Collapse of thick deepwater pipelines due to hydrostatic pressure

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Master thesis report
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Acknowledgments

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Abstract

The collapse-behaviour of pipes was to be studied by use of Finite Element modelling. Existing analytical expressions for collapse were evaluated and especially the one used in DNV-OS-F101 was decided to be studied in comparison with FE-model results. Parameters that may influence the collapse capacity and are not included in the analytical expressions –flattening, peaking, eccentricity, local wall thickness variation, material stress-strain curve, residual stresses - were defined and explained. A model was built in the Finite Element software package Abaqus v6.9.1 and several articles on collapse testing used to verify it. The aforementioned parameters were studied by use of sensitivity studies and the results shown and discussed. Effective thickness definitions for use in the DNV-formula and the DNV-yield stress criterion were discussed in the context of the results. The results seemed to indicate that the transition between the elastic and plastic range of the material stress-strain curve was of great importance. The results were discussed in the context of the different collapse-related parameters defined beforehand and some concluding remarks were made on possible further work related to these findings.
Nomenclature

\( D \)  
Nominal outer diameter

\( D_{\min} \)  
Minimum outer diameter

\( D_{\max} \)  
Maximum outer diameter

\( R_{\min} \)  
Minimum outer radius

\( R_{\max} \)  
Maximum outer radius

\( t \)  
Nominal wall thickness

\( t_{\min} \)  
Minimum wall thickness

\( t_{\max} \)  
Maximum wall thickness

\( t_{\text{avg}} \)  
Average wall thickness

\( t_{\text{mean}} \)  
Mean of average and minimum wall thickness

\( t_{\text{fun}} \)  
Wall thickness function (depending on \( e \))

\( D/t \)  
Outer diameter-to-thickness ratio

\( w \)  
Local wall thickness variation

\( f_0 \)  
Ovality

\( f_f \)  
Flattening

\( f_p \)  
Peaking

\( e \)  
Eccentricity

\( \sigma \)  
Stress

\( \varepsilon \)  
Strain

\( \sigma_y \)  
Yield stress

\( \sigma_{y,0} \)  
Nominal yield stress for variation of material properties

\( f_d \)  
Reduction factor for variation of material properties

\( p_c \)  
Critical/characteristic collapse pressure

\( p_{el} \)  
Elastic collapse pressure

\( p_p \)  
Plastic collapse pressure

\( p_0 \)  
Reference collapse pressure (at start of parameter variation)

\( E \)  
Young’s modulus

\( H \)  
Hardening modulus

\( v \)  
Poisson’s ratio

\( \alpha_{\text{fab}} \)  
Fabrication factor

\( \sigma_r \)  
Residual bending stress

\( f_r \)  
Application factor for residual stress

\( R_{p,01} \)  
Yield stress defined at 0.1% plastic strain

\( R_{p,02} \)  
Yield stress defined at 0.2% plastic strain

\( R_{p,03} \)  
Yield stress defined at 0.3% plastic strain

\( R_{p,05} \)  
Yield stress defined at 0.5% strain

\( \varepsilon_{y,DNV} \)  
Strain at yield for \( R_{p,05} \)

\( F_{\text{end_cap}} \)  
End cap-force

\( S \)  
Effective axial force

\( N \)  
True pipe wall force
<table>
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<th>Description</th>
</tr>
</thead>
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</tr>
<tr>
<td>$p_e$</td>
<td>External pressure</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Inner cross-sectional area</td>
</tr>
<tr>
<td>$A_e$</td>
<td>External cross-sectional area</td>
</tr>
<tr>
<td>$M$</td>
<td>Bending moment</td>
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1 Introduction

1.1 Background
Recent developments in the off-shore industry have seen the need to lay pipelines in ultra deep water. One such example is the Blue Stream pipeline delivering gas from Russia across the Black Sea to Turkey, of which more than half lies submerged at a depth of more than 2000 metres. Also of note is the planned Oman to India pipeline, reaching a maximum water depth of 3525 metres in the Dalrymple Trough.

One important failure mode of these pipelines is that of collapse followed by propagating buckling, caused by the external hydrostatic pressure which is a major design consideration for such deepwater applications. This is especially important during installation when the pipe is not pressurised and the external pressure is the dominating load.

1.2 Objective of the study
Existing collapse formulas are not without their limitations and it is therefore of interest to develop a FE (Finite Element)-model where the influences of model parameters not included in the existing formulas such as out-of-roundness, wall thickness variation and the material stress-strain curve can be studied. In the widely used standard “DNV-OS-F101: Submarine Pipeline Systems” [1] expressions for calculation of the collapse pressure exists, and the results from the simulations may then be compared to this standard.

1.3 Structure of thesis
In the first part of this thesis, some background on collapse is provided. Analytical formulations are presented and their limitations are discussed. Design parameters are also presented. Following this is the development of the FE-model, with all analysis-relevant information. Thereafter the sensitivity studies are presented with important parameters explained. In the section following this, the results from all these studies are shown and reflected on. Finally the findings are discussed and conclusions are made, followed by references and the appendix.
2 Background

2.1 Development of design equations
Design equations for calculation of the collapse pressure are developed in three steps.
- In the first step, all properties are known and assumed constant which leads to a
deterministic capacity equation.
- In the second step, structural reliability calculations are included. Here
characteristic variables (parameters to be included and their definition) and safety
factors are included.
- In the third step, the developed design equations are adjusted to industry practice.

In the following section, some analytical collapse capacity equations are presented.

2.2 Analytical formulations
The capacity equation used in [1] is the *Haugisma* equation defined as

\[
(p_c - p_{el}) \cdot (p_c^2 - p_p^2) = \frac{p_c p_{el} f_0 D}{t} .
\]  

(2.1)

Another example is the *Timoshenko* formula;

\[
(p_c - p_{el}) \cdot (p_c - p_p) = 3 p_c p_{el} f_0 \frac{D}{t} .
\]  

(2.2)

Finally, the *Shell* formula is defined as

\[
p_c = p_p p_{el} g (p_{el}^2 + p_p^2)^{\frac{1}{2}}
\]  

(2.3)

where

\[
g = \frac{(1 + p_p^2)^{\frac{1}{2}}}{(p_p^2 + f_p^2)^{\frac{1}{2}}}
\]  

(2.4)
\[ p = \frac{p_p}{p_{el}} \]  

(2.5)

\[ f = \left( 1 + \left( f_0 \frac{D}{t} \right)^2 \right)^{-\frac{1}{2}} - f_0 \frac{D}{t} \]  

(2.6)

In these equations, \( p_{el} \) is the elastic collapse pressure, \( p_{pl} \) is the plastic collapse pressure, \( D \) is the nominal outer diameter, \( t \) is the nominal wall thickness, \( f_0 \) is the ovality of the pipe cross-section and \( p_c \) is the characteristic collapse pressure.

The elastic and plastic collapse pressure are usually defined as

\[ p_{el} = \frac{2E(\frac{t}{D})^3}{1-\nu^2} \]  

(2.7)

\[ p_p = 2 \cdot \sigma_y \cdot \frac{t}{D}. \]  

(2.8)

In these equations \( E \) is Young’s modulus and \( \nu \) is Poisson’s ratio.

Out of these collapse formulas, it seems that the Shell formula is the most conservative one, the Timoshenko is the least conservative and the Haugsma formula is between them. As the \( D/t \) increases, all the formulas seem to approach the elastic collapse pressure, which is reasonable. For an ovality of 0.5\% and a yield stress of 413 MPa the collapse pressure is plotted for increasing \( D/t \) in Figure 1 together with the elastic and plastic collapse pressures.

![Figure 1 Collapse pressure for analytical formulas.](image-url)
2.3 Limitations

The aforementioned analytical formulations for the collapse capacity are not without their limitations. In particular they do not account for strain hardening behaviour of the pipe material. There is also no guidance on how to deal with a pipe with varying thickness around the cross-section. To account for variations in wall thickness around the cross-section it would be of interest to investigate if a characteristic wall thickness for use in [1] can be found to give an equal collapse capacity to that of a pipe with varying wall thickness. Also, the elastic collapse pressure is based on an assumption of thin-walled pipes and the plastic collapse pressure is based on an assumption of a uniform circumferential stress distribution throughout the wall thickness.

2.4 Design parameters

Collapse formulations typically include the following design parameters:

- \( \frac{D}{t} \) Diameter-to-thickness ratio of the pipe
- \( f_0 \) Cross-sectional ovality
- \( \sigma_y \) Yield stress.

Other parameters that affect the collapse capacity considered by FE-calculation in this thesis are:

- \( \sigma - \epsilon \) Stress-strain curve
- \( f_f \) Flattening
- \( f_p \) Peaking
- \( e\,w \) Wall thickness variation
- \( \sigma_r \) Residual stresses.

These are all described briefly in the following section.

\( \frac{D}{t} \) ratio

The diameter-to-thickness ratio is a major factor and is proportional to the collapse capacity for lower \( \frac{D}{t} \), where a lower ratio means a thicker and stronger pipe. However, pipes with these lower ratios require more material, become more expensive and are also more difficult to handle due to the weight.

Ovality

Ovality is one of the most important factors to consider when considering pipe collapse capacity. A high ovality drastically lowers the collapse pressure. This is due to the increase of projected area (resulting in a higher resultant force from the hydrostatic pressure) and the induced bending in the wall of the pipe. In Figure 2 this is shown between 0° and 270° for a number of pressures based on (2.9), derived in Appendix 7.
\[ M_1 - M_2 = 2 \int_0^{\pi/2} p r(\phi)^2 \cos(\phi) \sin(\phi) d\phi - p_e \frac{D_0^2}{4} \] (2.9)

Figure 2 Difference in bending moment due to initial ovality for different pressures.

The ovality is here defined as

\[ f_0 = \frac{D_{\text{max}} - D_{\text{min}}}{D} \] (2.10)

Here \( D_{\text{max}} \) and \( D_{\text{min}} \) are the maximum and minimum outer diameter respectively, with \( D \) being the nominal outer diameter. The oval shape that is implied here is obviously an idealised case, but in order to make sensitivity studies it is a simple and easy approach.

It is worth mentioning that another definition also exists (for API), defined as

\[ f_0 = \frac{D_{\text{max}} - D_{\text{min}}}{D_{\text{max}} + D_{\text{min}}} \] (2.11)

Yield stress

The point on the material stress-strain curve which governs initiation of plastic deformation is the yield stress. This yield stress is a way of characterising the entire non-linear stress-strain curve. Ideally, this single value should give a representative capacity for a large variation of materials.

The DNV-formula for local buckling utilises a yield stress for which the definition is a strain of 0.5% \((R_{0.05})\). It is of interest to see whether a more suitable definition may be found for use in the analytical expressions.
Material stress-strain curve
The stress-strain curve is very relevant for thick-walled pipes but less so for higher $D/t$ - these are independent of material strength. This is because a lower $D/t$ results in plastic deformation of the cross-section up to the point of collapse, while a higher ratio results in a mainly elastic collapse. Due to the deformation hardening and Bauschinger effect in the UOE forming process, a significant change in the characteristics of the stress-strain curve can be observed throughout the thickness of the pipe, and also to an extent around the cross-section. This was shown in [4]. In seamless pipes, these changes are not necessarily present in the formed pipe.

Flattening/peaking
Another type of imperfection is called flattening. This can typically be caused by the expanders of the UOE forming process or the crimping of the edges to be welded. Also of interest is peaking, which occurs typically at the weld of the pipe. These two types of imperfections – $f_p$ and $f_f$ – are defined similar to the ovality;

$$f_p = \frac{D_{\text{max}}}{D} - 1$$

$$f_f = 1 - \frac{D_{\text{min}}}{D}.$$  \hfill (2.12)

(2.13)

Since this comes from the minimum and maximum outer diameter being equal to the nominal diameter, it will make the different types of out-of-roundness imperfections easy to compare.

Wall thickness variation
The wall thickness variation is a statistical parameter which will depend on the pipeline fabrication process. There are two fundamental fabrication processes to consider – seamless pipes and the aforementioned UOE process. In the case of seamless pipes the wall thickness variation is caused by the punch being positioned off the centreline, shown in [3]. Then the wall thickness variation may be described as an eccentricity, defined as

$$e = \frac{t_{\text{max}} - t_{\text{min}}}{t_{\text{avg}}}.$$  \hfill (2.14)

Here $t_{\text{max}}$ and $t_{\text{min}}$ are maximum and minimum wall thicknesses respectively, and $t_{\text{avg}}$ is the average wall thickness value.

Another type of wall thickness variation is the case of a single imperfection. These kinds of local imperfections can be caused by the expansion step in the UOE process, between the expanders. For this kind of imperfection the wall thickness variation $w$ is defined as
\[ w = 1 - \frac{t_{\text{min}}}{t} \]  \hspace{1cm} (2.15)

In the case of the analyses, the nominal thickness was used as average thickness for the eccentric case in (2.14) so that the local wall thickness variation could be compared to this.

According to [1], a seamless produced linepipe’s weakest section may not be well represented by the minimum thickness value since it’s probably not present around the whole circumference. Instead, a larger thickness value may be used if it can be documented that this thickness value represents the lowest collapse capacity of the pipeline. It is therefore of interest to see if an effective wall thickness can be defined which represents this reduction of collapse capacity accurately.

Residual stresses
A typical problem with manufacturing pipes is the residual stresses that can sometimes be induced in the pipe. For example, in the manufacturing of UOE-pipes, the pipe is made up by submerged arc-welding, something which can cause large residual stresses in the axial direction of the pipe and cause a bending distortion. Of interest in collapse analyses however are the circumferential residual stresses. These bending residual stresses can be very large due to the massive amount of deformation that the plate is subjected to, especially where the plate is crimp pressed.

2.5 The UOE process
One of the most common methods of manufacturing a pipe is through the UOE process. The name UOE comes from the U-shape, O-shape and Expansion. It is summarised in Figure 3. A steel plate is first milled at the edges to accommodate the welding that will take place before expansion of the pipe. The edges are crimp-pressed to acquire a certain curvature, after which the plate is subjected to the U-punch, and horizontal rollers bend the plate into a shape that can be laid in the O-press. After compression in the O-press into a circular shape the pipe seam is welded through Submerged Arc Welding (SAW), and finally expanded to achieve as round a pipe as possible. This is illustrated in Figure 3 by use of a Finite Element model developed in Abaqus v6.9.
In [1] a fabrication factor $\alpha_{\text{fab}}$ was introduced when calculating the plastic pressure to take residual stresses and effect on the material stress-strain curve into account from the UOE process. This has been found experimentally to be approximately 0.85. Currently, this is being re-evaluated in a Joint Industry Project, where there is discussion if this can be raised by trading part of the expansion of the process for compression of the pipe. Of course, that means that there may be more substantial ovality in a pipe, but the material properties will not be as affected. This factor has also been adopted by API for UOE pipes, with the value 0.6/0.7.

### 2.6 Study

The limitations highlighted in section 2.3 were to be studied by FE-methods in the rest of the thesis in the context of the DNV-OS-F101 design criterion.
3 FE-Model

3.1 Features
A FE-model was to be developed in the Abaqus 6.9 Finite Element software package, with the following features:

- Modifiable geometry ($D, D/t, \text{length}$)
- Modifiable mesh
- Cross-sectional ovality
- Cross-sectional eccentricity
- Residual stresses
- Imperfections.

3.2 Model generation
The FE-model was generated directly through several Abaqus input-files with internal Abaqus keywords, as illustrated in Figure 4.

The basic cylinder representing the pipe was created with end nodes generated in circular arcs at each end of the model, the space between them filled with nodes at regular intervals, and this was then expanded into several layers. Five node layers were used as a default in this thesis.
From these node layers, elements of type C3D8R (8-node linear solid continuum elements with reduced integration) were generated, see Figure 5.

![Figure 5 Increasing mesh density through the thickness.](image)

Each layer was given a unique name, to enable different material properties and possible residual stresses throughout the thickness. For an ovalised pipe, the nodes were scaled an appropriate amount from the pipe centreline. In the case of an eccentricity the nodes of the inner surface were translated the required distance, and the nodes throughout the thickness were translated with a decreasing magnitude to avoid penetration of nodes into other elements. When generating imperfections locally, an Excel spreadsheet was generated with the appropriate node number and calculated translation for these. This was then implemented in the model through a separate input file as imperfections in the geometry.

### 3.3 End-cap effect

An important consideration in pipeline design is the end-cap effect, defined in (3.1) as

$$ F_{\text{end\_cap}} = p_i A_i - p_e A_e. $$

Here $A_i$ and $A_e$ are the inner and outer areas and $p_i$ and $p_e$ is the inner and external pressures, respectively. A related property is the effective axial force, defined in [1] as

$$ S = N - p_i A_i + p_e A_e. $$

where $N$ is the true axial force (the force acquired by integration of axial stresses in the pipe cross-section) of the pipe and $S$ is the effective axial force. If the pipeline is free to expand, this will result in a true axial force of (because only external pressure is considered in this thesis)
\[ N = -p_e A_e. \] (3.3)

To determine what the pressure to apply on the pipe model should be, simple equilibrium calculations can be made according to Figure 6.

![Figure 6 Effect of external hydrostatic pressure through the end-cap effect.](image)

It is obvious that the pressure to be applied on the end of the pipe - \( p' \) - to simulate the external pressure must now be (negative since it is a compressive load)

\[ p' = -\frac{p_e A_e}{A_e - A_i}. \] (3.4)

This pressure was applied at the end of the FE-model. For a pipe with ovality and the shape of an ellipse, the inner and outer areas are

\[ A_e = \pi R_{\text{max}} R_{\text{min}}. \] (3.5)
\[ A_i = \pi (R_{\text{max}} - t)(R_{\text{min}} - t). \] (3.6)

Here \( R_{\text{max}} \) and \( R_{\text{min}} \) are the maximum and minimum outer radiuses. Applying this recalculated pressure is an approximation of the reality, but the change in cross-sectional area up to the point of collapse is assumed to be small. This can easily be shown by extracting the section area from Abaqus, of which the relative difference of cross-sectional area between the collapsed and undeformed model turns out to be around 0.1%. There is also the beneficial effect of being able to allow radial displacement at the end of the model, which a physical end-cap would have prevented, resulting in a stiffer model. A more thorough explanation of the concept of the effective axial force can be found in [5].

### 3.4 Boundary conditions

The influences from the ends of the pipe model can be minimised as discussed in section 3.7.1. In most of the articles used to verify the model the test is performed with welded...
end-caps on the end, which constrains the radial displacement of the end nodes, but allows for expansion in the axial direction. For a model reflecting an infinite pipeline the radial degree of freedom is not constrained at one end. To achieve this radial and axial deformation simultaneously, a reference node is created in the centre of the pipe and the end surface is then coupled to this node with the appropriate degrees of freedom.

The different types of boundary conditions and couplings can be seen in Table 1 below.

<table>
<thead>
<tr>
<th>Type</th>
<th>Reference node constrained degrees of freedom</th>
<th>Constrained in coordinate system</th>
<th>Coupled degrees of freedom to surface</th>
<th>Coupling in coordinate system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capped End</td>
<td>1,2,4,5,6</td>
<td>Global</td>
<td>1,2,3,4,5,6</td>
<td>Cylindrical</td>
</tr>
<tr>
<td>Clamped End</td>
<td>1,2,3,4,5,6</td>
<td>Global</td>
<td>1,2,3,4,5,6</td>
<td>Cylindrical</td>
</tr>
<tr>
<td>Infinite pipeline</td>
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<td>Global</td>
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<td>Cylindrical</td>
</tr>
<tr>
<td>Plane strain</td>
<td>1,2,3,4,5,6</td>
<td>Global</td>
<td>2,3,4,5,6</td>
<td>Cylindrical</td>
</tr>
<tr>
<td>Plane stress</td>
<td>1,2,4,5,6</td>
<td>Global</td>
<td>2,4,5,6</td>
<td>Cylindrical</td>
</tr>
</tbody>
</table>

The degrees of freedom in the different coordinate systems are shown in Figure 7 together with their location (coincident) in the model.

![Figure 7 Degrees of freedom in Cartesian and cylindrical coordinate systems.](image-url)
The boundary condition for an infinite pipeline was used in all the sensitivity analyses. The plane stress and plane strain conditions were not used except to show the influence that allowing a pipe to expand had on the collapse pressure, shown in Figure 8. Of course, in the case of the plane stress, no end cap pressure was applied.

![Figure 8 Collapse pressure for the plane stress and plane strain boundary conditions.](image)

The other end of the pipe is given a symmetry condition in the axial ($z$-) direction, and the length of the pipe can then be halved, thereby drastically reducing the computation time without having to resort to a coarser mesh. The reason symmetry around the axial direction was not used to create a half-pipe is due to the boundary conditions. Since symmetry boundary conditions and kinematic couplings cannot be used at the same time on the same nodes, this would enable nodes to freely move in and out of the symmetry planes.

To prevent the pipe wall from penetrating the opposite side during the analysis, a self-contact (frictionless) surface was defined from the inner surface of the pipe.

### 3.5 Analysis type

Due to the high pressure involved in collapse of thick-walled pipes, collapse of the cross-section will involve plastic behaviour and large non-linear deformations. Since an eigenvalue analysis assumes a linear behaviour up to the point of collapse, the modified Riks algorithm was used instead. A comparison of the two methods for different diameter-to-thickness ratios can be seen in Figure 9.
Figure 9 Comparison of analysis results between eigenvalue analysis and Riks algorithm.

For this algorithm, all load magnitudes are assumed to vary with a single scalar parameter, and the basic idea is to find an equilibrium path in a space defined by the scalar loading parameter and the nodal variables. A detailed description of this is given in [6]. It is clear that the linear eigenvalue analysis yields a much too high collapse pressure compared to the Riks algorithm which takes plasticity into account. The two solution methods can be seen approaching the same results for higher $D/t$, but does not fully converge. This is probably due to the mesh size being insufficient for these very thin pipes – it has shown itself that higher $D/t$ models needs a more refined mesh.

For the analyses where it was important to capture the stress-strain material curve accurately the arc length increment in the Riks algorithm was given a maximum value. Since the differences between the strains to be extracted are quite small, not doing so could give erroneous values.

To allow the applied residual stresses to redistribute, an empty step has to be included in the beginning of the analysis, or there will be convergence problems.

A typical collapsed pipe model can be seen in Figure 10.
3.6 Results extraction

Determining the collapse pressure for the FE-model is quite a straightforward process. A typical load-displacement curve for a node at the collapsing cross-section is shown in Figure 11 below.

The displacement increases linearly at first, but a maximum point is reached, after which the pipe has to release strain energy to remain in equilibrium. The maximum value of the external pressure is then taken as the maximum value of the load proportionality factor, which the external pressure is scaled with. This will then be the collapse pressure of the pipe.
3.7 Verification analyses

3.7.1 Mesh convergence

In the model created, the number of nodes (and thereby elements) throughout the thickness, circumferentially around the cross-section and lengthwise along the pipe model can be specified. For the parameter studies the number of elements throughout the thickness was decided to be five.

The results of refining the mesh can be seen in Figure 12 below. Obviously, the result converges rapidly with an increasing mesh density.

![Figure 12 Effect on the collapse pressure of increasing the mesh density, D/t = 16.](image)

In most parametric studies, it was chosen to use 30 nodes lengthwise along the pipe and 40 nodes circumferentially, which was found to be a good combination of computation time and accuracy of results, in regards to collapse pressure. Increasing the mesh in the circumferential and lengthwise direction to 70 elements only yielded a small relative difference of the collapse pressure - roughly 1%. This showed to be a good choice for a $D/t$ up to 20. Above that, a finer mesh was needed. In the analyses a $D/t$ of 30 has been used, and the minimum mesh size for accurate results was here 50 nodes lengthwise and circumferentially. For the flattening and peaking ovality study 60 nodes around the circumference were used instead to be able to model their imperfections better.

An important issue was to determine the length of the model to minimize influence of the boundary conditions. According to [7] this length to diameter ratio should be no less than 7.5. Other suggestions also exist, such as that in [2], where it is specified that this should be at least 10, which was used in this thesis. An analysis was run with a ratio of 10 and compared to that of an analysis with a ratio of 20. The relative difference between the results (in this case the peak pressure) was insignificant; 0.8%.

Increasing the mesh in the thickness from five elements to six elements gave only a 0.5% relative difference in the result and five elements was deemed enough.
Also in the case of extracting the equivalent plastic strain a finer mesh was used for the DNV yield stress definition. This is because the relatively small difference in results in collapse pressure will still result in relative changes of the equivalent strain that cannot be neglected. The effect of an increasing mesh density can be seen in Figure 13, where the mesh node configuration defined as nodes circumferentially/nodes lengthwise.

![Figure 13 Effective plastic strain at collapse versus mesh configuration.](image)

The third configuration seems to be the best choice, to avoid excessive computation time.

### 3.7.2 Collapse pressure prediction

A number of articles were used for verification of the FE-model. These are summarised in Table 2, and the articles are found in [8], [9], [10] and [11].

<table>
<thead>
<tr>
<th>Article</th>
<th>OD (mm)</th>
<th>t (mm)</th>
<th>OD/t</th>
<th>Residual stress [MPa]</th>
<th>Max Ovality (%)</th>
<th>Max Eccentricity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. OMAE2003-37339 7549</td>
<td>325</td>
<td>18.37</td>
<td>17.7</td>
<td>177</td>
<td>0.2</td>
<td>9.7</td>
</tr>
<tr>
<td>2. OMAE2006-92173 Case 1 – HT</td>
<td>610</td>
<td>31.7</td>
<td>19.2</td>
<td>0</td>
<td>0.26</td>
<td>N/A</td>
</tr>
<tr>
<td>3. OMAE2006-92173 Case 1 – AR</td>
<td>610</td>
<td>31.7</td>
<td>19.2</td>
<td>N/A</td>
<td>0.26</td>
<td>N/A</td>
</tr>
<tr>
<td>4. OMAE2004-51569 EP25A</td>
<td>711.2</td>
<td>38.33</td>
<td>18.55</td>
<td>48.3</td>
<td>0.355</td>
<td>N/A</td>
</tr>
<tr>
<td>5. OMAE2004-51569 EP25B</td>
<td>711.2</td>
<td>38.31</td>
<td>18.72</td>
<td>137.9</td>
<td>0.191</td>
<td>N/A</td>
</tr>
<tr>
<td>6. Collapse Pressure Prediction of UOE pipes Group B</td>
<td>660.4</td>
<td>25.4</td>
<td>26</td>
<td>34</td>
<td>0.38</td>
<td>1.2</td>
</tr>
</tbody>
</table>

The simulations were run attempting to use the material points from the stress-strain curves of the articles. The results can be seen in Table 3 and Figure 14.
Table 3 Results from verification of model.

<table>
<thead>
<tr>
<th>Article Specimen</th>
<th>FE [MPa]</th>
<th>Article [MPa]</th>
<th>DNV-OS-F101 [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. OMAE2003-37339 7549</td>
<td>51.1</td>
<td>57.3</td>
<td>49.2</td>
</tr>
<tr>
<td>2. OMAE2006-92173 Case 1 – HT</td>
<td>49</td>
<td>45.5</td>
<td>44.2</td>
</tr>
<tr>
<td>3. OMAE2006-92173 Case 1 – AR</td>
<td>34.9</td>
<td>36.1</td>
<td>39.6</td>
</tr>
<tr>
<td>5. OMAE2004-51569 EP25B</td>
<td>47.92</td>
<td>50.5</td>
<td>44.1</td>
</tr>
<tr>
<td>6. Collapse Pressure Prediction of UOE pipes Group B</td>
<td>17.42</td>
<td>18.7</td>
<td>21.3</td>
</tr>
</tbody>
</table>

Figure 14 Comparison of results from articles, FE-simulations and DNV-OS-F101.

The spread of the results is partly due to the inexactness of trying to recreate the full material curves, but could also be because of the geometrical imperfections. Since there were no data given on full measurements on the pipes, an average (or when available a maximum) value was used for initial ovalities and eccentrics. The typical shapes of these were not mentioned either, so assumed ovalities and eccentrics were used. The average quotient between analysis and article result was 0.97. The assumed ovality may explain why almost all results were conservative; as will be shown an ovalised pipe will have a major influence on the collapse pressure. Despite the probable existence of local geometric variations the model can be concluded to give results in good agreement with those found by collapse testing.
4 Sensitivity studies

4.1 General

In order to study the limitations discussed previously, a set of sensitivity studies have been performed. For all studies, the Young’s modulus used was set to 200GPa, Poisson’s ratio $\nu$ was 0.3 and the outer diameter was set to 0.6m. An overview of the analyses is given in Table 4 and the individual sensitivities are discussed in the following section.

<table>
<thead>
<tr>
<th>Analysis, parameter variation</th>
<th>Material model</th>
<th>$D/t$</th>
<th>$f_0$ [%]</th>
<th>$\sigma_y$ [MPa]</th>
<th>$f_r$</th>
<th>$f_p$</th>
<th>e [%]</th>
<th>$f_r$</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ovality, $f_0$</td>
<td>RO (X60)</td>
<td>16,20,30</td>
<td>0.2-6 (0.2)</td>
<td>413</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Flattening, $f_f$</td>
<td>RO (X60)</td>
<td>16,20,30</td>
<td>-</td>
<td>413</td>
<td>0.2-1.2 (0.2)</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Peaking, $f_p$</td>
<td>RO (X60)</td>
<td>16,20,30</td>
<td>-</td>
<td>413</td>
<td>0.2-1.2 (0.2)</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Eccentricity, $e$</td>
<td>RO (X60)</td>
<td>14,16,20,30</td>
<td>0.5</td>
<td>413</td>
<td>-</td>
<td>-</td>
<td>0-90 (5)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Local wall thickness variation, $w$</td>
<td>RO (X60)</td>
<td>14,16,20,30</td>
<td>0.5</td>
<td>413</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0-50 (10)</td>
</tr>
<tr>
<td>$D/t$ ratio</td>
<td>RO (X60)</td>
<td>15-30 (1)</td>
<td>0.5,1.5,2.5</td>
<td>413</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Material stress-strain curve, $\sigma_y$</td>
<td>EPP</td>
<td>16,20,30</td>
<td>0.5</td>
<td>400-800 (100)</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BL ($H = 0.05E, 0.10E, 0.20E$)</td>
<td>16,20,30</td>
<td>0.5</td>
<td>400-800 (100)</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Lüder ($H = 0.05E, 0.10E, 0.20E$)</td>
<td>16,20,30</td>
<td>0.5</td>
<td>400-800 (100)</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>RO</td>
<td>16,20,30</td>
<td>0.5</td>
<td>400-800 (100)</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Material comparison, material grade</td>
<td>BL (X46,X60,X70,X80)</td>
<td>16,20,30</td>
<td>0.5</td>
<td>317,413,482,551</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RO (X46,X60,X70,X80)</td>
<td>16,20,30</td>
<td>0.5</td>
<td>317,413,482,551</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Varying material properties – thickness, $f_d$</td>
<td>EPP (X60)</td>
<td>16,20,30</td>
<td>0.5</td>
<td>413, $f_d$ 0-50 (5)</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BL (X60)</td>
<td>16,20,30</td>
<td>0.5</td>
<td>413, $f_d$ 0-50 (5)</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>RO (X60)</td>
<td>16,20,30</td>
<td>0.5</td>
<td>413, $f_d$ 0-50 (10)</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Varying material properties – cross-section, $f_d$</td>
<td>EPP (X60)</td>
<td>16,20,30</td>
<td>0.5</td>
<td>413, $f_d$ 0-50 (5)</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
The abbreviations are as follows:

- RO – Ramberg-Osgood material model
- EPP – Elastic-perfectly plastic material model
- BL – Bi-linear hardening model.

The parameters are given start and finish values, e.g. 0-10 with the increment in parenthesis. A comma means that the parameter study was run for all these values of that specific parameter. The material grades used are defined in [1].

### 4.1.1 Out of roundness

Three types of out of roundness imperfections have been evaluated, see Figure 15.

![Figure 15 Out of roundness types evaluated; ovality, flattening and peaking.](image)

**Ovality**
As mentioned earlier, a crucial parameter for the collapse resistance of pipelines is the ovality of the pipe cross-section. The FE-model was utilised to study the effect of an initial ovality on the collapse pressure. Because some of the models did not collapse properly until an imperfection was introduced, the first value of ovality to be induced was set to 0.2%.

**Flattening**
The model was also used to study the effect of a local flattening in the pipe. Imperfections were used to move specified nodes the correct distance to create a flattened part of the pipe. The circumferential extent of the imperfection was assumed to be 5% of the circumference of the pipe. In order to not give the pipe an unrealistic shape by displacing the flattened area too much, the maximum value of $f_f$ was set so 1.2%.
Peaking
Also a study of a peaking ovality was performed. The peak was generated by imperfections, as the flattening above. The same extent of the imperfection was assumed here.

4.1.2 Wall thickness variations
Two types of wall thickness variations were evaluated - see Figure 16, where $X_{t,\text{avg}}$ and $X_e$ are stochastic variables with a statistical distribution for the average wall thickness and eccentricity, respectively.

For the investigation of an eccentricity, a wide range of values were used, with an initial ovality of 0.5%.

For a local wall thickness variation, different extents and magnitudes were used and then compared to the results of an eccentricity.

4.1.3 $D/t$-ratio
A study of the diameter-to-thickness ratio was also performed, for different values of the ovality.

![Figure 16 Wall thickness variations.](image)
4.1.4 Material stress-strain curve sensitivity study

Investigating the influence of the material stress-strain behaviour on the collapse pressure is very important. Whether the pipe is heat-treated or not may change the characteristic of this material curve, see [11]. Four material curves were used in the simulations; an elastic-perfectly plastic curve, a Ramberg-Osgood curve, a curve with bi-linear hardening and a curve with a Lüder plateau, all shown in Figure 17.

![Elastic-perfectly plastic material curve.](image1)

![Material curve with bilinear hardening.](image2)

![Material curve with Lüder plateau.](image3)

![Ramberg-Osgood material curve.](image4)

**Figure 17 Material stress-strain curves used.**

**Elastic-perfectly plastic material model (EPP)**

An elastic-perfectly plastic material will have a stress-strain curve without any hardening behaviour at all. The yield stress does not change with increasing plastic strain.

**Bi-linear hardening material model (BL)**

A simple model for incorporating a hardening behaviour is the bi-linear hardening model, which assumes a constant slope of the hardening part of the stress-strain curve.
The slope of the plastic part of the material curve is defined as the hardening modulus $H$. The plastic strain at ultimate tensile strength was taken as a large number to avoid the material curve becoming flat in the analysis. This was also done for the Lüder model. The reason for not using the ultimate tensile strength as a point on the plastic curve is that it showed itself to give virtually identical results to the simulations made with the elastic-perfectly plastic one. A more significant hardening behaviour was desired in this part of the study to see the effect different hardening slopes have on the collapse pressure. In the simulations done with regards to the definition of yield stress, variation of material properties and residual stresses, the ultimate tensile strength was used to determine the slope of the hardening part of the material curve.

Lüder material model
The model with a Lüder plateau is very similar to bi-linear hardening, except that there is a plateau which offsets the hardening, in this case assumed to be a length of 1% strain. In reality this plateau is not flat, but this is assumed to be a good approximation.

Ramberg-Osgood material model (RO)
A common material model which ensures a smooth transition between the elastic and plastic part of the material curve is the Ramberg-Osgood model. The relation between stress and strain is shown in (4.1):

\[
\varepsilon = \frac{\sigma}{E} \left( 1 + \alpha \left( \frac{\sigma}{\sigma_t} \right)^{n-1} \right).
\]  

(4.1)

In this equation, $E$ is Young’s modulus, $\sigma$ is true stress, $\sigma_t$ is the true yield stress, $\varepsilon$ is true strain while $\alpha$ and $n$ are material parameters. The term engineering stress means a nominal stress for which the change in area over which the force is acting is small. A true stress however takes this into account, and is defined as

\[
\sigma = \sigma_e (1 + \varepsilon_e)
\]  

(4.2)

where, $\varepsilon_e$ is the engineering strain and $\sigma_e$ is the engineering stress.

For the analyses, the Ramberg-Osgood material curve points were created by using an excel sheet calculating the stress and logarithmic plastic strain for values of the yield stress and ultimate tensile strength. The ultimate tensile strength $\sigma_u$ – assumed at a strain of 10% - was defined as

\[
\sigma_u = \frac{\sigma_y}{0.9}.
\]  

(4.3)

A summary of the acquired values for different yield stress definitions is shown in Table 5.
Table 5 Yield stress values for different definitions and material models.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma_y$ [MPa]</th>
<th>$R_{p,01}$ [MPa]</th>
<th>$R_{p,02}$ [MPa]</th>
<th>$R_{p,03}$ [MPa]</th>
<th>$R_{t,05}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPP/Lüder</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>BL</td>
<td>400</td>
<td>400.5</td>
<td>400.9</td>
<td>401.4</td>
<td>401.4</td>
</tr>
<tr>
<td></td>
<td>500</td>
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</tr>
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<td>702.4</td>
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<td>800.9</td>
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<tr>
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<td>402</td>
</tr>
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<td>497</td>
<td>508</td>
<td>503</td>
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<td></td>
<td>600</td>
<td>582</td>
<td>603</td>
<td>616</td>
<td>603</td>
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<td></td>
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<td>714</td>
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<td>704</td>
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<tr>
<td></td>
<td>800</td>
<td>805</td>
<td>830</td>
<td>844</td>
<td>804</td>
</tr>
</tbody>
</table>

Element stresses and strains
Integration point stresses and strains were recorded from the elements at a position at 270° and transformed to a cylindrical coordinate system. Typical examples of circumferential ($\Phi$) stress-strain curves throughout the thickness can be seen in Figure 15 (here Ramberg-Osgood material model, grade X60).

![Figure 18 Typical stress-strain curves through the thickness.](image)

As seen, the outermost element first experiences compression (before collapse) and tension after collapse. This is due to the collapse shape, shown in Figure 19 with the selected elements of interest marked in red.
The material curve for the middle and inner elements of the element set are practically coincident up to the point of collapse, and at very large deformations the middle elements typically experiences tension instead of compression. Therefore, to ensure a smooth material curve, the inner element was selected, from which all strains and stresses were extracted.

To investigate whether a good collapse-related definition of the yield stress definition could be determined, the equivalent plastic strain and effective (von Mises) stress were recorded. The collapse capacity was then compared to that given by [1] with other definitions to be investigated - $R_{p,0.1}$ (0.1% plastic strain), $R_{p,0.2}$ (0.2% plastic strain) and $R_{p,0.3}$ (0.3% plastic strain), see Figure 20.

The tables with parameters for the yield stress and material model studies are shown in Table 4.
Comparison between bi-linear hardening and Ramberg-Osgood model
A set of simulations were also run to see how well the bi-linear hardening model and the Ramberg-Osgood material model coincide.

For modelling the bi-linear hardening model approximately as a Ramberg-Osgood model, it can easily be shown (see Appendix 8.3) that the intersection of the elastic and plastic line should be at a stress level of

\[ \sigma' = \sigma_y - H \cdot \varepsilon_{y,\text{DNV}} + \frac{\sigma_y - \varepsilon_{y,\text{DNV}} \cdot H}{E - 1} \cdot \varepsilon_y, \]

This is illustrated in Figure 21.

\[ (4.4) \]

Figure 21 Bi-linear hardening model and Ramberg-Osgood model.

In (4.4), \( \varepsilon_{y,\text{DNV}} \) corresponds to the definition of strain at yield also according to [1], at 0.5%. The resulting stress of course corresponds to zero plastic strain, and the final material point is obtained by using the Specified Minimum Tensile Strength.

4.1.5 Variation of stress-strain curves in the model
The parameters for the variation of the stress-strain curves are shown in Table 4.

Through thickness
To investigate the effect of varying material properties through the thickness the different layers in the model was assigned different material properties. Since it became clear that the Lüder model was coincident with the elastic-perfectly plastic model, it was omitted.

For the bi-linear hardening and Ramberg-Osgood models it was assumed that the entire material stress-strain curve decreased proportionally.

The maximum change of the material curve is taken to be that on the outside of the pipe, while the yield stress of the innermost part of the pipe remains unchanged. The material behaviour between these two layers varies approximately linearly.
Variation around cross-section
For a material stress-strain curve which varies around the cross-section (an angle $\Theta$) it is reasonable to assume a variation according to Figure 22.

![Figure 22 Variation of yield stress around the cross-section.](image)

To achieve a model with such a continuously varying yield stress would require an extremely fine mesh, and would be very tedious to implement in Abaqus. Instead, the cross-section was discretized into five parts according to Figure 23 in order to approximate this.

![Figure 23 Variation of cross-sectional yield stress in the model.](image)

Despite this not being exact, the general trend of a cross-sectionally varying yield stress should still be the same.

An illustration of this varying yield stress is given in Figure 24 where $f_i$ is a reduction factor, which the yield stress is multiplied with for a maximum reduced value and $\sigma_{y,0}$ is the nominal yield stress value.

An illustration of this varying yield stress is given in Figure 24 where $f_i$ is a reduction factor, which the yield stress is multiplied with for a maximum reduced value and $\sigma_{y,0}$ is the nominal yield stress value.
4.1.6 Residual stress

To simulate the existence of circumferential residual stresses, element stresses throughout the thickness were applied directly in Abaqus. Since these have to redistribute, the applied stresses in the input file will not be the actual residual stresses, but these can be extracted from the end of the first step later on. The stresses are applied as compressive, with a maximum value at the innermost layer and a minimum value at the outermost layer. This causes a slightly unsymmetric (due to eccentricity and ovality) bending stress distribution throughout the thickness of the pipe. The value to represent the residual stress is taken as the absolute value of the mean value of the inner-and outermost layer as

$$\sigma_r = \frac{\left|\sigma_{\text{outer}}\right| + \left|\sigma_{\text{inner}}\right|}{2}. \quad (4.5)$$

Here $\sigma_{\text{outer}}$ and $\sigma_{\text{inner}}$ are the stresses from the outer- and innermost layer of the pipe after redistribution of stresses.

The parameters for this study are tabled in Table 4, where $f_r$ is a residual stress factor, defined as

$$f_r = \frac{\sigma_{\text{r,applied}}}{\sigma_y}. \quad (4.6)$$

Here $\sigma_{\text{r,applied}}$ is the applied stress in the model, not the actual resulting residual stress which will have to be extracted from the results file.
5 Results

5.1 Out of roundness

In Figure 25 the actual and normalised collapse pressure against increasing initial ovality is shown together with results from [1].

These results indicate that a pipe with lower $D/t$-ratio is less sensitive to an initial ovality than one with a high ratio, which seems reasonable. The collapse pressures calculated from [1] for the lowest $D/t$ pipe seems to give increasingly conservative results when compared to the FE-analysis. For smaller ovalities, the analytical expression seems to fit quite well for lower $D/t$ and very well overall for higher $D/t$. There seems to be one area where the analytical expression yields unconservative results, however.

In Figure 26 the collapse pressure from flattening and peaking are shown together with the results of the ovality.

Figure 25 Influence of initial ovality on collapse pressure.

Figure 26 Influence of initial flattening and peaking on collapse pressure compared to ovality.
According to these results, ovality seems to be more influential on the collapse capacity than either flattening or peaking. But one thing to keep in mind is the definition of the three imperfections; with the definitions used flattening and peaking will mean twice the change in diameter than ovality since one parameter is kept constant.

5.2 Wall thickness variation

In Figure 27 the actual and normalised collapse pressure as a function of initial eccentricity is shown.

These results indicate that an eccentricity has only a minor effect on the collapse pressure for values below ten percent, and increases drastically only for unrealistic values.

From the normalised results, it can be seen that the sensitivity of a pipe to an eccentricity is independent of the $D/t$-ratio. When comparing these results to those found in [3], the ones found here show less dependence on $e$. This is because in the analysis performed here, an initial ovality was also included in the model, something that tended to reduce the influence of the wall-thickness variation.

For example, with an ovality of 0.1% the reduction of collapse pressure between eccentricities of 0% and 20% was 2.6%. For the same pipe, but with an ovality of 0.5% the same reduction in collapse pressure was 1.8%. These are not of the same magnitude as those in [3], but there the reference case was a perfectly round pipe. The conclusion that the $D/t$-ratio does not influence how sensitive the pipe is to a wall thickness variation was also found in the referred article.

From (2.14) it can be seen that (assuming an eccentric cross-section with average nominal wall thickness);

$$t_{\min} + \left(\frac{t_{\max} - t_{\min}}{2}\right) = t_0 \Rightarrow \frac{t_{\min}}{t_0} + \frac{e}{2} = 1 \Rightarrow \frac{t_{\min}}{t_0} = 1 - \frac{e}{2}. \quad (5.1)$$
Since a maximum allowed value of wall thickness variation is defined as a minimum wall thickness of 90% of the nominal wall thickness (representing an $e$ value of 20%), it is of interest to see the results for that specific value. It is shown in Table 6.

<table>
<thead>
<tr>
<th>$D/t$</th>
<th>$p_c/p_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.9768</td>
</tr>
<tr>
<td>20</td>
<td>0.9757</td>
</tr>
<tr>
<td>30</td>
<td>0.9792</td>
</tr>
</tbody>
</table>

To investigate if an effective wall thickness with regards to collapse can be determined, the predicted FE-results compared to those obtained by [1] from the minimum and average wall-thicknesses are shown in Figure 28 up to an eccentricity value of 20%. Values above this are not considered realistic and are therefore not included here.

Figure 28 Collapse pressure for different thickness definitions in DNV-OS-F101 and $D/t$, compared to analysis results.
In Table 7 the Coefficient of Variation and bias towards the DNV results are shown for the different thickness definitions used in [1]. CoV and bias are defined as

\[
CoV = \frac{\bar{x}}{\mu} \quad \text{(5.2)}
\]

\[
Bias = \mu - 1. \quad \text{(5.3)}
\]

Here \( \bar{x} \) is the standard deviation of the normalised results and \( \mu \) is the mean value.

Table 7 CoV and bias for comparison with DNV results with different wall thickness definitions.

<table>
<thead>
<tr>
<th>( t )-definition</th>
<th>( D/t = 14 ) CoV</th>
<th>( D/t = 14 ) Bias</th>
<th>( D/t = 16 ) CoV</th>
<th>( D/t = 16 ) Bias</th>
<th>( D/t = 20 ) CoV</th>
<th>( D/t = 20 ) Bias</th>
<th>( D/t = 30 ) CoV</th>
<th>( D/t = 30 ) Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{\text{min}} )</td>
<td>0.0343</td>
<td>0.0473</td>
<td>0.0418</td>
<td>0.0567</td>
<td>0.0614</td>
<td>0.0811</td>
<td>0.1079</td>
<td>0.1519</td>
</tr>
<tr>
<td>( t_{\text{avg}}(t) )</td>
<td>0.0109</td>
<td>0.0101</td>
<td>0.0096</td>
<td>0.0086</td>
<td>0.0099</td>
<td>0.0098</td>
<td>0.0087</td>
<td>0.0078</td>
</tr>
<tr>
<td>( t_{\text{mean}}(=\frac{t_{\text{min}} + t}{2}) )</td>
<td>0.0115</td>
<td>0.0173</td>
<td>0.0153</td>
<td>0.0221</td>
<td>0.0230</td>
<td>0.0314</td>
<td>0.0478</td>
<td>0.0655</td>
</tr>
<tr>
<td>( t_{\text{fun}}(=t_0 \exp(-e) \cdot (1+\sin(e))) )</td>
<td>0.0027</td>
<td>-0.0026</td>
<td>0.0005</td>
<td>-0.0003</td>
<td>0.0022</td>
<td>0.0012</td>
<td>0.0127</td>
<td>0.0118</td>
</tr>
</tbody>
</table>

With increasing eccentricity, the minimum thickness definition gives increasingly conservative results and the average thickness definition tends to do the opposite. Therefore, two definitions are also added in the interval \( t_{\text{min}} < t < t_0 \). Since the analyses did not always coincide completely with the analytical expressions the results were adjusted to these, so that the trend could be observed. To see the corresponding figures, see Appendix 8.4.

Since using the maximum wall thickness would yield unrealistic results – an increasing eccentricity would mean a higher wall thickness and therefore an increase of collapse pressure – it was not included here. Using the minimum thickness value does the opposite and gives increasingly over conservative results. The thickness definition as a mean value and the function definition seem to be the ones to take the reduction of collapse pressure into account without being too conservative or unconservative. The function also seems to work remarkably well - even better - although it is not based on any evidence.

To compare these results with those of the local wall thickness variation for the same value of \( e \) and \( w \) would not be appropriate, since the definitions are different. An eccentricity of 20% would give a minimum wall thickness of 90% of the nominal thickness. The same \( w \) value for this would be 10% since the maximum thickness is also the nominal thickness. By using (2.14) and (2.15) it is obvious that for the same minimum thickness

\[
t\left(1 - \frac{e}{2}\right) = t(1 - w) \Rightarrow w = \frac{e}{2}. \quad \text{(5.4)}
\]
These results are shown in Figure 29. Here only the $t_{\text{mean}}$ thickness definition is used when comparing the results to those given by [1].

![Figure 29 Collapse pressure as function of local wall thickness variation compared to analysis results and DNV-OS-F101 with a certain thickness definition.](image)

It appears that a local wall thickness variation is more dangerous with regards to the collapse pressure than an eccentricity. It would seem that for lower $D/t$ pipes the extent of the variation around the cross-section also matters more. This could be due to local stress concentrations which initiate yielding in those areas. The thickness definition $t_{\text{mean}}$ seems to be able to reasonably predict the collapse pressure when used in [1] for this kind of wall thickness variation. In real life, it should be pointed out that only values of $w$ up to 0.1 are realistic.

### 5.3 $D/t$ ratio

In Figure 30, the collapse pressure as function of the $D/t$ ratio can be seen for a few chosen initial ovalities, together with the corresponding collapse pressures according to [1].
There is some difference from the analyses when compared to the analytical results with the FE-model predicting a slightly lower collapse pressure for moderate \(D/t\) ratios \((D/t = 17-25)\) and low ovality. This is consistent with results in section 5.1.

### 5.4 Material stress-strain curve influence

In Figure 31 the collapse pressure for the different material models are shown against the yield stress for two \(D/t\). In this case, the hardening modulus \(H=0.05E\).

As could be expected, the result from the Ramberg-Osgood material model (which does not have an instantaneous shift from elastic to plastic deformation) shows the lowest collapse pressure, and is probably the most conservative of the material models. The elastic-perfectly plastic and Lüder models are coincident, with the hardening behaviour of the latter having no effect at all. This indicates that the area of interest on the material
stress-strain curve is the elastic-plastic transition region where neither the elastic-perfectly plastic or Lüder model has any hardening behaviour. The bi-linear hardening model yields the largest collapse pressures, and is presumably the least conservative of the models. For very large values of $H$, the hardening behaviour compared to the elastic-perfectly plastic model has more impact. For examples of this, see Appendix 8.2. An interesting trend is that for both cases the collapse pressure for the different material curves seems to be approaching the same value. The explanation to this is most likely the lesser importance of the hardening behaviour itself for higher yield stresses. For very high values the strain energy of the structure becomes largely elastic with little transition into the plastic area of the material stress-strain curve at collapse, making the different material models similar.

For a higher $D/t$ ratio the increase in collapse pressure is not as drastic since the elastic collapse pressure is of more importance.

In Figure 32 the normalised results are shown.

![Figure 32 Normalised collapse pressure as function of yield stress.](image)

The results indicate an almost linear increase of collapse pressure with an increasing yield stress for a lower $D/t$ pipe, and the Ramberg-Osgood model is actually the one most benefitting from this increase of a yield stress. Probably this is due to the stiffer elastic part of the material curve that results from a higher yield stress, and the collapse pressure will therefore be proportionally higher. The higher $D/t$ pipe is not affected as much by an increase of material strength, as can be expected.

The results of the comparison between the bi-linear hardening model and Ramberg-Osgood model can be seen in for the material grades used, defined in [1].
It is interesting to see here that the same trend mentioned above shows. The bi-linear hardening model and Ramberg-Osgood model can be seen approaching the same value for higher material grades. For higher $D/t$ pipes with material grades of X70 and above it seems that a bi-linear hardening material stress-strain curve is quite sufficient to properly model the material.

In Figure 34 typical hoop stress-strain material curves are shown for all four material models with the points of collapse marked for different $D/t$-ratios. The original material-curves specified in Abaqus are also shown. As before, $H=0.05E$ in both the case of the bi-linear hardening and Lüder model, and the amount of strain at the Lüder-plateau is 1%.
As seen, the hoop stress-strain curve does not coincide with the specified material curve. This is because the stress state in the pipe is not uniaxial, especially not with the compressive axial force resulting from the end cap-effect (and the restriction of axial displacement to that of the reference node). The compressive stresses resulting from this reduces the effective von-Mises stress and therefore a greater value of the hoop stress can be observed. Also quite obvious is that the elastic perfectly-plastic model and the Lüder material model are equivalent – collapse occurs at the flat section of the material curve, meaning that the hardening behaviour for the Lüder model is of no interest.

The resulting critical hoop strain at collapse for different material models and ratios can be seen in Figure 35. The material grade is X60.
The results above show that the material model resulting in the largest (compressive) hoop strain at collapse is the Ramberg-Osgood model, which seems reasonable for lower $D/t$. Since the elastic-perfectly plastic and Lüder material models proved themselves to be identical, only the elastic-perfectly plastic model was used in the subsequent analyses.

5.5 Definition of the DNV yield criterion

In the Ramberg-Osgood model previously, the ultimate tensile strength was defined as $\sigma_y/0.9$, but since this model is probably closest to reality a separate set of tests were also run where the ultimate tensile strength was defined as $\sigma_y/0.75$. This will cause a less stiff elastic-plastic transition of the material curve, as illustrated in Figure 36.

The effective (von Mises) stress and effective plastic strain are shown in Figure 37 for different values of the yield stress and material stress-strain curves.
As expected, there is very little difference between the elastic perfectly plastic and bi-linear hardening model. The Ramberg-Osgood model is the model most affected by an increased material strength, with a smaller amount of equivalent plastic strain for a higher yield stress. This is especially evident for the ultimate tensile strength defined as $\sigma_y/0.75$. But even in this case the results indicate that there are barely any results that indicate a collapse pressure at an equivalent plastic strain of more than 0.3%, and it is more common that it is around 0.2-0.25%. This would suggest that definitions above this are irrelevant in collapse analyses, but one should keep in mind that these results are simply the plastic strain at collapse, not necessarily having anything to do with the best definition of a collapse-related yield stress.

In the context, the collapse capacity of these two Ramberg-Osgood models are also of interest, and the result is shown in Figure 38, where the numbers 1 and 2 indicate $\sigma_y/0.9$ and $\sigma_y/0.75$ respectively.
For all but the largest $D/t$ there is a significant difference in collapse capacity between the two stress-strain curves even though they are defined with the same yield stress. This indicates that the characteristics of the transition range between the elastic and plastic range of the material stress-strain curve is of great importance for pipes with a dominating plastic collapse pressure. As before, for a higher $D/t$ the elastic behaviour is the most important one and there is only an insignificant difference between the two models.

The comparison between the analysis and the collapse pressure given by [1] is shown for the first Ramberg-Osgood material in Figure 39. Here, $R_{p,0.01}$ was chosen. For the full set of results for all models, see Appendix 8.5.

In Table 8 the Coefficient of Variation and bias towards the DNV-formula is shown for the different $D/t$-ratios. This is the same for the different yield stress definitions in the
case of the elastic-perfectly plastic material since the yield stress does not change with plastic deformation. Only $R_{p,01}$ is shown here.

Table 8 CoV and bias for different definition of the yield stress.

<table>
<thead>
<tr>
<th>Yield stress definition</th>
<th>Material model</th>
<th>$D/t = 16$ CoV</th>
<th>$D/t = 16$ Bias</th>
<th>$D/t = 20$ CoV</th>
<th>$D/t = 20$ Bias</th>
<th>$D/t = 30$ CoV</th>
<th>$D/t = 30$ Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{p,01}$</td>
<td>Elastic-perfectly plastic</td>
<td>0.0217</td>
<td>0.0931</td>
<td>0.0092</td>
<td>0.0568</td>
<td>0.0199</td>
<td>0.0758</td>
</tr>
<tr>
<td>$R_{p,01}$</td>
<td>Bi-linear hardening</td>
<td>0.0217</td>
<td>0.0928</td>
<td>0.0092</td>
<td>0.0565</td>
<td>0.0200</td>
<td>0.0758</td>
</tr>
<tr>
<td>$R_{p,02}$</td>
<td>Bi-linear hardening</td>
<td>0.0216</td>
<td>0.0918</td>
<td>0.0093</td>
<td>0.0559</td>
<td>0.0200</td>
<td>0.0756</td>
</tr>
<tr>
<td>$R_{p,03}$</td>
<td>Bi-linear hardening</td>
<td>0.0214</td>
<td>0.0908</td>
<td>0.0093</td>
<td>0.0553</td>
<td>0.0201</td>
<td>0.0755</td>
</tr>
<tr>
<td>$R_{p,03}$</td>
<td>Bi-linear hardening</td>
<td>0.0209</td>
<td>0.0917</td>
<td>0.0092</td>
<td>0.0557</td>
<td>0.0201</td>
<td>0.0756</td>
</tr>
<tr>
<td>$R_{p,01}$</td>
<td>Ramberg-Osgood – 1</td>
<td>0.0169</td>
<td>0.0630</td>
<td>0.0189</td>
<td>0.0220</td>
<td>0.0333</td>
<td>0.0548</td>
</tr>
<tr>
<td>$R_{p,02}$</td>
<td>Ramberg-Osgood – 1</td>
<td>0.0101</td>
<td>0.0324</td>
<td>0.0286</td>
<td>0.0042</td>
<td>0.0357</td>
<td>0.0506</td>
</tr>
<tr>
<td>$R_{p,03}$</td>
<td>Ramberg-Osgood – 1</td>
<td>0.0075</td>
<td>0.0154</td>
<td>0.0344</td>
<td>-0.0055</td>
<td>0.0369</td>
<td>0.0484</td>
</tr>
<tr>
<td>$R_{p,05}$</td>
<td>Ramberg-Osgood - 1</td>
<td>0.0090</td>
<td>0.0317</td>
<td>0.0372</td>
<td>0.0012</td>
<td>0.0378</td>
<td>0.0500</td>
</tr>
<tr>
<td>$R_{p,01}$</td>
<td>Ramberg-Osgood – 2</td>
<td>0.0232</td>
<td>0.0489</td>
<td>0.0251</td>
<td>-0.0037</td>
<td>0.0381</td>
<td>0.0358</td>
</tr>
<tr>
<td>$R_{p,02}$</td>
<td>Ramberg-Osgood – 2</td>
<td>0.0113</td>
<td>-0.0081</td>
<td>0.0430</td>
<td>-0.0374</td>
<td>0.0434</td>
<td>0.0277</td>
</tr>
<tr>
<td>$R_{p,03}$</td>
<td>Ramberg-Osgood – 2</td>
<td>0.0108</td>
<td>-0.0387</td>
<td>0.0544</td>
<td>-0.0548</td>
<td>0.0460</td>
<td>0.0235</td>
</tr>
<tr>
<td>$R_{p,05}$</td>
<td>Ramberg-Osgood – 2</td>
<td>0.0245</td>
<td>-0.0093</td>
<td>0.0591</td>
<td>-0.0428</td>
<td>0.0478</td>
<td>0.0265</td>
</tr>
</tbody>
</table>

It is hard to quantify a single definition which best seems to fit the results from this data. It seems that the definition which best fits the results depends on both the $D/t$ and the characteristics of the stress-strain curve. For the first case, the $R_{p,03}$ definition gives the least bias for the lowest $D/t$ pipe, but it does not give the least CoV. It also gives the lowest bias and CoV for the highest $D/t$. It should be kept in mind however that these values are calculated by numerical methods and the small differences between them may be due to numerical errors. In the second case the best choice for the lowest $D/t$ pipe seems to be the $R_{p,02}$ definition with both regards to bias and CoV.

The aim was to find a collapse-related yield stress independent of design parameters (such as $D/t$ and $f_0$) and stress-strain curve characteristics. Therefore the above results were used together with those from the ovality study and material comparison and normalised as above for each $D/t$. More parameters are then taken into account – desirable if the findings are to be useful in the standard. The results can be seen in Figure 40 and Table 9.
Figure 40 Comparison between analysis and DNV collapse pressure for different yield stress definitions.

Table 9 CoV and bias for different definition of the yield stress, all analyses.

<table>
<thead>
<tr>
<th>Yield stress definition</th>
<th>$D/t = 16$ CoV</th>
<th>$D/t = 16$ Bias</th>
<th>$D/t = 20$ CoV</th>
<th>$D/t = 20$ Bias</th>
<th>$D/t = 30$ CoV</th>
<th>$D/t = 30$ Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{p,01}$</td>
<td>0.0316</td>
<td>0.1040</td>
<td>0.0349</td>
<td>0.0508</td>
<td>0.0515</td>
<td>0.0515</td>
</tr>
<tr>
<td>$R_{p,02}$</td>
<td>0.0362</td>
<td>0.0615</td>
<td>0.0422</td>
<td>0.0177</td>
<td>0.0610</td>
<td>0.0352</td>
</tr>
<tr>
<td>$R_{p,03}$</td>
<td>0.0450</td>
<td>0.0376</td>
<td>0.0501</td>
<td>-0.0007</td>
<td>0.0670</td>
<td>0.0261</td>
</tr>
<tr>
<td>$R_{t,05}$</td>
<td>0.0410</td>
<td>0.0430</td>
<td>0.0496</td>
<td>0.0019</td>
<td>0.0670</td>
<td>0.0270</td>
</tr>
</tbody>
</table>

From this, it can be seen that definitions with increasing amount of plastic strain used in the analytical expression gives less conservative result (obviously). It can also be seen that there is quite a spread of results. This is because as seen in section 5.1 the analysis results with a lower $D/t$ for higher ovalities (which are included here) will be higher than the analytical results. Likewise, for moderate $D/t$ such as 20, the analysis results are lower than the analytical results for smaller ovalities. Thus, it is hard to quantify one single point on the material stress-strain curve to define a collapse-related yield stress. The definition to give the least bias is the $R_{p,03}$ definition, however.
5.6 Variation of material stress-strain curve

As mentioned previously, the UOE process has several effects on the collapse capacity of a pipe. Since the plate is subjected to very large forces and non-uniform plastic straining this will have an impact on the material properties of the pipe due to the Bauschinger effect. In Figure 41 the results from the study of a varied stress-strain curve through the thickness as described in section 4.1.5 can be seen for two values of $D/t$.

![Figure 41 Collapse pressure as function of through the thickness variation of material stress-strain curve.](image)

These results show a large dependence of the collapse pressure on the variation of the material properties. A small deviation was found in the first diagram – the mesh and the specific material properties causes the FE-solver to attempt a too large increment for an accurate result. When re-running this particular simulation, the result was different and more reasonable when limiting the arc length increment in Abaqus.

Similar to above, in Figure 42 collapse pressures are shown for material properties that vary around the cross-section, also described in section 4.1.5.

![Figure 42 Collapse pressures for material properties that vary around the cross-section.](image)
This cross-sectional variation appears to have more influence on the collapse pressure than the through-the-thickness variation. This may be because a reduction of material properties around the cross-section is more directly related to the collapse shape. By reducing the material strength around the cross-section it is easier for the pipe to assume a more oval shape, having the same effect on the collapse pressure as ovality. This is evident in the different collapse shape, as shown in Figure 43.

![Figure 43 Deformed model with cross-sectionally varying material properties.](image)

5.7 **Residual stress**

A new equilibrium is found after application of the residual stresses, with a stress distribution for example as shown in Figure 44 where $T$ is the distance from the centreline of the wall.

![Figure 44 Integration point bending stresses throughout pipe wall thickness.](image)
In Figure 45 the normalised collapse pressure as function of the residual bending stresses in the pipe is shown, with $\sigma_r$ being the mean value of the inner-and outermost stress value.

![Figure 45 Normalised collapse pressure for circumferential residual stresses.](image)

It can be seen that the influence of the circumferential bending residual stresses on lower $D/t$-pipes is minor. Pipes with higher $D/t$-ratios seem to be more sensitive for moderate ratios, but for very high $D/t$ pipes, the influence reduces once again. This is due to an even less influence of the plastic pressure and yielding of the material for pipes as thin as these.

The results are in good agreement with those calculated by Kyriakides in [2].
6 Discussion and conclusions

In conclusion it can be said that there are many parameters which influence the collapse capacity of a pipe. Some of these are governed by the fabrication process while others may be chosen already when designing the pipe. The most important geometrical imperfection has showed itself to be the ovality of the pipe cross-section. Due to the increase of projected area for the pressure to act on, a drastically reduced collapse pressure is the result of a more oval cross-section. The other out of roundness imperfections – flattening and peaking – are still important but do not affect the collapse capacity as much as ovality for the particular definition used in this thesis. It might be of interest to investigate other definitions of these also.

When it comes to wall thickness variation, an eccentricity has shown itself to have a small effect on the collapse pressure. These results were acquired when also including a small ovality of the pipe and when compared to a perfectly round pipe with only an eccentricity the influence may be slightly larger. Since no such thing as a perfectly round pipe exists this was not investigated here. Current practice in the industry is to use the minimum wall thickness of a pipe section in calculations to acquire conservative results. The possibility to define an effective wall thickness for use in [1] in the case of an eccentric pipe was also investigated, and using the definition of the mean value of minimum and average wall thickness seems to be a good mostly conservative choice. A mathematical function was also evaluated and showed itself to give surprisingly good results, but it is not based on anything and is therefore left for possible further analyses in this area.

The analysed types of local wall thickness variations were more detrimental to the collapse capacity of the pipe than the eccentric imperfection. How much more dangerous it is depends not only on the magnitude of the imperfection but also on the extent around the cross-section, at least for lower $D/t$ pipes. The influence increases with a higher magnitude but even for realistic values it is clear that these kinds of imperfections are important. The mean thickness described above gave good predictions, especially for lower $D/t$ pipes. For a higher $D/t$ the definition tends to yield overly conservative results while still following the trend.

The material stress-strain curve characteristic has shown itself to be a very important parameter in collapse analysis. The analyses performed in this thesis indicate (especially that of the two Ramberg-Osgood models) that the transition between the elastic and plastic range of the material curve has much influence on the collapse capacity. This is very clear when comparing the Ramberg-Osgood model with the more simplified elastic-perfectly plastic and bi-linear hardening models which assume an instantaneous initiation of plastic deformation. For the two Ramberg-Osgood models there is a significant difference in results even though the same yield stress was used, and the hardening behaviour is of little consequence. This could be kept in mind when considering heat treatment of a pipe – since the material stress-strain curves of a heat-treated pipe show a stiffer behaviour compared to that of a non heat-treated pipe the beneficial effect of heat treatment is quite clear.
It is hard to decide upon a single collapse-related yield stress since it appears to be dependent not only on the $D/t$ ratio of the pipe but also on the material characteristics. Tentatively, it seems that either $R_{p,02}$ or $R_{p,03}$ may be the best choices, depending on the material behaviour.

Suggested further work in this area could be to investigate the influence of the studied imperfections and how they affect the collapse pressure together. An example of this could be the study of an eccentric pipe with local wall thickness variations or a pipe with an initial ovality in combination with peaking and flattening. This could also be done to see the effects of these imperfections along the length of the pipe. With the FE-model developed, imperfections could be generated at specific desired positions along the pipe.

The FE-model is also easily modified to include bending of the pipe. It could then be used to study the effects of bending stresses on the collapse pressure, e.g. in the sag bend during installation. Some results of this have been acquired, but due to time and scope constraints these are not included in the thesis or discussed.
7 References


8 Appendix

8.1 Difference in bending moment for ovalised pipe

Consider a quarter of a pipe cross-section as shown in Figure 46 with unit length. For simplicity’s sake, assume that this has a mean diameter $D_0$. The outer pressure is $p_e$, the horizontal forces at the cut are $H_1$ and $H_2$, the vertical forces are $V_1$ and $V_2$ and the bending moments in the pipe wall are $M_1$ and $M_2$.

![Figure 46 Cross-section of quarter of a pipe.](image)

Let $D_0$ be the minimum diameter. By definition of ovality, the maximum diameter will then be $D_0(1+f_0)$ where $f_0$ is ovality.

Equilibrium equations:

\[ V_1 - V_2 = \frac{p_e D_0}{2} (1 + f_0) \]

\[ H_1 - H_2 = \frac{p_e D_0}{2} \]

To find $H_1$, $H_2$, $V_1$ and $V_2$ consider half of the pipe in Figure 47.
Equilibrium equations:

\[ \rightarrow 2H_2 = p_e D_0 \]
\[ \uparrow 2V_1 = p_e D_0 (1 + f_0) \]

This means that \( H_1 \) and \( V_2 \) are zero.

The moment equilibrium equation around (1):

\[
M_1 - M_2 - V_2 \frac{D_0}{2} (1 + f_0) + H_2 \frac{D_0}{2} - 2 \int_0^{\frac{\pi}{2}} p_e r(\phi)^2 \cos(\phi) \sin(\phi) d\phi = 0.
\]

The last term is the addition from the pressure acting on the pipe. It is found the following way.

The addition from the pressure needs to be integrated over the surface, see Figure 48.
Since the length $s$ is $\Phi r$, the addition to the bending moment will be
\[
\pi \int_0^\frac{\pi}{2} p_c r(\phi) \cos(\phi) r(\phi) \sin(\phi) d\phi - \pi \int_0^\frac{\pi}{2} p_c r(\phi) \sin(\phi) r(\phi) \cos(\phi) d\phi.
\]
These are from the vertical and horizontal components.

The moment equation can now be written as
\[
M_1 - M_2 = 2 \int_0^\frac{\pi}{2} p_c r(\phi)^2 \cos(\phi) \sin(\phi) d\phi - \frac{H_2}{2} \frac{D_0}{2} = 2 \int_0^\frac{\pi}{2} p_c r(\phi)^2 \cos(\phi) \sin(\phi) d\phi - p_c \frac{D_0^2}{4}.
\]
Here $r(\Phi)$ is the radius of an ellipse, defined as
\[
r(\phi) = \frac{a^2 b^2}{a^2 \sin^2(\phi) + b^2 \cos^2(\phi)},
\]
where
\[
a = \frac{D_0}{2} (1 + f_0),
\]
\[
b = \frac{D_0}{2}.
\]

The moment equation is integrated numerically. Of course, in the case of a perfectly circular pipe,
\[
r(\phi) = \frac{D_0}{2}.
\]

Then
\[
2 \int_0^\frac{\pi}{2} p_c \frac{D_0^2}{4} \cos(\phi) \sin(\phi) = p_c \frac{D_0^2}{4} \Rightarrow M_1 - M_2 = 0
\]
8.2 Results from yield stress study to illustrate hardening influence

![Figure 49 Collapse pressure as function of yield stress with different material models, $D/t=16$.](image)

![Figure 50 Collapse pressure as function of yield stress with different material models, $D/t=30$.](image)
Approximating a bi-linear model to a Ramberg-Osgood model.

Consider a stress-strain diagram as shown in Figure 51. The aim is to find the point of $\sigma'$. 

![Figure 51 Stress-strain diagram.](image)

Here, the following definitions apply:

$\varepsilon_{y,DNV} = 0.005$

$\varepsilon_{UTS} = 0.1$

In the plastic part the stress can be written as

$\sigma(\varepsilon) = \sigma_0 + H\varepsilon$

The hardening modulus $H$ is here

$H = \frac{\sigma_{UTS} - \sigma_y}{\varepsilon_{UTS} - \varepsilon_{y,DNV}}$

Now, the rest is simple:

$\sigma_y = \sigma_0 + H\varepsilon_{y,DNV} \Rightarrow \sigma_0 = \sigma_y - H\varepsilon_{y,DNV} \Rightarrow \sigma(\varepsilon) = \sigma_y + H(\varepsilon - \varepsilon_{y,DNV})$

Solve for the unknown:

$E\varepsilon_y = \sigma_y + H\varepsilon_y - H\varepsilon_{y,DNV} \Rightarrow \varepsilon_y = \frac{\sigma_y - H\varepsilon_{y,DNV}}{E - H}$

Finally:

$\sigma' = \sigma_0 + H\varepsilon_y = \sigma_y - H\varepsilon_{y,DNV} + \frac{\sigma_y - \varepsilon_{y,DNV}}{E - H} - 1$
8.4 Shifted results from thickness variation study

Figure 52 Collapse pressure adjusted to analytical results when calculating CoV and bias.
8.5 Results from yield stress definition study

Figure 53 Comparison between analysis and DNV collapse pressure, elastic-perfectly plastic material.
Figure 54 Comparison between analysis and DNV collapse pressure for the bi-linear hardening material.
Figure 55 Comparison between analysis and DNV collapse pressure for the Ramberg-Osgood model, first case.
Figure 56 Comparison between analysis and DNV collapse pressure for the Ramberg-Osgood model, second case.
8.6  Abaqus input file

8.6.1  Main input file

*heading
Input file for buckling analysis - Full model.

*parameter
**-----------------------------------------------PARAMETERS-----------------------------------------------
outer_radius = 0.3
ratio = 16
thickness = 2*outer_radius*(ratio**-1)
radius = outer_radius-thickness
length = 5*2*outer_radius
E = 200e+09
v = 0.3
**These mesh parameters determine how many nodes circumferentially and along the pipe the model shall have
node_circum = 50
node_length = 50
node_thick = 6
**node_circum no_of_eccetricity_nodes must be dividible by no_of_eccentricity_nodes
**-----------------------------------------------RESIDUAL STRESS-----------------------------------------------
**Insert values of the residual stress here for each layer
residual_stress_1 = 0
residual_stress_2 = 0
residual_stress_3 = 0
residual_stress_4 = 0
residual_stress_5 = 0
**--------------------------------------------DEFINITION OF OVALITY---------------------------------------
**Ovality definition according to the one used in DNV-OS-F101. Max and min radius, ovality, position (ratio) and number of elements defined.
**ovality_row defines the starting point (row) for the ovality in the pipe (when changing, the position will be of ratio ovality_row/node_length)
ovality = 0.006
ovality_row = 1
no_of_ovality_elements = node_length-1
scaling_factor_ovality_max = 1 + ovality*0.5
scaling_factor_ovality_min = 1 - ovality*0.5
**--------------------------------------------CALCULATION OF PRESSURE RATIO-------------------------------
--
outer_area = 3.14159*(outer_radius**2)*scaling_factor_ovality_max*scaling_factor_ovality_min
inner_area = 3.14159*(radius**2)*scaling_factor_ovality_max*scaling_factor_ovality_min
pressure_ratio = outer_area*((outer_area-inner_area)**-1)
pe = 1e+6
pe2 = pe*(pressure_ratio)
**--------------------------------------------DEFINITION OF ECCENTRICITY-------------------------------
**Eccentricity definition according to OMAE200337339. Eccentricity defined, and from this also the scale factors
eccentricity = 0.05
eccentricity_row = 1
no_of_eccentricity_elements = node_length-1
**--------------------------------------------MESH PARAMETERS & MESHING------------------------------------

*include, input=mesh_parameters.inp
*include, input=mesh.inp
8.6.2 **Mesh parameters input file**

**---------------------------------------------MESH PARAMETERS---------------------------------------------**

*parameter
radius_1 = radius
radius_2 = radius + thickness*0.2
radius_3 = radius + thickness*0.4
radius_4 = radius + thickness*0.6
radius_5 = radius + thickness*0.8
radius_6 = radius + thickness
radius = outer_radius-thickness
cn_shift_to_end = (node_length-1)*node_circum
node_incr_length = node_circum*node_thick
node_ang = -360.0/float(node_circum)
node_circum_1 = node_circum
node_circum_2 = node_circum*node_thick*node_length-(node_length-1)*node_circum
inner_node = 1
outer_node = (node_thick-1)*node_circum*node_length+1
node_length_int = node_length-1
node_thick_int = node_thick-1
node_circum_int = node_circum-1
thickness_int = node_length*node_circum
no_of_thickwise_elements = node_thick-1
no_of_lengthwise_elements = node_length-1
**Here all the corner nodes for the C3D8R elements are defined. Two sets for each layer due to the element "missing"**
element_node_1_1 = 1
element_node_1_2 = node_circum + 1
element_node_1_3 = node_circum + 1 + node_circum*node_length
element_node_1_4 = 1 + node_circum*node_length
element_node_1_5 = 2
element_node_1_6 = node_circum + 2
element_node_1_7 = node_circum + 2 + node_circum*node_length
element_node_1_8 = 2 + node_circum*node_length
element_node_2_1 = node_circum
element_node_2_2 = 2*node_circum
element_node_2_3 = 2*node_circum + node_circum*node_length
element_node_2_4 = node_circum + node_circum*node_length
element_node_2_5 = element_node_1_1
element_node_2_6 = element_node_1_2
element_node_2_7 = element_node_1_3
element_node_2_8 = element_node_1_4
first_element_no = 1
last_element_no = node_circum
coupling_node = node_length*node_circum*node_thick + 1
bending_node = coupling_node + 1

**-----------------------------------SELECTION PARAMETERS------------------------------------------------**
**These parameters are utilised when selecting nodes for the end sets**
**Define elements for capped end and surfaces**
surface_element_start_1_1 = 1
surface_element_start_1_2 = node_circum*node_length + 1
surface_element_start_1_3 = 2*node_circum*node_length + 1
surface_element_start_1_4 = 3*node_circum*node_length + 1
surface_element_start_1_5 = 4*node_circum*node_length + 1
surface_element_start_2_1 = node_circum*(node_length-2) + 1
surface_element_start_2_2 = node_circum*(2*node_length-2) + 1
surface_element_start_2_3 = node_circum*(3*node_length-2) + 1
surface_element_start_2_4 = node_circum*(4*node_length-2) + 1
surface_element_start_2_5 = node_circum*(5*node_length-2) + 1
surface_element_end_1_1 = node_circum
surface_element_end_1_2 = (node_circum) + node_circum*node_length
surface_element_end_1_3 = (node_circum) + 2*node_circum*node_length
surface_element_end_1_4 = (node_circum) + 3*node_circum*node_length
surface_element_end_1_5 = (node_circum) + 4*node_circum*node_length
surface_element_end_2_1 = node_circum*(node_length-1)
surface_element_end_2_2 = node_circum*(2*node_length-1)
surface_element_end_2_3 = node_circum*(3*node_length-1)
surface_element_end_2_4 = node_circum*(4*node_length-1)
surface_element_end_2_5 = node_circum*(5*node_length-1)
outer_surface_element_start = node_circum*node_length*(node_thick-1)-(node_circum*node_length) + 1
outer_surface_element_end = node_circum*node_length*(node_thick-1)-node_circum
displacement_node_1 = 1
displacement_node_2 = 11
strain_element_1 = 1
strain_element_5 = 1+int(float(4*node_circum*node_length))
limit_displacement = 0.1*outer_radius
layer_1_element_start = 1
layer_2_element_start = 1 + (node_length)*node_circum
layer_3_element_start = 1 + 2*(node_length)*node_circum
layer_4_element_start = 1 + 3*(node_length)*node_circum
layer_5_element_start = 1 + 4*(node_length)*node_circum
layer_1_element_end = (node_length-1)*node_circum
layer_2_element_end = (2*node_length-1)*node_circum
layer_3_element_end = (3*node_length-1)*node_circum
layer_4_element_end = (4*node_length-1)*node_circum
layer_5_element_end = (5*node_length-1)*node_circum

8.6.3 Mesh input file
**----------------------------------MESHING OF THE PIPE-------------------------**
**Nodes are generated in a cylindrical coordinate system, elements are generated**
*node, system=c
<inner_node>, <radius>, 0, 0, 0, 0
<outer_node>, <outer_radius>, 0

*ngen, line=c, nset=inner_circle
<inner_node>, <node_circum_1>, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0
*ngen, line=c, nset=outer_circle
<outer_node>, <node_circum_2>, 1, 0, 0, 0, 0, 0, 0, 0, 0

*nfill, nset=inner_circle, outer_circle, <node_thick_int>, <thickness_int>
*ncpy, new set=top, old set=bottom, shift, change number=chn shift to end
0, 0, 0, 0, <length>
0, 0, 0, 0, 0, 0, 0, 0, 0

*nfill, nset=between bottom, top, <node_length_int>, <node_circum>
*element, type=C3D8R
1, <element_node_1_1>, <element_node_1_2>, <element_node_1_3>, <element_node_1_4>, <element_node_1_5>, <element_node_1_6>, <element_node_1_7>, <element_node_1_8>

*elgen, elset=whole_pipe
1, <node_circum_int>, 1, 1, <no_of_lengthwise_elements>, <node_circum_int>, <node_circum_int>, <no_of_thickwise_elements>, <thickness_int>, <thickness_int>

*ngen, line=c, nset=inner_circle
<inner_node>, <node_circum_1>, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1.0
*ngen, line=c, nset=outer_circle
<outer_node>, <node_circum_2>, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1.0
*nfill, nset=bottom
inner_circle, outer_circle, <node_thick_int>, <thickness_int>

**-------------------------------------------------------------------END SURFACES--------------------------------------------**

**Define element set surface for capped ends and makes a surface.
*elset, elset=end_surface_1, generate
<surface_element_start_1_1>, <surface_element_end_1_1>, 1
<surface_element_start_1_2>, <surface_element_end_1_2>, 1
<surface_element_start_1_3>, <surface_element_end_1_3>, 1
<surface_element_start_1_4>, <surface_element_end_1_4>, 1
<surface_element_start_1_5>, <surface_element_end_1_5>, 1

*elset, elset=end_surface_2, generate
<surface_element_start_2_1>, <surface_element_end_2_1>, 1
<surface_element_start_2_2>, <surface_element_end_2_2>, 1
<surface_element_start_2_3>, <surface_element_end_2_3>, 1
<surface_element_start_2_4>, <surface_element_end_2_4>, 1
<surface_element_start_2_5>, <surface_element_end_2_5>, 1

*surface, name=end_surface_2, type=element
end_surface_2
*surface, name=end_surface_1, type=element
end_surface_1

**-------------------------------------------------------------------OUTER SURFACE------------------------------------------**

*elset, elset=outer_surface_elements, generate
<outer_surface_element_start>, <outer_surface_element_end>, 1
*surface, name=outer_surface, type=element
outer_surface_elements

**-------------------------------------------------------------------DEFINITION OF LOCAL SYSTEM FOR RESULTS------------------------**

**Define displacement sets and transform results to a cylindrical system
*transform, nset=displacement, type=c
0, 0, 0, 0, 0, 1

**-------------------------------------------------------------------DEFINITION OF DISPLACEMENT NODE SET-------------------------**
*nset, nset=displacement
<displacement_node_1>
<displacement_node_2>

**---------------------------------------DEFINITION OF STRAIN ELEMENT SET---------------------------------------**
*elset, elset=strain_element, generate
<strain_element_1>,<strain_element_5>,<thickness_int>

**---------------------------------------DEFINITION OF ELEMENT LAYERS---------------------------------------**
*elset, elset=layer_1, generate
<layer_1_element_start>,<layer_1_element_end>,1
*elset, elset=layer_2, generate
<layer_2_element_start>,<layer_2_element_end>,1
*elset, elset=layer_3, generate
<layer_3_element_start>,<layer_3_element_end>,1
*elset, elset=layer_4, generate
<layer_4_element_start>,<layer_4_element_end>,1
*elset, elset=layer_5, generate
<layer_5_element_start>,<layer_5_element_end>,1
8.6.4 Ovality and eccentricity node generation input file

**---------------------------------------------------OVALITY NODE GENERATION-------------------------------
*parameter
**Selects the nodes associated with appropriate rows for each layer and performs the ovalisation.
ovality_first_node = (ovality_row-1)*node_circum*node_thick+1
ovality_end_node = (ovality_row+no_of_ovality_elements)*node_circum*node_thick

**---------------------------------------------ECCENTRICITY PARAMETER GENERATION-------------------------------
---
eddcccddcdddddccddcdddccccccccccccccccccdddccddcddddccddcddddddcddccddcddcccccccccdddccddcddddccddcdddcdddccccccccccccccc

eccentricity_first_node_1 = (eccentricity_row-1)*node_circum+1
eccentricity_end_node_1 = (eccentricity_row+no_of_eccentricity_elements)*node_circum
eccentricity_first_node_2 = eccentricity_first_node_1 + node_circum*node_length
eccentricity_end_node_2 = eccentricity_first_node_2 + node_circum*node_length-1
eccentricity_first_node_3 = eccentricity_first_node_3 + node_circum*node_length
eccentricity_end_node_3 = eccentricity_first_node_3 + node_circum*node_length-1
eccentricity_first_node_4 = eccentricity_first_node_4 + node_circum*node_length
eccentricity_end_node_4 = eccentricity_first_node_4 + node_circum*node_length-1
eccentricity_first_node_5 = eccentricity_first_node_5 + node_circum*node_length
eccentricity_end_node_5 = eccentricity_first_node_5 + node_circum*node_length-1
eccentricity_first_node_6 = eccentricity_first_node_6 + node_circum*node_length
eccentricity_end_node_6 = eccentricity_first_node_6 + node_circum*node_length-1
translation_eccentricity_1 = eccentricity*thickness*0.5
translation_eccentricity_2 = (eccentricity*thickness)*(1-1*(5**-1))*0.5
translation_eccentricity_3 = (eccentricity*thickness)*(1-2*(5**-1))*0.5
translation_eccentricity_4 = (eccentricity*thickness)*(1-3*(5**-1))*0.5
translation_eccentricity_5 = (eccentricity*thickness)*(1-4*(5**-1))*0.5
translation_eccentricity_6 = (eccentricity*thickness)*(1-5*(5**-1))*0.5

8.6.5 Ovality and eccentricity generation input file

**-----------------------------------------------OVALITY GENERATION---------------------------------------
**Assigns ovality nodes to a set and scales them
*nset, nset=ovality_nodes, generate
<ovality_first_node>,<ovality_end_node>,1
*nmap, nset=ovality_nodes, type=scale
0,0,0
<scaling_factor_ovality_max>,<scaling_factor_ovality_min>,1.0

**-----------------------------------------------ECCENTRICITY GENERATION-------------------------------
**Assigns eccentricity nodes to a set and scales them
*nset, nset=eccentricity_nodes_1, generate
<eccentricity_first_node_1>,<eccentricity_end_node_1>,1
*nset, nset=eccentricity_nodes_2, generate
<eccentricity_first_node_2>,<eccentricity_end_node_2>,1
*nset, nset=eccentricity_nodes_3, generate
<eccentricity_first_node_3>,<eccentricity_end_node_3>,1
*nset, nset=eccentricity_nodes_4, generate
<eccentricity_first_node_4>,<eccentricity_end_node_4>,1
*nset, nset=eccentricity_nodes_5, generate
<eccentricity_first_node_5>,<eccentricity_end_node_5>,1
*nset, nset=eccentricity_nodes_6, generate
<eccentricity_first_node_6>,<eccentricity_end_node_6>,1
*nmap, nset=eccentricity_nodes_1, type=translation, definition=coordinates
0,0,0,0,1,0
<translation_eccentricity_1>
*nmap, nset=eccentricity_nodes_2, type=translation, definition=coordinates
0,0,0,0,1,0
8.6.6 Imperfections input file
**<Here all the nodal imperfections are input in a list>

8.6.7 Material input file
**Here, the material properties of the pipe are defined and assigned to the respective element layers/sets
*elset, elset=whole_pipe
layer_1,layer_2,layer_3,layer_4,layer_5
*material, name=Pipe
*density
7850
*elastic
<E>,<v>
*plastic, hardening=isotropic
**<Here hardening parameters are inserted>
*orientation, name=cylindrical_system, definition=coordinates, system=cylindrical
0,0,0,0,1
*solid section, elset=whole_pipe, material=pipe, orientation=cylindrical_system

8.6.8 Contact parameters and generation input file
**-----------------------------------------------CONTACT SURFACE PARAMETERS-------------------------------

*parameter
contact_element_start = 1
contact_element_end = node_circum*(node_length-1)
**-----------------------------------------------CONTACT GENERATION-------------------------------

*elset, elset=inner_surface, generate
<contact_element_start>,<contact_element_end>,1
*surface, name=inner_surface, type=element
inner_surface,s3
*surface interaction, name=inner_contact
*contact
*contact pair, interaction=inner_contact, type=surface to surface
inner_surface
8.6.9 Initial conditions and boundary conditions input file

**--------------------------------------------------INITIAL CONDITIONS-------------------------------------
**Here the initial hoop residual stresses are used as input
*initial conditions, type=stress
layer_1,0,<residual_stress_1>,0,0,0,0
layer_2,0,<residual_stress_2>,0,0,0,0
layer_3,0,<residual_stress_3>,0,0,0,0
layer_4,0,<residual_stress_4>,0,0,0,0
layer_5,0,<residual_stress_5>,0,0,0,0

**-----------------------------------------------------BOUNDARY CONDITIONS-------------------------------
**First, the node sets at the end of the pipe are defined. These will have symmetry in the z-direction
*boundary
bottom,ZSYMM
<coupling_node>,1,2,0
<coupling_node>,4,6,0

8.6.10 Coupling input file

**------------------------------------------COUPLING----------------------------------
*node, system=c
<coupling_node>,0,0,<length>
<bending_node>,0,0,0
*coupling, ref node=<coupling_node>, orientation=cylindrical_system, surface=end_surface_2, constraint
name=end_coupling
*kinematic
2,2
3,3
4,4
5,5
6,6

8.6.11 Analysis steps input file

**------------------------------------------MISC STEP-------------------------------------------------
*step, nlgeom=yes, name=Misc step
*static
*output, field, variable=preselect, frequency=1
*output, history, variable=preselect, frequency=1
*element output, elset=strain_element
le,e,s
*node output, nset=displacement
u
*node file, nset=displacement, global=no
u
*el file, elset=strain_element
le,e,s
*end step

**------------------------------------------BUCKLING STEP-----------------------------------------
**A surface pressure is applied
*step, nlgeom=yes, name=Buckling
*static, riks
*dsload
outer_surface,P,<pe>
end_surface_2,P,<pe2>
*output, field, variable=preselect, frequency=1
*output, history, variable=preselect, frequency=1
*element output, elset=strain_element
le,e,s
*node output, nset=displacement
u
*node file, nset=displacement, global=no
u
*el file, elset=strain_element
le,e,s
*end step