Corrections

Sangxia Huang

Chapter 6, page 97, Definition 6.1, line 3
The definition of a set $S \subseteq V$ being an independent set should be that “for all $f \in F, f \not\subseteq S$”, instead of “for all $f \in E, f \not\subseteq S$”.

Section 8.4, Theorem 8.12
There is a mistake in the proof of Theorem 8.12 in Section 8.4. The incorrect statement is on page 119, paragraph 2, line 6. The setting is that we have a matrix $M_v \in \mathbb{F}_2^{(m_r+1)\times(m_r+1)}$, and a subset of coordinates $S \subseteq [m_r+1]$. The claim in the thesis is that if a vector $y$ is in the column space of $M_v$, then the vector $y|_S$ is in the column space of $M_v|_S$, where $M_v|_S$ is the submatrix of $M_v$ where we only take the rows and columns that are in $S$, and $y|_S$ is the subvector where we only take the coordinates that are in $S$. This statement is not true in general, and it holds only when $\text{rank}(M_v) = \text{rank}(M_v|_S)$. The construction in Theorem 8.12 does not have this rank-preserving property and therefore the conclusion does not hold.

The manuscript (http://arxiv.org/abs/1504.03923) gives a solution to this problem. We replace Section 8.4 (in particular, Theorem 8.12) of the thesis with Section 4 (Theorem 4.8) of the manuscript. As a result, the size of the labels, $m_l$ and $m_r$, becomes $(\log n)^{5b+O(1)}$, as compared to $(\log n)^{(2+\omega(1))b}$ in the thesis, and the size of the bipartite graph becomes $2^{(\log n)^{5b+O(1)}}$, as compared to $2^{(\log n)^{2b+O(1)}}$ in the thesis. This makes the hardness of hypergraph coloring result worse. The new construction gives a quasi-NP-hardness of coloring 2-colorable 8-uniform hypergraphs of size $N$ with $2^{(\log N)^{1/10-\omega(1)}}$ colors, as compared to the $2^{(\log N)^{1/4-\omega(1)}}$ in the thesis.

The new construction is still fairly standard, and is very similar to the one used by Khot and Saket in the paper “Hardness of Coloring 2-Colorable 12-Uniform Hypergraphs with exp(\Omega(1) n) Colors” by Khot and Saket, published at FOCS ’14. The proofs in the remaining part of Chapter 8 is not dependent on this, and does not need to be changed other than the parameters that are affected by this issue.

The paper mentioned above by Khot and Saket gives a hardness of $2^{(\log N)^{c}}$ for $c \approx 1/20$ for hypergraph coloring and is the best previous hardness result. Thus the weaker theorems in the enclosed manuscript still gives the strongest known lower-bound.

I thank Rishi Saket for pointing out this mistake.