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Large eddy simulation of channel flow with and without periodic constrictions using the explicit algebraic subgrid-scale model

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We analyze the performance of the explicit algebraic subgrid-scale (SGS) stress model (EASSM) of Marstorp *et al.* (*J. Fluid Mech.*, vol. 639, 2009, pp. 403–432) in large eddy simulation (LES) of plane channel flow and the flow in a channel with streamwise periodic hill-shaped constrictions (periodic hill flow) which induce separation. The LESs are performed with the *Code_Saturne* which is an unstructured collocated finite volume solver with a second-order spatial discretization suitable for LES of incompressible flow in complex geometries. At first, performance of the EASSM in LES of plane channel flow at two different resolutions using the *Code_Saturne* and a pseudo-spectral method is analyzed. It is observed that the EASSM predictions of the mean velocity and Reynolds stresses are more accurate than the conventional dynamic Smagorinsky model (DSM). The results with the pseudo-spectral method were in general more accurate. In the second step, LES with the EASSM of flow separation in the periodic hill flow is compared to LES with the DSM, no SGS model and the highly resolved LES data of Breuer *et al.* (*Computers & Fluids*, vol. 38, 2009, pp.433–457) using the DSM. Results show that the mean velocity profiles, the friction and pressure coefficients, the length and shape of the recirculation bubble, as well as the Reynolds stresses are considerably better predicted by the EASSM than the DSM and the no SGS model simulations. It was also observed that in some parts of the domain the resolved strain-rate and SGS shear stress have the same sign. The DSM cannot produce a correct SGS stress in this case, in contrast to the EASSM.

Keywords: Large eddy simulation; Explicit algebraic subgrid-scale model; Periodic hill flow; Turbulence

1. Introduction

The assumption of an isotropic linear relation between the subgrid-scale (SGS) stress and resolved strain-rate tensors, used in eddy-viscosity models, is not valid for wall-bounded flows [1–6]. In fact, the mean alignment between the resolved strain-rate and SGS stress tensors is very poor in turbulent channel flow [7]. Several closures have been proposed to improve the geometrical representation of the SGS stress tensor in large eddy simulation (LES). Scale similarity [8] and nonlinear models [3–5, 9] based on a constitutive relation using the resolved strain- and rotation-rate tensors are examples of such closures. However, the better geometrical description of the SGS stress tensor by the scale similarity model does not always lead to improvements in the prediction of the resolved statistics due to the insufficient drain of energy by the model. Therefore, scale similarity models are usually accompanied by an eddy viscosity term in the form of a mixed model [10] to ensure a correct drain of energy. As for the Reynolds stresses in the Reynolds averaged Navier–Stokes (RANS) formalism [11–13], a general constitutive relation for the

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SGS stress tensor can be expressed in terms of the products of resolved strain- and rotation-rate tensors, which can be reduced using the Cayley-Hamilton theorem and the theory of invariants to five independent terms. The coefficients are functions of the combination of the invariants of the resolved strain- and rotation rate tensors [9]. This general formulation is not practically useful due to its complexity, therefore, various SGS models based on truncated constitutive relations have been proposed [3–5]. The explicit algebraic SGS stress model (EASSM) [3] is a nonlinear mixed model consisting of an eddy viscosity and a nonlinear term involving a combination of the resolved strain- and rotation-rate tensors. It is based on an explicit algebraic RANS model [14] and uses the weak equilibrium assumption to obtain the explicit algebraic form. The EASSM has been successfully applied to LES of channel flow with and without system rotation at various Reynolds numbers and directions of system rotation [6, 15]. It has also been combined with the explicit algebraic SGS scalar flux model to predict passive scalar transport [16].

Besides the incorrect geometrical representation of the SGS stresses, the SGS anisotropy is not properly modeled by eddy-viscosity type models. As a consequence, the resulting LES predictions become strongly resolution dependent [6]. This is more prominent at coarse resolutions, where the SGS anisotropy is appreciable. By contrast, the EASSM gives LES predictions that are more accurate and less resolution dependent, due to the proper modeling of the SGS anisotropy [6].

In this study, we aim at extending our previous investigation of the performance of the EASSM in turbulent plane channel flow [6] to LES of more complex flows. For this purpose, LES of turbulent channel flow with periodic hill-shaped constrictions and a homogeneous spanwise direction, called periodic hill flow hereafter, is carried out. This flow is challenging for any SGS model since it includes separation from a curved surface, a subsequent reattachment of the flow, interactions between the recirculation bubble and the free shear layer and irregular movements of the separation and reattachment lines in time and space. The periodic hill flow has been studied previously using LES, direct numerical simulation (DNS) and experiments [17–21]. The experimental study in [17] was performed at a bulk Reynolds number $Re_b = 60000$. The bulk Reynolds number and the geometry are different from those used in the current study. In [18], the performance of several SGS models at two relatively coarse grids were evaluated including a wall adapting local eddy-viscosity (WALE) model [22], a dynamic mixed model, constant coefficient and dynamic Smagorinsky models along with the effect of wall-functions on the results. The first highly resolved LES of the periodic hill flow at $Re_b = 10595$ was carried out in [19] using the dynamic Smagorinsky and WALE models using two different numerical schemes and a wall function on the upper wall of the channel. The two SGS models and numerical solvers gave similar predictions. In [20] this work is extended using the same numerical solver and the dynamic Smagorinsky model with a finer grid and a resolved boundary layer at the upper wall instead of a wall function. They also performed experiments at $Re_b = 10595$ and DNSs at lower Reynolds numbers. The bulk Reynolds number and the geometry of the periodic hill flow used in this paper is similar to the one in [18–21].

In LES of flows in complex geometries numerical methods with low-order spatial discretization schemes are often used. These methods introduce numerical dissipation which is typically of the same order as the SGS force [23]. Another common approach is to explicitly introduce numerical dissipation. In this study, contrary to our previous studies [3, 6, 15, 16], where an accurate pseudo-spectral Navier–Stokes solver was employed, we use a finite volume solver which has a second-order spatial accuracy and inherent numerical dissipation. We test the performance of the EASSM in LES of plane channel flow at $Re_\tau = 590$, based on friction velocity

and channel half height, at two resolutions and illustrate the effect of numerical dissipation in LES with and without SGS models by comparing the results with those of a LES with a pseudo-spectral method and the EASSM. Then LES of periodic hill flow with the EASSM is carried out with the *Code_Saturne* [24] and the results are compared to those of LES with no SGS model, dynamic Smagorinsky model and the well-resolved LES data of Breuer *et al.* [20].

2. Governing equations for LES and the SGS models

The governing equations for LES are obtained by filtering the Navier–Stokes and continuity equations. These, using the summation convention, read

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + F_i, \quad \frac{\partial \tilde{u}_i}{\partial x_i} = 0, \quad (1)$$

where $\tilde{u}_i$ and $\tilde{p}$ are filtered velocity and pressure, ρ is a constant density and F_i is a volume force. The SGS stress tensor

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j \quad (2)$$

has to be modeled to close the equations. The two SGS models employed in the present study are the explicit algebraic and dynamic Smagorinsky models.

2.1. Explicit algebraic subgrid-scale stress model (EASSM)

The EASSM [3] has the following formulation of the SGS stresses

$$\tau_{ij} = \frac{2}{3} K^{\text{SGS}} \delta_{ij} + \beta_1 K^{\text{SGS}} \tau^* \tilde{S}_{ij} + \beta_4 K^{\text{SGS}} \tau^{*2} (\tilde{S}_{ik} \tilde{\Omega}_{kj} - \tilde{\Omega}_{ik} \tilde{S}_{kj}), \quad (3)$$

which consists of an eddy viscosity (second term on the right-hand side) and a nonlinear term (last term on the right-hand side). In this formulation, $\tilde{S}_{ij}$ and $\tilde{\Omega}_{ij}$ are the resolved strain- and rotation-rate tensors

$$\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right), \quad \tilde{\Omega}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right), \quad (4)$$

τ^* is the time scale of the SGS motions and β_1 and β_4 are coefficients which determine the relative contribution of the eddy viscosity and nonlinear terms and are given by

$$\beta_1 = \frac{9}{4} c_1 \beta_4, \quad \beta_4 = -\frac{33}{20} \frac{1}{\left[(9c_1/4)^2 + \tau^{*2} |\tilde{\Omega}|^2 \right]}, \quad (5)$$

where $|\tilde{\Omega}| = \sqrt{2\tilde{\Omega}_{ij}\tilde{\Omega}_{ij}}$ is the norm of $\tilde{\Omega}_{ij}$. The present EASSM uses Yoshizawa's model [25] for K^{SGS}

$$K^{\text{SGS}} = c \tilde{\Delta}^2 |\tilde{S}|^2. \quad (6)$$

Here, $\tilde{\Delta} = \sqrt[3]{\Omega}$ (Ω is the volume of a computational cell), $|\tilde{S}| = \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}$ and c is dynamically determined using the Germano identity [26] with local averaging over the neighboring cells and is given as

$$c = \frac{1}{2} \frac{\widehat{\tilde{u}_k \tilde{u}_k} - \widehat{\tilde{u}_k} \widehat{\tilde{u}_k}}{\widehat{\tilde{\Delta}^2 |\tilde{S}|^2} - \tilde{\Delta}^2 |\tilde{S}|^2}, \quad (7)$$

where $\widehat{\cdot}$ represents the test filter, which is a top-hat filter (local averaging over the cells sharing a common face) with length scale, $\widehat{\Delta} = 3\tilde{\Delta}$. The model parameter c_1 is determined from the dynamic coefficient c and the SGS time scale, τ^* , with the inverse shear

$$c_1 = c'_1 \sqrt{c'_3} \frac{c^{1.25}}{(2C_s)^{2.5}}, \quad \tau^* = c'_3 \frac{1.5C_k^{1.5} \sqrt{c}}{2C_s} |\tilde{S}|^{-1},$$

where $c'_1 = 3.12$, $c'_3 = 0.91$, $C_k = 1.5$ is the Kolmogorov constant and $C_s = 0.1$, see [3, 7] for details. In LESs using the pseudo-spectral method the grid filter size is $\tilde{\Delta} = \sqrt[3]{\Omega}$. Test filtering is performed using a sharp cutoff filter in the homogeneous directions in Fourier space at a filter width $\widehat{\Delta} = \sqrt[3]{4}\tilde{\Delta}$.

2.2. Dynamic Smagorinsky model (DSM)

The DSM [26] has the following formulation

$$\tau_{ij} - \frac{2}{3} K^{\text{SGS}} \delta_{ij} = -2c_{\text{smag}} \tilde{\Delta}^2 |\tilde{S}| \tilde{S}_{ij} \quad (8)$$

where c_{smag} is computed dynamically using the Germano identity with the least square approximation [27]. Averaging of the dynamic coefficient is done over the neighboring cells. Test filtering is carried out with a top-hat filter (local averaging over the cells sharing a common face) with a length scale, $\widehat{\Delta} = 3\tilde{\Delta}$. The SGS kinetic energy, K^{SGS} , is not modeled but included in the pressure term in equation (1). The standard implementation of the DSM in the *Code_Saturne* uses $\tilde{\Delta} = 2\sqrt[3]{\Omega}$ [28] which hereafter is denoted as DSM-2. In this study, simulations are also performed with the DSM with $\tilde{\Delta} = \sqrt[3]{\Omega}$, hereafter denoted as DSM-1, for comparison with the EASSM simulations which also uses $\tilde{\Delta} = \sqrt[3]{\Omega}$ as the grid filter size.

3. Numerical method

Two numerical methods are used in the LESs. *Code_Saturne* (www.code-saturne.org) is used for channel and periodic hill flows. It is an unstructured collocated finite volume solver for incompressible flows [24], developed by Électricité de France (EDF), and has been employed extensively for simulations of industrial and academic flows [28–33]. A conservative form of the incompressible Navier–Stokes equations is solved by the code using a second-order central differencing in space and a second-order Crank–Nicolson scheme in time. The pressure-velocity coupling is based on a SIMPLEC algorithm with Rhie and Chow [34] interpolation to avoid odd-even oscillations.

A pseudo-spectral method is used for comparison of the EASSM results with those obtained from the *Code_Saturne* for the plane channel flow case. The pseudo-

Table 1. Summary of the channel flow simulations. EASSM/PS and EASSM/SAT: explicit algebraic SGS stress model with pseudo-spectral method and *Code_Saturne*, respectively; DSM-1: Dynamic Smagorinsky model with $\tilde{\Delta} = \sqrt[3]{\Omega}$ as the grid filter size, where Ω is the cell volume; DSM-2: Dynamic Smagorinsky model with $\tilde{\Delta} = 2\sqrt[3]{\Omega}$ as the grid filter size; NM: No SGS model.

Case	Subgrid-scale model	Numerical method	$N_x \times N_y \times N_z$	Δ_x^+	Δ_z^+	$\frac{\Delta_y^+}{(\min \sim \max)}$	Re_τ
1	EASSM	Pseudo-spectral	$64 \times 65 \times 64$	58	29	$0.71 \sim 28.9$	592
	EASSM	<i>Code_Saturne</i>	$64 \times 96 \times 64$	55	28	$0.28 \sim 32.1$	566
	DSM-1	<i>Code_Saturne</i>	$64 \times 96 \times 64$	53	26	$0.29 \sim 30.4$	535
	DSM-2	<i>Code_Saturne</i>	$64 \times 96 \times 64$	50	25	$0.25 \sim 27.0$	506
	NM	<i>Code_Saturne</i>	$64 \times 96 \times 64$	56	28	$0.34 \sim 32.5$	571
2	EASSM	Pseudo-spectral	$96 \times 97 \times 96$	38	19	$0.31 \sim 19.2$	586
	EASSM	<i>Code_Saturne</i>	$96 \times 96 \times 96$	38	19	$0.47 \sim 29.6$	582
	DSM-1	<i>Code_Saturne</i>	$96 \times 96 \times 96$	37	18	$0.45 \sim 28.6$	561
	DSM-2	<i>Code_Saturne</i>	$96 \times 96 \times 96$	35	17	$0.43 \sim 28.8$	531
	NM	<i>Code_Saturne</i>	$96 \times 96 \times 96$	39	19	$0.50 \sim 30.1$	591

spectral method [35] employs Fourier representation in wall-parallel directions (x and z) and Chebyshev representation in the wall-normal direction (y), using the Chebyshev–tau method. Aliasing errors are removed using the 3/2-rule [36]. The time integration is carried out with a four-step third-order Runge–Kutta scheme for the nonlinear terms and a second-order Crank–Nicolson scheme for the linear terms.

4. Plane channel flow simulations at $Re_\tau = 590$

Simulations are carried out at two resolutions referred to as case 1 and case 2, see table 1. In case 1 a different number of grid points in the wall-normal direction is used for the EASSM/PS than for the other models. However, this difference did not make considerable differences in the model performance for the results discussed in this paper. The table also shows the acronyms that are used in the paper. The simulations with *Code_Saturne* employ a tangent-hyperbolic grid-point distribution [37] in the wall-normal direction. Δ_x^+ , Δ_y^+ and Δ_z^+ are the grid spacings in the streamwise, wall-normal and spanwise directions, respectively, in wall units

The flow domain is a rectangular box with a streamwise and spanwise size of $2\pi\delta$ and $\pi\delta$, respectively, where δ is the channel half height. The bulk Reynolds number is $Re_b = U_b\delta/\nu = 10935$ and the friction Reynolds number of the corresponding DNS is $Re_\tau = 593$, where u_τ is the friction velocity. LES results with the EASSM are compared to the DNS results [16, 38] and the LES results with the dynamic Smagorinsky model (DSM) [26, 27].

Figure 1(a-c) shows the mean streamwise velocity profiles in wall units. In figure 1(a) different resolutions are separated by a shift in the ordinate direction. A close up of the logarithmic region is given in figure 1(b). In figure 1(c) different SGS model predictions are separated by a shift in the ordinate direction so that resolution effects for each SGS model can be assessed. The DSM-2 strongly under-predicts the wall shear stress at both resolutions, therefore, the mean velocity profiles in wall units are over-predicted. This is due to the unresolved small-scale turbulence close to the wall as a consequence of the large grid-filter size in this LES. By contrast, the DSM-1 which has a smaller grid-filter size predicts the wall shear and mean velocity profile in better agreement with the DNS data. At the coarse resolution (case 1), LES without a SGS model (NM) also over-predicts the mean velocity profile showing that the discretization errors dominate the true SGS dissipation because otherwise mean velocity profiles would have been under-predicted [6]. This over-prediction reduces with increasing resolution due to the smaller discretization error. The EASSM predictions are close to the NM results, showing that the

Figure 1. Mean velocity profiles in wall units $\langle u \rangle^+$. a) Case 1 and 2 are separated by a shift of 10 units in the ordinate direction for the clarity of the plot. b) Close up of the logarithmic region in figure (a). c) Different subgrid-scale models are separated by a shift of 10 units in the ordinate direction for the clarity of the plot. Arrows point to the direction of increasing resolution. —■— : EASSM/SP; —■— : EASSM/SAT; —○— : NM; —▲— : DSM-1; —△— : DSM-2; — : DNS. For the list of acronyms see caption of table 1.

Figure 2. Mean streamwise R_{uu}^+ (a), wall-normal R_{vv}^+ (b), spanwise R_{ww}^+ (c) and shear R_{uv}^+ (d) Reynolds stresses in wall units. Left half in each figure corresponds to the coarse resolution (case 1) and right half corresponds to the fine resolution (case 2). —■— : EASSM/PS; —■— : EASSM/SAT; —○— : NM; —▲— : DSM-1; —△— : DSM-2; — : DNS. For the acronyms, see the caption of table 1.

Figure 3. An x-y cross section of the geometry with the corresponding dimensions and the grid. Every second grid point is shown in the cross-flow direction.

model contribution to the SGS dissipation is small. The EASSM/SAT prediction approaches that of the DNS with increasing resolution in case 2. The EASSM/PS results are accurate at both resolutions and are close to the EASSM/SAT results at the fine resolution. Since in the EASSM/SP case the numerical scheme has a very low dissipation due to the spectral scheme, the EASSM prediction is more accurate than the EASSM/SAT one were the model contribution is affected by the numerical dissipation.

The streamwise Reynolds stresses in wall units, R_{uu}^+ , at the two resolutions are shown in figure 2(a). The EASSM predictions represent the resolved plus modeled Reynolds stresses, while for the DSM-1 and DSM-2 the modeled part is not available for the normal stresses. At the coarse resolution, all models, excluding the EASSM/PS, but including the NM clearly over-predict the near-wall peak of R_{uu}^+ , see figure 2(a). All predictions approach the DNS data with increasing resolution. The EASSM/SAT prediction is more accurate than the DSM-1, DSM-2 and the NM, and the EASSM/PS prediction has the best agreement with DNS. This illustrates the fact that the numerical methodology affects the LES results, even when the same SGS model is used.

The wall-normal Reynolds stresses in wall units, R_{vv}^+ , are shown in figure 2(b). At the coarse resolution all the models and the NM under-predict R_{vv}^+ . The DSM-2 prediction at this resolution shows a much larger under-prediction than the DSM-1, EASSM/SAT and the NM, which are similar. The EASSM/PS prediction is close to that of the DNS at both resolutions. The performance of the different models in prediction of the spanwise Reynolds stresses, R_{ww}^+ , is similar to that for the R_{vv}^+ , see figure 2(c).

5. Periodic hill flow

The flow geometry is shown in figure 3. It is periodically continued in the streamwise x-direction and thus comprises a channel with periodic hill-shaped constrictions on the lower wall. The streamwise and spanwise extents of the geometry are $L_x = 9.0h$ and $L_z = 4.5h$, where h is the height of the hill. The channel height in the unconstricted part is $L_y = 3.036h$ and the length of the hill is $L_h = 3.86h$. A variable volumetric force is used such that a constant mass flux is maintained and the bulk Reynolds number based on the bulk velocity at the inlet and hill height is $Re_b = 10595$. An elliptic grid generation technique [39] is used to generate a nearly orthogonal curvilinear grid in the x-y plane with clustering of the grid near the upper and lower walls to resolve the boundary layers. The grid in the

Table 2. Summary of the periodic hill flow simulations. Abbreviations are presented in table 1. The reference LES data is denoted as case REF. N_x , N_y and N_z are the number of cells in the streamwise, cross-stream and spanwise directions, respectively. Δt is the time step of the simulations. Mean separation and reattachment locations are denoted as $(\frac{x}{h})_{sep}$ and $(\frac{x}{h})_{reat}$, respectively.

Case	$N_x \times N_y \times N_z$	SGS model	$\Delta t/t_{bulk}$	$(\frac{x}{h})_{sep}$	$(\frac{x}{h})_{reat}$
EASSM		EASSM		0.22	4.42
DSM-1	$148 \times 156 \times 92$	DSM	4.0×10^{-3}	0.23	4.08
DSM-2		DSM		0.23	4.15
NM		—		0.24	4.24
REF	$280 \times 220 \times 200$	DSM	1.8×10^{-3}	0.19	4.69

Figure 4. Variation of the streamwise Δ_x^+ (a) and spanwise Δ_z^+ (b) resolutions at the lower wall in wall units. —■— : EASSM; —○— : NM; —▲— : DSM-1; —△— : DSM-2; — : REF.

streamwise direction is slightly clustered on the windward side of the hill using a tangent hyperbolic distribution to better resolve the sharp velocity gradients. Outside this area the grid is uniform in the streamwise direction. The distribution of the grid points in the spanwise direction is uniform. A summary of the simulation specifications as well as those of the reference LES data [20] are given in table 2. LESs are performed with the EASSM and DSM with different grid filter sizes, i.e. cases DSM-1 and DSM-2. A no SGS model (NM) LES is also carried out for comparison. The reference LES called REF is a well-resolved LES using the DSM-1.

Variation of the grid resolution in wall units in the streamwise and spanwise directions is given in figures 4(a) and (b). The grid resolution of the reference LES in wall units in the streamwise and spanwise directions are less than 30 in the whole channel. The resolution is much less than 30 wall units at the leeward side of the hill and the upper limit of 30 happens at the windward side of the hill. This reference LES is thus well resolved. We therefore assume that the values are accurate and can be used to validate the other cases. For the LESs carried out here, the grid resolution is less than 45 and 60 wall units in the streamwise and

Figure 5. Pressure coefficient C_p along the lower wall (a) and upper wall (b) and friction coefficient C_f along the lower wall (c) and the close up of the region $x/h < 5$ (d). — ■ — : EASSM; — ○ — : NM; — ▲ — : DSM-1; — △ — : DSM-2; — : REF.

spanwise directions, respectively.

5.1. Friction and pressure coefficients

Friction and pressure coefficients are shown in figure 5 (a–d). Due to the acceleration of the flow before the hill crest there is a sharp increase in the friction coefficient before the separation point, see figure 5(d). This peak is properly predicted in all cases. The mean flow separates at $x/h = 0.19$ in case REF, see table 2, where C_f becomes zero. Prior to the separation point, at the leeward side of the hill, the flow decelerates and the pressure coefficient increases, see figure 5(a). All cases predict a slightly delayed separation compared to case REF but case EASSM is the most accurate. After the separation, the flow in case REF tends to reattach at two locations; at $x/h = 0.75$ and at $x/h = 1.85$, where C_f locally increases, whereas the other cases do not show exactly this behavior due to the coarse resolution in this region. In cases DSM-1, DSM-2, EASSM and NM the flow almost reattaches at $x/h = 0.75$ where C_p reaches a plateau. While C_p prediction in case EASSM follows that of case REF very well in this region it is over-predicted in the other cases. The skin friction coefficient has a minimum between $x/h = 2.0$ and 3.0 where the reverse flow is the strongest. The location and magnitude of this minimum is accurately predicted in case EASSM. Its magnitude is under-predicted in cases NM and DSM-1 and over-predicted in case DSM-2 and its location is closer to the separation point in these cases. The reattachment point in case EASSM is close to the one in case REF, while the other cases predict a too early reattachment and therefore a too small separation bubble.

5.2. Streamlines of the mean flow

To give an overview of the main flow characteristics, mean streamlines are plotted for the various cases in figure 6(a–d). The streamlines are accompanied by contour plots of the mean resolved shear stress $\langle u'v' \rangle$. The flow separates almost at top of the hill with the mean separation at $x/h = 0.19$ for case REF and around $x/h = 0.22 \sim 0.24$ for the other cases, see table 2. The mean separation point is thus not very sensitive to the SGS model, which was also pointed out in [18]. In [19] using the same code as in case REF at a coarser resolution the separation point was at $x/h = 0.20$ indicating that it also has little sensitivity to the grid resolution.

In contrast, the mean reattachment point is sensitive to the SGS model. The shortest and longest reattachment lengths correspond to cases DSM-1 and EASSM, respectively. For case DSM-1 the mean reattachment point is at $x/h = 4.08$ and in case EASSM it is at $x/h = 4.42$, in better agreement with case REF in which it is at $x/h = 4.69$. The mean reattachment point observed in [19] is at $x/h = 4.56$ indicating that it is sensitive to the grid and its local variation of resolution.

The separation bubble consists of a strong shear layer with a large turbulence production, evident from the high shear stress, a reverse flow area with another free shear layer and a boundary layer. The streamlines from different cases are remarkably different. The streamlines from case EASSM show a separation bubble that is thicker and larger in the cross-stream direction compared to the other cases. The shallower angle of the streamlines in the upper free shear layer indicates a weaker entrainment of flow from outside the separation bubble. As will be shown later, the mean streamwise and cross-stream velocity profiles predicted in case EASSM are in better agreement with case REF than in the other cases. This is consistent with a close agreement between the streamlines in case EASSM and REF. The other cases predict a too high cross-stream velocity and a too low streamwise velocity in the free shear layer between $x/h = 0.1$ and 3.0 which confirms the squeezed shape of the separation bubble in these cases in comparison with case EASSM.

5.3. Mean velocity and Reynolds stresses

In this section, LES results of the mean velocity (figure 7), shear stresses (figure 8) and normal Reynolds stresses (figure 9), are discussed for cases EASSM, DSM-1, DSM-2 and NM and are compared to those of the well-resolved LES in case REF [20]. Results are presented at 10 downstream positions, i.e. $x/h = 0.05, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0$ and 8.0 . As in [19] results are mainly discussed at $x/h = 0.05, 2.0, 6.0$ and 8.0 , which are representative of the points before the separation on top of the hill, in the middle of the recirculation area, after the reattachment and on the windward side of the hill.

5.3.1. Position $x/h = 0.05$

This position is close to the hill crest where the flow is still attached. The boundary layer is about $0.1h$ thick and properly resolved by the grid. The flow at this point is much affected by the upstream turbulence. Especially, the normal Reynolds stresses show a history effect and a highly anisotropic behavior. The streamwise velocity has a near-wall peak due to the acceleration of the flow on the windward side of the hill and the cross-stream velocity is positive due to the flow contraction. The streamwise velocity predictions in all cases are comparable except for case DSM-1, which over-predicts the near-wall peak. As a consequence of the fixed bulk flow it under-predicts the streamwise velocity in the upper side of the channel $2.0 < y/h < 3.0$. These mis-predictions can also be observed at $x/h > 4.0$

Figure 6. Streamlines of the mean flow with contour plots of mean resolved shear stress in outer units $\langle u'v' \rangle / U_b^2$ for cases EASSM (a), DSM-1 (b), DSM-2 (c) and NM (d). Note that the streamline spacings are not uniform in all figures.

Figure 7. Upper figure: Mean streamwise velocity profiles in outer units $\langle u \rangle / U_b$. Lower figure: Mean cross-stream velocity profiles in outer units $\langle v \rangle / U_b$. — ■ — : EASSM; — ○ — : NM; — ▲ — : DSM-1; — ▽ — : DSM-2; — : REF. Velocity profiles are given at $x/h = 0.05, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0$ and 8.0.

and are due to the early reattachment of the flow and the quicker development of the lower boundary layer. Since the cross-stream velocity is an order of magnitude smaller than the streamwise velocity it is more difficult to predict accurately and the differences in the predictions are more pronounced. Case EASSM predictions of the cross-stream velocity coincide with case REF, while other cases slightly under-predict it outside the boundary layer.

The mean Reynolds shear stress $\langle u'v' \rangle$ profile has a peak in the boundary layer at the lower wall due to the high shear rate. From the lower wall up to the position of maximum streamwise velocity, the mean SGS shear stress $\langle \tau_{12} \rangle$ and the resolved one are negative, while the strain-rate is positive, hence, eddy viscosity models in cases DSM-1 and DSM-2 can predict $\langle \tau_{12} \rangle$ correctly. In this region cases EASSM and DSM-2 predict a similar $\langle \tau_{12} \rangle$, while case DSM-1 predicts a much smaller value than the other two cases. Case EASSM under-predicts $\langle u'v' \rangle$ in comparison with the other cases. Further away from the wall, outside the boundary layer $1.1 < y/h < 1.6$, the streamwise velocity gradient changes sign and becomes negative while $\langle u'v' \rangle$ stays negative, see also [19]. This is contrary to the eddy-viscosity concept resulting in a positive prediction of $\langle \tau_{12} \rangle$ in cases DSM-1 and DSM-2, in contrast to the negative $\langle u'v' \rangle$, correctly predicted by case EASSM. Due to the smaller length scale of the grid filter, the mis-prediction in case DSM-1 is smaller than in case DSM-2. Outside the boundary layer, case EASSM predicts $\langle u'v' \rangle$ more accurately than the other cases, which under-predict the peak around $y/h \approx 1.6$.

Contrary to the shear stresses, the normal stresses cannot be compared quantitatively since in the eddy-viscosity models the SGS kinetic energy is lumped in the pressure term in the Navier–Stokes equations and is thus not available. In the EASSM this term is modeled and therefore we add it to the normal stress predictions. Therefore, the total normal stresses from cases DSM-1 and DSM-2 cannot be exactly compared to those from case EASSM. It is reasonable to assume that in case REF, where the dynamic Smagorinsky is used, the SGS kinetic energy is small due to the very fine resolution. Therefore, the case EASSM results can be compared directly to those of case REF.

The streamwise normal stress $\langle u'u' \rangle$ has a peak in the boundary layer at the lower wall indicating a high production rate due to the large cross-stream gradient of the streamwise velocity. This peak is significantly over-predicted in cases DSM-1, DSM-2 and NM even though the model contribution is not added to the resolved stress, while in case EASSM it is well predicted. The flow at this position is strongly affected by the upstream flow conditions. Excellent predictions of the $\langle u'u' \rangle$ in case EASSM in the post-reattachment area and significant under-prediction in other cases is in accordance with this fact.

The cross-stream $\langle v'v' \rangle$ and spanwise $\langle w'w' \rangle$ stresses predicted in different cases are close to each other and agree reasonably well with case REF, except for $\langle v'v' \rangle$ in case NM, which is significantly under-predicted.

5.3.2. Position $x/h = 2.0$

This point lies in the middle of the recirculation region. It consists of the mean free shear layer extending from the hill crest, a reverse flow area below $y/h = 0.5$ which consists of another free shear layer and a boundary layer. In the reverse flow area, both mean streamwise and spanwise velocity profiles are well predicted in case EASSM, while other cases under-predict the streamwise component and significantly over-predict the cross-stream component. The same trend is also observed in and outside the upper free shear layer. Therefore, as it was observed earlier in section 5.2, streamlines are better predicted in case EASSM than in the other cases in this region, see figure 6.

An interesting observation is that in the boundary layer, below $y/h = 0.1$, while

the wall-normal gradient of the streamwise velocity is negative due to the reverse flow, $\langle u'v' \rangle$ is negative indicating an area dominated by viscous stresses, also observed in [19]. All models predict a positive $\langle \tau_{12} \rangle$ in accordance with the eddy-viscosity concept. Although these $\langle \tau_{12} \rangle$ predictions differ, different cases predict a similar $\langle u'v' \rangle$ while case EASSM prediction is slightly more accurate than the others.

The $\langle u'u' \rangle$ is over-predicted in all cases in the reverse flow area $y/h < 0.5$. Outside this region, the DSM-1 and DSM-2 results are closer to case REF than cases EASSM and NM, which over-predict $\langle u'u' \rangle$. On the other hand, $\langle v'v' \rangle$ is better predicted in case EASSM than in the other cases where the peak in the upper shear layer is under-predicted. The $\langle w'w' \rangle$ predictions are also more accurate in case EASSM, both in the reverse flow region and in the upper free shear layer. However, note that the SGS kinetic energy is added in case EASSM but not in cases DSM-1 and DSM-2.

5.3.3. Position $x/h = 6.0$

The mean flow reattachment point is at different locations for different cases and it varies from $x/h = 4.08$, for case DSM-1, to 4.42, for case EASSM, and is at $x/h = 4.69$ for case REF. Therefore, the point $x/h = 6.0$ lies after the reattachment point in all cases and is before the foot of the next hill. The mean velocity predictions are very sensitive to the differences in the location of the reattachment point. Case EASSM results are the most accurate but similar to case NM results. Case DSM-2 strongly over-predicts the mean streamwise velocity in the near-wall region, more than case DSM-1. Although the reattachment point in cases DSM-1 and DSM-2 is similarly predicted, the boundary layer in case DSM-2 develops faster and is thicker than for case DSM-1. The cross-stream velocity is still negative across the channel due to the wake formed after the recirculation area. After this point the cross-stream velocity changes sign before the flow reaches the foot of the next hill. Since the recirculation region is smaller in cases DSM-1 and DSM-2, their wake region is also smaller and the downward cross-stream velocity is also smaller than in the other cases.

Although $\langle \tau_{12} \rangle$ predictions vary considerably, the $\langle u'v' \rangle$ predictions are similar. However, case EASSM and DSM-2 predictions are slightly more accurate than the other cases. Case EASSM has the smallest $\langle \tau_{12} \rangle$ prediction among the cases which means that there are more resolved $\langle u'v' \rangle$ stresses in that LES compared to the other cases.

The $\langle u'u' \rangle$ prediction in case EASSM is in better agreement with case REF than in other cases outside the recirculation area. All models under-predict $\langle v'v' \rangle$ but case EASSM and DSM-1 results are more accurate.

5.3.4. Position $x/h = 8.0$

The streamwise velocity predictions in all cases agree well with case REF except in case DSM-2 where there are small deviations. The cross-stream velocity substantially increases due to the contraction in the geometry. All cases predict this velocity component very well except case DSM-2, which over-predicts the peak value.

The shear stress has a negative peak in the boundary layer due to the high streamwise gradient of the streamwise velocity caused by the acceleration of the flow. However, due to the eddy viscosity assumption, cases DSM-1 and DSM-2 fail to predict a positive $\langle \tau_{12} \rangle$, while the EASSM does produce a proper positive $\langle \tau_{12} \rangle$. Case EASSM predicts a more accurate $\langle u'v' \rangle$ outside the boundary layer where it is negative and has a peak around $y/h = 1.25$.

The $\langle u'u' \rangle$ prediction in case EASSM is in excellent agreement with case REF

Figure 9. Mean streamwise $\langle u'u' \rangle / U_b^2$ (a), cross-stream $\langle v'v' \rangle / U_b^2$ (b) and spanwise $\langle w'w' \rangle / U_b^2$ (c) Reynolds stress profiles in outer units at various downstream positions. —■— : EASSM; —○— : NM; —▲— : DSM-1; —△— : DSM-2; — : REF.

while other cases under-predict it. The $\langle v'v' \rangle$ is also better predicted in case EASSM while the $\langle w'w' \rangle$ predictions are similar in all cases.

5.4. The influence of the nonlinear term in the EASSM

The contraction of the nonlinear term $\beta_4 K^{\text{SGS}} \tau^{*2} (\tilde{S}_{ik} \tilde{\Omega}_{kj} - \tilde{\Omega}_{ik} \tilde{S}_{kj})$ in the EASSM with the symmetric $\tilde{S}_{ij}$ tensor is zero. Hence, it does not contribute to the SGS dissipation $-\tau_{ij} \tilde{S}_{ij}$ implying that it does not directly alter the resolved kinetic energy. However, it influences the vorticity dynamics through the dissipation term in the transport equation for the resolved enstrophy [40] and it also affects the individual SGS stress components. The contributions of the eddy viscosity $\beta_1 K^{\text{SGS}} \tau^* \tilde{S}_{ij}$ and nonlinear part of the EASSM to the mean SGS shear stress $\langle \tau_{12} \rangle$ are presented in figure 10(a-d) at $x/h = 0.05, 0.5, 1.0$ and 8.0 for $y/h < 1.5$. The figure indicates that the contribution of the nonlinear part is indeed significant in the near-wall region where it has an opposite sign to the eddy viscosity part, and in the free

Figure 10. Eddy viscosity and nonlinear part of the mean SGS shear stress $\langle \tau_{12} \rangle$ by the EASSM at $x/h = 0.05$ (a), $x/h = 0.5$ (b), $x/h = 1.0$ (c) and $x/h = 8.0$ (d) limited to $y/h < 1.5$. —■— : Eddy viscosity part, —□— : nonlinear part and — : total SGS shear stress.

shear layer where both parts have the same sign. Except at $x/h = 0.05$, the nonlinear part dominates the eddy viscosity part in the near-wall region. These plots indicate that the nonlinear part of the EASSM plays an important role in the shear dominated regions of the flow.

5.5. Turbulent kinetic energy and the eddy viscosity

The main role of a SGS model is to properly dissipate energy from the resolved scales in a volume averaged sense [41]. When low-order numerical methods with inherent numerical dissipation are used, this role becomes more complicated since the numerical dissipation may dominate the SGS force. This is more crucial in low Reynolds number flows, where there is no distinct inertial range and the numerical dissipation directly affects the large resolved scales. In this section, the mean turbulent kinetic energy and eddy viscosity are presented and discussed to evaluate the SGS models in the presence of numerical dissipation.

The mean turbulent kinetic energy $\langle K \rangle$ is given in figure 11(a). It is only for case EASSM that the SGS kinetic energy $\langle K^{\text{SGS}} \rangle$ is available and is added to the resolved one. In case REF, due to the fine grid used $\langle K^{\text{SGS}} \rangle$ is expected to be small so that case EASSM and REF predictions can be compared directly. For the other cases, a direct comparison is not possible. The mean turbulent kinetic energy is relatively well predicted in case EASSM in the whole channel. However, there are some small over-predictions of $\langle K \rangle$ in the recirculation area, mainly in the reverse flow region,

Figure 11. Mean turbulent kinetic energy in outer units $\langle k \rangle / U_b^2$ (a) and mean SGS eddy viscosity ν_{SGS} normalized by the kinematic viscosity (b). The turbulent kinetic energy profiles in cases DSM-1, DSM-2 and REF are only the resolved part. In case EA the resolved plus modeled turbulent kinetic energy is plotted. —■— : EASSM; —○— : NM; —▲— : DSM-1; —△— : DSM-2; — : REF.

and some small under-predictions at the windward side of the hill. The reason for these mis-predictions could be associated to the use of the weak equilibrium assumption in the EASSM, which assumes that the advection of the SGS stress anisotropy is negligible. This assumption has shown to be reasonably accurate in the mean sense and away from the solid walls [16]. However, in the recirculation area or the accelerating part of the boundary layer at the windward side of the hill this assumption may not be very accurate. Nevertheless, case EASSM results were still shown to be more accurate than those of cases DSM-1 and DSM-2 in this region.

In the channel flow simulations presented earlier, it was found that when the EASSM was used with the pseudo-spectral method, without significant numerical dissipation, the SGS kinetic energy K^{SGS} predicted by the Yoshizawa model in equation (6) was close to the filtered DNS data. On the other hand, when the EASSM was used with the second-order finite volume code, with significant numerical dissipation, the predicted K^{SGS} was much smaller than the filtered DNS data (results not presented). A lower K^{SGS} reduces the SGS dissipation predicted by the EASSM, i.e. $-0.5K^{\text{SGS}}\beta_1|\tilde{S}|^2$. As a consequence, the EASSM also has a smaller effective SGS eddy viscosity, see figure 11(b).

The mean SGS eddy viscosity defined as

$$\nu_{\text{SGS}} = \left\langle \frac{-\tau_{ij}\tilde{S}_{ij}}{|\tilde{S}|^2} \right\rangle, \quad (9)$$

normalized with the kinematic viscosity from the different cases is shown in figure 11 (b). The EASSM is less dissipative and has a smaller ν_{SGS} compared to cases DSM-1 and DSM-2. Hence, it allows for development of more resolved turbulence.

The eddy viscosity from case DSM-2 is roughly four times larger than that of case DSM-1, since ν_{SGS} is proportional to the square of the grid filter size. However, the overall performances of cases DSM-1 and DSM-2 are similar, as was shown earlier. In comparison, although the EASSM has a lower ν_{SGS} , its predictions of the resolved statistics were better than the other cases.

5.6. Turbulent structures

In figure 12, vortical structures are visualized using the Q -criterion [42] defined as

$$Q = \frac{1}{2} \left(\tilde{\Omega}_{ij} \tilde{\Omega}_{ij} - \tilde{S}_{ij} \tilde{S}_{ij} \right), \quad (10)$$

where iso-contours of $Q = 0.5$ are plotted. A positive value for Q highlights vorticity dominated areas. The iso-surfaces of Q are colored with the mean eddy viscosity and kinematic viscosity ratio for the EASSM, DSM-1 and DSM-2 cases and with the mean velocity magnitude in case NM.

Case EASSM predicts more small-scale vortical structures than cases DSM-1 and DSM-2, due to a lower eddy viscosity and dissipation rate. Case NM predictions are similar to case EASSM. The DSM-2 case predicts less small structures compared to the other cases because of a significantly larger SGS dissipation. Due to the lack of small scales one can easily identify the vortices due to the Kelvin–Helmholtz instability of the shear layer originating from the hill crest and their growth further downstream in the recirculation area [19]. In comparison with the other cases, one can also observe thicker and more elongated streaky structures aligned with the mean flow direction close to the upper wall.

6. Conclusions and outlook

LESs of turbulent plane channel flow and channel flow with periodic streamwise hill-shaped constrictions (periodic hill flow) were performed using the explicit algebraic subgrid-scale (SGS) stress model (EASSM) [3], the dynamic Smagorinsky model (DSM) and without a subgrid-scale model (NM). The LESs were carried out with the second-order finite volume *Code_Saturne* [24]. The EASSM is a non-linear mixed model which has an eddy viscosity and a nonlinear part containing $(\tilde{S}_{ik} \tilde{\Omega}_{kj} - \tilde{\Omega}_{ik} \tilde{S}_{kj})$ where $\tilde{S}_{ij}$ and $\tilde{\Omega}_{ij}$ are the resolved strain- and rotation-rate tensors. In LESs of plane channel flow, the results from the EASSM and NM showed the best agreement with the DNS data for the wall shear stress, while the EASSM presented the best predictions for the Reynolds stresses.

In the LESs of the periodic hill flow, the EASSM predictions were superior to the DSM and NM cases for the mean velocity profiles and Reynolds stresses. The pressure coefficient from the EASSM was also in excellent agreement with the reference data [20] at both walls, while in other cases it was over-predicted. In the second half of the recirculation zone, the friction coefficient was better predicted by the EASSM. The EASSM provided a smaller eddy viscosity and a lower SGS dissipation than the DSM and yet its predictions were better, which shows that the EASSM is able to better respond to the resolved anisotropies.

In contrast to the separation point, the reattachment point was found to be very sensitive to the SGS model. The EASSM prediction of the reattachment point was the most accurate, while the DSM predicted a too early reattachment and a too short separation bubble. Due to this early reattachment of the flow in case DSM and

Figure 12. Vortical structures visualized by iso-surfaces of Q coloured by the mean eddy viscosity divided by the kinematic viscosity (a–c) and coloured by the magnitude of the velocity (d). Cases EASSM (a), DSM-2 (b), DSM-1 (c) and NM (d).

Figure 13. Mean subgrid-scale shear stress in outer units $\langle \tau_{12} \rangle / U_b^2$ at $x/h = 0.05$ (a) and $x/h = 8.0$ (b) limited to $y/h < 1.5$. —■— : EASSM; —▲— : DSM-1; —△— : DSM-2.

NM, the velocity profiles and Reynolds stresses downstream of the reattachment point showed considerable differences to the reference data.

In the reverse flow region of the recirculation zone and in the boundary layer on the windward side of the hill, the sign of the SGS shear stress is the same as the velocity gradient, contrary to the eddy viscosity assumption and impossible to predict with the DSM. Figure 13(a) and (b) show two examples at $x/h = 0.05$ and $x/h = 8.0$ extracted from figure 8 and plotted for the near-wall region to clarify the differences in model predictions. It is observed that the DSM predicts a positive SGS shear stress at $x/h = 0.05$ outside the boundary layer, $y/h > 1.1$, due to the negative velocity gradient while the EASSM predicts this correctly. The significant positive shear stress predicted by the DSM leads to a higher SGS dissipation which negatively influences the initial development of the resolved turbulence in the subsequent shear layer, in contrast to the small shear stress predicted by the EASSM. Similar predictions are observed in the boundary layer at $x/h = 8.0$, where the DSM prediction of the shear stress is close to zero and the model is inactive, while the EASSM prediction is positive and physically more compatible with the resolved shear stress. Further investigation showed that the prediction of τ_{12} in the near-wall region by the EASSM is affected by the contribution of the nonlinear part of the model. Although this term does not contribute to the SGS dissipation of kinetic energy, it is known to influence the resolved enstrophy as well [43], which could have implications in the LES predictions of the periodic hill flow. This could be a topic of further research.

In LES of the periodic hill flow with the EASSM and DSM, with the same numerical method and flow geometry, at coarser resolutions, the EASSM gives also favourable results compared to the DSM and NM [44]. These studies show that the EASSM has a clear potential in prediction of separation and reattachment

processes, even in the presence of the numerical dissipation, which makes it suitable for LES of complex industrial flows using low-order commercial codes.

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