Transient Stability During Asymmetrical Faults

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MASTER THESIS

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TRANSIENT STABILITY DURING ASYMMETRICAL FAULTS

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This research project would not have been possible without the support of many people.

«
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Abstract

This research project has been conducted at RTE in order to study the transient stability after asymmetrical faults. When three-phase short-circuits occur in a network, almost all the electrical power is lost on the relevant line(s). Among all short-circuit types, it is the most drastic event and the issue has to be solved very quickly. But oddly, it is also the easiest problem to solve mathematically speaking. This comes from the fact that the system stays balanced, and equations can be simplified. However with line-to-ground faults this is no longer the case, and transient stability analysis becomes tricky.

Until now, unbalanced situations have not been studied much. Since this kind of trouble is less serious than losing all three phases, every protection devices on the network have been sized to counter three-phase faults in time and avoid severe consequences. They will then also work for one-phase problems.

Despite this, there is a desire from RTE to understand – physically and mathematically – what happens when one-phase faults occur, and it is the mission behind this master thesis. First, a mathematical theoretical model was derived to examine a network’s stability without running any simulation. Then, once simulation software programs were taken in hand, several tests were run on a very simplified network, and compared with the theory developed previously. Finally, these experiments were carried out on a much larger scale.

It is important to understand that, except for the theoretical model, all the results and conclusions in this document come from simulations. Even if a lot of tests and models led to them, these conclusions must be handled with care. The goal of this work was also to have a better understanding of unbalanced systems, of the Fortescue representation and thus, understand more clearly the parameters required by simulation tools like Eurostag© for future studies.
Sammanfattning


Fram tills nu har inte lösningar på obalanserade situationer studerats mycket. Eftersom denna typ avproblemat mindre allvarliga än att förlora alla tre faser, så har enheter på nätverket utformats för att motverka trefas-fel snabbt och undvika allvarliga konsekvenser. Enheterna kommer då också fungera för enfasproblem.


Det är viktigt att förstå att, utom den teoretiska modellen, kommer alla resultat i denna rapporten från simuleringar. Även om flera tester och modeller ledde fram till dem, ska dessa slutsatserhanteras varsamt. Målet med detta arbete var att få bättre förståelse för obalanserade system, representationen med symmetriska komponenter och därmed, få en klarare förståelse för parametrarna som krävs avsimuleringsverktyg så som Eurostag© för framtida studier.
This master thesis has been conducted for six months at RTE, Réseau de Transport d’Électricité, the French Transmission System Operator\(^1\).

RTE is a limited liability corporation founded in 2000. With more than 100,000 km of lines, it is Europe’s largest transmission system operator. Its main goal is to convey electricity from producers (nuclear, hydraulic, coal, wind or solar power plants) to consumers (retailers or industrial customers) through high-voltage lines (400 kV, 225 kV, 90 kV and 63 kV). The highest is used to route power on long distances (exit of power plants, interconnections across borders) while others are used to connect the transmission grid to the distribution one. This mission relies on three key points.

- **Supply-Demand balance management**: in order for the network to operate at fixed frequency (50 Hz), power groups have to adapt their production to the load required from customers. RTE makes sure all the produced power is consumed, and is always able to provide energy when needed.

- **Network operation**: power has to be conveyed along different lines in order to avoid overloads. The distribution of flows is supported by RTE and its regional dispatching units. They find the best path for electricity while ensuring network security.

- **Electricity market administration**: the company makes sure that every actors (producers, consumers and retailers) can intervene on the market to sell or buy energy. In real time RTE must be aware of the grid congestions to exchange with neighboring countries.

Inside RTE is the department responsible for systems expertise, DES\(^2\). This division takes care of parts of the Research and Development program. For always improving the technology used in the network (protections, breakers, etc.) and discover new solutions, RTE invests a lot in research. Several projects are currently developed: new methods to estimate lines capacities and increase trading, software development to simulate European interconnected grid, ... In this context it has been asked to analyze the transient stability happening during asymmetrical faults.

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\(^1\) TSO: Transmission System Operator

\(^2\) DES: Département Expertise Système
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Introduction

The goal of this thesis is to have a better understanding of unbalanced faults, and especially figure out what affects them and what are their consequences. A secondary objective is to get a better understanding of the Fortescue representation, especially in the software Eurostag©.

It has almost never been studied deeply at RTE. To set an example, protections on the networks such as circuit breakers or lightning arresters are dimensioned with three-phase faults studies. They must be able to counteract any network situation where three-phase short-circuits appear, but one- or two-phase faults are not considered. Usually if a breaker is fast enough to neutralize a three-phase fault, it will also do the trick for a line-to-ground one. Therefore this study is quite new for the company, which now has a strong desire to deeply analyze and understand this unbalanced phenomenon. As for the background, some papers have already investigated the two-node network model in order to mathematically analyze transient stability issues, but rarely with line-to-ground faults, and not as deeply as in this work.

Faults in an AC transmission system

In France, the main part of the electrical network is operated in AC-mode. Some DC-lines are used, mainly for interconnections with neighboring countries. This means that all the power is produced thanks to synchronous machines, usually rated at 20 kV (for nuclear power units, specially investigated in this study) and rotating at 50 Hz. Then it is transmitted by the TSO on high-voltage lines and cables (400 kV and 225 kV). Finally, the distributor lowers the voltage and distributes the power to customers.

A lot of problems and incidents can occur on transmission lines. In this study, only short-circuits on overhead lines are discussed. But several kinds of faults can be distinguished, as shown in Figure 1 [1]:

- **Three-phase faults (3φ):** All the three phases are short-circuited (D) and usually connected to the ground (E). They are the worst kind, since no power can go through the faulted line. If it is connected to a power unit, the problem is even more serious, as the group will have a lot of trouble to evacuate its power. Three-phase faults represent about 5% of total short-circuits. A classical example is when a tree falls down on a line.

- **Line-to-Line faults (LL):** They appear when two phases of a line are connected together (B). This problem is quite rare, representing only 5% of the cases, and usually happens after the ionization of air between two conductors. They will not be considered in this study.

- **Line-to-Ground faults (LG):** They appear when a single phase is connected to the ground (A), directly or through an impedance. This problem is the most common due to the high

---

3 OHL: Over Head Lines
distance (about 15 meters) between phases, with 80% of cases. It is the kind of short-circuit that will be studied here.

- **Double Line-to-Ground faults (DLG):** These faults are simply line-to-line faults connected to the ground (C). Standing for 10% of cases, they will not be considered in this study.

When one or several phases are connected to the earth, there is an impedance between the faulted line(s) and the ground. However, a drastic case is to consider the short-circuit “metallic”, meaning no impedance. Even if it is not realistic (lightning, trees and other faults makers have impedance) it is often considered as so because in that case, the short-circuit current is maximal, leading to maximal damage. This simplification represents the worst case scenario, and is called “bolted fault”. All along this study, short-circuits are considered bolted.

If the system stays symmetrical even with a short-circuit, the fault is called “symmetrical”. It is rarely the case (only with a three-phase fault), but is quite easier to compute mathematically. Otherwise, the faults are “asymmetrical”.

![Figure 1: Different kinds of faults on a 3-phase line](image)

<table>
<thead>
<tr>
<th>Seriousness ranking</th>
<th>Type of fault</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3φ</td>
<td>5%</td>
</tr>
<tr>
<td>2</td>
<td>DLG</td>
<td>10%</td>
</tr>
<tr>
<td>3</td>
<td>LL</td>
<td>5%</td>
</tr>
<tr>
<td>4</td>
<td>LG</td>
<td>80%</td>
</tr>
</tbody>
</table>

Table 1: Faults happening on 3-phase lines and occurrences [1]

After that, another distinction has to be made between transient and permanent faults. The first ones do not damage the network permanently and allow the line to be safely reconnected after a short period of time. In other words, if the TSO opens the faulted line, the short-circuit will disappear (ex: a flashover after a lightning strike). On the other side, permanent faults do not disappear when discharging the network (ex: a tree touching lines).
When a three-phase short-circuit is detected (about 10 ms after the beginning), the line is opened by the operation of a circuit breaker (in about 80 ms), and shortly after reconnected. If the fault is still present (permanent faults), the line is re-opened definitely in order for a human to manually repair it. In the case of a single phase fault, only the faulted phase is opened first, and reconnected. If it happened to be a permanent fault, all the three phases are disconnected definitely.

However in every simulation run here, it is considered that short-circuits disappear on their own, without any outages. The lines always stay connected, until a machine falls out of synchronism with the system. Actually lines are composed at least of two circuits, and often of four or six in parallel. In these cases, opening a line or not will not matter that much, so assuming self destructing faults will not give unrealistic results.

**Impact of faults and synchronization in an AC network**

In an AC network, synchronization is the process of matching the speed and frequency of a generator or other source to a running network [2]. An AC synchronous generator cannot deliver power to an electrical grid unless it is running at the same frequency as the network. If two segments of a grid are disconnected, they cannot exchange AC power again until they are brought back into exact synchronization.

The problem with short-circuits is the increasing speed of the nearby generators. During steady state, mechanical and electrical powers ($P_m$ and $P_e$) are equal, meaning that all the rotational energy from the rotor is transferred to the stator and evacuated as electricity. But during the fault, a fraction or the total of electrical energy is sent to the ground, meaning that $P_e$ decreases drastically. Therefore, the rotor cannot transfer all the power it is producing, and has to store it as kinetic energy, leading to acceleration. This question will be more deeply investigated in Section 2.1.

Then, if the short-circuit lasts too long, the rotor of one of the synchronous machines may accelerate too much and fall out of synchronism with the system. Consequently, the concerned generator would have to be separated from the grid, creating two isolated networks. In worst case scenarios when the isolation takes too long time, this may cause material damages or blackouts. The TSO must prevent these catastrophic situations. Studies about “Critical Time of Removal of faults” are conducted in order to design and size protections.

The Critical Time of Removal (CTR) of faults is the maximal duration of a short-circuit at a given location for a given network topology (global grid, injections of power, loads...) before at least one generator falls out of synchronism with the system. Typical values of CTR for symmetrical faults are in the range [100 ms; 180 ms]. For asymmetrical faults, it can vary between 250 ms and infinity. In fact, the network can sometimes regain stability during a line-to-ground fault, and then stay put no matter how long the line stays faulted. This phenomenon is described in Section 2.2.
Organization of the study

First part of this study conducts a theoretical analysis on critical times. Thanks to some simplification, simple mathematical equations are derived in order to find an expression of the critical time of removal of one-phase and three-phase faults. It is then interesting to study the impact of network parameters on it.

Next two simulation software programs used at RTE are introduced: Eurostag© and EMTP©. The user describes its network with physical quantities, and they are able to estimate output quantities such as voltages, current or frequencies. But they are quite different, do not use the same equations, the same parameters to model a network, and that is why their results are going to be compared. The goal is to check if, even with two distinct physical realities, they may converge toward similar results. This study has been conducted on very basic networks.

After checking that results given by Eurostag® are coherent with known reality, close to those from EMTP®, different parameters are modified in order to observe their influences. First classical settings are altered such as line length or input power. But the real purpose is to vary unknown parameters. These might be indefinite for the reason that RTE actually doesn’t know their value (whether it is because databases are too old to be trusted, because this parameter in question is rarely or never used or simply because the TSO doesn’t have access to it, like across borders). Otherwise, some (known) parameters are just simplified or approximated, and varying them allows to check if the approximation is acceptable (for example, nuclear groups’ auxiliaries are modeled so far as constant loads instead of motors).

Finally, a previous study conducted at RTE highlighted the fact that critical times of removal of one-phase faults is always higher than two times the critical time of removal of three-phase faults. Yet it has been tested on very simple networks, and the final goal of this work is to check and confirm this empiric law on a larger grid.
1. Literature review

1.1. Symmetrical components method (Fortescue representation)

The core of this study deals with line-to-ground faults. But with unbalanced systems it is common to move from a three-phase scheme to a Fortescue representation. It becomes mathematically easier to compute voltages, currents and impedances. The Fortescue transformation lies on the fact that any set of three unbalanced phasors can be expressed as the sum of three symmetrical sets of balanced phasors. They are called positive-sequence (notation “1” or “d” for direct), negative-sequence (“2” or “i” for inverse) and zero-sequence (“0”). They are respectively composed of three vectors of equal magnitude with a phase shift of ±120° (phase sequence abc), three vectors of equal magnitude with a phase shift of ±120° (phase sequence acb) and three vectors in phase with equal magnitude. [9] Figure 2 gives an example for voltages $V_a, V_b, V_c$.

![Symmetrical components](image)

**Figure 2: Symmetrical components**

Let’s set $a = e^{j \frac{2\pi}{3}}$. This operator causes a counterclockwise rotation of 120°. Therefore $a^3 = 1$ and $1 + a + a^2 = 0$. The goal is to find the three symmetrical components of the voltage introduced in Figure 2, such that

$$
\begin{align*}
V_a &= V_a^0 + V_a^1 + V_a^2 \\
V_b &= V_b^0 + V_b^1 + V_b^2 \\
V_c &= V_c^0 + V_c^1 + V_c^2
\end{align*}
$$

But by definition of symmetrical components,

$$
\begin{align*}
V_a &= V_a^0 + a^2 \cdot V_a^1 + a \cdot V_a^2 \\
V_b &= V_b^0 + V_b^1 + a \cdot V_b^2 \\
V_c &= V_c^0 + a^2 \cdot V_c^1 + a \cdot V_c^2
\end{align*}
$$

So

$$V^{abc} = A \cdot V_a^{012}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$
and

\[ V^{012}_a = A^{-1} \cdot V^{abc} \]  \hspace{1cm} (1.5)

\[ A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \]  \hspace{1cm} (1.6)

When one deals with a balanced system, \( V_b = a \cdot V_a \) and \( V_c = a^2 \cdot V_a \), and it turns out that

\[ \begin{align*}
V_0 &= \frac{1}{3} \cdot (1 + a + a^2) \cdot V_a \\
V_1 &= \frac{1}{3} \cdot (1 + a^2 + a^3) \cdot V_a \\
V_2 &= \frac{1}{3} \cdot (1 + a^2 + a^4) \cdot V_a
\end{align*} \]

So the equivalent representation is direct-sequence only.

The same method can be applied to currents of course. Thanks to this transformation, it is possible to deal with symmetrical components when running calculations. Unbalanced problems are then less difficult to be solved, and the obtained results are easily brought back into the classic phase-basis.

\subsection*{1.2. \( \pi \)-model for lines}

In the simulations performed in this thesis the overhead lines are represented by their \( \pi \)-model equivalent [10]. This model states that a line can be characterized as in Figure 3. This approximation lies on the Telegrapher’s equations, and is valid for line lengths less than a few hundred of kilometers. Beyond one must take into account the propagation phenomena in the line.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{pi_model.png}
\caption{\( \pi \)-model of a line}
\end{figure}

- The resistance per unit length is a function of the line design (cross-section area \( S \)) and material (resistivity \( \rho \)). If the temperature influence is neglected \((T = 20^\circ C)\),

\[ R = \frac{\rho}{S} \ [\Omega/km] \]  \hspace{1cm} (1.8)
The inductance per unit length depends on the distance between the three phases \((D_{ab}, D_{bc}, D_{ca})\). If \(r\) is the conductor radius, \(n\) the number of sub-conductors per phase and \(D = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ca}}\) the equivalent distance between phases, then

\[
L = \frac{\mu_0}{2\pi} \left( \ln \left( \frac{D}{r_{eq}} \right) + \frac{\mu_r}{4n} \right) \text{ [H/km]} \tag{1.9}
\]

The capacitor per unit length, equally distributed at the sending and receiving nodes, is

\[
C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln \left( \frac{D}{r} \right)} \text{ [F/km]} \tag{1.10}
\]

In order for this one-phase representation to be acceptable, the capacitors on each phase need to be equal. Yet transmission line conductors are rarely at the same height from the ground. If one of them is higher than the two others, its capacitance would be lower. To avoid this problem phases are shifted along the line path. At certain pylons, phase \(a\) takes the place of phase \(b\), \(b\) of \(c\) and \(c\) of \(a\), as in Figure 4. This way on average, all the phases have the same behavior.

![Figure 4: Transposition of a 3-phase line](image)

**Note:** because of the shunt capacitors, two \(\pi\)-models in series result in a \(\pi\)-model, but the equivalent components (resistance, inductance and capacitor) are not the sum of the components of the two lines. However in a first approach, it is possible to consider a line as purely inductive. This means \(R = 0\) and \(C = 0\) and then \(\pi\)-models can add-up.
2. Theoretical analysis – Equal Area Criterion

The purpose of this theoretical investigation is to find a mathematical relation between the rotor angle and the fault duration, with the aim of deriving an expression of the CTR. It relies on several simplifications and on a graphical study.

2.1. Overview with a three-phase fault

2.1.1. Equal Area Criterion

When a generator is connected to a line, there is a maximal amount of power that can be transmitted through it. By definition, if \( \phi \) is the phase shift between the voltage \( U \) and the current \( I \),
\[
P_{\text{active}} = U \cdot I \cdot \cos \phi
\]  
(2.1)

According to Figure 5 [3] and by neglecting the resistance,
\[
P_{\text{active}} = \frac{E \cdot U}{X} \cdot \sin \delta
\]

\[E\]
\[\delta\]
\[U\]
\[I\]

Figure 5: Phasor representation to compute the active power delivered by a synchronous generator

The active power delivered by the generator on a network depends on the source- and ending-voltage \( E, U \), the equivalent reactance \( X_{\text{eq}} = X_{\text{generator}} + X_{\text{transformer}} + X_{\text{line}} \) and the transport angle \( \delta \). The last one refers to the phase shift between the voltages at the two ending nodes, but corresponds also to the rotor angle. The transmitted power can be written
\[
P_e(\delta) = P_{\text{max}} \cdot \sin \delta
\]  
(2.2)

\[
P_{\text{max}} = \frac{E \cdot U}{X_{\text{eq}}}
\]  
(2.3)

Graph 1 shows the electrical and mechanical power transmitted on a line. Moreover, if \( H \) is the machine’s inertia (in s.) and powers are in pu (machine-basis), the equation controlling the rotor is
\[
\frac{d^2 \delta}{dt^2} = \frac{\omega_0}{2H} \cdot (P_m - P_e)
\]  
(2.4)
When steady-state is reached, \( P_m = P_e \). Therefore only two operating points are possible (corresponding to the intersection of the curves). But in order to be stable, the rotor angle has to be less than 90°. Finally, there is only one operating angle \( \delta_0 \) where electrical and mechanical powers are constant.

Let’s assume that a three-phase short-circuit is created at the ending node of the transformer at \( t = t_f \). All the electrical power is lost so \( P_e^f(\delta) = 0 \) (on Graph 2, operating point goes from A to B), but the mechanical can’t change that fast, and is supposed to be constant due to some regulation loop. This means that \( P_m \gg P_e \), and the synchronous machine accelerates; \( \delta \) increases (from B to C) according to Equation (2.4). After a certain duration \( (T_f) \), the fault is removed without any outage, meaning that the network topology is unchanged compared to steady-state. At the elimination time \( (t_e = t_f + T_f) \), the angle reached is named \( \delta_e \). As soon as the fault disappears, \( P_e = P_{max} \cdot \sin \delta_e \) (moving from C to D). From now on, electrical power is greater than mechanical input and the acceleration of the rotor angle decreases (while the rotor angle itself keeps increasing at first, from operating point D to E, to a value \( \delta_{max} \)). Then \( \delta \) decreases, until it crosses \( \delta_0 \) again. At this moment, \( P_m > P_e \) and it goes on and on. In a nutshell, the rotor angle will oscillate around \( \delta_0 \) (equilibrium point A). The explanation of this phenomenon is simple. During the acceleration phase, a lot of (kinetic) energy is stored in the machine, and has to be released. That is why the machine accelerates and slows down. Energies (stored and released) are visible in the graph, since they mathematically match the areas between \( P_m \) and \( P_e \).

However, this scenario relies on the fact that the machine doesn’t fall out of synchronism with the system. It is important for this study to find the critical angle \( \delta_{cr} \) corresponding to the maximal rotor angle at the end of the fault before losing synchronism. Graphically, it is clear that the generator will stay in synchronism with the system if and only if \( \delta_{max} < \pi - \delta_0 \). Otherwise, the deceleration of the machine will not compensate the kinetic energy stored during acceleration, and the rotor will increase indefinitely.
In critical situations, the area under $P_m$ and above $P_e$ (during fault) needs to compensate exactly for the area above $P_m$ and under $P_e$ (after fault). This method, called “Equal Area Criterion”, gives one equation to find the unknown variable ($\delta_{cr}$). Afterward it is quite easy to deduct the CTR $T_c$ from $\delta_{cr}$.

Following sections give some classical examples.

Graph 2: Power relative to rotor angle $\delta$ during a 3-phase fault

2.1.2. Simplifications and assumptions

To apply the Equal Area Criterion some assumptions have to be made:

- Mechanical power is kept constant during the study. This is acceptable because the time constants of mechanical outputs are much higher than electrical ones.
- Like usual, friction is neglected. Otherwise a term $D \cdot \frac{d\delta}{dt}$ would be added to Equation (2.4), and could make it hard to solve. Although it is a pessimistic simplification.
- In reality, electrical active power doesn’t become exactly zero, since some losses appear inside the transformer’s core and resistances. However they are neglected (pure inductive grid).
- Voltage at the sending node $V_s$ is assumed constant. Actually, it varies with $\delta$.
- Synchronous machines are modeled as voltage sources $E$ behind their transient reactance $X_d'$.
- Transformers are represented by their leakage reactance.
- Lines are purely inductive. Usually,

$$\frac{R_{line}}{X_{line}} \approx 10\%$$
2.1.3. Example 1: three-phase-fault on a one-line network

The first example computes the critical time of removal of a three-phase fault at the end of the transformer, where there is only one transmission line. The network is shown in Figure 6.

![Network of example 1](image1)

Figure 6: Network of example 1

2.1.3.1. Before the fault

Before the fault \((0 \leq t \leq t_f)\) during steady-state, the network can be modeled as in Figure 7. Since

\[
P_m = P_e = \frac{E \cdot U}{X_0} \cdot \sin \delta_0 = P_{\text{max}} \cdot \sin \delta_0
\]

(2.5)

where

\[
P_{\text{max}} = \frac{E \cdot U}{X_0}
\]

(2.6)

\[
X_0 = X_d' + X_t + X_l
\]

(2.7)

![Equivalent circuit diagram](image2)

Figure 7: Equivalent circuit diagram of Example 1 before the fault
Equation (2.4) becomes

\[
\frac{d^2 \delta}{dt^2} = 0
\]

\[
\frac{d\delta}{dt} = 0
\]

\[
\delta = \text{cst} = \delta_0
\]

It gives

\[
\delta_0 = \text{asin}\left(\frac{P_m}{P_{\text{max}}}\right)
\]

\[\text{(2.8)}\]

2.1.3.2. During the fault

A bolted short-circuit appears at the beginning of the line, at \(t = t_f\) and during a time \(T_f = t_e - t_f\). All the electrical power is lost and equations become

\[
\frac{d^2 \delta}{dt^2} = \frac{\omega_0}{2H} \cdot (P_m - 0) = \text{cst}
\]

\[
\frac{d\delta}{dt} = \frac{\omega_0 \cdot P_m}{2H} \cdot (t - t_f) + 0
\]

and the rotor angle is

\[
\delta(t) = \frac{\omega_0 \cdot P_m}{4H} \cdot (t - t_f)^2 + \delta_0
\]

\[\text{(2.9)}\]

Thus during a short-circuit, the rotor angle rises proportionally to \(t^2\!\!\!\!

2.1.3.3. After the fault

At \(t = t_e = t_f + T_f\), the elimination angle obtained is

\[
\delta_e = \frac{\omega_0 \cdot P_m}{4H} \cdot T_f^2 + \delta_0
\]

\[\text{(2.10)}\]

The rotor angle is now ruled by

\[
\frac{d^2 \delta}{dt^2} = \frac{\omega_0}{2H} \cdot (P_m - P_e(\delta))
\]

\[\text{(2.11)}\]

\[\frac{\omega_0 \cdot P_{\text{max}}}{2H} \cdot (\sin \delta_0 - \sin \delta)\]

This differential equation is non-linear, but it will not be necessary to solve it!
2.1.3.4. **Graphical computation**

The “critical angle” is \( \delta_{cr} = \delta_e \) such that the acceleration area from \( \delta_0 \) to \( \delta_{cr} \) equals the deceleration area from \( \delta_{cr} \) to \( \delta_{max} = \pi - \delta_0 \). This equality can be written

\[
\int_{\delta_0}^{\delta_{cr}} (P_m - 0) \, d\delta = \int_{\delta_{cr}}^{\pi - \delta_0} (P_{\max} \cdot \sin \delta - P_m) \, d\delta \quad (2.12)
\]

or thanks to Equation (2.5)

\[
(\delta_{cr} - \delta_0) \cdot P_{\max} \cdot \sin \delta_0 = P_{\max} \cdot \int_{\delta_{cr}}^{\pi - \delta_0} (\sin \delta - \sin \delta_0) \, d\delta
\]

And then,

\[
\delta_{cr} = \acos((\pi - 2\delta_0) \cdot \sin \delta_0 - \cos \delta_0) \quad (2.13)
\]

From that equality and using Equation (2.10), one can compute the critical time of removal

\[
T_c = \frac{4H \cdot (\delta_{cr} - \delta_0)}{\omega_0 \cdot P_m} \quad (2.14)
\]

2.1.4. **Interpretation**

It is interesting to observe which are the parameters that have impact on \( T_c \). The following approach is based on results from Example 1. By noticing that

\[
\delta_{cr} = f(\delta_0) \\
\delta_0 = f(P_m, P_{\max}) \\
P_{\max} = f(E, U, X_0)
\]

One can conclude that the critical time to eliminate a three-phase fault depends on:

- The injected power \( P_m \). When the mechanical power increases the network becomes less stable, and \( T_c \) decreases.
- The line voltages \( E \) and \( U \). With high voltages, it is possible to evacuate a lot of power and then stabilize the network, resulting in an increased critical time.
- The network impedance, reflected inside \( X_0 \). If this reactance increases, it becomes harder to transmit power, and \( T_c \) decreases.
The mechanical power and network impedance’s influences on the critical time of removal are presented in Graph 3, with classical values for parameters:

- \( X_d' = 0.509 \text{ pu} \) (20 kV / 1650 MVA)
- \( X_c = 0.154 \text{ pu} \) (400 kV / 1650 MVA)
- \( X_l = 0.01 \text{ pu} \) (400 kV / 100 MVA) (100 km line with a reactance of 0.16 Ω/km)
- \( E = 405 \text{ kV} / U = 415 \text{ kV} \)
- \( P_m = 1500 \text{ MW} \)
- \( H = 5.625 \text{ s} \) (launch time \( T_L = 2H = 11.25 \text{ s} \))
- \( f = 50 \text{ Hz} \)

One can see that line length linearly influences the critical time. After proceeding with a linear regression, it is found that \( T_c = 228 - 0.69 \times L_{\text{line}} \), with a coefficient \( R^2 = 0.999 \). On the other hand the curve of mechanical power input’s influence looks a lot like an inverse square root curve. This is because \( P_m \) doesn’t impact angles much, so according to Equation (2.14), \( T_c \) is proportional to \( 1/\sqrt{P_m} \).

**Graph 3: Influence of parameters \( P_m \) and \( X_{\text{line}} \) on the 3-phase CTR \( T_c \)**

**Note:** the location of the fault is not relevant in that example, because it has been approximated that all the electrical power is lost during the short-circuit.
2.2. Variant with an unbalanced fault

2.2.1. Equal Area Criterion

Now let’s consider a line-to-ground fault instead of a symmetrical one. Mechanical and electrical powers (before/after and during short-circuit) are plotted in Graph 4. At \( t = t_f \), operating point goes from A to B. Indeed this time, some of the power (roughly two third of it) can go through during the fault, and electrical power doesn’t fall down to zero. Therefore the machine will still store kinetic energy, but a lot less than with a three-phase short-circuit. Once the angle has increased (from B to C) and when the fault is cleared, operating point rises to D, and a similar situation as before appears. The rotor angle will increase and decrease, oscillating around equilibrium point A. Energies (stored and released) are again visible in the graph. It is clear that during a one-phase fault, less energy is stored, making the system more stable. In terms of critical times, this will lead to higher CTRs than before.

Graph 4: Power relative to rotor angle \( \delta \) during a 1-phase fault

More, a peculiar case can sometimes happen. If the input of mechanical power is not very high, the system is initially quite stable. If by any chance the electrical power during the fault \( P^f_e \) becomes higher than \( P_m \), this would mean that the rotor can slow down during the fault. It is even possible for the synchronous machine to regain an equilibrium position. This phenomenon is shown in Graph 5. When the short-circuit is created, electrical energy drops to \( P^f_e \). Then the rotor speed increases, and shortly after, \( P^f_e = P_m \). From this moment, the rotor can release its surplus of energy, and as before the system will oscillate around a new equilibrium point, at the intersection between mechanical power and fault electrical power. But the fault has not been cleared yet, and could never be! However for safety reasons, breakers will still open to eliminate the fault.
Theoretical analysis – Equal Area Criterion

Graph 5: Power relative to rotor angle \( \delta \) during a stable 1-phase fault

If the short-circuit is removed, electrical power will jump to its regular blue curve, leading to \( P_e(\delta_{eq}) > P_m \). The machine will slow down a little bit, and the rotor angle will slightly oscillate around its previous stationary value \( \delta_0 \).

2.2.2. Example 2: one-phase-fault on a one-line network

2.2.2.1. Symmetrical components

The three-phase fault is now replaced by a line-to-ground fault, causing asymmetries. The equivalent network in Fortescue representation is given in Figure 8. Several clarifications must be made:

- The generator only has a positive-sequence component. Since the grid is fed with balanced electromotive forces, those emf stand for a direct system, and their negative- and zero-sequence components are equal to zero.
- In the negative-sequence, transformer and line’s reactances are equal to the positive-sequence. This is because in passive symmetrical devices (lines, transformers among others), positive- and negative-sequence impedances are identical. However for the synchronous machine, the negative-sequence reactance should be a little bit smaller than the positive one. Nevertheless it is possible to consider them equal without skewing the results.
- In the zero-sequence, the generator is missing. Actually, generators’ transformers are \( \Delta\)-Yn coupled, so no zero-sequence current can flow pass the transformer up to the alternator. The transformer is represented only by its grounding impedance (usually 25 Ω or 40 Ω) [7].
2.2.2.2. Solving equations

Before and after the LG fault, the system is obviously balanced. The difference with Example 1 emerge during the time-frame $t_f \leq t \leq t_e$. During the fault (in a Fortescue representation) an impedance $\tilde{z}$ appears in the positive-sequence, between the fault location (point F) and the ground, giving $V_1(F) = \tilde{z} \cdot I_1$. Therefore only the positive-sequence is required, negative- and zero-sequences being unused. So now $\tilde{z}$ must be found. [4] With a line-to-ground fault on phase a, equations are

$$V_a = 0$$

$$I_b = I_c = 0$$

So according to Fortescue theory,

$$I_1 = I_2 = I_0$$

$$V_1 + V_2 + V_0 = 0$$

And thus,

$$V_1 = Z_2 \cdot I_2 + Z_0 \cdot I_0 = (Z_2 + Z_0) \cdot I_1$$

$$\tilde{z} = Z_2 + Z_0$$

where $Z_2$ and $Z_0$ are respectively the equivalent negative- and zero-sequence impedance’s components seen from point F. Figure 9 shows the equivalent circuit during the fault and the $Y\rightarrow\Delta$ transformation associated. Only $X_{12}$ is important so $X_{13}$ or $X_{23}$ will not be calculated. Now impedances are converted into pure inductances. The reactance between the generator and the infinite node is
Theoretical analysis – Equal Area Criterion

\[ X_{12} = X_1 + X_2 + \frac{X_1 \cdot X_2}{X_3} \quad (2.19) \]

\[
\begin{align*}
X_1 &= X_d' + X_t \\
X_2 &= X_l \\
X_3 &= X_2(F) + X_0(F)
\end{align*}
\]

Figure 9: Equivalent positive-sequence circuit diagram during a line-to-ground fault and Y→Δ transformation\(^4\)

Equivalent reactances seen from point F can be calculated, thanks to Figure 8, as follow:

\[ X_2(F) = \left( X_2^{GEN} + X_t \right) // X_l \quad (2.20) \]

\[ X_0(F) = X_0^{TYP} // X_0^{line} \quad (2.21) \]

A classical simplification can be made by assuming that \( X_2^{GEN} = X_d' \), meaning

\[ X_2(F) = X_1(F) = (X_d' + X_t) // X_l = \frac{(X_d' + X_t) \cdot X_l}{X_d' + X_t + X_l} = \frac{(X_d' + X_t) \cdot X_l}{X_0} \]

Now let’s set \( k = \frac{X_0(F)}{X_1(F)} \) [5]. By noticing that \( X_3 = (1 + k) \cdot X_1(F) \),

\[ X_{12} = X_0 + \frac{(X_d' + X_t) \cdot X_l}{(1 + k) \cdot X_1(F)} = \left( 1 + \frac{1}{1+k} \right) \cdot X_0 \quad (2.22) \]

---

\(^4\) Kennelly’s Theorem
Then, the electrical power during the fault is

$$P_e^f(\delta) = \frac{E \cdot U}{X_{12}} \cdot \sin \delta = \frac{k + 1}{k + 2} \cdot \frac{E \cdot U}{X_0} \cdot \sin \delta$$

And finally,

$$P_e^f(\delta) = \frac{k + 1}{k + 2} \cdot P_{\text{max}} \cdot \sin \delta \quad (2.23)$$

This means that the higher $k$ is the more stable the system becomes, since $P_e^f$ gets closer and closer to maximal power capacity. For information purposes $k$ increases when the equivalent zero- (resp. negative-) sequence reactance increases (resp. decreases). When solving the equal area equation one solves

$$\int_{\delta_0}^{\delta_{cr}} \left( P_m - \frac{k + 1}{k + 2} \cdot P_{\text{max}} \cdot \sin \delta_0 \right) \, d\delta = \int_{\delta_{cr}}^{\pi - \delta_0} (P_{\text{max}} \cdot \sin \delta - P_m) \, d\delta \quad (2.24)$$

And after simplification, if $\beta = \frac{k + 1}{k + 2}$

$$\delta_{cr} = \arccos \left( \frac{(\pi - 2\delta_0) \cdot \sin \delta_0 - (1 + \beta) \cos \delta_0}{1 - \beta} \right) \quad (2.25)$$

Furthermore the equation controlling the rotor angle during the short-circuit becomes

$$\frac{d^2 \delta}{dt^2} = \frac{\omega_0}{2H} \cdot \left( P_m - P_e^f(\delta) \right) = \frac{\omega_0 \cdot P_{\text{max}}}{2H} \cdot \left( \sin \delta_0 - \beta \cdot \sin \delta \right)$$

And to find the rotor angle curve, along with the CTR, one has to solve the non-linear differential equation

$$\frac{d^2 \delta}{dt^2} + \alpha \cdot \beta \cdot \sin \delta = \alpha \cdot \sin \delta_0 \quad (2.26)$$

$$\alpha = \frac{\omega_0 \cdot P_{\text{max}}}{2H} \quad (2.27)$$

$$\beta = \frac{k + 1}{k + 2} \quad (2.28)$$

$$k = \frac{X_0(F)}{X_1(F)} \quad (2.29)$$
2.2.3. Interpretation

Curves plotted in Graph 6 comes from Matlab©. Solutions to the non-linear differential equation are provided by the ordinary differential equation solver ode45.

In addition to investigate the influence of the line length and the mechanical power, the impact of the grounding reactance is studied. Trends are approximately identical to the ones observed with a three-phase fault: almost a linear influence of $L_{\text{line}}$ (linearity coefficient $R^2 = 0.98$) and a square root shape for $P_m$. The new variable is $X_0^{TP}$. It often takes pre-set values: the grounding can be direct, and $X_0^{TP} = 0 \, \Omega$; otherwise it is usually 25 $\Omega$ or 40 $\Omega$ [7]. The third graph shows its huge impact on critical times, quasi-linear with $T_c = 340 + 10 \times X_0^{TP}$, $R^2 = 0.997$. The benefits of grounding impedances are clear. They prevent the zero-sequence current’s component to flow back in the generator and stabilize the system. This will be deeper explained later on this document.

Graph 6: Influence of parameters $P_m, X_{\text{line}}$ and $X_0^{TP}$ on the LG fault CTR $T_c$
2.2.4. Similarities with a three-phase-fault on a two-line network

If a three-phase fault appears in the middle of one line in a two-line network, results are close to the one-phase fault model. If the fault still appears at the busbar right after the transformer, the situation has not changed compared to the one in Example 2. In that case both lines are faulted, and it would be equivalent to only consider one line with half of the line impedance. Thus the short-circuit is considered only on Line 1, at a certain distance $x$ from the busbar, as in Figure 10.

![Figure 10: Network of Example 2](image)

Before the fault, the system is equivalent to the one in the previous example but with half of the line impedance. So Formula (2.8) remains the same:

$$\delta_0 = \text{asin} \left( \frac{P_m}{P_{\text{max}}} \right)$$

with

$$P_{\text{max}} = \frac{E \cdot U}{X_0}$$

$$X_0 = X_d' + X_t + \frac{X_1}{2} \tag{2.30}$$

**Note:** with $n$ lines in parallel, the line reactance $X_1$ would simply be divided by $n$.

The differences appear here. Now some of the produced power can run through the healthy phases during the fault. Figure 11 shows successive transformations $\Delta \rightarrow Y$ and $Y \rightarrow \Delta$ applied in order to find the equivalent reactance $X_{12}$ between the generator and the infinite network.

The reactance between the generator and the infinite node is

$$X_{12} = X_1 + X_2 + \frac{X_1 \cdot X_2}{X_3}$$

Which gives after simplification

$$X_{12} = X_1 + \frac{1 + x}{x} \cdot (X_d' + X_t) \tag{2.31}$$
Therefore the electrical power during the short-circuit is

\[ P_e^f(\delta) = \frac{E \cdot U}{X_{12}} \cdot \sin \delta = \frac{X_0}{X_{12}} \cdot P_{\text{max}} \cdot \sin \delta \]  

(2.32)

And the equation controlling the rotor angle becomes

\[ \frac{d^2 \delta}{dt^2} = \frac{\omega_0}{2H} \cdot (P_m - P_e^f(\delta)) = \frac{\omega_0 \cdot P_{\text{max}}}{2H} \cdot \left( \sin \delta_0 - \frac{X_0}{X_{12}} \cdot \sin \delta \right) \]  

(2.33)

Like for a asymmetrical fault, one has to solve the non-linear differential equation below to find the rotor angle curve

\[ \dot{\delta} + \alpha \cdot \beta \cdot \sin \delta = \alpha \cdot \sin \delta_0 \]  

(2.34)

\[ \alpha = \frac{\omega_0 \cdot P_{\text{max}}}{2H} \]  

(2.35)

\[ \beta = \frac{X_0}{X_{12}} \]  

(2.36)
2.3. **Real life fault clearing**

Actually when a line is faulted, it is immediately disconnected. It is safer for the network and not that serious after all. Transmission lines are rarely alone, and usually at least three other parallel lines can still transmit power and take over the faulted line duty.

But if so, the grid’s topology changes between the two states “before” and “after the fault”. Indeed, the equivalent grid reactance increases a little and the electrical power curve after clearing the short-circuit is slightly below the maximal initial one. Graph 7 shows the new pattern of the rotor angle evolution. One can see that the new deceleration area is (slightly) smaller than before. This will decrease the critical time, but not much. To solve this new problem,

- Another parameter has to be introduced. $P_{3,\text{max}}$ is the maximal power after the clearing. Its value is totally known, and depends on the new line impedance $X_3$.

\[
P_{e,3}(\delta) = \frac{E \cdot U \cdot \sin \delta}{X_3} = P_{3,\text{max}} \cdot \sin \delta
\]

\[
P_{3,\text{max}} = \gamma \cdot P_{\text{max}}
\]

\[
\gamma = \frac{X_0}{X_3}
\]

- The maximal angle before falling out of synchronism is no longer $(\pi - \delta_0)$ but $(\pi - \delta_3)$, where

\[
\delta_3 = \arcsin \left( \frac{P_m}{P_{3,\text{max}}} \right)
\]

- The new critical angle is

\[
\delta_{cr} = \arccos \left( \frac{(\delta_{\text{max}} - \delta_0) \cdot \sin \delta_0 + \gamma \cdot \cos \delta_{\text{max}} - \beta \cdot \cos \delta_0}{\gamma - \beta} \right)
\]

- Meanwhile the CTR is calculated as before thanks to Equation (2.34).
Theoretical analysis – Equal Area Criterion

Graph 7: Power relative to rotor angle $\delta$ during a 3-phase fault with faulted line opened
3. Software

3.1. Getting started with Eurostag© and EMTP©

3.1.1. Global presentation

Two software programs are used to run simulations, Eurostag© (Version 5.1) and EMTP© (Version 4.3). The first one uses a non-constant simulation step time to solve equations. This means that when steady-state is reached, it will automatically increase the calculation step in order to save (precious) time. Eurostag© uses a phasor representation for physical parameters (voltage, current, etc). If the user wants to do unsymmetrical computations, he has to describe the network’s characteristics and components in the Fortescue representation (positive-, negative- and zero-sequence basis). Otherwise the software will do only symmetrical calculation, computing just one phase and deducting the two other by rotation of +/- 120°. On the other hand, EMTP© uses a three-phase representation and has a fixed calculation step. This implies that with this one, a choice has to be made between speed and precision. Moreover, this software is based on electromagnetic equation, and allows the user to observe electromagnetic transient phenomena. This is not possible with Eurostag©.

3.1.2. CTR search algorithm

In Eurostag©, the computation of the critical time of elimination of faults relies on the machines’ angles. To do so, the software is given a minimum and maximum fault length (usually 0 ms and 5 s), a precision (1 ms) and an observation frame (10 ms). Next, it will simulate the given network with a fault of 0 and 5 ms, and observe if at least one of the synchronous machines has fallen out of synchronism. The criterion is “at least one machine angle has exceeded ±180°”. It will then proceed by dichotomy on the fault length to find the CTR with the given precision.

EMTP© doesn’t have this kind of algorithm, so it has been chosen to recreate it manually. A batch of networks are simulated with various fault lengths. Angles are plotted, and as soon as one crosses 180°, the critical time is found.

3.1.3. Comparison

In Eurostag©, the speed reference is not fixed (to 50 Hz for example) but is equal to the center of gravity of all the speeds of the synchronous machines connected to the network, weighted by the product of their inertia and rated power. [6]

All the other rotational speeds are calculated in this rotating reference frame, so its value is essential to study short-circuits and critical times. However, in every simulations computed in this paper, one or several infinite nodes are connected. These are ideal nodes at which the voltage (magnitude and angle) is kept constant at all time. They are modeled with huge machines, with a large production of active power and an infinite inertia. Thus one can approximate that the speed reference is kept constant, and $\omega_{ref} = 50$ Hz. In EMTP®, it has been chosen to compute a rotor angle in degrees by the equation

$$\theta_{rotor}(t) = \int_0^t (\omega_{gen}(t) - \omega_{ref}) \cdot dt \times \frac{(100 \cdot \pi)}{2\pi}$$

where $\omega_{gen}$ is the speed of the generator considered, in per-unit. So with the approximation above,

$$\theta_{rotor}(t) = \int_0^t (\omega_{gen}(t) - 1) \cdot dt \times 18000$$

Moreover, it is important to note that the two simulation tools are not based on the same equations and do not use the same approximations. One drastic guess made by Eurostag® is to neglect the first derivative of the rotor magnetic flux inside synchronous machines. In reality this derivative is responsible for a slight slowdown of the rotor (during the first 10 ms) when a short-circuit appears, just before the rotor increases. This phenomenon is called “backswing” and has a direct effect on the critical time of elimination of faults. [12] A simple short-circuited network has been simulated to observe this behavior. The speed of the synchronous machine is plotted in Graph 8. The backswing is clearly visible on the right side (zoom of the left side). Because of this slow-down, the blue curve seems delayed compared to the red one. But actually it doesn’t matter for CTR calculation.

When a generator falls out of synchronism with the network, its speed gets out of control and accelerates indefinitely. However if the fault lasts $T_c - \epsilon$, the speed $\omega$ will be quickly damped. In terms of rotor angles, this means that just before the critical time, the angle stays at a reasonable value (e.g. a maximal value of $100^\circ$), far from the limit (which is $180^\circ$); but an increase of 1 ms in the fault length leads to angles of thousands of degrees instantaneously. The desynchronization happens almost always right from the first oscillation of the rotor speed (or angle). Thus the “exact” speed curve is not needed, and neglecting the backswing is acceptable.

Another significant difference between the two tools is that only EMTP® takes into account the 50 Hz transient oscillations that appear during a short-circuit. It can also be observed on Graph 8. As soon as the fault appears oscillations begin. Once the fault is cleared ($t = 1.12$ ms) they fade away. For the same reasons as before, this phenomenon will not impact the CTR.

---

\[ \text{pu: per-unit} \]
There is one final divergence between Eurostag© and EMTP©. During an unsymmetrical fault, the negative-sequence component of the current inside the synchronous generator induces a breaking torque

\[ T_{\text{brake}} = \frac{(R_2 - R_\alpha) \cdot |I_2|^2}{\omega} \]

(3.3)

where \( R_2 \) and \( R_\alpha \) are respectively the negative-sequence and the armature resistances, \( I_2 \) is the negative-sequence component of the rotor current and \( \omega \) is the rotor speed. But this induced torque is only taken into account in Eurostag©. [6] The impact of this torque is unclear for the moment.

3.1.4. Important note!

It is really important to understand that there is not one software better than the other. They both use “models”, and none of them stands for “reality”. In addition and for the same reasons, none of them is “more precise” than the other. EMTP© may consider all the electromagnetic transient phenomena, like the 50 Hz oscillations during short-circuits, but neglect the braking torque above. The reader must keep in mind these differences when comparing two “identical” simulations computed with Eurostag© and EMTP©. It will be natural not to find the same CTR in both of them; the goal is to find the same behavior in both of them, the same evolutions and influences, but not the same results.
3.1.5. Simulation times

The following survey was conducted on the 400 kV French network, with a 100 ms three-phase fault. Thanks to the non-constant simulation step time Eurostag© is very efficient. A load-flow procedure is instantaneous while a ten seconds dynamic simulation takes about 8 s. Conversely for the same network and with a step time of 100 μs, EMTP© needs 15 s to find the load-flow and more than 2 minutes and 30 seconds to compute the dynamic simulation.

<table>
<thead>
<tr>
<th>Description</th>
<th>Eurostag©</th>
<th>EMTP©</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load-Flow instant</td>
<td>instant</td>
<td>15''</td>
</tr>
<tr>
<td>1 s simulation</td>
<td>5''</td>
<td>30''</td>
</tr>
<tr>
<td>10 s simulation</td>
<td>8''</td>
<td>2'30''</td>
</tr>
<tr>
<td>1 min simulation</td>
<td>10''</td>
<td>11'30''</td>
</tr>
</tbody>
</table>

Table 2: Simulation times comparison between Eurostag© and EMTP©

3.2. Calibration of the two software programs

3.2.1. The network

To get started, the network considered is the simplest two-node one as shown in Figure 12.
The studied machine represents a 20 kV nuclear group of 1 650 MVA. Voltage and mechanical power regulations have been removed in order not to introduce non-linear phenomena and to simplify to the highest study. The transformer is rated at 20/405 kV and Δ-Yn coupled. Power is distributed to an ideal infinite node through a line, modeled with a pure reactance (π-model with \( R = 0 \) and \( C = 0 \)). The two LF devices are used to force the initial Load-Flow situation (PV-, PQ- or Slack-bus) to the software: the synchronous machine produces for example 1 500 MW and 0 MVAR, and the connection node of the infinite node is set as the slack bus. For now, iron losses in the transformer are neglected.

The same network has been implemented in Eurostag©. First step is to simulate and compare the two load-flow results. They are quasi-identical, with sometimes divergences around 0.1% (on about 1 000 MW, so a 1 MW difference). This is totally acceptable, and the small difference is mainly due to the models of transformer used in Eurostag© and EMTP© that differ.

Then critical times of removal of faults (three-phase and line-to-ground faults) are computed in both models. To go further some parameters are modified, and load-flows and CTRs are calculated and compared. These parameters are:

- The number of lines in parallel \( \in \{1 ; 2\} \). In this study, when several lines are available, a short-circuit only appear on one line.
- The line length \( \in \{10 ; 100 ; 500 \text{ km}\} \). It has been chosen to model a line as a pure inductance, with negative- and zero-sequence equal to the positive-sequence \( X_{\text{line}} = 0,16 \Omega/\text{km} = 0,0001 \text{ pu/km} \).
- The injected power from the generator \( \in \{500 ; 1 000 ; 1 500 \text{ MW}\} \).
- The location of the fault \( \in \{1\% ; 90\%\} \). The percentage represents the distance from the origin node (generator). 1% is critical since the generator can’t evacuate any power, and 90% is considered as a faraway fault.

### 3.2.2. Results and comments

Detailed results are given in Table 3 (Appendix A). One can see that the CTRs are very close in the two software programs. The main gaps appear when the injected power is low (500 MW). This can be explained by the fact that low amount of power tends to stabilize the network (less lost power during the fault), resulting in the increase of the CTR. With increased stabilities, the small differences between Eurostag© and EMTP© have more impact on the CTR, the lost of synchronism is less sharp and happens at different instants. Graph 9 shows the generator’s speed after a line-to-ground fault at 1% on a 10 km line, with 1 000 MW injected (simulation n°3). Even if the calculated times of removal of one-phase faults are different, the machine’s behaviors stay quite identical.
Moreover, these two tools do not require the same inputs. To set an example, Eurostag© asks for the negative- and zero-sequence components of the generator’s impedance \((R_2, X_2, R_0, X_0)\), while EMTP© only needs the zero-sequence component of the reactance \((X_0)\) and automatically fixes \(R_0 = R_a\).

This section tried to calibrate the two networks in order to be able to “compare the same thing”. The goal was not to obtain exactly the same results, but closely enough to confirm that the software tools used give a similar and realistic illustration of physical reality. It seems they do! Future simulations run in both programs will be compared to check if the influences of various parameters are consistent.
4. Influence of parameters on the Critical Time of Removal of faults

4.1. Classical parameters

Classical parameters refer to the length of the line (basically its impedance), the power the synchronous machine injects into the system and the location of the fault along the line. First these factors are studied on a small grid (two nodes) in Eurostag© and EMTP© in order to be compared with the theoretical model developed in previous sections. Then the simulations are run in a much bigger set of connections (French 400 kV) in order to verify if the simplest network is representative enough of the actual one, meaning if the conclusions drawn from a two-node grid can be applied to actual networks.

4.1.1. Two-node network

To begin with, the system is simplified to the highest, in order to compare the simulation results with the theory. Same grid as in Section 3.2.1 (Figure 12) is used. Now only one parameter at a time is altered to clearly observe its impact. Fixed parameters are chosen as follow:

- One line of 100 km.
- Mechanical input power $P_m = 1500$ MW.
- Close short-circuit (1% from sending node).

It is important to remind the reader that theoretical analysis has been conducted with heavy assumptions. The idea was to approximate the CTR for given networks and observe the global behavior. In particular the generator has been modeled as a perfect voltage source $E$ behind its direct axis transient reactance $X_d'$. But from now on, a classical model of synchronous machine is applied, and the output voltage $E$ is no longer kept constant. This will have effects on transmissible powers after the fault. Therefore theoretical curves are not plotted with the new ones, since it wouldn’t mean much. However global theoretical trend will be discussed.

With that in mind, following graphs include:

- The critical times of removal of three-phase faults computed by Eurostag© and EMTP©, along with the gap between the two.
- The critical times of removal of one-phase faults powered by Eurostag© and EMTP©, with the gap.
- Three-phase CTR opposed to one-phase CTR.

EMTP© results correspond to red triangles while blue dots stand for Eurostag© outcomes, both scaled on the left axis. The green curves linked to the right axis symbolize gaps between the two software programs.
Graph 10: 3-phase CTR with line length variation on a 2-node network

Graph 11: 1-phase CTR with line length variation on a 2-node network

Graph 12: 3-phase CTR with power injection variation on a 2-node network

Graph 13: 1-phase CTR with power injection variation on a 2-node network

Graph 14: 3-phase CTR with fault location variation on a 2-node network

Graph 15: 1-phase CTR with fault location variation on a 2-node network
The curves have been scaled to give consistent values of critical times of removal. A CTR of 2 s after a three-phase fault is not representative of any physical reality. Typically under 500 MW injected in Graph 12, results are not shown.

**Line length**

For starter, the influence of the line length after a three-phase (Graph 10) or a one-phase (Graph 11) fault is very close to theory. The linear regression was accurate at 99.9% and 98% for theoretical three-phase and line-to-ground CTRs respectively. With Eurostag©, this values become respectively 99.7% and 98.1%. More, results from the two tools are very close, not only from a point of view “influence on the CTR” but also because the times are quasi-identical. The gap between them never exceeds 10 ms. As expected if the line impedance increases, it becomes more and more difficult for the synchronous generator to evacuate power, and the stability decreases.

*Note:* a “step” phenomenon can be observed on EMTP© results. Critical times are constant for several consecutive line lengths and suddenly change. This explains the gap function’s serrated periodicity.

**Injected power**

Then the amount of power injected through the line is altered. Like in the theoretical analysis, the shape of the curves look a lot like $1/\sqrt{P_m}$. When the electrical power is high enough, results from Eurostag© and EMTP© are very much alike. Above 1 000 MW, the mean absolute gap is 5 ms for a three-phase fault and 15 ms with one faulted phase. But if the network doesn’t transport any power (or not much compared to its rated capacity), it is very stable. So short-circuit an unloaded line will not do much to the generator, and high critical times are obtained. In these cases, the gap between results of both tools tend to increase, because the values are huge.

**Fault location**

As for the fault location’s influence, results are as expected. If the short-circuit appears far from the generator, the machine will be able to evacuate a small amount of its power during the fault, recovering a little bit of stability ($P_c^f(\delta) \neq 0$). Both programs give equal results, but the step phenomenon is more visible, since an increase of 1% in the fault location may give the same CTR with a precision of 1 ms.

Globally both Eurostag© and EMTP© confirm the trends and results obtained with the theoretical approach, even with the strong approximation made. The first one seems to fall out of synchronism quicker than the second one, which is good since Eurostag is currently used to settle and adjust protections on the lines (breakers and disconnectors among others).
All the simulations results are put in a same graph in order to study the variation of the line-to-ground CTR relative to the three-phase CTR. A previous RTE note showed that one-phase CTRs are always at least two times higher than three-phase ones [5]. They conducted this study on a two-node network, with the same methodology as this one. In Graph 16, this empiric law is verified. Only two simulations (out of 400) do not respect this criterion, but they are absolutely not relevant, with line lengths of 500 km and critical times of a few milliseconds.

For likely situations, the factor 2 is clearly pessimistic. In a model with one line of 100 km and 1 500 MW injected,

\[ T_c(1\varphi) = 3 \times T_c(3\varphi) \]

However at the time, he didn’t have a Fortescue representation of a large network, so this law hasn’t been checked on realistic situations. This is done in the next part.
4.1.2. French 400 kV network

Now all the simulations are powered by Eurostag© on a 400 kV French network model realized by RTE in previous work. The goal is to check if switching to a larger and more realistic network changes a lot the results. Of course critical times to remove faults will be different since the whole topology has changed, but do the previous parameters have the same impact on stability?

With some Python scripts and a few batch treatments, the 400 kV French network has been converted into Fortescue representation in Eurostag©. At each step load-flows and behaviors of the system in symmetrical three-phase mode were checked (a Fortescue description should not change the software responses to balanced perturbations).

The chosen location for the study is well-located at an extremity of the grid (more critical situation with only one exit path for electricity), with a large nuclear group (1 650 MVA), so loosing it may have drastic outcomes. The data used in the situation studied in terms of generations and loads come from an actual Convergence situation (2013/9/9, 10 am). The system is quite stable at this moment, so in order to observe dangerous conditions and have the most pessimistic approach, some modifications are made:

- Only one line is kept at the exit of the generator. The reactor will then be very sensitive to short-circuits on its only exit way.
- Several lines are simply put away to make it hard to reach some regions.

Thanks to these destabilizations, the critical time for removing a three-phase fault right behind the group’s transformer falls from 259 ms down to 130 ms, much more plausible. With this new situation and like before, the length of the exit line is modified, along with the power produced at the nuclear site. Results are presented below, and figures on the next page.

Even when switching to a large real network, the length of the faulted line has a linear influence on the critical time of removal of both three-phase and one-phase faults. In the unsymmetrical case, $R^2 = 99.98\%$. Nonetheless a small shift appears in the case of a balanced fault around 120 km, decreasing the precision of the linear regression to 97%. This shift is due to a non linearity introduced by the LVA (“Limiteur de Vitesse et d’Accélération” for “Speed and Acceleration Limiter”), a regulation loop designed to protect the turbine by dropping the mechanical power when high accelerations are detected (Appendix C). For short lines (< 120 km), it is activated right after the fault appears whereas for longer lines, it is not. This can be seen on Graph 23, where the lines of respectively 120 km and 125 km are short-circuited. Both generator speeds and LVA activation signals are plotted. Moreover, an expansion of ten kilometers decreases three-phase CTRs of about 6 ms and 13 ms for one-phase CTRs. With the simpler network these values were respectively 4 ms and 13.2 ms.

The previous behavior has changed a little. Now the mechanical power is no longer kept constant, and the curve (proportional to $1/\sqrt{P_m}$) do not look like an inverse square root anymore. Still, critical times decrease with the power, but less and less quickly (page 39). So again, the produced power has a strong impact on stability.

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7 RTE’s software
Influence of parameters on the Critical Time of Removal of faults

As for the fault location’s influence, results do not change (page 39). Coefficients of linear regressions are still high, with $R^2 = 99.7\%$ with a line-to-ground short-circuit. But since the software gives times with a 1 ms precision, a step phenomenon appears again with a three-phase fault, dropping $R^2$ down to 97\%. On the actual line (about 25 km), when a substantial amount of power (1 500 MW) transits through it, a short-circuit right behind the transformer or at the ending side gives a gap of 7 ms (resp. 70 ms) for a three-phase (resp. one-phase) fault.

Finally once again, it is confirmed that in a pessimistic scenario where a short circuit appears near a big nuclear power plant like the one studied, all unsymmetrical critical times are at least two times higher than balanced ones. This is shown in Graph 24.

In conclusion, the results derived from the simulation of the French 400 kV network in Eurostag© are quite close from those obtained with a simpler grid. The influence of the classical parameters stays identical. Widening the network and adding more synchronous machines didn’t affect much the tests. This means among other things that only the closest part of the grid from the fault location impact the stability. If the power generation is suddenly increased in the North, the South generator will not fall out of synchronism. More, it is now clear that the most influent constraint on stability is the power flowing through the transmission lines. The location of the fault is also significant in a one-phase fault scenario.

But studied parameters, like the length of the line, are known. It is now necessary to investigate those for which the value is uncertain or even undetermined.
Graph 17: 3-phase CTR with line length variation on 400 kV network

Graph 18: 1-phase CTR with line length variation on 400 kV network

Graph 19: 3-phase CTR with power injection variation on 400 kV network

Graph 20: 1-phase CTR with power injection variation on 400 kV network

Graph 21: 3-phase CTR with fault location variation on 400 kV network

Graph 22: 1-phase CTR with fault location variation on 400 kV network

\[ y = -0.6298x + 148.82 \quad R^2 = 0.9724 \]

\[ y = -1.299x + 324.71 \quad R^2 = 0.9998 \]

\[ y = -0.259x + 487.27 \quad R^2 = 0.9944 \]

\[ y = -0.9002x + 1536.6 \quad R^2 = 0.9652 \]

\[ y = 0.0627x + 129.43 \quad R^2 = 0.9731 \]

\[ y = 0.7247x + 296.59 \quad R^2 = 0.997 \]
Graph 23: LVA activation function and speed, for line lengths of 120 and 125 km

Graph 24: 3-phase CTR relative to 1-phase CTR on a the 400 kV network
4.2. Unknown parameters

Some parameters are not well-known, or known without precision. This might be due to old databases. Another cause is that some parameters are rarely/never used, and therefore no one has ever created any accurate database, and if someday this parameter is required, a simple approximation will do the trick. These are the parameters one needs to study the impact on critical times of removal of faults, to confirm that classical estimations of their values are enough and not misleading.

4.2.1. Group transformer grounding

Group transformers are rated at 20/400 kV, Δ/Yn coupled, with a high rated power. This kind of coupling disconnects the group from the grid in the equivalent zero-sequence circuit, as described in Figure 13. Parts (a) and (b) show respectively the positive- and negative-sequence diagrams of the transformer. Part (c) corresponds to the zero-sequence when the neutral is grounded via an impedance $Z_n$, the case of an ungrounded neutral ($Z_n = \infty$) being part (d). [14]

![Figure 13: Positive- (a), negative- (b) and zero-sequence (c and d) diagrams of a Δ/Y transformer](image)

The goal of neutral grounding in the Y-side (secondary side for a step-up transformer) is to protect people and devices by controlling fault insulations. During an asymmetrical situation (fault, sudden change of load or production, etc.) the zero-sequence component of the current is not zero anymore and can flow back. In France grounding is mandatory.

Earthing a transformer has mainly two impacts. With a low reactance (for example with a direct grounding), line-to-ground short-circuit currents can become greater than three-phase short-circuit currents and may exceed the maximal value. On the contrary with a large reactance (or infinite if neutral is floating), one-phase faults create transient surges, also incompatible with the device. The challenge is to find a reactance value high enough to limit short-circuits currents and sufficiently low to avoid strong surges and lightning protection damages. In France the slot [25 ; 40 Ω] is chosen [7].
Influence of parameters on the Critical Time of Removal of faults

However this grounding impedance is only used for unbalanced studies, which are quite rare. So the values are not always up-to-date. The classical rule is to choose $25\ \Omega$ for nuclear groups transformers and $40\ \Omega$ with thermal or hydraulic groups.

This study is conducted on the French $400\ kV$ network, destabilized as before. All grounding impedances are varied from 0 to $40\ \Omega$ with a 1 $\Omega$ step. In addition, the number of evacuation lines between the group and the grid is modified. One, two or three lines are turned on. Numerical results are given for information in Table 4 (Appendix B). Graph 25 sums them up and linear regressions are added. Of course, the impact of the grounding is only studied on asymmetrical faults since it has no impact on three-phase analysis.

![Graph 25: Influence of the grounding reactance on the critical time of removal of 1-phase faults](image)

As told, this graph confirms that increasing the grounding impedance improves the stability and thus increases critical times. Once again, this parameter has a linear influence on the CTR, with a mean coefficient $R^2 = 99.7\%$. With one more evacuation line (red), critical times increase, but when another one is added (green), the difference is tiny. If the Y-side becomes ungrounded, $T_c = 6.58\ s$.

In reality when one looks in the database RXH, except for a few exceptions, the 25 and 40 $\Omega$ rule is globally complied. But sometimes, 24 or 26 $\Omega$ reactances can be used. According to the results, a difference of 1 $\Omega$ gives an error of 8 ms which is totally acceptable.

However, there are some exceptions in the actual network. Let’s assume the group’s transformer for example is directly earthed. If the mistake of choosing $25\ \Omega$ is made, $T_c = 535\ ms$ whereas it should be $T_c = 315\ ms$. This is a 40% error!

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RXH: RTE’s database for lines, cables and transformers parameters values.
Influence of parameters on the Critical Time of Removal of faults

**Note:** one can notice that the critical time found when changing the studied group’s transformer grounding to zero only is the same that the one obtained with all reactances equal to zero. This confirms that only “local grid” parameters around the fault truly impact stability. See Section 4.1.2.

To conclude, the influence of the grounding impedance on stability is quite significant. But actual values are codified and if the “right kind” of reactance is used, a small error on the true value is acceptable. The user must nevertheless watch for rare exceptions like direct earthing in some transformers.

### 4.2.2. Coupling between lines

When two lines are located close to each other, a mutual coupling effect between them will appear. This means that whenever a current flows in one line, another current is induced in the other one, and voltage drops occur. Figure 14 illustrates the coupling phenomena undergone by a conductor.

![Figure 14: All mutual coupling effects occurring on Circuit 1’s A phase](image)

Let’s consider the voltage drop on phase A in Circuit 1. The equation is

$$V_A = Z_{AA} \cdot I_A + \frac{(Z_{AB} \cdot I_B + Z_{AC} \cdot I_C)}{Circuit\ 1} + \frac{(Z_{AA} \cdot I_A + Z_{AB} \cdot I_B + Z_{AC} \cdot I_C)}{Circuit\ 2}$$  \hspace{1cm} (4.1)

It is simpler to assume that phase B and phase C have the same influence on phase A, thanks to transposition tricks. So $Z_{AB} = Z_{AC} := Z_M$ · Moreover, the distance between the two circuit is long enough to consider $Z_{AA} = Z_{Ab} = Z_{Ac} := Z_{m0}$. Therefore, Equation (3.3) boils down to

$$V_A = Z_{AA} \cdot I_A + Z_M \cdot (I_B + I_C) + Z_{m0} \cdot (I_a + I_b + I_c)$$  \hspace{1cm} (4.2)

It is clear that the influence of Circuit 2 only appears in the zero-sequence, since in both positive- and negative-sequence, the sum of the currents $I_a + I_b + I_c = 0$. So this coupling between OHL is only taken into account when unbalanced experiences are conducted, and has an effect on the zero-sequence only. Its value depends mainly on the distance between conductors. [8]
As seen before, only the portion of the grid located near the fault will have impact on stability. So couplings will be added to the model only for a few lines. Databases provide values for line coupling. Here it is decided to take those as reference, and to vary them between zero and twice this nominal value. That is why graph abscises go from 0 to 200%.

For the first test, the previous destabilized network is modified again, by adding a second line of evacuation with the purpose of coupling them. For the time being in order to observe more clearly the evolution of critical times, the group’s transformer grounding impedance is kept to 25 Ω. Now, two sets of lines are coupled: both lines of evacuation and two lines further. Simulations results are presented on the left side of Graph 26. One can see that taking line coupling into consideration tends to increase CTRs and stability. With the “true” coupling values, $T_c = 529$ ms. If an error of 100% (quite unlikely) is made, the CTR changes by only 12 ms.

Next, two lines are added at the end of the evacuation path to study the impact of coupling on a longer trajectory. This will also induce much higher CTRs. So in compensation, group’s grounding is decreased to 0 Ω (direct grounding). Graph 26 (right) shows the results of simulations when only one set of lines is coupled and then both. Several comments can be stated. First the red curve is above the blue one, simply because the second line is longer than the first evacuation path. As for the green curve, supposed to reflect the “factual” situation, it varies more or less linearly between 583 and 595 ms, with a critical time $T_c = 590$ ms. So a 100% mistake may lead to a 5 ms gap, which is nothing. The software can’t be actually precise to the millisecond.

In conclusion, the line coupling phenomenon has virtually no impact when unbalanced faults are involved. Therefore it might be unnecessary to implement it on Eurostag for this kind of transient stability studies, especially with large networks.

Graph 26: 1-phase CTR with mutual coupling variation on 400 kV network (test 1 on the left side, test 2 on the right side)
4.2.3. Loads in symmetrical representation

When switching the French 400 kV network into Fortescue representation, Eurostag® requires a negative- and zero-sequence component for the loads \([15]\). Starting from the symmetrical components theory presented in Section 1.1, let’s define the three-phase load matrix as

\[
Z_{abc} = \begin{bmatrix}
Z_s & Z_m & Z_m \\
Z_m & Z_s & Z_m \\
Z_m & Z_m & Z_s \\
\end{bmatrix}
\]  \( (4.3) \)

with \(Z_s\) the symmetrical load on each phase and \(Z_m\) the mutual impedance. From there,

\[
V_{abc} = Z_{abc} \cdot I_{abc}
\]

\[
A \cdot V_{012} = Z_{abc} \cdot A \cdot I_{012}
\]

and

\[
V_{012} = A^{-1} \cdot Z_{abc} \cdot A \cdot I_{012}
\]

After running the calculations, the load matrix in Fortescue is

\[
Z_{012} = \begin{bmatrix}
Z_s + 2 \cdot Z_m & 0 & 0 \\
0 & Z_s - Z_m & 0 \\
0 & 0 & Z_s - Z_m \\
\end{bmatrix}
\]  \( (4.4) \)

and in particular, \(Z_1 = Z_2 = Z_s - Z_m\).

But the mutual coupling for loads are not known. These loads can stand for actual consumptions as well as power drawing (at a transformer station for example, to represent a whole portion of the national grid at a lower voltage, not included in the simulation file). In the second case, it doesn’t have much sense to talk about mutual coupling between phases. When tested on a small 2-node grid, the impact of the load zero-sequence component was totally negligible.

For this parameter, the user should choose \(Z_m = 0\) and then enter in Eurostag® \(Z_1 = Z_2 = Z_0\).
Conclusion

This master thesis had several goals. The first one was to have a better understanding of line-to-ground faults in an AC system. Solving equations after a three-phase short-circuit is an easy thing to do, but as soon as unbalances come around, the problem gets trickier! This note presented how to get away with these issues by using the symmetrical components method.

First of all, a theoretical analysis were conducted in order to find critical times of removal of faults with a simplified network. When comparing these results with actual simulations, one could see that the results, and especially the global trends, were very much alike. However it was not possible to obtain exactly the same values for CTRs due to strong approximations that had to be made.

Then using Eurostag© and EMTP©, a lot of simulations have been run and several conclusions were drawn. Small two-node networks are quite representative of actual large ones. Indeed it is the local area of the grid that influences the most a generator. So even with a large network, it could be sufficient for some studies to only model a portion of the network, and replace the rest with equivalent impedances and injections. Classical results have been found: to set an example, stability decreases when the grid impedance or the electrical power flowing through the lines increase.

But the main part was to observe the impact of the unknown parameters. Except for grounding impedance, a small error on their value is not critical for the study. But with the wrong earthing reactance, one could make mistakes of several hundreds of milliseconds when dealing with CTR. However, since that value is codified, when one knows which kind of grounding he is dealing with, a small error on the grounding is harmless.

Beyond this first goal, another objective was to have a better knowledge of the parameters required by Eurostag© when using a Fortescue representation. This note can be used by anyone who wants to describe a network that way.

The reader must keep in mind that most of the conclusions found in this document come from simulations. Approximations are often made. So to apply these results one should check them first. For example, all fault clearings have been done without opening the lines, which is not the actual way of doing it.
### Appendix A: Results of software calibration tests

<table>
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<tr>
<th>N° sim</th>
<th>Nb. lines (km)</th>
<th>Line length (km)</th>
<th>Pinj (MW)</th>
<th>Fault loc. (%)</th>
<th>Gap max</th>
<th>Tc Eurostag (ms)</th>
<th>Tc EMTP (ms)</th>
<th>Gap (%)</th>
<th>Tc Eurostag (ms)</th>
<th>Tc EMTP (ms)</th>
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<td>301</td>
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<td>451</td>
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<td>∞</td>
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<td>640</td>
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</table>

**Table 3: Calibration and simulation results**

This table sums up the successive tests conducted in Section 3.2. Each row is a simulation, described by an id, the network parameters (number of lines, line length, injected power, fault location), the maximal gap between the two load-flow solutions and the CTRs (and their difference) on three-phase and one-phase fault.

Green cells show gaps between CTRs less than 10 ms for three-phase faults and 20 ms for line-to-ground faults. Conversely red cells indicates CTRs higher than 20 ms and 50 ms respectively.
Appendix B: Results of the transformer grounding’s influence analysis

### Table 4: Influence of grounding reactance on the critical time of removal

<table>
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<tr>
<th>Grounding (Ω)</th>
<th>$T_c$ (3φ)</th>
<th>Critical times after a line-to-ground short-circuit (ms)</th>
</tr>
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<tbody>
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<td></td>
<td>0 1 2 3 4 5 6 7 8 9 10</td>
<td></td>
</tr>
<tr>
<td>1 line</td>
<td>130</td>
<td>295 305 315 325 336 345 354 363 373 382 391</td>
</tr>
<tr>
<td>2 lines</td>
<td>135</td>
<td>311 321 332 343 353 363 373 382 392 400 410</td>
</tr>
<tr>
<td>3 lines</td>
<td>137</td>
<td>315 325 336 347 357 367 377 387 396 405 414</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>398</td>
<td>407</td>
<td>416</td>
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<td>427</td>
<td>436</td>
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<td>440</td>
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<td>28</td>
</tr>
<tr>
<td>516</td>
<td>522</td>
<td>529</td>
</tr>
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<td>539</td>
<td>546</td>
<td>554</td>
</tr>
<tr>
<td>543</td>
<td>550</td>
<td>558</td>
</tr>
</tbody>
</table>
Appendix B: Results of the transformer grounding’s influence analysis
Appendix C: LVA regulation loop

The LVA is a regulation loop used to protect nuclear and thermal groups in case of short-circuits [13]. When a high speed or derivative of speed (acceleration) is detected, this regulator generate an activation signal that decreases the input of mechanical power in the concerned synchronous generator. That way, this will slow down the rotor and delay (or if one is lucky avoid) the generator to fall out of synchronism.

It is more used with three-phase faults than with asymmetrical ones. Indeed, with a line-to-ground fault, power can still run through the line, so the acceleration of the rotor will be less important than in a balanced case.

Figure 15 presents the signal activation that controls the regulation loop.

![Figure 15: LVA (Limiteur de Vitesse et d’Accélération)]
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