Effects of Asymmetry and Other Non-Standard Excitations on Structural Dynamic Forced Response Analysis of Turbomachinery Flow-Path Components

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Graphic source: Marcu et al. [1]
Abstract

A recent paper [2] explored a potential deficiency in the single frequency harmonic response structural dynamic analysis approach typically used to assess a resonant condition in turbomachinery flow path components. This deficiency is prevalent in supersonic flow conditions when non-adjacent stages are present. The previous investigation demonstrated other excitation sources present in complex or supersonic flows could be missed using the typical analysis approach, in some cases leading to large under-prediction of structural response when compared with a baseline transient analysis. This paper presents the results of a follow on study, in which the inclusion of these effects in dynamic analyses is investigated. A representative rotor was created and analyzed for forced response characteristics when individual and combined unsteady content was present. A simple shell and beam 2D model was used to study the forced response behavior using transient and harmonic analyses. The results showed a significant contribution from non-integer forcing as well as from certain integer order forcing. A 3D model was also created for future analysis but did not display the sideband characteristics similar to flow seen in ref. [2].
Acknowledgements

The research presented in this paper took place at the University of Liège from February-June 2014 under the remote supervision of Nenad Glodic at KTH Stockholm and local supervision of Dr. Andrew Brown, a visiting scholar from NASA’s Marshall Space Flight Center (MSFC) and Duke University.

I would like to extend my sincerest gratitude to the EU Erasmus Mundus THRUST consortium, the universities involved (KTH Stockholm, Duke University, University of Liège and the Aristotle University of Thessaloniki), and the professors who taught our courses in person and remotely; thus granting us the opportunity and financial support to study the challenging field of aeromechanics with the available knowledge of experts from esteemed institutions on both sides of the Atlantic.

I would like to thank Nenad Glodic for taking on the challenging task of coordinating the entire program and teaching classes at the same time as supervising multiple Master’s students. I would also like to express my sincerest gratitude to Dr. Brown for his supervision and guidance in the course of the research and thesis writing process. In addition, I want to thank Preston Schmauch and Steven Delessio at NASA MSFC for their help with the numerical analysis concepts involved and conducting the CFD analysis without which this research would not have been possible.

And of course, I want to thank my parents, Olga and Steve for their support that got me to this point in my life!
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## Nomenclature

### Subscripts

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<thead>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Interval</td>
</tr>
<tr>
<td>i</td>
<td>Mode number</td>
</tr>
<tr>
<td>j</td>
<td>Any mode in modal coordinates</td>
</tr>
<tr>
<td>m</td>
<td>Factor in Tyler-Sofrin equation defining travelling wave behavior</td>
</tr>
<tr>
<td>n</td>
<td>Harmonic index</td>
</tr>
</tbody>
</table>

### Superscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Transpose</td>
</tr>
</tbody>
</table>

### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Amplitude</td>
</tr>
<tr>
<td>$A$</td>
<td>Prescribed excitation amplitude</td>
</tr>
<tr>
<td>$B$</td>
<td>Number of blades</td>
</tr>
<tr>
<td>$[C]$</td>
<td>Structural damping matrix</td>
</tr>
<tr>
<td>$e$</td>
<td>Euler's number</td>
</tr>
<tr>
<td>$F$</td>
<td>Force</td>
</tr>
<tr>
<td>${F}$</td>
<td>Applied load vector</td>
</tr>
<tr>
<td>$F(pf_0)$</td>
<td>Frequency domain transform (DFT)</td>
</tr>
<tr>
<td>$f(kt_0)$</td>
<td>Time domain data (DFT)</td>
</tr>
<tr>
<td>i</td>
<td>Imaginary number</td>
</tr>
<tr>
<td>$k$ (Fourier analysis)</td>
<td>Fourier series integer term</td>
</tr>
<tr>
<td>$k$ (Tyler-Sofrin)</td>
<td>Integer index for the number of vanes</td>
</tr>
<tr>
<td>$[K]$</td>
<td>Structural stiffness matrix</td>
</tr>
<tr>
<td>$[M]$</td>
<td>Structural mass matrix</td>
</tr>
<tr>
<td>n</td>
<td>Number of modes to be used</td>
</tr>
<tr>
<td>$N$ (in turbomachinery)</td>
<td>Multiple of /rev excitation</td>
</tr>
<tr>
<td>$N$ (in Fourier analysis)</td>
<td>Number of equal time intervals in discrete Fourier analysis</td>
</tr>
<tr>
<td>p</td>
<td>Fourier bin</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure field sampled by rotor</td>
</tr>
<tr>
<td>t</td>
<td>Instantaneous time</td>
</tr>
<tr>
<td>$T$</td>
<td>Final time</td>
</tr>
<tr>
<td>${u}$</td>
<td>Nodal displacement vector</td>
</tr>
</tbody>
</table>
\{u\} \quad \text{Nodal velocity vector}
\{u\} \quad \text{Nodal acceleration vector}
V \quad \text{Number of vanes}
y \quad \text{Modal coordinates}

**Abbreviations**

2D \quad \text{Two Dimensional}
3D \quad \text{Three Dimensional}
CFD \quad \text{Computational Fluid Dynamics}
DFT \quad \text{Discrete Fourier Transform}
FEM \quad \text{Finite Element Method}
INS \quad \text{Insignificant}
ITAR \quad \text{International Traffic in Arms Regulation}
MSFC \quad \text{Marshall Space Flight Center}
N/A \quad \text{Not Applicable}
ND \quad \text{Nodal Diameter}
Nt \quad \text{Newton}
NASA \quad \text{The National Aeronautics and Space Administration}
RPM \quad \text{Rotations Per Minute}
SDOF \quad \text{Single Degree of Freedom}
SLS \quad \text{Space Launch System}
SSME \quad \text{Space Shuttle Main Engine}

**Greek**

ζ \quad \text{Fraction of critical damping}
θ \quad \text{Blade location in radians}
\lambda \quad \text{Eigenvalue (natural frequency)}
σ \quad \text{Standard deviation}
φ \quad \text{Initial phase}
\{φ\} \quad \text{Eigenvector (mode shape)}
ω \quad \text{Natural frequency}
Ω \quad \text{Rotation speed in radians/second}
**Expressions**

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
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<tbody>
<tr>
<td>Bin</td>
<td>In a DFT, the discrete points in the frequency or spatial domain corresponding to a frequency (1D Fourier) or spatial wave shape (2D Fourier)</td>
</tr>
</tbody>
</table>
1 Background

1.1 Overview of Numerical Methods Used

In the following section, a brief overview will be given of the theory behind numerical methods used by the structural analysis software (ANSYS 14.5) in the course of the presented research. These algorithms are well established and their study is not the main focus of this thesis, so the theoretical review will be kept brief.

References [3], [4] and [5] provide detailed derivations of the equations, fundamental theory and methods used in the course of this investigation. The reader is urged to reference them if they are not familiar with the details of the theory discussed.

1.1.1 Fundamental Equation of Motion

The fundamentals of structural dynamic analysis using finite element methods all relate to the fundamental equation of motion presented here in matrix form (Equation 1) [3] [6] [5].

\[
[M]\ddot{u} + [C]\dot{u} + [K]u = F(t)
\]  

(Equation 1)

Where [5]:

- \([M]\) = Structural mass matrix
- \([C]\) = Structural damping matrix
- \([K]\) = Structural stiffness matrix
- \(\{\ddot{u}\}\) = Nodal acceleration vector
- \(\{\dot{u}\}\) = Nodal velocity vector
- \(\{u\}\) = Nodal displacement vector
- \(\{F(t)\}\) = Applied load vector at a point in time

Armed with this equation, matrix mathematics and time integration schemes, many complex dynamic problems can be solved using finite element methods [5] [7].

1.1.2 Modal Analysis

Modal analysis allows for the extraction of fundamental frequencies and mode shapes of a finite element structure using eigenvalue and eigenvector extraction from Equation 2 [3] [8] [5]. There are many different algorithms with the fundamental goal of eigenvalue and eigenvector extraction but with different computational efficiencies and advantages, for example the Block-Lanczos, one method used by ANSYS and discussed in ref. [9]. Additional algorithms used by ANSYS are introduced and can be studied in detail in ref. [5].

\[
[K]\varphi_i = \lambda_i [M]\varphi_i
\]  

(Equation 2)

Where [5]:

- \([M]\) = Structural mass matrix
- \([K]\) = Structural stiffness matrix
- \(\varphi_i\) = Eigenvectors (mode shapes)
- \(\lambda_i\) = Eigenvalues (natural frequencies)

The subscript \(i\) denotes the mode number. For each eigenvalue, there is an associated eigenvector describing the mode shape [3] [8].
1.1.3 Two Dimensional Fourier Analysis

Fourier analysis allows a signal to be decomposed from the time domain into frequency domain, thus providing a clear picture of the signal’s harmonic content. The Fourier series creates a representation of each signal from a superposition of a number of sinusoidal components [8] [3] [4].

A Discrete Fourier Transform (DFT) which is used in the course of this investigation allows for Fourier decomposition of any discrete signal assuming it is sampled at N equal time intervals from time t = T (final time) and is periodic with f(t) = f(t+T) [8] [4]. Equation 3 can be used to perform a DFT on a signal [8].

\[ F(pf_0) = \frac{2}{N} \sum_{k=0}^{N-1} f(kt_0)e^{-i2\pi pf_0kt_0}; \text{ } p = 1 ... N - 1 \]  

(3)

Where

\[ t_0 = \frac{T}{N} \]

\[ f_0 = \frac{1}{T} \]

A 2D DFT can be used in turbomachinery, since the rotor is cyclical. A 2nd Fourier decomposition of the Fourier magnitudes found using Equation 3 in each frequency bin at equal spatial intervals around the rotor can be done. This allows the analyst to determine which travelling wave mode shapes are present at each frequency [6].

A MATLAB routine was developed that performed a 2D Fourier analysis on a set of data and can be found in Appendix B. To validate the code, a heritage Fortran code developed at Rocketdyne was used as a baseline with known sets of input data and results. Thus, when the two codes agreed, the MATLAB code was considered to be validated. When using the MATLAB code, the analyst must remember that it is crucial to use the correct spatial and temporal interval numbers.

1.1.4 Mode Superposition Analysis

Mode superposition analysis allows the result of modal analysis (the natural frequencies and mode shapes of the structure) to be used to compute transient and harmonic responses with reduced computational time when compared to the direct solution of the equations of motion (Equation 1) [5].

The method is fundamentally similar across all solvers. The method used by the ANSYS 14.5 solver is similar to other programs and is outlined below:

First, a set of modal coordinates y can be defined using Equation 4.

\[ \{u\} = \sum_{i=1}^{n} \{\varphi_i\} y_i \]  

(4)

Where [5]:

\{\varphi_i\} = Mode shape of mode i

n = Number of modes to be used

Then, substituting Equation 4 into Equation 1 and performing extensive derivation (outlined in ref. [5]), the equation of motion becomes:

\[ \{\varphi_j\}^T [M]\{\varphi_j\}\ddot{y}_j + \{\varphi_j\}^T [C]\{\varphi_j\}\dot{y}_j + \{\varphi_j\}^T [K]\{\varphi_j\}y_j = \{\varphi_j\}^T \{F\} \]  

(5)

Where the subscript j represents any mode. Equation 5 can be simplified further to create Equation 6 [5].

\[ \ddot{y}_j + 2\omega_j \zeta_j \dot{y}_j + \omega_j^2 y_j = \{\varphi_j\}^T \{F\} \]  

(6)

Where [5]:
\[ \omega_j = \text{natural frequency of mode } j \]
\[ \zeta_j = \text{fraction of critical damping for mode } j. \]

This reduces the problem needed to be solved into a set of uncoupled single degree of freedom (SDOF) equations [6] [5].

From ref. [5]: “Since \( j \) represents any mode, [Equation 6] represents \( n \) uncoupled equations in the \( n \) unknowns \( y_j \). The advantage is that all the computationally expensive matrix algebra has been done in the eigensolver, and long transients may be analysed inexpensively in modal coordinates with [Equation 4].”

Thus, frequency response and transient response analyses can be solved using significantly less computational effort than required for direct solution methods [6] [5].

### 1.2 Rocket Propulsion Turbomachinery Overview

Rocket propulsion is a complex problem with often conflicting design requirements that must be balanced against each other to create a light weight optimized vehicle. This is quite evident in the delicate balancing act during the design of turbomachinery for liquid fueled rocket applications.

Low weight is paramount for all spacecraft components driving the need for smaller size. At the same time, the density and temperature of the propellants and oxidizer, the usually extremely high propellant feed rate into the engine’s thrust chamber as well as the propellant tank pressurization and design heavily influence the radial or axial pump impeller design and need to minimize cavitation risk. The pump design in turn influence the design of the turbine, from the required RPM and horsepower output to whether gearing or multiple turbines may be required. Vibrations from the engine, vehicle, pumps and driving turbine create a very complex dynamic environment for the turbopump [10].

Figure 1 is a simple schematic of a generic turbopump fuel feed system. A turbine driven by a gas generator is powering pumps that feed fuel and oxidizer to the combustor, which in turn expands the combustion gas out of the nozzle (also known as the thrust chamber). The exhaust from the turbine can be regeneratively used in the system or can provide cooling to the nozzle [11]. Figure 1 is just one example of a turbopump system, and the requirements of the engine and space vehicle weigh heavily on the correct design choice [10]. References [10] and [11] have excellent chapters on all the factors that drive turbomachinery system design and the different designs that have been used throughout the years.
As an example of the immense requirements taxing these systems, consider the Space Shuttle Main Engine (SSME). It differs from the Figure 1 schematic in that it has a separate turbine and gas generator driving each propellant pump. It requires a propellant flow rate into the combustor of 468 kg/sec. The oxygen pump must raise the liquid oxygen’s pressure to 55 MPa and the hydrogen pump must raise the liquid hydrogen’s pressure to 45 MPa. To drive the hydrogen pump, a 2 stage turbine running at 34,386 RPM was designed. To drive the oxygen pump, the separate 2 stage turbine ran at 37,263 RPM [11].

To minimize the number of stages and keep the weight of the components as light as possible while meeting such taxing requirements, the turbine is usually an impulse type. However, this power density comes at a price. Since the fluid velocities are much greater than in a reaction turbine (and often supersonic), the efficiency of the stage can be lower than seen in reaction turbines due to the unsteady nature of the flow and instability in the boundary layer [11]. Recent CFD analysis by Brown and Schmauch indicated the presence of significant flow unsteadiness in a supersonic turbine under development at NASA Marshal Spaceflight Center (MSFC) that could prove problematic in traditional structural dynamic analysis that takes into account only the dominant harmonic components [2]. Details of their investigation will be discussed in a subsequent section, as their findings are the main motivation for the research conducted in this thesis.

Examples of recent turbopump design and analysis methodology from both the United States and Europe will be discussed in Section 1.3. Two detailed papers will be summarized to give insight into recent design and analysis methodology.
1.3 Liquid Fuel Rocket Turbomachinery Analysis Methodology: Recent Examples

1.3.1 J-2X Engine

The NASA J-2X engine was originally under development for the Ares-1 rocket [1] and is now being applied to power the upper stage of the Space Launch System (SLS) rocket that would allow the Orion spacecraft to leave Earth orbit. It’s designed to produce an impressive 300,000 pounds (1,334,466 Nt) of thrust in a vacuum to achieve that goal using liquid oxygen and liquid hydrogen [12].

Its components, including the turbopump, were inspired by legacy systems from the Apollo era J-2 engine, while modern design analyses and materials were applied to boost the power output by ~25% and create a modern, efficient design [1] [12].

A comparison of the oxidizer and fuel pumps between the J-2 and J-2X can be seen in Figure 2 and Figure 3.

Figure 2: Baseline J-2 (left) and J-2X (right) fuel turbopump [1]

Figure 3: Baseline J-2 (left) and J-2X (right) oxidizer turbopump [1]

Both turbopumps are driven by 2-stage turbines, with the fuel pump supersonic and oxidizer pump subsonic. These heritage designs were subjected to modern modeling and analysis methods using NASA MSFC Phantom software for CFD analysis and ANSYS software for structural analysis [1].

The methodology first used cyclically symmetric sections for the analysis, but those methods were found to be deficient. Finally, full 3D grids were created to create an accurate flow field model for each turbine. By modifying the design, using harmonic and frequency response analyses of the flow fields, the engineers at Pratt and Whitney Rocketdyne and NASA MSFC were able to modify and improve the heritage design. For example, vane numbers were changed based on modal analysis and Campbell diagrams (such as Figure 4) in order to give greater margin from resonant conditions in the operating range [1].
By looking at the pressure and Mach number (Figure 5) distributions in the flow field and studying forcing over the blades in the frequency domain (Figure 6, showing reduction in off-resonant response due to nozzle bowing), informed design decisions were made in order to modify blade geometry and gaps between stages to reduce forced response [1].
Figure 6: “Rotor-1 unsteadiness with the baseline (left) and bowed (right) nozzle” [1]

1.3.2 Vulcain 2 Engine

Another example of a recent turbine design and analysis can be found in the liquid oxygen turbopump for the Vulcain 2 rocket engine powering the main cryogenic stage of the Ariane V rocket. The Vulcain 2 uses liquid hydrogen for the fuel and liquid oxygen for the oxidizer. Like the J-2X, it was designed to produce 300,000 lbf of thrust (1,334,466 N). [13]

The upgrade from the Vulcain to the Vulcain 2 stemmed from a desire to increase the original engine’s thrust by 20% [13] as well as its specific impulse and liquid oxygen to fuel ratio [14]. This necessitated a new design of the liquid oxygen turbine given the higher required oxidizer flow rate. The heritage Vulcain fuel turbopump design could remain unchanged, only requiring re-qualification at the new operating point. A cross-section of the oxidizer turbopump test article can be found in Figure 7 [14].

Figure 7: Vulcain 2 engine liquid oxygen turbine test article cross section [14]

The designer of the turbine, Volvo Aero, used many of the same techniques as the J-2X team. However, this was more of a clean slate design. Given a design point, prescribed gas properties created in the gas
generator, axial thrust and off design performance requirements, the initial turbine design could be carried out. A two stage impulse turbine was chosen, with blade numbers and shapes selected to fulfil requirements while minimizing cost and weight [14].

2D and multi-stage 3D flow simulations using the in-house program VOLSOL were used to optimize the blade design. Loads were estimated using 2D inviscid models. From the paper: “Loads corresponding to the eigen modes [were] transformed to Fourier coefficients and phase angles and used as input to forced response calculation[8]” [14].

When a test article (Figure 7) was instrumented and studied, the numerical models of the pressure distribution (Figure 9) agreed with the actual flow patterns (Figure 8).

![Figure 8: "Oil patterns seen from second rotor exit" [14]](image1)

![Figure 9: “Mach number distribution close to blade surface. Seen from second row exit” [14]](image2)

Internal pressure distribution and pressure ratios agreed closely with the CFD predictions (Figure 10). The numerical models thus produced a new, efficient and validated design.
1.4 Tyler-Sofrin Interaction

A key part to understanding the rest of this paper is a phenomenon akin to a sampling rate that can occur in turbomachinery. It was described by Tyler and Soffrin in 1961 [15] and has been experimentally demonstrated by Jay, MacBain, and Burns in 1984 [16]. This phenomenon can excite nodal diameter modes besides those of the per revolution excitation order exactly equal to the number of upstream or downstream flow distortions (those typically attributed to blades or stationary vanes).

Tyler and Soffrin described this phenomenon as pressure fluctuations in the reference plane due to rotor-stator interaction using Equation 7, where \( \sigma \) is the amplitude, \( \Omega \) is the shaft rotation speed in rad/sec, \( V \) the number of upstream vanes and \( B \) is the number of moving blades. A sinusoidal vane excitation can be seen from the stationary reference frame and sampled by the structure.

\[
P_{mn} = V\sigma_{mn}\cos[m(\theta - \frac{nB}{m}\Omega t) + \varphi_{mn}]
\]  

(7)

The subscript \( n \) is defined as the harmonic index and \( m \) is defined as Equation 8, where \( k \) is an integer index for the number of vanes. It can be -1, 0 or 1 for simple cases. Typically \( k \) can be assumed to be -1 [15].

\[
m = nb + kV
\]  

(8)

Given these assumptions, Tyler-Sofrin interaction creates a B-V wave pattern travelling around the disk at a speed determined by Equation 9 [15].

\[
TWSpeed \left( \frac{rad}{sec} \right) = \frac{nB\Omega}{m}
\]  

(9)

A simple way to think of this interaction without resorting to complex mathematics is as a sampling effect when the blades interact with the flow field [17]. Using a spreadsheet tool to demonstrate this effect, the generated Figure 11-Figure 13 use Equation 7 to create a simple sinusoidal excitation. Assuming that the excitation has a 35N frequency and the rotor disk exposed to the excitation field has 36 blades sampled as discrete points, it can be easily seen that the structure will sample this excitation as a 1ND travelling wave shape.
Of course, realistic blades are not perfectly discrete. When a blade width is considered (Figure 12), the 1ND wave can still be seen.

As a higher ND example with non-discrete blades (Figure 13), the 30N excitation causing a 6ND wave on the 36N rotor can be observed.
These waves are not stationary and will either be forward or backward travelling depending on the number of stators and blades (refer back to Equation 8).

To help visualize the travelling nature of the waves caused by the Tyler-Sofrin sampling effect, a MATLAB code was created that animates the interaction. It can be found in Appendix C and is ready for use.

2 Research Motivation: Brown – Schmauch NASA MSFC Investigation

Since this thesis is a direct follow up to the work of Brown and Schmauch, the results of their findings will need to be discussed in detail to understand the motivation for the additional investigation. In their paper, a potential deficiency in the current approach to turbomachinery structural analysis was explored (specifically, Fourier decomposition of a CFD generated flow field, the harmonic content of which at problematic frequencies is then applied to the structure to calculate the response), especially when the technique is applied to certain unsteady supersonic flow typical to multi-stage rocket propulsion turbomachinery [2].

Their investigation demonstrated that this method could potentially miss other excitation sources present in complex and supersonic flow inherent to supersonic impulse turbines of the type used for rocket propulsion, in some cases leading to large under-prediction of structural response when compared with a baseline transient analysis. To study this effect, a bladed disk was first modeled as a simple structure using shell elements for the disk and beam elements for the blades. It was found that contrary to expectations, the inclusion of randomness and variation from harmonic excitation did not lead to a large reduction in blade structural response, only small reductions were observed. A realistic 3D model and a flow field used in one of the MSFC engine programs were then created and a transient calculation were used as a baseline for comparison with the frequency response method. Results showed the frequency response method under predicted by 10% excitations where there was relatively little other content near the main harmonic (referred to as Fourier sidebands). However, for cases with substantial sidebands, frequency response methods could under predict the response by an astounding 600% [2].
In the flow field investigated, Brown and Schmauch identified the presence of non-integer order excitations beside the main excitation frequency (Figure 14). Since 4 revolutions were used for CFD analysis, the 296th Fourier frequency bin is the main excitation (74*4). The sidebands can be seen in the 295th (73.75) and 297th (74.25) bins.

It was hypothesized that this content is present due to the unsteadiness of the wakes and the effect of non-adjacent stages on the flow characteristics. The presence of these components increased the response of the transient system when compared to simple harmonic response analysis. The response showed wideband characteristics, so it was suggested that additional modes may have been excited by the sidebands [2].

In this thesis, the flow field was revisited to further look at the frequency content in the flow. If the 2D Fourier decomposition (Figure 15) of a leading edge node near the tip is examined, additional content around the main excitation can be clearly seen. In Figure 15, the X axis is the frequency bin in terms of per/revolution and the Y axis is the excited spatial wave. For proprietary and ITAR (International Traffic in Arms Regulations) reasons, the exact number of excitations cannot be discussed. Nonetheless, it still can be discussed that there are additional strong +/-1N integer order sidebands present that the node sampled as +/-1 ND travelling waves near the main excitation, consistent with the effects discussed in section 1.4. It is also interesting to note the presence of +/-4N integer order sidebands that exist in the same travelling wave pattern as the main excitation and do not follow the sampling pattern discussed in section 1.4. These sidebands were not noticed in the original Brown-Schmauch investigation [2] and were discovered once the flow field was revisited for this thesis.

At this time, it was only possible to obtain the 2D decomposition of the complete flow field over one revolution (4 revolutions were generated through CFD). That decomposition did not spatially show non-integer effects that were observed in the temporal decomposition plotted in Figure 14 [2]. However, the non-integer components could still manifest themselves as the increased presence of integer order sidebands in the case when one revolution is decomposed, as they are sampled by the Fourier algorithm as lumped in the frequency bins closest to their true frequency.

The +/-1N sidebands could be attributed to many things, among them wake instability and the effects of non-adjacent stages before the stator. The +/-4N sidebands are a bit more tricky to pin down on an exact cause and could be due to several different flow instability phenomena.

At the time of this thesis’ writing, additional literature related to this specific type of flow field phenomena has not been found.
3 Numerical Modeling

The first step in analyzing the structural effects of the flow phenomena discussed in section 2 was to create a simple 2D model to which similar forcing could be applied in a controlled and narrow banded manner so that various effects could be isolated. The design of the model had to keep in mind future 3D design and analysis as forward work. Work on both 2D and 3D modelling occurred in parallel.

3.1 Criteria for Blisk Dimensions and Number of Blades, Nozzles and Vanes

The model did not require a full aeromechanical design process. It needed to be a simple academic model that would allow for the isolated excitation of sidebands and for the structural response to be studied. The design of the 2D model also had to keep in mind future 3D analysis, keeping the number of blades as low as realistically possible, so that a follow on 3D model may be more tractable for transient analysis given limited computational resources.

3.1.1 Tyler-Sofrin Tables for Design

The number of modeled excitations from upstream and downstream nozzles and vanes had to be decided upon first. After several design iterations, it was found that 36 blades with 35 upstream nozzles and 32 downstream vanes would be a good assumption that would allow for a number of discernable travelling waves excited by both upstream and downstream stators (if present). As seen in Table 1 and Table 2, this arrangement would allow for different ND waves to be created by nozzles and vanes from 1st order
interaction with the rotor in addition to different patterns that could also be excited by higher order effects. In that way, the contribution of each interaction could be easily identified.

Table 1: Tyler-Sofrin interaction due to upstream nozzles

<table>
<thead>
<tr>
<th>Upstream Nozzle Multiples</th>
<th>35</th>
<th>70</th>
<th>105</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade Multiples</td>
<td>36</td>
<td>1</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>N/A</td>
<td>2</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>108</td>
<td>N/A</td>
<td>3</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>144</td>
<td>N/A</td>
<td>N/A</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2: Tyler-Sofrin interaction due to downstream vanes

<table>
<thead>
<tr>
<th>Downstream Vane Multiples</th>
<th>32</th>
<th>64</th>
<th>96</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade Multiples</td>
<td>36</td>
<td>4</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>N/A</td>
<td>8</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>108</td>
<td>N/A</td>
<td>12</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>144</td>
<td>N/A</td>
<td>N/A</td>
<td>16</td>
</tr>
</tbody>
</table>

3.2 2D Model

The model was created with the following criteria:

- A small, but computationally efficient model allowing for rapid transient and harmonic analyses.
- 2D blades modeled as rectangular beams approximating realistic blade moments of inertia.
  - For the 3D model, the shape of the blades would be created following standard design procedure at MSFC (discussed in section 5.2). The only set requirements being blade span and disk thickness.
- Shaft treated as a fixed support.
- Disk and blade dimensions allowing for significant disk participation in the vibration (large disk diameter relative to blade span).

After several design iterations, the dimensions were set to the following:

- Disk diameter: 0.69 m
- Shaft diameter: 0.138 m
- Disk thickness: 0.07 m
- 2D beam element cross section for blade approximation:
  - Length: 0.07 m
  - Thickness: 0.065m (close to disk thickness, slightly less in the 2D case in order to create moments of inertia closest to the 3D blades designed in parallel).
  - Width (aligned with circumference): 0.0155 m
3.2.1 2D Mesh Overview

The disk mesh was created out of 2268 2nd order shell elements. Each blade was created as a beam with 10 2nd order beam elements. A brief overview will be given of each element type, but since the elements are well established and not the focus of this investigation, in depth theoretical discussion on the subject will not be included and is available in the cited reference.

3.2.1.1 Disk Shell Elements

The outer edge of the disk was split so that the shell element nodes would coincide with the connected beam element nodes. The elements used were the SHELL281 8-node shell elements with 6DOF at each node (Figure 17) that are designed for analysis of thin to moderately-thick shell structures [5].
3.2.1.2 **Blade Beam Elements**

The beam elements used were BEAM189 (Figure 18), a higher order beam element consisting of 3 nodes, using Timoshenko beam theory, allowing for shear-deformation effect (although that would not be needed in this analysis) [5].

3.2.1.3 **Beam to Disk Connection**

The beams were connected to the disk by means of MPC184 multipoint constraint elements. These elements simply link the degrees of freedom of nodes at the intersection between elements (in this case the beam (blade) and shell (disk) elements) [5]. Different constraint options are possible for use in ANSYS. In order to model a blisk, the intersection was treated as a rigid link between the overlapping nodes’ degrees of freedom.
3.2.2 Material Properties

The material properties of the 2D model can be seen in Table 3. These properties are taken from the ANSYS 14.5 materials library for a representative titanium alloy [5]. Although the material was not important to this investigation (just that it is uniform in the whole model), it is useful to keep the material properties as close to reality as possible.

Table 3: 2D model material properties

<table>
<thead>
<tr>
<th>Material Properties: Titanium Alloy (ANSYS Library)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Young's Modulus:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Poisson's Ratio:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Bulk Modulus:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Shear Modulus:</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density: 4620 kg/m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus: 9.60E+10 Pa</td>
</tr>
<tr>
<td>Poisson's Ratio: 0.36</td>
</tr>
<tr>
<td>Bulk Modulus: 1.14E+11 Pa</td>
</tr>
<tr>
<td>Shear Modulus: 3.53E+10 Pa</td>
</tr>
</tbody>
</table>

Given the beam element dimensions and the material properties, the resulting moments of inertia for each blade were calculated to be:

- $I_x$: 0.000121241 kg*m$^2$
- $I_y$: 0.000139569 kg*m$^2$
- $I_z$: 0.000247763 kg*m$^2$

Constant damping ratio of $\zeta = 0.01$ was assumed to simplify transient and harmonic analyses.

3.2.3 Modal Analysis Results

A total of 150 modes were calculated for use in the mode superposition analyses. Two blade mode families were extracted. The $1^{st}$ 13 nodal diameter modes are shown in Figure 20 in the format of a modified SAFE/ZZENF style diagram [17]. The mode shapes are also tabulated in Appendix A $1^{st}$ 13 Nodal Diameter Disk Modes alongside their graphical representation for the reader’s reference. The first mode family contains no blade flex (rigid body blade motion, disk participation in vibration) while the $2^{nd}$ mode family contains a $1^{st}$ blade flex mode in addition to the disk modes.

Two straight lines can also be seen on the charts. They represent the forcing frequencies and nodal diameter shapes of potential travelling waves that could be present in addition to the main excitation at a specific operating point if the flow contains additional up or downstream integer order forcing and assuming the structure samples them as Tyler-Sofrin interactions described in section 1.4. Negative travelling waves are those that would be present if the additional excitation was at a lower per rev frequency than the disk’s blade.
number (36 blades in this case) while the positive travelling waves are those that would be present if the additional excitation is at a higher per rev frequency than the disk’s blade number. If those lines intersect with the mode fundamental shape and frequency on the plot, that means the specific forcing would then excite an additional mode at its fundamental frequency and shape. This effect could greatly increase the structure’s forced response.

For the remainder of this investigation, the 1ND mode in the 1st flex family was chosen as the primary mode to excite for sideband effect analysis. The 35N excitation signal was generated by a spreadsheet tool using a rotor RPM that coincides with the 1ND mode’s fundamental structural frequency of 2359.8 Hz. The same spreadsheet tool was also used in the sideband analysis to create the forcing functions.

From Figure 20, it can be seen that if the operating point is chosen such that the primary upstream 35N forcing excites the 1ND 1st blade flex mode at its fundamental shape and frequency of 2359.8 Hz (red circle), if additional integer order excitations are present in the forcing function, other modes will not be excited. This is desirable if the aim is to investigate only off resonant sideband effects on structural response, but this condition is not true for all operating points and rotor-stator combinations encountered in turbines.

![Modal Frequencies of Structure and Possible Single Operating Point Excitations](image)

**Figure 20**: Mode and excitation diagram with possible forward and backward travelling waves caused by additional excitations around main 2359.8 Hz excitation. 36 blade rotor with 35 upstream nozzles

For example, one may look at a different structure and operating point (Figure 21) in which the main excitation is instead 46N (due to, for instance, 46 upstream stators) and the rotor remains with 36 blades. Let’s assume then that the turbine RPM is at an operating point such that the 10ND no blade flex mode is excited at its natural frequency (2914.4 Hz) and shape due to interaction with the 46N excitation.
Figure 21: Mode and excitation diagram with possible forward and backward travelling waves caused by additional excitations around main 2914.4 Hz excitation. 36 blade rotor with 46 upstream nozzles

It can be seen from Figure 21 at that specific operating point (red circle), additional possible travelling waves sampled by the structure (if additional integer order excitations are present due to sidebands or downstream stators, such as 47N) will be close to the natural frequencies and mode shapes of modes in the “no blade flex” family (the 47N forcing mentioned would excite an 11ND mode). On the other hand, a 25N additional excitation will also cause an 11ND travelling wave, however it will be much below the structure’s 11ND fundamental frequency and would not elicit a resonant response.

It can also be seen that an additional 37N excitation at that operating point will be close to exciting a 1ND mode in the 1st blade flex family at its natural frequency and mode shape.

Thus, this turbine design at the described operating point along with the presence of sideband content or downstream excitation could excite additional travelling wave modes at their fundamental shapes and frequencies simultaneously to the primary excitation, leading to higher and more wideband structural response. Obviously, this is a situation that should be avoided. The charts developed and discussed in this section would be a useful tool in the initial assessment of structural response at a certain operating point and whether additional excitations present in the flow could excite another mode at its fundamental shape and frequency.

3.3 Two Dimensional Model Cases Tested

To attempt and replicate the response observed by Brown and Schmauch due to forcing with sidebands, two steps in the analysis were taken. The first step in this procedure required the performance of a dynamic response analysis using an excitation history explicitly defined in time and space (a transient analysis) to serve as a baseline. The second step was then to compare that transient analysis results to a harmonic frequency response analysis which was run at the amplitudes of the main excitation and each prescribed sideband component.

3.3.1 Excitation Function

To excite each blade, a forcing function was created in an Excel tool using Equation 10, multiple instances of which were added together for every additional excitation source that was included in each analysis in order to create the complete forcing functions.

\[
F = A \cos[V(\theta - \Omega t) + \varphi]
\]  

(10)
A is defined as the prescribed excitation amplitude, \( V \) is the number of excitations (nozzles or stators), \( \theta \) is the blade location in radians, \( \Omega \) is the rotational speed of the rotor in rad/sec, \( t \) is time in sec and \( \varphi \) is the initial phase.

The forcing function was applied perpendicular to the plane of the structure to each blade at a time step of 1.413E-05 seconds (allowing for 30 samples evenly spaced for each vane passage at the 1ND modal frequency of 2359.8 Hz). The forcing was created for a total of 6 complete rotations of the blisk.

For the ANSYS Newmark time integration algorithm, approximately 20 time steps per cycle of highest frequency of interest are desired in order to accurately capture the response. Since the highest secondary excitation frequency analyzed is 41.75N, or 2814.9 Hz, the selected time step allows for ~25.14 time steps at the highest frequency of interest.

![Figure 22: Effect of integration time step on period elongation [5]](image)

The initial phase (\( \varphi \)) was kept at 0 to create maximum interference between the harmonic 35N nozzle passage frequency and the sideband excitations.

The end time of each revolution can be seen in Table 4.

<table>
<thead>
<tr>
<th>Revolutions</th>
<th>End Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4832E-02</td>
</tr>
<tr>
<td>2</td>
<td>2.9664E-02</td>
</tr>
<tr>
<td>3</td>
<td>4.4495E-02</td>
</tr>
<tr>
<td>4</td>
<td>5.9327E-02</td>
</tr>
<tr>
<td>5</td>
<td>7.4159E-02</td>
</tr>
<tr>
<td>6</td>
<td>8.8991E-02</td>
</tr>
</tbody>
</table>

### 3.3.2 2D Blisk Cases Tested

In order to isolate the off-resonant effect of each sideband identified in the original investigation on the response, each significant sideband present in the excitation was tested under the condition that no additional modes were excited close to or at resonance (aside from the main 35N excitation). All sideband cases tested can be seen in Table 5. First each sideband’s effect was tested individually in addition to the
main excitation before the forcing cases in which the combination of all sidebands present was tested. It was hypothesized that the non-integer sidebands close to the main excitation will have the highest effect due to their proximity to the resonance frequency.

The magnitudes of the sidebands used in testing were proportional to the main excitation as seen in the temporal and 2D Fourier decompositions of the MSFC CFD data discussed in section 2 [18]. The main excitation was chosen as a cosine wave with an amplitude of 400 Nt (Newtons) distributed evenly over each blade and applied in plane to the blisk (case 3). The +/- 4N sidebands were observed to be in some cases relatively small (~15% of the main excitation) and in other cases a bit larger (~35-40%). Two cases were thus run with those sidebands differing in magnitude: 50Nt sidebands at 12.5% of the main 400Nt excitation and 150Nt sidebands at 37.5% of the main excitation. All other sidebands were kept at 50N, or 12.5% of the main excitation.

One must keep in mind that the MSFC data indicated the +/- 4N (2629.491/2090.109 Hz) integer order sidebands, unlike the other excitations that followed the spatial aliasing rules laid out by Tyler-Sofrin, were acting at the same nodal diameter as the main excitation. In order to simulate that effect, Equation 11 was used to generate their forcing functions.

\[ F = A \cos[(V \pm 0.25N)\theta - V \Omega t + \phi] \]  

Table 5: Tested sideband cases

<table>
<thead>
<tr>
<th>Case #</th>
<th>35N (2359.8 Hz) Main Excitation Magnitude (Newtons)</th>
<th>+/- 0.25N (2376.656/2342.944Hz) Non-Integer Sideband Magnitude (Newtons)</th>
<th>+/- 1N (2427.223/2292.377 Hz) Integer Sideband Magnitude (Newtons)</th>
<th>+/- 4N (2629.491/2090.109 Hz) Integer Sidebands Magnitude (Newtons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>481</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>50</td>
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<td>0</td>
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<tr>
<td>5</td>
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<td>0</td>
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<tr>
<td>6</td>
<td>400</td>
<td>0</td>
<td>0</td>
<td>50</td>
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<td>7</td>
<td>400</td>
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<td>9</td>
<td>400</td>
<td>50</td>
<td>50</td>
<td>150</td>
</tr>
</tbody>
</table>

The motivation for case 2 is that it is the same excitation amplitude (481N) as that shown in Figure 49 of section 4.3.3, the 2D Fourier decomposition amplitude of a signal with non-integer 0.25N sidebands taken over just 1 revolution leading to only 1N frequency resolution. It is in the interest of this investigation to determine if the response will be underestimated if only one revolution is analyzed and only the dominant harmonic is taken into account.

Case 1 is a simple addition of the non-integer sideband amplitude to the main excitation amplitude (500Nt). It is in the interest of this investigation to see how these simple excitations compare to a case with non-integer sidebands and whether they can be used as good analogues for complex flow.

In addition to the main excitation (35N) + sideband cases in Table 5, two other cases were created: one where two modes were excited at once (henceforth referred to as case 10) and one where the off resonant
sidebands of case 9 were combined with the resonant modes of case 10 (this is henceforth referred to as case 11). The presence of a simultaneously excited mode could arise in a real blisk structure due to downstream excitation or sidebands that are close to the spatial aliasing shape and frequency of adjacent modes (as discussed in section 3.2.3 and seen in Figure 21).

Since Figure 20 shows that at the decided upon operating speed, number of blades, nozzles and vanes, there are no additional integer effects possible that would excite a mode near its natural frequency and shape, an excitation was created specifically to excite the 2ND 1st flex blade mode near its natural frequency of 2808.3 Hz. Equation 12 was used to generate the 2ND excitation wave near its structural natural frequency, using a signal frequency of 41.75N, or 2814.9 Hz and an amplitude of 150 Nt.

$$F = A\cos[(V - 3.75)\theta - V\Omega t + \varphi]$$  (12)

Figure 23 provides for an easy way to visualize the frequency and nodal diameter wave number of the sidebands as they relate to the natural frequencies of the modes surrounding them. It can be seen that the sidebands around the 35N excitation are not close to exciting any other modes at their resonance. The 41.75N signal specifically created to excite an additional mode will excite the 2ND travelling wave mode when present.

One must also take into account the fact that these different ND waves will be travelling at different speeds as discussed in Equation 9, section 1.4 and [15]. This difference seen in Table 6 will play a role in the interaction between the different excited wave responses as they travel around the disk, especially if one wave is forward travelling and the other backward travelling. The pattern peak frequency is simply the frequency of the sideband excitation. The negative in the 41.75N row, angular velocity and pattern peak frequency columns indicate that the pattern is travelling backwards and opposite to the pattern created by the 35N upstream vanes.
Table 6: Speed and frequency of the travelling waves excited by the main excitation and sidebands

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Pattern</th>
<th>Angular Velocity (rad/sec)</th>
<th>Angular Velocity Difference from 35N (rad/sec)</th>
<th>Pattern Peak Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#N</td>
<td>#ND</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>1</td>
<td>14827.06</td>
<td>0.00</td>
<td>2359.80</td>
</tr>
<tr>
<td>34.75</td>
<td>1</td>
<td>14721.15</td>
<td>-105.91</td>
<td>2342.94</td>
</tr>
<tr>
<td>35.25</td>
<td>1</td>
<td>14932.97</td>
<td>105.91</td>
<td>2376.66</td>
</tr>
<tr>
<td>36</td>
<td>0</td>
<td>0.00</td>
<td>-14827.06</td>
<td>0.00</td>
</tr>
<tr>
<td>34</td>
<td>2</td>
<td>7201.72</td>
<td>-7625.35</td>
<td>2292.38</td>
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<tr>
<td>41.75</td>
<td>-2</td>
<td>-8843.28</td>
<td>-23670.34</td>
<td>-2814.90</td>
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</tbody>
</table>

4 Forcing Functions and Simulation Results: Time and Frequency Domain

It is useful to look at both the time domain plots and frequency domain Fourier decomposition plots of the forcing functions to gain an understanding of their characteristics. Seeing the forcing function’s time domain plots provides a qualitative overview of its characteristics while the 2D Fourier decomposition demonstrates which frequency content and traveling waves are present that may interact with and excite the structure.

To measure the structure’s response, the out of plane (perpendicular) displacement of each blade (beam) tip was recorded at each step in the transient analysis. This provided for a metric of the response that could be compared between each test case. During initial data analysis, the 3σ values of the response were used as metrics for comparison, but the narrow bandedness of the excitation and response proved that the maximum displacement was a better metric. For each case, the maximum response values out of all blades were used for comparison.

The 2D Fourier decomposition of the tip response from all blades helps enumerate the response of modes and traveling waves and show the contribution of each sideband to the structure’s response.

For each test case, the time domain plots of the excitation and response will first be discussed, followed by discussions of the excitation and response 2D Fourier decomposition.

4.1 Case 1

Case 1 uses a simple 500 Nt excitation. An addition of two 50 Nt sidebands on top of the 400Nt main forcing at a single frequency, this case was created in order to test how this simplification would differ from the true case with non-integer sidebands.

4.1.1 Time Domain Plots

First, the excitation and response of this case are studied in the time domain to gain a qualitative understanding.

4.1.1.1 Excitation

From Figure 24 and Figure 25, the excitation can be seen as a simple cosine wave over the entire interval. This is analogous to a case where 35 nozzles upstream of the rotor create a sinusoidal pressure field with an amplitude of 500Nt seen by the disk.
Figure 24: Time history of excitation on blade 1 over 6 revolutions, test case 1

Figure 25: Snapshot of excitation on blade 1, test case 1

4.1.1.2 Response

The out of plane response of blade 1 due to the case 1 excitation can be seen in Figure 26. There is a noticeable transient startup effect present in the 1st 2 revolutions. In order to decide when convergence is reached, a simple routine was developed in MATLAB that finds local maxima and calculates the percentage difference from the previous local maxima (code found in Appendix D). Since the excitation signal is perfectly sinusoidal, there should ideally be no difference. As a criteria, an average of less than 1% difference was decided to be considered converged.
Figure 26: Blade 1 tip displacement over 6 revolutions, test case 1

Figure 27 shows the blade response for revolutions 3-6. Visually, the response appears to have stabilized, with only a small instability still present in the third revolution, but such qualitative judgments are not sufficient to establish convergence.

Figure 27: Blade 1 tip response for last 4 revolutions, test case 1

Figure 28 shows the convergence of the maximum displacement points using the routine discussed in Appendix D. After 1 revolution (~0.01496 sec.), the model was not quite converged. After 2 revolutions (~0.02969 sec.), the peak response percent difference averaged less than 1% and convergence was considered to be achieved. Therefore, it was decided to use revolutions 3-6 as converged solutions for data extraction for all transient analyses in the following sections. From the rev 3-6 data in this case, the maximum response for blade 1 was found to be 7.5202E-05 m and the 3σ response to be 6.4147E-05 m.
Figure 28: Peak convergence over case 1 transient run

### 4.1.2 Fourier Decomposition

As an additional test of both the Fourier decomposition program and the signal generation spreadsheet, the clean cosine excitation from case 1 was decomposed over 4 revolutions using the 2D Fourier analysis methodology described in section 1.1.3.

#### 4.1.2.1 Excitation

Figure 29: Temporal Fourier decomposition of 4 revolutions, blade 1 excitation, test case 1

It can be seen from Figure 29 that there is only one excitation of 500 Nt present at the 35N frequency. When the 35N temporal bin is decomposed spatially in Figure 30, a forward traveling 1ND wave can be seen at an amplitude of 500 Nt.
This initial proof of concept test with a steady sinusoidal excitation was repeated using case 2 and 3 excitation amplitudes and produced similar results (same frequency content but different amplitudes of 481 N in case 2 and 400 N in case 3), confirming the validity of the signal generation tools used.

### 4.1.2.2 Response

The Fourier decomposition of the case 1 blade 1 tip response (Figure 31) demonstrates that the structure responds at the 35N frequency with an amplitude of $7.16 \times 10^{-5}$ m, similar to the time history amplitude.

Figure 32 demonstrates the spatial characteristic of the structural response at the 35N frequency bin. The plot confirms that when the 36 blade blisk is excited by a clean 35N signal at the resonant frequency, the structure will respond with a 1ND travelling wave mode at that same frequency.
This gave another vote of confidence and confirmation with regards to the forcing function signal generation methodology, ANSYS numerical modeling algorithm and analysis methods used.

Figure 32: Spatial Fourier decomposition of 4 revolutions, blade 1 response, 35N bin, test case 1

An interesting phenomenon can also be observed when looking at Figure 31 near the 6.25N frequency. A response 2 orders of magnitude less than the main response can be seen (zoomed in view plotted in Figure 33). 6.25 N corresponds almost exactly to the no blade flex, 1ND mode (the mode’s natural frequency is 416.07 Hz, approximately 6.17N). This demonstrates that the presence of the 1ND travelling wave mode also excited the lower frequency mode, albeit at a much smaller amplitude. This finding raised a question: will the presence of sidebands also increase this response and will it be greater relative to the main response?

Figure 33: Temporal Fourier decomposition of 4 revolutions, blade 1 response, test case 1, zoomed in on 6.25N region
This phenomenon could also be the result of the transient start up effects still present in the structure (seen in Figure 27). Even though the <1% convergence criteria was met, the presence of small residual transient start-up effects could manifest itself as these lower frequency responses. It's interesting to see how this effect will vary in the presence of sidebands and additional excited modes.

Nonetheless, the examination of the spatial response at 6.25+/-0.25N was warranted to determine which travelling waves were excited at those frequencies. Looking at bins 6 (Figure 34), 6.25 (Figure 35) and 6.75 (Figure 36), it can be seen that forward and backward travelling 1ND waves are present in all 3 bins, with the forward travelling waves having slightly larger magnitudes than the backward travelling waves.

Since the 6.25N bin is closer to the 6.17N natural frequency of the lower 1ND mode, the responses are, as expected, higher than in the 6N and 6.5N bins. The spatial decompositions of each temporal bin will be compared in all test cases, with graphics posted only in the case of significant difference from these plots. Otherwise, the data will be tabulated in order to save space.

Figure 34: Spatial Fourier decomposition of 4 revolutions, blade 1 response, 6N bin, test case 1
4.2 Case 2 and 3

Cases 2 and 3 employ simple sinusoidal forcing functions. The case 2 amplitude of 481 Nt is extracted when only one revolution of case 4 with non-integer sidebands is subjected to Fourier decomposition. The case 3 400 Nt clean sinusoidal excitation is the baseline case for this investigation.

4.2.1 Excitation

As listed in Table 5, the excitation in these two cases is a simple cosine wave as in case 1, with the only difference being the amplitude (481 Nt in case 2 and 400 Nt in case 3). Thus, no plots are necessary.
4.2.2 Response

The response of the structure to the clean case 2 and 3 excitation will be discussed in the time and frequency domains.

4.2.2.1 Time History Response

For case 2 and 3, the responses were the same visually as in case 1, but with different amplitudes, so the figures are omitted. For case 2, the maximum response for blade 1 was found to be $7.2344 \times 10^{-5}$ m and the $3\sigma$ response to be $6.4147 \times 10^{-5}$ m. For case 3, the maximum response for blade 1 was found to be $6.0161 \times 10^{-5}$ m and the $3\sigma$ response to be $5.3344 \times 10^{-5}$ m.

Case 3 was established as the “clean flow” baseline, where the structure is excited by a simple sinusoid from the upstream nozzles with an amplitude of 400 Nt.

4.2.2.2 Fourier Decomposition

The Fourier decompositions of these cases are visually identical to case 1, but with different amplitudes for the main response, so the plots are omitted from this analysis.

Case 2 elicits a response in the temporal (35N) and spatial (1ND) bins of an amplitude of $6.64e-05$m. Around the 6.25 bin, as expected, the case 2 temporal decomposition showed a smaller response than in case 1 as did the spatial decompositions. The magnitudes are tabulated in Table 7.

<table>
<thead>
<tr>
<th>Temporal Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Bin #</td>
</tr>
<tr>
<td>Blade 1 Response (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>-1ND Wave Magnitude (m)</td>
</tr>
</tbody>
</table>

Case 3 Fourier decomposition shows a lower response of $5.73e-05$ m from both the temporal (35N) and spatial (1ND) bins as expected. In the neighborhood of the 6.25N response, the behavior temporally and spatially is the same as in case 1, but with a lower amplitude. The amplitudes are tabulated in Table 8.
Table 8: Temporal and spatial decomposition of response near 6.25N frequency, case 3

<table>
<thead>
<tr>
<th>Temporal Decomposition</th>
<th>6N</th>
<th>6.25N</th>
<th>6.5N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Bin #</td>
<td>6N</td>
<td>6.25N</td>
<td>6.5N</td>
</tr>
<tr>
<td>Blade 1 Response (m)</td>
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<td>8.84e-08</td>
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</table>

<table>
<thead>
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<th>Spatial Decomposition</th>
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<th>-1ND Wave Magnitude (m)</th>
</tr>
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<tr>
<td></td>
<td>2.48e-07</td>
<td>1.74e-07</td>
</tr>
</tbody>
</table>

### 4.3 Case 4

Case 4 is the first case where sideband content was introduced. The main 400 Nt excitation was joined by +/- 0.25N sidebands to create the overall forcing function.

#### 4.3.1 Time Domain Plots

First, the time domain plots of the excitation allow a qualitative understanding of the forcing function.

#### 4.3.1.1 Excitation

From Figure 37, it can be seen that the excitation signal exhibits a low frequency beating phenomenon over 6 revolutions, with a single beat taking several revolutions. This beating cannot be easily noticed over a short snapshot of the excitation (Figure 38). The unsteadiness in the wakes could produce a similar phenomenon, resulting in the data seen in [2].
4.3.1.2 Response

The time history response of this case plotted in Figure 39 shows that the response closely follows the beating present in the excitation. The maximum response and the maximum $3\sigma$ response become amplified. The proximity in frequency of the non-integer effects to the main excitation frequency and the structure’s natural frequency has a strong effect on the response, increasing the maximum response to $6.9785E-05$ m and the $3\sigma$ response of the maximum responding blade to $5.3344E-05$ m.

The cases discussed so far all highlight the narrow-bandedness of both the excitation signal (it is explicitly prescribed after all) and the responses, unlike a real flow field that would have much more noise and unsteadiness. Due to that fact, it was concluded that the $3\sigma$ response is not necessary as a performance metric in the discussion and the maximum displacement was used as an accurate metric for comparison.
4.3.2 Fourier Decomposition 4 Revolutions

As with the previous cases, the Fourier decomposition of the signal into the frequency domain allows for a deeper understanding of the forcing function and response. First, all 4 revolutions were decomposed.

4.3.2.1 Forcing

Looking at the temporal Fourier decomposition of the first blade forcing function over 4 revolutions, a clear picture can be seen, with the main excitation and sidebands visible (rounding is used in Figure 40, where the non-integer sideband frequencies are displayed rounded to the nearest tenth).

Figure 39: Blade 1 tip response for last 4 revolutions, test case 4

Figure 40: Temporal Fourier decomposition of 4 revolutions, blade 1 excitation, test case 4
A clear excitation can also be seen spatially for the 1ND traveling wave caused by the 35N frequency in Figure 41.

![Graph 1](image1)

Figure 41: Spatial Fourier decomposition of 4 revolutions, 35N bin, test case 4

What is interesting to note is that when the non-integer bins are decomposed spatially (Figure 42 and Figure 43), while most of the energy was present at a 1ND mode, there is also some wideband lower amplitude content in the wave shapes surrounding this peak. This suggests that non integer sidebands can excite several traveling waves at once, albeit at low amplitudes. Nonetheless, if there is a presence of natural modes at or near those frequencies and shapes in the structure, this excitation could prove problematic.

![Graph 2](image2)

Figure 42: Spatial Fourier decomposition of 4 revolutions, 34.75N bin, test case 4
4.3.2.2 **Response**

The temporal Fourier decomposition (Figure 44) indicated that in addition to the main response at the resonant peak, the structure responded approximately an order of magnitude less in the frequencies of the non-integer sidebands, with the 34.75N sideband eliciting a slightly larger response than the 35.25N sideband.

Figure 45 shows a clean 1ND response present in the 35N bin, which is expected from the main excitation and resonant frequency.
When the non-integer bins are spatially decomposed (Figure 46 and Figure 47), one can notice that in addition to the 1ND wave, a -1ND backward travelling wave is excited as well as a 0ND shape. This suggests that non-integer sidebands can excite a relatively wide band of travelling wave shapes around the main excitation, as suggested in section 4.3.2.1.

It must also be noted that in addition to the main 1ND response, the non-integer sidebands also elicit a response in the +/- 6ND and +/- 7ND travelling waves, albeit orders of magnitude smaller than the +/- 1 ND waves excitation. This is an interesting phenomenon and was not expected. The response of the structure to non-integer sideband content is not trivial.
Figure 46: Spatial Fourier decomposition of 4 revolutions, 34.75N bin, test case 4

Figure 47: Spatial Fourier decomposition of 4 revolutions, 35.25N bin, test case 4
Looking at the temporal response around 6.25N, it was observed that the presence of sidebands increases the response of the mode around the 6.25N frequency. The Fourier decomposition figures are similar to Figure 33-Figure 36 from section 4.1.2.2, so they will not be reproduced. Rather, the response magnitudes of interest will be tabulated in Table 9. It can be seen that the presence of sidebands increases the response in that frequency range when compared to the clean excitation response seen in case 3.

While this response is much smaller in magnitude when compared to the response around the main excited mode, this is nonetheless an interesting finding.

Table 9: Temporal and spatial decomposition of response near 6.25N frequency, case 4

<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
<th>1ND Wave Magnitude (m)</th>
<th>-1ND Wave Magnitude (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6N</td>
<td>5.31e-07</td>
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<tr>
<td>6.25N</td>
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</tr>
<tr>
<td>6.5N</td>
<td>2.89e-07</td>
<td>2.03e-07</td>
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<table>
<thead>
<tr>
<th>Frequency Bin #</th>
<th>6N</th>
<th>6.25N</th>
<th>6.5N</th>
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</thead>
<tbody>
<tr>
<td>Blade 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response (m)</td>
<td>1.92e-07</td>
<td>3.47e-07</td>
<td>1.05e-07</td>
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</tbody>
</table>

**4.3.3 Fourier Decomposition, 1 Revolution**

In order to demonstrate how only taking a Fourier decomposition of a single revolution impacts the analyst’s understanding of the flow field, the 1st converged revolution (revolution 3) was decomposed using the 2D Fourier routine.

**4.3.3.1 Forcing**

In Figure 48, it can be seen that unlike the clear picture seen for the 4 revolution case in section 4.3.2, the energy is distributed between the main 35N excitation and the 34N and 36N sidebands, since only a 1N resolution is possible in the Fourier analysis.

It must also be noted that since the excitation is a traveling wave around the blisk, the temporal decomposition of the excitation at each blade will differ. It will also differ depending on which revolution is decomposed, since the beating phenomenon modulates the amplitude of the forcing signal over a time frame spanning several revolutions.
Figure 48: Temporal Fourier decomposition of 1 revolution, blade 1 excitation, test case4

Spatially, the 35N excitation was split up into a dominant 1ND wave in Bin 35 (Figure 49), with small excitations orders of magnitude less present at 2ND and 0ND.

Figure 49: Spatial Fourier decomposition of 1 revolution, 35N bin, test case 4

In bins 34 and 36 (Figure 50 and Figure 51, respectively), low amplitude but wideband spatial excitation can be seen. With only one revolution analyzed, it would appear as if quite a wideband excitation would be present in the integer order sidebands. Energy from the non-integer sidebands “leaks” into the integer order sidebands creating this effect. Thus, an analyst taking only one revolution would not have an accurate understanding of the flow field affecting the structure and the exact forcing frequencies.
Therefore, if the standard approach of taking the Fourier decomposition of one revolution was used to calculate the harmonic or transient response in this unique forcing case, the result would not be accurate.

Naturally, it is not possible to predict beforehand which sidebands will be present, especially in the non-integer case. While increasing the number of revolutions will increase the accuracy of the simulation, a point of diminishing returns will be reached at which the required computational resources will not lead to a much more accurate solution. This opens a potential future topic for investigation.

### 4.4 Case 5

This case studies the effect of +/-1N integer order 50 Nt sidebands on the structure’s response.
4.4.1 Time Domain Plots

First, the excitation and response are analyzed as temporal plots in order to gain a qualitative understanding of the signals present in the excitation and structural response.

4.4.1.1 Excitation

It can be seen in Figure 52 that the beat phenomenon becomes more high frequency and pronounced when \(1N\) integer order sidebands are applied to the excitation signal.

![Temporal Wave on Blade 1, 6 revolutions](image1)

Figure 52: Time history of excitation on blade 1 over 6 revolutions, test case 5

![Temporal Wave on Blades 1, 6 revolutions](image2)

Figure 53: Snapshot of excitation on blade 1, test case 5

4.4.1.2 Response

In Figure 54, it’s interesting to note that unlike in Case 4, the response is generally sinusoidal, but an additional small undulation can be seen in the response. Since the \(+/-1N\) integer sidebands are farther away from the resonant frequency, the effect is not as great as the one caused by the \(+/- 0.25N\) sidebands in case...
4. The maximum peak response was found to be 6.0856E-05 m, only a slight increase over the case 3 baseline. It can be qualitatively seen that the +/-1N integer order sidebands cause only slight unsteadiness in the response as the off resonance sidebands interact with the structure.

![Time History of Blade 1 Response: Case 5](image)

Figure 54: Blade 1 tip response for last 4 revolutions, test case 5

4.4.2 Fourier Decomposition

A frequency domain analysis offers us deeper insight into the excitation and response characteristics.

4.4.2.1 Excitation

Since the main excitation and the sidebands are integer order in this case, it can be easily seen that the excitations are in the correct bins from Figure 55, unlike in cases with non-integer sidebands.

![Temporal FFT algorithm, 1st location File Magnitude](image)

Figure 55: Temporal Fourier decomposition of 4 revolutions, blade 1 excitation, test case 5
The spatial decomposition shows that the 35N excitation (Figure 56) excites a 1ND wave, the 34N sideband (Figure 57) creates a smaller 2ND wave and the 36N sideband (Figure 58) excites a 0ND shape, as expected from Tyler-Sofrin [15]. These bins show that the excitation is present in a very orderly manner in each expected frequency bin. In subsequent sections in which only integer order sidebands are analyzed, the excitation will be discussed but the Fourier decomposition will not be plotted to avoid redundancy.

Figure 56: Spatial Fourier decomposition of 4 revolutions, 35N bin, test case 5

Figure 57: Spatial Fourier decomposition of 4 revolutions, 34N bin, test case 5
4.4.2.2 Response

The temporal Fourier decomposition of the blade 1 response (Figure 59) indicates that the main response came from the 35N mode. The response due to the sidebands was much lower, by 2 orders of magnitude. Understandable, as the 1N sidebands are farther from any mode’s natural frequency.

The spatial decomposition of the 35N signal did not reveal any surprises, with the 1N mode responding at the 35N frequency with a similar magnitude to case 3.
Interestingly, the 34N response involved not only the 2ND response, but the 1ND, albeit at 3 orders of magnitude less than the 35N response. This suggests that the 1N sideband can add energy into the 1ND mode.

The 36N bin showed a response at the 0ND shape, indicating a more narrowband response than the 34N case. The 0ND shape excited also would not exhibit travelling wave behavior.
In the 6.25N neighborhood, the response looked to be very similar to the case 3 response. Since the +/-1N sidebands are farther away from the resonant mode, the main effect on the 6.25N response appears to be from the 1ND, 35N mode.

### Table 10: Temporal and spatial decomposition of response near 6.25N frequency, case 5

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<thead>
<tr>
<th>Temporal Decomposition</th>
<th>Frequency Bin #</th>
<th>6N</th>
<th>6.25N</th>
<th>6.5N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade 1 Response (m)</td>
<td>Blade 1 Response (m)</td>
<td>1.57e-07</td>
<td>2.86e-07</td>
<td>8.71e-08</td>
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<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
<th>1ND Wave Magnitude (m)</th>
<th>4.56e-07</th>
<th>8.2e-07</th>
<th>2.48e-07</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1ND Wave Magnitude (m)</td>
<td>3.19e-07</td>
<td>5.74e-07</td>
<td>1.74e-07</td>
<td></td>
</tr>
</tbody>
</table>

### 4.5 Case 6

Case 6 studies the effect of the +/-4N, 50 Nt sidebands that did not follow Tyler-Sofrin spatial aliasing rules and instead were present at the same nodal diameter (1ND) as the 35N excitation.

#### 4.5.1 Time Domain Plots

As with the previous cases, the time domain plots allow for a qualitative look at the excitation and response.

##### 4.5.1.1 Excitation

In case 6, the beating of the excitation signal is much more frequent than in case 5 and can be noticed in a small snapshot of the signal as well (Figure 64). The +/-4N sidebands would introduce this off-resonance interference in the signal at the 1ND shape travelling at a different speed than the main excitation wave (see Table 6), although it’s not expected to have as great an effect on the response as the non-integer sidebands.
4.5.1.2 Response

The response exhibits a steady, almost sinusoidal characteristic similar to the response observed in case 5. There can be seen low amplitude, high frequency fluctuations in the response due to the signal beating and travelling wave interaction with a maximum response of $6.2060 \times 10^{-5}$ m. This is similar to the perturbation shown in the signal, although much less pronounced. Even though the travelling waves in 1ND $+/-4N$ are not at resonance, they periodically come into phase with the main resonant excitation, possibly causing the effect seen in Figure 65.
4.5.2 Fourier Decomposition

Frequency domain analysis offers further insight into the effect of these sidebands.

4.5.2.1 Excitation

Fourier decomposition of this case yields no interesting plots, with sidebands clearly seen at +/- 4N from the main excitation at their prescribed amplitude. Therefore, figures will be avoided in this section. Spatially, all excitations fall on the main 1ND mode in order to replicate the MSFC data where the +/- 4N excitations fall on the main excitation’s ND wave number (as shown in Figure 15).

4.5.2.2 Response

The plots of the Fourier decomposition are not present in this section, since they simply show the clean response in each bin as in Case 5. The magnitudes of the responses are instead tabulated in Table 11.

Table 11: Temporal and spatial magnitudes of response Fourier decomposition, case 6

<table>
<thead>
<tr>
<th>Frequency Bin #</th>
<th>35N</th>
<th>31N</th>
<th>39N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade 1 Response (m)</td>
<td>5.73e-05</td>
<td>5.48e-07</td>
<td>6.96e-07</td>
</tr>
<tr>
<td>1ND Wave Magnitude (m)</td>
<td>5.73e-05</td>
<td>5.52e-07</td>
<td>6.93e-07</td>
</tr>
</tbody>
</table>

It can be noticed that while the primary response of the 1ND mode is in the 35N bin, there are additional response components present in the 31N and 39N bins. They are 2 orders of magnitude less than the main response but when the main response and the sideband travelling waves momentarily come into phase, the response would increase to result in the maximum value discussed in Section 4.5.1.2.
Table 12: Temporal and spatial decomposition of response near 6.25N frequency, case 6

<table>
<thead>
<tr>
<th>Frequency Bin #</th>
<th>6N</th>
<th>6.25N</th>
<th>6.5N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade 1 Response (m)</td>
<td>1.99e-07</td>
<td>3.63e-07</td>
<td>1.11e-07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1ND Wave Magnitude (m)</th>
<th>2ND Wave Magnitude (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.72e-07</td>
<td>1.03e-06</td>
</tr>
<tr>
<td></td>
<td>3.99e-07</td>
<td>7.18e-07</td>
</tr>
</tbody>
</table>

When the response around the 6.25N frequency is tabulated (Table 12), it can be observed that the responses are greater than in case 3. This leads to the possibility that merely the presence of the 1ND forcing elicits a response at the lower frequency 1ND mode possibly during the transient startup effects still present in the structure, albeit at a much lower amplitude than the primary excited 1ND mode.

4.6 Case 7

Case 7 analyzes the same sideband structure as in case 6, but this time the sideband amplitude is increased to 150 Nt.

4.6.1 Time Domain Plots

The time domain plots in this case show interesting effects in both the excitation and response.

4.6.1.1 Excitation

The time domain plots show the same high frequency beating phenomenon as seen in case 6, except it has a much stronger effect on the forcing function, as can be seen from Figure 66 Figure 67. The larger amplitude of the sidebands dramatically changes the forcing function when compared with case 6.

Figure 66: Time history of excitation on blade 1 over 6 revolutions, test case 7
4.6.1.2 Response

The unsteady response due to the beating phenomenon is stronger than in case 6, leading to an increase of the maximum response to $6.5857 \times 10^{-5}$ m. Similar observations regarding the flow can be made as in case 6; that is, the interference between the main forcing and the off resonance sideband travelling wave leads to an increase in the maximum structural response.

4.6.2 Fourier Decomposition

The Fourier decomposition of this case shows similar frequency domain characteristics to case 6, with the only difference being magnitude.
4.6.2.1  **Excitation**  
In the interest of saving space, the Fourier plots are avoided, since they are exactly the same in nature as in cases 5 and 6, with each prescribed excitation falling into its correct bin and nodal diameter.

4.6.2.2  **Response**  
As with case 6, the plots of the Fourier components are avoided to save space. All the responses at the main sidebands can be found in Table 13. When compared to case 6, an increase in the off-resonant +/-4N sideband response is seen.

<table>
<thead>
<tr>
<th>Frequency Bin #</th>
<th>35N</th>
<th>31N</th>
<th>39N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade 1 Response (m)</td>
<td>5.73e-05</td>
<td>1.64e-06</td>
<td>2.08e-06</td>
</tr>
</tbody>
</table>

Table 13: Temporal and spatial magnitudes of response Fourier decomposition, test case 7

The response around the 6.25N frequency shown in Table 14 demonstrates a marked increase in the response when compared to case 6 and even greater when compared to the baseline case 3. This further demonstrates that the presence of 1ND travelling waves at one frequency can increase the response in 1ND modes at other frequencies, possibly during transient startup.

<table>
<thead>
<tr>
<th>Temporal Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Bin #</td>
</tr>
<tr>
<td>Blade 1 Response (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ND Wave Magnitude (m)</td>
</tr>
</tbody>
</table>

Table 14: Temporal and spatial decomposition of response near 6.25N frequency, case 7

4.7  **Case 8**  
This case is the first where all sidebands are combined. The 400 Nt main excitation is surrounded by 50 Nt non-integer +/-0.25N sidebands, 50 Nt +/-1N sidebands and 50 Nt +/-4N sidebands (that do not follow Tyler-Sofrin, but instead fall on the same 1ND shape as the 35N excitation).

4.7.1  **Time Domain Plots**  
Understandably, in this case the time domain plots of the excitation and response become more complex.

4.7.1.1  **Excitation**  
From Figure 69 and Figure 70, it can be seen that there is a strong and varied beating present in the excitation when all sidebands are added into the forcing function. The non-integer sidebands excite the overall large
beating pattern as seen in case 4 and the integer sidebands contribute to the additional unsteadiness as seen in cases 5-7.

Figure 69: Time history of excitation on blade 1 over 6 revolutions, test case 8

Figure 70: Snapshot of excitation on blade 1, test case 8

4.7.1.2 Response

The time history of the response shows the same large beating present as in case 4. The presence of the other integer order sidebands add on top of that the unsteadiness in the response as seen in cases 5-8. This causes the peak response to increase to 7.1311E-05 m. It can be assumed that the response is dominated by the effects of the non-integer sidebands, as will be later confirmed by the Fourier decomposition.
4.7.2 Fourier Decomposition

Frequency domain analysis of this case offers a clearer picture of the combined sideband effects on the structural response.

4.7.2.1 Excitation

The Fourier plots for this case are avoided for redundancy and to save space. Since this is an artificially created excitation, all sidebands and the main driver are present at their designated frequencies. When those frequency bins are decomposed spatially, the correct nodal diameter waves are shown at their respective amplitudes. While the integer sidebands fit neatly in their bins, the non-integer sidebands in the excitation exhibit the same wideband characteristics as seen in section 4.3.2.1.

4.7.2.2 Response

The temporal Fourier decomposition (Figure 72) of this case shows that the largest response is present in the main 35N frequency bin, with smaller responses in the non-integer sidebands and smaller still responses in the integer order sideband frequencies.
As in the previously discussed cases, a clean 1ND response is seen in the 35N bin (Figure 73).

The 4N integer order sidebands demonstrate a response in their respective bins (Figure 74 and Figure 75) at the 1 ND travelling wave shape in a similar fashion to cases 7 and 8.
The 34N integer order sideband elicited small responses in the 2ND and 1ND shapes (Figure 76) similar to the response seen in case 5.
The 36N sideband causes a response in the 0 nd mode (Figure 77), a similar response to case 5.

The non-integer order sidebands (Figure 78 and Figure 79) show response components similar to those seen in case 4. The major travelling waves excited are the forward travelling 1ND waves with smaller responses seen as backward travelling 1ND modes. Much as in case 4, the non-integer sidebands also seem to elicit a small response in the +/- 6 and 7 ND travelling waves.
In addition to the plots, it’s helpful to look at all the responses collected in one table (Table 15). The responses that are insignificant or nonexistent compared to the others in each case are labeled “INS”.

From the Fourier decomposition of the response, it can be seen that the full signal can be described as a combination of individual travelling waves of a similar magnitudes to the ones in the cases where only one set of sidebands is present. The increase in the peak response appears due to the interaction of these travelling waves with each other, leading to larger unsteadiness and response.
Table 15: Temporal and spatial magnitudes of response Fourier decomposition, test case 8

<table>
<thead>
<tr>
<th>Temporal Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Bin #</td>
</tr>
<tr>
<td>Blade 1 Response (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>2ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>-1ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>0ND Wave Magnitude (m)</td>
</tr>
</tbody>
</table>

The response in the neighborhood of the 6.25N frequency shows magnitudes higher than those in the previous cases. The cumulative effect of the non-integer sidebands combined with the +/-4N 1ND sidebands causes an increase in the response of other 1ND mode, possibly during transient start up effects.

Table 16: Temporal and spatial decomposition of response near 6.25N frequency, case 8

<table>
<thead>
<tr>
<th>Temporal Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Bin #</td>
</tr>
<tr>
<td>Blade 1 Response (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>-1ND Wave Magnitude (m)</td>
</tr>
</tbody>
</table>

4.8 Case 9

Case 9 is similar to case 8 in that all sidebands are present in the excitation. The only difference is the increase of the +/-4N sidebands to 150 Nt.

4.8.1 Time Domain Plots

The time domain plots of this case show a much stronger effect on the excitation and response due to the increase of the +/- 4N sideband magnitude.
4.8.1.1 Excitation

Case 9 excitation exhibited similar behavior to case 8, but the beating caused by the large 4N sidebands was much more pronounced.

![Temporal Wave on Blade 1, 6 revolutions](image)

Figure 80: Time history of excitation on blade 1 over 6 revolutions, test case 9

4.8.1.2 Response

The time history of the response showed similar characteristics to case 8. The response shows an increase in the unsteady peaks due to the increase in the 4N sidebands. This leads to an increase of the peak response to 7.3898E-05 m.
Figure 82: Blade 1 tip response for last 4 revolutions, test case 9

4.8.2 Fourier Decomposition

Frequency domain analysis in this case shows characteristics close to case 8, with the main difference being the response due to the +/-4N sidebands.

4.8.2.1 Excitation

The Fourier decomposition plots will not be shown here in the interest of saving space. They simply demonstrated each excitation magnitude at its proper frequency bin and at the correct #ND travelling wave.

4.8.2.2 Response

In the interest of saving space and avoiding redundancy, the figures for the 2D Fourier response are omitted as they are generally the same as shown in case 8 with different response magnitudes. The magnitudes can be found collected in Table 17. The most notable difference between case 9 and case 8 is the magnitude of the +/-4N sideband response. Note that an entry of “INS” means the response is insignificant or nonexistent.
Table 17: Temporal and spatial magnitudes of response Fourier decomposition, test case 9

<table>
<thead>
<tr>
<th>Frequency Bin #</th>
<th>35N</th>
<th>31N</th>
<th>39N</th>
<th>34N</th>
<th>36N</th>
<th>34.75N</th>
<th>35.25N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade 1 Response (m)</td>
<td>5.72e-05</td>
<td>1.64e-6</td>
<td>2.08e-6</td>
<td>1.84e-07</td>
<td>9.45e-07</td>
<td>7.19e-06</td>
<td>3.46e-06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>2ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>-1ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>0ND Wave Magnitude (m)</td>
</tr>
</tbody>
</table>

A good way to qualitatively visualize this frequency domain distribution is as a contour plot (Figure 83). In the plot, it’s obvious that the largest response component is at the 35N frequency and a 1ND shape, but around it, one can notice the responses at the non-integer sideband frequencies. The response at the 1N sideband frequencies are weaker in comparison to the non-integer and +/- 4N frequencies.
The increase in the magnitude of the +/-4N sidebands caused an increase from the case 8 response in the neighborhood of the 6.25N frequency bin. This again added further credence to the idea that modes of a particular nodal diameter can be excited by the presence of an excitation in that nodal diameter family at a different frequency, even if that excitation may be a transient and not a steady state phenomenon.

Table 18: Temporal and spatial decomposition of response near 6.25N frequency, case 9

<table>
<thead>
<tr>
<th>Temporal Decomposition</th>
<th>Frequency Bin #</th>
<th>6N</th>
<th>6.25N</th>
<th>6.5N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade 1 Response (m)</td>
<td></td>
<td>3.14e-07</td>
<td>5.68e-07</td>
<td>1.72e-07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
<th>1ND Wave Magnitude (m)</th>
<th>8.8e-07</th>
<th>1.58e-06</th>
<th>4.78e-07</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1ND Wave Magnitude (m)</td>
<td>6.13e-07</td>
<td>1.1e-06</td>
<td>3.34e-07</td>
</tr>
</tbody>
</table>
4.9 Case 10

This case aimed to study the response of the structure in the presence of 2 modes simultaneously excited at their fundamental frequencies and shapes. In addition to the main excited 1ND mode at 35N, a second 2ND mode was excited by a 150 Nt forcing at a 41.75N frequency.

4.9.1 Time History

The time history of this case showed a much greater amplitude in the excitation and response.

4.9.1.1 Excitation

The time history of the excitation showed a high frequency beating pattern over the 6 revolution period, due the two strong forcing sources at resonance, yet far apart in frequency, interacting with one another.

![Temporal Wave on Blade 1, 6 revolutions](image)

Figure 84: Time history of excitation on blade 1 over 6 revolutions, case 10

![Temporal Wave on Blade 1, 6 revolutions](image)

Figure 85: Snapshot of excitation on blade 1, case 10
4.9.1.2 *Response*

The time history of the response indicated the response following a similar beating characteristic to the excitation. Since both modes were excited at resonance, eliciting large responses, it is understandable that the response signal would look similar to the excitation signal. The two travelling waves at resonance would respond to the excitation and interaction between them would occur leading to the pattern seen. The maximum blade response value was found to be higher than the baseline at 7.3347E-05 m.

![Time History of Blade 1 Response: Case 10](image)

Figure 86: Blade 1 tip response for last 4 revolutions, case 10

4.9.2 *Fourier Decomposition*

In the frequency domain, the excitation analysis offers no surprises, but the decomposition of the response shows a few interesting findings.

4.9.2.1 *Excitation*

The temporal and 2D Fourier decomposition of the signal (Figure 87- Figure 89) demonstrated that the excitations are in their proper frequency bins (note that 41.75 is rounded to 41.8 only for display in Figure 87).
Figure 87: Temporal Fourier decomposition of 4 revolutions, case 10

Figure 88: Spatial Fourier decomposition of 4 revolutions, 35N bin, case 10
4.9.2.2 Response

The temporal and 2D Fourier decomposition (Figure 90 - Figure 92) demonstrated that the responses from the two modes fell neatly into their respective Fourier bins.

Figure 89: Spatial Fourier decomposition of 4 revolutions, 41.75N bin, case 10

Figure 90: Temporal Fourier decomposition of 4 revolutions, blade 1 response, case 10
When looking at the lower frequency modal response temporally, an interesting picture emerges (Figure 93). In addition to the response in the neighborhood of 6.25N, a small peak can be seen at 7.75N. From Appendix A, it can be noticed that the lower frequency 2ND mode lies at 532.32 Hz, or 7.90N. It can thus be hypothesized that, similar to the 1ND modes, the presence of the higher frequency 2ND mode near 41.75N excited the lower mode at 7.90N, as seen in Figure 93 in the 7.75N bin. As with the previous cases of this lower frequency response, this could attributed to transient startup effects still present in the structure.
Temporal Fourier decomposition of 4 revolutions, blade 1 response, test case 10, zoomed in on 6.25N region

In the immediate neighborhood of 6.25N, the temporal response is of similar magnitude to case 3. However, the spatial decomposition of the bins of interest (Figure 94 - Figure 97) show a slightly different picture than cases 1-9 where only one resonant mode was excited. The presence of smaller +/-2N spatial waves can be observed in addition to the +/-1N waves (the magnitudes of which are similar to case 3).

Figure 93: Temporal Fourier decomposition of 4 revolutions, blade 1 response, test case 10, zoomed in on 6.25N region

Figure 94: Spatial Fourier decomposition of 4 revolutions, blade 1 response, 6N bin, test case 10
Figure 95: Spatial Fourier decomposition of 4 revolutions, blade 1 response, 6.25N bin, test case 10

Figure 96: Spatial Fourier decomposition of 4 revolutions, blade 1 response, 6.5N bin, test case 10

Figure 97 shows that the spatial characteristic of the 7.75N bin response is dominated by +/-2N waves, with a small contribution from the +/-1N waves. The higher frequency 41.75N 2ND excitation can thus be seen to produce a response at the lower frequency similar to the 35N 1ND excitation. Since the 6.25N and 7.75N modes are near in frequency, the +/- 1N and +/- 2N waves can be seen in both bins.
4.10 Case 11

This case is the ultimate combination of sidebands and simultaneous resonant modes. The sidebands from case 9 are joined by the additional resonant mode of case 10. This is the case that is expected to show the greatest response.

4.10.1 Time History
The time history of this case shows interesting characteristics, both in the excitation and in the response.

4.10.1.1 Excitation
The presence of the strong wideband forcing produces a signal that does not lend itself to an easy description. However, it could be observed when comparing this case with the case 9 and 10 forcing functions that the presence of the sidebands creates a similar pattern to case 9, while the additional 41.75N frequency content has a strong effect on the shape of the signal, creating the high frequency beating phenomenon similar to that seen in case 10.
4.10.1.2 Response

The combination of the strong excitation frequency content elicited quite an interesting response in the structure. The large beating pattern present due to non-integer sidebands (as seen in cases 4, 8 and 9) is dominating the response in addition to strong unsteady and high frequency response peaks excited by the additional resonant mode. Since the two resonant modes are travelling around the disk at different speeds and directions (a difference higher than the one between sidebands and the 35N excited mode), the response peaks can be assumed to occur when the positive interference between the two waves is at the maximum.
4.10.2 Fourier Decomposition

The frequency domain analysis of this case offers a deeper understanding of the combined effect of the sidebands and the additional excited mode on the structure’s response.

4.10.2.1 Excitation

The Fourier decomposition plots of the forcing function are omitted in this case, since as in all previous cases, the forcing content is assigned and is extremely narrow band for each superimposed forcing function. Therefore, the Fourier decomposition shows all of the prescribed forcing functions in their respective frequency bins. The spatial decomposition shows the same magnitude for the travelling waves in their respective frequency and spatial bins as seen in cases 9 and 10.

4.10.2.2 Response

The plots of the Fourier decomposition will not be inserted into this section to avoid redundancy due to the narrow bandedness of the response. The components of the response are tabulated in Table 19. It can be then seen that like in all previous cases, the response is comprised of temporal and spatial components of similar magnitude to all previous cases. The difference in the maximum response seen in section 4.10.1.2 from all other cases can then be understood as the result of interaction of these travelling waves, whose amplitude and phase interaction are not only governed by the spatial aliasing discussed in section 1.4, but also by the system stiffness, mass and damping matrices, causing complex interactions between the travelling waves.
Table 19: Temporal and spatial magnitudes of response Fourier decomposition, test case 11

<table>
<thead>
<tr>
<th>Frequency Bin #</th>
<th>35N</th>
<th>31N</th>
<th>39N</th>
<th>34N</th>
<th>36N</th>
<th>34.75N</th>
<th>35.25N</th>
<th>41.75N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade 1 Response (m)</td>
<td>5.72e-05</td>
<td>1.64e-6</td>
<td>2.08e-6</td>
<td>1.84e-07</td>
<td>9.45e-07</td>
<td>7.19e-06</td>
<td>3.46e-06</td>
<td>1.34e-05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>2ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>-2ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>-1ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>0ND Wave Magnitude (m)</td>
</tr>
</tbody>
</table>

The lower frequency response around 6.25N showed behavior and amplitude similar to that seen in case 9. The 7.75N response was a bit more complex. While the response magnitude was slightly higher than in case 10, Figure 101 shows that unlike in that case, the spatial characteristic in the 7.75N bin shows many travelling waves present that contribute to the overall response. It appears that in addition to the 41.75N excitation having an effect, the presence of the sidebands around the 35N excitation (especially the 39N sideband, as it is close in frequency to the 41.75N excitation) leads to a greater response and spatial content in the 7.75N bin.

Table 20: Temporal and spatial decomposition of response near 6.25N frequency, case 11

<table>
<thead>
<tr>
<th>Temporal Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Bin #</td>
</tr>
<tr>
<td>Blade 1 Response (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ND Wave Magnitude (m)</td>
</tr>
<tr>
<td>-1ND Wave Magnitude (m)</td>
</tr>
</tbody>
</table>

-87-
4.11 Frequency Response Analysis Results: All Test Cases

Frequency response harmonic analyses were performed at the main excitation and sideband frequencies with proper phasing for the forces around the disk in order to represent the travelling wave mode shapes excited. The non-integer sidebands were phased as 1ND waves with a 45 Nt magnitude from the excitation signal Fourier decomposition seen in Case 4 (Figure 42 and Figure 43). The maximum responses of the blade tip are shown below in Table 21 and can be visualized in Figure 102.

It is not surprising that the cases where modes are excited at resonance (35N and 41.75N) show the greatest harmonic response, the non-integer sidebands show the 2nd greatest response and the other off resonance integer order sidebands are orders of magnitude less in the maximum response magnitude.

<table>
<thead>
<tr>
<th>Frequency (#N)</th>
<th>Force (Nt)</th>
<th>Response (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>500</td>
<td>7.5984E-05</td>
</tr>
<tr>
<td>35</td>
<td>481</td>
<td>7.3096E-05</td>
</tr>
<tr>
<td>35</td>
<td>400</td>
<td>6.0787E-05</td>
</tr>
<tr>
<td>31</td>
<td>50</td>
<td>5.2846E-07</td>
</tr>
<tr>
<td>39</td>
<td>50</td>
<td>7.2163E-07</td>
</tr>
<tr>
<td>31</td>
<td>150</td>
<td>1.5859E-06</td>
</tr>
<tr>
<td>39</td>
<td>150</td>
<td>2.1689E-06</td>
</tr>
<tr>
<td>34.75</td>
<td>50</td>
<td>5.2997E-06</td>
</tr>
<tr>
<td>35.25</td>
<td>50</td>
<td>6.1770E-06</td>
</tr>
<tr>
<td>34</td>
<td>50</td>
<td>1.5753E-07</td>
</tr>
<tr>
<td>36</td>
<td>50</td>
<td>9.8386E-07</td>
</tr>
<tr>
<td>41.75</td>
<td>150</td>
<td>1.5916E-05</td>
</tr>
</tbody>
</table>
4.12 Results Summary

After going through each case individually in detail, the results must be summarized and compared.

4.12.1 Maximum Response Discussion

Figure 103 plots the maximum transient displacements for every case and the percent difference of each case from the case 3 baseline. From this plot, several observations can be made. The clean cosine wave cases (1, 2 and 3) produce strong responses due to the fact that they are simply resonant excitations with different amplitudes.

In case 4, the non-integer sidebands cause the maximum response out of all the individual non-resonant sideband cases, increasing it by 16%. Case 2 was tested to see if exciting the structure at the amplitude shown during 2D Fourier decomposition of the non-integer excitation over 1 revolution only (section 4.3.3) would lead to the same results, but case 2 ended up over predicting the case 4 response by 3.67%, leading to a conservative estimate, but did not over estimate by a large margin. This is understandable, since pure resonance with a clean excitation signal effects the largest response. Case 1 at 500 Nt amplitude produces an even greater response, over predicting the case 4 sideband response by 7.8%.

The 1N integer sidebands (case 5) had a small, nearly negligible effect on the max response, while the 4N sidebands had a slightly stronger effect, likely due to the fact that even though they were off resonance, they traveled at a 1ND pattern around the blisk, same as the main excitation. Case 7 (150 Nt 4N sidebands) caused a 9.47% increase in the response when compared to the baseline, meaning that large sidebands of that nature have an appreciable effect on the maximum response and should be considered if they appear during flow field analysis.

When all sidebands are introduced into the forcing function, it’s likely that the dominant increase is due to the non-integer sideband contribution, since the case 8 maximum response is only slightly higher than case 4 maximum response. The 4N sidebands likely account for most of that difference. When they are increased to 150N in case 9, the response is appreciably higher, further demonstrating their effect.
When 2 modes are excited simultaneously (case 10), the response is much higher than the case 3 baseline, leading to an increase of 21.92%. This is understandable, since even though the two modes are of different ND families, resonance nonetheless produces two strong travelling waves around the disk.

When the forcing function includes sidebands in addition to another resonant excitation (case 11), the response is dramatically increased (44%). This would be the most troubling case, as standard analysis practice would lead to drastic under prediction of the structure’s response.

**Figure 103: Maximum response comparison between all cases**

### 4.12.2 Fourier Spatial Magnitude Discussion

In Figure 104 (showing the magnitude of the 1ND spatial wave excited by the main resonant 35N frequency) the component can be seen to be the same for all cases where the main excitation is kept constant in the forcing function (recall Cases 1 and 2 are simply increased 35N excitations).

**Figure 104: 35N Fourier bin spatial magnitude comparison between all cases**
Now let’s examine all the cases where non-integer sidebands are present. One can easily see from Figure 105 and Figure 106 that the magnitude of each spatial wave excited by the non-integer effects stays essentially the same in all cases tested. Very minor variance is present.

Figure 105: Magnitude of 0.25N non-integer sideband excited 34.75N spatial waves

Figure 106: Magnitude of 0.25N non-integer sideband excited 35.25N spatial waves

Looking at Figure 107 and Figure 108, it can be observed that fact remains true for the 1ND sidebands as well. The small exception being the 1ND mode response from the 34N sideband that slightly varies in each case. It must be noted that this wave does not follow Tyler-Sofrin interaction rules and is much smaller in magnitude than all the other responses to the point of being close to insignificant.
Figure 107: Magnitude of 1N integer sideband excited 34N spatial waves

Figure 108: Magnitude of 1N integer sideband excited 36N spatial wave

Figure 109 Figure 110 for the 4N sidebands indicates similar behavior. Cases where the sidebands are 50Nt (6 and 8) show the same response magnitudes as each other. The same observation can be made when the sideband magnitude increases to 150Nt (cases 7, 9 and 11).
Lastly, a strong response from the -2ND mode can be seen in the cases where two modes are excited simultaneously (case 10 and 11), as expected (Table 22). The magnitude of that response remains the same in both cases.

Table 22: Magnitude of 41.75N spatial wave

<table>
<thead>
<tr>
<th>Case #</th>
<th>-2ND</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.34E-05</td>
</tr>
<tr>
<td>11</td>
<td>1.34E-05</td>
</tr>
</tbody>
</table>
When the 6N (Figure 111), 6.25N (Figure 112) and 6.5N (Figure 113) responses are compared across all cases, clear trends emerge. When the main sinusoidal excitation changes in magnitude (cases 1-3), the magnitude of the response in the lower frequency changes as well. The presence of non-integer sidebands (case 4) increased the response. What’s interesting to note is that the +/-4N sidebands (case 6 and 7) had a greater effect than the non-integer sidebands, a different trend from the observed effect near the 35N frequency. The cases with all sidebands (case 8 and 9) show an even greater increase from the baseline while the presence of the additional 41.75N excitation did not have a great effect on these frequency bins.

Figure 111: 6N bin spatial response: all cases

Figure 112: 6.25N bin spatial response: all cases
Figure 113: 6.5N bin spatial response: all cases

On the other hand, the response in the 7.75 bin did not follow as easily of a quantifiable pattern. The presence of sidebands seems to reduce the response in the 2ND shapes while increasing the 1ND wave response. Note that the responses in the 6.5N and 7.75N neighborhood may simply be artifacts of the transient start-up. Nonetheless, they should be noted in this investigation.

Figure 114: 7.75N bin spatial response: cases 10 and 11

Looking at all results, one can see that despite the varying loading in each case, the forcing from the main excitation and sidebands caused nearly the same response in their specific frequency and spatial bins in each case where that specific excitation was constant. Since all responses are travelling at different speeds around the blisk and not necessarily at the same speed as their respective excitations, how these responses interact with each other is not trivial. But could a simple approximation be used? For example, if all spatial mode
response magnitudes are added together, would it be a good approximation for the maximum transient response? The reality turns out to be quite a bit more complicated than that hypothesis.

Figure 115 demonstrates that complication, since no clear pattern can be seen. In most cases, simple addition under predicts the response. The cases where it does appear to be close are cases with non-integer sidebands (4, 8 and 9). Case 11 where another mode is excited at resonance along with other sideband content is also closely approximated, unlike case 10 with 2 pure resonances. In Case 11 it can be seen that simple addition is fairly close to the measured maximum response.

The non-integer sidebands are close to the main excitation in frequency and contribute to the increased response by producing forward travelling waves near the resonant 35N main excitation and excite those travelling waves in the structure at nearly the same wave speed as the main excitation. Due to this fact, the interaction between them and the main response is likely simpler than in other cases.

![Comparison of maximum transient response with the addition of individual spatial component magnitudes](image)

**Figure 115:** Comparison of maximum transient response with the addition of individual spatial component magnitudes

### 4.12.3 Estimation of Transient Response Using Harmonic Components

The next question was whether an accurate representation of the transient response with sideband content can be attained using addition of harmonic components.

This was done in Figure 116, where each harmonic component calculated and tabulated in section 4.10 was added for each case according to its excitation signal content.

In cases 1-3, a steady difference of 1.04% can be seen. This demonstrates that the transient response to a resonant excitation is close to the harmonic response value as expected.

When only non-integer sidebands are involved (case 4), the harmonic addition method provides a decent approximation of the full response.
When 1N sidebands (case 5) are studied, the addition method slightly over predicts the response seen, but the difference is not much greater than the difference already present between simple excitation cases and its harmonic analysis (case 1-3).

What’s interesting to note is that in the case of 4N sidebands, the response is actually under predicted slightly by the harmonic addition method, especially when large sidebands are assumed (case 7). Since these sidebands are present at the same ND wave as the main excitation, the interaction between them causes the larger response when they come in and out of phase. The simple addition of harmonic components does not fully predict the interaction between the modes.

In cases 8 and 9, the non-integer and 1N sidebands cause the transient response to be over predicted if harmonic magnitudes are added. This is however tempered by the presence of large 4N sidebands (case 9) that cause that percentage to decrease. This can be understandable, as harmonic analysis assumes a clean harmonic excitation when in reality, the complex forcing functions excite a wide band of travelling waves at different speeds interacting with each other, as discussed in section 4.12.1. In reality, the interaction is more complex than a simple addition of magnitudes.

For the cases with 2 resonant modes at different NDs, the simple addition method over predicts the response by 4.58 % in case 10 and 7.38 % in case 11. The simple addition method does not account for the complex interaction between two different resonant mode shapes travelling at different speeds and frequencies around the disk. It does however provide a metric that could be used to determine which sideband excitation seen in the flow may be problematic for structural response.

![Comparing Harmonic Magnitude Addition and Max Transient Response](image)

Figure 116: Comparing addition of harmonic components with the maximum transient response

### 4.12.4 Brown-Schamuch Results Discussion

A similar approach was taken by Brown and Schmauch in the course their investigation. Now that the numerical results of the follow on investigation have been discussed, they can be compared to the results of Brown-Schmauch’s 3D analysis (completely recreated in Table 23 lists the displacements of several nodes on the blade surface when subjected to a full transient and harmonic analyzes (74N excitation exciting a 5ND mode). Harmonic
Table 23 and Table 24 for a 5ND and 12ND analyzed mode, respectively) [2].

Table 23 lists the displacements of several nodes on the blade surface when subjected to a full transient and harmonic analyses (74N excitation exciting a 5ND mode). Harmonic analysis was performed at the main and non-integer sideband frequencies. It was discovered that the addition of harmonic responses at sideband frequencies to the main harmonic response varied across the different nodes in the accuracy of its prediction of the transient response [2]. This suggested additional complex 3D effects from the sidebands on the blade’s response.

Table 23: Results for frequency and transient analysis for full airfoil solid model excitation of 5ND mode

<table>
<thead>
<tr>
<th>Node</th>
<th>Transient Response Theta Displacement</th>
<th>74N Frequency Response Magnitude Displacement</th>
<th>74N Frequency Response Theta Displacement</th>
<th>Error 74N Freq Resp from Transient Theta</th>
<th>73.8N Frequency Response Theta Displacement</th>
<th>74.2N Frequency Response Theta Displacement</th>
<th>Peak of sum of 74N, 73.8N, and 74.2N Error sum Freq Resp from Transient Theta</th>
</tr>
</thead>
<tbody>
<tr>
<td>113894</td>
<td>.00380</td>
<td>.00358</td>
<td>.00358</td>
<td>6%</td>
<td>.000658</td>
<td>.000732</td>
<td>.00415</td>
</tr>
<tr>
<td>94577</td>
<td>.00403</td>
<td>.0037</td>
<td>.00366</td>
<td>9%</td>
<td>.000606</td>
<td>.000668</td>
<td>.00421</td>
</tr>
<tr>
<td>39041</td>
<td>.00398</td>
<td>.00358</td>
<td>.00353</td>
<td>11%</td>
<td>.000694</td>
<td>.000772</td>
<td>.00414</td>
</tr>
<tr>
<td>52320</td>
<td>.004</td>
<td>.00345</td>
<td>.00357</td>
<td>11%</td>
<td>.000666</td>
<td>.000740</td>
<td>.00415</td>
</tr>
<tr>
<td>83711</td>
<td>.0042</td>
<td>.00355</td>
<td>.00352</td>
<td>16%</td>
<td>.000695</td>
<td>.000664</td>
<td>.00409</td>
</tr>
</tbody>
</table>

Table 24 shows the effects from a 57N excitation of a 12ND mode on the structural response when compared to the frequency response analysis. Large under predictions were observed for all nodes examined. Even lumping non-integer effects into the 57N bin (“Frequency Response 57N, 1 rev, 57th bin” column) only slightly decreases the discrepancy. Significant content could also be seen at the 64 N and 63 N bins, suggesting that the complexity of the 3D flow excited multiple nodal diameter mode shapes. For a full understanding of these phenomena, the reader is encouraged to read their paper in full [2].

Table 24: Results for frequency and transient analysis for full airfoil solid model excitation of 12ND mode

<table>
<thead>
<tr>
<th>Location</th>
<th>Node</th>
<th>Transient Response, theta, 5, revs, excitation 40882.5Hz</th>
<th>Frequency Response 57N, 5 revs, 285th bin (5*57N)</th>
<th>Error from transient Frequency Response 57N, 1 rev, 57th bin</th>
<th>Frequency Response bin64 at 45903Hz theta disp</th>
<th>Error from transient Frequency Response bin64 at 45903Hz theta disp</th>
<th>Frequency Response bin 63 at 45186</th>
</tr>
</thead>
<tbody>
<tr>
<td>Close to mid-chord</td>
<td>35378</td>
<td>7.25E-04</td>
<td>2.02E-04</td>
<td>-72%</td>
<td>4.55E-04</td>
<td>-37%</td>
<td>6.50E-05</td>
</tr>
<tr>
<td>Close to mid-chord</td>
<td>36585</td>
<td>7.35E-04</td>
<td>2.00E-04</td>
<td>-73%</td>
<td>4.64E-04</td>
<td>-37%</td>
<td>4.17E-05</td>
</tr>
<tr>
<td>Mid-chord</td>
<td>82462</td>
<td>7.75E-04</td>
<td>3.20E-04</td>
<td>-59%</td>
<td>1.85E-04</td>
<td>-76%</td>
<td>1.16E-04</td>
</tr>
<tr>
<td>Close to mid-chord</td>
<td>11022</td>
<td>7.35E-04</td>
<td>2.80E-04</td>
<td>-62%</td>
<td>1.83E-04</td>
<td>-75%</td>
<td>5.01E-05</td>
</tr>
<tr>
<td>close to mid-chord</td>
<td>93537</td>
<td>5.60E-04</td>
<td>4.09E-04</td>
<td>-27%</td>
<td>1.76E-04</td>
<td>3.29E-05</td>
<td>1.76E-04</td>
</tr>
<tr>
<td>Largest transient of mid-chord</td>
<td>86507</td>
<td>3.81E-04</td>
<td>2.60E-05</td>
<td>-93%</td>
<td>1.01E-04</td>
<td>4.30E-05</td>
<td>1.01E-04</td>
</tr>
<tr>
<td>Mid-chord</td>
<td>97375</td>
<td>3.70E-04</td>
<td>7.40E-05</td>
<td>-80%</td>
<td>6.16E-05</td>
<td>3.03E-05</td>
<td>6.16E-05</td>
</tr>
</tbody>
</table>
The reader can note similarities in the numerical results. What are not accounted for, however are the 3D effects such as blade-flow interaction.

### 5  3D Model Construction for Future Analysis

In the course of this investigation, a 3D model was also created to potentially analyze the response due to forcing caused by a transient 3D flow field and compare responses to flow fields with and without sidebands.

#### 5.1 3D Model Dimensions and Material Properties

The diameter and thickness of the disk were kept the same as the 2D model:

- Disk diameter: 0.69 m
- Shaft diameter: 0.138 m
- Disk thickness: 0.07 m

The height of the blades was fixed at 0.07 m, but the rest of the blade was designed by an engineer and CFD specialist at MSFC following the procedure in the following section.

The material properties for the model were kept the same as in the 2D case (Table 3, section 3.2.2).

#### 5.2 Blade Design and CFD Flow Field Generation

Keeping the blade count and geometry constant as design requirements, a run speed was picked that would give proper bulk flow angles for a supersonic turbine. However, the blade counts were too low for the radius used, so the blade loading coefficients were off [19].

This resulted in not getting the expected flow turning and a lot of flow separation (a bad turbine design, but ok for structural dynamics theoretical work), due to the fact that geometric constraints were used to create a fictitious turbine that would have fluid drivers with known spatial content [19].

For flow field generation, the surface meshes were generated at MSFC in ANSA cad/grid package and the volume mesh was generated with AFLR3 (Advancing Front Local Reconnection), a code developed at Mississippi State University [19].

The blades were imported into ANSYS using the IGES (Initial Graphics Exchange Specification) format. Lessons learned regarding that process can be found in Appendix E.

The blades were meshed in ANSYS 14.5 with a less dense grid for structural analysis. Due to the IGES file import, the faces were fragmented and had to be merged before meshing.

After the completion of the meshing, the pressure and suction side nodes and elements were exported to MSFC where the data from the much finer CFD mesh was interpolated onto the structural surface mesh. Data was output as both harmonic content for frequency response analysis and 2000 transient time steps over one revolution for the transient analysis. Additional details regarding steps for CFD data import can be found in Appendix F. A small issue with the ANSYS code is that surface loads cannot be used in mode superposition analyses. Several workarounds are possible, one possible method is discussed in Appendix G.

As an examination of the flow field, its harmonic components were studied at several nodes on one blade. The real inefficiency of the flow in the turbine then became apparent. From the suction side (Figure 117) and pressure side (Figure 118) it can be seen that flow unsteadiness is present in addition to the main 35N excitation. However, sideband content is not present due to the absence of non-adjacent stage as in [2].
This structure and flow was thus found not suitable for this investigation. A realistic multi-stage turbine would be needed to truly study this phenomenon. One simulation would have to be run without taking into account non-adjacent stage effects and one simulation with non-adjacent stages modeled, the effects of the unsteadiness and sidebands on a 3D structure, especially the blades could thus be analyzed.
5.3 3D Mesh

The mesh was constructed with perfect cyclic symmetry in the disk and blade meshes in mind so that pure structural response phenomena could be studied. The model with the surface mesh shown can be seen in Figure 119.

The surfaces were all mapped as quadratic elements and the interiors were mapped as hexahedron dominant. The specific element used in ANSYS 14.5 was SOLID186, a 2nd order element that can take the shape of a tetrahedron, pyramid or prism [5].

Figure 119: 3D model with surface mesh displayed

Figure 120: SOLID186 Element [5]
5.3.1 Blades

After much refinement, each blade had an identical mesh comprised of 3418 nodes and 598 elements. The minimum Jacobian ratio was 1.0055 and maximum of 6.4184 with 1.570 as the average, exhibiting a good quality of the mesh [7]. From Figure 121, it can be seen that across the full range of Jacobian ratios, the mesh is dominated by the hexahedral element (referred to as Hex20) while only a small portion of the blade mesh are prisms (referred to as Wed15). The quality of the mesh for the blades was decided to be very good based on these findings [7].

![Figure 121: Jacobian ratio vs. number of elements, individual blade mesh](image)

5.3.2 Disk

The disk was created out of 101808 nodes and 21600 elements. The Jacobian ratio ranged from 1.03 to 1.13, and dominated by only hexahedrons (as seen in Figure 122). This mesh was determined to be of very high quality [7].
5.3.3 Blade – Disk Connection

The meshes of the blades and disk were connected with rigid surface to surface contact elements. The disk was the target, using TARGE170 elements on the surface where the blades came in contact with the disk [5].
For the blades, the bottom was defined as CONTA174 elements (Figure 124). Using these elements, loads could be interpolated across the dissimilar meshes [5]. It must be noted however that the mesh used will not have accurate stress results in the immediate area of the contact. However, stress would not be necessary as a performance metric in a study of sideband response, so it was not deemed to be detrimental to this investigation.
Even though the 3D model was created, in its current iteration it was not useful to this investigation. However, it can offer a springboard for a further course of study, such as analysis of a production model with and without non-adjacent stages modeled to study the effect of these complex excitations.

6 Conclusion

This paper presents an investigation into a phenomenon recently discovered by Brown and Schmauch [2] in the course of flow analysis of a supersonic turbine intended for spacecraft propulsion. Their investigation uncovered that the complex nature of the flow in such a device due to the presence of multiple non adjacent stages created unsteadiness that manifested itself as sidebands around the main forcing frequency. These sidebands became evident when 2D Fourier analysis was performed on the unsteady pressure history spanning multiple revolutions taken at the blade surface nodes.

The sidebands had a unique distribution. There was significant sideband content in the non-integer bins immediately surrounding the main forcing frequency as well as 1N integer sidebands. Once the flow field was re-examined in the course of this investigation, 4N integer order sidebands that did not obey the spatial aliasing described by Tyler and Sofrin [15] were discovered. These sidebands instead existed spatially at the same nodal diameter as the main forcing function.

A follow on investigation was thus conducted to determine the effects of these forcing sidebands on structural response individually and in combination. The transient response of a 2D model comprised of shell elements for the disk and beam elements for the blades to allow for rapid analysis was used for this purpose. The investigation used ANSYS 14.5 structural FEM solver and modelled 4 stabilized revolutions using a mode superposition transient analysis method. Several simple to use and understand tools were developed for this investigation using MATLAB and Excel. They include a 2D Fourier analysis routine that may take as input single timestep files, or a matrix of data; a routine that demonstrates Tyler-Sofrin interaction; a routine to determine transient stabilization and a spreadsheet that generates forcing for use in 2D analysis.

The analysis revealed that the non-integer order sidebands (0.25N) had the greatest effect by increasing the maximum response, while the 1N sidebands did not elicit a strong reaction. The 4N sidebands had a surprising contribution, increasing the maximum response of the structure by a significant percentage. The cases where all sidebands were present showed an increase comparable to that seen in [2]. The main contribution came from the non-integer order sidebands with 4N sidebands playing a lesser role.

It was also found that if a forcing function with sideband content was modeled, but only one revolution was subjected to Fourier decomposition, with the harmonic content from the main frequency bin subsequently applied to harmonic analysis, the response could be slightly over predicted (this strongly depends on which revolution is analyzed). If, on the other hand, multiple revolutions were subjected to Fourier decomposition and only the harmonic content from the main frequency bin was used for harmonic analysis, the response could be greatly under predicted.

An interesting finding also demonstrated that the presence of 1ND forcing at the 35N frequency also produced a much smaller response in the lower 6.25N frequency 1ND mode. This response was also amplified by the presence of sidebands, mainly those of 0.25N and 4N. This suggests that the mere presence of a travelling wave of one shape produces a small, off resonant response in all modes of the same shape present in the structure. This can be either a physical or numerical effect, possibly caused by transient startup effects.

To no surprise, it was also found that the presence of 2 simultaneously excited modes at their fundamental frequencies and shapes greatly increased the disk’s structural response, even more so if sidebands are also present. Therefore, if structural analysis of a turbine is to be conducted, it is useful to first look at the harmonic content present in the flow field and create plots similar to those developed for this study (Figure 20 and Figure 21 from Section 3.2.3) that would allow the analyst to first assess if sideband content or other
forcing content present could excite additional modes at their fundamental shapes and frequencies at certain operating points before proceeding with continued analysis and design.

It was found that the transient response did not lead itself to easy prediction using simple addition of responses from harmonic analyses or individual component response magnitudes due to the complex nature of travelling wave mode interaction.

Lastly, a 3D model was created. The 3D supersonic flow field generated for this structure at NASA MSFC did not have non adjacent stage effects and thus, similar sideband content was not observed in the flow. Several useful methods for the particular software used were catalogued, thus providing a possible springboard for future studies.

7 Recommended Forward Work

The results presented in this paper offer a springboard to continue in depth analysis of the dynamic response due to flow unsteadiness (sidebands) caused by non-adjacent stages in supersonic turbomachinery similar to the forcing identified in Brown-Schmauch [2]. The 2D model provided fundamental insight into the effects of individual sidebands on the disk response using simplified blades. However, with blades modeled as simple beams, the 3D nature of the flow and the effects of the flow unsteadiness on blade response cannot be studied.

A follow on investigation could use a real world design such as the turbine modeled in [2], using multi-revolution CFD and compare the flow field, pressure distribution on the blade surfaces and structural response of the same structure with and without non-adjacent stages modeled to assess the 3D effects of non-adjacent stages.

The unsteady pressure distribution on the blades could be compared between the two cases. It would be interesting to investigate how the presence of sidebands due to multiple stages could affect the blade modal response and fluid-structure coupling.

Additionally, a follow on investigation could continue to study the interaction between travelling waves around the disk at different speeds and magnitudes and assess if a simple mathematical model of the interaction can be created.
### Appendix A 1st 13 Nodal Diameter Disk Modes

<table>
<thead>
<tr>
<th>Disk ND</th>
<th>1</th>
<th>Disk ND</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade Mode</td>
<td>Rigid (no flex)</td>
<td>Blade Mode</td>
<td>1st flex</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>416.07</td>
<td>Frequency (Hz)</td>
<td>2359.8</td>
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<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>Blade Mode</td>
<td>Rigid (no flex)</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>532.32</td>
</tr>
</tbody>
</table>

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</tr>
</thead>
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<tr>
<td>Blade Mode</td>
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</tr>
<tr>
<td>Frequency (Hz)</td>
<td>2808.3</td>
</tr>
<tr>
<td>Disk ND</td>
<td>3</td>
</tr>
<tr>
<td>---------</td>
<td>-----</td>
</tr>
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<td>Blade Mode</td>
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<tr>
<td>Frequency (Hz)</td>
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<td>Blade Mode</td>
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<td>Frequency (Hz)</td>
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<tbody>
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<td>Blade Mode</td>
<td>1st flex</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>3473.5</td>
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</tbody>
</table>

<table>
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<th>4</th>
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</thead>
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<td>Blade Mode</td>
<td>1st flex</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>4180.7</td>
</tr>
<tr>
<td>Disk ND</td>
<td>Blade Mode</td>
</tr>
<tr>
<td>---------</td>
<td>------------------</td>
</tr>
<tr>
<td>Disk ND</td>
<td>Rigid (no flex)</td>
</tr>
<tr>
<td>Disk ND</td>
<td>1st flex</td>
</tr>
<tr>
<td>Disk ND</td>
<td>Rigid (no flex)</td>
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<tr>
<td>Disk ND</td>
<td>1st flex</td>
</tr>
<tr>
<td>Disk ND</td>
<td>7</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
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<td>Blade Mode</td>
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</tr>
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<td>Frequency (Hz)</td>
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</table>

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Appendix B 2D Fourier Analysis MATLAB program

%This program will create a fourier decomposition of data, %both in the %temporal and spatial domains

%The data must be arranged in files (a separate file for each %measured location) %containing the time and value of interest or a matrix of all %values

%Created as a part of the THRUST Spring 2014 thesis work by %Anton Gagne, %KTH Stockholm, supervised by Dr. Andrew Brown (Duke, NASA %MSFC)

close all
clc

%%%%%%%%%%%%%%%%%%%%%%%%%%The part below needs to be filled in by the user%%%%%%%%%%%%%%%%%%%%%%%%%%
%to initialize all rotor parameters

%Number of temporal harmonics
tharm = 300;

%RPM of the rotor
RPM = 4045.371429;

%shaft frequency
Nsh = RPM/60;

%Number of revolutions recorded
nrev = 4;

%Root: root directory where signals are stored
root = 'C:\Users\Anton Gagne\Desktop\Thesis\Fourier Files\Rotating_probes';

%Base name of signal file
base = 'phist_';

%Number of measurement points/nodes (enter by user)
npoint = 36;

%Number of spatial intervals
nspatint = 36;

%Number of time histories for the signal (signal dependent)
nhist = 4201;

%Number of intervals
Nint = 4200;

%%%%%%%%%%%%%%%%%%%%Calculation past this point%%%%%%%%%%%%%%%%%%%%%

%bin resolution
bres = 1/nrev;

%fourier resolution
fourres = Nsh/nrev;

%%%%%%%%%%%%%%%%%%%%%%%%The following section will differ depending on your input data
%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%The following is used if your data is in an individual file for
%%%%%each time step (otherwise comment out this section)

%Create pressure array by reading in all files
%NOTE: In this case, the 1st column is the time and second the data, this
%may change depending on your data
for i = 1:npoint
    temp = load([root,base,num2str(i),'.dat']);
    pres(:,i) = temp(:,2);
end

%Time array extracted from 1st file
%NOTE: This may change depending on the file
p1 = load([root,base,'1.dat']);
t = p1(:,1);

dt = t(2)-t(1);

%%%%%End of section for individual file data read-in (if applicable)

%%%%%If the data for analysis is not in individual time step files, please
%%%%%enter a matrix of data with the proper dimensions into
%the workspace
%%%%%and name it "pres"

%time step Entered (if data is in matrix format, comment out otherwise)
\[ dt = 1.4125490860807E-05; \]

%%%Subsequent sections of the code are calculations

%Frequency data and array calculated

\[ T = dt*Nint; \quad \text{%Total time} \]
\[ f0 = 1/T; \quad \text{%Frequency interval} \]
\[ F = \text{bres}*(0:1:(Nint)); \quad \text{%Frequency array} \]

%Nyquist frequency
\[ F_{nyq} = F(1:(Nint/2)); \]

%Initializing the array for the temporal fourier transform
\[ \text{tempfour} = \text{zeros}(Nint, \text{npoint}); \]

%Loop over all measurement points
\[ \text{for ind = 1:npoint} \]
\[ \quad \text{%initializing array for temporal transform} \]
\[ \quad \text{tempdft} = \text{zeros}(Nint,1); \]
\[ \quad \text{%Extracting pressure for current point from pressure array} \]
\[ \quad \text{temppres} = \text{pres}(:,\text{ind}); \]
\[ \quad \text{%Simple counter for indexing the sum values} \]
\[ \quad \text{count} = 0; \]
\[ \quad \text{%Loop over all time intervals to calculate the full array of fourier} \]
\[ \quad \text{%components (could be changed to tharm to save computation time)} \]
\[ \quad \text{for p=1:Nint;} \]
\[ \quad \quad \text{count} = \text{count} +1; \]
\[ \quad \quad \text{%DFT sum formulation} \]
\[ \quad \quad \text{for k=0:Nint-1;} \]
\[ \quad \quad \quad \text{tempdft}(p)=\text{tempdft}(p)+\text{temppres}(k+1)*(2/Nint)*\exp(-1i*2*pi*p*k*(1/(Nint))); \]
\[ \quad \quad \end\]
\[ \quad \end\]
\[ \quad \text{%Pasting the temporal fourier column into the results matrix} \]
\[ \quad \text{tempfour(:,ind)} = \text{tempdft}; \]
\[ \end\]
%Calculating spatial nyquist point
Spatsize = size(pres(1,:));
Snyq = Spatsize(2)/2;

%The DFT array without the DC component, up to the max number of temporal harmonics (for possible troubleshooting)
filttempfour = tempfour(1:tharm);

%Extracting the temporal data only for the 1st file
firstfile = tempfour(:,1);
filtfirstfile = firstfile(1:tharm); %First file to the temporal harmonic

%Real values of the 1st file DFT
realfirstfile = real(firstfile);
realfiltfirstfile = realfirstfile(1:tharm);

%Imaginary values of the 1st file DFT
imagfirstfile = imag(firstfile);
imagfiltfirstfile = imagfirstfile(1:tharm);

%Real value of full temporal fourier matrix
Real = real(tempfour);

%-imaginary values of the full temporal fourier matrix
Imag = -imag(tempfour);

%Initializing zero matrices for the calculation of spatial DFT
RealImagSpat = zeros(Nint, 2*Snyq+1); %Real values of the transform of the imaginary temporal components
ImgImagSpat = zeros(Nint, 2*Snyq+1); %Imaginary values of the transform of the imaginary temporal components
ImagRealSpat = zeros(Nint, 2*Snyq+1); %Imaginary values of the transform of the real temporal components
RealRealSpat = zeros(Nint, 2*Snyq+1); %Real values of the transform of the real temporal components
DFTRealComp = zeros(Nint, 2*Snyq+1);
DFTImgComp = zeros(Nint, 2*Snyq+1);

%Wave number from - to +spatial nyquist
wavenumber = (-Snyq):Snyq;

%Loop over all temporal harmonics
for ind = 1:tharm

    %Initializing zero real and imaginary arrays
spatdftreal = zeros(2*Snyq+1,1);
spatdftimag = zeros(2*Snyq+1,1);

% Extracting real and imaginary values for the current spatial bin
realspat = Real(ind,:);
imagspat = Imag(ind,:);

count = 0; % Counter for array indexing

%DFT from - to + spatial nyquist points
for p=(-Snyq):(Snyq);

    count = count + 1;
    
    % & a nested for loop for the sum expression

    % DFT for real values
    for k=0:nspatint-1;
        spatdftreal(count)=spatdftreal(count)+(1/(nspatint))*realspat(k+1)*exp(-1i*2*pi*p*k*(1/(nspatint)));
    end

    % DFT for imaginary values
    for k=0:nspatint-1;
        spatdftimag(count)=spatdftimag(count)+(1/(nspatint))*imagspat(k+1)*exp(-1i*2*pi*p*k*(1/(nspatint)));
    end

% Pasting the real and imaginary arrays into the full data matrix
DFTRealComp(ind,:) = spatdftreal;
DFTImagComp(ind,:) = spatdftimag;

% Split into two parts for troubleshooting, but now identical. The
% negative imaginary value of the real component DFT
ImagRealSpat(ind,1:Snyq) = -imag(spatdftreal(1:Snyq));
ImagRealSpat(ind,(Snyq+1):(2*Snyq+1)) = -imag(spatdftreal((Snyq+1):(2*Snyq+1)));

% Real value of the imaginary component DFT
RealImagSpat(ind,:) = real(spatdftimag);
% Real value of the real component DFT
RealRealSpat(ind,:) = real(spatdftreal);

% Split into two parts for troubleshooting, but now identical. The
% negative imaginary value of the imaginary component DFT
ImagImagSpat(ind,1:Snyq) = -imag(spatdftimag(1:Snyq));
ImagImagSpat(ind,(Snyq+1):(2*Snyq+1)) = -imag(spatdftimag((Snyq+1):(2*Snyq+1)));
end

% Combining all terms to form final spatial values
% real(realtransform) - imag(imagtransform) = realarray
FullRealSpat = RealRealSpat - ImagImagSpat;
% imag(realtransform) + real(imagtransform) = imagarray
FullImagSpat = (ImagRealSpat + RealImagSpat);

% Spatial magnitude: (realarray^2 + imagarray^2)^(1/2)
MagSpat = (FullRealSpat.^2 + FullImagSpat.^2).^(1/2);

%%%%%%%%%%%%%%%%%%%%%%%%%% Plotting below w/this point%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Plot magnitude of 1st file temporal transform for comparison and
% troubleshooting
figure(1)
hold on
title('Temporal FFT algorithm, 1st location File Magnitude')
xlabel('Frequency Multiples of N (Shaft Frequency)')
ylabel('Amplitude')
bar(Fnyq(2:tharm+1),abs(filtfirstfile))
hold off

figure(2)
hold on
title('Temporal FFT algorithm, 1st location File Real Component')
xlabel('Frequency Multiples of N (Shaft Frequency)')
ylabel('Amplitude')
bar(Fnyq(2:tharm+1),realfiltfirstfile) 
hold off

figure(3)
hold on
title('Temporal FFT algorithm, 1st location File Imaginary Component')
xlabel('Frequency Multiples of N (Shaft Frequency)')
ylabel('Amplitude')
bar(Fnyq(2:tharm+1),imagfiltfirstfile)
hold off

% Asking if you want to plot any spatial bins
spatyn = input('Do you want to print out a spatial bin? (y/n) ','s');

% Plot spatial bins of interest

count = 0;
while spatyn == 'y'

    count = count + 1;
    binnum = input('What # bin do you want to print out? ');
    binnumult = binnum * nrev;

    figure(count+3)
    hold on
    title(['Bin',num2str(binnummult*bres),'N'])
    xlabel('Spatial Wave')
    ylabel('Amplitude')
    bar(wavenumber, MagSpat(binnummult,:))
    hold off

    more = input('Do you want more spatial bins? (y/n) ','s');
    if more == 'n'
        break
    end
end

% This portion here to debug and plot file outputs from the DFT program used
% at MSFC with a reference data set (1st file temporal transform for comparison)
% It is presented for reference and was used during the program's original
% creation

% Asking if results need to be plotted (y/n)
% presyn = input('Do you want to plot some of prestons results? ' ,'s');
% if presyn == 'y'
load([root,'file_1_harm_bin_mag.dat'])
figure(count+4)
hold on
title('Temporal Fourier Transform magnitude, Prestons 1st file')
xlabel('Frequency')
ylabel('Magnitude')
bar(file_1_harm_bin_mag(:,2))
hold off

load([root,'file_1_harm_bin_re_im.dat'])
figure(count+5)
hold on
title('Temporal Fourier Transform real, Prestons 1st file')
xlabel('Frequency')
ylabel('Magnitude')
bar(file_1_harm_bin_re_im(:,2))
hold off

figure(count+6)
hold on
title('Temporal Fourier Transform imaginary, Prestons 1st file')
xlabel('Frequency')
ylabel('Magnitude')
bar(file_1_harm_bin_re_im(:,3))
hold off

end
Appendix C Tyler-Sofrin Interaction Travelling Wave Mode Demonstration MATLAB program

%This program calculates an example of a the Tyler-Sofrin (T-S) traveling wave mode for a simple example blade - vane interaction

%Created as a part of the THRUST Spring 2014 thesis work by
%Anton Gagne,
%KTH Stockholm, supervised by Dr. Andrew Brown (Duke, NASA
%MSFC)

clear all
close all
clc

FPS = 2; %Enter desired frames per second for animation

intref = 3000; %Number of intervals for vane field
dtheref = (2*pi)/intref; %Number of reference intervals in angles
bld = 36; %Number of blades
dthe = (2*pi)/bld; %delta for the angle (rad)
V = 35; %number of vanes
phi = 0; %wave phase
Amn = 1; %amplitude
n = 1; %harmonic index
RPM = 1935; %RPM
omega = RPM * ((2*pi)/60); %angular velocity (rad/sec)
tsn = 1; %time for the snapshot graph
dT = 0.1; %dt for the TWM calculation
TF = 2; %Final time for the TWM calculation

%Creating a snapshot of the pressure field to compare only the vane effects
%with T-S effects
pstat = [];
pstatVane = [];

thetatot = (0:dthe:(2*pi)); %Loop over all thetas from 0 to 2*pi
for theta = 0:dthe:(2*pi)
    pstat = [pstat, Amn*cos(n*V*(theta-omega*tsn)+phi)]; %blade only
end

thetabld = (0:dtheref:(2*pi)); %Loop over all thetas from 0 to 2*pi
for theta = 0:dtheref:(2*pi)
\[ p_{\text{statVane}} = [p_{\text{statVane}}, \text{Amn}\cos(nV(\theta - \omega t_{\text{sn}})+\phi)]; \quad \% \text{blade only} \]
end

\% Graphing the rotor field snapshot
figure(1)
hold on
plot(thetatot, pstat, '-o')
plot(thetabld, pstatVane, 'g', 'LineWidth', 2)
xlabel('Theta location (radians)')
ylabel('Pressure field')
title(['Pressure field at time = ', num2str(tsn)])
legend('Sampled Excitation', 'Vane Only Pressure Field')
hold off

\% Creating the traveling wave mode animation
P = [];
PVane = [];
Ttot = (0:dT:TF);
for T = 0:dT:TF
    pstatTWM = [];
pstatVaneTWM = [];
    for theta = 0:dthe:(2*pi)
        pstatTWM = [pstatTWM, \text{Amn}\cos(nV(\theta - \omega T)+\phi)]; \% Full
    end
    for theta = 0:dtheref:(2*pi)
        pstatVaneTWM = [pstatVaneTWM, \text{Amn}\cos(nV(\theta - \omega T)+\phi)]; \% Vane
    end
    P = [P, pstatTWM'];
PVane = [PVane, pstatVaneTWM'];
end

\% Calculating the speed of the traveling waves
\% blades only
[C, I] = findpeaks(P(:,3));
[C2, I2] = findpeaks(P(:,5));
thet1 = thetatot(I);
theta2 = thetatot(I2);
dtheta = theta2 - thet1;
dthetaavg = mean(dtheta);
speed = dthetaavg / dT;

disp('The speed of the blade only wave is = (rad/sec) ')
disp(speed)
%Including TS interaction

[CTS,ITS] = findpeaks(PVane(:,3));
[C2TS,I2TS] = findpeaks(PVane(:,4));
thet1TS = thetabld(I TS);
thet2TS = thetabld(I2TS);
dthetTS = thet2TS - thet1TS;
dthetavgTS = mean(dthetTS);
speedTS = dthetavgTS / dT;

disp('The speed of the TS wave is = (rad/sec) ')
disp(speedTS)

%Calculating ratio of the speeds
ratio = speedTS/speed;

disp('The ratio between the TS wave and the regular wave is = ')
disp(ratio)

ratio2 = speedTS/omega;

disp('The ratio between the TS wave and the shaft speed is = ')
disp(ratio2)

%Animation of the TWM
counter = 0;
for i = 0:dT:TF
    figure(2)
    counter = counter + 1;
    plot(thetatot, P(:,counter), '-o', thetabld, PVane(:,counter), 'g')
    xlabel('Theta location (radians)')
    ylabel('Pressure field')
    title('Traveling Waves')
    legend('Sampled Excitation', 'Vane Only Pressure Field')
    F(counter) = getframe;
end
movie(F,20,FPS)
Appendix D: Transient Convergence Determination
MATLAB Program

% This is a quick routine that takes the periodic signal’s maxima and compares it with the previous, to find when the solution converges

% Created as a part of the THRUST Spring 2014 thesis work by Anton Gagne,
KTH Stockholm, supervised by Dr. Andrew Brown (Duke, NASA MSFC)

[pks, locs] = findpeaks(bld1);

ptime = time(locs);

percentdiff = []; for i = 2: (length(pks))
    percentdiff(i) = (abs((pks(i) - pks(i-1)))/pks(i-1)) * 100;
end

figure(1)
plot(ptime(2: length(pks)), percentdiff(2: length(pks)), '-o', 'MarkerSize', 2)
title('Peak Convergence Over Time')
xlabel('Time (sec)')
ylabel('% Difference From Previous Peak')
Appendix E IGES File Import Lesson Learned

The .igs format files have a header with 2 unit flags which all import algorithms use.

In the case of this investigation, for both of the files originally sent, the header looked like Figure E1. Note how both flags are set in inches (highlighted).

```
1H,,1H;,,34H/u/home1/pachman/new_21/rotor.igs,4HANSA,6H14.1.0,32,38,6, G 1
308,24,,1,,12HIN,1,1,,15H20140319.143212,7.874015748031496D-06,0,,11,G 2
1,15H20140319.143212,,; G 3
```

Figure E1: IGES header with unit flag set to inches

The flags had to be changed to centimeters and the import into ANSYS worked with the correct dimensions.

```
1H,,1H;,,34H/u/home1/pachman/new_21/rotor.igs,4HANSA,6H14.1.0,32,38,6, G 1
308,24,,1,,10,2HIN,1,1,,15H20140319.143212,7.874015748031496D-06,0,,11,G 2
1,15H20140319.143212,,; G 3
```

Figure E2: IGES header with unit flag set to centimeters

There is another major headache that was noted. The alignment of every row in the file has to be perfect. Since one flag was changed from 1 to 10, each line had to be manually changed to delete a space and move a comma. Otherwise, the import failed. (Notepad++ is really good for editing all files with a similar format).
Appendix F Method for Transferring Surface Nodes from ANSYS Workbench to NASA MSFC for CFD Data Interpolation

Here described is a simple method that was used for exporting nodes and elements from ANSYS workbench to NASA MSFC for CFD data interpolation. It’s not the most elegant, but it worked when data was needed to be quickly transferred.

It was required to export wetted area nodes, their coordinates and the surface elements (quads in the case of this 3D structure). The same origin was used in both structural and CFD models to make the interpolation easy.

The surface of the blades and disk were all defined for quad meshing in the ANSYS structural model. The 3D elements were mostly hexahedrons.

The following steps were taken:

1. Create a named selection on the wetted area of the blades (select the faces), can break it out into pressure and suction side (Pressure Blades and Suction Blades as defined in this model), or create a named selection for the full wetted area that selects both pressure and suction side of the blades (that’s actually easier and would be recommended). Doing so removes some complication from the process such as double bookkeeping of leading and trailing edge nodes).

2. Apply a unit pressure to the wetted surface named selection (the analysis defined in the model tree can be harmonic, or any that requires surf154 elements. It’s not important though, since this is only used to generate the batch file from which nodes and elements can be extracted). What is important here is not the actual pressure, but the fact that the pressure object in the tree will skin the surface with surf154 elements over the 3D elements at the same nodes. In this case, since 2nd order hex 3D elements were used, the surface thus becomes 2nd order quads with 8 nodes on each quad (4 corners and 4 midpoint nodes).

3. Create a batch file by going to the toolbar and selecting tools->write input file

The batch file can be opened in any text editor. Notepad++ works well due to its speed and ease of use.

Near the top of the file, all of the model’s nodes will be listed along with their x, y, and z coordinates. This section can be copied and imported into Excel or MATLAB.
Next, the named selection can be searched for in the batch file. The section below the name will list all node numbers in that selection. In Figure F3, “Pressure Blades” named selection can be seen. This section can be copied and exported into another sheet in the Excel workbook, or a matrix in MATLAB. To make the use of vlookup in Excel easier, all nodes could be placed into one column after pasting the data into a workbook.

With these two matrices, the x, y and z coordinates of the wetted nodes can be extracted by the user’s preferred method in Excel or MATLAB.

In this case, one worksheet was used where the wetted nodes were kept in a column. From there, vlookup was used to find that node number in another worksheet containing all nodes and their coordinates. Thus
the x, y and z coordinates for each wetted node were located. This resulted in a final worksheet containing the wetted nodes and their coordinates.

Next, the surface quads (or tris) can be extracted from the nodes. This is where the pressure application comes in.

In the batch file, the heading for the section that applies pressure loading will look like Figure F4.

First, this section creates surf 2D elements over every surface of the 3D elements where pressure is applied. In this case, this was 2nd order quads comprised of 8 nodes. The data is arranged in the following way: The 1st column is the # of the surface element created by ANSYS (not too important for this purpose). The next important data points are the last columns clustered together, which will depend on what mesh is used at the structural model wetted area. These columns list the node numbers of the surface elements.

The data will be comprised of several element blocks, denoted as:

Eblock,##,,,###

This is not too important, but just as a point of reference. In summary, these last columns can be used to create one worksheet or matrix that lists all wetted surface elements and the nodes they are comprised of.

After the completion of this procedure, the following data was sent to MSFC:

- One space delimited .txt file containing the wetted nodes, with node number in the 1st column followed by the x, y and z coordinate (all in meters).

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<td>-0.02078460507</td>
<td></td>
<td></td>
</tr>
<tr>
<td>316</td>
<td>0.12890374130</td>
<td>0.41397455570</td>
<td>-0.01715955287</td>
<td></td>
<td></td>
</tr>
<tr>
<td>317</td>
<td>0.12500339780</td>
<td>0.41406935020</td>
<td>-0.01369244222</td>
<td></td>
<td></td>
</tr>
<tr>
<td>318</td>
<td>0.12086956500</td>
<td>0.41416256568</td>
<td>-0.01038740872</td>
<td></td>
<td></td>
</tr>
<tr>
<td>319</td>
<td>0.11676316520</td>
<td>0.41426564550</td>
<td>-0.00729133520</td>
<td></td>
<td></td>
</tr>
<tr>
<td>320</td>
<td>0.11237870260</td>
<td>0.41436177610</td>
<td>-0.00465892237</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Figure F5: Example of .txt file containing all wetted nodes
One text document containing the surface elements, with the 1st column being an arbitrary assigned number for each element followed by 8 columns containing the node numbers for each 2nd order quad element.

1  665  680  932  666  2880  2909  2881  2879
2  664  701  680  665  2878  2908  2880  2877
3  663  722  701  664  2876  2970  2879  2875
4  662  743  722  663  2874  3011  2876  2873
5  661  764  743  662  2872  3052  2874  2871
6  660  785  764  661  2870  3093  2872  2869
7  659  806  785  660  2868  3134  2870  2867
8  658  827  806  659  2866  3175  2868  2865
9  657  848  827  658  2864  3216  2866  2863
10 656  869  848  657  2862  3257  2864  2861
11 655  890  869  656  2860  3298  2862  2859
12 654  911  890  655  2858  3339  2860  2857
13 653  932  911  654  2856  3380  2858  2854

Figure F6: Example of .txt. document containing surface elements

This allowed the CFD data to be interpolated onto the nodes of the structural ANSYS model.
Appendix G Possible Method for Applying Surface Forces in Mode Superposition Transient Analyses Using ANSYS Version 14.5

The following is one of the methods that could be used to allow for mode superposition transient analysis in ANSYS 14.5 using surface forces. Pseudo-code in the ANSYS APDL language is shown to get the user started.

1. In the modal analysis portion of the APDL code, set up a loop over all nodes where pressure will be applied, such as:
   - DO Nodes 1st (surface) to last (surface)
     - SFDELE
     - SF,1,PRES,1
     - SOLVE ---->Load Vector
   - ENDDO

2. In the transient portion of the analysis, create tables for each node of the load varying over the entire time range.
   - LVSCALE, (1st surface node), %TABLE 1%
   - LVSCALE, (2nd surface node), %TABLE 1%
   - Create similar tables for all surface nodes

3. Solve
Bibliography


