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Optimal Steering for Double-Lane Change Entry Speed Maximization

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This study introduces a method for estimating the vehicle’s maximum entry speed for an ISO3888 part-2 double-lane change (DLC) test in simulation. Pseudospectral collocation in TOMLAB/PROPT calculates the optimal steering angle that maximizes the entry speed. The rationale is to estimate the vehicle’s performance in the design phase and adapt the tuning to improve DLC ratings. A two-track vehicle dynamics model (VDM) employing non-linear tires, suspension properties and a simplified Dynamic Stability and Traction Control (DSTC) system was parameterized as a 2011 T5 FWD Volvo S60 using in-field tests and its corresponding kinematics and compliance (K&C) measurements. A sensitivity analysis on the parameters revealed certain trends that influence the entry speed, which can be varied from 69.4 up to 73.3 km/h when adapting certain vehicle features. To evaluate the method, the generated optimal steering control inputs for the simulated S60 were applied on the actual car motivating the further development of the method.

Vehicle dynamics: steering, brake, tire, suspension, optimization, simulation

1. INTRODUCTION

Tuning a vehicle, and more specifically the DSTC, involves physical vehicle testing; a time consuming and costly procedure. It is normally performed in an early phase in the development process where only prototype vehicles are available. The corresponding vehicle’s performance from the tuning is rated by independent organizations, such as the EuroNCAP, using tests such as the ISO 3888 part-2 DLC (c.f. Fig. 1). EuroNCAP assesses the DSTC by performing a series of tests where the steering and yaw behaviour can be simultaneously evaluated [1]. The DSTC and the vehicle's handling are also rated subjectively [2] [3].

![Fig. 1 ISO 3888 part-2 double-lane change; instance from testing.](image)

Although the aforementioned methods are often used for handling rating, they are sometimes characterized unsuitable for objective assessment of the vehicle’s performance, because the driver is involved in the control loop [4]. Objectivity can be ensured by examining solely the vehicle’s behaviour. Substituting test drivers by a controller, which would generate the optimal steering inputs for achieving maximum entry speed, would enable the definition of an objective performance metric [5] and a tool to assess the vehicle's handling, early in the development process.

It is envisioned that in an effort to improve development efficiency, promote safety and reduce prototype vehicles, the DSTC tuning in future vehicles will be achieved using computer-aided-engineering (CAE) tools. This is expected to reduce cost and lead-time, facilitate objective assessment of the car's safety and offer numerically optimized tuning sets; better and safer cars for the road.

2. OPTIMAL STEERING INPUT GENERATION

The optimal steering input generation can be formulated as an optimal control problem, with the objective to maximize the vehicle’s entry speed while satisfying the vehicle dynamics and DLC path constraints. The augmented objective function of this problem can be given as:

$$J = -V_x\big|_{x=0} + W_\delta \int_{t=0}^{t=t_{final}} \delta^2 dt + W_{\psi} \int_{t=0}^{t=t_{final}} \psi^2 dt$$

(1)

where $V_x$ denotes the vehicle’s longitudinal speed, $t_f$ the time needed to complete the manoeuvre, $\delta$ the steering rate, $\psi$ the yaw rate and $W_\delta, W_\psi$ are weighting factors. The objective function aims to minimize the negative entry speed $-V_x\big|_{x=0}$ (corresponding to maximization of the entry speed) and
is augmented with 3 energy related terms \( (\dot{\delta}^2, \dot{\psi}^2, \tau_f) \) so as to regulate the optimal steering input. This approach is motivated by the numerical difficulties that arise when solving a boundary value problem associated with the original “bang-bang” optimal control problem. It can be noticed that the obtained auxiliary optimal control with the augmented objective function approaches the original optimal control problem as \( W_{\delta}, W_{\psi} \) and \( W_{\tau_f} \) approach zero. The solution of the initial optimal control problem is then reduced to the solution of a sequence of auxiliary optimal control problems.

2.1 Optimization method

The infinite dimensional optimal control problem defined above is converted into a finite dimensional optimization problem using a direct transcription method and the resulting optimization problem is solved using TOMLAB/PROPT [6] in Matlab. PROPT uses a Gauss pseudospectral collocation method for solving the optimal control problem, meaning that the solution takes the form of a polynomial, and this polynomial satisfies the differential algebraic equations (DAE) and the path constraints at the collocation points.

2.2 Path constraints

![Image](image.png)

Fig. 2. ISO 3888 course, part-2 obstacle avoidance manoeuvre (DLC).

\[
\begin{align*}
1 + \frac{A}{2} & \left ( 1 + \frac{X - 25.2}{a_{tr}} \right ) - \frac{A}{2} \\
1 + \frac{C}{2} & \left ( 1 - \frac{X - 36.8}{a_{tr}} \right ) - 1 + C \\
1 + \frac{B}{2} & \left ( 1 + \frac{X - 12.3}{a_{tr}} \right ) \\
\frac{A}{2} + \frac{1 + B}{2} & \left ( 1 - \frac{X - 48.5}{a_{tr}} \right ) - 1 - B
\end{align*}
\]

\( (2) \)

The DLC track is shown in Fig. 2. The track boundaries are described with mathematically smooth functions, since discontinuities are not recommended optimization problems [6]. Equation (2) describes the lateral position boundary; the terms \( A, B \) and \( C \) correspond to the notation used in Fig. 2 while \( a_{tr} \) defines the smoothness of each corner. The body dimensions (four sides of the car) were discretised into body-points subject to the track constraints.

2.3 Complexity built-up hierarchy

The optimization method involved an iterative process starting with a centreline guess of a point mass, with the results used as the guess to the next more sophisticated VDM.

2.3.1 Initial guess

The first optimization problem started from a guess of the manoeuvre completion time \( t_g = 3.05 \) s and 15% percentage drop of the entry speed \( v_g \). The states of the vehicle were then estimated according to eq. (3) to (7).

The subscript \( g \) stands for guess.

\[
X_g = \frac{6 \cdot t_g}{t_g} \quad (3)
\]

\[
Y_g = \frac{A + B + 2}{B + C + 2} \left ( \frac{25.2}{a_{tr}} \right ) + \frac{A}{2} - \frac{C}{2} \left ( \frac{36.8}{a_{tr}} \right ) - 1 + \frac{B}{2} \left ( \frac{12.3}{a_{tr}} \right ) - 1 + \frac{B}{C} \left ( \frac{48.5}{a_{tr}} \right ) - 1 - B
\]

\( (4) \)

\[
v_{x_g} = \frac{6 \cdot t_g}{t_g} \left ( 1 + \frac{v_g}{100} - 1 \right ) \quad (5)
\]

\[
v_{y_g} = \frac{6 \cdot t_g}{t_g} \left ( 1 + \frac{v_g}{100} \right ) \quad (6)
\]

\[
\psi_g = \frac{\tan^{-1} \left ( \frac{v_g}{t_g} \right )}{3} \quad (7)
\]

2.3.2 Point mass model

The VDM used for the 1\textsuperscript{st} optimization step is a point mass model with acceleration and body-dimension constraints. The kinematics equations used are given from (8) to (13).

\[
\begin{align*}
\dot{X}_g &= \hat{X} \\
\dot{Y}_g &= \hat{Y} \\
a_x &= a_x \\
a_y &= a_y \\
\psi &= \frac{\tan^{-1} \left ( \frac{v_g}{t_g} \right )}{3} \\
\dot{V}_g &= V \cdot \cos(\psi) \\
\hat{V}_g &= V \cdot \sin(\psi) \\
a &= a_x^2 + a_y^2
\end{align*}
\]

\( (8) \) to \( (13) \)

The point mass model assumes that the vehicle’s longitudinal velocity \( V_x \) will always be tangent to the trajectory (10) (00 slip angle) and that the resultant velocity will be constant throughout the manoeuvre (13). The acceleration constraints derive from the vehicle’s technical specifications: time from 0 to 100 km/h (\( t_{0-100} \)) in 6.6 s and brake distance from 100 to 0 km/h in 37 m (\( d_{100-0} \)).

\[
\begin{align*}
-\frac{(100/3.6)}{2 \cdot a_{max}} & \leq a_x \leq \frac{100}{t_{0-100}} \text{m/s}^2 \\
|a| & \leq 0.1 \text{ m/s}^2 \\
|\psi| & \leq a_g = 8.9 \text{ m/s}^2
\end{align*}
\]

\( (14) \) to \( (16) \)

The optimal solution search was performed sequentially with \( n = 20, 40, 60 \) and 90 collocation points [7] with the weight factors \( W_\delta = 0.05, W_{\tau_f} = 0.4 \) and \( W_{\psi} = 0.1 \) for eq. (1).

2.3.3 Single-track model (STM) with linear tires

A 3 degree-of-freedom (DOF) (translational and yaw motion) STM [8] was used as the next VDM. The STM assumes small angles, lateral tire forces which are linearly dependent on their slip angles and longitudinal aerodynamic drag force; the equations of motion can be found in the literature [9, pp. 29, 97]. The steering angle \( \delta \) constitutes the sole control variable for the optimization. The solution search was performed with \( n = 30, 50, 60 \) and 80 collocation points sequentially with cost function of eq. (1) and the same weight factors with the point mass model. Furthermore, the corresponding variable constraints are given in Table 1.

<table>
<thead>
<tr>
<th>Variable constraints</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_x \geq 10 )</td>
<td>Longitudinal speed (m/s)</td>
</tr>
<tr>
<td>(-20 \leq V_y \leq 20)</td>
<td>Lateral speed (m/s)</td>
</tr>
</tbody>
</table>
2.3.4 STM with non-linear tires

The results from the STM with linear tires constitute the initial guess for a STM with non-linear tires. The tire model used here is a simplified Magic Formula [10] (c.f. eq. (17)) where D, C and B are the peak value factor, shape factor and stiffness factor respectively. $F_{zi}$ is the normal load on the axle; the indices $i, j$ in the rest of the paper stand for $i$: $f$ (front) $r$ (rear); $j$: $l$ (left) $r$ (right). The product $BCD \cdot F_{zi}$ represents the cornering stiffness $C_i$ on the axle.

$$ \mu(s) = \frac{1}{\sin(C \tan^{-1}(Bs))} $$

(17)

The resultant tire slip $s_i$ for each tire was defined as in eq. (18). $V_{ix}$ and $V_{iy}$ are the $x$ and $y$ velocity components on the tire frame, $s_{ix}$ and $s_{iy}$ (19) the corresponding tire slips and $r$ the wheel radius.

$$ s_i = \sqrt{s_{ix}^2 + s_{iy}^2} $$

(18)

$$ s_{ix} = \frac{V_{ix} \cdot \alpha_t r_i}{\alpha_t r_i} $$

$$ s_{iy} = \frac{V_{iy} \cdot \alpha_t r_i}{\alpha_t r_i} $$

(19)

$$ \mu_{ix} = \frac{\alpha_t r_i}{\alpha_t r_i}, \mu_{iy} = \frac{-\alpha_t r_i}{\alpha_t r_i} $$

(20)

Eq. (20) calculates the tire’s x and y friction coefficients. The front $F_{zf}$ and rear $F_{zr}$ normal forces at the tires are calculated with eq. (21) with $\frac{mh}{L} a_x$ being the longitudinal acceleration induced load transfer, due to the height $h$ of the CG. The tire forces at each tire frame are given in eq. (22).

$$ F_{zf} = \frac{mg}{L} + \frac{mh}{L} a_x, F_{zf} = \frac{mg}{L} + \frac{mh}{L} a_x $$

$$ F_{zi} = \mu_{ix} F_{zf}, F_{yi} = \mu_{iy} F_{zf} $$

(21)

(22)

The dynamical equations for the model are given through (23) to (29). In (23) to (29), $m$ is the vehicle’s mass, $I_x$ its moment of inertia around the vertical axis (yaw inertia), $I_{yw}$ the moment of inertia of each wheel and front $T_f$ and rear $T_r$ the applied torque [9] [11]. The objective function (1) was the same as in the linear tires’ case in the STM using once again an iterative solution strategy with increasing number of collocation points; $n = 50, 60$ and $80$. The constraints changed and/or added to the linear STM constraints (c.f. Table 1) appear in

Table 2. Constraints for the non-linear STM model.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10/r \leq \omega_{ij} \leq 80/r$</td>
<td>$i, j$ wheel’s rotational speed rad/s</td>
</tr>
<tr>
<td>$\omega_{r1} \leq 1.2V_s$</td>
<td>Indirect limitation on $i$ (front/rear) wheel slip</td>
</tr>
</tbody>
</table>

2.3.5 Two-track model

The final VDM comprised of a 7-DOF VDM (longitudinal, lateral and yaw movement and the 4 wheels’ rotational dynamics) including roll (static load transfer), non-linear tires (’87 Magic formula [12]) with transient effects (tire-relaxation [13]) and suspension properties (lateral force compliance, camber change, roll steer and roll stiffness [13]) as well-as-a simplified DSTC implementation. A DSTC was modelled as a yaw rate error controller utilizing trigonometric functions for approximating its discontinuous behaviour; discontinuities are not recommended in optimization problems [6].

The current optimization step was initialized using the non-linear STM result as a start guess and the same weight factors and objective function as with the non-linear tires STM. The adapted constraints appear in Table 3.

Table 3. Constraints for the full vehicle model.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10/r \leq \omega_{ij} \leq 80/r$</td>
<td>$i, j$ wheel’s rotational speed rad/s</td>
</tr>
<tr>
<td>$\omega_{r1} \leq 1.2V_s$</td>
<td>Indirect limitation on $i, j$ wheel slip</td>
</tr>
</tbody>
</table>

2.3.5.1 Wheel kinematics; roll kinematics, roll steer and lateral force compliance

The vehicle’s centre-of-gravity (CG) lies at a certain height above the ground changing the wheels’ normal load and roll angle. Eq. (30) to (33) calculate the induced load transfer [14, p. 683] and static roll angle $\phi$ (33); $K_f/K_r$ are the front/rear roll stiffness$^*$, $e_i/e_r$ is the height of the roll centre at the front/rear and $h_e$ is the distance of the CG from the roll axis.

$$ F_{stf} = \frac{mg b}{L} \cdot \frac{m}{L} a_x - \frac{m}{L} G_{front} a_y $$

$$ F_{sre} = \frac{mg b}{L} \cdot \frac{m}{L} a_x - \frac{m}{L} G_{rear} a_y $$

(30)

with $G_{front} = \left( K_f + K_r - \frac{mg h_e}{b} \right) b$ $G_{rear} = \left( K_f + K_r - \frac{mg h_e}{b} \right) b$

(31)

$^*$ The term roll stiffness includes not only the stiffness imposed by antroll bars, but also from the suspension geometry, springs, frame, and all the factors that contribute to the axle’s total roll stiffness in general.
with 

\[
\delta_{ref} = \left( \frac{h_v K_v}{K_f + K_r - mg h_e} + f \right) \frac{f_{ax} + b_{ey}}{L}
\]

\[\varphi = \frac{m h_e}{K_f + K_r - mg h_e} a_y\]

(32)

(33)

During cornering, the front wheel angle can change due to a) roll steer induced from body roll motion and b) due to lateral force compliance steer induced by the suspension’s compliance to lateral forces applied at the tire-road contact [14] [15] [16].

Table 4. Roll steer coefficients \( \frac{\delta_i}{\varphi} \text{[deg/deg]} \) for the Volvo S60.

<table>
<thead>
<tr>
<th>Left wheel</th>
<th>Right wheel</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front axle</td>
<td>-0.135</td>
<td>-0.111</td>
</tr>
<tr>
<td>Rear axle</td>
<td>-0.00</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

The ratio of the induced wheel angle over the corresponding roll angle is the roll steer coefficient \( \frac{\delta_i}{\varphi} \) (steering angle \( \delta \) function of the roll angle \( \varphi \)). The roll angle is positive when the vehicle leans to the right as seen from the rear. The roll steer coefficient can be measured using kinematics and compliance (K&C) tests; the values used for the Volvo S60 are shown in Table 4 (the mean value of the right and left wheel’s roll steer was used for the front and rear axles). The values depict that during cornering the front wheels steer outwards with respect to the curve; the rear wheels have negligible roll steer. For the front axle, a negative roll steer coefficient results in an understeer effect and the opposite applies for the rear axle [15]. The change in the steering angle due to roll steer is calculated with (34).

\[
\delta_{ref} = \frac{\delta_i}{\varphi}, \delta_{ref} = \frac{\delta_i}{\varphi}
\]

(34)

The lateral force compliance steer can be regarded as the wheel steering angle change when a lateral force is applied at a) the tire-ground contact patch at the wheel centre \( (X = 0) \) and b) at a distance of \( X = 30 \text{ mm} \) behind the centre of the tire-ground contact patch. The distance \( X = 30 \text{ mm} \) is an approximate value for a typical tire’s pneumatic trail at small slip angles [13]. For small slip angles/linear tire region the pneumatic trail is almost constant. For larger slip angles/non-linear tire region the pneumatic trail reduces [14] [13] [10]. The wheel steering angle change will therefore depend on the distance from the contact patch centre where the lateral force will be applied. The lateral force compliance steer coefficient \( LF_{cij} \) is a function of the pneumatic trail and in principle interpolates linearly the lateral force compliance steer [deg/kN] between its value for \( X = 0 \text{ mm} \) and \( X = 30 \text{ mm} \). The resultant formula is given in Table 5.

Table 5. Lateral force compliance steer coefficient \( LF_{cij} \).

<table>
<thead>
<tr>
<th>Left wheel</th>
<th>Right wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front axle</td>
<td>-0.135</td>
</tr>
<tr>
<td>Rear axle</td>
<td>-0.00</td>
</tr>
</tbody>
</table>

1 Even though this is undesirable, it is a very common characteristic of most of the suspension and steering systems, which depends on their geometry [6, 9].

2 The word behind here indicates the direction that is opposite to the tire’s longitudinal travelling direction at the tire frame’s coordinate system.

The front and rear axle lateral force compliance steer is given in (35) and is the mean of the left and right wheel of the corresponding axle (36). The tire’s pneumatic trail is calculated as in (17).

\[
\delta_{cij} = \frac{L_{f_{cij}} - L_{r_{cij}}}{2}
\]

(35)

\[
\delta_{c} = \frac{\delta_{c_{left}} + \delta_{c_{right}}}{2}
\]

(36)

2.3.5.2 Tire lateral dynamics and camber thrust

A tire will typically require half to one rotation to build its steady state lateral force [9]; this distance can be referred as the relaxation length \( L_{relax} \); this transient behaviour can be modelled through the first order differential eq. (37) [9, p. 429] where \( \tau \) is a time constant and \( f_{yy} \) is the steady state value of the lateral force for a given slip angle \( \alpha \). The time constant is related to the relaxation length as in (38) where \( V_z \) is the tire’s longitudinal velocity. According to [10], the higher the slip angle, the shorter the relaxation length becomes.

\[
\tau = \frac{L_{relax}}{V_z}
\]

(37)

(38)

During cornering the camber angle \( \varepsilon \) of the wheel with respect to the body changes; the camber angle gain \( \frac{\delta_{c_{ij}}}{\varphi} \) with respect to body roll for the S60 is given in Table 6.

Table 6. Camber gain per roll angle \( \frac{\delta_{c_{ij}}}{\varphi} \text{[deg/deg]} \).

<table>
<thead>
<tr>
<th>Left wheel</th>
<th>Right wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front axle</td>
<td>+0.243</td>
</tr>
<tr>
<td>Rear axle</td>
<td>+0.452</td>
</tr>
</tbody>
</table>

The camber thrust, the lateral force due to tire camber angle, derives from the wheel’s camber-inclination angle \( \gamma \) relative to the ground; the left and right angle \( \gamma_{ij} \) is calculated with (40).

\[
\epsilon_{ij} = \frac{\epsilon_{lj}}{\varphi} \epsilon_{ij} = \frac{\epsilon_{lj}}{\varphi}
\]

(39)

\[
\gamma_{ij} = \gamma_{jl} \gamma_{ij} = \gamma_{jl}
\]

(40)

\[
F_{yij}(a, t) = \left( F_{yij}(a) - F_{yij}(a, t) \right) \frac{V_{yij}}{V_{relax}}
\]

(43)

\[
m_{ij}(\psi_j - \psi_0) = (F_{x_{ij}} + F_{y_{ij}}) \cos \delta_i + (F_{r_{ij}} + F_{r_{c_{ij}}}) \cos \delta_i - (F_{y_{ij}} + F_{r_{ij}} + F_{r_{c_{ij}}}) \sin \delta_i - (F_{y_{ij}} + F_{r_{ij}} + F_{r_{c_{ij}}}) \sin \delta_i - \frac{1}{2} \rho A C_d V_z^2
\]

(44)

\[
m_{ij}(\psi_j + \psi_0) = (F_{y_{ij}} + F_{r_{ij}}) \cos \delta_i + (F_{r_{ij}} + F_{r_{c_{ij}}}) \cos \delta_i - (F_{y_{ij}} + F_{r_{ij}} + F_{r_{c_{ij}}}) \sin \delta_i - (F_{y_{ij}} + F_{r_{ij}} + F_{r_{c_{ij}}}) \sin \delta_i + f_{yy}(t) \cos \delta_i
\]

(45)

\[
I_{\psi_j} = \frac{1}{2} I_{\psi_j}
\]

(46)
The equations for the two-track model are given through (42) to (47) with \( m \) the vehicle’s mass, \( \omega_i \) the moment of inertia of each wheel, \( F_{yi} = g(q) \) the steady state lateral force (23) of the \( i,j \) wheel for a given slip angle \( \alpha \), \( W_i, W_j \) the distance of the right/left wheel from the axle’s centreline and \( T_{ij} \) the drive/brake torque on the \( i,j \) wheel respectively.

### Dynamic stability and traction control; DSTC

The DSTC logic has been implemented using the DSTC braking logic is summarized below:

1. rear left ESC\(_{CI}^r\): \( \psi_d > 0 \) and \( \psi_e < 0 \).
2. rear right ESC\(_{CR}^r\): \( \psi_d < 0 \) and \( \psi_e > 0 \).

Oversteer, thus brake:

- 3) front left ESC\(_{CI}^l\): \( \psi_d < 0 \) and \( \psi_e < 0 \).
- 4) front right ESC\(_{CR}^r\): \( \psi_d > 0 \) and \( \psi_e > 0 \).

The DSTC logic has been implemented using continuous functions as in eq. (50) to (53).

\[ \psi_e = \psi - \psi_d \]

\[ \psi_d = \frac{\dot{\psi}}{\dot{R}} = \frac{(f + b) - \max^2(\frac{F_{ea} - bC_{a}}{2C_{arg}f} + f)}{f + b} \]

The DSTC braking logic is summarized below:

1. rear left ESC\(_{CI}^r\): \( \psi_d > 0 \) and \( \psi_e < 0 \).
2. rear right ESC\(_{CR}^r\): \( \psi_d < 0 \) and \( \psi_e > 0 \).
3. front left ESC\(_{CI}^l\): \( \psi_d < 0 \) and \( \psi_e < 0 \).
4. front right ESC\(_{CR}^r\): \( \psi_d > 0 \) and \( \psi_e > 0 \).

The DSTC logic has been implemented using continuous functions as in eq. (50) to (53).

\[ \psi_e = \frac{1}{4} \left( 1 + \tanh \left( \frac{\psi_e}{a} \right) \right) \left( 1 + \tanh \left( \frac{\psi_d}{a} \right) \right) \]

\[ \psi_d = \frac{1}{4} \left( 1 + \tanh \left( \frac{\psi_d}{a} \right) \right) \left( 1 + \tanh \left( \frac{\psi_e}{a} \right) \right) \]

\[ T_{ij} = \frac{1}{4} \left( 1 + \tanh \left( \frac{\psi_d}{a} \right) \right) \left( 1 + \tanh \left( \frac{\psi_e}{a} \right) \right) \]

\[ T_{ij} = \frac{1}{4} \left( 1 + \tanh \left( \frac{\psi_e}{a} \right) \right) \left( 1 + \tanh \left( \frac{\psi_d}{a} \right) \right) \]

The threshold braking torque at individual wheel is given by (54); it is a function of the \( \psi_e \) (when the error is greater than the threshold \( \psi_t \), \( T_{init} \) is a linear increase gain, \( T_{ij} \) the increase factor and \( a \) the smoothness factor (c.f. Fig. 3).

Fig. 3 DSTC torque characteristics; \( \psi_e = 2^o, T_{init} = 200 \text{Nm}, T_{ij} = 5 \) and \( a = 0.005 \).

### Two-track model optimization process

The optimal solution search is performed in 6 steps:

1. Set the start and finish point at the middle (\( Y \) position) of the 2 lines that define the lateral constraints of the track. The solution search was performed sequentially with \( n = 40, \) 60 and 80 collocation points.
2. Solve the problem again using as a starting point the solution of the previous problem, without constraining the start (\( Y \) position) to the middle: \( n = 30, \) 50 and 80.
3. Add suspension-wheel kinematics and camber thrust (c.f. 2.3.5.1 and 2.3.5.2); \( n = 40, \) 60 and 80.
4. Add DTSC (c.f. 2.3.5.3); \( n = 20, \) 25, 40, 60 and 80.
5. Add tire lateral transient behaviour-dynamics (c.f. 2.3.5.2); \( n = 30 \) and 50, \( W_{tf} (1) \) is set to 0.6.
6. Finally the solution is sought again incorporating all the above features but with increased number of body-points (c.f. 2.2); \( n = 30, \) 50, 60, 70 and 90.

### Parameterization

The vehicle’s nominal parameterization (c.f. Table 7) derives from Volvo’s corresponding K&C measurements and design data. The vehicle’s yaw inertia was estimated with the empirical formula \( I_z \approx 0.46 \text{mL}^2 \) yielding a value of \( I_z = 3500 \text{kgm}^2 \).
optimization process was run multiple times from different starting values, i.e. guesses, such that the case of local optima around a global one would be considered. Fig. 4 illustrates the model measured and simulated results using the STM after the optimization process.

<table>
<thead>
<tr>
<th>Table 7. VDM nominal data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter description</td>
</tr>
<tr>
<td>Model name</td>
</tr>
<tr>
<td>Mass (driver + equipment) (m)</td>
</tr>
<tr>
<td>Yaw moment of inertia (L_y)</td>
</tr>
<tr>
<td>Body: length/width (length/width)</td>
</tr>
<tr>
<td>Wheelbase/trackwidth (L/t)</td>
</tr>
<tr>
<td>Distance of CG from front/rear axle (f/b)</td>
</tr>
<tr>
<td>Height CG/roll centre front/rear (b/δ_f/δ_r)</td>
</tr>
<tr>
<td>Roll stiffness: front/rear (K_f/K_r)</td>
</tr>
<tr>
<td>Steering: maximum angle/rate (θ_max/θ_max)</td>
</tr>
<tr>
<td>Roll steer: front/rear (δ_f/δ_r)</td>
</tr>
<tr>
<td>Tires: pneumatic trail at linear range/relaxation (t_f/δ_p)</td>
</tr>
<tr>
<td>Tires-wheels</td>
</tr>
<tr>
<td>Wheel radius (r)</td>
</tr>
<tr>
<td>Magic formula: B/CG D</td>
</tr>
</tbody>
</table>

3. DLM OPTIMIZATION RESULTS

3.1 Sensitivity analysis

Table 8 shows the maximum entry speed achieved for different tuning setups by changing the front $K_f$ and rear $K_r$, roll stiffness, the front $δ_f$ and rear $δ_r$, camber gain, camber stiffness $C_y$ and the front $δ_f$ and rear $δ_r$ roll steer coefficient.

<table>
<thead>
<tr>
<th>Table 8. Maximum entry speed (km/h) with tire lateral dynamics disabled (42) and DSTC enabled while changing (multiplied with a gain factor $G$) one feature value and keeping ($G = 1$) the rest at their nominal values Table 7. $G$ ranges from 0.25 (25%) up to 3 (300%). The entry speed with the nominal values was 70.8 km/h, the greatest was $72.39$ km/h and the smallest was $69.75$ km/h.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Features</td>
</tr>
<tr>
<td>$G$</td>
</tr>
<tr>
<td>$K_f$</td>
</tr>
<tr>
<td>$C_y$</td>
</tr>
<tr>
<td>$δ_f$</td>
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<tr>
<td>$δ_r$</td>
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<tr>
<td>$δ_f$</td>
</tr>
<tr>
<td>$δ_r$</td>
</tr>
</tbody>
</table>

3.2 Four optimization cases

Enabling the tire lateral dynamics and disabling the DSTC and by selecting the feature value which gave the maximum (Max) and minimum (Min) entry speed per row in Table 8, as well as the nominal (Nom) features yielded 69.78, 64.16 and 68.60 km/h of entry speed correspondingly. Those three are illustrated in conjunction with a fourth optimization through Fig. 5 to Fig. 9 which had the nominal feature values (Table 7) and both DSTC and tire dynamics enabled (Nom DSTC) which yielded 71.99 km/h. Fig. 10 displays the trajectory and DSTC related results from the Nom DSTC case.

![Fig. 5 Front wheels’ steering angle δ (top), lateral position Y (middle) and resultant speed (RMS of longitudinal and lateral speed) (bottom) for the optimization cases described in 3.2. The DSTC is disabled for the Max, Nom and Min cases and is enabled for the Nom DSTC case. Although all four cases had similar trajectories Y, the optimal steering angle δ and velocity profile were considerably different; the Nom DSTC case has the highest entry speed (71.99 km/h).](image)

![Fig. 6 Vehicle’s roll angle φ (top), resultant lateral acceleration (RMS of longitudinal and lateral acceleration) (middle) and yaw rate ψ (bottom) for the optimization cases described in 3.2. The DSTC is disabled for the Max, Nom and Min and is enabled for Nom DSTC case. The Min yielded the greatest in magnitude roll angle due to the small roll stiffness value. The resultant acceleration and yaw rate start to considerably differ at approximately X = 20 m.](image)

![Fig. 7 Camber angles γ_f (40) for the optimization cases described in 3.2. The DSTC is disabled for the Max, Nom and Min and enabled for Nom DSTC case. The Min yielded the greater in magnitude camber values due to the small roll stiffness value and highest roll angles correspondingly (c.f. Fig. 6).](image)
and \( \psi_d \) and braking torques \( T_{ij} \) from the DSTC. Individual vehicle frame in the trajectory subplot is 0.25 s apart from its neighbours; the vehicle develops high side-slip angle values especially at the end of the manoeuvre. The DSTC brakes the rear left wheel to compensate for small in magnitude \( \psi \) in the 1st left turn, then the front right wheel to compensate for the increased in magnitude \( \psi \) and so on according the logic described in 2.3.5.3.

4. DISCUSSION

This paper proposed a method for generating the optimal steering control which maximizes a vehicle’s entry speed for the ISO3888 part-2 double-lane change (DLC) manoeuvre (c.f. Fig. 1). The proposed method involves an iterative process, starting from a centreline guess with the results being used as guess to the next more sophisticated vehicle dynamics model (VDM); the final VDM comprised of a 7-DOF VDM, non-linear tires with transient effects and suspension properties as-well-as a simplified dynamic stability and traction control (DSTC). The optimal control problem is converted into a finite dimensional optimization problem using a direct transcription method and is solved using TOMLAB/PROPT [6] in Matlab.

4.1 Real vehicle testing and validation

The same steering robot (SR60) and car used to estimate certain vehicle parameters (c.f. 2.4) were used to evaluate the method. The maximum entry speed achieved with the SR60, by manually tuning its control parameters for the DLC manoeuvre (a long iterative process necessitating multiple DLC runs without ensuring optimality), was 74.34 km/h and 65.88 km/h with and without DSTC respectively. It is hypothesized that a better tuning could have yielded higher entry speeds (the 1st author of the paper achieved more than 68 km/h in wet-driving with the DSTC disabled). The optimization results (c.f. 3.2) were 71.99 km/h and 70.80 km/h respectively. For comparison purposes, online reports for the DLC manoeuvre suggested maximum entry speed for the: Volvo S60 Polestar (STCC Racecar) 78 km/h, Audi A4 TDI 72 km/h, BMW 320d 74 km/h, BMW 335i AWD 75 km/h; the online sources are not verified and the authors do not take responsibility for the validity of the measurements.

To evaluate the feedforward-control potentials of the method, the generated optimal steering control input for the nominal simulated S60 was applied with the steering robot to the actual car. This method was expected to be fallible; any discrepancy between the model and the real vehicle (oversimplified DTSC etc.) and the testing surface would accumulate error. Multiple runs with the same steering angle and velocity profile yielded different trajectories suggesting that the current method has to be extended-improved.

4.2 Future work

To improve realism, the VDM complexity as-well-as the modelled DSTC (c.f. 2.3.5.3) should become more sophisticated. Still though, despite the fact that the a sophisticated VDM can realistically model the tires, the vehicle’s suspension characteristics as-well-as the DSTC functionalities, the final judgement of the vehicle is done with physical testing; this work will utilize the optimal steering input generated in a feed-forward
manner and employ a feedback controller which will compensate for the modelling errors and external disturbances during testing. Real vehicle tests will be used to verify its performance.

4.3 Rationale of this work and vision

It is envisioned that manufactures in an effort to increase efficiency, promote safety and reduce prototype vehicles will tune and develop various dynamic related systems (DSTC, steering system, suspension etc.) using CAE tools. Besides the reduced cost and lead-time, CAE will facilitate objective assessment of the car’s safety and numerically optimized tuning sets; safer and better cars for the road.

5. REFERENCES


