Preprint

This is the submitted version of a paper presented at *IEEE Jordan Conference on Applied Electrical Engineering and Computing Technologies (AEECT) 3-5 Nov 2015.*

Citation for the original published paper:

In: IEEE
http://dx.doi.org/10.1109/AEECT.2015.7360585

N.B. When citing this work, cite the original published paper.

© 2015 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

Permanent link to this version:
http://urn.kb.se/resolve?urn=urn:nbn:se:kth:diva-176654
Multiphase Unbalanced Power Flow and Fault Analysis of Distribution Networks with High Penetration of Inverter-Interfaced DERs

Hossein Hooshyar* and Luigi Vanfretti†
*KTH Royal Institute of Technology
Stockholm, Sweden
Statnett SF
Oslo, Norway
hossein.hooshyar@ee.kth.se

Abstract— The traditional power flow and fault analysis methods fail to meet the requirements in both performance and accuracy aspects in the distribution network applications. This is due to the unbalanced multiphase nature of the distribution network and also due to the emerging penetration of renewable generation at the distribution level. This paper proposes a comprehensive method for power flow solution and fault analysis of multiphase unbalanced distribution networks with high penetration of inverter-interfaced DERs. Also, the self-protection scheme employed for the inverter of the DERs is also formulated in the proposed method. Performance of the proposed method has been assessed by simulations on a sample distribution network.

Keywords— distribution network; distributed energy resources (DER); fault analysis; power flow; PSCAD/EMTDC.

I. INTRODUCTION

The unbalanced multiphase characteristics of the distribution network together with the high penetration of distributed energy resources (DER) impose additional requirements to the traditional power flow and fault analysis methods. The Gauss-Seidel and Newton-Raphson techniques, and the traditional fault analysis methods such as sequence networks fail to meet performance and accuracy requirements for distribution network analysis studies. In particular, the assumptions resulting in the simplifications used in these standard methods are not valid in distribution networks [1,2].

New methods for power flow and fault analysis have been proposed in literature [1-7]. [3-5] propose solutions based on actual three-phase equations; however, system matrices are constructed in such way that power injection from DERs cannot be included. More advanced methods with the ability of considering DERs have been introduced in [1,6] but they are all valid for radial or, at most, weakly meshed networks.

This paper proposes a new comprehensive method performing both power flow and fault analysis for distribution networks of any kind of structure that are highly penetrated by inverter-interfaced DERs. The multiphase unbalanced nature of the distribution networks is also considered in the proposed method.

The paper begins with an introduction on the modeling principles in Section II. In Section III, the proposed method is introduced. Section VI evaluates the performance of the proposed method by comparing the results obtained from these methods and the results obtained from the PSCAD simulation on a test case. Conclusions are drawn in Section V.

II. MODELING PRINCIPLES

A. Basics

Fig. 1 shows a generic three phase line section. As shown in the figure, each phase is modeled by a PI section ($Z_{sr}$, $C_{sr}$) and is coupled to other phases through mutual impedances ($Z_{ab}$, $Z_{ac}$). The current of the line section, $I_{sr}$, flows from the sending node, $s$, to the receiving node, $r$.

To each of these nodes, an inverter-interfaced DER and/or loads of different types, listed as follows, may be connected:

- $Z_{ci}$: Constant impedance load.
- $I_{cc}$: Constant current load.
- $I_{cp}$: Constant power load. The constant power load current is calculated as $I_{cp} = S / V$, where $S$ is the load rating and $V$ is the node voltage to which the load is connected.
- $I_{der}$: Inverter-interfaced DER. The inverter is designed to push the power available from the DER ($P_{der}$) to the power grid and also, in some cases, to exchange reactive power ($Q$) with the power grid as an ancillary service. So the current to be injected from the inverter for a specific active and reactive power is dependent on its terminal voltage ($V$) [2,8]. However, if this current gets to be higher than the maximum current rating of the inverter, inverter limits the current at its maximum level. So the inverter current can be described as $I_{der} = f(P_{der}, Q, V)$.

This work was supported in part by the FP7 IDE4L project funded by the European Commission, the STandUp for Energy Collaboration Initiative and by Statnett SF, the Norwegian TSO.
B. Formulation

We start by deriving a KVL equation as illustrated by the solid arrow in Fig. 1. The voltage drop across the phase ‘a’ of the line section can be determined as:

\[
V^a - V^a_r = Z^a \sum I^a + Z^a_{DER} I^a_{DER} + Z^a_{CP} I^a_{CP} - I^a_{SCP} \quad (1)
\]

On the other hand, \( V^a \) and \( V^a_r \) can be obtained from the following equations:

\[
V^a = \left( Z^a_{ct} \parallel Z^a \right) \left( \sum I^a + I^a_{DER} - I^a_{SCP} \right) \quad (2)
\]

\[
V^a_r = \left( Z^a_{ct} \parallel Z^a \right) \left( I^a_{DER} + I^a_{SCP} - \sum I^a_{n} \right) \quad (3)
\]

where \( Z^a_{ct} \) and \( Z^a \) represent the total capacitive impedances connected to nodes \( s \) and \( r \), and are calculated as:

\[
Z^a_{ct} = \frac{1}{\sum_{i} C_i \frac{1}{j \omega}} \quad (4)
\]

Now by replacing \( V^a \) and \( V^a_r \) in (1) by (2) and (3), we get to the following equation:

\[
-Z^a_{ct} \sum I^a + \left( Z^a_{ct} \parallel Z^a \right) Z^a_{DER} I^a_{DER} + Z^a_{CP} I^a_{CP} - I^a_{SCP} = 0
\]

Following the same logic, (6) and (7) give the same relation for phases ‘b’ and ‘c’, respectively.

\[
-Z^a_{ct} \sum I^b + \left( Z^a_{ct} \parallel Z^a \right) Z^b_{DER} I^b_{DER} + Z^b_{CP} I^b_{CP} - I^b_{SCP} = 0
\]

\[
-Z^a_{ct} \sum I^c + \left( Z^a_{ct} \parallel Z^a \right) Z^c_{DER} I^c_{DER} + Z^c_{CP} I^c_{CP} - I^c_{SCP} = 0
\]

Note that the unknown variables in (5) to (7) are the line currents, \( I^a, I^b, I^c \). As indicated in the equations, the contributions of DERs, constant power loads, and constant current loads in the power flow are considered by current sources whereas those of the constant impedance loads are considered by impedances. The currents of DERs and constant power loads are determined by the models introduced in the previous section assuming that the terminal voltages, \( V \), are known. It is worth mentioning that the line section parameters are the only essential elements of the derived equations, i.e., all other parameters (DER and/or loads) can be simply set to zero in case they don’t exist.

Also, note that the currents directions can be determined arbitrarily. In this study, the directions of currents are assumed to be from the nodes with the lower numbers to the nodes with the higher numbers.

Deriving the same type of equations, for all line sections of a distribution network with \( N \) lines, results in a system of \( 3N \) linear equations with \( 3N \) variables where \( k_i \) is the number of the phases of the \( i \)th line section. These equations will be used in the next section to form the system matrices.

III. THE PROPOSED COMPREHENSIVE METHOD FOR POWER FLOW AND FAULT ANALYSIS

A. Formation of System Matrices

The equations derived in the previous Section are put into matrix form as expressed in (8). Note that in this equation, the state variables are the currents of the line sections.

\[
\begin{bmatrix} Z_{bus} \end{bmatrix} \begin{bmatrix} I_{line} \end{bmatrix} = \begin{bmatrix} Z \end{bmatrix}
\]

where, for a system with \( N \) line sections, \( Z_{bus} \) is a symmetric \( N \times N \) block matrix containing the system impedances, \( I_{line} \)
Fig. 2. Structure of the system matrices.

is a \(N \times 1\) block matrix containing the line section currents as the system state variables, and \([Z_I]\) is a \(N \times 1\) block matrix containing the information of the loads and DERs connected to the line sections. Fig. 2 illustrates the structures of the matrices.

The line sections can be numbered in an arbitrary order. A building algorithm for these matrices can be developed as follows. Note that, in all equations throughout this paper, the numeric superscripts of the parameters represent the arbitrary number of the phases and should not be confused with mathematical power.

- \([Z_{bus}]\): As shown in Fig. 2, \([Z_{bus}]\) consists of \(N\) blocks of rows and \(N\) blocks of columns. The diagonal and off-diagonal blocks should be constructed as follows:

  **Diagonal blocks:** Each diagonal block belongs to a line section and is sized as \(k_1 \times k_1\). (9) shows how to construct the diagonal block of a line section with \(k_i\) phases. (10), (11), and (12) are examples showing the diagonal blocks of a single-phase, a two-phase, and a three-phase line section, respectively.

\[
[Z_{bus}]_{\theta,\phi}^{(i)} = \\
\begin{bmatrix}
Z_{c_{\theta}}^{(i)} \parallel Z_{e_{\phi}}^{(i)} + Z_{w_{\phi}}^{(i)} + Z_{c_{\phi}}^{(i)} & \cdots & Z_{w_{\phi}}^{(i)} \\
\vdots & \ddots & \vdots \\
Z_{w_{\phi}}^{(i)} & \cdots & Z_{c_{\phi}}^{(i)} + Z_{w_{\phi}}^{(i)} + Z_{c_{\phi}}^{(i)} & \parallel Z_{c_{\phi}}^{(i)}
\end{bmatrix}
\]  
(9)

\[
[Z_{bus}]_{\theta,\phi}^{(i)} = [Z_{c_{\theta}}^{(i)} || Z_{e_{\phi}}^{(i)} + Z_{w_{\phi}}^{(i)} + Z_{c_{\phi}}^{(i)} || Z_{c_{\phi}}^{(i)}]
\]  
(10)

\[
[Z_{bus}]_{\theta,\phi}^{(i)} = \\
\begin{bmatrix}
Z_{c_{\theta}}^{(i)} \parallel Z_{e_{\phi}}^{(i)} + Z_{w_{\phi}}^{(i)} + Z_{c_{\phi}}^{(i)} & \cdots & Z_{w_{\phi}}^{(i)} \\
\vdots & \ddots & \vdots \\
Z_{w_{\phi}}^{(i)} & \cdots & Z_{c_{\phi}}^{(i)} + Z_{w_{\phi}}^{(i)} + Z_{c_{\phi}}^{(i)} & \parallel Z_{c_{\phi}}^{(i)}
\end{bmatrix}
\]  
(11)

\[
[Z_{bus}]_{\theta,\phi}^{(i)} = [Z_{c_{\theta}}^{(i)} || Z_{e_{\phi}}^{(i)} + Z_{w_{\phi}}^{(i)} + Z_{c_{\phi}}^{(i)} || Z_{c_{\phi}}^{(i)}]
\]  
(12)

\[
\begin{bmatrix}
Z_{c_{\theta}}^{(i)} + Z_{w_{\phi}}^{(i)} + Z_{c_{\phi}}^{(i)} & \cdots & Z_{w_{\phi}}^{(i)} \\
\vdots & \ddots & \vdots \\
Z_{w_{\phi}}^{(i)} & \cdots & Z_{c_{\phi}}^{(i)} + Z_{w_{\phi}}^{(i)} + Z_{c_{\phi}}^{(i)} & \parallel Z_{c_{\phi}}^{(i)}
\end{bmatrix}
\]

Off-diagonal blocks: The off-diagonal blocks, \((i, j)^a\) and \((j, i)^b\), sized as \(k_i \times k_j\) and \(k_j \times k_i\) respectively, refer to the interconnection between the \(i^a\) and the \(j^b\) line sections and should be constructed as shown in (13). If there is no interconnection between the \(i^a\) and the \(j^b\) line sections, the corresponding \((i, j)^a\) and \((j, i)^b\) blocks are null matrices.

\[
[Z_{bus}]_{\theta,\phi}^{(i, j)^a} = [Z_{bus}]_{\theta,\phi}^{(j, i)^b} = 0
\]  
(13)

where subscript \(cn\) refers to the common node of \(i^a\) and \(j^b\) line sections. Note that the matrix entries in (13), \(Z_{cl_{-cn}}\) and \(Z_{cn_{-c}}\), are the constant impedance load and the total capacitive impedance at \(cn\), respectively. The ‘±’ sign is ‘+’ if the line sections are interconnected via sending or receiving sides at the \(cn\); otherwise, i.e. one side is sending and the other side is receiving, it’s ‘−’. Equations (14), (15), and (16) are examples showing the off-diagonal blocks corresponding to the connections between a single-‘a’-phase and a three-phase line section, a double-‘ab’-phase and a three-phase line section, and two three-phase line sections, respectively.

\[
[Z_{bus}]_{\theta,\phi}^{(i, j)^a} = [Z_{bus}]_{\theta,\phi}^{(j, i)^b} = [Z_{cn}^{(i)} || Z_{c_{cn}}^{(j)}]
\]  
(14)

\[
[Z_{bus}]_{\theta,\phi}^{(i, j)^a} = [Z_{bus}]_{\theta,\phi}^{(j, i)^b} = [Z_{cn}^{(i)} || Z_{c_{cn}}^{(j)}]
\]  
(15)

\[
[Z_{bus}]_{\theta,\phi}^{(i, j)^a} = [Z_{bus}]_{\theta,\phi}^{(j, i)^b} = [Z_{cn}^{(i)} || Z_{c_{cn}}^{(j)}]
\]  
(16)

Finally note that the building algorithm of \([Z_{bus}]\) is not based on the system topology, therefore the method can be used for both radial and meshed distribution networks.
• \([\text{Iline}]\): \([\text{Iline}]\) is a vector of \(N\) blocks as shown in Fig. 2. Each block, sized as \(k_x \times 1\), contains the currents of the \(i^a\) line section as indicated in (17):

\[
[\text{Iline}]_{k_x,1} = \begin{bmatrix} I_{s}^{a} \\ I_{b}^{a} \end{bmatrix}
\]

(17)

• \([\text{ZI}]\): As shown in Fig. 2, \([\text{ZI}]\) is a vector of \(N\) blocks. Each block, sized as \(k_x \times 1\), contains the information of the loads and DERs connected to the \(i^a\) line section as indicated in (18):

\[
[\text{ZI}]_{k_x,1} = \begin{bmatrix} (Z_{sc}^{a} || Z_{f}^{a}) (I_{sDER}^{a} - I_{sCC}^{a} - I_{f}^{a}) - (Z_{sc}^{a} || Z_{r}^{a}) (I_{sDER}^{a} - I_{sCC}^{a} - I_{r}^{a}) \\ (Z_{sc}^{a} || Z_{f}^{a}) (I_{bDER}^{a} - I_{bCC}^{a} - I_{f}^{a}) - (Z_{sc}^{a} || Z_{r}^{a}) (I_{bDER}^{a} - I_{bCC}^{a} - I_{r}^{a}) \end{bmatrix}
\]

(18)

B. Incorporating Fault in System Matrices

Fig. 3 shows a generic fault, occurring at the arbitrary node \(s\). As shown in the figure, applying a fault at \(s\) corresponds to adding the line section, \(si\). This leads to addition of a diagonal block, sized as \(k_x \times k_x\), as the \((N+1)\)th diagonal block entry of \([\text{Zbus}]\), as shown in Fig. 4. Also, for each line sections, connected to the faulted node \(s\), two off-diagonal blocks are added to the \((N+1)\)th row and the \((N+1)\)th column. Fig. 4 shows the added off-diagonal blocks, sized as \(k_x \times k_x\) and \(k_x \times k_y\), assuming that the \(i^a\) line section is connected to the faulted node \(s\). Determination of \(k_x\) and construction of the added blocks are done differently for grounded and ungrounded faults, as discussed below.

• Grounded faults: Following the same logic, used in Section II.B, deriving KVL equation for phase ‘a’, as shown by solid arrows in Fig. 3, leads to (19) if a single-‘a’-phase to ground, (20) if a double-‘ab’-phase to ground, and (21) if a three-phase to ground fault is applied.

\[
-(Z_{sc}^{a} || Z_{f}^{a}) \sum_{i=0}^{N-1} I_{i}^{a} + (Z_{sc}^{a} || Z_{r}^{a}) (R_{f}^{a} + R_{r}^{a}) I_{f}^{a} = \]

\[
(Z_{sc}^{a} || Z_{f}^{a}) (I_{sDER}^{a} - I_{sCC}^{a} - I_{f}^{a})
\]

(19)

\[
-(Z_{sc}^{a} || Z_{f}^{a}) \sum_{i=0}^{N-1} I_{i}^{a} + (Z_{sc}^{a} || Z_{r}^{a}) (R_{f}^{a} + R_{r}^{a}) I_{f}^{a} +
\]

\[
R_{f}^{a} I_{f}^{a} = (Z_{sc}^{a} || Z_{f}^{a}) (I_{sDER}^{a} - I_{sCC}^{a} - I_{r}^{a})
\]

(20)

Fig. 3. Applying a generic fault at an arbitrary node.

\[
[\text{Zbus}] = \begin{bmatrix} \ldots & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & 0 & \ldots \\ \ldots & \ldots & \ldots \\ 0 & 0 & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix}
\]

\[
[\text{Iline}] = \begin{bmatrix} \ldots & \ldots & 1 \\ \ldots & \ldots & \ldots \\ 1 & 1 & \ldots \\ \ldots & \ldots & \ldots \\ 1 & 1 & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix}
\]

Fig. 4. Extension of system matrices as a result of applying a fault.
\[-(Z_{scf}^a \parallel Z_{cfa}^a) \sum_{i=1}^{s} I_i^a + (Z_{scf}^a \parallel Z_{ef}^a + R_f^a) I_f^a + R_f^a I_f^a + R_f^a = (Z_{scf}^a \parallel Z_{cfa}^a) (I_{nder}^a - I_{acc}^a - I_{scp}^a) + (Z_{scf}^a \parallel Z_{ef}^a + R_f^a) I_f^a + R_f^a I_f^a + R_f^a\] 

(21)

Similar equation to those shown by the solid arrows in Fig. 3 can be derived for other phases if any type of grounded faults is applied. Rearranging the derived equations into the form of matrix, results in the additional diagonal and off-diagonal blocks to be constructed as follows:

\[
[Zbus]_{k_f,s}^{(N+1,4)} = 
\begin{bmatrix}
Z_{scf}^a \parallel Z_{cfa}^a^1 + R_f^a + R_f^a & \ldots & R_f^a \\
\vdots & \ddots & \vdots \\
R_f^a & \ldots & Z_{scf}^a \parallel Z_{cfa}^a + R_f^a + R_f^a \\
\end{bmatrix}
\]

(22)

\[
[Zbus]_{k_f,s}^{(N+1,4)} = \left(\begin{bmatrix}
Z_{scf}^a \parallel Z_{cfa}^a \\
\vdots \\
0 \\
\end{bmatrix} \right) = 
\begin{bmatrix}
\pm (Z_{scf}^a \parallel Z_{cfa}^a) & \ldots & 0 \\
0 & \ddots & 0 \\
0 & \ldots & \pm (Z_{scf}^a \parallel Z_{cfa}^a) \\
\end{bmatrix}
\]

(23)

where \(k_f\) is the number of faulted phases. As the faulted node \(s\) is always the sending side for the fault branch \(sn\), the ‘+’ sign is ‘+’ if the \(i^{th}\) line section is interconnected via its sending side; otherwise, it’s ‘−’.

Equations (24) and (25), (26) and (27), and, (28) and (29) are examples showing the diagonal and off-diagonal blocks for a single-‘a’-phase to ground, a double-‘ab’-phase to ground, and a three-phase to ground fault, respectively. In all of these examples, it’s assumed that the \(i^{th}\) line section, connected to the faulted node, is a three-phase line section.

\[
[Zbus]_{k_f,s}^{(N+1,4)} = [Z_{scf}^a \parallel Z_{cfa}^a + R_f^a + R_f^a]
\]

(24)

\[
[Zbus]_{k_f,s}^{(N+1,4)} = \left(\begin{bmatrix}
Z_{scf}^a \parallel Z_{cfa}^a \parallel Z_{cfa}^a \parallel Z_{cfa}^a \\
\vdots \\
0 \\
\end{bmatrix} \right) = \begin{bmatrix}
\pm (Z_{scf}^a \parallel Z_{cfa}^a) & \ldots & 0 \\
0 & \ddots & 0 \\
0 & \ldots & \pm (Z_{scf}^a \parallel Z_{cfa}^a) \\
\end{bmatrix}
\]

(25)

\[
[Zbus]_{k_f,s}^{(N+1,4)} =
\begin{bmatrix}
Z_{scf}^a \parallel Z_{cfa}^a + R_f^a + R_f^a & R_f^a \\
R_f^a & Z_{scf}^a \parallel Z_{cfa}^a + R_f^a + R_f^a \\
\end{bmatrix}
\]

(26)

\[
[Zbus]_{k_f,s}^{(N+1,4)} = \left(\begin{bmatrix}
Z_{scf}^a \parallel Z_{cfa}^a \\
\vdots \\
0 \\
\end{bmatrix} \right) = 
\begin{bmatrix}
\pm (Z_{scf}^a \parallel Z_{cfa}^a) & \ldots & 0 \\
0 & \ddots & 0 \\
0 & \ldots & \pm (Z_{scf}^a \parallel Z_{cfa}^a) \\
\end{bmatrix}
\]

(27)

\[
[Zbus]_{k_f,s}^{(N+1,4)} = \left(\begin{bmatrix}
Z_{scf}^a \parallel Z_{cfa}^a \\
\vdots \\
0 \\
\end{bmatrix} \right) = 
\begin{bmatrix}
\pm (Z_{scf}^a \parallel Z_{cfa}^a) & \ldots & 0 \\
0 & \ddots & 0 \\
0 & \ldots & \pm (Z_{scf}^a \parallel Z_{cfa}^a) \\
\end{bmatrix}
\]

\[
[Zbus]_{k_f,s}^{(N+1,4)} = 
\begin{bmatrix}
Z_{scf}^a \parallel Z_{cfa}^a + R_f^a + R_f^a & R_f^a & \ldots \\
\ldots & \ldots & \ldots \\
R_f^a & R_f^a & Z_{scf}^a \parallel Z_{cfa}^a + R_f^a + R_f^a \\
\end{bmatrix}
\]

(28)

\[
[Zbus]_{k_f,s}^{(N+1,4)} = \left(\begin{bmatrix}
Z_{scf}^a \parallel Z_{cfa}^a \\
\vdots \\
0 \\
\end{bmatrix} \right) = 
\begin{bmatrix}
\pm (Z_{scf}^a \parallel Z_{cfa}^a) & \ldots & 0 \\
0 & \ddots & 0 \\
0 & \ldots & \pm (Z_{scf}^a \parallel Z_{cfa}^a) \\
\end{bmatrix}
\]

(29)

(30) and (31) show the added blocks to \([Iline]\) and \([Zf]\), respectively.

\[
[Iline]_{k_f,s}^{(N+4)} = 
\begin{bmatrix}
I_f^1 \\
\vdots \\
I_f^s \\
\end{bmatrix}
\]

(30)

\[
[Zf]_{k_f,s}^{(N+4)} = 
\begin{bmatrix}
(Z_{scf}^a \parallel Z_{cfa}^a) (I_{nder}^a - I_{acc}^a - I_{scp}^a) \\
\vdots \\
(Z_{scf}^a \parallel Z_{cfa}^a) (I_{nder}^a - I_{acc}^a - I_{scp}^a) \\
\end{bmatrix}
\]

(31)

**Ungrounded faults:** Similar to the grounded-faults, KVL equations can be derived between the faulted phases as shown by the dotted arrows in Fig. 3. Rearranging the derived equations into the form of matrix, results in (32) to (35) showing the added blocks if ungrounded phase-to-phase fault is applied, and (36) to (39) showing the added blocks if ungrounded three-phase fault is applied. Note that, for ungrounded faults, \(k_f\) is equal to the number of faulted phases minus 1. This is because, in this type of faults, the number of independent fault current variables is always equal to the number of faulted phases minus 1.

\[
[Zbus]_{k_f,s}^{(N+1,4)} = \begin{bmatrix}
Z_{scf}^a \parallel Z_{cfa}^a + R_f^a + R_f^a + Z_{scf}^a \parallel Z_{cfa}^a \\
\vdots \\
(Z_{scf}^a \parallel Z_{cfa}^a) (I_{nder}^a - I_{acc}^a - I_{scp}^a) \\
\end{bmatrix}
\]

(32)

\[
[Zbus]_{k_f,s}^{(N+1,4)} = \begin{bmatrix}
(Z_{scf}^a \parallel Z_{cfa}^a) (I_{nder}^a - I_{acc}^a - I_{scp}^a) \\
\vdots \\
(Z_{scf}^a \parallel Z_{cfa}^a) (I_{nder}^a - I_{acc}^a - I_{scp}^a) \\
\end{bmatrix}
\]

(33)

\[
[Iline]_{k_f,s}^{(N+4)} = \begin{bmatrix}
I_f^1 \\
\vdots \\
I_f^s \\
\end{bmatrix}
\]

(34)

\[
[Zf]_{k_f,s}^{(N+4)} = \begin{bmatrix}
(Z_{scf}^a \parallel Z_{cfa}^a) (I_{nder}^a - I_{acc}^a - I_{scp}^a) \\
\vdots \\
(Z_{scf}^a \parallel Z_{cfa}^a) (I_{nder}^a - I_{acc}^a - I_{scp}^a) \\
\end{bmatrix}
\]

(35)
As shown in the figure, the network solution, proposed solution of a distribution network with high penetration of DERs and loads, introduced in Section II, to update the values of the currents of DERs and constant power loads. This iteration runs until convergence is reached. Note that the characteristics of constant impedance and constant current loads do not need to be updated as they are represented with constant values in the network solution.

The algorithm, illustrated in Fig. 6, gives the power flow solution and can also be used for the fault analysis as the fault can be modeled in system matrices. However, the values given for fault analysis are valid only for the first moments after the fault occurrence. This is because the DERs employ self-protection systems which disconnect them from the grid under such conditions. The disconnection time for different DERs is not the same, as for each DER, it depends on DER’s distance to the fault and also the type of the employed self-protection system. Hence, as a result of DERs disconnecting at different times from the grid, the fault currents will be varying [2]. Thus, in order to be able to determine such a fault current profile, the proposed algorithm is extended, as shown in Fig. 7. As shown in the figure, the response of the DER self-protection is emulated in each iteration in order to determine the DERs that will be disconnected by their protection system, and how long it will take for the protection system to disconnect them. This loop is repeated until the DER protection does not disconnect any more DER.

IV. TEST CASE

To evaluate the performance of the proposed methods on a multiphase unbalanced distribution network, the IEEE 34 node test feeder is used as the test case in this study. Since the original test feeder contains only radial sub-feeders, a number of lines have been added to create a meshed distribution network. Fig. 8 shows the modified test feeder. It’s assumed that each customer has a PV system (as DER) that can generate up to its maximum load of the customer unit (100% penetration). For this study, this system is simulated using PSCAD in order to get detailed time domain responses and to use them for comparison with the proposed method.

The PV system is simulated by adopting a typical model, which uses average models for the converters, and includes maximum power point tracking [9]. The PVs’ self-protection system is IEEE 929 compliant [10].
network solution from (8) and constant power load.

\[ I = I + \Delta t \]

End

Table I.

| Node | Phase | \( |V| \) p.u. (Simulation) | \( |V| \) p.u. (Proposed Method) | \( \angle V \) deg. (Simulation) | \( \angle V \) deg. (Proposed Method) |
|------|-------|---------------------------|---------------------------------|----------------------------|-----------------------------------|
| 802  | a     | 0.9974                    | 0.9974                          | -0.0333                    | -0.0333                           |
|      | b     | 0.9976                    | 0.9976                          | -120.0505                  | -120.0505                         |
|      | c     | 0.9978                    | 0.9978                          | 119.9542                   | 119.9542                          |
| 816  | a     | 0.9124                    | 0.9124                          | -0.8528                    | -0.8528                           |
|      | b     | 0.9568                    | 0.9568                          | -120.2738                  | -120.2736                         |
|      | c     | 0.9214                    | 0.9214                          | 119.6667                   | 119.6667                          |
| 862  | a     | 0.8607                    | 0.8607                          | -0.9947                    | -0.9969                           |
|      | b     | 0.9315                    | 0.9315                          | -119.9604                  | -119.9592                         |
|      | c     | 0.8486                    | 0.8486                          | 119.946                    | 119.9459                          |

Max. Dif. = 0.0000%  Max. Dif. = 0.2212%

Table II.

| Node | Phase | \( |V| \) p.u. (Simulation) | \( |V| \) p.u. (Proposed Method) | \( \angle V \) deg. (Simulation) | \( \angle V \) deg. (Proposed Method) |
|------|-------|---------------------------|---------------------------------|----------------------------|-----------------------------------|
| 802  | a     | 0.999                      | 0.999                           | 0.0647                     | 0.0654                            |
|      | b     | 0.9989                     | 0.9989                          | -119.9514                  | -119.9507                         |
|      | c     | 0.999                      | 0.999                           | 120.0505                   | 120.0505                          |
| 816  | a     | 0.9697                     | 0.9702                          | 2.3249                     | 2.3444                            |
|      | b     | 0.9859                     | 0.9861                          | -118.9451                  | -118.9369                         |
|      | c     | 0.9746                     | 0.9751                          | 122.203                    | 122.2142                          |
| 862  | a     | 0.9542                     | 0.955                           | 3.8982                     | 3.9259                            |
|      | b     | 0.9818                     | 0.9822                          | -118.1724                  | -118.1625                         |
|      | c     | 0.9575                     | 0.9584                          | 124.5343                   | 124.5512                          |

Max. Dif. = 0.0940%  Max. Dif. = 1.0819%

Note that when the PVs generation are at the maximum level, the fault currents will have two different values, as shown in Table IV. This is because, due to low system voltage during the fault, the PVs' self-protection disconnect the PV systems from the grid 0.1s after the fault occurrence which, in turn, results in a step in the fault current profile. Fig. 9 shows such changes in the fault current profile for a three-phase fault occurring at node 862 with fault resistance of 3Ω. As shown in the figure, the transients captured by the simulation results are not included in calculated results. Also, the simulations results contain the inherent delay of the RMS measuring blocks which does not exist in the results obtained from the proposed method.

It is worth noting that the proposed method and also the test feeder have been coded in MATLAB to calculate the presented power flow and fault analysis results.

V. CONCLUSION

This paper proposed a new comprehensive method for power flow solution and fault analysis of multiphase unbalanced distribution networks with high penetration of inverter-interfaced DERs. The proposed method can be used for distribution networks of any kind of structure, i.e. radial and meshed. Also, the self-protection scheme employed for the
inverter of the DERs can be formulated in the proposed method. As shown in the paper, the maximum difference between the results calculated by the proposed method and obtained from the simulations is less than 1.4%, indicating that the method provides quite accurate estimates with low computational burden when contrasted to PSCAD simulation.

### Table III. Fault Current Values (No PV-Night)

<table>
<thead>
<tr>
<th>Node</th>
<th>Phase</th>
<th>If-3LG (kA) (Simulation)</th>
<th>If-3LG (kA) (Proposed Method)</th>
<th>If-LG (kA) (Simulation)</th>
<th>If-LG (kA) (Proposed Method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>802</td>
<td>a</td>
<td>16.1057</td>
<td>16.105</td>
<td>12.7462</td>
<td>12.7457</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>17.3865</td>
<td>17.3857</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>16.4279</td>
<td>16.4272</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>816</td>
<td>a</td>
<td>0.4941</td>
<td>0.4941</td>
<td>0.3982</td>
<td>0.3982</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.7308</td>
<td>0.7309</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>0.5176</td>
<td>0.5176</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>862</td>
<td>a</td>
<td>0.4867</td>
<td>0.4868</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.2667</td>
<td>0.2666</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*Max. Dif. = 0.0375% Max. Dif. = 0.0039%

### Table IV. Fault Current Values (Max. PV-Noon)

<table>
<thead>
<tr>
<th>Node</th>
<th>Phase</th>
<th>If-3LG (kA) (Simulation)</th>
<th>If-3LG (kA) (Proposed Method)</th>
<th>If-LG (kA) (Simulation)</th>
<th>If-LG (kA) (Proposed Method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>802</td>
<td>a</td>
<td>3.8895</td>
<td>3.8894</td>
<td>3.7889</td>
<td>3.7888</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>3.9861</td>
<td>3.9861</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>3.9636</td>
<td>3.9635</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>816</td>
<td>a</td>
<td>0.4483</td>
<td>0.4483</td>
<td>0.3725</td>
<td>0.3725</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.6473</td>
<td>0.6474</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>0.4728</td>
<td>0.4728</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>862</td>
<td>a</td>
<td>0.2383</td>
<td>0.2383</td>
<td>0.2035</td>
<td>0.2035</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.4467</td>
<td>0.4468</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>0.2524</td>
<td>0.2524</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*Max. Dif. = 1.3169% Max. Dif. = 1.3779%

Fig. 9. Fault current profile for a three-phase fault at node 862 with $R_f = 3 \Omega$.

A. References


[9] Model available on the website of ECEE department at the University of Colorado at Boulder (http://ece Engineering and Environmental Sciences Department).