Use of Experiments, Computations and Models for HCF Design

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Abstract

In order to perform accurate HCF predictions, certain requirements must be fulfilled. Examples of such requirements are that the HCF model used describes the fatigue phenomenon in an accurate way, that the material parameters in the model are determined with certain accuracy and that the FE mesh used for the HCF prediction is fine enough.

Today, in the gas turbine industry, HCF design is typically performed by use of a deterministic point stress method in conjunction with a stress invariant based HCF model. This design method does not reflect the non-local stochastic fatigue behavior of the material and often leads to over-conservative design, which has a negative effect on the performance of the machine. There is a need for models with improved prediction accuracy.

In Paper A, it is investigated which mesh properties that are important in order to reach convergence in the fatigue prediction for a compressor blade when the volume based Weibull model is used. This is done by subjecting a blade to a steady load due to rotation, and an amplitude load applied as a uniform pressure to the blade’s pressure side, and then change the mesh properties and measure the change in computed fatigue risk. The investigation shows that the number of elements through the blade’s thickness is the most important mesh property.

In Paper B, it is investigated which test strategy that should be used in order to perform accurate estimations of the material parameters in multiaxial HCF models by use of as few tests as possible when different types of scatter is present, and when the cost to perform fatigue tests is included. It is shown that performing tests on few stress ratios located far away from each other is the best strategy.

In Paper C, the prediction accuracy for the volume based Weibull- and $V^*$ models, as well as for the point stress method, is investigated. The models are fitted to experimental results obtained from rotating bending tests performed for single notched cylindrical 12 % Cr-steel specimens of different sizes. The tests are designed such that at a certain stress level the stress on a cross section of a specimen has the same shape for all specimen sizes, but different gradients. Therefore, the highly loaded volume differs between the specimens. It is seen that the smallest total error in fatigue prediction is obtained for the $V^*$-model, and that the $V^*$-model provides the best fatigue prediction at low stress levels.
**Sammanfattning**

För att kunna genomföra noggranna HCF analyser måste ett antal olika förutsättningar vara uppfyllda. Exempel på detta är att den HCF modell som används beskriver utmattningsfenomenet på ett bra sätt, att materialparameterarna i modellen bestäms med en viss noggrannhet samt att det FE nät som används för HCF beräkningen är tillräckligt fint.


I **Artikel B** genomförs en undersökning av hur utmattningsprov skall genomföras på bästa sätt för att kunna bestämma materialparametrar i fleraaxliga utmattningsmodeller med så få prov som möjligt. Olika typer av spridning påverkar provresultaten och kostnaden för att utföra proven inkluderas. Undersökningen visar att prov bör genomföras på få spänningskvoter som är lokaliserade långt ifrån varandra.

Preface

The work in this thesis has been performed at the Department of Solid Mechanics, KTH. The financial support from the Swedish Energy Agency Authority through the TurboPower initiative is gratefully acknowledged. The cooperation with GKN Aerospace and Siemens Industrial Turbomachinery AB is also acknowledged.

First of all I would like to express my deepest gratitude to my supervisor Professor Mårten Olsson for his encouragement, excellent guidance and genuine support.

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All specimens used in the experiments have been manufactured by Messrs. Yngve Lindvall, Kurt Lindqvist and Göran Rådberg. The experimental realization of this work has been assisted by M.Sc.s. Martin Öberg, Hans Öberg and Veronica Wåtz.

Finally, I would like to thank my friends and my family for their support.

Stockholm, January 2015
Daniel Sandberg
List of appended papers

**Paper A**  
*FE-mesh effect of the volume based weakest-link fatigue probability applied to a compressor blade*  
Salar Sadek, Daniel Sandberg and Mårten Olsson  
*Proceedings of ASME Turbo Expo 2012, ASME Paper GT2012-69852*

**Paper B**  
*On the optimal choice of experiments for parameter determination in multiaxial HCF-criteria*  
Daniel Sandberg and Mårten Olsson  
*International Journal of Fatigue 61 (2014) 315-324*

**Paper C**  
*An investigation of the prediction accuracy for volume based HCF models using scaled geometries and scaled loading*  
Daniel Sandberg and Mårten Olsson  
*Internal report No. 568, Department of Solid Mechanics, KTH Royal Institute of Technology, Stockholm*
In addition to the appended papers, the work has resulted in the following TurboPower Deliverable Reports and presentations:

**Development and validation of numerical tools for prediction of HCF**
Daniel Sandberg
Presented at *TurboPower Program Conference 2011*, Chalmers, Gothenburg, Sweden

**Att bestämma HCF-parametrar med få experiment**
Daniel Sandberg
Presented at *Svenska mekanikdagar 2011*, Chalmers, Gothenburg, Sweden

**Numerical model – The probabilistic HCF post-processor AROMA-PF**
Daniel Sandberg and Salar Sadek

**Test data – Statistical evaluation of the test results obtained from uniaxial fatigue tests performed for 12 % Cr-steel and Titanium Ti-6-4**
Salar Sadek and Daniel Sandberg

**Development and validation of a numerical tool for prediction of HCF**
Daniel Sandberg and Salar Sadek
Presented at *UTMIS network meeting 2012*, SP, Borås, Sweden

**Validation – Prediction accuracy for local multiaxial HCF models and validation of the FE mesh requirements for a compressor blade**
Salar Sadek and Daniel Sandberg

**Input of respective work task to design tool and final synthesis report**
Salar Sadek and Daniel Sandberg

**Improved methods for more accurate HCF assessment**
Daniel Sandberg
Presented at *TurboPower Program Conference 2013*, LTH, Lund, Sweden

**Tuned and ranked fatigue models – The prediction accuracy for the family of VPF models compared to the Weibull model**
Salar Sadek and Daniel Sandberg

**An investigation of transferability of the weakest-link volume model using scaled geometries**
Daniel Sandberg and Mårten Olsson
Presented at *International Conference on Fatigue Damage of Structural Materials X 2014* (poster), Hyannis, USA

**Fatigue test data – Statistical evaluation of test results obtained from rotating bending tests performed for 12 % Cr-steel**
Salar Sadek and Daniel Sandberg

**An investigation of the prediction accuracy for volume based HCF models using scaled geometries and scaled loading**
Daniel Sandberg and Mårten Olsson
Presented at *UTMIS network meeting 2015*, ABB, Karlskrona, Sweden
Contribution to the papers

**Paper A**
Secondary author, performed some of the computations and part of the writing. Salar Sadek was the main author. Interpretation of the results together with Salar Sadek and Mårten Olsson.

**Paper B**
Principal author, performed all simulations and experiments. Interpretation of the results together with Mårten Olsson.

**Paper C**
Principal author, performed all the experimental work and computations. Interpretation of the results together with Mårten Olsson.
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Paper A

Paper B

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1. Introduction

HCF failure of turbomachinery components is very serious and costly, and may lead to substantial damage, injury and even death. This is true mainly in operating machines, but also during testing of new engine designs. Failure in turbomachinery in aero planes may lead to great harm. Failure in stationary turbomachinery, for production of electricity, leads to large societal losses. Therefore, there is a need for models and methods that can be used for accurate HCF-predictions already at an early design stage.

A current development is to consider the variation of the design set-up. The purpose of this is to estimate the probability of failure. If it is possible to model the probability of failure in an appropriate way, the sources of failure risk can be identified and managed. This is of most importance in order to reach different goals, such as high efficiency, low weight, low total cost, etc.

One example of gas turbine components subjected to HCF is compressor- and turbine blades, see Figure 1. The blades are subjected to vibrations due to the periodic excitation from the surrounding flow. The critical situations from a fatigue point of view occur when the frequency of the excitation forces coincides with an eigenfrequency for the structure.

![Figure 1. Overview of the gas turbine Siemens SGT-700, taken from [1].](image)

HCF design of Turbomachinery compressor- and turbine blades includes several technical areas, such as aero-forcing, aero-damping, frictional damping and material fatigue, see Figure 2. From the aero-elastic and structural damping computations, the stress history that acts in the blades is obtained. The obtained stress history is then used in a HCF-model and the fatigue risk of the component is computed.
1.1 The COMP project

The work presented in this thesis is conducted within the COMP project, which is part of the Turbo Power initiative [1]. The COMP project is performed in close cooperation between Swedish gas turbine industry and several Departments at KTH. The overall goal in COMP is to develop and validate a computational tool that covers all the technical disciplines involved in the design chain, see Figure 2, and that can be used to perform HCF prediction for components subjected to aerodynamically induced vibrations. Important factors for the methods developed in each technical discipline are of course the prediction accuracy, but also the computational time.

The COMP project is divided into four work packages (WPs), with the aim that each work package is specialized in a particular technical discipline. The work packages are: WP1 Synthesis, WP2 Aero-forcing and aero-damping, WP3 Structural damping and WP4 Material fatigue. The present work is performed within WP4 Material fatigue.

1.2 State-of-the-art in HCF design

In the gas turbine industry, traditional HCF design is performed by use of the deterministic point stress approach. A multiaxial HCF model is used to transform the multiaxial stress history $S(x, t)$ into a fatigue effective scalar stress field, $\sigma_{\text{eff}}(x)$. The
effective stress field $\sigma_{\text{eff}}(x)$ describes the severity of the loading from a fatigue point of view at each point $x$ in the component. Usually, due to its short computational time and simplicity, a stress invariant based Sines-similar multiaxial HCF model [3] is used to compute $\sigma_{\text{eff}}(x)$. Fatigue failure for the component is predicted if

$$\sigma_{\text{eff}}(x) = \eta \sigma_{\text{VM,amp}}(x) + \alpha \sigma_{\text{h,mean}}(x) \geq \sigma_{\text{eff},c},$$

at any point $x$ in the component. The stress measure $\sigma_{\text{VM,amp}}$ is the amplitude of the von Mises stress that acts at point $x$ during a load cycle, $\alpha$ is a material parameter that controls the influence of the mean hydrostatic stress $\sigma_{\text{h,mean}}$ on the effective stress $\sigma_{\text{eff}}$. The parameter $\sigma_{\text{eff},c}$ is a critical effective stress (the fatigue limit) and $\eta$ is a safety factor used to account for different type of uncertainties. The material parameters $\alpha$ and $\sigma_{\text{eff},c}$ are determined from fatigue experiments. When the point stress method is used, the fatigue risk for the component is predicted by only the effective stress that acts at the maximum stressed point. In the deterministic point stress approach, a component is considered safe from fatigue if the maximum stressed point is stressed below $\sigma_{\text{eff},c}$ for the chosen value of $\eta$, see Figure 3.

**Figure 3.** The traditional point stress deterministic HCF design method is used in gas turbine industry. The component is considered as safe if all material points are stressed below $\sigma_{\text{eff},c}$ for the chosen value of $\eta$. In the Figure, $\eta = 2$.

Transferability of HCF models from one geometry to another is crucial. One example of this is the transferability from specimen to component. It has been shown that the transferability for point stress method is very limited [4-6] which results in poor fatigue predictions. To compensate for the lack of a good description of the fatigue phenomenon that results from the use of a deterministic point stress method, unnecessarily high safety factors are used in the fatigue design. This leads to overly conservative design which has a negative effect on the efficiency of the turbine. Use of a fatigue model that provides an improved physical description of the fatigue phenomenon will enable more appropriate blade designs. This leads to reduced safety margins, without increasing the HCF risk, which will have a positive
effect on the turbine’s efficiency. Therefore, there is a need for develop and evaluate other types of HCF-models, that gives an improved physical description of the fatigue phenomenon, and that can be used in turbomachinery HCF design instead of the traditional deterministic point stress method.

1.3 Requirements for good HCF design

It is well-known that not only the stress in the maximum stressed point controls a component’s risk of fatigue failure. Actually, fatigue cracks that lead to final failure may initiate at any point stressed above a certain threshold stress $\sigma_{th}$, i.e. all points stressed above $\sigma_{th}$ contribute to the component’s total failure probability. Generally, the larger the highly stressed region is, the higher the failure probability becomes. Especially for compressor blades, that may be subjected to large regions with moderate peak stresses, see Figure 3 and 4, this volume effect is important to consider in HCF design. Also, the stress gradient is an important factor for the fatigue risk.

![Figure 4. Two examples of the effective stress field that acts in the compressor blades for two different vibrational modal shapes. Fatigue cracks may initiate at any point stressed above a certain threshold stress $\sigma_{th}$.](image)

This implies that in order to reach increased accuracy in the fatigue prediction,

In order to perform a good HCF prediction, several requirements must be fulfilled, e.g.:

- It is necessary to have a good description of the load history that the component is subjected to and of the component geometry. Both load history and component geometry typically include uncertainties.
- The HCF model must account for stress multiaxiality and mean stress effects.
To account for the volume- and gradient effect, the HCF model must somehow be based on the effective stress field’s distribution within the component, i.e. the model must not only be based on the stress that acts at the maximum stressed point.

To account for the variability in material properties, a HCF model based on a probabilistic approach must be used. HCF design based on a probabilistic approach is a completely new way of thinking compared to the traditional deterministic design. Instead of applying a safety factor, the variation in fatigue strength is included in the model and the component is directly designed against a sufficiently low failure probability, e.g. $p_f = 10^{-4}$.

The HCF models include material parameters that must be estimated from fatigue tests. Due to scatter in test results, an increased number of tests gives improved parameter estimation. However, since performing fatigue tests is expensive and time-consuming, it is of interest to perform the type of tests that contribute with much information from as few tests as possible, i.e. a good test strategy is required.

The FE model used for stress computation should describe the physical problem in a proper way and the finite element mesh must be fine enough.

The computational applicability for the HCF model must be good. The model should be straightforward to use in conjunction with FEM and the computational time cannot be too long.

In this thesis, the focus is on investigation and evaluation of:

- The important mesh properties in the FE model in order to reach convergence in the fatigue probability computation for a compressor blade [10].
- Prediction accuracy for probabilistic HCF models [12].

2. Experimental work

To enable HCF prediction and HCF model validation, the relevant material parameters used in the HCF models must first be estimated from experiments. In this work, the HCF properties for 12 % Cr-steel and the Titanium alloy Ti-6-4, two typical gas turbine materials, have been determined. In total five different types of fatigue tests have been performed: Fatigue tests in uniaxial loading at the stress ratios -1, 0.1, 0.55 and 0.65, and fatigue tests in rotating bending. Both smooth and notched specimen geometries have been used, see Table 1 and Figure 5.
Table 1. Fatigue tests performed in the COMP project (X: performed, -: not performed).

<table>
<thead>
<tr>
<th>Type of test</th>
<th>Stress ratio</th>
<th>Smooth specimen geometry</th>
<th>Notched specimen geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cr-steel</td>
<td>Ti-6-4</td>
</tr>
<tr>
<td>Uniaxial tension</td>
<td>-1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>(-compression)</td>
<td>0.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.65</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Rotating bending</td>
<td>-1</td>
<td>-</td>
<td>X</td>
</tr>
</tbody>
</table>

Figure 5. The experimental setup in a rotating bending test performed for a single notched 12 % Cr-steel specimen.

At each stress level, in order to estimate the failure probability $p_f$, several tests have been conducted. In order to obtain a good picture of the HCF properties for both materials, a large number of HCF tests (in total about 250 tests) have been performed.

3. Probabilistic HCF models

The most well-known probabilistic HCF model is the weakest-link volume model developed by Weibull [7]. In a weakest-link model, a component is divided into a number of sub-volumes, and the failure probability for each sub-volume is computed based on the stress that acts in that sub-volume. The term weakest-link stems from the assumption that as soon as one sub-volume in the component fails, the whole
component fails. The failure probability for a component is obtained from an integration of the effective stress field over the component volume $V$,

$$p_f = 1 - \exp\left[-\frac{1}{V_0} \int_V \left(\frac{(\sigma_{\text{eff}}(x) - \sigma_{\text{th}})}{\sigma_u}\right)^b dV\right],$$

where the characteristic fatigue strength $\sigma_u$, the threshold stress $\sigma_{\text{th}}$ and the Weibull exponent $b$ are material parameters related to the arbitrary chosen reference volume $V_0$. In [8], it is shown that the transferability in $p_f$ for the Weibull model between different notched specimen geometries is not very good, especially for lower and higher $p_f$-values.

Another example of a volume based weakest-link HCF model is the $V^*$-model [9]. This probabilistic model belongs to the family of VPF models, which all are developed in the COMP project. The failure probability is computed according to

$$p_f = 1 - \exp[-q(V^* - V_{\text{th}})],$$

where $q$ is a material parameter, $V_{\text{th}}$ is a threshold volume and $V^*$ is the volume in which the effective stress is above the threshold stress $\sigma_{\text{th}}$. The parameters $q$, $V_{\text{th}}$ and $\sigma_{\text{th}}$ are determined from fatigue tests.

4. The HCF post-processor AROMA-PF

AROMA-PF is a HCF post-processor developed in the COMP project. The aim with AROMA-PF is to implement HCF models. This includes already existing models but also completely new models that have been developed, such as the family of VPF models [9]. The models should be ready for direct use in probabilistic HCF design, and the program is delivered to the involved industrial partners.

The code is written in Matlab, and the input is FEM results based on a dynamic analysis of the blades that, in turn, comes from simulation of the fluid flow simulation of the compressor. The input can be generated from any commercial FEM software. The main output is the effective stress field, computed by use of any of the implemented local HCF models, and the failure probability for the analyzed component, computed by use of either the volume based weakest-link model developed by Weibull or the $V^*$-model. Also, input files that can be used in the FE software Ansys in order to plot the effective stress field and the failure probability density $p'_f$ given by the Weibull model over the component are generated during a run in AROMA-PF. The structure of AROMA-PF is presented in Figure 6.
By performing the HCF analysis for several stress amplitudes, the failure probability distribution curves can be plotted. In order to “resolve” the $p_f$-curve for lower failure probabilities, the logarithm of $p_f$, $\log(p_f)$, can be used when plotting.

The simplest method to use for the volume integration in the Weibull- and $V^*$-models is to use the stress that acts at the elements’ centroid, and the corresponding element volumes. In most commercial FEM softwares, this data can be easily exported to external files. When using this integration method, it is assumed that the stress is constant throughout each finite element. This integration method is used in the mesh sensitivity investigation performed in [10].

In AROMA-PF, the volume integration in the Weibull- and the $V^*$-models are performed by use of the stress that acts at the elements’ integration points and standard Gauss integration. This integration method is a more accurate volume integration method than the centroid stress integration method, since it means a higher resolution of the stress field in the volume integration. Therefore, a lower number of finite elements need to be used in order to reach convergence in $p_f$. The more advanced volume integration is implemented for all types of standard finite elements with quadratic shape functions. It is used in the HCF model prediction accuracy investigation in [12].
5. Results

Based on the fatigue test results obtained for smooth Ti-6-4 specimens, it is shown that the most optimal strategy to use for HCF testing in order to determine material parameters in multiaxial HCF models when different type of scatter and the cost to perform the tests are taken into account is to perform tests on few stress ratios located far away from each other.

The relevant material parameters in the Weibull- and the \( V^* \)-models as well as in the point stress method, based on a Weibull distributed critical effective stress, have been fitted to experimental data for Cr-steel specimens of three sizes (small, medium and large) subjected to rotating bending, see Figure 7. The relevant material parameters have been fitted to all eight experimental data points shown in Figure 7. By comparing fatigue test results for Cr-steel to the models it is seen that the \( V^* \)-model provides the smallest error between experiments and predictions, and the most accurate fatigue prediction at low stress levels. The \( V^* \)-model is therefore favoured for design purpose.

![Figure 7](image)

**Figure 7.** Failure probability according to the Weibull (W)- and the \( V^* \)-models, and the point stress method, as function of the maximum stress in the notch root. The models are applied to single notched cylindrical Cr-steel specimens of different sizes subjected to rotating bending.

The experimental failure probabilities obtained from the testing, which are used to quantify the prediction capabilities for the HCF-models, typically range \( 0.1 \leq p_f \leq 0.7 \). This is far away from the low \( p_f \)-values typically used in design. However, experimental estimation of a failure probability at for example \( 10^{-4} \) is impossible from a practical point of view due to the large number of experiments that must be
performed. Also, it is important to keep in mind that for the applications considered in this thesis, the loading is not deterministic, it is stochastic. Therefore, the model prediction accuracy for the whole $p_f$-curve given in Figure 7 is important, not only for very low $p_f$-values.

The mesh sensitivity analysis performed in [10] shows that the number of elements used through the thickness of the compressor blade is the most important mesh property for reaching convergence in the HCF prediction. The fatigue prediction is not very sensitive to the in-plane mesh properties.

6. Conclusions

The purpose of this work has been to investigate and evaluate the (i) important mesh properties in the FE-model for reaching convergence in the fatigue probability computation for a compressor blade, (ii) find the most beneficial test strategy for HCF-testing in order to determine material parameters in local multiaxial HCF models and (iii) evaluate the prediction capacity for the probabilistic Weibull- and $V^*$-models.

The results show that the most important mesh property is the number of elements used through the blade’s thickness. In order to obtain good parameter estimations in multiaxial HCF models by use of as few tests as possible, tests should be performed at few stress ratios located far away from each other. It is also shown that if fatigue tests are performed in a high quality laboratory, the scatter in material properties is the main reason for scatter in the parameter estimations. Finally, it is concluded that the volume based $V^*$-model is favoured for design purpose.
7. Summary of appended papers

**Paper A: FE-mesh effect of the volume based weakest-link fatigue probability applied to a compressor blade**

In order to obtain a converged fatigue prediction for a component, in this case a compressor blade, it is important that the finite element mesh is fine enough. By use of a load case where a compressor blade is subjected to a steady load due to rotation, and an amplitude load in form of a uniform pressure applied to the blade’s pressure side, it has been investigated which mesh properties that are important for performing a good fatigue prediction for the blade.

The weakest-link volume model suggested by Weibull is used in the investigation. The volume integration is performed by use of the stress that acts at each element’s centroid (interpolated from the Gauss points) along with the corresponding element volumes. The advantage with this integration method is its simplicity. By changing the mesh properties of the blade, it is shown that the number of elements used through the thickness of the blade is more important than the blade’s in-plane mesh properties.

**Paper B: On the optimal choice of experiments for parameter determination in multiaxial HCF-criteria**

The relevant material parameters used in local HCF-models must be determined from experiments. Performing a higher number of tests increases the probability to estimate parameter values that are close to the true values. But, on the other hand, performing fatigue tests is very costly, whereby there is an interest to perform the type of tests that enables good parameter estimations by use of as few tests as possible. By use of the results obtained from HCF testing of smooth Ti-6-4 specimens, and taking the cost to perform fatigue tests and different types of scatter into account, it has been investigated how tests should be performed in order to estimate the material parameters in multiaxial HCF-models in the most effective way.

The investigation is limited to tests performed in the Haigh plane. The results show that decreasing the number of stress ratios that tests are performed for, but instead increase the number of tests performed at each stress ratio, is more effective than performing few tests at many stress ratios.
**Paper C: An investigation of the prediction accuracy for volume based HCF models using scaled geometries and scaled loading**

The HCF-prediction capabilities for the volume based weakest-link model suggested by Weibull and the $V^*$-model are investigated, and compared to the point stress method. The models are tuned to HCF experimental results obtained for cylindrical single notched 12 % Cr-steel specimens of different sizes loaded in rotating bending. The tests are designed such that at a certain stress level, independent of the specimen size, the specimens are subjected to a stress field that has the same shape on a specimens’ cross section, but a different gradient. Thus, the maximum stress in the notch is the same but the highly loaded volume differs between the specimens. The total error between experiments and model predictions are computed. The smallest total error is obtained for the $V^*$-model. The $V^*$-model also provides the most accurate fatigue prediction at low stress levels. Based on the results, the $V^*$-model is favoured for design purpose.

**8. Future work**

The main advantage of the stress invariant based HCF models is the short computational time. However, since these types of models do not include directional information about the stress state they may not be very suitable to use for components subjected to non-proportional stressing, which is the case for many components such as compressor blades. Therefore, it should be investigated if it is possible to somehow augment or further develop for example the Sines model in such way so it can appreciate rotating principal stresses, but at the same time keep the advantage with a short computational time.

In this work, only uncertainties in the material properties is dealt with. In reality, several types of uncertainties are present such as in material properties, loading, manufacturing, component dimensions, etc. They all affect the final failure probability for the component. The future work will include development and investigation of methods for reliability based design when accounting for different types of uncertainties, not only uncertainty in the material. This is also in line with the current direction of HCF design development in industry.
9. Bibliography


