Numerical Investigation of the Sensitivity of Forced Response Characteristics of Bladed Disks to Mistuning

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ABSTRACT

Two state of the art finite element reduction techniques previously validated against the direct finite element method, one based on classical modal analysis and another based on component mode synthesis, are applied for efficient mistuned free vibration and forced response analysis of several bladed disk geometries. The methods are first applied to two test cases in order to demonstrate the differences in computational efficiency as well as to validate the methods against experimental data. As previous studies have indicated, no noticeable differences in accuracy are detected for the current applications, while the method based on classical modal analysis is significantly more efficient. Experimental data (mistuned frequencies and mode shapes) available for one of the two test cases are compared with numerical predictions, and a good match is obtained, which adds to the previous validation of the methods (against the direct finite element method).

The influence of blade-to-blade coupling and rotation speed on the sensitivity of bladed disks to mistuning is then studied. A transonic fan is considered with part span shrouds and without shrouds, respectively, constituting a high and a low blade-to-blade coupling case. For both cases, computations are performed at rest as well as at various rotation speeds. Mistuning sensitivity is modelled as the dependence of amplitude magnification on the standard deviation of blade stiffnesses. The finite element reduction technique based on classical modal analysis is employed for the structural analysis. This reduced order model is solved for sets of random blade stiffnesses with various standard deviations, i.e. Monte Carlo simulations. In order to reduce the sample size, the statistical data is fitted to a Weibull (type III) parameter model. Three different parameter estimation techniques are applied and compared. The key role of blade-to-blade coupling, as well as the ratio of mistuning to coupling, is demonstrated for the two cases. It is observed that mistuning sensitivity varies significantly with rotation speed for both fans due to an associated variation in blade-to-blade coupling strength. Focusing on the effect of one specific engine order on the mistuned response of the first bending modes, it is observed that the mistuning sensitivity behaviour of the fan without shrouds is unaffected by rotation at its resonant condition, due to insignificant changes in coupling strength at this speed. The fan with shrouds, on the other hand, shows a significantly different behaviour at rest and resonant speed, due to increased coupling under rotation. Comparing the two cases at resonant rotor speeds, the fan without shrouds is less or equally sensitive to mistuning than the fan with shrouds in the entire range of mistuning strengths considered.

This thesis' scientific contribution centres on the mistuning sensitivity study, where the effects of shrouds and rotation speed are quantified for realistic bladed disk geometries. However, also the validation of two finite element reduction techniques against experimental measurements constitutes an important contribution.

PREFACE

This thesis is mainly based on the following publications:

Enclosed in the appendix:

- I. Myhre, M.; Moyroud, F. M.; Fransson, T. H.; 2003, "Numerical investigation of the sensitivity of forced response characteristics of bladed disks to mistuning", Proceedings of ASME TURBO EXPO, GT2003-38007.

Not enclosed in the appendix:

- II. Myhre, M.; 2002, "Bladed disk mistuning – a literature survey", Technical Report KTH/HPT 02/06, Chair of Heat and Power Technology, Royal Institute of Technology, Stockholm, Sweden, 2002.

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NOMENCLATURE

a	Weibull (Type III) location parameter	
b	Weibull (Type III) scale parameter	
c	Weibull (Type III) shape parameter	
c	Damping	[Ns/m]
d	Mistuning parameter	
d	Vector of mistuning parameters	
D	Number of degrees of freedom	
E	Modulus of Elasticity	[N/m ²]
f	Frequency	[Hz]
F	Force	[N]
h	harmonic of mistuning pattern	
i	Imaginary unit	
k	Stiffness	[N/m]
K	Elemental stiffness matrix	
m	Mass	[kg]
n	Wave number / number of nodal diameters	
N	Number of blades	
P	Probability operator	
Q	Objective function	
r	Engine order	
s	Level of localization	
t	Time	[s]
u	Amplitude	[m]
u	Vector of complex amplitude components	[m]
V	Volume	[m ³]
X	Vector of unknown mass and stiffness deviations	

Greek letters

α	Integer constant	
β	Interblade phase angle	[rad]
ρ	Density	[kg/m ³]
ε	Stiffness mistuning parameter	
θ	Circumferential (bladed disk) angle	[rad]
ν	Poisson's ratio	
ω	Angular frequency	[rad/s]
Ω	Rotation speed	[rad/s]
ξ	Modal damping factor	

Subscripts

0	Reference sector
c	Coupling
clamp	Clamped
cub	Cube
cyl	Cylinder
exp	Experimental

j	General counting index
n	Blade index
num	Numerical

Superscripts

max	Maximum value during a period of vibration
mis	Mistuned
T	Transpose
tun	Tuned
*	Hermitian conjugate

Abbreviations

ANN	Artificial Neural Networks
B	Bending mode
CBSR	Craig and Bampton substructuring and reduction method
CC	Characteristic Constraint
CCM	Characteristic Constraint modes
CFP	Closed Form Perturbation
CMS	Component Mode Synthesis
DOF	Degrees of Freedom
E	Edge mode
EO	Engine Order
F	Flexing mode
FE	Finite Element
FEM	Finite Element Method
FMM	Fundamental Mistuning Model
FRF	Frequency Response Function
GFLOP	Giga Float Operations Per Second
HCF	High Cycle Fatigue
HP	High Pressure
INSAL	Institut National des Sciences Appliquées, Lyon, France (Laboratoire de Mécanique des Structures)
IRMS	Improved Random Modal Stiffness
LDV	Laser Doppler Vibratometer
LMT	Limit distribution / three parameter model
LP	Low Pressure
LS	Least Squares (Linear Regression)
MAC	Modal Assurance Criterion
ME	Moment Estimation
ML	Maximum Likelihood
MLE	Maximum Likelihood Estimation
MR	Modal decomposition / reduction method
ND	Nodal Diameter
NN	Neural Networks
NS	No (without) Shrouds
NSC	National Supercomputer Center
ODS	Operating Deflection Shape
P	Plate mode
PS	Part-span Shrouds

RMS	Random Modal Stiffness
RMS	Root Mean Square
ROM	Reduced Order Model
rpm	Rotations Per Minute
SBM	Single Blade Mistuning
SMART	Secondary Modal Analysis Reduction Technique
SNAC	Swedish National Allocations Committee
SNM	System Nominal Modes
STD	Standard Deviation
T	Torsion mode

1 INTRODUCTION

1.1 General background

Gas turbine technology

Gas turbine technology is widely used in aircraft-, marine- and space-propulsion engines, as well as for stationary energy conversion in power plants. Due to the vast extent of gas turbine applications world wide, major research efforts have been devoted to improvements of durability, efficiency and production costs. Compromises between these requirements have to be made, demanding extensive knowledge on the engine operation. Particularly in the propulsion sector, requirements of low weight-to-thrust ratios as well as safety and reliability have driven engine designs to its edge. Figure 1.1 shows an example of an aircraft engine.

Figure 1.1: EJ200 turbofan engine from the Eurojet consortium (Rolls-Royce, FiatAvio, ITP and MTU) powering the Eurofighter Typhoon. “LP” and “HP” denote “Low Pressure” and “High Pressure”, respectively. Courtesy Rolls-Royce, plc.

Vibration problems in gas turbine engines

Much attention has been devoted to the problem of vibrations of turbomachinery blades, which may lead to High Cycle Fatigue (HCF) failure. Such failures are sudden events which cannot be readily detected and monitored and therefore represent a threat to the engine safety and reliability. Turbomachinery vibrations are commonly divided in self-excited vibrations and forced vibrations.

Self-excited vibrations

Vibration motion of a blade set off by small aerodynamic or mechanical disturbances causes an unsteady pressure field on the blade surface. This pressure field may damp the blade such that the vibrations vanish, or it may excite the blade such that the vibrations escalate rapidly. The latter course of event is commonly referred to as self-excited vibrations, aeroelastic instability or flutter. Turbomachinery flutter is to a large extent governed by the flow velocity, as well as the natural frequencies and mode shapes of the rotor. If the flow velocity exceeds a critical value, drastic increases in vibration amplitudes may occur.

Forced vibrations

Unsteady pressure fields on blade surfaces may also arise from external sources. Such blade excitations may result in forced- or resonant- vibrations. The most common forced vibration problems are caused by the relative motion between rotating and non-rotating parts, i.e. flow phenomena where the excitation frequency is synchronized with the rotation speed of the rotor (often referred to as synchronized vibration). Such vibrations may arise due to inlet distortions originating at the air intake (e.g. inlet struts), blade row interactions and flow distortions due to the burners. Non-synchronized forced vibrations may also occur in an engine due to flow phenomena, which are not synchronized with the rotor speed, such as rotating stall, vortex shedding etc. However, only synchronized vibrations are discussed in this thesis.

Resonant vibrations occur if the frequency and shape of the unsteady pressure field coincides with the natural frequency and shape of a structural mode of the turbomachinery rotor. Resonant vibration regions are typically illustrated by means of a Campbell diagram, showing the variation of excitation- and natural- frequencies with the rotor speed. Figure 1.2 shows an example Campbell diagram. Resonant conditions are indicated as circles.

Mistuning

In order to prevent HCF, the high performance bladed-disks used in today's turbomachines must meet strict standards in terms of aeroelastic stability and resonant response levels. One structural characteristic that can significantly impact on both these areas is that of mechanical mistuning. Such mistuning of turbomachinery rotors or bladed disks may be defined as random blade-to-blade geometric and structural variations that may occur during the manufacturing process and as a consequence of in-service wear. However, blades may also be mounted in a particular order based on their individual properties, in order to obtain certain effects. This is commonly referred to as intentional mistuning or detuning.

Mistuning results in blade-to-blade variations in natural frequency and mode shape. This in turn affects the free vibration and forced response of the bladed disk assemblies. It has been shown that while mistuning has a beneficial effect on flutter, it may have an undesirable effect on the forced response through an increase in the maximum amplitude experienced by some blades. Thus, the ability for the bladed disk designer to accurately predict and understand the effects of mistuning on forced response levels is crucial.

Figure 1.2: Campbell diagram for a first stage turbine rotor with stiff wheel. Adapted from Jay and Fleeter (1987).

1.2 Problem Formulation

In order to incorporate vibration risk assessment of turbomachinery blades in an early design phase, the designer needs numerical tools to enable reliable predictions at a satisfactory computational cost. Structural analysis of industrial bladed disks is typically (or exclusively) performed using the finite element method (FEM). Such analysis of mistuned bladed disks is computationally expensive, as the entire bladed disk must be modelled. “Cyclic symmetry”, which is efficiently exploited in tuned bladed disk analysis, cannot be used since the blades are not identical. Further, due to the statistical nature of mistuning, the treatment of a large number of mistuning configurations may be necessary in order to obtain statistical data. These aspects have motivated researchers to develop “reduced order models” in order to reduce the computational costs while maintaining a high accuracy. Although many such models have been validated against direct FEM, they need to be validated also against experimental measurements, in order to verify that they capture the essential physical phenomena.

Although the above-mentioned aspects regarding reduced order models are treated in the current work, the main focus is put on the underlying physical mechanisms governing the adverse effects of mistuning. Numerical

(parameter) studies of the effects of mistuning have often been performed with discrete mass-spring-damper models with a few DOF per sector. These simple models have the advantage to make the problem computationally tractable while capturing the essential features. Thus, they are well suited for simple parameter studies. However, it can be difficult to correlate the results obtained for such simple models with actual bladed disks. Thus, it is necessary to focus on more realistic bladed disk geometries.

1.3 Structure of the present work

Chapter 2 provides a review of basic bladed disk terminology and tuned vibration characteristics. Chapter 3 aims to present the current state of the art in the field of mistuned blade disk assemblies. Chapter 4 states the objectives of the present work and the approach to achieve them. Chapter 5 gives a description of the numerical methods applied in this work. The results of the present work are provided in chapter 6, 7 and 8. Chapter 6 and 7 present a comparison of two numerical methods for mistuning analysis (Comparisons with experimental measurements are included in the latter). Chapter 8 constitutes the main part of the work, namely a mistuning sensitivity study. Finally, chapter 9 and 10 give conclusions and aspects regarding future work, respectively.

2 TERMINOLOGY AND VIBRATION CHARACTERISTICS OF TUNED BLADED DISKS

2.1 Basic terminology

Hardware and geometry

A *bladed disk* or a *bladed disk assembly* consists, as the name implies, of a number of blades mounted on a disk. If the bladed disk can be divided in a number of identical sectors with identical inter-connections, it is said to be *cyclic symmetric*. The number of such cyclic sectors is typically equal to the number of blades on the disk. However, each sector may also consist of several blades, e.g. if the blades are mounted on the disk in packets.

Bladed disks are sometimes designed with *shrouds*, i.e. the blades are interconnected at part or full span. Shrouds may be continuous or non-continuous, i.e. divided between each blade. See e.g. Figure 8.1 as an example of a cyclic symmetric bladed disk with part span shrouds.

In bladed disk assemblies, the disk and possible shrouds act as a structural coupling device between the blades. This is commonly referred to as *blade-to-blade coupling*, *inter-blade coupling* or *internal coupling*.

Basic structural dynamics

All structures have a propensity to vibrate (in the absence of any continuing excitation) at certain frequencies - referred to as *natural frequencies* or *eigenfrequencies*. When a structure vibrates at one of its natural frequencies, a “freeze-frame” of the displaced shape is called a *mode shape*. A mode shape may contain one or several *nodal lines*, i.e. lines where the structure is at rest (see Figure 2.3 and Figure 2.4). A set of mode shapes and natural frequencies constitute the *free vibration* characteristics of the structure.

When a structure is excited at one of its natural frequencies, the structure may respond very violently – a phenomenon referred to as *resonance*. Such a resonant *forced response* constitutes an *operating deflection shape* (ODS), which is not necessarily the same as the corresponding mode shape. Note that resonance only occurs if the shape of the exciting force field coincides with the corresponding mode shape to some extent. E.g., if a (point) force is applied at a nodal line of a structure, no response is obtained from the corresponding mode shape.

Damping is the phenomenon whereby energy is dissipated from a structure when it is moving. The effect of damping on resonant response is illustrated in Figure 2.1. In general, when a structure is excited, one or several mode shapes may respond. The forced response of a system is dependent on the system's mode shapes and natural frequencies, the force field working on the system, as well as the level of damping.

Figure 2.1: Principle sketch of the effect of damping on a frequency – response curve.

2.2 Vibration characteristics of tuned bladed disks

The sections below aim to give an overview of the vibration characteristics of bladed disk assemblies and how they are influenced by various features. Parts of the information are taken from an AGARD manual (Platzer and Carta (1988)) and material presented by (Jacquet-Richardet (1997)).

Classification of single-blade modes

The mode shapes of a single cantilevered / clamped blade may be classified according to certain characteristic blade motions. Figure 2.2 illustrates such a classification of the first order modes (1F, 1E, 1T and 1P) for a simple plate geometry clamped at its root.

Figure 2.2: Basic blade mode shapes, represented by displacements. Adapted from Fransson et al (2002).

The flexing mode, F , and the edge mode, E , are both often referred to as *bending* (B) modes. For simple plate models, classification may also be made in terms of the number of nodal lines (lines constituting zero displacement) p transverse to the blade span and the number of nodal lines q longitudinal to the blade span, respectively. Figure 2.3 illustrates such a classification by (q,r) . E.g., the first-, second-, and third- bending modes have 0-, 1-, and 2- nodal lines transverse to the blade span and no nodal lines longitudinal to the blade span, and are represented by $(0,0)$, $(0,1)$ and $(0,2)$ as illustrated in the figure. Note that such a classification is more complicated for real blade geometries in turbomachinery, as the corresponding mode shapes are complex and often occur as combinations of the above-mentioned basic modes.

Figure 2.3: Basic mode shapes, represented by nodal lines. Adapted from Jacquet-Richardet (1997).

Classification of bladed disk modes

Cyclic symmetric bladed disk assemblies exhibit certain well-defined types of vibration modes. These modes may be characterised by the number of equally spaced diametral nodal lines, n , typically referred to as *nodal diameters*. The maximum number of nodal diameters, which can occur, is $N/2$ if N is even and $(N-1)/2$ if N is odd, where N is the number of blades on the disk. In the discussion below, N is assumed to be even for convenience. Further, bladed disk modes may be characterized by the number of circumferential nodal lines, referred to as *nodal circles*.

As the number of blades in a bladed disk increases, the cyclic symmetric structure will be more and more axi-symmetric in nature. For an axi-symmetric structure, the above-mentioned nodal lines will occur as perfectly straight lines and circles (see Figure 2.4 a). This is not the case for cyclic symmetric structures, such as bladed disks (see Figure 2.4 b). Still, the classification according to nodal diameters and nodal circles is often used and is indeed appropriate.

Figure 2.4: Example of mode shapes represented by nodal lines of (a) an axisymmetric structure and (b) a cyclic symmetric structure. Adapted from Jacquet-Richardet (1997).

The mode shapes corresponding to $n = 0$ and $n = N/2$ occur as *single* modes. For all other nodal diameters, however, the mode shapes occur as double modes, i.e. pairs of mode shapes with identical frequency (if gyroscopic effects are neglected, see below). The modes constituting such a mode pair are orthogonal, i.e. they are identical but rotated 90° relative to each other. Thus, they can combine to form a mode with any circumferential orientation.

Double modes may also combine into forward- or backward- *travelling wave modes*, i.e. modes where the nodal diameters are rotating on the bladed disk. In such a case, each cyclic symmetric bladed disk sector features the same modal displacement, but there is a constant phase lag between adjacent

sectors of the assembly. This phase lag is typically referred to as the inter-blade phase angle β , and can take the discrete values

$$\beta = \pm \frac{2 \cdot \pi \cdot n}{N}, n = 0, \dots, N/2 \quad (2-1)$$

Note that the modes corresponding to $n = 0$ and $n = N/2$ actually constitute single standing wave modes, rather than travelling wave modes.

Bladed disk systems tend to have natural frequencies that fall into closely grouped clusters, where each cluster corresponds to a particular mode family, and the closely spaced frequencies correspond to different nodal diameter modes in the bladed disk. For example, the lowest frequency modes often correspond to the 1B mode family. Structures with closely spaced natural frequencies are often said to have high *modal density*.

Forcing fields and modal response

Bladed disks in turbomachinery are usually subjected to excitations of the “engine order” type, i.e. the force amplitudes acting on different blades are identical but there is a fixed phase shift between forces on adjacent blades of the assembly. Thus, for a tuned assembly, all blades will vibrate with the same amplitude.

A forcing field will excite a particular mode shape, only if the forcing field and the mode in question coincides both in frequency and shape. Thus, each n nodal diameter mode shape can only be excited by the following engine orders, r (α is an integer constant):

$$r = \alpha N \pm n \quad (2-2)$$

Damping

There are two main damping mechanisms related to bladed disk vibration, aerodynamic damping and mechanical damping, respectively. Aerodynamic damping is typically treated separately from the structural analysis. Mechanical damping can further be divided in

- Structural damping or material hysteresis
- Dry friction damping

Structural damping is typically quite low compared to friction damping. Dry friction may occur in various joints / connections, such as at the blade roots, under-platform dampers, shroud- and lacing wire- connections or at the upper platform. Figure 2.5 shows a principle sketch of where friction damping may occur on a blade. Figure 2.6 shows a sketch of different under-platform damper designs.

The contact forces in these joints generally increase with rotation speed due to the centrifugal effects. This is also the case for shroud connections, even if

these interfaces may be parallel to the centrifugal force, since the blades will un-twist and the shroud connections will get more and more stuck.

Figure 2.5: Principle sketch of where friction damping may occur on a blade. Adapted from Fransson et al (2002).

Figure 2.6: Principle sketches of different under-platform damper designs. Adapted from Fransson et al (2002).

Structural coupling and modal density

As mentioned above, the disk and possible shrouds introduce structural coupling between the blades. This coupling occurs by means of an interaction between the blade mode shapes and the disk / shroud mode shapes. Coupling typically decreases with increasing number of nodal diameters of the bladed disk modes. Thus, as the number of nodal diameters increases, the bladed disk modes approach single-blade (clamped) modes. Mode shapes with zero or a few nodal diameters are more dominated by the disk motion, and are often referred to as disk modes. Further, the magnitude of such coupling is to a large extent governed by the natural frequencies of the different components.

Note also that a low level of coupling is typically associated with high modal density, i.e. the frequencies of the modes of each mode family are grouped in

isolated clusters. High coupling on the other hand gives more distributed frequencies within each mode family.

Frequency veerings

Two different modes with the same number of nodal diameters, which have close natural frequencies is a phenomenon commonly referred to as a *frequency veering*. This may provide the combination of high coupling strength and high modal density, which is critical to the mistuned response behaviour and will be discussed further in the next chapter. Figure 2.7 shows an example plot of natural frequencies versus nodal diameter. Frequency veerings are indicated by circles.

Figure 2.7: Example of frequency veerings. Adapted from Yang and Griffin (2001).

Effect of various parameters on free and forced vibrations

Effect of disk and shroud stiffness

Coupling typically decreases with increasing disk or shroud stiffness. In the limiting case of a perfectly rigid disk, no coupling occurs and single blade modal analysis may be used.

Effect of shrouds

Shrouds are used in turbomachinery designs in order to provide additional stiffness of the blades, as well as to provide friction damping at the shroud interfaces. The span-wise location of part-span shrouds and the angle of the contact interfaces are often used to control the free vibration characteristics. Further, it is worth mentioning that while the natural frequencies in a mode family increase monotonously with the number of nodal diameters for non-shrouded bladed disks, this is not necessarily the case for shrouded bladed disks.

Effect of rotation speed

During operation, bladed disks in turbomachinery are subjected to high rotation speeds. This has two counteracting effects on the natural frequencies of the assembly. *Stress stiffening*, also called *geometric stiffening* or *initial stress stiffening*, is the stiffening of the structure due to its pre-stressed state caused by centrifugal forces. Stress stiffening is thus related to potential energy. *Spin softening* occurs because the distance of each point of the structure to the centre of rotation varies with the vibrational motion, and is thus related to kinetic energy. Stress stiffening typically has the strongest effect, and thus, the natural frequencies of bladed disk assemblies typically increase with increasing rotation speed (when temperature is constant). Figure 2.8 illustrates the effect of stress stiffening and spin softening. Note that the effect of spin softening only (curve C in Figure 2.8) is larger than what one may expect for a typical bladed disk assembly.

The *gyroscopic* effect causes the frequencies of the different mode pairs (see above) to split or separate from each other. This effect is usually neglected when dealing with bladed disks, but may be significant in the case of blade-disk-shaft assemblies.

Effect of temperature

Natural frequencies decrease with increasing temperatures because of a reduction in Young's modulus of elasticity. Due to relatively large operating temperatures in turbines, frequency drops can be greater than 10%. Thus, even when considering stress stiffening, the natural frequencies of turbine bladed disks generally decrease with rotation speed, because of the associated rise in temperature (see e.g. Figure 1.2).

Effect of engine order

The natural frequencies of a bladed disk typically increase with increasing number of nodal diameters in the respective mode shapes. Thus, increasing the engine order excitation, r , increases the resonance frequency, as a higher nodal diameter mode will be excited. Considering a bladed disk with N blades, this is only true for $r \leq N/2$, however, as $N/2$ is the maximum possible number of nodal diameters. For higher engine orders, i.e. $r > N/2$, the resonance frequency will decrease with increasing engine order.

Figure 2.8: Effect of stress stiffening and spin softening on fan blade natural frequency, (A) no effects, (B) stress stiffening only, (C) spin softening only and (D) the combined effect. Adapted from Jacquet-Richardet (1997).

3 STATE OF THE ART - BLADED DISK MISTUNING

3.1 Introduction

Mechanical mistuning was first identified from a number of experimental investigations done in the early 70's (Ewins (1969, 1973, 1976)). Subsequently, numerous research efforts have been devoted to the mistuning problem. Due to the vast amount of material, several surveys have also been published (Ewins (1991), Srinivasan (1997), Slater et al (1999)). Current research efforts concerning forced response of mistuned rotors may be divided into five main areas:

1. Study of the underlying mistuning mechanisms with the goal of understanding which factors influence high sensitivity to mistuning.
2. Development of reduced order modelling techniques to achieve a compromise between reasonable computational efficiency and accuracy.
3. Development of techniques for approximations of statistics of the forced response.
4. Development of techniques for optimisation of the mistuning problem, i.e. to maximize or minimize the forced response.
5. Identification of mistuning characteristics from experimental data.

3.2 Mistuning mechanisms

Introduction

The ultimate interest regarding fatigue problems in blade disks are the stress levels. However, researchers commonly base their studies on response amplitudes. Even though there is no linear or other simple relation between amplitude and stress levels between different mistuning patterns, amplitudes still yield a qualitative indication of stress levels. High amplitudes give high stresses and vice versa. Further, the effect of mistuning on forced response levels are commonly measured as amplitude magnification, i.e. the ratio of mistuned to tuned response amplitude at the highest responding blade. Even though other blades may vibrate with smaller amplitudes, this is of little importance, as it is the highest responding one, which is likely to fail due to high cycle fatigue.

Note that numerical studies of the effects of mistuning on the free vibrations and forced response of bladed disks have often been performed with discrete mass-spring-damper models with a few DOF per sector (see appendix A for an example). Such simple models have the advantage to make the problem computationally tractable while capturing the essential features. However, it is important to keep in mind that these models may be difficult to correlate with actual bladed disks. Thus, some of the more recent studies consider also full

finite element models (FEM). Mistuning is typically introduced by varying the blade stiffnesses.

Frequency splitting

A number of numerical investigations (e.g. Ewins (1973), Afolabi, (1985) Ewins and Han (1984)) have shown that blade mistuning results in splitting of the double nodal diameter modes of bladed disks (see chapter 2.2) into modes with different frequencies. The split natural frequencies are close, but the respective modes can no longer combine into a single sinusoidal wave.

Mode Localization

The phenomenon of distorted mode shapes was first reported by Ewins (1969) as “complex modes of vibration”, before being identified and examined as “localized modes” by Wei and Pierre (1988a). Figure 3.1 shows examples of tuned and mistuned modes of a bladed disk assembly represented by relative displacement of each blade. Note the severely localized modes shown in Figure 3.1b where, in fact, modal displacements are localized around a limited set of consecutive blades. Localized / distorted modes are composed of a series of regular nodal diameter components, which may be identified through a Fourier analysis.

Figure 3.1: Modes of (a) tuned and (b) mistuned rotors, represented by relative displacement of each blade. (The tuned modes are represented as travelling waves.) Adapted from Srinivasan (1997).

Blade-to-blade coupling and modal density

Blade-to-blade coupling strength (or more specifically, the ratio of mistuning strength to coupling strength) has been identified as the key parameter governing mode localization (Wei and Pierre (1988a)). Mode localization

increases with decreasing levels of inter-blade coupling. Various authors have later verified this dependence, e.g. a recent experimental investigation made by Judge et al (2001). Wei and Pierre also concluded that bladed disks with high modal density are more susceptible to mode localization than those with widely spaced modes.

Rotation speed

The effect of rotation speed on mode localization has been demonstrated by e.g. Moyroud et al (2002). While severely localized modes of a shrouded fan were observed at rest, the same modes were significantly less localized at the upper limit of the operating range (8000 rpm).

Quantification of localization

Rivas-Guerra and Mignolet (2001) provided an assumption free estimator of localization in terms of the number of blades, s , whose mistuning affects the forced response of a central blade. This is the basis of the partial mistuning model (chapter 3.3).

Amplitude magnification

Since mistuned mode shapes in general consist of several nodal diameter components, as described above, they can be excited by several engine orders. This results in the appearance of multiple peaks in the frequency response curves. Figure 3.2 shows an example of response amplitude versus excitation frequency for a mistuned bladed disk, compared to its tuned counterpart. Numerous mistuned response peaks are observed, in contrast to the tuned case, where only the modes with number of nodal diameters corresponding to the applied engine order respond (see chapter 2.2).

Figure 3.2: Example of tuned and mistuned response versus excitation frequency. Adapted from Petrov et al (2002).

When a mistuned mode shape is excited, the resulting vibration energy can be concentrated on a few blades, leading to stress levels considerably higher than those predicted on the associated tuned assembly. In the literature, there are frequent predictions of amplitude magnification levels of around 2 for fluctuations of the blade properties by only 1-2% (Ewins (1991)). In general, amplitude magnification levels are influenced by numerous parameters, involving the geometry of the bladed disk and operating conditions. Blade-to-blade coupling strength (through disk or shrouds), mistuning strength, modal density, characteristic blade motion (mode shape) and damping are important parameters.

Blade-to-blade coupling and modal density

Whereas mode shape localization is known to increase monotonically with decreasing levels of inter-blade coupling, the mistuning effect on forced response has often been found to feature a local maximum at a moderately low coupling level (e.g. Wei and Pierre (1990)). This has been explained by an energy augmentation mechanism, where the blade experiencing the maximum amplitude can draw on the energy being fed to the other blades in the assembly (Ottarson and Pierre (1995)). In order to obtain large amplitude magnifications, the inter-blade coupling must be small enough to yield localized modes but sufficiently strong so that the blade around which vibrations are being localized can receive energy from neighbouring blades. Later investigations have shown that while the mean and standard deviations of the maximum amplitude magnification on a disk exhibit peaks as described above, the maximum possible amplitude magnification *increases monotonically* with coupling (Rivas-Guerra and Mignolet (2001)).

Mode localization increases with increasing modal density, and thus, keeping other factors constant, also amplitude magnification can be expected to increase with increasing modal density.

Using the localization estimator, s , described above, Rivas-Guerra and Mignolet found that the highest amplitudes of response are always associated with a high level of localization (low mistuning width). Figure 3.3, which shows a scatter plot of amplitude versus the value of s , illustrates this phenomenon. Note, however, that a high level of localization not necessarily implies large amplitudes.

Frequency veerings

It is evident from the discussion above that the maximum possible amplitude magnification increases both with increasing coupling and increasing modal density. However, coupling and modal density are highly dependent on each other. High coupling typically gives low modal density and vice versa. Thus, the combination of both high modal density and coupling can only occur when two different modes with the same number of nodal diameters have close natural frequencies. This is observed in the frequency versus nodal diameter plot, when two different mode families approach and “veer” away from each other, commonly referred to as a frequency veering (see e.g. Figure 2.7). Large amplitude magnifications may be achieved at such veerings.

Figure 3.3: Scatter plot of the width of partial mistuning required to achieve an accuracy of 10% on the amplitude of response of the blades of 5 randomly mistuned disks. Adapted from Rivas-Guerra and Mignolet (2001).

Effect of mistuning strength

The dependence (slope) of amplitude magnification on mistuning strength may be used as a measure of the sensitivity to mistuning. Using the standard deviation of blade stiffnesses / frequencies as a measure of mistuning strength, several studies have demonstrated that amplitude magnification tends to exhibit a peak value at a relatively low standard deviation of mistuning (1-2%) ((Ewins (1969), MacBain and Whaley (1984), Wei and Pierre (1990), Ottarson and Pierre (1995), Castanier and Pierre (1997,1998)). That is, amplitude magnification increases with increasing mistuning up to a certain level, but a further increase in mistuning actually results in lower magnifications. This indicates that bladed disks are very sensitive to mistuning around the tuned conditions, but that this sensitivity decreases with increasing mistuning strength. This phenomenon has been explained (Ottarson and Pierre (1995)) by the energy augmentation mechanism described above. In cases where an increase in mistuning leads to decreased mistuning sensitivity, additional mistuning will prevent the augmentation of vibration energy in the localized blade for much the same reason that it causes localization in the first place – by preventing the propagation of energy-carrying waves to the localized blade. Figure 3.4 shows an example of this phenomenon.

Quantification of the maximum amplitude magnification factor

Quantification of the maximum possible amplitude magnification in a bladed disk is a complex task. Predictions of the maximum made by various authors differ significantly, as the answer is extremely case specific. However, Whitehead (1966) showed analytically that the maximum factor by which forced vibration of blades can increase due to mistuning is given by:

Figure 3.4: Statistical estimations of amplitude magnification versus standard deviation of random mistuning strength for a 1-engine-order excitation of a 29-blade industrial compressor rotor. Adapted from Castanier and Pierre (1998).

$$\frac{1}{2}(1 + \sqrt{N}) \quad (3-1)$$

Whitehead (1976) corrected this expression, before claiming (Whitehead (1998)) that the original expression was in fact correct. The expression depends only on the number of blades on the disk, N . However, certain conditions are required to obtain this maximum factor, e.g. the level of damping must be small compared to mistuning and coupling strengths (Whitehead (1998)).

Whitehead's expression for the maximum possible amplitude magnification has been compared to numerous numerical and analytical studies in the literature. Using a simplified bladed disk model (appendix A), Rivas-Guerra and Mignolet (2001, 2002) and Kenyon et al (2002) obtained a good match for certain cases. The latter reference concludes that while equation 3-1 is valid for engine orders 0 and $N/2$, the expression found by Whitehead (1976), where N is replaced by $N/2$, is valid for all other engine orders. Kenyon et al also found that the maximum amplitude magnification occurs when the harmonic components of a distorted mode superimpose in a certain manner, such that a balance between the generalized force and localization is achieved (see chapter 3.5 about the method).

Intentional mistuning

The phenomenon of a peak in amplitude magnification with respect to mistuning strength has lead researchers to investigate whether intentional mistuning (sometimes referred to as “detuning”) could be introduced into the bladed disk design in order to reduce the adverse effects of random mistuning. Ewins (1980) discussed the possible advantages of grouping the blades into “packets” of shrouded blades. Griffin and Hoosac (1984) considered an “alternate mistuning” pattern, i.e. placing blades with high and low frequencies alternatingly around the disk.

Most recent studies focus on harmonic mistuning, i.e. certain blade properties follow a harmonic (sinusoidal) pattern around the bladed disk (Castanier and Pierre (1997,1998), Slater and Blair (1998), Kenyon and Griffin (2000, 2001)). Such a pattern is explicitly described by the harmonics number (h) and the mistuning amplitude. Note that frequency splitting will occur when the number of nodal diameters is any integer multiple of $h/2$ (Kim et al (2000)). Castanier and Pierre (1997,1998) found that for some intentional mistuning harmonics, the amplitude magnification factor exhibits a peak at a relatively low value of the intentional mistuning amplitude (similar to the case of random mistuning). Thus, for amplitudes of intentional mistuning higher than that corresponding to the response peak, the rotor may show less sensitivity to random mistuning. In general, it was found that intentional mistuning could greatly reduce the rotor’s sensitivity to mistuning, particularly in the range of random mistuning where large amplitude magnification is observed for the tuned design. Figure 3.5 shows an example of the beneficial effect of harmonic mistuning. In this case, mistuning harmonics 3 – 8 are the most beneficial. Also non-harmonic mistuning patterns have been shown to yield a beneficial effect on mistuning sensitivity (Slater and Blair (1998), Choi et al (2001)).

Identification of the most responding blade

El-Bayomi and Srinivasan (1975) and later Griffin and Hoosac (1984) concluded that the blades with the cantilever frequencies close to the coupled blade-disk resonance frequency usually respond the greatest. On the contrary, Ewins and Han (1984) found that blades with extreme mistune are most likely to vibrate with the greatest amplitudes. Attempts have been made to identify the most responding blade according to the Fourier coefficients of the eigenvectors (Afolabi (1985b, 1988)) and the blade-to-blade coupling level (Wei and Pierre (1988b)). Sanliturk et al (1992) concluded that there is no general rule for identifying the most responding blade.

Multi-stage coupling

Bladh et al (2001d) explored the effects of multi-stage coupling on the dynamics of bladed disks with- and without- mistuning. The authors concluded that multi-stage analysis may be required when excitations are expected to fall near frequency veering regions (see section above), or when the sensitivity to blade mistuning is to be accounted for.

(a)

(b)

Figure 3.5: Statistical estimations of amplitude magnification versus standard deviation of random mistuning strength for a 1-engine-order excitation of 29-blade industrial compressor rotor, which is intentionally mistuned (prior to the random mistuning). Adapted from Castanier and Pierre (1998).

3.3 Reduced order models

Introduction

In order to reduce the large computational cost associated with FE (finite element) modelling of mistuned bladed disks, techniques to reduce the size of the FE models - often referred to as reduced order models, are essential. Two main classes of reduced order models have evolved, based on Component Mode Synthesis (CMS) and classical modal analysis respectively. The principles of the two methods are similar, as both methods employ tuned modes to reduce the finite element matrices. CMS methods employ modes of the individual substructures, however, rather than modes of the entire structure. Both these branch methods, as well as several other proposed reduction techniques, are described and compared below.

Method based on classical modal analysis (Modal reduction technique)

Yang and Griffin (2001) applied classical modal analysis to mistuned systems, where the mistuned modes are represented in terms of a limited sum or subset of “nominal” system modes, and thus labelled by the authors as the SNM method (Subset of Nominal Modes). This technique has later been referred to as e.g. the mistuning projection method (Bladh et al (2001a,b)) and the modal reduction (MR) technique (Moyroud et al (2002)). The latter term (MR) will be used here.

The assumption that the mode shapes of the mistuned bladed disk can be expressed as a linear combination of a set of tuned mode shapes is justified by the assumption (Bladh et al (2001a)) that any admissible disk shape, no matter how spatially localized, may be realized by a linear combination of its harmonic shapes in cylindrical co-ordinates if all harmonics 0 through P are included in the model, where P is the highest possible harmonic ($N/2$ if N is even and $(N-1)/2$ if N is odd).

A fundamental step of the method is to define a subset of nominal modes, and thereby introducing an approximation. Naturally, the accuracy increases with increasing number of nominal modes used in the representation. However, it has been shown (Yang and Griffin (2001)) that neglecting nominal modes with remote natural frequencies results in errors, which are inversely proportional to the frequency difference (between the remote modes and the mode of interest). Thus if the natural frequencies of a family of tuned blade modes are densely populated and isolated from the frequencies of other modes, the corresponding mistuned modes may be well predicted using only this family of modes in the modal basis.

Component mode synthesis techniques (CMS)

The main item distinguishing the different CMS approaches is the type of modal basis used. Important requirements for the modal basis are that the modes should be linearly independent, and that the limit of a complete set of modes in the basis should yield the exact solution relative to the parent FE model (i.e. it should span the complete deformation space of the FE model).

Receptance technique

The receptance method (Menq et al (1986), Yang and Griffin (1997)) expresses the DOF of each substructure (disk / blades) in terms of the DOF of its interfaces, which yields a significant model reduction. However, the substructure's modes have to be free at the disk-blade interfaces, which is undesirable (Yang and Griffin (1997)). Further, the number of DOF of the ROM can still be quite large, depending on the number of nodes at the blade interface.

CBSR

The Craig and Bampton method (Craig and Bampton (1968)) has been reformulated specifically for the analysis of mistuned bladed disks by various authors, e.g. Moyroud et al (2002) and Bladh et al (2001a,b). The modified method will be referred to here as the CBSR (Craig and Bampton substructuring and reduction) technique. The CBSR method employs two sets of modes to represent the motion of each component: First, a truncated set of normal dynamic modes of vibrations with the DOF at component interfaces held fixed, and second, a complete set of static constraint modes induced by successive unit deflection of each interface DOF while all other interface DOF are held fixed. The CBSR method yields a robust and highly reliable ROM. However, the retained physical interface DOF may lead to impracticably large CMS models when using highly detailed parent FEM models.

REDUCE

Castanier et al (1997) introduced a component mode synthesis method, "REDUCE", without constraint (static) modes in the modal basis (similar to the method presented by Benfield and Hruda (1971)). The authors claim to have constructed physically meaningful constraint modes by means of a modal analysis with massless blades. Because massless blades have no inertia, they will follow the motion of the disk, but will not add artificial natural frequencies. The method was validated against FEM by Kruse and Pierre (1996a,b). Bladh et al (1999) extended the technique to turbomachinery rotors with shrouded blades.

SMART

Bladh et al (2001a,b) proposed to perform a full-scale secondary modal analysis on the already reduced CBSR model (described above). This method, named by the authors as the secondary modal analysis reduction technique (SMART) may in principle be applied to any intermediate model. However, the authors chose the CBSR method, since it gives direct access to the blade modal properties. Bladh et al (2001c) adapted the method for multi-stage rotors.

CCM

In contrast to a *full-scale* secondary modal analysis, such as in the SMART approach, Castanier and Pierre (2001) proposed to perform a *partial* secondary modal analysis on the partitions of the CBSR mass and stiffness (reduced) matrices that correspond to the constraint modes. The result of such an analysis is a new set of constraint modes, referred to as characteristic

constraint modes or CC modes, which represent the characteristic motion of the interfaces. Bladh et al (2001c) adapted the method for multi-stage rotors.

Adaptive perturbation technique

Lin and Mignolet (1997) formulated an adaptive perturbation approach, based on a partitioning of the modal impedance matrix into blocks associated with natural frequencies that are either close to or far from the excitation frequency. The modal basis consists of the mode shapes of the *tuned* system when the coupling terms are large and the mode shapes of the *decoupled* system when the coupling terms are small (with respect to mistuning strength). Rivas Guerra et al (2001) modified the “large-coupling” adaptive perturbation technique, by improving the basis of modes with natural frequencies close to the excitation frequencies.

Partial mistuning model

Mignolet et al (2000a,b) found that both qualitative and quantitative aspects of forced response of weakly coupled bladed disks can be accurately predicted by considering only a few consecutive blades as mistuned (from 3 to 9), referred to by the authors as a partial mistuning model.

An exact reduced order model

Petrov et al (2000a, 2002) presented a reduction technique without introducing any approximations compared to the full FEM model. The system equations are solved only for a subset of *active* coordinates, i.e. those where mistuning is applied and those where forced response levels are of interest. This is achieved by expressing the response of the mistuned system as a function of the tuned response, the FRF matrix of the tuned system and a mistuning matrix.

A fundamental mistuning model

Feiner and Griffin (2002) introduced a so-called fundamental model of mistuning (FMM), which is applicable when only a single mode family is excited and strain energy in that family’s modes are primarily in the blades. The method is based on the tuned system frequencies and the mistuned blade-alone frequency deviations, which explicitly provides the mistuned system modes and frequencies.

The Artificial Neural Network approach

Recent publications (e.g. Peng and Yang (2000) and Lecce et al (2002)) have focused on the application of Artificial Neural Networks (ANN) for mistuned forced response predictions. During a “training phase”, the ANN tries to learn the connection between the chosen input (a set of mistuning parameters) and the chosen output (e.g. amplitude magnification). Once trained, the network should be able to make output predictions for any input parameters.

Comparison of the methods

The reduced order models described above have been assessed and compared in several publications. Key items for comparison are naturally accuracy and computational efficiency. The FE models employed in these studies range from simple test cases to industrial bladed disks. Further,

mistuning is typically modelled as offsets in blade stiffness. Note that the ANN approach is not included in the discussion below, since few results are available in the literature at the present time.

Comparison of accuracy

Accuracy is typically compared in terms of mistuned mode- and forced response representation. Most of the above-mentioned methods have been shown to produce results in good or excellent agreement with direct FEM. It is worth giving some additional comments, however:

- The receptance technique does not show good agreement with direct FEM when modes from two different families, which are too close in frequency, are excited simultaneously (Yang and Griffin (1997)).
- The MR and CBSR techniques have been demonstrated to capture adequately strong mode localization, compared to direct FEM (Moyroud et al (2002)).
- The MR, CBSR and SMART techniques have been shown to exhibit comparable or *improved* accuracy compared to REDUCE (Bladh et al (2001b)).

Comparison of computational efficiency

Computational efficiency may be compared in terms of theoretical count of floating-point operations or simply the CPU time required for the calculation. However, the computational efficiency is highly dependent on certain key features of the models. Important such features are (a) how mistuning is introduced into the model and (b) how the size of the ROM depends on the size of the parent FEM. These features are presented for the different models in Table 3.1.

	<i>Mistuning projection domain</i>	<i>Size dependence between FEM and ROM</i>
<i>MR</i>	Physical	Independent
<i>Receptance technique</i>	Physical / Modal	Proportional to the #of interface DOF
<i>CBSR</i>	Physical / Modal	Proportional to the #of interface DOF
<i>REDUCE</i>	Physical / Modal	Independent
<i>SMART</i>	Physical / Modal	Independent
<i>CCM</i>	Physical / Modal	Independent
<i>Partial Mistuning</i>	Physical	Inverse proportional to the level of localization
<i>Exact ROM</i>	Physical	Proportional to the # of mistuned and “interesting” DOF
<i>ANN</i>	--	Independent

Table 3.1: Comparison of various reduced order models (ROM).

The modal reduction technique (MR) suffers a high computational cost due to carrying out the mistuning projections in the physical domain. On the other hand, the size of the ROM is independent of the size of the parent FEM, which makes the method attractive. Moyroud et al (2002) showed that the MR method was far superior to the CBSR method in computational efficiency. Note, however, that mistuning projection was carried out in the physical domain for both methods. The SMART approach has been shown to be exceptionally fast (Bladh et al (2001b)), since it features both FEM size independence and the capability to introduce mistuning in the modal domain. The same yields the CCM approach. Bladh et al further concluded that the MR, CBSR and SMART approaches were superior to the REDUCE approach in terms of computational efficiency.

Shroud friction constraints

When analysing bladed disks with non-continuous shrouds, it is important to capture the shroud friction phenomenon satisfactory. CMS approaches are attractive in this context since each sector / blade of the bladed disk may be treated as separate substructures, and thus, a friction model may be incorporated at the connection interfaces.

3.4 Statistical techniques

Introduction

The determination of statistical properties of the forced response of randomly mistuned bladed disks is an especially important problem. Such analysis typically involves estimation of the distribution of the following three response amplitudes (Mignolet et al (1999)):

- a) Amplitude of a typical blade at a given excitation frequency.
- b) Amplitude of the maximum responding blade on the disk at a given excitation frequency.
- c) Amplitude of the maximum responding blade on the disk over a frequency sweep.

The most straightforward method to obtain such statistics is by means of Monte Carlo simulations, i.e. to compute the response for a large number of mistuning configurations with a given statistical distribution. However, this is a computationally expensive approach. The sample size (number of random mistuning patterns) required to obtain a fairly good prediction of the probability density function of response amplitudes is large, typically in the order of $10^4 - 10^5$ (see e.g. Rivas Guerra and Mignolet (2001)). Especially, prediction of the high amplitude tail of the distribution, which is important in the present context, requires a large sample size. Thus, large efforts have been devoted to finding more efficient methods for statistical mistuned response predictions, as described below.

Parametric / Non-parametric approaches

The estimation of the distribution of a random variable can be accomplished in two different ways (Mignolet et al (1999)). Non-parametric approaches can be used to obtain the required distribution $p(x)$ for certain or all values of x through independent computations. As an example, the histogram obtained from Monte Carlo simulations provides a non-parametric estimate of the distribution. On the other hand, a parametric approach relies on the specification of a physically justified model of the distribution that involves a small number of unknown parameters (typically 1 – 6). Then, additional information, typically based on a relatively small sample size, is used to estimate the appropriate values of these parameters. This approach is well applicable to experimental programs where experimental data is typically limited to only a few disks / mistuning configurations, as well as random (Monte Carlo) numerical simulations where the number of required random configurations may be significantly reduced.

Available methods

Table 3.2 shows a summary of available techniques for the determination of the statistics of the forced response of mistuned bladed disks. Focusing first on the response statistics of a typical blade, the CFP (Closed Form Perturbation) method (Mignolet and Lin (1993)) has been found to be best applicable for off-resonant excitations and / or disks with a small number of blades. The Rayleigh type distribution (Sinha (1986) and Sinha and Chen (1989)) is limited to very high blade-to-blade coupling situations, where it is fairly reliable. The three parameter- or limit distribution (LMT)- model is shown to be reliable in the entire range of blade-to-blade coupling levels (Mignolet et al (2001a), Mignolet and Hu (1998)). Note that the three parameters may be calculated directly, i.e. without Monte Carlo simulations or similar.

Level	Non-parametric approaches	Parametric approaches	
		<i>Distribution model</i>	<i>Parameter estimation</i>
Typical blade	Closed form perturbation	Rayleigh-type	Third order perturbation
	Monte Carlo simulations	Three parameter model (LMT)	Cumulant closure
Maximum response at given excitation frequency	Monte Carlo simulations	Weibull-type III	ME, LS, MLE
Maximum response in a frequency sweep	Monte Carlo simulations	Weibull-type III	ME, LS, MLE

Table 3.2: Summary of available techniques for the determination of the statistics of forced response of mistuned bladed disks. Adapted from Mignolet et al (1999).

Focus is now put on the techniques to estimate the statistics of the maximum responding blade. It has been demonstrated that the statistics of the maximum amplitude on a bladed disk (either at a fixed excitation frequency or in a frequency sweep) can be accurately represented by a Weibull (type III) parameter model (Mignolet et al (1999), Castanier and Pierre (1997, 1998), Mignolet et al (2000a,b)). Figure 3.6 shows an example of such Weibull predictions. The model has been found accurate in the entire range of blade-to-blade coupling strengths (Mignolet et al (1999)). Correlation of statistical data to such a Weibull distribution model involves the estimation of the three Weibull parameters a , b and c . Mignolet et al (2000a,b) applied a moment estimation (ME) technique, where the mean, standard deviation and skewness of the Weibull distribution are matched to those of the sample data. Castanier and Pierre (1997, 1998) relied on a reasonable approximation of a , and used a least squares (LS) linear regression to find b and c . For the LS technique, the upper magnification limit predicted by Whitehead (1966, 1998), is typically used as an approximation of a . Applying the LS parameter estimation technique, sample sizes as small as 50–100 have been shown to introduce errors of the 99th percentile of the cumulative probability distribution, which are smaller than 5% compared to a sample size of 10^5 (Castanier and Pierre (1997, 1998)).

Figure 3.6: Probability distribution of the maximum response in a frequency sweep obtained by Monte Carlo simulations (MC) and Weibull type III approximation. Adapted from Rivas Guerra et al (1999).

3.5 Optimisation techniques

Introduction

Knowledge of the largest possible amplitude magnification caused by mistuning, as well as the arrangement of blades that are the most favourable, and those which are the most dangerous, is very important in design practice. The statistical treatment of mistuning, as described in the previous chapter, gives valuable information about such aspects. Even the large amplitude limit of the distribution, i.e. the tail of the probability density function, can be predicted quite accurately. However, these methods cannot give exact predictions of the maximum possible response amplitude, even though reasonable estimates can be obtained. E.g. for Monte Carlo simulations, you can never be sure that you covered the worst possible mistuning configuration, regardless of the population size considered. Thus, efforts have been devoted to formulating an optimisation problem.

The optimisation problem

A bladed disk mistuning configuration can be represented by a mistuning vector, $\mathbf{d}$, containing a number of mistuning parameters, d_j . The mistuning parameters typically constitute blade material properties or cantilevered natural frequencies, so that the number of elements in $\mathbf{d}$ is equal to the number of blades, N .

Finding the mistuning vector, such that the maximum (or minimum) stress- or vibration- amplitudes are obtained is an optimisation problem. The objective function of such a problem is the stress- or vibration- amplitude. For search of the worst / best mistuning patterns, the objective function has to be maximized / minimized respectively. Considering the amplitude, u , the objective function Q for a certain mistuning vector $\mathbf{d}$ is given as follows:

$$Q(\mathbf{d}) = u^{\max} \quad (3-2)$$

$u^{\max}$ is searched over all points of the bladed disk, over all excitation frequencies in a given range and over all time instants during a vibration period. The following constraint (if any) is applied to the mistuning vector:

$$d^- \leq d_n \leq d^+ \quad (3-3)$$

It is evident that the maximum amplitude $u^{\max}$ is a function of the mistuning vector $\mathbf{d}$, where $\mathbf{d}$ spans a N -dimensional space. The objective is thus to find the location in this N -dimensional space, where u exhibits its global maximum value. Considering a disk with only two blades, this is easily illustrated by finding the global maximum on a surface.

Available methods

The problem of searching for the worst and best mistuning patterns was first formulated as an optimisation problem by Petrov (1993,1994). A later attempt was made by Sinha (1997). More recent efforts involve the works by Petrov and Ignilin (1999), Petrov et al (2000a,b) and Petrov and Ewins (2001). In

these latter approaches, the optimisation problem is based on the sensitivity coefficients of the objective function with respect to the mistuning parameters. It has been demonstrated that such approaches are superior to Monte Carlo simulations (where each mistuning configuration is chosen randomly instead of learning from the previous iteration). A further reduction in computational costs may be achieved by introducing response surface techniques, which fit low order polynomials to function and gradient values.

Based on the observation by Rivas-Guerra and Mignolet (2001) that the largest amplitude magnification always occurs for a high level of localization, the authors successfully performed an optimisation routine based on the partial mistuning model (chapter 3.3), gradually increasing the number of mistuned blades. Accurate predictions of maximum magnification levels were in fact obtained by considering only 3-7 blades to be mistuned, and thereby reducing the size of the optimisation problem significantly.

Kenyon et al (2002) developed an expression for the maximum forced response due to mode shape distortion in terms of the participation of the tuned modes in the mistuned response (using the SNM approach described in chapter 3.3). Thus, the mistuned mode, which leads to the maximum response, can easily be identified. Then a mistuning configuration that will produce the identified mode can be explicitly calculated.

3.6 Identification of mistuning characteristics from experimental data

It is of great importance to be able to characterize mistuning patterns from experimental data. Naturally, it is impossible to identify FE matrices directly from experiments. Instead, FE models are commonly correlated to certain experimental data, e.g. clamped blade-alone frequencies. This is an indeterminate problem, however, since there is no explicit set of stiffness and mass matrices corresponding to such data. Several methods have been proposed in the literature on how to accurately estimate the fluctuations in the model parameters, i.e. stiffness and mass terms. The method proposed by Mignolet and Lin (1997), which relies on forced response measurements of consecutive blades on a series of bladed disks, recovered the structural properties quite accurately. The required measurements are relatively expensive, however. Two other approaches are the random modal stiffness approach (RMS) and the maximum likelihood method (ML).

Random modal stiffness (RMS) approach

In the RMS approach, the mass matrix is taken equal to its tuned counterpart while the stiffness matrix is found by matching the blade-alone frequencies. A further assumption is that mode shapes of the blades are unaffected by mistuning. The RMS approach is computationally quite attractive. However, Mignolet et al (2001a,b) demonstrated that the reliability of forced response of mistuned systems predicted by this method varies substantially (leading to underestimation of forced response), depending in particular on the level of blade-to-blade coupling, excitation characteristics, etc. This was explained by

the fact that matching precisely the blade-alone frequency leads to slightly erroneous estimates of the bladed disk modal characteristics.

Maximum likelihood (ML) method

In the ML method, both the mass and stiffness matrices are chosen by maximizing the joint probability function $p_{\mathbf{X}}(\mathbf{X})$, where the vector $\mathbf{X}$ contains the unknown mass and stiffness deviations, given the observed values of the natural frequencies. It is assumed that the stiffnesses and masses are normally distributed, which is justified by noting that the forced response of mistuned bladed disks depends only slightly on the shape of the distribution of blade properties (Rivas-Guerra et al (1999)). Mignolet et al (2001a,b) revealed that the ML method is most reliable when the assumed distribution of the blade parameters is accurate. However, even when this model is vastly in error, predictions are generally much better than the RMS prediction.

Improved random modal stiffness (IRMS) approach

Mignolet et al (2001a,b) introduced an improved random modal stiffness (IRMS) approach. The IRMS approach is based on the maximum likelihood principle but yields a mistuning model similar to that of the RMS technique, and thus provides a bridge between the two above described approaches. The random modal stiffnesses are selected to match the behaviour of the blade as part of a tuned assembly of similar blades rather than the blade alone frequencies, as in the RMS approach. As expected, the IRMS formulation led to errors in the forced response prediction that were smaller than for RMS although both of these approaches are similar in that they do not include mistuning effects on either mass or mode shape.

4 OBJECTIVES AND APPROACH

4.1 Objectives

The objective of the present work can be divided into the following four items:

1. Obtain increased knowledge of the influence of blade-to-blade coupling and rotation speed on the sensitivity to mistuning, for real bladed disk geometries.
2. Obtain an assessment of existing methods for the prediction of mistuned response statistics.
3. Obtain a validation of two reduction techniques against experimental measurements.
4. Obtain a comparison of two finite element reduction techniques for the prediction of mistuned forced response amplitudes.

Item 1 is the main focus of the current work. As charted in the previous chapter, most previous analyses on this topic focus on generic parameter studies using simple lumped parameter models, and the few which consider realistic bladed disk models typically do not involve variations in geometric parameters, such as the effects of shrouds, or operating conditions, such as rotation speed. Emphasis is also put on evaluating certain available statistical methods to obtain mistuned response statistics (item 2).

4.2 Approach

Two state of the art finite element reduction techniques, one based on classical modal analysis and another based on component mode synthesis, are employed and compared for the structural analysis of two test geometries. The first geometry, a cyclic symmetric plate, is used in order to obtain a comparison of forced response amplitudes predicted by the two methods (objective 4 above). The two methods are then validated against experimental measurements existing for a test blisk within a European project, ADTurBII (objective 3 above).

One of the respective methods is then chosen for the main part of the work – a detailed mistuning sensitivity study (objectives 1 and 2 above), modelling sensitivity as the dependence of amplitude magnification on the standard deviation of blade stiffnesses. A transonic fan is chosen for the analysis, and is considered with part span shrouds and without shrouds, respectively, constituting a high and a low blade-to-blade coupling case. For both cases, computations are performed at rest as well as at various rotation speeds. The

chosen reduced order model is solved for sets of random blade stiffnesses with various standard deviations, i.e. Monte Carlo simulations. In order to reduce the sample size, the statistical data is fitted to a Weibull (type III) parameter model. Three different parameter estimation techniques are applied and compared, in order to assess their accuracy and applicability for small sample sizes.

5 APPLIED NUMERICAL METHODS

5.1 Structural analysis

In order to reduce the required computational costs associated with the current work, two state of the art finite element reduction techniques are applied, the MR and CBSR reduction techniques, respectively (see chapter 3.3). A thorough description and validation of both methods against direct FEM is available in Moyroud et al (2002) and Moyroud (1998). Therein, perfect agreement in mistuned natural frequencies, mode shapes and forced response with direct FEM is documented for several test geometries, including the shrouded fan considered in the current study. The algorithms used in this work consists of the following steps:

MR technique

1. Cyclic symmetric modal analysis on the reference sector of the tuned bladed disk. The mode shapes are obtained in travelling wave representation on the reference sector.
2. Transform the mode shapes from travelling wave- to real-valued-representation for each sector.
3. Reduce the finite element matrices of the mistuned bladed disk, using the tuned mode shapes.
4. Solve the system of modal equations (for forced response analysis, a generalized force vector is included).
5. Compute the physical displacement vector.

CBSR technique

1. Calculate the Craig and Bampton substructure modes (static and dynamic) on the reference sector of the tuned bladed disk.
2. Reduce the mistuned finite element matrices of all N substructures of the mistuned bladed disk, using Craig and Bampton substructure modes.
3. Assemble the N reduced matrices.
4. Solve the reduced system of equations (for forced response analysis, a reduced force vector is included).
5. Compute the physical displacement vector.

The above-mentioned techniques are integrated in a research FE code from LMSt-INSAL (named "CORIODYN"), which solves the structural static and dynamic equations of motion of cyclic symmetric blade-disk-shaft assemblies with a Galerkin finite element method. Spin softening, stress stiffening and gyroscopic effects are included. The program data structure is based on the concept of pseudo-dynamic allocation, which allows to efficiently store arrays with a minimum of memory space. The code is further documented in (Moyroud (1998), Henry and Ferraris (1984) and Jacquet-Richardet et al (1996)).

The implementation of the described reduction techniques is especially tailored for Monte Carlo simulations. Thus, effort has been made in order to

minimise the computational cost for each mistuning configuration. Especially, the calculation of modal matrices can be computationally expensive. This cost is reduced by calculating the reduced (modal) matrices from the element matrices, rather than from the assembled matrices. The reduced matrices are first calculated for the tuned configuration. Then, for a given mistuning configuration, contributions to the reduced matrices are evaluated for the limited set of FE elements, which are mistuned. This procedure enables an efficient loop over all mistuning configurations.

5.2 Modelling mistuning

For real bladed disks, mistuning arises from blade-to-blade material property- and / or geometrical- variations, which in turn lead to blade-to-blade variations in frequency and / or mode shape. Thus, it is evident that a certain blade can be mistuned in an indefinite number of ways, which complicates realistic numerical modelling. In this work, mistuning is introduced in two different ways:

Blade stiffness mistuning

For the simple test case treated in chapter 6, as well as for the sensitivity study described in chapter 8, mistuning is introduced by perturbing the stiffness (K) of a certain number of blades by a scaling factor ε , leading to blade frequency variations (blade mode shapes not affected). Note that the stiffness, K , is the sum of elastic stiffness as well as the stress stiffening- and spin softening- terms, so that mistuning is applied to all these terms. The disk on the other hand is considered tuned. Thus, the selected mistuning pattern can be described as follows,

$$\{K^{mis}\}_n = (1 + \varepsilon_n)\{K^{tun}\}_0, n = 0, \dots, N-1 \quad (5-1)$$

where $\{K^{tun}\}_0$ and $\{K^{mis}\}_n$ are the elemental stiffness matrices on the reference blade 0 of the tuned bladed disk, and on the blade n of the mistuned bladed disk, respectively. N is the number of blades on the disk, and ε_n is the mistuning parameter for blade number n . A particular mistuning pattern is thus given by a set of N parameters, which are set in a statistical manner using a random number generator.

Blade mass mistuning

For the test blisk treated in chapter 7, mistuning is introduced by adding a point mass to each blade tip, which constitutes a simple model of the bolts and washers, which were mounted on the corresponding test piece.

5.3 Modelling of forcing fields

A simple engine order force field (see chapter 2.2) is applied in the current work by means of one point force working on each blade. Thus, the complex

force F at a node located at circumferential position θ , is determined by the following equation:

$$F(\theta, t) = F_a \exp[i r (\Omega t \pm \theta)] \quad (5-2)$$

where F_a is the force amplitude, r is the engine order, Ω is rotation speed, t is time and i is the imaginary unit. A forward (resp. backward) travelling wave is obtained by applying a positive (resp. negative) angle, θ .

5.4 Identification of resonant peak amplitudes

For lightly damped structures, such as bladed disks, amplitudes vary drastically close to resonant peaks. In order to make sure that the exact peaks are found, the respective search is formulated as an optimisation problem in the current work. The maximum amplitude is found by means of a combination of the golden section method and parabolic interpolation (see e.g. Forsythe et al (1977) for a description of these methods). The method of Brent (1973), which enables the best possible exploitation of the above-mentioned methods, is used. Note that the golden section search is designed to handle the worst possible case of function maximization. If the function is nicely parabolic near to the maximum, the parabola fitted through any three points speeds up the respective optimisation process.

Such optimisation requires the respective peaks to be bracketed by means of three excitation frequencies, such that the response at the mid frequency is higher than at the other two frequencies (thereby verifying that a peak exists). In order to avoid time-consuming frequency sweeps in the current computations, the peaks are bracketed by means of natural frequencies. Due to damping, the resonant frequencies are generally slightly lower than the corresponding natural frequencies. However, it was found that a search range of $\pm 0.5\%$ around each mistuned frequency was sufficient to capture the peaks efficiently. A few examples of application of the current optimisation technique is shown in Appendix B. Note that there is not necessarily a separate response peak for each single mistuned natural frequency, which results in certain response predictions in off resonance regions. However, the current analysis focuses on the maximum response in a frequency sweep, which is always captured.

The maximum amplitude on the entire disk is recorded at the highest peak of the frequency response curve. It may be worth specifying exactly how the maximum amplitude on the disk is found. Each component of the nodal displacement vector can have its own phase, which results in an elliptical orbit of a node during vibration. Because of that, the maximum displacement at a node cannot be obtained simply as the sum of the squares of the amplitudes of all its coordinate components. Instead, the resulting amplitude can be obtained by the following expression (Petrov and Ewins (2001)):

$$u^{\max} = \sqrt{\frac{1}{2}(u^* u + u^T u)} \quad (5-3)$$

where u is the vector of complex amplitudes of displacement components, $*$ represents the Hermitian conjugate and T is the transpose. This expression is thus evaluated at each node in order to find the maximum amplitude on the entire disk.

5.5 Statistical analysis

Parametric distribution model

Focus is put on the amplitude of the maximum responding blade on the disk over a frequency sweep, as this is most critical to the life of the bladed disk. Mistuned amplitudes are non-dimensionalized with the respective tuned amplitudes to yield the amplitude magnification factor. The modal reduction (MR) technique is solved for sets of random blade stiffnesses with various standard deviations, i.e. Monte Carlo simulations. In order to reduce the sample size, the statistical data is fitted to a Weibull (type III) parameter model (see chapter 3.4). Applied to mistuned response statistics, the Weibull probability density function and cumulative probability distribution may be written as follows (Castanier and Pierre (1997)):

$$f(x; a, b, c) = \frac{c(a-x)^{c-1}}{b^c} \exp\left[-\left(\frac{a-x}{b}\right)^c\right] \quad (5-4)$$

$$F(x; a, b, c) = \exp\left[-\left(\frac{a-x}{b}\right)^c\right] \quad (5-5)$$

where a is the location parameter, b is the scale parameter and c is the shape parameter. The location parameter (a) is particularly significant since it represents the largest possible amplitude. Note that this Weibull model is a “switched” version of the Weibull model often used in the statistics community, i.e. $(x - a)$ has been replaced by $(a - x)$ in order to introduce an upper limit rather than a lower limit of the response.

Three parameter estimation techniques are applied and compared in the current work. The ME- and LS- parameter estimation techniques (see chapter 3.4) are both considered. For the LS technique, the upper magnification limit predicted by Whitehead (1966, 1998) is used as an approximation of a . In order to investigate the sensitivity of the results on the value of a , an attempt is also made to increase its value by 50%. Further, the Maximum Likelihood Estimation (MLE) technique developed by Qiao and Tsokos (1995) is applied for comparison with the above-mentioned methods. While all the considered techniques are expected to give the same result for a sufficiently large sample size, this is not necessarily the case for a small sample size, such as 100.

Generation of sample mistuning parameters

For the simple test case treated in chapter 6 (cyclic symmetric plate with 20 blades), as well as for the sensitivity study described in chapter 8 (industrial fan with 30 blades), the mistuning parameters are obtained by means of a random number generator.

For the cyclic symmetric plate, 2000 parameters (20 blades times 100 mistuning configurations) with a uniform probability distribution (equal probability for all parameter magnitudes) within a tolerance of $\pm 10\%$ are created. The statistics of the resulting population are as follows: Minimum and maximum values of -9.98% and $+9.99\%$, as well as a mean of 0.099% .

For the fan geometries, 3000 parameters (30 blades times 100 mistuning configurations) are created for each standard deviation of mistuning. Figure 5.1 shows the statistics of the mistuning parameter obtained with the random number generator plotted versus the intended standard deviation.

Figure 5.1: Statistics of the mistuning parameters applied for the mistuning sensitivity analysis in chapter 8.

It is evident that the real standard deviation of each sample set is fairly close to the intended value, i.e. the curve labelled *STD* appears as an

approximately straight line. The mean is fairly close to zero in all cases, as expected. Looking at the extreme (minimum and maximum) values, these lines also show near linear trends from zero to approximately 50%. I.e. at a standard deviation of blade stiffnesses of 12%, the corresponding maximum offset is 50% (approximately 25% offset in blade frequency). This is a quite large deviation from the tuned stiffness / frequency, which would seldom happen in a real bladed disk. The possibility of such cases happening cannot be excluded, however, and it is considered appropriate to consider such a range of mistuning strengths in the current work.

6 COMPARISON OF TWO REDUCTION TECHNIQUES

6.1 Introduction

A thorough comparison of the MR and CBSR techniques (see chapter 3.3 and 5.1) was made by Moyroud (1997) and Moyroud et al (2002). Therein, focus was put on the capability of the methods of representing mistuned mode shapes and natural frequencies accurately, considering certain deterministic mistuning configurations. Here, focus is put on forced response amplitudes produced by a set of statistical mistuning configurations. Note that this is not intended as a validation of the methods, as this has already been performed, but rather a demonstration of the differences in efficiency.

6.2 Bladed disk model

In order to enable a comparison between the two reduction techniques applied in the current work at a minimum computational cost, a simple test case is used, as shown in Figure 6.1. The geometry is a cyclic symmetric plate, which can be viewed as a simplified model of an unshrouded bladed disk. The FE model consists of 400 H20 elements and 3600 nodes. The model is representative for a bladed-disk, while being small enough to allow fast calculations and fast post-processing. The material properties, geometry and boundary conditions are given in Table 6.1.

<i>Material properties:</i>	
Modulus of elasticity (N/m ²)	1.096E+11
Density (kg/m ³)	4430
Poissons ratio	0.35
<i>Geometry:</i>	
Thickness (m)	0.003
Inner / outer radius of the disk (m)	0.10, 0.13333
Radius of the blade tip (m)	0.20
Number of sectors	20
<i>Boundary conditions:</i>	Clamped at inner radius

Table 6.1: Data for the cyclic symmetric plate.

The current FE model is relatively coarse. In fact, the model is not converged, i.e. increasing the mesh density has an effect on the natural frequencies of the structure. However, using this simple model fulfils the objectives of the present work at a minimized computational cost. In order to verify this assumption, all computations performed in this chapter have been redone with a converged mesh. This did not have any effect on the conclusions, which are made.

Figure 6.1: FE model of simple bladed disk (cyclic symmetric plate).

6.3 Computation parameters

Size of the reduced models

The accuracy of mistuned frequencies, mode shapes and response amplitudes predicted by the two reduction techniques are dependent on the size of the modal basis. The following choice has been made for the current calculations. For the MR analysis, the modal basis consists of 80 full assembly modes (the first four mode families). For the CBSR analysis, the modal basis consists of 30 static- and 14 dynamic- modes per substructure / sector. Thus, the reduction in DOF (degrees of freedom) compared to direct FEM is 99% for the MR method and 91% for the CBSR method respectively. This is consistent with the work performed by Moyroud et al (2002).

Statistical probability distribution and tolerances

A population of 100 mistuning configurations are treated, consisting of random mistuning parameters with a uniform probability distribution within a given tolerance range. Each mistuning configuration consists of 20 random parameters, one for each sector. A bladed-disk mounted on an engine is constrained by a fairly tight manufacturing tolerance. Blade stiffnesses would vary with a maximum of - say $\pm 5\%$ from the nominal value. On an operating engine, the situation is different as blades can be damaged or cracked for example due to HCF. In this work, a stiffness mistuning tolerance of $\pm 10\%$ is chosen. Note that in this analysis, mistuning is applied to the entire blade-disk sectors rather than the blades alone.

Forced response parameters

For simplicity, the test case in this work is subjected to a 0 EO excitation, i.e. the excitations of all blades are in phase. This is achieved by means of a point

force located at the edge of each blade tip. Such a 0 EO excitation is not representative for real engine conditions, but constitutes a simple test case. This is not expected to put any constraints on the results or conclusions that are obtained. The rotation speed is 0 in all calculations. A frequency sweep of 100-350 Hz, with a 1 Hz step is performed. Modal damping, ξ , is set to 1% for the current computations.

6.4 Results

Mistuned natural frequencies

Figure 6.2 shows natural frequencies versus mistuning configuration for the first mistuned mode. As expected, there is an excellent agreement between the MR and CBSR methods. The mean mistuned frequency is 250 Hz compared to the tuned value of 253 Hz, and it is evident from the figure that most calculated frequencies are below the tuned frequency. This is due to the fact that the system mistuned natural frequencies are more sensitive to negative than positive blade stiffness perturbations (Moyroud et al (2002)).

Figure 6.2: Frequency of the mistuned bladed disk versus mistuning configuration for the first mistuned mode, obtained with MR and CBSR.

Forced response amplitudes

Figure 6.3 shows the maximum amplitude throughout the disk versus mistuning configuration for the MR and CBSR reduction techniques, respectively. There is a good agreement between the two methods for all

configurations. The largest maximum amplitude is about 0.49 mm for the MR and CBSR methods respectively. The maximum amplitude magnification is thus approximately 1.8, compared to Whitehead's prediction for a 20-bladed disk (Equation 3-1) of 2.7, which is not unreasonable. Note that it cannot be expected to find the maximum possible amplitude magnification by means of 100 random mistuning configurations. Further, Equation 3-1 is expected to occur for the limit case of zero damping and high blade-to-blade coupling, which does not correspond to the case considered in this chapter.

Figure 6.3: Maximum amplitude versus mistuning configuration, obtained with MR and CBSR.

6.5 Comparison of computational costs

Computer hardware

The computer used for the calculations presented here is an SGI – Origin 3000 machine consisting of 96 processors with 96GB shared memory. Each processor has a 400 MHz clock rate and a peak performance of 1.0 GFLOP/second. However, the current calculations have been performed with 1 processor only.

Comparison of MR and CBSR

Table 6.2 shows the CPU time for the analysis methods / types involved in the calculations described above. For 100 mistuning configurations, the free vibration calculations take a total of 17 min for the MR method and 28 min for

the CBSR method. The forced response calculations with 401 excitation frequencies take a total of 2h 45 min for the MR method and 18h 47 min for the CBSR method. The MR method is significantly faster than the CBSR method. Considering that the accuracy in numerical predictions is the same in both cases, the MR method is preferable.

Analysis type	MR (80 DOF)	CBSR (880 DOF)
Free vibration		
Static + cyclic symmetric modal analysis	2.55 s	2.55 s
Full assembly modal analysis	10.4 s	16.9 s
Nb. of modes obtained	80	140
CPU time / mode	0.13 s	0.12 s
Forced response		
Nb. of excitation frequencies	401	401
CPU time / frequency	0.247 s	1.687 s

Table 6.2: Comparison of CPU time.

Efficiency of the CBSR method

The efficiency of the CBSR method is highly dependent on the number of left and right hand side boundary nodes of the reference sector, as this determines the number of static modes required in the modal basis. The cyclic symmetric plate considered in this work has very few such boundary nodes (30) compared to the total amount of nodes per substructure (209). This low inter-sector coupling makes the CBSR method more attractive compared to cases with an increased number of boundary nodes.

Computational optimisation

It is important to emphasise that the optimised calculation procedure for reduced matrices, as described in chapter 5.1, has no advantage in the calculations performed in this case. For a given mistuning configuration, contributions to the reduced matrices are evaluated for the limited set of FE elements, which are mistuned. However, since all the elements of the bladed disk are mistuned in each configuration, the complete matrices must be calculated each time. Thus, the Monte Carlo simulations performed here represent a “worst case” regarding computational efficiency. In cases where a limited set of finite elements is mistuned (e.g. mistuned blades on a tuned disk), the efficiency will be higher.

6.6 Summary

Two finite element reduction techniques previously validated against direct FEM, the Craig and Bampton substructuring and reduction method (CBSR) and the Modal Reduction technique (MR), are applied to predict mistuned natural frequencies and resonant amplitudes of 100 random mistuning configurations on a simple test case. As previous studies have indicated, no noticeable differences in accuracy are detected for the current applications, while the modal reduction technique is significantly more efficient.

7 VALIDATION AGAINST EXPERIMENTS

7.1 Introduction

This chapter aims to compare mistuned natural frequencies and mode shapes predicted by the MR and CBSR reduction techniques (see chapter 3.3 and 5.1) with experimental measurements performed on a test-blisk (labelled here as blisk2). The measurements were performed at Imperial College (London, England) within the European project ADTurBII (Aeromechanical Design of Turbine Blades II). The experimental campaign, and thus also the numerical analysis presented in this chapter, focuses on the first bending mode family. This mode family is relatively isolated from the higher order modes of the blisk considered, such that interaction with the higher order modes can be expected to be limited. It is evident that for other cases, where e.g. frequency veerings may occur, the effects of mistuning may be different compared to the observations made in this chapter, due to a higher degree of coupling between different mode families, etc.

Natural frequencies of the blisk were measured by impact testing, where the response of one blade (tip) was measured by LDV (directed parallel to the axis of rotation). Mode shapes were measured by a circular scan LDV (blade tips only). See reference (Stanbridge et al (2002)) for more details about the experimental method. It should also be noted that the initial state of the test piece was checked and found to be satisfactory tuned. This was done by rotating a disordered mistuning pattern by 3 blades, and verifying that the changes in mode shapes were insignificant.

Note that although the ultimate goal of bladed disk designers is the prediction of mistuned forced response amplitudes, the prediction of frequencies and mode shapes still gives an indication of the capabilities of the methods. However, comparisons of forced response amplitudes, which involve the effects of damping and forcing fields, are necessary to complete the validation. This is left for future work, as such experimental results on this geometry are not available at the present time.

7.2 Bladed disk model

Figure 7.1 shows the FE model of blisk2. Only one cyclic symmetric sector is modelled. Increasing the number of elements by 100% results in negligible changes in natural frequencies (less than 0.1%), thereby verifying mesh convergence. Note that there is a small gap between the shrouds of adjacent blade-disk sectors. The test piece is clamped to its shaft by an expanding mandrel in its bore, and thus, the corresponding (surface) nodes of the FE model are treated accordingly. The material of the test piece is unhardened carbon (tool) steel with nominal material properties $E = 210$ GPa, $\rho = 7800$ kg/m³, $\nu = 0.3$.

However, the density has been modified according to measurements (2.6% decrease), and the stiffness has been correlated to yield a matching of numerical and measured tuned natural frequencies (2.5% decrease). The blade tip holes existing on the test piece (for mounting mistune masses) have been incorporated into the model by decreasing the density of the blade tips corresponding to the metal removed (affected volume indicated in Figure 7.1).

Figure 7.1: FE model of blisk2 (one cyclic sector). Blade tip holes (for mounting mistune masses) are modelled as a density decrease of the indicated volume.

The updated density is found by the following simple relation:

$$\rho_{cub} = \rho \left(1 - \frac{V_{cyl}}{V_{cub}} \right) \quad (7-1)$$

where ρ is the density of the blisk, ρ_{cub} is the modified (artificial) density of the blade tip volume V_{cub} , and V_{cyl} is the volume of the cylindrical hole. This procedure is justified by the fact that the current analysis focuses on the first bending modes only. The current simplified model may introduce errors in the modal characteristics of the torsion modes due to a different rotational inertia compared to the real test piece, but is not expected to affect the respective bending modes.

Figure 7.2 shows the deviation between numerical and experimental tuned frequencies (f_{num} and f_{exp}) of the 2-12 ND first bending modes, before ("original") and after update of the stiffness of the FE model (The frequencies of the 0- and 1- ND modes have not been measured). The frequency deviation is computed as follows:

$$\frac{(f_{num} - f_{exp})}{f_{exp}} \cdot 100\% \quad (7-2)$$

The curves labelled “Clamped” constitute the operating conditions, as described above, while the curves labelled “Free” are obtained without any clamped boundary conditions. For the clamped case, it is evident that while there is a fairly good match between numerical and experimental results for the 3-12 ND modes (less than 0.5% deviation), a significant frequency deviation occurs for the 2-ND mode. However, when considering the bore free, a good agreement between numerical and experimental frequency is obtained also for the 2 ND mode.

Figure 7.2: Deviation between numerical and experimental frequencies of the 2-12 ND first bending modes.

Thus, it may be assumed that the deviations observed for the clamped case is due to non-perfect clamping at the bore of the experimental test piece, which leads to decreased frequencies compared to the numerical “perfectly clamped” case. Figure 7.3 shows a comparison of numerical and experimental tuned natural frequencies for the first four mode families (values are scaled due to confidentiality). The frequency deviations for the 1T modes follow the same trend as for the 1F modes, which was discussed above. The trend observed for the 2F modes is somewhat different, however, as the predicted frequencies for the lower (2-5) ND modes are significantly lower than the measured values. The reasons for these differences are unclear.

Figure 7.3: Comparison of numerical and experimental tuned natural frequencies (values are scaled) for the first four mode families (a). Deviations in numerical frequencies compared to the experimental values are shown in (b).

However, considering the good match for the other modes, this deviation is not considered to be important in the present context. The natural frequencies of these (2F) modes are well separated from the 1B modes, and thus, it can be expected that their influence on the mistuned 1B modes is negligible.

Further, Figure 7.4 shows the first tuned mode family represented as relative axial displacement versus radius of a chosen blade-disk-sector, obtained by FE model and measurements (Radial scale not shown due to confidentiality). A good match is obtained for all modes, which indicates a good quality of the current FE model. Note that the measured mode, which matches the predicted 1-ND mode has been experimentally identified as a 2-ND mode, and that no measurements matches the predicted 2-ND mode. This can be due to the fact that the clamping did not put sufficient constraints during measurements of the 2-ND mode (see discussion above regarding the natural frequency of the 2-ND mode).

Considering that the deviations observed for the lower nodal diameter modes are assumed to be due to non-perfect clamping of the test piece, the current match between predicted and measured tuned frequencies and mode shapes is considered to be satisfactory for the subsequent mistuning analysis.

Figure 7.4: The first tuned mode family represented as relative axial displacements versus radius of a chosen blade-disk-sector, obtained by FE model and measurements. Radial scale not shown due to confidentiality.

7.3 Modelling mistuning

In the experiments, washers of different weight are mounted on each blade tip with a bolt. This is introduced in the FE model by means of point masses at the centre of each blade tip surface. These point masses vary between 0 and 1.7 grams and are distributed according to the experimental configurations: Single blade mistuning, harmonic mistuning (harmonic 4, 8 and 12), as well as a random mistuning pattern. The random mistuning pattern is of course a deterministic pattern, but is of a “disordered” nature.

7.4 Size of the reduced order models

The accuracy of mistuned frequencies, mode shapes and response amplitudes predicted by the two reduction techniques are dependent on the size of the modal basis. For the MR technique, modes with remote frequencies compared to the frequencies of the modes of interest are not expected to give any significant modal contribution (see chapter 3.3). Keeping this in mind, Figure 7.3 indicates that the 2F modes are insignificant. Further, the modal contribution of the 1E and 1T modes can be expected to be limited. However, noting that the increase in computational cost associated with including additional mode families in the modal basis for the current deterministic mistuning analysis, all the four mode families shown in Figure 7.3 are included for convenience. For the CBSR technique, 14 dynamic modes per substructure have provided good results for several geometries (Moyroud et al (2002)), and it was decided to make an attempt with such a modal basis also for the current analysis.

Thus, for the MR analysis, the modal basis consists of 96 full assembly modes (twin mode shapes included). For the CBSR analysis, the modal basis consists of 396 static modes and 14 dynamic modes per substructure / sector, giving a total of 9840 modes. Thus, the reduction in DOF (degrees of freedom) compared to direct FEM (260640 DOF) is 99.96% for the MR method and 96.23% for the CBSR method, respectively.

7.5 Results

Mistuned frequency predictions obtained with the MR- and CBSR- FE reduction techniques are nearly identical (less than 0.01% difference). The same yields the predicted mistuned mode shapes, as a MAC (Modal Assurance Criterion) number of 1 was computed for the first 24 mistuned modes of the first mode family. Thus, only one set of frequencies and mode shapes (MR results) is included in the analyses below. Notations “SBM”, “sin4”, “sin8”, “sin12” and “ran” will be used for the defined mistuning patterns (see chapter 7.3).

Comparison of mistuned frequencies

Focus is here put on the frequency *splits* for each mode pair. Figure 7.5 shows comparisons of numerical and experimental frequency splits (in Hz) for different mistuning patterns. It is evident from the figure that a good agreement is obtained. Some aspects of the presented plots should be pointed out, however. First, the deviations between numerical and experimental frequency predictions appear to be relatively large for the “SBM” mistuning pattern. However, the total added mass is relatively small in this case, which gives correspondingly small frequency splits. Thus, small deviations in experimental conditions compared to the assumed conditions, may explain the current deviations. Note that Moyroud et al (2002) demonstrated a perfect match in frequency and mode shape for both reduction techniques compared to direct FEM, when considering relatively strong mistuning of one blade only. Further, a very good match is obtained for the other mistuning patterns, which supports the validation of the current methods. For the “sin4” and “random” mistuning configurations, certain relatively large frequency splits predicted by the numerical models are not shown at all for the experiments. This is due to the fact that the respective frequencies were not captured by the measurements (they are indeed expected to exist, however). On the other hand, certain small frequency splits are observed for the real test piece, where the numerical models predict no splits at all. In the numerical model, mistuning is introduced as point masses, as described above. This does not describe the real situation perfectly, as the different combinations of empty blade tip holes, holes with bolts and washers of different sizes etc, introduce effects which are not captured by the simplified numerical mistuning model.

Note that the numerical and experimental frequency deviations compared to their tuned counterparts (not shown here for brevity) also match well. Focusing on the absolute frequency values, the agreement between numerical predictions and experiments, the matching corresponds to the observations made for the tuned system, i.e. deviations less than 0.5% for the 3-12 ND modes.

It is important to emphasize that, based on previous validations, the applied reduction techniques are assumed to be nearly as accurate as the parent FEM. Thus, it is the author's opinion that the current comparison should be viewed more in terms of FEM versus experiments rather than reduction techniques versus experiments. In fact, the FEM itself constitutes a number of idealizations (boundary conditions, material properties etc.) compared to the real test piece, which may explain the deviations observed in Figure 7.5.

Figure 7.5: Frequency splits predicted by MR and CBSR compared to measurements for different mistuning patterns.

Comparison of mistuned mode shapes

In this section, numerical predictions and experimental measurements of mistuned mode shapes resulting from the “random” mistuning pattern are compared. Since focus is put on the first (bending) mode family, the comparison is made in terms of blade tip displacements, i.e. there is only one measurement point on each blade. Figure 7.6 - Figure 7.8 show the results obtained for mistuned modes number 4-12, 14-20, as well as 24. Note that modes number 13 and 21-23 could not be captured in the experimental campaign, and are thus not included in the current figures.

The level of mode distortion / localization increases with increasing mistuned mode number. This can be expected, and is due to the fact that the modal density of the tuned modes increases with increasing number of nodal diameters (see chapter 3.2), which results in a higher level of modal interaction for the mistuned blisk. In fact, mistuned modes number 4-9 or so (corresponding to the tuned 2-4 nodal diameter modes) are relatively undistorted (appearing as approximate sinusoidal waves). Mistuned mode number 24, on the other hand, is highly distorted. In this case, displacements are to a large extent localized around 5 consecutive blades only.

It is evident that there is a good match between predictions (shown as small circles) and measurements (shown as lines) for all the considered mode shapes. Note particularly the excellent match for the highly distorted / localized modes 10-24. Surprisingly, the largest deviations are observed for modes number 6 and 9, which are relatively undistorted. No reasonable explanation has been found for this.

7.6 Summary

Two finite element reduction techniques previously validated against direct FEM, the Craig and Bampton substructuring and reduction method (CBSR) and the Modal Reduction technique (MR), respectively, are validated against experimental measurements. A good match in mistuned frequencies and mode shapes are obtained, which adds to the validation of the methods. No noticeable differences in accuracy between the MR and CBSR techniques are detected.

Figure 7.6: Mistuned mode shapes number 4-11, predictions by the MR technique (circles) and measurements (lines).

Figure 7.7: Mistuned mode shapes number 12-13 and 15-20, predictions by the MR technique (circles) and measurements (lines).

Figure 7.8: Mistuned mode shape number 24, predictions by the MR technique (circles) and measurements (lines).

8 MISTUNING SENSITIVITY STUDY

8.1 Introduction

The objective of this chapter is to obtain increased knowledge about the effect of blade-to-blade coupling strength on the sensitivity of real bladed disk geometries to mistuning. A detailed mistuning sensitivity study is presented, where the blade-to-blade coupling strength is altered by varying the rotation speed of both a shrouded and an unshrouded rotor. Emphasis is also put on a comparison of different techniques to obtain mistuned response statistics. The statistics of the maximum amplitude magnification in a frequency sweep is obtained by means of the methods described in chapter 5. Based on the findings in the two previous chapters, that the MR technique gives as accurate results as the CBSR method at a significantly lower computational cost, only this technique will be considered here.

8.2 Bladed disk models

The current work focuses on a transonic fan belonging to a modern aero-engine, labelled here as “DCAHM”. The fan is considered with continuous part span shrouds (DCAHM-PS) and without shrouds (DCAHM-NS), respectively. Figure 8.1 shows the DCAHM-PS fan. The DCAHM-NS fan has the same disk and blades, the shrouds being removed.

The rotor has 30 blades with an aspect ratio of 3.4. Material properties, geometrical parameters and boundary conditions are listed in Table 8.1. The shrouds of the DCAHM-PS fan are located at 60% of the blade span. These shrouds are modelled as continuous, i.e. with fully stuck shroud interfaces. Modal damping, ξ , is set to 1% for the current computations. The DCAHM-PS rotor exhibits many of the characteristics of modern high-performance turbomachinery designs. The structural and aeroelastic behaviours of the tuned DCAHM-PS rotor have previously been studied in Moyroud et al (1996), Jacquet-Richardet et al (1997) and Moyroud (1998). A study of the sensitivity of the second mistuned mode of the rotor to mistuning strength and centrifugal stiffening was included in Moyroud et al (2002).

8.3 Computation parameters

Engine order and mode shapes

Analysis of the entire range of engine orders is outside of the scope of this work, and focus is thus put on one specific engine order. A 3-EO excitation is chosen for the current analysis. (The 3rd engine order line is the first, with resonant crossings within the operating range for both geometries.) For the mistuned system, the 3-EO excitation can excite any mode, in contrast to the tuned system, where it can only excite 3-ND modes. The current work focuses

Figure 8.1: Finite Element model of the DCAHM-PS transonic fan with continuous part span shrouds, for one sector only (a) and the complete bladed disk (b).

<i>Material properties (titanium):</i>	
Modulus of elasticity (N/m ²)	1.1·10 ¹¹
Density (kg/m ³)	4430
Poissons ratio	0.35
<i>Geometry:</i>	
Number of blades	30
Average radius, hub and tip (m)	0.195 and 0.509
Average blade length (m)	0.314
Average chord (m)	0.090
Stagger angle, hub and tip (deg)	10.8 and 57.2
Aspect Ratio (-)	3.4
Hub-Tip Ratio (-)	0.38
Twist (deg)	46.4
<i>Boundary conditions:</i>	Clamped at inner radius

Table 8.1: Data for the DCAHM-PS and DCAHM-NS fans.

on the effect of the 3-EO on the first (bending) mode family. The excitation field is modelled as an axially directed point force on the leading edge of each blade tip. The location of the force was based on an inspection of the tuned 3-ND mode for the two geometries (DCAHM-NS and DCAHM-PS), which showed maximum in axial displacement components at the leading edges for both cases. A force amplitude of 0.01 N is applied.

It is important to emphasize that the choice of analysing only the 3-EO case puts relatively large restrictions on a possible generalization of the results and conclusions, which are made in this chapter. For other engine orders, results may be different. Especially for higher order modes and in possible frequency veering regions, the mistuning sensitivity behaviour is likely to be different compared to the case considered here. The decision to analyse only the 3-EO case is mainly due to computational cost considerations. However, it is also considered as a natural starting point to look at a relatively simple case, without frequency veerings or other complications. A possible extension of the analysis presented here is thus left for future work.

Frequency sweeps and rotation speed

For the tuned cases, the crossing of the frequency of the 3-ND mode and the 3-EO line occurs at approximately 6000 rpm for the DCAHM-PS fan and 1500 rpm for the DCAHM-NS fan (see Appendix C). The frequency - response curves around these crossings will feature one single response peak. When the rotors are mistuned, on the other hand, it is evident that the frequency - response curves will feature multiple response peaks (see chapter 3.2) at a range of rotation speeds around the respective crossings. However, the respective mistuned resonant peaks are expected to fall within a *relatively* limited speed range, with correspondingly modest variations in the effect of rotation on the respective modal characteristics. Thus, in the current work, the computation of all peaks in a given frequency – response curve are based on the free vibration characteristics of the respective rotors at a fixed rotation speed, i.e. 6000 rpm for the DCAHM-PS fan and 1500 rpm for the DCAHM-NS fan.

Further, as this chapter aims to investigate the effects of rotation speed on the mistuned response behaviour, a series of rotation speeds (at which the free vibration characteristics are computed) are considered for both rotors. These speeds are 0 rpm, 1500 rpm, 3000 rpm, 4500 rpm and 6000 rpm, respectively. Note that the additional speeds cannot be identified as actual crossings in the Campbell diagram. However, this procedure makes it possible to study the effects of rotation speed on the mistuning sensitivity behaviour for a particular mode shape and engine order.

The computational procedure may be summarized as follows. Using the MR reduction technique (chapter 5.1), the mistuned response is obtained by means of the tuned mode shapes, as well as the FE matrices, where rotational contributions to the FE stiffness matrix are included. In this work, these inputs to the MR technique are first computed for the specific rotation speeds defined above, before extracting the maximum resonant peak for each mistuning configuration by means of the MR technique. As described in

chapter 5.4, accurate estimation of the response peaks are found by means of optimisation of the respective frequency - response curve around each mistuned natural frequency. Such optimisation is performed for the first 30 mistuned natural frequencies of the DCAHM-NS rotor, (corresponding to the first mode family) and the first 31 mistuned natural frequencies of the DCAHM-PS rotor (including also the first mode of the second mode family). Note that the first frequency of the DCAHM-PS rotor is significantly lower than the frequencies of the other modes in the first mode family, which indicates that this mode will not yield any significant mistuned response for a 3-EO excitation. Appendix B gives a few examples of frequency – response curves obtained by frequency sweeps, compared to response peaks obtained by curve optimisation.

The excitation field is rotating relative to the rotor in all cases. A backward travelling wave excitation is applied, i.e. travelling in the opposite direction to the rotation of the fan. Note that the rotation speed of the excitation field equals the rotation speed of the fan only for the resonant crossings indicated above (6000 rpm for the DCAHM-PS fan and 1500 rpm for the DCAHM-NS fan), and further, only at the position in the frequency sweep corresponding to the tuned 3-ND mode.

Modal basis

Figure 8.2 and Figure 8.3 show natural frequencies versus the number of nodal diameters for the first four mode families of the DCAHM-PS rotor (0 rpm and 6000 rpm) and DCAHM-NS rotor (0 rpm, 1500 rpm and 6000 rpm). The natural frequencies of the DCAHM-NS fan are densely populated, both at rest and at rotation. Recalling the findings related to the modal basis of the modal reduction technique (see chapter 3.3), it could be expected that a rather limited modal basis would be sufficient for the DCAHM-NS geometry. The natural frequencies of the DCAHM-PS fan are much more distributed, and it could be expected that a larger modal basis is required. Appendix C shows Campbell diagrams for the two fans, which indicate that the considerations above are valid in the entire range of rotation speeds considered, although modal density generally decreases with rotation speed.

In Moyroud et al (2002), 120 full assembly modes in the modal basis (four first mode families) resulted in an excellent agreement with direct FEM for the DCAHM-PS rotor. As the required size of the modal basis is expected to be smaller for the DCAHM-NS rotor compared to the DCAHM-PS rotor, this was used as a starting point for both geometries. An attempt was made for both cases to reduce the modal basis to contain only 60 modes (two first mode families). This was done by comparing the maximum amplitude on the disk for 100 random mistuning configurations (applying a standard deviation of blade stiffnesses of 2%). For the DCAHM-NS rotor, a maximum deviation of 0.08% was detected, thereby justifying the smaller modal basis. For the DCAHM-PS rotor, the maximum error was significantly larger, 22%. Thus, the modal basis applied in the current work consists of 120 (resp. 60) full assembly modes for the DCAHM-PS (resp. DCAHM-NS) rotor. The CPU time required for the analysis of 100 random mistuning configurations is approximately 10 hours for the DCAHM-NS rotor and 12 hours for the DCAHM-PS rotor, respectively.

Figure 8.2: Natural frequencies versus nodal diameter for the first four mode families of the DCAHM-PS rotor, at rest and at 6000 rpm.

Figure 8.3: Natural frequencies versus nodal diameter for the first four mode families of the DCAHM-NS rotor, at rest, 1500 rpm and 6000 rpm.

8.4 Comparison of parameter estimation techniques

Figure 8.4 - Figure 8.7 show the sensitivity of the DCAHM-PS and DCAHM-NS fan geometries to mistuning, at rest and 6000 rpm, respectively. Note that the scales of the axis are different for the two rotors. The 99th percentiles of the cumulative probability distribution are shown, i.e. there is a 99- percent probability of obtaining amplitudes below the respective curves. A comparison of predictions obtained with the ME-, LS-, and MLE- Weibull parameter estimation techniques are shown. Recall from chapter 5.5 that the amplitude magnification predictions are based on estimations of the three Weibull distribution parameters a , b and c . The location parameter (a) is particularly significant since it represents the largest possible amplitude. Note that for the LS technique, this parameter (a) is preset to an approximated value (3.2 for the curves labelled “LS” and 4.9 for the curves labelled “LS,mod”).

Increasing the approximation of the location parameter, a , by 50% (from 3.2 to 4.9) for the LS technique, leads to a maximum increase in magnification of 1.9% for the DCAHM-PS rotor and 0.2% for the DCAHM-NS rotor. Thus, it is evident that the LS technique is relatively insensitive to changes in this parameter, especially in the latter case. The LS technique generally predicts higher magnification values than the other techniques. This can be expected, since the location parameter, a , is preset to a much higher value than the values predicted by the other methods. The ME technique generally gives the lowest magnification predictions, and the MLE technique typically gives predictions somewhere in between. Note, however, that the differences are small (less than 5%).

Certain difficulties were experienced with the ME technique. For four of the sample data (DCAHM-PS at rest, 3% STD (standard deviation), DCAHM-NS at rest/1500 rpm, 8% STD, as well as DCAHM-NS at 6000rpm, 5% STD), no solution was obtained. This may be an indication that the sample size is not sufficiently large for this technique. For the same curve points, the MLE technique shows a jump from the main curve trend (particularly for DCAHM-PS at rest, 3% STD and DCAHM-NS at rest, 2.5% STD). An investigation of the sample data was made in order to search for an explanation to the experienced complications. For some of the respective sample sets, certain magnification values are considerably higher than the rest of the population. However, this occurs also for some of the sample sets where no problems were experienced. Thus, no definite conclusion could be made regarding the reasons for the problems. However, the LS technique appears to be the most robust method for statistical mistuned response predictions involving relatively small sample sizes.

Figure 8.8 - Figure 8.11 show the location parameter a , applied / predicted by the LS-, ME-, and MLE- parameter estimation techniques. The maximum magnification observed for each sample set is also included for comparison. It is evident that the predictions made by the ME- and MLE- techniques are fluctuating, which indicates a poor repeatability for the current sample size. This is particularly the case for the ME-technique, which also gives very large

Figure 8.4: Sensitivity of the DCAHM-PS fan to mistuning at 0 rpm.

Figure 8.5: Sensitivity of the DCAHM-NS fan to mistuning at 0 rpm.

Figure 8.6: Sensitivity of the DCAHM-PS fan to mistuning at 6000 rpm.

Figure 8.7: Sensitivity of the DCAHM-NS fan to mistuning at 6000 rpm.

Figure 8.8: Location parameter of the Weibull (Type III) probability distribution, DCAHM-PS at 0 rpm.

Figure 8.9: Location parameter of the Weibull (Type III) probability distribution, DCAHM-NS at 0 rpm.

Figure 8.10: Location parameter of the Weibull (Type III) probability distribution, DCAHM-PS at 6000 rpm.

Figure 8.11: Location parameter of the Weibull (Type III) probability distribution, DCAHM-NS at 6000 rpm.

parameter predictions for certain sample data – as high as 3.9 and 7.1 for the DCAHM-PS rotor (outside the plot limits). Note again that no solution was obtained for certain of the sample data with this method, as mentioned above.

The maximum possible amplitude magnification may in fact be assumed to be independent of the STD of mistuning. Even if the standard deviation of mistuning is very low, extreme cases may still occur. Thus, to preset the maximum magnification level to a fixed value for all STD's of mistuning, as for the LS technique, can be considered appropriate for the current analysis. Note, however, that the magnification limit can be expected to depend on the level of blade-to-blade coupling, as discussed further below.

8.5 Validation of sample size and optimisation technique

The sample size is increased from 100 to 1000 for one chosen value of mistuning strength (at 2% STD) for the DCAHM-NS rotor at rest (indicated in Figure 8.5 as large rectangles), and it is evident that this has a negligible effect on magnification predictions for all methods (less than 1.4%). As there is no reason to believe that the effect on the other curve points should be any different, this is considered as a validation of the sample size of 100, which leads to a significant computational saving compared to larger sample sizes. A constant excitation frequency step of 0.1Hz is applied for one chosen value of mistuning strength (at 2% STD) for the DCAHM-PS- and DCAHM-NS-rotors, respectively (indicated in Figure 8.4 and Figure 8.5 as large triangles), in order to compare with the optimisation technique. The results are nearly identical, although the optimisation technique gives slightly higher values, as expected.

8.6 Mistuning sensitivity analysis

As above, the analyses here focus on the 99th percentile cumulative probabilities of amplitude magnification. Figure 8.12 shows the sensitivity of the DCAHM-PS and -NS fans to mistuning at 0 rpm, 1500 rpm, 3000 rpm, 4500 rpm and 6500 rpm, respectively. All curve points are obtained with the LS parameter estimation technique. Focus is first put on the sensitivity of the DCAHM-PS fan (*with* shrouds) and the DCAHM-NS fan (*without* shrouds) at rest. The sensitivity of amplitude magnification to mistuning is relatively high near the tuned value for both geometries. This sensitivity flattens out and vanishes at higher mistuning strengths, consistent with previous findings in the literature (e.g. Castanier and Pierre (1997, 1998)). A peak of the 99th percentile curves is apparent in both cases, although this peak is not very distinct for the DCAHM-NS case. The peaks occur at standard deviations of blade stiffnesses of approximately 2% and 5% for the two cases, respectively, which is also consistent with previous findings. Predictions of peaks in the range (1-2)% are typical. Note, however, that some authors have used the STD of blade *frequencies* as a measure of mistuning strength rather than the STD of blade *stiffnesses*. E.g. 1% and 2% STD of the former correspond to approximately 2% and 4% STD of the latter. Also the amplitude magnifications

observed are consistent with previous findings, although comparisons are difficult between different cases. E.g., Castanier and Pierre (1997, 1998) predicted peak values of the 99th percentile in the range 1.5 – 2.0, compared to approximately 1.3 and 2.3 in the current cases. Note that the theoretical limit predicted by Whitehead (1966, 1998) for a blade disk with 30 blades is 3.2, and is expected to occur for the case of zero damping and for the 0 and 15 ($N/2$) nodal diameter modes. Thus, the theoretical limit in the present case ($EO = 3$ and a modal damping of 1%) can be expected to be lower than the value of 3.2. In fact, the maximum magnification factor derived by Whitehead (1976) and Kenyon et al (2002) for a 3EO excitation is 2.4 (see chapter 3.2), which constitutes a good match with the current results.

The phenomenon of a peak or flattening out of mistuning sensitivity has previously been explained (Ottarson and Pierre (1995)) by an energy augmentation mechanism. In cases where an increase in mistuning strength leads to decreased mistuning sensitivity, additional mistuning will prevent the propagation of energy-carrying waves to the localized blade (see also chapter 3.2).

Figure 8.12: Sensitivity of the DCAHM-PS and -NS fans and to mistuning at 0 rpm, 1500 rpm, 3000 rpm, 4500 rpm and 6000 rpm.

It is evident from Figure 8.12 that amplitude magnification tends to show an increasing trend at the upper limit of the mistuning strengths considered. E.g. for DCAHM-NS at rest, this is the case for mistuning strengths within the range of approximately $0.06 < \text{STD} < 0.08$. Noting that the mistuning strengths in these regions are relatively large (compared to the mistuning strengths where the magnification peaks occur), the observed trend cannot be explained by the traditional mistuning effects, such as mode localization or the energy augmentation mechanism through blade-to-blade coupling. Rather, it may be assumed that the observed phenomenon is simply due to increased variations in the mistuned natural frequencies. As the standard deviation of mistuning increases, lower and lower mistuned natural frequencies are obtained. Excluding traditional mistuning effects, amplitude will increase with decreasing natural frequency. This assumption will be further justified in chapter 8.9.

8.7 Influence of shrouds

Focus is now put on the *differences* between the two cases at rest (Figure 8.12, curves indicated by circles). Although the sensitivity to mistuning is fairly similar for both cases near the tuned condition, the sensitivity flattens out and vanishes at a higher standard deviation of mistuning for the DCAHM-PS rotor (*with* shrouds) compared to the DCAHM-NS rotor (*without* shrouds), leading to a significantly larger peak amplitude magnification in the former case. This behaviour can be expected and may be explained as follows. While mode localization has been shown to decrease with increasing coupling, this is not necessarily the case for amplitude magnification. Studies have indicated that the mean and standard deviation of the amplitude magnification factor exhibits a peak at an intermediate level of coupling, while the maximum possible factor *increases* monotonically with coupling (see chapter 3.2). In this light, it can thus be expected that the 99th percentile of amplitude magnification in fact exhibits a peak at an intermediate level of coupling for a given STD of mistuning, while the peak 99th percentile of amplitude magnification over a *range* of mistuning strengths increases monotonically with coupling. Coupling is larger for the DCAHM-PS rotor compared to DCAHM-NS, and thus, amplitude magnifications are also larger, i.e., the low coupling for DCAHM-NS does not enable vibration energy to be transported efficiently to the maximum responding blade.

It has been shown that it is the *ratio* of mistuning strength to blade-to-blade coupling strength, rather than coupling alone, which is the key parameter governing mode localization (see chapter 3.2). Blade-to-blade coupling is larger for the DCAHM-PS rotor compared to DCAHM-NS, and thus, a larger mistuning strength is required to obtain the same mistuning to coupling ratio. This explains the qualitative observation of the shift in the peak location towards a higher mistuning strength for the DCAHM-PS rotor.

The estimation of all amplitude magnifications are based on the tuned amplitude in the respective cases, and the tuned amplitude of the DCAHM-PS rotor is 69% lower than for DCAHM-NS. Thus, even if the peak amplitude

magnification is higher in the former case, the absolute mistuned amplitude peak is lower (45%).

8.8 Influence of rotation speed

Focus is now put on the two fan geometries under rotation (still Figure 8.12). Rotation has a significant effect on mistuning sensitivity for both cases. For both rotors, the peak amplitude magnification is shifted to higher mistuning strengths as the rotation speed increases (from 0 rpm to 6000 rpm), i.e. it occurs at a higher standard deviation of mistuning. The peak locations are shifted from approximately 5% to 9% for the DCAHM-PS rotor and from approximately 2% to 6% for the DCAHM-NS rotor. These shifts can be explained by increased coupling levels under rotation - a phenomenon, which can be spotted as spreading of frequency lines in the Campbell diagram for increasing rotation speed. This will be further discussed and justified in chapter 8.10. However, when coupling increases, a larger mistuning strength is required in order to obtain the same mistuning to coupling ratio as for the cases at rest.

Looking at the peak amplitude magnification level, the effect of rotation is different for the two cases. For the DCAHM-NS rotor, peak magnification increases from approximately 1.3 to 1.6. This is similar to the observation made for the comparison of the two cases at rest, i.e. increased peak magnification due to increased coupling. For the DCAHM-PS rotor, on the other hand, peak magnification is nearly unchanged, compared to the case at rest. Amplitude magnification is closer to the theoretical limit (2.4 or 3.2, see chapter 8.6) for DCAHM-PS compared to DCAHM-NS, and smaller effects of increased coupling may be expected as we approach this limit. It is also important to emphasize that blade-to-blade coupling through the shrouds and disk may behave differently. Above, it was argued that the 99th percentile of amplitude magnification exhibits a peak with respect to blade-to-blade coupling for a given STD of mistuning. The current results indicate that coupling for the DCAHM-PS rotor (resp. DCAHM-NS) is lower (resp. higher) than the value corresponding to such a peak. Note that this observation is not valid in regions where amplitude magnification decreases with mistuning strength (above approximately 5% for the DCAHM-PS rotor).

As expected, the effect of rotation speed on coupling, and thus on mistuning sensitivity, is not linear. It is evident that the results obtained at 1500 rpm are nearly identical to the results obtained at rest. This is because the natural frequencies (and thus also coupling) are nearly unaffected by such a low speed. Thus, the resonant speed of the DCAHM-NS rotor (1500 rpm, when considering the first bending mode) has little effect on its sensitivity to mistuning. As the resonant speed of the DCAHM-PS rotor is significantly higher, the effect on mistuning sensitivity is correspondingly larger.

As above, it should be noted that the estimation of all amplitude magnifications are based on the tuned amplitude in the respective cases. Rotation speeds of 1500 rpm and 6000 rpm result in 22% and 84% reduction in tuned amplitudes for the DCAHM-NS rotor compared to the case at rest,

implying similar reductions in absolute mistuned amplitude peaks. The same comment can be made for the DCAHM-PS rotor, where the rotation speed of 6000 rpm results in a reduction in tuned amplitude of 54%. Comparing the two cases at their resonant speeds, the absolute mistuned amplitude peak is significantly lower for DCAHM-PS compared to DCAHM-NS (68%).

8.9 Analysis of sample result sets

Before elaborating further on the mistuning sensitivity behaviour observed above, it is important to understand the nature of the underlying sample results sets. Recall that Monte Carlo simulations with a sample size of 100 were performed for each standard deviation of mistuning. The two sections below aim to illustrate the nature of these results.

Amplitude magnification versus mistuned mode number

In order to investigate the level of modal interaction, focus is put on the mistuned mode numbers, which constitute the maximum amplitude magnification for each mistuning configuration. These mistuned mode numbers are simply found by comparing the excitation frequencies with the natural frequencies and identifying the best match. Because of damping, resonant excitation frequencies typically move relative to the corresponding natural frequencies. However, as this deviation is low compared to the natural frequency separation in all cases, this procedure is found to be satisfactory. Figure 8.13 - Figure 8.16 show the respective plots for the DCAHM-PS and DCAHM-NS fans at 0 rpm and 6000 rpm.

Looking first at the DCAHM-PS rotor at 0 rpm (Figure 8.13), it is evident that mistuned modes number 6 and 7 (corresponding to the tuned 3-ND modes) give the highest amplitude magnifications for all mistuning configurations up to a STD of approximately 5%. This is expected, as the natural frequencies of the DCAHM-PS rotor are fairly distributed (i.e. low modal density), thereby limiting modal interaction. At higher mistuning strengths, however, modal interaction occurs to a larger and larger extent, moving maximum amplifications towards lower mistuned mode numbers. Note that the first mistuned mode never constitutes the maximum amplitude magnification. This is because its natural frequency is very separated from the other natural frequencies, and thus does not interact to a significant extent with the other modes, which leaves it relatively unaffected by a 3-EO excitation.

Focusing next on the DCAHM-NS rotor at 0 rpm (Figure 8.14), the behaviour is quite different. In this case, the natural frequencies are much more densely populated than for the DCAHM-PS rotor, and thus, a larger amount of modal interaction takes place. In fact, the maximum amplitude magnifications are distributed over a large range of modes even at the lowest mistuning strength. As mistuning strength increases, modal interaction increases even further, and the maximum magnification move more and more towards lower mode numbers, as was observed for the DCAHM-PS rotor. At high mistuning strengths, maximum magnifications are to a large extent centred on the first mistuned mode.

Figure 8.13: DCAHM-PS at 0 rpm – Maximum amplitude magnification for each mistuning configuration, plotted against corresponding mistuned mode number.

Figure 8.14: DCAHM-NS at 0 rpm – Maximum amplitude magnification for each mistuning configuration, plotted against corresponding mistuned mode number.

Figure 8.15: DCAHM-PS at 6000 rpm - Maximum amplitude magnification for each mistuning configuration, plotted against corresponding mistuned mode number.

Figure 8.16: DCAHM-NS at 6000 rpm - Maximum amplitude magnification for each mistuning configuration, plotted against corresponding mistuned mode number.

Rotation (at 6000 rpm) has a significant effect on the modal response for both rotors compared to the cases at rest. For the DCAHM-PS rotor (Figure 8.15), the behaviour is similar to the case at rest, but shifted to higher mistuning strengths. Now, maximum amplitude magnification is centred on mistuned mode number 7 and 8 (corresponding to the tuned 3-ND mode) up to higher mistuning strengths. Note that the tuned 0-ND mode of the second mode family occurs as tuned mode number 6, and that mistuned mode number 6 gives the highest response magnification for certain mistuning configurations from a STD of approximately 5%. A further spread in the highest responding modes does not occur before a STD of approximately 9-10%.

The effect of rotation speed on the DCAHM-NS rotor (Figure 8.16) is significantly different compared to DCAHM-PS. At low mistuning strengths, maximum amplitude magnification is centred on the high-order modes, typically mistuned mode number 25 and up-wards. As mistuning strength increases, maximum magnifications move more and more towards lower mistuned mode numbers, as was observed above.

Amplitude magnification versus excitation frequency

Figure 8.17 - Figure 8.20 show the maximum amplitude magnification in a frequency sweep plotted versus the corresponding excitation frequency. At the lowest mistuning strength (0.5%), the maximum amplitude magnifications occur at a quite narrow band of excitation frequencies for all cases. This can be expected, as such a low mistuning typically results in modest deviations in natural frequencies. For the DCAHM-PS rotor, the natural frequencies are relatively distributed, but the response is exclusively centred on mistuned mode number 6 and 7, as described above, explaining the narrow frequency band. For, the DCAHM-NS rotor, a larger extent of modal interaction takes place, but as the natural frequencies are very densely populated (which is the reason for high modal interaction in the first place), the result is again, a narrow frequency band.

When gradually increasing the standard deviation of mistuning, amplitude magnifications increase substantially, while generally maintaining a relatively narrow frequency band. When magnification reaches a maximum, a gradually spreading of excitation frequencies is observed. This can be explained by increased modal interaction, i.e. maximum amplification levels are spread over a larger and larger range of mistuned modes, as described above, and further, increased mistuned frequency offsets compared to their respective tuned values. Together with this frequency spread, a general decrease in amplitude magnification is observed, consistent with the observations made in chapter 8.6, i.e., a decrease in mistuning sensitivity is obtained for sufficiently large mistuning strengths.

When increasing the mistuning strength further, an increase in magnification levels are obtained, as mentioned in chapter 8.6. Interestingly, the corresponding curve points occur more and more along what appears to be a straight line. This phenomenon is particularly apparent for DCAHM-NS at rest (Figure 8.18), for $0.06 < \text{STD} < 0.08$. Noting that peak amplitude magnification

Figure 8.17: DCAHM-PS at 0 rpm - Maximum amplitude magnification for each mistuning configuration, plotted against corresponding excitation frequency.

Figure 8.18: DCAHM-NS at 0 rpm - Maximum amplitude magnification for each mistuning configuration, plotted against corresponding excitation frequency.

Figure 8.19: DCAHM-PS at 6000 rpm - Maximum amplitude magnification for each mistuning configuration, plotted against corresponding excitation frequency.

Figure 8.20: DCAHM-NS at 6000 rpm - Maximum amplitude magnification for each mistuning configuration, plotted against corresponding excitation frequency.

for this case occurs for a mistuning strength of $STD=0.02$, the mistuning strengths within the range $0.06 - 0.08$ are relatively large, such that the increasing magnification trend cannot be explained by mode localization or energy augmentation effects. Rather, it may be assumed that the observed phenomenon is simply due to increased variations in the mistuned natural frequencies, as mentioned in chapter 8.6. Considering a tuned bladed disk excited by a given engine order excitation field, the amplitude is inverse proportional to its (uniform) modulus of elasticity E . Thus, excluding variations in any other properties, a given resonant amplitude is inverse proportional to the square of the natural frequency of the corresponding (tuned) mode shape. When mistuning is introduced, this relation is naturally not valid anymore. However, for sufficiently high mistuning strengths such that the traditional mistuning effects can be excluded, as mentioned above, the amplitude of the mistuned bladed disk can be expected to be largely governed by the magnitude of its natural frequencies. Thus, the trends observed above are actually not linear, but rather, amplitude can be assumed to be inverse proportional to the square of the resonant frequency. This is evident from Figure 8.21, which shows scatter plots of amplitude magnification versus corresponding excitation frequency for DCAHM-PS, $STD = 0.12$ at 0 rpm (a) and 6000 rpm (b), as well as DCAHM-NS, $STD = 0.08$ at 0 rpm (c) and 6000 rpm (d).

Figure 8.21: Scatter plots of amplitude magnification versus corresponding excitation frequency for DCAHM-PS at 0 rpm (a) and 6000 rpm (b) ($STD = 0.12$), as well as DCAHM-NS at 0 rpm (c) and 6000 rpm (d) ($STD = 0.08$). Trend lines showing amplitude inverse proportional to the square of the resonant frequency have been added (The proportionality constant differs between the curves).

These plots are also shown in Figure 8.17 - Figure 8.20. Here, however, trend lines have been added, showing amplitude as inverse proportional to the square of the resonant frequency (Note that the proportionality constant differs between the curves). Particularly for DCAHM-NS at rest, the scattered points tend to line up nicely along the given trend line. The same phenomenon is also apparent for the other cases, just that increased coupling shifts all the observed trends to higher mistuning strengths, especially for the two cases at rotation. It may be assumed, however, that increasing the mistuning strength even further for these cases would result in a similar clear trend as observed for DCAHM-NS at rest.

Thus, the observations made in chapter 8.9, that amplitude magnification levels exhibits an increasing trend at very high mistuning strengths, has been explained.

8.10 Coupling estimations and correlation of results

In order to make a quantitative comparison of the cases above, a measure of blade-to-blade coupling is required. The ratio of the blade-alone frequency to the bladed disk system frequency of the mode of interest is often used as a measure of blade-to-blade coupling (Srinivasan (1997)). For the DCAHM-NS rotor, the blade-alone frequency can be found simply by clamping the blade at its root. For the DCAHM-PS rotor, this is more complicated, as there is blade-to-blade coupling both through the disk and through the shrouds.

In order to obtain estimates of blade-to-blade coupling, which are comparable between the two cases considered here, it is proposed to define total blade-to-blade coupling (through disk and shrouds) as

$$coupling(n) = \frac{f(n+1) - f(n)}{f(n)} \quad (8-1)$$

where f is natural frequency and n is the number of nodal diameters. However, which modes should the coupling estimations focus on? Even though the current study focuses on a 3-EO excitation, the resulting mistuned forced response patterns consist of several nodal diameter components, and thus, the corresponding blade-to-blade coupling levels will typically be affected by each component. Figure 8.22a shows estimated coupling for the 0-14 nodal diameter modes for DCAHM-PS and DCAHM-NS at 0 rpm and 6000 rpm. Note that for some nodal diameters, coupling exhibits negative values. This is especially the case for DCAHM-PS, nodal diameter 8 and higher. However, the corresponding absolute values are relatively low. Coupling is expected to be dominated by the lower nodal diameter modes and focus is thus put on these modes. Table 8.2 shows estimated coupling values for the 0-3 ND modes.

Coupling is significantly higher for DCAHM-PS compared to DCAHM-NS due to the shrouds for all modes included in Table 8.2. Above, it was argued that

for the given operating conditions and computation parameters, coupling increases with rotation speed for both fan geometries. In Table 8.2, coupling values, which *increase* with rotation speed appear in *italic*. Coupling for the 1-3 ND modes of the DCAHM-PS rotor increases significantly with increasing rotation speed, due to stress stiffening. This is consistent with the observation in Moyroud et al (2002), which showed that mode localization decreased significantly with increasing rotation speed for the DCAHM-PS rotor. For the DCAHM-NS rotor, coupling increases with rotation speed for the 0-1 ND modes.

Case	Speed (rpm)	Coupling			
		0 ND	1 ND	2 ND	3 ND
Mode #		1	2/3	4/5	6/7
DCAHM-PS	0	0.89939	<i>0.00440</i>	<i>0.02986</i>	<i>0.02840</i>
	6000	0.86127	<i>0.01691</i>	<i>0.05836</i>	<i>0.04833</i>
DCAHM-NS	0	<i>0.00649</i>	<i>0.00012</i>	0.00063	0.00050
	1500	<i>0.00793</i>	<i>0.00024</i>	0.00053	0.00041
	6000	<i>0.03005</i>	<i>0.00273</i>	0.00010	0.00001

Table 8.2: Estimates of blade-to-blade coupling for the 0-3 ND, 1st bending modes, obtained by equation 8-1.

An attempt is now made to compare the current coupling estimations with coupling based on the clamped blade alone frequencies, as mentioned above, which may be defined as follows:

$$coupling(n) = \frac{f_{clamp} - f(n)}{f(n)} \quad (8-2)$$

Note that this defines a frequency deviation rather than a frequency ratio, and is used in order to enable a comparison with the mistuning strength estimations used in the current work. For the DCAHM-NS rotor, the value of f_{clamp} is obtained by clamping the blade at its root. For the DCAHM-PS rotor, the value of f_{clamp} is obtained by clamping the blades at the root and at the blade-shroud interface – constituting a perfectly rigid shroud case. Figure 8.22b shows the estimated values. Looking at the variation of coupling with the number of nodal diameters, the trend observed for the DCAHM-NS rotor correspond well with Figure 8.22a. The trend observed for the DCAHM-PS rotor, however, is somewhat different, especially for nodal diameter 1.

The natural frequencies of the DCAHM-PS rotor are fairly distributed (i.e. low modal density), which limits modal interaction. In fact, it was observed that mistuned modes number 6 and 7 (corresponding to 3 nodal diameters in the tuned case) gave the maximum amplitude magnification for all mistuning configurations up to a standard deviation of 5% for 0 rpm and 8% for 6000

Figure 8.22: Coupling for the DCAHM-PS and DCAHM-NS rotor at 0 rpm and 6000 rpm, estimated by (a) Equation 8-1 and (b) Equation 8-2.

rpm, indicating that the coupling level may be well represented by coupling for the 3-ND mode. The natural frequencies of the DCAHM-NS rotor are much more densely populated than for the DCAHM-PS rotor, and thus, a larger amount of modal interaction can be expected. Following along the analysis above, the maximum amplitude magnifications were centred on lower mistuned mode numbers (typically mistuned mode number 1). An exception to this trend was observed for 6000 rpm – near the tuned condition (up to approx. 1.5% STD) where the maximum amplitude magnifications were centred on much higher mode numbers. However, based on the main trend observed, and keeping the above-mentioned exception in mind, it was decided to base coupling estimation for the DCAHM-NS rotor on the 0-ND mode.

Figure 8.23 shows the above results (including only the LS parameter estimation technique), plotted as amplitude magnification versus mistuning-to-coupling ratio. Coupling estimations are obtained with (a) equation 8-1 and (b) by equation 8-2. Considering first Figure 8.23a, it is observed that the peak or flattening out of mistuning sensitivity occurs at approximately the same mistuning-to-coupling ratios for all cases. The curves for the DCAHM-PS rotor are remarkably similar, due to the unchanged magnification levels. The curves for the DCAHM-NS rotor are shifted vertically, due to the different magnification levels, as discussed in chapter 8.7. Note particularly the behaviour for the case at 6000 rpm, near the tuned condition, showing a very steep curve trend. This region corresponds to the exception mentioned in chapter 8.9, where maximum amplitude magnifications were centred on higher mode numbers. Here, lower blade-to-blade coupling can be expected, which would decrease the curve slope in the respective region. Figure 8.23b gives a poorer match in mistuning-to-coupling ratios, at which the peak or flattening out of mistuning sensitivity occurs.

It is important to emphasize that due to the uncertainties in the blade-to-blade coupling strengths, the current results should be considered as a qualitative (rather than quantitative) demonstration of the effects of coupling on the respective peak locations. In particular, it is evident that the high level of modal interaction for the DCAHM-NS fan complicates the mistuned response behaviour significantly compared to DCAHM-PS, and that the corresponding analysis is more difficult in this case.

8.11 Summary

Mistuning sensitivity is modelled as the dependence of amplitude magnification on the standard deviation of blade stiffnesses. The modal reduction technique is solved for sets of random blade stiffnesses with various standard deviations. In order to reduce the sample size, the statistical data is fitted to a Weibull (type III) parameter model. Three different parameter estimation techniques are applied and compared: a least squares linear regression-, a moment estimation- and a maximum likelihood- technique. The least square method appears to be the most robust method for statistical mistuning analysis with small sample sizes.

Figure 8.23: Sensitivity of the DCAHM -NS and -PS fans to mistuning at various rotation speeds, plotted as amplitude magnification versus mistuning-to-coupling ratio obtained by equation (a) 8-1 and (b) 8-2. Obtained with the LS parameter estimation technique.

The key role of blade-to-blade coupling, as well as the ratio of mistuning to coupling, is demonstrated for a fan geometry, considered with- and without-shrouds, respectively. The fan with shrouds features a significantly higher amplitude magnification due to mistuning compared to the fan without shrouds, due to higher blade-to-blade coupling. It is also found that amplitude magnification is largely dependent on the rotor speed, due to an associated variation in blade-to-blade coupling strength. Focusing on the resonant speeds of the respective fans, the fan without shrouds is less or equally sensitive to mistuning than the fan with shrouds in the entire range of mistuning strengths considered.

The current results indicate that the 99th percentile amplitude magnification exhibits a peak with respect to blade-to-blade coupling for a constant standard deviation of mistuning. However, considering the standard deviation of mistuning variable, the 99th percentile amplitude magnification increases monotonically with increasing blade-to-blade coupling. Increased coupling tends to shift the mistuning strength at which a peak in amplitude magnification occurs towards higher values. The study indicates that the respective peak occurs at approximately the same mistuning-to-coupling ratio, however. Note that the current study focused exclusively on the effect of one specific engine order on the mistuned response of the first bending modes of the airfoils.

8.12 Significance of the results

Manufacturing tolerances are often maintained by means of measuring clamped “blade-alone” frequencies of certain blade samples. High manufacturing precision, as well as tight tolerances typically lead to relatively small standard deviations of blade frequencies - say 1%, corresponding to 2% STD of blade stiffnesses. During service, on the other hand, anything may occur, indicating that a significantly larger mistuning range must be considered – at least the range considered here (up to 8% and 12% standard deviation of mistuning, for the two fan geometries, respectively).

In this light, some comments can be made regarding the fan geometries with and without shrouds at their respective resonant rotor speeds. (The other speeds are not considered here, as they are not representative of engine conditions.) Very close to the tuned condition (up to a STD of blade stiffnesses of approximately 0.5%), the mistuning sensitivity is fairly similar for both cases. However, in the remaining range of mistuning strengths, the fan *without* shrouds is less sensitive to mistuning than the fan *with* shrouds. This is particularly important when considering in-service wear, but is also relevant regarding manufacturing tolerances.

Note that this is not an attempt to argue for or against shrouds in bladed disk design. Shrouds are introduced both to prevent flutter and forced response, and a number of considerations typically leaves the designer without much choice. However, the current findings give a contribution to the increased physical understanding of the mistuning phenomenon.

8.13 Limitations of the results

In the current study, focus is put on low-order modes and one specific engine order of the excitation field. For higher-order modes, where frequency veerings between families of modes may occur, mistuning effects may be different. Further, mistuning effects vary between different engine orders. It is also necessary to emphasize that the mistuning problem is very case specific, and thus, it is difficult to read across different bladed disk designs. Hopefully, more general results will be achieved in the future. At the present time, however, a case specific analysis must be performed for each bladed disk design.

The effects of damping are not studied in the current work. The part span shrouded fan was considered with fully stuck shroud connections, and the same modal damping factor as for the fan without shrouds was used. A real shrouded fan will typically experience rubbing at the shroud contact surfaces, introducing friction damping, which affects the mistuned forced response behaviour.

While the current analysis gives valuable results regarding the statistics of mistuned amplitude magnification, the study does not involve the maximum possible amplitudes, which may occur. While statistical data give valuable insight into the mistuning phenomenon, the maximum possible amplitude magnification values should also be considered in bladed disk design.

9 CONCLUSIONS

9.1 Comparison and validation of FE reduction techniques

Two state of the art finite element reduction techniques previously validated against direct FEM, the Craig and Bampton substructuring and reduction method (CBSR) and the Modal Reduction technique (MR), respectively, are compared and validated against experimental measurements. As previous studies have indicated, no noticeable differences in accuracy are detected for the current applications, while the modal reduction technique is significantly more efficient. Experimental data (mistuned frequencies and mode shapes) available for one of the two test cases are compared with numerical predictions, and a good match is obtained, which adds to the validation of the methods.

9.2 Mistuning sensitivity study

The influence of blade-to-blade coupling and rotation speed on the sensitivity of bladed disks to mistuning is studied. Focus is put on the first vibrational (bending) mode of the airfoils and a 3-EO excitation. A transonic fan is considered with part span shrouds and without shrouds, respectively, constituting a high and a low blade-to-blade coupling case. For both cases, computations are performed at rest as well as at various rotation speeds, including the speeds corresponding to resonant crossings in the Campbell diagram. Mistuning sensitivity is modelled as the dependence of amplitude magnification on the standard deviation of blade stiffnesses. The modal reduction technique is employed for the structural analysis. This reduced order model is solved for sets of random blade stiffnesses with various standard deviations, i.e. Monte Carlo simulations. In order to reduce the sample size, the statistical data is fitted to a Weibull (type III) parameter model. Three different parameter estimation techniques are applied and compared: a least squares linear regression-, a moment estimation- and a maximum likelihood- technique. The least squares method appears to be the most robust method for statistical mistuning analysis with small sample sizes.

The key role of blade-to-blade coupling, as well as the ratio of mistuning to coupling, is demonstrated for the two cases. Focusing on the effect of one specific engine order on the mistuned response of the first bending modes, it is observed that mistuning sensitivity varies significantly with rotation speed due to an associated variation in blade-to-blade coupling strength. Increased coupling under rotation tends to shift the mistuning strength at which a peak in amplitude magnification occurs towards higher values.

Focusing on the resonant speeds of the respective fans, it is observed that the mistuning sensitivity behaviour of the fan without shrouds is unaffected by rotation at its resonant condition (1500 rpm), due to insignificant changes in

coupling strength at such a low speed. The fan with shrouds, on the other hand, shows a significantly different behaviour at rest and resonant speed (6000 rpm), due to increased coupling under rotation. Comparing the two cases at resonant rotor speeds, the fan without shrouds is less or equally sensitive to mistuning than the fan with shrouds in the entire range of mistuning strengths considered.

The current results indicate that the 99th percentile amplitude magnification exhibits a peak for with respect to blade-to-blade coupling for a constant standard deviation of mistuning. However, considering the standard deviation of mistuning variable, the 99th percentile amplitude magnification increases monotonically with increasing blade-to-blade coupling. Increased coupling tends to shift the mistuning strength at which a peak in amplitude magnification occurs towards higher values. The study indicates that the respective peak occurs at approximately the same mistuning-to-coupling ratio. Note, however, that the current study focused exclusively on the effect of one specific engine order on the mistuned response of the first bending modes of the airfoils.

10 FUTURE WORK

10.1 Comparison and validation of FE reduction techniques

The validation against experiments achieved in this work is not completed. Amplitude magnification levels, as well as detailed mistuned blade-to-blade response patterns must be considered. Indeed, such results will be available within the project ADTurBII in the near future, both for the test blisk considered in the present work, as well as for a realistic turbine bladed disk geometry. Thus, interesting comparisons between numerical predictions and measurements can be made. Damping as well as the nature of the experimental forcing fields may play an important role for the forced response patterns, and increased understanding of these phenomena will hopefully be obtained.

10.2 Mistuning sensitivity study

The mistuning sensitivity study performed in this work should be extended to include different mode shapes and engine orders. The effects of damping need to be investigated, especially for the shrouded fan where rubbing at the shroud interfaces typically affects the forced response characteristics significantly. Further, several other geometries should be included in the study, involving both turbine- and compressor- rotors.

It would also be very interesting to extend the analysis to yield maximum possible amplitude magnifications, in addition to the magnification statistics. Recent publications have focused on optimisation techniques to yield such results (e.g. Petrov and Ewins (2001)). Mistuning sensitivity can be modelled as optimised amplitude magnification versus mistuning tolerances, and a comparison of such results with the results obtained in this work can be made.

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APPENDICES

Appendix A: Example of simple bladed disk model

Figure A.1: Example of a single degree of freedom per blade disk model, connected by a blade-to-blade coupling stiffness k_c . m_j , k_j and c_j are blade mass, stiffness and damping respectively, and F_j is the force on blade j . Adapted from Rivas-Guerra (2001).

Appendix B: Example frequency response curves

Figure B-1: Example of resonant amplitudes for the DCAHM-PS fan, predicted by a regular frequency sweep (0.1 Hz step) and amplitude optimisation (peaks bracketed by means of natural frequencies).

Appendix C: Campbell diagrams

Figure C-1: Campbell diagram for the DCAHM-PS rotor.

Figure C-2: Campbell diagram for the DCAHM-NS rotor.

Appendix D: PUBLICATION I

Myhre, M.; Moyroud, F. M.; Fransson, T. H.; 2003

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