem to appear at the higher one.

Knowing the rotation speed and number of fan blades, the discrete frequencies can be identified as originating from either the rotation frequency and its harmonics, or the blade passing frequency and its harmonics. According to Figure 3.8 and 3.9, Table 3.1 lists the order of highest power at the discrete frequencies for the measured 5-bladed mixed flow fan.

<table>
<thead>
<tr>
<th>Type</th>
<th>Zero order</th>
<th>First order</th>
</tr>
</thead>
<tbody>
<tr>
<td>1'st Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>2'nd Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>3'rd Rot</td>
<td></td>
<td>O, X</td>
</tr>
<tr>
<td>4'th Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>5'th Rot + 1'st BPF</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>6'th Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>7'th Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>8'th Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>9'th Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>10'th Rot + 2'nd BPF</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>11'th Rot</td>
<td>X</td>
<td>O</td>
</tr>
<tr>
<td>12'th Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>13'th Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>14'th Rot</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>15'th Rot + 3'rd BPF</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>16'th Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>17'th Rot</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>18'th Rot</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>19'th Rot</td>
<td>O, X</td>
<td></td>
</tr>
<tr>
<td>20'th Rot + 4'th BPF</td>
<td>O, X</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: The order of highest power regarding the discrete frequencies of the mixed flow fan. Rot = Rotation frequency, BPF = Blade Passing Frequency, O = 3187 rpm, X = 4085 rpm.
Note that for some higher order harmonics of the rotation or blade passing frequency, the difference between the level of the zero and first order is not big, according to Figure 3.8 and 3.9. That explains why there is a shift between the zero and first order for the 11’th and 19’th order harmonics when the rotation speed changes. However, according to Table 3.1, the assumption of the sound power of a certain order being higher than the others is a unique property of each fan-type that is related to its geometry, and therefore not dependent on rotation speed, seem to be true. Otherwise the results from the two cases wouldn’t coincide to this high extent.

The next measured fan is a Papst 7218N, which is a 5-bladed pure axial fan.

Concerning the broadband component, the predominance of the first order is even clearer here. This can be expected since the radial component of the flow on the outlet is rather small for the pure axial fan and consequently as well the random dynamic blade forces in the radial direction. Hence, random dynamic forces in the axial direction will dominate and give rise to the broadband noise. The sound power of the first order is to a higher extent the highest at the discrete frequencies comparing with the mixed flow fan.

From a discrete integration using the measured sound pressure autospectra at the different measurement points, the total sound power of the fan can be calculated. In Figure 3.11 and 3.12 the total power is compared to the sum of the zero, first and second order powers according to Equation 3.7.
Figure 3.11: Total sound power compared to the sum of the zero, first and second order powers for the mixed flow fan running at 4085 rpm.

Figure 3.12: Total sound power compared to the sum of the zero, first and second order powers for the pure axial fan running at 2287 rpm.
In both cases, the difference between the two curves is significant. One possible cause to the difference is the truncation of Equation 3.7 to only three terms, the zero, first and second order terms. However, based on 18 measurement points instead of 9, it was also possible to calculate the sound power of order 3, 4 and 5. As the levels of order 2-5 decreases continuously with increasing order number, and in addition are significantly lower than the zero and first order levels in the frequency range of interest, the power of the orders \( n \geq 6 \) will most probably continue to decrease. Consequently, the truncation is not the main cause to the difference.

The complex sound pressure is measured according to Equation 3.10. Due to this, only the part of the sound pressure at point \( m \) which is coherent with the sound pressure measured at the reference point, is present in the complex pressure. Therefore, additional sound pressures at point \( m \), which are generated by the fan but not coherent with the sound pressure at the reference point, will be “filtered out” of the complex pressure. As the sound power of the respective orders is based on the complex pressure, their sum will always be less than the total sound power, which is based on the measured autospectra. The amplitude of the autospectra will always be higher because of the contribution from non-coherent sources within the source region. However, the difference between the total power and the coherent power is not that big at the discrete frequencies where most of the energy is concentrated. Hence, at these frequencies all sources within the source region are more or less coherent to each other.

Now, if a dipole would serve as a free space model of the fan, the power of the first order alone should equal the total power. But, this is not the case. Consequently, in order to capture the total sound power of the fan in a free space model, the model need to be more complex.

Based on the measured sound pressure autospectra, the free space directivity of the two measured fans is plotted in Figure 3.13 and 3.14 respectively. The frequency range is limited to the range where most of the total power is captured in the discrete frequencies, i.e., the frequencies where all sources within the source region are more or less coherent to each other.
From looking at Figure 3.13 and 3.14, it’s obvious that the directivity pattern at these frequencies is different from that produced by a dipole. The overall directivity is more of omnidirectional character, which of course in the spherical harmonics expansion show up as the zero order having the highest sound power at certain discrete frequencies.
An omnidirectional directivity pattern is produced by a monopole centred at the origin. However, according to Chapter 3.1 monopole radiation is not significant for the low to medium speed fans considered here. Modelling an axial fan with a monopole would therefore go against the physical noise generation mechanism of the fan. Instead, the physical explanation to these results must be found elsewhere.

3.3.2. Physical Explanation To The Omnidirectional Behaviour

Consider a distribution of dipole sources along the fan blades with corresponding dipole-axes normal to the fan blade curvature. Because the rotation of the impeller the dipole-axes will rotate around the symmetry axis of the fan and because the blade curvature there will be certain angles between the dipole-axes and the symmetry axis. At the frequencies where all dipoles are coherent to each other, they can be replaced by a single rotating dipole. This concept of a rotating dipole has by Gutin [8] been proven to be valid concerning noise from a propeller, and in this case the difference between a propeller and a fan is not that big. Now it will be proven that a rotating dipole can give rise to an omnidirectional directivity pattern.

The strength and direction of a dipole is represented by its complex dipole force, \( \tilde{F}_0 = F_0(t) \).

With spherical coordinates, the projection of the force on the \( x, y \) and \( z \) axis is equal to

\[
\begin{align*}
F_x &= F_0 \sin \theta_0 \cos \varphi_0 \\
F_y &= F_0 \sin \theta_0 \sin \varphi_0 \\
F_z &= F_0 \cos \theta_0
\end{align*}
\]

Equations 3.11

Introduce a rotation of the force round the \( z \)-axis: \( \varphi_0 = \varphi_0(t) = \Omega t \), where \( \Omega \) is the constant angular frequency of the rotation. Substituting the rotation into Equations 3.11 gives

\[
\begin{align*}
F_x &= F_0 \sin \theta_0 \cos(\Omega t) \\
F_y &= F_0 \sin \theta_0 \sin(\Omega t) \\
F_z &= F_0 \cos \theta_0
\end{align*}
\]

Equations 3.12
The total sound pressure of three separate but coherent dipole sources fixed along the $x$, $y$ and $z$-axis respectively, is according to [3] given by

$$p(t) = p_x(t) + p_y(t) + p_z(t) = \frac{e^{-ikr}}{4\pi r} \left( \frac{1}{r^2} + \frac{ik}{r} \right) (F_x x + F_y y + F_z z),$$  \hspace{1cm} \text{Equation 3.13}

where $F_x$, $F_y$ and $F_z$ are the respective dipole forces.

Substituting Equations 3.12 into Equation 3.13 means that the three separate and fixed dipole forces are replaced by the components of the rotating dipole force. Replace also the cartesian coordinates in Equation 3.13 with spherical coordinates, according to

$$x = r \sin \theta \cos \phi$$
$$y = r \sin \theta \sin \phi$$
$$z = r \cos \theta$$

and the resulting sound pressure at the fixed observation point $(r, \theta, \phi)$ is given by

$$p(r, \theta, \phi, t) = \frac{F_0 e^{-ikr}}{4\pi r} \left( \frac{1}{r^2} + \frac{ik}{r} \right) \begin{pmatrix} \sin \theta \cos \phi \sin \theta_0 \cos(\Omega t) + \\ \sin \theta \sin \phi \sin \theta_0 \sin(\Omega t) + \\ \cos \theta \cos \theta_0 \end{pmatrix},$$  \hspace{1cm} \text{Equation 3.14}

where the parameter $\theta_0$ denote the angle of the rotating force to the $z$-axis.

With a stationary rotation of the dipole, the squared rms value of the sound pressure is given by

$$\tilde{p}^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T p^2(t) \, dt = \lim_{T \to \infty} \frac{1}{T} \int_0^T (\text{Re}[p(r, \theta, \phi, t)])^2 \, dt.$$  \hspace{1cm} \text{Equation 3.15}

If the force is a harmonic force, $F_0 = \tilde{F}_0 e^{i\omega t}$, and the near field contribution to the sound pressure according to Equation 3.14 is neglected, the squared rms value according to Equation 3.15 becomes

$$\tilde{p}^2 = \frac{k^2 |\tilde{F}_0|^2}{32\pi^2 r^2} \left( \cos^2 \theta \cos^2 \theta_0 + \frac{1}{2} \sin^2 \theta \sin^2 \theta_0 \right), \text{ if } \Omega \neq 0.$$  \hspace{1cm} \text{Equation 3.16}

As Equation 3.16 show no dependence on $\phi$ it means that the radiation from the rotating dipole is axially symmetric and consequently in this respect in full agreement with the expected behaviour of an axial fan.
In order to prove that there are angles \( \theta_0 \) where the directivity is omnidirectional, the following equation is solved

\[
\frac{\partial \rho^2}{\partial \theta} = 0. \tag{Equation 3.17}
\]

Carrying out the derivation gives

\[
\frac{k^2|\hat{F}_0|^2}{32\pi^2r^2} \sin \theta \cos \theta \left( \sin^2 \theta_0 - 2 \cos^2 \theta_0 \right) = 0. \tag{Equation 3.18}
\]

In order for Equation 3.18 to equal zero for all \( \theta \), the parenthesis must equal zero. Solving \( \sin^2 \theta_0 - 2 \cos^2 \theta_0 = 0 \) gives two solutions: \( \theta_0 = 54.7, 144.7 \) [deg]. This means that the directivity is omnidirectional if the angle of the rotating dipole force to the \( z \)-axis is equal to one of these angles. If \( \theta_0 \) take other values, the produced directivity pattern will be more similar to pure dipole patterns.

3.4. CONCLUSIONS

It’s of major importance that a source model represents the nature of noise generation of the real source in a proper way in order to get the right response from the structure. Therefore, when modelling an axial fan, only dipole sources will be concerned.

In the above analysis, it’s been concluded that the radiation of an axial fan predominantly is of dipole character even in free field conditions. Therefore, the single axial dipole model may work in the entire frequency range of interest for the cabinet example. However, from the analysis certain shortcomings of the single axial dipole model appear. One is the inability of the single axial dipole model to capture the total power of the fan. Neither can it produce the directivity measured at certain discrete frequencies. These shortcomings can partly be eliminated if the fan is modelled by a rotating dipole instead. However, a model based on a rotating dipole would be complex and therefore require rather complicated measurements. Therefore, in order to minimise the measurement and calculation efforts needed, simple models are to prefer.

In practise most fans are more or less In-duct mounted. This holds also for the axial fans in the present cabinet example. Therefore, the error when the simple single axial dipole model is used in an application not strictly In-duct, is probably rather small. Here are the reasons to why this is the case.

Concerning the broadband component of the noise the spherical harmonics expansion showed that it’s predominantly of the first order, i.e., of dipole type. As the high frequencies of the axial fan spectrum mainly is of broadband character, the error at these frequencies will be small if the total power of the fan is captured in the single
axial dipole. By “total power” is meant that the contribution from coherent as well as non-coherent sources, is included in the power. Consequently, there is an additional assumption that the non-coherent sources also are of dipole type and acting in the same direction as the axial dipole. The small error at high frequencies is because the diminishing importance of direction of excitation when the mode density in the duct system increases and approaches the character of a reverberation field. Hence, at high mode density the most important property of the model is that it captures the total power of the fan.

The low frequency part of the fan spectrum is to a higher extent dominated by discrete frequencies, where most of the power is concentrated to the blade passing frequency. As shown in Chapter 3.3.2, it’s possible to model the fan by a rotating dipole at these frequencies. However, at low frequencies is the direction of excitation not that important because only plane waves can propagate through the duct system anyway, and these are equally well excited by the single axial dipole. At most In-duct applications is the cut-on frequency of the first higher order mode above the tonal region of the fan spectrum, with the size of fans considered here. Hence, if the total power of the fan is captured in the single axial dipole model even at low frequencies, the error is probably relatively small.

Consequently, the single axial dipole model will have its largest shortcomings in the mid-frequency region where the mode density is low and therefore the direction of excitation important. The decision on a fan model for the cabinet example is despite of the latter shortcoming, the single axial dipole model.

In order to develop a model of an axial fan suitable when the fan is mounted in a structure of arbitrary shape and size, further work must be done where the rotating dipole will serve as the starting-point.
4. INSTALLATION EFFECTS

The airborne sound power generated by a fan application, is dependent on: Point of operation on the fan-curve, in/outlet flow conditions to the fan and acoustic influence from the structure in terms of reflection or absorption of the acoustic energy generated by the fan. The status of the above installation effects can in each application be expected to be different.

One application is the test-rig used when measuring sound power of a fan according to a certain standard. Another is the present application in which the fan is involved, e.g., the NABUCCO cabinet. In order to measure values of sound power in the test-rig that shall correspond to the values achieved in the present application, the same conditions concerning point of operation and in/outlet flow conditions must be present. The acoustic influence from the structure of the present application is included in the airborne transfer function (ACF) of the structure. The ACF is further considered in Chapter 5. However, the acoustic influence of the fan test-rig must be corrected for in order to achieve values of sound power that corresponds to the single axial dipole model of the fan.

4.1. ISO 10302

The international standard ISO 10302 was developed during the 1980’th to allow airborne sound power measurements of small air-moving devices, e.g., small axial fans.

![Figure 4.1: The ISO 10302 test-rig.](image)

The test-rig is a wooden-frame box covered with thin plastic in order to make it transparent to sound. The insertion loss of a properly manufactured test-rig is within ±1.5 dB in each one-third-octave-band. The fan is mounted to a rubber cloth in order to support it and avoid vibrations spreading from the fan to the test-rig. By adjusting the slider it is possible to change the point of operation along the fan-curve of the tested
fan. Hence, among the factors determining the airborne sound power of an axial fan, the test-rig only allow control of the operating point.

4.2. BAFFLING EFFECT

In [9] the radiation of sound from a loudspeaker (dipole source) located in a rigid finite circular baffle has been investigated analytically. The investigation show that the radiation pattern of a loudspeaker in a finite baffle start to differ from that of a dipole at rather low frequencies. In the ISO 10302 test-rig the tested axial fan, which also is a source of dipole type, is mounted to a rubber cloth. Because this rubber cloth, which from acoustic point of view is a rigid surface in contrary to the thin plastic covering the rest of the box, a baffling effect, similar to that of a loudspeaker in a circular baffle, will also take place for the fan. Taking the dimensions of the rubber cloth: \( W \times H = 1.2 \times 0.6 \ m^2 \) and turn it into an equivalent diameter of a circular baffle:

\[ d_{\text{eqv}} = \sqrt{4 \cdot W \cdot H / \pi} = 0.957 \ m \]

and the arrangement of a loudspeaker mounted to the test-rig will radiate as a dipole only for frequencies \( f \leq 114 \ Hz \) according to [9]. At higher frequencies the wavelength becomes approximately equal to or smaller than the equivalent diameter. The rubber cloth then begins to approach an infinite baffle and the radiation simultaneously approach that of a monopole near an infinite plane. This change in radiation will cause a higher sound power measured when the dipole source (i.e., the fan) is mounted to the test-rig compared to when it’s free. Consequently, this increase of measured sound power must be adjusted for in order to get the correct sound power of a free space single axial dipole, which is the chosen model of an axial fan. In order to be able to do the adjustment, the baffling effect of the test-rig must be measured.

When the sound power of an axial fan is measured according to ISO 10302, no ducts are attached to the fan. Therefore, the fan behaves like in free space conditions except for the effects due to baffling. In Chapter 3.3, it was proven that at certain harmonics the best model of an axial fan in free space conditions, is a rotating dipole. Because of the rotation, the baffling effect of the test-rig at these harmonics can not be measured using a loudspeaker. Since, if a loudspeaker were attached to the test-rig it would only produce a dipole fixed in the normal direction of the rubber cloth. Instead, the baffling effect is unique for each type of axial fan because the angle of the rotating dipole to the symmetry axis most probably differs for different types of axial fans. If the angle is small there will be more influence from the baffle compared to when the angle is large. With a large angle the rotating dipole is almost in parallel with the baffle and therefore not affected by it. Consequently, the baffling effect must be measured using the axial fan of interest as sound source.
An establishment of the baffling effect for a certain type of axial fan requires two measurements, which shall be performed in a semi-anechoic room. The first one is narrow-band measurement of the sound power generated by the fan when it’s mounted to the test-rig and operating at the maximum volume flow available at the chosen voltage supplied to the motor. This point of operation is achieved by opening the slider completely. At the second measurement the same fan is mounted on a stand, therefore operating at maximum volume flow, and located at the same place as when it was mounted to the test-rig in the previous measurement. The test-rig shall then be removed. In order to achieve the same flow conditions when the fan is mounted on the stand as when it’s mounted to the test-rig, a thin acoustically transparent plastic is attached to the fan. The purpose of the plastic is to prevent flow from going directly from the outlet of the fan to the inlet. The second measurement is also a narrow-band measurement of sound power and performed at the same voltage supplied to the fan as in the first measurement. The baffling effect is then given by

\[ \Delta L_W = L_{W,\text{test-rig}} - L_{W,\text{stand}}, \]  

Equation 4.1

where \( L_{W,\text{test-rig}} \) is the sound power measured with the fan mounted to the test-rig and \( L_{W,\text{stand}} \) the sound power measured with the same fan mounted on a stand and thin plastic attached to it.

4.2.1. Baffling Correction

The measured baffling effect, \( \Delta L_W \), of the mixed flow fan is shown in Figure 4.2.

In Chapter 3.3.1, the spherical harmonics expansion showed that the broadband component of the noise produced by the fan predominantly is of the first order, i.e., of dipole character. In Figure 4.2, the baffling effect is very clear at low frequencies concerning the broadband part of the noise, a fact that support the possibility of modelling the broadband noise with a single axial dipole at these frequencies. At higher frequencies there is almost no baffling effect at all concerning the broadband noise. The explanation to this is to be found in Chapter 3.1, namely the distribution of dipole sources over the fan blades which in free space conditions and in conjunction with high frequencies, makes only part of the dipoles directed normal to the baffle and therefore fully affected by it. The spherical harmonics expansion also showed that the order of highest sound power at the discrete frequencies varied depending on the origin of a particular frequency. At the blade passing frequency (\( \approx 340 \) Hz) for example, the zero order is the highest. In Figure 4.2, there is almost no effect from the baffle at the blade passing frequency. Consider the rotating dipole explanation to the
zero order appearance of certain discrete frequencies and this result is in agreement with what’s expected.

![Graph showing frequency vs. 
Lw[dB] for the baffling effect and corresponding baffling correction for the mixed flow fan.](image)

**Figure 4.2**: The baffling effect and corresponding baffling correction for the mixed flow fan.

Figure 4.2 also show a baffling effect at some first order discrete frequencies which is different from that of the broadband noise despite the first order character of the broadband noise. This contradictory appearance is most probably due to measurement errors caused by too high frequency resolution. Since the baffling effect is established from two separate measurements, it’s impossible to have the fan running at exactly the same rotation speed in both measurements. Therefore, large errors appear quickly due to the frequency shift if the frequency resolution is too high. Another possible cause is that at some of the discrete frequencies determined to be of the first order, the sound power of the zero order is only slightly lower. Therefore, a dip will occur in the baffling effect.

In order to establish a general correction curve for the baffling effect of a certain fan, it’s not possible to use the measured baffling effect immediately. This because the rotation speed of the fan changes with the point of operation on the fan-curve and therefore also the frequency of the discrete frequency components. The peaks and dips in the measured baffling effect, which are caused by the discrete frequencies of the fan, would consequently be applied to the wrong frequencies if the point of operation is different from the one used when measuring the baffling effect. Hence, it’s only possible to get a general baffling correction curve for the first order frequencies and
it’s simply obtained from fitting a piecewise straight line to the broadband component of the noise, according to Figure 4.2. By this, errors due to frequency shift are eliminated when the baffling correction is applied to a fan running at an arbitrary rotation speed.

4.3. ACOUSTICALLY TRANSPARENT DUCTS
As described in Chapter 3.1 is the sound generation of an axial fan dependent on the in and outlet flow conditions to the fan. Disturbance of the flow is usually caused by the duct system in which the fan is mounted. Irregularly shaped ducts will most probably cause a non-uniform flow profile and turbulence due to flow-separation at sharp edges. When measuring sound power of a fan in the test-rig it’s therefore desirable to simulate the flow conditions present in the real application (e.g., the cabinet) as close as possible. These conditions can be simulated by building exact copies of the duct system in the case of point, and attach them to the test-rig. It’s not necessary to make the ducts very long since flow disturbances close to the fan influences most. However, a duct system with acoustically rigid walls would not only affect the flow conditions but also the radiation impedance seen from the fan. As the radiation impedance seen by the fan in the structure already is included in the airborne transfer function (ACF), the ducts, providing the right boundary conditions for the flow, must be acoustically transparent in order not to incorporate the radiation impedance twice.

![Acoustically transparent ducts](image)

Figure 4.3: Acoustically transparent ducts simulating the flow conditions in the cabinet.

The acoustically transparent ducts in Figure 4.3 are manufactured of thin steel bars shaped as the geometry of the duct system. The same type of thin plastic as used in the test-rig is then stretched between the steel bars in order to constitute acoustically transparent duct walls. The effects of these ducts are shown in Figure 4.4.
Figure 4.4 shows that the effect of the inlet duct is small, except for the heystack-effect about the blade passing frequency and its harmonics referred to in Chapter 3.1. The on the whole small effect is probably because the inflow condition still is pretty good even with the duct attached. However, the outlet duct seems to have larger effect on the sound generation in terms of increased broadband levels. Some flutter in the plastic of the outlet duct was present during the measurement but it’s not likely that it alone explains the significantly higher broadband levels. The levels of the discrete frequencies are about the same with and without the ducts in the entire frequency range measured. This indicates that the ducts truly are acoustically transparent. Otherwise, reflections from duct walls and the end of the duct would create standing wave patterns inside the ducts resulting in amplification or attenuation of the levels relatively the levels generated without ducts. As the acoustically transparent ducts seem to affect the flow conditions very much the same way as rigid ducts do, but not affecting the radiation impedance seen from the fan, the conclusion is that the ducts fulfil what’s required from them.

As the flow conditions is of great importance to the sound generation of an axial fan, it’s advisable to try to simulate these when measuring sound power according to the ISO 10302 standard. In practise, it’s not possible to simulate the exact flow conditions of every possible design of a duct system. Instead, as a suggestion for the

![Figure 4.4: Sound power levels of the mixed flow fan (U = 48V, ∆P = 137 Pa) with and without acoustically transparent ducts.](image-url)
future, three typical cases can be measured by the fan manufacturer and chosen from by the customer.

Most fan installations fit into one of the typical cases in Figure 4.5. Case A is the most common installation where the cabinet serves as an example. Case B is sometimes used when a fan provides fresh air from outdoors to indoors. An example of case C is the mounting of the ventilating fan in personal computers. If the customer is able to identify his installation among these three typical cases then the sound power data received probably is more in accordance with the actual sound generation of the fan.

4.3.1. Flow Interaction Between Two Fans

Sometimes the capacity of one fan isn’t enough but it’s also impossible to use a larger fan. In those cases two or more smaller fans may be used. If two fans are mounted close to each other (e.g., side by side) some flow interaction between the two fans can be expected if they don’t have separate in and outlet ducts. If flow interaction is present, the sound power of two fans running with separate in and outlet ducts would differ from that when the in and outlet ducts are common for the two fans. Possible flow interaction can be investigated using acoustically transparent ducts.

Figure 4.5:

Figure 4.6: Left: Separate inlet ducts to the fans. The duct wall common to the fans is removable and thereby allowing for investigation of flow interaction. Right: The same set-up for the outlet.
By making the ducts acoustically transparent, the radiation impedance seen from the fans will not change when the duct wall in the middle, which is common to the fans, is removed. Consequently, only the effect of flow interaction is investigated when the duct wall is removed. The length of the inlet ducts is five times the diameter of a single fan. As an axial fan is very sensitive to flow-disturbances especially at the inlet, the ducts are made that long in order to separate the inflows completely. The length of the outlet ducts is equal to the diameter of a single fan. They can be made this short because the velocity of the radial outlet flow component has its highest value close to the outlet of the fan, and therefore also the importance of flow interaction.

Figure 4.7 shows that there is no significant effect due to flow interaction at either the inlet or the outlet side. From flow point of view does the side by side mounting of the fans constitute a sort of “worst case” compared to other possible ways of mounting the fans. Therefore the conclusion can be drawn that flow interaction between fans is generally of no importance to the sound power generated. This result is quite useful. Since, if two fans will be used in a certain application, there is no need to measure the sound power of both fans running together in the test-rig in order to take account of possible effects due to flow interaction. Instead, each fan can be measured individually.

![Figure 4.7: Effects due to flow interaction. (Mixed flow fans, $U = 48\text{V}$, $\Delta P = 160\text{ Pa}$)](image_url)

I: Inlet duct wall. O: Outlet duct wall
4.3.2. **Acoustic Correlation Between Two Fans**

If two fans of the same type are fed by the same voltage, therefore running at approximately the same rotation speed, and operating at the same point on their common fan-curve, some degree of acoustic correlation between the fans can be expected.

Now, the same set-up used to investigate possible flow interaction can also be used to determine the degree of acoustic correlation between two fans. The first measurement performed was measurement of sound power when both the left and right mixed flow fans were fed by the same voltage \( U = 48 \, \text{V} \) and the slider adjusted so that \( \Delta P = 160 \, \text{Pa} \). This point of operation is to the right of the saddle point referred to in Figure 3.6, and therefore the level of the broadband sound is not that high. Still though, it wasn’t possible to hear any beating effects at the discrete frequencies. During the measurement both the inlet and outlet removable duct walls was present in order to separate the flows to each fan. In the next two measurements, the sound power of each fan was measured separately. When the left fan alone was measured at \( U = 48 \, \text{V} \) and \( \Delta P = 160 \, \text{Pa} \), the outlet of the right fan was blocked but the duct walls still present in order to have the same flow conditions as in the first measurement. Then the right fan alone was measured in the same way. The degree of acoustic correlation between the two fans can now be determined from the difference in sound power between the two fans running together and the total sound power achieved if the individual sound power of each fan are added as if they are mutually uncorrelated. The mathematical formulation read

\[
\Delta L_W = L_{W_{L\&R}} - 10 \log \left( 10^{L_{W_{L}}/10} + 10^{L_{W_{R}}/10} \right),
\]

where index L and R denote the left and right fan respectively. The result in Figure 4.8 show upon a negligible degree of acoustic correlation between the two fans, as \( \Delta L_W \approx 0 \) in the entire frequency range. When changing the point of operation to above the saddle point, the degree of correlation remain very low. The reason to the low degree of correlation despite the fans are of the same type, fed by the same voltage and operating at the same point on the fan-curve, is because the fans are not running at exactly the same rotation speed. This result therefore implies that two fans in practice can be considered as mutually uncorrelated and no regard to the phase of the single axial dipole modelling the fan have to be taken into account when determining its source strength (CSS).
4.4. CALCULATION OF SOURCE STRENGTH

As concluded in Chapter 3.4 can the acoustic behaviour of a more or less In-duct axial fan be modelled by a single dipole. The source strength of a dipole is given by its dipole force, which have magnitude, phase and direction. According to Chapter 4.3.2 it’s not necessary to take the phase of the force into account. The direction of the force is according to Chapter 3.2 parallel with the direction of flow, i.e., directed along the symmetry axis of the fan.

In ordinary textbooks, e.g. [3], it is possible to find the following relationship between the sound power level generated by a dipole in free space, $L_{W,FS}$, and the squared rms value of its dipole force

$$\bar{F}^2 = \frac{12\pi\rho c W_{\text{ref}} 10^{L_{W,FS}/10}}{k^2},$$  

Equation 4.3

where $W_{\text{ref}}$ is the reference sound power. Usually $W_{\text{ref}} = 10^{-12}$ W is used when converting sound power into sound power level.

When the sound power level of an axial fan is measured in the ISO 10302 test-rig, the same point of operation on its fan-curve, and the same flow conditions as in the real application, has to be simulated in order to measure the right sound power level. The point of operation is easily simulated by adjusting the slider and the flow conditions by attaching acoustically transparent ducts to the test-rig. The transparent
ducts will not change the radiation impedance seen from the fan but still there will be an increase of sound power due to the baffling effect. As the sound power level inserted in Equation 4.3 shall correspond to that of a dipole in free space, the increase of sound power caused by the baffle must be subtracted from the measured sound power. As pointed out in Chapter 4.2.1, the baffle affects the broadband component and some discrete frequency components differently and therefore a selective application of the baffling correction curve is necessary.

The method now to be proposed is based on dividing the measured sound power level into two parts, a zero order part and a first order part. The zero order part contains only the discrete frequencies, which in the spherical harmonics expansion turned out to have the highest power at the zero order. Table 3.1 identifies these frequencies for the mixed flow fan. In terms of power, only the first rotation frequency and first blade passing frequency have to be concerned. This is for the simplicity of industry partners. By assuming that these frequencies remain of the zero order even at operating points different from the somewhat unusual operating point used in the spherical harmonics expansion, the same frequencies can be identified in the measured sound power spectrum and put into the zero order part.

The remaining part of the measured spectrum now contain mainly first order frequencies, both broadband and discrete. From physical aspects the first order part must be a continuous spectrum, which it’s not if the zero order frequencies simply is “cut out” of the measured spectrum in order to form the zero order part. Therefore the broadband (first order) level between the left and right marks on each side of a zero order harmonic, is interpolated in order to create a continuous first order part, according to Figure 4.9. In order to equal the measured sound power level when the sound power levels of the zero and continuous first order parts are added, the level of the zero order harmonic between the marks is calculated. Consequently, its peak level will be lower than the peak level of the measured spectrum at the harmonic. Finally, in this simple example, two different spectra have been created according to Figure 4.10.
Figure 4.9: Interpolation of the first order level between the two marks on opposite side of a zero order harmonic. The level of the zero order harmonic is then calculated given that the levels of the zero and first order parts added shall equal the measured level.

Figure 4.10: The zero and first order spectra.
As the zero order harmonics are not affected by the baffle but the first order frequencies are, the free space sound power level, which shall be inserted in Equation 4.3, is calculated from

$$L_{W,FS} = 10 \log \left( 10^{\frac{L_{W,ZO}}{10}} + 10^{\frac{L_{W,FO} - \Delta L_W}{10}} \right),$$

Equation 4.4

where $L_{W,ZO}$ is the zero order part, $L_{W,FO}$ the first order part and $\Delta L_W$ the baffling correction curve.

This very simple and therefore practically usable method of adjusting for the baffling effect, has shown to give results accurate enough at least in the cabinet example.

To sum up, the airborne source strength (CSS) of an axial fan is equal to the dipole force given in Equation 4.3 where the free space sound power level is given by Equation 4.4.

4.4.1. Discussion On A Possible Modification Of The ISO 10302 Test-rig

With the chosen single axial dipole model of an axial fan it's still a little bit too complicated to take account of the baffling effect when calculating the source strength. If there would be no baffling effect, the measured sound power would be directly transferable into dipole force according to Equation 4.3. Within the NABUCCO project there has been discussions on how this can be realised. One possible solution that came up is to mount the fan to the test-rig by a number of springs. The function of the springs is to support the fan and to prevent vibrations spreading from the fan to the test-rig. With this modification the rubber cloth can be removed and replaced by the thin acoustically transparent plastic in order to seal the box. The fan will now experience basically free field conditions, since the thin plastic will not act as a baffle.
5. TRANSMISSION PATH

Noise generated at the source point is transmitted through the structure to finally reach a receiver point outside the structure. It is obvious though, that the noise can find its way to the receiver point through numerous transmission paths through the structure. However, if a source and a receiver point are decided upon, it is possible to establish a single transfer function, including all possible transmission paths, for each type of noise excitation at the source. From evaluation of different transfer functions, corresponding to different source points, noise reduction can then be practised.

5.1. ESTABLISHMENT OF AN AIRBORNE TRANSFER FUNCTION

As concluded in Chapter 2.2, only the airborne part of the noise generated by the axial fans considered here, has to be taken into account. The airborne source strength is generally represented by the complex dipole force of the single dipole modelling the fan. The relation between force at the source point to sound pressure at the receiver point, is therefore given by

\[ \hat{p}_r = H \hat{F}_s, \]

Equation 5.1

where \( H \) is the complex transfer function representing all airborne transmission paths through the structure. As complex forces and transfer functions only are of interest if several correlated sources are acting, there is no need for them to be complex in this case. Since as concluded in Chapter 4.3.2, two fans in practice can be considered as mutually uncorrelated and therefore the phase has no meaning. Uncorrelated sources allow the following mathematical treatment of Equation 5.1

\[ |\hat{p}_r|^2 = |H|^2 |\hat{F}_s|^2 \Leftrightarrow \tilde{p}_r^2 = |H|^2 \tilde{F}_s^2. \]

Equation 5.2
This formulation turns out to be very advantageous. Because on the right hand side it’s possible to identify the source strength, $\tilde{F}^2$, of the fan and on the left hand side the resulting sound pressure corresponding to the hearing experience.

By rewriting Equation 5.2, the transfer function is given by

$$|H|^2 = \frac{\tilde{P}^2}{F^2}. \tag{Equation 5.3}$$

Unfortunately would a calculation or measurement of this transfer function require application of a point-force to the fluid (air), which is a difficult task to perform.

5.1.1. Lyamshevs Reciprocity Relation

In 1960 Lyamshev [10] presented the following reciprocity relationship for elastic media excited by a point-force

$$\hat{P}_1 = \frac{\hat{q}_2}{F_2}. \tag{Equation 5.4}$$

As air at rest behaves like a linear elastic medium at sound pressure levels below 130 dB, the above relationship is applicable to airborne sound. Consequently, if point 1 and 2 are located somewhere in the same medium, Equation 5.4 state that the relation between sound pressure at point 1 to force at point 2 is equal to the relation between particle velocity at point 2 to volume velocity at point 1.

The absolute value squared of Equation 5.4 equals

$$\left| \frac{\hat{P}_1}{F_2} \right|^2 = \left| \frac{\hat{q}_2}{Q_1} \right|^2 \iff \frac{\tilde{P}^2}{F^2} = \frac{\tilde{q}^2}{Q^2}. \tag{Equation 5.5}$$

Equation 5.5 implies that the transfer function in Equation 5.3 can be determined in two different ways. If point 1 equal the receiver point and point 2 the source point, the left hand side of Equation 5.5 is equal to Equation 5.3. As mentioned above does a direct determination of the transfer function require a point-force applied to the fluid, which is difficult. Instead, it’s more convenient to use the right hand side of Equation 5.5, since particle velocity and volume velocity both are possible to measure and generate. Hence, the established transfer function (ACF) for the airborne noise is given by

$$ACF = |H|^2 = \frac{\tilde{v}^2}{\tilde{Q}^2}. \tag{Equation 5.6}$$
5.2. CALCULATION AND MEASUREMENT OF THE AIRBORNE TRANSFER FUNCTION

When the transfer function according to Equation 5.6 is calculated, the best choice is probably direct determination, i.e., let point 1 equal the receiver point and point 2 the source point. In a Finite Element Model of the structure, the particle velocity at the source point can be generated by oscillation of a rigid plane circular surface of the same area as the projected area of the fan blades. This simplified physical model of an axial fan in free space is acting as a dipole point source in the frequency range where the wavelength exceed the diameter of the surface.

The oscillating surface shall be located at the plane in the duct system where the plane through the centre of the impeller is located. Therefore, the direction of the velocity will coincide with the direction of the dipole force usually representing the excitation from the fan. The location and nature of the excitation is important to the response of the structure, especially at low frequencies.

If a prototype or other physical structure exists, it is also possible to measure the transfer function. Volume velocity can be generated by an omnidirectional loudspeaker, which as long as the wavelength exceeds its dimensions can be considered as a point source. The transfer function is measured reciprocally by locating the omnidirectional loudspeaker at the receiver point and measure the resulting particle velocity at the source point. As concluded in Chapter 2.2, not only the direct airborne transmission path must be concerned for, but also radiation from the plates. However, in the reciprocal measurement both of these paths will be included. This because the sound emitted from the loudspeaker will not reach the source point only through the duct openings directly, but also from inducing vibrations in the plates. The vibration of the plates then spreads into the structure and finally radiates sound reaching the source point.
The 1-dimensional equation of motion is relating a harmonic particle velocity to
sound pressure through

$$\rho_0 \frac{\partial \hat{v}_z}{\partial t} = -\frac{\partial \hat{p}}{\partial z} \Rightarrow \hat{v}_z = -\frac{1}{i\rho_0 \omega} \frac{\partial \hat{p}}{\partial z}. \quad \text{Equation 5.7}$$

The derivative of $\hat{p}$ with respect to $z$ can be approximated from sound pressures at
two equally spaced points, 1 and 2, on opposite side of the source point, as illustrated
in Figure 5.3.

![Figure 5.3: Definition of measurement points.](image)

By introducing this approximation, the particle velocity at the source point will be
given by

$$\hat{v}_s = \hat{v}_z \approx \frac{1}{i\rho_0 \omega \Delta z} (\hat{p}_1 - \hat{p}_2). \quad \text{Equation 5.8}$$

Equation 5.8 introduces the possibility of using microphones instead of a particle
velocity transducer, as such transducers are rare. The distance $\Delta z$ between the
microphone positions has to be small in order to be able to approximate the particle
velocity at high frequencies. If the distance $\Delta z \leq \lambda/6$ is used, where $\lambda$ is the
wavelength of the highest frequency of interest, the error in the approximation is
acceptable in the frequency range of interest.

By use of Equation 5.8, the transfer function in Equation 5.6 is given by

$$|H|^2 = \frac{\hat{v}_s^2}{Q^2} \approx \frac{1}{\rho_0^2 \omega^2 (\Delta z)^2} \left| \frac{\hat{p}_1}{Q} - \frac{\hat{p}_2}{Q} \right|^2 = \frac{1}{\rho_0^2 \omega^2 (\Delta z)^2} |H_1 - H_2|^2. \quad \text{Equation 5.9}$$

As the transfer functions $H_1$ and $H_2$ has to be complex, a reference is required
during the measurements. The most easily accessed reference available is the voltage,
$\hat{U}$, feeding the omnidirectional loudspeaker. Therefore, an appropriate reformulation
of the transfer function read

$$H_i = \frac{\hat{p}_i}{Q_i} = \frac{H_{up,i}}{H_{uQ}}, \quad i = 1, 2. \quad \text{Equation 5.10}$$
Because the distance $\Delta z$ between the microphones has to be small, the phase difference between the points is small at low frequencies. Therefore, large errors at low frequencies may appear if part of the phase difference between the points is introduced by the microphones or the measurement system themselves. However, as the phase reference is taken from the voltage feeding the loudspeaker, the transfer functions can be measured separately. If then only one microphone is used to measure both $H_{up_1}$ and $H_{up_2}$, the above phase error is eliminated.

5.2.1. Calibration Of An Omnidirectional Loudspeaker

$H_{UQ}$, in Equation 5.10, is a transfer function between voltage fed to the omnidirectional loudspeaker and the resulting volume velocity produced. This transfer function is achieved from calibration of the loudspeaker.

In anechoic (free field) conditions, the sound pressure generated by a monopole (point) source is given by

$$\hat{p}_m = \frac{i \rho \omega \Omega e^{-ikr}}{4\pi r}.$$  \hspace{1cm} \text{Equation 5.11}

Dividing both sides of Equation 5.11 with $\hat{U}$ and the desired transfer function is given by

$$H_{UQ} = \frac{4\pi e^{ikr}}{i \rho \omega} \hat{p}_m = \frac{4\pi e^{ikr}}{i \rho \omega} H_{up_n}.$$  \hspace{1cm} \text{Equation 5.12}

When calibrating the omnidirectional loudspeaker, it shall be fed by a random noise signal. The transfer function, $H_{up_n}$, between voltage at the loudspeaker to sound pressure at a point on a certain radius from the loudspeaker, is then measured in anechoic conditions. As extraneous noise can be expected at the microphone, the $H_1$ estimate shall be used.

In order to cover a wide frequency range, two different types of omnidirectional loudspeakers had to be used. The left picture in Figure 5.4 show the calibration of a dodecaheder and the right picture the calibration of a pipe.

![Figure 5.4: Left: Calibration of a dodecaheder. Right: Calibration of a pipe.](image-url)
In order to check in which frequency range the respective loudspeakers act as monopole sources, a simple test was done during the calibration. As the directivity of a monopole source is omnidirectional, no change in the sound pressure shall be measured if the loudspeaker is slightly rotated around its axle. In practice especially the dodecahedron have some directivity and therefore the upper frequency limit of the dodecahedron acting as a monopole source can easily be detected from the frequency where the measured sound pressures start to deviate from each other. As this deviation also will show up in \( H_{up} \), it can be viewed instead.

Figure 5.5 show the results for the dodecahedron. Four measurements were taken and between each measurement the dodecahedron was slightly rotated. As deviations between the measurements show up at approx. 1 kHz, this frequency constitute the upper frequency limit for the dodecahedron to serve as a monopole source. In Figure 5.6 the results from the same measurement on the pipe is shown. As the pipe has a lack of power at frequencies lower than approx. 500 Hz, this frequency constitute the lower frequency limit for the pipe.

Figure 5.5: Upper frequency limit for the dodecahedron to serve as a monopole source.

Figure 5.6: Lower frequency limit for the pipe to serve as a monopole source.
5.2.2. Measurement Of The Airborne Transfer Function Of The Cabinet

In order to evaluate the applicability of NST to the cabinet example, a simpler case than the original case with two axial fans was needed in order to eliminate possible effects due to interaction between the fans. Consequently, the cabinet door was redesigned in order to fit only one fan. Therefore, two cases exist: the One Fan Case and the Two Fans Case.

Independently of the degree of correlation between two fans, the two corresponding transfer functions of the Two Fans Case can be measured individually and this because the phase reference is taken from the voltage feeding the loudspeaker. Hence, in the following, the measurement of a single transfer function will be treated.

When measuring the transfer function, the fan must be removed in order to be able to locate the microphone positions according to Figure 5.3. Because of the design of the fan housing, and the high degree of acoustic transparency through it even when the fan is operating, the passive acoustic properties of an axial fan is about equal to those of a pipe of the same length and diameter as the fan housing [11]. As the length of the fan housing is short (approx. 5 cm), the additional impedance added by the presence of the short “pipe” is negligible. The error in the transfer function from measuring without the fan (or a short pipe) present, is therefore also negligible.

As the mixed flow fan has most of its sound power concentrated to rather low frequencies, the main interest will be below 2 kHz. In order to get a good approximation of the particle velocity in this frequency range, the distance between the microphone positions becomes \( \Delta z = 3.0 \, \text{cm} \), according to the previous text. Figure 5.8 shows the set-up used to locate the microphones at the defined positions inside the duct system from the outside.

Figure 5.7: Left: One Fan Case. Right: Two Fans Case.

Figure 5.8: Microphone set-up.
As Lyamshevs reciprocity relation and reciprocal measurements in general only are valid without flow, no external flow can be applied when measuring the transfer function. However, in the present application the Mach number is small and the “no flow” condition therefore approximately satisfied. The absence of flow will result in the transfer function being more resonant compared to the “true” transfer function, since flow usually introduces damping in the system. Despite the absence of flow, extraneous noise can be expected at the microphone and therefore the \( H_1 \) estimate shall be used when measuring \( H_{u_p_1} \) and \( H_{u_p_2} \). As the phase information in \( H_{u_p_1} \) and \( H_{u_p_2} \) is important to the approximation of the particle velocity, the measurements must be performed in an environment where a direct field exist. Consequently, measurements can be performed in anechoic or semi-anechoic conditions but not in reverberant conditions.

The transfer function(s) to two different receiver points was measured for both the One Fan Case and the Two Fans Case. Receiver point 1 is located at \((X_r, Y_r, Z_r) = (0.3, 1.5, 1.5) \, [m]\) relatively the local coordinate system in Figure 5.1, and receiver point 2 at \((X_r, Y_r, Z_r) = (1.6, 1.5, 1.5) \, [m]\). In order to cover the entire frequency range of interest, both types of omnidirectional loudspeakers was used during the measurement.

![Figure 5.9: Left: The transfer function to receiver point 1 is measured using the dodecaheder. Right: The corresponding measurement performed with the pipe.](image)

By using the data from the dodecaheder measurement from 100 Hz-1 kHz and the pipe data from 1 kHz and above, the transfer function(s) were calculated according to Equation 5.9 and 5.10. Figure 5.10 shows the achieved transfer functions from the left and right fan to receiver point 1. Please observe that the accuracy in the transfer functions is lower above 2 kHz because of the distance \( \Delta z \) that was used. These transfer functions shows how resonant a transfer function could be because of the complex geometry of the duct system and partially because of the absence of flow.
Figure 5.10: Two Fans Case. The transfer functions from the left and right fan to receiver point 1.
6. PREDICTION

The Sound Power Flow Model is the most frequently used prediction model when it comes to prediction of sound power emitted from a duct system where an axial fan is the source of sound. In this model, it is assumed that the fan sound power entering the duct is a function only of the particular fan used and its operating parameters, and is not influenced by the acoustic properties of the duct. The duct, in turn, is assumed adequately describable in terms of its acoustic power dissipation.

The proposed measures of fan source strength and properties of the duct system, described in Chapter 4 and 5 respectively, are in some respect similar to those of the sound power flow model as they are determined separately from each other. However, the main difference between the models is that the sound is described by field variables in the proposed model instead of its power. This description allows for effects of standing waves or resonances to be taken into account. The shorter the duct system is or the less amount of absorbent material inside it, the larger are the effects of standing waves or resonances because the low propagation-damping of the sound wave. The main drawback with the model is that it’s limited to more or less In-duct fans only.

6.1. PREDICTION ACCORDING TO THE NST MODEL

Figure 2.11 shows the NST model of the cabinet. The model provides help in formulating the mathematics needed to predict the sound pressure at the receiver point.

The sound pressure generated at the receiver point is for a single fan given by Equation 5.2. Because the fans are mutually uncorrelated, the superposed total sound pressure at the receiver point is given by

$$\bar{p}_i^2 = \sum_{n=1}^{N} \hat{F}_n^2 |H_n|^2,$$

Equation 6.1

where $\hat{F}_n^2$ is the individual fan source strength (CSS$_n$) and $|H_n|^2$ its corresponding transfer function (ACF$_n$). Here $N$ takes two values, 1 or 2, which correspond to the One Fan Case and the Two Fans Case respectively.
6.1.1. *One Fan Case*

The best way of evaluating the predicted results is by comparison to the measured values. Hence, the sound pressure level generated by the cabinet was measured in anechoic conditions at the two receiver points when the mixed flow fan (Figure 5.7) was supplied by either 35 V or 56 V. In order to get information about the point of operation on the fan-curve, the static pressure difference was measured at the same time. With acoustically transparent ducts attached to the inlet and outlets sides (Figure 4.3), the sound power of the same type of fan was measured in the ISO 10302 test-rig and converted into source strength following the procedure described in Chapter 4.4. The predicted sound pressure then equals the source strength times the transfer function, according to Equation 6.1. In Figure 6.1-6.3, the predicted results are obtained from measured transfer functions.

![Graph showing predicted and measured sound pressure levels at receiver point 1. (U=35V, ΔP=90Pa).](image-url)
Figure 6.2: One Fan Case. Predicted and measured sound pressure levels at receiver point 1. (U=56V, ΔP=158Pa).

Figure 6.3: One Fan Case. Predicted and measured sound pressure levels at receiver point 2. (U=56V, ΔP=158Pa).
Figure 6.1-6.3 shows that the agreement between measured and predicted levels in general is quite good. The modal behaviour of the prediction is similar to the measured at low frequencies, which supports the chosen dipole model of the fan. The overall lower predicted broadband levels are most likely because the flow conditions in the cabinet door are worse than those simulated with the transparent ducts. Higher measured broadband levels could also be due to flow-induced noise. This type of noise is not included in the NST model and therefore not part of the prediction. At the 1.6 kHz third-octave, a discrepancy from the measurements shows up in all predictions. A possible cause to this could be lack of power from the pipe loudspeaker when measuring the transfer function. According to Figure 5.6, the pipe has a dip around this frequency. Another cause could be that the mid-frequency region, where the mode density is low, is entered, and as concluded in Chapter 3.4, it was expected that the single axial dipole probably wouldn’t provide the excitation necessary in this frequency region.

At high frequencies, the predictions show a much more resonant behaviour compared to the measurements. The cause to this can be found in the transfer function. As no flow can be present during the reciprocal measurement of the transfer function, the smoothing of the transfer function normally caused by damping effects due to flow, will not take place. Despite the fact that the measured transfer function is of low accuracy at high frequencies, due to the spacing between the microphone positions, the prediction is still quite good.

The transfer function is quite resonant also at low frequencies as Figure 5.10 show. This means that large discrepancies may appear very quickly, especially at the harmonics, if the rotation speed of the fan is different when it’s measured in the test-rig from when it’s running in the application. In Figure 6.1-6.3, a small shift in frequency can be observed at the harmonics due to this, and part of the errors at the harmonics thereby explained. As the prediction improves with the change of receiver point between Figure 6.2 and 6.3, discrepancies especially at the harmonics can also be explained by the sensitivity to location of the microphone when measuring sound pressure in anechoic conditions.

A simulation of the emitted sound power from the cabinet has been made by the cabinet manufacturer. In this case, the transfer function is calculated from a 3-dimensional FEM model of the cabinet door. The excitation from the fan was modelled according to Figure 5.2 at the calculation of the transfer function. The dipole force of a real mixed flow fan was then multiplied by the calculated transfer function in order to simulate the emitted sound power. As the sound pressure level was measured in anechoic conditions, it’s possible to compare the character of the simulated sound power to the character of the measured sound pressure level.
Figure 6.4 shows that the characters are very similar to each other. The major difference is that the simulation shows a more resonant behaviour at high frequencies, but this is because no damping was introduced in the FEM model. This promising result opens up for great possibilities to calculate the transfer function from a computer model instead of measuring it on a prototype. Thereby a lot of time and money can be saved in the noise reduction work.

6.1.2. Two Fans Case

In the Two Fans Case, two mixed flow fans of the same type are mounted in the cabinet door (Figure 5.7). In this case, the only results evaluated are those at receiver point 1. When measuring the sound pressure level at this point, both fans were supplied by either 48 V or 56 V. No difference in the static pressure difference was measured between the fans. Hence, both fans were operating at the same point of operation on the fan-curve but the point did of course change with the voltage supplied.

In order to try the idea of simulating the flow conditions according to one of the three typical cases shown in Figure 4.5, the sound power of the mixed flow fan was measured in the ISO 10302 test-rig with acoustically transparent ducts attached according to type case A, as this case correspond to the mounting of the fan in the cabinet door. The “duct-chain” used is the left one of the ducts shown in the left picture of Figure 4.6. As both fans in the cabinet are of the same type and operating at
the same point of operation, just a single fan was measured. Hence, after conversion to source strength, according to the by now familiar procedure, the same source strength was used for both fans in the prediction formula. However, two separate transfer functions, corresponding to the left and right fan, was measured and used in the predicted results in Figure 6.5 and 6.6.

The agreement between measured and predicted levels seems to be slightly better here compared to the One Fan Case. The most probable explanation to the improved prediction at the broadband levels is the following. In the One Fan Case the point of operation on the fan-curve is to the right of the saddle point and in the Two Fans Case it’s to the left of it. To the left of the saddle point, the broadband part of the fan noise is significantly higher than to the right of it. This means that the source strength used in the Two Fans Case contains a significantly higher broadband part and therefore additional broadband noise, e.g., flow induced noise, will play a less important role. As otherwise exactly the same type of discrepancies appear in the Two Fans Case as in the One Fan Case, the analysis of the One Fan Case results also apply to the Two Fans Case.

Figure 6.5: Two Fans Case. Predicted and measured sound pressure levels at receiver point 1. (U=48V, ΔP=185Pa).
Figure 6.6: Two Fans Case. Predicted and measured sound pressure levels at receiver point 1. (U=56V, ΔP=215Pa).
7. SUMMARY

A prediction of the exterior sound pressure level generated by a cabinet containing a radial fan and two axial fans have been carried out according to the Noise Shaping Technology (NST). According to NST, the noise source is described by one or several noise descriptors, CSSs, and the corresponding transmission paths through the structure described by one or several ACFs. Preliminary measurements on the cabinet showed that only the airborne sound generated by the axial fans had to be considered. Therefore, an airborne CSS and a corresponding airborne ACF for small axial fans had to be established.

Previous research [4, 5, 6] has shown that a single equivalent dipole source directed in the axial direction of the fan will serve as an acoustic model of an In-duct axial fan in the plane wave region. However, in the cabinet example the axial fans are not strictly In-duct and therefore the single axial dipole model may break down too early in frequency because lack of excitation provided in other directions. In order to quantify the degree of excitation in different directions, the radiation of an axial fan in free space was examined using a spherical harmonics expansion of the pressure field generated by the fan. Analysis of the measured results shows that the radiation predominantly is of dipole character even in free space conditions and therefore the single axial dipole model may work in the entire frequency range of interest for the cabinet example. However, further analysis show upon inability of the single axial dipole model to capture the entire sound power generated by the fan. It also shows upon the inability of the single axial dipole to generate the directivity pattern that the axial fan generates at certain discrete frequencies. In order to take account of these shortcomings, a more complex free field model based on a single rotating dipole has been proposed for the future. However, despite the shortcomings, the single axial dipole model was chosen anyway. Since, in the cabinet example, the axial fans are more or less In-duct mounted and therefore the importance of the shortcomings not that big. However, a necessary condition is that the total power of the fan is captured in the single axial dipole.

The sound power of small axial fans is measured according to the international standard ISO 10302, which allow sound power as a function of static pressure load on the fan, to be measured. However, when an axial fan is mounted in a structure, other installation effects appear which also changes the sound generation of the fan. An axial fan is very sensitive to the in and outlet flow conditions. Therefore, when measuring the sound power of a fan in the ISO 10302 test-rig, flow conditions corresponding to those in the real structure must be simulated. This can be done by the use of acoustically transparent ducts. The ducts need to be acoustically transparent since the acoustical installation effect on the fan is already taken care of in the
airborne transfer function (ACF) of the structure. The test-rig itself does however exert a certain acoustical installation effect on the fan. As the sound generation of an axial fan is of dipole type, there will be a baffling effect from the rubber clot supporting the fan in the test-rig. Due to the baffling effect, the measured sound power from the fan increases at low frequencies. Because the source strength of the single axial dipole model shall correspond to that of a dipole in free space, the “extra” power due to the baffling must be withdrawn from the sound power measured in the test-rig. A simple but practical method of how to convert the measured sound power into dipole force, which is the airborne CSS, is proposed where the baffling effect is taken into account.

The corresponding airborne ACF is a transfer function from dipole force at the source point, which is the point in the structure where the single axial dipole modelling the fan is located, to sound pressure at a receiver point located somewhere in the exterior of the structure. As it’s difficult to apply a point-force to a fluid, the use of Lyamshev’s reciprocity relation allow the transfer function to be measured reciprocally by generating acoustic volume velocity at the receiver point and measure the resulting particle velocity at the source point. A necessary condition for reciprocal measurements to be valid in a fluid is that the fluid must be at rest. However, in the cabinet example the Mach number is low and therefore the condition approximately satisfied.

From the use of the CSSs and ACFs, the sound pressure level at some receiver points was predicted and compared to measurements in two cases. The One Fan Case correspond to a single axial fan of mixed flow type mounted in the cabinet, and the Two Fans Case to two axial fans of the same type. The overall agreement is quite good in both cases but certain discrepancies occur in the mid-frequency region where the modal density is low. This is most probably because lack of excitation in different directions from the single axial dipole modelling the fan.

In order to improve the prediction especially in the mid-frequency region, the fan model needs to be more complex. A suggestion for future research is therefore to investigate the possibility of modelling a free space fan on the basis of a rotating dipole.
8. REFERENCES


