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Postprint

This is the accepted version of a paper presented at *Forty-Eight Asilomar Conference on Signal, Systems and Computers, November 2-5 2014, Pacific Grove, CA, USA*.

Citation for the original published paper:

Stathakis, E., Rasmussen, L. K., Skoglund, M. (2014)

Characterization of outage performance for cognitive relay networks with mixed fading.

In: *Forty-Eight Asilomar Conference on Signal, Systems and Computers: November 2-5 2014* (pp. 84-84).

<http://dx.doi.org/10.1109/ACSSC.2014.7094729>

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:kth:diva-159154>

Characterization of Outage Performance for Cognitive Relay Networks with Mixed Fading

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Abstract—We consider a dual-hop underlay cognitive radio network with a single transceiver pair, which utilizes an amplify-and-forward relay to establish end-to-end communication. The secondary nodes, i.e., the transmitter and the relay, obey transmit power constraints, which are imposed within the underlay paradigm, to guarantee that the instantaneous peak interference at the primary receiver will not exceed a certain threshold. Each of the secondary communication links contains a line-of-sight component whereas the external links, to the primary receiver, are subject to Rayleigh fading. For this system model, we analyze the outage probability, investigate asymptotic cases and outline some interesting variations of the proposed setup. Finally, we demonstrate the accuracy of the obtained mathematical expressions via numerical simulations, which corroborate our theoretical findings.

I. INTRODUCTION

The unprecedented growth of generated data and wireless traffic has injected new challenges in the fields of signal processing and communications. These challenges have stimulated new research paths, partially steered in the direction of improving spectrum utilization via changes in the allocation policies, which are driven by the cognitive radio (CR) paradigm [1]. Systems within the CR framework are entities that interact with the wireless environment through a continuous cycle of side-information acquisition and decisions. Information-theoretic studies [2], [3] have flagged the potential gains stemming from dynamic spectrum management strategies such as: i) *opportunistic spectrum access* (interweave); and ii) *spectrum sharing* (underlay/overlay) [4]. Spectrum sharing strategies promise higher achievable rates, yet such designs are challenging since one has to manage the generated interference, which degrades the quality-of-service (QoS) of the licensed (primary) system and thus needs to be controlled. This control is realized by imposing constraints on the unlicensed (secondary) system design such as limiting the transmit power to avoid excessive interference.

An early information-theoretic study of the underlay CR system capacity, under average receive-power constraints, was carried out in [5] while its Gaussian counterpart was analyzed in [6]. The well-documented merits of (non-)regenerative relaying strategies [7], in increasing reliability, motivated extensions of the decode-and-forward (DF) [8] and the amplify-and-forward (AF) [9] schemes to the underlay framework. A capacity-achieving power allocation scheme for AF CR relay

systems was considered in [10]. Common denominator in the extensive literature for CR relay systems is the assumption of shadowed communication environments. However, the trend in latest deployed systems is oriented towards very small-scale operation, e.g., femtocells [11]. The propagation environment, in such architectures, is likely to contain line-of-sight (LoS) components, thus the stochastic model for characterizing the relevant network quantities has to be updated [12]. A multi-antenna scheme, tailored to femtocell statistics and with rate-optimized dynamic power allocation was analyzed in [13], yet the analysis pertained to single-hop networks. To the best of the authors' knowledge, LoS fading has been investigated for conventional dual-hop AF networks [14] but it has escaped attention, so far, for AF/DF CR relay networks.

This gap motivates our study, which has direct application to the emerging cognitive femtocell architectures [15]. In particular, we consider a dual-hop AF relay network where the secondary nodes have to obey peak interference constraints, which limit the instantaneous interference to the primary system below a fixed threshold. The secondary links are subject to LoS fading, i.e., they are modeled as Rician [16], whereas the links to the primary receiver experience Rayleigh fading. A key for analyzing the AF scheme is the entanglement, in the stochastic characterization, of the non-central $\mathcal{F}$ -distribution [17] that arises by recasting the Rician/Rayleigh model by means of the Gaussian distribution with (non-)zero mean. Capitalizing on this mathematical insight, we provide a closed-form result for the outage behavior of our network, which subsumes the result of [9]. We analyze the impact of the Rician factor and discuss variations of our setup. The experimental results are congruent with our theoretical analysis, showing very good agreement with our obtained outage expressions.

The paper is organized as follows. In Section II we briefly overview the CR AF scheme whereas in Section III we characterize its outage performance and analyze some relevant aspects. Numerical simulations are provided in Section IV and, finally, we conclude this work in Section V.

II. SYSTEM MODEL

Consider the fundamental underlay CR relay network model, depicted in Figure 1. In this setup, a secondary source SU_S communicates individual information symbols, to its intended destination SU_D in two time-slots, via a half-duplex non-regenerative AF relay SU_R . The relay normalizes the received signal to unit power and retransmits the weighted signal without any further processing. The underlay paradigm dictates that communication of the secondary network obeys certain constraints, which are imposed such that a baseline

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The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement number 228044.

Fig. 1. The basic cognitive relay network.

primary QoS can be guaranteed. In the scenario under consideration, these constraints are realized by limiting the SU_S and the SU_R transmission powers such that the instantaneous peak interference, which is inflicted at the primary receiver PU_D , does not exceed a predefined threshold Q .

We assume that each of the SU_S - SU_R (SR) and SU_R - SU_D (RD) propagation environments contains a line-of-sight component, i.e., each link experiences Rician fading. The channels of the secondary system are modeled as $h_\nu \triangleq \sqrt{\frac{K_\nu}{K_\nu+1}} h_\nu^{\text{LoS}} + \sqrt{\frac{1}{K_\nu+1}} h_\nu^{\text{nLoS}}$ where $\nu \in \{sr, rd\}$ indicates the corresponding link, h_ν^{LoS} is a fixed quantity that determines the channel mean and satisfies $|h_\nu^{\text{LoS}}| = 1$, K_ν is the Rician factor and $h_\nu^{\text{nLoS}} \sim \mathcal{CN}(0, 1)$ is the non-LoS (nLoS) fading component. The external links SU_S - PU_D (SP) and SU_R - PU_D (RP) are subject to Rayleigh fading and the respective channels are modeled as $h_\mu \sim \mathcal{CN}(0, \sigma_\mu^2)$, where $\mu \in \{sp, rp\}$. We assume that the secondary nodes, source and relay, have perfect knowledge of their associated cross-link channel powers $|h_{sp}|$ and $|h_{rp}|$. Moreover, all stochastic processes in our setup are assumed to be independent.

Using the above definitions one can briefly describe the AF scheme as follows. In phase (slot) one, the relay receives the following signal from the source node

$$y_r = h_{sr} \sqrt{P_s} s + n_r, \quad (1)$$

where $n_r \sim \mathcal{CN}(0, \sigma_1^2)$ is additive white noise, s is the unit-power information symbol and $P_s \triangleq Q/|h_{sp}|^2$ is the source transmit power, obeying the interference constraint (IC). In phase two, the relay multiplies y_r with $G \triangleq (\sqrt{P_s}|h_{sr}|^2 + \sigma_1^2)^{-1}$ and sends the normalized version of its input, which yields the following signal y_d at node SU_D ,

$$y_d = h_{rd} \sqrt{P_r} G y_r + n_d, \quad (2)$$

where $n_d \sim \mathcal{CN}(0, \sigma_2^2)$ is additive white noise and $P_r \triangleq Q/|h_{rp}|^2$ is the relay transmit power, normalized such that the IC is satisfied. Expanding Eq. (2) in terms of Eq. (1) yields the end-to-end (ETE) propagation model.

III. PERFORMANCE ANALYSIS

A. Characterization of Outage Probability

An outage event occurs when the instantaneous signal-to-noise ratio (SNR), or equivalently the instantaneous mutual information, drops below the decoding threshold $x \triangleq (2^R - 1)$, where R is the transmission rate. Prior to characterizing the ETE SNR γ we need some definitions. Letting $\sigma_{sr}^2 \triangleq \frac{0.5}{K_{sr}+1}$, $\sigma_{rd}^2 \triangleq \frac{0.5}{K_{rd}+1}$ we can further define $c_1 \triangleq \frac{Q}{\sigma_1^2} \frac{\sigma_{sr}^2}{\sigma_{sp}^2}$, $c_2 \triangleq \frac{Q}{\sigma_2^2} \frac{\sigma_{rd}^2}{\sigma_{rp}^2}$ and $\gamma_{sr} \triangleq \frac{\sigma_{sp}^2 |h_{sr}|^2}{\sigma_{sr}^2 |h_{sp}|^2}$, $\gamma_{rd} \triangleq \frac{\sigma_{rp}^2 |h_{rd}|^2}{\sigma_{rd}^2 |h_{rp}|^2}$. Using these definitions we can express the SNR of the SR and RD links

as $\tilde{\gamma}_1 \triangleq c_1 \gamma_{sr}$ and $\tilde{\gamma}_2 \triangleq c_2 \gamma_{rd}$, respectively. Finally, plugging the latter quantities in the expression for the ETE SNR of a single AF relay system yields $\gamma = \frac{\tilde{\gamma}_1 \tilde{\gamma}_2}{\tilde{\gamma}_1 + \tilde{\gamma}_2 + 1}$ [7].

To obtain the stochastic characterization of γ one needs to integrate over several random variables. However this task can be significantly eased by exploiting the following key observation; capitalizing on the fact that all channels are independent we can readily establish that $\gamma_\nu \sim \mathcal{F}_{2,2}(\lambda_\nu, 0)$, i.e., both γ_{sr} and γ_{rd} follow a non-central $\mathcal{F}$ -distribution with non-centrality parameters $\lambda_{sr} = 2K_{sr}$ and $\lambda_{rd} = 2K_{rd}$, respectively. Using this insight we derive the outage probability for the CR relay system, as given by the following proposition.

Proposition 1. Let $\delta(t)$ be the Kronecker delta, i.e., $\delta(t) = 1$ for $t = 0$ and zero otherwise. We then define the quantities $\delta_{c_1} \triangleq \delta(c_1 - 1)$, $\delta_m \triangleq \delta(k - m - 1)$ and their complementary counterparts $\tilde{\delta}_{c_1} \triangleq 1 - \delta_{c_1}$, $\tilde{\delta}_m \triangleq 1 - \delta_m$. Using these definitions, the outage probability of the CR relay network with LoS fading for the secondary links and nLoS fading for the secondary-to-primary links is expressed by Eq. (3) (shown at the top of the next page).

Proof. Given in Appendix B. $\square$

To retrieve the ETE SNR density function one needs to differentiate Eq. (3) but the result is not presented herein due to space limitations. We note that Eq. (3) contains two infinite series, indexed by n and m , respectively. For practical implementation purposes, each of these summations has to be truncated to T_ν terms, where ν is defined as in Section II. To claim a closed-form solution, one needs to supplement Eq. (3) with convergence analysis of the truncation error, which inevitably arises in this case. Based on [13, Proposition 3] it can be established that by truncating each of the infinite series, involved in Eq. (3) to T_ν terms, such that $T_\nu > eK_\nu$, the truncation error will decay exponentially¹ in proportion to $(\frac{eK_\nu}{T_\nu})^{T_\nu}$. Thus, depending on the value of the channel Rician factor K_ν one can choose appropriately the associated index T_ν such that the resulting truncated series yields a value that is very close to the actual one.

B. Extensions and Special Cases

Here, we discuss briefly several aspects of the system under consideration and highlight some interesting extensions. For notational convenience we define the *nominal* signal-to-noise ratio (NSNR) of each hop as $\rho_1 \triangleq Q/\sigma_1^2$ and $\rho_2 \triangleq Q/\sigma_2^2$, for the SR and RD links, respectively.

1) *Performance at high-SNR:* To characterize the high-SNR performance we let, for simplicity, both links' NSNR scale in the same way, i.e., we assume that $\rho_1, \rho_2 \propto \rho$ for some ρ that is sufficiently large such that the following two conditions are satisfied: i) $\rho \gg \max\{K_{sr}, K_{rd}\} + 1$ (A1); and ii) also $\rho \gg x$ (A2). Based on these two assumptions we derive the following approximate expression $P_{\text{out}}^{\text{high}}(x)$ for the outage performance in the high-SNR regime,

$$P_{\text{out}}^{\text{high}}(x) \approx \sum_{l=1}^2 e^{-K_l} \frac{x}{c_l} + \frac{(x+1)x}{c_1 c_2} (e^{-K_{sr}} + e^{-K_{rd}}). \quad (4)$$

¹In fact the decay rate can be larger, since [13, Proposition 3] establishes an upper-bound on the truncation error.

$$\begin{aligned}
P_{\text{out}}(x) = & e^{-(K_{sr}+K_{rd})} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{K_{sr}^n}{n!} \frac{K_{rd}^m}{m!} \left\{ \left(\frac{x}{x+c_2} \right)^{m+1} + \delta_{c_1}(m+1) \left[\frac{x(c_2-1)}{c_2(x+1)} \right]^{n+1} \sum_{k=0}^{n+1} \binom{n+1}{k} \frac{1}{(c_2-1)^k} \right. \\
& \left[\delta_m \ln \left(\frac{c_2}{x} \right) + \tilde{\delta}_m \frac{1}{[m-k+1]} \left[1 - \left(\frac{x}{x+c_2} \right)^{m-k+1} \right] \right] + \tilde{\delta}_{c_1} \frac{(m+1)}{(1-c_1)^{n+1}} \left[\frac{1}{m+1} \left[1 - \left(\frac{x}{x+c_2} \right)^{m+1} \right] \right] + \\
& \sum_{k=1}^{n+1} \binom{n+1}{k} \left((-c_1)^k \left(\frac{x}{x+c_2} \right)^{m+1} \sum_{l=0}^{k-2} \frac{\Gamma(k-1-l)}{\Gamma(k)} \frac{\Gamma(m+2+l)}{\Gamma(m+2)} \left[\frac{c_2(x+1)}{(c_1-1)(x+c_2)} \right]^{l+1} + \right. \\
& \left. \left[-\frac{c_1(x+1)}{x+c_1} \right]^k \sum_{l=0}^m \frac{\Gamma(k+l)}{\Gamma(k)} \frac{\Gamma(m+1-l)}{\Gamma(m+2)} \left[\frac{(c_1-1)x}{c_2(x+c_1)} \right]^l + \frac{(-1)^k \Gamma(m+k+1)}{\Gamma(k)\Gamma(m+2)} \frac{[c_1 c_2(x+1)]^k [(c_1-1)x]^{m+1}}{[(c_1+c_2)x+c_1 c_2-x]^{m+k+1}} \right. \\
& \left. \left. \left[\ln \left[\frac{(x+c_1)(x+c_2)}{x(x+1)} \right] + \sum_{l=1}^{m+k} \binom{m+k}{l} \frac{(-1)^l}{l} \left[2 - \left(-\frac{(c_1-1)x}{c_2(x+c_1)} \right)^l - \left(-\frac{c_2(x+1)}{(c_1-1)(x+c_2)} \right)^l \right] \right] \right] \right\} \quad (3)
\end{aligned}$$

Proof. The assumptions (A1) and (A2) are needed to guarantee that $\min\{c_1, c_2\} \gg \max\{x, 1\}$. We exploit the latter condition to approximate each summation term, associated with each pair (m, n) , with its dominant arguments. To conclude the proof, we observe that the dominant summation term corresponds to the pair $(n, m) = (0, 0)$. $\square$

Recalling that c_1 and c_2 are proportional to ρ , we infer from Eq. (4) that the outage probability is inversely proportional to ρ , hence the diversity order is one, as expected. It is important to note here that the “high-SNR” regime depends on the value of the Rician factor. As K_ν increases the nominal SNR of the associated link should increase accordingly to ensure that condition (A1) is satisfied. Thus, the outage probability is influenced by two components and might exhibit heterogeneous performance patterns. Moreover, if the secondary links are asymmetric, e.g., if conditions (A1), (A2) are satisfied only for the SR hop then the RD link becomes the bottleneck and its performance will be the limiting factor.

2) *Lifting channel model statistics:* Although in this work we investigate the basic scenario, which corresponds to Rician fading, one can plug more general models in our system and carry out similar analysis. To see this intuitively, let us generalize the Rayleigh distribution to a central chi-square $\chi_{2M_\mu}^2(0)$ (for the nLoS case) and the Rician distribution to a non-central chi-square $\chi_{2N_\nu}^2(\lambda')$ (for the LoS case), where M_μ, N_ν are integers, both greater than one and the subscripts μ and ν are defined as in Section II. Under these premises the induced SNR random variables will be non-central $\mathcal{F}$ -distributed of higher order, i.e., $\gamma_\nu \sim \mathcal{F}_{2N_\nu, 2M_\mu}(2N_\nu K_\nu, 0)$. To lift the result of Proposition 1 in higher order (M_μ, N_ν) one can follow a line-of-proof similar to that in Appendix B, but with more involved algebraic steps. Moreover, the truncation indices would now be chosen to satisfy $T_\nu > eN_\nu K_\nu$ to guarantee that the truncation error decays exponentially.

3) *Impact of Rician factor:* By definition, the Rician factor determines the contribution of the specular component relatively to its shadowed counterpart. The higher K_ν gets the less “random” the channel becomes, i.e., the density gets more concentrated around its mean, and vice versa. For single-hop networks it is known that sufficiently large K_ν , i.e., almost no fading, corresponds to the most favourable scenario, in terms of outage. We extrapolate these insights and claim that

our CR network will benefit from higher K_ν . To give some mathematical intuition for the induced outage gain, we note that the infinite series, which is involved in Eq. (B1) and is indexed by n is decreasing in K_{sr} . To be more precise, it can be shown that the following series

$$\begin{aligned}
S_n(K_{sr}) = & e^{-K_{sr}} \sum_{n=0}^{\infty} \frac{K_{sr}^n}{n!} \left[\frac{(z+1)x}{x(1-c_1) + (c_1+x)z} \right]^{n+1} \\
= & e^{-\frac{(z-x)c_1 K_{sr}}{(z+1)x + (z-x)c_1}} \left[\frac{(z+1)x}{(z+1)x + (z-x)c_1} \right]
\end{aligned}$$

is strictly monotone decreasing in K_{sr} , for any $z > x$. Note that $S_n(K_{sr})$ is present in Eq. (B1), *prior to the integration step*, yet its monotonicity properties are not affected since integration is with respect to $z \in [x, \infty)$. Due to the symmetry of γ , relative to γ_{sr} and γ_{rd} , we can claim the same monotonicity properties with respect to K_{rd} . Thus, the stronger the LoS paths get the more reliable the system will be.

4) *Special cases:* From our setup and its generalized counterpart, described in Section III-B2, we can extract several special cases. When the LoS components are extremely weak, i.e., when K_{sr} and K_{rd} attain sufficiently small values, then we obtain the following result.

Corollary 1. *Let the Rician factors K_{sr}, K_{rd} , be sufficiently small. Then the outage probability $P_{\text{out}}^{\text{weak}}(x)$ of the system, described in Section II, is given by the formula shown at the top of the next page.*

Proof. For sufficiently small values of K_{sr} and K_{rd} the terms that correspond to the indices $n = m = 0$ will dominate the double summation of Eq. (3). In fact we can show that the approximation error of Eq. (B1),² induced by neglecting the terms for $n \geq 1$ and $m \geq 1$, is upper-bounded by the product $K_{sr}K_{rd}$ which is negligible according to our assumptions. Similarly, we can show that $F_{\gamma_{rd}}(\frac{x}{c_2}) \approx (1 - K_{rd})\frac{x}{x+c_2}$. The result then follows by retaining the relevant terms from Eq. (3) and carrying out straightforward algebraic steps.³ $\square$

When both links, SR and RD, are shadowed, i.e., when

²We hasten to add here that the term $(m+1)I_0$, which appears in Eq. (B1), is smaller than one. This is exploited to upper-bound the approximation error $I_a \triangleq \sum_{n \geq 1} \sum_{m \geq 1} (K_{sr}^n/n!)(K_{rd}^m/m!)(m+1)I_0^{(n,m)}$.

³The simplified expressions that correspond to δ_{c_1} and its complementary event $\tilde{\delta}_{c_1}$ are unified in this case.

$$P_{\text{out}}^{\text{weak}}(x) = \frac{(1 - K_{rd})x}{c_2 + x} - (1 - K_{sr} - K_{rd}) \left\{ \frac{(1 - c_2)x}{[(c_1 + c_2)x + c_1c_2 - x]} \frac{c_2}{(c_2 + x)} + \frac{c_1c_2x(x+1)}{[(c_1 + c_2)x + c_1c_2 - x]^2} \ln \left[\frac{x(x+1)}{(c_1 + x)(c_2 + x)} \right] \right\}$$

$(K_{sr}, K_{rd}) = (0, 0)$, then from the expression for $P_{\text{out}}^{\text{weak}}$ we retrieve [9, Eq. (5)] that corresponds to pure Rayleigh fading. For the generalized model, described in Section III-B3, zeroforcing the Rician factors yields the equivalent Nakagami- m fading models for integer values of the parameter m .

5) *Multiple relays*: In the presence of multiple relays, the secondary system can improve its outage performance by employing a proper cooperative scheme. A solution that was devised for conventional non-CR networks, involving minimum resource allocation without compromising the maximum diversity order, is the opportunistic relay selection [19] where only the relay with the strongest ETE SNR is activated. This strategy can be intuitively extended into the CR relay network, though the outage analysis is more involved, due to the coupling of the different ETE SNR variables. However, based on results, which are tailored to simpler channel models [10], we conjecture that the system will still attain full diversity order, equal to the number of relays, in our scenario.

IV. NUMERICAL RESULTS

In this section we investigate the impact of various system parameters into the outage performance. In all scenarios the channel mean is fixed to $h_{\nu}^{\text{LoS}} = (\sqrt{2}/2)(1 + i)$ and the results are plotted against the RD link nominal SNR ρ_2 . The values of the truncation indices T_{sr} and T_{rd} , given in Table I, were chosen judiciously, based on the Rician factor, to serve a dual purpose; i) guarantee good approximation of the infinite series; and ii) avoid excessive running time.

Figure 2 illustrates the outage performance for the case where both hops have identical statistical properties and equal nominal SNR values. We observe that when the specular component is weak, i.e., when K_{ν} is in the range of $[-5\text{dB}, 0\text{dB}]$, then the system experiences only marginal gain with increasing K_{ν} . However, in the region where the LoS component dominates, i.e., $K_{\nu} > 0\text{dB}$, there is drastic improvement. In particular, the system greatly benefits when the Rician factor increases from 5 to 10dB, e.g., the NSNR gain to achieve an outage probability 10^{-2} is approximately 6dB. In all cases, we observe perfect match of the curves generated using Eq. (3) and those generated through numerical simulations.

It follows from Figure 2 that Eq. (4) is a good proxy for high-SNR outage performance since it approximates well the generic formula (3), especially for low values of the parameter K_{ν} . This result supports our statement that the diversity order is one. In line with the intuitive arguments of Section III-B1, we observe that the effective high-SNR regime, where performance is characterized by the NSNR exponent, is shifted depending on the value of K_{ν} . It is noteworthy that

when $K_{\nu} = 10\text{dB}$ there exists an intermediate NSNR regime ([6dB, 18dB]) where the outage probability decays faster than with the inverse NSNR. This implies that for moderate NSNR values it is K_{ν} that mainly drives the outage performance and the system greatly benefits from large Rician factors.

In Figure 3 we show the impact of the NSNR asymmetry between the two hops, through two different sets of curves. The first set corresponds to $\rho_1 = 0\text{dB}$, i.e., the SR link is the bottleneck while the second set corresponds to $\rho_1 = 20\text{dB}$, hence in this case the RD link is the bottleneck. It is seen that the system exhibits an outage floor when the first hop is the limiting factor. The outage performance is very poor and only infinitesimal improvement can be achieved with increasing K_{sr} and K_{sd} . On the contrary, when $\rho_1 > \rho_2$ the system does not suffer any degradation within the regime [0dB, 20dB] and fully harvests the gains stemming from higher K_{ν} .

Figure 4 illustrates the outage performance for equal NSNRs and dissimilar (K_{sr}, K_{rd}) pairs. One artefact is that the CR system does not experience an outage threshold with increasing NSNR, as in Figure 3, and is able to attain always full diversity. This is not surprising, since the specular components only affect the dispersion of the links' SNR around its mean, yet this dispersion is not influential when low probability events are considered. We further observe that the system does not benefit from a high K_{sr} except for the case when both Rician factors K_{sr} and K_{rd} are high, i.e., when $K_{sr} = K_{rd} = 10\text{dB}$. For any other pair (K_{sr}, K_{rd}) we see that imbalance between the links' LoS strength biases performance strongly towards the smaller K_{ν} . Individual increments of either K_{sr} or K_{rd} induce marginal performance enhancement in the "weak LoS" region, where $\min\{K_{sr}, K_{rd}\} \leq 0\text{dB}$, but offer significant benefits in the "strong LoS" region.

V. CONCLUSIONS AND DISCUSSION

In this paper we investigated a dual-hop CR system with a single relay that employs AF. Our scenario is relevant to the case of small-scale architectures, such as femtocells, where it is likely that links will contain a LoS path between the end nodes. For this setup we derived the outage probability and assessed numerically the impact of different network aspects in the overall performance. The simulated results were in perfect agreement with our theoretical derivations.

Our investigation revealed a manifold interplay between the nominal SNR of the individual links, their associated Rician factor and the outage probability. We highlight the substantial performance improvement, exhibited by the CR network, when the LoS component dominates. Thus, we conclude that the outage behavior is enhanced compared to shadowed environments. The current setup can be readily modified to account for the case where the SP and RP links operate in LoS environments. Derivation of the exact probability-of-error may be involved but one can exploit tight bounds. An interesting extension involves the integration of transmit power constraints in the model to address more practical designs.

$K_{\nu}(\text{dB})$	-10	-5	0	5	10	15
T_{ν}	10	10	10	25	50	100

TABLE I
CHOICE OF THE TRUNCATION INDEX FOR VARIOUS VALUES OF THE
RICIAN FACTORS.

Fig. 2. Comparison of numerical and theoretical outage probability for transmission rate $R = 1\text{ bpcu}$ ($x = 1$). The (SR,SP) and (RD,RP) pairs have identical statistical properties, i.e., $\rho_1 = \rho_2, c_1 = c_2$ and $K_{sr} = K_{rd}$ (shown as LoS). The thick dotted curves correspond to high-SNR (Eq. (4)).

Fig. 3. Comparison of numerical and theoretical outage probability for fixed ρ_1 , rate $R = 1\text{ bpcu}$ ($x = 1$) and $K_{sr} = K_{rd}$ (shown as LoS).

APPENDIX

A. Some Identities on the Gauss Hypergeometric Function

In this section we provide some results pertaining to the Gauss hypergeometric function ${}_2F_1(a, b; c|x)$ [20, Eq. (15.1.1)], for the special case when the pochhammer arguments a, c are positive integers and b is a non-negative integer. These results will be used to simplify the ${}_2F_1(\cdot)$ function, wherever it arises in the subsequent analysis. The first result follows by setting $b = 0$ in [20, Eq. (6.6.8)]

$$F(1, c; c+1|x) = cz^{-c}B(c; 0|x), \quad (\text{A1})$$

Fig. 4. Comparison of numerical and theoretical outage probability for different (K_{sr}, K_{rd}) pairs, $\rho_1 = \rho_2$ and rate $R = 1\text{ bpcu}$ ($x = 1$). The dotted curves correspond to theoretical results generated according to Eq. (3).

where $B(c; 0|z)$ is the incomplete Beta function [21, Eq. (8.391)]. We use Eq. (A1) to obtain the following representation,

$$F(1, c; c+1|x) = \frac{c}{x^c} \left\{ \sum_{l=1}^{c-1} \binom{c-1}{l} \frac{(-1)^l}{l} [1 - (1-x)^l] - \ln(1-x) \right\}. \quad (\text{A2})$$

We further use the following two identities,

$${}_2F_1(a, 1; a+b+1|z) = \frac{(x-1)^b}{(a+b)} \frac{{}_2F_1(1, a+b; a+b+1|x)}{B(a, b+1)} + \sum_{l=0}^{b-1} \frac{B(a+l, b-l)}{B(a, b+1)} \Gamma(b+1)(x-1)^l \quad (\text{A3})$$

$${}_2F_1(a, a+b; a+b+1|z) = \frac{{}_2F_1(1, a+b; a+b+1|z)}{(-1)^{1-a}B(a, b+1)(a+b)} + \frac{1}{(1-x)^{a-1}} \sum_{l=0}^{a-2} \frac{B(a-1-l, b+1+l)}{B(a, b+1)} (x-1)^l. \quad (\text{A4})$$

These identities are proven via recursive application of Eqs. (15.2.15) and (15.2.18) from [20], which relate ${}_2F_1(a, b; c|z)$ to its contiguous functions. Eqs. (A3) and (A4) can be further simplified by plugging (A2) into the right-hand-side (RHS) of their respective expressions.

B. Proof of Proposition 1

Let $F_{\gamma_\nu}(x)$ denote the distribution function of the $\mathcal{F}$ -distributed random variable γ_ν . Starting from the definition of γ (see Section III-A) one can determine its distribution function $F_\gamma(x)$ as follows [18, Eq. (9)]

$$F_\gamma(x) = \int_0^x \Pr\left[\bar{\gamma}_1 \geq \frac{x(z+1)}{z-x} \middle| z\right] f_{\bar{\gamma}_1}(z) dz$$

$$\begin{aligned}
I_0^{(n,m)} &= \frac{1}{(1-c_1)^{n+1}} \sum_{k=0}^{n+1} \binom{n+1}{k} \left[\frac{c_1 c_2}{x} \frac{(x+1)}{(c_1-1)} \right]^k \int_{\frac{x}{x+c_2}}^1 \frac{t^{k+m}}{\{1 - [(c_1 x + c_2 x + c_1 c_2 - x)/(c_1 - 1)x]t\}^k} dt \\
&= \frac{1}{(1-c_1)^{n+1}} \sum_{k=0}^{n+1} \binom{n+1}{k} \left[\frac{c_1 c_2}{x} \frac{(x+1)}{(c_1-1)} \right]^k \frac{1}{m+k+1} \left\{ \left[\frac{-(c_1-1)x}{c_2(x+c_1)} \right]^k {}_2F_1 \left[k, 1; k+m+2 \middle| \frac{(c_1+c_2)x + c_1 c_2 - x}{c_2(x+c_1)} \right] \right. \\
&\quad \left. - \left[\frac{x}{x+c_2} \right]^{k+m+1} {}_2F_1 \left[k, k+m+1; k+m+2 \middle| \frac{(c_1+c_2)x + c_1 c_2 - x}{(c_1-1)(x+c_2)} \right] \right\} \quad (B2)
\end{aligned}$$

$$+ \int_x^\infty \Pr \left[\tilde{\gamma}_1 \leq \frac{x(z+1)}{z-x} \middle| z \right] f_{\tilde{\gamma}_2}(z) z = F_{\gamma_{rd}} \left(\frac{x}{c_2} \right) + I,$$

where the second summand is represented as

$$I = e^{-(K_{sr}+K_{rd})} \sum_{n=0}^\infty \sum_{m=0}^\infty \frac{K_{sr}^n}{n!} \frac{K_{rd}^m}{m!} (m+1) I_0^{(n,m)}. \quad (B1)$$

The term I_0 corresponds to the integral shown below,

$$\begin{aligned}
I_0^{(n,m)} &= \int_x^\infty \left[\frac{(z+1)x}{x(1-c_1) + (c_1+x)z} \right]^{n+1} \frac{c_2 z^m}{(c_2+z)^{m+2}} dz \\
&= \int_{\tilde{x}}^1 \left[\frac{x + (c_2-1)xt}{x(1-c_1) + (c_1x + c_2x + c_1c_2 - x)t} \right]^{n+1} t^m dt,
\end{aligned}$$

where $\tilde{x} \triangleq \frac{x}{x+c_2}$ and the second equality follows by the change of variables $t = z/(z+c_2)$ and some algebraic steps. To proceed with a simplified solution of the abovementioned integral we shall distinguish two cases:

i) $c_1 = 1$: In that case the integral is evaluated as

$$\begin{aligned}
I_0^{(n,m)} &= \left[\frac{x(c_2-1)}{c_2(x+1)} \right]^{n+1} \sum_{k=0}^{n+1} \binom{n+1}{k} \frac{1}{(c_2-1)^k} \\
&\quad \left\{ \delta_m \ln \left(1 + \frac{c_2}{x} \right) + \tilde{\delta}_m \frac{1}{m-k+1} \left[1 - \left(\frac{x}{x+c_2} \right)^{m-k+1} \right] \right\},
\end{aligned}$$

where the coefficients δ_m and $\tilde{\delta}_m$ are defined in the proposition statement.

ii) $c_1 \neq 1$: In this case, we manipulate the initial integral I_0 and transform it as follows,

$$I_0 = \frac{1}{(1-c_1)^{n+1}} \int_{\tilde{x}}^1 \left[1 + \frac{\frac{c_1 c_2}{x} \frac{(x+1)}{(c_1-1)} t}{1 - \frac{(c_1 x + c_2 x + c_1 c_2 - x)}{(c_1-1)x} t} \right]^{n+1} t^m dt.$$

To obtain the solution we expand the term inside the integral by means of the binomial summation formula and then utilize the integral representation [20, Eq. (15.3.1)] of the ${}_2F_1(\cdot)$ function and its linear transformation [20, Eq. (15.3.1)] to obtain the end result in Eq. (B2). The Gauss Hypergeometric functions that appear in Eq. (B2) can then be represented in terms of more elementary function by means of Eqs. (A3),(A4). Finally, to obtain Eq. (3) we unify the above cases using δ_{c_1} , $\tilde{\delta}_{c_1}$ and merge the two terms, which are present in the expression for $F_\gamma(x)$.

We note here that, although the Gauss Hypergeometric function ${}_2F_1(\cdot)$ is accepted as a closed-form solution for the associated integral, the value of obtaining simpler representations is demonstrated in the implementation stage. The underlying reason is that evaluation of the elementary functions, which appear in Eq. (3), leads to more accurate and numerically stable results than evaluation of Eq. (B2), which might result in undesired imaginary residuals.

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