Progressive failure analyses of concrete buttress dams

INFLUENCE OF CRACK PROPAGATION ON THE STRUCTURAL DAM SAFETY

CHAORAN FU & BJARTMAR PORRI HAFLIÐASON
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Abstract

Concrete buttress dams are the most common type of concrete dams for hydropower production in Sweden. Cracks have been observed in some of them. However, only limited research has been made concerning the influence of these cracks on the structural dam safety. In conventional analytical stability calculations, a concrete dam is assumed to be a rigid body when its safety is verified. However, when cracks have been identified in a dam structure, the stability may be influenced and hence the information of cracks may need to be included in the stability calculations.

The main aim of this project is to study how existing cracks and further propagation of these cracks, influence the structural dam safety. Another important topic was to study suitable methods to analyse a concrete dam to failure. In addition, a case study is performed in order to capture the real failure mode of a concrete buttress dam.

The case study that has been studied is based on a previous project presented by Malm and Ansell (2011), where existing cracks were identified in a 40 m high monolith, as a result from seasonal temperature variations. Two similar models are analysed where one model is defined with an irregular rock-concrete interface, and the other with a horizontal interface.

Analyses have been performed on both an uncracked concrete dam but also for the case where information regarding existing cracks, from the previous project, have been included in order to evaluate the influence of cracks on the dam safety. The finite element method has been used as the main analysis tool, through the use of the commercially available software package ABAQUS. The finite element models included nonlinear material behaviour and a loading approach for successively increasing forces called overloading, when performing progressive failure analyses.

The results show that existing cracks and propagation of these resulted, in this case, in an increased structural safety of the studied dam. Furthermore, an internal failure mode is captured. The irregular rock-concrete interface has a favourable effect on a sliding failure and an unfavourable effect on an overturning failure, compared to the case with the horizontal interface.

Based on the results, the structural safety and the failure mode of concrete buttress dams are influenced by existing cracks. Although an increased safety is obtained in this study, the results do not necessarily apply for other monoliths of similar type. It is thus important that existing cracks are considered in stability analyses of concrete buttress dams.

Keywords: concrete buttress dams, cracked concrete, failure modes, safety factor, finite element analysis, nonlinear material properties
Sammanfattning


Huvudsyftet med detta projekt är att studera hur befintliga sprickor och dess propageringen påverkar dammsäkerheten. Ett annat viktigt syfte är att studera lämpliga metoder för att analysera en betongdamm till brott. Dessutom, genomförs en fallstudie i syfte att analysera ett verkligt brottförlopp av en lamelldamm.

Fallstudien som utförs i detta projekt, baseras på ett tidigare projekt utfört av Malm and Ansell (2011), där befintliga sprickor identifierades i en monolit på 40 m som ett resultat av temperaturvariationer. Två modeller med snarlik geometri har analyserats, där den ena är definierad med en med oregelbunden kontaktyta mellan berg och betong och den andra med en horisontell kontaktyta.

Analyserna har utförts på dels en sprucken damm men även där information om befintliga sprickor från det tidigare projektet beaktas, i syfte att jämföra inverkan av sprickor på dammsäkerheten. Finita element metoden har använts som verktyg vid dessa analyser, genom det kommersiellt använda programmet Abaqus. De finita element modellerna inkluderar icke-linjära material egenskaper hos betong och armering samt baseras på en metod för successiv belastning, som kallas ‘overloading’, vid analys av brottförloppet.

Resultatet visar att befintliga sprickor och propageringen av dessa i detta fall kan leda till ökad säkerhet hos den studerade dammen jämfört mot fallet utan beaktande av sprickbildning. Utöver detta fängas även ett inre brottmod. Den oregelbundna kontaktytan mellan betongen och berget har en gynnsam effekt vid ett glidbrott men en gynnsam inverkan vid ett stjälpningsbrott, i jämförelse med fallet med en horisontell kontaktyta.

Baserat på dessa resultat så påverkas dammens säkerhet och brottförloppet hos lamelldammen utav befintliga sprickor. Även om en ökad säkerhet fas i denna studie är det inte säkert att detta stämmer för andra monoliter av samma slag. Dock är det viktigt att hänsyn tas till befintliga sprickor i stabilitets analyser av lamelldammar.

Nyckelord: betong, lamelldammar, uppsprucken betong, brottmoder, säkerhetsfaktor, finita element analyser, icke-linjära material modeller
Preface

The research presented in this project has been carried out from January to June 2015 at SWECO Energuide AB in collaboration with the Division of Concrete Structures, Department of Civil and Architectural Engineering at the Royal Institute of Technology (KTH). The project was initiated by Dr. Richard Malm who also supervised the project, together with Ph.D. candidate Daniel Eriksson and Adjunct Prof. Erik Nordström.

We would like to express our sincere gratitude to Dr. Richard Malm for his guidance, encouragement and invaluable advice throughout the project. Furthermore, we are grateful to Ph.D. candidate Daniel Eriksson for taking time to help and support, whenever needed.

We also wish to thank Adjunct Prof. Erik Nordström for his advise and input to the project.

Alongside our supervisors, we would also like to thank Johan Nilsson for giving us the opportunity to carry out the work at SWECO Energuide AB.

Stockholm, June 2015

Bjartmar Þorri Haflíðason and Chaoran Fu
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Chapter 1

Introduction

1.1 Background

Buttress dams are the most common type of the concrete dams in Sweden. According to Cederström (1995), there are around 1000 dams in Sweden linked to hydropower production, 200 of which are classified as large dams, i.e. higher than 15 m. The total number of large concrete dams in Sweden is 44, where 27 of them are buttress dams. This is according to a computerised version of the World Register of Dams, unavailable to the authors, nevertheless published by Douglas et al. (1998). A typical Swedish concrete buttress dam is shown in Figure 1.1.

![Figure 1.1: Example of a typical Swedish concrete buttress dam, photo by Christer Vredin (Sweco).](image)

The majority of the Swedish dams were built from the 1950s to 1970s. (Cederström, 1995) Cracks have been detected in some of the dams from the time of construction, and additional cracks have also been observed over time. Cracks due to restrained
shrinkage during cooling, following casting, are well known in large concrete structures. Cracks can also be caused by high stresses from loads in the serviceability state, including seasonal temperature variations, hydrostatic pressure and ice loads. (Malm and Ansell, 2011)

Limited research has been made regarding the influence of cracks on the global dam safety, although the cracks may result in potential failure planes within the dam body. Nowadays, design and calculation models are normally based on highly simplified methods. According to RIDAS, the Swedish power companies guidelines for dam safety, the general procedure in the structural design of a buttress dam, is to verify its global stability only considering overturning and sliding failure separately, and consider the dam acting as a rigid body. (Svensk Energi, 2011)

It is of great importance to perform stability analyses with existing cracks, since these cracks could cause possible instability. It is likely that a real failure mode may be a result of combinations of the theoretical modes, or an internal failure of the monolith.

Numerical models based on the finite element method, have been used to simulate and identify the cause of cracking and their location in Swedish buttress dams. Malm and Ansell (2011) suggested further research, aiming to calculate the safety of existing concrete buttress dams, including the existing cracks.

1.2 Aims and scope

The main aim of this project was to capture a real failure mode of a cracked concrete buttress dam and detect how the existing cracks effect the structural dam safety. This project only focused on Swedish buttress dams, however, it could also provide a basis for analyses of other similar buttress dams worldwide. Furthermore, the results could also give an important insight for flooding analyses regarding dam failures.

The following research questions were sought to be answered in this project.

- How is the safety of a buttress dam influenced by existing cracks and propagation of these cracks?
- How does a real dam failure occur in a cracked concrete buttress dam?
- How should the progressive failure be simulated?
- How does the use of a horizontal rock-concrete interface, instead of an irregular interface, influence the results of stability calculations?

The limitations of this project are presented below.

The only loads considered were the following static loads; hydrostatic pressure, uplift pressure at the rock-concrete interface and ice load. Furthermore, the uplift pressure was defined according to RIDAS, where it is assumed to only act on the frontplate of the monolith with a linearly decreasing pressure distribution. Thus, potential opening in the rock-concrete interface is not taken into account and uplift pressure
is not allowed to act on the bottom of the buttress wall. Water penetration into opening cracks in the monolith was also excluded.

As the main aim of the project was to analyse the influence of existing cracks on the safety of a monolith, the rock foundation was assumed to be of a linear elastic material in all analyses, thus excluding potential failure modes in the rock mass, e.g. crushing failure and internal sliding failure.

According to RIDAS, a common procedure was to include rock bolts in old concrete dams in Sweden, to connect the frontplate to the rock foundation. The rock bolts were intended to give an additional safety, although the influence of them could not be accounted for in stability calculations due to difficulties in verifying their strength. Thus, a concrete dam should be stable without considering any rock bolts. Due to that restriction, set by RIDAS, as well as unknown conditions of rock bolts, rock bolts were excluded in this project.

An additional limitation is the idealisation of the analysed structure, which is unavoidable when a numerical model is constructed. In this project, the geometry of the monolith analysed, was simplified to some degree, as well as the location of existing cracks and the geometry of the foundation. However the simplifications made were assumed to have limited influence on the results.

1.3 Outline

The content of the included chapters are summarised below to give an overview of the structure of this report.

In Chapter 2, the general theory and background of concrete buttress dams are presented. The failure calculations for design are also presented along with the design loads considered.

In Chapter 3, information regarding the theoretical background for numerical analyses and nonlinear material behaviour of concrete is summarised.

In Chapter 4, a brief description of a dam monolith used as case study in this project is given, followed by some important information from previous research.

In Chapter 5, a full description of numerical models used in this project is given.

In Chapter 6, the results from all analyses, i.e. the most suitable loading approach and influence of existing cracks, are presented along with discussion.

In Chapter 7, the conclusions of the study in this project are presented, followed by suggestions for further research.
Chapter 2

Concrete buttress dams

2.1 Principles of design

A concrete buttress dam consists of multiple concrete monoliths, placed side by side and separated by contraction joints. Each monolith has two connected structural elements, a relatively slender inclined frontplate which is exposed to the hydrostatic pressure and a buttress wall which supports the frontplate and transfer the hydrostatic forces to the foundation. The inclination of the frontplate results in increased stability due to the additional vertical hydrostatic pressure while the relatively slender frontplate and buttress results in a relatively low uplift pressure in comparison with gravity dams. (Ansell et al., 2007)

There are several types of buttress dams world wide. The type which is the most common in Sweden is generally called slender buttress dam (lamelldamm in Swedish).
The largest Swedish buttress dams are up to 40 m high and can consist of up to 100 monoliths. The buttresses of the highest monoliths are around 30 m to 35 m wide at the foundation with a thickness up to 2 m while the frontplate is 8 m to 10 m wide and with a varying thickness, from about 2.5 m at the foundation to 1.0 m at the dam crest. (Ansell et al., 2010)

The main advantage of building a buttress dam is the lower amount of concrete needed compared to a gravity dam, as buttress dam requires less than 50% of the concrete needed for gravity dam of the same height. Consequently, buttress dams are suitable on low quality foundation that would not be able to support the weight of a gravity dam, however higher pressure is expected due to a smaller contact area. Despite of savings in concrete, a buttress dam is not necessarily less expensive than a gravity dam due to increased work amount and materials regarding formwork and reinforcement. (Bergh, 2014) (Ansell et al., 2007)

2.1.1 RIDAS, the Swedish guidelines for dam safety

In Sweden, buttress dams are designed within the framework of RIDAS, the Swedish power companies guidelines for dam safety. The guidelines are applicable in design of new dams as well as for verification of old dams and are based on normal practice in design of dams worldwide. (Svensk Energi, 2011)

According to RIDAS, a concrete buttress dam should be designed and analysed with all reasonable loads and load combinations acting on the dam. A verification should be made, not only considering global stability due to overturning or sliding, but also local failure in the materials. The failure modes are further described in Section 2.3.

Three different load cases should be taken into consideration for global stability verification; normal, exceptional and accidental loads. In terms of local analyses, serviceability limit state (SLS), ultimate limit state (ULS) and accidental load cases should be considered.

2.1.2 Fundamental principles of stability analyses

Structural safety is commonly determined using the concept of safety factor, $sf$, where the structural resistance, $R$, and the action, $S$, shall fulfil the following condition in Equation (2.1).

$$ S \leq \frac{R}{sf} $$

The safety factor is determined based on experimental observations, experience as well as economical- and political considerations to provide sufficient safety of the structure. (Westberg, 2010)
2.2 Loads

A concrete dam must withstand various types of loads. According to RIDAS, the loads which should be considered in Sweden are the following:

− Self-weight.
− Hydrostatic pressure due to both head- and tailwater.
− Uplift pressure.
− Ice load.
− Earth pressure.
− Traffic loads.
− Loads due to temperature variations, shrinkage and creep.

In addition, other loads which should be accounted for where applicable, are especially earthquake loads and sediment loads. (Westberg, 2010)

A description of the loads considered in this project, can be found in the following subsections.

2.2.1 Self-weight

The self-weight of a dam is normally the dominant stabilising force. In design of new dams, the density of reinforced concrete, \( \rho_c \), should be taken as 2300 kg/m\(^3\), unless results from material testing give a different value. When analysing older dams, the density should be obtained with material testing. (Svensk Energi, 2011)

The total weight of a dam is calculated using Equation (2.2).

\[
F_g = \rho_c \ g \ V_c
\]  

(2.2)

where,

\( F_g \) is the resulting gravity force of the dam structure.
\( g \) is the gravitational acceleration, \( g = 9.81 \text{ m/s}^2 \).
\( V_c \) is the volume of the reinforced concrete.
2.2.2 Hydrostatic pressure

The hydrostatic pressure, \( p_w \), is normally the dominant external static force acting on large dams. Hydrostatic pressure, both upstream and downstream should be considered, by using the most unfavourable combinations of the two. (Svensk Energi, 2011)

The hydrostatic pressure is calculated by using Equation (2.3).

\[
p_w(y) = \rho_w g y
\]  

(2.3)

where,

\( \rho_w \) is the density of water, \( \rho_w = 1000 \text{ kg/m}^3 \).

\( y \) is the depth below the water level, m.

2.2.3 Uplift pressure

The uplift pressure acting on a buttress dam is, according to RIDAS, assumed to decrease linearly from the maximum headwater hydrostatic pressure, to the tailwater hydrostatic pressure.

If the thickness of the buttress is lower than approximately 2 m, the uplift pressure acting on the buttress can be taken as the tailwater pressure only. If the thickness is greater than this, the pressure distribution can be assumed to vary linearly from the heel to the toe of the monolith, see Figure 2.2. This criterion applies only to the centre line of the dam as the water pressure on all edges from the downstream side is equal to the tailwater pressure.

![Uplift pressure distribution, according to RIDAS.](image)

Figure 2.2: Uplift pressure distribution, according to RIDAS.
2.3. Failure modes

The distribution of the uplift pressure can be calculated with help of Equation (2.4).

\[ p_u(x) = \rho_w g \left( H - \frac{H - h}{L} x \right), \quad 0 \leq x \leq L \]  

(2.4)

where,

- \( p_u(x) \) is the uplift pressure at location \( x \), as shown in Figure 2.2.
- \( L \) is the thickness of the frontplate (\( L_{fp} \)) or the total width (\( L_{tot} \)).
- \( H \) is the upstream water head.
- \( h \) is the downstream water head.

### 2.2.4 Ice load

Ice load on dams is generated by a variation in volume of an already built up ice cover at a reservoir. With rising temperatures, the volume of the ice increases and vice versa. With decreasing volume, cracks will be formed in the ice cover, which are instantly filled with freezing water. An increased temperature will push the ice cover, causing horizontal pressure from ice covers to the surroundings. The magnitude of the pressure depends on the thickness of the ice cover, the duration of high temperatures and intensity of the rising temperature. (Bergh, 2014)

According to RIDAS, the magnitude of the ice load should be based on geographical location, altitude and local conditions at and around the dam. In Sweden, the magnitude of the ice load can normally be assumed with a range of 50 kN/m to 200 kN/m. As a guideline, the magnitude can be taken as 50 kN/m at low elevations in the south of Sweden, 100 kN/m up to a degree of latitude crossing Stockholm and 200 kN/m elsewhere. The ice cover thickness can then be assumed to be 0.6 m south of Stockholm and 1.0 m north of Stockholm.

Increased ice load may occur, e.g. if the opposite bank to the dam is steep or the thickness of the ice cover is large. If large openings appear in the ice cover close to the dam, the ice load is redistributed, resulting in increased loads outside of the openings. If the dam face is inclined where the ice load is acting, it can be decreased.

The ice pressure is assumed have a triangular distribution over the thickness of the ice cover with the highest pressure at the design water level.

### 2.3 Failure modes

As emphasised by Westberg (2010), failure modes are of major importance in design of dams. In order to determine the safety of a dam with certainty, all possible failure modes have to be known. The failure of a dam should be determined by the governing mode of failure. If the design is based on failure modes that are not the most essential, there is a risk that the outcome of the analyses will not reflect the real behaviour.
CHAPTER 2. CONCRETE BUTTRESS DAMS

According to RIDAS, three failure modes are considered in the design of concrete dams:

i) Overturning
   - of the whole monolith as a rigid body.

ii) Sliding
    - along the rock-concrete interface,
    - along joints in the rock mass,
    - along joints in the concrete.

iii) Material failure

In the literature, other modes of failure are also presented. According to Fishman (2009), a failure mode, named limit overturning is the most likely failure mode to occur for retaining concrete structures founded on rock without dangerous discontinuities. Another potential failure mode, just mentioned in Ansell et al. (2007), is an overturning failure of internal parts of a cracked concrete dam.

2.3.1 Overturning

According to RIDAS, two conditions have to be satisfied when considering overturning failure.

The safety factor for overturning should not fall below the recommended values. The safety factor is calculated according to Equation (2.5). The safety of the structure is the ratio between stabilising moments and overturning moments. The axis of rotation should be carefully chosen considering the strength of the structure and the foundation. With high quality foundation the axis of rotation is normally assumed to be located at the toe of the monolith.

\[
s_{fo} = \frac{M_s}{M_o}
\]  

(2.5)

where,

\( s_{fo} \) is the safety factor for overturning.

\( M_s \) is the sum of stabilising moments.

\( M_o \) is the sum of overturning moments.

Safety factors recommended by RIDAS are presented in Table 2.1.

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Normal</th>
<th>Exceptional</th>
<th>Accidental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety factor, ( s_{fo} )</td>
<td>1.50</td>
<td>1.35</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Table 2.1: Safety factors for overturning according to RIDAS, (Svensk Energi, 2011).
In addition to the condition described above, the resultant of all vertical forces should be located within a certain area of the foundation. In the case of normal loads, the vertical resultant must be located within the core limit, i.e. inside the mid third of the foundation area, preventing tensile stresses to occur in the foundation. This condition is especially important for buttress dams due to an increased risk of leaking if the frontplate is not pressed against the foundation.

In the case of exceptional loads, the vertical resultant is allowed to be located outside the core area, provided it fall inside the middle 60% of the foundation area. This condition allows tensile stresses to arise close to the heel of the dam. The guidelines propose that full uplift pressure shall be assumed in the tensile area. The appearance of the discussed areas are visualised in Figure 2.3.

The size of the core area is determined based on the Navier’s equation, with the aim, as previously mentioned, of preventing tensile stresses to occur in the foundation ($\sigma(x) < 0$ in Equation (2.6)). (Bergh, 2014)

$$\sigma(x) = -\frac{N}{A} - \frac{M_0}{I_0} x$$  \hspace{1cm} (2.6)

where,

$\sigma(x)$ is the stress at a specific point of interest.
N is the vertical resultant.
A is the total area of the rock-concrete interface.
$I_0$ is the second moment of area of the rock-concrete interface.
$M_0$ is the moment due to the vertical resultant ($R_V \cdot e$).

### 2.3.2 Sliding

As stated above, according to RIDAS, the risk for sliding should be assessed for the rock-concrete interface, weak planes within the foundation as well as for joints in the concrete.
The safety against sliding is based on the criteria that horizontal forces can be transferred from the dam to the foundation without a failure. The sliding stability calculations are based on the Mohr-Coulomb model, where the maximum allowed tangential stress, $\tau$, is estimated as

$$\tau \leq c + \sigma_n \tan \phi$$

(2.7)

where,

- $c$ is the cohesion.
- $\sigma_n$ is the effective normal stress towards the sliding surface.
- $\phi$ is the friction angle between the adjacent materials.

If the tangential stresses and normal stresses are integrated over the sliding plane, Equation (2.7) becomes

$$T \leq c A + N \tan \phi$$

(2.8)

where,

- $T$ is the resultant force acting parallel to the sliding plane.
- $N$ is the resultant force acting perpendicular to the sliding plane.
- $A$ is the contact area.

By using this expression, it is assumed that the ultimate shear capacity is reached at every point on the sliding surface. For ductile materials, this could be true, but sliding planes are, in practice, considered to be brittle or semi-brittle. (Bergh, 2014)

In RIDAS the cohesion is normally neglected, i.e. $c = 0$. By introducing the friction coefficient, $\mu = \tan \phi$, Equation (2.8) can be written as

$$\mu = \frac{T}{N}$$

(2.9)

Equation (2.9) form the bases for the criterion defined in RIDAS (Equation (2.10)), where the safety factor for sliding, $s_f$, is applied as a reduction of the failure value of the friction coefficient, $\tan \delta = 1$.

$$\mu = \frac{T}{N} \leq \mu_{all} = \frac{\tan \delta}{s_f}$$

(2.10)

Where $\mu_{all}$ is the allowed value for the friction coefficient. The allowed friction coefficients are shown in Table 2.2 which are only applicable for dams founded on rock of good quality.
### 2.3. FAILURE MODES

Table 2.2: Allowed friction coefficients and safety factors for sliding according to RIDAS.

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Normal</th>
<th>Exceptional</th>
<th>Accidental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety factor, $s_f$</td>
<td>1.35</td>
<td>1.10</td>
<td>1.05</td>
</tr>
<tr>
<td>Friction coefficient, $\mu_{all}$</td>
<td>0.75</td>
<td>0.90</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Johansson (2005) studied the common practice in Sweden, when selecting the friction coefficient before the implementation of RIDAS, as the guidelines do not indicate the origin of the failure value used. His conclusion is that the failure value is based on experience gained under decades of construction, where the allowable friction coefficient of $\mu_{all} = 0.75$ were interpreted (while no weak planes were found in the rock foundation) as a control for sliding failure in the foundation as well as for sliding in the rock-concrete interface.

**Along the rock-concrete interface**

The monolith is constructed directly on the rock surface. The self-weight of the dam structure should be able to withstand the sliding forces from hydrostatic pressure and others loads. The rock-concrete interface is critical with regard to sliding failure. The dam is built that the shear resistance of the contact should increase to a certain level to improve the stability of the structure Fishman (2009). However, the interface has really poor resistance for overturning moments. The contact between monolith and rock mass is rather easy to open during failure if large overturning moments exist. In order to prevent the overturning failure, grouted rock anchors are usually applied to the dam structure to increase the resistance for overturning moments.

The rock anchors shall not be accounted for in the dam stability calculation when the new dam is under construction. However, it is advantageous to insert the coarse rock anchors ($\phi = 25$ mm to $32$ mm) as additional security, which was often performed on older dams. Bolt connections are also used sometime to increase the stability against sliding if sliding was the main problem. (Svensk Energi, 2011)
Along joints in the rock mass

Sliding could also happen along the joints in the rock mass. The shearing through the rock mass will always occur. According to Gustafsson et al. (2008), the shear strength of rock mass can be described with the Mohr-Coulomb failure criterion,

\[ T = c_m A_{cm} + N' \phi_m \]  \hspace{1cm} (2.11)

where,

- \( T \) is the shear capacity of the rock mass.
- \( A_{cm} \) is intact area of the rock mass.
- \( c_m \) is the cohesion of the rock.
- \( N' \) is the sum of the normal force reduced with respect to uplift force.
- \( \phi_m \) is the friction angle of the rock mass.

The shear strength of joints is then critical to sliding. The Mohr-Coulomb criterion is not sufficient to describe the strength in joints. The Mohr-Coulomb can only provide a linear description of the shear strength, while the exponential curve of the real failure shear strength is more favourable to simulate with a bi-linear approach. The details about bi-linear approach for exponential curve are presented in Section 3.1.1.

The material property of rock should be examined and analysed to satisfy the requirement regarding the sliding failure.

Along joints in the concrete

The concrete structure would also suffer risk in sliding failure with itself, the shear resistance could also be described by Mohr-Coulomb criterion. The shear strength in the concrete part would then be critical for this type of safety factor.

The concrete shear friction has a friction coefficient of \( \mu = 0.5 \) as FIB (2013) suggested. The monolith concrete is reinforced fully in all area. The amount of reinforcement is then controlled the shear strength in joints. However, the shear capacity is dependant on both roughness of the cracked surface and the strength of the reinforcement.

2.3.3 Material failure

Material failure will occur if stresses in the dam or the foundation exceeds the ultimate strength of the material. Stresses inside the dam body are often calculated based on Navier’s equation, see Equation (2.6). Compressive- or shear strength of concrete or rock is generally not exceeded in dams of a height below 100 m. Tensile stresses are however more likely to be exceeded, especially at the upstream side of the frontplate of concrete buttress dams. (Bergh, 2014).
Chapter 3

Numerical modelling of concrete

In this chapter, various aspects related to numerical modelling of concrete are presented. Several different material models have been developed over the years which describe the structural behaviour of concrete. This chapter however, will focus on those material models available in the finite element program Abaqus.

3.1 Nonlinear behaviour of concrete

Concrete is a composite material which consists mainly, approximately 60% to 70%, of aggregates that are glued together with hydrated cement paste. Aggregates are a mixture of particles of varying grain size, usually obtained from rock material. In normal concrete, i.e. with water-cement ratio above 0.4 and high quality aggregates, the strength of hardened concrete is influenced by the cement paste and the bond between cements and aggregates. Concrete is generally assumed to be homogeneous and isotropic in design and in numerical analyses. (Ansell et al., 2013)

Concrete can resist large compressive stress, however the tensile strength of concrete is much lower, normally only around a tenth of the compressive strength. Concrete has different behaviour when subjected to high or low pressure, where a brittle behaviour occurs when subjected to a low pressure and a plastic behaviour occurs when subjected to high pressure in uni-axial stress state. (Björnström et al., 2006)

3.1.1 Uni-axial stress

Compression

From experimental observations, the results have shown that concrete has nonlinear behaviour under uni-axial compressive stresses. Figure 3.1 shows the typical failure mechanism for concrete under uni-axial compression. The stress-strain curve can be subdivided into three stages by the points (a) to (d).
Concrete is considered to act linear-elastic, when subjected to low compressive stresses, as shown between point (a) and (b) in Figure 3.1. At this stage, only crack due to incomplete bond exists. Micro cracks will not propagate during this stage. Thus, the stress-strain curve is approximately linear. Point (b) is reached after approximately 30% of the compressive strength. The stiffness of the concrete will start to decrease in the macroscopic scale between point (b) and point (c). The nonlinear behaviour in this stage has minor effects on the stress-strain relationship. (Mang et al., 2003)

After point (c), increased loading will result in formation of visible cracks. The residual stresses will also reduce due to the material failure.

The nonlinear stress-strain relation can for instance be calculated based on EC 2 (2004) as shown in Equation (3.1),

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta} \tag{3.1}$$

where,

- $\eta$ is the ratio of actual compressive strain compared to strain at peak stress, $\eta = \varepsilon_c/\varepsilon_{c1}$.
- $\varepsilon_c$ is the compressive strain in the concrete.
- $\varepsilon_{c1}$ is the compressive strain in the concrete at the peak stress $f_c$.
- $k$ is a factor describing the actual stress compared to the compressive strength, $k = 1.0 \frac{E_{cm}}{f_{cm}} |\varepsilon_{c1}|/f_{cm}$.
- $\sigma_c$ is the compressive stress in the concrete.
- $f_{cm}$ is the mean value of concrete cylinder compressive strength.

**Tension**

Concrete is normally assumed to be a linear-elastic material until it reaches the tensile strength, although some minor plastic deformation occurs. (Eriksson and Gasch, 2011) In Figure 3.2, the stress-strain relationship for a concrete specimen subject to a deformation controlled tensile loading is illustrated.
As shown in Figure 3.2, the stress-strain curve is approximately linear until stress in specimen reach around 70% of the tensile strength. At this stage, the pre-existing cracks remain stable. After passing the tensile strength, these cracks will start to form and develop, which lead to the reduction of stiffness as shown in Figure 3.2 (d). The stress-strain curve consequently become nonlinear. The observable crack ($\geq 0.1 \text{mm}$) will start to propagate perpendicular to the tensile stress direction at the final stage of the crack opening. (Björnström et al., 2006)

Tension softening

The concrete cracking in finite element analyses is performed by introducing a crack opening law which is often referred to tension softening. The tensile strength $f_{ct}$ of the material determines the crack initiation and the fracture energy $G_f$ is used to describe how the crack propagates. Fracture energy $G_f$ is a material property that describes the energy consumed when an unit area of a crack is completely opened, which is defined as the area under the tension softening curve. (Karihaloo, 2003)

The most simple solution to introduce the crack opening law is to apply an approximation, where the softening is assumed to be linear. The linear tension softening model is calculated according to Equation (3.2).

$$\omega_c = 2 \frac{G_f}{f_t} \quad (3.2)$$

The bi-linear estimation is a good numerical simulation to apply the tension softening. The bi-linear relationship proposed by Hillerborg et al. (1976) is commonly used. This bi-linear approach is a good simulation of the exponential curve by Cornelissen et al. (1985), both shown in Figure 3.3.
CHAPTER 3. NUMERICAL MODELLING OF CONCRETE

(a) Bi-linear function.

(b) Exponential function.

Figure 3.3: Bi-linear (Hillerborg et al., 1976) and exponential (Cornelissen et al., 1985) tension softening model.

For the exponential model proposed by Cornelissen et al. (1985) follows the expression,

$$\frac{\sigma}{f_t} = f(\omega) - \left(\frac{\omega}{\omega_c}\right)f(\omega = \omega_c) \tag{3.3}$$

in which:

$$f(\omega) = (1 + \left(C_1 \frac{\omega}{\omega_c}\right)^3 \exp(-C_2 \frac{\omega}{\omega_c} \tag{3.4})$$

where,

- $\omega$ is the crack opening displacement.
- $\omega_c$ is the crack opening at which stress no longer can be transferred, $\omega = 5.146 \frac{f}{f_t}$ for normal density concrete.
- $C_1$ is the material constant which $C_1 = 3$ for normal density concrete.
- $C_2$ is the material constant which $C_2 = 6.93$ for normal density concrete.

### 3.1.2 Multi-axial stress

The behaviour of concrete under multi-axial stress is different from the behaviour of the uni-axial stress. Figure 3.4 shows the failure development for concrete and cracking that corresponds to the bi-axial loading.
3.1. NONLINEAR BEHAVIOUR OF CONCRETE

Figure 3.4 illustrates the yield criteria for concrete under bi-axial loading. The tensile cracking occurs in the first, second and fourth quadrant where it is subjected to tensile stresses. The crack grows perpendicular to the principal tensile stress. The state of simultaneous compression and tension reduces the tensile strength. The third quadrant describes the bi-axial compression condition. The compressive strength increases significantly under bi-axial compression; up to 25% of the uni-axial compressive strength. (Malm, 2006)

When subjected to tri-axial compressive stresses, the mode of failure involves either tensile fracture parallel to the maximum compressive stress or a shear mode of failure. The strength and ductility of concrete under tri-axial compression increases significantly compared to the state under uni-axial compression. (Wight and MacGregor, 2000)

As shown in Figure 3.5, the cylinder is subjected to constant lateral fluid pressure $\sigma_3$, while the longitudinal stress, $\sigma_1$ is increased until failure. In cases with high confining pressures ($\sigma_3 = 4090$ psi, 28 MPa), the compressive failure is much more brittle than the corresponding lower confining pressures ($\sigma_3 = 550$ psi, 3.8 MPa).
3.2 Constitutive material models for concrete

In this section, an introduction to fracture mechanics, plasticity theory and damage theory for concrete is given. The constitutive models implemented in the finite element program is presented in Section 3.2.4.

3.2.1 Basic failure mode

The concrete failure modes are usually described as three different modes which can be seen from Figure 3.6. Mode I is the tensile failure, Mode II is the shear failure and Mode III is the tear failure. For concrete, Mode I is the most common type of crack growth and it could in some case occur in its pure form. The other modes are rarely obtained in their pure form. Combinations of the different modes often occur, and in concrete it is usually a combination of Mode I and II. (Malm, 2006)
3.2.2 Plasticity theory

The plasticity theory is generally used to describe ductile materials, like metals, however it could also be applied to brittle materials under specific situations. Plasticity theory has been applied to describe compressive behaviour of concrete successfully, where Karihaloo (2003) had several different examples illustrated. Most of the classical theories have described concrete as a brittle material, but nowadays researches have intend to treat the concrete with the means of plasticity theory. (Lubliner et al., 1989)

Yield and failure function

Concrete can exhibit a significant volume change when subjected to severe inelastic states. The point on the stress-volumetric strain diagrams in Figure 3.7a indicate the limit of elasticity, the point of inflexion in the volumetric strain, the bend-over point corresponding to the onset of instability or localisation of deformation, and the ultimate load. As shown in Figure 3.7b, the failure surface "expand", i.e., the reserves of strength that concrete has from the moment its elastic limit is reached until it completely ruptures. (Karihaloo, 2003)

![Figure 3.7: The yield and failure surface from Karihaloo (2003).](image)

(a) Volumetric strain change in bi-axial compression. 
(b) Typical loading curve under bi-axial stresses.

The yield criterion is described as a yield function for bi-axial and multi-axial behaviour. The von-Mises yield criterion is the most famous one which normally applies to steel material. Concrete and other brittle materials, mainly depend on the Mohr-Coulomb and the Drücker-Prager criteria, which both can be expressed as,

\[ F(\sigma) = c \] (3.5)

where,

\[ c \] is the cohesion.
\( F(\sigma) \) is the function that is homogeneous in the first degree of the stress components.

However, both criteria have poor correlation with experimental data for concrete and geo-materials. Figure 3.8 shows three common criteria along with the experimental data from Kupfer et al. (1969). The combination of Drucker-Prager and Mohr-Coulomb criteria was developed by Lubliner et al. (1989), the criterion is quadratic in octahedral shear stress (or, equivalently in \( \sqrt{J_2} \) the second invariant of the stress deviator) and linear in the mean normal stress (or in \( I_1 \), the first invariant of stress); the third invariant enters through the polar angle \( \beta \) in the deviatoric plane. With these forms, the meridians in the \( \sigma_1, \sigma_2, \sigma_3 \) space are curved. The failure surface tends to a circular cylinder as \( I_1 \to \infty \). However the high pressure region is excluded. Then, the failure criterion could use Equation (3.6) to fit the experimental data. (Lubliner et al., 1989)

\[
F(\sigma) = \frac{1}{1-\alpha} \left[ \sqrt{3J_2} + \alpha I_1 + \beta <\sigma_{\text{max}} > -\gamma < -\sigma_{\text{max}} > \right] \tag{3.6}
\]

where \( \alpha, \beta, \gamma \) are dimensionless constant, the details about the calculation of these constant is introduced in Section 3.2.4.

![Figure 3.8: Failure criterion for bi-axial stress state illustrated for plane stress state. From Jirasek and Bazant (2002).](image)

**Hardening**

The uni-axial stress-strain relationship, shown in Figure 3.1, indicates that the compressive stress could increase after the nonlinear strain start to appear. The strain is normally regarded as small and finite, thereby the strain rate tensor can be decomposed into an elastic part and a plastic part.

\[
\dot{\varepsilon} = \dot{\varepsilon}^{el} + \dot{\varepsilon}^{pl} \tag{3.7}
\]

where,

\( \dot{\varepsilon} \) is the total strain rate.
\( \dot{\varepsilon}^{el} \) is the elastic part of the strain rate.

\( \dot{\varepsilon}^{pl} \) is the plastic part of the strain rate.

Plasticity theory allows a description of the dependence of strain in the material on its history through the introduction of an internal scalar variable, here defined as scalar hardening parameter \( \kappa \) as expressed in Equation (3.8).

\[
d\kappa = f(d\varepsilon^{pl})
\]  

(3.8)

The dependence of the yield function \( f(\sigma, \kappa) \) on the loading history through the scalar hardening parameter \( \kappa \) can only expand or shrink but not translate or rotate in the stress space. Such type of hardening is called isotropic hardening, irrespective of the work-hardening or the strain-hardening hypothesis. (Karihaloo, 2003)

Isotropic hardening is normally considered to be a suitable model for concrete material. For isotropic hardening, the yield surface will expand with the increasing stress in all directions with the same shape and origin. In Figure 3.9, the initial yield surface expand to the subsequent surface after hardening. Isotropic hardening is defined in Equation (3.9),

\[
f(\sigma_{ij}, \kappa) = f_0(\sigma_{ij}) - \kappa = 0
\]  

(3.9)

where,

\( \sigma_{ij} \) is the second order stress tensor.

\( \kappa \) is the hardening parameter.

Equation (3.9) defines that the initial yield function will change in size as the hardening parameter \( \kappa \) changes.

![Figure 3.9: The yield surface for isotropic hardening.](image)

Kinematic hardening is another common hardening rule, which means that the Bauschinger effect is modelled. The Bauschinger effects is a phenomenon which plastic deformation of a material increases the yield strength in the direction of
plastic flow and decreases the yield strength in the opposite direction. The yield surface will keep the same shape and size and only translate as a rigid body in stress space.

\[ f(\sigma_{ij}, \kappa) = f_0(\sigma_{ij} - \alpha_{ij}) = 0 \]  

(3.10)

where,

\[ \alpha_{ij} \]

is the stress which known as back-stress or shift-stress.

Equation (3.10) suggests the scaling hardening parameter \( \kappa \) in this case is the stress \( \alpha_{ij} \), the initial yield surface coordinates \( \sigma_1 \) and \( \sigma_2 \) will shift to the axes of \( \alpha_{ij} \). Figure 3.10 describes the translation of the yield surface due to hardening. Once the stress reaches the initial yielding point, the yield surface will translate to the subsequent yield surface.

![Figure 3.10: The yield surface for kinematic hardening.](image)

Some other hardening rules can also be used, for example, a combination of the kinematic and isotropic rule. This approach will have more hardening parameters.

**Flow rule**

The connection between the stress-strain relationship and the yield surface is the flow rule.

The flow rule represents the direction of the inelastic deformations in classical plasticity. The concrete will suffer significant volumetric change in the plastic stage. This change in volume, caused by plastic distortion, can be reproduced by using adequate plastic potential function \( G \), as defined in Equation (3.17). (Lubliner et al., 1989) The details for the function is describe in Section 3.2.4.

In the associative flow rule, the plastic flow develops along the normal direction to the yield surface. The flow rule follow the yield criterion as mentioned in Section 3.2.2. (Malm, 2006) The other approach is the non-associative flow rule, the plastic flow rule and the yield surface do not coincide in this approach. The flow direction is not normal to the yield surface. (Gálvez et al., 2002)
3.2.3 Damage theory

Continuum damage mechanics are usually applied to describe the nonlinear behaviour of concrete. The progressive evolution of micro-cracks and nucleation and growth of voids are represented in concrete damage models by a set of state variables which alter the elastic and plastic behaviour of concrete at macroscopic level. In practical implementation, the concrete damage model is very similar to the plasticity theory from Section 3.2.2. (Karihaloo, 2003)

The damage model of the total stress-strain relationship is defined by Equation (3.11),

\[ \sigma = D^s : \varepsilon \]  

where,
\( \sigma \) is the stress tensor.
\( D^s \) is the stiffness tensor represent the stiffness of the undamaged model.
\( \varepsilon \) is the strain tensor.

Equation (3.11) distinguishes itself from classical nonlinear elasticity by a history dependence, which is presented through a loading-unloading function \( f \), which vanishes upon loading, and is negative otherwise. For damage growth, \( f \) must remain zero for an infinitesimal period, so it will have the additional requirement, \( f = 0 \). The damage theory is completed by specifying the appropriate evolution equations for the internal variables. (de Borst, 2002)

**Isotropic damage model**

The isotropic damage model specialise the total stress-strain relationship shown in Equation (3.11) into following form in Equation (3.12),

\[ \sigma = (1 - d) \ D^s : \varepsilon \]  

where,
\( d \) is the damage variable grows from zero at an undamaged state to one as a complete loss of integrity.

Equation (3.12) includes the degradation of the initial shear modulus and the initial bulk modulus with separate scalar damage variables \( d_1 \) and \( d_2 \). In an isotropic model, the degradation of the secant shear stiffness \( (1 - d_1) \ G \) and the secant bulk moduli \( (1 - d_2) \ K \) occur in the same manner during damage growth, i.e., \( d \equiv d_1 = d_2 \). This means the Poisson’s ratio of the material remains unchanged during damage growth and leads to the expression in Equation (3.12). (de Borst, 2002)
Damage-coupled plasticity theory

The damage strain from damage theory is not a permanent strain. It is fully recovered after unloading unlike the equivalent plastic strain. The plasticity theory should include to describe the permanent crack due to the growth of micro-cracks. (Malm, 2006)

A coupled damage plasticity model could use follow equation developed by Ju (1989).

\[
\sigma = (1 - d) D^0 : (\varepsilon - \varepsilon^p)
\]  

(3.13)

According to the effective stress concept the plastic yield function is formulated in terms of effective stress. The effective stress is calculated according to Equation (3.14) Malm (2006),

\[
\tilde{\sigma} = \frac{\sigma}{1 - d}
\]

(3.14)

3.2.4 Constitutive model for concrete in ABAQUS

Since concrete is the major material in a concrete buttress dam structure, the analyses results will depend on the concrete material properties. The constitutive material model for describing the nonlinear behaviour of concrete in ABAQUS are described below.

ABAQUS offers three constitute models for different purposes. The smeared cracked model can be used in the implicit solver for cases where the concrete is subjected to essentially monotonic straining. The brittle cracking model can perform in the explicit solver. Tensile cracking of the concrete dominates the model behaviour while the compressive behaviour is assumed to be elastic. The concrete damaged plasticity model can be applied in both implicit and explicit solvers. The model considers the nonlinear behaviour of the concrete in both the tensile and the compressive parts. The model is designed for applications for concrete subjected to both arbitrary and cyclic loading approaches. (Dassault Systèmes, 2014)

The concrete damage plasticity model in ABAQUS is based on the development by Lubliner et al. (1989) with the modification by Lee and Fenves (1998). It is defined to have a tension softening behaviour based on a crack-opening law and fracture energy. Damage theory is associated with the failure mechanics and therefor result in a reduction in the elastic stiffness. The stress-strain relationship in ABAQUS under uni-axial compression and tension loading are,

\[
\sigma_t = (1 - d_t) E_0 (\varepsilon_t - \varepsilon^p_t)
\]

(3.15a)

\[
\sigma_c = (1 - d_c) E_0 (\varepsilon_c - \varepsilon^p_c)
\]

(3.15b)

where,
3.2. CONSTITUTIVE MATERIAL MODELS FOR CONCRETE

\( \sigma_t \) is the stress in tension.
\( \sigma_c \) is the stress in compression.
\( d_t, d_c \) is the scalar degradation factor in tension & compression.
\( E_0 \) is the initial elastic stiffness.
\( \varepsilon_p^c, \varepsilon_p^t \) is the plastic strain in compression and tension respectfully.

Figure 3.11 shows the tensile response for uni-axial loading of concrete in the model. Concrete has a linear-elastic behaviour before reaching the yielding strength. Then, the strength of the concrete will decrease after passing the yielding stress, see the Function (3.15a).

![Figure 3.11: Tensile response for uni-axial loading of concrete, from Dassault Systèmes (2014).](image)

Figure 3.12 shows the compressive response. The plastic stage is also described by a reducing stiffness function, see Function (3.15b).

![Figure 3.12: Compressive response for uni-axial loading of concrete, from Dassault Systèmes (2014).](image)

The yield criterion is based on an invariant function of the state of stress which
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takes the form as shown in Equation (3.16), The yield function is controlled by the hardening variables \( \varepsilon_p^p \) and \( \varepsilon_t^p \).

\[
F(\bar{\sigma}, \bar{\varepsilon}^p) = \frac{1}{1 - \alpha} \left( \bar{q} - 3 \alpha \bar{p} + \beta \left( \bar{\varepsilon}^p \right) \left( \hat{\bar{\sigma}}_{\text{max}} \right) - \gamma \left( \hat{\bar{\sigma}}_{\text{max}} \right) \right) - \bar{\sigma}_c \left( \bar{\varepsilon}_c^p \right) \leq 0 \tag{3.16}
\]

Figure 3.13: Bi-axial yield surface in the constitutive model concrete damaged plasticity. From Dassault Systèmes (2014).

where,

\( \alpha \) a dimensionless coefficient \( \alpha = \frac{f_b 0 - f_c 0}{2 f_b 0 - f_c 0} \) where \( 0 \leq \alpha \leq 0.5 \).

\( \bar{p} \) is the hydrostatic pressure stress, which is a function of the first stress invariant \( I_1 \)
\[ \bar{p} = -I_1/3 = -(\sigma_{11} + \sigma_{22} + \sigma_{33})/3. \]

\( \bar{q} \) is the Mises equivalent effective stress
\[ \bar{q} = \sqrt{\frac{3}{2} S : S} = \sqrt{3 J_2} \]
\[ J_2 = \sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11} \sigma_{22} \] for bi-axial loading and \( S \) is the effective deviatoric stress tensor \( S = \bar{\sigma} + \bar{p} I \).

\( f_{c0} \) is the initial uni-axial compressive yield stress.
\( f_{b0} \) is the initial equibiaxial compressive yield stress.
\( f_{t0} \) is the uni-axial tensile stress at failure.
\( \beta \) is a dimensionless coefficient
\[ \beta = \frac{\dot{\bar{\sigma}}_c(\bar{\varepsilon}_c^p)}{\dot{\bar{\sigma}}_t(\bar{\varepsilon}_t^p)} (\alpha - 1) - (\alpha + 1) . \]
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\( \hat{\sigma}_c(\tilde{\varepsilon}_c) \) is the effective compressive cohesion stress.

\( \hat{\sigma}_t(\tilde{\varepsilon}_t) \) is the effective tensile cohesion stress.

The typical experimental values of the ratio \( \frac{f_{b0}}{f_{c0}} \) for concrete are in the range from 1.10 to 1.16, yield values of \( \alpha \) between 0.08 and 0.12. (Lubliner et al., 1989)

The concrete damaged plasticity model uses a non-associative flow rule. The flow potential \( G \) is described by the Drücker-Prager hyperbolic function, shown in Equation (3.17).

\[
G = \sqrt{(\varepsilon f_{b0} \tan \psi)^2 + \bar{q}^2} - \bar{p} \tan \psi
\]  

(3.17)

where,

- \( \varepsilon \) is the eccentricity, which defines the rate at which the plastic potential function approaches the asymptote. Increasing value of the \( \varepsilon \) provides more curvature to the low potential.

- \( \psi \) is the dilation angle, measured in the p-q plane at high confining pressure.

The flow potential is illustrated in Figure 3.14. The flow potential approaches a straight line when the eccentricity closes to zero. If the dilation angle is equal to the material inner friction angle, then the flow rule becomes associative. (Dassault Systèmes, 2014)

![Figure 3.14: The Drücker-Prager hyperbolic plastic potential function in the meridional plane. From Dassault Systèmes (2014) and Malm (2006).](image-url)
Figure 3.15 illustrates a uni-axial load cycle assuming the default behaviour, where $\Gamma_t = 0$ corresponds to no recovery as load changes from compression to tension and $\Gamma_c = 1$ corresponds to a complete recovery as the loading changes from tensile to compressive.

![Uni-axial load cycle with the stiffness recovery factors $\Gamma_t = 0$ and $\Gamma_c = 1$, from Dassault Systèmes (2014) and Malm (2006).]

Experimental observations in most quasi-brittle materials, including concrete, show that the compressive stiffness is recovered upon crack closure as the load changes from tension to compression. On the other hand, the tensile stiffness is not recovered as the load changes from compression to tension, once crushing micro-cracks have developed. This behaviour, which corresponds to $\Gamma_t = 0$ and $\Gamma_c = 1$, is the default value used by ABAQUS. (Dassault Systèmes, 2014)

### 3.3 Quasi-static analyses

Several different algorithms implemented in numerical model to solve the finite element problem. The implicit and explicit solvers have its own advantages. The implicit method is often applied to linear problem if the deformation of the model is relatively small and the convergence is easy to obtain. The explicit solver on the other hand, is more capable to solve the large structure with complicated contact and large deformation.

The explicit dynamic procedure is originally developed to model high-speed impact events, i.e., car crush or missile impact to a concrete structure. However, with special consideration, the quasi-static analyses are used to solve a "static" problem with a true dynamic procedure. The quasi-static method transfers the normal static problem to a dynamic problem within the same equilibrium. For this method, the acceleration has relatively small affects on the final result compare to normal static problem.
3.3 QUASI-STATIC ANALYSES

3.3.1 Explicit time integration

Explicit solver allows fixed or automatic time incrementation with the global time estimator, which means the explicit solver reform the calculation in really small time increments. Explicit integration has two methods, Classical Central Differences and Half-Step Central Differences. Half-Step Central Differences is based on the implementation of an explicit integration rule along with the use of diagonal element mass matrix (lumped mass matrix). The equation of motion is shown below as Equation (3.18). (Cook et al., 2007)

The integration method uses the known acceleration values from current time step and set up to calculate the velocity at the mid-increment, then the result is applied to calculate the final displacement in final step.

\[
\begin{align*}
\mathbf{u}^{(i+1)} &= \mathbf{u}^{(i)} + \Delta t^{(i+1)} \mathbf{\dot{u}}^{(i+\frac{1}{2})} \\
\mathbf{\dot{u}}^{(i+\frac{1}{2})} &= \mathbf{\dot{u}}^{(i-\frac{1}{2})} + \frac{\Delta t^{(i+1)} + \Delta t^{(i)}}{2} \mathbf{\ddot{u}}^{(i)} \\
\mathbf{\ddot{u}}^{(i)} &= \mathbf{M}^{-1}(\mathbf{F}^{(i)} - \mathbf{I}^{(i)})
\end{align*}
\] (3.18)

where, 
- \(\mathbf{u}\) is the displacement.
- \(\dot{\mathbf{u}}\) is the velocity.
- \(\ddot{\mathbf{u}}\) is the acceleration.
- \(\Delta t\) is the incremental time.
- \(i\) refers to the increment number.
- \(\mathbf{M}\) is a "lumped" element mass matrix.
- \(\mathbf{F}\) is the applied load vector.
- \(\mathbf{I}\) is the internal force vector.

The suitable time increment is critical for the calculation. If the time increment is too small, the computation will become costly; if the time increment is too large, the integration fails. Equation (3.19) often used to determine the \(\Delta t_{\text{min}}\) in whole numerical model.

\[
\Delta t_{\text{min}} \leq \frac{L_{\text{min}}}{c}
\] (3.19)

where,
- \(\Delta_{\text{min}}\) is the minimum increment time in the model.
- \(L_{\text{min}}\) is the minimum element dimension of the mesh.
- \(c\) is the wave speed of the material.
The main drawback of the explicit method is that the method is conditionally stable. The maximum time increment must be less than a critical time of the smallest transition times for a dilatational wave to cross any element in the meshed model. (Sun et al., 2000)

The critical time increment sets up the upper boundary in order to ensure the calculation is stable, which related to the material density and the characteristic element length. It could be expressed as Equation (3.20).

$$\Delta t_{cr} = \frac{L^e}{c_d}$$

(3.20)

where,

- $t_{cr}$ is the critical time increment of the calculation.
- $L^e$ is the characteristic length of the element (Not the real length).
- $c_d$ is the current, effective dilatational wave speed of s of the material.

The dilatational wave speed in a linear-elastic material is defined according to Equation (3.21).

$$c_d = \sqrt{\frac{E \left(1 - \nu\right)}{\rho \left(1 + \nu\right) \left(1 - 2\nu\right)}}$$

(3.21)

where,

- $E$ is the Young’s modulus of the material.
- $\rho$ is the density of the material.
- $\nu$ is the Poisson’s ratio.

### 3.3.2 Loading rate

The loading rate is critical in quasi-static simulation, since it affects the inertial forces. In static problems, the loading is considered to be static, which means in the natural time scale, the velocity of the loading is zero. The quasi-static analysis should gain the result which is most closed to the static case. Quasi-static analyses require a loading which is as smooth as possible. Since the sudden movement would induce inaccurate solution with high influence from dynamic effects. ABAQUS provides a smooth amplitude curves to reduce the calculation time and still gives insignificant dynamic effects. (Dassault Systèmes, 2014)

### 3.3.3 Mass scaling

In quasi-static simulations, the natural time scale is generally not important. The mass scaling is often used in analyses by artificially increasing the mass of the model to increase the stable time increment to achieve efficiency.
3.3. QUASI-STATIC ANALYSES

Mass scaling could help the analyses be able to perform calculation economically without increasing the loading rate. In quasi-static analyses, the convergence at some element may need smaller time increment and consume more attempts for convergence. By increasing the mass scaling factor, the influence of elements with small time increment are adjusted by increasing density or mesh size in the critical region.

ABAQUS provides two different methods for mass scaling, the fixed mass scaling and the variable mass scaling. The two methods can be applied separately, or they can be applied together to define an overall mass scaling strategy. Mass scaling can also be applied globally to the entire model or, alternatively, on a specific set of elements. (Dassault Systèmes, 2014)

3.3.4 Energy balance

In quasi-static analyses, the external work caused by external forces should be equal to the internal energy. It should also be constant during the analyses. The total energy should be constant, however, in numerical model $E_{total}$ is only approximately constant, generally with an error of less than 1%.

$$E_{total} = E_I + E_V + E_{FD} + E_{KE} + E_{IHE} - E_W - E_{PW} - E_{CW} - E_{MW} - E_{HF} \quad (3.22)$$

where,

- $E_{total}$ is the total energy, generally with an error of less than 1%.
- $E_I$ is the internal energy.
- $E_V$ is the dissipated energy.
- $E_{FD}$ is the frictional energy.
- $E_{KE}$ is the kinetic energy.
- $E_{IHE}$ is the internal heat energy.
- $E_W$ is the work done by external loads.
- $E_{HF}$ is the external heat energy through external fluxes.
- $E_{PW}, E_{CW}, E_{MW}$ is the work done by contact penalties, constraint penalties, propelling added mass.
For a simple tensile uni-axial loading test, the loading rate within certain limits would give almost non kinetic energy as shown in Figure 3.16.

Figure 3.16: Energy history for a quasi-static tensile test. From Dassault Systèmes (2014).

If the simulations are performed as a quasi-static analysis, the work done by external forces should be approximately equal to the internal energy. Also, the kinetic energy should generally be restricted to a limit that less than 5%-10% of the internal energy. (Dassault Systèmes, 2014)
Chapter 4

Case study: A concrete buttress dam

The main goal of this report was to study how the safety and the failure mode of a common type of Swedish concrete buttress dam is affected by internal cracks. The models used in this project were defined to represent a typical Swedish concrete buttress dam. The geometry and the crack pattern of the studied monolith was essentially based on the previous project presented by Malm and Ansell (2011).

In this chapter, general information about the models used for the analyses is presented. The information includes the geometry of the monolith and the location and dimension of assumed existing cracks, material properties, design loads and lastly, previous modelling of similar buttress dams.

4.1 Geometry

The geometry of the analysed monolith is visualised in Figures 4.1 and 4.2a. In this report, two models were considered; one with an irregular rock-concrete interface and the other with a horizontal interface. Henceforth, the two models are referred to as Model A and Model B, respectively. Model A represent actual geometrical conditions and Model B have a simplified geometry, which is a common practice to simplify the analytical stability calculations.

The frontplate has a width of 8 m and a constant thickness of 1.2 m, for the vertical part, whereas the inclined part has a varying thickness from 1.2 m to 2.6 m at the heel. The buttress has a constant thickness of 2.0 m. The frontplate and buttress are rigidly connected. The dam crest has a width of 4 m. An inspection gangway is passing through the monolith approximately 9 m above the ground. The design water level is 1.75 m below the dam crest.

The reinforcement in the models are arranged as shown in Figure 4.2b and is applicable to both models. The reinforcement consists of horizontal and vertical bars with three different diameters, 19 mm, 25 mm and 31 mm, with a concrete cover of 50 mm. Two layers of 19 mm bars are located in the segment between the frontplate and the buttress, placed perpendicular to the frontplate.
CHAPTER 4. CASE STUDY: A CONCRETE BUTTRESS DAM

Figure 4.1: Model A. Irregular rock-concrete interface.

Figure 4.2: Model B. Horizontal rock-concrete interface.

The monolith is assumed to be founded on good-quality granite where the frontplate is embedded into the rock foundation.
4.2 Material properties

The material properties of concrete, rock and steel considered in this project are shown in Table 4.1. The concrete is assumed to be of quality C25/30 in accordance with EC 2 (2004) in all parts of the monolith. The reinforcement consists of standard ribbed steel bars of Swedish type Ks400s, with material properties obtained from Ljungkrantz et al. (1994). The properties of the rock foundation were obtained from Björnström et al. (2006). The fracture energy of the concrete is calculated in accordance with FIB (2013). If a property not apply to a material it will be denoted with "$\cdot\$".

Table 4.1: Assumed material properties.

<table>
<thead>
<tr>
<th></th>
<th>Concrete C25</th>
<th>Rock</th>
<th>Reinf. 19 mm to 25 mm</th>
<th>Reinf. 31 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density [kg/m$^3$]</td>
<td>2300</td>
<td>2300</td>
<td>7800</td>
<td>7800</td>
</tr>
<tr>
<td>Elastic Modulus [GPa]</td>
<td>30</td>
<td>60</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Tensile strength [MPa]</td>
<td>2.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Compressive strength [MPa]</td>
<td>28</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Yield strength [MPa]</td>
<td>-</td>
<td>-</td>
<td>370</td>
<td>350</td>
</tr>
<tr>
<td>Ultimate strength [MPa]</td>
<td>-</td>
<td>-</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Ultimate strain [mm/mm]</td>
<td>-</td>
<td>-</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>Fracture energy [Nm/m$^2$]</td>
<td>131.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

4.3 Existing cracks

The monolith has several existing cracks where the major reason for these cracks are due to temperature variation as presented by Malm and Ansell (2011). In this project, the existing cracks were included in the model to study their influence on the dam safety.

Figure 4.3 shows the position and distribution of existing cracks in a side view.

- Crack 1 goes through the frontplate.
- Crack 2 goes through the frontplate and the buttress wall.
- Crack 3 goes through the frontplate and extend into the buttress.
- Crack 4 goes through the buttress, crossing the inspection gangway.
- Crack 5 goes through the buttress, close to the monolith toe.
CHAPTER 4. CASE STUDY: A CONCRETE BUTTRESS DAM

4.4 Design loads

In this project, the design loads were calculated according to RIDAS, see Section 2.2. The loads considered and their values are presented in Figure 4.4 and Table 4.2. Only the case with normal loads were considered, calculated with design water level and full ice load.

Figure 4.4: Distribution of design loads considered in the analyses. Applicable to Model A and B.
Table 4.2: External loads considered in the analyses

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Model B</th>
<th>Model A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design loads</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrostatic pressure</td>
<td>kPa</td>
<td>0 to 374.7</td>
</tr>
<tr>
<td>Ice load</td>
<td>kN/m</td>
<td>200</td>
</tr>
<tr>
<td>Uplift pressure</td>
<td>kPa</td>
<td>0 to 374.7</td>
</tr>
</tbody>
</table>

The uplift pressure is considered to be acting on the frontplate only. As presented in Section 2.2.3, the limit for extending the distribution of the uplift pressure to the toe of the buttress is unclear and is left for the designer to decide. In this project, the width of the buttress is 2 m, thus uplift pressure on the buttress was excluded. As there is no tail water surrounding the monolith on the downstream side ($h = 0\,\text{m}$, see Figure 2.2), the uplift pressure goes down to 0 kPa downstream of the frontplate.

In the stability calculations and when comparing results from linear finite element analyses to the stability calculations, it was assumed that the frontplate is not embedded into the rock foundation and the uplift is acting vertically, see Figure 4.4(a).

The geographical location of the dam is assumed to be in the north of Sweden, which according to RIDAS gives an ice load of 200 kN/m and an ice thickness of 1.0 m.

The volume of the monolith for the two models are $V_{c,A} = 1908\,\text{m}^3$ and $V_{c,B} = 1967\,\text{m}^3$. The self-weight was calculated using Equation (2.2).

$$F_{g,A} = 43.0\,\text{MN} \text{ and } F_{g,B} = 44.4\,\text{MN}$$

4.5 Previous modelling

Many research projects have been performed to analyse concrete buttress dams with cracks in linear and nonlinear analyses. Björnström et al. (2006) aimed to apply advanced fracture mechanic models to dam structures with linear elastic numerical analyses. It also aimed to identify whether deformations and stresses caused by seasonal variations of temperatures could initiate cracks in a concrete dam. The result has shown a good agreement between analytical calculations and observations on a real concrete buttress dam. The calculation showed how cracks occur and what direction these cracks will take. However, the nonlinear modelling techniques could not be implemented in this project to simulate crack growth.

A following project presented by Ansell et al. (2007) used a two-dimensional numerical model consisted of triangular continuum elements and nonlinear material properties for concrete. The analyses were performed on a fictitious buttress dam structure, with the measurement from a typical Swedish dam. The model had
nearly 6000 triangular elements with almost 53,000 degrees of freedoms. The result has shown that it is possible to use nonlinear analyses to describe and follow the initiation and propagation of cracks found in situ on the studied type of concrete buttress dam.

A project presented by Malm et al. (2013) has modelled the Storfinnforsen hydropower dam with the crack pattern obtained in Ansell et al. (2010). The finite element model was defined with three node shell element (S3R) with a mesh size of 0.5 m. The reinforcement was included in the model with an additional identical mesh with a minimal thickness of shell elements. This mesh is then tied to the mesh with the predefined cracks. The project performed different benchmark test to compare different constitutive models for concrete in ABAQUS. The result showed that the concrete damaged plasticity model with a seam crack approach gave good results compared to the measurements. The numerical models were used to simulate the displacement and crack width for long time analysis.
Chapter 5

Finite element models for case study

In this chapter, a description of the finite element models used for the case study is given. The finite element models were based on the general information presented in Chapter 4. In addition to the general information, this chapter includes for example; more detailed information regarding nonlinear material properties for both concrete and reinforcement, information about how the loads and the boundary conditions were modelled and how the progressive failure analyses were performed.

5.1 Geometry

Figure 5.1 shows the geometry of the monolith and rock foundation for Model A and Model B, where existing cracks have been included. In all of the finite element models, parts of the rock foundation were included, both in order to study the influence of the rock-concrete interface and also to represent the elasticity of the foundation. Figure 5.2 shows the layout of the reinforcement in the two models.

(a) Model A.       (b) Model B.

Figure 5.1: The geometry of the two finite element models, with existing cracks.
5.2 Nonlinear material properties

Concrete

The nonlinear behaviour of the concrete was calculated based on the method of the hardening rule and tension softening for compression and tension, respectively. These methods are further described in Section 3.1.1. The tensile behaviour used the tension softening to represent the behaviour based on the bi-linear approach, seen in Figure 5.4.

![Figure 5.3: Compressive behaviour used for concrete model.](image)
5.2. NONLINEAR MATERIAL PROPERTIES

Tensile damage was also provided where the input information is shown in Table 5.1. According to a study performed by Eriksson and Gasch (2011), a damage parameter equal to 0.95 fits the experimental data quite well. The damage is then defined as a 95\% reduction of the stiffness for a crack displacement of $2.176 \times 10^{-4}$ m as shown in Figure 5.4.

The displacement is calculated based on the bi-linear approach method described in Section 3.1.1.

Table 5.1: Tensile damage variables and corresponding displacement.

<table>
<thead>
<tr>
<th>Damage Parameter [-]</th>
<th>Displacement [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.95</td>
<td>$2.2 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Reinforcement

The nonlinear material behaviour of the reinforcement was assumed to be represented by the behaviour of structural steel. The stress-strain curve used in this project was according to BSK 07 (2007), see Figure 5.5.
The stress-strain relationship is calculated according to Equation (5.1).

\[
\varepsilon_1 = \frac{f_{yd}}{E_d} \quad \text{(5.1a)}
\]

\[
\varepsilon_2 = 0.025 - 5 \frac{f_{ud}}{E_d} \quad \text{(5.1b)}
\]

\[
\varepsilon_3 = 0.02 + 50 \frac{f_{ud} - f_{yd}}{E_d} \quad \text{(5.1c)}
\]

\[
\varepsilon_{\text{max}} = 0.6 A \quad \text{(5.1d)}
\]

where,

- \( E_d \) is the design value of the modulus of elasticity.
- \( f_{yd}, f_{ud} \) is the yield strength and the ultimate strength, respectively.
- \( A \) is the rupture strain.

The stress-strain relationship for the reinforcement is shown in Figure 5.6.

![Figure 5.6: Uni-axial behaviour used for the reinforcement.](image)

The reinforcement was assumed to be perfectly bonded to the concrete, i.e. bond-slip was excluded.

### 5.3 Defining existing cracks

Existing cracks can be introduced in various ways in a finite element model. In this project, the cracks were defined as seam cracks. The seam crack approach is a discrete crack approach which defines an edge or a face with overlapping nodes that can separate during an analysis. The procedure of the seam crack approach is shown in Figure 5.7.
The seam cracks were defined with hard contact in the normal direction for compression, i.e. preventing overclosure of nodes. The hard contact was also defined so that it allowed for separation due to tensile forces, i.e. allowed for crack opening.

The existing cracks can not propagate by allowing these cracks to be extended to uncracked regions. The propagation was obtained by the nonlinear analyses with the concrete damaged plasticity model.

## 5.4 Applying loads

In this project, a load-control loading system was used in all the analyses, which means the load can only increase. A shortcoming of the load-control loading system is that the response, after the peak load is reached, can not be obtained as unloading is not allowed. For the same reason, a crack plateau will be obtained when a crack is initiated. This behaviour is shown in Figure 5.8. The figure also shows the response of using a deformation-controlled loading system, which would result in a drop after initiation of a crack as well as after a peak load is reached. (Malm, 2006)
The hydrostatic and the uplift pressure were applied as a distributed pressure over the loading surface with a linear variation, in accordance with Section 4.4. In the same section the ice load has a triangular pressure distribution over 1.0 m, with the maximum pressure at the design water level. The pressure resultant is thus 0.33 m below the design water level. In the finite element models, the ice load was defined as an evenly distributed pressure with a magnitude of 300 kPa with a height of 0.67 m, thus having the same location and magnitude of the pressure resultant.

**Progressive failure analyses**

Progressive failure analysis is a methodology used in numerical analyses to force a structure to failure, in order to determine the failure mechanism of a structure and to estimate the safety factor. This method is commonly used in analyses of bridges and nuclear power plants. (Nordström et al., 2015)

In numerical analyses of concrete dams, two different methods are generally considered, one is to increase loads until failure, called the overloading method and the other method is to presume material degradation, called strength reduction.

When applying strength reduction, the strength of a chosen material is reduced until failure occur, meanwhile keeping the design loads constant.

The overloading method is more commonly used compared to the strength reduction method, primarily because of its simplicity in implementation.

When using the overloading method, the design loads are first applied, followed by a successive increase of the magnitude of the loads until the structure fails. The load magnitude is defined as the ratio between the applied load and the design load and the safety factor is equal to the load magnitude at failure, see Equation (5.2).
## 5.4. Applying Loads

\[
sf = \frac{\text{Load}}{\text{Design load}} = 1 + \lambda, \quad \lambda \geq 0
\]  

(5.2)

Where \( \lambda \) is a successively increasing load factor. This corresponds to fundamental principles of stability analyses, presented in Section 2.1.2.

### Increasing hydrostatic pressure

When it comes to dam analyses, two different methods are usually considered when increasing the hydrostatic pressure, called increasing density and increasing overflow. (Oliver et al., 2006) These two methods are applied with a load-controlled loading system, presented in the beginning of this section.

The two methods are based on the same concept, i.e. magnifying the design pressure using \( \lambda \). When the method of increasing density is used, the water head is kept constant while the density of water, i.e. the pressure, is increased. However, when using increasing overflow the water head is raised and the water level can exceed the crest of the dam. The two procedures are illustrated in Figure 5.10.

![Figure 5.10: Generally considered methods to increase the hydrostatic pressure. Reproduction from Oliver et al. (2006).](image)

The main difference of the two methods is that the position of the resultant force change in the increasing overflow method compared to a constant position of the resultant with increasing density. As a result, the two methods will give different outcome in safety calculations. (Nordström et al., 2015) It should be mentioned that the resultant can only vary between \( \frac{1}{3}H \) and \( \frac{1}{2}H \) (measured from the foundation), as can be seen in Figure 5.10b. It should also be mentioned that the load above the design water level is not included in the method of increasing overflow.
CHAPTER 5. FINITE ELEMENT MODELS FOR CASE STUDY

An important factor, regarding increasing hydrostatic pressure in overload analyses, is how the increased load should be applied on inclined frontplates. By dividing the pressure into horizontal and vertical components, see Figure 5.11, it can be seen that the vertical component gives stabilising effects in concern to the generally assumed failure modes, i.e. overturning and sliding failure. Four different loading approaches are thus possible for the overloading method when the frontplate is inclined;

- increasing density,
- increasing density, only the horizontal component,
- increasing overflow,
- increasing overflow, only the horizontal component.

Figure 5.11: Hydrostatic pressure on an inclined frontplate.

5.5 Interface and boundary conditions

The rock-concrete interface is modelled by an interaction function to simulate the connection property between two surfaces.

Boundary conditions were applied to the rock foundation. As Figure 5.12 shows, the boundary conditions were defined so that the displacement perpendicular to each side has been constrained. The monolith is free to move in all direction.
The interface inside cracks were modelled with a friction coefficient of $\mu = 0.5$ according to the description in Section 2.3.2.

5.6 Mesh

This project includes four different models in total; Model A and Model B with and without cracks. All models were meshed separately. The finite element models were defined with 3D solid elements with eight nodes. The three dimensional elements have the advantage to have stress transferring in the third axis. Figure 5.13 shows the meshed finite element model for Model A with cracks. The mesh of all the models are shown in Appendix A.1.

The reinforcement was included in models with cracks. The reinforcement was meshed with 3D truss element with two nodes. Table 5.2 shows the general element length for all models and the total number of degrees of freedom (DOF).

<table>
<thead>
<tr>
<th>Model</th>
<th>Monolith C3D8R</th>
<th>Rock foundation C3D8R</th>
<th>Reinforcement T3D2</th>
<th>D.O.F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without cracks Model A</td>
<td>0.5</td>
<td>1</td>
<td>-</td>
<td>230 000</td>
</tr>
<tr>
<td>Without cracks Model B</td>
<td>0.35</td>
<td>1</td>
<td>-</td>
<td>160 000</td>
</tr>
<tr>
<td>With cracks Model A</td>
<td>0.4</td>
<td>1</td>
<td>0.4</td>
<td>311 000</td>
</tr>
<tr>
<td>With cracks Model B</td>
<td>0.35</td>
<td>1</td>
<td>0.4</td>
<td>320 000</td>
</tr>
</tbody>
</table>
5.7 Reducing analysis time

As mentioned in Section 3.3, the nonlinear analyses would require significant CPU time in the explicit analysis. It is critical to reduce the analysis time needed without affecting the accuracy of the result. Variable mass scaling was used in this project to reduce the CPU time in all quasi-static analyses. In these models, the density of the critical elements was increased to obtain a stable time increment equal to $\Delta t = 3.0 \times 10^{-5}$ s or higher.
Chapter 6

Failure analyses

Several finite element analyses, using both linear and nonlinear material models, have been performed to provide adequate information to answer the research questions. These include convergence analyses and sensitivity analyses, performed to obtain sufficient mesh size regarding efficiency and precision, investigate the influence of different interaction properties etc. General information about the finite element models are presented in Chapter 5, while other specific information will be given along with the presentation of the analyses below.

All the analyses were performed with the use of the overloading method, presented in Section 5.4, with the loads presented in Section 5.4. Failure was assumed to occur when the ultimate load for a monolith was reached. One can see that the largest displacements appear at the crest, independent of the mode of failure. To detect if internal failure modes were occurring, displacements at suitable reference points in the front plate were also monitored.

At first, analytical stability calculations were performed to provide a basis for the numerical analyses. Thereafter, several numerical analyses were conducted to determine which loading approach was the most suitable when increasing the hydrostatic pressure, see Section 5.4. Next, failure analyses were performed, using linear material properties, without introducing the existing cracks. Finally, the influences of existing cracks were investigated both using linear and nonlinear material properties for the concrete.

The failure analyses were performed for both Model A and Model B, both with and without the assumed existing cracks.

In several analyses, a pure overturning failure was of interest in order to compare with the failure modes used in the stability calculations. To force a pure overturning failure, the friction in the rock-concrete interface was increased significantly to eliminate sliding. A real failure was defined as the failure of a model when friction coefficient was $\mu = 0.75$. 
6.1 Analytical stability calculations

Analytical stability calculations were performed according to RIDAS, see Section 2.3. The results were used as a reference for the finite element analyses. The resultant forces and lever arms for the design loads considered, are shown in Figure 6.1. The axis of rotation was assumed to be located at the toe of the monolith for both models. The results are shown in Table 6.1.

![Figure 6.1: Resultant forces and lever arms used for the analytical stability calculations.](image)

<table>
<thead>
<tr>
<th>Sliding</th>
<th>Overturning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu \leq 0.75 ), ( s_f_s \geq 1.35 )</td>
<td>( s_f_o \geq 1.5 )</td>
</tr>
<tr>
<td>( \mu = \frac{T}{N} ), ( s_f_s = \frac{\tan \delta_g}{\mu} ), ( s_f_o = \frac{M_s}{M_o} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Results from the analytical stability calculations.

The results showed that the risk of a pure sliding failure is governing in both Model A and Model B. When the safety factor for sliding was calculated for Model A, the advantage of irregularities in the rock-concrete interface was not taken into account. Thus, the value presented for Model A does not necessarily represent the real safety factor for sliding.

Due to the difference in geometry of the two models, one should notice that the loads acting on the models were not identical regarding magnitude and lever arm.
6.2 INCREASING HYDROSTATIC PRESSURE

This leads to a clear change of the safety factor, especially for overturning failure, which is 13\%.

When compared to the allowable safety factors recommended in RIDAS, see Tables 2.1 and Table 2.2, the safety factor for sliding is close to the limit for normal loads while the safety against overturning is well above the allowable value of 1.5.

6.2 Increasing hydrostatic pressure

Analyses were performed to obtain the most suitable loading approach for increasing the hydrostatic pressure. These analyses were only conducted using Model B without exiting cracks and with linear elastic material properties. The loads considered were according to Figure 4.4a, as the results had to be comparable to the stability calculations presented in Section 6.1. Interaction between the frontplate and the rock foundation was excluded.

The analyses were performed in accordance with Section 5.4, using four different approaches;

- increasing density,
- increasing density, only the horizontal component,
- increasing overflow,
- increasing overflow, only the horizontal component.

To be able to compare the obtained results to the failure modes used in the stability calculations, the properties of the rock-concrete interface had to be modified accordingly. To obtain a pure sliding failure, no separation was allowed, meanwhile using the failure value for the friction coefficient, \( \tan \delta_g = 1 \), recommended in RIDAS (see Section 2.3.2). However, to force a pure overturning failure, the friction coefficient was assumed as \( \mu = 5.0 \). The results from the analyses are presented in Figure 6.2 and Table 6.2.

<table>
<thead>
<tr>
<th></th>
<th>( sf_s )</th>
<th>( sf_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stability calculations</td>
<td>1.35</td>
<td>1.97</td>
</tr>
<tr>
<td>FE analyses</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Increasing density</td>
<td>1.86</td>
<td>&gt; 8</td>
</tr>
<tr>
<td>Increasing density, horiz.</td>
<td>1.33</td>
<td>1.94</td>
</tr>
<tr>
<td>Increasing overflow</td>
<td>1.86</td>
<td>3.84</td>
</tr>
<tr>
<td>Increasing overflow, horiz.</td>
<td>1.33</td>
<td>1.70</td>
</tr>
</tbody>
</table>
When the results of the finite element analyses are compared to the analytical stability calculations, presented in Section 6.1, it can be seen that the results obtained using the approach of only increasing the horizontal component when increasing density, shows the best agreement among the four approaches. As stated in Section 5.4, increasing the pressure, independent of load direction, results in increased stability, both in case of overturning and sliding failure, as seen for instance in Table 6.2.

In addition, an important reason for using this model where the interaction between the frontplate and rock foundation was excluded, is that erroneous results were avoided. As the material properties used in the analyses are linear-elastic, crack propagation cannot take place where tensile stresses exceed the ultimate tensile strength of the concrete. The tensile stresses in the model with and without the interaction between the frontplate and rock foundation can be visualised in Figure 6.3. The ultimate tensile strength of the concrete is 2.2 MPa, see Section 4.2.
6.3 LINEAR ANALYSES WITHOUT EXISTING CRACKS

As a result of the increased level arm of the pressure resultant, the increasing overflow approach gives a lower safety factor considering overturning. Furthermore, for the same reason, larger crest displacement occurs due to larger deformations in the monolith.

Based on the results, it can be concluded that the most suitable approach is to only increase the horizontal pressure component with increasing density of the water. This approach was thus used for all the following numerical analyses.

6.3 Linear analyses without existing cracks

To investigate the behaviour of the monolith as a rigid body with the two different geometrical models, further analyses were performed only considering linear material properties, without introducing the existing cracks.

An important thing to notice is, that henceforth, if not otherwise stated, a friction coefficient $\mu = 0.75$ was used, compared to $\mu = \tan \delta_g = 1.00$ in Section 6.2. As a consequence, the safety factor obtained when sliding failure occur, cannot be compared to the recommended values in RIDAS. The obtained values will however be useful to compare different analyses in the section.

The results of the analyses are first presented for the two models separately, followed by comparison of the obtained results.
Model A

In addition to capture the real failure of the monolith using friction coefficient of \( \mu = 0.75 \), analyses were also run with a friction coefficient of \( \mu = 5.0 \) to force a pure overturning failure.

The failure modes and the distribution of the contact pressure at failure can be seen in Figures 6.4 and 6.5.

![Failure modes of Model A](image)

(a) Real failure, \( \mu = 0.75 \). \( sf = 1.64 \). (b) Pure overturning failure, \( \mu = 5.0 \). \( sf = 1.74 \).

Figure 6.4: Failure modes of Model A, using linear material properties. Displacement scale factor: 50.

![Contact pressure in the rock-concrete interface](image)

(a) Real failure, \( \mu = 0.75 \). \( sf = 1.64 \). (b) Pure overturning failure, \( \mu = 5.0 \). \( sf = 1.74 \).

Figure 6.5: Contact pressure in the rock-concrete interface.

The axis of rotation is located slightly to the left of the rightmost peak of the rock surface when a real failure is simulated as shown in Figure 6.5a. As presented later, the real failure is a combination of sliding and overturning failure, resulting in separation between the toe of the monolith and the rock foundation as the whole
monolith is lifted. For a pure overturning failure, Figure 6.5b, the axis of rotation occurs at the toe. The obtained safety factor for a pure overturning failure correspond to the value obtained in the analytical stability calculations, see Table 6.1. The difference of safety factors for the two analyses can be explained with the help of Figure 6.6, where analytical stability calculations were performed with varying locations of the axis of rotation. According to the figure, the safety factor decreases linearly from 1.75 to 1.65 by moving the axis of rotation from the toe, towards the heel, by a distance of 1.3 m. This is verified by the safety factors obtained in the finite element analyses.

![Figure 6.6: (a) Zoomed in view of the dam toe at failure. (b) Variation of the safety factor with increasing distance from the axis of rotation to the toe.](image)

As discussed in Section 2.3.2, the value of the friction coefficient can vary, although, commonly considered to be between $\mu = 0.75 - 1.00$. To investigate the influence of the friction coefficient on the safety and failure mode of the monolith, multiple analyses were performed with varying friction coefficient. The results can be seen in Figures 6.7 and 6.8.

![Figure 6.7: Comparison of failure modes using varying friction coefficient, $\mu$. Displacement scale factor: 10.](image)
By investigating the behaviour of the monolith, it was found that sliding failure was the governing failure mode when the friction coefficient was \( \mu < 0.7 \). A pure overturning failure was obtained when using \( \mu = 5.0 \); no horizontal displacement occurred at the toe. With a friction coefficient between \( \mu = 0.70 \) and \( \mu = 1.25 \) the obtained failure was a combination of overturning and sliding failure. To be able to withstand the design loads, the friction coefficient had to be higher than \( \mu = 0.36 \).

**Model B**

The results from the analytical stability calculations, presented in Section 6.1, showed that stability against sliding is dominating the failure behaviour. As shown above, the real failure mode of Model A was a combination of sliding and overturning failure. In order to get comparable results for Model B, the analyses were also run with a high friction coefficient. For the same reason as presented in Section 6.2, the interaction between the front plate and rock foundation was not included. The results can be seen in Figures 6.9 and 6.10.
6.3. LINEAR ANALYSES WITHOUT EXISTING CRACKS

(a) Real failure, $\mu = 0.75$. $sf = 1.01$.
(b) Pure overturning failure, $\mu = 5.0$. $sf = 1.94$.

Figure 6.10: Failure modes for Model B, using linear material properties. Displacement scale factor: 50.

As the friction coefficient was $\mu = 0.75$ to obtain a real failure, the monolith barely was able to withstand the design loads. As shown in Table 6.2, the safety factor for pure overturning agreed well with the analytical stability calculations.

The relationship between friction coefficient and safety factor for Model B is shown in Figure 6.11, where the influence of varying friction coefficient on the safety factor can be seen. Sliding failure is governing for commonly considered values of the friction coefficient. The curve for sliding failure was obtained, using Equation (2.9), while a pure overturning failure was forced by using $\mu = 5.0$. To be able to withstand the design loads, the friction coefficient in the rock-concrete interface had to be equal or higher than $\mu = 0.74$, while $\mu = 0.36$ was sufficient for Model A.

Figure 6.11: Model B. Relationship between friction coefficient and safety factor.
Comparison

In Figure 6.12, a comparison is made of the crest displacement, for the four analyses presented above. For Model A, the difference between real failure and a pure overturning failure is modest, while it is significant for Model B.

![Figure 6.12: Relationship between crest displacement and increasing loads for the analyses without existing cracks.](image)

6.4 Influence of existing cracks

In this section, the existing cracks are introduced, see Figure 4.3a to study their influence on the safety of the monolith and the mode of failure. These analyses were performed, both with linear and nonlinear material properties for the concrete. The reinforcement is also introduced with its nonlinear material properties.

When using linear material properties for the concrete and nonlinear properties for the reinforcement, crack propagation is not allowed, while the reinforcement can yield and even fail if its ultimate strength is reached. With this kind of analysis, critical areas related to crack initiation and concrete crushing can be observed, by looking at the tensile and compressive stresses in the concrete.

The nonlinear analyses were performed using the quasi-static method with the Abaqus/Explicit solver, see Section 3.3. Using nonlinear analyses with the concrete damage plasticity model, potential crack propagation could be simulated. The representation of crack propagation can be observed by plotting the tensile damage parameter.

The results of the analyses are first presented for the two models separately, followed by comparison of the results.

Model A

The results from the linear and nonlinear analyses performed with Model A, including existing cracks are presented below. Figure 6.13 show the obtained failure of the
model, with and without existing cracks.

(a) Linear analysis. $sf = 1.76$.  
(b) Nonlinear analysis. $sf = 1.70$. Also showing the obtained crack pattern.

Figure 6.13: Failure modes of Model A with existing cracks. Displacement scale factor: 50. $\mu = 0.75$.

When performing quasi-static analyses, it is important that dynamic effects do not influence the results, as discussed in Section 3.3. Figure 6.14a shows that the ratio between kinetic energy and internal energy is minimal up to a load magnitude of 1.68, where the curve starts rising and exceeds a recommended limit of 5% at a load magnitude of 1.70. As unloading is not allowed for a load-controlled loading system, the increasing dynamic effects indicates instability in the model and that the ultimate load has been reached. The rapid increment of the crest displacement at the same load magnitudes gives a further indication that the ultimate load has been reached, see Figure 6.14b.
CHAPTER 6. FAILURE ANALYSES

1.1 Chapter 6

1.2 Failure Analyses

1.3 Introduction

1.4 Objectives

1.5 Methodology

1.6 Results

1.7 Discussion

1.8 Conclusions

The crack pattern obtained during the occurrence of high dynamic effects can not be verified. The crack propagation can be visualised in Appendix A.2. It can be seen that the failure of the model is due to crushing of the concrete close to the toe of the monolith, which does not necessarily represent the real failure, again due to high dynamic effects from the load-controlled loading procedure. Crack pattern at failure can be seen in Figure 6.15.

Severe vertical cracking took place in the frontplate at a load magnitude of 1.57 without causing any significant dynamic effects. The horizontal reinforcement, in the upstream area of the frontplate does not yield at any magnitude, showing that the frontplate can withstand the load at failure, see Figure 6.16.
6.4. INFLUENCE OF EXISTING CRACKS

Figure 6.15: Nonlinear analyses. Tensile damage at failure, \( sf = 1.70 \). Displacement scale factor: 30.

Figure 6.16 shows the plastic strain in the reinforcement at failure. The reinforcement has not reached its ultimate strength at any location although it is yielding in all pre-existing cracks. It can also be observed that crack 4 shows the largest crack width of the existing cracks at failure.

(a) Red areas indicate plastic strain. (b) Upper limit is set equal to the ultimate strain of the reinforcement.

Figure 6.16: Plastic tensile strain in the reinforcement at failure, \( sf = 1.70 \). The view is cut through the centre of the monolith. Displacement scale factor: 50.

The safety is higher in the model with existing cracks than in the model without the cracks, as seen in Figure 6.17. According to Figure 6.13a, the failure mode of the
model, using linear material properties, is similar to the failure mode of Model A without cracks, see Figure 6.4a. The main difference that appeared when comparing the two models is how the load is transferred from the monolith to the foundation. By comparing Figure 6.18 and Figure 6.5, which show the distribution of contact pressure in the models, it can be seen that for the model with cracks, the contact pressure is distributed over a larger area and all the way to the peak of the toe.

![Figure 6.17: Model A. Relationship between crest displacement and increasing loads.](image)

![Figure 6.18: Model A. Contact pressure in the rock-concrete interface at failure.](image)

To determine the actual failure mode of the model, when using nonlinear material properties, the relative crack openings of cracks 3 and 4 have been studied. As seen in Figure 6.19, crack 3 starts to open at a load magnitude of 1.57 and continues until failure is reached (see also Appendix A.2). Crack 4 propagates through the frontplate at a load magnitude of 1.68 with a rapid opening, indicating a brittle failure. Based on this results, the failure appears to be an internal overturning failure of the intact part of the monolith, above crack 4.
6.4. INFLUENCE OF EXISTING CRACKS

Model B

It has been observed that sliding failure is governing for Model B and the monolith was barely able to withstand the design loads. In Figure 6.3b, it can be seen that tensile stresses in the monolith did not reach the ultimate strength of the material when design loads were applied. Thus, it was not of interest to include nonlinear material properties. However, analysis with linear properties were performed to investigate the influence of existing cracks. Figure 6.20 shows the real failure of Model B with existing cracks.

![Figure 6.20: Real failure of Model B with existing cracks, \( \mu = 0.75 \). \( sf = 1.10 \).](image)

From Figure 6.21, it can be seen that the stability of the monolith is slightly increased by introducing the cracks. The most likely reason for the increased stability is that in the model with cracks, unlike in the model without cracks, the frontplate is embedded to the rock foundation and the reinforcement crossing the lowest crack is working as an anchorage.
Analyses were conducted to investigate the failure mode of the model by forcing a pure overturning failure, as for the model without existing cracks in Section 6.3. The analyses were performed both using linear and nonlinear material properties. Figure 6.20 shows the pure overturning failure of Model B with existing cracks, both for linear and nonlinear analyses.

In Figure 6.22a, it can be seen that the model is acting as two separate blocks divided by crack number 2. By forcing a pure overturning failure, large concentrated compressive forces appeared at the toe of the two blocks as shown in Figure 6.23.
6.4. INFLUENCE OF EXISTING CRACKS

Figure 6.23: Model B. Contact pressure in the rock-concrete interface for pure overturning failure. $\mu = 5.0$.

Figure 6.24 indicates that the results of the nonlinear analyses are not affected by dynamic effects. The analysis was aborted due to wave speed problems, as can be seen when visualising the development of crack propagation, in Appendix A.3. Wave speed problems have occurred in nonlinear analyses, during this project, when the compressive strength of the concrete has been reached.

Figure 6.25 shows that the stability against pure overturning is reduced when existing cracks are included. The behaviour obtained both with linear and nonlinear analysis was more or less identical until the nonlinear analysis was aborted at load magnitude of 1.65.
Comparison

A comparison of obtained safety factors with the different analyses are shown in Table 6.3.

Table 6.3: Obtained safety factors for all the analyses.

<table>
<thead>
<tr>
<th></th>
<th>Model A</th>
<th></th>
<th>Model B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overt.*</td>
<td>Real**</td>
<td>Sliding</td>
<td>Overt.*</td>
</tr>
<tr>
<td>Stability calculations</td>
<td>1.75</td>
<td>-</td>
<td>-</td>
<td>1.96</td>
</tr>
<tr>
<td>Without cracks - Linear analyses</td>
<td>1.74</td>
<td>1.64</td>
<td>1.01</td>
<td>1.94</td>
</tr>
<tr>
<td>With cracks - Linear analyses</td>
<td>-</td>
<td>1.76</td>
<td>1.10</td>
<td>1.81</td>
</tr>
<tr>
<td>With cracks - Nonlinear analyses</td>
<td>-</td>
<td>1.70</td>
<td>-</td>
<td>1.65</td>
</tr>
</tbody>
</table>

* Pure overturning failure, $\mu = 5.0$

** Real failure, $\mu = 0.75$

The most interesting results were that the safety factor for real failure is increased for both of the models, when existing cracks are introduced.

The pure overturning failure load is lower for the model with irregular rock-concrete interface (Model A) than with a horizontal interface. However, sliding failure occurs at a lower load magnitude in the model with horizontal rock-concrete interface (Model B), as expected.

When analysing the models with existing cracks, the results show that using linear material properties gave higher safety factor compared to using nonlinear properties. This is due to the fact that material failure is not accounted for with linear material properties, neither crack propagation nor material failure due to overstressing.

When the results from the pure overturning failure analyses were compared to the models without cracks, insignificant difference was obtained. In the linear finite element analyses, elastic deformations are included, compared to the assumption of
a rigid body movement of the monolith, in the analytical calculations. This means that assuming rigid body movement of the monolith is not influencing the accuracy of the calculations to any reasonable extent.
Chapter 7

Conclusions and further research

7.1 Conclusions

In this project, a case study was carried out, in order to investigate the influence of cracks on the structural dam safety and failure mode of a monolith in a concrete buttress dam. Multiple analyses have been performed to answer the research questions, both in terms of the behaviour of the monolith and also to acquire suitable methods for progressive failure analyses, using the finite element method.

The results of analytical calculations in Section 6.1 showed that the monolith fulfils the stability requirements, recommended in RIDAS. Using a horizontal rock-concrete interface increased the stability regarding a pure overturning failure compared to the model with an irregular interface.

The failure mode of a concrete buttress dam is either a pure sliding failure or a pure overturning failure according to RIDAS. However, the real failure of the finite element model without existing cracks and defined with an irregular rock-concrete interface, was a combination of sliding and overturning failure, as shown in Section 6.3. Finite element analyses were essential to identify the real failure mode and provided a different safety factor compared to the analytical stability calculations.

7.1.1 Influence of existing cracks

Finite element analyses have been performed with linear and nonlinear material properties for the models including existing cracks. The main difference of these two approaches, is that the influence of further crack propagation, is only possible to account for with nonlinear analyses. The results, using linear material properties, indicate only where the first stages of crack propagation will take place. The results from the linear analyses are thus only valid up to a load magnitude where crack propagation takes place. As shown in Section 6.4, the obtained safety factors with linear analyses are higher for both of the models, compared to the safety factor obtained from nonlinear analyses for both sliding and overturning.
A beneficial consequence of allowing crack propagation, with the use of nonlinear material behaviour in finite element analyses, is the degradation in stiffness and subsequent redistribution of forces. In Section 6.4, it was shown that crack opening led to redistribution of stresses, which increased the safety of the monolith, compared to the results obtained with the model without existing cracks.

The failure mode was captured when an internal failure occurred, with an overturning failure of the intact part of the monolith, above the topmost crack.

The case studied does not necessarily represent similar monoliths. However, the results showed that existing cracks have significant influence on the safety factor and the failure mode of the monolith. It is thus important that existing cracks and propagation of these cracks are considered in stability analyses of concrete buttress dams.

### 7.1.2 Simulating progressive failure

The progressive failure was simulated by the overloading method. The first step, concerning progressive failure analyses, was to obtain the most suitable loading approach to increase hydrostatic pressure with the overloading method. Results from four different loading approaches were compared with the analytical calculations. As discussed in Section 6.2, the most suitable approach was to increase only the horizontal pressure component by increasing the density of the water. As a part of the frontplate in the monolith is inclined, increasing the vertical pressure component, resulted in unrealistic estimation of the safety factor.

A load-controlled loading system was used to apply the loads in all the finite element analyses. As presented in Section 5.4, unloading of the monolith is not possible after its ultimate load is reached. Thus, the continuing behaviour of the monolith could not be captured, after the ultimate load was reached. However, the method was applicable to obtain the magnitude of the ultimate load, i.e. the safety factor, and also to investigate the behaviour of the monolith until the ultimate load was reached. For the nonlinear analyses, when crack opening takes place, it is not really of interest to capture the behaviour after the ultimate load is reached, as leakage of water will take place, resulting in water pressure inside of the cracks.

### 7.1.3 Influence of horizontal rock-concrete interface

Analytical stability calculations are generally performed with a horizontal rock-concrete interface, in order to simplify calculations. Using finite element analyses to perform stability calculations gives an opportunity to include detailed geometry, without increasing the complexity of the analysis. In Section 6.3, finite element analyses were performed, to investigate the influence of using horizontal interface on the safety and failure mode of the monolith, compared to an irregular interface.

With the analytical stability calculations, it was already shown that pure sliding was the governing failure mode for the model with a horizontal interface. In the finite
element analyses, pure overturning failure for this model, was only possible to obtain, if a sliding failure was fully prevented, i.e. by increasing the friction coefficient up to unreasonable values. This indicated that a sliding failure always occurred for the friction coefficient between $\mu = 0.75$ and $\mu = 1.0$. It was also shown, that the friction coefficient had to be higher than 0.74 to fulfil the requirements defined by RIDAS.

The friction coefficient was critical to the failure mode of the model with the irregular interface. With the friction coefficient between $\mu = 0.75$ and $\mu = 1.0$, it was shown that the failure was a combination of overturning and sliding failure.

A significant difference in the safety factor for a pure failure was obtained, when the two models were compared. By introducing an irregular rock-concrete interface, the safety factor was increased from $sf = 1.01$ to $sf = 1.64$. This showed, it may be of great importance to perform numerical analysis with more accurate geometry for the rock-concrete interface to obtain the real failure mode and safety factor of a concrete buttress dam.

### 7.2 Further research

The behaviour of the monolith after reaching the ultimate load, was not captured in this project since a load-controlled loading system was used to apply the loads. In order to investigate the continuing behaviour of the monolith and give an important insight for flooding analyses, it would be of interest to study other approaches to apply the increased density method without using load-controlled loading system. One possible approach would be to use a deformation-controlled loading system, after the ultimate load has been reached.

When loads are increased, crack opening takes place in the frontplate. One limitation in this project was that water penetration into opening cracks was excluded, which could be included in further research. This is most likely of importance if an accurate description of the failure behaviour, i.e. the response after reaching the ultimate load, is studied.

In future studies, analyses with strength reduction in the reinforcement could be studied and compared to the overloading method to find the most suitable loading method for nonlinear stability analyses of dam structures.

In this project, a potential failure in the rock foundation was excluded. In further research, the material behaviour of the rock mass and planes of weaknesses could be included in the stability analyses. This would allow a potential failure in the foundation to be captured.

The material model, concrete damaged plasticity in ABAQUS, was applied in this project to simulate crack propagation. However, other approaches like smeared crack approach could also be applied to get comparison between different material models.
Finally, it would be of interest to investigate other types of dam monoliths with similar kind of analyses to examine whether the method used in this project could provide a basis for numerical stability analyses of concrete dams.
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Appendix A

Figures
A.1 Mesh of the finite element models.

(a) Model A, without the existing cracks.  
(b) Model A, with the existing cracks.  
(c) Model B, without the existing cracks.  
(d) Model B, with the existing cracks.

Figure A.1: Mesh of the finite element models.
A.2 Nonlinear analysis of Model A, tensile damage.

Figure A.2: Tensile damage during nonlinear analysis of model A. The value below each figure represent the load magnitude.
A.3 Nonlinear analysis of Model B, tensile damage.

Figure A.3: Tensile damage during nonlinear analysis of model B. The value below each figure represent the load magnitude.