STRENGTH AND DEFORMABILITY OF FRACTURED ROCKS

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PhD Thesis
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DEDICATION

To
my parents and parents-in-law
and
my lovely wife
SUMMARY

Knowledge about the strength and deformability parameters of fractured rocks is one of the crucial issues for design, construction, operation, and performance/safety assessments of surface and subsurface structures in civil and mining engineering. The fractured rock masses consist many non-uniform and non-regular fractures of varying sizes, orientations and locations with complex fracture system geometry and large uncertainties. However, the proper characterization of these properties of fractured rocks is impossible directly at present using the current laboratory test facilities that are designed for testing small intact rock samples. Large-scale in-situ field tests are not practically feasible due to difficulties in the definition and control of the initial and boundary loading conditions at representative elementary volume (REV).

This thesis developed a systematic numerical modeling framework to simulate the stress-deformation and coupled stress-deformation-flow processes by performing uniaxial and biaxial compressive numerical experiments on fractured rock models with considering the effects of different loading conditions, different loading directions (anisotropy), and coupled hydro-mechanical processes for evaluating strength and deformability behavior of fractured rocks. A stochastic analysis was performed to quantify the variations of strength and deformability of fractured rock, using multiple realization models of the fracture system geometry.

By using code UDEC of discrete element method (DEM), a series of numerical experiments were conducted on two-dimensional (2D) discrete fracture network models (DFN) at an established REV based on realistic geometrical and mechanical data of fracture systems from field mapping at Sellafield, UK. The obtained stresses and strains results from the numerical experiments were used to represent the stress-strain behavior of fractured rocks as a function of confining pressure, and to estimate the equivalent directional Young’s modulus and Poisson’s ratio as two important deformation parameters. The results were used to fit the Mohr-Coulomb (M-C) and Hoek-Brown (H-B) failure criteria, represented by equivalent material properties defining these two criteria.

The results demonstrate that strength and deformation parameters of fractured rocks are dependent on confining pressures, loading directions, water pressure, and mechanical and hydraulic boundary conditions. Fractured rocks behave non-linearly, represented by their elasto-plastic behavior with a strain hardening trend. Analysis of the stress-deformation of fractured rocks with the axial stress and axial velocity loading conditions shows that there are differences between strength curves and strength parameters under these loading conditions. The results obtained from the rotated DFN models indicate that strength and deformability of fractured rocks are direction-dependent, vary with the loading conditions. Therefore, the directional variations (anisotropy) of strength and deformability of fractured rocks must be considered in practice. The numerical results of modeling fluid flow in fractured rocks under hydro-mechanical loading conditions show an important impact of water pressure on the strength and deformability parameters of fractured rocks, due to the effective stress phenomenon, but the values of stress and strength reduction may or may not equal to the magnitude of water pressure, due to the influence of fracture system complexity. The results of stochastic analysis indicate that the strength and deformation properties of fractured rocks have
ranges of values instead of fixed values, hence such analyses should be considered especially in cases where there is significant scatter in the rock and fracture parameters. These scientific achievements can improve our understanding of fractured rocks’ hydro-mechanical behavior and are useful for the design of large-scale in-situ experiments with large volumes of fractured rocks, considering coupled stress-deformation-flow processes in engineering practice.
تجربه دقیق برآمردهای مقاومتی و تغییر شکل‌پذیری توده سنگ‌های دره‌دار بکی از موضوعات سیاسی مهم و ضروری در مرحله طراحی، ساختن، اجرای انتخابی و کاربردی از سوی دو نظریه سطحی و زیرزمینی می‌باشد. به علت اینکه سنگ‌های دره‌دار اغلب در شرایط مختلف تنش کار دارند، تغییرات پارامترها و تغییرات شکل توده سنگ ممکن است باعث تغییرات در توانایی آزمایشگاهی می‌شوند. نظریه سطحی آزمایشگاهی ممکن است در کنار این نظریه باشد.

نتایج این تحقیق نشان می‌دهد که مقاوت و تغییرات شکل‌پذیری توده سنگ‌های دره‌دار در حال واقعیتی است و بر پایه عملیاتی و اصولی ممکن است باشد. در این تحقیق، نماینده سیستم مهندسی بررسی نشان می‌دهد که در تعریف شکل‌پذیری توده سنگ‌های دره‌دار، تغییرات پارامترها و تغییرات شکل توده سنگ‌های دره‌دار باعث تغییرات در رفتار الستوپلاستیک و روند کرنش سخت‌شدن می‌شوند. مقایسه در این تحقیق نشان داده است که تغییرات پارامترها و تغییرات شکل توده سنگ‌های دره‌دار باعث تغییرات در رفتار الستوپلاستیک و روند کرنش سخت‌شدن می‌شوند.
مقاوتي و تغییر شکل پذیری توده سنگ‌های درزدار نشان داده شده است. در این موارد، به دلیل بی‌چیدگی گی ذرات سیستم درزه های موجود در توده سنگ و تاثیرات آنها، مقادیر کاهش نشگری مقاومت نشگری توده سنگ‌های درزدار ممکن است برای مقدار قوی‌تری مقاومت با مقادیر فشار بپرسد. نتایج تحلیل‌های آماری نیز نشان می‌دهد که مقادیر بدست آمده از پارامترها مقاویت و تغییر شکل پذیری توده سنگ‌های درزدار به جای یک مقادیر در یک محدوده متغیر می‌باشد، از آن‌رو این چنین تحلیل‌هایی پایین در کارهای عملی انتحار شود، به ویژه در مواردی که پراکندگی زیادی در خصوص مقادیر مکانیکی و هندسی پارامترهای سنگ و درز وجود دارد.

دستاوردهای علمی بدست‌آمده از این تحقیق می‌تواند منجر به بهبود کارهای پیشرفته در تولید و استفاده از میکروب‌های مکانیکی و هندسی، بهبود کارکرد و کیفیت محصولات صنعت سیستم‌های درزدار شود. نتایج این تحقیق نشان می‌دهد که بهترین راه حل‌ها برای کاهش تغییر نتیجه‌های مکانیکی و هندسی پارامترها در سنگ‌های درزدار نشان می‌دهد. نتایج این تحقیق نشان می‌دهد که بهترین راه حل‌ها برای کاهش تغییر نتیجه‌های مکانیکی و هندسی پارامترها در سنگ‌های درزدار نشان می‌دهد.

کلمات کلیدی: توده سنگ‌های درزدار، آزمایش‌های عملی، روش آزمایش‌های عملی، سنگ‌های درزدار، ناهماهنگی سنگ، شکست، مدل‌های تصادفی، نارسایی.
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First and above all, praises and thanks to God for life and providing me this opportunity and ability to do this research work, with a great inspiration and stimulus in times of despair.

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Last but as it is usually stated, absolutely not least; I would of course also like to take the opportunity to warmly thank and appreciate my parents and parents-in-law for their material and spiritual support in all aspects of my life, who have encouraged me along the way. I am most grateful to my lovely wife, Asieh, for her love, unconditional support, and the greatest patience during the past years. Without her encouragement and understanding this achievement would have not been possible. Dear Asieh, this success belongs to both of us!

Majid Noorian-Bidgoli
Stockholm, November 2014
LIST OF PAPERS

This PhD thesis is based on the following five papers, which are referred in the text of the thesis by their Roman numbers (I-V) and are appended at the end of the thesis.


The following works are not appended in the thesis, but a partial list is provided below.


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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\Delta u_n$</td>
<td>Normal displacement increment (mm)</td>
</tr>
<tr>
<td>$\Delta \sigma_n$</td>
<td>Normal stress increment (MPa)</td>
</tr>
<tr>
<td>$\sigma_{i}^m$</td>
<td>Measured values of $\sigma_1$ for the $i^{th}$ rotated model</td>
</tr>
<tr>
<td>$\sigma_{i}^p$</td>
<td>Predicted values of $\sigma_1$ for the $i^{th}$ rotated model</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>Chi-Squared goodness-of-fit test</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>Pressure difference (MPa)</td>
</tr>
<tr>
<td>$\Delta \sigma_y$</td>
<td>Axial compressive stress load increment (MPa)</td>
</tr>
<tr>
<td>2D</td>
<td>Two-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>3DEC</td>
<td>Three-dimension Distinct Element Code</td>
</tr>
<tr>
<td>ANN</td>
<td>Artificial Neural Networks</td>
</tr>
<tr>
<td>BB</td>
<td>Barton-Bandis</td>
</tr>
<tr>
<td>c</td>
<td>Cohesion (MPa)</td>
</tr>
<tr>
<td>C,P</td>
<td>Confining Pressure (MPa)</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>D</td>
<td>Density (Kg/m$^3$)</td>
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<tr>
<td>DEM</td>
<td>Discrete Element Method</td>
</tr>
<tr>
<td>DFN</td>
<td>Discrete Fracture Network</td>
</tr>
<tr>
<td>DIANE</td>
<td>Discontinuous, Inhomogeneous, Anisotropic, and Not Linearly Elastic</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus (GPa)</td>
</tr>
<tr>
<td>$E_i$</td>
<td>Expected frequency</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite Dexterity Method</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>GP</td>
<td>Genetic Programming</td>
</tr>
<tr>
<td>GSI</td>
<td>Geological Strength Index</td>
</tr>
<tr>
<td>H-B</td>
<td>Hoek-Brown</td>
</tr>
<tr>
<td>HM</td>
<td>Hydro-mechanical</td>
</tr>
<tr>
<td>k</td>
<td>Number of bin or interval</td>
</tr>
<tr>
<td>$k_n$</td>
<td>Normal stiffness of fracture (GPa/m)</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Shear stiffness of fracture (GPa/m)</td>
</tr>
<tr>
<td>m</td>
<td>Parameter of the Hoek-Brown failure criterion</td>
</tr>
<tr>
<td>M-C</td>
<td>Mohr-Coulomb</td>
</tr>
<tr>
<td>MRM</td>
<td>Mining Rock Mass Rating</td>
</tr>
<tr>
<td>n</td>
<td>Number of data pairs</td>
</tr>
<tr>
<td>N</td>
<td>Rock mass number system</td>
</tr>
<tr>
<td>$O_i$</td>
<td>Observed frequency</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Distribution Function</td>
</tr>
<tr>
<td>PFC2D</td>
<td>Two-dimensional Particle Flow Code</td>
</tr>
<tr>
<td>PFC3D</td>
<td>Three-dimensional Particle Flow Code</td>
</tr>
<tr>
<td>Q</td>
<td>Tunneling quality index</td>
</tr>
<tr>
<td>R</td>
<td>Correlation coefficient</td>
</tr>
<tr>
<td>REV</td>
<td>Representative Elementary Volume</td>
</tr>
<tr>
<td>RMi</td>
<td>Rock Mass index</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Squared Error</td>
</tr>
<tr>
<td>RQD</td>
<td>Rock Quality Designation</td>
</tr>
<tr>
<td>s</td>
<td>Parameter of the Hoek-Brown failure criterion</td>
</tr>
<tr>
<td>THMC</td>
<td>Thermo-Hydro-Mechanical-Chemical</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>--------</td>
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</tr>
<tr>
<td><strong>UCS</strong></td>
<td>Uniaxial Compressive Strength (MPa)</td>
</tr>
<tr>
<td><strong>UDEC</strong></td>
<td>Universal Distinct Element Code</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Major principal stress at failure or elastic strength (MPa)</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>Minor principal stress or confining pressure (MPa)</td>
</tr>
<tr>
<td>$\sigma_{ci}$</td>
<td>Uniaxial compressive strength of the intact rock (MPa)</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Normal stress (MPa)</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>Confining pressure (MPa)</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Axial compressive stress (MPa)</td>
</tr>
<tr>
<td>$\tau_{\text{max}}$</td>
<td>Shear strength (MPa)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Friction angle (°)</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Laplace integral</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Dilation angle (°)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mean value or scale parameter of the distribution</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation value or location parameter of the distribution</td>
</tr>
</tbody>
</table>
Abstract

This thesis presents a systematic numerical modeling framework to simulate the stress-deformation and coupled stress-deformation-flow processes by performing uniaxial and biaxial compressive tests on fractured rock models with considering the effects of different loading conditions, different loading directions (anisotropy), and coupled hydro-mechanical processes for evaluating strength and deformability behavior of fractured rocks. By using code UDEC of discrete element method (DEM), a series of numerical experiments were conducted on discrete fracture network models (DFN) at an established representative elementary volume (REV), based on realistic geometrical and mechanical data of fracture systems from field mapping at Sellafield, UK. The results were used to estimate the equivalent Young's modulus and Poisson’s ratio and to fit the Mohr-Coulomb and Hoek-Brown failure criteria, represented by equivalent material properties defining these two criteria.

The results demonstrate that strength and deformation parameters of fractured rocks are dependent on confining pressures, loading directions, water pressure, and mechanical and hydraulic boundary conditions. Fractured rocks behave nonlinearly, represented by their elasto-plastic behavior with a strain hardening trend. Fluid flow analysis in fractured rocks under hydro-mechanical loading conditions show an important impact of water pressure on the strength and deformability parameters of fractured rocks, due to the effective stress phenomenon, but the values of stress and strength reduction may or may not equal to the magnitude of water pressure, due to the influence of fracture system complexity. Stochastic analysis indicates that the strength and deformation properties of fractured rocks have ranges of values instead of fixed values, hence such analyses should be considered especially in cases where there is significant scatter in the rock and fracture parameters. These scientific achievements can improve our understanding of fractured rocks’ hydro-mechanical behavior and are useful for the design of large-scale in-situ experiments with large volumes of fractured rocks, considering coupled stress-deformation-flow processes in engineering practice.

Key words: Fractured crystalline rocks; Numerical experiments; Discrete element methods (DEM); Discrete fracture network (DFN); Representative elementary volume (REV); Coupled hydro-mechanical processes; Anisotropy; Effective stress; Failure criteria; Stochastic realizations.

1. Introduction

1.1. Background and motivation

Strength and deformation parameters define the mechanical behavior of fractured rock masses and are the key governing issues required for design, construction, operation, and performance/safety assessments of surface and subsurface structures in and on rock masses in rock engineering, especially for rock engineering projects of importance for energy resources and environment protection issues, such as underground nuclear waste repositories, gas/oil/water storage caverns and geothermal reservoirs.
The crystalline rock masses are fractured media and complex materials in nature, which consist essentially of intact rock matrix (block) and rock fractures (discontinuities). Due to the complex geometry of the fracture systems, the crystalline rock masses are largely discontinuous, inhomogeneous, anisotropic, and not linearly elastic (DIANE) materials (Hudson and Harrison, 1997). Fractured rock masses are usually weaker, more deformable, and highly anisotropic compared with intact rocks.

Generally, mechanical and hydraulic properties of rock matrices and fractures, and geometric characteristics of the fracture systems dominate the strength and deformation behavior of fractured rocks. In this thesis, the terms of fractures and discontinuities are treated as the same general term representing the systems of joints, faults and fault zones, for simplicity, unless specific definitions are requested for special problems considered.

The geometrical properties of the fracture systems (such as size, orientation, and location), dominate processes of fluid flows in either liquid or gas phases (such as water, oil, natural gases and air), and localized in-situ conditions of stresses near fracture intersections, since the fracture networks in fractured rocks usually serve as the main flow channels or conduits for the movement of water through rock masses. Hence, presence of fracture systems leads to a significant level of uncertainty in fractured rock behavior and makes working conditions more difficult.

It should be noted that, to obtain realistic results for strength and deformation behaviors of fractured rocks, large volumes of rock containing fractures should be tested at desired stress levels, in theory. In practice, however, the presences of various discontinuities in fractured rocks, the inherent complexity of their geometrical parameters, and the difficulties for estimation of their geomechanical and geometrical properties, make it difficult to measure directly mechanical properties of fractured rocks in laboratory conditions.

Conventional rock mechanics laboratory tests of intact rock samples of small volumes cannot provide information about the strength and deformation behaviors of fractured rock masses that include many fractures of varying sizes, orientations and locations at larger scales. On the other hand, large-scale in-situ tests of fractured rock masses are economically costly and often not practical in reality at present, such as definitions of the initial and boundary conditions of the to be tested volumes. Therefore, alternative approaches are needed for estimation of the strength and deformability of fractured rock masses.

Nowadays, numerical methods are able to calculate strength and deformability of fractured rocks with flexibility, considering the interactions between the intact rock matrix and fractures, by representing different mechanical and geometric features of the fractures and the intact rock matrices. Among available methods, the discrete element method (DEM) is a very attractive method that simulates explicitly, complex geometry of discrete fracture
network (DFN) models, with simple or complicate constitutive models of rock fractures and rock matrix (Jing and Stephansson, 2007).

During the past decades, prediction of the strength and deformability of fractured crystalline rocks have not been investigated systematically and this important issue remains unclear. The available researches were most often conducted on the intact rocks without fracture, or on rock masses containing a single joint set or regular and isotropic fracture system geometry under simple assumptions, which are often not adequate representation of realistic rock mass geometry.

The study presented in this thesis provides some fundamental insights for better understanding of the strength and deformation behavior of fractured rocks, through a systematic numerical methodology, by using realistic fracture system information from field mapping, so that more realistic representation of complex fracture system geometry of fractured rocks can be considered.

1.2. Objective

Due to the complicated structure of fractured rock masses and lack of accurate knowledge on distribution of fracture system geometry in nature, a complete and reliable conceptual understanding on the strength and deformability behavior of fractured rocks is still not possible without support of large scale laboratory tests of samples of fractured rocks of volumes not smaller that their respective REV (representative essential volume). However, numerical modeling approaches, especially DEM, can provide useful, although more generic rather than practical estimations on important issues affecting mechanical processes of fractured crystalline rocks, such as anisotropy of strength and deformability and effects of water pressure on them, at a REV level. This is useful since laboratory tests for such samples of huge volume is not possible at present or in the near future, but such knowledge is still required in rock engineering practice. This consideration is the motivation of the research presented in this thesis.

A numerical modelling methodology was developed to perform a fundamental study on strength and deformability of fractured rocks, as an overall objective of this research. For a series of two-dimensional (2D) numerical experiments conducted, a number of stress-deformation-flow analyses were conducted to estimate the strength parameters and water pressure effects on mechanical behavior of fractured rocks.

The specific objectives of the thesis are summarized as follows:

- Development a reliable numerical predicting platform for logical representation of complex and realistic fracture system geometry to calculate the representative strength and deformability of fractured rocks at REV level, in a generic sense.

- Estimation of two equivalent elastic deformation parameters of the fractured rocks, namely Young’s modulus and Poisson’s ratio,
as functions of lateral confining pressure during elastic deformation stages.

- Determination of equivalent strength parameters of the fractured rocks, using two popular strength failure criteria, namely Mohr-Coulomb (M-C) and Hoek-Brown (H-B), and their differences during fitting processes of strength envelopes.

- Investigation of the effects of two most used loading conditions, namely axial load and axial velocity, to understand possible differences when testing large volumes of fractured rocks with fracture and intact rock matrix of different constitutive behaviors, since such tests have not been performed yet.

- Study of effects of loading directions on strength and deformability of fractured rocks, for investigating anisotropy of strength and deformation behavior of fractured rocks considered, due to the fact that only numerical modeling, at least in practice at present, can provide such a systematic numerical testing, but such a systematic study has not been reported.

- Investigation of the effects of water pressure on strength and deformation parameters of fractured rocks through a coupled stress-flow processes modeling so that the conditions for validity of conventional definitions of effective stress can be evaluated, which has not been attempted before.

- In addition, a set of 50 multi-fracture system realizations were conducted for a statistical analysis of uncertainty of strength and deformability of fractured rocks, which will be a helpful approach for the design and performance assessments of rock engineering project.

It should be noted that since fractured rocks under low confining pressures usually show complex behaviors of deformability, such as higher values of equivalent Poisson's ratio larger than 0.5, a set of confining pressure of small magnitudes were assumed to check such special phenomena, since this is an important issue when large volume tests of fractured rocks in laboratory conditions.

1.3. Thesis layout

In order to get a clear overview of the thesis, based on the five papers mentioned in the preface, short descriptions of the contents in each chapter are summarized as follows:

Chapter (1): a general background of this research and motivations briefly described.

Chapter (2): a comprehensive literature review, as the current state-of-the-art and outstanding issues on behavior of fractured rocks, and also estimation methods and factors on the strength and deformability of fractured rocks are presented.

Chapter (3): a systematic numerical procedure is established to predict strength and deformability parameters and stress-strain behavior of fractured rocks (Paper I).

Chapter (4): presentation of a numerical modeling on the influence of different loading conditions on compressive strength and
deformation behavior of fractured rocks are conducted when axial velocity and axial stress controlled loading conditions (Paper I, II). Chapter (5): presentation of a fundamental study on anisotropy of strength and deformability of a typical fractured rock sample, with variations under different loading directions of rotating computational models of fracture systems (Paper III).

Chapter (6): presentation of a numerical methodology to simulate the coupled stress-deformation-flow processes for evaluating the water pressure effects on strength and deformability of fractured rocks and assessing the validity of the classical effective stress concept to fractured rocks (Paper IV).

Chapter (7): presentation of a stochastic analysis with multiple realizations of the fracture system geometry to estimate uncertainty of strength and deformability of fractured rocks (Paper V).

Chapter (8): some important issues relating to the current study are discussed.

Chapter (9): a summary of main conclusions and scientific achievements of the conducted work are drawn. Also, some recommendations and topics for further research are proposed, based on the outstanding issues that remain to be addressed in future.

2. LITERATURE REVIEW

2.1. Fractured rock masses

As mentioned in introduction, knowledge of the fractured rock masses behavior is one of the major tasks in rock mechanics and rock engineering. For such studies, it is extremely important to understand and describe the structure of the rock masses.

Rock masses are never isotropic materials in nature due to the existence of inherent discontinuities such as joints, bedding planes, veins, folds, fissures, cleavages, faults, and sheared zones that control their hydro-mechanical behaviors.

In this thesis, for simplicity the term ‘fracture’ and ‘fractures’ are adopted as the general term for all types of discontinuities of rocks, unless specified separately.

Because of the difficulty of in-situ geological surveys of fractures inside the rock masses, fractures are usually described as assemblages, and classified into sets. Fractures belong to the same set run almost parallel to each other, but may have different sizes, shapes or hydro-mechanical properties from one fracture to another. Figure 1 shows a fractured rock mass, containing multiple fracture sets and varying trace lengths. Two or more fractures sets that intersect at certain angles form a fracture system (Cai and Horii, 1992).

The main geometrical properties of the fractures in rock masses that have been used in numerical modeling are aperture (or opening) as the perpendicular width of an open fracture, trace length as the intersections of fractures with an observation surface.
such as a rock face, orientation defined by its dip angle and dip direction or strike, location defined by the coordinate system adopted in mapping practice, and density as the number of fractures per unit area or volume. These geometric parameters usually are derived from field mapping measurements and core or borehole logging data during site investigation procedures.

Fractured rock masses are often considered as anisotropic, due to the mainly existence of non-uniform or non-regularly fracture system geometry. The presence of one or several sets of fractures in a rock mass creates anisotropy in its response to loading conditions (Amadei and Savage, 1989). Amadei (1996) pointed out the importance of anisotropy of rock masses and interactions among anisotropy, stress, deformability and strength of a rock mass containing a regular fracture set. However, there is a need to study anisotropy of strength and deformability of fractured rocks more systematically, when complex fracture system geometry needs to be considered.

2.2. Factors on strength and deformation behavior of fractured rocks

Strength and deformation behavior of fractured rock mass depend on many factors, such as hydraulic and mechanical properties of rock matrices, fracture geometry system and hydro-mechanical properties of fractures. Behaviors of fractured depend also upon inherent morphological, geological and environmental factors (Sridevi and Sitharam, 2000) of the rock masses concerned.

The fractures present in the rock mass play a critical role in the hydro-mechanical behavior of fractured rocks. The complex
geometry of the fracture systems causes issues of uncertainty in rock engineering, and is a major challenge for evaluating the mechanical behavior of fractured rock masses (Oda, 1983).

Based on experimental and numerical studies on some rocks (Gutierrez et al. 2000; Odedra et al. 2001; Tang et al. 2004; Wang 2006; Talesnick and Shehadeh 2007; Bäckström et al. 2008; Wang et al. 2013), it was found that generally the presence of water reduces strength of the rocks and affects the deformation behavior of the rocks. Hence, the fluid flow through rock fractures affects strength and deformation behavior of fractured rock mass.

Rutqvist and Stephansson (2003) and Hudson et al. (2005) highlighted some important aspects about the hydro-mechanical (HM) couplings in fractured rocks. There are several attempts in literature that considered the influence of water on mechanical properties and deformability of fractured rock masses (Goodman and Ohnishi 1973; Noorishad et al. 1982; Barton et al. 1985; Oda 1986; Bruno and Nakagawa 1991; Vlastos et al. 2006; Yuan and Harrison 2006; Zhang et al. 2007), but most of their models contained simple or regular fractured systems without considering the impact of fracture systems on the applicability of the effective stress concept for fractured rocks under different loading conditions.

Terzaghi (1923) defined the effective stress concept to describe the deformation behavior of water saturated soil, as the total stress minus the pore-water pressure, based on the results of experiments on the strength and deformation of soils. The effective stress is a function of the total or applied stress and the pressure of the fluid in the pores of the soil, known as the pore pressure or pore water pressure (Brady and Brown 2004). The concept has been studied for rocks (Brace and Martin 1968; Nur and Byerlee 1971; Robin 1973; Carroll 1979; Walsh 1981; Bernabe 1986; Boitnott and Scholz 1990; Bluhm and Boer 1996; Oka 1996), with conclusions that the Terzaghi effective stress concept may not hold true universally, especially for fractured rock masses. However, validity of this concept for fractured crystalline rocks has not been adequately discussed in literature, and has not been thoroughly tested and verified in laboratory or field experiments to the knowledge of the author.

It is recognized that sizes or scales of the models defined for the problems in hands is an important issue in characterization of rock masses (Fig. 2). Min and Jing (2003) and Baghbanan and Jing (2007) numerically demonstrated that the mechanical and hydraulic properties of fractured rock masses are strongly dependent on scale. The behavior of the rock mass is dependent on the relative scales between the problem domain and the rock blocks formed by the fractures. Realistic representation of the fracture geometry is important for selecting an adequate model size that is representative of the overall behaviors of the fractured rock mass concerned. For this purpose the REV concept (Fig. 3), which is defined as the minimum volume (or a range) of a sampling size
Figure 2. The classic diagram showing the transition from an isotropic intact rock to a heavily fractured rock mass with increasing sample size (Hoek, 1983).

Figure 3. Representative elementary volume (REV) concept (Long et al., 1982).
beyond which the mechanical and hydraulic properties of the sampling size remain essentially constant (Long et al., 1982), should be applied for studies of coupled stress-deformation-flow analysis of fractured rocks.

2.3. Estimation methods of strength and deformability of fractured rocks

Generally, the available approaches to estimate of the strength and deformability of fractured rock masses can be divided into two broad categories, namely direct and indirect methods.

2.3.1. Direct methods

Direct methods, as a quick and a simple measurement method, are in fact the experimental investigations, including standard laboratory and in-situ field rock mechanic tests.

As reported in the literature (Goldstein et al., 1966; Hayashi, 1966; Brown, 1970a-b; Brown and Trolley, 1970; Einstein and Hirschfeld, 1973; Reik and Zacaz, 1978; Heuze, 1980; Broch, 1983; Yoshinaka and Yamabe, 1986; Tsourelis and Exadaktylos, 1993; Ramamurthy and Arora, 1994; Yang and Huang, 1995; Aydan et al., 1977; Kulatilake et al., 1997; Chen et al., 1998; Yang et al., 1998; Kulatilake et al., 2001a-b; Ajalloeian and Lashkaripour, 2000; Talesnick et al., 2001; Asef and Reddish, 2002; Rawling et al., 2002; Nasseri et al., 2003; Tiwari and Rao, 2006a-b; Gercek, 2007; Prudencio and Van Sint Jan, 2007; Tiwari and Rao, 2007; Gonzaga et al., 2008; Sharma et al., 2008; Singh and Singh, 2008; Kulatilake, 2009; Rao and Tiwari, 2011; Wang et al., 2011; Wasantha et al., 2011; Hagara et al., 2012; Ghazvinian and Hadei, 2012; Cho et al., 2012; Maji and Sitharam, 2012; Moomivand, 2013; Wasantha et al., 2014), major laboratory studies have been performed on rock specimens containing fractures, specifically on unrealistic artificial rock samples of small volumes and containing artificial fractures formed by plaster of Paris or concrete (artificial rock-like materials), to evaluate of strength and deformation parameters of the tested models representing fractured rock masses. Generally, nonlinear deformation was confirmed by the results of these experimental tests.

However, due to the fact that such small-scale samples were not representatives of the real rock masses containing fractures of varying sizes, orientations and locations at larger scales, laboratory tests on samples of small volumes cannot be a proper method to estimate strength and deformation parameters of fractured rocks. At present, standard laboratory tests are suitable only for determining physical properties of the intact rocks of small volumes, or rock samples containing only one fracture of small sizes. Therefore, to obtain a reasonable understanding of the overall behavior of fractured rock masses, large volumes of samples of rock mass containing natural fracture networks of complex geometry should be tested at desired stress levels, in theory. Such tests, however, are almost impossible to be carried
out in conventional laboratory facilities today, but are possible by using direct in-situ field tests.

There are several in-situ field tests, using loading techniques such as plate jacking, plate loading, flat jack, pressure chamber and Goodman jack (Bieniawski, 1968; Pratt et al., 1972; Bieniawski and Heerden, 1975; Bieniawski, 1978; Rowe, 1982; Rutqvist et al., 1992; Singh, 2011). However, although in-situ field tests are the most realistic ways to study this matter, it is usually very difficult to control the initial and boundary (loading) conditions for such tests and are time-consuming, economically costly, and often not practical in reality for rock engineering practice at present. Also, they usually involve uncertainties in results, due to the effects of hidden fractures that cannot be accurately mapped and monitored during tests and interpretation of results.

2.3.2. Indirect methods

Indirect methods commonly include empirical, analytical and numerical methods.

2.3.2.1 Empirical methods

One of the most widely used and simple indirect methods for estimating strength and deformability of fractured rocks is the empirical methods. In practice, there are at present three types of empirical methods for this purpose:

a) Jointing index and joint factor methods

Both of these methods are based on laboratory test data of intact and fractured specimens. Jointing index is defined as an index of the ratio of sample length to fracture spacing or number of blocks contained in the sample. Also, joint factor is defined as a factor that relates the strength ratio to the joint frequency, joint orientation, and joint strength. Therefore, using these methods require extensive task to estimate the information about joint frequency, joint orientation, and joint strength, that this is very time consuming and costly (Aydan et al., 1997; Zhang, 2010).

b) Rock mass classification systems

These methods, as popular and easy-to-use methods used in engineering practice, are based on the engineering experiences obtained from the past projects. In these methods, rock mass properties are linked to representative rock mass classification indexes that reflect the overall rock mass quality. Over the years, many rock mass classification systems have been developed, including the rock quality designation (RQD) (Deere, 1967), the rock mass rating (RMR) (Bieniawski, 1976), the tunneling quality index (Q) (Barton et al., 1974; Barton 2002), geological strength index (GSI) (Hoek et al., 1995), the rock mass index (RMI) (Palmstrom, 1996), and a geo-engineering classification (Ramamurthy, 2004). Also, some systems have been developed by modification of existing ones. For example, the mining rock mass rating (MRMR) system developed by modifying the RMR system (Laubscher, 1990), the rock mass number (N) system as a modified Q-system (Goel et al., 1996), and a new general empirical approach
for the prediction of rock mass strengths (Dinc, 2011). A review and comprehensive study of the different rock mass classification systems can be found in Singh and Seshagiri Rao (2005), Edelbro et al. (2007), and Zhang (2010).

Although these methods have been applied for estimating strength and deformability of rock masses, such as reported in Marinos and Hoek (2000), Cai et al. (2004), Sonmez et al. (2004), Zhang and Einstein (2004), and Cai et al. (2007), there are still some disadvantages. Besides that sufficient experience and related knowledge are needed when these methods are used, the main shortcoming of rock mass classification systems is that it lacks a proper mathematical platform to generate representative parameters for establishing constitutive models of the fractured rocks, so that the second law of thermodynamics should not be violated, since complex geometry properties of a rock mass cannot be satisfactorily represented in this method quantitatively with a proper mathematical logic. However, the method enjoys its wide applicability for engineering design.

c) Empirical strength failure criteria

For fractured rock masses, strength criteria established by pure theoretical approaches do not exist. Empirical strength failure criteria must be developed and applied, and are equations based on analyzing existing data of strength of different rock mass types. For the past years, a number of different empirical rock strength failure criteria have been developed, which describe the relations between the principal stresses or between the shear stress and the normal stress acting on a defined surface in the rock or rock mass system. Sheory (1997) has studied some of the most commonly used strength failure criteria. The two best-known and widely used empirical failure criteria are the Mohr-Coulomb (M-C) and Hoek-Brown (H-B) failure criteria.

The M-C criterion is a linear failure criterion (Fig. 4) that can be expressed as:

\[ \tau_{\text{max}} = c + \sigma_n \tan \varphi \]  

(Eq. 1)

Where:
- \( \tau_{\text{max}} \): is the shear strength,
- \( \sigma_n \): is the normal stress,
- \( c \): is the cohesion, and
- \( \varphi \): is the internal friction angle.

The M-C failure criterion also can be expressed in terms of principal stresses as:

\[ \sigma_1 = \frac{2c \cos \varphi}{1 - \sin \varphi} + \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3 \]  

(Eq. 2)
Figure 4. The Mohr-Coulomb strength failure criterion. a) shear failure on plane ab, b) strength envelope of shear and normal stresses, and c) strength envelope of principal stresses (Zhao, 2000).

Where:

\[ \sigma_1 \]: is the major principal stress at failure or elastic strength, and

\[ \sigma_3 \]: is the minor principal stress or confining pressure.

The M-C failure criterion can be applied for both intact rocks and rock masses, with the parameter \( c \) and \( \phi \) changes representing the effects of intact rock properties and fractures on the overall equivalent strength of the fractured rock mass concerned.

The H-B criterion is a nonlinear failure criterion (Fig. 5) that was proposed for failure of intact rocks and especially rock masses (Hoek and Brown, 1997; Hoek and Diederichs, 2006; Brown, 2008). It can be expressed in terms of principal stresses (Hoek and Brown, 1980; Hoek et al., 2002) as:

\[
\sigma_1 = \sigma_3 + \sigma_{ci} \left( m \frac{\sigma_3}{\sigma_{ci}} + s \right)^{0.5}
\]  

(Eq. 3)

Where, \( \sigma_{ci} \) is the uniaxial compressive strength (UCS) of the intact rock, and \( m \) and \( s \) are two parameters not constant, but variables depending on the direction of weakness plane, \( s=1 \) for intact rock.

Several attempts have been made to modify M-C (Singh and Singh, 2012) and H-B (Hoek et al. 1992; Hoek, 1998; Saroglou and Tsiambaos, 2008) failure criteria to eliminate some of the deficiencies that should be considered when using of them.

### 2.3.2.2 Analytical methods

Analytical methods attempt to calculate mathematically strength and deformability of fractured rocks from the strength and deformation properties of fractures and intact rock matrix. Analytical methods are very useful because they provide results that can highlight the impacts of the most important issues or variables that determine the solution of a problem, when the assumptions made for deriving the analytical solutions are realistic enough for the problems concerned.
Efforts have been carried out to find analytical solutions for obtaining equivalent elastic moduli (Salamon, 1968; Morland, 1976; Gerrard, 1982; Fossum, 1985; Kemeny and Cook, 1986), deformability (Zhang, 2010), mechanical behavior (Singh, 1973a-b; Oda, 1983; Amadei and Savage, 1989; Chappell, 1989), strength (Amadei, 1988; Bekaert and Maghous, 1996; Single et al., 1998; Trivedi, 2010; Zhang et al., 2012), and constitutive models (Oda, 1982, 1984; Wu, 1988; Cai and Horii, 1992; Liu et al., 2009; Wang and Huang, 2009) of fractured rocks for cases of simple and often persistent and orthogonal fracture system geometries.

However, analytical methods are applicable only with simple and regular fracture system geometry, due to simplifying assumptions needed. These limitations make this approach impossible for fractured rocks containing complex fracture systems. Also, these methods do not consider the interaction between the fractures and the blocks divided by the fractures, which may have significant impacts on the overall behavior of rock masses, due to the reason that the intersections of the fractures are often the locations with the largest stress and deformation gradients, damage and failure.

2.3.2.3 Numerical modeling methods

Numerical modeling methods have been used extensively for a wide variety of applications in solving rock engineering problems. In a broad sense, numerical methods can be classified into the continuum and discontinuum methods (Jing, 2003).

Numerical methods can be used to calculate strength and deformability of well characterized fractured rocks with more flexibility, by representing different mechanical and geometric
features of the fractures and the intact rock matrices. With almost daily improvements to efficiencies of numerical solution methods and increase of computing power, numerical modeling methods have been developed to estimate the strength and deformability of fractured rocks by using various discrete and continuum modeling methods.

Among the various existing numerical modeling methods, the finite element method (FEM), as a numerical continuum method, is one of the most widely applied numerical method for studying strength (Kulatilake, 1985; Zerstsalov and Sakaniya, 1994,1997; Sridevi and Sitharam, 2000; Pouya and Ghoreychi, 2001; Niu et al., 2010; Yan et al., 2014), mechanical behavior (Cai and Horii, 1993; Sitharam et al., 2001; Maghous et al., 2008; Chen et al., 2011; Sagong et al., 2011; Wang et al., 2011; Pei-Feng et al., 2012; Sun et al. 2012; Zhang et al. 2012), and hydro-mechanical behavior (Noorishad et al., 1992) of fractured rock masses.

The finite difference method (FDM), as a numerical continuum method, was also applied to such research, such as Wang (2005) for studying of fracture effects on strength and deformation behavior of rocks, Sainsbury et al. (2008) for investigating the scale effects on rock mass strength, and Sitharam (2009) for determination of equivalent mechanical properties of fractured rocks. Since the FEM and FDM models are based on an overall continuum material assumption, effective and reliable considerations of effects of a large number of fractures of different sizes, orientations and behaviors are still difficult.

The Discrete Element Methods (DEM), as numerical discontinuum methods, are very attractive methods that simulate models of discrete systems of particles or blocks, and was introduced by Cundall (1971) and further developed by Cundall and co-workers (Lemos et al., 1985; Lorig et al., 1986; Cundall, 1988; Hart et al., 1988). A comprehensive presentation of the DEM can be found in Jing and Stephansson (2007). However, in the presence of very high densities of fractures, DEM models become very demanding for computational capacities.

DEM is a powerful technique to perform stress analyses for blocky rock masses formed by fractures, since its advantage of explicit representations of both the fracture system geometry and constitutive behaviors of fractures and intact rock matrix.

Currently, there are in general two commercial DEM codes suitable for modeling fractured rocks, namely the codes UDEC (Itasca UDEC, 2004) and 3DEC (Itasca 3DEC, 2007) for 2D and
3D problems of block systems and the PFC2D (Itasca PFC2D, 2008) and PFC3D (Itasca PFC3D, 2008) codes for particle flow simulations for granular material problems.

There are particle modeling studies by using PFC2D or PFC3D as reported in literature, such as recent work by Park et al. (2006) to investigate the mechanical behavior of rock masses, Zhang et al. (2007) to study strength of fractured rock mass, Cundall et al. (2008) to assess the size effect of rock mass strength, and Chiu et al. (2013) to study the anisotropic behavior of fractured rock mass. However, it should be noted that the grain sizes of rock materials are very small and are most often at micro-meter scales, the particle flow modeling is only practicable for small size domains due to the difficulties in realistic representations of fractures for both their geometry and constitutive behaviors in PFC models. Therefore, discrete particle systems are not suitable to apply for large scale problems of fractured rock masses that can be most realistically represented as block-fracture systems.

In the discrete rock blocks systems, the fractured rock mass is modeled as an assemblage of rigid or deformable blocks and fractures are considered as distinct interfaces representing interactions between contacting blocks. Therefore, it is very suitable to study block-fracture interactions by effectively calculating the strength and deformability of fractured rocks under different boundary conditions.


There are also methods based on artificial intelligence, such as artificial neural networks (ANN) and genetic programming (GP) can also be used to evaluate the strength and deformability of fractured rocks. These approaches are computational methods in machine learning field for non-linear regression problems, and can provide descriptive and predictive capabilities. For this reason, they have been applied to rock parameter identification and engineering activities (Jing and Hudson, 2002). Maji and Sitharam (2008) and Arunakumari and Latha (2008) used this method to predict of elastic modulus, and stress-strain behavior of fractured rock mass, respectively. Although, these methods have already been applied to the variety of subjects in rock mechanics and rock engineering, they have not yet provided an alternative to conventional modelling, due to the fact that they cannot reliably estimate parameters outside its range of training parameters, and lack of adequate theoretical basis for verification and validation of the techniques and their outcomes.
3. **Numerical Modeling the Strength and Deformability of Fractured Rocks**

As a basis to quantitatively predict the strength and deformability parameters of fractured rocks under compression, this chapter focuses on the numerical modeling processes of fractured rocks for the stress-deformation analyzing. Firstly, several DFN models with different sizes are generated to represent the fractured rock based on an established REV, and then the generated geometry models are used to create the DEM models for performing the numerical experiments of different typical laboratory compression tests.

### 3.1. Methodology

A systematic two-dimensional (2D) numerical modeling platform based on the DEM approach was developed to create a numerical predictive tool for studying strength and deformation behaviors of the fractured models during stress-deformation analysis.

Figure 6 shows all stages were used for the stress-deformation analyzing in this study. As one can see, the numerical process starting with generating the DFN models to represent the complex geometry system of the fractured rocks. A DFN realization constructs by using a Monte Carlo simulation process and geometric parameters of fractures such as, length, location, and orientation. In this study, geometric parameters for generating fracture network realizations were based on the field mapping results of a site characterization in the Sellafield area, undertaken by the United Kingdom (Nirex, 1997). The basic information of the four sets of fractures used is shown in the table 1, as reported in Min and Jing (2003).

In this stage, it should be noted that the sizes of DFN models must not be less than its REV of the models concerned. Min and Jing (2003) and Min et al. (2004) carried out numerical studies to establish elastic compliance tensor and permeability tensor for fractured rock masses, by investigating the scale-dependent equivalent permeability of fractured rock at the Sellafield site, Cumbria, England. Their results showed that an acceptable REV scale is above 5m × 5m for the fracture systems with constant apertures, for both elastic compliance tensor and permeability tensor of the concerned fractured rock as an equivalent continuum.

<table>
<thead>
<tr>
<th>Fracture set</th>
<th>Dip/Dip direction</th>
<th>Fisher constant</th>
<th>Fracture Density</th>
<th>Mean Trace Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8/145</td>
<td>5.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>88/148</td>
<td>9</td>
<td></td>
<td>4.6</td>
</tr>
<tr>
<td>3</td>
<td>76/21</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>69/87</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Table 1. Fracture parameters used for the DFN generation (Min and Jing, 2003).*
Therefore, three square-shaped DEM models of fracture systems were generated with side length of 2m × 2m, 5m × 5m, and 10m × 10m, respectively, as extracted from the center of an original parent model of fracture system, based on the same fracture system model data as was used in Min and Jing (2003). The DEM model with the size of 2m × 2m was used only for demonstrating the different results obtained when model sizes are less than 5m × 5m, and the DEM model with the size of 10m × 10m was used to ensure the validity of the 5m × 5m REV. Figure 7 shows the DFN models generated with different sizes.

Based on procedure presented in the figure 6, in the next step, the DFN models were used to create the DEM models with an internal discretization of finite difference elements, using the UDEC code. Before performing the analyses, the DFN models were regularized by deleting the isolated fractures and dead-ends, so that the resultant fractures were all connected and each fracture contributes to form two and just two opposing surfaces on two adjacent blocks. Figure 8 shows a DEM model of 2m × 2m in size before and after the fracture system regulation as an example.
(I) General assumptions

In the final stage, a few numerical experiments of typical laboratory compression tests were performed to determine, generically, the compressive strength and deformation parameters of the fractured rock, as equivalent properties at the established REV.

1. The numerical model was defined in a two-dimensional (2D) space for a generic study.
2. Simulations were performed under quasi-static plane strain conditions for deformation and stress analysis, without considering the effects of gravity.
3. Fractured rock was a hard rock mass, containing rock matrix and fracture.
4. Rock matrix was a linear, isotropic, homogeneous, elastic, and impermeable material.
5. The fractures follow an ideal elasto-plastic model of an M-C model in the shear direction and a hyperbolic behavior in the normal direction based on Bandis’ law (Bandis et al., 1985), without considering strain-softening.
6. The initial aperture of fractures (without stress) was a constant.
7. Strain-softening with continuous loading was not considered since the peak stress at the elasto-plastic deformation process was required and the model behavior cannot be considered as an equivalent continuum behavior with continued strain-softening behavior.
8. Partial cracking and complete crushing of rock blocks during loading processes were not considered, due to limitations of the current version of the UDEC code that does not have the ability to consider block cracking.
It is noted that, these assumptions are necessary for a generic study, since our aim was to establish a numerical platform for predicting strength and deformability of fractured rocks, not application for site-specific case studies. Also, the coupled hydro-mechanical effects on the fractures were neglected in this part of the study.

(2) Constitutive model

There are many types of constitutive models in the UDEC that can be used for intact rock and fractures. In the normal direction, the stress-displacement relation is assumed to be linear and governed by the normal stiffness ($k_n$) such that (Itasca UDEC, 2004):

$$\Delta \sigma_n = -k_n \Delta u_n$$  \hspace{1cm} (Eq. 4)

Where:
- $\Delta \sigma_n$: is the effective normal stress increment, and
- $\Delta u_n$: is the normal displacement increment.

In this study, a simplified Barton-Bandis (BB) model, as a reasonable representative of the physical response of fractures that is displacement-weakening response, was adopted such that the stress-displacement relation was assumed to be nonlinear.

3.2. Numerical experiment

3.2.1. Model establishment

In this study, the UDEC code was used to perform numerical compressive tests on the DEM models. The basic information about the intact rock, the granite matrix, and mechanical properties of fractures that were used for modeling in UDEC is shown in the table 2. This information was based on the laboratory test results reported in the Sellafield site investigation, which was used in Min and Jing (2003).
Table 2. Material properties of intact rock and fractures (Min and Jing, 2003).

<table>
<thead>
<tr>
<th>Type</th>
<th>Mechanical properties</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>Density (D)</td>
<td>2500</td>
<td>Kg/m³</td>
</tr>
<tr>
<td></td>
<td>Young’s modulus (E)</td>
<td>84.6</td>
<td>GPa</td>
</tr>
<tr>
<td></td>
<td>Poisson’s ratio (ν)</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Uniaxial compressive strength (UCS)</td>
<td>157</td>
<td>MPa</td>
</tr>
<tr>
<td>Fracture</td>
<td>Initial normal stiffness (Kₙ)</td>
<td>434</td>
<td>GPa/m</td>
</tr>
<tr>
<td></td>
<td>Shear stiffness (Kₛ)</td>
<td>434</td>
<td>GPa/m</td>
</tr>
<tr>
<td></td>
<td>Friction angle (φ)</td>
<td>24.9</td>
<td>(°)</td>
</tr>
<tr>
<td></td>
<td>Dilation angle (Ψ)</td>
<td>5</td>
<td>(°)</td>
</tr>
<tr>
<td></td>
<td>Cohesion (C)</td>
<td>0</td>
<td>MPa</td>
</tr>
<tr>
<td></td>
<td>Aperture for zero normal stress (maximum)</td>
<td>65</td>
<td>µm</td>
</tr>
<tr>
<td></td>
<td>Residual aperture at high stress (minimum)</td>
<td>1</td>
<td>µm</td>
</tr>
<tr>
<td></td>
<td>Shear displacement for zero dilation</td>
<td>3</td>
<td>mm</td>
</tr>
</tbody>
</table>

3.2.2. Simulation procedure

In this study, similar to the standard compression test on the axisymmetric small intact rock samples in laboratory, a series of numerical experiments to simulate the uniaxial and biaxial compression tests, were conducted on the DEM models.

Figure 9 shows the typical physical set-up and boundary conditions of uniaxial (Fig. 9a) and biaxial (Fig. 9b) compression tests, respectively. For both uniaxial and biaxial compression tests, the bottom of the DEM models was fixed in the y-direction and an axial compressive stress load (σᵧ) was applied on the top of the DEM model. For the uniaxial compression tests, the two vertical sides of the DEM model were kept as free surfaces. While in the biaxial compression tests, varying confining pressure (σₓ) of 0.5 MPa, 1 MPa, 1.5 MPa, 2 MPa, 2.5 MPa and 3 MPa, respectively, was applied laterally on the two vertical boundary surfaces of the model.

According to the numerical compressive experiments procedure in the figure 10, the DEM models were loaded sequentially with a constant and very small axial compressive stress load increment (Δσᵧ), equal to 0.05 MPa, in every loading step of calculation in the vertical direction, the same as conventional uniaxial or biaxial loading tests on intact rock samples.

During axial loading on the DEM models of the fractured rock concerned, both rock matrix and fractures will deform or be displaced, governed by the equations of motions of the rock blocks and constitutive models, material parameters for rock matrix and fractures, and the initial and boundary conditions.
In order to keep a servo-controlled loading condition, a new FISH program was developed and inserted in the UDEC model to simulate a standard servo-controlled test similar to the standard servo-controlled tests of small intact rock samples in laboratory, to

**Figure 9. Typical set-ups and boundary conditions for numerical experiments.**

(a) set-up for the uniaxial compression tests, and (b) set-up for the biaxial compression tests.

![Diagram](image)

**Figure 10. The procedure used for the DEM numerical compressive experiments.**
minimize the influence of inertial effects on the response of the model, by setting the upper and lower limits for unbalanced forces. Cyclic loading rate was kept in a range of maximum and minimum unbalanced forces in UDEC program to avoid sudden failure of the DEM models during cycles of the uniaxial and biaxial tests.

The axial compressive stress loading process was controlled by a velocity monitoring scheme during simulation. The velocities (in both x- and y-directions) at a number of carefully specified monitoring points were checked to ensure that they become zero or very close to zero at the end of every loading step so that a quasi-static state of equilibrium of the model was reached under the applied boundary conditions, since the simulated tests should be quasi-static tests for generating static behaviors of the models.

In order to using the velocity monitoring technique, checking equilibrium state, and obtaining required displacement and stress parameters at the end of each loading step during loading compression tests, a grid of monitoring points was defined into the DEM models. Figure 11 shows a DEM model with the size of 10m × 10m and positions of monitoring points. Six parallel sampling lines within each model were placed in both x- and y-directions, with the same distance in between. Therefore, thirty-six points were defined at intersections of the horizontal and vertical monitoring lines. These points plus one point at the center of the DEM model were the monitoring points in this study.

Figure 12 shows an example for curves of velocity versus time in the x- and y-directions at 6 selected monitoring points located on two horizontal and vertical lines within a DEM model during a few loading compression tests. It can be observed that the values of velocities at the defined monitoring points became very close to zero at the end of every loading step.

![Figure 11. Position and numbering of the monitoring points into the DEM model with size of 10m × 10m.](image)
Strength and deformability of fractured rocks

Figure 12. Curves of velocity versus time in x- and y-directions at the six monitoring points. The numbers of the monitoring points and their locations in the DEM model with size 10m × 10m are shown in figure 11.

In addition to monitoring the vertical and horizontal velocities, vertical and horizontal displacements (y- and x- displacements), and normal and shear stresses ($\sigma_{yy}$, $\sigma_{xx}$, and $\tau_{xy}$) were monitored, using the same velocity monitoring grid, at all monitoring points at each loading step during the uniaxial and biaxial compression tests. The deformation and stress of each DEM model were evaluated in order to calculate the average normal stress and strain values of the tested model in the x- and y-directions, which were then used to evaluate the equivalent strength and deformability parameters when different strength criteria were adopted. The average stresses and strains were computed by taking the average values obtained from the monitoring points by using the FISH algorithm, the programming language embedded within the UDEC code.

It should be noted that the equivalent strength and deformability of the fractured rocks, as an equivalent continuum, were the concern of research, not the complete constitutive model of the fractured rock as an equivalent continuum under any stress paths. Therefore, the loading needs to be stopped when the peak strength of the model was reached, without model collapse or appearance of very large shear displacements along the fractures or large block motion, which will make the equivalent continuum assumption of the fractured rock invalid, and the homogenization (averaging) for equivalent parameter evaluation could not be applied.

3.3. Results

3.3.1. Deformability of the fractured rock

(1) Deformation behavior of the fractured rock

The stresses and strains obtained from the numerical experiments, as the axial stresses versus axial strains curves, were used to investigate of deformation behaviors of fractured rocks after the models reached their peak strengths.
The numerical test results of deformation behaviors of the fractured rock models under uniaxial compression tests for the DEM models with varying sizes is shown in the figure 13. These results show behaviors of the DEM models change with increase in the model size, but the change becomes insignificant between models of the size of 5m × 5m and 10m × 10m. Therefore, the model of the established REV size of 5m × 5m is adequate for evaluating the equivalent strength and deformability of fractured rock concerned.

Figures 14 and 15 show the results of numerical biaxial tests with different confining pressures of 0.5 MPa, 1 MPa, 1.5 MPa, 2 MPa, 2.5 MPa, and 3 MPa for the DEM models with sizes of 5m × 5m, and 10m × 10m, respectively. It can be seen that the DEM models deform linearly and elastically at axial stresses below the yield strength, depending on the confining pressure. Further compression leads to inelastic deformation up to the peak strength. With increasing of confining pressure, the strength of the DEM models increases and the stress-strain curves follow an elasto-plastic behavior with a strain hardening trend.

Generally, our models did not show strain-softening, since we need to stop loading when the peak strength of the model was reached in order to maintain a physical basis of equivalent continuum assumption of the rock mass concerned. Strain softening may occur with continued loading.

(2) Deformation parameters of the fractured rock

The Young’s modulus and Poisson’s ratio are two important deformability parameters of the rock mass. In this study, based on the stress-strain curves obtained, the Young’s modulus (E) was calculated as the averaged local slope of the stress-strain curves of the DEM models during the stage of elastic deformation (Eq. 5), and the Poisson’s ratio (ʋ) was calculated as the ratio of the mean transverse strain to the mean axial strain of the DEM models (Eq. 6).

\[ E = \frac{\bar{\sigma}_y}{\bar{\varepsilon}_y} \quad \text{(Eq. 5)} \]

\[ \nu = \frac{\bar{\varepsilon}_x}{\bar{\varepsilon}_y} \quad \text{(Eq. 6)} \]

Where \( \bar{\sigma}_y \) is the mean value of axial stress in the elastic limit, \( \bar{\varepsilon}_x \) and \( \bar{\varepsilon}_y \) are the mean values of lateral and axial strain, respectively.

The equivalent directional Young’s modulus of the fractured rock concerned is shown in the figure 16, as functions of lateral confining pressure and model size. One can see a gradual increase of Young’s modulus with increase of confining pressure, but the effect of DEM model sizes is not very significant. Also, compared with the magnitude of Young’s modulus for the intact rock (see Table 2), the Young’s moduli of the fractured rock models are less than the Young’s modulus of the intact rock.
Strength and deformability of fractured rocks

Figure 13. Stress-strain curves for the DEM models with varying size under uniaxial compression conditions.

Figure 14. Stress-strain curves for the DEM model with size of 5m × 5m under different confining pressures.

Figure 15. Stress-strain curves for the DEM model with size of 10m × 10m under different confining pressures.
Figure 16. Equivalent directional Young’s modulus for the DEM models with two sizes under different confining pressures (Young’s moduli at zero confining pressure are 43 and 72 MPa for the two models with sizes of 5m × 5m and 10m × 10m, respectively).

Figure 17. Equivalent Poisson’s ratio for the DEM models with two sizes under different confining pressures.

Figure 17 shows the equivalent Poisson’s ratio of the fractured rock concerned, as functions of lateral confining pressure and size of the model. The Poisson’s ratio of the fractured rock decreases gradually with increase of confining pressure and size of DEM models, but the magnitude of Poisson’s ratio for fractured rocks is much larger than that for intact rock.
The general trends of the curves are the same and converge when the confining pressure reaches 3.0 MPa, but with a considerable difference between the two models of different sizes after the confining pressure is larger than 1.5 MPa. The obtained results of larger Poisson’s ratio indicated that fractures affected the deformability of the rock mass much more significantly than that of strength, so that care should be taken when developing constitutive models of fractured rocks as equivalent continua.

3.3.2. Strength of the fractured rock

(1) Strength envelops

In order to study of fractured models strength, the obtained pairs of major (σ₁) and minor (σ₂) principal stresses from the numerical experiments were used to fit the Mohr-Coulomb (M-C) and Hoek-Brown (H-B) strength failure criteria. Based on the equations 7 and 8, the M-C and H-B criteria can be expressed in terms of normalized principal stresses. For the M-C criterion, we can have:

\[
\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin \varphi}{1 - \sin \varphi} + \left(\frac{2c \cos \varphi}{1 - \sin \varphi}\right) \frac{\sigma_3}{\sigma_3}
\]  
(Eq. 7)

and for the H-B criterion, we can have:

\[
\frac{\sigma_1}{\sigma_3} = 1 + \sigma_{cl} \left(\frac{m}{\sigma_{cl}}\right)^{0.5} \frac{\sigma_3}{\sigma_3}
\]  
(Eq. 8)

The curve fitting results, as normalized strength versus normalized confining pressure with the M-C and H-B failure criteria were shown in the figure 18, for the DEM models of 5m × 5m and 10m × 10m in size, under different confining pressures. Both the M-C and H-B strength envelops made acceptable fitting to the numerical data, with insignificant difference between them, despite the fact that the M-C criterion is a linear and H-B criterion is a nonlinear one.

(1) Strength parameters

The equivalent strength parameters for the two criteria, namely cohesion (c) and friction angle (φ) of the M-C criterion, and m and s for the H-B criterion, derived from the fitting to the strength criteria, are given in the table 3. The results clearly show that the differences between strength parameter values of the DEM models are basically minor between model sizes of 5m × 5m and 10m × 10m. The difference between the correlation coefficient root values (R) of two strength envelops is also insignificant.
Figure 18. Strength envelopes for the DEM models in the normalized principal stress space. left) with model size of 5m × 5m, and right) with model size of 10m × 10m.

3.4. Summery discussions

The basic DFN and DEM models and 2D numerical procedure of simulating the both uniaxial and biaxial laboratory tests on the small intact rock samples and stress-deformation analyzing in fractured rocks were presented, followed by a systematic numerical methodology developed for the first time to predict the strength behavior and deformability parameters of fractured rocks.

It is noted that, due to the lack of measured data support from laboratory or in-situ experiments of testing volumes not less than the REV sizes of the granite rocks at the Sellafield site, this study has been performed in a generic form in nature and the results have only conceptual values. Although the numerical methodology presented requires much more computing time compared with that used by the empirical and analytical methods, but it is a suitable and flexible numerical approach to predict the behaviors and properties of fractured rocks that cannot be obtained by conventional laboratory tests using small intact rock samples.

Table 3. Equivalent strength parameters of the M-C and H-B failure criteria.

<table>
<thead>
<tr>
<th>Size of Model</th>
<th>Mohr-Coulomb</th>
<th>Hoek-Brown</th>
<th>Hoek-Brown</th>
<th>Hoek-Brown</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cohesion (MPa)</td>
<td>Friction angle (°)</td>
<td>Correlation coefficient</td>
<td>m</td>
</tr>
<tr>
<td>5m × 5m</td>
<td>0.1727</td>
<td>28.3</td>
<td>0.9950</td>
<td>0.0591</td>
</tr>
<tr>
<td>10m × 10m</td>
<td>0.1596</td>
<td>28.2</td>
<td>0.9948</td>
<td>0.0572</td>
</tr>
</tbody>
</table>
The main conclusions and outstanding issues from the numerical results obtained are summarized as follows:

1) Strength and deformation parameters of fractured rocks are dependent upon confining pressure. Strength of fractured rocks increases with increase of confining pressure. Deformation parameters of fractured rocks, including the Young’s modulus and Poisson’s ratio, change significantly with confining pressures.

2) Deformability behavior of fractured rocks follows an elastoplastic behavior with a strain hardening trend over the concerned range of stress.

3) Both the M-C and H-B criteria give fair estimation of the compressive strength of the fractured rock concerned. However, the H-B criterion is in essence a nonlinear failure envelope and is more flexible for modeling different fracture system geometries and stress conditions.

4) The strength and deformability behavior of fractured rocks are the important issues that must be readily understood for constitutive model development of fractured rocks whose properties have significant dependence on understanding of stress and size effect.

5) Adequate quantitative knowledge of fracture system geometry and their mechanical behaviors play a significant role for designing future physical tests for estimating the strength and deformability of fractured rocks, very different and much more challenging compared with testing intact rock samples.

4. EFFECTS OF LOADING CONDITIONS ON STRENGTH AND DEFORMABILITY OF FRACTURED ROCKS

The axial stress loading and axial velocity loading are two commonly applied boundary/loading conditions for performing laboratory tests on the small rock samples. In this chapter, the possible differences caused by such loading conditions in the strength and stress-strain behaviors of fractured rocks are studied. In fact, this study is answer to this question that how these two testing conditions may affect results when a large volume of fractured rocks has not been investigated in conventional laboratory test environments, since practical difficulties make such physical tests not feasible.

4.1. Methodology

Based on the systematic numerical modeling approach presented in the previous chapter, the effects of loading conditions on strength and deformability of fractured rock masses were investigated, when both axial stress and axial velocity loading conditions were applied for a series of numerical compressive uniaxial and biaxial compression experiments on two-dimensional fracture network models containing a large number of fractures of varying sizes, locations and orientations, created by using of stochastic discrete fracture network (DFN) method, based on the
realistic geometrical and mechanical data of fracture systems from field mapping with the same testing model geometry and size. Similar to the simulation procedure presented in the previous chapter, a new boundary condition applied to the DEM models with the same testing model geometry. Hence, besides the axial compressive stress loading condition, the constant normal axial velocity boundary condition also applied in the y-direction at the top boundary of the models for performing uniaxial and biaxial numerical experiments (Fig. 19) on the two DEM models with the size length of 5m × 5m and 10m × 10m. The downward velocity loading conditions applied were similar to the previously mentioned axial stress boundary condition. The bottom of the DEM model was fixed in the y-direction and the numerical experiments were performed by varying the confining pressure.

4.2. Results

4.2.1. Deformability of the fractured rock
(1) Deformation behavior of the fractured rock
Figures 20 and 21 show the obtained stress-strain curves from doing several biaxial numerical experiments by adapting a constant normal axial velocity boundary condition, with different confining pressures of 0.5 MPa, 1 MPa, 1.5 MPa, 2 MPa, 2.5 MPa and 3 MPa, for two DEM models with size of 5m × 5m and 10m × 10m, respectively.

Figures 22 and 23 compare the obtained stress-strain curves using the axial velocity boundary condition (solid line) and the axial stress boundary condition (dashed line) with different confining pressures, for two DEM models of 5m × 5m and 10m × 10m, respectively. As one can clearly see, there are differences between stress-strain curves obtained when using axial velocity and axial stress loading conditions. Generally the average axial stress obtained from the axial velocity loading condition is higher compared to that from axial stress loading condition.

(2) Deformation parameters of the fractured rock
For studying deformability of fractured DEM models under the two considered loading conditions, the equivalent directional

Figure 19. Different boundary conditions applied for the uniaxial (a), and biaxial (b) compression numerical experiments.

(a) Uniaxial compression test  (b) Biaxial compression test with confining pressure
Figure 20. Stress-strain curves for the DEM model of 5m × 5m with applying constant axial velocity loading condition, under different confining pressures.

Figure 21. Stress-strain curves for the DEM model of 10m × 10m with applying constant axial velocity loading condition, under different confining pressures.

Young’s modulus was calculated as the local slope of the stress-strain curves of the DEM models during the stage of elastic deformation, as functions of lateral confining pressure. Figure 24 compares the distribution of equivalent directional Young’s modulus for the DEM models of 5m × 5m and 10m × 10m with the axial velocity boundary condition (solid line) and the axial stress boundary condition (dashed line) with different confining pressures. As one can clearly see, gradual increase of Young’s modulus with increase of confining pressure, and also higher Young’s modulus under axial velocity loading condition than that under axial stress loading condition. Also, the magnitude of Young’s modulus for fractured rocks is less than the Young’s modulus for intact rock under both loading conditions.
4.2.2. Strength of the fractured rock

(1) Strength envelops

For investigating strength of the DEM model with the size of 5m × 5m under the two considered loading conditions, the normalized version of the Mohr-Coulomb (M-C) and Hoek-Brown (H-B) criteria (Eqs. 7 and 8) were used as the relations between the obtained pairs of major and minor principal stresses from numerical experiments during the curve fitting process.
Strength and deformability of fractured rocks

Figure 24. Comparison of the equivalent directional Young’s modulus for the DEM models with size of 5m × 5m and 10m × 10m, under different confining pressure conditions, between using the axial velocity loading condition (solid lines) and the axial stress loading condition (dashed lines).

Figure 25 shows a comparison between the strength envelope fitting results for the DEM model of 5m × 5m, as normalized strength versus normalized confining pressure with M-C and H-B failure criteria, under the axial velocity loading (dashed lines) and axial stress loading (solid lines) conditions and with different confining pressures. One can see acceptable fitting quality to both M-C and H-B strength envelops, with the H-B failure criterion provided a better fitted non-linear strength curve from the numerical results.

(1) Strength parameters

Table 4 compares the equivalent strength parameters and the correlation coefficients for the curve fitting derived from the fits to two strength criteria, between the axial stress and the axial loading conditions of the DEM model with size 5m × 5m. The results show different values of cohesion friction angle, m and s representing M-C and H-B failure criteria, respectively. The difference may not be significant in terms of magnitudes of the parameter values, but significant for showing that for fractured rocks, testing conditions, including loading conditions, and sample volumes and fracturing status, play an important role.

4.3. Summary discussions

In this section, the presented effects of two different popular loading conditions, namely axial stress and axial velocity, on strength and deformability properties of fractured rocks is systematically examined using a series of numerical compressive experiments on the DEM models.
Figure 25. Compression of strength curves in the normalized principal stress space for the DEM model of 5m × 5m, with M-C and H-B strength failure criteria under axial velocity loading (dashed lines) and axial stress loading (solid lines) conditions.

The obtained results show, in addition to existence difference between stress-strain behaviors, there are differences between strength curves and strength parameters of fractured rocks generated by the axial stress and axial velocity loading conditions were applied to samples of fractured rocks of same large volumes with the same large numbers of fractures, when both M-C and H-B failure criteria were assumed. Also, the results show that a higher average axial stress and higher directional Young’s modulus under axial velocity test condition than that under axial stress condition.

Table 4. Compression of equivalent strength parameters of both M-C and H-B criteria for two loading conditions concerned of the DEM model with size 5m × 5m.

<table>
<thead>
<tr>
<th>Loading conditions</th>
<th>Mohr-Coulomb</th>
<th>Hoek-Brown</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cohesion (C (MPa))</td>
<td>Friction angle (°)</td>
</tr>
<tr>
<td>Axial Stress</td>
<td>0.1727</td>
<td>28.3</td>
</tr>
<tr>
<td>Axial Velocity</td>
<td>0.1930</td>
<td>32.6</td>
</tr>
</tbody>
</table>
The results obtained present one step forward for deepening our current understanding of the strength and deformability of fractured rocks. In essence, this part of this research demonstrates that for testing fractured rocks of large volumes with large number of fractures, loading condition is an important issue since it may generate different results due to the presence of fractures inside the tested volumes.

Therefore, the effects of different loading conditions should be carefully considered in the design and interpretation of results for in-situ experiments with large volumes of fractured rocks. This is especially important when designing large scale in-situ experiments in fractured rocks are considered.

5. **Effects of Loading Directions on Strength and Deformability of Fractured Rocks**

Based on the understanding of effects of loading directions on the strength and deformation behavior of fractured rocks, this chapter presents a numerical study to evaluate quantitative demonstration of anisotropy or variations of strength and deformability properties of the rotated fractured rock models with respect to vary loading directions, due to its impact on design, safety and performance assessment of rock engineering projects.

Anisotropy is a major research topic in earth sciences. Fractured rock mass is often considered as an anisotropic material because mainly it contains the fracture system geometry that is usually not uniformly or regularly distributed, and its behavior may change with loading directions. Hence, through this study try to answer this question that how significant such change, which is an important issue and remains unclear.

5.1. **Methodology**

In this part of this research, based on the methodology presented in the first chapter, a series of stress-deformation analysis on the rotated DEM models were performed to check anisotropy of the strength and deformability of fractured rocks. According to the figure 26, the stress-deformation analysis was conducted by utilizing a systematic numerical uniaxial and biaxial test procedure, for observing the anisotropic behaviors of the rotated fractured rock models.

The rotated DFN models were defined by rotating a primary geometrical DFN model extracted from a larger DFN model (Fig. 27), from 0° to 180° with an interval of 30° in an anti-clockwise direction (Fig. 28). The size of the square-shaped DFN models was 5m × 5m that is equal to the accepted REV size.

Based on similar way presented in the chapter 1, the computational models were established through the numerical experiments for simulating conventional uniaxial and biaxial compression tests on the rotated DEM models. The boundary conditions were the same as previous work, so an incrementally increased axial load was applied on the top of the rotated DEM model in the vertical
direction, followed by a continued process of iteration (cycling), with the same stress increment of 0.05 MPa, until a quasi-static equilibrium state of the models was reached.

For each rotated model, the numerical tests were composed by seven loading steps, through applying a set of confining pressures of 0 MPa (representing a uniaxial loading test), 0.5 MPa, 1.0 MPa, 1.5 MPa, 2.0 MPa, 2.5 MPa and 3.0 MPa, respectively. Under each confining pressure, iterative axial loading (cycling) continued with the fixed increment until a peak stress at the end of elasto-plastic deformation process of the DEM models was reached. In total, 42 UDEC models (6 rotational directions and 7 loading steps) were simulated.

It should be noted that the equivalent strength and deformability of the fractured rocks, as equivalent continua, were the concern of research, not the complete constitutive model of the fractured rock concerned. Therefore, the loading needs to be stopped when the peak strength of the model was reached, for homogenization (averaging) of the equivalent strength parameter evaluations.

5.2. Results

5.2.1. Deformability of the fractured rock

(1) Deformation behavior of the fractured rock

The average values of the normal stress and strain components of each rotated DEM model, as the corresponding equivalent values from the monitoring points, were calculated at the end of each loading step, and plotted as stress-strain curves for deriving peak strength and the elastic deformability parameters.
Figure 27. Extracting a DEM model with size of $5m \times 5m$ from the center of the original parent model and its rotating in various direction angles from $0^\circ$ to $180^\circ$ with a $30^\circ$ interval.

Figure 28. Fracture system geometry of the rotated DEM models in various direction angles of $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, and $150^\circ$. 
Figures 29 and 30 shows the curves of axial stresses versus axial strains for the rotated DEM models for the uniaxial and biaxial numerical experiments, as functions of the six orientation angles of the models, respectively. The results for the uniaxial loading (Fig. 29) are exceptional since, unlike testing small samples of intact rock materials, the model of a large number of blocks reached the peak strength much more quickly with larger strains with no lateral confining pressure, thus it is not appropriate to be presented in the same plot as the others.

Generally, it can be seen from the numerical results that the DEM models deform linearly and elastically during the initial loading stage with axial stresses below the yield strength (the stress point where the change of elastic to plastic deformation process starts on the stress-strain curves), whose magnitudes depend on the confining pressure. Afterwards, continued axial compression then leads to inelastic deformation up to the peak strength. With increase of lateral confining pressure, the strengths of the DEM models increase and the stress-strain curves follow an elasto-plastic behavior with a strain hardening trend. Strain-softening may appear if continued axial loading was applied.

Although the general deformation behavior of the DEM models had similar trends with rotation of the fractured rock models, the magnitude of peak stress and slope of stress-strain curves before reaching the yielding strength varied with rotation angles, indicating existence of anisotropy of strength and deformability, which are mathematically analyzed in the next section.

![Figure 29. Stress-strain curves for the rotated DEM model with size of 5m x 5m in various direction angles of 0°, 30°, 60°, 90°, 120°, and 150°, without confining pressure.](image-url)
Figure 30. Stress-strain curves for the rotated DEM model with size of 5m × 5m in various direction angles of 0°, 30°, 60°, 90°, 120°, and 150°, and under different confining pressures of 0.5 MPa, 1 MPa, 1.5 MPa, 2 MPa, 2.5 MPa, and 3 MPa.
(2) Deformation parameters of the fractured rock
The stress-strain curves were used to evaluate Young’s modulus and Poisson’s ratio, as two deformation parameters of the rotated DEM models in the similar pervious ways.

Figure 31 shows variations of the equivalent directional Young’s modulus as functions of rotation angles of the models with varying confining pressures. As can be seen, although the equivalent Young’s modulus of the fractured rock models increases gradually with the increase of confining pressures, it changes slightly with model rotation angle. Due to small values of Young’s modulus near the origin of the coordinate frame, the values of Young’s modulus at zero confining pressure are presented.

Variations of the equivalent Poisson’s ratio as functions of rotation angles of the models with varying confining pressures is shown in the figure 32. As can be seen the equivalent Poisson’s ratios of the models decrease generally with the increase of confining pressures, but with moderately more variations with model rotation angle.

5.2.2. Strength of the fractured rock
(1) Strength envelops
The stress-strain curves were used to obtain strength envelops of the rotated DEM models, after the models reached their peak strength, using the curve fitting technique based on the Mohr-Coulomb (M-C) and Hoek-Brown (H-B) strength failure criteria.

The curve fitting results, as normalized strength versus normalized confining pressure with M-C and H-B failure criteria are shown in the figure 33 using data of the rotated DEM models under different confining pressures.

Figure 31. Distribution of equivalent Young’s modulus for the rotated DEM model with size of 5m × 5m in various direction angles, under different confining pressures (Young's moduli at zero confining pressure are 43, 30.7, 31.5, 35.5, 36.6 and 37.6 MPa for 0º, 30º, 60º, 90º, 120º, and 150º direction angles, respectively).
Correlation coefficients are presented in the table 5 for each rotation angle. The fitting qualities of M-C and H-B strength envelopes were acceptable, but the M-C criterion was better for the numerical results.

For a more quantitative comparison between M-C and H-B failure criteria, a root mean squared error (RMSE) index was used as an indication of the differences between strength values predicted by two criteria and measured numerical values. The values of RMSE can be calculated as:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\sigma_{1,i}^p - \sigma_{1,i}^m)^2}$$  \hspace{1cm} (Eq. 9)

Where $\sigma_{1,i}^p$ and $\sigma_{1,i}^m$ are the predicted and measured values of $\sigma_1$ for the $i^{th}$ rotated model, and $n$ is the number of data pairs that is equal to 7, one pair for uniaxial and six pairs for biaxial numerical tests in this study.

RMSE values of the M-C and H-B failure criteria for all of model rotation angles are given in the table 5, which shows that in all of the rotated models considered, the M-C criterion showed lower RMSE magnitudes relative to that of the H-B criterion, and hence M-C strength envelope fits better with the overall mechanical behavior of the fractured rock model. This observation, however, is still subjective to the modeling conditions of this study, and conclusions may change when different fracture system geometry and mechanical properties are used.
Figure 33. Strength curves for the rotated DEM models with size of 5m × 5m in the normalized principal stress space. a) M-C, and b) H-B failure criterion.

(2) Strength parameters

Figures 34-37 show the variations of equivalent cohesion ($c$), friction angle ($\phi$), as two strength parameters of M-C criterion, and $m$ and $s$ as two strength parameters of H-B criterion, of the rotated DEM model with the size of 5m × 5m, respectively.

The numerical results show a certain degree of anisotropy of the cohesion when the M-C criterion was used (Fig. 34). The frictional angle changes also insignificantly with model rotations (Fig. 35). The small changes of the friction angle obtained were due to insignificant changes of the slope angles of the fitted linear curves in the figure 33-a, based on the M-C criterion.

Table 5. RMSE and correlation coefficient (R) values of the M-C and H-B criteria in prediction of normalized strength.

<table>
<thead>
<tr>
<th>Rotation angle of DEM model (°)</th>
<th>RMSE</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M-C</td>
<td>H-B</td>
</tr>
<tr>
<td>$\theta$=0</td>
<td>0.284</td>
<td>0.555</td>
</tr>
<tr>
<td>$\theta$=30</td>
<td>0.393</td>
<td>0.826</td>
</tr>
<tr>
<td>$\theta$=60</td>
<td>0.249</td>
<td>0.730</td>
</tr>
<tr>
<td>$\theta$=90</td>
<td>0.447</td>
<td>0.986</td>
</tr>
<tr>
<td>$\theta$=120</td>
<td>0.199</td>
<td>0.694</td>
</tr>
<tr>
<td>$\theta$=150</td>
<td>0.481</td>
<td>0.755</td>
</tr>
<tr>
<td><strong>Average value</strong></td>
<td>0.342</td>
<td>0.758</td>
</tr>
</tbody>
</table>
The equivalent parameters $m$ and $s$ of the H-B criteria show significant directional dependencies (Figs. 36 and 37), especially in the directions of 150º, and 90º.
In summary, one may conclude that the equivalent strength of the models is significant, whether either M-C or H-B criteria are adopted, and major changes occur when model oriented to 0, 30, 90 and 150 degrees, respectively. This is a clear contrast compared with elastic deformation parameters. The main reason is that the non-linear plastic deformation caused significant irreversible anisotropic distributions of stress and displacement fields that were largely controlled by the fracture system geometry.

5.3. Summary discussions

In this part of the research, anisotropy of strength and deformability of fractured rocks was investigated by using a predictive numerical methodology. The developed mathematical approach provided a proper and workable tool for evaluating the impacts of anisotropy of strength and deformability of fractured rocks, especially when the host rocks are hard crystalline rocks such as granites.

Strength and deformability of fractured rocks are directional dependence. The numerical results show significant anisotropy of strength and elastic deformability of fractured rocks models, which vary with the loading conditions. The main reason for such anisotropic feature of strength and deformation behaviors are the complex fracture system geometry that was not regular and isotropic, so that the model behaved differently for different model rotations, even their loading conditions was identical.

Both the M-C and the H-B criteria provided acceptable fitting to the numerical results of strength, with M-C criterion yielding better estimation of the RMSE values for engineering practices. More investigations are needed for understanding the physical reasons for the differences.

Therefore, anisotropies of strength and deformability of fractured rocks are important issues for design and performance of rock engineering projects, since the relative directions of in-situ stresses (determining the loading directions), fracture system geometry (selecting the directional variation of stress distribution) and rock
engineering projects (such as choosing axis of tunnels relative to not only in-situ stress directions but also orientations of dominating fracture sets) may have a significant impact on the safety and performance of the rock engineering structures located in fractured rock masses.

6. Effects of Water Pressure on Strength and Deformability of Fractured Rocks

Fluid flow and rock deformation in fractured crystalline rocks are important subjects in many rock engineering fields such as the geological disposal of radioactive wastes, oil and geothermal energy reservoirs, and CO₂ sequestration and storage, where impact of the engineering on environment is one of the most critical issues (Jing et al., 2013).

Based on outstanding issues drawn from the previous chapters, in this chapter the study was extended to consider water pressure effects on the strength and deformability of the fractured rock, and to deepen our understanding of the validity of the effective stress concept, by the DEM models for the same fracture system model used in the previous chapters.

6.1. Methodology

In this part of the study, the numerical experiments for coupled stress-flow simulation were conducted step by step following the modeling logic displayed in the figure 38, as a conceptual approach, to evaluate water pressure effects on strength and elastic deformability of fractured rocks, after building the deformed DEM model.

The fracture network model (DFN) adopted and discretization scheme were the same as the DEM model used in the previous chapters, as a square-shaped model with the size of 5m × 5m based on the accepted REV size. Figure 39 shows the DEM geometry model before (Fig. 39a) and after (Fig. 39b) the fracture system regulation.

![Figure 38. Typical procedure for the numerical coupled stress-flow analysis in a fractured rock.](image)
As shown in the figure 39a, a grid of 36 monitoring points was defined at intersections of the six parallel horizontal and six vertical lines, with the same spacing. These points plus one point at the center of the DEM model were the monitoring points for stress, displacement, velocity, and water pressure in this study. Also, as shown in the figure 39b, for prevention of large deformation and failing of corner blocks, and also for having uniform water flow through fractured rock, a number of parallel artificial fractures with zero friction angle and cohesion were added on two horizontal sides of the DEM model.

Using UDEC code for simulating the coupled stress-flow, the Cubic law was assumed for water flow in smooth parallel plates of fractures with rock blocks and water assumed to be impermeable and incompressible, respectively, for the purpose of simplicity. Also, water flow was allowed only in the connected fractures. The mechanical properties of the rock matrix and fracture are the same as those used in the first chapter, as defined in the table 2.

According to figure 40, two loading stages were concerned for coupled stress-flow analyzing, concerning purely mechanical process and coupled mechanical and water flow processes, respectively. The boundary conditions adapted during performing these stages are shown in the figure 41.

The first stage simulates the deformation behavior of the DEM model during pure mechanical loading conditions without water flow. The stress-deformation analysis was carried out to generate the results for evaluating strength and deformability parameters without water pressure effects, so that comparison with results with the water pressure effects, as obtained from the second task, could be performed.
As shown in the figure 41, in the first stage, a constant and very small axial load increment ($\Delta \sigma_y$) was applied on the top of the DEM model in the vertical direction, as mechanical loading conditions (Fig. 41b), while bottom of the DEM model was fixed in the y-direction for simulating the laboratory uniaxial and biaxial compression tests. The two vertical sides of the DEM model were kept as free surfaces in the uniaxial test, while in the biaxial test varying confining pressure ($\sigma_x$), equal to 0.5 MPa, 1 MPa, 1.5 MPa, 2 MPa, 2.5 MPa, and 3 MPa, respectively, was applied laterally on the two vertical boundary surfaces of the model.

In continue this stage, according the process presented in the first chapter (Fig. 10), the DEM model was loaded sequentially with a constant stress increment of 0.05 MPa, until a quasi-static

**Figure 40. Main stages for a numerical stress-deformation-flow coupling analysis procedure.**

**Figure 41. Typical boundary conditions for the numerical experiments. a) Setup for the biaxial compression tests, b) Mechanical boundary conditions, and c) Hydraulic boundary conditions.**
equilibrium state of the model was reached. Equilibrium state during loading process of the DEM model and the velocities (in the both x and y directions) at a number of monitoring points (Fig. 39a) were checked to ensure that they become zero or very close to zero at the end of every loading step, and the resultant deformed DEM models at the end of the loading cycling process generated stress-strain behavior of the fractured rock concerned, represented by the DEM model.

The second stage was to simulate the coupled water flow and stress processes with additional water pressures applied at the two vertical lateral boundaries (Fig. 41c), with the top and bottom boundaries sealed hydraulically. At this stage, water flow through the deformed DEM models under mechanical loading was simulated under the specified hydraulic boundary conditions, representing a horizontal hydraulic gradient as illustrated in the figure 41c. Water flow is governed by the Cubic Law with a pressure gradient between the two lateral boundaries of the DEM model (Fig. 41c), which is given by:

\[ \Delta p = p_2 - p_1 \]  
(Eq. 10)

Therefore, when a pressure difference (\( \Delta p \)) exists between two vertical sides of the DEM model, water flow will take place (Fig. 41c). This hydraulic condition allows water to flow horizontally, from the right side to the left side of the model, at all coupled hydro-mechanical simulation stages with different confining pressures.

In this research, a constant water pressure gradient equal to 0.001 MPa, with \( P_1 = 1.0 \) MPa and \( P_2 = 0.999 \) MPa, respectively, was used for evaluating the influence of water pressure on strength and deformability of fracture rocks. The hydraulic loading remained constant during each mechanical loading stage (by changing confining pressure), until the DEM model reaches steady state flow and mechanical equilibrium. The state of water flow was controlled by checking the in-flow and out-flow rates at the ends of the model until a steady state flow was obtained.

The choice of such a small hydraulic pressure gradient is the requirement for a basically uniform water pressure field of 1.0 MPa over the whole DEM model for strength and parameter evaluations. The values of confining pressure on the two lateral boundaries were lower than the water pressure during the two early loading steps of the numerical experiments, in order to check if the effective stresses in x-direction were still generally compressive. Also, this relatively low water pressure gradient is within the range of the water pressure gradient for in-situ conditions in the shallow depth of the Earth’s crust, such conditions existing in low and intermediate radioactive waste repository considerations in a number of European countries.
6.2. Results

6.2.1. Deformability of the fractured rock

(1) Deformation behavior of the fractured rock

Using the normal stresses and strains obtained in the x and y directions at the end of each loading step, for the both stages and from all monitoring points, the mean values of stress and strains were computed by using a FISH function developed in UDEC. Water pressure effects on deformation behavior of the DEM model representing the fractured rock concerned were presented by plotting the numerical test results as stress-strain curves.

A comparison for the deformation behaviors of the DEM models, between pure mechanical deformation curves without water pressure (solid lines) and deformation curves with combined stress and water pressure boundary conditions of horizontal hydraulic pressure (dashed lines) under different confining pressure conditions is shown in the figure 42.

As shown in the figure 42, the DEM model deformed linearly and elastically before the axial stresses approached the elastic limit in the absence of water (solid line curves), but the slopes of the stress-strain curves within these ranges also changed slightly with different confining pressures. Further compression led to inelastic deformation up to the peak plastic strength. With increasing confining pressure, the axial stresses of the DEM models increased, and the stress-strain curves followed an elastic-plastic behavior with slight strain hardening. Also, the DEM models had the same nonlinear stress-strain curves of elastic-plastic behavior in the present of water (dashed line curves), but with a significant reduction of axial stress and peak strength of the stress-strain curves, and the curves representing pure mechanical and coupled stress-flow behaviors were generally parallel in trend.

It should be noted that a sharp increase of the stress-strain curve of the DEM model with a confining pressure of 1.0 MPa and the horizontal hydraulic pressure (dashed blue line curve) occurred. The reason for this sharp change was local interlocking of DEM model blocks, as examined below.

Figure 43 shows a comparison between major principal stress contours of the DEM model under 1.0 MPa confining pressure without water (dry condition, Fig. 43a, b) and with water (wet condition, Fig. 43c, d) at two specified points, representing the starting point (point 1) and finishing point (point 2) defining the sharp change interval, when considering water pressure. It can be seen from the figure 43 that while stress distributions were approximately the same at the two corresponding specified points of the curve without water (Fig. 43a, b), stress distribution is quite similar in both contour shape and magnitudes. However, between point 1 (Fig. 43c) and point 2 (Fig. 43d) for the wet condition, both shape and magnitude of the stress distribution changed with higher magnitude at point 2, with a higher stress concentration around a few number of fracture intersections caused by interlocking of DEM model blocks, and caused by water pressure.
Figure 42. Comparison of stress-strain curves between mechanical analysis (solid lines) and hydro-mechanical analysis (dashed lines), with different confining pressure conditions and under horizontal hydraulic pressure (from right to left).

This effect indicates that in reality, water pressure may cause local stress distribution patterns as the combined contributions of local fracture geometry and water pressure, and one may need to be concerned when evaluating large-scale in-situ tests of stress and water flow behavior of fractured rocks.

In following this section, more numerical experiments were performed to investigate effect of hydraulic conditions on deformation behaviors of fractured rock, and to test the reliability of the numerical modeling developed, by adapting the hydraulic-mechanical boundary condition (Case 2) rather than the previous mechanical-hydraulic boundary condition (Case 1). In Case 2, the hydraulic conditions adapted before applying axial compressive stress condition, as against Case 1 that the hydraulic conditions adapted after applying axial compressive stress condition.

Figure 44 compares the obtained stress-strain curves using the Case 1 (solid line) and Case 2 (dashed line) loading condition orders with different confining pressures. Numerical results show insignificant differences between the obtained stress-strain curves. Therefore, applying hydraulic boundary conditions before or after...
Figure 43. Comparison between major principal stress contours (unit: Pa) of the DEM model under 1.0 MPa confining pressure. (a-b) with dry condition, and (c-d) with wet condition for two specified points, namely before interlocking (point 1) and after interlocking (point 2).

applying axial compressive stress loading condition has no significant effect on the deformation behaviors of fractured rock. Choosing the order actually depends on the physical experiment requirements, such as sealing of water during the mechanical loading.

(2) Deformation parameters of the fractured rock
Based on the stress-strain curves obtained at the end of coupled hydro-mechanical analysis with application of hydraulic conditions after applying axial compressive stress, the equivalent directional Young’s modulus and Poisson’s ratio were calculated as a function of water pressure effects and confining pressures.

Figure 45 shows the comparison between the variations of the equivalent directional Young’s modulus for the DEM model without and with water pressure, under different confining pressures. The values of Young’s modulus decreased slightly when a water pressure was present compared with that without water pressure. The discrepancy was moderate when confining pressure was smaller than 1.5 MPa, but insignificant afterwards. Slightly larger overall horizontal displacements or larger strain rates of the DEM model caused a slight decrease of the slopes of the stress-strain curves, confining pressures were below 1.5 MPa, and water flow with a pressure of 1.0 MPa was involved. Generally, the magnitude of the directional Young’s modulus for both cases with and without water is much less than the Young’s modulus of intact rock.

A comparison between the variations of the Poisson’s ratio for the DEM model without and with water pressure under different confining pressures shows that the values of Poisson’s ratio decrease gradually with increase of confining pressure for both
cases with and without water (Fig. 46). The Poisson’s ratio increases slightly with water pressure. The reason for this difference can be the effect of parallel directions of the horizontal water flow direction and confining pressure. Generally, the magnitude of Poisson’s ratio for both cases with and without water is much larger than that for intact rock. The obtained results of larger Poisson’s ratio indicated that water flow affected the deformability of the fractured rock in some confining pressure levels.

Figure 45. The equivalent directional Young’s modulus for the DEM model without and with water pressure, under different confining pressures (The Young’s modulus at zero confining pressure are 43 MPa, and 11 MPa for DEM model without and with water pressure, respectively, which cannot be more clearly presented in this figure due to their very small magnitudes).

Figure 46. The equivalent values of Poisson’s ratio for the DEM model without and with water pressure under different confining pressures.
Figure 47 shows the normalized flow rates in each fracture intersecting at the outlet flow side (left) of the DEM model with increasing confining pressure, illustrating the flow pattern changes with respect to the stress changes. Flow rates were normalized with respect to the mean flow rates (total flow rate divided by the number of fractures) at the boundary of the DEM model. One can see from these figures that the changes in flow-rate patterns in the lower confining pressure conditions, e.g., with confining pressure equal to 0.5 and 1.0 MPa, are more significant than that with confining pressures equal to or higher than the water pressure.

Figure 47. Normalized flow rates in each fracture intersecting the left vertical boundary of the DEM mode under different confining pressures.
6.2.2. **Strength of the fractured rock**

(1) Strength envelops

Mohr-Coulomb (M-C) and Hoek-Brown (H-B) strength failure criteria were used for estimating the strength envelops and equivalent strength parameters of the DEM model, with (wet) and without (dry) considering water pressure conditions. The curve fitting results, as normalized strength versus normalized confining pressure, with M-C and H-B criteria were shown in the figure 48. Generally, based on correlation coefficient (R) obtained for two failure envelopes (Table 6), the quality of fitting to the M-C and H-B strength envelopes were acceptable, with the M-C criterion showing a slightly better correlation.

(2) Strength parameters

Table 6 represents the comparison between the regressed strength parameters of both criteria with and without considering water pressure conditions. The results clearly show a general reduction of DEM models’ strength parameters of both M-C and H-B criteria, except s as parameter of H-B criterion. The material constant parameters of H-B failure criteria, namely m and s, seem to be more sensitive to the presence of water flow.

6.2.3. **Effective stress behavior**

As mentioned in the introduction, the effective stress defined as the total stress minus the pore-water pressure. Therefore, one of the main reasons for the significant differences observed of some strength parameters in this study can be the effective stress, which induced by both direction of water flow and magnitude of water pressure, represented by the specified hydraulic boundary conditions due to the existence of the complex fracture system.

![Figure 48. Strength curves for DEM models in the normalized effective stress space without (solid red line) and with (dashed blue line) water pressure condition. left) M-C criterion, and right) H-B criterion.](image-url)
Table 6. Equivalent strength parameters of M-C and H-B failure criteria.

<table>
<thead>
<tr>
<th>Water pressure (MPa)</th>
<th>Mohr-Coulomb</th>
<th>Hoek-Brown</th>
<th>Hoek-Brown</th>
<th>Hoek-Brown</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cohesion C (MPa)</td>
<td>Friction Angle φ (°)</td>
<td>Correlation Coefficient R</td>
<td>Parameters of Hoek-Brown m, s</td>
</tr>
<tr>
<td>Δp=0</td>
<td>0.1727</td>
<td>28.3</td>
<td>0.9950</td>
<td>0.0591</td>
</tr>
<tr>
<td>Δp=0.001</td>
<td>0.0616</td>
<td>22.9</td>
<td>0.9853</td>
<td>0.0233</td>
</tr>
</tbody>
</table>

Tables 7 and 8 show the calculated elastic limit and peak plastic strength values versus confining pressure in dry and wet conditions and differences between them, calculated using results in elastic and plastic deformation ranges, respectively. The results show that a water pressure of 1.0 MPa caused a reduction in strength values of 1.15-2.24 MPa, much larger than the water pressure applied at the lateral boundaries, with a larger mean difference for peak plastic strength. Therefore, the effective stress behavior is quite different from its classical definitions in Terzaghi and Biot’s poroelasticity theory (since the Poisson’s ratio is generally larger than 0.5), due to the influence of fracture system complexity, direction of water pressure conditions applied and complex stress distribution in the tested volumes, especially at intersections of fractures.

6.3. Summery discussions

The systematic numerical approach was provided to study of water pressure influences on strength and deformation behaviors of the fractured rocks with considering the effective stress concept through the coupled stress-deformation-flow analyses. The following outcomes are the several interesting insights that may lead to a better understanding about this matter and a useful tool for the design of large-scale in-situ experiments on coupled stress-flow processes of fractured rocks:

1. Water pressure and confining pressure play a significant role in deformability parameters of fractured rocks. The trends of the elastic modulus differ more significantly than those of Poisson’s ratio with differences between water pressure and mechanical confining pressures.

2. Strength of fractured rocks is dependent on hydraulic boundary conditions and mechanical confining pressures. Unlike the confining pressure that generally causes strength increase of fractured rocks, water pressure generally causes strength decrease of fractured rock, due to the effective stress phenomenon, but the values of stress and strength reduction may or may not equal to the magnitude of water pressure, due to the influence of fracture system complexity.
Table 7. Calculated elastic limit values versus confining pressures in dry and wet conditions and differences between them, with obtained results in the elastic deformation range.

<table>
<thead>
<tr>
<th>Confined pressure (MPa)</th>
<th>Elastic limit (MPa)</th>
<th>Difference between dry and wet conditions (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dry condition</td>
<td>Wet condition</td>
</tr>
<tr>
<td>0.5</td>
<td>1.23</td>
<td>0.486</td>
</tr>
<tr>
<td>1.0</td>
<td>2.5</td>
<td>0.974</td>
</tr>
<tr>
<td>1.5</td>
<td>3.73</td>
<td>2.18</td>
</tr>
<tr>
<td>2.0</td>
<td>5.05</td>
<td>3.45</td>
</tr>
<tr>
<td>2.5</td>
<td>6.34</td>
<td>4.73</td>
</tr>
<tr>
<td>3.0</td>
<td>7.6</td>
<td>5.96</td>
</tr>
</tbody>
</table>

Table 8. Calculated peak plastic strength values versus confining pressure in dry and wet conditions and differences between them, with obtained results in the plastic deformation range.

<table>
<thead>
<tr>
<th>Confined pressure (MPa)</th>
<th>Peak plastic strength (MPa)</th>
<th>Difference between dry and wet conditions (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dry condition</td>
<td>Wet condition</td>
</tr>
<tr>
<td>0.5</td>
<td>1.53</td>
<td>0.968</td>
</tr>
<tr>
<td>1.0</td>
<td>3.33</td>
<td>2.18</td>
</tr>
<tr>
<td>1.5</td>
<td>5.37</td>
<td>3.13</td>
</tr>
<tr>
<td>2.0</td>
<td>6.15</td>
<td>4.70</td>
</tr>
<tr>
<td>2.5</td>
<td>7.39</td>
<td>5.96</td>
</tr>
<tr>
<td>3.0</td>
<td>8.59</td>
<td>7.37</td>
</tr>
</tbody>
</table>

3. Both the M-C and H-B criteria give fair estimations of the peak compressive strength of fractured rock.

4. The concept of effective stress may still be applicable for fractured crystalline rocks, but may differ quite significantly with that usually applied in soil mechanics or theories of poroelasticity or poroplasticity, with a larger difference between total and effective stress than water pressure.

The theories of classical poroelasticity or poroplasticity with small deformation assumptions are not applicable if the equivalent Poisson’s ratio is larger than 0.5. Therefore, one should be cautious when applying the classical effective stress concept to fractured rock media and when the finite deformation assumption may need to be considered.
7. **Statistical Analysis of Strength and Deformability of Fractured Rocks**

Strength and deformation properties of fractured rocks are characterized by numerous uncertainties such as the existing uncertainty in the geometrical characterization of fractures. On the other hand, the inherently complex and irregular nature of fracture system geometry and available limitation on the current direct laboratory and in-situ field observations/measurements make this task very difficult. In these cases, the stochastic methods are more realistic than the deterministic methods for investigating the unpredictable behavior and mechanical properties variation of the fractured rock mass.

In this chapter, a stochastic analysis was presented to quantify the distributions of strength and deformability of a fractured rock mass through a series of systematic numerical experiments, using multiple realizations of stochastic DFN models at a REV level.

### 7.1. Methodology

Since the aim of this part of the research was an accurate prediction of strength and deformability of fractured rocks, it is necessary to generate a large number of stochastic DFN models to reduce uncertainty. Hence, a set of 50 square-shaped DFN realizations were generated with side length of 5m × 5m, as extracted from the center of an original parent model of the fracture system.

An independent DFN generator was developed in the MATLAB workspace for generating the DFN realizations, based on the method developed by Min and Jing 2003. It is able to create the geometric parameters of stochastic fractures, according to the fracture parameter distribution defined, using the Monte Carlo simulation technique. In this study, the fracture location, orientation, and length are assumed to follow a Poisson, Fisher, and power law distribution, respectively, based on the field mapping results in Sellafield area that were four identified sets of fractures (Table 1). Figure 49 shows the geometry of fracture systems after fracture regularization for the fifty DFN models. The DFN models generated look similar (since they obey the same probabilistic functions of their parameters) but different in fracture system geometry (since random variation was assigned by the Monte Carlo simulation algorithms).

In the next step, the resultant multiple DFN models were used to stochastic stress-deformation analysis in the same of the methodology presented in the first chapter. Therefore, 350 numerical experiments were conducted in this research, including 50 uniaxial experiments and 300 biaxial experiments.

In the following, the mean values of stress and strain were calculated using the displacement and stress components obtained from 36 monitoring points of a regular network inside of the DEM models at intersections of the six parallel horizontal and six vertical lines, with the same spacing.
The numerical mean values calculated of the normal stress and strain components for each realization of the DEM model were used to evaluate the distribution of strength and deformability of fractured rocks using the axial stress-strain curves.

In the final, the appropriate statistical distributions are fitted (based on the some common standard distributions such as normal, lognormal, exponential, and etc.) on the strength and deformation parameters of fractured rock by using the Chi-Squared goodness-of-fit test with 95% of confidence level. The Chi-Squared statistic is defined as:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$  \hspace{1cm} (Eq. 11)

Where $O_i$ and $E_i$, respectively, are the observed and expected frequency for the $i$th, and $k$ is number of bin or interval. $E_i$ for the bin $i$ calculated by:

$$E_i = F(x_2) - F(x_1)$$  \hspace{1cm} (Eq. 12)

Where $F$ is the cumulative distribution function (CDF) of the probability distribution being tested, and $x_1$ and $x_2$ are the limits for bin $i$. The CDF and PDF (probability distribution function) were used to describe the uncertainty of the stochastic strength and deformation parameters in this study.

### 7.2. Results

#### 7.2.1. Distribution of deformability behavior

The curves of mean values of axial stresses versus axial strains for the fifty DEM model realizations with the 0.5 MPa, 1 MPa, 1.5 MPa, 2 MPa, 2.5 MPa, and 3 MPa confining pressures, was illustrated in the figures 50-55, respectively.

It can be seen from the figures 50-55 that, the DEM models deformed linearly and elastically during the initial loading stage at axial stresses below initial yield strength, after which the deformation behavior changed from elastic to plastic. The slopes of the elastic stage of these curves changed slightly with different confining pressures before this elastic limit defined by the initial yielding strength. Further axial compression led to inelastic deformation up to the peak plastic strength, where the loading stopped.

Also, the obtained stress-strain curves show that with the increase of confining pressure, the average axial stress and strength of fractured rocks increases, and that the deformation behavior follows elasto-plastic behavior with a strain-hardening trend for all of the DEM model realizations. By continued loading, strain-softening may occur, but it is not required since the constitutive model of the rock mass is not included in this research.
Although the general deformation behavior of fifty realizations has the similar trends, but the magnitude of peak stress at the end of loading and the slope of elastic regions of stress-strain curves before reaching the initial yielding strength vary, due to changing the fracture system geometry, and differences in the number and size of blocks during the generating multiple realization process.

Figure 56 shows the distribution of the peak of compressive strength curves for fifty realizations of DEM models under different confining pressures. The blue dotted lines are the lower and upper bounds obtained of peck strength. Generally, besides the peak strength for each realization of the DEM models, the distance between two bounds of peak strength increased with the increasing confining pressures. This show that the variations range of peak strength of fractured rocks is dependent to confining pressure. With increasing confining pressure, the strain rate becomes less and thus fractured rock becomes stronger and stiffer and requires a higher value of axial load to deform it. The strain rate variations are more persistent in the lower confining pressure and vice versa, hence it causes the stress jump upon varying levels with the wider range when high confining pressure.

7.2.2. Distribution of deformation parameters

Figures 57 and 58 show the variation of equivalent directional Young’s modulus and Poisson’s ratio for fifty realizations of the DEM models under different confining pressures, respectively. The blue dotted lines are the lower and upper bounds obtained for each parameter. The variation ranges of the Young’s modulus are approximately constant, while the variation ranges of the Poisson’s ratio decrease with increase confining pressures. This is due to that variation of curve slops are insignificant in each confining pressure, hence variation range of Young’s modulus of DEM models is constant under different confining pressures. Also, the deformation rate of the DEM models decreases under loading, hence the variation range of Poisson’s ratio decreases when high confining pressure applied.
Figure 50. Axial stress-strain curves of fifty DEM model realizations under 0.5 MPa confining pressures.

Figure 51. Axial stress-strain curves of fifty DEM model realizations under 1.0 MPa confining pressures.
Figure 52. Axial stress-strain curves of fifty DEM model realizations under 1.5 MPa confining pressures.

Figure 53. Axial stress-strain curves of fifty DEM model realizations under 2.0 MPa confining pressures.
Figure 54. Axial stress-strain curves of fifty DEM model realizations under 2.5 MPa confining pressures.

Figure 55. Axial stress-strain curves of fifty DEM model realizations under 3.0 MPa confining pressures.
Figure 56. Variations of the peak of strength curve for fifty realizations of the DEM models under different confining pressures.

It can be seen that unlike the equivalent values of Young’s modulus (Fig. 57), the equivalent values of Poisson’s ratio decreased with increasing of confining pressures for each realization (Fig. 58).

In addition, the obtained numerical results from each realization of the DEM models compared with the deformation parameters of the intact rock (see Table 2) show that the Young’s moduli for fractured rocks is less than the Young’s modulus of the intact rock and the Poisson’s ratio of fractured rocks is much larger than that for intact rock.

Based on data obtained from the fifty numerical experiments on the DEM models, the frequency distribution of the equivalent directional Young’s modulus and Poisson's ratio for the DEM models with 0.5 MPa confining pressure are illustrated in the figures 59 and 60, as an example. Similar frequency distributions were obtained under different confining pressures.

It was found that the PDF and CDF of the equivalent directional Young’s modulus (E) under different confining pressures follow approximately the standard normal distribution that are given by the equations, respectively:

\[
f(E) = \frac{\exp\left(\frac{-1}{2}\left(\frac{E-\mu}{\sigma}\right)^2\right)}{\sigma\sqrt{2\pi}} \quad (\text{Eq. 13})\]

and,

\[
F(E) = \Phi\left(\frac{E-\mu}{\sigma}\right) \quad (\text{Eq. 14})
\]
Figure 57. Variations of the equivalent directional Young’s modulus for fifty realizations of the DEM models under different confining pressures.

Figure 58. Variations of the equivalent Poisson’s ratio for fifty realizations of the DEM models under different confining pressures.

Also, the PDF and CDF of the equivalent Poisson’s ratio ($\nu$) under different confining pressures follow quite well a lognormal distribution that are given by the equations, respectively:

$$f(\nu) = \frac{\exp\left(-\frac{1}{2}\left(\frac{\ln(\nu) - \mu}{\sigma}\right)^2\right)}{(\nu)\sigma\sqrt{2\pi}} \quad (Eq. 15)$$

$$F(\nu) = \Phi\left(\frac{\ln(\nu) - \mu}{\sigma}\right) \quad (Eq. 16)$$
Figure 59. Frequency of the equivalent directional Young’s modulus for fifty realizations of the DEM models under 0.5 MPa confining pressures.

Figure 60. Frequency of the equivalent Poisson’s ratio for fifty realizations of the DEM models under 0.5 MPa confining pressures.

Where $\Phi$ is the Laplace integral, $\mu$ is the scale parameter or mean value, and $\sigma$ is the location parameter or standard deviation value of the distribution.

Figure 61 shows the PDFs together with the typical best-fitted curve of the equivalent directional Young’s modulus and Poisson's ratio of the fractured rock concerned with 0.5 MPa, 1 MPa, 1.5 MPa, 2 MPa, 2.5 MPa, and 3 MPa confining pressures as the normal and lognormal distribution, respectively, for showing of the how fitting curve to the data.
Figure 6.1. Probability distributions and the typical curve fitted of the equivalent Young’s modulus (a), and Poisson’s ratio (b) of the fractured rock with various confining pressures.
The CDFs of the equivalent directional Young’s modulus and Poisson’s ratio for fifty realizations of DEM models under different confining pressures are illustrated in the figures 62 and 63, respectively. It can be seen that, while the CDFs of both parameters show a very similar behavior with the different confining pressures, the CDFs of Young’s modulus (Fig. 62) were migrated to the right side (larger values) and the CDFs of
Poisson's ratio (Fig. 63) were migrated to the left side (smaller values) with increasing the confining pressure.

Tables 9 and 10 summarize the results of the statistical analysis and parameters of Chi-Squared goodness-of-fit test for the equivalent Young’s modulus and Poisson's ratio with different confining pressures, respectively. It can be seen that, unlike the equivalent Young’s modulus, the mean value (μ) and standard deviation (σ) value of the equivalent Poisson's ratio decrease with increasing the confining pressure.

7.2.3. Distribution of strength parameters

(1) Strength envelops

Figure 64 shows the distribution of the curve fitting results, which contains the results of 350 numerical experiments, as normalized strength versus normalized confining pressure with M-C (Fig. 64a) and H-B (Fig. 64b) failure criteria of fifty realizations of the DEM model to represent the equivalent strength parameters of the fractured rock concerned. The blue dotted lines are the liner and non-liner lower and upper bounds obtained for M-C and H-B strength envelopes, respectively.

Table 9. Statistical best-fitted distribution characteristics of the equivalent Young’s modulus with different confining pressures.

<table>
<thead>
<tr>
<th>Confining pressures (MPa)</th>
<th>Minimum</th>
<th>Maximum</th>
<th>σ</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>28.04</td>
<td>36.22</td>
<td>2.1937</td>
<td>32.576</td>
</tr>
<tr>
<td>1</td>
<td>37.65</td>
<td>47.19</td>
<td>2.4677</td>
<td>42.956</td>
</tr>
<tr>
<td>1.5</td>
<td>45.60</td>
<td>54.85</td>
<td>2.4853</td>
<td>50.634</td>
</tr>
<tr>
<td>2</td>
<td>51.52</td>
<td>60.96</td>
<td>2.3927</td>
<td>56.192</td>
</tr>
<tr>
<td>2.5</td>
<td>56.30</td>
<td>65.82</td>
<td>2.3465</td>
<td>60.858</td>
</tr>
<tr>
<td>3</td>
<td>60.42</td>
<td>70.14</td>
<td>2.3643</td>
<td>64.782</td>
</tr>
</tbody>
</table>

Table 10. Statistical best-fitted distribution characteristics of the equivalent Poisson's ratio with different confining pressures.

<table>
<thead>
<tr>
<th>Confining pressures (MPa)</th>
<th>Minimum</th>
<th>Maximum</th>
<th>σ</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.555</td>
<td>0.855</td>
<td>0.1148</td>
<td>0.682</td>
</tr>
<tr>
<td>1</td>
<td>0.528</td>
<td>0.794</td>
<td>0.1138</td>
<td>0.648</td>
</tr>
<tr>
<td>1.5</td>
<td>0.498</td>
<td>0.737</td>
<td>0.1103</td>
<td>0.603</td>
</tr>
<tr>
<td>2</td>
<td>0.462</td>
<td>0.699</td>
<td>0.1078</td>
<td>0.562</td>
</tr>
<tr>
<td>2.5</td>
<td>0.446</td>
<td>0.630</td>
<td>0.1021</td>
<td>0.529</td>
</tr>
<tr>
<td>3</td>
<td>0.423</td>
<td>0.586</td>
<td>0.0911</td>
<td>0.495</td>
</tr>
</tbody>
</table>
Strength and deformability of fractured rocks

Figure 64. Distribution of strength envelopes for fifty realizations of the DEM models in the normalized principal stress space. a) M-C, and b) H-B.

(2) M-C strength parameters

Based on the strength envelopes obtained of the M-C failure criterion (Fig. 64a), the normal distribution was exhibiting the best fit to both friction angle (Fig. 65) and cohesion (Fig. 66) of fractured rock concerned. The CDF together with the typical best-fitted curve of the equivalent friction angle and cohesion of fractured rock illustrated in the figures 67 and 68, respectively, for showing of the how fitting curve to the data.

Table 11 summarizes results of the statistical analysis and parameters of Chi-Squared test for both equivalent strength parameters of M-C strength criterion, namely cohesion and friction angle. The cohesion ranges widely from 0.025 MPa to 0.389 MPa with a mean value of 0.2012. Also, friction angle ranges narrowly from 23.68° to 34.85° with a mean value of 28.86 degree.

(3) H-B strength parameters

Based on the strength envelopes obtained of the H-B failure criterion (Fig. 64b), the lognormal distribution was exhibiting the best fit to m (Fig. 69) and s (Fig. 70) parameters of the H-B failure criterion of fractured rock concerned.

Table 11. Statistical best-fitted distribution characteristics of the equivalent strength parameters of M-C strength criterion.

<table>
<thead>
<tr>
<th>Strength parameters</th>
<th>Minimum</th>
<th>Maximum</th>
<th>σ</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion (MPa)</td>
<td>0.025</td>
<td>0.389</td>
<td>0.0969</td>
<td>0.2012</td>
</tr>
<tr>
<td>Friction Angle (°)</td>
<td>23.679</td>
<td>34.855</td>
<td>2.5746</td>
<td>28.86</td>
</tr>
</tbody>
</table>
Figure 65. The PDF and typical curve fitted of the equivalent friction angle of the fractured rock.

Figure 66. The PDF and typical curve fitted of the equivalent cohesion of the fractured rock.

The CDF together with the typical best-fitted curve of the both equivalent parameters of H-B failure criterion illustrated in the figures 71 and 72, for showing of the how fitting curve to the data. Table 12 shows the results of the statistical analysis and parameters of Chi-Squared goodness-of-fit test for both equivalent strength parameters of H-B strength criterion, namely m and s. The values of m changes from 0.0390 to 0.1074 with a mean value of 0.0672. Also, the values of s changes from 3.140e7 to 1.539e5 with a mean value of 5.693e^6 that is close to zero.
Strength and deformability of fractured rocks

Figure 67. The CDF and typical curve fitted of the equivalent friction angle of the fractured rock.

Figure 68. The CDF and typical curve fitted of the equivalent cohesion of the fractured rock.

Table 12. Statistical best-fitted distribution characteristics of the equivalent strength parameters of H-B strength criterion.

<table>
<thead>
<tr>
<th>Strength parameters</th>
<th>Minimum</th>
<th>Maximum</th>
<th>σ</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>0.0390</td>
<td>0.1074</td>
<td>0.0181</td>
<td>0.0672</td>
</tr>
<tr>
<td>s</td>
<td>3.140 e⁻⁷</td>
<td>1.539 e⁻⁵</td>
<td>3.929 e⁻⁶</td>
<td>5.6937 e⁻⁶</td>
</tr>
</tbody>
</table>
Figure 69. The PDF and typical curve fitted of the $m$ parameter of the H-B for fractured rock.

Figure 70. The PDF and typical curve fitted of the $s$ parameter of the H-B for fractured rock.

7.3. Summery discussions

A systematic numerical procedure was presented to estimate uncertainties in measuring fracture systems of fractured rock masses and predict variations of strength and deformation parameters of fracture rocks, using 50 realizations of stochastic discrete fracture network (DFN) models at the REV established.

A stochastic analysis was performed to quantify the distributions of strength and deformability a fractured rock mass, using the 350 uniaxial and biaxial compression numerical experiments.
The Chi-Squared goodness-of-fit test was used to frequency and probability and cumulative distribution functions (PDF-CDF) of the strength and deformability of fracture rocks distributions. The variations and the best-fitted distribution functions of strength and deformability parameters were estimated statistically, as follows:

1) The Young’s modulus and Poisson’s ratio during elastic deformation stages have normal and lognormal distributions, respectively,

2) Both the friction angle and cohesion derived from Mohr-Coulomb (M-C) strength criterion obey normal distributions,
3) The m and s parameters of Hoek-Brown (H-B) strength criterion have lognormal distributions. The results obtained also indicate that generally variations of strength and deformation parameters of the fractured rocks are dependent upon confining pressure. Therefore, the effects of confining pressure can be important in evaluating the mechanical properties of fractured rocks.

8. Discussion

Despite the fact that the numerical modeling of the fractured rocks requires much more computing time compared with that used by the empirical and analytical methods, they can consider irregular and complex fracture system geometry. The comprehensive modeling procedure presented in the previous chapters is in fact a combination of both DFN and DEM methods, which can be used as a suitable and flexible numerical approach to predict the mechanical behaviors and properties of fractured rocks that cannot be obtained by conventional laboratory tests using small intact rock samples and in-situ tests at present. This is acceptable when the size of DFN models is at or larger than the REV scale. The variations observed in the strength and deformation parameters of fractured rocks (Papers I-V) reflect the confining pressure dependence of the mechanical behaviors and properties of the fractured rock mass. The findings show that the effect of confining pressure is important for evaluating mechanical properties of fractured rocks. The complex and nonlinear mechanical behaviors of fractured rocks under low confining pressures, such as higher values of equivalent Poisson’s ratio larger than 0.5, should be considered for design and perform an assessment of the large volume tests of fractured rocks in the laboratory or field conditions.

The difference observed between the stress-strain behaviors of fractured rocks when axial stress and axial velocity loading conditions were applied (Paper I and II) demonstrates the effects of loading conditions in determining the strength and deformation parameters of fractured rocks. The main reasons causing such difference is the shear dilation effect on normal stress of fractures. As can be seen in the figure 73 higher normal stress of fractures was induced during shear under constant velocity conditions, due to the dilation angle of the fractures that it assumed equal to 5° in this research. There may be other reasons, such as block interlocking and stress concentration at fracture intersections, which cannot appear when testing samples of intact rock materials of small volumes, which may also contribute to such differences locally, but play a less important role compared with shear dilation effects on normal stress of fractures. This subject remains an important issue for further investigations.
The main reason for the significant anisotropy observed on strength and elastic deformability of fractured rocks (Paper III) with the loading directions is the complex fracture system geometry that was not regular and isotropic in this research, so that the model behaved differently for different model rotations, even if their loading conditions were identical.

Figures 74 and 75 show distributions of fracture orientation angles with lengths more than 1 m and 2 m, respectively, as rose diagrams. One can see from these figures that numbers of fracture in the direction of 90, 30 and 150 degrees are larger than that in other directions, especially when length of fractures equals to 2 m (Fig. 75). Because the main fractures of longer lengths plays more significant roles for stress-displacement behavior of the tested models, therefore, anisotropy in the strength and deformability begins to be more significant for the models of rotated to these directions, due to the fact that these major fractures are the major weakness planes that cause the major stress changes in the respective directions, with respect to the loading directions.

Figure 76 shows the iso-value contours of the minor principle stress of the DEM model with rotation angles of 150° and 90° with respect to the horizontal x-direction, respectively. Significant variation of stress distribution occurred, due to the different

Figure 73. Normal stress versus shear displacement curves of direct shear tests of rock fractures under different system stiffness (Skinas et al., 1990).
orientations of fractures relative to the directions of axial load and lateral confining pressures. In conclusion, the fractured rock displays anisotropic behavior in strength and deformability, depending on fracture system geometry and loading directions.

The effective stress induced by both direction of water flow and magnitude of water pressure is an important issue to estimate the strength and deformability of fractured rocks. The applied hydraulic boundary conditions for the coupled hydro-mechanical analysis may cause significant differences in some strength parameters, but moderate effects on elastic deformation parameters of fractured rock models (Paper IV).
Figure 76. Distribution of minor principle stress contours (unit: Pa) of the DEM model with a) 150° rotation, and b) 90° rotation.

Figure 77 shows the distribution of major principle stress contours of the DEM models without (left column) and with (right column) water pressure condition, as effective stresses under different confining pressures, respectively. Significant variation of stress distribution occurred, due to applying a constant water pressure and lateral confining pressures. The amounts of major principle stress decrease with water pressure. From the figure 77a, one can see that no significant tensile stress was observed in the model when the confining pressure is 0.5 MPa.

The strength and deformation properties of fractured rocks have ranges of values instead of fixed values (Paper V). The main source of such uncertainties comes through largely unknown fracture system properties, so that stochastically generated DFN realizations showing different fracture system geometry and block numbers, has to be applied as a tool for uncertainty quantification. Therefore, the parameter variability must be taken into account in the constitutive model development for fractured rocks. In such cases, stochastic analysis is a useful way to estimate how the strength and deformability parameters of fractured rocks vary within what kind of limitations, which will be helpful for the design and performance assessments of rock engineering projects.

9. **Concluding remarks and future work**

9.1. **Main conclusions and scientific achievements**

Numerical modeling is an essential requirement for understanding and predicting strength and deformability of fractured rock masses, due to the results of small-scale laboratory experiments are not representative of fractured rock masses containing large number of fractures, and also non practical and high costs of large-scale in-situ experiments in reality at present.
This thesis deals with the development of a systematic numerical predicting platform, for a fundamental study, to estimate the representative strength and deformability of fractured rocks at REV level when the host rocks are hard crystalline rocks such as granites. A number of stress-deformation and coupled stress-deformation-flow analyses were carried out to evaluate the impacts of static loading condition (mechanics effect), loading direction...
(anisotropy effect), and water pressure (hydro-mechanics effect) on strength and deformation parameters and also statistical analysis of uncertainty of strength and deformability of fractured rocks, using a series of two-dimensional (2D) numerical experiments. The main conclusions are summarized based on main findings in the following aspects:

1. Generally, strength and deformability of fractured rocks are dependent on confining pressures, loading directions, water pressure, and mechanical and hydraulic boundary conditions. Deformation behavior of fractured rocks is nonlinear over the concerned range of stress and follows an elasto-plastic behavior with a strain hardening trend in all of cases considered in this study.

2. The strength and deformation parameters of fractured rocks, represented by Young's modulus and Poisson's ratio, change significantly with confining pressures. While the strength and the equivalent directional Young’s modulus of fractured rocks increases, the equivalent Poisson's ratio of fractured rocks decreases with increase of confining pressure.

3. The numerical results demonstrate that in addition to difference between stress-strain behaviors, there are differences between strength curves and strength parameters of fractured rocks tested under axial stress and axial velocity loading conditions. A higher average axial stress and higher directional Young’s modulus under axial velocity test condition than that under axial stress condition was observed, due to the effects of shear dilation of fractures.

4. The calculated results indicate that strength and deformability of fractured rocks are directional dependent, which vary with the loading conditions. Anisotropies of strength and deformability of fractured rocks are important issues for design and performance of rock engineering projects, since the relative directions of in-situ stresses (relevant to determining the loading directions), fracture system geometry (relevant to directional variation of stress distribution) and rock engineering projects (such as choosing axes of tunnels relative to not only in-situ stress directions but also orientations of dominating fracture sets) may have a significant impact on the safety and performance of the rock engineering structures located in fractured rock masses.

5. Water pressure plays a significant role in strength and deformability parameters of fractured rocks. While the equivalent directional Young’s modulus of fractured rocks decreased slightly with water pressure, the Poisson’s ratio increases slightly when a water pressure was present compared with that without water pressure. Water pressure generally causes strength decrease of fractured rock, due to the effective stress phenomenon, but the values of stress and strength reduction may or may not equal to the magnitude of water pressure, due to the influence of fracture system complexity.

The concept of effective stress may still be applicable for fractured crystalline rocks, but may differ quite significantly with that usually
applied in soil mechanics or theories of poroelasticity or poroplasticity, with a larger difference between total and effective stress than water pressure. The theories of classical poroelasticity or poroplasticity with small deformation assumptions are not applicable if the equivalent Poisson’s ratio is larger than 0.5 when confining pressure is small.

6. The results obtained from stochastic analysis indicate that the strength and deformation properties of fractured rocks have ranges of values instead of fixed values. The Young’s modulus and Poisson’s ratio during elastic deformation stages have normal and lognormal distributions, respectively. Both the friction angle and cohesion derived from M-C strength criterion obey normal distributions. Also, m and s as two strength parameters of H-B failure criterion have lognormal distributions. However, above findings are site-specific and applicable only for the specific geological conditions considered in this research.

7. Both the M-C and H-B criteria give fair estimates of the compressive strength of the rock concerned for almost all cases tested in this study. In contrast to the M-C criterion that is a linear failure envelope with better estimation of the RMSE values for some issues, the H-B criterion is a nonlinear failure envelope and is more flexible for modeling different fracture system geometries and stress conditions.

The results in the thesis are useful when designing of large-scale in-situ experiments with large volumes of fractured rocks considering coupled stress-deformation-flow processes. In such cases, knowledge on fracture system geometry and their mechanical behaviors play a significant role to design future physical tests for estimating the strength and deformability of fractured rock masses, which will be very different and much more challenging compared with a testing intact rock sample of small volumes. Also, the effects of different loading conditions should be carefully considered for designing and result interpretation for large scale in-situ experiments. Due to the directional dependence of strength and deformability of fractured rocks, proper site investigations for in-situ stress and fracture system geometry and hydro-mechanical behaviors are crucial for design and performance/safety assessments of underground engineering works in fractured rock mass. In addition, the directional variations of strength and deformability of the fractured rock mass concerned must be treated properly with respect to the directions of in-situ stresses. In the current situation, many in-situ experiments were conducted in underground tunnels or caverns with disturbances to the local stress field (usually with stress releasing in the direction of excavated free face) and significant water flow situation changes (usually drainage), so that maintaining proper initial and boundary conditions of the large volume of rock mass samples to be tested become important. Therefore, comprehensive numerical modeling needs to be integrated with physical experiments from the start to the end of such projects.
9.2. Recommendations for future study

Although the results obtained from this research lead to some important insights in strength and deformability of fractured rocks, with some assumptions to simplify the problems, still some open questions as outstanding issues remain for continued improvements in future studies as follows:

1. Although it is almost impossible at present, the large-scale laboratory or in-situ field experiments on large volumes of rock mass samples containing many fractures should be tested to verify the validity of the modeling approaches pursued and their results.

2. The numerical approach developed can be extended to three-dimensional (3D) cases for more realistic simulations to eliminate the limitations caused by the assumption of 2D space under plane strain conditions, with significant increase of computing capacity required.

3. A full coupled thermo-hydro-mechanical-chemical (THMC) analysis could be conducted for more complete prediction of strength and deformability of fractured rocks, with additional chemical and temperature effects and fluid transfer processes considered.

4. Since this research was based on an assumption that the initial aperture of fractures has a constant value for simplicity, more modeling is needed to do numerical experiments with consideration of heterogeneous fracture apertures distributions when initial aperture of fractures is not constant but correlated to other fracture properties like length.

5. The partial cracking and complete crushing of rock blocks during loading processes and the isolated fractures, non-persistent fractures, and fracture dead-ends cannot be considered at present, due to current limitations of the current version of the UDEC code, and proper representations of their effects can be an issue for future work, especially when hydraulic and chemical processes need to be included.

6. In order to further improve the understanding of behavior fractured rock masses, it is necessary to conduct sensitivity studies of uncertainties in terms of the effects of hydro-mechanical and geometry parameters of rock matrix and fractures on strength and deformability of fractured rocks.

7. It would be of great interest to investigate the dynamic loading effect on strength and deformability of fractured rocks through dynamic analysis for proper characterization of fractured rocks under dynamic conditions, such as earthquakes, land sliding, blasting or explosions.

8. Development and improvement of failure criteria for fractured rocks with consideration of uncertainties in fracture system geometry and properties is needed to establish more reliable constitutive models and safety assessment for rock engineering in fractured rocks.
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