



**KTH Architecture and
the Built Environment**

Aspects of probabilistic serviceability limit state design of dry deep mixing

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Doctoral Thesis
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Preface

The work presented in this thesis was conducted between January 2009 and July 2014 at the Division of Soil and Rock Mechanics, Department of Civil and Architectural Engineering, Royal Institute of Technology (KTH), Stockholm, Sweden. The work was supervised by Professor Stefan Larsson, Head of the Division of Soil and Rock Mechanics.

Sincere thanks go to the Development Fund of the Swedish Construction Industry (SBUF) and to the Swedish Transport Administration (Trafikverket), whose financial support made this work possible.

I would also like to thank my supervisor, Stefan Larsson, who has supported this work with his knowledge and never-ending enthusiasm. Further acknowledgments are directed to my colleagues at the Division of Soil and Rock Mechanics.

Finally, I would like to thank my daughter Greta, for the joy she brings me and my parents, Sture and Ann-Charlotte, for their assistance. Without you, this work would never have been possible.

Stockholm, June 2015

Niclas Bergman

Summary

An expanding population and increased need for infrastructure increasingly necessitate construction on surfaces with poor soil conditions. To facilitate the construction of buildings, roads and railroads in areas with poor soil conditions, these areas are often improved by means of foundation engineering. Constructions that are fairly limited in scope are often founded on shallow or deep foundations. However, these methods are relatively expensive and thus not applicable for large-scale constructions like roads and railroads. A cost-effective way to deal with poor soil conditions is to use ground improvement. This thesis deals with a ground improvement method called deep mixing (*DM*) using lime-cement columns.

Lime-cement columns are manufactured by pushing a mechanical mixing tool to the desired depth, with the tool then rotated and retracted while a lime-cement binder is distributed into soil, forming lime-cement columns. Because of the complex mixing process and inherent soil variability, soil improved by *DM* shows high variability with respect to strength and deformation properties. Due to this high variability, it is difficult to predict the properties in advance; it is therefore important to verify the properties after installation. In Sweden, this is normally done using the column penetration test (*KPS*) method.

Current design praxis considers evaluated mean values in the design, and the effect of variability and uncertainties is dealt with by using a sufficiently high total factor of safety. A more rational approach for dealing with the effect of variability and uncertainties on the reliability of a mechanical system is to include them as parameters in the design model. This can be done by using reliability-based design (*RBD*). A major incentive for using *RBD* is that lower variability in design properties produces higher design values. This is important since it encourages contractors to improve their manufacturing methodologies because *RBD* allows more homogenous columns to be assigned higher design values. Reliability-based design is also in line with Eurocode 7, which states that the selection of the characteristic values for geotechnical parameters shall take the variability of the measured property values into account.

The first part of this doctoral thesis deals with test methods and quantification of the strength variability of soil improved by lime-cement columns. Tip resistances from three different test sites using three different penetration test methods – the cone penetration test, the column penetration test and the total-sounding test – are analysed and quantified in terms of means, variances and scale of fluctuations. The second part introduces *RBD* in serviceability limit state (*SLS*) design, using First Order Reliability Methods (*FORM*) and Monte-Carlo simulations.

Summarizing the most important findings and conclusions from this study:

- The scale of fluctuation was estimated to be 0.2-0.7 m and 0-3 m in the vertical and horizontal direction, respectively.
- The relation between cone tip resistances measured using the cone penetration test and column penetration test does not correspond to the cone factors proposed in previous studies and in the Swedish Design Guidelines.
- The agreement between the column penetration test and total-sounding test was found to be “good enough”. It is therefore suggested that the total-sounding test be used as a

complement to the column penetration test in evaluating the average strength properties of a group of medium- and high-strength lime-cement columns.

- Reliability-based design is a rational approach to incorporate strength and deformation parameter variability with an *SLS* design.

Sammanfattning

Med en växande population och infrastruktur, ökar behovet av att bebygga områden med dåliga grundläggningsförhållanden. För att möjliggöra byggandet av byggnader, vägar och järnvägar på dessa områden används olika typer av grundläggningsmetoder. Byggnationer med relativt liten utbredning kan ofta grundläggas med platta på mark eller pålning. Dessa grundläggningsmetoder är dock relativt dyra och därmed inte lämpliga för utbredda konstruktioner som vägar och järnvägar. Ett kostnadseffektivt alternativ till att handskas med dåliga grundläggningsförhållanden är olika jordförstärkningsmetoder. Denna avhandling behandlar jordförstärkningsmetoden djupstabilisering med kalk-cementpelare.

Kalk-cementpelare tillverkas genom att ett roterande blandningsverktyg trycks ner i jorden och ett bindemedel bestående av kalk och cement matas ut under omrörning. Variationerna i hållfasthets- och deformationsegenskaperna blir ofta stora på grund av den komplexa blandningsmekanismen samt variationerna i den naturliga jorden. På grund av de stora variationerna i hållfasthets- och deformationsegenskaperna är det svårt uppskatta dessa egenskaper innan tillverkning. Det blir således viktigt att man i efterhand kontrollerar sina antaganden avseende dessa egenskaper. I Sverige är den huvudsakliga provningsmetoden kalkpelarsondering.

Gällande dimensioneringsmetodik använder utvärderade medelvärden vid dimensionering där inverkan av variationer och osäkerheter hanteras med en tillräckligt stor säkerhetsfaktor. Ett mer rationellt tillvägagångssätt att ta hänsyn till inverkan av variationer och osäkerheter på säkerheten i en konstruktion, är att ta med dem som parametrar i designmodellen. Detta görs möjligt genom sannolikhetsbaserad dimensionering. Ett av de främsta incitamenten till införandet av sannolikhetsbaserad dimensionering är att lägre variationer i en egenskap leder till ett högre dimensionerande värde. Detta är väsentligt eftersom det uppmuntrar entreprenörer till att utveckla sina produktionsmetoder då sannolikhetsbaserad dimensionering tillåter att mer homogena pelare ges högre dimensionerande värde. Ett ytterligare incitament till införandet av sannolikhetsbaserad dimensionering är att den uppfyller kraven i Eurocode 7 som gör gällande att man vid utvärderingen av karakteristiska värden ska ta hänsyn till variationerna hos den uppmätta parametern.

Den första delen av denna doktorsavhandling behandlar testmetoder och kvantifiering av variationer i hållfasthetsparametrar i jord förstärkt med kalk-cement pelare. Spetstrycket från tre olika testmetoder, kalkpelarsonden, CPT-sonden samt Jb-totalsonden, utförda i kalk-cementpelare på tre olika testplatser, kvantifieras avseende medelvärden, varianser samt fluktuationsavstånd. Den andra delen introducerar sannolikhetsbaserad dimensionering för bruksgränsstadiet, med metoder som First Order Reliability Methods (*FORM*) och Monte-Carlo simuleringar.

De viktigaste upptäckterna och slutsatserna från denna studie kan summeras enligt:

- Fluktuationsavståndet uppmättes till 0.2-0.7 m i vertikalled och 0-3 m i horisontalled.
- Förhållandet mellan uppmätta spetstryck från CPT-sonden och kalkpelaresonden överensstämmer inte med de bärighetsfaktorer som föreslagits i tidigare studier och i svensk standard.
- Överensstämmelsen mellan kalkpelaresonden och Jb-totalsonden var tillräcklig för att Jb-totalsonden ska kunna användas som ett komplement till kalkpelarsonden för att uppskatta medelhållfastheten i en grupp med hårda och medelhårda kalk-cementpelare.

- Sannolikhetsbaserad dimensionering är en rationell metod för att inkludera variationer i hållfasthets- och deformationsparametrar i dimensionering av bruksgränsstadiet.

List of publications

This doctoral thesis is based on work presented in the following publications.

Appended papers:

- Paper I Al-Naqshabandy, M. S., Bergman, N. and Larsson, S., 2012. Strength variability in lime-cement columns based on cone penetration test data. Published in *Ground Improvement* 165 (1), 15-30.
<http://dx.doi.org/10.1680/grim.2012.165.1.15>.
Al-Naqshabandy and Bergman performed the analyses in parallel. Al-Naqshabandy, Bergman and Larsson jointly wrote the paper.
- Paper II Bergman, N., Al-Naqshabandy, M. S. and Larsson, S., 2013. Variability of strength and deformation properties in lime-cement columns evaluated from CPT and KPS measurements. Published in *Georisk* 7(1), 21-36.
<http://dx.doi.org/10.1080/17499518.2013.763571>.
Bergman performed the analyses and wrote the paper. Al-Naqshabandy contributed valuable comments. Larsson contributed writing and valuable comments.
- Paper III Bergman, N. and Larsson, S., 2013. Comparing column penetration and total-sounding data for lime-cement columns. Published in *Ground Improvement* 167 (4), 249-259.
<http://dx.doi.org/10.1680/grim.12.00019>.
Bergman performed the analyses and wrote the paper. Larsson contributed writing and valuable comments.
- Paper IV Bergman, N., Ignat, R. and Larsson, S., 2013. Serviceability limit state design of lime-cement columns – a reliability-based design approach. Published in *Proceedings of the 4th International Symposium on Geotechnical Safety and Risk (ISGSR2013)*, Hong Kong.
<http://dx.doi.org/10.1201/b16058-64>.
Bergman performed the analyses and wrote the paper. Ignat contributed valuable comments. Larsson contributed writing and valuable comments.
- Paper V Larsson, S. and Bergman, N., 2014. Probabilistic design of dry deep mixing using an observational approach. Published in *Ground Improvement* ahead of print.
<http://dx.doi.org/10.1680/grim.14.00011>.
Larsson performed the analyses and wrote the paper. Bergman designed the FORM model and contributed valuable comments.
- Paper VI Bergman, N., Johansson, F. and Larsson, S., 2015. Probabilistic serviceability limit state design approach for dry deep mixing. Re-submitted to *Soils and Foundations* in April 2015.
Bergman performed the analyses and wrote the paper. Johansson contributed valuable comments. Larsson contributed writing and valuable comments.

Connecting publications:

Fransson, J., 2011. A study of the correlation between soil-rock sounding and column penetration test data. KTH, Master Thesis.
Bergman supervised the work.

Ehnbom, V. and Kumlin, F., 2011. Reliability-Based Design of Lime-Cement Columns based on Total Settlement Criterion. KTH, Master Thesis.
Bergman supervised the work.

Contents

Preface.....	iii
Summary	v
Sammanfattning.....	vii
List of publications.....	ix
Chapter 1 – Introduction	1
1.1 Background.....	1
1.2 Previous studies.....	3
1.3 Scope and scientific contribution of the present research	4
1.4 Outline of thesis	6
Chapter 2 – Quality control	9
2.1 General	9
2.2 Test methods.....	9
2.2.1 Column penetration test	10
2.2.2 Cone penetration test	10
2.2.3 Total-sounding test.....	10
Chapter 3 – Statistical analyses	13
3.1 Spatial variability	13
3.1.1 Mean.....	13
3.1.2 Variance.....	13
3.1.3 Scale of fluctuation.....	13
3.2 Variance reduction factor.....	14
3.3 Correlation and agreement	15
Chapter 4 – Uncertainties and their impact on the evaluation of the design value	17
4.1 General	17
4.2 Uncertainties in deep mixing.....	18
4.3 Evaluation of design value.....	19
Chapter 5 – Serviceability limit state design of deep-mixed soils.....	21
5.1 Total settlement	21
5.2 Post-construction settlements	22
5.3 Column stress	23
Chapter 6 – Reliability-based design	25
6.1 First order reliability methods.....	25
6.2 Monte-Carlo simulations.....	26

6.3	Probabilistic Deep-Mixing design in practice	26
Chapter 7 – Summary of appended papers		29
7.1	Paper I.....	29
7.2	Paper II.....	30
7.3	Paper III.....	31
7.4	Paper IV	32
7.5	Paper V	33
7.6	Paper VI	34
Chapter 8 – Results and discussion		35
8.1	Test sites.....	35
8.2	Penetration test methods	35
8.3	Distribution of test data	36
8.4	Uncertainties and the coefficient of variation	36
8.5	Scale of fluctuation.....	36
8.6	Variance reduction factor.....	37
Chapter 9 – Conclusions and future research		39
References.....		41
Appended papers		49

Chapter 1 – Introduction

1.1 Background

An expanding population and growing need for infrastructure increasingly necessitate construction on surfaces with poor soil conditions. To facilitate the construction of buildings, roads and railroads in areas with poor soil conditions, these areas are often improved by means of ground improvement, or by shallow or deep foundations. Constructions that are fairly limited in scope are often founded on shallow or deep foundations. However, these methods are relatively expensive and thus not applicable to large-scale constructions like roads and railroads. A cost-effective way to deal with poor soil conditions is to use ground improvement methods. This thesis deals with a ground improvement method called deep mixing (*DM*) using lime-cement columns.

Deep mixing using lime-cement columns is a ground improvement method developed simultaneously in the Scandinavian countries and Japan during the 1970s (Boman and Broms 1975; Broms 1984; Terashi and Juran 2000; Larsson 2005a). The method is mainly applicable to soft soils like clay, silt and peat and improves the strength and deformation properties of the soil. Columns are manufactured by pushing a mechanical mixing tool to the desired depth. The mixing tool is then rotated and retracted while a binder is distributed into the soil, forming columns. Deep mixing can be subdivided into two groups depending on how the binder is distributed (Topolnicki 2004; Larsson 2005a). The method commonly used in Sweden is known as the dry method and uses compressed air to distribute the dry binder powder into the soil. Another category of *DM* methods is the wet mixing method, where the binder, normally cement, is mixed with water prior to installation. In this thesis, the dry mixing method is studied. Figure 1(a) shows typical machinery for manufacturing lime-cement columns, and Figure 1(b) shows two mechanical mixing tools.

Because of the complex mixing process and inherent soil variability, soil improved by *DM* shows high variability with respect to strength and deformation properties (Larsson 2005a). Due to the high variability, it is difficult to predict the properties in advance; it is therefore important to verify the properties after installation. In Sweden, this is normally done using the column penetration test method (*KPS*) (Axelsson and Larsson 2003; TK Geo 2011).



Figure 1: (a) Typical machinery for manufacturing lime-cement columns; (b) Pin drill and Swedish standard mixing tool (courtesy of Skanska AB; Larsson et al. 2005a).

Current design praxis uses deterministic mean values for design, and uncertainties in evaluating the mean are incorporated in a single value represented by a partial factor or total factor of safety. This means that a high-quality column with low-strength variability is assigned a design value equal to that of a low-quality column with high-strength variability, provided that the average strength of the columns is equal. One problem with this design approach is that it does not promote improvement in column quality since there is nothing to be gained from this, at least from a manufacturer's point of view. A more rational design approach would be to incorporate the uncertainties as parameters in the design model. This can be done by introducing reliability-based design (*RBD*). The reliability-based design approach promotes the development of manufacturing methodologies since it assigns relatively higher design values to high-quality columns. Furthermore, *RBD* is in line with Eurocode 7 (Eurocode 7: Geotechnical design – Part 1: General rules 2004), which states that the selection of

characteristic values of geotechnical parameters shall take the variability of the property values measured into account.

1.2 Previous studies

Although *RBD* is rarely used in practice, the need for it in *DM* has been identified by several authors. This section gives a brief overview of previous studies published in journals, at conferences or in theses, addressing *RBD* of soil improved by *DM*.

Honjo (1982) was the first to address the need for *RBD* in *DM*. Honjo proposed a probabilistic failure model taking the variability of the soil into account in design. Statistical methods were used to quantify the compressive strength and variability of soil improved by *DM*. The scale of fluctuation was evaluated by means of an autocorrelation function. It was concluded that the coefficient of variation of the unconfined compressive strength of the stabilized soil ranges between 0.21 and 0.36, regardless of the average strength. Furthermore, the scale of fluctuation in a vertical direction is influenced by factors such as in-situ soil properties, binder content and mixing conditions.

Filz and Navin (2006) presented the concept of *RBD* in the ultimate limit state (*ULS*) design of column-supported embankments. Reliability-based design is recommended for a *DM* project, mainly since it accounts for the significant variability in deep-mixed materials, but also since it permits rational development of statistically based design specifications. Furthermore, the study presents a coefficient of variation of unconfined compressive strength from nine deep-mixing projects in the U.S. ranging between 0.34 and 0.79.

Further studies addressing *RBD* in *ULS* design of soil improved by *DM* are presented by Kitazume (2004), Navin (2005), Terashi and Kitazume (2009), Kasama et al. (2009), Adams et al. (2009) and Al-Naqshabandy and Larsson (2013).

Huang et al. (2015) compare the results from a simple one-dimensional probabilistic method with the results from a probabilistic finite element method (*PFEM*) in both *ULS* and *SLS*. The study concludes that the simple one-dimensional probabilistic method can be used in reliability-based design for *DM* soils.

Chen et al. (2014) presented a statistical framework for strength prediction in *DM* based on a statistical analysis of a large quantity of sample measurements in a series of centrifuge model tests.

Huang et al. (2013) conducted a preliminary study of the system redundancy of dry soil mix columns.

Yong (2013) examined the spatial variability of *DM* soils including deterministic trends, stochastic fluctuation and positioning error in placing columns. Furthermore, parametric studies were conducted on how the random variation in material properties affects large scale behavior using a 3D random finite element method.

Kasama et al. (2012) presented a reliability assessment for the undrained bearing capacity of a surface strip foundation based on the results of a probabilistic study. The results showed how the bearing capacity was related to the coefficient of variation and correlation length scale in both shear strength and unit weight.

Srivastava and Sivakumar Babu et al. (2012) presented a framework which considered variability and its implication on the design strength of *DM* soils. The framework provided a mathematical basis for handling variability and brought rationality in the decision-making.

Sivakumar Babu et al. (2011) illustrated the use of a reliability analysis for unconfined compressive strength of soil improved by *DM*. They concluded that reliability-based analyses provide a rational choice of design strength values.

Navin and Filz (2005) analysed thirteen data sets of unconfined compressive strength for deep-mixed materials constructed using the wet and dry method. Their analysis showed that strength data tend to fit a log-normal distribution. Values of the coefficient of variation of the unconfined compressive strength ranged from 0.34 to 0.74. Analyses of the spatial correlation indicated a scale of fluctuation of 12 m for the wet method, while for the dry method no scale of fluctuation could be detected. Furthermore, a moderate correlation was found between the unconfined compressive strength and the elastic modulus.

Larsson et al. (2005a) and (2005b) investigated the influence of a number of factors in the installation process on the strength variability of lime-cement columns. The retrieval rate and the number of mixing blades were found to have a significant impact on variability, while rotational speed, binder tank air pressure and the diameter of the outlet hole were insignificant.

The variability of soil improved by *DM* has further been studied by Hedman and Kuokkanen (2003), Larsson et al. (2005c), Larsson and Nilsson (2009), Al-Naqshabandy (2012), Jian (2012) and Namikawa and Koseki (2013).

While a number of papers about *RBD* in *DM* have been published, only a few studies address *RBD* in *SLS* design (Zheng et al. 2009 and Huang et al. 2015). Furthermore, a number of papers address inherent and spatial variability in *DM*, although other sources of uncertainties (such as measurement, statistical and model transformation uncertainties) have not been considered.

The Federal Highway Administration's recently published design manual for deep-mixing (Bruce et al. 2013) suggest that variability can be taken into account by performing reliability analyses.

1.3 Scope and scientific contribution of the present research

The first part of this project, which was presented in my licentiate thesis, dealt primarily with test methods and the quantification of strength variability. The second part of the project, presented in this doctoral thesis, dealt with the implementation of *RBD* in *SLS* design. Figure 2 gives an overview of the appended papers and their main topics.

The scope and scientific contribution of this study can be summarized as follows:

- Contributed to the empirical knowledge about strength variability in soil improved by *DM* and its influence in determining the design value using *RBD*.
- Investigated the influence of different test methods (the cone penetration test and column penetration test) on the quantification of means, variances and scale of fluctuations.
- Investigated the possibility of using the total-sounding test method to assess the strength of soil improved by *DM*.

- Presented a probabilistic serviceability limit state design approach for dry deep mixing.

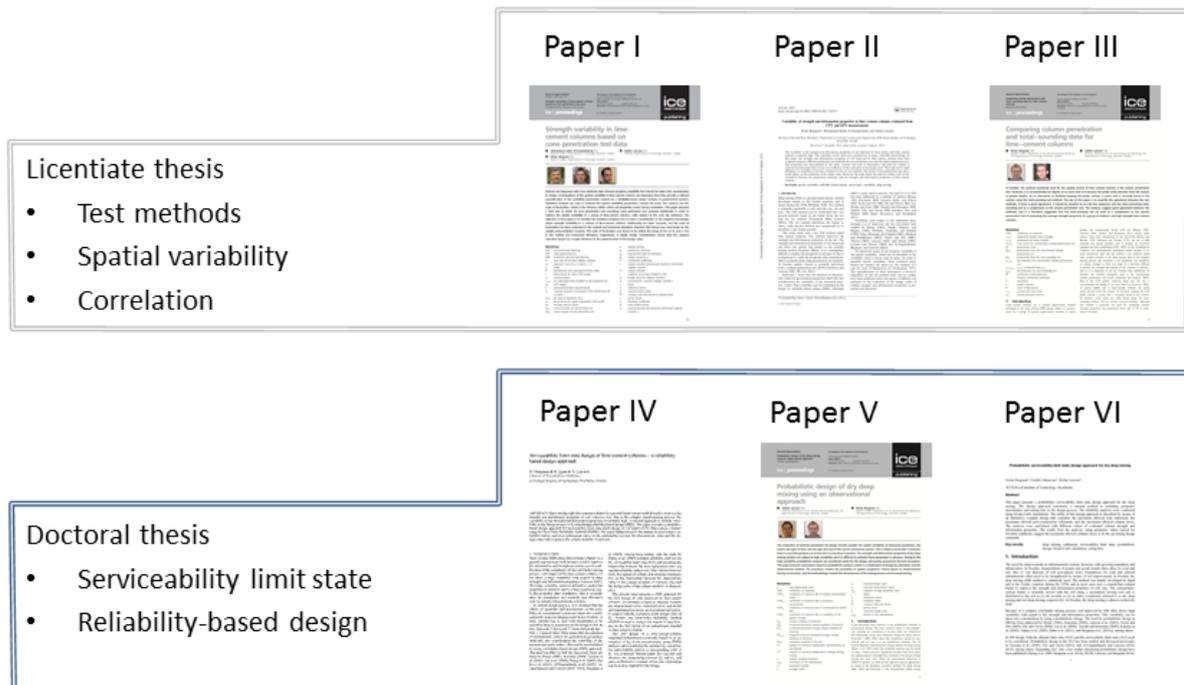


Figure 2: Overview of the appended papers and their main topics.

1.4 Outline of thesis

The purpose of this thesis is to provide a brief presentation of uncertainties in general and of uncertainties in deep-mixed soils in particular. The thesis also presents methods for quantification of the design properties of deep-mixed soils and the concept of reliability-based serviceability limit state design is presented. Other models and methods used throughout the appended papers are described in each paper separately.

The thesis consists of an introductory section in which the background and objectives of this study are presented. A summary of the literature survey is presented, including major findings and conclusions from previous work.

Chapter 2 – Quality control

This chapter gives an introduction to current Swedish quality control methodology. It also presents the penetration test methods used in this study.

Chapter 3 – Statistical analyses

Using *RBD*, a statistical quantification of the mean value and uncertainties related to the evaluation of the mean value is essential. This chapter presents the statistical analyses used in this study. The concept of variance reduction is introduced and correlation and agreement analyses are explained.

Chapter 4 – Uncertainties and their impact on the evaluation of the design value

Using *RBD*, the impact of uncertainties on the determination of the design value is significant. This chapter gives an introduction to uncertainties in general and to uncertainties related to *DM* in particular.

Chapter 5 – Serviceability limit state design of deep-mixed soils

This chapter gives an introduction to *SLS* design of deep-mixed soils.

Chapter 6 – Reliability-based design

This chapter gives an introduction to the concept of *RBD*. An example is given of how *RBD* can be incorporated in the serviceability limit state design of soil improved by *DM*.

Chapter 7 – Summary of appended papers

This chapter gives a brief summary of the appended papers.

Chapter 8 – Results and discussion

This chapter presents and discusses the results from this study.

Chapter 9 – Conclusions and future research

This chapter summarizes the major conclusions from this study and gives suggestions for future work related to this study.

Chapter 2 – Quality control

In this study, the quality of lime-cement columns was studied using three different penetration test methods. This chapter gives an introduction to current Swedish quality control methodology and describes the three different penetration test methods used in this study.

2.1 General

Because of the complex mixing process, variability in column strength properties is normally very high, which is why it is difficult to predict the quality of the columns in advance (Larsson 2005a). The quality of lime-cement columns is governed by several factors, such as the rheology of the soil and binder, stress conditions in the soil, the geometry of the mixing tool and its retrieval rate. Although the influence of these factors on the quality of the lime-cement column has been investigated by Larsson et al. (2005a, 2005b), it is not considered in practice. Consequently, it is important to test the quality of the columns after installation. In Sweden, 1% or at least four of the columns are tested after installation (TK Geo 2011; AMA Anläggning 10).

2.2 Test methods

In Sweden, the most frequently used penetration test method is the column penetration test (*KPS*). Internationally, a wide range of field test methods have been used, such as the reversed column penetration test (*OKPS*), cone penetration test (*CPT*), standard penetration test (*SPT*), rotary sounding test (*RPT*) and pressure meter test (*PMT*) (Porbaha 2002). In this study, data from three different test methods – the column penetration test, cone penetration test and total-sounding test (*Jbt*) – were analysed.

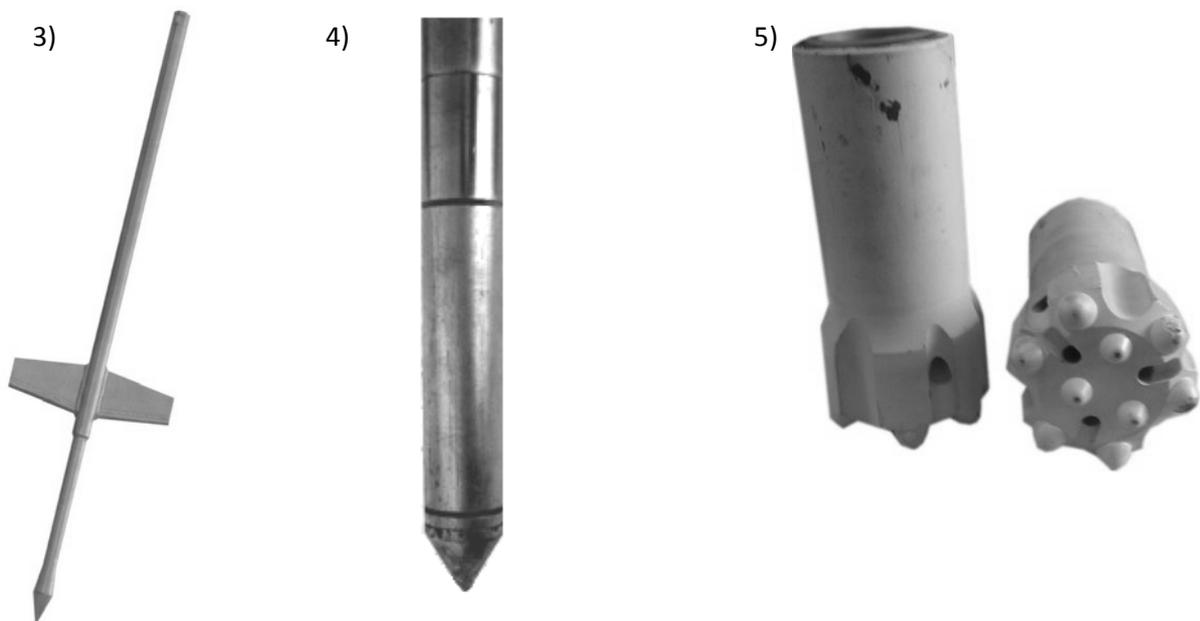


Figure 3: The column penetration test (*KPS*) (courtesy of Geotech).

Figure 4: The cone penetration test (*CPT*).

Figure 5: The total-sounding (*Jbt*) bore bit (courtesy of Geotech).

2.2.1 Column penetration test

The column penetration test (*KPS*) was developed in the 1980s by Torstensson (1980a, 1980b) and is the most frequently used penetration test method in Sweden today for quality control of lime-cement column properties. The test is executed by pushing a cylindrical penetrometer with two horizontal vanes, or probe (see Figure 3), down into the center of the column, while continuously recording the penetration force (Q_{KPS}). Tests are normally performed according to Swedish guidelines (TK Geo 2011, Larsson 2006). The probe is pushed into the column at a constant rate of penetration of 20 mm/sec. To obtain a good representation of the column, the probe should be as wide as possible and preferably 100 mm smaller than the column diameter (Axelsson and Larsson 2003). Because of the relatively large size of the *KPS*-probe, it is recommended for depths of no more than 8 m (Larsson 2006). At greater depths and in high-strength columns, the probe easily deviates from the column. To facilitate the verticality of the *KPS*-probe, a center hole can be bored in the column. In so doing, the penetration depth of the *KPS*-probe may be increased to 12-15 m (Ekström 1994). The column penetration test can be improved by attaching the *KPS*-probe to a cone penetration test (*CPT*). This improvement is important since it enables *KPS* to distinguish bar friction from penetration resistance ($q_{c,KPS}$), where bar friction can be as large as $q_{c,KPS}$ in stabilized soil (Larsson 2005a). From $q_{c,KPS}$ the column undrained shear strength (c_u) can be evaluated using the following empirical relation:

$$c_u = \frac{q_{c,KPS}}{N_{k,KPS}} \quad (1)$$

where $N_{k,KPS}$ is the cone factor for *KPS*. According to Swedish guidelines, $N_{k,KPS}$ should be set to 10. However, values of $N_{k,KPS}$ ranging from 10 to 20 have been suggested by several authors (Halkola 1999; Axelsson 2001; Wiggers and Perzon 2005; Liyanapathirana and Kelly 2011).

2.2.2 Cone penetration test

The cone penetration test (*CPT*) is a penetration test method used internationally to test improved soil (Halkola 1999; Larsson 2005a, 2005b; Puppala et al. 2005a, 2005b). In the cone penetration test in this study, a cylindrical electronic test probe was used whose cone tip measured 1000 mm². As in *KPS*, the *CPT* probe (Figure 4) is driven into the column at a constant rate of penetration of 20 mm/sec. The penetration resistance ($q_{c,CPT}$) is measured continuously and c_u can be evaluated using the following empirical relation (Lunne et al. 1997):

$$c_u = \frac{q_{c,CPT} - \sigma_{v0}}{N_{k,CPT}} \quad (2)$$

where $N_{k,CPT}$ is the cone factor for *CPT* and σ_{v0} is the total vertical soil stress. Values of $N_{k,CPT}$ ranging from 15 to 23 have been suggested by several authors (Tanaka et al. 2000; Porbaha 2001; Puppala et al. 2005b).

2.2.3 Total-sounding test

The Swedish total-sounding test (*Jbt*) method is a modification of the Norwegian total-sounding test method (SGF 2006). It was primarily designed to measure bedrock level and to determine the existence of large boulders and has been used successfully to locate and map the extent of quick clay formations (Lundström et al. 2009; Solberg et al. 2011). *Jbt* has also been used to evaluate lime-cement column strength properties (Nilsson and Forssman 2004; Jelusic and Nilsson 2005). Tsukada et al. (1998) used the rotary penetration test (Porbaha 2002), a test method similar to *Jbt*, to evaluate

the strength of improved soil. The total-sounding test method is a rotary penetration test where a vertical force is applied to a rotating drilling rod. Standard equipment is a 57 mm drill bit (Figure 5) attached to a 44 mm drilling rod. The rod is driven into the center of the lime-cement column at a rate of penetration of 20 mm/s and with a rotational speed of 25 rpm, while continuously recording the penetration force (Q_{Jbt}). In addition to the tip penetration resistance ($q_{c,Jbt}$), Q_{Jbt} also includes drill rod bar friction. This is an important factor to consider since drill rod bar friction may constitute a large part of Q_{Jbt} in improved soil.

Chapter 3 – Statistical analyses

Using *RBD*, a statistical quantification of the mean value and uncertainties related to the evaluation of the mean value is essential. This chapter presents the statistical analyses used in this study. Furthermore, the concept of variance reduction is introduced, and correlation and agreement analyses are explained.

3.1 Spatial variability

An important measure of soil variability is spatial variability. Spatial variability can be described as the variability of a mean value in space. In order to quantify spatial variability, three statistical measures are needed – the mean, the variance and the scale of fluctuation.

3.1.1 Mean

The arithmetic mean (\bar{x}) is a numerical measure to describe a set of data. It is defined as the sum of the observations divided by the sample size. It is defined by the following formula:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad (3)$$

where x_i is the i^{th} observation and n the number of observations.

3.1.2 Variance

The most common measure of the variation of a set of data is the sample variance (s^2). It measures the degree to which the actual values differ from the mean and is defined by the following formula:

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \quad (4)$$

It can also be quantified as the coefficient of variation (*COV*), which is defined by the following formula:

$$COV = \frac{\sqrt{s^2}}{\bar{x}} \quad (5)$$

3.1.3 Scale of fluctuation

The scale of fluctuation (θ) is an important measure in evaluating spatial variability and can be described as the distance within which a measured parameter shows a relatively strong correlation (Vanmarcke 1977). The occurrence of θ has a significant impact on the evaluation of the mean. If a series of measurements lie closer than θ , we can expect that the average of the measurements is probably higher or lower than the average of the soil layer tested. The scale of fluctuation is commonly evaluated using variograms or autocorrelation functions (*ACF*). In the present study, θ was evaluated from the sample *ACF*, which is the variation of the autocorrelation coefficient ($\rho'(k)$):

$$\rho'(k) = \frac{c_k}{c_0} \quad (6)$$

where c_k is the autocovariance at lag number k and c_0 is the autocovariance at lag distance 0. c_k is defined by:

$$c_k = Cov(q_c(z_i), q_c(z_{i+k})) = E[(q_c(z_i) - \bar{q}_c)(q_c(z_{i+k}) - \bar{q}_c)] \quad (7)$$

where k is the lag distance, $q_c(z_i)$ is the tip resistance at depth z_i , $i = 0, 1, \dots, n-1$ and \bar{q}_c is the mean tip resistance.

By fitting a theoretical ACF ($\rho(k)$) into the sample ACF , the one-dimensional θ can be evaluated by (Vanmarcke 1983):

$$\theta = 2 \int_0^{\infty} \rho(k) dk \quad (8)$$

Five theoretical models are widely used in analyzing geotechnical data as shown in the table below (Table 1) (Jaksa 1999; Phoon 2003). Due to best fit and the relatively limited data, the binary noise model was used in this study.

3.2 Variance reduction factor

The effect of spatial variability on the determination of the design value can be dealt with by means of spatial averages, in this study represented by the average tip resistance over a depth or volume. The variance reduction factor (Γ^2) is dependent on θ and the scale of scrutiny (L), that is, the size of the mechanical system of failure domain, where a small θ and a large L are attributes that contribute to a reduction in variability. Vanmarcke (1977) defines Γ^2 in the one-dimensional case as:

$$\Gamma^2(L_x) = \frac{2}{L_x} \int_0^{L_x} \left(1 - \frac{k}{L_x}\right) \rho(k) dk \quad (9)$$

where L_x is the size of the average length of the domain size, k is the separation distance and $\rho(k)$ is the normalized autocorrelation function. Assuming separate correlation structures, the three-dimensional Γ^2 is defined as:

$$\Gamma_{xyz}^2 = \frac{2 \cdot 2 \cdot 2}{L_x \cdot L_y \cdot L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[\left(1 - \frac{x}{L_x}\right) \left(1 - \frac{y}{L_y}\right) \left(1 - \frac{z}{L_z}\right) \rho(x) \rho(y) \rho(z) \right] dy dx dz \quad (10)$$

The use of Γ^2 will be further described in section 4.2.

Table 1: $\rho(k)$ is the theoretical autocorrelation function, k is the lag number, and c , m , b , d and a are model constants (decay factors).

Autocorrelation model	Equation
Binary noise	$\rho(k) = \begin{cases} 1 - c k & k \leq 1/c \\ 0 & \text{otherwise} \end{cases}$
Single exponential	$\rho(k) = \exp(-m k)$
Squared exponential	$\rho(k) = \exp(-bk)^2$
Cosine exponential	$\rho(k) = \exp(-d k) \cos(d k)$
Second-order Markov	$\rho(k) = (1 + a k) \exp(-a k)$

3.3 Correlation and agreement

In order to investigate the possibility of using *Jbt* to assess the strength of soil improved by *DM*, the correlation and agreement between *Jbt* and *KPS* are analysed.

Correlation analysis is a widely used tool for quantifying the relation between two or more sets of data. A commonly used measure is the Pearson product-moment correlation coefficient. It gives a measurement of linear correlation and can be estimated by the sample correlation coefficient (r) according to:

$$r = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right) \quad (11)$$

where n is the number of data in a sample, x and y are two sets of data, \bar{x} and \bar{y} are the mean values, and s_x and s_y are the sample standard deviation of the respective sets. It can be shown that the value of r is always between -1 and 1. A value of $r = 1$ implies a perfect positive linear relation between x and y , while $r = -1$ implies a perfect negative linear relation. A value of $r = 0$ implies that there is no linear relation between x and y .

Correlation analysis not is always a good measure of agreement between two sets of data. There will be a perfect correlation if the data scatter plot follows any straight line, but there will be perfect agreement only if the data scatter plot follows the line of perfect equality (Figure 6).

The agreement between two sets of data can be visualized by Tukey mean-difference plots (Tukey 1977) (Figure 7), where the differences between data points are plotted against their average values. However, the extent to which the two measurements can differ without having a significant impact on the evaluation of column undrained shear strength will be a question of judgment. The Tukey mean-difference plot is only meaningful for two similar sets of test data, that is, with the same physical dimensions and expressed in the same units.

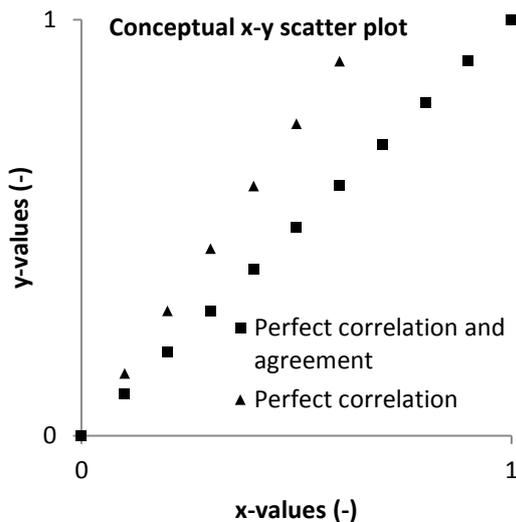


Figure 6: Perfect correlation is obtained if the data scatter plot follows any straight line; perfect agreement will be obtained only if the data scatter plot follows the line of perfect equality.

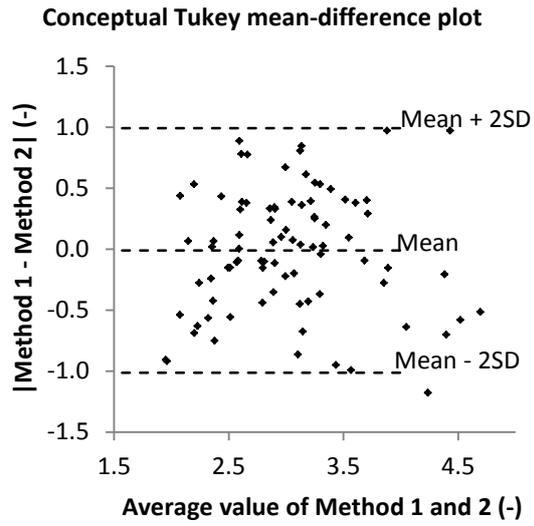


Figure 7: Conceptual Tukey mean-difference plot, where differences between data points are plotted against their average values.

Chapter 4 – Uncertainties and their impact on the evaluation of the design value

4.1 General

Geotechnical engineers face many sources of uncertainties in the design process (Phoon and Kulhawy 1999a, 1999b; Baecher and Christian 2003). Design parameters are often evaluated from field and laboratory tests using empirical relations. Figure 8 categorizes the different sources of geotechnical uncertainties. Geotechnical uncertainties can be described as either aleatory or epistemic. Aleatory uncertainties are those associated with randomness, or are modeled as caused by chance. In geotechnical engineering, data scatter from laboratory and field tests is often modeled as caused by chance. Furthermore, data scatter from tests is considered to be caused by natural variability in the soil and measurement errors. Epistemic uncertainties, commonly known as knowledge uncertainties, are associated with a lack of information or knowledge about processes and physical laws that limits our ability to model the real world. Transformation or model errors and statistical errors are examples of epistemic uncertainties. Transformation or model errors are often associated with the accuracy and validity of empirical relations, such as Equations 1 and 2. Statistical errors are associated with the precision with which model parameters can be estimated, and are governed by available test data. In this study, uncertainties are quantified by means of *COV*.

In Sweden, the effect of parameter uncertainties on the design of geotechnical constructions has been studied previously by Olsson (1986), Alén (1998), Stille et al. (2003), Al-Naqshabandy (2012), Müller (2013), among others.

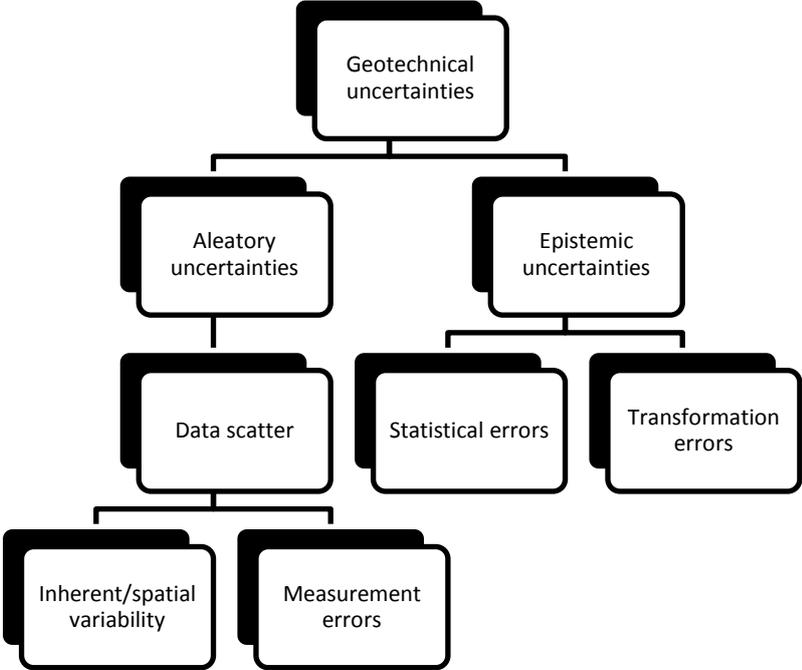


Figure 8: Classification of different sources of geotechnical uncertainties (after Baecher and Christian (2003)).

4.2 Uncertainties in deep mixing

For natural soils, soils improved by deep mixing have a relatively high inherent variability. The high variability is mainly caused by the complex mixing process and natural variability of the unimproved soil (Larsson 2005a). Larsson (2005a), Burke and Sehn (2005), Navin and Filz (2005) and Kasama and Zen (2009) present COV evaluated from compression tests of soil improved with DM ranging from 14 to 76%.

Another major source of uncertainties in DM is transformation errors. Deformation properties, such as undrained shear strength, are often evaluated from their empirical relation with the cone tip resistance of a penetration test method. In Equations 1 and 2, the relation is governed by a cone factor ($N_{k,KPS}$ and $N_{k,CPT}$). However, the wide range of cone factors suggested for the two methods introduces further uncertainties into the evaluation.

When evaluating the average column undrained shear strength ($\bar{c}_{u,col}$), uncertainties can be modeled as stochastic variables, representing quotas of the parameter measured:

$$\bar{c}_{u,col} \propto \bar{q}_c \cdot (\eta_w \cdot \eta_m \cdot \eta_{st} \cdot \eta_{tr}) \quad (12)$$

where \bar{q}_c is the average tip resistance, η_w is the uncertainty associated with spatial variability, η_m is the uncertainty associated with measurement errors, η_{st} is the uncertainty associated with statistical errors, and η_{tr} is the uncertainty associated with transformation errors. The quotas are assumed to be normally distributed with an expected value and a standard deviation according to:

$$\eta_w \in N(1, COV_w) \quad (13)$$

$$\eta_m \in N(1, COV_m) \quad (14)$$

$$\eta_{st} \in N(1, COV_{st}) \quad (15)$$

$$\eta_{tr} \in N(1, COV_{tr}) \quad (16)$$

where COV_w is the coefficient of variation associated with inherent variability, COV_m is the coefficient of variation associated with measurement errors, COV_{st} is the coefficient of variation associated with statistical errors, and COV_{tr} is the coefficient of variation associated with transformation errors.

The uncertainty of a product of stochastic variables can be approximated by the square root of the sum of the squared COV of individual stochastic variables (Goodman 1960; Jaksa et al. 1997). Consequently, the total uncertainty ($COV_{\bar{c}_u}$) in determining the design value, evaluated from mean tip resistances (\bar{q}_c), can be defined as:

$$COV_{\bar{c}_u} = \sqrt{COV_{w,\bar{q}_c}^2 + COV_{m,\bar{q}_c}^2 + COV_{st,\bar{q}_c}^2 + COV_{tr,\bar{q}_c}^2} \quad (17)$$

Based on penetration test data, the COV of individual sources of uncertainties is given by:

$$COV_{w,\bar{q}_c}^2 = (COV_{q_c}^2 - COV_{m,q_c}^2) \cdot \Gamma^2 \quad (18)$$

$$COV_{m,\bar{q}_c}^2 = \frac{COV_{m,q_c}^2}{N} \quad (19)$$

$$COV_{st,\bar{q}_c}^2 = (COV_{q_c}^2 - COV_{m,q_c}^2) \cdot \frac{1}{N} \quad (20)$$

where COV_{q_c} is the evaluated coefficient of variation of tip resistance (q_c), COV_{m,q_c} is the coefficient of variation associated with random measurement noise, and N is the number of uncorrelated tests with respect to \bar{q}_c .

Combining Equations 17–20, Equation 17 can be re-written as:

$$COV_{\bar{c}_u}^2 = (COV_{q_c}^2 - COV_{m,q_c}^2) \left(\frac{1}{N} + \Gamma^2 \right) + \frac{COV_{m,q_c}^2}{N} + COV_{tr,\bar{q}_c}^2 \quad (21)$$

4.3 Evaluation of design value

Uncertainties are included as design parameters in the evaluation of the design value using an *RBD* methodology. For normally distributed variables, the design value can be calculated as (Thoft-Christensen and Baker 1982):

$$x_d = \bar{x} + \alpha\beta\sqrt{s^2} \quad (22)$$

where \bar{x} is the sample mean, s^2 is the sample variance, α is the sensitivity factor that describes the significance of the variable for the mechanical system, and β is the required reliability index.

Based on penetration test data, the normalized design value ($c_{u,d}/\bar{c}_u$) can be evaluated as:

$$c_{u,d}/\bar{c}_u = 1 + \alpha\beta \cdot \sqrt{COV_{\bar{c}_u}^2} \quad (23)$$

For log-normally distributed variables, Equation 23 can be re-written as:

$$c_{u,d}/\bar{c}_u = \exp \left[-\frac{1}{2} \ln(1 + COV_{\bar{c}_u}^2) + \alpha\beta \sqrt{\ln(1 + COV_{\bar{c}_u}^2)} \right] \quad (24)$$

Here, the values of $COV_{\bar{c}_u}^2$ are given by the statistical analyses presented in section 4.2, β is given by standards, and α is evaluated from reliability analyses, which are described further in Chapter 6.

Chapter 5 – Serviceability limit state design of deep-mixed soils

Over the years, several studies of consolidation and settlements in deep-mixed soils have been published (Broms 1999, Bergado et al. 1999, Lin and Wong 1999, Baker 2000, Lorzeno and Bergado 2003, Alén et al. 2005, Yin and Fang 2006, Miao et al. 2008, Zheng et al. 2009, Chai and Pongsivasathit 2010, Chai et al. 2010, Venda Oliveira et al. 2011, Banadaki et al. 2012, Horpibulsuk et al. 2012, Jiang et al. 2013, Lu et al. 2013, Jiang et al. 2013, Muntohar et al. 2013, Pongsivasathit et al. 2013, Kamash and Han 2014, Yapage et al. 2014, Zheng et al. 2014, Huang et al. 2015).

In *SLS* design of deep-mixed soils, one is normally constrained by:

1. The maximum allowed total settlements.
2. The distribution of settlements with time.
3. The maximum allowed column stress (which is a restriction with present settlement model).
4. The maximum allowed differential settlements.

In this study, (1) – (3) are considered in design. Although it is an important and problematic topic, the analysis of the maximum allowed differential settlements are not included in the design framework. Differential settlements occur when adjacent areas inherit large differences in strength and deformation property values. The occurrence of these areas is difficult to predict. It is therefore the authors' belief that differential settlements due to spatial variability should not be taken into account in *SLS* design since they may result in considerable cost increases. When more data are available, differential settlements can be treated by means of probabilistic analyses with a system perspective on the occurrence of local spatial differences in strength and deformation properties.

Current design methodology is further described in SGF (2000), Larsson (2006) and TK Geo (2011).

5.1 Total settlement

The total settlement (S_{tot}) in a soil improved by *DM* can be expressed by:

$$S_{tot} = S_e + S_c + S_s \quad (25)$$

where S_e is the elastic settlement caused by elastic deformations of soils without any change in moisture content, S_c is the primary consolidation settlement which is the result of a volume change caused by expulsion of pore water, and S_s is the secondary consolidation or creep caused by plastic adjustment of soil fabrics. In this thesis, only S_c is considered and will from now on be referred to as S_{emb} .

For the sake of simplicity, a simple settlement model that is easy to understand was used:

The settlement (S_{emb}) of an embankment founded on normally consolidated clay improved by end-bearing lime-cement columns can be described by (Broms 1979, EuroSoilStab 2002, TK Geo 2011, Bruce et al. 2013):

$$S_{emb} = \sum \frac{h_f \cdot q}{\alpha \cdot E_{col} + (1-\alpha) \cdot M_{clay}} \quad (26)$$

where h_j is the height of layer j , q is the additional strain, a is the area ratio for the lime-cement columns, E_{col} is the elastic modulus of the columns, and M_{clay} is the oedometer modulus of the clay. The elastic modulus of the lime-cement column is normally not measured in situ. It is instead assumed to be a function of the c_u evaluated and is assessed using (TK Geo 2011):

$$E_{col} = 13 \cdot c_u^{1.6} \quad (27)$$

Evaluating E_{col} from c_u introduces further transformation errors, which have to be considered in *RBD*.

The advantage of starting with a simple settlement model is that it makes it easier to focus on the reliability-based design methodology rather than on the complexity of the settlement model itself.

5.2 Post-construction settlements

The total allowed settlement is normally an important design constraint. An equally important constraint is however the distribution of settlements over time. One can easily understand that it is preferable to realize the main part of the total settlement within the time frame of the construction, when maintenance and fixes are relatively cheaper to carry out.

Equation 26 describes the consolidation settlement due to an increase in effective vertical stress. This settlement does not occur instantly with the applied load, but is a slow process depending on the decrease rate of excessive pore water pressure. This decrease rate is time-dependent and can be described by analogy to the consolidation of soils, improved by prefabricated vertical drains, using the function $U(t)$ (Baker 2000, TK Geo 2011):

$$U(t) = 1 - \exp[-T_v \cdot t] \quad (28a)$$

where t is the elapsed time and T_v is a time factor defined as:

$$T_v = \frac{-2 \cdot c_{h,block}}{R^2 \cdot f(n)} \quad (28b)$$

$$f(n) = \frac{n^2}{n^2 - 1} \cdot \left[\ln(n) - 0.75 + \frac{1}{n^2} \cdot \left(1 - \frac{1}{4n^2} \right) \right] + \left[\frac{n^2 - 1}{n^2} \cdot \frac{1}{r^2} \cdot \frac{k_{h,soil}}{k_{v,col}} \cdot L_{drain}^2 \right] \quad (28c)$$

where $c_{h,block}$ is the area-weighted horizontal coefficient of consolidation of the improved soil (block) defined as (Alén et al. 2006):

$$c_{h,block} = \frac{k_{h,soil} \cdot [\bar{M}_{soil} \cdot a + (1-a) \cdot \bar{E}_{col}]}{\gamma_w} \quad (28d)$$

where $k_{h,soil}$ is the horizontal hydraulic conductivity for the virgin soil, $k_{v,col}$ the vertical hydraulic conductivity of the column and γ_w the unit weight of water, R the column radius of influence defined as $0.55 \cdot cc_{pel}$, and cc_{pel} the column center-to-center distance, n the quota R/r and r the column radius, and L_{drain} the length of the column assuming a single drainage path.

Figure 9 presents the conceptual behavior of column strain plotted with elapsed time where t_0 represents the time of installation of the columns, $t_{\Delta\sigma}$ represents the time for the loading of the columns, and t_{EOC} represents the time for the end of the construction. In the calculations performed

within the scope of this study, 90% of the settlements were assumed to be realized within the time frame of the construction.

5.3 Column stress

The third design constraint considered in this study is the maximum allowed additional column stress ($\Delta\sigma_{max,col}$). Exceeding $\Delta\sigma_{max,col}$ could potentially give cause to local failures and large unexpected deformations. This design procedure is based on the Rankine theory of lateral earth pressure as (TK Geo 2011):

$$\Delta\sigma_{max,col} = \frac{2 \cdot \cos(\phi_{col})}{1 - \sin(\phi_{col})} \cdot c'_{col} + \frac{1 + \sin(\phi_{col})}{1 - \sin(\phi_{col})} \cdot \sigma'_{h,col} - \sigma'_{v,0,col} \quad (29a)$$

$$\sigma'_{h,col} = \sigma'_{h,0,soil} + K_0 \cdot \Delta\sigma'_{v,soil} \quad (29b)$$

where c'_{col} is the column cohesion, ϕ_{col} the column angle of friction, $\sigma'_{h,col}$ the horizontal effective stress acting on the column, and $\sigma'_{v,0,col}$ the vertical effective columns stress prior to loading. Further, $\sigma'_{h,0,soil}$ is the horizontal effective soil stress prior to loading, $\Delta\sigma'_{v,soil}$ the increase in vertical effective soil stress due to loading, and K_0 the coefficient of active earth pressure at rest.

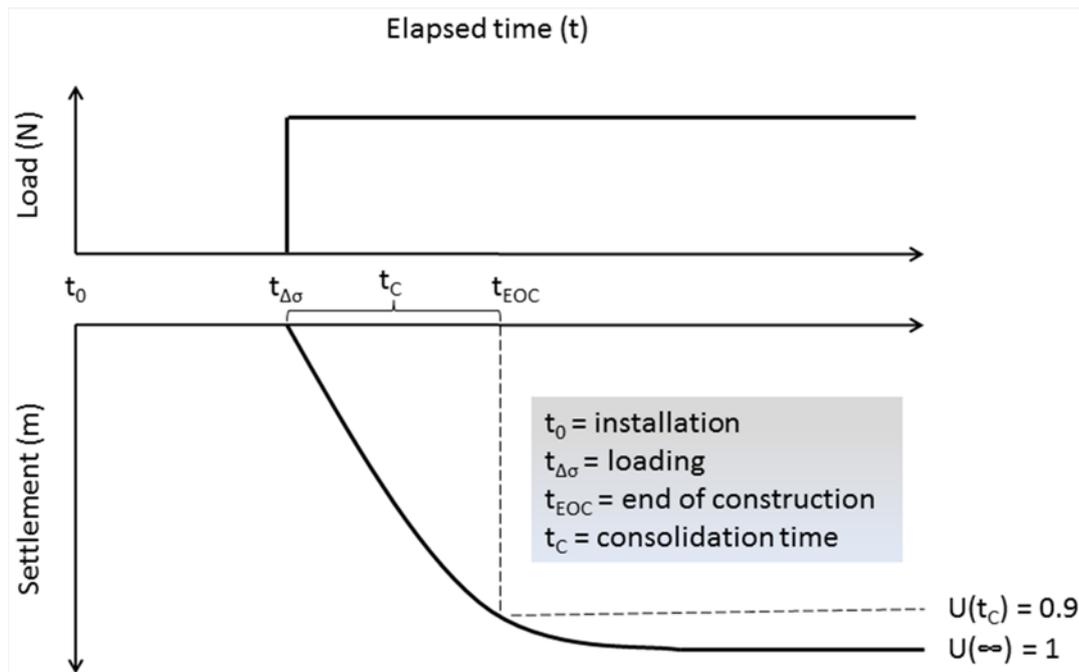


Figure 9: Conceptual behavior of column strain plotted with elapsed time.

Chapter 6 – Reliability-based design

In geotechnical engineering, soil properties are in general dealt with in a deterministic way. Mean values are often considered for design, and the effect of variation and fluctuation in these values is represented by a partial total factor of safety. A more rational approach to dealing with the variability and fluctuation of design properties is to use reliability-based design (*RBD*). Probabilistic or reliability-based design is not however a new research topic in geotechnical engineering. Numerous papers have been published on the topic in recent decades. Examples include:

Tang et al. (1976) presented a risk-based design method for slope stability incorporating uncertainties in the evaluation of the reliability of a given design.

Vanmarcke (1977) introduced the concept of variance reduction due to spatial variability in geotechnical engineering.

Fenton et al. (2003) reported on the reliability of a serviceability limit state design of a strip footing with respect to the soil's variance and scale of fluctuation.

Phoon and Kulhawy (2008) discussed the application of a probabilistic model for performing reliability-based design at the serviceability limit state.

Müller et al. (2014) presented a study on an extended multivariate approach for uncertainty reduction in the assessment of undrained shear strength in clays.

Prästings et al. (2014) presented a study on the observational method related to the design of an embankment.

Several books have also been published on the topic in recent years. Baecher and Christian (2003) and Phoon (2008) are examples of two frequently cited books.

There are several ways of carrying out a reliability analysis. In this study, the Hasofer-Lind method, also known as the first order reliability method (FORM), and Monte-Carlo simulations were used. This chapter describes how a deterministic design methodology can be incorporated in an *RBD* methodology.

6.1 First order reliability methods

Reliability analysis is an attempt to quantify how close a system is to failure (Baecher and Christian 2003). Failure in *SLS* can be defined as an unacceptable difference between expected and observed performance. To analyse the reliability of a geotechnical structure, a limit state function ($g(X)$) is defined as $g(X) = 0$. In *SLS* design, $g(X)$ can be defined as:

$$g(X) = \delta_{max} - \delta(x_1, x_2, x_3 \dots) = 0 \quad (30)$$

where δ_{max} is the maximum settlement allowed and $\delta(X)$ is the settlement assessed from design properties x_1, x_2, \dots, x_n . $G(X) > 0$ indicates acceptable differences between expected and observed performance. By combining Equations 22, 26 and 30, the performance function can be re-written as:

$$g(E_{col}, M_{clay}) = \delta_{max} - \sum h_j \cdot \frac{q}{a \cdot (\mu_{E_{col}} - \alpha_{E_{col}} \cdot \beta \cdot \sigma_{E_{col}}) + (1-a) \cdot (\mu_{M_{clay}} - \alpha_{M_{clay}} \cdot \beta \cdot \sigma_{M_{clay}})} \quad (31)$$

where $\mu_{E_{col}}$ is the mean value of E_{col} , $\alpha_{E_{col}}$ is the evaluated sensitivity factor, $\sigma_{E_{col}}$ is the reduced standard deviation of E_{col} , $\mu_{M_{clay}}$ is the mean value of M_{clay} , $\alpha_{M_{clay}}$ is the evaluated sensitivity factor, and $\sigma_{M_{clay}}$ is the reduced standard deviation of M_{clay} . In this example, q is considered to be deterministic. The reliability index is associated with the probability of failure and is determined by standards. The sensitivity parameter is given by an iterative process described by Rackwitz and Fiessler (1978) and Baecher and Christian (2003) and is defined as:

$$\alpha_{x_i} = \frac{\left(\frac{\partial g}{\partial x_i}\right)}{\sqrt{\sum \left(\frac{\partial g}{\partial x_i}\right)^2}} \quad (32)$$

where $\left(\frac{\partial g}{\partial x_i}\right)$ is the partial derivate of $g(X)$ with respect to failure point x_i .

The derivation of Equation 31 shows the relative simplicity of combining an *RBD* methodology with an established deterministic design methodology.

6.2 Monte-Carlo simulations

The computational power of modern computers has made different simulation techniques available to us in a way that was not possible before. The Monte-Carlo simulation technique simulates the outcome of a limit state function including one or several stochastic variables, e.g.

$$g(X, Y) = \delta_{max} - \sum h_j \cdot \frac{q}{a \cdot (X) + (1-a) \cdot (Y)} \quad (33)$$

where X and Y are the stochastic variables $X \in \log N(\mu_{E_{col}}, \sigma_{E_{col}})$ and $Y \in \log N(\mu_{M_{clay}}, \sigma_{M_{clay}})$. By realizing Equation 33, a large number (N) of times, the probability of $g(X, Y) < 0$ can be calculated.

6.3 Probabilistic Deep-Mixing design in practice

In *DM*, the initial design is often based on strength and deformation parameter values evaluated from lab tests or assumed based on previous experience. Once the columns have been installed, the strength and deformation parameter values are evaluated in the field and the design is updated accordingly. Figure 10 presents the workflow of a design approach facilitating this design methodology.

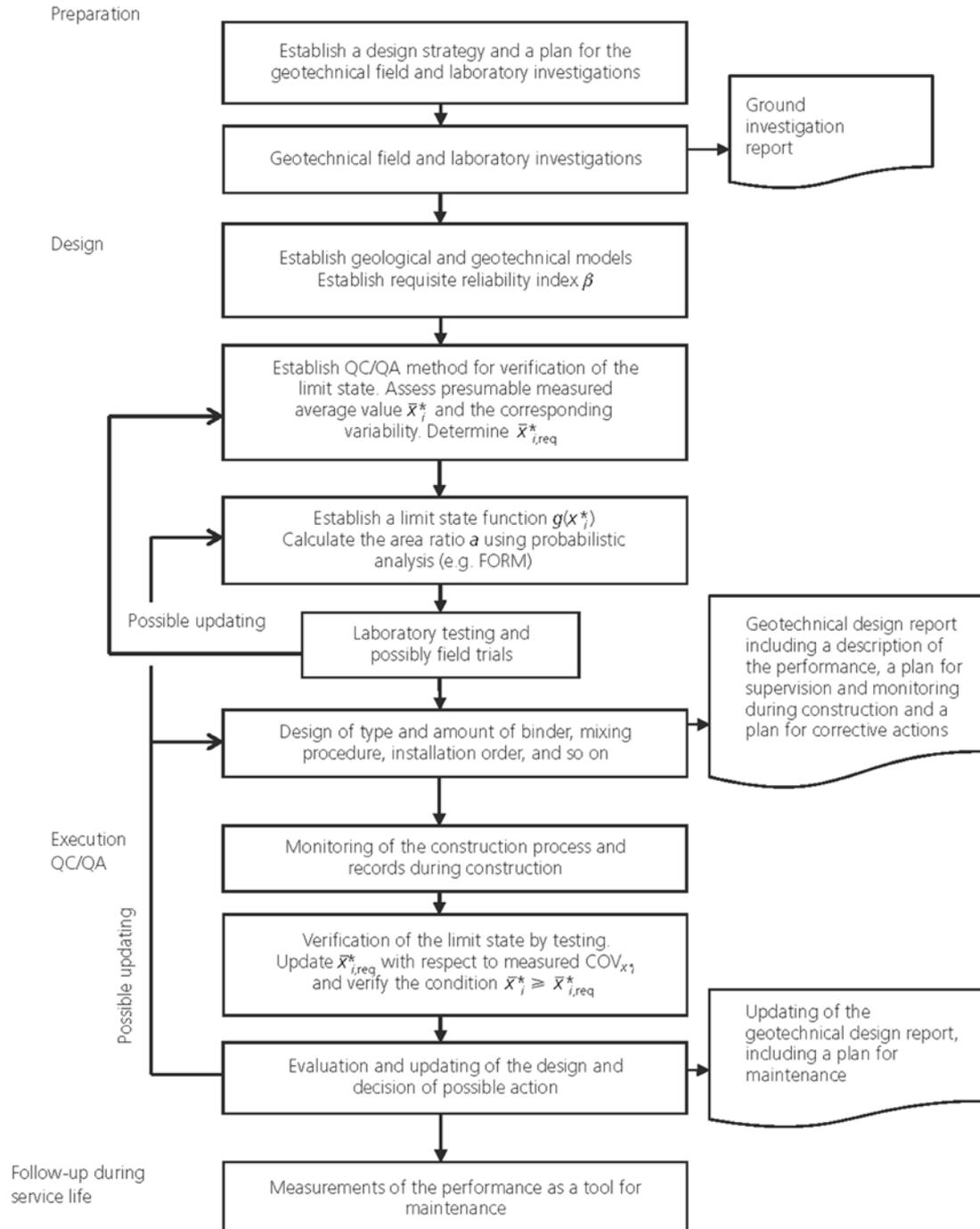


Figure 10: Workflow for the design and verification process of Deep-Mixing (from Larsson and Bergman 2014).

Chapter 7 – Summary of appended papers

This thesis is based on six papers, which have been published in or submitted to international scientific peer review journals or conferences. The following chapter is a summary of these papers.

7.1 Paper I

Strength variability in lime-cement columns based on cone penetration test data

Mohammed Salim Al-Naqshabandy, Niclas Bergman and Stefan Larsson

Published in Ground Improvement 165(1): 15-10, 2012

Deep-mixing is an internationally accepted ground improvement method for improving the engineering properties of soft soils. In Sweden, the dry method, also known as the lime-cement column method, is almost exclusively used. Because of a complex soil-binder mixing process, deep-mixed soils often show high variability in strength and deformation properties. As a consequence, it becomes difficult to predict the engineering properties of the lime-cement column in advance. It is therefore important to verify these properties after installation. In Sweden, this is normally done using the column penetration test. According to Swedish practice, 1% of the columns should be tested after installation. There is, however, no guideline that governs how these columns should be tested with regards to spatial variability, which can be described as the variability of a mean value in space.

Reliability-based design is a rational approach to incorporating parameter uncertainties into the design process. The aim of this paper is to describe the statistical parameters needed to quantify the uncertainties of soil improved by deep mixing. These parameters – the mean, the variance and the scale of fluctuation, which can be described as the distance within a soil property showing a relatively strong correlation – are all prerequisites for reliability-based design.

This study is based on 30 cone penetration tests in soil improved by deep mixing. The test site was located at Lidatorp on Road 73, 50 km south of Stockholm. It was part of a large road development project involving 500,000 m³ of improved soil. The test site itself measured 15 x 15 m and included 312 lime-cement columns. The columns measured 7-8 m in length and 0.8 m in diameter. Of these columns, 30 were chosen randomly for the tests.

Test data were quantified by means and variances using basic statistics and by the scale of fluctuation using variograms. The most important findings of this paper can be summarized as:

- The scale of fluctuation was estimated to be 0.2-0.7 m and 0-3 m in the vertical and horizontal direction, respectively. The spacing between the tests should therefore exceed 3 m in order to attain statistically independent samples.
- A simple design consideration was carried out to show the potential influence of the variance reduction factor in determining the design value. It showed that in the case of high spatial variability, the variance reduction factor had a significant impact on the evaluation of the design value.

7.2 Paper II

Variability of strength and deformation properties in lime-cement columns evaluated from CPT and KPS measurements

Niclas Bergman, Mohammed Salim Al-Naqshabandy and Stefan Larsson

Published in Georisk 7(1): 21-36, 2013

This paper evaluates the strength variability in soil improved by deep mixing using two different test methods: the column penetration test and the cone penetration test. The aim of this study is to examine the impact of each method in assessing the design value using reliability-based design. Firstly, the variability or uncertainties of the test data were quantified by means, variances and scale of fluctuation. Secondly, the strength and deformation properties of the lime-cement columns were evaluated using empirical models. In so doing, further uncertainties were introduced into the evaluation of the strength and deformation properties. The paper also presents a rational approach for how to quantify the total variability in the evaluation of the average strength and deformation properties of lime-cement columns. The uncertainties were divided into four categories: variability associated with spatial variability, variability associated with statistical uncertainties, variability associated with random measurement noise, and variability associated with model and transformation errors.

The study is based on 30 column penetration tests and 30 cone penetration tests executed in lime-cement columns in Kista, 10 km north of Stockholm. The test site itself measured 15 x 15 m and contained 225 lime-cement columns. The columns measured 6 m in length and 0.6 m in diameter. Of these columns, 30+30 were chosen randomly for the tests. To validate parts of the findings from Kista, 12 additional tests were executed at a second test site on Lidingö.

The most important findings of this paper can be summarized as:

- This study shows small differences in the variability of test data, using the two different test methods.
- A simple design consideration demonstrates the impact of different uncertainties in assessing the design value. Uncertainties associated with model and transformation errors are shown to have the most significant impact on the evaluation of the design value.
- The results from the analyses suggest that the relationship between measured cone tip resistances from the cone penetration test and the column penetration test does not correspond to the empirical cone factors proposed in previous studies and in the Swedish Design Guidelines.
- Reliability-based design is recommended for both contractors and clients, since it promotes improvement in manufacturing methodologies and design models.

7.3 Paper III

Comparing column penetration and total-sounding data for lime-cement columns

Niclas Bergman and Stefan Larsson

Published in Ground Improvement 167 (4), 249-259.

In Sweden, the penetration test method commonly used for tests in lime-cement columns is the column penetration test. Because of the relatively large size of the test probe, it is recommended for depths of no more than 8 m. At greater depths and in high-strength columns, the probe easily deviates from the column. To facilitate the verticality of the probe, a center hole can be bored in the column. This is usually done using the total-sounding test method. Consequently, two sets of test data are often produced for each column. The aim of this paper is to quantify the agreement between the two methods. If a good agreement is found, it should be possible to replace the column penetration test with the less expensive and less time-consuming total-sounding test.

The study is based on 38 column penetration tests and 38 total-sounding tests executed at two different test sites. The correlation and agreement between the test data from the two different methods were analysed using Pearson product-moment correlation coefficients and Tukey mean-difference plots.

A good enough agreement between the two methods was found. Thus, it is suggested that the total-sounding test be used as a complement to the column penetration test in evaluating the average strength properties of a group of medium- and high-strength lime-cement columns. In this study, however, the tests were executed in medium- and high-strength columns. Accordingly, this study has not been able to quantify the agreement in the low-strength interval (undrained shear strength < 150 kPa). The impact of the discrepancies between the methods should also be assessed for each design, since discrepancies that are considered acceptable in one design might be unacceptable in another. The total-sounding test method should not be used to evaluate the undrained shear strength of individual columns or to evaluate the strength of low strength columns.

7.4 Paper IV

Serviceability limit state design of lime-cement columns – a reliability-based design approach.

Niclas Bergman, Razvan Ignat and Stefan Larsson

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Hong Kong, December 2013

Deep mixing with lime-cement columns is a ground improvement method used to improve the strength and deformation properties of soft cohesive soils. Due to the complex manufacturing process, the variability in the strength and deformation properties is normally high. A rational approach to including variability in the design process is to introduce reliability-based design. This paper presents a reliability-based design approach for serviceability limit state design of soil improved by lime-cement columns using the First-Order Reliability Method. The paper further presents the impact of uncertainties, distributions, reliability indices and area replacement ratios on the relationship between the characteristic value and the design value with respect to the column modulus of elasticity.

Figure 11 shows the outcome of the probabilistic analyses. The quotient between the characteristic value and the design value (E_k/E_d) is plotted against the total uncertainties ($COV_{\bar{E},TOT}$) with different reliability indices (β) and distributions. The figure shows how the quotient increases with increasing uncertainties. For small uncertainties, the impact of different distributions (normal or log-normal) and of β is small, but becomes substantial as the uncertainties increase. In serviceability limit state design, Eurocode 0 suggests beta = 1.5. It is the authors' belief, however, that a wider range of beta could be allowed for structures whose potential failure is of minor consequence. In such cases, beta should be decided by balancing the cost of making higher quality columns and additional tests against the cost of structural maintenance.

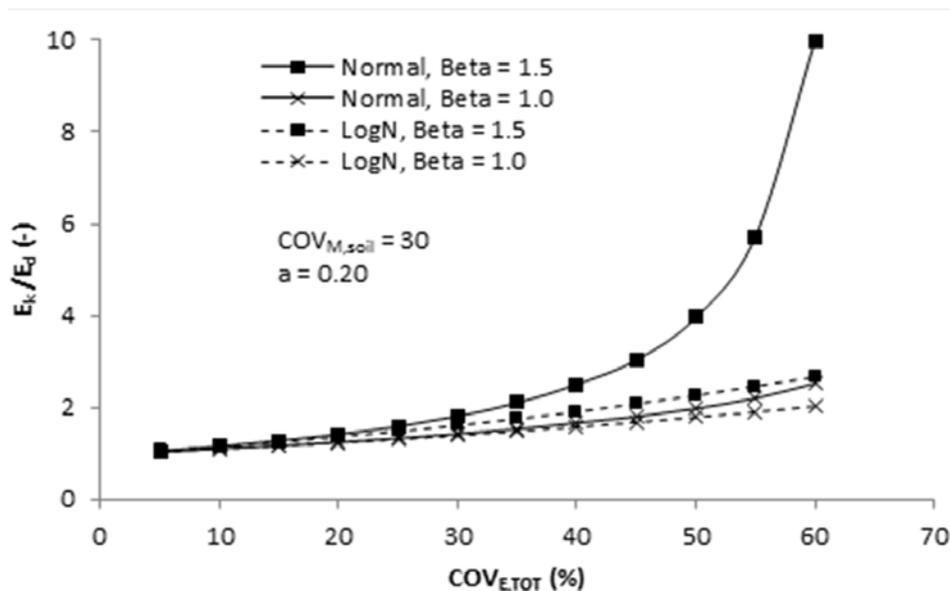


Figure 11: The quotient E_k/E_d plotted against $COV_{\bar{E},TOT}$ with different values of β . (from Bergman et al. (2013)).

7.5 Paper V

Probabilistic design of dry deep mixing using an observational approach

Stefan Larsson and Niclas Bergman

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This paper presents a deep-mixing design and verification procedure, based on probabilistic analysis, which is a combination of design by calculation and the observational method.

Probabilistic analysis has come to be used more frequently in geotechnical engineering since it is considered to better account for design parameter variability. Furthermore, when the design properties are hard to predict, an observational design approach is appropriate.

In deep-mixing design, the initial design criterion is often based on column strength and deformation property values evaluated from lab tests or assumed based on previous experience. Once the columns have been installed, the strength and deformation parameter values are evaluated in the field and the design is updated accordingly.

The design procedure presented in this paper gives a detailed example of probabilistic deep-mixing design in practice using a combination of design by calculation and the observational method. The utility of the proposed design procedure is shown by an illustrative example. Even though probabilistic analyses have the advantage of including parameter variability in the design model, the paper concludes that when probabilistic analyses become highly subjective, they provide little improvement of the safety assessment relative to the use of constant safety factors.

7.6 Paper VI

Probabilistic serviceability limit state design approach for dry deep mixing

Niclas Bergman, Fredrik Johansson and Stefan Larsson

Re-submitted to *Soils and Foundations* in April 2015

This paper presents a probabilistic serviceability limit state (SLS) design approach for dry deep mixing. The design approach constitutes a rational method of including parameter uncertainties and curing time in the design process. The reliability analyses were conducted using Monte-Carlo simulations. The utility of this design approach was described by means of an illustrative example design.

In SLS design, the engineer is normally constrained by:

- The maximum allowed total settlements.
- The distribution of settlements with time.
- The maximum allowed column stress with respect to bearing capacity.
- The maximum allowed differential settlements.

In the present paper, points (1)-(3) are considered in the proposed design approach.

The analyses were performed with different values of evaluated column strength and deformation properties using design parameter values typical for Swedish conditions.

Figure 12 shows the outcome of an example design where the minimum area replacement ratio (a) is plotted against the total uncertainty in the evaluation of the column modulus of elasticity (COV_E) using different values of consolidation time (t_c). In general, the designs were governed by the maximum allowed column stress. An exception is a design allowing short consolidation time using columns with small values of COV_E .

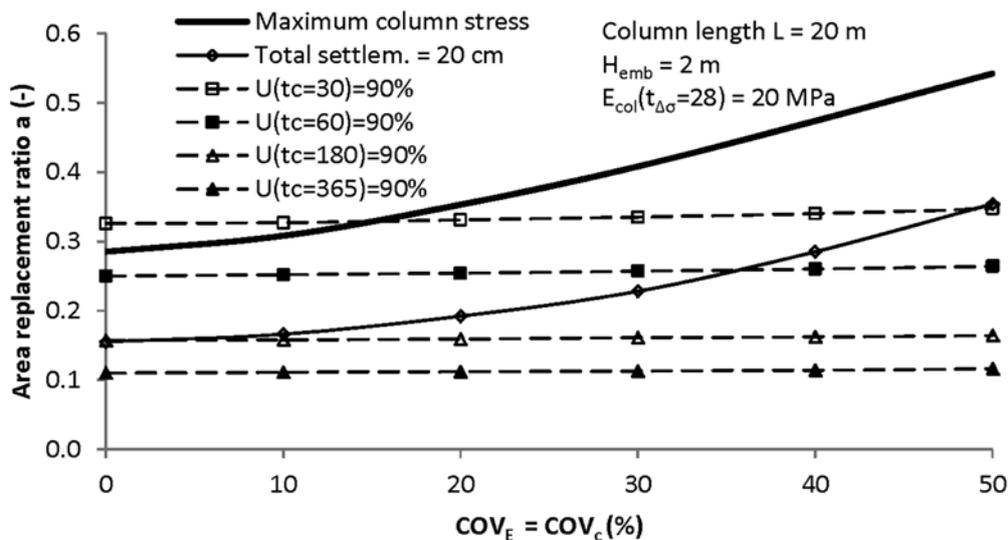


Figure 12: Example design of a column improved soil (from Bergman et al. (2014)).

Chapter 8 – Results and discussion

The following section is a summary and discussion of the findings from the present study.

8.1 Test sites

The spatial variability parameters, coefficient of variation (COV), and scale of fluctuation (θ) were evaluated from penetration test data at two different test sites: Lidatorp and the E18 European highway, both located in the vicinity of Stockholm. As a consequence, the quantification of the spatial variability parameters is only valid for areas close to Stockholm and with similar geotechnical conditions. In order to internationalize this study, a quantification of the spatial variability parameters needs to be conducted at a number of different test sites with varying geotechnical conditions.

With the purpose of comparing the correlation and agreement between the different penetration test methods, a third test site was established on the island of Lidingö, to the east of Stockholm.

8.2 Penetration test methods

The field tests were conducted using three different penetration test methods: the cone penetration test (*CPT*), the column penetration test (*KPS*) and the total-sounding test (*Jbt*). At Lidatorp, 30 *CPT* soundings were performed within a 15 m x 15 m area. On the E18, 30 *KPS* and 30 *Jbt* were performed within a 16 m x 16 m area. At the Lidingö site, 12 *CPT*, 12 *KPS* and 12 *Jbt* were performed.

One of the objectives of this study was to investigate the influence of different test methods (the *CPT* and the *KPS*) on the quantification of means, variances, and scale of fluctuation. The relationship between the undrained column shear strength and the *CPT* and *KPS* penetration resistances are governed by empirical relationships. The agreement between the undrained column shear strength evaluated from the *CPT* and *KPS*, respectively, were evaluated and found to be poor, that is, the relationship between cone tip resistances measured in the *CPT* and the *KPS* does not correspond to the cone factors proposed in previous studies and in the Swedish Design Guidelines. Based on this study, it can be concluded that there is an obvious need for further work on the correlation between penetration resistance and strength and deformation properties in lime-cement columns, including work on the calibration of test equipment. The present study also shows small differences in the variability of test data using the two different test methods. If the impact of variability on the evaluation of the design value is considered independently of method accuracy, the choice of test method becomes unimportant. Finally, as will be further discussed and explained in sections 8.5 and 8.6, the impact of the scale of fluctuation, evaluated from the two different methods, will have an insignificant influence on the *SLS* design.

Further, the correlation and agreement between the *KPS* and the *Jbt* were analysed in order to investigate the possibility of using the *Jbt* as a complementary field control method for lime-cement columns. In this study, a good enough agreement was found between *Jbt* and *KPS*, and it is suggested that *Jbt* be used as a complement to *KPS* in evaluating the average strength of a group of medium- and high-strength lime-cement columns. On sites where relatively good agreement between the methods can be shown, the number of *KPS* can be reduced. It is the author's belief that

the use of Jbt can result in a more cost-effective testing methodology. However, due to the high variability in test data, Jbt should not be used to evaluate single point values. The correlation and agreement analyses were performed on test data from medium- and high-strength columns (undrained shear strength > 150 kPa). Consequently, the study has not been able to quantify the correlation and agreement in the low-strength interval (undrained shear strength < 150 kPa).

8.3 Distribution of test data

In this study, the Kolmogorov-Smirnov test was used to detect normality or log-normality. Some samples were found to be normally distributed, some to be log-normally distributed. Others were found to be neither, or something in-between. In the probabilistic analyses, test data were assumed to be log-normally distributed, primarily because the distribution is strictly non-negative.

8.4 Uncertainties and the coefficient of variation

In this study, uncertainties in the evaluation of the design value are discussed and categorized according to their origin. It was shown that the evaluation of the design value was strongly influenced by uncertainties due to transformation or model errors; consequently, it is recommended that these uncertainties be considered in the *RBD* of soil improved with lime-cement columns. A major source of transformation errors is the empirical relationships between tip resistance and column undrained shear strength. Therefore, it is important to calibrate the penetration test methods with a standardized method and, in so doing reduce the transformation errors.

At the Lidatorp test site, the *COV* with respect to *CPT* tip resistance ranged from 0.22 to 0.67. At the E18 test site, the *COV* with respect to *CPT* and *KPS* tip resistances ranged from 0.18 to 0.59 and 0.19 to 0.47, respectively. The wide range of *COV* indicates high variability in lime-cement columns. This variability is most likely due to the complex mixing process and to the inherent variability of the unimproved soil. Further, the evaluated variability corresponds well to the range reported in previous studies.

8.5 Scale of fluctuation

At the Lidatorp test site, using the *CPT* method, the vertical and horizontal scale of fluctuation ranged from 0.2 to 0.7 m and from 0 to 3 m, respectively.

At the E18 test site, the vertical scale of fluctuation evaluated from *CPT* and *KPS* measured 0.4 m and 0.6 m, respectively. The horizontal scale of fluctuation was evaluated at three depths for both *CPT* and *KPS*. At a depth of 2.5 m below ground, *CPT* data indicated a horizontal θ of 3.5 m. However, due to the scatter in the sample *ACF*, the evaluated horizontal θ is questionable. Furthermore, no indication of a horizontal θ could be found at any other depth or by using *KPS*.

The values of the vertical scale of fluctuation evaluated in this study are consistent with those reported in previous studies, summarized by Al-Naqshabandy et al. (2012), and they are therefore considered to be reliable.

It is the author's belief that the horizontal scale of fluctuation originates primarily in the mixing process rather than being a property inherited from the virgin soil. If the horizontal scale of

fluctuation originates in the mixing process, it should be possible to reduce it by improving the manufacturing methodology.

Furthermore, to fulfill the requirement of uncorrelated samples, tests should be separated by a distance greater than the scale of fluctuation.

8.6 Variance reduction factor

The variance reduction factor (I^2) is proportional to the product of the sizes of the three one-dimensional scales of fluctuation and inversely proportional to the product of the sizes of the three one-dimensional failure domains. Accordingly, the study concludes that mechanical systems which are much larger relative to the evaluated scale of fluctuation have relatively small values of I^2 ; the impact of I^2 on the *SLS* design of lime-cement columns is therefore considered to be very small.

8.7 Reliability-based/probabilistic design

All of the models used in this study are accepted and obtained from published peer reviewed journal papers, design guidelines, and standards or geotechnical handbooks. By definition, models are simplifications of reality and are thus associated with model errors, which represent the inconsistency between the reality and the outcome of the models. However, none of these models errors has previously been quantified in any published study. Accordingly, they were assumed for the probabilistic analyses conducted in this study.

The settlement model used in this study (Equation 26) is a weighted mean, equal strain model (Voigt model) where the modules are formulated from Hook's law. Even though apparently simple, the model has proven to be valid for a design using end-bearing lime-cement columns. In a study by Jiang et al. (2013), this simplified model was compared with a 3D *FEM* (Finite Element Method) model and was found to give a conservative estimate of the settlement; more precisely, it slightly overestimated the settlement by 10% or less.

Generally, high-order complexity models include a relatively higher number of design parameters compared to simpler models. These additional design parameters bring further uncertainties to the probabilistic analyses. The potential gain in accuracy of the high-order models may be nullified by the increased uncertainties as they are brought into a probabilistic design methodology, as shown by Müller and Larsson (2013).

Numerous papers on numerical analyses of the serviceability limit state using *FEM* have been published over the past decade (Omine et al. 1999, Vogler and Karstunen 2009, Chai et al. 2010, Venda Oliveira et al. 2011, Banadaki et al. 2012, Horpibulsuk et al. 2012, Jiang et al. 2013, Muntohar et al. 2013, Pongsivasathit et al. 2013, Yong 2013, Kamash et al. 2014, Yapage et al. 2014, Zheng et al. 2014, Huang et al. 2015). It is, however, reasonable to question the utility and contribution of *FEM* in probabilistic design. The results of *FEM* analyses are often highly dependent on subjective model assumptions such as boundary conditions and the size and shape of the mesh, which introduces additional uncertainties into the analyses. Since probabilistic analyses are highly sensitive to uncertainties, the impact of these additional uncertainties on the design needs to be considered before combining probabilistic design with *FEM* analyses.

With previous reasoning in mind, the present study advocates the use of relatively simple design models in probabilistic design in cases with limited knowledge about the uncertainties in the design parameters. The governing design parameters should be treated as stochastic variables and efforts should be made to thoroughly quantify their mean value and variability. The outstanding design parameters should be treated in a deterministic way and cautious estimates of their design value should be used.

The study concludes that reliability-based/probabilistic design is a rational approach to including strength and deformation parameter variability in *DM* design. If it is adapted and presented in the context of its particular area of application, the methodology can become a useful design aid. The methodology should also appeal to both contractors and clients as it encourages improvement in manufacturing methodologies as well as in design models. However, although probabilistic analyses have the advantage of including parameter variability in the design model, the study concludes that when probabilistic analyses become highly subjective, they provide little improvement of the safety assessment relative to the use of constant safety factors. The subjective parts of the design therefore need to be minimized. A challenging but important task is the quantification of parameter uncertainties. Only when this has accomplished can the reliability-based/probabilistic design approach be considered a credible design option.

Chapter 9 – Conclusions and future research

The following section is a summary of the major findings and conclusions from the present study. Future related research is also suggested.

The major findings and conclusions from this doctoral project can be summarized as:

- The scale of fluctuation was estimated to be 0.2-0.7 m and 0-3 m in the vertical and horizontal direction, respectively. The impact of spatial variability on the *SLS* design of *DM* was however shown to be small.
- The relationship between cone tip resistances measured using the cone penetration test and the column penetration test does not correspond to the cone factors proposed in previous studies and in the Swedish Design Guidelines.
- The agreement between the column penetration test and the total-sounding test was found to be “good enough”. It is therefore suggested that the total-sounding test be used as a complement to the column penetration test in evaluating the average strength properties of a group of medium- and high-strength lime-cement columns.
- Reliability-based/probabilistic design is a rational approach to incorporating strength and deformation parameter variability with an *SLS* design. As a necessity for the credibility of the methodology, a quantification of the parameter uncertainties needs to be carried out.

Future research considered to be beyond the scope of this doctoral project:

- Due to the strong influence of transformation and model errors, it is important to minimize the magnitude of these errors. Accordingly, it is recommended that the column penetration test be calibrated using a standardized method and, in so doing, reduce the transformation errors.
- This study has not been able to quantify the correlation and agreement between *Jbt* and *KPS* in low-strength columns (undrained shear strength < 150 kPa). Accordingly, further research on the correlation and agreement between the two methods is needed.
- Because of the significant influence of *Jbt* bar friction on the magnitude of the total penetration force, there is a need for a standardized method for *Jbt* bar friction assessment.
- Although it is an important and problematic topic, design constraints related to differential settlements were not treated thoroughly in this study. Hence, it is suggested for future research.
- This study did not quantify nor consider any possible correlations between the design parameters. Accordingly, further research on this topic is needed.

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Appended papers