Thermal Bridges
Efficient Models for Energy Analysis in Buildings

by

Guofeng Mao

Dissertation
THERMAL BRIDGES

Efficient Models for Energy Analysis in Buildings

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GUOFENG MAO

FIRST EDITION
Akademisk avhandling som med tillstånd av Kungliga Tekniska Högskolan framlägges till offentlig granskning för avläggande av teknisk doktorsexamen den 5 juni 1997 kl 13:00 i collegiesalen, KTH, Valhallavägen 79, Stockholm. Avhandlingen försvaras på svenska.

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To my son Robin
Preface to this thesis

This thesis is a stage report that presents only one part of the results from my six years' work at the Division of Building Technology, Department of Building Sciences at Kungliga Tekniska Högskolan (KTH) - the Royal Institute of Technology in Stockholm, Sweden.

The thesis is mainly based on the results obtained in a research project entitled "Köldbryggor i byggnadskonstruktioner. Beräknings- och informationsmodeller (Thermal bridges within building constructions. Models of calculations and information)" financed by the Swedish Council for Building Research (BFR), which is gratefully acknowledged.

At the same time, a cooperation project entitled "Insulated sheet metal structures" between KTH and VTT (Technical Research Center in Finland) had been carried out. One part of the results from this project is included in the Appendix 7. Herewith, I would like to express my sincere thanks to VTT for financing the project and the Finnish colleagues, Mr. Jyri Nieminen and Mr. Ilpo Kõuhia for a pleasant research cooperation atmosphere.

Regarding to the results included in Appendix 3 - Dynamic calculation of thermal bridges, I would like to acknowledge Vattenfall's "Energistipendium (Energy reward)" of 1992. Thanks are due to my colleague, Dr. Per Levin as a project leader of "Project - majrovägen" for part financing on the Appendix 1.

I am very grateful to my supervisor Professor Gudni Jóhannesson, head of the Division of Building Technology at KTH in Stockholm. His encouragement and constructive advice over the years have been of great value.

Many thanks are also due to all friends at the Department of Building Sciences, KTH in Stockholm especially to our secretary Ms Ninni Bodin. I would like to thank my colleagues in the Nordic countries, Germany, Canada and the Netherlands for their patient understanding, helpful comments and kind supports during the international activities.

Gratitude is due to my Tsinghua University in Beijing, China for the splendid technical discipline I can get in China, and to my supervisor Prof. Cao Bolin who is the paramount specialist within combustion especially the Circulating Fluidized Bed Combustion (CFBC) area where I had my master of engineering degree in 1989.

Last but not least, I would like to express my deep appreciation to my mother and father and my younger brother in Beijing, China, and Ms. Lingling Qi for their considerate support during my six-year study in Sweden.

Stockholm, April 25, 1997

Guofeng Mao
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VI. Measurement on thermal performance of thermal bridges
Introduction

Background

This thesis is based on the summary of project orientated researching results. The relevant projects are:

- A research project entitled *Köldbryggor i byggnadskonstruktioner. Beräknings- och informationsmodeller (Thermal bridges within building constructions. Models of calculations and information)* is financed by the Swedish Council for Building Research (BFR).

- A cooperation project entitled *Insulated sheet metal structures* between KTH and the Technical Research Center in Finland (VTT).


- *Project - majrovägen* is financed by the Swedish Council for Building Research (BFR).

- *IEA task XIII - Advanced solar low energy buildings* is financed by the Swedish Council for Building Research (BFR).

Scope

<table>
<thead>
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<th>Thermal bridges are studied for</th>
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Summary of the appendixes

I. Thermal bridges in super insulated constructions and their role in the energy balance

In this paper, experiences from one- and two dimensional heat flux calculations for three Swedish multi-family buildings and four single-family houses are demonstrated. Both the one dimensional multi-layer components related to U-values and the details, thermal bridges, are considered for the evaluation of total heat transfer through the building envelopes. The effect of thermal bridges within the three new Swedish multi-family buildings was found to increase the predicted total energy use between 2 and 21%, or the conduction heat loss through the building envelope between 5 and 39%. While the effect of thermal bridges for the single-family houses was found to increase the predicted energy bills between 21% and 47%, compared with one dimensional calculations as generally performed by building designers. This means that results based on one dimensional calculation only are generally not accurate enough for prediction of the energy use for the whole building envelope. The relevant building codes and standards are also briefly discussed in the last part of this paper.

II. The modified λ-value method for estimating linear thermal transmittances of details within building constructions

In this paper, a modified λ-value method is introduced for estimating linear thermal transmittance of thermal bridges. The so called λ-value method has been widely used for certain types of thermal bridges like insulated timber framed walls for a long time.
In order to implement this method into other types of thermal bridges like metal stud walls, junction constructions, a limiting zone for the transversal heat flow around the thermal bridge is identified. Beside the zone, heat flow through the construction is supposed to be one dimensional. The λ-value method is applied inside the zone. The extra heat flow due to the thermal bridge is studied by changing the relatively limit of the zone to give a safe and relatively accurate linear thermal transmittance, comparing with the result by two dimensional computer calculation.

III. Dynamic calculation of thermal bridges

Thermal bridges play a relatively larger roll on heat loss with increasing thermal insulation level of building envelope. In this paper, multi-dimensional dynamic models are set up to predict the thermal performances of thermal bridges using a frequency response method. The thermal bridge is divided into finite meshes where the nodal temperatures are expressed in the form of complex quantities that are related to amplitude and phase lag of the variations of the temperature. By considering the heat balance for the nodes, the real and imaginary parts of the nodal temperatures can be solved simultaneously. A PC computer program is developed to calculate the dynamic responses of temperatures and heat flows within thermal bridges, and has been utilized for thermal bridges such as a light weight metal stud wall, a heavy weight wall - intermediate floor junction and a wall foundation with ground coupling.
IV. The $\omega$-RC transform for thermal bridges

In this paper a method referred to as the $\omega$ - RC transform is developed to obtain simplified dynamic models for thermal bridges. The resulting model is an equivalent serial coupling of thermal resistances (R) and capacitances (C) of thermal bridges that can be implemented in the same manner as one dimensional constructions into dynamic simulation programs to include the impact of thermal bridges in the energy balance for a whole building system. The process of the RC circuit method is that first a dynamic numerical calculation of temperatures and heat flows for the selected thermal bridge is performed for a range of frequencies in the frequency domain; and then an equivalent one dimensional RC circuit called the $\Pi$-link is optimized to give the same steady state response and the optimum dynamic response. The application of the $\omega$ - RC transform in this paper is exemplified with an insulated steel profile structure, an external wall/intermediate floor junction and a wall foundation with ground coupling. The accuracy of the $\Pi$-link RC circuit is examined in the whole frequency domain. Discussions on the dynamic parameter - the total capacitance and other types of RC circuit links are briefly given.

V. Laboratory measurement on dynamic thermal performance of a thermal bridge

Laboratory measurement on dynamic thermal performance of a thermal bridge at Division of Building Technology, KTH in Stockholm is presented in this paper. To arrange the measurement and analyze the measured results, it requires dynamic multi-dimensional modelling that can describe the dynamic behaviours of thermal bridges
in a simple and accurate way. A network method with a \(\Pi\)-link representing the dynamic parameters for a thermal bridge, thermal resistance and thermal capacitance, is developed to predict, arrange and analyze the measured results of a thermal bridge. The dynamic thermal parameters - Resistance, \(R\) and Capacitance, \(C\) are derived from a dynamic multi-dimensional modelling under frequency domain using the so called \(\omega\) - RC technique. By using a \(\Pi\)-link network related to the thermal bridge i.e. a ‘white box’, the measurement on a thermal bridge can be both measured and the measured results can be analyzed in a robust and accurate way. This is demonstrated by analyzing the measured thermal performances of an insulated concrete sandwich structure with wooden studs. This study is orientated to combine modelling and measurement to get quick and reasonable measured results for thermal bridges, and this procedure can be also implemented into measurement on thermal bridges in-situ. Discussions on the deviations on the results between the modelling and the whole measurement are given in the last part of this paper.

VI. Measurement on thermal performance of thermal bridges

The paper deals with the measurements on thermal bridges in situ. Due to the complexity of geometry, the non uniform properties of materials and boundary conditions, this is a very difficult task. In this paper, a methodology for the measurement on thermal bridges in situ is discussed. In order to analyze the measured results, the method used is to combine measurements of temperatures and surface heat flow rates with calculation methods. From an introductory study, a comparison between measurement and calculation on thermal performance of insulated steel metal structures is demonstrated. Measurements on thermal bridges from newly built
multi-family houses are analyzed in order to compare the pattern of heat flow to the calculated steady state properties, and furthermore the dynamic properties are compared to multidimensional calculations in the frequency domain which are carried out with a newly developed computer program.

Conclusions

• Thermal bridges play relatively large role for energy balance both for super-insulated multi-family buildings and single-family houses.

• Related to building design, the modified λ-value method for estimating linear thermal transmittances of details within building constructions is an efficient method.

• For research on dynamic thermal performance of thermal bridges, the calculation method related to the frequency response method in Appendix 3 is a suitable method for research work.

• The RC network method is effective both for computer simulations of the whole building system and for planning of measurement on thermal bridges.

• The RC network method for effective in-situ/laboratory measurements on dynamic performance of thermal bridges has been tested with promising results, and should be evaluated for more cases.
Appendix I
Thermal bridges in super insulated constructions and their role in the energy balance

Guofeng Mao, Per Levin

Division of Building Technology, Kungliga Tekniska högskolan, S 100 44 Stockholm, Sweden

Abstract

In this paper, experiences from one- and two dimensional heat flux calculations for three Swedish multi-family buildings and four single-family houses are demonstrated. Both the one dimensional multi-layer components related to U-values and the details, thermal bridges, are considered for the evaluation of total heat transfer through the building envelopes. The effect of thermal bridges within the three new Swedish multi-family buildings was found to increase the predicted total energy use between 2 and 21%, or the conduction heat loss through the building envelope between 5 and 39%. While the effect of thermal bridges for the single-family houses was found to increase the predicted energy bills between 21% and 47%, compared with one dimensional calculations as generally performed by building designers. This means that results based on one dimensional calculation only are generally not accurate enough for prediction of the energy use for the whole building envelope. The relevant building codes and standards are also briefly discussed in the last part of this paper.

Keywords: U-values; Details; Thermal bridges; Energy use; Building code and standard
1. Introduction

Energy efficient and healthy buildings as targets have been prompted by research, revision of building code, design, workmanship and maintain enhance over the last two decades. At the design phase, normally only the one dimensional multi-layer components are calculated for conventional energy efficient buildings regarding to the energy balance. This means that thermal performance is enhanced by adding extra insulation layers and the phenomena of heat loss through the construction are only related to the thermal transmittance, namely U-values. The effects of details with two or three dimensional heat transfer features (i.e. thermal bridges) on total heat loss through the constructions are normally not included in the design phase. With increasing demand for energy saving and comfort, details play a relatively larger role both for the total heat loss and the comfort. In this paper, experiences from three super insulated Swedish multi-family buildings and four single-family houses taken from an IEA task on advanced solar low energy houses are demonstrated. Both the 1-D multi-layer components and the details with 2-D heat conduction features are considered for the evaluation of total heat loss through the building envelopes. For super insulated energy efficient buildings planned from 1990’s, it is necessary to include the details at the design stag, and the relevant building codes are briefly discussed in this paper.

2. Constructions of the three winning multi-family buildings

2.1 Background for the three super insulated Swedish energy efficient multi-family buildings

A design competition, held by the City of Stockholm and the Swedish Council for Building Research took place in 1990. The competition was a part of the efforts made by the City of Stockholm to promote energy efficient and healthy multi-family buildings. One purpose of the city’s Energy Programme can be
realised through full scale projects. Efficient energy use will help to reduce disruption to the environment, the need for new, expensive energy plants, and, not least, the energy bill for the inhabitants of Stockholm.

The competition entries were predicted to have a much lower energy use for heating than multi-family houses built according to the current Swedish building code. One reason for this is the special attention that was paid by the competitors to avoid thermal bridges in the building construction details.

Three winners of the 16 entries to the competition were awarded by the competition jury with a site allocation in a southern suburb of Stockholm. This means that they had the permit to build about 60 apartments each on adjacent lots. These entries were those which were judged to best fulfil the objectives of the Programme as a whole. The objectives were:

- Healthy indoor climate and good thermal comfort
- Low purchased energy requirement for heating and domestic hot water
- Efficient electrical system solutions and efficient electrical apparatus
- Problem-free operation

The paper firstly discusses the importance of the thermal bridges for the three Swedish multi-family houses (the three winning houses), both compared to the overall thermal loss and to older Swedish multi-family buildings.

The constructions of the winning buildings are briefly described in the following. The descriptions of heating and ventilation systems can be found in Levin and Rolén [1]. Thermal bridges are denoted by circles in all the constructions. These thermal bridges include a roof/wall interface, a joint construction between an external wall and floor structure, a joint construction between a window frame and wall structure and an external wall/foundation interface [2].
2.2 Svenska bostäder/BPA

Fig. 1 shows a cross section of the building construction for the awarded Svenska bostäder/BPA entry, which was chosen for the best forced air heating system, the quality plan, low energy use and good solutions for efficient electricity use.

Fig. 1 Cross section of the building construction for the awarded Svenska bostäder/BPA entry. Circles denote the thermal bridges [2].
2.3 HSB/Ohlsson & Skarne

This entry was chosen because of its own production of electricity and user-friendly building services together with low energy use and efficient electricity use. A cross section of the building construction is shown in Fig. 2.

![Diagram of building construction]

Fig. 2 Cross section of the building construction for the awarded HSB/Ohlsson & Skarne entry. Circles denote the thermal bridges [2].
2.4 NCC-Stockholm

The third winner was selected for its high quality in construction and problem-free operation and fine-tuning of existing technology. It also has low energy use. A cross section of the building construction is given in Fig. 3.

Fig. 3 Cross section of the building construction for the awarded NCC-Stockholm entry. Circles denote the thermal bridges [2].
3. The four single-family houses of the fifteen IEA advanced solar low energy houses

3.1 Background

As part of an international collaboration within the framework of Task XIII - Advanced Solar Low Energy Buildings of the IEA-SHAC (Solar Heating & Cooling Programme of the International Energy Agency), fifteen houses from twelve countries have been built or are being built to achieve relatively low energy consumption without compromising comfort [3].

Two solar low energy houses among the 15 houses are chosen to make comparisons with 2 typical wooden and brick (masonry) houses as reference houses on the heat loss for the whole house envelopes. These comparisons are carried out both with and without consideration of the impact of details.

The predicted energy bills for the four houses are taken from the reference [4] which is a result of the international collaboration. It should be noted that it is not the goal of this evaluation to calculate the exact heat balance of the houses, but rather to illustrate the relative impact of the details on house performance. Some simplifications of the buildings were made, for instance the typical wooden and brick houses are assumed that they have the same floor plans and building dimensions as the Waterloo and Rottweil houses respectively. Thus the differences between the houses are due to the U-values of the 1-D multi-layer components and the details with 2-D heat conduction features.

3.2 Descriptions of the IEA houses and the details

3.2.1 Reference Brick House

Brick construction is used in central and southern Europe. A typical brick building construction is shown in Fig 4. The walls are constructed with low-conductivity brick. For structural reasons the floors need to be constructed of medium density concrete. To reduce heat loss at wall floor junctions, 25 mm of
polyurethane foam insulation is used. The roof is a wood frame assembly with 120 mm of mineral wool insulation.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Layer} & \text{Description} & \text{Width} & \text{R-Value} \\
\hline
\text{Roof} & & & 0.33 \\
1 & Roofing Tile & - & - \\
2 & Lattice & 2.4 & - \\
3 & Water Barrier & - & - \\
4 & Wood (15\%) & 16.0 & 0.14 \\
5 & Mineral Wool (85\%) & 12.0 & 0.04 \\
6 & Vapour Barrier & - & - \\
7 & Lattice/Air Space & 2.4 & - \\
8 & Gypsum Board & 1.3 & 0.23 \\
\hline
\text{Exterior Wall} & & & \\
9 & Mineral Wool & 7.0 & 0.04 \\
10 & Brick & 11.5 & 0.7 \\
11 & Wood & 12.0 & 0.14 \\
12 & Polyurethane Foam & 2.5 & 0.025 \\
13 & Concrete & 25.0 & 2.1 \\
14 & Plaster & 2.5 & 0.87 \\
15 & Porous Brick & 30.0 & 0.16 \\
16 & Plaster & 1.5 & 0.35 \\
17 & Concrete & 27.5 & 2.1 \\
18 & Polyurethane Foam & 2.5 & 0.025 \\
\hline
\text{Window} & & & 2.80 \\
19 & Wood Frame & & \\
20 & Glazing & & \\
21 & Glazing and Frame & & \\
\hline
\text{Basement Ceiling (Connection)} & & & 2.80 \\
22 & Brick & 11.5 & 0.7 \\
23 & Air Space & 3.0 & - \\
24 & Polyurethane Foam & 2.5 & 0.025 \\
25 & Concrete & 21.5 & 2.1 \\
26 & Screedfloor & 4.0 & 1.4 \\
27 & Mineral Wool & 3.0 & 0.04 \\
28 & Concrete & 16.0 & 2.1 \\
29 & Polyurethane Foam & 2.5 & 0.025 \\
30 & Plaster & 1.5 & 0.35 \\
31 & Porous Brick & 24.0 & 2.1 \\
\hline
\text{Below Grade Wall} & & & 0.52 \\
32 & Asphalt Bitumen & - & - \\
33 & Plaster & 2.5 & 0.87 \\
34 & Porous Brick & 36.5 & 0.21 \\
35 & Plaster & 1.5 & 0.35 \\
\hline
\text{Basement Slab} & & & 0.44 \\
36 & Screedfloor & 4.0 & 1.40 \\
37 & Polystyrene & 8.0 & 0.04 \\
38 & Water Proofing & 0.2 & - \\
39 & Concrete & 12.0 & 2.1 \\
40 & Concrete & 5.0 & 2.1 \\
\hline
\end{array}
\]

Fig. 4. Cross section of the building construction for the Reference Brick House. Details (calculated): external wall/intermediate floor; external wall/window; below-grade wall/basement floor [4].
3.2.2 Reference Wooden House

Wood-frame construction is used in North America and Northern Europe. Fig 5 shows a typical interior wall insulation system: wood stud on 400 mm centres with fibreglass batts in between. The basement floor is uninsulated. In warm climates, the basement walls are not insulated or the building is built slab-on grade. The U-values for the below-grade components are to the exterior of the assembly and exclude any insulating effects of the ground.

<table>
<thead>
<tr>
<th>Layer</th>
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<th>U-value</th>
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<tr>
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<td>Cellulose Insulation</td>
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<td>5</td>
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<td>0.21</td>
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<td>26</td>
<td>Waterproofing</td>
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Fig. 5. Cross section of the building construction for the Reference Wooden House. Details (calculated): external wall/roof; external wall/intermediate floor; external wall/window; below-grade wall/basement floor [4].
3.2.3 Rottweil House from Germany

The 'ultra low energy house' is a duplex located in Rottweil (50 km south of Stuttgart). The building opens like a fan with maximum southern exposure (containing about 90% of the glazing) and minimum northern exposure. A multi-storied sunspace is embedded in the south side of each house. The roof has 340 mm of insulation and walls have 400 mm. The window glazing is filled with xenon and achieves a U-value of 0.5 W / m²K (see Fig. 6).

<table>
<thead>
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</tr>
</tbody>
</table>

Fig. 6. Cross section of the building construction for the Rottweil House. Details (calculated): external wall/intermediate floor; external wall/window; wall/grade level floor [4].
3.2.4 Waterloo House from Canada

The Waterloo region single-family green house has a well-insulated and airtight envelope (see Fig. 7). Above-grade walls are framed with wooden I-beams and below-grade walls use a precast concrete system requiring only half the concrete of typical foundations. The triple-glazed windows feature insulated fibreglass frames. The house include reused wood flooring, refurbished bathroom fixtures, recycled gypsum board, carpets made from plastic pop bottles, and formaldehyde and VOC-free paints, glues and fabrics.

Fig. 7 Cross section of the building construction for the Waterloo House from Canada. Details (calculated): external wall/roof; external wall/window; below-grade wall/basement floor [4].
4. The impact of the details on the total energy use for the three new Swedish multi-family buildings

In order to investigate how the thermal bridges affect on the total energy use of the new Swedish multi-family buildings, calculations without consideration of the effect of thermal bridges were made according to the new Swedish building code [5]. The results, which are given in section 4.1, are used as bases for the comparison with consideration of the effect of thermal bridges, and with the older Swedish multi-family buildings.

4.1 The total energy use without consideration of the details

Overall building heat transfer coefficient for each building is calculated as basis number. This coefficient includes: the sum of thermal transmittance (U-value) and surface area (A) products for the building envelope. Table 1 shows the calculated results for the three winning buildings.

Table 1

U-values for the three Swedish multi-family buildings. The thermal bridges are not included.

<table>
<thead>
<tr>
<th>Component</th>
<th>Svenska bostäder/BPA</th>
<th>HSB/Ohlsson &amp; Skarne</th>
<th>NCC-Stockholm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>U</td>
<td>UA</td>
</tr>
<tr>
<td></td>
<td>m²</td>
<td>W/m²K</td>
<td>W/K</td>
</tr>
<tr>
<td>Roof</td>
<td>866.7</td>
<td>0.18</td>
<td>156.0</td>
</tr>
<tr>
<td>Wall</td>
<td>2425.6</td>
<td>0.18</td>
<td>436.6</td>
</tr>
<tr>
<td>Floor</td>
<td>874.7</td>
<td>0.18</td>
<td>157.4</td>
</tr>
<tr>
<td>Window</td>
<td>669.6</td>
<td>1.83</td>
<td>1225.4</td>
</tr>
<tr>
<td>Door</td>
<td>2.3</td>
<td>1.13</td>
<td>2.6</td>
</tr>
<tr>
<td>Total UA</td>
<td>1978.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Total Energy use for each building depends on building construction and building service system. Table 2 shows the predicted results of energy use for each building without consideration the effect of thermal bridges.

Table 2

Total energy use without consideration of the thermal bridges [2].

<table>
<thead>
<tr>
<th>Energy Use (kWh/m²)</th>
<th>Svenska bostäder/BPA</th>
<th>HSB/Ohlsson &amp; Skarne</th>
<th>NCC-Stockholm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heating</td>
<td>66.90</td>
<td>26.79</td>
<td>50.28</td>
</tr>
<tr>
<td>Domestic electricity use</td>
<td>27.93</td>
<td>34.74</td>
<td>31.67</td>
</tr>
<tr>
<td>Building electricity use</td>
<td>17.75</td>
<td>24.18</td>
<td>17.67</td>
</tr>
<tr>
<td>Total kWh/m²</td>
<td>112.58</td>
<td>85.71</td>
<td>99.62</td>
</tr>
</tbody>
</table>

One annotation is that the U-value of window is the total U-value. Therefore, the type of thermal bridge, like a joint construction between a window frame and wall structure, is not included in section 4.2.

4.2 The total energy use with consideration of the details

Excess building heat transfer coefficients, for instance so called linear thermal transmittances i.e. Ψ-values for the details with 2D heat flow features, for the effect of thermal bridges are calculated. For each thermal bridge geometry, excess heat transfer coefficient is determined by: summing the heat transfer caused by unit temperature difference) along the interior surface, and subtracting the products of U-value (from section 4.1, Table 1) and the width of the thermal bridge geometry. The meaning of this coefficient is the additional heat transfer due to the thermal bridge.

Details with two-dimensional heat flow features are called linear thermal bridges. They are chosen from the drawings of the buildings shown in this paper. Ψ-values for these linear thermal bridges within the three different Swedish multi-family buildings are calculated. The details of the four single-family IEA solar low energy houses are treated in the same way, which will be described in chapter 5. All of the
calculations on the details in this paper are based on internal dimensions, and the PC program GF2DIM [6] is used to perform the calculations. A more detailed description on Ψ-values can be found in EN ISO 10211-1 [7] and Burch et al [8].

Table 3 gives the Ψ-values for each thermal bridge geometry. Total Ψ⁺l-values for the whole building, which are obtained by multiplying the linear thermal transmittance, Ψ-value, by the length l (distance into page in the two-dimensional drawings of the thermal bridges given in Fig. 1-3) of the thermal bridge geometry in the building, are given in the next column. The percentage of increasing on the building heat transmission through the building envelopes due to the thermal bridge is given in the last column for each building.

Table 3
Ψ-values for the details and their impact on the total U·A-values for the whole buildings for the three Swedish multi-family houses.

<table>
<thead>
<tr>
<th>Detail</th>
<th>Svenska bostäder/BPA</th>
<th>HSB/Ohlsson &amp; Skarne</th>
<th>NCC-Stockholm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ψ</td>
<td>ΣΨ l</td>
<td>%</td>
</tr>
<tr>
<td>W/mK</td>
<td>W/K</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Roof/Wall</td>
<td>0.227</td>
<td>63.56</td>
<td>3.21</td>
</tr>
<tr>
<td>External wall/Floor</td>
<td>0.0034</td>
<td>3.81</td>
<td>0.19</td>
</tr>
<tr>
<td>External wall/Foundation</td>
<td>0.102</td>
<td>28.56</td>
<td>1.44</td>
</tr>
<tr>
<td>Total</td>
<td>95.93</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

A comparison on the total energy use due to the consideration of the details for the three Swedish multi-family houses and the increasing percentage of total energy use in each building caused by the details are given in Table 4.
Table 4

Comparison total energy use without and with consideration of thermal bridges [2].

<table>
<thead>
<tr>
<th></th>
<th>Total Energy Use, kWh/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Excluding Thermal Bridges</td>
</tr>
<tr>
<td>Svenska bostäder/BPA</td>
<td>113</td>
</tr>
<tr>
<td>HSB/Ohlsson &amp; Skarne</td>
<td>86</td>
</tr>
<tr>
<td>NCC-Stockholm</td>
<td>99</td>
</tr>
</tbody>
</table>

5. Summary of the impact of the details on the total heat loss for the four single-family IEA houses

The procedure used in chapter 4 is used to predict the impact of the details on the energy bills for the four single-family IEA houses. The calculated $\Psi$-values for the thermal bridges in the four houses are given in Table 5.

Table 5
$\Psi$-values for the details in the four single-family IEA houses.

<table>
<thead>
<tr>
<th>House</th>
<th>$\Psi$-values for the details, W/mK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>wall/roof</td>
</tr>
<tr>
<td><strong>Brick:</strong></td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>-</td>
</tr>
<tr>
<td>Rottweil</td>
<td>-</td>
</tr>
<tr>
<td><strong>Wooden:</strong></td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>0.067</td>
</tr>
<tr>
<td>Waterloo</td>
<td>0.044</td>
</tr>
</tbody>
</table>

One annotation is that due to different types details for the four single-family IEA houses, some results like the details of wall/roof are not listed in the above table.

Table 6 summarises the results of the analysis for the Rottweil house and the reference brick house, and the results for the Waterloo house and the reference wood frame house.
The predicted energy bills for the four IEA houses, with and without consideration of the details. The heat balance was calculated for the period October 1 to March 31 with a constant interior temperature of 20 °C and monthly average weather conditions [4].

<table>
<thead>
<tr>
<th>House type</th>
<th>Energy bill (no details) kWh</th>
<th>Energy bill (with details) kWh</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Brick</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rottweil</td>
<td>3236</td>
<td>4070</td>
<td>26</td>
</tr>
<tr>
<td>Reference</td>
<td>11995</td>
<td>14553</td>
<td>21</td>
</tr>
<tr>
<td><strong>Wooden</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Waterloo</td>
<td>4576</td>
<td>6730</td>
<td>47</td>
</tr>
<tr>
<td>Reference</td>
<td>12132</td>
<td>17051</td>
<td>41</td>
</tr>
</tbody>
</table>

6. Discussion and conclusion

6.1 The three Swedish multi-family buildings

6.1.1 The effect of details on the total energy use

With consideration the effect of thermal bridges on the new Swedish multi-family buildings, the predicted total energy use increases between 2% and 21% for the three Swedish multi-family. This means that a generally valid coefficient for the effect of the thermal bridges on the total energy use for different buildings can not be given. For the building construction design, more attention should be put on the interface of external wall/foundation and the joint construction between external wall/floor to further reduce the effect of thermal bridges.
As for the detail of window/external wall, one project performed by one of the authors and adopted in [9] shows that the calculated $U/L$ value of this detail is 30.4 W/K. While the total $U$-$A$ value is 407.9 W/K, which means that only the detail of window/external wall can increase about 7% of the total heat transmission through the whole building envelope.

6.1.2 Comparison with the conventional Swedish Multi-family Houses

Comparison with the older Swedish multi-family houses is given in Fig. 8. The average predicted total energy use for the competition houses is about 30% less than for the houses in the Stockholm project built in the mid-1980-ies, due to reducing the effect of thermal bridges, lower electricity use for building services and lower domestic electricity use.

![Graph showing energy use](image)

Fig. 8 Total energy use for multi-family buildings in Stockholm (recalculated to standard year). The two bars on the right are: measured results from the Stockholm project and the average predicted results for the competition winners in which the thermal bridges are not included. data from Levin et al. (1993).
6.2 The four single-family IEA houses

The predicted results on the four single-family IEA houses show that

1. The two advanced solar low energy houses (Rottweil and Waterloo) have significant low energy bills, if compared with the relevant reference houses.

2. The thermal bridges play a relatively larger role on the energy bills for the two advanced solar low energy houses, if compared with the relevant reference houses. This indicates that 1 D components come firstly into consideration by architect and construction designer, and then details should be examined carefully. This because that the details can also lead to other problems, if they were not included at the design phase.

6.3 Brief discussion on the relevant building codes

The building codes related to details are given in CEN’s proposals [7] and [10]. These regulations provide the definitions on the linear and point thermal transmittances and give PC program as calculation methods. The Swedish building codes are relatively simple and accurate like λ-value method on wooden stud component [11] and network method on steel stud component [12].

6.4 Conclusions

The conclusions from the experiences on the super insulated energy efficient buildings are that:

- The effect of thermal bridges within the three Swedish multi-family buildings was found to increase the predicted total energy use between 2 and 21%, or the transmission heat loss through
the building envelope between 5 and 39%. While the effect of thermal bridges for the four single-family IEA houses was found that the predicted energy bills increased between 21% and 47%, compared with one dimensional calculations.

- Details should be considered together with 1 D components for the whole building constructions, and be evaluated from case to case. This is not only for the more accurate predicted results on the energy use for the whole building, but also providing useful data for the whole building system for instance for the parts of ventilation and indoor air etc. This is more important for super insulated energy efficient buildings.

- This paper shows the method for implementing thermal bridges into calculation of the energy balance, and gives the reader a quantitative insight into the relevance of thermal bridges for exemplified super insulated constructions. This method is limited for the steady state calculations, i.e. at the very early design phase.

- For the relevant building codes, the thermal transmittance calculation should include the detail parts. Linear or point thermal bridges (i.e. details with 3D heat flow features) should be performed systematically for the whole building construction.

- To perform the evaluation on details' effect on the whole building construction, either PC programs or simple calculation method with comparable accuracy can be used.

In order to consider details at design phase in a quick and accurate way, a series of simple calculation methods for different types of details are developed by the authors, and this will be deal with in a separate paper.
Acknowledgement

The support by the Swedish Council for Building Research (BFR) is gratefully acknowledged. Thanks are due to Prof. Gudni Jóhannesson, head of the Division of Building Technology at KTH for his guidance.

REFERENCES


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Appendix II
The Modified $\lambda$-value Method for Estimating Linear Thermal Transmittances of Details within Building Constructions

Mao, G. and G. Jóhannesson
Division of Building Technology
Kungliga Tekniska Högskolan
S 100 44, Stockholm, Sweden

1. Introduction

Thermal bridges are parts of building construction in which heat flows are not one dimensional and distributions of internal surface temperature are non-uniform. Problems associated with thermal bridges are a significant increase of energy loss and often severe condensation of moisture at interior surfaces. With increasing insulation thickness, thermal bridges play a relatively larger role for energy consumption. In order to evaluate the influence of thermal bridges at the design stage, simplified calculation methods are required for building designers. These methods should be easy to use and understand for designers and give a safe and relatively accurate value, when compared with the results from other more complicated methods. Based on this aim, a modified $\lambda$-value method has been developed at the Division of Building Technology, Kungliga Tekniska Högskolan in Stockholm to estimate the linear thermal transmittance of thermal bridges. This paper presents case studies of determination of linear thermal transmittance using the modified $\lambda$-value method for timber framed wall, metal stud wall and junction construction with two dimensional heat transfer features.
2. The modified $\lambda$-value method

The so called $\lambda$-value method has been used for types of thermal bridges like timber framed walls for a long time. It has also been assumed that the method was not suitable for constructions made of materials with large differences in thermal conductivities. In order to make this method suitable for other types of thermal bridges like metal stud walls, junction constructions, a limiting zone for the transversal heat flow around the thermal bridge is defined. Beside the zone, heat flow through the construction is supposed to be one dimensional. The original $\lambda$-value method is then applied within the defined zone. The extra heat flow due to the thermal bridge is studied for varying width of the defined influencing zone to give a safe and relatively accurate linear thermal transmittance, comparing with the results by two dimensional computer calculations.

2.1 Linear thermal transmittance of thermal bridges - $\psi$-value

In order to estimate the extra heat loss caused by a thermal bridge, the linear thermal transmittance ($\Psi$-value) is used. For the construction including a thermal bridge, the linear heat flow rate is given by eq. 1:

$$L = \left( \sum_{j=1}^{n} U_j \cdot 1_j + \Psi \right) \cdot (\theta_i - \theta_e)$$  

(1)
The calculation of \( \Psi \)-value can be divided into 3 steps for a two dimensional computer calculation, and the result is used to check the \( \Psi \)-value calculated by the modified \( \lambda \)-value method:

**Step 1:** From a drawing of a thermal bridge, the \( U_j \)-values should be determined firstly. It is also necessary to state which dimensions \( l_j \) (e.g. internal or external) are being used because for several types of thermal bridges the resulting linear thermal transmittance depends on this choice. Internal dimensions are used in this paper to perform the new \( \lambda \)-value method.

**Step 2:** A two-dimensional steady-state calculation is used for determining the total heat flow rate i.e. \( L \)-value of the thermal bridge according to the drawing. In this paper, a 2-D steady-state PC program named GF2DIM has been used to calculate the heat flows through the thermal bridges.

**Step 3:** \( \Psi \)-value can be calculated by the equation:

\[
\Psi = \frac{L - \left( \sum_{j=1}^{n} U_j \cdot l_j \right) \cdot (\theta_i - \theta_x)}{(\theta_i - \theta_x)}
\]  

(2)

### 2.2 The modified \( \lambda \)-value method for calculation of linear thermal transmittance (\( \psi \)-value)

The procedure of the new \( \lambda \)-value method for calculation of linear thermal transmittance (\( \psi \)-value) can also be divided into 3 steps:
Step 1: Same as the Step 1 for a two dimensional computer calculation, $U_j$-value for the homogenous parts should be determined firstly:

$$U_j = \frac{1}{R_{si} + \sum_{i=1}^{n} \frac{d_i}{\lambda_i} + R_{se}} \quad (3)$$

Step 2: The original $\lambda$-value method is used for a zone that heat flow dominated by the thermal bridge. The limits of the zone are given in equation (4) which is based on the theory for heat flow along a cooling flange:

Fig. 1. Definition of distance (b) between thermal bridge and surrounding 1-D geometry (Jóhannesson and Andersson 1989).

$$b = \max \left\{ \frac{c \cdot \sqrt{\lambda_i \cdot d_i \cdot R_{si}}}{c \cdot \sqrt{\lambda_e \cdot d_e \cdot R_{se}}} \right\} \quad (4)$$

The total width of the zone for the thermal bridge in Fig. 1 is $2b + b_{bb}$. The $\lambda$-value method for this thermal bridge can be written as follows:
For layer $i$: $$\lambda'_i = \frac{(2b \cdot \lambda_i + b_{ib} \cdot \lambda_{ib})}{(2b + b_{ib})}$$

For layer iso: $$\lambda_{iso}' = \frac{(2b \cdot \lambda_{iso} + b_{ib} \cdot \lambda_{ib})}{(2b + b_{ib})}$$ weighting the $\lambda$-values \hspace{1cm} (5)

For layer $e$: $$\lambda_e' = \frac{(2b \cdot \lambda_e + b_{ib} \cdot \lambda_{ib})}{(2b + b_{ib})}$$

Both the construction and the thermal bridge can consist of an arbitrary number of layers with different $\lambda$ values.

The thermal resistance of the thermal bridge including the limiting zone is then calculated as:

$$R_{tb} = R_{si} + d_i/\lambda_i' + d_{iso}/\lambda_{iso}' + d_e/\lambda_e' + R_{se}$$ \hspace{1cm} (6)

**Step 3:** The linear thermal transmittance ($\Psi$-value) of the thermal bridge is then calculated by eq. 7, which is valid for thermal bridge covered by the internal surface area (see Fig. 1, U-value is calculated from eq. 3 for the 1-D part).

$$\Psi = \frac{2b + b_{ib}}{R_{tb}} - (2b + b_{ib}) \cdot U$$ \hspace{1cm} (7)

For a junction construction (see Fig. 2), the width of the zone for a thermal bridge is also $2b + b_{ib}$. The junction construction can be converted into a plain construction with different $R_{si}$ for calculation of thermal resistance of the thermal bridge. The equivalent internal surface resistance $R_{si}'$ for the plain construction can be approximately calculated as follows:
Fig. 2. Junction construction (a) can be converted into a plain construction (b) with different $R_{si}$.

\[
R_{si'} = \frac{2b + b_{tb}}{2b + \frac{\lambda_{tb}}{R_{si}}} + \frac{\frac{\lambda_{a} \cdot R_{si}}{b_{tb}}}{\sqrt{0.33 + \frac{\lambda_{a} \cdot R_{si}}{b_{tb}}}}
\]  

(8)

The thermal resistance of the thermal bridge including the limiting zone is then calculated as:

\[
R_{tb} = R_{si'} + \frac{d_i}{\lambda_{a}'} + \frac{d_{iso}/\lambda_{iso}'}{d_{iso}/\lambda_{iso}'} + \frac{d_e/\lambda_e'}{R_{se}}
\]  

(9)

The linear thermal transmittance ($\Psi$-value) of the thermal bridge is then calculated under Step 3 (see Fig. 2, $U$-value is calculated from eq. 3 for the 1-D part):

\[
\Psi = \frac{2b + b_{tb}}{R_{tb}} - 2b \cdot U
\]  

(10)
Note: The procedure has for simplicity been shown for a homogenous thermal bridge with symmetrical flanking elements. The method is equally valid for asymmetrical thermal bridges where the influencing zone can have different lengths on each side and for different material layers in the thermal bridge as well. If the attached wall or floor is non homogenous, a safe value can be reached by using the highest $\lambda$ value in the attached construction in eq. 8.

2.3 Case studies using the $\lambda$-value method for calculation of $\Psi$-values

Case 1: Timber framed wall

![Diagram of timber framed wall](image)

- Gypsum board 9 mm $\lambda = 0.21$ W/mK
- Insulation 150 mm $\lambda = 0.037$ W/mK
- 150*50 mm wood stud $\lambda = 0.14$ W/mK
- Gypsum board 13 mm $\lambda = 0.23$ W/mK

Fig. 3. Timber framed wall used for the case study.

Case 2: Metal stud wall - C sheet metal profile with non metal surface cladding

![Diagram of metal stud wall](image)

- Gypsum board 9 mm $\lambda = 0.21$ W/mK
- Insulation 150 mm $\lambda = 0.037$ W/mK
- 150*50*1 mm C (stainless steel) $\lambda = 20$ W/mK
- Gypsum board 13 mm $\lambda = 0.23$ W/mK

Fig. 4. Metal stud wall used for the case study.
The difference between case 1 and case 2 is that the wooden stud in case 1 is replaced by a C sheet metal profile in case 2.

**Case 3: Junction construction**

![Diagram of junction construction](image)

Fig. 5. Junction construction used for the case study.

The results of $\Psi$-values calculated by the new $\lambda$-value method are listed in Table 1, and also are compared with the $\Psi$-values calculated by 2-D computer program.
Table 1. The results of $\Psi'$-values calculated by the new $\lambda$-value method for the 3 cases.

<table>
<thead>
<tr>
<th>The Modified $\lambda$-value Method</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: $U$-value calculation [W/m²K]</td>
<td>0.231</td>
<td>0.231</td>
<td>0.221</td>
</tr>
<tr>
<td>Calculation of $b$ [m] with $c = 1$</td>
<td>0.02</td>
<td>0.02</td>
<td>0.149</td>
</tr>
<tr>
<td>Calculation of equivalent $\lambda$-value [W/mK]</td>
<td>$11.128^{1/0.259^2}$</td>
<td>0.707</td>
<td></td>
</tr>
<tr>
<td>Calculation of $R_{si'}$ (for Case 3) [m²K/W]</td>
<td>-</td>
<td>-</td>
<td>0.134</td>
</tr>
<tr>
<td>Calculation of $R_{tb}$ [m²K/W]</td>
<td>1.865</td>
<td>0.841</td>
<td>0.514</td>
</tr>
<tr>
<td>Step 3: $\Psi'$-value of the thermal bridge [W/mK]</td>
<td>0.0275</td>
<td>0.0862</td>
<td>0.9030</td>
</tr>
<tr>
<td>$\Psi'/\Psi'_{comp}$ (comparison with 2-D program)</td>
<td>1.02</td>
<td>1.16</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Check safe and accurate $\Psi'$-values

- If $c = 0.5$
  - $b$ [m] | 0.01 | 0.01 | 0.074 |
  - $\Psi'/\Psi'_{comp}$ | 1.00 | 1.08 | 0.93 |
- If $c = 2$
  - $b$ [m] | 0.04 | 0.04 | 0.297 |
  - $\Psi'/\Psi'_{comp}$ | 1.05 | 1.26 | 1.38 |

1: for the insulation layers with the thickness of 1 mm, corresponding to the thickness of the flange of the metal profile.

2: the thickness of the rest part of insulation, i.e. 148 mm.
3. Discussion and conclusion

3.1 The size of the chosen zone influenced by the thermal bridge

The constant $c$ in eq. 4 determines the size of the zone influenced by the thermal bridge. Fig. 6 shows the relations of the ratio of $\psi/\psi_{comp}$ and the constant $c$ for the three thermal bridges. These curves begin with the constant $c$ equal to zero that leads to the U-value method which gives a lower limit to the U-values for constructions including thermal bridges. These curves end where the zone covers the whole construction that leads to the original $\lambda$-value method. The middle point of the straight line means the average result from the values of the U-value and the original $\lambda$-value method (Isolerguiden 1990 and EN ISO 6946). For a timber framed wall and metal stud wall, the whole size of the construction is the width of 600 mm. For the junction construction, the whole size of the construction is the height of 2.2 m (i.e. $1 + 0.2 + 1$ m).

Fig. 6. Relations of the ratio of $\psi/\psi_{comp}$ and the constant $c$ for the three thermal bridges.
The results of linear thermal transmittances ($\psi_{comp}$) from two dimensional computer calculations are based on the whole size of these constructions. A conclusion is that the original $\lambda$-value method can not be directly used for the whole size of the construction, even for wooden thermal bridges. The area included in a thermal bridge calculation will affect the resulting accuracy.

3.2 Safe and relatively accurate linear thermal transmittances for the thermal bridges

This paper shows some case studies of the application of the modified $\lambda$-value method. By limiting the area of thermal bridge with a constant of one (see Fig. 6 and 7), this method gives relatively accurate results of the linear thermal transmittances of thermal bridges, if compared with 2-D computer calculations. For safe values a constant $c$, slightly larger than one has to chosen for some thermal bridges.

The linear thermal transmittance ($\psi$-value) of the thermal bridge to be added to the homogenous $U$-value of the internal surface. For instance, the thermal transmittance of the whole size of the construction for case 1 - timber framed wall is 0.277 W/m²K calculated by eq. 7, and for case 2 - metal stud wall is 0.375 W/m²K.
Fig. 7. Comparison between the new $\lambda$-value method with 2-D computer calculations.

This method can be used for designers at very early design stage to take care of the thermal bridge problem. And it can also be used for consulting work and university's undergraduate students as an easy exercise associated with building physics. More examples are needed to evaluate this method, and this will be given in a separate paper.

Acknowledgement

The support by the Swedish Council for Building Research (BFR) is gratefully acknowledged.
Nomenclature

b  the chosen limit for the thermal bridge calculations (m).
c  is a constant which governs the accuracy and safety of the method.
d  thickness (m).
L  linear heat flow rate of a thermal bridge from a 2-D calculation [W/m].
l  the length over which the value U applies [m].
R  thermal resistance [m²K/W].
U  thermal transmittance [W/m²K].

Greek symbols

λ  thermal conductivity [W/mK].
θ  air temperature [°C or K].
Ψ  linear thermal transmittance of a thermal bridge [W/mK].

Subscripts

e  external e environment.
i  internal i environment.
iso  insulation material.
j  the jth part of component.
se  external surface.
si  internal surface.
tb thermal bridge.

Superscript

' equivalent.

References


Isolerguiden 1990. Swedisol och Pelle Thorsén AB.


Dynamic calculation of thermal bridges

Guofeng Mao, Gudni Jóhannesson
Division of Building Technology, Kungliga Tekniska Högskolan, S 100 44 Stockholm, Sweden

Abstract

Thermal bridges play a relatively larger roll on heat loss with increasing thermal insulation level of building envelope. In this paper, multi-dimensional dynamic models are set up to predict the thermal performances of thermal bridges using a frequency response method. The thermal bridge is divided into finite meshes where the nodal temperatures are expressed in the form of complex quantities that are related to amplitude and phase lag of the variations of the temperature. By considering the heat balance for the nodes, the real and imaginary parts of the nodal temperatures can be solved simultaneously. A PC computer program is developed to calculate the dynamic responses of temperatures and heat flows within thermal bridges, and has been utilised for thermal bridges such as a light weight metal stud wall, a heavy weight wall - intermediate floor junction and a wall foundation with ground coupling.

Keywords: Thermal bridges; Frequency response method; Multidimensional heat flow

1. Introduction

Thermal bridges can be divided into two groups: thermal bridges within the building components and thermal bridges at joints between the building components. They have two important consequences compared with those of the unbridged structure [1]:

1. a change in heat flow rate, and
2. a change in internal surface temperature.

Levin and Mao [2] studied the effect of thermal bridges on predicted total energy use (the sum of heating, building electricity use and domestic electricity use) of three winning building constructions among 16 entries to a design competition, which was held by the City of Stockholm and the Swedish Council for Building Research in 1990. The results of this study are that even if the thermal bridges effects of windows and doors were excluded:

(a) the predicted total energy use was found to increase between 2 and 21%,
(b) the heat flow through the building envelope increased between 5 and 39%, and
(c) especially the connection between external walls and the foundation requires further attention to reduce additional heat loss.

The investigation of the effect of thermal bridges on energy consumption shows that there is a great need to include thermal bridges in simulations of building thermal performance and indoor climate. A reference to comparison between simulation programs by Poel [3] reveals that all of the building energy simulation programs assume the heat transfer through the building constructions as one dimensional heat transfer.

For thermal bridges themselves, dynamic calculation methods with analytical solutions are rather rarely found, and most of them are two dimensional (2-D) dynamic solutions for simplified geometry of thermal bridges. Hagentoft [4] developed 2-D dynamic calculation methods on the heat loss to the ground limited to periodic heat loss and step change of outdoor temperature. Krarti [5,6] derived the Interzone Temperature Profile Estimation (ITPE) technique to solve two dimensional periodic heat transfer from slab-on-grade floors with vertical and horizontal insulation accounting for the existence of a water table underneath the slab.

On the other hand, there are plenty of numerical dynamic calculation computer codes for thermal bridges. For instance, a list with more than 20 most common programs is given by Blomberg [7].

Andersson and Jóhannesson [8] developed the frequency response method for multi-dimensional non-steady linear heat flow problems. The main advantages of this method are that:

1. by considering the dynamic feature in the frequency domain in steady of the time domain, huge time series can be consolidated into a more manageable form with Fourier series.
2. the results can be further utilised to optimise the parameters of simplified models such as RC-networks, see also Mao and Jóhannesson [9].

Based on this method, a PC computer program has been developed at the Division of Building Technology, Kungliga Tekniska Högskolan (KTH) in Stockholm. This paper presents the frequency response method and gives applications of this method on a light weight construction consisting a metal stud wall, a heavy weight floor junction and a wall foundation with ground coupling.
2. The frequency response method

Dynamic heat conduction for each frequency \( \omega \) within building construction can be expressed as:

\[
ad \tilde{\theta} = \omega \tilde{\theta}
\]

(1)

The above equation is based on two approximations: One is that the thermal diffusivity is independent of time, the whole system is a linear system. The results of the dynamic heat flows or temperatures can be superposed. Another is that variation of the boundary condition is in the form of periodic variation under each frequency. Each dynamic variation within the system, for instance temperature, heat flow and the response factor, can be expressed in the same form as the temperature listed in equation (1). More detailed expressions can be found in Johannesson [10].

The most straightforward solution of equation (1) is by dividing the constructions into finite elements and then solving the heat balance for the temperatures in the centre nodes of the cells. A heat balance for the cell in Fig. 1 gives:

\[
s_1 (\tilde{\theta}_1 - \tilde{\theta}_0) + s_2 (\tilde{\theta}_2 - \tilde{\theta}_0) + s_3 (\tilde{\theta}_3 - \tilde{\theta}_0) + s_4 (\tilde{\theta}_4 - \tilde{\theta}_0) = \frac{d\theta}{dt}
\]

(2)

where

\[
s_j = \frac{d\theta_j}{dx_j} \quad \text{(for } j = 1,3)\)
\[
s_j = \frac{d\theta_j}{dx_j} \quad \text{(for } j = 2,4)\)

The temperature \( \tilde{\theta} = \tilde{\theta} \cdot e^{i(\omega t+\phi)} \) can also be written as the following form:

\[
\tilde{\theta} = (u + iv) \cdot e^{i\omega t}; \quad u = \theta \cos \phi, \quad v = \theta \sin \phi
\]

(3)

By substituting equation (3) into equation (2), we can get two equations, one for the real and one for the imaginary part. The resulting equation system for the cell numbered 0 becomes in matrix form:

\[
\begin{bmatrix}
\Sigma_{i,j}^4 s_j & -d\omega \rho c \\
\end{bmatrix}
\begin{bmatrix}
u_0 \\
v_0
\end{bmatrix} =
\begin{bmatrix}
\Sigma_{i,j}^4 s_j v_j \\
\Sigma_{i,j}^4 v_j
\end{bmatrix}
\]

(4)

The computational technique used in the PC program is simultaneous over-relaxation with Chebyshev acceleration method (Press et al. [11]). The computational time for each frequency is of the same order of magnitude as for the steady state solution.

Since we are mainly interested in the dynamic heat flow and temperature on internal surfaces, for the following analysis, two factors are used: 

Admittance of a building component (A) is defined as the ratio between the variation of surface heat flow and inside air temperature when the outside air temperature equals to zero.

\[
A = \frac{\bar{Q}_s}{\bar{\theta}_i} \quad (\bar{\theta}_c = 0)
\]

(5)

Transmittance of a building component (T) is defined as the ratio between the variation of surface heat flow and outside air temperature when the inside air temperature equals to zero.

\[
T = \frac{\bar{Q}_s}{\bar{\theta}_c} \quad (\bar{\theta}_i = 0)
\]

(6)

At a given surface, the heat flow \( \bar{Q}_s \) can be expressed as a sum of the responses to given temperatures \( \bar{\theta}_i \) and \( \bar{\theta}_c \) at the boundaries under each frequency \( \omega \), i.e.

\[
\bar{Q}_s = A \cdot \bar{\theta}_i + T \cdot \bar{\theta}_c
\]

(7)

where

Fig. 1. Discrete representation of the two-dimensional non-steady state heat flow problem.
\[ A = \gamma_A \cdot e^{i \phi_A} ; T = \gamma_T \cdot e^{i \phi_T} \]

The results of amplitudes and phases for admittance and transmittance are plotted in Bode-diagram known from automatic control theory, see Fig. 3.

3. Applications

3.1 Metal stud wall

Insulated metal stud walls as building envelope systems are currently being constructed in a number of new office buildings and multifamily houses in the Nordic countries. The main benefits of insulated metal stud walls are that there is less risk of moisture and fire damage, and it is not difficult to control the drying out time of the structures compared with wooden or masonry structures.

The thermal conductivity of steel is several orders of magnitude larger than the surrounded insulation material and therefore, a metal stud in an insulated structure is a thermal bridge. This leads to a significant effect on the heat loss of the structure, and at the same time there is a risk of inside surface condensation.

The effect of thermal bridges may be reduced by simple structural means, for instance by perforating the metal profiles of the structure or by sufficient thermal breaks in the structure.

Below, a part of the results from a joint project between the Technical Research Centre of Finland (VTT) and KTH is presented. One of the lightweight insulated structures with sheet metal stud is used here for the application of the above method. More detailed information on the design and thermal performance of insulated sheet metal structures can be found in Mao and Jóhannesson [12] and Niemien et al [13].

3.1.1 Description of a metal stud wall

The construction studied here is single C profile (Fig. 2). Gypsum boards are located on the top side with 9 mm thickness and at the bottom side with 13 mm thickness. Insulation material is located at the middle part of the structure with 150 mm thickness. The central distance between two profiles is 600 mm.

![Fig. 2. Insulated sheet metal structure with C profiles.](image)

3.1.2 Dynamic calculation of the metal stud wall

In order to study the influence of the thermal bridge on heat loss of the structure, one dimensional multi-layer structure without metal profile and the current structure are calculated using the same PC program. For one dimensional multi-layer structures, their analytical solutions are given by Carslaw and Jaeger [14], which are in the form of matrices.

The material properties used for the calculations are listed in Table 1. The width of the structure for the calculations is 600 mm which is the central distance of two metal profiles. The internal surface resistance is 0.13 m²K/W and the external surface resistance is 0.04 m²K/W.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \lambda ) (W/mK)</th>
<th>( \rho ) (kg/m³)</th>
<th>c (J/kgK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gypsum (top side)</td>
<td>0.21</td>
<td>800</td>
<td>1050</td>
</tr>
<tr>
<td>Insulation</td>
<td>0.037</td>
<td>20</td>
<td>1030</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>20</td>
<td>7800</td>
<td>460</td>
</tr>
<tr>
<td>Gypsum (bottom side)</td>
<td>0.23</td>
<td>800</td>
<td>1050</td>
</tr>
</tbody>
</table>

The series of frequencies used in the calculations and the corresponding period length in the time domain are given in Table 2. From the physical point of view, the steady state solution can be reached when the frequency goes down to zero.

<table>
<thead>
<tr>
<th>Frequency domain ( \omega = 2\pi f ) (rad/s)</th>
<th>Time domain t</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \omega ) (steady state)</td>
</tr>
<tr>
<td>1.99238 \times 10^{-7}</td>
<td>1 year</td>
</tr>
<tr>
<td>2.42407 \times 10^{-6}</td>
<td>1 month</td>
</tr>
<tr>
<td>1.03889 \times 10^{-5}</td>
<td>1 week</td>
</tr>
<tr>
<td>7.27221 \times 10^{-5}</td>
<td>1 day</td>
</tr>
<tr>
<td>1.45444 \times 10^{-4}</td>
<td>12 hours</td>
</tr>
<tr>
<td>5.81777 \times 10^{-4}</td>
<td>3 hours</td>
</tr>
</tbody>
</table>

The dynamic results in the form of the amplitude and the phase lag of admittance and transmittance are listed in Table 3 and plotted in Fig. 3 in form of a Bode-diagram for the one dimensional multi-layer structure. The admittances and transmittances present the heat flows through the structure under different time periods for indoor air and outdoor air oscillations.
Table 3
The results of amplitude and phase lag of admittance and transmittance for the one dimensional multi-layer structure.

<table>
<thead>
<tr>
<th>Time From Table 2</th>
<th>Admittance</th>
<th>Transmittance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_A$ (W/m²K)</td>
<td>$\phi_A$ (°)</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.231</td>
<td>-180.0</td>
</tr>
<tr>
<td>1 y</td>
<td>0.231</td>
<td>-179.5</td>
</tr>
<tr>
<td>1 m</td>
<td>0.233</td>
<td>-173.4</td>
</tr>
<tr>
<td>1 w</td>
<td>0.260</td>
<td>-153.7</td>
</tr>
<tr>
<td>1 d</td>
<td>0.861</td>
<td>-112.9</td>
</tr>
<tr>
<td>12 h</td>
<td>1.632</td>
<td>-112.6</td>
</tr>
<tr>
<td>3 h</td>
<td>4.597</td>
<td>-137.2</td>
</tr>
</tbody>
</table>

Fig. 3. Amplitude and phase lag of admittance and transmittance for the one dimensional multi-layer structure. The amplitude is given as the ratio between the current amplitude and the steady state amplitude.

For the current thermal bridge structure, the dynamic results like amplitude and phase lag of admittance and transmittance are listed in Table 4 and plotted in Fig. 4.

Table 4
The results of amplitude and phase lag of admittance and transmittance for the current structure.

<table>
<thead>
<tr>
<th>Time From Table 2</th>
<th>Admittance</th>
<th>Transmittance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_A$ (W/m²K)</td>
<td>$\phi_A$ (°)</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.353</td>
<td>-180.0</td>
</tr>
<tr>
<td>1 y</td>
<td>0.353</td>
<td>-179.6</td>
</tr>
<tr>
<td>1 m</td>
<td>0.354</td>
<td>-175.7</td>
</tr>
<tr>
<td>1 w</td>
<td>0.374</td>
<td>-162.1</td>
</tr>
<tr>
<td>1 d</td>
<td>0.915</td>
<td>-120.2</td>
</tr>
<tr>
<td>12 h</td>
<td>1.677</td>
<td>-116.9</td>
</tr>
<tr>
<td>3 h</td>
<td>4.595</td>
<td>-138.9</td>
</tr>
</tbody>
</table>

Fig. 4. Amplitude and phase lag of admittance and transmittance for the current thermal bridge structure. The amplitude is given as the ratio between the current amplitude and the steady state amplitude.

Several conclusions can be drawn from the results for the structures:
(a) the steady state results are obviously different for the two structures. The thermal transmittance (the so called U-value) for the one dimensional multi-layer structure is 0.231 W/m²K which is the result of amplitudes of admittance or transmittance under steady state. For the current thermal bridge structure with a metal stud, the total thermal transmittance is 0.353 W/m²K which is about 53% more heat loss than the former structure.
(b) the steady state results for the interior heat flow can approximately be reached for a period of one day for outdoor air oscillation. This leads to some useful information for the preparation and analysis of measurements.
(c) the amplitudes of admittance and transmittance become gradually equal with increasing time period (i.e. with decreasing frequency), and finally reach the steady state result. With decreasing time period, the amplitude of admittance has a larger magnitude than the amplitude of transmittance. Thus, the admittance dominates the heat flow through internal surface if the amplitudes of temperatures at internal and external sides are the same at same frequency.
(d) if this construction is exposed to an external diurnal temperature variation, the internal heat flow will have a maximum. The amplitude reduction compared to steady state will be insignificant and the delay of maximum heat flow at the interior surface will be a little more than an hour.
(e) the dynamic processes of the two structures are quite similar which can be seen from the dimensionless curves of amplitude and the curves of phase lags in Fig. 3-4. Under high frequency, the amplitudes of admittances are very close which means that thermal bridge does not affect the admittance significantly.
3.2 Heavy weight wall - intermediate floor junction

A typical floor junction used in Swedish multi-family houses is shown in Fig. 5. The material properties used in the calculation are listed in Table 5.

To perform the dynamic calculation of the thermal bridge, three special assumptions have been made:

(a) cut-off planes are located at a distance about

\[ 2\sqrt{d_{\text{concrete}} \cdot \frac{\lambda_{\text{concrete}}}{R_{\text{SI}}}} \text{ [17]} \]

apart from the floor. Cut-off planes are adiabatic.

(b) the variations of temperatures inside the two rooms are assumed the same.

(c) the external surface resistance (R_e) is 0.04 m²K/W, and the internal surface resistances (R_i) for the two rooms are the same value of 0.13 m²K/W.

![Fig. 5. Schematic drawing of the calculated floor junction. The cover layer over the insulation layer on the floor is not shown in the drawing, which is not included in the calculation. All the dimensions are in mm, and the labels of a-d are materials listed in Table 5.](image)

---

### Table 5

<table>
<thead>
<tr>
<th>Material</th>
<th>( \lambda ) (W/mK)</th>
<th>( \rho ) (kg/m³)</th>
<th>( c ) (J/kgK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. concrete</td>
<td>1.7</td>
<td>2300</td>
<td>900</td>
</tr>
<tr>
<td>b. sealant</td>
<td>1.0</td>
<td>1800</td>
<td>950</td>
</tr>
<tr>
<td>c. insulation</td>
<td>0.04</td>
<td>50</td>
<td>800</td>
</tr>
<tr>
<td>d. insulation</td>
<td>0.06</td>
<td>50</td>
<td>800</td>
</tr>
</tbody>
</table>

Two structures are calculated by using the same PC program. One structure is the current floor junction. Another structure is one dimensional multi-layer external wall which consists of 120 mm concrete (inside layer), 150 mm insulation (middle layer) and 70 mm concrete (outside layer). With the same material properties given in Table 5 and the same boundary conditions, the results for the heat flow through the 1-D multi-layer external wall are listed in Table 6 and plotted in Fig. 6. While, the calculated results for the heat flow through the floor junction are given in Table 7 and plotted in Fig. 7.

### Table 6

The results of amplitude and phase lag of admittance and transmittance for the 1-D multi-layer structure.

<table>
<thead>
<tr>
<th>From</th>
<th>Amplitude</th>
<th>Phase lag</th>
<th>Amplitude</th>
<th>Phase lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \gamma_a ) (W/mK)</td>
<td>( \phi_a ) (°)</td>
<td>( \gamma_T ) (W/mK)</td>
<td>( \phi_T ) (°)</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.211</td>
<td>-180.0</td>
<td>0.211</td>
<td>0.0</td>
</tr>
<tr>
<td>1 y</td>
<td>0.216</td>
<td>-169.5</td>
<td>0.211</td>
<td>-0.6</td>
</tr>
<tr>
<td>1 m</td>
<td>0.536</td>
<td>-118.2</td>
<td>0.210</td>
<td>-7.2</td>
</tr>
<tr>
<td>1 w</td>
<td>1.986</td>
<td>-116.6</td>
<td>0.197</td>
<td>-30.1</td>
</tr>
<tr>
<td>1 d</td>
<td>5.179</td>
<td>-159.6</td>
<td>0.062</td>
<td>-130.1</td>
</tr>
<tr>
<td>12 h</td>
<td>5.454</td>
<td>-168.4</td>
<td>0.023</td>
<td>-9.1</td>
</tr>
<tr>
<td>3 h</td>
<td>5.809</td>
<td>-174.0</td>
<td>0.001</td>
<td>-15.8</td>
</tr>
</tbody>
</table>

![Fig. 6. Amplitude and phase lag of admittance and transmittance for the one dimensional multi-layer structure. The amplitude is given as the ratio between the current amplitude and the steady state amplitude.](image)
Table 7: The results of amplitude and phase lag of admittance and transmittance for the current structure.

<table>
<thead>
<tr>
<th>Time</th>
<th>Amplitude $\gamma_A$ (W/mK)</th>
<th>Phase lag $\phi_A$ ($)</th>
<th>Amplitude $\gamma_T$ (W/mK)</th>
<th>Phase lag $\phi_T$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t=\infty$</td>
<td>0.246</td>
<td>-180.0</td>
<td>0.246</td>
<td>0.0</td>
</tr>
<tr>
<td>1 y</td>
<td>0.253</td>
<td>-167.0</td>
<td>0.246</td>
<td>-0.8</td>
</tr>
<tr>
<td>1 m</td>
<td>0.754</td>
<td>-115.5</td>
<td>0.244</td>
<td>-9.5</td>
</tr>
<tr>
<td>1 w</td>
<td>2.757</td>
<td>-120.7</td>
<td>0.217</td>
<td>-38.6</td>
</tr>
<tr>
<td>1 d</td>
<td>6.080</td>
<td>-161.0</td>
<td>0.047</td>
<td>-143.8</td>
</tr>
<tr>
<td>12 h</td>
<td>6.523</td>
<td>-167.7</td>
<td>0.016</td>
<td>-16.2</td>
</tr>
<tr>
<td>3 h</td>
<td>7.140</td>
<td>-173.5</td>
<td>0.001</td>
<td>-11.0</td>
</tr>
</tbody>
</table>

Fig. 7. Amplitude and phase lag of admittance and transmittance for the current thermal bridge structure. The amplitude is given as the ratio between the current amplitude and the steady state amplitude.

Several conclusions can be drawn from the results of heat flows through the heavy weight wall - intermediate floor junction and the 1-D multi-layer structure:

(a) the steady state results are different for the two structures. The heat flow through the one dimensional multi-layer structure is 0.211 W/mK which is the results of amplitudes of admittance or transmittance under steady state (see Table 6). For the current thermal bridge structure of floor junction, the heat flow is 0.246 W/mK which is about 20% more heat loss than the 1-D structure.

(b) the steady state results for the interior heat flow can approximately be reached for a period of one month for outdoor air oscillation. This is the main difference on dynamic thermal performance between light weight structure and heavy weight structure, and this gives some useful information for the preparation and analysis of measurements.

(c) if this construction is exposed to an external diurnal temperature variation, the amplitude reduction of the internal heat flow compared to steady state will be significant and the delay of maximum heat flow at the interior surface will be about ten hours.

(d) the dynamic processes of the two structures are quite similar which can be seen from the dimensionless curves of amplitude and the curves of phase lags for both admittance and transmittance. Under high frequency, the amplitudes of transmittances are very close which means that thermal bridge does not affect the transmittance significantly.

3.3 Ground coupling

Heat loss to the ground from a building can reach about 30% of the total heat loss due to transmission in a cold climate [4]. Ground coupling plays also an important roll in buildings with low mass but well insulated walls [18], where the floor and ground underneath are the only thermal mass available for heat storage.

Ground heat loss is a two or three dimensional dynamic heat transfer problem that can be calculated by numerical methods or by simplified methods based upon analytical solution for simple configurations. Three dimensional steady state numerical method has been proposed as an international standard [1] to calculate floor surface temperatures and heat loss. An explicit simplified calculation method for the steady state and annual periodic variations of ground heat loss is given in another proposal of international standard [19]. More information on ground heat loss, especially on analytical or semi-analytical methods, can be found in [20-25].

In this paper, a part of research results on ground heat loss from a report [26] is presented to demonstrate application of the frequency response method.

3.3.1 Ground coupling - the configuration

The configuration of the ground coupling is based on the benchmark test case of IEA Task 8 [27], i.e. the shoebox case. The length of the shoebox has been extended to infinite long to get a two dimensional reference case (see Fig. 8).
3.3.2 Dynamic calculation of the ground coupling

The material properties of the ground coupling are listed in Table 8. The internal surface resistance is 0.13 m²K/W and the external surface resistance is 0.04 m²K/W. The series of frequencies used in the calculations and the corresponding period length in the time domain are given in Table 9, which has more low frequencies due to ground. The calculated results are listed in Table 10 and plotted in Fig. 9.

Table 8

Thermal conductivity ($\lambda$), density ($\rho$) and specific heat capacity ($c$) used for the calculations [26].

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda$ (W/mK)</th>
<th>$\rho$ (kg/m³)</th>
<th>$c$ (J/kgK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. timber</td>
<td>0.14</td>
<td>650</td>
<td>1200</td>
</tr>
<tr>
<td>b. insulation</td>
<td>0.04</td>
<td>50</td>
<td>800</td>
</tr>
<tr>
<td>c. earth</td>
<td>1.4</td>
<td>1900</td>
<td>1700</td>
</tr>
</tbody>
</table>

Table 9

The series of frequencies used in the calculations and relations to the time domain in case of ground coupling.

<table>
<thead>
<tr>
<th>Frequency domain $\omega = 2\pi/t$ (rad/s)</th>
<th>Time domain $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(steady state)</td>
</tr>
<tr>
<td>9.96190 $\times 10^{-2}$</td>
<td>20 years</td>
</tr>
<tr>
<td>1.99238 $\times 10^{-3}$</td>
<td>10 years</td>
</tr>
<tr>
<td>3.98476 $\times 10^{-4}$</td>
<td>5 years</td>
</tr>
<tr>
<td>9.96190 $\times 10^{-4}$</td>
<td>2 years</td>
</tr>
<tr>
<td>1.99238 $\times 10^{-5}$</td>
<td>1 year</td>
</tr>
<tr>
<td>2.42407 $\times 10^{-6}$</td>
<td>1 month</td>
</tr>
<tr>
<td>1.03889 $\times 10^{-5}$</td>
<td>1 week</td>
</tr>
<tr>
<td>7.27221 $\times 10^{-5}$</td>
<td>1 day</td>
</tr>
<tr>
<td>1.45444 $\times 10^{-4}$</td>
<td>12 hours</td>
</tr>
<tr>
<td>5.81777 $\times 10^{-4}$</td>
<td>3 hours</td>
</tr>
</tbody>
</table>

Table 10

The results of amplitude and phase lag of admittance and transmittance for the current structure.

<table>
<thead>
<tr>
<th>Time</th>
<th>Admittance</th>
<th>Transmittance</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>$\gamma_A$ (W/m²K)</td>
<td>Phase lag $\phi_A$ (°)</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.266</td>
<td>-180.0</td>
</tr>
<tr>
<td>20 y</td>
<td>0.288</td>
<td>-173.6</td>
</tr>
<tr>
<td>10 y</td>
<td>0.303</td>
<td>-172.2</td>
</tr>
<tr>
<td>5 y</td>
<td>0.322</td>
<td>-169.9</td>
</tr>
<tr>
<td>2 y</td>
<td>0.369</td>
<td>-167.0</td>
</tr>
<tr>
<td>1 y</td>
<td>0.417</td>
<td>-166.9</td>
</tr>
<tr>
<td>1 m</td>
<td>0.548</td>
<td>-171.4</td>
</tr>
<tr>
<td>1 w</td>
<td>0.592</td>
<td>-162.8</td>
</tr>
<tr>
<td>1 d</td>
<td>1.348</td>
<td>-127.6</td>
</tr>
<tr>
<td>12 h</td>
<td>2.325</td>
<td>-128.5</td>
</tr>
<tr>
<td>3 h</td>
<td>4.567</td>
<td>-154.3</td>
</tr>
</tbody>
</table>

Fig. 9. Amplitude and phase lag of admittance and transmittance for the current thermal bridge structure. The curve of amplitude is the ratio between the current amplitude and the amplitude under steady state.

The calculations of the ground coupling problems show that:
(a) steady state heat transfer is not reached even for a period of 20 years.
(b) for annual variation of outside air temperature, the magnitude of heat transfer through the ground is only about 25% of steady state heat transmittance through the ground for the steady state solution.
(c) for daily variation of room air temperature, the magnitude of heat loss to the ground is about 5 times of the steady state admittance. The results give information on the possibility of a daily control strategy. With the extension of this dynamic model including heat source, an electric floor heating system has been simulated and compared with measurements [28].

It should be mentioned that the differences between the current calculated results and the results using the surrounding geometry from the CEN standard [1] are about 1%.

4. Conclusions

In this paper, multi-dimensional dynamic models are set up to predict the thermal performances of thermal bridges using a frequency response method. The thermal bridge is divided into finite meshes where the nodal temperatures are expressed in the form of complex quantities that are related to the amplitude and the phase lag of the variations of the temperature. By considering the heat balance for nodes, the real and imaginary parts of the nodal temperatures can be solved simultaneously. A PC computer program has been developed to calculate the dynamic responses of temperatures and heat flows within thermal bridges, and has been utilised for thermal bridges such as a light weight metal stud wall, a heavy weight wall -intermediate floor junction and a floor slab on the ground.

The advantage of the frequency response method is that it can give a thorough description of the dynamic thermal performance of thermal bridges, and gives a fast solution for the dynamics of the ground coupling problem. This is very useful for the preparation or analysis of measurements considering of the type of structures.

The output of the frequency response method gives the basis for producing simplified equivalent RC networks for thermal bridges to be used in building simulations [9].

Acknowledgement

The support by the Swedish Council for Building Research is gratefully acknowledged.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>admittance of a building component (W/m² K)</td>
</tr>
<tr>
<td>a</td>
<td>thermal diffusivity (m²/s)</td>
</tr>
<tr>
<td>c</td>
<td>specific heat capacity (J/kg K)</td>
</tr>
<tr>
<td>dx</td>
<td>dimension in the x-direction (m)</td>
</tr>
<tr>
<td>dy</td>
<td>dimension in the y-direction (m)</td>
</tr>
<tr>
<td>i</td>
<td>√-1</td>
</tr>
<tr>
<td>Q</td>
<td>density of heat flow rate (W/m²)</td>
</tr>
<tr>
<td>R</td>
<td>thermal resistance of material layer (m² K/W)</td>
</tr>
<tr>
<td>s</td>
<td>conductance between two nodes (W/m K)</td>
</tr>
<tr>
<td>t</td>
<td>transmittance of a building component (W/m² K)</td>
</tr>
<tr>
<td>t</td>
<td>time (s)</td>
</tr>
<tr>
<td>u</td>
<td>real part of the complex quantity (°C)</td>
</tr>
<tr>
<td>v</td>
<td>imaginary part of the complex quantity (°C)</td>
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Greek symbols

<table>
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<th>Definition</th>
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<tbody>
<tr>
<td>Δ</td>
<td>Laplace operator</td>
</tr>
<tr>
<td>φ</td>
<td>phase lag (°)</td>
</tr>
<tr>
<td>γ</td>
<td>amplitude of the response factor (W/m² K)</td>
</tr>
<tr>
<td>λ</td>
<td>thermal conductivity (W/m K)</td>
</tr>
<tr>
<td>θ</td>
<td>Celsius temperature (°C)</td>
</tr>
<tr>
<td>ρ</td>
<td>density (kg/m³)</td>
</tr>
<tr>
<td>ω</td>
<td>angular velocity (=2πf, f is frequency) (rad/s)</td>
</tr>
</tbody>
</table>

Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>A</td>
<td>admittance</td>
</tr>
<tr>
<td>e</td>
<td>external</td>
</tr>
<tr>
<td>i</td>
<td>internal</td>
</tr>
<tr>
<td>j</td>
<td>number of nodes</td>
</tr>
<tr>
<td>s</td>
<td>surface</td>
</tr>
<tr>
<td>T</td>
<td>transmittance</td>
</tr>
</tbody>
</table>

Superscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>~</td>
<td>temperature in form of complex number</td>
</tr>
<tr>
<td>^</td>
<td>amplitude of temperature</td>
</tr>
</tbody>
</table>

References

1004, Lund, Sweden.


Appendix IV
The $\omega$ - RC transform for thermal bridges

Guofeng Mao, Gudni Jóhannesson

Division of Building Technology, Kungliga Tekniska Högskolan, S 100 44 Stockholm, Sweden

Abstract

In this paper a method referred to as the $\omega$ - RC transform is developed to obtain simplified dynamic models for thermal bridges. The resulting model is an equivalent serial coupling of thermal resistances (R) and capacitances (C) of thermal bridges that can be implemented in the same manner as one dimensional constructions into dynamic simulation programs to include the impact of thermal bridges in the energy balance for a whole building system. The process of the RC circuit method is that first a dynamic numerical calculation of temperatures and heat flows for the selected thermal bridge is performed for a range of frequencies in the frequency domain; and then an equivalent one dimensional RC circuit called the $\Pi$-link is optimised to give the same steady state response and the optimum dynamic response. The application of the $\omega$ - RC transform in this paper is exemplified with an insulated steel profile structure, an external wall/intermediate floor junction and a wall foundation with ground coupling. The accuracy of the $\Pi$-link RC circuit is examined in the whole frequency domain. Discussions on the dynamic parameter - the total capacitance and other types of RC circuit links are briefly given.

Keywords: Thermal bridges; Frequency response method; $\omega$ - RC transform; Resistance; Capacitance
1. Introduction

The effect of thermal bridges within 3 examples of new Swedish multi-family buildings was found to increase the predicted total energy use between 2 and 21%, or the transmission heat loss through the building envelope between 5 and 39% [1]. It is also reported in an IEA Task 13 working document that because of the thermal bridges for the relatively well insulated buildings dealt with, the transmission heat loss through the building envelopes increased between 6 and 22%, compared with one dimensional calculations as generally performed by building designers [2]. This means that results, based on 1-D calculation only, are generally not correct enough for prediction of the energy consumption for the whole building system.

In order to implement thermal bridges into building construction and system simulation, simplified expressions for the thermal performances of thermal bridges are required. A method referred to as the $\omega$-RC transform is developed to obtain simplified dynamic models for thermal bridges. The resulting model is an equivalent serial coupling of thermal resistances (R) and capacitances (C) of thermal bridges that can be implemented in the same manner as one dimensional constructions into dynamic simulation programs to include the impact of thermal bridges in the energy consumption of whole building systems. The process of the RC circuit method is that first a dynamic numerical calculation for the selected thermal bridges is performed for a range of frequencies in the frequency domain; and then the results are used to optimise an equivalent one dimensional RC circuit called the $\Pi$-link.

Andersson and Jóhannesson [3] applied the $\omega$-RC transform to two-dimensional heat flow problem using a T-link RC circuit for ground coupling. Mao and Jóhannesson [4] further developed this method for the dynamic thermal bridges using a $\Pi$-link RC circuit.

For implementation of an RC circuit of thermal bridge into building system simulation, Akander et al [5] evaluated how a thermal bridge affects the energy consumption and room air temperature in a room by using a M-link RC circuit.
In this paper, the $\omega$ - RC transform for optimising $\Pi$-link RC circuits of thermal bridges is presented. An insulated steel profile structure, an external wall/intermediate floor junction and a wall foundation with ground coupling are studied by this technique. The results of frequency responses using the optimal resistances (R) and capacitances (C) under frequency domain are compared with original simulated results by finite difference method for the thermal bridges.

The accuracy of the $\Pi$-link RC circuit is examined in the whole frequency domain, a steady state term is introduced as one example on how to utilise this method for practice and discussions on the dynamic parameter - the total capacitance and other types of RC circuit links are briefly given.

2. The $\omega$ - RC transform for thermal bridges

The process of the $\omega$ - RC transform for thermal bridges is in two steps:

*Step 1: The frequency response method* is used for the calculation temperatures and heat flows of thermal bridges in the frequency domain using the finite difference method. The thermal bridge is divided into rectangular elements. The nodal temperatures placed at the centre of the elements are expressed in form of complex quantities that are related to amplitude and phase lag of an oscillation for each frequency. Forming a heat balance for the elements, the amplitude and phase lag of the nodal temperatures can be solved simultaneously. In the frequency domain, the real climate data at the boundaries can be expressed as a superposition of an average value and several periodic oscillations at different frequencies. The dynamic thermal performance of the thermal bridge can then be expressed as a combination of the responses at these frequencies. In this step, two important results for the responses for each frequency are defined as: the admittance and the transmittance.
The Admittance of a building component \((Y)\) is defined as the ratio between the variation in the interior surface heat flow rate and inside air temperature when the outside air temperature equals to zero. A building component means a complete building element (wall, floor, or roof) or a factory made component intended to form part of such elements.

\[
Y = \frac{\delta \Phi}{\delta \theta} \quad \text{(when } \delta \theta_e = 0) \tag{1}
\]

The Transmittance (or Dynamic Thermal Transmittance) of a building component \((T_D)\) is defined as the ratio between the variation in the interior surface heat flow rate and outside air temperature when the inside air temperature equals to zero.

\[
T_D = \frac{\delta \Phi}{\delta \theta_e} \quad \text{(when } \delta \theta_i = 0) \tag{2}
\]

For a given surface, the dynamic heat flow rate \(\delta \Phi\) can be expressed as a sum of the responses to given temperatures \(\delta \theta_i\) and \(\delta \theta_e\) at the boundaries under each frequency \(\omega\), i.e.

\[
\delta \Phi = Y \cdot \delta \theta_i + T_D \cdot \delta \theta_e \tag{3}
\]

where \(Y = r_Y \cdot e^{i \cdot \phi_Y}\), \(T_D = r_{TD} \cdot e^{i \cdot \phi_{TD}}\).

The results of amplitudes and phases for admittance and transmittance are plotted in a Bode-diagram. The frequency response method and applications of the method are given in Mao and Jóhannesson [7].
Step 2: The $\omega$-RC transform for optimising a $\Pi$-link RC circuit for a thermal bridge (see Fig. 1).

Fig. 1. A thermal bridge is represented by a $\Pi$-link RC circuit that also can be used for a one-dimensional building component.

The first Resistance ($R$) and the Capacitance ($C$) for a building component shown in Fig. 1 are determined by the Admittance ($Y$), which gives its real part ($Re$) for the resistance $R$, and its imaginary part ($Im$) for the capacitance $C$. The values of $R$ and $C$ are obtained under $T = 24$ hour period oscillation i.e. $\omega$ as $7.27 \times 10^{-5}$ rad/s. This is because that for building simulation dealing with time-varying conditions, especially the periodically varying condition where the main excitation is of period 24 hour (see also reference [6]).

$$R = -Re\left(\frac{1}{Y}\right); \quad C = \frac{1}{Im\left(\frac{1}{Y}\right) \cdot \omega}$$

(4)

The total Resistance ($R_{tot}$, $R_{st} = -1/Y$ or $R_{st} = 1/T_p$) of a thermal bridge is obtained when the frequency ($\omega$) equals to zero i.e. the steady state solution is reached.
The Resistance \((R')\) and the Capacitance \((C')\) for a building component are mainly determined by the transmittance \((T_D)\), from which \(R'\) is given in eq. (5) and \(C'\) can be obtained from eq. (6):

\[
R' = (R_{ex} - R) / 2
\]

\[
T_D = K - \frac{L \cdot I}{J}; \quad Y = \frac{L}{J}
\]

\[
\begin{bmatrix} I & J \\ K & L \end{bmatrix} = \begin{bmatrix} 1 & -R' \\ 0 & 1 - i\omega C \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & -R' \\ 0 & 1 - i\omega C' \end{bmatrix}
\]

After some mathematical exercises, the capacitance \(C'\) can be given by eq. (7), which is the function of the frequency.

\[
C' = \frac{\text{Re}(T_D) - \text{Re}(Y) + \text{Im}(Y) \cdot \omega \cdot R \cdot C}{\omega \cdot \{\omega \cdot C \cdot R'[1 - R \cdot \text{Re}(Y)] - (R + R') \cdot \text{Im}(Y)\}}
\]

The optimal frequency for calculation of the capacitance \(C'\) varies with type of construction. Generally, the capacitance \(C'\) should be calculated at a higher frequency for light weight constructions, and at a lower frequency for heavy weight constructions. This will be demonstrated after some applications of this technique for different types of structures.

By representing thermal bridges as a series of resistances and capacitances, the thermal bridges can be implemented into the simulation of the whole building system. In this way, the dynamic thermal performances of large systems including thermal bridges can be studied in the time domain.
3. Applications of the $\omega$ - RC transform

3.1 Three types of thermal bridges

An insulated steel profile structure, an external wall/intermediate floor junction and a wall foundation with ground coupling are modelled by the $\omega$ - RC transform technique (see Fig. 2-4). The material properties are given in Table 1-3.

![Diagram of insulated sheet metal structure with C profiles]

Fig. 2. Insulated sheet metal structure with C profiles [7].

Table 1

Thermal conductivity ($\lambda$), density ($\rho$) and specific heat capacity ($c$) used for the insulated steel stud construction [7].

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda$, (W/mK)</th>
<th>$\rho$, (Kg/m³)</th>
<th>$c$, (J/kgK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gypsum (top side)</td>
<td>0.21</td>
<td>800</td>
<td>1050</td>
</tr>
<tr>
<td>Insulation</td>
<td>0.037</td>
<td>20</td>
<td>1030</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>20</td>
<td>7800</td>
<td>460</td>
</tr>
<tr>
<td>Gypsum (bottom side)</td>
<td>0.23</td>
<td>800</td>
<td>1050</td>
</tr>
</tbody>
</table>
Fig. 3. Schematic drawing of a floor junction used for the calculation. All the dimensions are in mm, and the labels of a-d are materials listed in Table 2 [7].

Table 2
Thermal conductivity ($\lambda$), density ($\rho$) and specific heat capacity ($c$) used for the external wall/intermediate floor construction [7].

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda$, (W/mK)</th>
<th>$\rho$, (Kg/m$^3$)</th>
<th>$c$, (J/KgK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. concrete</td>
<td>1.7</td>
<td>2300</td>
<td>900</td>
</tr>
<tr>
<td>b. sealant</td>
<td>1.0</td>
<td>1800</td>
<td>950</td>
</tr>
<tr>
<td>c. insulation</td>
<td>0.04</td>
<td>50</td>
<td>800</td>
</tr>
<tr>
<td>d. insulation (in polythene)</td>
<td>0.06</td>
<td>50</td>
<td>800</td>
</tr>
</tbody>
</table>
Fig. 4. Configuration of the ground coupling and boundary conditions. a-c are materials listed in Table 3 [7].
Table 3

Thermal conductivity ($\lambda$), density ($\rho$) and specific heat capacity ($c$) used for the ground coupling [7].

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda$, (W/m·K)</th>
<th>$\rho$, (kg/m$^3$)</th>
<th>$c$, (J/kg·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. timber</td>
<td>0.14</td>
<td>650</td>
<td>1200</td>
</tr>
<tr>
<td>b. insulation</td>
<td>0.04</td>
<td>50</td>
<td>800</td>
</tr>
<tr>
<td>c. earth</td>
<td>1.4</td>
<td>1900</td>
<td>1700</td>
</tr>
</tbody>
</table>
3.2 The $\omega$ - RC transform for the thermal bridges

3.2.1 Step 1: Results of the frequency response method

The frequency response method [7] is used for calculations of dynamic performance including the steady-state solution for the thermal bridges. The angular frequencies ($\omega$) and the period lengths (T) in the corresponding time domain are listed in Table 4. In principle, the real steady state solution can only be reached under the frequency as zero.

Table 4

The series of frequency domain used in the calculations and relations to time domain.

<table>
<thead>
<tr>
<th>Angular frequency, $\omega$ ($=2\pi/T$) (rad/s)</th>
<th>Period length in the corresponding time domain, T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\infty$ (real steady state)</td>
</tr>
<tr>
<td>$9.96190 \times 10^{-9}$</td>
<td>20 years*</td>
</tr>
<tr>
<td>$1.99238 \times 10^{-8}$</td>
<td>10 years*</td>
</tr>
<tr>
<td>$3.98476 \times 10^{-8}$</td>
<td>5 years*</td>
</tr>
<tr>
<td>$9.96190 \times 10^{-8}$</td>
<td>2 years*</td>
</tr>
<tr>
<td>$1.99238 \times 10^{-7}$</td>
<td>1 year</td>
</tr>
<tr>
<td>$2.42407 \times 10^{-6}$</td>
<td>1 month</td>
</tr>
<tr>
<td>$1.03889 \times 10^{-5}$</td>
<td>1 week</td>
</tr>
<tr>
<td>$7.27221 \times 10^{-5}$</td>
<td>1 day</td>
</tr>
<tr>
<td>$1.45444 \times 10^{-4}$</td>
<td>12 hours</td>
</tr>
<tr>
<td>$5.81777 \times 10^{-4}$</td>
<td>3 hours</td>
</tr>
</tbody>
</table>

*: only for the case of ground coupling
In all of the calculations, the external surface resistance \( (R_{se}) \) is chosen as 0.04 m\(^2\)K/W, and the internal surface resistance \( (R_{in}) \) is 0.13 m\(^2\)K/W. The results like amplitudes and phase lags of the Admittance and the Transmittance for the thermal bridges are listed in Table 5, which is taken from [7]. The results in the first row are the real steady state solutions for the thermal bridges.

Table 5

The results of amplitude \((r)\) and phase lag \((\phi)\) of admittance and transmittance for the insulated steel stud wall, the external wall/intermediate floor junction and the ground coupling [7].

<table>
<thead>
<tr>
<th>Time</th>
<th>Insulated steel stud wall</th>
<th>External wall/floor junction</th>
<th>Ground coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Admittance  Transmittance</td>
<td>Admittance  Transmittance</td>
<td>Admittance  Transmittance</td>
</tr>
<tr>
<td></td>
<td>(r_Y \quad \phi_Y \quad r_{\tau_a} \quad \phi_{\tau_D} )</td>
<td>(r_Y \quad \phi_Y \quad r_{\tau_a} \quad \phi_{\tau_D} )</td>
<td>(r_Y \quad \phi_Y \quad r_{\tau_a} \quad \phi_{\tau_D} )</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.353 -180.0 0.353 0.0</td>
<td>0.246 -180.0 0.246 0.0</td>
<td>0.266 -180.0 0.266 0.0</td>
</tr>
<tr>
<td>20 y</td>
<td>0.288 -173.6 0.209 -22.6</td>
<td>0.303 -172.2 0.169 -27.2</td>
<td>0.322 -169.9 0.139 -31.7</td>
</tr>
<tr>
<td>10 y</td>
<td>0.369 -167.0 0.100 -45.2</td>
<td>0.417 -166.9 0.068 -56.5</td>
<td>0.548 -171.4 0.008 -112.0</td>
</tr>
<tr>
<td>5 y</td>
<td>0.354 -175.7 0.353 -0.6</td>
<td>0.754 -115.5 0.244 -9.5</td>
<td>0.592 -162.8 0.001 -14.7</td>
</tr>
<tr>
<td>2 y</td>
<td>0.374 -162.1 0.353 -2.6</td>
<td>2.757 -120.7 0.217 -38.6</td>
<td>1.348 -127.6 2.6 \times 10^{-3} -71.8</td>
</tr>
<tr>
<td>1 y</td>
<td>0.355 -179.6 0.353 -0.1</td>
<td>0.253 -167.0 0.246 -0.8</td>
<td>0.417 -166.9 0.068 -56.5</td>
</tr>
<tr>
<td>1 m</td>
<td>0.548 -171.4 0.008 -112.0</td>
<td>0.592 -162.8 0.001 -14.7</td>
<td>1.348 -127.6 2.6 \times 10^{-3} -71.8</td>
</tr>
<tr>
<td>1 w</td>
<td>0.915 -120.2 0.348 -18.2</td>
<td>6.080 -161.0 0.047 -143.8</td>
<td>2.325 -128.5 1.6 \times 10^{-3} -132.8</td>
</tr>
<tr>
<td>1 d</td>
<td>1.677 -116.9 0.332 -36.0</td>
<td>6.523 -167.7 0.016 -16.2</td>
<td>4.567 -154.6 9.3 \times 10^{-7} -129.7</td>
</tr>
</tbody>
</table>
3.2.2 Step 2: The \( \omega - RC \) transform for the thermal bridges

The total thermal resistances of the thermal bridges are given in Table 6, which are reciprocal of the results listed in the first row in Table 5.

Table 6
The total thermal resistances of the thermal bridges.

<table>
<thead>
<tr>
<th>Thermal bridges</th>
<th>Calculated surface, ( m^2 )</th>
<th>Total thermal resistance, ( R_{\text{tot}} ) [K/W]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insulated steel stud wall</td>
<td>( 1 \times 0.6 = 0.6 )</td>
<td>2.833</td>
</tr>
<tr>
<td>External wall/intermediate floor</td>
<td>( 1 \times 0.85 = 0.85 )</td>
<td>4.065</td>
</tr>
<tr>
<td>Ground coupling</td>
<td>( 1 \times 6.0 = 6.0 )</td>
<td>3.759</td>
</tr>
</tbody>
</table>

The Resistance (R) and the Capacitance (C) in the \( \Pi \)-link RC circuit are calculated according to eq. (4). The R and C are obtained under \( T = 24 \) hour i.e. 1 day period oscillation. Table 7 shows the calculated R and C values for the thermal bridges.

Table 7
The Resistance (R) and Capacitance (C) in the \( \Pi \)-link RC circuit for the thermal bridges.

<table>
<thead>
<tr>
<th>Thermal bridges</th>
<th>Resistance, ( R ) [K/W]</th>
<th>Capacitance, ( C ) [J/K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insulated steel stud wall</td>
<td>0.550</td>
<td>14588.0</td>
</tr>
<tr>
<td>External wall/intermediate floor</td>
<td>0.156</td>
<td>256800.2</td>
</tr>
<tr>
<td>Ground coupling</td>
<td>0.453</td>
<td>23395.9</td>
</tr>
</tbody>
</table>

The Resistance \( (R') \) in the \( \Pi \)-link RC circuit is calculated by \( (R_{\text{tot}} - R)/2 \). The Capacitance \( (C') \) is calculated according to equation (7). The optimal \( C' \) values for the thermal bridges are listed in Table 8,
in which the optimal $C'$ value for the insulated steel stud wall is obtained under 1 day time period, 1 month time period for the external wall/intermediate floor and 1 year time period for the ground coupling.

Table 8

The optimal Resistance ($R'$) and Capacitance ($C'$) in the Π-link RC circuit for the thermal bridges.

<table>
<thead>
<tr>
<th>Thermal bridges</th>
<th>Resistance, $R'$ [K/W]</th>
<th>Capacitance, $C'$ [J/K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insulated steel stud wall</td>
<td>1.142</td>
<td>1594.0</td>
</tr>
<tr>
<td>External wall/intermediate floor</td>
<td>1.955</td>
<td>75894.6</td>
</tr>
<tr>
<td>Ground coupling</td>
<td>1.653</td>
<td>10693603.3</td>
</tr>
</tbody>
</table>

3.2.3 Comparison with the results from the frequency response method

The results of admittances and transmittances calculated by the Π-link using eq. (6) for the three thermal bridges are compared with the results from the frequency response method in Fig. 5-7 in form of Bode-diagrams.
Fig. 5. Amplitude and phase lag of admittance and transmittance for the insulated steel stud wall. The amplitude is given as the ratio between the current amplitude and the steady state amplitude.

(o: numerical simulated results by the frequency response method; +: results of the RC circuit method).
Fig. 6. Amplitude and phase lag of admittance and transmittance for the external wall/floor junction.

The amplitude is given as the ratio between the current amplitude and the steady state amplitude.

(o: numerical simulated results by the frequency response method; +: results of the RC circuit method).
Fig. 7. Amplitude and phase lag of admittance and transmittance for the ground coupling. The amplitude is given as the ratio between the current amplitude and the steady state amplitude. 

(o: numerical simulated results by the frequency response method; +: results of the RC circuit method).

4. Discussions and Conclusions

4.1 The application of Π-link RC circuit for one dimensional multilayer constructions

The $\omega$ - RC transform technique using Π-link can be used for one dimensional multilayer constructions. In order to make a comparison with the related thermal bridges, an one dimensional multilayer construction without metal stud and an one dimensional multilayer construction without the floor junction but with the same height 0.85 m are used in this paper.
Step 1: Frequency response method. For one dimensional multilayer constructions, analytical solutions for the responses under each frequency are given by Carslaw and Jaeger [8]. The analytical solutions for the two one dimensional constructions are listed in Table 9, which is taken from [7].

Table 9

The results of amplitude (\( r \)) and phase lag (\( \phi \)) of the admittance and the transmittance for the relevant 1 D multilayer wall without metal stud, and the relevant 1 D multilayer external wall without floor junction [7].

<table>
<thead>
<tr>
<th>Time</th>
<th>1 D multilayer wall without steel stud</th>
<th>1 D multilayer wall without floor junction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Admittance</td>
<td>Transmittance</td>
</tr>
<tr>
<td></td>
<td>( r_Y )</td>
<td>( \phi_Y )</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.231</td>
<td>-180.0</td>
</tr>
<tr>
<td>1 y</td>
<td>0.231</td>
<td>-179.5</td>
</tr>
<tr>
<td>1 m</td>
<td>0.233</td>
<td>-173.4</td>
</tr>
<tr>
<td>1 w</td>
<td>0.260</td>
<td>-153.7</td>
</tr>
<tr>
<td>1 d</td>
<td>0.861</td>
<td>-112.9</td>
</tr>
<tr>
<td>12 h</td>
<td>1.632</td>
<td>-112.6</td>
</tr>
<tr>
<td>3 h</td>
<td>4.597</td>
<td>-137.2</td>
</tr>
</tbody>
</table>

Step 2: The \( \omega - RC \) transform using \( \Pi \)-link RC circuit. By repeating the process mentioned above, the total thermal resistances of the 1 D constructions are given in Table 10, which are reciprocal of the results in the first rad in Table 9. The Resistance (R) and the Capacitance (C) in the \( \Pi \)-link RC circuit
are calculated according to eq. (4). The R and C are obtained under \( T = 24 \) hour i.e. 1 day period oscillation (see Table 10). The \textit{Resistance (R')} in the \Pi-link RC circuit is calculated by \( (R_{\text{tot}} - R)/2 \). The \textit{Capacitance (C')} is calculated according to equation (7). The optimal C' values for the 1 D multilayer walls are also listed in Table 10, in which the optimal C' value for the 1 D multilayer wall without metal stud is obtained under 1 day time period and 1 week time period for the external wall without intermediate floor junction.

Table 10

The total thermal resistance (\( R_{\text{tot}} \)), the resistances (R and R') and the capacitances (C and C') for the relevant 1 D multilayer constructions.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 D multilayer wall without a metal stud</td>
<td>4.329</td>
<td>0.452</td>
<td>1.939</td>
<td>12852.6</td>
<td>746.2</td>
</tr>
<tr>
<td>1 D multilayer wall without the floor junction</td>
<td>4.739</td>
<td>0.181</td>
<td>2.279</td>
<td>204308.8</td>
<td>3639.6</td>
</tr>
</tbody>
</table>

Comparison with the results from the frequency response method: The results of admittances and transmittances calculated by the \Pi-link using eq. (6) for the 1 D multilayer constructions are compared with the results from the frequency response method in Fig. 8-9 in form of Bode-diagrams.
Fig. 8. Amplitude and phase lag of admittance and transmittance for the relevant 1 D multilayer construction without metal stud. The amplitude is given as the ratio between the current amplitude and the steady state amplitude.

(o: analytical results by the frequency response method; +: results of the RC circuit method).
Fig. 9. Amplitude and phase lag of admittance and transmittance for the relevant 1 D multilayer external wall without floor junction. The amplitude is given as the ratio between the current amplitude and the steady state amplitude.

(o: analytical results by the frequency response method; +: results of the RC circuit method).

4.2 Discussion on the optimal parameter $C'$

As mentioned in the part of the $\omega$ - RC transform for thermal bridges, the capacitance $C'$ is a function of the angular frequency $\omega$ which can be calculated from eq. (7). The best value of $C'$ is obtained with the consideration of the deviations of the results on the admittances and transmittances between frequency response method and the II-link RC circuit using the $\omega$ - RC transform technique. When using the frequency response method as the first step, analytical solutions on admittances and transmittances are available for one dimensional multilayer structures as given in the part of 4.1, and
numerical solutions on admittances and transmittances are solved for thermal bridges as shown in this paper. If the superscript \( F \) stands for the results of admittances and transmittances by the frequency response method and the superscript \( N \) stands for the results of admittances and transmittances by the \( \Pi \)-link using the \( w \)-RC transform, the relative deviations between frequency response method and the \( \Pi \)-link using the \( \omega \)-RC transform technique are given in eq. (8), which are also the function of frequency (see also Akander [9]).

\[
\varepsilon_Y = \left| \frac{Y^F - Y^N}{Y^F} \right| \quad \text{(for Admittance)} \quad \text{and} \quad \varepsilon_T = \left| \frac{T_D^F - T_D^N}{T_D^F} \right| \quad \text{(for Transmittance)} \tag{8}
\]

The deviations under each frequency for the external wall/intermediate floor junction and the relevant 1 D multilayer wall without floor junction are demonstrated in Figure 10-11. From the results of the deviations, the optimal capacitances \( C' \) are chosen for the thermal bridge for the time period 1 month and for the relevant 1 D multilayer external wall for the time period 1 week.
Fig. 10. The deviations of transmittances and admittances are the functions of the capacitance $C'$ and the frequency. The thicker line is the results of the optimal capacitance $C'$ that will be chosen. This figure stands for the case of an external wall/intermediate floor junction.

Fig. 11. The deviations of transmittances and admittances are the functions of the capacitance $C'$ and the frequency. The thicker line is the results of the optimal capacitance $C'$ that will be chosen. This figure stands for the case of an one dimensional multilayer external wall without floor junction.

Same procedure can be applied for the other building components, the deviations of admittances and transmittances between frequency response method and the II-link RC circuit using the $\omega$ - RC
transform for all of the components are summarised in Table 11 related to the best capacitances ($C'$) under each frequency, and plotted in Figure 12-14.

Table 11

The deviations of admittances and transmittances related to the optimal capacitances $C'$ under each frequency for the building components used in this paper.

<table>
<thead>
<tr>
<th>Time</th>
<th>1 D external wall no floor junction</th>
<th>external wall/floor junction</th>
<th>1 D wall no steel profile</th>
<th>insulated steel profile wall</th>
<th>ground coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_1$</td>
<td>$\varepsilon_\tau_1$</td>
<td>$\varepsilon_1$</td>
<td>$\varepsilon_\tau_1$</td>
<td>$\varepsilon_1$</td>
</tr>
<tr>
<td>20 y</td>
<td>0.007</td>
<td>0.002</td>
<td>0.025</td>
<td>0.009</td>
<td>0.0003</td>
</tr>
<tr>
<td>10 y</td>
<td>0.086</td>
<td>0.645</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 y</td>
<td>0.086</td>
<td>0.812</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 y</td>
<td>0.067</td>
<td>0.890</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 y</td>
<td>0.065</td>
<td>0.780</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 m</td>
<td>0.029</td>
<td>0.030</td>
<td>0.095</td>
<td>0.112</td>
<td>0.005</td>
</tr>
<tr>
<td>1 w</td>
<td>0.028</td>
<td>0.128</td>
<td>0.107</td>
<td>0.381</td>
<td>0.030</td>
</tr>
<tr>
<td>1 d</td>
<td>0.006</td>
<td>0.839</td>
<td>0.009</td>
<td>0.683</td>
<td>0.231</td>
</tr>
<tr>
<td>12 h</td>
<td>0.020</td>
<td>1.936</td>
<td>0.054</td>
<td>1.219</td>
<td>0.396</td>
</tr>
<tr>
<td>3 h</td>
<td>0.075</td>
<td>4.798</td>
<td>0.120</td>
<td>1.292</td>
<td>0.626</td>
</tr>
<tr>
<td>optimal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C'$ under T = 1 week</td>
<td>3639.6 [Ws/mK]</td>
<td>75894.6 [Ws/mK]</td>
<td>746.2 [Ws/m²K]</td>
<td>1594.0 [Ws/m²K]</td>
<td>1.069 x 10^7 [Ws/m²K]</td>
</tr>
<tr>
<td>$C'$ under T = 1 month</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C'$ under T = 1 day</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C'$ under T = 1 day</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 12. The deviations of admittances and transmittances between the frequency response method and the Π-link RC circuit using the $\omega$ - RC transform. The cases shown in this figure are: an external wall/intermediate floor junction (the lines with '+' ) and the relevant 1 D multilayer wall without floor junction (the lines with 'o').

Fig. 13. The deviations of admittances and transmittances between the frequency response method and the Π-link RC circuit using the $\omega$ - RC transform. The cases shown in this figure are: an insulated steel stud wall (the lines with '+') and the relevant 1 D multilayer wall without steel stud (the lines with 'o').
Fig. 14. Deviations of admittances and transmittances between frequency response method and the Π-link RC circuit using the ω - RC transform. The case shown in this figure is: a foundation with ground coupling.

The results show that the deviations of transmittances are generally larger than the deviations of admittances especially under shorter time periods i.e. under higher frequencies. One thing that should be kept in mind is that the magnitude of the amplitude of transmittance is much lower than the magnitude of the amplitude of admittance. Therefore, the admittances dominate the process of heat transfer under higher frequencies which are calculated with reasonable accuracy by the Π-link using the ω - RC transform under the whole frequency domain. Under lower frequencies (i.e. longer time periods), the deviations of transmittances are at the same level as the deviations of the admittances except in the extreme case like the ground coupling. In the case of ground coupling, the admittance dominates the process of heat transfer under the whole frequency domain that leads to the requirement on higher accuracy for the admittance. The Π-link using the ω - RC transform for the ground coupling can meet the requirement (as shown in Figure 14). When comparing with the accuracy for the other building components, the admittances for the ground coupling calculated by the Π-link has less accurate values under lower frequencies.
4.3 Discussion on the total capacitance

The capacitances in the Π-link RC circuit in this paper are parameters for dynamic modelling of the building components. They are the functions of frequency after the details of building components are known. The optimal capacitances are solved to reach the minimised deviation under the whole frequency domain. The total capacitance (i.e. $C + C'$) in the Π-link is not required to equal to the heat capacity of a building component which is normally calculated by sum of the production of specific heat capacity with the dimension for each material used in the building component. The total capacitances and the heat capacities for the 1 D multilayer building components used in the paper are given in Table 12.

Table 12
Comparison between the total capacitance ($C + C'$) in the Π-Link and the heat capacity for 1 D multilayer building components.

<table>
<thead>
<tr>
<th>Building component</th>
<th>The total capacitance ($C + C'$) for the model</th>
<th>The heat capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 D multilayer wall without steel stud</td>
<td>13598.8 [J/K]</td>
<td>19510.0 [J/K]</td>
</tr>
<tr>
<td>1 D multilayer wall without floor junction</td>
<td>207948.4 [J/K]</td>
<td>339405.0 [J/K]</td>
</tr>
</tbody>
</table>

This inequality between the total capacitance and the heat capacity is also confirmed by Davies [6]. The condition on the equality between a 1-D continuum and its equivalent discrete presentation with the thermal resistance and capacitance ($R$ and $C$) in a Π-link circuit is described in the appendix, and examples of some building materials are given.

4.4 Brief discussion on other kinds of RC circuits using the $\omega$ - RC transform
There are other possible kinds of RC circuits like the T-link, the M-link and so on for simplifying the dynamic performance for the same building component. The Resistances and Capacitances in more complex RC circuits can also be determined by using the $\omega$-RC transform technique. In general, with an increased number of nodes of a RC circuit, a better accuracy for the whole frequency range can be obtained.

In reality, thermal bridges exist more or less in every building. For practical applications, by implementing the Π-link circuit for a thermal bridge into the 1 D construction model for a normal building, result on dynamic thermal performances of whole building construction can significantly be improved, if compared with the result of that only 1 D construction model is used for the building. In order to examine the accuracy on the dynamic thermal performance of the whole building construction, a prerequisite is that the 1 D construction model should be examined firstly. More details on 1 D model using different types of RC-link can be found in [9].

4.5 Conclusions

In this paper a method using $\omega$ - RC transform is developed obtain model parameters for thermal bridges. The purpose of this method is to establish an equivalent linear model with thermal resistance (R) and capacitance (C) for thermal bridges to be implemented into one dimensional dynamic simulation programs for buildings which can correctly model the transmission loss including thermal bridges. The process of the method is that: to perform dynamic numerical calculation for selected thermal bridge under frequency domain; to set up an equivalent one dimensional RC circuit called Π-link optimised by using $\omega$ - RC transform. The applications of the RC circuit method in this paper are an insulated steel stud wall, an external wall - intermediate floor junction and a wall foundation with ground coupling and the relevant one dimensional multilayer structures.
The optimisation on the capacitance $C'$ in the $\Pi$-link due to the transmittance is carried out under the whole frequency domain, by minimising the deviation on the admittance and transmittance between the results from frequency response method and the $\Pi$-link RC circuit using the $\omega$ - RC transform.

The total capacitance in the $\Pi$-link circuit for 1 D multilayer component is compared with the heat capacity normally used for a continuum. The condition on the equality between the desecrate dynamic parameter and the relevant continuum is given in the part of appendix, and can be applied for dimensioning building components for some building materials.

Different kinds of RC circuits can be set up to meet the different requirements of the building system by using the $\omega$ - RC transform technique. For practical applications, by implementing the $\Pi$-link circuit for a thermal bridge into the 1 D construction model for a normal building, result on dynamic thermal performances of whole building construction can significantly be improved, if compared with the result of that only 1 D construction model is used for the building. In order to examine the accuracy on the dynamic thermal performance of the whole building construction, a prerequisite is that the 1 D construction model should be examined firstly.

The optimal parameters obtained in the $\Pi$-link for the thermal bridges can be implemented into building simulation codes when thermal bridges are taken into account or can be used for analysing the measured results for thermal bridges in real time series.

Acknowledgement

The support by the Swedish Council for Building Research is gratefully acknowledged.

Nomenclature

$C$ capacitance of a building component (J/K)
c  specific heat capacity (J/kgK)
I  substitutive complex variable used in eq. (6)
Im imaginary part of complex number
i $\sqrt{-1}$
J substitutive complex variable used in eq. (6)
K substitutive complex variable used in eq. (6)
k coefficient used in the appendix (1/m)
L substitutive variable used in eq. (6), or the thickness for a 1 D continuum in the appendix
R resistance of a building component (K/W)
Re real part of complex number
r amplitude of admittance or transmittance (W/K)
T time period corresponding to each frequency (s)
$T_d$ transmittance of a building component (W/K)
t time (s)
Y admittance of a building component (W/K)

Greek symbols

$\varepsilon$ relative deviation
$\Phi$ heat flow rate (W)
$\phi$ phase lag (°)
$\lambda$ thermal conductivity (W/mK)
$\theta$ Celsius temperature (°C)
$\rho$ density (kg/m³)
\( \omega \) angular frequency \((=2\pi/T)\) (rad/s)

**Subscripts**

D dynamic
e external
i internal
se external surface
si internal surface
\( T_D \) transmittance
tot total resistance
Y admittance
\( = \) real steady state

**Superscripts**

C analytical solution for a 1 D continuum in the appendix
F frequency response method
N \( \Pi \)-link RC circuit method
T-link solution obtained from a T-link circuit in the appendix
- temperature or density of heat flow rate in form of complex number, dynamic variable
' dynamic parameter for a part of Resistance and Capacitance
References


Appendix: The relation between a one dimensional continuum and its equivalent discrete presentation of thermal resistance and capacitance

Part I: The condition on equality between the heat capacity and the total capacitance in a Π-link circuit

A one dimensional continuum and its equivalent discrete presentation of thermal resistance and capacitance with a Π-link circuit are shown in Fig. A1. With consideration of unit of area perpendicular to the paper, the heat capacity of the 1-D continuum with the thickness L is calculated as: \( \text{L \rho c} \) in [J/K]. The total thermal capacitance \( (C + C') \) in the Π-link circuit is the function of frequencies when the dimension and material properties are determined. Analytical solution of the total capacitance is given as [8]:

\[
C + C' = \frac{\lambda \cdot k \cdot [\cosh(kL) \cdot \sin(kL) + \sinh(kL) \cdot \cos(kL)]}{\omega}
\]  

(A.1)

where

\[
k = \sqrt{\frac{\omega \cdot \rho \cdot c}{2 \cdot \lambda}}
\]
Fig. A1. 1-D continuum (a) and its equivalent discrete presentation of thermal resistances (R and R') and capacitances (C and C') in a Π-link circuit (b).

The \( \cosh(kL) \), \( \sinh(kL) \), \( \cos(kL) \) and \( \sin(kL) \) can also be expressed as:

\[
\cosh(kL) = 1 + \frac{(kL)^2}{2!} + \frac{(kL)^4}{4!} + \ldots \quad (kL < \infty)
\]

\[
\sinh(kL) = kL + \frac{(kL)^3}{3!} + \frac{(kL)^5}{5!} + \ldots \quad (kL < \infty)
\]

\[
\cos(kL) = 1 - \frac{(kL)^2}{2!} + \frac{(kL)^4}{4!} - \ldots \quad (kL < \infty)
\]

\[
\sin(kL) = kL - \frac{(kL)^3}{3!} + \frac{(kL)^5}{5!} - \ldots \quad (kL < \infty)
\]

When \( kL < 0.182 \), we have the simple solutions of all of the above functions with the maximum absolute error less than 0.001. The ratio between the total capacitance and the heat capacity is given in Fig. A.2 to show this condition.

\[
\cosh(kL) = 1 + \frac{(kL)^2}{2!}
\]
\[ \sinh(kL) = kL \]
\[ \cos(kL) = 1 - \frac{(kL)^2}{2!} \]
\[ \sin(kL) = kL \]  \hspace{1cm} (A.3)

\[ \frac{(C + C')}{L \rho c} \]

Fig. A2. The ratio between the total capacitance and the heat capacity as a function of \( kL \).

The equation of (A.1) for calculation of the total capacitance can be rewritten as:

\[ C + C' = \frac{\lambda \cdot k}{\omega} \left[ 1 + \frac{(kL)^2}{2} \right] \cdot (kL) + \left[ 1 - \frac{(kL)^2}{2} \right] \cdot (kL) \]
\[ = 2 \frac{\lambda \cdot L}{\omega} \cdot k^2 = \frac{2 \lambda \cdot L}{\omega} \cdot \frac{\omega \rho c}{2\lambda} = L \rho c \]  \hspace{1cm} (A.4)

This equals to the heat capacity for a 1-D continuum. The time period \( T (\omega = 2\pi/T) \) for the equality can be easily derived from \( kL < 0.182 \) as:

\[ T > 95 \frac{\rho \cdot c}{\lambda} \cdot L^2 \]  \hspace{1cm} (A.5)
The application of the eq. (A.5) for some building materials is listed in Table A.1.

### Table A.1

The minimum time period $T_{\text{min}}$ for the equality for some building materials for dimension.

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Insulation</th>
<th>Gypsum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda=1.7 \text{ W/mK}, \rho=2300 \text{ kg/m}^3, c=900 \text{ J/kgK}$</td>
<td>$\lambda=0.037 \text{ W/mK}, \rho=20 \text{ kg/m}^3, c=1030 \text{ J/kgK}$</td>
<td>$\lambda=0.23 \text{ W/mK}, \rho=800 \text{ kg/m}^3, c=1050 \text{ J/kgK}$</td>
</tr>
<tr>
<td>L (m)</td>
<td>$T_{\text{min}}$ (days)</td>
<td>L (m)</td>
</tr>
<tr>
<td>0.05</td>
<td>3.5</td>
<td>0.05</td>
</tr>
<tr>
<td>0.10</td>
<td>14</td>
<td>0.10</td>
</tr>
<tr>
<td>0.15</td>
<td>30.</td>
<td>0.15</td>
</tr>
<tr>
<td>0.25</td>
<td>54</td>
<td>0.20</td>
</tr>
<tr>
<td>0.25</td>
<td>84</td>
<td>0.25</td>
</tr>
<tr>
<td>0.30</td>
<td>120</td>
<td>0.30</td>
</tr>
</tbody>
</table>

---

Part 2: The relation between analytical solution on transmittance for a 1D continuum and the transmittance representing by relevant discrete thermal resistance and capacitance in a T-link circuit

Analytical solution on transmittance ($T_D^C$) for a 1D continuum is given in eq. (A.6), the superscript $c$ stands for the continuum.

$$T_D^C = \frac{kL(1+i)}{R_{\text{tot}} \cdot \sinh[kL(1+i)]}$$ (A.6)

where

$$R_{\text{tot}} = \frac{L}{\lambda}$$
In order to demonstrate the $\omega \cdot RC$ transform method in a easy way, the transmittance ($T_{D}^{\text{link}}$) representing by the relevant discrete thermal resistance and capacitance in a T-link circuit (see Fig. A3) is given as:

$$T_{D}^{\text{link}} = \frac{1}{R_{\text{tot}} + i \cdot \omega \cdot C \cdot R \cdot R'}$$  \hspace{1cm} (A.7)

where

$$R + R' = R_{\text{tot}}$$

Obviously, the term of $\omega \cdot C \cdot R \cdot R'$ in the T-link circuit is only the function of frequency when the material properties and dimension of the 1 D continuum are known, and can be expressed as:

$$\omega \cdot C \cdot R \cdot R' = \frac{R_{\text{tot}}}{2} \cdot \left[ \frac{\cosh(kL) \cdot \sin(kL) - \sinh(kL) \cdot \cos(kL)}{kL} \right] = \text{Im}\left(\frac{1}{T_{D}^{\text{c}}}\right)$$  \hspace{1cm} (A.8)

Similarly, the real part of $\frac{1}{T_{D}^{\text{c}}}$ is written as:

$$\text{Re}\left(\frac{1}{T_{D}^{\text{c}}}\right) = \frac{R_{\text{tot}}}{2} \cdot \left[ \frac{\sinh(kL) \cdot \cos(kL) + \cosh(kL) \cdot \sin(kL)}{kL} \right]$$  \hspace{1cm} (A.9)
Fig. A3. 1-D continuum (a) and its equivalent discrete presentation of thermal resistance (R and R') and capacitance (C) in a T-link circuit (b) for demonstration of the ω - RC transform method in an easy way.

Therefore, the ratio between $T_D^{T\text{-link}}$ and $T_D^C$ can be written as:

$$\frac{T_D^{T\text{-link}}}{T_D^C} = \frac{1}{1 + T_D^C \left[ R_{\text{tot}} - \text{Re}\left(\frac{1}{T_D^C}\right) \right]} \quad (A.10)$$

After some mathematical exercises, the ratio of the amplitudes between $T_D^{T\text{-link}}$ and $T_D^C$ is obtained as:

$$\frac{T_D^{T\text{-link}}}{T_D^C} = \sqrt{1 + \frac{[\sinh(kL) \cdot \cos(kL) + \cosh(kL) \cdot \sin(kL)]^2 - 4 \cdot (kL)^2}{[\cosh(kL) \cdot \sin(kL) - \sinh(kL) \cdot \cos(kL)]^2 + 4 \cdot (kL)^2}} \quad (A.11)$$

The result of the eq. (A.11) is given in a Bode-diagram (see Fig. A4) as a function of kL.
Fig. A4. A Bode-diagram on the ratio of the amplitudes between $T_{D}^{T\text{-link}}$ and $T_{D}^{C}$ as a function of $kL$.

From Fig. A4, we can see that T-link is not so good when $kL > 3$. Same method can be used for error control for admittance, and for other types of RC link circuits.

Some conclusions can be given as follows:

- The capacitance in the T-link circuit equals to the total capacitance in the II-link circuit.
- Under a critical frequency corresponding to $kL \approx 1$, the transmittance calculated by the T-link circuit approximately equals to the transmittance for the 1 D continuum, and the capacitance in the T-link will approximately be equal to the heat capacity ($Lpc$) for the 1 D continuum.
Appendix V
Laboratory Measurements on Dynamic Thermal
Performance of a Thermal Bridge

Guofeng Mao, Tekn Lic
Division of Building Technology
Department of Building and Building Sciences
Kungliga Tekniska Högskolan
S 100 44 Stockholm, Sweden

Abstract

Laboratory measurement on dynamic thermal performance of a thermal bridge at Division of Building Technology, Kungliga Tekniska Högskolan in Stockholm is presented in this paper. To arrange the measurement and analyse the measured results, it requires dynamic multi-dimensional modelling that can describe the dynamic behaviours of thermal bridges in a simple and accurate way. A network method with a Π-link representing the dynamic parameters for a thermal bridge, thermal resistance and thermal capacitance, is developed to predict, arrange and analyse the measured results of a thermal bridge. The dynamic thermal parameters - Resistance, R and Capacitance, C are derived from a dynamic multi-dimensional modelling under frequency domain using the so called $\omega$ - RC technique. By using a Π-link network related to the thermal bridge i.e. a ‘white box’, the measurement on a thermal bridge can be both measured and the measured results can be analysed in a robust and accurate way. This is demonstrated by analysing the measured thermal performances of an insulated concrete sandwich structure with wooden studs. This study is orientated to combine modelling and measurement to get quick and reasonable measured results for thermal bridges, and this procedure can be also implemented into measurement on thermal bridges in-situ. Discussions on the deviations on the results between the
modelling and the whole measurement are given in the last part of this paper to make a check on the individual part of the whole procedure.

*Keywords:* thermal bridge, laboratory measurement, Π-link network, thermal resistance, thermal capacitance, temperature, heat flow density.

1. **Introduction**

Thermal bridges are parts of building components, where heat loss is increased and internal surface temperature is relatively low. Basically, thermal bridges can be divided into two groups: thermal bridges between the building components and thermal bridges at joints between the building components [1], see Fig. 1. The characteristics of thermal performance of thermal bridges are two or three dimensional, due to non-uniform components or variation of geometry of the structure.

Since 1970's, either calculation methods or measurement methods on thermal performance of thermal bridges have been developed with consideration of the role of thermal bridges on energy consumption and durability of a building construction. But the developments of the two types of methods seem in parallel, especially for dynamic behaviour of thermal bridges.

The normal routine for measuring on thermal bridges whatever laboratory or in-situ measurement was that huge cost was put in the measuring system: large number of sensors were used in order to get 'qualified' measured results. Computational modelling was used after the measurement to analyse the measured results. Sometimes, data were borrowed from measurements were used to verify the modelling or some well known commercial simulation programs were used to analyse the measured results. This leads to the problem that the researcher can not see the clear picture on the content of the modelling or those famous programs. This is more or similar to a pilot flying
an aeroplane with a *black box*. Expert will eventually find the black box is, if a horrible disaster occurs. It is late.

The reason for the parallel developments may be due to the complexity of heat transfer features of a individual thermal bridge and time consumption for handling such problems. It requires dynamic multi-dimensional modelling that can describe the dynamic behaviours of thermal bridges in a simple and accurate way for arranging the measurements and analysing the measured results.

![Thermal bridges diagram](image)

**Fig.1** Thermal bridges, (a)-thermal bridge within building component; (b)-thermal bridge at joint between building component [1].

In this paper, a network method with a Π-link representing the dynamic thermal parameters for a thermal bridge, *Resistance* and *Capacitance*, is developed to analyse the measured results of a thermal bridge. The dynamic parameters - Resistance, \( R \) and Capacitance, \( C \) are derived from a dynamic multi-dimensional modelling under frequency domain using the so called \( \omega \)-RC technique [2-4]. By using a Π-link network related to the thermal bridge, both the arrangement of the measurement and the measured results on heat flows of a thermal bridge can be handled in a robust and accurate way. This is demonstrated by analysing the measured thermal performances.
of a thermal bridge: an insulated concrete sandwich structure with wooden studs. This study is orientated to combine measurement and modelling to get quick and reasonable measured results for thermal bridges, and this procedure can be also implemented into measurement on thermal bridges in-situ [5]. Discussions on the deviations on the results between the modelling and the whole measurement are given in the last part of this paper to make a check on the individual part of the whole procedure.

2. A new methodology for measurements on the dynamic thermal performance of a thermal bridge and an application

2.1 The methodology

In order to measure thermal performance of thermal bridges accurately, some factors, such as the geometry of thermal bridges, convective and radiative heat transfer boundary conditions, non stationary variations of temperatures, ageing effect of material properties, air leakage related to workmanship, organised air movement through the thermal bridge and measuring systems etc., should be taken into account [6]. In general, it is necessary to combine the measurements of temperatures and surface heat flow densities with calculation methods to analyse the measured results. During these years, the capability of PC computer is enhanced so rapidly that researchers in the building and environment areas are gradually getting benefited both on their efficient work and on the inspirations from the world round using information technology. Therefore, efficient and qualified projects are today’s topic where object or project orientated analyse and measurement will be the task for the researchers. In this paper, a new methodology of measurement on dynamic thermal performance of a thermal bridge is shown in Fig. 2.
The methodology is explained as follows:

- *To find* a thermal bridge from the drawings of a building, a *qualified* judgement is required.
- *To calculate* thermal performance of the thermal bridges, a *white box* will be given in advance.
- *To measure* the thermal bridge, a measuring system is *individually* chosen.
- *To make* a comparison with calculated results, the *white box* together with the measuring system are examined. The new improvements for the *white box* and the system are obtained from the measurements.

2.2 The application for measurement on a thermal bridge - The *pre*

processing of the whole measurement

In this paper, results on dynamic variations of temperature and heat flow of a thermal bridge and a comparison with dynamic modelling are presented. This study is carried out in the laboratory at
the Division of Building Technology at Kungliga Tekniska Högskolan in Stockholm in the middle of 1995. The purpose of the study is to find out an effective method for determining dynamic thermal performances of thermal bridges. The principle for this measurement is that to use the measuring equipment as less as possible to get quick and reasonable results if the measurement is planned very well by the aid of a white box. This study is a part of the whole research programme on building physics at the Division.

### 2.2.1 The measured thermal bridge in the laboratory

An insulated concrete sandwich structure with wooden studs is used as the measured object under the whole measuring process in the laboratory, see Fig. 3. The thermal bridge is the planar type of thermal bridge. This type of building component is prefabricated and can be used for partition wall, external wall or floor structure of apartments.

![Diagram of the measured object](image)

**Fig. 3** The measured object in the laboratory - a thermal bridge: an insulated concrete sandwich structure with wooden studs.

### 2.2.2 The network method for dynamic modelling on thermal performance of thermal bridges - a white box

A dynamic modelling on thermal performance of thermal bridges developed at the Division of building technology at KTH is used for comparing with the results of the measurement. An RC
(resistance and capacitance) II-link is used for representing the dynamic thermal performance of a thermal bridge. The dynamic parameters of resistance R and capacitance C are obtained optimally from simulating the thermal bridge under frequency domain, so called $\omega$-RC transform. More detailed information on the network method can be found in [2-4]. The application of the network method on the thermal bridge is demonstrated as follows:

1. **Two-dimensional dynamic calculations under frequency domain for the thermal bridge:**

- **Geometry** of the thermal bridge is shown in Fig. 3.

- **Thermal proprieties** (heat conductivity, heat capacity and density) of the building materials used in the simulation are listed in Table 1.

Table 1. The thermal properties of the building materials used in the simulation.

<table>
<thead>
<tr>
<th>Material</th>
<th>Heat Conductivity $\lambda$ [W/m·K]</th>
<th>Heat Capacity $c$ [J/Kg·K]</th>
<th>Density $\rho$ [kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>1.7</td>
<td>2200</td>
<td>2200</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0.039</td>
<td>1470</td>
<td>40</td>
</tr>
<tr>
<td>Mineral wool</td>
<td>0.036</td>
<td>750</td>
<td>23</td>
</tr>
<tr>
<td>Air cavity</td>
<td>R=1.6 [m$^2$K/W]</td>
<td>1000</td>
<td>1.2</td>
</tr>
<tr>
<td>Wooden stud</td>
<td>0.14</td>
<td>1880</td>
<td>550</td>
</tr>
<tr>
<td>Gypsum board</td>
<td>0.22</td>
<td>840</td>
<td>740</td>
</tr>
</tbody>
</table>

- **Thermal boundary conditions** for energy balance of the dynamic simulation. Standard surface thermal resistances $R_{si}$ as 0.13 $m^2$ K/W and $R_{se}$ as 0.04 $m^2$ K/W are used for the inside and outside surfaces, and the heat flow rates through the symmetrical lines which divide the whole construction into a basic component shown in Fig. 3.
2. *Dynamic results on the total heat flow the thermal bridge - The Admittance and Transmittance are shown in Table 2.*

Table 2. The results of amplitude (r) and phase lag (\(\phi\)) of admittance and transmittance for the thermal bridge. These results are related to surface-to-surface temperature variations.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Frequency (\text{rad/s})</th>
<th>Admittance (r_\gamma) (\phi_\gamma)</th>
<th>Transmittance (r_\tau) (\phi_\tau)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state</td>
<td>0.</td>
<td>0.285 (-180.0)</td>
<td>0.285 (0.0)</td>
</tr>
<tr>
<td>8 days</td>
<td>(9.0903 \times 10^{-6})</td>
<td>0.308 (-159.0)</td>
<td>0.282 (-10.0)</td>
</tr>
<tr>
<td>2 days</td>
<td>(3.6361 \times 10^{-5})</td>
<td>0.538 (-126.2)</td>
<td>0.242 (-36.0)</td>
</tr>
<tr>
<td>24 hours</td>
<td>(7.2722 \times 10^{-5})</td>
<td>0.923 (-115.1)</td>
<td>0.185 (-58.7)</td>
</tr>
<tr>
<td>8 hours</td>
<td>(2.1817 \times 10^{-4})</td>
<td>2.285 (-109.3)</td>
<td>0.096 (-114.0)</td>
</tr>
<tr>
<td>1 hour</td>
<td>(1.7453 \times 10^{-3})</td>
<td>13.549 (-111.8)</td>
<td>0.002 (-44.8)</td>
</tr>
</tbody>
</table>

3. *The white box for the thermal bridge for the dynamic thermal performance using a RC network, see Fig. 4.*
Fig. 4 The *white box* is representing for the dynamic thermal performance for the thermal bridge using a RC network.

This *white box* is set up for arranging the individual measurement on the thermal bridge in an cost-efficient way and also will be used for a comparison with the measured results on the dynamic thermal performances of the thermal bridge.

2.2.3 Description of the climatic simulator

During the measurement process, the thermal bridge is located between at the middle of a climatic simulator, which has a cold room and a warm room, see Fig. 5.

Fig. 5 The climatic simulator used for the whole measuring process in the laboratory.
2.2.4 The thermocouples and heat flow meters

This is the pilot study on the measurement procedure for dynamic thermal performances of thermal bridges, using thermocouples for temperature measurements and heat flow meters for heat flow measurements.

Temperatures on the surfaces of the thermal bridge are measured by a series of thermocouples. The type of thermocouples is T. Heat flow meters are used to measure heat flow through the surfaces of the thermal bridge, and the diameter of these heat flow meters is φ 75 mm.

2.2.5 The data logging system

A DT100 Data Logger is used for collecting the results on temperature and heat flows from the sensors and saving these results in a PC computer file.

3. Measurement processing and the results on the dynamic thermal performances of the thermal bridge

This measurement was carried out in the laboratory at the Division of Building Technology, Kungliga Tekniska Högskolan in Stockholm in the middle of July 1995. The measurement was arranged according to the predicted dynamic calculations under different frequencies, especially under a series of relatively higher frequencies. This arrangement was aimed at obtaining the thermal performance in a quick and relatively accurate way, if compared with traditional measurement procedures. Besides, the method developed here has been used for measuring thermal bridges in-situ as a pilot study [5].
3.1 Calibrations of the thermocouples

Total 16 thermocouples are calibrated using the ice point as the reference junction at the begin of the calibration. By opening the ice bottle, the temperature increases after absorbing heat from the ambient air. The calibrated results are given in Fig. 6, which shows the variation of temperatures under more than 2 days.

Fig. 6 Calibrations of the sixteen thermocouples under the variation of temperature.

3.2 Calibrations of the heat flow meters

A hot plate apparatus is used for calibrating of 4 heat flow meters. Fig. 7 gives the relation between heat flow rates and the temperature difference between two plates using a *hot plate*.
3.3 Measurement on temperatures and heat flow rates of the thermal bridge

The arrangement of thermocouples and heat flow meters is shown in Fig. 8. The distance between horizontal measuring points is 300mm, and the distance between vertical measuring points is 600mm. The time interval of collecting results is 15 minutes.
3.4 The dynamic variations of the temperatures

Fig. 9 gives the results of a period of 3 days’ air temperatures in the cold room and warm room in the climatic simulator. For the measured surface temperatures are under the same time series, namely all of the results are original recordings with the time interval of 15 minutes. The un-continuous points shown in the following figures are due to the artificial control: turn on the system in the morning when the author was coming for the daily research work and turn off the system at afternoon when the author finished the daily work. This strategy used here will be examined in-situ measurement.

![Graph showing temperature variations over time](image)

Fig. 9 Air temperatures at the cold room and warm room in the climatic simulator.

The relevant temperatures at the different points of the surfaces are given in Fig. 10-13.
Fig. 10 Surface temperatures at the warm room in the climatic simulator. Horizontal direction, top position: \( o \) - the middle point; \( + \) - the right point; \( * \) - the left point.

Fig. 11 Surface temperatures at the cold room in the climatic simulator. Horizontal direction, top position:

\( o \) - the middle point, corresponding to the middle point at the warm room;

\( + \) - the left point, corresponding to the right point at the warm room;

\( * \) - the right point, corresponding to the left point at the warm room.

The difference between Fig. 11 and Fig. 12 is that the results on the temperature are in horizontal direction in Fig.11, and the results on the measured temperature are in vertical direction in the following figure.
Fig. 12 Surface temperatures at the warm room in the climatic simulator. Vertical direction, middle position: o - the middle point; + - the top point; * - the bottom point.

Fig. 13 Surface temperatures at the cold room in the climatic simulator. Vertical direction, middle position: o - the middle point; + - the top point; * - the bottom point.
3.5 The measured dynamic performances of the heat flow density

Fig. 14 shows the heat flows through the surfaces at the four points using the 0.75 mm heat flow meters.

Heat flow density, q in W/m²K

![Heat flow density graph]

Time (×15 minutes)

Fig. 14 The measured heat flow densities the surfaces at the four points on the thermal bridge. The heat flow meters are located on the warm side in the climate simulator.

4. Comparison with the simulated results on the total heat flow through the thermal bridge by the white box - The pro processing of the measurement

4.1 The application of the white box on the dynamic variation of the total heat flow through the thermal bridge

The dynamic thermal parameters, the resistance (R) and the capacitance (C), for the thermal bridge in the white box is used for:
• Predicting the dynamic thermal variations of the thermal bridge in real time series. This idea was initiated and the method was developed by Jóhannesson [1] for one dimensional multi-layer homogenous building constructions. Jóhannesson studied systematically ten kinds of the typical Swedish building components: three external walls from heavy weight wall, medium weight wall to light weight wall, three different types of roofs, two internal walls and two type of intermediate floors which are flanked with external walls. The term, Active heat capacity, from this study was defined and studied systematically for building components with 1 D heat conduction features. The methodology on modelling of thermal bridges was also given, and at that time implementation the methodology into the study on the details of dynamic thermal performance for thermal bridges was called alchemy. Even so, Andersson and Jóhannesson [2] demonstrated the application of the procedure for heat and moisture transfer within a foundation using a T-link network.

• Arranging the positions of sensors according to the predicted results for reducing the cost on the amount of expensive heat flow meters. At the same time, some weight factors can be obtained for the total heat flow prediction or the measurement.

• Giving the predicted total heat flow both for steady state solution, for instance the total thermal transmittance through the whole thermal bridge and the dynamic thermal performance of the thermal bridge, if the RC network is optimised for the thermal bridge used in reality.

• Examining the white box when it is compared with the measured results.
• Implementing into whole building system including HVAC etc., in real time series, for instance the network method developed by Mao and Jóhannesson [3] has been preliminarily implemented in the whole building system [4].

4.2 A comparison between the measured and simulated total thermal resistance \( R \) for building design

The total thermal resistance, \( R \) through the surfaces of the thermal bridges is used for obtaining the total thermal transmittance for the thermal bridge. For 1 D multi-layer homogenous building component, so called U-value is represented for the total thermal transmittance. For a thermal bridge, the total thermal transmittance is the result of the part of U-value plus the 2 D part of linear thermal transmittance and plus the 3 D part of point thermal transmittance if the 3 D heat flow feature exists. Anyway, the total thermal resistance \( R \) is the parameter for heat conduction under steady state, i.e. for building design.

The total thermal resistance, \( R \), is handled by:

• Calculating the time average temperature differences on the warm side and cold side.

• Summarising the total temperature difference using the zone weighting method which can be considered as the same factors for the total heat flow prediction under the steady state, see below part.

• Calculating the time average total heat flow through the surface of the component of the thermal bridge, \( Q \). Here, the \( Q \) value is the weighted sum of the four measured heat flow rates times the effective area represented by the heat flow meter. The sub zone weighting
are calculated from the simulation according to the distribution of heat flow rates along the surface of the thermal bridge. If two sub zone weighting factors are used, the continuous distribution of heat flow rates can be divided into two zones where each zone can be considered as an 1 D multi-layer homogenous sub-component. In this way, the measured results are normally very close to the simulated. In general, the two sub zone weighting factors can be obtained from the simulations. An example on how to divide a component with 2 D heat conduction feature i.e. the thermal bridge in a simple way is given in [7]. Table 3 gives a comparison on the total thermal resistance, R between the measured value and the calculated value from the dynamic computer simulation under steady state conditions.

Table 3. The total thermal resistance, R, for the measured thermal bridge.

<table>
<thead>
<tr>
<th>The total thermal resistance, R [m²·K/W]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
</tr>
<tr>
<td>3.50</td>
</tr>
</tbody>
</table>

4.3 The thermal capacitance C in the white box

The total thermal capacitance, C of the thermal bridges in the white box is used for studying the dynamic thermal performance of the thermal bridge under real time series. In order to obtain the optimal values for representing the dynamic thermal parameters for thermal bridges, the so called ω-RC transform developed by Mao and Jóhannesson [3] is used to studied the response of the thermal dynamic features under frequency domain, especially under relatively higher frequency areas for the purpose of time saving on the whole measuring process. When the ω-RC transform procedure is performed for thermal bridges, simple RC network with relatively good accuracy is recommended by the author for easy checking of the thermal capacitance will be installed in the
white box. Here, a Π-link RC network is used for the content of the white box which is shown in Fig. 2, and the dynamic thermal parameters in the white box are given in Table 4.

Table 4. The thermal resistance, $R$, and thermal capacitance, $C$, in the white box for dynamic thermal performance of the thermal bridge using a Π-link RC network.

<table>
<thead>
<tr>
<th></th>
<th>$R$</th>
<th>$R'$</th>
<th>$C$</th>
<th>$C'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[m$^2$·K/W]</td>
<td>[m$^2$·K/W]</td>
<td>[J/m$^2$·K]</td>
<td>[J/m$^2$·K]</td>
<td></td>
</tr>
<tr>
<td>0.558</td>
<td>1.561</td>
<td>14009.3</td>
<td>18144.7</td>
<td></td>
</tr>
</tbody>
</table>

$R_{tot} = R + 2R' = 3.68$

$C_{tot} = C + C' = 32154$

The RC network can be used for calculating dynamic heat loss through the whole thermal bridge using three nodes Π-link with the optimal values in Table 4. Same procedure has been used for in-situ measurement on thermal bridges [5].

5. Discussion and Conclusion

5.1 Uncertainty factors due to the measuring system

In a normal way, it is necessary to analyse the deviations due to the whole measuring system which is the classical issues for measurements and independent of the white box. The accuracy of the measurements depends on the group of the errors which are summarised in Table 5. For this measurement process which is aimed at the implementation into in-situ measurement process, all of the data listed Table 5 are rearranged from [7].
Table 5. The deviations related to the measuring systems [7].

<table>
<thead>
<tr>
<th>Deviations related to measurement accuracy</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>a - Calibration inaccuracy of the heat flow meter</td>
<td>2%</td>
</tr>
<tr>
<td>b - Calibration inaccuracy temperature sensors</td>
<td>1%</td>
</tr>
<tr>
<td>c - Inaccuracy in the reading of the output of the heat flow meter</td>
<td>0.5%</td>
</tr>
<tr>
<td>d - Inaccuracy in the reading of the output of the temperature sensors</td>
<td>0.05 K</td>
</tr>
</tbody>
</table>

2. Deviations related to measurement sensors:

| Inaccuracy due to the heat flow meter                                         | 4%         |
| Inaccuracy in the position of the temperature sensor                          | 2%         |

5.2 Uncertainty factors due to the white box

In general, errors caused by the analysis of non steady state temperatures and heat flow rate should be less than 2% which is required by the [7].

The RC network introduced in this paper leads to the construction of a white box, which can be modified tills the agreement between the modelling and the measurements seems good.

5.3 Uncertainty factors due to the measured object - the thermal bridge

Typical additional systematic errors due to moisture effects for such type of thermal bridge are up to 5%. Cares should be taken on ageing and air leakage problems which are difficult to predict.
5.4 The conclusion

From this pilot study on the measurement on the dynamic thermal performance of a thermal bridge, we have the conclusion:

- **New methodology** on measurement on dynamic thermal performance of thermal bridges is developed for predicting, arranging and controlling the measurement at a relatively less cost and with good accuracy.

- **Difference** between traditional measurement procedure on thermal bridge and the new methodology is that a *white box* with optimal dynamic thermal performance, thermal resistance and thermal capacitance, are set up in advance and then compared and modified using the results from the measurements.

- **A thermal Bridge** which was measured at the Laboratory at the Division of Building Technology KTH has been used as an example in this paper to demonstrate this new procedure, The measurement on temperatures and heat flows is carried out using thermocouples and heat flow meters in a climate simulator. The measured results are compared with the simulated, and the steady state deviation for the thermal resistance is about 5%.

- **Deviations** related to the whole measuring system, the white box and the measured thermal bridge are discussed. In this way, all uncertainty factors except the competence of the person carrying out the modelling and the measurements can be estimated.
Acknowledgements: The economical support from the Swedish Council for Building Research (BFR) is gratefully acknowledged. Thanks are due to Prof. Gudni Jóhannesson, the head of the Division for his encouragement and guidance.

6. Symbols

C  capacitance of a construction layer [J/m²·K]
c  specific heat capacity [J/kgK]
R  thermal resistance between parallel boundaries [m²·K/W]
r  amplitude of admittance or transmittance [W/m²·K]
T_D transmittance of a building component [W/m²·K]
Y  admittance of a building component [W/m²·K]

Greek symbols

ϕ  phase lag (°)
λ  thermal conductivity (W/mK)
θ  Celsius temperature (°C)
ρ  density (kg/m³)
ω  angular frequency (=2π/T) (rad/s)

Note: The symbols used for the components in an RC network are in most applications not related to area but are valid for boundaries with an arbitrary geometry.
Subscripts

e \quad \text{external}

i \quad \text{internal}

se \quad \text{external surface}

si \quad \text{internal surface}

T_b \quad \text{transmittance}

tot \quad \text{total resistance}

Y \quad \text{admittance}

Superscripts

\sim \quad \text{temperature or density of heat flow rate in form of complex number, dynamic variable}

\:\quad \text{dynamic parameter for a part of Resistance and Capacitance}

7. References


Appendix VI
Measurement on thermal performance of thermal bridges

Guofeng Mao, MSc*
Gudni Jóhannesson, Professor*

KEYWORDS: Thermal bridges, temperature, heat flux, measurement, calculation

1. INTRODUCTION
Thermal bridges are parts of building construction where heat flows are not one dimensional and distributions of internal surface temperature are non-uniform. Problems associated with thermal bridges are often severe condensation of moisture at interior surfaces and a significant increase of energy loss. With increasing insulation thickness, thermal bridges play a relatively larger role both for energy consumption and quality assessment of buildings. This paper deals with the measurements on thermal bridges in situ. The purpose of this study is to get quantitative data relating to the effects of thermal bridges in built house and to evaluate the reality of calculation methods.

1.1 A Process for In Situ Measurement on Thermal Bridges
In order to in situ measure thermal performance of thermal bridges accurately, some factors, such as the geometry of thermal bridges, convective and radiative heat transfer boundary conditions, non stationary variations of outdoor or indoor temperatures, ageing effect of material properties, air leakage and measuring equipments etc., should be taken into account (Jóhannesson, 1979 and Mao, 1995). In general, it is necessary to combine measurements of temperatures and surface heat fluxes with calculation methods to analyse the measured results. A process of in situ measurement on thermal bridges is:

   to go through drawings of the building chosen for measurement;
   to find thermal bridges parts from the drawings;

* Division of Building Technology, Department of Building Sciences, Kungliga Tekniska Högskolan, S 100 44 Stockholm, Sweden
to calculate thermal performance of the thermal bridges;
to measure the thermal bridges in situ;
to make a comparison with calculated results.

1.2 An Introductory Study on Insulated Steel Metal Structures

1.2.1 Description of the Structures. Results from a joint program between VTT and KTH are briefly presented in this paper. More detail information can be found in Nieminen et al (1995). The insulated steel metal structures studied were single C or Z-profiles, perforated webs and crossed bars design in different combinations (Figure 1). Gypsum boards are located on the top side with 9 mm thickness and at the bottom side with 13 mm thickness. Insulation material is located at the middle part of the structure with different thicknesses of 150 mm, 175 mm or 250 mm. The central distance between two profiles is 600 mm.

![Diagram](attachment:image.png)

Figure 1 Insulated sheet metal structures used in the study:
(a) - single C-profiles with perforated webs;
(b) - single C-profiles; (c) - cross bars with C and Z-profiles;
(d) - cross bars with C-profiles (C 100 * 50 * 1 mm with perforated web).
1.2.2 Calculation Methods. A 3-dimensional PC computer program using finite difference method has been developed at the Division of Building Technology, KTH in Stockholm for calculating of temperature, heat flow distribution within the structures and overall thermal transmittances (i.e. overall U-values) of the structures (Mao and Jóhannesson, 1994). The calculated overall U-values for the structures are given in Table 1 (see Ut). At the same time, the U-values for the structures without any profiles are also listed in Table 1 for the comparison with the studied structures with sheet metals.

<table>
<thead>
<tr>
<th>Material</th>
<th></th>
<th>Material</th>
<th></th>
<th>Material</th>
<th></th>
<th>Material</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>gypsum board (inside)</td>
<td>0.23</td>
<td>gypsum board (outside)</td>
<td>0.21</td>
<td>insulation (Rock wool)</td>
<td>0.037</td>
<td>sheet metal (steel)</td>
<td>55</td>
</tr>
<tr>
<td>sheet metal (stainless steel)</td>
<td>20.</td>
<td>The thermal conductivities used in calculations:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Network method: Jóhannesson and Áberg (1981) developed an analytical method to calculate the effect of thermal bridges caused by sheet metal. The principle of the network method is that the heat flow through a sheet metal construction is divided into flow paths, based on physical considerations. For each flow path, a resistance is derived analytically. These resistances are combined into a network. The network simulates the thermal performance of the construction. The overall U-value of the construction is given by calculating the total resistance of the network. For cross bar sheet metal structures, Jóhannesson and Mao (1996) developed a similar network method for calculation the overall U-value with 3-dimensional heat conduction feature. The results of overall U-values for the structures are given in Table 2.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Structure</th>
<th>Structure</th>
<th>Structure</th>
<th>Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure (a)</td>
<td>Structure (b)</td>
<td>Structure (c)</td>
<td>Structure (d)</td>
<td></td>
</tr>
<tr>
<td>Ut (W/m²-K)</td>
<td>0.268</td>
<td>0.347</td>
<td>0.329</td>
<td>0.214</td>
</tr>
<tr>
<td>U (W/m²-K) (1-D)</td>
<td>0.231</td>
<td>0.231</td>
<td>0.200</td>
<td>0.142</td>
</tr>
</tbody>
</table>

Table 2 The overall U-values of the structures by the network method.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Structure</th>
<th>Structure</th>
<th>Structure</th>
<th>Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ut (W/m²-K), network method</td>
<td>0.274</td>
<td>0.347</td>
<td>0.334</td>
<td>0.214</td>
</tr>
<tr>
<td>Ut (W/m²-K), from Table 1</td>
<td>0.268</td>
<td>0.347</td>
<td>0.329</td>
<td>0.214</td>
</tr>
<tr>
<td>Remarks</td>
<td>2-D network</td>
<td>2-D network</td>
<td>3-D network</td>
<td>3-D network</td>
</tr>
</tbody>
</table>
1.2.3 Measurement with Heat Flow Meters. An example of the measurement procedure for structure (a) is given in Figure 2. The measured and calculated overall U-value of structure (a) are listed in Table 3.

![Diagram of heat flow meter measurement procedure]

**Figure 2** An example of the heat flow meter measurement procedure.

**Table 3** Measured and calculated overall U-value of the structure (a).

<table>
<thead>
<tr>
<th>Method</th>
<th>$U_t$ (W/m²·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KTH/3-D</td>
<td>0.268</td>
</tr>
<tr>
<td>Heat flow meter</td>
<td>0.270</td>
</tr>
</tbody>
</table>
2. IN SITU MEASUREMENT ON THERMAL BRIDGES - AN EXAMPLE

2.1 Description of a Measured Thermal Bridge

A measured thermal bridge from a newly built multi-family house located at Sollentuna (about 15 km north of Stockholm) is shown in Figure 3. This thermal bridge is towards the north. Heat flow meters with φ75 mm and thermocouples are used to measure heat fluxes and temperatures of the thermal bridge. At the same time, indoor and ambient air temperatures are measured. The arrangements of heat flow meters and thermocouples are demonstrated in Figure 3. The measurement was carried out over a period of three weeks between November and December of 1995. A PC computer is used to record continuously the results from the heat flow meters and thermocouples every fifteen minutes. The material properties of the thermal bridge are listed in Table 4 for calculations. The calculations are made with uniform surface resistances.

![Figure 3](image-url)  
**Figure 3**  Schematic drawing of a measured thermal bridge and layout of heat flow meters and thermocouples.

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal Conductivity λ [W/m-K]</th>
<th>Density ρ [kg/m³]</th>
<th>Heat Capacity c [J/kg K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>1.7</td>
<td>2300</td>
<td>900</td>
</tr>
<tr>
<td>Insulation</td>
<td>0.056</td>
<td>50</td>
<td>800</td>
</tr>
<tr>
<td>Gravel</td>
<td>1.4</td>
<td>1700</td>
<td>1400</td>
</tr>
<tr>
<td>Soil</td>
<td>2.3</td>
<td>1900</td>
<td>1700</td>
</tr>
</tbody>
</table>

Table 4  Material properties of the thermal bridge for calculations.
2.2 Measured Results and Comparison with Calculations

2.2.1 Steady State Thermal Performance of the Thermal Bridge. As the first step, the thermal transmittances (U-values) excluding thermal bridges for the external wall and the foundation are derived from the measurement and calculation. The measured and calculated U-values are given in Table 5.

Table 5  
<table>
<thead>
<tr>
<th></th>
<th>External Wall</th>
<th>Foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated U-value [W/m²·K]</td>
<td>0.20</td>
<td>0.18</td>
</tr>
<tr>
<td>Measured U-value [W/m²·K]</td>
<td>0.21</td>
<td>0.19</td>
</tr>
</tbody>
</table>

The second step is to get quantitative data related to the thermal bridge. For a thermal bridge with two dimensional heat flow feature, linear thermal transmittance (i.e. $\Psi$-value, W/m·K) is used to describe the extra heat loss for a considered thermal bridge (prEN ISO 10211-1, 1994). The results of calculated and measured $\Psi$-value for the above thermal bridge are listed in Table 6.

Table 6  
<table>
<thead>
<tr>
<th>$\Psi$-value, [W/m·K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated</td>
</tr>
<tr>
<td>Measured</td>
</tr>
</tbody>
</table>

2.2.2 Dynamic Thermal Performance of the Thermal Bridge. The measured air and external wall inner surface temperatures are shown in Figure 4, in which we can see that the temperature near the corner is about 4 °C lower than the temperature one meter apart from the corner. This relates to significant differences of surface heat fluxes. The measured external wall surface heat fluxes are compared with two dimensional calculations in the frequency domain which are carried out with a newly developed computer program (Mao and Johannesson, 1993; Mao, 1995). The results of calculated and measured external wall surface heat fluxes are given in Figure 5. The surface heat flux near the corner is about four times the surface heat flux one meter apart from the corner.
Figure 4  The measured air and surface temperatures:
point 4 - on the external wall inner surface, 1 m above the foundation;
point 5 - on the external wall inner surface, 0.3 m above the foundation;
point 6 - on the external wall inner surface, 0.08 m above the foundation.

Figure 5  The measured and calculated heat fluxes on external wall inner surface:
location a - 1 m above the foundation;
location b - 0.3 m above the foundation;
location c - 0.08 m above the foundation.
3. CONCLUSION

The results in this paper show that sheet metal frame structures can be insulated to meet a wide range of overall U-values for external constructions. The computer programs available are very adequate tools to be used in the design process, and simplified methods are an important complimentary tool in the practical design process. A necessary precondition for successful use of computer modelling is that the actual thermal conductivities of the metallic parts should be measured. Furthermore, the combined use of single surface heat flow meters, temperature measurements and computer calculations gave promising results that this technique can be used to measure the thermal performance of insulated sheet metal structures and multi-family house in situ.

4. REFERENCES


