State-based Priorities for Tournaments in Wireless Networked Control Systems

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Abstract

We introduce a state-based distributed prioritization mechanism for a sensor associated with a dynamical system to access a wireless network, when multiple such systems share the same network link. The priorities, designated *Attention Factors*, are assigned by each sensor to its data packets, based on the measurements of the system state. The Attention Factor represents a quantized value of the minimum risk in not transmitting a given measurement. The Attention Factors from different sensors are evaluated and allotted slots, in a distributed manner, using a dominance-based protocol called *tournaments*. Packets with the same Attention Factor in a tournament collide, and are lost. We analytically evaluate the probability of a successful transmission using this access mechanism. We also find an upper bound for the estimation and control performance of a system using tournament access, which shows the benefits of using state-based priorities. The proposed tournament mechanism is implemented on the IEEE $802.15.4$ standard protocol stack, and evaluated in a hardware-in-the-loop experimental setup.

I. Introduction

A. Motivation

Wireless sensing and actuating networks consist of many sensors that monitor physical systems and transmit the collected data to a Data Processing Unit (DPU) across a shared network.

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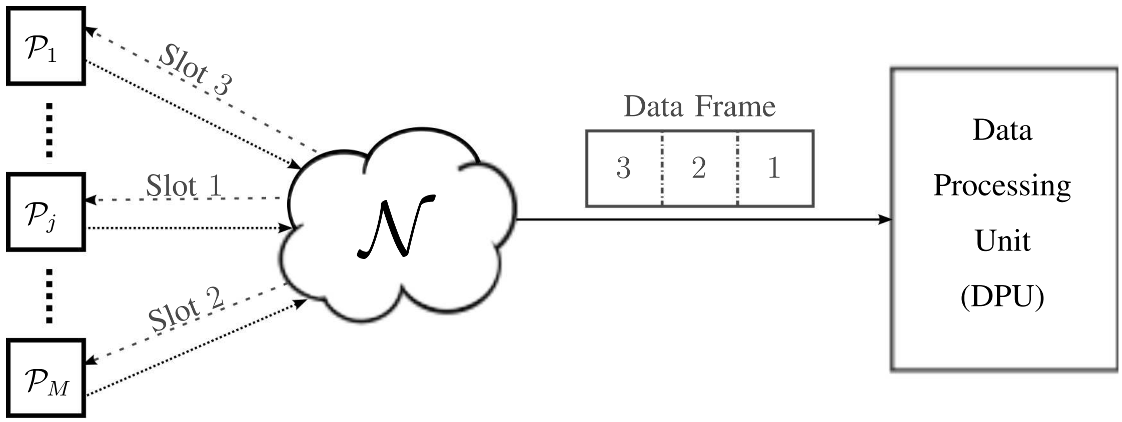


Fig. 1. An overview of a multiple access network ($\mathcal{N}$) of plants ($\mathcal{P}_j$), $j \in \{1, \ldots, M\}$, with prioritized access in the MAC. The plant sensors communicate a priority to the network, which evaluates it distributedly, and suitably orders data packets in the frame.

In typical sensor networks, the DPU may track the state of the physical system. In actuating networks, the DPU may issue a control signal to regulate the state of the physical system. In either case, the sensing link from each system to the DPU belongs to a shared network, as depicted in Fig. 1. Wireless networks are broadcast mediums, and simultaneous transmissions on the same link interfere with each other, causing collisions and packet losses. To prevent collisions, a multiple access protocol is used to arbitrate access to the shared network. Designing a multiple access protocol that provides sufficient reliability for control systems in the network, while retaining ease of implementation on wireless systems, is a challenge [1], [2].

Wireless actuating networks differ from sensor networks due to the consequences of delayed control action on critical dynamical processes [3], [4]. The communication infrastructure for such a network must meet harsher time constraints, in proportion to how far the state or measurement is from the nominal value. At the same time, the sensing paradigm continues to apply to actuating networks; implying that there are many nodes in the shared network. Thus, any multiple access protocol must be scalable with network size.

The two most commonly used multiple access mechanisms for wireless networks are Time Division Multiple Access (TDMA) [5] and Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) [6], due to ease of implementability. TDMA provides guaranteed access at the cost of delay, while CSMA/CA reduces delay by randomizing access [5]. For large networks, TDMA could result in large delays, and CSMA/CA could randomly lose important packets [5].

Dynamic and/or distributed scheduling mechanisms have been proposed for wired networks and control systems, but these are not robust to the vagaries of the wireless medium, and suffer the possibility of disruptions due to losing requests and tokens [7], [8].

A prioritized access mechanism addresses most reliability concerns. In the absence of a prioritized access mechanism, a sensor containing critical information could be blocked by a sensor containing regular or non-critical measurements. Static priorities are easy to implement, but often inefficient. A fire detector in a chemical plant is likely to be assigned a higher priority than a temperature sensor monitoring operational levels, but the temperature measurements at most time instants are likely to contain more important information regarding operation of the plant as compared to routine (safe) measurements from the fire detectors. Dynamic priorities based on information in the current measurements solves the problem, but these are hard to assign and evaluate in a wireless network.

B. Contributions

A protocol that allows for prioritization of data based on the current measurement, in a distributed setting, would meet all the above requirements. Previous attempts at introducing priorities within CSMA/CA include arbitration of interframe space or contention window differentiation, such as in IEEE 802.11e [9]. This protocol still results in random access, but with prioritized access probabilities. It also does not allow for sufficient priority levels, as will be required when the priorities are based on the current measurement. In this paper, we introduce a prioritized access scheme with transmission slots reserved for nodes that win a tournament. The idea of a tournament to resolve contention based on static priorities is already prevalent in the literature, e.g., the CAN Bus Protocol [10] and its recent adaptation to wireless networks in WiDOM [11]. However, the priority mechanism in our proposal is dynamic, and priorities are assigned to data packets, not to nodes.

The main contribution of this paper consists of a method of assigning measurement-based priorities, and evaluating the resulting network and system performance. A node evaluates the criticality of the current measurement to be transmitted to the controller or monitoring unit and assigns an appropriate priority. The priority is a measure of the attention that a packet requires from the controller, and hence called *Attention Factor*. These priorities are evaluated in a distributed manner, through a tournament. The reliability and delay of such an access mechanism is analyzed. This analysis is verified through simulations. We also find an upper-bound for the resulting estimation and control costs in the network.

In addition, we provide an implementation of the tournaments on low-rate wireless devices using IEEE $802.15.4$ [12] compliant radios. This is meant as proof-of-concept, to illustrate that the proposed mechanism is compatible with current radio transceivers, and can be implemented on current off-the-shelf hardware. We explore the throughput-tradeoffs in the implementation, and include these effects in our analysis. The protocol is implemented as part of the IEEE $802.15.4$ medium access control standard [12], which serves as the basis for Zigbee [13]. We also present experimental results from a hardware-in-the-loop simulation of a wireless tank process, with the modified protocol implemented and executed on real wireless sensor nodes.

C. Related Work

The idea of using the state or measurement of a physical system to determine channel access has been prevalent for some time now [14]–[16]. The deviation in the state from the nominal value was used to determine a priority in Try-Once-Discard (TOD) [14]. Deadbands around the nominal value were used to limit the use of the channel in [15], [16]. Both these ideas have given rise to many related works, which we discuss below.

Maximum error first is the prioritization principle used in TOD, to guarantee input-to-state stability for deterministic systems with disturbances. The implementation of the original idea was centralized, and required a network coordinator to collect and compare errors from the various physical processes in the network. This contention-free implementation has been extended to include effects of packet losses in [17]. Recently, a distributed implementation for this protocol has been conceived and successfully implemented [18], but without evaluating the robustness of the implementation to information loss. In contrast, this paper deals with state-based priorities for stochastic processes, where the priorities are allocated and evaluated in a distributed manner. Moreover, the emphasis in this paper is on performance and not only stability. Finally, the analytical results presented in this paper incorporate the information loss that occurs while using the multiple access protocol in a wireless medium.

A state-based priority can be viewed as an M-ary extension of binary-valued events. Consequently, many of the analytical results in this paper build on results from the literature in stochastic event-triggered systems [19]. Structural results for the closed-loop system that motivate the use of policies such as maximum-error-first have been explored in [20], [21]. The network interactions that result from state-based access methods introduce correlations between exogenous processes, as pointed out in [22], [23]. The state-based policy presented in this paper circumvents these complications, while retaining the benefits of using the state to determine channel access. A related work from the event-triggered literature is [24], which explores a dynamic utilization policy for the TDMA slots of the IEEE $802.15.4$ protocol.

The outline of the paper is as follows. We formulate the problem in Section II. We present the Attention Factor formulation and the tournament access mechanism in Section III, and analyze the performance of this protocol in Section III-C. We explain the implementation of tournaments in Section IV, motivate the choice of many parameters in the protocol. Finally, we illustrate the simulation and experimental results in Section V.

II. Problem Formulation

We consider a network of M processes and their respective sensors, which communicate over a shared channel, as shown in Fig. 1. Access to the network is determined by a state-based priority, and the priorities are evaluated in a distributed manner. Sensors that secure access transmit their packets in the chosen order. Each system in this network views the rest of the network through the model depicted in Fig. 2. The blocks in this figure are explained below.

Plant: Each plant $\mathcal{P}_j$, for $j \in \{1, \dots, M\}$ has state dynamics given by

$$x^j_{k+1} = Ax^j_k + Bu^j_k + w^j_k \,, \tag{1}$$

$$y^j_k = Cx^j_k + v^j_k \,, \tag{2}$$

where the state $x^j_k \in \mathbb{R}^n$ and the measurement $y^j_k \in \mathbb{R}^m$. The initial state x^j_0, the process noise w^j_k and the measurement noise v^j_k are i.i.d. zero-mean Gaussians with covariance matrices R_0, R_w and R_v, respectively. If the physical process is not part of a control loop, there is no control term in the state equation, i.e., $B = 0$. This discrete time model is defined with respect to a sampling period T for each plant, and the sampling instants are generated by a synchronized network clock. The different plants in the network are driven by exogenous noise processes. We comment on an extension to heterogeneous networks in Section III.

Kalman Filter: A Kalman filter (KF) is implemented in each sensor to provide an estimate

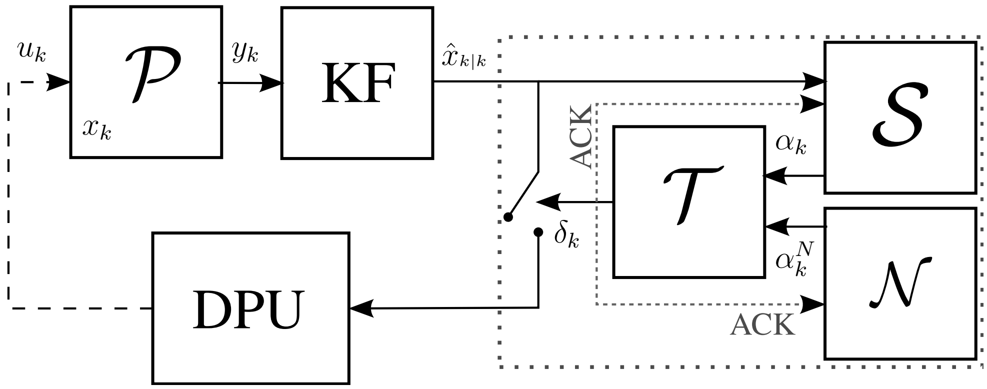


Fig. 2. A mathematical model of a single system in the network. The system itself consists of a plant ($\mathcal{P}$), a Kalman filter (KF) at the sensor, a state-based priority synthesizer ($\mathcal{S}$) and a Data Processing Unit (DPU) across the network. The priority α_k is used to determine access to the shared network. The rest of the traffic sources in the network are abstracted away into the network block ($\mathcal{N}$), and this block produces an aggregate priority α_k^N, a vector of all the other priorities in the network. The priorities are evaluated in the Tournament block ($\mathcal{T}$), and $\delta_k \in \{0, 1\}$ indicates if the current packet is successfully transmitted across the network or not, respectively.

$\hat{x}^{s,j}_{k|k}$, where the superscript 's' denotes the estimator at the sensor, and is given by

$$\hat{x}^{s,j}_{k|k} = \hat{x}^{s,j}_{k|k-1} + K_{f,k} e^j_k \ , \ \hat{x}^{s,j}_{k|k-1} = A\hat{x}^{s,j}_{k-1|k-1} + Bu^j_{k-1} \ , \tag{3}$$

where $K_{f,k}$ denotes the Kalman gain, e^j_k denotes the innovation in the measurement, and $\hat{x}^{s,j}_{k|k-1}$ denotes the predicted estimate. The innovation is defined as

$$e^j_k = y^j_k - C\hat{x}^{s,j}_{k|k-1} \ . \tag{4}$$

The Kalman gain is defined as $K_{f,k} = P^s_{k|k-1} C^T R^{-1}_{e,k}$, where $R_{e,k} = CP^s_{k|k-1}C^T + R_v$ is the covariance of the innovation e^j_k. The prediction error covariance is given by $P^s_{k|k-1} = AP^s_{k-1|k-1}A^T + R_w$ and the filtered error covariance is given by $P^s_{k|k} = P^s_{k|k-1} - K_{f,k}R_{e,k}K^T_{f,k}$.

State Based Priorities: There is a local scheduler $\mathcal{S}$, situated in the sensor node, between the plant and the network, which calculates the state-based priority, α^j_k, of the data packet. This block is formulated using a policy f, as given by

$$\alpha^j_k = f_k(\omega^{s,j}_k) \ , \tag{5}$$

where, $\omega^{s,j}_k \in \Omega^{s,j}_k$ and $\Omega^{s,j}_k$ is the σ-algebra generated by the information set at the scheduler, given by $\mathbb{I}^{s,j}_k = \big\{\{\hat{x}^{s,j}\}^{k|k}_{0|0}, \{y^j\}^{k-1}_0, \{\alpha^j\}^{k-1}_0, \{\delta^j\}^{k-1}_0, \{u^j\}^{k-1}_0\big\}$. The notation $\{c\}^b_a := \{c_a, \ldots, c_b\}$, for $a \leq b$.

Network: The network $\mathcal{N}$ generates other traffic, with an aggregate priority $\alpha_k^{N,j}$, which denotes a vector of all the other priorities in the network.

Tournament Block: The tournament $\mathcal{T}$ resolves contention between multiple simultaneous channel access requests. The channel access indicator $\delta_k^j \in \{0,1\}$ is given by

$$\delta_k^j = \mathcal{T}(\alpha_k^j, \alpha_k^{N,j}) \ . \tag{6}$$

DPU: The DPU receives $z_k^j = \delta_k^j \hat{x}_{k|k}^{s,j}$, and utilizes the estimate of the state $\hat{x}_{k|k}^{s,j}$, when it receives it, in monitoring, control or detection applications. With no loss of generality, we assume that the DPU contains an estimator followed by a controller. The estimate at the DPU is denoted $\hat{x}_{k|k}^{c,j}$, where the superscript 'c' indicates the controller, and is given by

$$\hat{x}_{k|k}^{c,j} = \delta_k^j \hat{x}_{k|k}^{s,j} + (1-\delta_k^j)\hat{x}_{k|\tau_k}^{s,j} \ , \tag{7}$$

where τ_k is the time index of the last received packet. The estimation cost is given by the average estimation error variance at the DPU,

$$J_E = \mathrm{tr}\{\mathbf{E}[P_{k|k}^{c,j}]\} \ , \tag{8}$$

where $P_{k|k}^{c,j}$ is the estimation error covariance at the controller and $\mathrm{tr}\{\cdot\}$ is the trace operator.

The controller implements a policy g_k, defined on the σ-algebra generated by the information set of the controller, given by $\mathbb{I}_k^{c,j} = \left\{\{z^j\}_0^k, \{\delta^j\}_0^k, \{u^j\}_0^{k-1}\right\}$. The control policy is typically chosen to minimize the infinite horizon Linear Quadratic Guassian (LQG) control cost, given by

$$J_C = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} \mathbf{E}\left[(x_k^j)^T Q_1 x_k^j + (u_k^j)^T Q_2 u_k^j\right] \ , \tag{9}$$

where the weighting matrices, Q_1 and Q_2, are non-negative and positive, respectively.

In this paper, we present a formulation for the prioritization policy f_k and the tournament mechanism $\mathcal{T}$. We identify properties of the resulting access mechanism, and provide an analytical expression for the probability of a successful transmission $\mathbf{P}(\delta_k^j = 1)$. We also characterize the estimation and control costs, J_E and J_C, respectively. In addition to the above theoretical investigations, we design a modification to the existing IEEE $802.15.4$ MAC standard, to include tournaments, and implementing the resulting protocol on state-of-the-art low-rate wireless devices with IEEE $802.15.4$ compliant radios. We validate the analysis using a wireless networked control experimental setup.

III. Protocol Design and Analysis

We now present the main theoretical results in this paper. We begin by introducing a formulation for the priorities, and identifying some of its properties. Then, we present tournaments as a mechanism to evaluate priorities in a distributed manner. We also present a network-level performance analysis of the resulting access mechanism. We use the results of this analysis to evaluate upper bounds for the estimation and control costs for systems that use this access mechanism.

A. Attention Factor

In this section, we derive an expression for the state-based priorities and in doing so, identify the scheduling policy f_k in (5). The priorities are assigned by each sensor node to its own data packet, in isolation from the rest of the network.

The Attention Factor is an adaptive priority designed to call the attention of the DPU to the current data in the node, and to connote a penalty in not being able to transmit this data. At some time $k-1$, the j^{th} sensor delivers a measurement y^j_{k-1} to its local estimator (KF), which computes the estimate $\hat{x}^{s,j}_{k-1|k-1}$. Let us assume that the node is successful in sending this estimate to the DPU over the network. The estimator at the DPU (7) can generate future estimates as $\hat{x}^{c,j}_{k|k} = \hat{x}^{s,j}_{k|k-1}$,$\hat{x}^{c,j}_{k+1|k+1} = \hat{x}^{s,j}_{k+1|k-1}$, and so on. The motivation for allocating channel resources to the sensor in order to deliver the next packet to the DPU, is the innovation e^j_k in the measurement y^j_k, given by (4). The risk in not being able to deliver this packet can be assigned a value equal to a function of the difference between the predicted estimates $\hat{x}^{s,j}_{k+1|k-1} = A\hat{x}^{s,j}_{k|k-1} + Bu^j_k$ and $\hat{x}^{s,j}_{k+1|k} = A\hat{x}^{s,j}_{k|k} + Bu^j_k$. In fact, the quantity $\hat{x}^{s,j}_{k+1|k-1} - \hat{x}^{s,j}_{k+1|k}$, is an indicator of the minimum risk in not delivering the current packet, as the risk will only be larger if the packet at time $k-1$ did not reach the DPU. We have used the predicted estimate in the formulation so far to emphasize process dynamics and make the Attention Factor sensitive to unstable systems. To further emphasize the dynamics, the prediction horizon need only be extended further more.

Since we deal with estimation and control in this paper, let us look at a distortion-like function of the quantity described above.

Definition 3.1 (Minimum Risk Indicator): The increase in the sample variance of the prediction error due to not receiving a packet at time k is denoted $\Delta P^j_{\text{samp},k}$ and given by

$$\Delta P^j_{\text{samp},k} = f_{\text{MRI}}(y^j_k) := \text{tr}\{(\hat{x}^{s,j}_{k+1|k} - \hat{x}^{s,j}_{k+1|k-1}) \cdot (\hat{x}^{s,j}_{k+1|k} - \hat{x}^{s,j}_{k+1|k-1})^T\}\ . \tag{10}$$

$\Delta P^j_{\text{samp},k}$ is an empirical quantity based on knowledge of the measurement y^j_k, and can be rewritten as $\Delta P^j_{\text{samp},k} = \text{tr}\{AK_{f,k}e^j_k(e^j_k)^T K^T_{f,k}A^T\}$.

A prioritized transmission scheme based on the quantity $\Delta P^j_{\text{samp},k}$ should ideally result in

$$\delta^j_k = \mathcal{T}_{\text{ideal}}(\{\Delta P^i_{\text{samp},k}\}^M_{i=1}) := \begin{cases} 1 & j = \arg\max_i \Delta P^i_{\text{samp},k} \ , \\ 0 & \text{otherwise} \ , \end{cases} \tag{11}$$

for the j^{th} sensor node. Thus, the node with the largest priority gets to transmit. We use this idealized tournament to inspect the merits of the prioritization mechanism. First, note that the expected value of the minimum risk indicator can be shown to be proportional to the minimum increase in the variance of the prediction error at the DPU due to not possessing information about the measurement y^j_k, i.e.,

$$\mathbf{E}[\Delta P^j_{\text{samp},k}] = \text{tr}\{AK_{f,k}\,\mathbf{E}[e^j_k(e^j_k)^T]K^T_{f,k}A^T\} = \text{tr}\{P^s_{k+1|k-1} - P^s_{k+1|k}\} \ .$$

Thus, the quantity $\Delta P^j_{\text{samp},k}$ can be said to be proportional to the increase in the per sample variance of the prediction error at the DPU. Then, let $\Delta P_{\text{sys},k} \triangleq \sum^M_{j=1} \Delta P^j_{\text{samp},k}$ be the net increase in the sample variance of the prediction error due to not possessing information about measurements $\{\boldsymbol{y^j_k}\}^M_{j=1}$ from all the nodes in the network at time k. Using $\Delta P_{\text{sys},k}$ as an optimization index, we find that the resulting channel access mechanism is an optimal scheduling policy in a resource constrained network. In addition, the prioritization mechanism results in a few desirable system-level properties, such as separation between designs of the scheduler, observer and controller, and a minimum mean-squared error (MMSE) estimate with a simple, Kalman filter-like recursive update. All of these are compiled in the lemma below.

Proposition 3.1 (Properties of the Minimum Risk Indicator): For the network of M systems given by (1)–(6), with $f_k = f_{\text{MRI}}(y^j_k)$ for $1 \leq j \leq M$ as in (10) and $\mathcal{T} = \mathcal{T}_{\text{ideal}}(\{\Delta P^i_{\text{samp},k}\}^M_{i=1})$ as in (11), it holds that:

i. The resulting scheduling policy minimizes $\Delta P_{\text{sys},k}$.
ii. The estimate (7) minimizes the mean-squared estimation error.
iii. The DPU estimates are correlated, but the network traffic remains independent.
iv. The optimal control policy for a closed-loop system in this network is certainty equivalent.

Proof: To prove Claim i, let us define the ratio $\eta^j_k \triangleq \Delta P^j_{\text{samp},k}/\Delta P_{\text{sys},k}$. In the prioritized access scheme in (11), the channel is allotted to the data packet which maximizes $\Delta P^j_{\text{samp},k}$. This also maximizes η_k^j, which implies that

$$\max_j \ \eta_k^j = \max_j \ \frac{\mathrm{tr}\{(\hat{x}^{s,j}_{k+1|k} - \hat{x}^{s,j}_{k+1|k-1}) \cdot (\hat{x}^{s,j}_{k+1|k} - \hat{x}^{s,j}_{k+1|k-1})^T\}}{\sum_{j=0}^{M} \mathrm{tr}\{(\hat{x}^{s,j}_{k+1|k} - \hat{x}^{s,j}_{k+1|k-1}) \cdot (\hat{x}^{s,j}_{k+1|k} - \hat{x}^{s,j}_{k+1|k-1})^T\}} \ .$$

Thus, the data packet (measurement) which results in a maximum reduction of the sample variance of the prediction error is allotted channel access. Thus, priorities based on $\Delta P^j_{\text{samp},k}$ result in a sample-wise optimal scheduling strategy given limited communication resources.

We now prove Claim ii. At any time k, let τ_k denote the time index of the last received packet by the j^{th} controller. Then, the MMSE estimate $\mathbf{E}[x_k^j|\mathbb{I}_k^{c,j}]$ is given by

$$\mathbf{E}[x_k^j|\mathbb{I}_k^{c,j}] = \underbrace{A^{k-\tau_k}\hat{x}^{s,j}_{\tau_k|\tau_k} + \sum_{n=1}^{k-\tau_k} A^{n-1}Bu^j_{k-n}}_{\hat{x}^{s,j}_{k|\tau_k}} + \mathbf{E}[x_k^j - \hat{x}^{s,j}_{k|\tau_k}|\{\boldsymbol{\delta^j}\}^k_{\tau_k+1} = 0]$$

$$= \mathbb{1}_{\{\delta_k^j=1\}}\hat{x}^{s,j}_{k|k} + \mathbb{1}_{\{\delta_k^j=0\}}\hat{x}^{s,j}_{k|\tau_k} = \hat{x}^{c,j}_{k|k} \ ,$$

where the second expression above is obtained by virtue of the symmetric prioritization policy. The term $\mathbf{E}[x_k^j - \hat{x}^{s,j}_{k|\tau_k}|\{\boldsymbol{\delta^j}\}^k_{\tau_k+1} = 0]$ vanishes when $\delta_k^j = 0$ due to the symmetric function of the innovations chosen for f_{MRI} in (10). Thus, the estimate in (7) is indeed the MMSE estimate.

To see how Claim iii holds, note that $\mathbf{E}[x_k^j|\mathbb{I}_k^{c,j}]$ is determined by δ_k^j, the channel access indicator. Since only a fixed number of nodes can access the channel, the variables $\{\delta_k^j\}_{j=1}^M$ are correlated. Thus, the network interaction induces a correlation in the estimates $\mathbf{E}[x_k^j|\mathbb{I}_k^{c,j}]$ across the network. However, the network traffic is determined by the priorities assigned to the nodes, which are functions of the independent innovations process. Thus, our choice of f_{MRI} in (10) results in independent traffic, despite the network interactions.

The choice of the scheduling policy f_k at each node is a symmetric function of the innovations process, and thus, the closed loop system is free of a dual effect [25]. This implies that the optimal control policy for a linear quadratic cost is certainty equivalent, as stated in Claim iv. ■

Thus, our choice of the prioritization policy f_{MRI} results in many desirable properties, both for the network and the system. Importantly, f_{MRI} results in independent network traffic due to its dependence on the innovations e_k^j alone, simplifying the analysis and design considerably. A prioritization policy based on the *entire* prediction error, $(\hat{x}^{s,j}_{k+1|k} - \hat{x}^{s,j}_{k+1|\tau_k})$, rather than on e_k^j alone, might well result in better estimation and control costs. However, analyzing and designing such a scheme for a network of systems would certainly be more complex, and we comment on this later. Furthermore, our optimization index $\Delta P_{\text{sys},k}$ is a greedy index, emphasizing the current increase in per sample variance rather than the expected increase. Nevertheless, such a choice can still result in considerable benefits in terms of the estimation and control performance, as indicated by Theorem 3.7.

The tournament mechanism $\mathcal{T}_{\text{ideal}}$ is impractical as it compares real-valued priorities. To implement a prioritized access scheme, we require a fixed resolution priority. We obtain this by quantizing the minimum risk indicator in (10) to give us the Attention Factor, denoted as α_k^j.

Definition 3.2 (Attention Factor): For a dynamical system given by (1), the state-based priority $\alpha_k^j \in \mathbb{Z}$ for $1 \leq j \leq M$ is defined as

$$\alpha_k^j = \text{round}\left(\text{tr}\{AK_{f,k}e_k^j(e_k^j)^T K_{f,k}^T A^T\} \cdot \frac{A_{\text{max}}}{P_{\text{s,max}}}\right) , \tag{12}$$

where $0 \leq \alpha_k \leq A_{\text{max}}$ for some integer $A_{\text{max}} \in \mathbb{Z}$, and $P_{\text{s,max}} = \text{tr}\{\kappa K_{f,k} R_{e,k} K_{f,k}^T \kappa^T\}$ is the maximum tolerable increase, as defined by $\kappa \in \mathbb{Z}^+$, in sample variance of the prediction error for a node attached to a plant with identity system matrix ($A = I$).

The constant κ is used by the sensor node to dictate its own tolerance limits and influence the increase of α with deviating measurements. This constant determines the probability mass function (PMF) of the Attention Factor for the entire network, and we comment on its selection in later sections. Note that the merits of a prioritization scheme based on $\Delta P_{\text{samp},k}^j$ (Proposition 3.1) are unaffected by the transformation to α_k^j for sufficiently large values of A_{max}. Thus, (12) determines our choice of f_k in (5) at each node.

B. Evaluating Priorities Using Tournaments

Now that adaptive priorities have been assigned to the data packets, there remains the task of designing an arbitration policy to resolve contention. In other words, how should the data packets exchange priorities and decide who gets to transmit? We use tournaments as a distributed access mechanism that evaluates priorities in a non-coordinated manner and identify the function $\mathcal{T}$ in (6) corresponding to this mechanism.

Consider the frame structure presented in Fig. 3. There are N_{T} transmission slots in each frame, and sensors that wish to transmit in this frame must generate an Attention Factor as described in Section III-A. The data frame begins with N_{T} tournaments, followed by the same

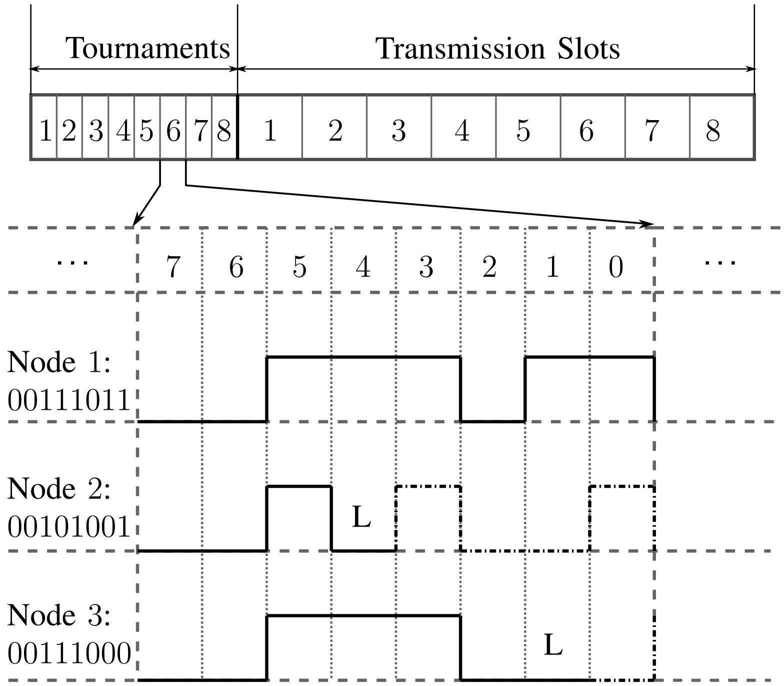


Fig. 3. The Frame Structure of the tournament access mechanism, with a tournament comprising of 8 priority bits, for each of the $N_{\mathrm{T}} = 8$ transmission slots. An example of a tournament is depicted here, with three nodes competing for the sixth transmission slot. Nodes 2 and 3 lose the tournament to node 1, and the symbol L depicts the priority bit during which the tournament is lost. Node 1 gets to transmit its data packet in the sixth transmission slot.

number of transmission slots. To transmit in one of the slots in this frame, a node must win the corresponding tournament. During the tournament, qualifying packets transmit their Attention Factors, starting with the most significant bit. Nodes transmit a suitably chosen pulse for a bit of value one and remain silent during the zero bit. As wireless transceivers cannot transmit and receive at the same instant, nodes can listen during the zero bits. A busy channel indicates that they have lost the tournament. The packet(s) with the largest Attention Factor wins the tournament.

As the Attention Factors are assigned by each node, more than one packet can have the same Attention Factor and win the tournament. Multiple winners are not aware of each other, and cause a collision. Using the same mechanism as in CSMA/CA, nodes are aware of a collision by the lack of an acknowledgment (ACK). Fig. 3 illustrates the concept of a tournament between three nodes with Attention Factors of 59, 41 and 56 respectively. Nodes 2 and 3 lose the tournament after transmitting the fourth and first bits of their priorities, respectively, as they hear a busy channel during their recessive bits. Node 1 wins the tournament and transmits in the corresponding slot.

Thus, using this access mechanism, a node can either lose a tournament or win a tournament. A node that wins a tournament can either collide in the transmission slot or successfully transmit its data. We now define the tournament function $\mathcal{T}(\alpha_k, \alpha_k^N)$.

Definition 3.3 (Tournament Access Mechanism): Let the N_{T} highest values of Attention Factors, selected from the Attention Factors of all the nodes in the network, $\alpha_k^1, \dots, \alpha_k^M$, be given by the set $\mathcal{A}_k = \{\bar{\alpha}_1, \dots, \bar{\alpha}_{N_{\mathrm{T}}}\}$, where $\bar{\alpha}_1 > \bar{\alpha}_2 > \dots > \bar{\alpha}_{N_{\mathrm{T}}}$. Each of these values may be the Attention Factor corresponding to one or more nodes in the network. It is clear that the values in the set $\mathcal{A}_k$ win the tournament, but if any of these values occur more than at one node, then, the corresponding packets are lost due to a collision. Thus, only the Attention Factors corresponding to unique values from the set $\mathcal{A}_k$ succeed in transmitting a packet. Let us denote the set of nodes that win a tournament at time k by $\mathfrak{W}_k$, and the set of nodes that successfully transmit in a tournament by $\mathfrak{T}_k$. We write these sets as

$$\begin{aligned} \mathfrak{W}_k &:= \{j : \alpha_k^j \in \mathcal{A}_k\} \,, && \text{for } 1 \leq j \leq M \,, \\ \mathfrak{T}_k &:= \{j : \alpha_k^j \neq \alpha_k^s \; \forall \, s \in \mathfrak{W}_k \setminus j\} \,, && \text{for } j \in \mathfrak{W}_k \,. \end{aligned} \tag{13}$$

If $\mathfrak{A} = \{1, \dots, M\}$, then clearly, the set of nodes that lose a tournament is given by $\mathfrak{L}_k := \mathfrak{A} \setminus \mathfrak{W}_k$, and the set of nodes that collide in a tournament is given by $\mathfrak{C}_k := \mathfrak{W}_k \setminus \mathfrak{T}_k$. Thus, the tournament function is given by

$$\mathcal{T}(\alpha_k^j, \alpha_k^{N,j}) = \begin{cases} 1 & j \in \mathfrak{T}_k \\ 0 & \text{otherwise} \,. \end{cases} \tag{14}$$

C. Analysis of the tournament access mechanism

We now analyze the performance of a node that uses the Attention Factor in the tournament access mechanism. There are two performance metrics we identify - the probability of a successful transmission, and the distribution of the delay since the last successful transmission. These expressions will come of use in later sections to evaluate the estimation/control performance of each system in the network. Our analysis is presented for a homogenous network of M systems, but it will be shown that this analysis extends to networks with different types of processes. The analysis presented below depends on the statistical description of the system variables, which are identical in a homogenous network. Thus, we skip the index j in this section.

An important property of the Attention Factor is that we can find the PMF of each node's Attention Factor. For the general model described in Section II, the innovations $e_k \in \mathbb{R}^m$ may be a vector. The sum of unnormalized squared Gaussian variables ($\mathrm{tr}\{AK_{f,k}e_k e_k^T K_{f,k}^T A^T\}$) with unequal variances has a multivariate Gamma-type distribution, as discussed in [26]. Then, the PMF is given by

$$\mathbf{P}(\alpha_k = a) = \begin{cases} \Phi(0.5\frac{P_{\text{s,max}}}{A_{\text{max}}}) & a = 0 \\ \Phi((\alpha+0.5)\dfrac{P_{\text{s,max}}}{A_{\text{max}}}) - \Phi((\alpha-0.5)\dfrac{P_{\text{s,max}}}{A_{\text{max}}}) & 0 < a < A_{\text{max}} \\ 1 - \Phi((A_{\text{max}} - 0.5)\frac{P_{\text{s,max}}}{A_{\text{max}}}) & a = A_{\text{max}} \end{cases} , \tag{15}$$

where, $\Phi(\cdot)$ is the cumulative distribution function corresponding to the multivariate Gamma-type distribution of $\mathrm{tr}\{AK_{f,k}e_k e_k^T K_{f,k}^T A^T\}$. If $e_k \in \mathbb{R}$, or if $\mathbf{E}[e_k e_k^T] = \sigma^2 I$, a diagonal matrix with equal values on the diagonal, then the above distribution becomes a first-order or higher-order Chi-Squared distribution, respectively.

It is easy for each node to characterize the probability of another node in the network with Attention Factor less than (p_{L}), less than or equal to (p_{LE}) and greater than (p_{G}) itself. In a homogenous network, these quantities are given by

$$p_{\text{L}}(\alpha) = \sum_{a<\alpha} \mathbf{P}(a) \ , \quad p_{\text{LE}}(\alpha) = \sum_{a\leq\alpha} \mathbf{P}(a) \ , \quad p_{\text{G}}(\alpha) = 1 - p_{\text{LE}}(\alpha) \ . \tag{16}$$

Let us denote $p_{\text{C},n}(\alpha)$ as the probability that n nodes with Attention Factors greater than α collide in a single slot. This can be found as

$$p_{\text{C},n}(\alpha) = \sum_{a=\alpha+1}^{A_{\text{max}}-1} (\mathbf{P}(a))^n \ . \tag{17}$$

A combinatorial analysis using the above quantities gives us the probability of a node winning, successfully transmitting, losing or colliding in a tournament comprising of a number of tournament slots. We now present these results.

Theorem 3.2: For a homogenous network of M systems given by (1)–(6), with f_k as given in (12) and $\mathcal{T}$ as defined in (14), the probability of successful transmission in N_{T} tournament slots for each node is given by

$$p_{\mathfrak{T}} := \mathbf{P}(\delta_k = 1) = \sum_{\alpha_k} \mathbf{P}(T_{N_{\text{T}},M-1}|\alpha_k)\, \mathbf{P}(\alpha_k) \ , \tag{18}$$

where $\mathbf{P}(T_{N_\mathrm{T},M-1}|\alpha_k)$ is the conditional probability of a node with Attention Factor α_k succeeding in transmission in N_T slots against $M-1$ other nodes, as given in (20).

Proof: Let $\mathbf{P}(W_{N_\mathrm{T},M-1}|\alpha)$ denote the conditional probability of a node with Attention Factor α winning a tournament in N_T slots against $M-1$ other nodes. It is given by

$$\mathbf{P}(W_{N_\mathrm{T},M-1}|\alpha) = \sum_{n=0}^{N_\mathrm{T}-1} C_n^{M-1} p_\mathrm{G}^n(\alpha) p_\mathrm{LE}^{M-1-n}(\alpha) + \sum_{n=N_\mathrm{T}}^{M-1} C_n^{M-1} \left(\sum_{c=1}^{\bar{c}} \mathbf{P}(G^c_{n,n-N_\mathrm{T}+1+c,c})(\alpha) \right) p_\mathrm{LE}^{M-1-n}(\alpha) \ , \tag{19}$$

where $C_k^n = \frac{n!}{(n-k)!k!}$ denotes the binomial coefficient. The term $\mathbf{P}(G^c_{n,l,c})(\alpha)$ denotes the probability of n nodes with Attention Factors greater than α, out of which at least l nodes suffer up to c collisions and $\bar{c} = \min(\lfloor n/2 \rfloor, N_\mathrm{T}-1)$ denotes the maximum number of collisions that can occur among n nodes in $N_\mathrm{T}-1$ slots. The first term in (19) states that there can be only up to $N_\mathrm{T}-1$ packets with Attention Factors greater than α and that the rest must have Attention Factors less than or equal to α. This term does not take into account any collisions that might have occurred. The second term computes the probability of winning a tournament in N_T slots, given that $1 \le c \le \bar{c}$ collisions have occurred before the node with Attention Factor α gets to transmit.

Thus, there can be more than $N_\mathrm{T}-1$ packets with Attention Factors greater than α, as long as at least $n - N_\mathrm{T} + 1 + c$ of these additional packets collide. When all the additional packets collide in the same slot, we have

$$\mathbf{P}(G^c_{n,n-N_\mathrm{T}+2,c=1})(\alpha) = C^n_{n-N_\mathrm{T}+2} \, p_{\mathrm{C},n-N_\mathrm{T}+2}(\alpha) \, p_\mathrm{G}^{N_\mathrm{T}-2}(\alpha) \ .$$

This term requires $n - N_\mathrm{T} + 2$ nodes with Attention Factors greater than α to collide, as given by (17), and the other $N_\mathrm{T}-2$ nodes to have an Attention Factor greater than α. It includes cases where some or all of the $N_\mathrm{T}-2$ nodes collide in the same slot as the $n - N_\mathrm{T} + 2$ other nodes, or collide in other slots, or transmit without collisions. When the number of additional packets exceeds two, i.e., $n - N_\mathrm{T} + 2 > 2$, these packets can also collide over multiple slots, giving rise to expressions for $\mathbf{P}(G^c_{n,l,c})(\alpha)$ for $c > 1$. A general expression for any c can be found using combinatorial ideas, but this expression is quite complex. However, these terms can be neglected when a collision is sufficiently rare.

The conditional probability of losing tournaments in all N_T slots against $M-1$ packets is then given by $\mathbf{P}(L_{N_\text{T},M-1}|\alpha) = 1 - \mathbf{P}(W_{N_\text{T},M-1}|\alpha)$. The conditional probability of succeeding in transmission in N_T slots against $M-1$ packets is obtained by replacing the term $p_\text{LE}^{M-1-n}(\alpha)$ in (19) by $p_\text{L}^{M-1-n}(\alpha)$, as given by

$$\begin{aligned}\mathbf{P}(T_{N_\text{T},M-1}|\alpha) = &\sum_{n=0}^{N_\text{T}-1} C_n^{M-1} p_\text{G}^n(\alpha) p_\text{L}^{M-1-n}(\alpha) \\ &+ \sum_{n=N_\text{T}}^{M-1} C_n^{M-1} \left(\sum_{c=1}^{\bar{c}} \mathbf{P}(G^c, n, n-N_\text{T}+2, c)(\alpha) \right) p_\text{L}^{M-1-n}(\alpha) \, . \end{aligned} \quad (20)$$

This expression differs from (19) by requiring that the other packets have Attention Factors strictly less than the value α. Finally, the conditional probability of a collision under these circumstances is given by $\mathbf{P}(C_{N_\text{T},M-1}|\alpha) = \mathbf{P}(W_{N_\text{T},M-1}|\alpha) - \mathbf{P}(T_{N_\text{T},M-1}|\alpha)$. We can then define the probability of a successful transmission ($p_\mathfrak{T}$) for each node in the network using (18). ■

We can now use this expression to characterize delay. We define the delay d_k suffered by a node as the number of sampling periods since the last successful transmission. Let τ_k denote the time-index of the last successful transmission. This can be defined as $\tau_k := \max\{t : \delta_t = 1\}$, for $-1 \leq t \leq k$ and $\delta_{-1} = 1$. Then, it follows that $d_k := k - \tau_k$.

Corollary 3.3: For a homogenous network of M systems given by (1)–(6), with f_k as given in (12) and $\mathcal{T}$ as defined in (14), the probability distribution of the delay d_k for each node is given by

$$\mathbf{P}(d_k = d) = p_\mathfrak{T}(1 - p_\mathfrak{T})^{d-1} \, . \quad (21)$$

Proof: The Attention Factor α_k is based on the innovations e_k, which is a white process. The results of Theorem 3.2 show that the outcomes of the tournaments can be treated as independent. Now, we use the fact that a successful transmission after a delay of d sampling instants implies a transmission failure for $d-1$ sampling instants. Using the expression for $p_\mathfrak{T}$ from Theorem 3.2, we obtain (21). ■

Extension to Heterogenous Networks: An analysis of a heterogenous network can be performed along similar lines, though it yields more elaborate expressions. This is because the terms in (16)–(17) must be defined individually for each process. Consequently, the probability of each possible combination of nodes must be computed, as these are no longer equal for different combinations[1]. However, if the PMF of the Attention Factors of different processes are designed to be alike, then the above analysis applies directly to heterogenous networks as well.

D. Estimation and Control Performance

We now make use of the performance analysis results of the previous section in order to evaluate the estimation and control performance. In this section as well, we skip the index j for the same reason as before. We begin with the estimation cost J_E in (8).

In the tournament access mechanism, the DPU can gain information about the statistics of the state even if the data packet is not received. This is because the conditional probabilities of losing or colliding in a tournament alter the probability density of the innovations. The altered probability density of the innovations can be written as

$$\psi(\bar{e}_k|\delta_k = 0) = \sum_{\alpha_k=0}^{A_{\max}-1} \psi(\bar{e}_k|\alpha_k) \frac{(1 - \mathbf{P}(T_{N_{\mathrm{T}},M-1}|\alpha_k))}{1 - p_{\mathfrak{T}}} \,, \tag{22}$$

where $\bar{e}_k = AK_{f,k}e_k$ and $\psi(\cdot)$ is the probability density function of $\bar{e}_k$, which is a multivariate normal distribution. Thus, the innovations no longer appear to be normally distributed to the DPU. In fact, if the DPU can 'listen' to the values of the priorities broadcast during the tournaments, then it has even more information to obtain a better posterior description of the innovations.

The posterior mean of the innovations cannot be improved in the above manner, due to the choice of a symmetric policy f_k in (12). However, the posterior variance of the innovations, and consequently of the estimation error, is altered. Finding an exact expression for this variance is difficult, as the density in (22) is non-Gaussian. Instead, we find an upper bound for the variance. The upper bound is the variance of the estimation error corresponding to a Bernoulli packet loss process with loss probability $(1-p_{\mathfrak{T}})$. To see why, note that we can design the tournament access mechanism to ensure that the conditional probability of transmitting a packet is an increasing function of the Attention Factor. Then, the conditional probability of transmission is a non-decreasing function of the magnitude of the innovations. This will naturally result in a lower variance than a uniform probability of transmission across all Attention Factors. We show this formally below, beginning with the following property that reflects our design choice.

[1]For example, let $C_\ell(M,n)$ denote the set of indices of the ℓ^{th} combination of n of M nodes in a heterogenous network. The probability of n of M nodes in this network generating Attention Factors greater than α is given by $\sum_{\ell=1}^{C_n^M} \prod_{j\in C_\ell(M,n)} p_{\mathrm{G},j}(\alpha)$. The corresponding expression for a homogenous network is simply $C_n^M p_{\mathrm{G}}^n(\alpha)$.

Property 3.1 (Non-increasing tail of the Attention Factor distribution): The distribution of the Attention Factor, $\mathbf{P}(\alpha_k = a)$, is said to possess the *non-increasing tail property* if $\mathbf{P}(\alpha) \leq \mathbf{P}(\alpha - 1)$ for all $\alpha > 0$.

This is not a surprising property to obtain from a Gaussian innovations process, if it were not for the finite range of the Attention Factor. As the $\mathbf{P}(\alpha_k = A_{\text{max}} - 1)$ in (15) is equal to the probability of the tail of the distribution, this value might not confirm with the above property. However, there is always a suitable value of κ in (12), for which the above property can be made to hold. Furthermore, such a value of κ is indeed a desirable design for the Attention Factor, as it reduces the probability of a collision for the highest value of Attention Factor.

Impact of κ: The impact of κ can be noted from the Attention Factor distributions presented in Fig. 4. The Attention Factors in this example are calculated for a process with parameters $A = 1$, $B = 1$, $C = 1$ and all initial variances equal to one. The maximum value of the Attention Factor is $A_{\text{max}} = 256$. The parameter $\kappa = 2.25$ ensures Property 3.1, whereas $\kappa = 1$ does not, as shown in Fig. 4. In general, lower values of κ result in a peak at the higher end of the probability distribution of the Attention Factor, while higher values of κ under utilize the range of the Attention Factor. Critical nodes must set as low a value for κ as possible to generate packets with high attention values for small deviations in measurements.

We now present some consequences of well designed state-based priorities, with negligible collision probabilities.

Lemma 3.4: For a policy f_k that results in a non-increasing tail for the Attention Factor distribution, as defined in Property 3.1, the conditional probability of winning $\mathbf{P}(W_{N_{\text{T}},M-1}|\alpha)$ is a non-decreasing function of α as the probability of collision goes to zero.
The proof for this lemma is presented in Appendix A.

Lemma 3.5: For a policy f_k that results in a non-increasing tail for the Attention Factor distribution, as defined in Property 3.1, the conditional probability of a successful transmission $\mathbf{P}(T_{N_{\text{T}},M-1}|\alpha)$ is a non-decreasing function of α as the probability of collision goes to zero.
When the probability of collisions is reduced to zero, a node that wins the tournament directly gets to transmit. In other words, $\mathbf{P}(T_{N_{\text{T}},M-1}|\alpha) = \mathbf{P}(W_{N_{\text{T}},M-1}|\alpha)$. Thus, Lemma 3.4 directly gives us the above result. Both the above results can also be shown to hold for non-zero collision probabilities, with additional constraints on the PMF of the Attention Factor. In the examples we

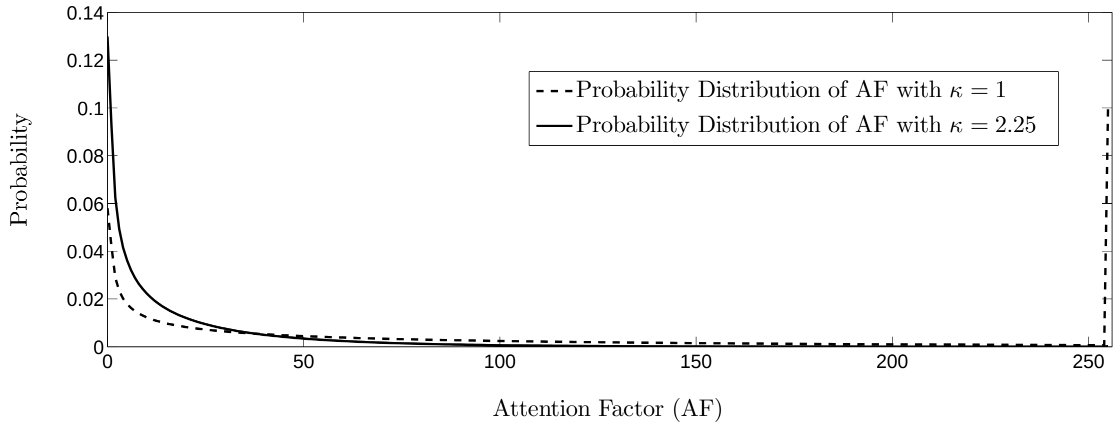


Fig. 4. This plot compares the empirically obtained probability distributions of the Attention Factor for different values of κ. For $\kappa = 1$, the distribution given by the dashed line does not satisfy Property 3.1, as can be seen from the value of $\mathbf{P}(\alpha = A_{\text{max}} - 1)$. However, for $\kappa = 2.25$, the solid line distribution clearly satisfies this property.

present in Section V-A, the above results clearly hold despite a non-zero probability of collisions.

An immediate consequence of Lemma 3.5 is given below.

Lemma 3.6: For a homogenous network of M systems given by (1)–(6), with a policy f_k that results in a non-increasing tail for the Attention Factor distribution, as defined in Property 3.1, and a tournament $\mathcal{T}$, as defined in (14) that results in negligible collision probabilities, the posterior variance of the innovations is less than its a priori value.

The proof of this lemma is presented in Appendix A. We are now ready to present the main result of this section, which highlights the benefits of using state-based priorities.

Theorem 3.7: For a homogenous network of M systems given by (1)–(6), with a policy f_k that results in a non-increasing tail for the Attention Factor distribution, as defined in Property 3.1, and a tournament $\mathcal{T}$, as defined in (14), that results in negligible collision probabilities, it holds that:

i. The variance of the estimation error at the DPU can be bounded from above as

$$J_E \leq \text{tr}\{P_{\text{loss}}(p_{\mathfrak{T}})\} \;, \tag{23}$$

where $P_{\text{loss}}(p_{\mathfrak{T}})$ is the estimation error covariance obtained with a Bernoulli packet loss process of probability $1 - p_{\mathfrak{T}}$ [27].

ii. The LQG control cost at the DPU can be bounded from above as

$$J_C \leq J_{\text{loss}}(p_{\mathfrak{T}}) \ , \tag{24}$$

where $J_{\text{loss}}(p_{\mathfrak{T}})$ is the LQG cost obtained with a Bernoulli packet loss process of probability $1 - p_{\mathfrak{T}}$ [27], [28].

Proof: We use the fact that the variance of the innovations as seen by the DPU is less than its a priori value, shown in Lemma 3.6. Furthermore, from Proposition 3.1, we know that $\text{tr}\{AK_{f,k}\,\mathbf{E}[e_k e_k^T]K_{f,k}^T A^T\} = \text{tr}\{P_{k+1|k-1} - P_{k+1|k}\}$. Thus, the reduction in variance implies that

$$\text{tr}\{P^c_{k|k-1} - P^c_{k|k}\} \leq \text{tr}\{P^s_{k|k-1} - P^s_{k|k}\} \ , \tag{25}$$

following a successful transmission at time $k-1$, where $P^c_{k|k} = P^s_{k|k}$ as this indicates a successful transmission at time k as well. Thus, we have the desirable inequality $\text{tr}\{P^c_{k|k-1}\} \leq \text{tr}\{P^s_{k|k-1}\}$, following a successful transmission at time $k-1$. In fact, for any burst of non-transmissions of length $\ell \geq 0$, the inequality $\text{tr}\{P^c_{k+\ell|k-1}\} \leq \text{tr}\{P^s_{k+\ell|k-1}\}$ holds due to the evolution of the prediction error covariance, even if we assume that there are no further reductions in variance due to using the tournament access.

Now, note that a Bernoulli packet loss process does not provide any information about the innovations, and hence, the variance at the DPU is always given by $\text{tr}\{P^s_{k|\tau_k}\}$, where τ_k is the time index of the last received packet. Thus, the above inequalities imply that the variance of the DPU with tournament access can be upper-bounded by the variance of the DPU with a Bernoulli packet loss process. Using these inequalities in the expression for the average estimation error variance gives us the desired result for estimation. We have

$$\begin{aligned}\text{tr}\{\mathbf{E}[P^c_{k|k}]\} &= \sum_{d=0}^{\infty} \text{tr}\{P^c_{k|k-d}\} \cdot \mathbf{P}(d_k = d) \\ &\leq \sum_{d=0}^{\infty} \text{tr}\{P^s_{k|k-d}\} \cdot \mathbf{P}(d_k = d) = \text{tr}\{P_{\text{loss}}(p_{\mathfrak{T}})\} \ ,\end{aligned} \tag{26}$$

where the average estimation error at any time k can be obtained by marginalizing over the delay due to the independent packet transmission process at each time instant.

Due to the Certainty Equivalence Property from Proposition 3.1, the inequality for the control cost follows from the inequality for the estimation error variance and is given by (24). ■

Any non-state-based random access mechanism can be modelled as a Bernoulli packet loss channel, with suitable assumptions on the operating time scales of the system and the protocol.

Thus, the above result indicates that a well-designed tournament with state-based priorities can outperform any random access mechanism that results in the same probability of transmission, for the estimation and control costs considered in this paper.

Extension to Packet Erasures due to a Lossy Medium: In the work presented so far, we have implicitly assumed that the physical medium does not drop packets. However, this may not be the case in general. If we model the losses due to the physical medium using a Bernoulli process with the probability of loss denoted by p_{loss}, then the probability of a successful transmission will be altered to be

$$\hat{p}_{\mathfrak{T}} := (1 - p_{\text{loss}}) \sum_{\alpha_k} \mathbf{P}(T_{N_{\text{T}},M-1} | \alpha_k) \, \mathbf{P}(\alpha_k) \ , \tag{27}$$

where the conditional probability of transmission $\mathbf{P}(T_{N_{\text{T}},M-1}|\alpha_k)$, for $0 \leq \alpha_k \leq A_{\text{max}}$, is multiplied by the compliment of the loss probability. Thus, note that p_{loss} is indiscriminate of the Attention Factor of the data packet, whereas the probability of transmission that we obtain in the above analysis varies with the Attention Factor. By substituting the probability of transmission $p_{\mathfrak{T}}$ with $\hat{p}_{\mathfrak{T}}$ in the above analysis, we can include the effects of losses from the physical layer in evaluating the control and estimation performance.

IV. Tournament Access Mechanism

We implement the tournament access mechanism as part of the current IEEE $802.15.4$ MAC [12] standard. The IEEE $802.15.4$ physical and medium access control layers are used in some of the proposed protocols for industrial wireless communication, e.g., WirelessHART [29] and ISA 100.11a [30]. This protocol is particularly efficient for sensing applications as it provides a hybrid MAC layer that integrates both guaranteed time slots and contention-based slots in a single scheme. However, it is not optimized for wireless networked control systems as transmissions can only take place through random access and/or be dynamically scheduled, while incurring a fixed delay. The scheduled transmissions are delayed by at least one beacon interval, and can only be scheduled when guaranteed slots are available. This feature in the protocol was meant for slow monitoring applications that require a few continuous guaranteed transmissions, such as video or voice. Finally, adding prioritized access makes this protocol suitable for wireless control applications. Thus, the tournament access mechanism we propose is complimentary to existing features of this protocol. The compatibility of the proposed protocol stack with the IEEE

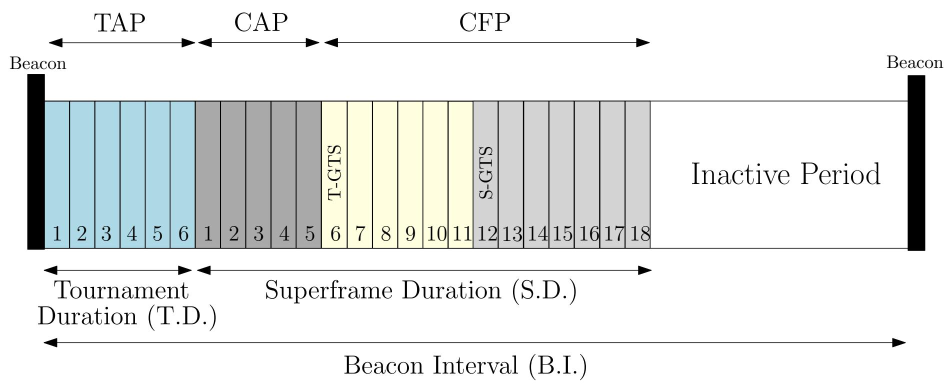


Fig. 5. Example of a superframe structure of the tournament-based IEEE 802.15.4 MAC. The tournament occurs during the TAP, while the transmission of the winner nodes is assigned to the T-GTSs, during the CFP. Scheduled communication for other network nodes is performed during the S-GTS, while best-effort transmissions occur during the CAP.

802.15.4 is desirable since it represents a de-facto standard at physical and MAC layer for sensor network solutions.

We focus on the beacon-enabled mode MAC specified in the standard. In such a setup, a centralized coordinator node, the Network Manager (NM), is responsible for synchronizing and configuring all the nodes. The synchronization and configuration messages take place periodically at each beacon message which defines the time bounds of the superframe structure. The Beacon Interval (BI) denotes the superframe length. The BI is further divided into active and inactive periods, as shown in Fig. 5. The active period has a time interval defined by the Tournament Duration (TD) and the Superframe Duration (SD). The TD and SD can be divided in a maximum of 32 equally sized slots each. The length of the active period is given by the sum of the TD and SD, in which each can have a maximum of 32 equally sized slots. The TD contains the Tournament Access Period (TAP), which comprises of multiple tournament slots. In each tournament slot, nodes transmit priorities and receive acknowledgements if they are winners of the tournaments.

The SD contains the Contention Access Period (CAP) and the Collision Free Period (CFP). During the CAP, nodes transmit best effort messages using CSMA/CA. The CFP is intended to provide real-time guaranteed service, by allocating Guaranteed Time Slots (GTS) to the nodes using a TDMA scheme. In our protocol, two types of GTS slots are defined: Tournament GTS (T-GTS), where the transmissions of tournament winner nodes take place, and Standard GTS

(S-GTS), which can be scheduled by the NM for communication between specific nodes, in a TDMA fashion. An inactive period is defined at the end of the active period so that the network nodes and the network manager enter a low-power mode and save energy.

In the current implementation, we allow for the different channel access mechanisms to be used. Particularly, nodes which lose a tournament may be allowed to transmit their information in a best-effort manner during the CAP at the current superframe, or may be also scheduled for transmission by the NM in S-GTSs in the following superframe CFP. Such mechanisms are application specific and are left to be defined by the user.

A. Tournament access mechanism implementation

The implementation was performed for the Telos wireless platform [31]. These nodes are equipped with a Texas Instruments MSP430 16-bit, 8 Mhz microcontroller with 48 kB of Flash and 10 kB of RAM memory, 250 kbps 2.4GHz Chipcon CC2420 IEEE $802.15.4$ compliant radio and on-board sensors. The operating system used is TinyOS [32]. The implementation of the protocol is based on the IEEE $802.15.4$ MAC TinyOS implementation [33]. The code for these experiments is available for download at [34]. The implementation of the complete protocol requires 30 kB of memory.

The tournament access mechanism is implemented according to the tournament defined in Section III-B. Each sensor node computes its Attention Factor according to (12) (decimal number), converting it afterwards to a binary sequence. The implementation is then carried out in the following manner. For a value of '1', an unmodulated carrier pulse is transmitted by the CC2420 radio. When a recessive bit (value '0') is present, the node refrains from transmitting an unmodulated carrier pulse, and detects unmodulated carrier pulses transmitted by other devices. This action is performed by the Clear Channel Accessment (CCA) mechanism of the CC2420 radio. When the CCA is issued, the average Received Signal Strength Indicator (RSSI) value is measured for a duration of $128\mu s$. Then, this value is compared to a pre-defined threshold in order to decide if the radio channel is busy or idle.

B. Tournament access mechanism validation

Several tests have been performed to validate the proposed tournament access mechanism. The results are summarized in Table I. The tests were performed indoors. In each test a NM is

TABLE I

TOURNAMENT ACCESS MECHANISM VALIDATION

Number of nodes	topology	distance to coordinator (meters)	false positive %
4	circle	5	0
4	circle	2.5	0
2	line	15	0.7

deployed in addition to the number of nodes specified in the table.

In the circle topology, all nodes were placed in a room at the same distance from the NM and spaced 90 degrees apart. In the line topology, the nodes were placed in a long corridor stretch, with the NM node in the middle. The devices priority was fixed through all tournaments and set to $\{160, 72, 37, 32\}$ for the 4 node case, and $\{160, 72\}$ in the 2 node case. A total number of 60000 tournaments was performed for each setup. As the results show, the tournament mechanism has no errors for short distances. However, a small percentage of false positives is verified for the case of a distance of 30 meters between the nodes.

V. Results

We provide two types of experimental results in this section. We first begin by validating our analysis in Section III with simulation results. Next, we present experimental results obtained from testing our protocol on state-of-the-art low power wireless devices.

A. Verification of Analytical Results using Simulations

We now present the results obtained from a Monte Carlo simulation, and use these to validate the analysis in Section III. For this experiment, we simulated a network of $M = 20$ control systems that use attention-based tournaments in Matlab. We considered the process to be controlled as a first order plant with a scalar measurement. We chose the parameters in (1) to be $A = 1$, $B = 1$, $C = 1$, with all variances set to one. Additionally, we used $Q_1 = 1$ and $Q_2 = 1$ in the infinite horizon LQG cost in (9). Each sensor in the network generated a packet to transmit, and these packets vied for $N_{\text{T}} = 10$ tournament slots. The maximum value of the Attention Factor $A_{\text{max}} = 256$, which is sufficiently large to prevent frequent collisions while achieving sufficient throughput. The parameter $\kappa = 2.25$ is selected to ensure Property 3.1, as shown in Fig. 4.

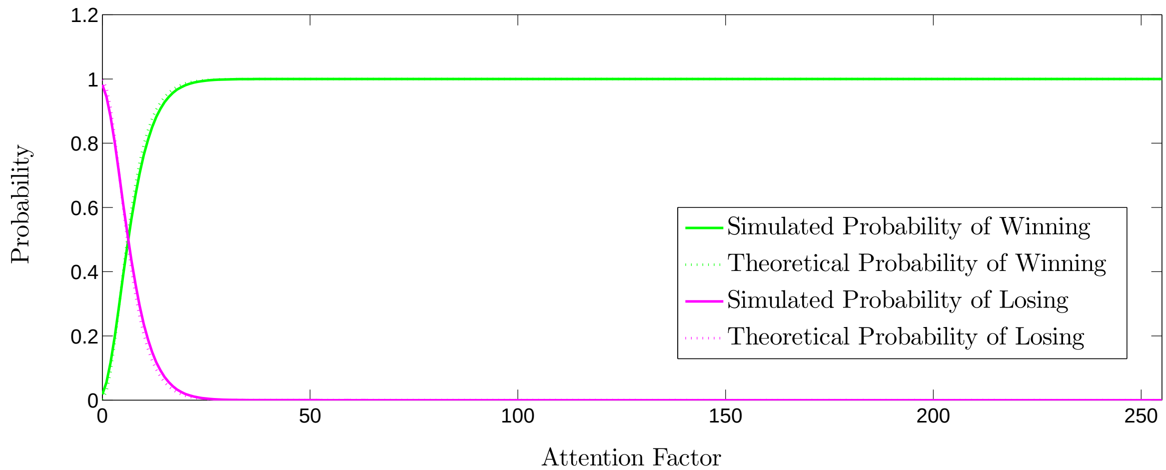


Fig. 6. The conditional probabilities of winning and losing in the tournament access mechanism are plotted above. Note that the simulated probabilities of these complimentary events match the analytical values. The probability of winning is a non-decreasing function of the Attention Factor, as shown in Lemma 3.4.

Recall that lower values of κ result in a peak at the higher end of the probability distribution of the Attention Factor, while higher values of κ under utilize the range of the Attention Factor.

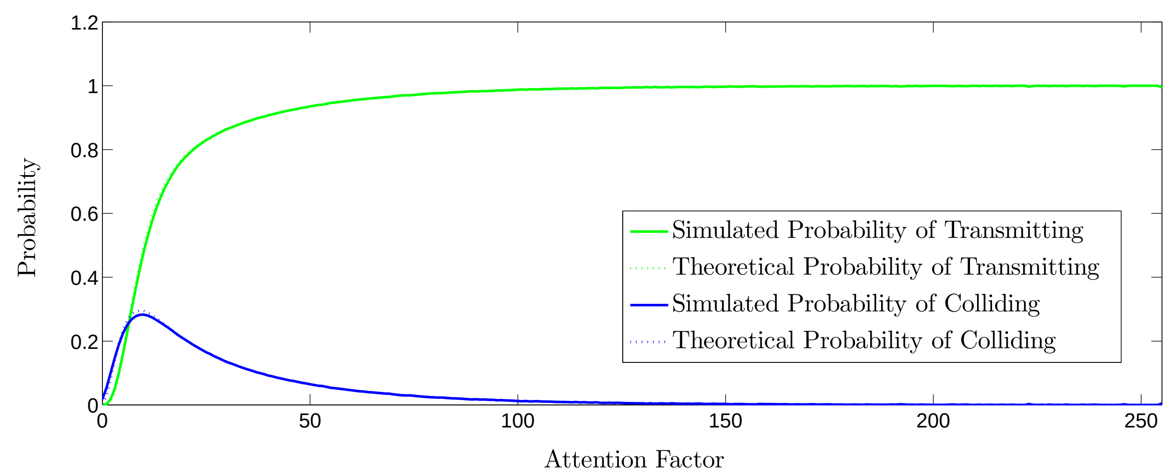


Fig. 7. The conditional probabilities of transmitting successfully and colliding in the tournament access mechanism are plotted above. Note that the simulated probabilities match the analytical values. The probability of transmitting is a non-decreasing function of the Attention Factor, as shown in Lemma 3.5.

The simulated results matched the analysis closely, as shown in Figs. 6 and 7. Thus, neglecting terms that accounted for more than two collisions in (19) and (20) is justifiable when the probability of a collision is small. The conditional probabilities of winning and transmitting successfully are almost 1 for packets with high attentions. Furthermore, these plots show the non-decreasing nature of the conditional probabilities of winning and transmitting successfully, verifying the results of Lemmas 3.4 and 3.5, respectively. The peak in the conditional probability of collision (Fig. 7) can be explained from the PMF of the Attention Factor, which indicates that there are few packets with large Attention Factors. These are most likely to win the tournament in the first few slots and transmit without collision. Hence, the curve falls to nearly 0 for high values of α. Packets with lower values of Attention Factor mostly win the tournament in the last few slots, and since there are many such packets, collisions are very likely. Finally, the packets with very low values of attention do not win the tournament often, and hence the probability of collision is low for these values.

In Table II, we present a comparison of the estimation and control costs, J_E and J_C, for three setups. The entries in the first column, titled Tournament Access, correspond to values obtained from our Monte Carlo simulations of the network of systems described in this section using the tournament access mechanism along with Attention Factors for priorities. The average probability of transmission is found to be $p_{\mathfrak{T}} = 0.4403$ from the simulations. The entries in the second column, denoted Packet Loss, corresponds to the same network of systems using a multiple access mechanism or a physical medium that suffers from a Bernoulli packet loss process, with a probability of loss equal to $1 - p_{\mathfrak{T}}$. The third column corresponds to the network of systems and a packet loss scenario termed as 'ideal' because the probability of transmission is equal to the ratio of the available number of slots to the number of processes in the network, $N_{\mathrm{T}}/M = 0.5$. The entries in the packet loss columns are calculated using the expressions for the average estimation error variance in (26) and for the LQG cost from [35]. The control cost expression uses the estimation error variance found in (26). The values from the tournament access and packet loss columns in this table confirm the results from Theorem 3.7, showing that a well designed tournament access mechanism outperforms an agnostic packet loss mechanism that results in the same average probability of transmission. Furthermore, the values from the ideal packet loss column indicate that even a throughput achieving access mechanism cannot outperform the tournament access mechanism, despite the collisions in the latter mechanism.

TABLE II

A COMPARISON OF ESTIMATION AND CONTROL COSTS

	Tournament Access	Packet Loss	Ideal Packet Loss
Average Transmission Probability	$p_{\mathfrak{T}} = 0.4403$		$p_{\mathfrak{T}} = 0.5$
Average Estimation Cost $\mathbf{E}[P^c_{k\|k}]$	0.9765	1.8894	1.618
Average Control Cost J	0.2576	0.3524	0.3252

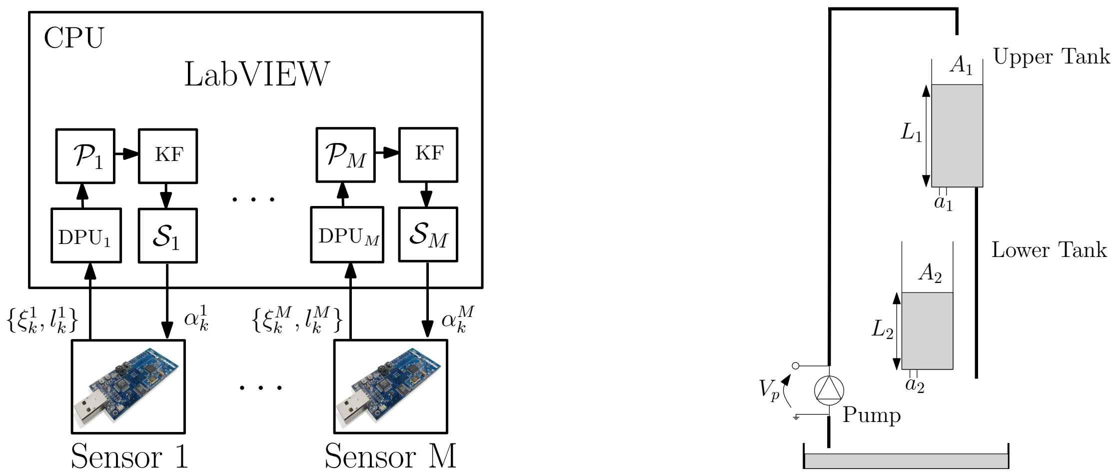
B. Experimental Results

We performed hardware-in-the-loop simulations of a network of wireless control systems, to provide a proof of concept and an evaluation of the protocol on current technology. The control system was simulated in LabVIEW, while the tournaments were executed on wireless sensor nodes. Using this setup, we performed optimal reference tracking of the level of water in two simulated double tank processes over a shared wireless network. We begin with a description of this setup, and then present the results obtained from our experiments.

Experimental Setup: The experimental setup is depicted in Fig. 8(a). The control systems, each comprising of a double tank process, sensor, controller and actuator, were simulated in a MATLAB environment within LabVIEW. Corresponding to each control system, a wireless sensor node, connected to the computer using the serial port, implemented the tournament access protocol. The Attention Factors were generated by the simulated control systems and sent to the sensor nodes. Then, the nodes conducted N_{T} tournaments, and returned the tournament outcomes, along with the tournament slot numbers when successful, to the computer. After discarding any tournament slots with multiple winners, the control systems closed the loop for the successful nodes in the simulation environment.

Protocol Parameters: The data packets from these systems vie for $N_{\mathrm{T}} = 1$ tournament slot, with an 8-bit priority field, thus resulting in $A_{\max} = 256$. We found that $\kappa = 7.5$ ensures a good design. We selected the beacon interval BI as $937.5\,\mathrm{ms}$ with a slot duration of $20\,\mathrm{ms}$, and the tournament slot was of duration $11.8\,\mathrm{ms}$. The sampling period h of the sensor node was set to the value of the BI, i.e., $h = 937.5\,\mathrm{ms}$, to ensure that the node attempts to transmit once per superframe.

Double Tank Process: The double-tank process [36] consists of a water basin and two tanks of uniform cross sections, along with a pump (the actuator) and pressure sensors, as depicted in



(a) Hardware-in-the-loop experimental setup

(b) The double tank process

Fig. 8. The double tank process, sensor, controller and actuator of each of the M control systems are simulated in LabVIEW. The tournaments are conducted in M wireless nodes that communicate with LabVIEW. The sensors send the Attention Factor (α_k^j) to their corresponding wireless node, and the wireless nodes conduct tournaments. They report whether they have won ($\xi_k^j \in \{0, 1\}$), and if so, the tournament slot number that they have won ($l_k^j \in \{1, \ldots, N_\mathrm{T}\}$).

Fig. 8(b). Water is pumped from the basin to the upper tank. Outlets in both tanks allow water to flow from the upper to the lower tank, and back to the basin. Pressure sensors placed under each tank provide measurements of the water levels in the tanks. The double-tank system is modelled as a nonlinear system in continuous time, for which we find a discretized, linearized model, with parameters

$$A = \begin{bmatrix} 0.9200 & 0 \\ 0.0775 & 0.9409 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.2734 \\ 0.0113 \end{bmatrix}, \qquad R_w = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R_v = R_w\ , \qquad R_0 = R_w\ .$$

Optimal Controller Design: The control objective is to optimally track a constant reference ($r = 50$) for the water level in the lower tank by adjusting the pump voltage accordingly. To perform reference tracking for the lower tank level, we augment the state with the integral of the reference error, given by $i_{k+1} = i_k + h\left(r - \begin{bmatrix} 0 & 1 \end{bmatrix} y_k\right)$. However, the resulting augmented system has correlated process and measurement noise. This implies that certainty equivalence will no longer hold for an LQG cost, as explained in [37]. However, separation still holds, and

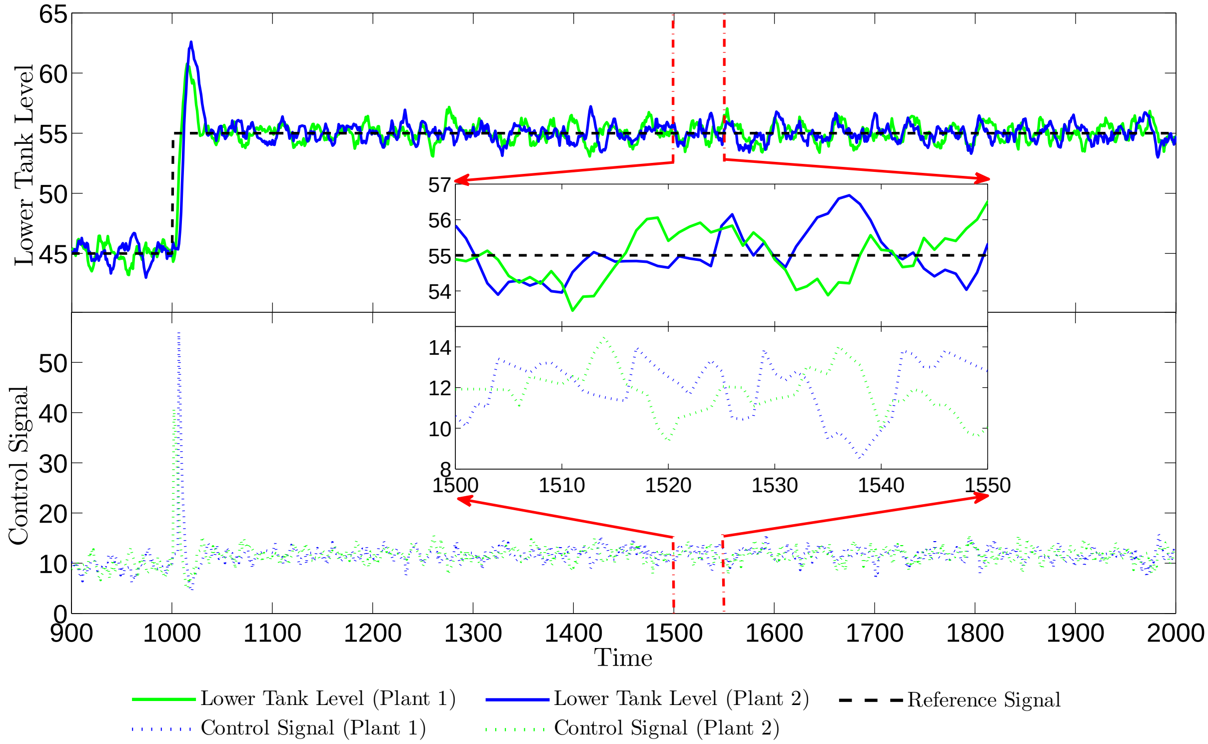


Fig. 9. A trace of the lower tank levels and controls, for both control systems, as obtained from the hardware-in-the-loop experiment. Notice that the control signal does not always correspond to the state, due to intermittent transmissions. Also, the reference tracking is successful for both stochastic systems, despite the two systems sharing a single slot for transmission.

thus, a sufficient statistic comprising of the filtered estimate and the innovations process, can be used to define an optimal control policy [37]. In the context of tournaments, the transmitted packet now contains the innovations process in addition to the filtered estimate. The cost function penalizes divergence from the steady state mean values for the state and control, with weighting matrices given by $Q_1 = C^T I C$ and $Q_2 = I$.

Experimental Results: We carried out more than $60,000$ tournaments on this setup and found that the average probability of transmission for each node was 0.4625, matching our analytical predictions quite well. Furthermore, around 1.12% of the tournaments resulted in errors due to execution on wireless motes. Despite this, the reference tracking was successful, as depicted in Figure 9. A trace of the lower tank levels and control signals of both plants are shown in this figure. This trace indicates the intermittent nature of feedback control that is obtained using attention-based tournaments. The controls follow the state only when there is a transmission.

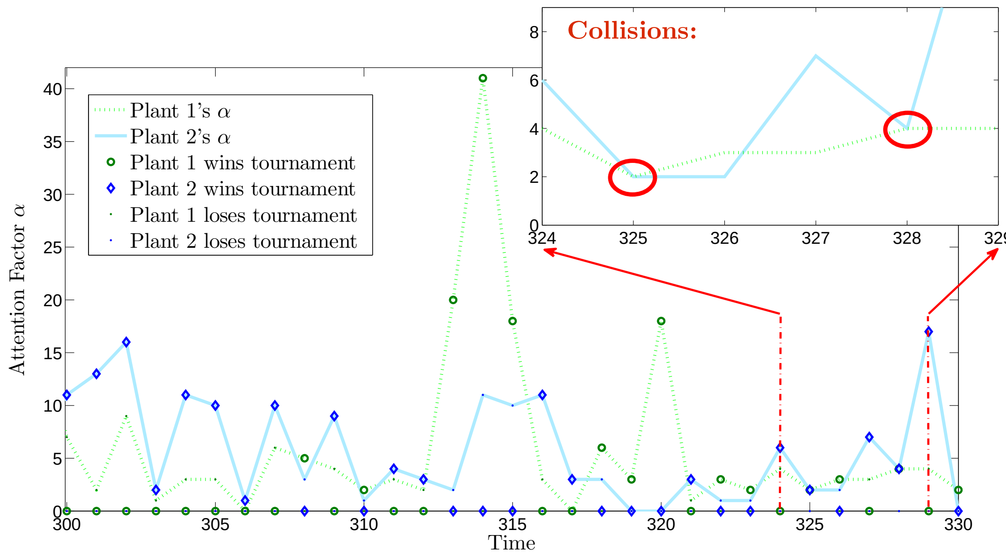


Fig. 10. A trace of the Attention Factors α_k^j and the corresponding wins and losses in the tournaments, for both control systems in our hardware-in-the-loop experiment. The Attention Factors are obtained by quantizing a distortion-like function of the innovations (12). Note that two collisions are imminent from this trace.

When there is no transmission, the control signals are generated in an open-loop manner. The performance of the tournaments on the sensor nodes can be validated from Figure 10. Here, we depict a trace of the Attention Factors of both nodes and the corresponding tournament outcomes. Notice that the highest Attention Factor always wins, as desired in tournaments. Also, this trace shows two time instants when both Attention Factors were equal. This result in both nodes winning the tournament, but none of them transmitting because these data packets result in collisions.

Accounting for Packet Losses: Tournaments conducted on hardware sometimes result in unexpected outcomes. A node that loses a tournament might still transmit because it may not have heard the winning node due to a poor channel. Similarly, a node with the highest Attention Factor may not realize that it has won a tournament due to communication noise. These anomalies are unavoidable in a wireless network. Thus, we incorporate these errors into our analysis of the protocol in (27) by modelling them as packet losses from a Bernoulli process. A comparison of the analytical and simulated values of the probability of transmission, with packet losses is depicted in Figure 11. The close match we obtain validates our analysis. We also compare these

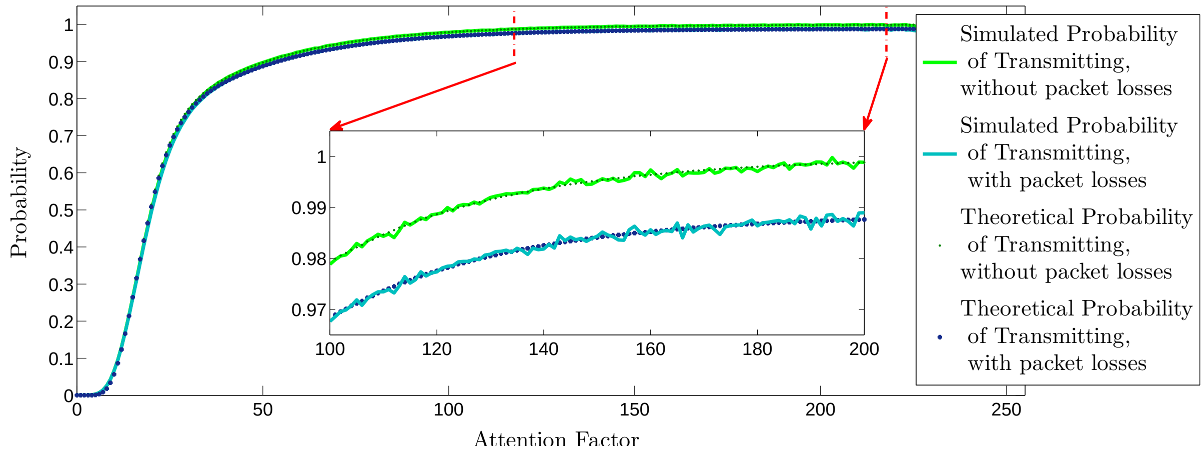


Fig. 11. An analysis of our hardware-in-the-loop experiments tells us that 1.12% of the tournament slots were lost due to errors in the execution of the tournaments on a wireless network. By modelling these errors as a Bernoulli process with loss probability 0.0112, we can include these real world errors in our analysis. A comparison of the theoretical and simulated values of the probability of transmission, with and without packet losses is depicted in this plot, showing that the performance of tournaments is not significantly affected by execution errors.

curves to the probability of transmission curves we would have obtained, were there no packet losses. Clearly, the difference is quite small, thus proving that tournaments with Attention Factors for priorities are robust to losses in the wireless medium.

VI. Conclusions

We design and analyze a distributed prioritized access mechanism using tournaments and state-based priorities called Attention Factors. There are two main merits of the tournament access mechanism. First, the priorities are assigned and evaluated in a distributed manner, thus rendering it suitable for wireless networks. Second, the state-based priorities result in better estimation and control performance than agnostic access methods, as shown by our results. A major concern in such a study is often the implementability of the proposed protocol. We implement tournaments on wireless nodes and verify our analytical results with experiments using these nodes. In conclusion, the proposed modification to IEEE $802.15.4$ introduces priorities to a wireless network protocol, making it suitable for estimation and control of physical systems over wireless networks.

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APPENDIX

PROOF OF LEMMA 3.4

Proof: When the probability of collision is reduced to zero, the first term in (19) gives us an exact expression for the conditional probability of winning a tournament. We show the result using properties of the binomial terms in the sum. First, note that $p_{\mathrm{LE}}(\alpha) \geq p_{\mathrm{LE}}(\alpha - 1)$. Then, to show that the conditional probability of winning a tournament is a non-decreasing function of α, it is sufficient to show that

$$\frac{d}{d\Delta}\left(\sum_{n=0}^{N_{\mathrm{T}}-1} C_n^{M-1}(p_g - \Delta)^n (p_{le} + \Delta)^{M-1-n}\right) > 0 \ ,$$

where $p_g = p_{\mathrm{G}}(\alpha)$ and $p_{le} = p_{\mathrm{LE}}(\alpha)$. If the above inequality can be shown to hold for a continuous variable, such as Δ, then the non-decreasing nature of the conditional probability of winning will also hold for discontinuous jumps in p_{LE} at Attention Factors lesser than α.

The above derivative can be rewritten as

$$\sum_{n=0}^{N_{\mathrm{T}}-1} C_n^{M-1}\Big(- n(p_g - \Delta)^{n-1}(p_{le} + \Delta)^{M-1-n} + (M-1-n)(p_g - \Delta)^n (p_{le} + \Delta)^{M-2-n}\Big)$$

$$= \sum_{n=0}^{N_{\mathrm{T}}-2} (p_g - \Delta)^n (p_{le} + \Delta)^{M-2-n}\Big((M-1-n)C_n^{M-1} - (n+1)C_{n+1}^{M-1}\Big)$$

$$+ C_{N_{\mathrm{T}}-1}^{M-1}(p_g - \Delta)^{N_{\mathrm{T}}-1}(p_{le} + \Delta)^{M-1-N_{\mathrm{T}}} \ ,$$

where the second equation is obtained by collecting similar terms from the first one. Now, notice that $(M-1-n)C_n^{M-1} = (n+1)C_{n+1}^{M-1} = (M-1)!/((M-n-2)! \cdot n!)$. Thus, the derivative simplifies to the last term of the second equation above, which is always positive. This proves that for $N_{\mathrm{T}} < M-1$, the conditional probability of winning a tournament is always a non-decreasing function of α. ■

Tournaments with Collisions: For the purpose of this proof, we consider only the first term in (19). The contribution of the second term is negligible due to the required number of collisions. However, using a similar argument, the proof can be extended to show that the above result holds even when the second term is taken into account by requiring the probability of the Attention Factor to fall sufficiently fast, which is satisfied for PMFs obtained from multivariate Gamma-type distributions or Chi-squared distributions, such as the one in (15).

PROOF OF LEMMA 3.6

Proof: The a priori variance of $\bar{e}_k$ is given by $\sigma^2_{\bar{e},k} = \text{tr}\{AK_{f,k}R_{e,k}K^T_{f,k}A^T\}$, where $R_{e,k}$ is given in (4). We denote the variance of the posterior distribution in (22), as $\sigma^2_{DPU,k}$. We can find expressions for both the variances as

$$\sigma^2_{\bar{e},k} = \sigma^2_{\bar{e},k,1} + \cdots + \sigma^2_{\bar{e},k,A_{\max}} \; ,$$

$$\sigma^2_{DPU,k} = \sigma^2_{\bar{e},k,1}\frac{(1-\mathbf{P}(T_{N_{\text{T}},M-1}|\alpha_k = 0))}{1-p_{\mathfrak{T}}} + \cdots + \sigma^2_{\bar{e},k,A_{\max}}\frac{(1-\mathbf{P}(T_{N_{\text{T}},M-1}|\alpha_k = A_{\max}-1))}{1-p_{\mathfrak{T}}} \; ,$$

where $\sigma^2_{\bar{e},k,a} = \int_{\Delta_{a-1}\leq|\bar{e}|<\Delta_a} |\bar{e}|^2\psi(\bar{e})d\bar{e}$, for $1 \leq a \leq A_{\max}$. The thresholds of the symmetric scheduling policy are denoted $\{\Delta\}_0^{A_{\max}}$, where $\Delta_0 = 0$ and $\Delta_{A_{\max}} = \infty$. Now, the variance of the posterior distribution can be rewritten as

$$\begin{aligned} \sigma^2_{DPU,k} &= \sigma^2_{\bar{e},k} + \sigma^2_{\bar{e},k,1}\rho(0) + \cdots + \sigma^2_{\bar{e},k,A_{\max}}\rho(A_{\max}-1) \\ &\leq \sigma^2_{e,k} + \max\left(\sigma^2_{e,k,1}\rho(0) + \cdots + \sigma^2_{e,k,A_{\max}}\rho(A_{\max}-1)\right) \; , \end{aligned}$$

where $\rho(a) = \frac{(1-\mathbf{P}(T_{N_{\text{T}},M-1}|\alpha_k=a))-(1-p_{\mathfrak{T}})}{1-p_{\mathfrak{T}}}$. The maximum value of the latter terms can be found by evaluating the integrals at their upper boundaries, such as $\max\sigma^2_{\bar{e},k,a} = \Delta^2_a\mathbf{P}(\alpha_k = a-1)$. However, not all of these terms are positive. This can be seen from Lemma 3.5, as $(1-\mathbf{P}(T_{N_{\text{T}},M-1}|\alpha_k = a-1)) > (1-p_{\mathfrak{T}})$ for small Attention Factors and vice versa. Thus, there is a value $\bar{a} \in \{0,\ldots,A_{\max}-1\}$, such that $\rho(a)$ for $a \leq \bar{a}$ are positive or zero and $\rho(a)$ for $a > \bar{a}$ are negative. The maximum value of the negative terms are found by evaluating the integrals at their lower boundaries. Doing so, we obtain

$$\begin{aligned} \sigma^2_{DPU,k} \leq \sigma^2_{\bar{e},k} + \Big(&\Delta^2_1\mathbf{P}(\alpha_k = 0)\rho(0) + \cdots + \Delta^2_{\bar{a}}\mathbf{P}(\alpha_k = \bar{a}-1)\rho(\bar{a}-1) \\ &- \Delta^2_{\bar{a}}\mathbf{P}(\alpha_k = \bar{a})\rho(\bar{a}) - \cdots - \Delta^2_{A_{\max}-1}\mathbf{P}(\alpha_k = A_{\max}-1)\rho(A_{\max}-1)\Big) \; . \end{aligned}$$

Now, the increasing order of the thresholds $\Delta_1 \leq \cdots \leq \Delta_{A_{\max}-1}$ implies that we can upper bound the terms in parenthesis in the above expression as

$$\begin{aligned} \sigma^2_{DPU,k} &\leq \sigma^2_{\bar{e},k} + \Delta^2_{\bar{a}}\Big(\mathbf{P}(\alpha_k = 0)\rho(0) + \cdots + \mathbf{P}(\alpha_k = \bar{a}-1)\rho(\bar{a}-1) \\ &\qquad - \mathbf{P}(\alpha_k = \bar{a})\rho(\bar{a}) - \cdots - \mathbf{P}(\alpha_k = A_{\max}-1)\rho(A_{\max}-1)\Big) \\ &= \sigma^2_{\bar{e},k} + 0 \; , \end{aligned}$$

where it is easy to check that the terms in the inner bracket sum to zero. From this inequality, it is simple to deduce that the posterior variance of the innovations is also less than its a priori value $\sigma_{e,k}^2 = \mathrm{tr}\{R_{e,k}\}$. Thus, the non-decreasing probability of transmission leads to a lower variance than would have resulted otherwise. ■