Digital Motion Control Techniques for Electrical Drives

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1. Introduction to the thesis

It is believed that engineers, who were mainly trained in Power Electronics have developed the area of Motion Control. Motion control area is a result of applying control theory to power electronics. Currently it is a fairly matured field, which is almost three decades old. The research on application of microprocessors for electrical drive control in 1970’s laid the foundation stone for the Motion Control area [1]. The historical development of the Motion Control field is shown in Figure 1.1.

Though initiated by power electronic engineers, current status of motion control is such that conventionally trained power electronic engineers will probably require following complementary courses in motion control to grasp the development in this area. Now this field is an extremely competitive one. Nowadays, power electronics and motion control can be treated as two distinct fields that emerge as the technology advances.

Power electronics is mainly the transformation of electric power. High speed (in component switching), energy efficiency, environmental friendliness are keywords in the on going researches in this area. Achieving complete control over the motion is the fundamental idea of motion control. Obviously, this incorporates multiple fields as a typical system consists of not only electrical devices, but also mechanical components etc. Thus, motion control demands more advanced control strategies with higher degree of reliability. In most of the motion control problems, the processes under consideration are electrical machines controlled by some power electronic equipment.

The aim of this thesis is to present several digital motion control techniques that could be applied in the area of electrical drives. Before going deeper into the contents of this thesis, it is useful to have a perspective on the gradual development of digital motion control area, its present situation and the future development. From the initial introduction, it is already clear by now that motion control area is a result of the union of power electronics and automatic control theory (see Figure 1.1).

The invention of thyristors can be considered as the foundation stone of the area of power electronics. This triggered the development of sophisticated commutation circuits, as the thyristor does not have turn-off capability. This is the first phase of the development of power electronic area.
Power electronic area went through an extensive expansion during the second stage of the development, when large and small fast-switching power devices and microprocessors were introduced into the market in mid 70’s. The introduction of microprocessors into the field enhanced the possibilities of incorporating a vast amount of control theory in power electronic systems.

The key feature in the third phase of the development is the high-frequency power electronic devices. Device size can be made smaller by increasing the switching frequency. The introduction of Digital Signal Processors (DSPs) in place of microprocessors was another big step forward in this phase, which can be located in the late 80’s. The three phases of the development of power electronics are shown in Figure 1.2.

![Figure 1.2: Development of power electronics through three major phases](image)

The immediate interest of this thesis is not to go deep into the motion control problems from the power electronics point of view. Instead the approach will be to contribute to the digital motion control area from the applied control theory. Thus, the past and the recent trends of automatic control will be presented in greater detail.

### 1.1 Automatic control - past and present

Automatic control systems can now be found everywhere in day to day life, both in industry and domestic applications. The field by now (up to the beginning of the new millennium) is about 50 years old and is today well established. The development of automatic control is closely connected to the industrial revolution. Typical examples are the inventions of new power sources and new production techniques. Newly invented power sources had to be properly controlled, while new production techniques had to be kept operating smoothly with high product quality.

The centrifugal governor is one classical example from this era of automatic control, which has become a symbol of the field. This was mainly applied in the steam power generation and is still used in some industrial applications. Some other typical application examples from the early stages of automatic control are also worth mentioning here. Industrial process control and automation that occurred in parallel to the invention of governors is one such example. The classical PID controller still widely used in the industry is a result of this. Ship steering was another application problem, which later led the way to the invention of autopilots. Autopilots in flight control were another development that took place as early as the beginning of the century. In the meantime another
breakthrough came from the telecommunication area. This was the invention of feedback power amplifier, which has better linear operation characteristics.

With this brief description it is clear that the field of automatic control developed simultaneously as a result of attempting to solve some application problems in several different areas. The information sharing in between the application areas was not so effective. This clearly reflects from the following quotation in [2].

“Routh and Hurwitz were not aware of each other’s contributions and neither knew about the fundamental work on stability by Liyapunov”

The Second World War also motivated intensive research and development of automatic control techniques in the United States and across Europe including former Soviet Union.

1.1.1 Digital control - the second wave

Space missions started in Russia with the Sputnik in 1957 and later in the United States made a break through in automatic control. It was the first time that the computers were used for control applications [2]. Thus, physical quantities such as current, voltage, pressure, temperature etc., treated in analog form before, could then be treated as numerical figures inside a computer. This brought about an extensive flow of ideas from mathematics into the field of automatic control. The scientists were also farsighted in using advanced mathematics extensively in formulating design strategies.

This brought up new concepts such as optimisation, stochastic control, identification, adaptive control, predictive control and various other control methods.

1.1.2 Current status

The field of automatic control today is a well-established technology, which has a wide range of application areas. Some of them worth mentioning are

- Generation and distribution of electricity
- Process control in industries
- Manufacturing and robotics
- Medical area
- Various means of transportation
- Structural stabilisation and control of heating, ventilation and air conditioning in buildings
- Materials
- Instruments
- Entertainment

Whatever the application area, the approach of a control engineer nowadays to the particular problem is governed by a number of disciplines established inside the automatic control field itself. They can be briefly described as follows.

(a) Systems theory

All real systems that have to be dealt within automatic control problems can be classified into one of the following system descriptions. These are, linear systems, non-linear systems, stochastic systems, discrete-time systems, time delay systems, distributed parameter systems and decentralised systems. To describe the system behaviour, measures such as stability, observability, controllability, robustness, sensitivity, system structure etc are used.
(b) Modelling and identification

Mathematical models of systems are very important in any automatic control application. If the modelling is completely based on the physics of the particular system, it is called white box modelling. Else, it can be done by conducting experiments, which is called black box modelling. Grey box modelling is the method that uses the physics and experiments in the process. With the maturity of the systems theory, a new sub-field within automatic control called system identification was born. Today it is a well-established discipline inside automatic control area.

(c) Design

For a given process, finding a suitable controller configuration that can tackle either the servo problem or the regulation problem by following a systematic approach, is called as controller design. A lot of creativity has been shown in this area by control engineers. Most of the available methods can now be found in the form of standard textbooks and are also taught at undergraduate and postgraduate levels.

(d) Learning and adaptation

Processes that are being controlled have the tendency to change their properties and thereby dynamics with ageing. Systems that learn about such changes automatically and re-tune the controllers are very handy to have in many applications that need higher reliability throughout their operating life. Learning and adaptation is the answer for such application problems from the automatic control community.

(e) Computing and simulation

Automatic control has been tightly linked to computing throughout its development. Analog computing that was used in the beginning was replaced by digital computing later. Today there are so many mathematical computation tools specially meant for control system analysis and simulation. Examples of such tools are Matlab/Simulink, Matrixx. Some computer algebra software tools such as Maple, Mathematica etc. are also available.

(f) Implementation

This again is a very important aspect in control engineering. Implementation issues form the bridge between the theory and real application. Several practical problems that may sometimes be impossible to incorporate in the theoretical study will have to be overcome at the implementation stage. These implementation aspects are rarely addressed in highly theoretical approaches to problems in automatic control area. However, such research work has a higher possibility of producing novel theoretical contributions to the field. Yet, most of the practical problems associated with the implementation of the methods are not theoretically dealt with in many cases. It is again appropriate to quote one of the great scientists in automatic control, Karl Åström from [2].

"Many important aspects on implementation are not covered in textbooks. A good implementation requires knowledge of control systems as well as certain aspects of computer science.

It is necessary that we have engineers from both fields with enough skills to bridge the gap between the disciplines. Typical issues that have to be understood are windup, real-time kernels, computational and communication delays, numerics and man machine interfaces. Implementation of control systems is far too important to be delegated to a code generator."
Lack of understanding of implementation issues is in my opinion one of the factors that has contributed most to the notorious GAP between theory and practice."

This thesis is meant to be a contribution that emphasises control aspects all the way down to the implementation stage, starting from the basic control theory.

(g) Commissioning and operation

Right after a control system has been designed and implemented, it has to be commissioned. Several other interesting problems may occur at this stage. Treating these issues is frequently taken up in today’s control engineering forums. In fact, a new sub-area has emerged, known as fault detection and diagnosis.

Following the above brief overview of the development of the area of automatic control so far, a quick look at available standard controller design methods will be presented in the next section.

1.1.3 Brief overview of the available methods

As the automatic control area developed and especially after discrete time control systems were initiated, a steady flow of advanced mathematics poured into controller design. Thus one can find several control design approaches that have also developed as independent sub-fields within the automatic control domain. Some of them to mention are,

- Lead-lag compensation
- PID control
- Optimal control
- Adaptive control
- Robust control
- Non-linear control
- Model predictive control

These areas are very well developed today so that it is relatively easy to find good quality textbooks and publications covering each control approach [3, 4, 5, 6, 7, 8, 9, 10]. It must also be mentioned here that except for the basic control methods such as lead-lag compensation and PID control, all other methods mentioned above require a digital control platform for implementation. In fact, the whole automatic control area developed initially through analog controllers. However, even for simple motion control problems, introduction of digital control can give rise greater flexibility and better performance (even though with some limitations) [11]. A detailed comparison of merits and demerits of the two technologies giving more emphasis on motion control area can be found in [12].

The present status of motion control will be discussed in the next section.

1.2 Present status of motion control

As explained earlier, motion control has emerged from two already established fields, power electronics and automatic control theory. This makes motion control a very competitive area, covering every motion system from micro-sized to macro-sized. It has to deal with different types of actuators, mainly electrical. Some of the electrical actuators are electromagnets, DC/AC servomotors, linear motors etc.

Since motion is a mechanical quantity, any motion control system has got a mechanical component in the process of interest. As an example, in robot manipulators even if an electrical motor is the
actuator, the complete system consists of several mechanical parts that are moving and the dynamics of all components in motion have to be taken into account. The recent trend in this type of applications is to consider the dynamics of the complete system as one model in the control design [1]. This has reduced the gap between the electrical and mechanical design considerations of a particular motion control system. Developments in system identification methods have also contributed in promoting this approach. This is because the use of system identification methods makes it easier to find the input output model of the complete Electro-mechanical system (i.e. electrical actuator and mechanical components). A typical modern motion control system is shown in Figure 1.3, which elaborates this combined concept graphically.

![Figure 1.3: A typical modern motion control system](image)

However, there are two major difficulties that modern motion control faces. One is the robustness of the control algorithms against the parameter variations of the complete system. Unlike in the case of electrical actuators, there can be other mechanical parameters such as friction, damping, wind resistance etc. that might also change with time and can have an impact on the overall controller performance. Thus the degree of parameter variation is much higher in the actual motion control application. On-line parameter estimation and adaptation is one solution to this problem.

The other difficulty is how to make the control system intelligent so that it learns from its own past experience or from the experience of an expert. Auto tuning methods [13], adaptive control techniques [6], fuzzy and neural network control systems [25] have shown some positive progress towards this direction.

With this broad overview of the history and the present status of digital motion control, it is now suitable to narrow down the focus of the discussion on to the two motion control problems related to electrical drives, which is the main topic in this thesis.

1.3 Two motion control problems under consideration

As mentioned earlier, the aim of this thesis is to present several digital motion control techniques applicable to the electrical drives area, while tackling two different applications. These applications are active magnetic bearings and high-speed permanent magnet synchronous motors. Each application carries two different motion control problems. These are precision motion control, which is applied for precise eccentric rotor positioning using active magnetic bearings and sensorless
control, which is used in sensorless control of high-speed permanent magnet synchronous motors. Both control problems are demanding and are often met by control engineers in a wide range of applications.

1.3.1 Precision motion control

Typical high precision applications include multi-axis co-ordinate measuring machines and machining of optical components [14, 15, 16]. High precision systems usually suffer from mechanical resonance that limits their closed loop bandwidths. This puts very high demands on the performance of high precision servo systems. Therefore, special digital motion control algorithms are required in order to achieve expected accuracy and speed [17, 18].

However, the actuator used in high-precision motion control applications does not have to be a servomotor. Magnetic levitation is also used in some high precision applications. One example is the magnetically levitated high accuracy positioning system presented in [19]. There, the order of positioning accuracy expected is about 30 nm. Actively controlled magnetic levitation is used to achieve the expected precise motion. Similar precision motion control applications employing magnetic levitation can be found in [20] and [21]. In such motion control applications, electromagnets are used as the actuators for the mechanical system.

Magnetic levitation principle is used in electrical drives to replace the mechanical bearings of the rotor and levitate it in air during rotation. Such electromagnetic actuators are called Active Magnetic Bearings (AMB). These are based on the idea that magnetic levitation of the rotor is actively controlled in these bearings. In the first application that is considered in this thesis, such AMBs are used to move the rotor to arbitrary radial positions in the air gap. This is done to study the acoustic noise phenomena in electrical machines that originate from rotor eccentricities in a separate project. This motion control problem will be briefly explained in section 1.4.

1.3.2 Sensorless motion control

The whole concept of feedback control is based on sensing the controlled variable (or state) of the process and feeding that information back to the controller. The task of a sensor or a transducer is to accept energy from one part of the system and emit it in a different form to another part of the system. One simple example is a thermocouple that converts heat energy into electrical energy by inducing a potential difference between the two bimetallic junctions [26]. All such sensors add some noise component into the measurement and cause some degree of deterioration in the measurement. The use of sensors needs additional cabling in the system also. Better sensors always means more cost, which will add up to the overall cost of an industrial product. Thus, reducing the number of sensors used in a control system is of utmost importance and has been a promising research area in recent years.

With the introduction of computer control to this area, there has been a vast influx of mathematics into automatic control. This has enabled modelling of the processes that are controlled and estimating some states using models running in parallel to actual processes. Such estimates can always replace direct measurement from sensors and thereby eliminate some sensors from the system.

Eliminating the shaft mounted speed sensor of an electrical drive is a very important achievement in a variable speed industrial drive. Speed sensorless operation of almost all types of AC drives (Induction, Permanent magnet synchronous, DC, Brushless DC, Switched reluctance etc.) has been an interesting area of research for a long time. This has been sighted as one of the future trends in the
new millennium in [22]. The second application considered in this thesis is sensorless control of Permanent Magnet Synchronous Motors (PMSMs) for high-speed applications. This will also be briefly outlined in section 1.5.

1.4 Active magnetic bearings for precise eccentric rotor positioning

The first motion control application considered in this thesis will be very briefly outlined in this section. The basic concept of Active Magnetic Bearing (AMB) can be described with the help of the suspended globe example. The idea here is shown in Figure 1.4 and the task is to stabilise the position of the metal sphere and maintain it at a specified distance below the electromagnet by means of a position feedback control system. This concept can be extended to make Active Magnetic Bearings. The purpose of AMBs is basically to replace the mechanical bearings on the rotor shaft of an electrical machine to achieve complete levitation of the rotating body. This requires two radial bearings for the stability in the radial direction and one axial bearing for the stability in the axial direction.

![Figure 1.4: Basic concept of AMB – "The Suspended Globe"](image)

An accurate method of positioning the rotor in the stator bore is essential for the study of the effect of rotor eccentricity on acoustic noise in a standard electrical machine. One way of doing this is to use eccentric rings with mechanical bearings to achieve the required eccentricity [23]. Instead of using mechanical bearings with eccentric rings, AMB actuators can be employed to carry out the noise study, provided that they are capable of precisely positioning the rotor shaft at any predefined eccentric position. Such a system essentially is superior to eccentric ring system mainly because AMB is able to mechanically de-couple the rotor from the stator. When controlled digitally, AMB system can fulfil a lot of put forward demands by the acoustic noise study of electrical machines.

Digital motion control techniques that can be incorporated in elevating the rotor from the resting position to the centre of the air gap, eccentric positioning of it and maintaining the eccentric position will be presented under the first motion control application.

1.5 Sensorless control of PMSMs for high speed applications
"Sensorless control of PMSMs for high-speed applications" is the second motion control problem that will be dealt with in this thesis. Conventional PMSM drives employ a shaft-mounted encoder or a resolver to identify the rotor flux position. This maintains the synchronism that is an essential requirement in this type of drives. But on the other hand, in most applications the presence of an encoder or a resolver causes several disadvantages. They are additional cabling cost, a higher number of connections between motor and controller, noise interference and reduced robustness. These are the reasons that raise the need to develop sensorless control schemes for PMSM drive systems.

Elimination of the direct rotor position and speed measurement means that one obviously has to use as much information as possible contained in supply voltages and currents of the motor in order to extract the rotor position information. This can be done by having suitable sensors fixed closer to the control and power electronics (inverter). Thus, the number of cable connections between the control hardware and the motor is reduced to a minimum of 3 (three supply lines). In fact, this can be used as a definition for sensorless operation of an AC machine. It can be given as follows.

“If complete variable speed operation of an AC drive is achieved, while keeping the three power conductors as the only galvanic connection between the motor and the control electronics, that operation can be called as sensorless operation of the particular drive.”

The concept is graphically depicted in Figure 1.5. Different sensorless control strategies for all types of AC drives have been suggested in many publications. However, less computationally heavy, parameter insensitive and more robust sensorless control algorithms are still in demand.

In the second application considered in this thesis, a sensorless control algorithm originally suggested in [24] will be further investigated and later modified to have tracking capability of speed ramps. In addition to this, most of the implementation issues regarding the commissioning of a sensorless controlled PMSM drive will be theoretically dealt with. Thus, the outcome of this work will be a complete sensorless control drive strategy. In fact, most of the concepts that are presented here on
issues such as current control, speed control, output saturation of controllers and compensation for inverter non-idealities can be considered as general for many types of variable speed AC drives.

1.6 Overview of the thesis

As mentioned before, this thesis gives its emphasis on control aspects all the way down to the implementation stage, starting from the basic control theory. The typical viewpoint one must have on this approach of the thesis is further clarified from the following quotation of Karl Åström found in [2].

“A long experience with journals and conferences has shown that it is very difficult to get good applications papers. The engineers, who really know about the applications, do not have the time or the permission to publish. Many of those who write have only a superficial knowledge about the applications. This sends distorted signals in all directions. There are occasional efforts with special issues of journals, where really good applications papers sometimes appear. We need those badly for better education of the next generation of control engineers.”

This thesis has two parts. The first part is on the digital control of active magnetic bearings for precise eccentric rotor positioning. The second part is on sensorless control of permanent magnet synchronous motors for high-speed applications. Both these application examples can be seen as two different test platforms used to verify the digital motion control strategies presented in the thesis. Hence, the main theme of the thesis is presenting several digital motion control techniques that can be applied not only in the two application examples discussed here, but also in many other motion control applications with suitable modifications. All motion control techniques are suggested as contributing factors in achieving the demanded control objectives in each application. Due to this reason, the two application examples will be presented as separate application reports under the title “Digital Motion Control Techniques for Electrical Drives”.

The thesis will branch out after this introduction to the thesis and the application example, “Digital Control of Active Magnetic Bearings for Precise Eccentric Rotor Positioning” will be the first part. All the work done under the AMB application will be presented in this part of the thesis.

Research on “Sensorless Control of Permanent Magnet Synchronous Motors for High-Speed Applications” is the second part of the thesis. This part of the thesis will present important information on how to implement a variable speed AC drive system practically according to the specifications. A long list of practical problems in an AC drive has been studied under this work. Thus, this application example does not limit its scope merely to sensorless control and the information given can be very useful for variable speed AC drives in general.
Reference


Part I

Digital Control of Active Magnetic Bearings for Precise Eccentric Rotor Positioning
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### List of symbols

- $M$ - total mass of the rotor (kg)
- $M_1$ - equivalent mass of rotor (kg)
- $J_1$ - equatorial mass moment of inertia (kgm$^2$)
- $d$ - journal diameter (m)
- $l_{12}$ - distance between radial AMBs (m)
- $z_1$ - distance to AMB 1 from the end of the rotor (m)
- $z_2$ - distance to AMB 2 from the end of the rotor (m)
- $t$ - pole width (m)
- $p$ - number of poles of a radial AMB stator
- $l$ - length of the radial AMB stator (m)
- $\Delta\alpha$ - angle of a pole arc (rad)
- $m$ - number of parallel branches in the electrical circuit
- $K_l$ - inductance factor (Hm)
- $\delta$ - air gap (m)
- $k_p$ - poles number factor
- $r$ - resistance/winding of radial AMB (Ω)
- $w$ - turns/winding of axial AMB
- $\mu$ - permeability (H/m)
- $r_a$ - inner radius of inner pole - axial bearing (m)
- $r_p$ - inner radius of outer pole - axial bearing (m)
- $R_a$ - outer radius of inner pole - axial bearing (m)
- $R_p$ - outer radius of outer pole - axial bearing (m)
- $i_{1c}$ - bias current of upper coil at zero eccentricity (A)
- $i_{2c}$ - bias current of lower coil at zero eccentricity (A)
- $I_{1c}$ - bias current of left winding of the axial bearing (A)
- $I_{3c}$ - bias current of right winding of the axial bearing (A)
- $C_A$ - power amplifier gain
- $C_i$ - current feedback gain
- $C_T$ - position sensitivity (V/m)
- $C_a$ - negative stiffness due to effect of axial bearing (N/m)
- $C_r$ - negative stiffness due to radial bearing (N/m)
- $\beta_{21}$ - ratio of bias currents
- $\beta$ - bias current ratio factor
1. Introduction

Bearings are closely linked with all types of mechanical movement. The primary task of a bearing is to ensure smooth, low friction motion of the moving rigid body to the required degree of freedom, while withstanding the forces exerted on it from the rigid body. Mechanical engineers have developed various types of mechanical bearings to suit different applications. The word mechanical here means that the bearing of interest always has a frictional contact with the moving body and the contacting surfaces need lubrication to minimize wear and tear, friction, noise and heating due to friction. This type of mechanical bearing is disadvantageous in applications, where clean and dry environments are preferred as well as high speeds of motion is required. One example is the electrical motors used in modern textile industries, where rotational speeds of around 100,000 rpm have to be achieved. The development of electromagnetic technology, together with automatic control theory has resulted in alternative solutions such as magnetic bearings. The use of magnetic forces to overcome the forces exerted by the moving mechanical body is the prime idea of magnetic bearings. In the early stage, repulsive force between similar poles (north-north or south-south) of two carefully designed permanent magnets were used to keep the moving body away from the surface that will have to withstand the reaction forces. This concept is called passive magnetic bearings. The technology developed further and the passive magnets were replaced by electromagnets, which have varying force-current characteristics for this purpose. This enabled the engineers to have active control over the current flowing in the electromagnets used in the bearings. The ability to have active control over the performance gave these bearings the name “Active Magnetic Bearings”.

The concept of “suspended globe” that lead the way to Active Magnetic Bearing (AMB) was presented in the beginning of this thesis (see Figure 1.4 of the “Introduction to the thesis”). Since the force-current characteristic of the electromagnet has a quadratic relationship, the problem becomes a non-linear one. However, a simple controller design based on a system linearised about the required operating point can still handle this stability problem [19].

The extension of the concept to replace the mechanical bearings of an electrical machine is graphically depicted in Figure 1.1. Each radial bearing has two axes (X, Y) for rotor movement. Thus, a control loop is required for each axis to achieve complete stability in the radial direction. This demands four independent control loops are required for the two radial bearings. Another control loop is required for the axial bearing. A complete motor drive with AMBs will therefore have five independent control loops to operate the three AMBs. Both analog and digital control strategies can be used to stabilise the rotor shaft at the centre of the air gap. The objective of this part of the thesis is to present several digital motion control
techniques that can be used to elevate the rotor and position it precisely at a required position (can also be eccentric) in the air gap.

### 1.1 Use of AMB as an actuator

The history of using AMBs as actuators to accomplish specific tasks other than just replacing the normal mechanical bearings has a long history. Due to feedback control, the bearing currents have direct reactions to the external forces acting on the rotor. This makes it possible to gather information on the forces acting on the rotor of a certain drive system by monitoring the bearing currents. The ability to generate forces acting on the rotor through active control is also a big advantage of using AMBs as actuators. Another important advantage is the possibility of de-coupling the rigid radial connection between the stator and the rotor, which normally exists in a conventional electrical machine with mechanical bearings. A lot of work done has been reported, where the above properties of the AMBs have been exploited to achieve particular goals. Some examples of such approaches are the analysis of the dynamic response of a rotor [1] and the use of magnetic bearings as a force generator to simulate various forces acting on a rotor under working condition [2].

The main goal for the development of the digitally controlled AMB presented in this report is to develop a high performance AMB system to be used in the study of acoustic noise in a standard induction machine. The special features mentioned above makes AMB attractive for such studies.

#### 1.1.1 AMB actuator for noise study – the approach of this work

The study of the effect of rotor eccentricity on acoustic noise in a standard electrical machine requires an accurate method of positioning the rotor in the stator bore. One such method is to use eccentric rings with mechanical bearings to achieve the required eccentricity. This method is discussed in [3]. Instead of using mechanical bearings with eccentric rings, AMB actuators can be employed to carry out the noise study. The AMB based test rig is superior to eccentric ring system for the following reasons:

(a) **AMB is able to mechanically de-couple the rotor from the stator.**

Since complete levitation of the rotor is achieved, there will not be any mechanical contact between the machine housing and the rotor. This is meant by “mechanical de-coupling”.

(b) **AMBs offer the possibility to investigate the effect of the stiffness of the rotor suspension system on acoustic noise (possibility of varying stiffness).**

A conventional mechanical bearing also has some finite stiffness of the rotor suspension, which depends mainly on the material properties used and the construction. Hence, the stiffness can not be varied. On the contrary, stiffness of the rotor suspension system of a magnetic bearing varies, if the controller parameters are changed. This feature can be exploited to do the above study.

(c) **AMBs offer the possibility to achieve static eccentricity as well as dynamic eccentricity.**

Using appropriate digital motion control techniques, it is possible to force the mechanical axis of the rotor to be at a pre-defined eccentricity (\(X_e, Y_e\)), while the machine is rotating. Such an eccentricity is called as static eccentricity. It is also possible to let the rotor follow a circular eccentric path (of radius \(\sqrt{X_e^2 + Y_e^2}\)) during rotation. This is called a dynamic eccentricity.
These advantages highly motivate the use of AMB as an actuator for the study of acoustic noise phenomena in standard electrical machines.

1.2 Brief overview of this work

The main objective of this work is to develop a test rig for the acoustic noise study of standard induction machines. To achieve this objective, the possibility of developing a digital signal processor based digital control system will be investigated. The objective of this approach is to provide a user-friendly environment for the noise study [48, 49, 50]. The AMB system is initially modeled using first principles with the help of manufacturer data. Later, controller designs are carried out in the simulation level, before implementing them practically. Successful implementation of the digital controller opens up several opportunities for further research. One of them is the model validation through system identification tests. Under this, the mathematically developed AMB model is validated in both time domain and frequency domain by conducting system identification experiments. Periodic disturbances occur in the position feedback signals, when the motor is rotated. An adaptive periodic disturbance cancellation method is employed to overcome this problem. This further improves the performance of the complete control strategy. Thus, the digitally controlled AMB system offers several capabilities that can lead to very important findings in the area of acoustic noise in standard induction machines.

1.2.1 Layout of the report

This part of the thesis has seven chapters.

Chapter 2 is allocated to give the details of the AMB system used in this work. It also explains the digital signal processing environment used for the controller design and implementation.

Chapter 3 describes the modeling of the components in AMB system including the controller electronics. This leads to build up a simulation model of the complete AMB control system.

Chapter 4 presents the design of the digital controller and the introduction of anti wind-up and bumpless transfer techniques to overcome the problems encountered in the implementation stage.

Chapter 5 focuses on the model validation of the AMB actuator from system identification experiments.

Chapter 6 presents the periodic disturbance cancellation method used together with a description of how to improve the performance of the existing analog controller of the AMB test rig using an outer digital control loop.

Chapter 7 contains some concluding remarks with suggestions for further continuation of research in this area.
2. Experimental set-up

As mentioned in the introduction, the aim of this work is to develop a test rig for the acoustic noise study of standard induction machines. The task of the AMBs is to replace the normally used mechanical bearings in order to achieve the mechanical de-coupling between the rotor and the stator, while having controllability over the dynamic stiffness of the rotor suspension. It is therefore important to keep the overall construction of the test motor as close as possible to the construction of a standard induction machine. The solution is to use a standard induction machine with modifications to the end shields to accommodate the magnetic bearings. Modifications to the induction motor were made by an AMB manufacturer (Pskov Engineering Company in Pskov, Russia). The modified system has a complete analog control system provided by the manufacturer. However, due to several advantages that will later be explained in Chapter 4 under digital controller design, it was decided to replace the analog control system using a digital signal processor based digital controller. Necessary modifications to the hardware electronics to accommodate the digital control system were made in the laboratory. In this chapter, the important information about the induction motor and the AMBs used to replace the conventional mechanical bearings will be given. Also presented will be the details of the digital signal processing system employed.

2.1 Specifications of the induction motor

The ratings of the induction machine are as follows,

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>MBT 160 L by ABB (Asea Brown Boveri Ltd.)</td>
</tr>
<tr>
<td>No. of poles</td>
<td>4</td>
</tr>
<tr>
<td>Rated power</td>
<td>15 kW</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1453 rpm</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>380/660 V</td>
</tr>
<tr>
<td>Frequency</td>
<td>50/60 Hz</td>
</tr>
<tr>
<td>Rotor weight</td>
<td>35 kg (before modifications)</td>
</tr>
<tr>
<td>Air gap</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>Air gap flux</td>
<td>0.93 T</td>
</tr>
</tbody>
</table>

Figure 2.1 Induction motor with magnetic bearings
The end shields of the motor are replaced by new end shields that contain magnetic bearings. Complete levitation of the rotor is achieved by means of 2 radial bearings and one axial bearing as explained in the first chapter. Figure 2.1 shows the appearance of the motor with modified end shields.

### 2.2 Brief description of the AMB system

Each newly designed end shield accommodates one radial bearing and one half of the axial bearing. The rotor package is left as it is in the standard machine in order to preserve the same electromagnetic characteristics between the stator and rotor. The rotor shaft is modified (elongated) in order to accommodate for the laminations of the two radial bearings and the inductive position sensors. Symmetry of load distribution is preserved during the modification of rotor shaft. This has enabled the designer to use radial AMBs with similar specifications at both ends. Detailed mechanical drawings regarding the modified rotor and the AMBs together with the specifications of the magnetic bearings provided by the manufacturer can be found in Appendix A. Only the physical arrangement of the system will be presented here with the aid of some photographs. A description about the control electronics and the modifications made to accommodate the digital signal processing system will also follow.

#### 2.2.1 Modified end shields and rotor shaft

![Modified end shield](image)

*Figure 2.2 Modified end shield*

Each end shield contains the stator of one complete radial bearing. The complete stator of the axial bearing has two annular electromagnets. Each of them is fitted on the two end shields. The stators of the inductive position sensors incorporated in sensing the radial and axial motions are also fitted in the end shields. Figure 2.2 shows the inside view of an end shield.
The rotor shaft is modified to accommodate the laminations of the radial AMBs and position sensors. Figure 2.3 shows the inside view of the new rotor shaft.

### 2.2.2 Control electronics

It is important to have an overview about the hardware electronics used to control the AMB system and the modifications made for it. Analog control system provided by the manufacturer for the AMBs is powered by ±50 V supply voltage. The control electronics have been modified so that the user can disable the analog PID controller from the system and connect the digital control system whenever required. This has enabled the user to control the same process either by analog or digital controller, depending on the requirement. Block diagram of the complete control loop to control one axis of a radial bearing or the complete axial bearing is shown in Figure 2.4. The digital controller block shown in Figure 2.4 is implemented in a DSP system. This is explained in detail in Section 2.3.

![Figure 2.3 Modified rotor shaft](image)

**Figure 2.3** Modified rotor shaft

Each control loop in this system is assumed to be independent in two ways. When a single bearing is considered, a movement along X-axis of the AMB does not influence the position sensor signal of Y-axis (i.e. no cross coupling). The other assumption is any movement that take place at one AMB does not influence the position sensor signals of the other bearing. With these assumptions, it is possible to implement five independent control loops for the
control purpose of the complete AMB system (two control loops for each radial bearing and one loop for the axial bearing). Apart from these five independent analog control loops, hardware electronics required to generate the excitation carrier frequency for the inductive position sensors [18] must also be included in the system. All these are fabricated on a single circuit board and delivered as a compact box shown in Figure 2.5.

![Hardware electronics for the AMB system](image)

The front panel that can be seen in the figure is for the connection of I/O channels of the DSP system and one can also see the switches used for the isolation of the analog controllers. On the top panel one can find the taping points to measure all bearing currents, position sensor signals and the reference inputs for position control loops.

The above brief introduction of the test rig will now be followed by a description of the digital signal processing system used for the control purpose.

### 2.3 Digital Signal Processing System

Selection of the Digital Signal Processing (DSP) system for the AMB application was done giving emphasis to the following important aspects:

- Sampling frequency going to be used for the AMB application
- Performance of the A/D and D/A converters
- Convenient programming platform
- Ability to perform real-time tuning of controller parameters
- On-line monitoring capability of variables in the control algorithm
- Possibility for future expansions

Typical sampling frequencies used for AMB applications fall around 10 kHz per each control loop. A/D converters without internal iterative digital filters (known as Delta-Sigma technique) were preferred in order to avoid group delays of the sampled signals due to internal filtering. Five independent control loops of the AMB system required five A/D and
Digital Control of Active Magnetic Bearings...

D/A channels. Hardware system was selected to meet those demands. The high-level programming platform of the DSP, Matlab\textsuperscript{TM}/Simulink\textsuperscript{TM}/Real-Time Interface (RTI) that will be explained in the Section 2.3.2 proved to be a very effective and user friendly software environment. The same system was later used in the research on sensorless control of permanent magnet synchronous motors with some expansions. Thus, giving emphasis on future expansion possibilities proved to be a farsighted move. Hardware environment will be explained in the following section.

2.3.1 Hardware configuration of the DSP system

The basic arrangement and the specifications of the system will be presented in this section [8,9,10]. Figure 2.6 shows the configuration diagram of the hardware set-up. The digital control system used to control the five loops in the AMB system is implemented in a DSP system, which is based on Texas Instrument TMS320C40 processor running at 50 MHz. An Analog to Digital Conversion (ADC) board with 5 independent channels and a Digital to Analog Conversion (DAC) board with 6 channels are also employed. The host PC is equipped with a Pentium 100 MHz processor and 16 MB RAM. The important specifications are given below.

(a) DS1003 DSP board with TMS320C40

- 50 MHz
- 512 KB Memory

(b) DS2001 ADC board

- 5 parallel 16 bit ADC channels with simultaneous sampling
- 5 µs conversion time
- ±5 V & ±10 V software programmable input ranges
- 1 MΩ input impedance

(c) DS2101 DAC board

- 6, 16 bit DAC channels
- ±5 V, ±10 V, 0...10 V software programmable output ranges

![Figure 2.6: Hardware arrangement](image-url)
± 5 mA max. output current

2.3.2 Software environment for the DSP system

The software environment for the DSP system offered by dSPACE™ together with Matlab™ and SIMULINK™ is a powerful one as it has a high level controller design, automatic code generation, real-time implementation and experimental data analysis. Software modules from three companies are associated with the overall environment. The complete set of software modules is listed below,

- TI cross C compiler  
  *Texas Instruments Inc.*
- RTI40  
  *dSPACE*
- TRACE40W  
  *dSPACE*
- COCKPIT40W  
  *dSPACE*
- MTRACE40  
  *dSPACE*
- MLIB  
  *dSPACE*
- MATLAB  
  *MathWorks*
- SIMULINK  
  *MathWorks*
- RTW (Real Time Workshop)  
  *MathWorks*

The complete software-programming platform is easily explained with the aid of Figure 2.7. For programming the DSP, the user can either use C language, which is more conventional or Matlab™/SIMULINK™/RTI platform [11, 12]. In the second method, the controller can be built in SIMULINK™ platform by using the SIMULINK™ block models together with the additional blocks provided in RTI software by dSPACE™. This programming process becomes much faster. Since the second option was used in this case, Matlab™/SIMULINK™/RTI platform will be explained in detail.

![Figure 2.7: Complete software environment](image)

Once a controller is tuned for the mathematical model of the real process in the simulation level, it can directly be used to build up the real time controller block model. The dSPACE™ has provided another block library called DSLIB that contains block models of all I/O devices. Therefore SIMULINK™ model for the real time controller is a combination of
appropriate I/O devices from DSLIB and the controller block extracted from simulation model built in SIMULINK™. This SIMULINK™ model is then used to generate the C code for the DSP. This is done by a combination of software tools. They are Real Time Workshop™ (RTW) [13], Real Time Interface (RTI) [14] and Texas Instruments C compiler [15].

2.3.3 On-line parameter tuning using COCKPIT™

There can be modeling errors between the real process and the mathematical model used for simulation, which demands fine-tuning of real time controller parameters. This is done in Windows environment using a software tool called COCKPIT™ [16]. It can be operated on-line, while the real time process is being controlled by the DSP.

The dSPACE’s COCKPIT™ program is an instrument panel, which provides graphical output and interactive modification of variables, for any application running on a DS1003 DSP board. It is possible to modify or display all variables represented as single-precision floating point or integer variables in the processor board’s memory. Output is performed with instruments like numeric displays, gauges, or moving bars. Variables can be modified with sliders, various buttons, or numeric input from the keyboard. COCKPIT™ gives the user a pool of such controls, which can be used to define an application-specific layout. However, the COCKPIT™ program is not intended for real-time data acquisition purposes like the dSPACE’s TRACE™ module [17] (discussed in the next section), i.e. DSP access is not performed at fixed time steps.

![Figure 2.8: Implementation of COCKPIT on the DSP](image)

When used, COCKPIT™ downloads some DSP code to the processor board, which stores data corresponding to each COCKPIT™ instrument used in a table on the DSP memory. This table can be read by COCKPIT™ as a block and the values of data are then assigned to the respective controls. The DSP routine also executes write operations as soon as one is requested by a COCKPIT™ control. A block data can be transferred much faster to the host PC. This is faster than trying to access many randomly distributed memory locations. Other programs like TRACE™, which need accessing the DSP memory, while running parallel to
Cockpit™ are also benefited from this feature. Figure 2.8 graphically shows how the Cockpit™ mechanism that communicates with the algorithm running in the DSP.

2.3.4 On-line monitoring using TRACE™

When a digital control scheme is implemented, it is of great importance to be able to monitor the variables in the control algorithm, which is running in the DSP. For example to check whether the integrator windup is taking place, one has to have the ability to see the variation of the integrator output value of a digital PID controller. This requirement is met by another software tool called TRACE™ [17]. TRACE™ is capable of capturing the variation of an assigned variable over a defined period and displaying it on the host PC. If a later analysis is also required, then TRACE40 has an acquisition mode, by which one can save data as *.mat files in the host PC. Analysis can be carried out later using Matlab™ and its toolboxes.

TRACE™ has the following features:

- Free-running or level-triggered mode with pre- and post-trigger
- Distinction between variables of several data types including static and dynamic data
- Variable TRACE™ capture sampling rates (down sampling)
- Automatic repetitive TRACE™ capture (automatic TRACE™)
- Endless TRACE™ capturing without data gaps (ONLY for slow processes)
- Freely configurable order, size and position of plots
- Presentation of signals as y-t or x-y plots

The TRACE™ mechanism is somewhat similar to that of Cockpit™. TRACE™ also has two different states on the DSP. First there is the inactive state, when TRACE™ is not performing data acquisition. The active state is entered, when TRACE™ has started data capture and the DSP is collecting the data in real-time. TRACE™ uses the DSP memory to install a piece of DSP code and to store the traced data until they are transferred to the host computer. Figure 2.9 shows the mechanism.

**Figure 2.9: Implementation of TRACE on the DSP**
The main limitation of this data acquisition method is the on-board memory size of the DSP board. Free memory space of the DSP board after the program down loading, is used to store the data temporarily. Once the acquisition is completed over the defined time interval, data is transferred to the hard disk of the PC through ISA bus. This technique imposes a limitation on the acquisition time length based on the free memory space. In other words, one has to consider the sampling period, memory occupied by the digital control algorithm and the number of variables required to be stored, when deciding the acquisition period. This is graphically shown in Figure 2.10.

![Figure 2.10: Memory limitations for TRACE on the DSP](image)

This is a brief overview of the DSP system used for the AMB application example in this thesis.
3. Modelling of the components in the AMB system

When working with real-time control systems it is essential that the design engineer does some preliminary design and simulation studies on any control strategy implemented on the real system. With such an approach one can always get rid of very basic problems that may occur such as instabilities due to wrong closed loop pole placement etc. System modelling becomes a key word in this respect. In order to build up a reliable mathematical model for the AMB actuator both the mechanical rotor suspension system and the hardware electronics have to be modelled. This chapter explains about the mathematical modelling of AMB actuator. A preliminary model validation will be made at the end of the chapter.

Generally speaking, the AMB is a Multi-Input Multi-Output (MIMO) system, where there are five inputs and five outputs. One approach to develop the mathematical model is to take the interaction of inputs on different outputs into consideration and develop a full MIMO model. In addition to the interactions among the five actuators of the AMB, there is also the presence of vibration modes of the shaft as reported in [3]. These modes are observed, even when the motor shaft is not rotating. Since the preliminary investigations with the existing analog control system shows that it is sufficient to control the five-input five-output system as 5 individual loops, instead of developing the full MIMO system, a Single Input Single Output (SISO) model for each actuator will be developed mathematically in this chapter. Developing the model for a single actuator was done based on the following basic assumptions.

1. The interactions (or coupling) from the other actuators are negligible
2. The effects of the vibration modes of the shaft are negligible.
3. The shaft is not rotating and hence the additional periodic disturbances due to whirling can be neglected.

3.1 Mathematical model of the rotor suspension system

Derivation of the mathematical model of rotor suspension system is a complex task. This is closely related with the design process of a particular AMB system. The modelling procedure must start with basic electromagnetic circuit equations, taking into account mechanical dimensions of the AMB stator and rotor. This process involves several equations based on empirical results also. The particular procedure is fully described starting from the first principles in [18]. With the basic assumptions made in the beginning of this chapter, modelling of one axis of the rotor suspension system of a radial AMB can be done considering the electromagnetic circuit shown in Figure 3.1. The procedure given in [18] was repeated to obtain the mathematical model of the rotor suspension, since it was not readily available in the information provided by the manufacturer. Appendix B describes all the key equations used in modelling and presents numerical state space models of each AMB actuator.

Due to the non-linear force-current characteristics of the electromagnets, the original model for the electromagnetic rotor suspension system becomes non-linear. This equation is linearised about the stable operating point, which is the centre of the air gap of the AMB stator that have the co-ordinate (0,0) with respect to (X, Y) axis system shown in Figure 1.1 of the introduction. In this section only the linearised state-space equation for one AMB actuator (see Figure 3.1) will be given.
The corresponding linearised state space description for the above rotor suspension (see Appendix B for the derivation) system can be given as

\[
\begin{bmatrix}
  0 & 1 & 0 & \frac{h_1}{M_1} & 0 \\
  0 & 0 & -\frac{\beta h_1}{L_1} & -\frac{(r+C_A C_c)}{L_1} & 0 \\
 0 & \frac{C_r + C_a}{M_1} & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
y \\
i_v \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
\frac{\beta C_A}{L_1} \\
\end{bmatrix}
U_c'
\]

\[y(t) = \begin{bmatrix}
C_T & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
y \\
i_v \\
\end{bmatrix}.
\]

where

- \(y\) - small displacement of rotor from the centre (equilibrium) of the air gap (m)
- \(i_v\) - small deviation from bias of the current in the equivalent single magnet (A)
- \(C_r\) - negative stiffness due to radial bearing (N/m)
- \(C_a\) - negative stiffness due to effect of axial bearing (N/m)
- \(h_1\) - current stiffness of upper electromagnet (N/A)
- \(M_1\) - equivalent mass of rotor (kg)
- \(\beta\) - bias current ratio factor
- \(r\) - resistance/winding of radial AMB (Ω)
- \(C_A\) - power amplifier gain
- \(C_i\) - current feedback gain
- \(L_1\) - actual inductance (after correction for leakage) (H)
- \(C_T\) - sensitivity of position sensor (V/m).

This linearised state space equation is the basis for the model-based analysis of the rotor suspension system throughout this part of the thesis. In all simulations this basic equation is used, while introducing non-linearities like saturation of power amplifiers, limits of the rotor movement due to finite air gap and so on. All simulation and experimental results that will be presented throughout this part of the thesis will be for the X-axis of AMB 1 (denoted as AMB1_X in the Appendix B). The corresponding state space equation with numerical values computed according to Appendix B is

\[
\text{Figure 3.1: Single magnetic bearing actuator}
\]
The negation of the control signal input in (3.2) is to represent a negation that takes place in the hardware electronics.

### 3.2 Mathematical model of the hardware electronics

The electromagnetic rotor suspension system described in the previous section, together with complete hardware electronics associated, form the closed loop control system for the particular AMB actuator. Mathematical modelling of hardware electronics is associated with tracing the complete electronic circuit of the AMB controller and building up the continuous domain models of each circuit module. Since the information received from the AMB manufacturer was very limited, it was necessary to trace the complete electronic circuit and to model each component separately. This was also necessary for two reasons. The first reason was that it enabled building up a more accurate simulation model of the existing system. Secondly, it was essential to have an accurate model of the hardware electronics, since some of the circuit modules (position feedback system and power amplifiers) were used in the digital controller implementation also. Appendix C gives the details of electronic circuit modules associated, together with the component values for one control loop. The axis system, which is inclined to horizontal by 45°, has enabled the designer to use identical components for all four radial bearing control loops. The fine-tuning is done by means of some potentiometers located in the position feedback module and controller module. Figure 3.2 shows a complete AMB actuator control loop in modular form.

All these hardware electronic modules are mathematically modelled using the information about component values in Appendix C. It is now assumed that all low-pass filters in the system have much higher cut-off frequencies than the cut-off frequency of the rotor suspension system. Then the significant modules that must be included in the complete closed loop system model reduce to the following:

(a) **Analog PID controller**

The analog PID controller has the transfer function given by

\[
G_{PID}(s) = 5.909 + \frac{433.22}{s} + \frac{(5.909)k.s}{s + 27272},
\]

where \(k(<1)\) represents a potentiometer attenuation of the derivative part that facilitates final tuning of the system. For the actuator AMB1_X, the value of \(k\) is 0.7.

(b) **Scale factor for the reference input**

At the summing point of the feedback signal with reference, some scaling of the reference signal takes place. This value is computed to be 0.2267.
All these models of the significant components can be put together now to formulate the closed loop system model of one AMB actuator for further analysis and simulation purposes.

### 3.3 Closed loop model of the complete system

The next step of the modelling process is to build-up the closed loop model of the complete system. This is done by combining the linear state space model for the rotor suspension system together with the analog PID controller and the input scaling. This closed loop system is graphically depicted in Figure 3.3.

To comply with the digital controller design and the programming of the DSP using Matlab™/SIMULINK™/RTI platform explained in the previous chapter, a SIMULINK™ model of the closed loop AMB actuator having the same structure was developed. This model was used for all simulation studies done afterwards. In fact, for the digital controller design only the analog controller of this model was replaced from the digital controller.

### 3.4 Preliminary validation of the mathematical model

The aim of the analysis presented so far is to build up a reliable mathematical model, which can later be used for the design and simulation purposes of the digital control strategies. It is therefore interesting at this stage to do some comparison between the actual AMB system and the mathematical model derived. This section is devoted for that purpose.
3.4.1 Frequency response of the PID controller

With the modifications done to the hardware electronics it was possible to isolate the PID controllers of each AMB actuator. Shown in Figure 3.4 (a) is the frequency response plot of the PID controller of AMB_X1 experimentally obtained using a dynamic signal analyser. The Figure 3.4 (b) shows the same response analytically obtained using the mathematical model of the PID controller described by (3.3).

![Frequency response plots](image)

(a) Experimental

(b) From the mathematical model

**Figure 3.4:** Frequency response of the analog PID controller
It can be seen that the analytical results closely follow the experimental results except at very high frequencies of the order of 10 kHz. Gain drop in the experimental frequency response around that frequency range can be due to the cut-off limits of the electronic components used. This has not been modelled mathematically.

3.4.2 Frequency response of the closed loop system

The closed loop frequency response of the real system and the mathematical model can also be compared. This was obtained experimentally using a dynamic signal analyser, while the AMB was
operating. The result is shown in Figure 3.5 (a). The continuous domain closed loop transfer function for the system shown in Figure 3.3 can be derived using (3.2), together with (3.3) taking into account the hardware negation (see Figure 3.3) also. Frequency response obtained analytically is shown in Figure 3.5 (b).

Close matching can be observed between experimental and analytical results. Phase lag of $-180^0$ in the experimental phase plot is due to the hardware negation of the position sensor output signal that was mentioned in Section 3.1. Notches present between 100 Hz and 200 Hz can be due to natural vibration modes of the rotor [3] that are not actively controlled. It was observed the fact that those vibration modes were excited, while recording these observations.

Due to obvious limitations in an analog control system, the model validation had to be limited to these two tests. Further tests on model validation requires facilities such as recording of input output data to the process. Thus the basic digital controller design had to be done based on this mathematical model. This design procedure will be presented in the next chapter.
4. Design of digital controller

Designing a controller for an open loop unstable non-linear system such as the AMB actuator is a complicated task. One can always simplify this problem by considering the linearised model of the plant about the required operating point. For all design and simulation examples in this chapter the linearised AMB model presented in Chapter 3 (Equation (3.2)) was used. The objective of this chapter is to gradually develop the final digital motion control strategy that is capable of elevating the rotor from rest up to the centre of the air gap and maintaining that position [48, 49, 50].

4.1 Motivation for digital control of AMB

Being an inherently unstable system, the AMB needs to be stabilised using a feedback control system for its proper operation. A well-tuned analog PID controller can be used to achieve such stabilisation like in the case of the original AMB system explained before. Although the analog controller can be used to stabilise the rotor shaft at a location in the air gap, it does not give enough flexibility to include additional features into the control system. This is because an analog controller consists of pre-tuned fixed hardware. Some of the features that are preferred to have in a test rig for acoustic noise study can be described as follows:

(a) **Study of noise with varying static and dynamic stiffness**

All control loops in an AMB system perform integral action to enable them to follow a given step set-point change without steady state errors. This makes it difficult to measure the stiffness of a magnetic bearing suspension, because the steady state error of a system controlled with integral action is ideally zero. This problem can be looked upon in two ways. The integrator action of the control system can be disabled and the radial force to displacement characteristics of the rotor can be measured using the rest of the controller. This is called as static stiffness. Alternatively, one could define a quantity called dynamic stiffness, which can be estimated from the level of deviation of the system from the setpoint, when subjected to an impulsive disturbance [4]. With a digital controller it is easy to measure and vary both these stiffness quantities, while still keeping the closed loop system within the stable region.

(b) **Study of noise with varying rotor eccentricity (eccentric rotor positioning)**

When a digital control scheme is employed, it is easier to vary the rotor position on-line, while the machine is rotating. With such a system, it is even possible to synchronise the measurements taken for noise study with the preferred eccentric motion of the rotor.

(c) **Emulate complex rotor motions in the air gap**

Different drive systems can impose different time varying (some times periodic) radial forces on the rotors. If the elasticity of the mechanical bearings and end shields is taken into account, these radial forces can cause the rotor to follow orbital motions in the air gap. The effect of these motions on acoustic noise can be easily studied by emulating these motions (by making appropriate setpoint changes on-line) using the digital control system.
(d) Introduce periodic disturbance cancellation to achieve high positioning accuracy during rotation

When the rotor levitated with AMBs is rotating, periodic disturbances are introduced to the position sensor signals of radial bearings. This is due to the non-collocation of the geometric and gyroscopic axes of the rotor shaft. It can cause the rotor to follow orbital paths inside the air gap. Hence, positioning accuracy deteriorates. This demands some form of a periodic disturbance cancellation technique in the controller to achieve precise positioning during rotation. Such techniques can easily be implemented on a digital control platform.

These factors highly motivate the need for incorporating a digital control system instead of the original analog controller.

4.2 Overview of the design problem

Digital control of AMB is considered as a difficult task. One reason for this is the handling of noise in the hardware electronics. In usual AMB hardware electronics, there can be oscillator stages coming in for the modulation of position sensor signals and in switched mode power amplifiers. These oscillators can generate a lot of periodic harmonic noise components. In addition to this, the position sensor output signal can contain a lot of noise. This demands careful design of filtering techniques for the digital control system, to eliminate aliasing effects.

At the same time, levitation of a rigid body is not a straightforward task. When the rotor is levitated in the air gap, high frequency components in the control signal output can always excite the rigid body modes and bending modes of the rotor, causing instability due to mechanical vibrations [3]. Some of these vibration modes also depend on the stiffness of the closed loop rotor suspension system. Since this stiffness is dependent on the parameters of the controller, the design becomes complicated. The main difficulty is to obtain an accurate mathematical model describing all these vibration modes. This is almost impossible, since the vibrations change their characteristics based on so many other factors affecting rotor dynamics. One way to resolve this is to do a reasonable digital controller design based on the mathematical model discussed in Chapter 3 and then follow it up with real-time hand tuning until a stable operating point without rotor vibrations is reached.

Since the open loop AMB system is unstable, one has to totally depend on the mathematical model (3.2) described in Chapter 3 and it is not even possible to validate that model further or build up more accurate models using additional experiments, without successfully commissioning the digital controller. Thus, achieving complete rotor levitation with the digital control system was considered as the first objective.

4.2.1 Basic criteria for controller design

The aim of the controller design was to produce a flexible test platform for magnetic noise study of the induction machine under eccentric conditions. It was therefore important that the start-up of the levitation and positioning to a given rotor eccentricity could be achieved by the designed controller. The digital motion control techniques developed were aimed at fulfilling these requirements. In summery, the following basic criteria for the controller were considered.
(a) Stability criterion

This is the ability of the controller to reach a given arbitrary rotor position in the air gap, while the rotor is not rotating. This criterion arises since the AMB system has non-linear characteristics in its domain of operation.

(b) Performance criterion

This is the ability of the controller to maintain the given rotor position under the disturbances occurring, when the rotor is rotating.

Even though the linearized model of the complete system is available after the modelling process described in Chapter 3, one straightforward way to design a digital controller was to discretize and implement the existing analog PID controller. This approach was preferred, since it was not possible to verify the validity of the derived mathematical model (3.2) further. In the following sections of this chapter, an explanation is given on the method of discretization used. Also explained will be the other practical problems encountered in implementing this basic digital PID controller and the techniques that were used to overcome those problems. The design approach is the same for all four axes. The results for the X-axis of the AMB_1 (AMB1_X) will be presented as a verification of the techniques.

4.3 Discretization of the analog PID

The s-domain transfer function of the analog controller $G_{PID}(s)$ obtained from the mathematical modelling explained in Chapter 3 can be given in the form (see Equation (3.3))

$$G_{PID}(s) = K_p + \frac{K_i}{s} + \frac{K_d}{s + a}.$$  \hfill (4.1)

where $K_p$, $K_i$ and $K_d$ are the proportional integral and derivative gains respectively. One important point to note here is the practical implementation of the derivative part of the controller. Instead of having pure derivative (just a zero at the origin), in the analog design a pole at $s = -a$ has been introduced. Thus, the gain of the derivative action has been limited to low frequencies. Its purpose is to restrict the controller from giving large amplifications to high frequency noise content of the error signal [22].

Discrete approximation of a continuous time controller can be done using several methods [22, 29]. Since the PID regulator is so simple, there are some special methods that are used. The following is a popular approximation that is very easy to derive.

**Proportional part** $P(t)$:

$$P(t) = K_p e(t).$$  \hfill (4.2)

This requires no approximation since it is a purely static part.

**Integral part** $I(t)$:

The integral part expressed in continuous time is

$$I(t) = K_i \int_0^t e(s)\,ds.$$  \hfill (4.3)
This can be approximated by the forward-rectangular approximation [29] as
\[ I(k) = I(k - 1) + K_T e(k - 1) . \] (4.4)

\( T \) here is the sampling time. The corresponding operator description \((q^{-1})\) can be given in the form
\[ I(q) = K_T \frac{q^{-1}}{1 - q^{-1}} . \] (4.5)

**Derivative part with filtering \( D(t) \):**

The derivative part can be expressed as
\[ \frac{dD(t)}{dt} + aD(t) = K_D \frac{de(t)}{dt} . \] (4.6)

This is approximated by taking backward difference [22] and can be given in the form
\[ \frac{D(k) - D(k - 1)}{T} + aD(k) = K_D \frac{e(k) - e(k - 1)}{T} . \] (4.7)

The corresponding operator description is
\[ [(1 + aT) - q^{-1}]D(k) = K_D (1 - q^{-1})e(k) , \] (4.8)

which simplifies to
\[ D(k) = \frac{K_D}{(1 + aT)} \left[ (1 - q^{-1}) \frac{e(k)}{1 - \frac{q^{-1}}{1 + aT}} \right] . \] (4.9)

Thus, the final discrete PID controller in pulse transfer function form is given by
\[ G_{PID}(q) = K_P + K_T \frac{q^{-1}}{1 - q^{-1}} + \frac{K_D}{(1 + aT)} \left[ \frac{(1 - q^{-1})}{1 - \frac{q^{-1}}{1 + aT}} \right] . \] (4.10)

Equation (4.10) gives the discrete version of the basic analog PID controller that has been implemented. At this point it will be interesting to mention some of the practical aspects that need to be taken into account, when realising a digital PID. In the work with analog controllers it has been found advantageous not to let the derivative part act on the command signal. This will stop the derivative part of the controller from responding to step changes of the command signal. Later it was also found suitable to let only a fraction \( b \) of the command signal act on the proportional part [22].

The control law after doing the above modifications can be given as
\[ u(k) = K_P \left[ by_{ref} (k) - y(k) \right] + K_T \frac{q^{-1}}{1 - q^{-1}} \left[ y_{ref} (k) - y(k) \right] - \frac{K_D}{(1 + aT)} \left[ \frac{(1 - q^{-1})}{1 - \frac{q^{-1}}{1 + aT}} \right] y(k) , \] (4.11)

where \( y_{ref} (k) \) and \( y(k) \) are the command and measurement signals respectively. The term \( b \) is an attenuation constant.
4.4 Hand tuning of the PID controller

The structure of the basic digital PID controller implemented is given by Equation (4.10). However, when the analog controller parameters obtained by mathematical modelling were directly converted and implemented as a digital PID, vibrations were observed and the system was very unstable. It was therefore necessary to do some real time tuning of the controller parameters. Since each axis of the AMB system can be operated independently, the best way to do the hand tuning was to keep all other axes operating with the analog controllers, while a particular axis is tuned. The COCKPIT™ software environment was very useful for this real-time parameter tuning. The main features that were expected to achieve from this tuning were,

1. Smooth elevation of the rotor from rest to the centre of the AMB stator
2. A stiff enough suspension, while the rotor is at the centre.

![Figure 4.1: Closed loop poles in the z plane](image)

The variation of the closed loop poles of the suspension system as the proportional, derivative and integral gains change was studied with the aid of the mathematical model (3.2). These variations are important, when deciding towards which direction a certain parameter had to be varied.

The mathematical model for the AMB_X1 is used for this analysis and the analog controller was discretized as explained in the previous section. Closed loop system poles with digital PID parameters obtained by the direct conversion of analog PID parameters are 0.9981, 0.9513 + j(0.10791), 0.9513 – j(0.10791), 0.5333 and 0.1893. Figure 4.1 shows the locations of the poles in the z-plane.

The corresponding digital PID parameters are, $K_p = 5.909$, $K_i = 100$ and $K_D = 31.7$. The variation of the closed loop poles as $K_p$, $K_D$ and $K_i$ are varied will be investigated here. The ranges of $K_p$, $K_D$ and $K_i$ are as follows (see (4.10)):

1. $K_p$ is varied between 1 and 10.
2. $K_D$ is varied between 30 and 180.
3. $K_i$ is varied between 43 and 4330.
4.4.1 Variation of the proportional gain

As can be seen Figure 4.2 (a) presents the variation of all five poles of the closed loop system. The two leftmost poles do not show a significant variation. An enlarged view of the poles that show a dominant variation is given in Figure 4.2 (b).

From the pole plots it can be seen that by decreasing $K_p$ down to a certain range, it is possible to achieve better damping.

\[\text{Figure 4.2: Movement of closed loop poles with } K_p\]

4.4.2 Variation of the derivative gain

What is presented here is the movement of the closed loop poles as the derivative gain is varied in the range mentioned above. As before, Figure 4.3 (a) shows all the poles and (b) is an enlargement.

\[\text{Figure 4.3: Movement of closed loop poles with } K_d\]

As can be seen from the above figure the damping can be increased by increasing $K_d$, which is the expected behaviour. More important however is that the two complex conjugate poles play a dominant role in this case also. The pole closer to origin does not show a considerable variation, while the other two poles on the real axis remain on the same axis, though moving slightly.
4.4.3 Variation of the integral gain

As can be seen from Figure 4.4 below, the poles are less sensitive to the variation of the integral gain. In practice also it could be observed that the integral gain only affects the time taken for the correction of the steady state error. Figure 4.4 (a) and (b) show the corresponding variation.

![Figure 4.4: Movement of closed loop poles with $K_I$](image)

The hand tuning experience was not completely successful. The main drawback was that the stiffness of the suspension was not sufficient, when the controller was tuned to obtain a smooth elevation. The system also became unstable, when attempts were made to increase the stiffness by manipulating the proportional and derivative gains.

Elevation of the rotor from rest was done by applying a step change in reference (the typical step change here is from -0.4 mm - width of the air gap - to 0 mm). Output saturation of the controller can occur, when applying such step changes to a controller tuned to give high stiffness in the rotor suspension system. It is well known that this type of controller output saturation lead to oscillations and instabilities in closed loop control systems [22]. The oscillations observed, when the rotor was elevated with stiff controller parameters, were assumed to be due to integrator windup problem. Under these circumstances it was concluded that this problem in elevating the rotor with stiff controller parameters could be eliminated by introducing a suitable integrator anti-windup scheme. This is discussed in detail in the next section.

4.5 Integrator anti-windup scheme for AMB controller

Many controller designs are carried-out assuming that there are no non-linearities between the controller output $u$ and the process input $v$. But in practice such non-linearities occur often due to,

- Limitations on the control signal and / or limitations on the speed of the control signal
- Switching between controllers
- Loss of connection between the controller output and the process input

If a controller is used neglecting such non-linearities, it may give rise to deterioration of the control performance in many ways. One of them is the windup problem. One way to deal with this problem is to take into account these non-linearities at the design stage itself. Still it is of wide interest to
investigate the possibilities of using controllers designed without considering these non-linearities, while handling the windup problem with some other techniques. This approach is preferred, since the controller design can then be performed considering a linear process.

A typical controller with output saturation is shown in Figure 4.5 and it can be seen that a mismatch between the controller output $u(t)$ and the process input $v(t)$ (this is the output from the saturator) may occur, if the control signal exceeds the limits of the saturator (i.e. $|u(t)| > u_r$). Desired control signal will no longer apply on the process and the expected output variation of the process will slowdown as a result. This is known as the windup problem. When this happens, the usual feedback path of the closed loop system breaks and special care has to be taken to bring the control signal back to the realisable range (i.e. $|u(t)| \leq u_r$) as the process output gradually reaches the setpoint.

![Figure 4.5: A typical controller with output saturation](image)

In the literature it can be seen that some control engineers have treated this windup problem as a global one, where the solutions suggested can be applied to general type of controllers and general type of non-linearities [20, 21, 22, 23]. Some of the general anti-windup compensators are

1. General incremental form
2. AWC based on actual process input
3. Model based AWC
4. Conditioning technique
5. Generalised anti-windup compensator.

The generalised anti-windup compensator suggested by Åström and Wittenmark [22] is of wide interest. In [20] it is shown that all other anti-windup compensators are special cases of the generalised anti-windup compensator.

One of the most common controller module that can be subjected to windup is the integrator that is employed in a PID controller or in any other controller. In any real time control system there can be certain circuit modules that show non-linear input output characteristics as the input signal exceeds a certain limit. For example, a power amplifier stage implemented with OPAMPs biased with $\pm 15$ V will reach saturation as the input goes closer to the biasing voltages. This will slow down the required change of the controlled variable, expected by the controller at higher output levels. The result is an uncompensated error in the output that causes the integrator to reach very high values. When this occurs, the system has to run with a large error of opposite sign to bring the integrator output back to a low value. Some times it may even become impossible to bring the integrator back to low values (especially in the case of digital control algorithms, where this can cause numerical over flows). This phenomenon is called integrator windup and it can cause severe deterioration of performance of the controllers. It is therefore important to have an overview on the different types of
integrator anti-windup techniques that are presented in the literature. Here again [20] shows that all
the methods mentioned can be derived from the generalised anti-windup compensator in [22].
However, it is very important to keep in mind the fact that lack of anti-windup on any other state of
the controller (e.g. differentiator filter state in a PID controller) may deteriorate the controller
performance as well.

It is recommended to follow [20, 21, 22, 23, 30, 31] to gather more information on different
integrator anti-windup methods that are currently used. The particular anti-windup technique used in
this case is the tracking algorithm [47].

4.5.1 Tracking algorithm applied to AMB controller

Prime idea of an integrator anti-windup method is to reset the integrator, when the control signal
reaches the output saturation. Thus, the stability of the integrator is guaranteed, while control signal
remains saturated, causing a discontinuity in the feedback path. This resetting mechanism may
change with the particular anti-windup method used. The strategy here is to reset the integrator only
by a factored amount of the difference between the computed controller output $u(t)$ and the
saturated output $v(t)$. When applied to the PID controller designed for AMB (given in pulse transfer
function form in (4.10)), resulting controller equation can be given as

\[
    u(k) = K_p e(k) + \frac{q^{-1}}{1-q^{-1}} [K_i T e(k) + K_{awc} (v(k)-u(k))] + K_d \left(\frac{1-q^{-1}}{1+(1-aT)}\right) e(k),
\]

where $K_{awc}$ is the anti-windup gain. The new controller structure is given in Figure 4.6.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{tracking_algorithm.png}
\caption{Functional block diagram form of tracking algorithm}
\end{figure}

When the actuator of the system is under saturation, the integrator starts acting freely. By feeding
back the term $(v(t)-u(t))$ to the integrator, which is otherwise marginally stable, is made to be more
stable (integrator pole is moved into the unit circle from its normal location at $q = 1$). This effective
closed loop around the integrator during control signal saturation is further elaborated in Figure 4.7.
This stable closed loop made around the integrator has the pulse transfer function given by

\[ G_{awc}(q) = \frac{K_{awc} q^{-1}}{1 - (1 - K_{awc}) q^{-1}}, \quad (4.13) \]

which clearly shows the movement of the integrator pole from 1 to \((1-k_{awc})\). This stable system always attempts to bring the controller output \(u(t)\) equal to the saturator output \(v(t)\) \((v(t)\) will be \(\pm u_r\) during an output saturation). The dynamics of the system can be changed by using \(k_{awc}\) as a tuning parameter to move the integrator pole in the desired direction. Since the control output \(u(t)\) tracks the saturated control signal, the method is called the “tracking algorithm”.

4.5.2 Verification by simulation

The new digital controller structure with anti-windup compensation (4.12) was implemented and verified in the simulation level. The state space system description in (3.2) was used as the process here with the controller in (4.12). The limits of the controller output saturation was set to \(\pm 10\) V in order to emulate the saturation that can occur at the DAC stage of the digital control system. Corresponding controller parameters were \(K_p = 4.43\), \(K_d = 40\), \(K_i = 0.02\), and \(k_{awc} = 0.00451\). Simulation results are shown in Figure 4.8 below to illustrate the performance of the anti-windup scheme.

The reduction of overshoot and the settling time of the measurement output are the improvements, when the anti-windup compensation is active. This was implemented in real-time and the digital controller was still unable to elevate the rotor from the rest without causing mechanical vibrations. The main reason was the excitation of the mechanical vibration modes of the rotor during this sudden lifting process [3]. This situation demands some form of a smooth elevation process that does not excite these mechanical vibration modes. One solution is to use gain-scheduled control as described in [32]. However, the observations made during the hand tuning process of the PID controller were useful in deciding the solution for this particular application. Since it was possible to elevate the rotor with a controller that shows low stiffness, the motivation here was to use one controller (with low stiffness) for the elevation process and later change over to another controller that has high stiffness. This kind of controller switching is called as “Bumpless Transfer” in automatic control [22]. Application of this technique enabled to establish a start-up method for the AMB actuator described in the next section.
4.6 Start-up method based on bumpless transfer

The start-up of the AMB system is defined here as, the elevation of the rotor from its resting position on touch down bearings up to the centre of the air gap of radial AMB stators. This needs the proper operation of 4 control loops in case of the 2 radial AMBs. During this elevation operation the rotor passes through a region, in which force to current characteristics are non-linear [19]. These non-linearities along with the bending modes of the rotor causes a lot of vibration oriented problems. At this point it is of interest to look at how the designer of the analog controller has overcome these problems. Figure 4.9 shows the typical variation of position sensor signal of one axis of the AMB as the start-up switch of the analog controller is switched on.
Figure 4.9: Start-up with analog controller

This clearly shows a time delay in activating the integrator that removes the steady state error. Therefore, the elevation is done using only the proportional and derivative parts of the controller and the correction of the steady state error is done later by activating the integrator.

4.6.1 Suggested start-up procedure for the digital controller

As mentioned earlier, the non-linearities along with the bending modes of the rotor shaft cause a lot of vibration oriented problems during the elevation process of the AMB system. Additionally, since the DSP system was connected to the hardware electronics by isolating the existing analog controller, it was observed that electromagnetic noise also creates problems that lead to excite rigid body modes of the suspension system. Therefore it was decided to try a method, which will elevate the rotor step-by-step, by switching on each control loop of the complete AMB system one after the other.

Another practical difficulty to overcome was to achieve enough stiffness of the rotor suspension system. By trial and error, it was seen that controller parameters that correspond to high enough stiffness of rotor suspension system lead to oscillations in the control loops during start-up. Under these circumstances the solution attempted was to do the elevation smoothly with controller parameters corresponding to a lower stiffness and then to hand over the control to a high stiffness controller. Additionally, the method used in the analog controller was also adopted and the complete elevation was designed in two steps. The sequence of tasks that had to be done during switching ON of each radial bearing was:

1. Switch ON one control loop with low stiffness controller parameters (slow PD controller of one axis).
2. Switch ON the control loop also with similar parameters of the other axis.
3. Compensate the steady state error by switching ON integral parts of the two control loops.
4. Transfer the control from low stiffness controller to high stiffness controller.

This sequence has to be repeated for the other AMB also.

4.6.2 Implementation of the start-up technique
A parallel combination of two PID controllers is used to overcome the start-up problem explained so far. The Figure 4.10 gives the details of complete controller structure for one closed loop.

The steps of elevating the rotor along one axis are given below. Assume all switches are open in the beginning. The two-way switch S4 at the output is set to CTL 1.

1. Close S1 - AMB is elevated with PD-1.
2. Close S2 - Integrator of CTL 1 becomes active. Rotor is brought to the centre with zero steady state error.
3. Close S3 - CTL 2 functions as a full PID and starts tracking CTL 1 (explained in Section 4.6.3).
4. Change S4 to 2 - Process is handed over to CTL 2.

This completes the elevation procedure. PID parameters of CTL 1 are tuned to get fairly slow closed loop characteristics. This guarantees the smooth lifting of the rotor without exciting its vibrating modes. Then the control is switched to CTL 2, which has faster closed loop characteristics. This switching between controllers has to be done without causing any disturbance to the closed loop system. This is achieved by employing the bumpless transfer technique, which is explained in the next section.

4.6.3 Bumpless transfer

Bumpless transfer is required in many control applications. One example is, when it is necessary to change the control of a certain system from manual to automatic [21,22]. In the present case
Design of digital controller

However, the bumpless transfer is employed, when the control of AMB is switched from one PID controller to another. The parameters of the two controllers are not identical and hence the closed-loop behaviours under the two controllers are different. The bumpless transfer concept is explained with the aid of Figure 4.10.

The idea here is to keep the outputs of both active and inactive controllers at the same value during the switching instant of the controllers. Then the controlled process will not experience a discontinuity in the input signal and hence the transferring process between the controllers will be smooth (i.e., bumpless). Assume that S3 in Figure 4.10 is switched ON, while CTL 1 is active and CTL 2 is inactive. Then there is a stable closed loop almost similar to the closed loop around the integrator shown in Figure 4.7. The difference here from the loop shown in Figure 4.7 is that the set-point of this loop is the output of CTL 1 and the saturator is not active.

Due to the action of this closed loop, output of CTL 2 will track the output of CTL 1. Therefore, the control can now be switched to CTL 2 without disturbing the controlled process. The dynamics of the tracking ability changes with varying gain $k_{awc2}$. However, when the process is controlled by CTL 2, $k_{awc2}$ will act as the gain of the linear feedback anti-windup compensator for CTL 2 as described in (4.12). Therefore, the value of $k_{awc2}$ is a compromise between the expected tracking ability of CTL 2 for bumpless transfer and performance of the anti-windup compensator for CTL 2. For the bumpless transfer fast tracking using higher values of $k_{awc2}$ is preferred. However, this may not always be the best suitable value for the anti-windup mechanism.

The position sensor signal variation during start-up with the structure shown in Figure 4.10 is given in Figure 4.11.

Figure 4.11: Position sensor signal during start-up (0.2 s/div, 2V/div)

Figure 4.12 shows the two controller outputs made from the data stored using on-line data acquisition technique of the DSP system. The figure shows how CTL 2 (U2) starts tracking the output of CTL 1 (U1) as S-3 is closed approximately after 0.8 sec. The above mentioned switches are implemented as software switches and are operated through the PC using the COCKPIT™ instrument panel.

With the implementation of this method, it was possible to elevate the rotor smoothly and changeover to a controller with high stiffness afterwards. The final controller parameters used for elevation and steady state operation will be presented in Section 4.7 followed by further analysis.
4.7 Final controller parameters and closed loop stability

In Section 4.4, variation of closed loop poles of the digital controller as $K_P$, $K_D$, and $K_I$ are varied independently was presented. In the sections followed, other practical problems that could arise to prevent successful implementation of a digital controller and some remedies that could be taken to overcome them were also discussed. Table 4.1 shows the hand tuned controller parameters for the two PID structures CTL 1 and CTL 2.

<table>
<thead>
<tr>
<th>Controller</th>
<th>$K_P$</th>
<th>$K_D$</th>
<th>$K_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTL 1</td>
<td>3.16</td>
<td>31.7</td>
<td>0.01</td>
</tr>
<tr>
<td>CTL 2</td>
<td>4.43</td>
<td>40</td>
<td>0.02</td>
</tr>
</tbody>
</table>

*Table 4.1: Hand tuned final controller parameters*

It is now interesting to see how the closed loop poles of the overall system (closed loop system is obtained using the tuned controller parameters and the mathematical model in (3.2)) varies as the AMB passes through the start-up procedure. Table 4.2 shows the closed loop poles during each state that the system passes during the start-up procedure. These are namely, the elevation with PD controller of CTL 1, correction of steady state error with PID controller of CTL 1, after bumpless transfer to PID controller of CTL 2.

<table>
<thead>
<tr>
<th>Controller states</th>
<th>Closed loop poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD controller of CTL 1</td>
<td>(0.9462 + j0.0657), (0.9462 - j0.0657)</td>
</tr>
<tr>
<td></td>
<td>0.5421</td>
</tr>
<tr>
<td></td>
<td>0.1897</td>
</tr>
<tr>
<td>PID controller of CTL 1</td>
<td>0.9961</td>
</tr>
<tr>
<td></td>
<td>(0.9481 + j0.0640), (0.9481 - j0.0640)</td>
</tr>
<tr>
<td></td>
<td>0.5422</td>
</tr>
<tr>
<td></td>
<td>0.1897</td>
</tr>
<tr>
<td>PID controller of CTL 2</td>
<td>0.9946</td>
</tr>
<tr>
<td></td>
<td>(0.9303 + j0.0769), (0.9303 - j0.0769)</td>
</tr>
<tr>
<td></td>
<td>0.5875</td>
</tr>
<tr>
<td></td>
<td>0.1788</td>
</tr>
</tbody>
</table>
In Figure 4.13 (a) and (b), closed loop poles of the AMB system are shown for the above mentioned effective controller states. Figure 4.15 (a) shows the variation of all closed loop poles and (b) shows an enlarged view of the poles that shows significant variations.

This analysis verifies the fact that the experimentally obtained tuned controller parameters result in a stable system with the mathematical model (3.2). This motivates a comparative study of time and frequency domain responses of the mathematical model and the experimental set-up with the two controllers tuned (Table 4.1).

### 4.8 Simulation and Experimental results

It is of wide interest at this stage to present some simulation and experimental results as a performance evaluation of the digital controller under non-rotating condition of the rotor shaft. For the simulation studies in this section the mathematical model in (3.2) is used. The analog PID controller (3.4) is replaced from the digital PID controllers tuned as described in this chapter. Similar to this, in the real system analog controller was isolated and the DSP system was used as the controlling element with the tuned digital controllers.

#### 4.8.1 A small step change in set-point

The idea of eccentric rotor positioning addressed here is to move the rotor shaft away from the centre of the air gap, while preserving the stability (see the stability criterion discussed in Section 4.2). The ability of the designed digital controller to do this is demonstrated by the experiment. Figure 4.14 (a) and (b) shows simulation and experimental results respectively for a positive step change of reference input by 1V (corresponding approximately to 0.02 mm off the centre). The results are with controller CTL2, of which the parameters may be found in Table 4.1.
Figure 4.14: Response to a positive step change (0V to +1V) in reference

Figure 4.15 (a) and (b) show the results of a similar test for a negative step change of reference input by 1V. To get an idea about the actual rotor movement in the air gap due to these step changes in reference, one must take into account the fact that the sensitivity of the position sensors is 52,000 V/m (i.e. corresponding eccentric rotor movement is approximately 0.02 mm).

The goal, when tuning the controllers was to achieve as high dynamic stiffness as possible in order to achieve better performance of the controllers during rotation. This has however, led to overshoots in the step response as shown in Figures 4.14 and 4.15. Since a two step elevation process has been suggested, it is still possible to move the rotor to the required eccentricity smoothly (i.e. coarse movement using the PD controller and the fine movement by switching on the integral action). Thus, any mechanical rotor vibrations due to these overshoots can be avoided.

4.8.2 Performance under an impulsive disturbance

Deviation of the rotor shaft from the stable operating point due to an impulsive disturbance gives an idea about the dynamic stiffness of the controller used. The two oscilloscope traces in Figure 4.16 (a) and (b) shows the system response under an impulsive disturbance with controllers CTL1 and CTL2 respectively.
The difference in settling time under the two controllers is an interesting feature to note. Since the controller CTL1 is de-tuned to give a smooth elevation to the rotor shaft, it shows a longer settling time and also a larger deviation from the operating point.

Due to practical difficulties in mathematically modelling the applied impulse, simulations for this test were not carried out.

![Figure 4.16: Performance under an impulsive disturbance, 20ms/div, 0.5 V/div](image)

### 4.8.3 Frequency response of the closed loop system

Using the same method as in Section 3.4, it is possible to measure the frequency response of the closed loop system experimentally using the dynamic signal analyser. Figure 4.17 shows the results with the two controllers.

![Figure 4.17: Experimental frequency responses of closed loop system](image)

Corresponding analytical results are shown in Figure 4.18.
A phase lag of \(-180^0\) in the experimental phase plots is due to a hardware negation of the position sensor output signal. Just as in the case of the closed loop frequency response with the analog controller (see Figure 3.5 of Chapter 3), notches of the closed loop frequency response plots due the vibration modes can be observed in the experimental responses with digital controllers also.

These results show that the mathematical model derived from the first principles (3.2) has reasonable matching with the real system behaviour. In fact this is a significant achievement since this modelling involved all components in the rotor suspension system (Namely, electromagnetic circuit, mechanical data of the rotor, power electronic components, electronic components in the position sensing and
control system). All these information may have some uncertainty due to measurement accuracy, tolerance in electronic components etc. However, it is clear from the results that the high-frequency modes of the rotor suspension system have not been accurately modelled by (3.2) (in fact they were neglected in this modelling process). Ironically, they are the main cause of instabilities in the AMB system.

However, these results also show that the digital motion control strategy proposed here is now capable of fulfilling the stability criterion described in Section 4.2. This opens up two possibilities for further research. One is the research on validating the mathematical model in (3.2) and building better models for the AMB actuator using suitable system identification experiments. The other approach is to investigate on additional motion control strategies that could be used with this controller to satisfy the performance criterion explained in Section 4.2. Both these approaches will be dealt with in the following chapters of this part of the thesis.
5. Model Validation and improved model for AMB

Validation of the mathematical model (3.2) was limited due to the obvious limitations in the analog control system in Chapter 3. A digital control strategy was then developed to stabilise the rotor in the air gap in Chapter 4. The new closed loop system with the digital controller offers more flexibility and additional features for conducting experiments for model validation. These features include the possibilities of exciting the AMB actuator with a desired excitation signal and recording input output data. These added features make the AMB test rig an ideal platform for model validation and system identification experiments. Prime objectives of this chapter are to present the model validation strategies used and describe the system identification experiments carried out to build an improved model. The experiments conducted in time and frequency domain for validating the mathematical model will be first described followed by the results of the model validation. Afterwards, the system identification experiment conducted will be explained. This is followed by the development of the improved model. The improved model will also be validated towards the end of the chapter.

5.1 Closed loop AMB system description

The general linearised mathematical model (3.1) of the rotor suspension system was explained in Chapter 3 under some basic assumptions (see Section 3.1). Those basic assumptions are still valid for the work described in this chapter also.

Since the digital control system was designed to operate at 10 kHz, any response data obtained through the DSP system will be sampled at that frequency. Thus, any validation of the mathematical model derived for the AMB1_X actuator (continuous time description is given by (3.2)) must be done at this sampling rate. The simplest approach therefore is to obtain the discrete-time linear state space model of (3.2) at 10 kHz. This can easily be done using scientific computation software such as Matlab and the discrete state space description corresponding to (3.2) is given by

\[
\begin{align*}
\mathbf{A}_d(k+1) &= \begin{bmatrix} 1.0005 & 0.0001 & 0 \\ 9.3601 & 1.0001 & 0.0008 \\ -3.2392 & -0.5941 & 0.3651 \end{bmatrix} \mathbf{y}(k) + \begin{bmatrix} 0.0 \end{bmatrix} U_c(k) \\
\mathbf{B}_d(k+1) &= \begin{bmatrix} 0.0002 \end{bmatrix} \\
\mathbf{C}_d(k+1) &= \begin{bmatrix} 0.2169 \end{bmatrix} \\
y(k) &= \begin{bmatrix} 52000 & 0 & 0 \end{bmatrix} \mathbf{y}(k) + \begin{bmatrix} 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \end{bmatrix} \mathbf{i}_r(k) \\
\end{align*}
\]

The frequency response of the above discrete time open loop AMB model is shown in Figure 5.1. The poles of the process are at \(q=0.3658, q=0.9693\) and \(q=1.0305\) and the zeros of the process are at \(q=-0.204\) and \(q=-2.968\). Thus, the AMB process is both open loop unstable and non-minimum phase.

This open loop process was controlled by the designed digital PID controller with tracking type windup compensator given by Equation (4.12) in Chapter 4. Corresponding controller parameters can be found in Table 4.1. However, the controller given by (4.12) needs to be modified, when used for analysis. Due the sampling delay, or the unit sample time delay in sending out the control
signal to the D/A converter, an additional delay term needs to be introduced to the controller. The modified equation is can be given as

$$u(k) = q^{-1} \left[ K_p e(k) + \frac{q^{-1}}{1-q^{-1}} [K_T e(k) + K_{ac} (v(k) - u(k))] + \frac{K_D}{(1+aT)} \frac{(1-q^{-1})}{1-(1+aT)} e(k) \right]. \quad (5.2)$$

However, instead of adding this delay to the controller, one could also include this in the process model as an input delay. In fact, in this chapter that is how it is treated, as this delay do exist between the measured process output and the computed control signal.

![Image](image1.png)

**Figure 5.1:** Open loop frequency response of the AMB1_x actuator

The closed loop system has some scaling factors in addition to the AMB actuator module and the digital controller module. One scaling factor is right at the reference input. There is attenuation and amplification at the ADC and DAC conversion stages respectively. The schematic block diagram of the closed loop system in the transfer function form, together with these additional details is shown in Figure 5.2.

![Image](image2.png)

**Figure 5.2:** Detailed schematic block diagram of the closed loop system in transfer function form with appropriate scaling and bias terms
The bias terms indicated in the above diagram along with the measured variable, is for the adjustments needed, when validating the model responses with the experimental responses. The system shown above is the one used for all the model validation tests. Different validation tests will be discussed in the following section.

5.2 Experiments for Model Validation

The mathematical model developed using the basic principles in Chapter 3 is validated both in time domain and frequency domain. As the open-loop AMB is unstable, the response data of the system are obtained from closed loop experiments. The digital controller described in the previous section is used to maintain the AMB stable, while the experiments were conducted.

5.2.1 Time domain validation - Closed Loop Step Response

For an open loop unstable system like this, there is no bounded step response. A step is given to the set point $w$, and the process output $y$ is measured. The step response of the real system and the step response of the simulated process with the same controller are shown in Figure 5.3. Clearly there seems to be a considerable mismatch between the step responses. It is also noteworthy that the step response of the real system, which shows damped oscillations, does not have the symmetric oscillation characteristics, as one would expect from a linear system. On the other hand, this confirms the fact that the magnetic forces in the air gap has non-linear characteristics.

\[ G_{yw} \]

\[ G_{uw} \]

**Figure 5.3:** Comparison of closed loop response of the real system and the mathematical model, both using the same digital controller for a step change in set point

5.2.2 Frequency domain validation - Frequency Response

To verify the mismatch that occurred in the closed loop step responses, the model was validated in the frequency domain. The sinusoidal responses of the closed loop system was obtained by giving a sinusoidal signal of known frequency, amplitude and phase, to the reference input $w$. Once steady state is reached, a few periods of the process output $y$, control input $u$ and the reference signal $w$ are stored for processing. The phase and amplitudes of $u$ and $y$ are computed using either least square approximation or using any non-linear optimisation technique. The frequency responses of the transfer function between $w$ and $y$, i.e. $G_{yw}$, and of the transfer function between $u$ and $w$, i.e. $G_{uw}$ are computed by
\[
\begin{align*}
|G_{yw}(e^{j\omega})| &= \left| \frac{A_y(\omega)}{A_w(\omega)} \right| \\
\arg\left(G_{yw}(e^{j\omega})\right) &= \phi_y(\omega) - \phi_w(\omega) \\
\end{align*}
\]

and

\[
\begin{align*}
|G_{uw}(e^{j\omega})| &= \left| \frac{A_u(\omega)}{A_w(\omega)} \right| \\
\arg\left(G_{uw}(e^{j\omega})\right) &= \phi_u(\omega) - \phi_w(\omega) ,
\end{align*}
\]

where \(A_w, A_y, A_u\) are the amplitudes and \(\phi_w, \phi_y, \phi_u\) phases obtained from the sine test for a particular frequency for \(w, y\) and \(u\) respectively.

Once these two closed loop frequency responses are computed, the open loop frequency response or the frequency response of the transfer function \(G_P\) is computed from

\[
\begin{align*}
|G_P(e^{j\omega})| &= \left| \frac{G_{yw}(e^{j\omega})}{G_{uw}(e^{j\omega})} \right| \\
\arg\left(G_P(e^{j\omega})\right) &= \frac{\arg\left(G_{yw}(e^{j\omega})\right)}{\arg\left(G_{uw}(e^{j\omega})\right)} .
\end{align*}
\]

The open loop frequency response from the sine tests and the response with the mathematical model (Equation (3.2)) with additional time delay of one sampling period are shown in Figure 5.4. Note that the additional time delay needs to be added to the mathematical model to account for the computation delay. Although the amplitude response seems to match well, the phase obtained from the sine tests has an additional phase shift.

\[\text{Figure 5.4: Comparison of open loop frequency response of the real system and the mathematical model}\]

The closed loop frequency responses from the model and from the experiments are shown in Figure 5.5. One could observe that the frequency response obtained for lower frequencies are reasonably good, while the ones for high frequencies seem to fluctuate a lot. There are a number of reasons for such behaviour. First, at high frequencies, the sinusoidal signal measured has a low signal to noise ratio. In addition, at these frequencies, some of the vibration modes of the motor shaft are also
invoked, and they too corrupt the measurements. In fact, to avoid harmful vibrations occurring at high frequencies, the amplitudes of the input sinusoidal were reduced. Despite the fact that attempts were made to reduce the noise and some of the vibrations, using time varying tuned filters, the final frequency response still seem to show fluctuations.

![Comparison of closed loop frequency response of the real system and the mathematical model - both with the same controller](image)

*Figure 5.5: Comparison of closed loop frequency response of the real system and the mathematical model - both with the same controller*

It is clear that, even in the frequency domain, the mathematical model does not match well with the reality. This discrepancy is natural, as the mathematical model was developed using only manufacturer's data and some measured data. As a reasonable process model is important for the controller design, in the next section a model is developed using system identification techniques.

5.3 System Identification

The mathematical model for the AMB system developed in Chapter 3, using basic principles does not match with either the experimental step response or the frequency response. Hence, it was decided to carry out system identification experiments and to develop a model using the resulting. This section describes the experiments conducted for identification and the methods used to estimate the models.

5.3.1 Identification Experiments

The identification experiments were conducted in closed loop. The closed loop system and consequently the open loop system is perturbed around a normal operating point by introducing a randomly varying set point to provide sufficient persistent excitation to guarantee the identifiability of all the parameters. The randomly varying set point $w$, process-input $u$ and process output $y$ were collected for the identification. To ensure identifiability, experiments were also conducted with both rapidly and slowly varying set point changes, and also with two different controllers, one that gives very slow closed loop dynamics and the other giving slightly faster dynamics. Furthermore, some experiments were also conducted with two controllers operating in parallel and switching between them at specified time instants.
5.3.2 Closed Loop Identification Methods

When the process input output data are collected under closed loop experiments, there is a fundamental problem due to the possible correlation between the process input and the unmeasurable noise. Many methods that perform well with open loop data tend to fail, when used with closed loop data due to this reason. Thus, there has been a lot of interest in the last three decades, on closed loop system identification, and there are a large number of methods available today in the literature. An early survey on system identification methods for closed loop systems is given in Gustavsson et al. [33], while some of the latest surveys on this area are Gevers [34] and Van den Hoff and Schrama [35]. A comprehensive study of closed-loop identification in the prediction error framework is given in Forssell and Ljung [36].

The identification methods available in the literature for closed loop data falls in to three main categories. They are (i) Direct approach (ii) Indirect approach and (iii) Joint input-output approach.

In this report, the direct approach for system identification has been considered. In this approach, the process input-output data obtained from a closed loop experiment is used directly in a standard identification method, as if the data is not generated through a feedback system. Furthermore, the study in this part of the thesis has also been confined to prediction error based methods. In the following subsection the prediction error methods suitable for unstable open loop system will be discussed.

5.3.3 Prediction Error Method

The prediction error based identification methods are well documented in Ljung [37]. In general, there is an opinion that the prediction error methods, which works very well in model identification, have difficulties when used for the direct identification of open loop unstable systems, if the model structures are either output error form or Box-Jenkins form. However, instead of using the natural predictor form of the respective model structure, a stable predictor similar to the observer concept in state space form can be developed to handle the unstable systems. In this section, the prediction error method is presented similar to the ideas presented in [38], for developing the Matlab system identification routines.

Under the non-rotating condition and for small perturbations around the centre of the magnetic bearing, which would not excite much of the vibration modes, it is possible to assume that the open loop model has a output error model structure in the form

\[ y(k) = \frac{B}{F} u(k) + e(k), \quad (5.6) \]

where \( B \) and \( F \) are polynomials of \( q^{-1} \) and \( B/F \) is the process transfer function of interest, \( e \) is random noise process. However, as the output error model is a special case of the Box-Jenkins model, the following Box-Jenkins model will be considered instead.

\[ y(k) = \frac{B}{F} u(k) + \frac{C}{D} e(k), \quad (5.7) \]

where \( C \) and \( D \) are polynomials of \( q^{-1} \) that will be used for presentation. The natural predictor for this model is given by

\[ \hat{y}(k) = \frac{DB}{FC} u(k) + \frac{C - D}{C} y(k) \quad (5.8) \]
or in the recursive form as
\[ C \hat{F}(k) = F[k - D]y(k) + DBu(k). \tag{5.9} \]

One of the difficulties with the above predictors, when the process is unstable is that the polynomial \( F \) has roots outside the unit circle and consequently the predictor become unstable. As the predictor provides the variables for computation of the prediction errors for the identification algorithm, the unstable predictor would result in unbounded signals.

To overcome this difficulty one needs to define a stable predictor that can be used to generate the predictions. One approach is to use a state space representation of the process in the innovation form by
\[
\begin{align*}
  x(k+1) &= Ax(k) + Bu(k) + Ke(k) \tag{5.10} \\
  y(k) &= Cx(k) + Du(k) + e(k), \tag{5.11}
\end{align*}
\]
where \( A, B, C, D \) and \( K \) are matrices of appropriate dimensions and matrix \( A \) represents the process noise dynamics as well as the noise dynamics. \( e(t) \) which is called the innovations and represent the random noise present in the form of process and measurement noise.

The natural predictor \( \hat{y}(t) \) for the above model is directly formulated as
\[
\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K[y(k) - c\hat{x}(k) - Du(k)] \tag{5.12} \\
\hat{y}(k) = C\hat{x}(k) + Du(k). \tag{5.13}
\]

For this predictor to be stable the gain vector \( K \) has to be chosen or computed such that the eigenvalues of \((A+KC)\) are all within the unit circle.

If the process is of \( n^{th} \) order, then for a SISO system, the matrices \( A,B,C,D \) and \( K \) provide totally \( n^2 + 4n \) unknown parameters. But for an \( n^{th} \) order process, for identifiability the number of unknown parameters must be confined to \( 4n \). To achieve this the observer canonical form for the above state space representation can be used (see [37]).

If the prediction error is defined as
\[ e(k) = y(k) - \hat{y}(k) = y(k) - c\hat{x}(k) - Du(k), \tag{5.14} \]
the general prediction error criterion can be expressed as follows (see [37]).
\[
V_N(\theta, Z^N) = \frac{1}{N} \sum_{i=1}^{N} l(e(i, \theta), \theta), \tag{5.15}
\]
where \( Z^N \) is the input-output data matrix of length \( N \), \( l(.,.) \) and is an appropriately chosen function of the prediction error, \( \theta \) is the parameter vector containing all the \( 4n \) unknown parameters of the model. The above function cannot in general be minimised analytically. However, when \( l \) is a quadratic function, minimisation can be carried out analytically for some model structures, e.g. ARX, ARMAX.

In this work numerical optimisation has been considered, for the case of SISO system and for the quadratic criterion given by
\[
V_N(\theta, Z^N) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} e^2(i, \theta). \tag{5.16}
\]
The minimisation of this cost function numerically can be done using a number of different approaches. One approach is to use search techniques that do not need the derivatives of the function \( V(\Theta) \). For example the simple search algorithm originally proposed by Spendley et al. [39] and modified later by Nelder and Mead [40] can be conveniently used as this algorithm does not require either the analytical derivatives or the computed derivatives.

In this work, gradient search methods as proposed in Ljung [37] have been used and in a similar manner they are implemented in Matlab® software (see [38]). The standard parameter update equation for the gradient search methods is given by

\[
\vec{\Theta}(i+1) = \vec{\Theta}(i) + \alpha f(i),
\]

where \( f(i) \) is a search direction based on \( V(\Theta(i)) \), and \( \alpha \) is a positive constant, which decides the decrease in the value of \( V \).

For the Newton algorithms the search direction \( f(i) \) is given by

\[
f(i) = \left[V''(\vec{\Theta}(i))\right]^{-1}V'(\vec{\Theta}(i)).
\]

(5.18)

As \( V \) is such that it is not easy to find the analytical derivatives \( V'(\vec{\Theta}(i)) \) and \( V''(\vec{\Theta}(i)) \), the approach used here is to numerically compute \( V'(\vec{\Theta}) \). Since

\[
V_N'(\Theta, Z^N) = -\frac{1}{N} \sum_{i=1}^{N} \psi(t, \Theta) \frac{1}{2} \varepsilon^2(i, \Theta),
\]

(5.19)

the task is to numerically compute the gradient \( \frac{\partial \psi(t, \Theta)}{\partial \Theta} \), which is the gradient of \( \hat{y}(\Theta) \) with respect to \( \Theta \). Once \( \psi(t, \Theta) \) is computed the Hessian matrix \( \left[V''(\vec{\Theta}(i))\right] \) can be computed using the approximate relationship

\[
V_N''(\Theta, Z^N) = \frac{1}{N} \sum_{i=1}^{N} \psi(t, \Theta)\psi^T(t, \Theta) = H_N(\Theta).
\]

(5.20)

With this computed gradients the update mechanism is given by

\[
\vec{\Theta}_N(i+1) = \vec{\Theta}_N(i) - \mu_N(i)[-H_N(\Theta)(i)]^{-1}V_N'(\vec{\Theta}(i), Z^N).
\]

(5.21)

There are many mechanisms one can employ to compute gradients numerically. It basically depends on a perturbation of a single parameter around the current estimate of the parameter and computing the deviation in the prediction \( \hat{y}(k) \) to compute the derivative vector \( \psi(k, \Theta) \).

**Remarks on the Stable Predictor**

The state space approach described above will result in the equivalent Box-Jenkins model, with a stable predictor, even when the process is unstable. The estimator gain vector \( K \) in this case is also estimated in the optimisation algorithm similar to the other process parameters. However, one does not need to estimate the gain vector if the assumed model is of the output error form. In the stochastic framework, the steady state Kalman filter gains for an estimator for a process that has only measurement noise as in output error form, is zero. However, if the gains are zero, the predictor
will become unstable provided the process is unstable. Thus, for estimating output error models using the state space form, one can assume an artificial covariance matrix for the process noise and compute either time-varying estimator gains or steady state gains to be used for producing the predictions. Here, one does not need to compute the gains in the optimisation algorithm. For any given current estimate of the parameter vector, the estimator gains can be obtained by either solving the matrix Riccati difference equations for time varying gains or matrix Riccati equation for steady state gains. One could also use the process noise and measurement noise covariances as tuning parameters to improve the predictions. In fact, these artificial noise covariances can be chosen such that there will be only a single parameter available for convenient tuning. On the other hand, instead of computing the gains in a stochastic framework, a predictor can be developed based on an observer. An appropriate observer can be derived based on pole placement, for example using Ackermann's formula. To deal with the noise in the assumed output error model, the observer can be tuned with different observer pole placements. Here the only reason for using the observer is to obtain a stable predictor.

**5.4 Identification Results and Discussion**

As mentioned earlier, various identification experiments were conducted with the closed loop AMB system. The models were built basically using the prediction error approach described in the previous section. Both simplex search method and the gradient methods for the optimisation of the cost function have been tried. Studies were also done using different ways of dealing with the stable predictor. For example the stable predictor gains were either computed from the optimisation or by using the observer concept as described in the previous section.

In this section, an identification model based on prediction error method, where the predictor gains were also estimated in the optimisation, will be presented. The initial values of the process parameters for the optimisation were obtained using the ordinary least square method.

**5.4.1 Identified Process Model**

As the mathematical model derived using the basic principles in Chapter 3 was a 3rd order model; the aim was to identify a third order model. A model that was decided as one of the best obtained in the validation process is the following 3rd order model, which is described in the transfer function form as

\[
\bar{G}(q) = 10^{-3} q^{-1} \frac{0.12 + 0.232 q^{-1} - 0.5627 q^{-2}}{1 - 2.69 q^{-1} + 2.3796 q^{-2} - 0.6897 q^{-3}}. \tag{5.22}
\]

The zeros of the system are at \( q_{\text{zero}} = -3.313 \) and \( q_{\text{zero}} = 1.402 \). The poles of the system are at \( q_{\text{pole}} = 1.094 \), \( q_{\text{pole}} = 0.9697 \) and \( q_{\text{pole}} = 0.6909 \). Thus, it can be seen that the system is unstable and non-minimum phase. It is also noteworthy that the unstable system pole is quite close to the unstable pole of the mathematical model given by equation (5.1). In developing the above model the predictor gains \( K \) have also been identified. Here only the process transfer function has been considered assuming it has the output error form (5.6).

**5.4.2 Model Validation**

The validation of the process model given above, and also the other candidate models that were developed were not straightforward as just investigating the residuals of the process output and the model output for a given set of experimental data. First of all the investigation of residuals needed the
observer form of the identified model for generating the process output. As the gains of the stable predictor was already available such validations could be easily carried out for the open loop system. However, the correlation investigations of the residuals generated from the open loop system may give false information as the input data used for generating the residuals are correlated with the measured process output because of the data was collected from closed loop experiments. While taking the minimum of $V_{\text{ss}}(\theta)$ as a means to select the some of the candidate models, the final validation was carried out by comparing the following three responses.

1. Open loop frequency response
2. Closed loop step response
3. Closed loop frequency response

Before discussing the validation process it is worth to mention that the frequency responses of the real system, both open loop and closed loop as shown in Figure 5.4 and Figure 5.5 respectively, have the problem of fluctuations in the higher frequencies. This indeed makes the validation process difficult, and demanded simultaneously looking at the results of all the three above responses to make an intelligent decision. It is after such a process that the above model was decided, and the comparison of the above responses for the real system and the identified model will be presented here.

The open loop frequency response of the real AMB system and model identified above is shown in Figure 5.6. Both the gain plot and the phase plots match very well for the low frequencies, and match in an average sense for the higher frequencies. The AMB model is then validated in the closed loop. The closed loop validation is carried out by considering the same digital controller for both the real system and the identified model.

![Figure 5.6](image.png)

**Figure 5.6:** Comparison of open loop frequency response of the real AMB system and model identified above in (5.25)

The responses of these two systems for a step change in the set point are compared in Figure 5.7. Although the identified model has captured most of the dynamics of the real system, the real system is still less damped than the identified model. Figure 5.8 shows comparison of closed loop frequency
responses. The closed loop response (both gain and phase) seems to match very well in the low frequencies and in the peak frequency. For higher frequencies the matching is on the average. The higher damping in the identified model as shown in Figure 5.7 is expected from the less sharpness in the frequency response near the peak.

![Comparison of responses of the two systems for a step change in the set point](image1)

**Figure 5.7:** Comparison of responses of the two systems for a step change in the set point

![Comparison of closed loop frequency responses](image2)

**Figure 5.8:** Comparison of closed loop frequency responses

In the beginning of this chapter, it was mentioned that one of the basic assumption that was made both in mathematical model building and experimental validation is that the shaft is not rotating and hence the additional periodic disturbances due to whirling can be neglected. However, under rotating conditions unbalance in the rotor causes a lot of periodic disturbances that affect the closed loop control system through position sensor feedback. This problem will be thoroughly investigated in the next chapter and a cancellation method will be suggested and tested in real-time.
6. Periodic disturbance cancellation

The analog PID controller provided by the manufacturer (3.3) as well as the digital PID controller (4.10) designed in Chapter 4 were tested for their capability of meeting the performance criterion explained in Section 4.2 of Chapter 4. When the rotor rotates, while it is elevated by any of these controllers, some periodic disturbances are introduced to the position sensor feedback signals of the radial bearings. This is due to whirling of the rotor shaft and the prime demand of the noise study (precise eccentric positioning) is still not accomplished due to this reason. Objective of this chapter is to first explain the nature of these disturbances followed by a discussion on the possible reasons for their presence. An adaptive periodic disturbance cancellation method will then be presented, which is capable of cancelling periodic disturbances of this nature. Using the cancellation method to improve the performance of the digital controller will be illustrated. Later in the chapter the same method will be applied as an outer digital control loop for the original analog controller to improve its performance.

6.1 About periodic disturbances in AMB control loop

![Graphs showing periodic disturbances](image)

**Figure 6.1:** (a) DFT of one sensor signal, (b) y-t plot of the two sensor outputs, (c) x-y plot of the two sensor outputs (with the digital controller).
It was illustrated that the stability criterion described in Section 4.2 can be met from the digital controller structure presented in Chapter 4. It is however interesting to see what happens to the performance, when the machine is rotated.

Figure 6.1 show the $y-t$ and $x-y$ traces of the two position sensor signals and the Discrete Fourier Transform (DFT) of one position sensor signal of AMB 1, when the machine is rotated from a 30 Hz supply on no load. Figure 6.1 (a) and (b) show the path that the rotor follows at a vertical section, where the two position sensors of AMB_1 are located. Figure 6.1 (c) shows that there are peaks in the DFT of the position sensor signal at 15, 30, 45, 60, 75, 90 Hz (with negligible amplitude for still higher order harmonics). There can be two reasons for this behaviour, which are described in Section 6.1.1 and 6.1.2 respectively.

### 6.1.1 Non collocation of the geometric and gyroscopic axes of the rotor

When a rigid body is levitated and rotated, the axis of rotation will be its gyroscopic axis. In the case of a symmetrical body like the rotor shaft, this axis needs not to be coinciding with the axis of symmetry or the geometric axis. The Figure 6.2 shows this non-collocation problem of the rotor. This can be due to inaccurate machining of the rotor as well as the non-homogeneity of the material.

![Non co-location of the geometric and gyroscopic axes of the rotor](image)

**Figure 6.2: Non collocation of the geometric and gyroscopic axes of the rotor**

### 6.1.2 Non collocation of the axes of AMB stators and the axis of machine stator

There are three stators involved in the case of the radial stability of a rotor levitated by AMBs. They are the stators of the two AMBs and the stator that contains the three-phase windings of the induction machine. Due to alignment difficulties the central axes of the three stators may not coincide. This can exert eccentric rotating force waves originated from the stator of the induction machine on the rotor shaft. These forces can make the rotor deviate slightly from its stable centre position. The presence of the 15 Hz peak, while driven from a 30 Hz supply (4-pole machine) is a good indication of the above phenomena. Figure 6.3 shows this no-collocation graphically.
The periodic disturbances generated due to those reasons continue to exist within the closed control loop due to the incapability of the existing controller to provide high gain at these disturbance frequencies (i.e. dynamic stiffness is not sufficient). The existence of these periodic disturbances is clearly a degradation of performance, when the rotor is rotating. Thus, the demand arises for suitable periodic disturbance cancellation techniques. The main reason for this is the positioning accuracy required in magnetic noise study. The digital controller must be capable of limiting the deviation of the rotor from a given set-point under rotational conditions (i.e. controller must offer high dynamic stiffness at these frequencies). The technique incorporated to reject these periodic disturbances is discussed in the forthcoming Section.

6.2 Periodic disturbance cancellation technique

Periodic disturbances occur not only in magnetic bearing systems but also in many other engineering applications. Eccentricity of the track on a magnetic disk, torque pulsation created by the cogging torque (in permanent magnet machines) and detent torque (in stepper motors) are some of the examples that require periodic disturbance cancellation techniques. In some cases, the frequency of disturbance is not precisely known. In many others, the fundamental frequency of the disturbance originates from some variable that is independently regulated and easily measurable. Unless the disturbance is purely sinusoidal, the harmonics also have to be compensated for. However, because of low-pass properties of physical systems, at most a handful of harmonics needs to be considered in general (from Figure 6.1 it is clear that this is the case for the AMB application also).

At this point, it is worth while introducing the word repetitive controller. The purpose of a repetitive controller is either to reject a periodic disturbance or to track a periodic reference signal. Design of repetitive controllers is a vast area [41, 42, 43]. Several methods are available for the rejection of sinusoidal disturbances, and they can be easily extended to the case, when several sinusoidal components are present [20, 25]. The most common approach is based on the Internal Model Principle (IMP) proposed by Frances and Wonham [44], which states that a model of the disturbance generation system must be included in the feedback system. The other method is the Adaptive Feedforward Cancellation (AFC), based on a totally different concept [26]. In this method, the disturbance is simply cancelled at the input of the plant by adding the negative of its value at all times. Since the cancellation technique used here falls into the second category, the basic concept of AFC method will be further explained.
6.2.1 Adaptive Feedforward Cancellation (AFC)

If a linear time-invariant plant \( P(s) \) is perturbed by an input disturbance \( d(t) \) of the form
\[
   d(t) = A \sin(\omega t + \phi) = a_1 \cos(\omega t) + b_1 \sin(\omega t),
\]
the basic principle behind AFC approach is to generate exactly the same disturbance \( d(t) \) adaptively and use the generated disturbance signal to cancel the original disturbance \( d(t) \), at the process input. The principle of feedforward cancellation is shown in Figure 6.4 in block diagram form. According to this strategy, the control is selected to be
\[
   u(t) = \Theta_1 \cos(\omega t) + \Theta_2 \sin(\omega t).
\]
so that the disturbance is completely cancelled, when the parameters \( \Theta_1 \) and \( \Theta_2 \) have the nominal values
\[
   \Theta_1^* = -a_1 \text{ and } \Theta_2^* = -b_1.
\]
The problem is to find a suitable parameter adjustment mechanism so that the parameters \( \Theta_1, \Theta_2 \) are estimated to be equal to \( \Theta_1^*, \Theta_2^* \) and the disturbance is cancelled completely. Thus, an adaptive mechanism is needed.

![Diagram](image)

**Figure 6.4: Controller based on the Adaptive Feedforward Cancellation principle**

With the vector definitions
\[
   \mathbf{\Theta} = \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} \cos(\omega_1 t) \\ \sin(\omega_1 t) \end{bmatrix} \text{ and } \mathbf{\Theta}^* = \begin{bmatrix} -a_1 \\ -b_1 \end{bmatrix},
\]
the plant output can be written as
\[
   y = P(\mathbf{\Theta} - \mathbf{\Theta}^*)^T \mathbf{w}.
\]
This expression falls within the standard framework of adaptive control theory [45]. Thus a possible update law [24, 26, 27] for the adaptive parameter is
\[
   \dot{\mathbf{\Theta}} = -\gamma \mathbf{w} \quad \text{or} \quad \dot{\Theta}_1 = -\gamma \cos(\omega_1 t) \quad \dot{\Theta}_2 = -\gamma \sin(\omega_1 t).
\]
where \( g > 0 \) is an arbitrary parameter called the adaptation gain. Explained above is the very basic concept of AFC. The historical background of the particular AFC strategy will follow.

### 6.2.2 Historical development of AFC used in AMB

The method used here was first suggested by Bodson et al. in [24], which was originally used to overcome a similar type of a problem in a high performance magnetic disk drive. Basic idea of this particular adaptation technique was first presented by Chen and Paden in [27] to reduce the torque-ripple in step motors. That was the first systematic approach to torque-ripple reduction via adaptive control. It is interesting at this point to briefly describe their method and the results that were achieved. The idea was extremely simple. In order to cancel the torque-ripple term, the addition of some ripple to the input current of the motor was suggested. The actual torque was assumed to be unknown and a set of shape functions (which was used as the regressor vector) were chosen, the coefficients of which were tuned adaptively. When this controller was implemented in real time with a low-gain PD controller to cancel the first torque-ripple harmonic, it was possible to observe some interesting results. The first torque-ripple harmonic was reduced dramatically. In addition to that a reduction of the third harmonic was also observed. An explanation to this behaviour is however not given in [27] as the authors do not cover the theory of the phenomenon with sufficient depth. However, it has been suggested that this is due to un-modelled non-linearities in the plant.

Bodson et al. in [26] went further into this issue and showed that the rejection of the higher-order harmonics can occur even if the plant is linear. In other words, an adaptive algorithm designed to cancel the first harmonic may be capable of reducing the amplitude of the third harmonic as well. The scheme, somehow, is capable of generating harmonics of order higher than those, for which it was designed. The generation of harmonics in the adaptive algorithm is found to be due to the time-variation of the adaptive parameters and can be explained using modulation arguments from standard signals and systems theory. In [26] Bodson et al. go further to prove that there is harmonic generation in adaptive systems analytically.

### 6.2.3 Discrete time AFC for the AMB

In the case of AMB the requirement is to cancel a periodic disturbance of known frequency and its harmonics. The discrete time feedforward scheme is shown in Figure 6.5 in block diagram form. The idea here is to estimate the Fourier coefficients of a periodic disturbance of known frequency in real-time.

It is assumed that the unknown disturbance consists of a sum of sinusoids of known frequencies and can be expressed in discrete domain as

\[
d(k) = \sum_{i=1}^{N} \left( a_i(k) \cos \left( \frac{2\pi ik}{N} \right) + b_i(k) \sin \left( \frac{2\pi ik}{N} \right) \right),
\]

(6.7)

where \( N \) is the number of samples per cycle. Even if the disturbance is entering the process from another point, it can always be reflected to the input of the process. The output of the system is

\[
y(k) = P(q^{-1}) \left[ u(k-1) - d(k-1) \right],
\]

(6.8)

where \( P(q^{-1}) \) is the transfer function of the plant. It is desired to remove the disturbance observed at the output of the system by forming a control input that exactly cancels the disturbance. This control input is selected to be
\[ u(k) = \sum_{i=1}^{n} \left( \hat{a}_i(k-1) \cos \left( \frac{2\pi i(k-1)}{N} \right) + \hat{b}_i(k-1) \sin \left( \frac{2\pi i(k-1)}{N} \right) \right). \] (6.9)

Exact cancellation is achieved, when
\[ \hat{a}_i = a_i \quad \text{and} \quad \hat{b}_i = b_i. \] (6.10)

The adaptive laws to adjust estimates \( a_i \) and \( b_i \) use the approximate update laws
\[ \hat{a}_i(k) = \hat{a}_i(k-1) - g_i y(k) \cos \left( \frac{2\pi i k}{N} + \phi_i \right) \] (6.11)
\[ \hat{b}_i(k) = \hat{b}_i(k-1) - g_i y(k) \sin \left( \frac{2\pi i k}{N} + \phi_i \right), \] (6.12)

where \( y(k) \) is the deviation of the measurement output from its zero setpoint (otherwise \( y(k) \) must be replaced by \((y_{sp} - y(k))\)) where \( y_{sp} \) can be any realisable setpoint. The gains \( g_i, \quad i = 1...n \) and the phase shifts \( \phi_i \) have to be appropriately chosen for stability and convergence respectively. In fact, a detailed analysis of the stability properties of the algorithm can also be found in [24].

The description above explains the basic theory behind the particular AFC method. A systematic approach was followed at the design stage for the AMB application. In the initial stages the Matlab\textsuperscript{TM} working environment was extremely useful in determining the frequency components in the periodic disturbance (use of DFT etc.) [11, 28]. First, this adaptive cancellation scheme was
Periodic disturbance cancellation

implemented in SIMULINK™ and tested with the closed loop simulation model consisting of the process (3.2) and the digital controller (4.10).

Since the periodic disturbance contains several harmonics, it was observed that a tuned filter, which is to extract the particular frequency to be eliminated, reduces the bias of the estimates. Therefore, a tuned filter (discussed in the Section 6.3) was also implemented and inserted before the disturbance cancellation stage in the simulation model [20]. The performance of the Adaptive Periodic Disturbance Cancellation (APDC) with pre-filtering was also tested in the simulation level. Later, it was implemented in real-time [49, 50].

6.3 Tuned filter for improved adaptation

As discussed in the beginning of this chapter, the periodic signal that has to be dealt with consists of several harmonics. In addition, the position measurement also contains noise. To be able to cancel periodic signals of different frequencies, it is necessary to implement adaptation loops tuned to each frequency in a parallel structure. Under these circumstances, it is important to extract the particular harmonic out of the measurement, which contains noise and periodic signals of other frequencies, before it is sent through the particular adaptation loop. This type of pre-filtering makes the convergence of coefficients faster (This will be demonstrated from simulation results in the following section). Bodson et al. do not mention this kind of pre-filtering with the adaptation algorithm in [24] and hence this is a new contribution to the APDC technique, resulting from this work.

In [20] a method of estimating a periodic signal corrupted by noise has been presented. When the disturbance measurements are corrupted by noise, the filtering problem in hand is to obtain an estimate of the periodic signal \( d(t) \) (see (6.1)) from the measurement \( d_m(t) \), which is corrupted by noise \( e(t) \) according to,

\[
d_m(t) = d(t) + e(t)
\]

(6.13)

The above filtering problem, also known as the line enhancement problem, has extensively been studied in the signal processing literature. The ideal solution to the problem in relation to this work is the tuned filter obtained from the notch filter for a single sinusoidal component. In other words, if \( L_N(q^{-1}) \) is the notch filter, then the tuned filter \( L_T(q^{-1}) \) is given by,

\[
L_T(q^{-1}) = 1 - L_N(q^{-1})
\]

(6.14)

The particular filter will be explained in the next section.

6.3.1 Tuned (or Notch) filter for a single frequency

The near optimal time invariant tuned filter to estimate the signal \( d(t) \), which consists of a single sinusoidal disturbance at \( \omega \) from a noisy measurement \( d_m(t) \) is given by

\[
\hat{d}(k) = L_T(q^{-1})d_m(k)
\]

(6.15)

where

\[
L_T(q^{-1}) = 1 - \frac{1 - 2 \cos(\omega T_s)q^{-1} + q^{-2}}{1 - 2\alpha \cos(\omega T_s)q^{-1} + \alpha^2 q^{-2}} = 1 - \frac{H(q^{-1})}{H(\alpha q^{-1})}
\]

(6.16)

according to [46]. The tuning coefficient \( \alpha (0 < \alpha < 1) \) here determines the sharpness of the tuning (the degree of noise filtering). This means that the bandwidth of the tuned filter becomes narrower as
\( \alpha \) is brought closer to 1 (but not equal to 1). However, it can be seen from the simulation results presented later in the Section 6.3.2 that the transient time needed to reject the initial errors becomes larger as \( \alpha \) gets closer to 1. Hence, selecting \( \alpha \) is a compromise between larger time constant in the transient Vs the degree of noise filtering. A simple approach to get the full benefit of this behaviour is to use a smaller \( \alpha \) at the beginning to reduce the transient time constant and then increase \( \alpha \) to achieve good noise rejection [20]. This on-line tuning of \( \alpha \) is not done in this case and a suitable value for \( \alpha \) is chosen by simulation.

The block diagram form of the tuned filter (6.16) is shown in Figure 6.6 and this was implemented as a SIMULINK™ model and used in front of the APDC block to extract the particular periodic disturbance required to be cancelled. This was later used in the real-time digital controller block model.

\[ \text{Figure 6.6: Near optimal tuned filter} \]

It must be noted here that the inclusion of this filter changes the closed loop poles of the system. Hence, the stability analysis in [24] on the APDC technique will no longer be valid. Thus, further analysis on the closed loop stability of the APDC modified with tuned filter is recommended for future research.

6.3.2 Performance of Tuned Filter by simulation

Figure 6.7 shows the performance or the extraction quality of the tuned filter as \( \alpha \) is moved towards 1. As mentioned in the previous section it can be seen from the results that the transient time constant of the filter increases as \( \alpha \) increases.

6.4 Simulation study of the cancellation technique

The adaptive algorithm was tested at the simulation level in a step-by-step manner so that the situation in real-time is reached as close as possible. Simulations were started with ideal conditions (i.e. without any noise in the measurement signal). Later the results for a measurement signal corrupted with noise will be presented. Finally the effect of tuned filter on adaptation will be presented. The important aspects to investigate at the simulation level are the convergence of the coefficients \( \hat{a} \) and \( \hat{b} \), with and without noise and the speed of cancellation of the periodic disturbance as the acceleration factor \( g \) is varied. Apart from these aspects it is also interesting to see the effect of a step change of the set-point on convergence of the coefficients (\( \hat{a} \) and \( \hat{b} \)) and the effect on them due to a fluctuation of the disturbance frequency. The closed loop simulation model consisting of the process (3.2) and the digital controller (4.10) was used for the simulation study.
The steady state position set-point for the AMB actuator was adjusted to 0 (i.e. the AMB is at the centre of the air gap).

![Performance of Tuned Filter](image)

**Figure 6.7: Performance of Tuned Filter**

### 6.4.1 Cancellation technique without noise
Figure 6.8: Cancellation with $g = 0.001$

Here the cancellation of a disturbance of 15 Hz (fundamental component) without adding the measurement noise to the system is presented. Two sets of results are given that will show the effect of increased adaptation gain. Simulation results in Figure 6.8 and 6.9 show that the rate of convergence of the adaptive scheme increases as the adaptation gain is increased. One should however bear in mind the fact that the change of $g$ can make an impact on the overall stability of the system as it can very much affect the closed loop system poles. However, in all the simulation and also in real time tests the value $g = 0.005$ will be used. In fact, analytical limitation on the maximum value of $g$ has been found in [24].

Figure 6.9: Cancellation with $g = 0.005$

6.4.2 Performance with measurement noise and pre-filtering
Simulation is now brought closer to the real situation by adding white noise to the measurement output. These results show the effect of noise on the convergence of the coefficients. The solution suggested for this case was the introduction of pre-filtering, using the special tuned filter discussed in the previous section. Simulation results are shown in Figure 6.10. The tuned filter discussed in Section 6.3 was introduced to extract the sinusoidal disturbance (with a filter coefficient of $\alpha = 0.95$) and the filtered signal was fed to the cancellation algorithm. From the simulation results shown in Figure 6.11, it can be seen that the coefficients in this case have a better convergence. However, it must be noted here that the transient time of the
cancellation can vary due to the settling time of the notch filter (see Figure 6.7) and in fact, it can be observed in Figure 6.11.

6.4.3 Step change in set-point while APDC is active

Another interesting aspect to investigate is the robustness of the feedforward algorithm for changes in position set-point of the rotor. One way to check this is to simulate a step change in set-point and see how it affects the performance of the cancellation technique, i.e. to investigate whether the algorithm still works properly and also to investigate how the adaptation coefficients are disturbed.

According to Figure 6.12 (a) and (b) the adaptation coefficients are disturbed by the step change but settle down later to the originally converged values. This is because the periodic disturbance of the system does not undergo any changes although the set-point was changed. Obviously the final values of the adaptation coefficients are determined by the parameters of the periodic disturbance in the system. Figure 6.12 (c) shows that the cancellation is satisfactorily done even after the step change.

![Figure 6.12: Step change in reference while cancellation is ON](image)

6.4.4 Fluctuation of disturbing frequency while APDC is active

Another test for the robustness is the performance of the algorithm under frequency fluctuations. This is important since the rotating machines tend to undergo speed fluctuations due to both, changes in load and fluctuations in supply frequency. It is extremely important, when using AMB that the overall system (including the APDC technique) remains stable during a fluctuation of the rotational speed of the machine. Otherwise there is the risk that the rotor can crash-land on the emergency bearings, while still rotating. Obviously, this could lead to devastating consequences. Figure 6.13 shows simulation results, when the frequency of the sinusoidal disturbance undergoes a fluctuation. Here the change simulated starts at $t = 1.5$ s and the frequency drops to 14 Hz. It returns to 15 Hz at $t = 5.5$ s.
The adaptation coefficients show some interesting behaviour during frequency fluctuation. It can be seen that the adaptation coefficients ($\hat{a}$ and $\hat{b}$) fall into limit cycles during the frequency fluctuation and the overall performance of the APDC shows somewhat deterioration. This observation will later be verified by the real time tests carried out. In fact it can be observed that the overall system remains stable during a frequency fluctuation (see Section 6.5.4). Further investigation in this regard is required to come to any concrete conclusion regarding the maximum allowable frequency fluctuation while preserving overall stability.

The overshoot visible in the beginning of the frequency fluctuation is due the fact that the frequency was changed from 15 Hz to 14 Hz instantaneously. This is due to the practical difficulties in simulating a gradual frequency change in SIMULINK™. A practical frequency fluctuation will however not cause such an overshoot in the measurement output. This was observed during real-time tests and will be addressed further in the following section with some more real-time test results.

### 6.5 Real-time implementation of APDC technique

With the promising results obtained from the simulations, the next step was to implement the disturbance cancellation technique in real-time in the DSP and test it. Once again the user friendly programming environment enabled the cut and paste of disturbance cancellation block onto the digital controller block model to build up the complete controller. The dSPACE™ software environment was useful in performing the implementation in a step-by-step manner. The procedure followed will now be described.

#### 6.5.1 Real-time emulation of whirling of the rotor shaft

Any failures of real time tests of the cancellation technique can make the rotor to crash land on the emergency bearings while rotating, thereby causing mechanical damage. This can also damage the stator winding of the induction machine unless the supply is switched off immediately after a sudden stop of the rotor (the back emf will instantaneously vanish). It was therefore necessary to be assured of the proper functionality of the algorithm, before the tests under rotation could be carried out.
Thus the whirling of the rotor shaft was emulated in real-time. All that was required to test the cancellation algorithm was to introduce sinusoidal disturbance in the position measurements of an AMB. To achieve this, two sinusoidal signals of suitable amplitude and $90^\circ$ phase shift were added to the set-point inputs of the control loops of the two axes of an AMB. When the sinusoidal signals were switched ON using the COCKPIT control panel, it was possible to see that the rotor was following a circular orbit in the air gap. Then the cancellation technique was switched ON. The initial tuning of the adaptation gain was also done with this mechanism and fixed to $g = 0.005$ as mentioned before. The following were the steps of the real-time simulation.

1. Elevate the rotor following the steps given in Section 4.6.
2. Switch ON the two sinusoidal signals (15 Hz and $90^\circ$ out of phase) to the two set-point inputs of the control loops.
3. Observe the circular movement of the rotor in the air gap.
4. Activate the cancellation algorithm.
5. Repeat this, while tuning the adaptation gain to obtain a stable and fast enough cancellation performance.

The above test assured that the real-time controller of the actual AMB plant remains stable, while the cancellation technique activates and reaches steady state.

In case of the AMB application, the requirement is to cancel a periodic disturbance of known frequency and its harmonics. The reason for that is the particular electrical machine with AMBs is a standard industrial one and the magnetic noise study is directly focused on induction machines working at industrial speeds (i.e. 1453 rpm for a 4 pole machine at 50 Hz). It should however be mentioned here that all real time test results presented here were recorded, when the machine was supplied from a supply frequency of 30 Hz. This was done to avoid possible mechanical damage to the rotor shaft, if the tests were done at full speed. Some interesting results obtained from the real-time tests will now be presented.

6.5.2 Cancellation of 15 Hz sinusoidal signal

For comparison purposes the DFT of one position measurement before cancellation is presented in Figure 6.14. This is the spectrum, when the machine is rotated from a supply frequency of 30 Hz. As can be seen from the figure, the first harmonic observed in the position measurement is at 15 Hz.
The cancellation process of the 15 Hz sinusoidal disturbance will now be presented. Figure 6.15 (a) shows a y-t plot of one position measurement during the convergence of APDC technique. Figure 6.15 (b) and (c) show the x-y plots of the two position measurements of one bearing, during the convergence process and after it has fully converged. From the y-t and x-y plots it can be seen that the rotor settles down to a new orbit, which is defined by the remaining harmonics in the position measurements as the fundamental is cancelled.

\[ \hat{a} \quad \hat{b} \]

\[ (0.5 \text{ s/div}, 50 \text{ mV/div}) \quad (50 \text{ ms/div}, 50 \text{ mV/div}) \]

\[ (a) \text{ y-t plot during convergence} \quad (b) \text{x-y plot during convergence} \]

\[ (c) \text{x-y plot after cancellation has settled} \quad (50 \text{ ms/div}, 50 \text{ mV/div}) \]

\textit{Figure 6.15: Cancellation of the first harmonic}

The other important variation of interest here is the convergence of the coefficients (\( \hat{a} \) and \( \hat{b} \)) in (6.9). Figure 6.16 below shows these variations. The oscillatory nature of the adaptation coefficients can be due to the presence of some DC bias in the position measurement fed into the cancellation technique. Even though the set-point is zero (rotor positioned at the centre), there can be a small bias term due to some offsets in the hardware electronics which causes this oscillatory nature of the adaptation coefficients.
The DFT plot of the position sensor signal after APDC is activated and converged gives a clear picture about the frequency components remaining. This is shown in Figure 6.17 and it can be seen that the 15 Hz fundamental has been suppressed successfully by the APDC technique. A comparison between the DFT of position measurement in Figure 6.17 with Figure 6.14 will give a clear understanding about the degree of cancellation of the 15 Hz fundamental.

6.5.3 Step response while rotating

The machine was rotated and the 15 Hz fundamental disturbance was cancelled using the APDC technique. While the cancellation is active, a step disturbance of 1 V was given to the set-point input of one control loop and the variation of the measurement output is shown as an oscilloscope trace in Figure 6.18. Due to memory limitations of the DSP system, it was not possible to have a TRACE™ capture of this variation. The deviation in the negative direction is due to a negation taking place in the hardware electronics.
6.5.4 Performance under frequency fluctuation

The supply frequency to the induction machine was dropped to 29.6 Hz, while the cancellation technique was active for 15 Hz fundamental. This effectively reduced the first harmonic disturbance to 14.8 Hz. Figure 6.19 (a) shows the DFT of the position measurement during the frequency fluctuation. This may be compared with Figure 6.17 to understand the resulting degree of performance degradation. Figure 6.19 (b) shows the x-y plot of the new trajectory of the rotor. When compared the trajectory of the rotor in Figure 6.19 (b) with the trajectory shown in Figure 6.15 (c), it can be seen that the rotor deviates more from the set-point in the latter case.
One could observe from the DFT shown in Figure 6.19 that the 14.6 Hz spike emerging as the cancellation becomes weak due to fluctuation of frequency. Figure 6.20 shows the changes in the coefficients ($\hat{a}$ and $\hat{b}$) in (6.9) during the fluctuation (this may be compared with the corresponding variations in simulations in Figure 6.13 (a) and (b)).

6.5.5 Closed loop frequency response

Actual measurement of the closed loop frequency response of the system including the APDC technique also gave some interesting results. The control algorithm along with the cancellation technique was implemented in the DSP. The AMB was controlled from the DSP. A Dynamic Signal Analyser with Swept-Sine measurement facility was employed for the task. The sinusoidal signal generated by the Dynamic Signal Analyser was sampled from another ADC channel and the sampled value was added to the set-point input of one digital control loops for the AMB. Then the position measurement was tapped and fed to the spectrum analyser for the estimation of frequency response. The measured frequency response is shown in Figure 6.21.
**Figure 6.21: Measured frequency response**

The measured frequency response shows a notch at 15 Hz, which corresponds to the tuned frequency of the APDC technique. This also helps in a way to predict the behaviour of the scheme under a frequency fluctuation. Since the amplitude response shows continuity in the vicinity of 15 Hz, what could be expected, when the periodic disturbance deviates slightly from 15 Hz is a reduction of the amount of periodic disturbance cancellation. Hence the possibility of a disastrous instability can be ruled out. In fact, the simulation results presented in Section 6.4.4 and the experimental results presented in Section 6.5.4 justify this argument.

**6.6 Outer digital control loop for the analog controller**

![Diagram of outer digital control loop](image)

**Figure 6.22: Outer digital control loop for the analog controller**

As mentioned in the beginning of this chapter, the same type of whirling of the rotor shaft can be observed, when the rotor is elevated from the analog control system and rotated. The digital control system can now be used to build up an outer digital control loop for the original analog controller of the AMB actuator. Purpose of this outer digital control loop is to facilitate the APDC technique as such a feature is not available with the analog control
(a) Before cancelling  
(b) 15 Hz cancelled
Periodic disturbance cancellation

(c) 30 Hz cancelled

(d) 45 Hz cancelled
system. Cancellation technique having the same structure was employed. Figure 6.22 shows the block diagram of the system that was implemented.

First simulations were performed to study the potential of the method. An approach similar to the one that was done in the case of the digital controller was followed and it was possible to cancel several harmonics since the computational overhead on the DSP was less (control of AMB actuators is not performed by the DSP in this case). Figure 6.23 presents a series of DFT plots to illustrate the cancellation of periodic disturbances from 15 Hz up to 75 Hz, while the machine is rotated by 30 Hz supply voltage.

As in Section 6.5.5, a closed loop frequency response measurement was also taken and Figure 6.24 shows the results. Since parallel cancellation blocks were implemented to cancel up to 75 Hz (fifth harmonic), it is possible to see a series of notches in the amplitude response corresponding to the tuned frequency of each APDC block.

Figure 6.23: Cancellation of harmonics (15...75 Hz)
In fact, this analog-digital cascade control system becomes a very powerful tool for the acoustic noise study using induction motor with AMBs. Since it is possible to cancel almost all significant periodic disturbance components, very high positioning accuracy can be achieved. Thus the system will meet both the stability and performance criteria described in Section 4.2 of Chapter 4. The implementation shown here is a clear indication that performance of the AMB system can be improved by the introduction of the digital control. This in a way also indicates the superiority of digital control over analog control.

From the results that were presented throughout this chapter, it is now clear that the APDC technique provides a robust method to improve the positioning accuracy of the rotor shaft under rotating condition. Before winding up this chapter, some comments must be made on this method. One big disadvantage of this APDC algorithm is the vast amount of memory required by the algorithm. This is because, cancellation of periodic disturbance signals of different frequencies requires parallel implementation of the same APDC structure (one APDC block for each frequency). Apart from this the inability to track the disturbing frequency automatically is also another disadvantage. Hence, the method as it is applies only for periodic disturbances with known frequency. If the algorithm has the auto frequency tracking facility, then its use could have been more generalised.
7. Conclusions and future work

As mentioned in the beginning of this report, the main objective of this work was to develop a flexible test rig for the acoustic noise study of standard induction machines. The idea of developing a digital control platform was to provide a user-friendly environment for the noise study of standard induction motors as the original analog control system has some obvious limitations. In the process of developing the control system this research has made some significant contributions to the Active Magnetic Bearing control area.

7.1 Main contributions of this work

Some of the important contributions will be highlight in this section.

(a) Start-up method for AMB

The AMB actuator is a non-linear system and most of the controller designs are done based on the linearized model around the centre point of the air gap. When the AMB is inactive, the rotor is always at rest on the emergency bearings - a point relatively far away from the point of linearization. This makes the start-up of the AMB and elevation of the rotor to the centre point of the air gap, a difficult task. The start-up procedure that has been suggested in this report can be easily implemented to accomplish this difficult task successfully. In fact, the whole procedure can be automated by using appropriate time delays between each action taken, so that the user will be able to start-up the complete system just by activating a single trigger signal.

(b) Mathematical modelling and system identification

The work done on modelling the complex AMB system to reasonable accuracy and building the improved model later using system identification experiment and validating both models is also a clear contribution. This in fact addresses the demanding research area of “Identification of Open Loop Unstable Non-linear systems”.

(c) Precise eccentric rotor positioning through APDC

The rotor unbalance due to the eccentricity causes periodic disturbances in the position sensor signals and rotor tends to deviate (whirling) from the set-point. The APDC technique suggested here is capable of cancelling these periodic disturbances and forcing the rotor to remain at the eccentric set-point during rotation. From the result presented, it is clear that the positioning accuracy can be improved more and more as the number of harmonics that are cancelled is increased. The limitations here are the available memory of the DSP to accommodate the code and the execution time. One method that can be used here is to execute the APDC blocks as low priority tasks at slower sampling rate than the sampling rate for the AMB control loops.

(d) Improvements to the original APDC technique

The introduction of the notch filter (6.16) to the AFC method originally suggested in [24] can also be highlighted as a new contribution to this technique. This pre-filtering has proved to be helpful in achieving better convergence of the algorithm.
(e) **Concept of analog and digital controllers in cascade**

The concept of using the inner analog control loops for positioning and outer digital control loops for periodic disturbance rejection will be very effective in the acoustic noise study. In addition to periodic disturbance rejection, the digital control loop in cascade can be used to implement additional functions such as on-line set-point changes of the AMB actuators. Thus, most of the features that were initially lacking in the analog control system can be offered to the user with this cascaded system.

### 7.2 Future work

The research work that has been reported here opens up several interesting directions for further investigation. It is important to mention some of those possibilities for future research related to this work before concluding this part of the thesis. They will be listed and briefly explained below.

(a) **Digital controller design based on improved model**

In Chapter 5 an improved model of the AMB actuator was developed using the system identification experiments. It could be seen from the model validations presented at the end of Chapter 5 that the high frequency modes of the rotor suspension system have been modelled up to a certain extent in this new model. It will be interesting to design a digital controller using this improved model and test it on the real system to see whether the new controller can elevate the rotor without exciting the vibration modes of the rotor.

(b) **Stability analysis of the APDC technique with notch filter**

Since it changes the closed loop poles of the system, the introduction of the notch filter (for pre-filtering) demands a new stability analysis different from [24]. Performing this analysis and obtaining modified limitations for the adaptation gain $g$ will be a major contribution to the AFC algorithm. Another interesting to investigate is the capability of the APDC algorithm to reducing the third harmonic of its tuned frequency (this has been reported in [27]) with the notch filter. Since the notch filter will not pass through the third harmonic, one could expect the APDC to lose that capability with this new modification.

(c) **Magnetic noise study with the digital controller**

This has to be carried out as a separate research as magnetic noise study of induction motors is a wide area, which involves a lot of analytical computations. The developed digital motion control scheme is capable of changing the dynamic stiffness of the rotor suspension system. Therefore, interesting results can be obtained on the magnetic noise phenomena of electrical machines using this test rig. Specially, since the rigid coupling between the stator housing and the rotor is no longer there due to the use of magnetic bearings, future research in this area can give important observations on how the magnetic noise is linked to the coupling between the rotor and the stator.


Reference


Appendix A : Manufacturer data for the AMB system

Information provided by the manufacturer of the AMB system will be mentioned here.

A.1 Parameters of the induction motor

The set-up is equipped with the production type of an induction motor MBT 160L manufactured by the firm ABB (ABB - Asea Brown Boveri Ltd.)

A.1.1 The main parameters of the motor

Important parameters of the motor are tabulated in Table A.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value with unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>15 kW</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1453 rpm</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>380/660 V</td>
</tr>
<tr>
<td>Rated current</td>
<td>30.4/15 A</td>
</tr>
<tr>
<td>Frequency</td>
<td>50/60 Hz</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>256.8 mm</td>
</tr>
<tr>
<td>Air gap diameter</td>
<td>165 mm</td>
</tr>
<tr>
<td>Lamination length</td>
<td>170 mm</td>
</tr>
<tr>
<td>Air gap length</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>Air gap induction</td>
<td>0.93 T</td>
</tr>
<tr>
<td>Rotor mass</td>
<td>35 kg</td>
</tr>
<tr>
<td>Typical expected eccentricity of the rotor</td>
<td>25% or 0.1 mm</td>
</tr>
<tr>
<td>Magnetic force (design value) calculated</td>
<td>1078 N</td>
</tr>
</tbody>
</table>

*Table A.1: Parameters of the induction motor*

A.1.2 Schematic drawings of the motor parts

Some mechanical drawings describing the parts of the modified system are shown here.

*Figure A.1: Configuration of the shaft*
A.1.3 Modified rotor dimensions

Total mass 40 kg

Moment of inertia (experimental) 0.56 kg\( \cdot \)m\(^2\)

Distance between the AMB centers 350 mm.

Figure A.2: Configuration of the rotor

Figure A.3: General view of the motor with AMBs

Figure A.4: Directions of control channels
A.2 Specifications of the active magnetic bearings

Specifications of the AMBs are given here

A.2.1 Stator core dimensions:

![Configuration of the radial AMB](image1)

![Configuration of the axial AMB](image2)

**Figure A.5: Configuration of the radial AMB**  
**Figure A.6: Configuration of the axial AMB**

A.2.2 Parameters of the radial AMB

Parameters of the radial AMB are given in Table A.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value with unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of poles</td>
<td>8</td>
</tr>
<tr>
<td>Number of turns per pole</td>
<td>92</td>
</tr>
<tr>
<td>Connection of coils in winding is</td>
<td>Series</td>
</tr>
<tr>
<td>Diameter of wire</td>
<td>1.1 mm</td>
</tr>
<tr>
<td>Resistance of winding</td>
<td>1.2 Ω</td>
</tr>
<tr>
<td>Rated current</td>
<td>2.5 A</td>
</tr>
<tr>
<td>Maximum current</td>
<td>5 A</td>
</tr>
<tr>
<td>Carrying capacity</td>
<td>1000 N</td>
</tr>
<tr>
<td>Outer diameter of stator</td>
<td>148 mm</td>
</tr>
<tr>
<td>Air gap diameter</td>
<td>80 mm</td>
</tr>
<tr>
<td>Lamination length</td>
<td>40 mm</td>
</tr>
<tr>
<td>Pole thickness</td>
<td>16 mm</td>
</tr>
<tr>
<td>Air gap length</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>Inductance in central position</td>
<td>38 mH</td>
</tr>
</tbody>
</table>

**Table A.2: Parameters of the radial AMB**
Appendix A

A.2.3 The main parameters of axial AMB

Parameters of the axial AMB are given in Table A.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value with unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of turns</td>
<td>140</td>
</tr>
<tr>
<td>Diameter of wire</td>
<td>0.71 mm</td>
</tr>
<tr>
<td>Rated current</td>
<td>2.5 A</td>
</tr>
<tr>
<td>Maximum current</td>
<td>5 A</td>
</tr>
<tr>
<td>Carrying capacity</td>
<td>400 N</td>
</tr>
<tr>
<td>Air gap length</td>
<td>0.5 mm</td>
</tr>
</tbody>
</table>

*Table A.3: Parameters of the radial AMB*

A.2.4 Position sensor characteristics

*Figure A.7: Eccentric value versus reference signal*
Appendix B : Mathematical Modelling of AMB Actuator

Appendix B gives all numerical calculations involved in deriving the linearized state space models of the four rotor suspension systems. Manufacturer’s information from Appendix A is also used in this. Detailed derivations of the equations can be found in [18].

B.1 Derivation of the mathematical model of the AMB actuators

The equivalent mass of the rotor, which is the component of the rotor weight that has to be handled by a particular bearing is given by

\[ M_1 = \frac{1}{l_{12}^2}(J_1 + M_2 z_2^2). \]  

(B.1)

The inductance factor is

\[ K_L = \frac{p w^2}{8 m^2} \cdot \mu_0 \cdot d \cdot l \cdot \Delta \alpha. \]  

\[ \Delta \alpha = \frac{t}{d / 2}. \]  

(B.2)

Main inductance of the electromagnet is found from

\[ L = k_L. \]  

(B.3)

Actual inductance with the correction for leakage inductance is

\[ L_1 = k_1 \cdot L. \]  

(B.4)

Constant external force that has to be handled by each electromagnet (component from the self weight) is given by

\[ Q_c = -\frac{M g}{2}. \]  

(B.5)

Control voltages for bias condition \( u_{1c}, u_{2c} \) is given by

\[ u_{1c} = r \cdot i_{1c} \]
\[ u_{2c} = r \cdot i_{2c}. \]  

(B.6)

What is meant by bias condition here is the average control voltages of the upper and lower electromagnets, when the rotor is stabilised at the centre of the air gap. Ratio of bias currents (currents corresponding to the above situation) is

\[ \beta_{21} = \frac{i_{2c}}{i_{1c}}. \]  

(B.7)

Bias current ratio factor is defined by,

\[ \beta = 1 + \beta_{21}^2. \]  

(B.8)

Current stiffness \( h_1, h_2 \) is found from,
Negative stiffness $C$, of the rotor suspension system is given by,
\[
C = \frac{k_p^2.L(i_{1c}^2 + i_{2c}^2)}{\delta^2}. \tag{B.10}
\]

One magnetic actuator of the AMB is shown in Figure B.1.

\[U' \quad \text{AMPLIFIER} \quad (C_A) \quad U \]

\[\beta_{21} \quad \text{Figure B.1: One magnetic actuator} \]

The basic non-linear equations for the plant are
\[
\dot{\mathbf{x}} = \frac{1}{M_1} F(i_1, i_2, y) + \frac{M_a(y)}{M_1 l_{12}} + \frac{1}{M_1} Q(t)
\]
\[
\left( \frac{k_i}{\delta - k_p y} + L_i \right) \dot{x}_1 + \frac{k_i k_p}{\delta - k_p y} x_1 + r_i = u_1
\]
\[
\left( \frac{k_i}{\delta + k_p y} + L_i \right) \dot{x}_2 + \frac{k_i k_p}{\delta + k_p y} x_2 + r_i = u_2,
\]
where, $F(i_1, i_2, y)$ is the force by the two electromagnets and $M_a(y)$ is the effect of axial bearing. $i_1$, $i_2$ are the instantaneous currents in the top and bottom electromagnets and $u_1$, $u_2$ are the respective instantaneous voltages. The two equations for them are shown below.

\[
F(i_1, i_2, y) = \frac{k_L k_p}{2} \left[ \frac{i_1^2}{(\delta - k_p y)^2} - \frac{i_2^2}{(\delta + k_p y)^2} \right]
\]
\[
& \quad \tag{B.12}
\]
\[
M_a(y) = \frac{1}{2} \left( \frac{\partial^2 L_a}{\partial y^2} \right)_0 \left( i_{1c}^2 + i_{2c}^2 \right) \frac{1}{l_{12}} y.
\]
\( I_{1c}, I_{3c} \) are the bias currents of axial AMB and

\[
\left( \frac{\partial^2 L_{kk}}{\partial \phi_x^2} \right)_0 = \left( \frac{\partial^2 L_{kk}}{\partial \phi_y^2} \right)_0 = \frac{\omega^2 \mu_0 S}{8} \frac{r_a^2 + r_b^2 + R_a^2 + R_b^2}{2}.
\] (B.13)

Then the equations are linearised around the equilibrium point \( y=0 \) and the result is

\[
\begin{align*}
\ddot{y} &= \frac{C_r + C_a}{M_1} y + \frac{h_1}{M_1} i_{1v} - \frac{h_2}{M_1} i_{2v} + \frac{Q(t)}{M_1} \\
\ddot{i}_{1v} &= -\frac{h_1}{L_1} \ddot{y} - \frac{r}{L_1} i_{1v} + \frac{1}{L_1} u_{1v} \\
\ddot{i}_{2v} &= \frac{h_2}{L_1} \ddot{y} - \frac{r}{L_1} i_{21v} + \frac{1}{L_1} u_{21v},
\end{align*}
\] (B.14)

where \( i_{1v}, i_{2v} \) are small deviations of bearing currents from \( i_{1c} \) and \( i_{2c} \) respectively. In usual arrangements both power amplifiers are driven from one control signal, \( Uc' \) in this case. When the equations are modified to include \( Uc' \) a state space description given below is obtained

\[
\begin{bmatrix}
\dot{y} \\ \dot{\xi}_y \\ \dot{\xi}_{1v} \\ \dot{\xi}_{2v}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & \frac{h_1}{M_1} & 0 \\ 0 & \frac{h_1}{L_1} & -(r+C ACM_1) & 0 \\ 0 & \frac{h_2}{L_1} & 0 & -(r+C ACM_1) \\
0 & 0 & 0 & \frac{r}{L_1}
\end{bmatrix}
\begin{bmatrix}
y \\ \xi_y \\ i_{1v} \\ i_{2v}
\end{bmatrix}
+ \begin{bmatrix}
0 \\ 0 \\ 0 \\ \frac{r}{L_1}
\end{bmatrix}
\begin{bmatrix}
Q(t) \\ U_{1d}(t) \\ U_{2d}(t)
\end{bmatrix}
\]

\[
y(t) = \begin{bmatrix} C_T & 0 & 0 & 0 \\ 0 & \xi_y & i_{1v} & i_{2v} \end{bmatrix}
\] (B.15)

where \( Q(t), U_{1d}(t), U_{2d}(t) \) represent the disturbance terms of corresponding states. The linear transformation \( i_v = i_v - \beta_{21} i_{2v} \) where \( \beta_{21} = \frac{i_{2c}}{i_{1c}} \) (the ratio of bias currents) is used to reduce the system down to order three. The new system description is
Appendix B

This linearised state space set of equations is the basis for the mathematical model of the rotor
suspension system. In all simulations this basic set of equations are used while introducing non-
linearities like saturation of power amplifiers, limits of the rotor movement due to finite air gap and so
on.

When the typical values for the two radial AMBs are substituted in the equations B.1 to B.16 to
obtain the values shown in Table B.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AMB1_x</th>
<th>AMB1_y</th>
<th>AMB2_x</th>
<th>AMB2_y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$ (kg)</td>
<td>15.05</td>
<td>15.05</td>
<td>14.69</td>
<td>14.69</td>
</tr>
<tr>
<td>$K_L$ (Hm)</td>
<td>$13.61 \times 10^6$</td>
<td>$13.61 \times 10^6$</td>
<td>$13.61 \times 10^6$</td>
<td>$13.61 \times 10^6$</td>
</tr>
<tr>
<td>$L_1$ (mH)</td>
<td>34.025</td>
<td>34.025</td>
<td>34.025</td>
<td>34.025</td>
</tr>
<tr>
<td>$L_2$ (mH)</td>
<td>38.108</td>
<td>38.108</td>
<td>38.108</td>
<td>38.108</td>
</tr>
<tr>
<td>$Q_c$ (N)</td>
<td>-142.42</td>
<td>-142.42</td>
<td>-139.8</td>
<td>-139.8</td>
</tr>
<tr>
<td>$i_{c1}$ (A)</td>
<td>2.218</td>
<td>2.298</td>
<td>2.294</td>
<td>2.214</td>
</tr>
<tr>
<td>$i_{c2}$ (A)</td>
<td>1.592</td>
<td>1.588</td>
<td>1.578</td>
<td>1.534</td>
</tr>
<tr>
<td>$u_{c1}$ (V)</td>
<td>2.742</td>
<td>2.972</td>
<td>2.753</td>
<td>2.657</td>
</tr>
<tr>
<td>$u_{c2}$ (V)</td>
<td>1.935</td>
<td>1.927</td>
<td>1.894</td>
<td>1.841</td>
</tr>
<tr>
<td>$\beta_{21}$ (fixed)*</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$h_1$ (N/A)</td>
<td>179.2</td>
<td>182.1</td>
<td>180.3</td>
<td>174.02</td>
</tr>
<tr>
<td>$h_2$ (N/A)</td>
<td>121.68</td>
<td>125.88</td>
<td>124.03</td>
<td>120.57</td>
</tr>
<tr>
<td>$C_r$ (N/m)</td>
<td>$1.417 \times 10^6$</td>
<td>$1.368 \times 10^6$</td>
<td>$1.408 \times 10^6$</td>
<td>$1.317 \times 10^6$</td>
</tr>
</tbody>
</table>

Table B.1: Typical values for the two AMBs

- It must be noted here that the manufacturer has fixed $\beta_{21}$ to be at 1 on the contrary to the
definition in the theoretical derivation.
State space models of the two radial AMBs are given in Table B.2

<table>
<thead>
<tr>
<th>Bearing</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMB1_x</td>
<td>$\begin{bmatrix} 0 &amp; 1 &amp; 0 \ 9.36 \times 10^4 &amp; 0 &amp; 11.93 \ 0 &amp; -9425 &amp; -10067 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0 \ 3441 \end{bmatrix}$</td>
<td>$[52000 \ 0 \ 0]$</td>
</tr>
<tr>
<td>AMB1_y</td>
<td>$\begin{bmatrix} 0 &amp; 1 &amp; 0 \ 9.95 \times 10^4 &amp; 0 &amp; 12.03 \ 0 &amp; -9499 &amp; -10067 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0 \ 3441 \end{bmatrix}$</td>
<td>$[52000 \ 0 \ 0]$</td>
</tr>
<tr>
<td>AMB2_x</td>
<td>$\begin{bmatrix} 0 &amp; 1 &amp; 0 \ 9.58 \times 10^4 &amp; 0 &amp; 12.27 \ 0 &amp; -9463 &amp; -10067 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0 \ 3441 \end{bmatrix}$</td>
<td>$[52000 \ 0 \ 0]$</td>
</tr>
<tr>
<td>AMB2_y</td>
<td>$\begin{bmatrix} 0 &amp; 1 &amp; 0 \ 8.97 \times 10^4 &amp; 0 &amp; 11.85 \ 0 &amp; -9133 &amp; -10067 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 \ 0 \ 3441 \end{bmatrix}$</td>
<td>$[52000 \ 0 \ 0]$</td>
</tr>
</tbody>
</table>

*Table B.2: State space models of rotor suspension systems in the 2 AMBs*
Appendix C : Modelling of Hardware Electronics

Appendix C gives all information about the hardware electronics associated with the control of AMBs. The Figure C.1 shows the modular form of hardware electronics. All four de-coupled closed loop controllers have the same structure.

Figure C.1: Complete hardware electronics in modular form

In the following sections s-domain transfer functions will be obtained for each important circuit modules in numerical form using the component values traced from the hardware.

C.1 Low-pass filter and summer at set-point

The tracking error circuit, which consists of the set-point scaling, measurement filtering at summer is shown in Figure C.2. $V_r$ is the set-point, $V_f$ is the measurement and $V_o$ is the output. For the OPAMP circuit shown in Figure C.2 following nodal equation can be written

\[
\frac{V' - V_f}{R_1} + \frac{V' - 0}{R_3} + \frac{V' - V_o}{R_4} + \frac{V' - 0}{(1/C_i s)} = 0 \quad (C.1)
\]

\[
\frac{0 - V_r}{R_2} + \frac{0 - V'}{R_3} + \frac{0 - V_o}{(1/C_i s)} = 0. \quad (C.2)
\]

From the above two equations, the output voltage $V_o$ can be obtained in terms of $V_f$ and $V_r$ as

\[
V_o = \frac{Q_r(s)}{P(s)} V_r + \frac{Q_f(s)}{P(s)} V_f, \quad (C.3)
\]

where

\[
Q_r(s) = -(R_1 R_4 + R_3 R_4 + R_4 + R_1 R_3 R_4 C_i s) \\
Q_f(s) = -R_2 R_3 \\
P(s) = R_1 R_2 R_3 R_4 C_i C_2 s^2 + R_2 (R_4 + R_3 R_4 + R_1 R_3) C_2 s + R_1 R_2. \quad (C.4)
\]
Figure C.2: Low-pass filter and summer module at setpoint

$V_r$ is the set-point input that can be assumed time invariant with respect to $V_f$, which is the feedback signal from the position sensor. Therefore, a simplification can be made by putting $s = 0$ in $Q_r(s)/P(s)$, which will result in just a scalar multiplication factor with $V_r$. It is obvious that $V_f$ is subjected to both low-pass filtering and scaling at this summing point.

With the component values shown in Figure C.3, the output for DC signals

$$V_0 = -0.2267V_r - V_f.$$  

(C.5)

Figure C.3: Demodulation, summation with low-pass filtering of position feedback signal
Block diagram of the summer, low-pass filtering and the position feedback can be given as in Figure C.4.

\[
\frac{2.13 \times 10^8}{s^2 + 22723s + 2.13 \times 10^8}
\]

**Figure C.4:** Summer, low-pass filtering and the position feedback

### C.2 Analog controller

Controller structure in Figure C.5 consists of two parallel paths and OPAMP addition at the output. Integral action takes place in one branch and the derivative action with filtering takes place in the other branch. Even though there is no clear proportional path final expression for the controller transfer function carries a proportional quantity. The following equations describe the transfer function of the controller.

**Integrator path:**

\[
V_{int} = - \left[ \frac{R_2}{R_1} + \frac{1}{R_1 C_1 s} \right] V_{err}
\]  \hspace{1cm} (C.6)

**Derivative path:**
\[ V_{\text{der}} = -\left[ \frac{R_4C_2s}{1 + R_3C_2s} \right] V_{\text{err}} \]  \tag{C.7}

**Summer output:**

\[ V_{\text{out}} = -R_0 \left[ \frac{V_{\text{int}}}{R_5} + \frac{kV_{\text{der}}}{R_6} \right] \]  \tag{C.8}

Where \( 0 < k < 1 \) represents the attenuation taking place by a tuning pot at the derivative output. By substituting for \( V_{\text{int}} \) and \( V_{\text{der}} \) one can obtain the following final expression for the overall controller module.

\[
V_{\text{out}} = \left[ \frac{R_1R_4}{R_1R_3} + \left( \frac{R_7}{R_1R_3C_1} \right) \frac{1}{s} + \frac{R_1R_2C_2}{R_6(1 + R_3C_2s)} k \cdot s \right] V_{\text{err}}
\]  \tag{C.9}

Block diagram of the controller for the existing component values is shown in Figure C.6.

![Block diagram of the controller](image)

**Figure C.6: Analog controller**

### C.3 Current feedback of the power amplifier

The differential amplifier arrangement shown in Figure C.7 gives the following input output relationship.

\[
V_{\text{out}} = R_4 \left[ \frac{R_2}{R_2 + R_3} \left( \frac{1}{R_1} + \frac{1}{R_4} \right) V_i - \frac{1}{R_1} V_2 \right]
\]  \tag{C.10}

The particular combination of resistors used in the circuit has the following relationships.

\[ R_3 = R_4 = 10, R_1 = 10, R_2 = R \]  \tag{C.11}

This reduces the expression to

\[ V_{\text{out}} = 10(V_i - V_2) \].  \tag{C.12}
But with the bridge arrangement typical to switched mode power amplifiers, average value of $(V_1 - V_2)$ is $rI_{av}$ where $I_{av}$ is the average current through the winding. Therefore the average current through the winding is sensed as a voltage signal by means of this arrangement (since $r = 0.1$ Ohm, $V_{out} = I_{av}$ from Equation C.12). Current feedback along with low-pass filtering is shown in Figure C.8.

The low-pass filter stages just before the summing point are shown in Figure C.9.
C.4 Other circuit modules

Apart from the circuit modules discussed so far in detail, the following modules also play an important role in the control loop.

1. Position sensor (52000 V/m - manufacturer information & experimentally measured)
2. Power amplifier (experimentally measured DC gain 11.84)
Part II

Sensorless Control of Permanent Magnet Synchronous Motors for High-Speed Applications
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List of symbols

Unless otherwise stated, the quantities are in per-unit

$u_k(t), \ k = a,b,c$ - instantaneous phase voltage

$i_k(t), \ k = a,b,c$ - instantaneous phase current

$U_k(t), \ k = a,b,c$ - amplitude of sinusoidal phase voltage

$\phi_k(t), \ k = a,b,c$ - phase difference in each phase voltage

$\psi_k, \ k = a,b,c$ - flux linkage in each phase

$u_S$ - stator voltage space vector in stasionary reference frame

$K$ - scaling constant for co-ordinate transformation

$u_\alpha$ - real component of $u_s(t)$ in stationery reference frame

$u_\beta$ - imaginary component of $u_s(t)$ in stationery reference frame

$T_{32}$ - 3 phase to 2 phase transformation matrix

$T_{23}$ - 2 phase to 3 phase transformation matrix

$u$ - stator voltage space vector in synchronous reference frame

$i$ - stator current space vector in synchronous reference frame

$\hat{i}$ - estimated staotor current space vector in synchronous reference frame

$\tilde{i}$ - error between $i$ and $\hat{i}$ defined as $i - \hat{i}$

$u_d(t)$ - real component of $u(t)$ in synchronous reference frame

$u_q(t)$ - imaginary component of $u(t)$ in synchronous reference frame

$i_d(t)$ - real component of $i(t)$ in synchronous reference frame

$i_q(t)$ - imaginary component of $i(t)$ in synchronous reference frame

$\theta$ - actual rotor position

$\hat{\theta}$ - estimated rotor position

$\tilde{\theta}$ - rotor position estimation error defined as $\theta - \hat{\theta}$

$T_{SR}$ - stationery to synchronous co-ordinate transformation matrix

$T_{RS}$ - synchronous to stationery co-ordinate transformation matrix

$\psi_m$ - flux due to permanent magnets of PMSM

$L_{aa}$ - average air gap inductance per phase of PMSM stator

$L_l$ - leakage inductance per phase of PMSM stator

$L_g$ - maximum amplitude of inductance variation as the rotor rotates

$R_s$ - stator resistance per phase

$L_d$ - $d$-axis inductance

$L_q$ - $q$-axis inductance

$T$ - electromagnetic torque produced

$n_p$ - pole pair number

$J$ - inertia of the rotor
\( B \) - mechanical damping constant of the rotor
\( T_l \) - load torque
\( \omega \) - actual rotor speed (same as electrical angular frequency)
\( \hat{\omega} \) - estimated rotor speed
\( \bar{\omega} \) - speed estimation error defined as \( \omega - \hat{\omega} \)
\( \omega_{ref} \) - speed set-point
\( \omega_e \) - speed error defined as \( \omega_{ref} - \omega \)
\( G_p \) - linear matrix transfer function describing motor current dynamics - actual
\( \hat{G}_p \) - estimated \( G_p \)
\( G_c \) - internal model controller matrix transfer function
\( F \) - final IMC controller matrix transfer function
\( F_d \) - final IMC controller transfer function for \( d \)-axis
\( F_q \) - final IMC controller transfer function for \( q \)-axis
\( G_{cl} \) - closed loop matrix transfer function of current dynamics
\( \alpha \) - closed loop bandwidth of current dynamics
\( T_{sn} \) - normalised sampling time
\( T_s \) - real sampling time (s)
\( i_{dref} \) - set-point for \( d \)-axis current
\( i_{qref} \) - set-point for \( q \)-axis current
\( G_M \) - transfer function describing speed dynamics (\( \omega \) versus \( i_{qref} \) characteristics)
\( K_M \) - motor torque constant
\( \hat{G}_M \) - estimated \( G_M \)
\( G_{CM} \) - internal model controller for speed
\( F_M \) - final IMC controller for speed control
\( G_{clm} \) - closed loop transfer function of speed dynamics
\( \alpha_s \) - closed loop bandwidth of speed dynamics
\( U_{MAX} \) - voltage limit of the inverter
\( I_{MAX} \) - rated current limit of the motor
\( \omega \) - reference variables
\( y \) - output variables
\( u \) - desired controlling variables
\( v \) - desired controller state variables
\( \omega' \) - realizable reference variables
\( u' \) - realizable (actual) controlling variables
\( v' \) - realizable controller state variables
\( \varepsilon_d \) - \( d \)-axis voltage error
\[ \varepsilon_q \] - \textit{q-axis} voltage error

\[ \rho \] - closed loop bandwidth of the adaptive observer

\[ T_{d-on} \] - turn-on delay of IGBT (s)

\[ T_{d-off} \] - turn-off delay of IGBT (s)

\[ T_d \] - effective dead-time (s)

\[ T_{dc} \] - dead-time introduced by the controller (s)

\[ T_{i-on} \] - ideal on-time of IGBT (s)

\[ T_{a-on} \] - actual on-time of IGBT (s)

\[ I_{Sk} \quad k = A, B, C \] - output current from one inverter pole

\[ V_{Sk} \quad k = A, B, C \] - output voltage from one inverter pole

\[ R_T \] - conducting resistance of IGBT

\[ V_{T-on} \] - on-time voltage drop across IGBT

\[ V_T \] - total voltage drop across IGBT

\[ R_D \] - conducting resistance of freewheeling diode

\[ V_{D-on} \] - on-time voltage drop across freewheeling diode

\[ V_D \] - total voltage drop across freewheeling diode

\[ U_{DC} \] - half the DC-link voltage - instantaneous

\[ U_{DC,n} \] - half the DC-link voltage - nominal

\[ R_C \] - per phase equivalent resistance representing core loss

\[ R_e \] - per phase equivalent frequency dependent resistance

\[ L_e \] - per phase equivalent frequency dependent inductance

\[ R_d \] - equivalent resistance along \textit{d-axis} representing active losses

\[ R_q \] - equivalent resistance along \textit{q-axis} representing active losses
1. Introduction

The motor drive has been highlighted as one of the most common actuator in a motion control system [4]. Many complex motions can be derived from the basic rotational motion of an electrical machine. In fact, one way of defining motion control is “the application of high-performance servo drives to rotational or translational control of torque, speed and/or position” [1]. In most applications variable-speed drives are capable of providing improved performance, productivity, energy efficiency and better controllability of overall industrial processes [2, 3]. Induction Machine (IM) has been the “work horse” in industry since many years. Thanks to the invention of Field Oriented Control (FOC) method [1, 18], IM was able to take the lead and play a key role in all aspects of motion control as a high-performance drive. However, Permanent Magnet AC machine (PMAC) is at present emerging as a good contender to the Induction Machine in the motion control area. Main feature in the basic construction of a PMAC machine is having permanent magnets mounted on the rotor to provide air gap flux. Since no magnetising current is necessary in these machines due to the presence of permanent air gap flux, the dynamical response of the PMAC is usually faster than the IM. The machine has higher efficiency and higher power density ($kW/kg$), which are beneficial for application in motion control area [5].

There can of course be different variants of PMAC machines. Permanent Magnet Synchronous Motor (PMSM), which is one variant of PMAC machines, is the focus of this part of the thesis. When the stator windings of the PMAC machine is distributed in several slots to produce a sinusoidally distributed stator flux in the air gap, the machine is called a Permanent Magnet Synchronous Machine (PMSM) [5]. Being a synchronous machine direct rotor position sensing is sufficient to obtain the rotor flux location and hence the control of PMSMs becomes less computationally heavy (when compared to IM, where position of the rotor flux must be estimated using an algorithm [1]).

Since accurate rotor position information is essential for the closed loop control of PMSMs, these drives usually employ a shaft-mounted encoder or a resolver to measure the rotor flux position. In most applications the presence of an encoder or resolver causes several disadvantages due to additional cabling cost, higher number of connections between motor and controller, noise interference and reduced robustness. These are the reasons that motivate the need to develop sensorless control schemes for PMSM drive systems. The objective of this part of the thesis is to present several digital motion control techniques that can be incorporated to realise a complete sensorless control strategy for PMSM drives in high-speed range.

1.1 Sensorless control drive structure

A fundamental definition for a sensorless control AC drive was established in the introduction of this thesis. The most general implementation structure according to that definition was given in Figure 1.5 of the “Introduction to the thesis”. The complete sensorless controller implementation structure in this particular application example is slightly different. The line voltage measurement is a tedious task, when the switched voltages are involved. This is the typical situation in the case of an inverter fed machine. Therefore, the approach here is to skip the line voltage measurement and contend with the sampled line currents and DC-link voltage. Thus, the sensorless control drive structure used in this application example is shown in Figure 1.1.
This method of implementation reduces the number of A/D converter channels required for the digital control implementation and hence the overall cost. In fact, the number of line current measurement can also be reduced to two instead of three in a real industrial implementation. Due to the symmetry the third current can be calculated as the negative sum of the two already sampled currents. This will finally reduce the A/D channel requirement in a commercial application down to two.

1.2 Brief overview of this work

As mentioned earlier, this part of the thesis is presented as one digital motion control application out of the two discussed in this thesis. The main focus is the “Sensorless Control of Permanent Magnet Synchronous Motors for High-Speed Applications”. This part of the thesis includes even other implementation aspects of a well performing AC drive. Apart from the sensorless adaptive observers for speed and rotor position estimation presented in Chapter 7, this part of the thesis covers many other aspects that are general to all AC drive systems. Rather than using ad hoc methods, automatic control theory is applied to overcome implementation difficulties. Fast current control and reliable speed control are key words in any variable speed AC drive system. Current and speed control methods suggested in this work together with solutions for the output saturation in the controllers can be applied to any variable speed AC drive with suitable modifications. Compensation methods for inverter non-idealities can also be highlighted as general to all inverter fed three-phase loads. The reliability of performance of the suggested control strategies is further strengthened by machine parameter estimation and its usage for improved performance discussed towards the end of this part.

1.2.1 Layout of the report

This part of the thesis has ten chapters. A brief chapter-by-chapter overview is presented here.

- Chapter 2:

In this chapter, the most essential information on space vector theory and basic equations of PMSMs are presented. Vector control of PMSMs will be discussed towards the end.
• **Chapter 3:**

The PMSM test rig used to implement all the control strategies will be described in this chapter. Two back-to-back connected PMSMs (one as the motor and the other as the load) of same specifications will be used in the test rig. Both machines will be controlled by separate digital control systems.

• **Chapter 4:**

Current control methods for PMSMs in synchronous frame will be discussed in this chapter. An investigation on the performance of basic internal model based current controller will be made. The effect of sampling delay in discrete-time control systems will be thoroughly investigated. Predictive observer based solution and an improved internal model controller will be suggested as solutions.

• **Chapter 5:**

Speed controller design is the key issue in Chapter 5. Different controller structures that can be used for electrical machines will be discussed. Controllers that are capable of tracking constant set-points as well as ramp-like set-points will be presented.

• **Chapter 6:**

In a typical AC drive, the current controller and speed controller form a cascaded system with the current control loop being the inner loop and the speed controller being the outer loop. The problem of output saturation of both these controllers will be the focus of this chapter. Some novel concepts to the variable speed drive control area will be presented in this chapter.

• **Chapter 7:**

The adaptive observer for speed and rotor position estimation of the PMSM under sensorless operation will be discussed in this chapter. A computationally efficient method based on previous work suggested in [6] will be further investigated using non-linear system analysis. A modified algorithm capable of tracking speed ramps will also be suggested.

• **Chapter 8:**

Being a non-ideal device, the inverter has many drawbacks. Dead-time between the IGBT switching, resistive voltage drop of the switching components and the DC-link voltage fluctuations have been identified as the most problematic non-idealities. Analysis of the adverse effects of these problems and compensation methods will be the focus of Chapter 8.

• **Chapter 9:**

Machine parameters such as stator resistance and inductance of any AC machine varies with temperature and operating frequency. Characterising the frequency dependent machine parameter variations in this PMSM and suggesting some methods to use the obtained variations for improved performance will be presented in this chapter.

• **Chapter 10:**

Correlating the results, conclusions and suggestions on future research based on this work will be made in this chapter.
2. Basics of Permanent Magnet Synchronous Motors

The history of important use of permanent magnets goes down to the time, when the compass was invented in China and exported to Europe around 1200 AD. Since then, the coercive strength \((kA/m)\), remanent flux density \((T)\) and maximum energy product \((kJ/m^3)\) of the permanent magnets have been increasing with the invention of each new material \([7, 8]\). At the same time, the prices have been dropping, which is a significant motivating factor for the increased usage of Permanent Magnet (PM) machines in industrial applications. PM machine construction today, is a well-developed technology. Information about different machine constructions is often found in the literature \([7]\). Control strategies required for different machine types can also differ from each other. The methods and conventions used for mathematical modeling of different PM machines can be different as well. The main focus of this work is on the PMSMs that have symmetrically distributed three-phase stator winding to produce sinusoidal flux distribution in the air gap \([7]\). The aim of this chapter is to briefly present the basic transformations and model equations for PMSMs that will be used throughout this part of the thesis. The structure of the vector controller \([14]\), which is the most essential component in an AC drive, controlled in synchronous frame, will also be described towards the end of the chapter.

2.1 Variants of PMSMs

Information about the principles and different basic constructions of PM machines can be found in \([7]\). Since the main focus in this part of the thesis is on PMSMs, it is interesting to brief on the front end of this technology. With what has been presented in \([7]\) as the fundamental principle, several new designs have come-up for a wide spectrum of applications.

One variant is to have a slot-less stator by introducing what is called an air gap winding \([9]\). Removing the stator slots gives rise to smoother stator flux distribution in the air gap. This reduces the cogging torque and also the eddy current losses on the rotor surface. Removal of teeth leads to increased copper area, making it possible to increase the (power/volume) ratio further. As an example the slotless PMSM design in \([9]\) has a rated power output of 80kW with a nominal speed of 12 000 rpm and is intended for a high-speed screw-compressor drive.

Integration of the power electronic driver (inverter) and the motor produces an “integral motor”, as it is commonly known in the electrical machine industry. Concept of integration has the advantages of high load efficiency, easy installation and commissioning, less space requirements, reduced Electromagnetic Compatibility (EMC) problem etc. Integrated induction motors has quite a long history. Recent work has been reported on integrated PMSMs also \([10]\). The higher (power/volume) ratio, which is a typical characteristic to PMSMs, makes them ideal for integral motors for high power applications (The integral motor in \([10]\) is a 15 kW one for the normal industrial operating speed range of \(\pm 0-1500\) rpm).

An application of a PMSM with switched stator windings, as a traction drive is discussed in detail in \([8]\). By connecting the stator winding sectors in different combinations (classical Y and \(\Delta\)) using electronic switches, operation above base speed has been achieved. This is somewhat similar to the operation in the field weakening range (above base speed) of an induction motor. Unlike in the case of an induction motor, the rotor flux is permanently built-up by the magnets of a PMSM. This gives the advantage of not having to deal with slower
rotor flux dynamics as in the case of induction motor. The application in [8] exploits this particular characteristic of PMSMs.

A slightly different PMSM machine construction applied in hybrid electric vehicles is discussed in [11] and [12]. Construction of the machine differs from the fact that the stator winding is located on the inner rotor and the permanent magnets are located on the inner surface of the outer rotor (again called “rotor” as the machine is based on a “the double rotor concept”). The unit works as an integrated energy transducer and can work both in motor mode and generator mode.

2.2 Summary of space vector representation of three-phase systems

Having a clear understanding about the space vector representation of three-phase systems is important for the study of variable speed AC drives. If not clearly understood, this representation is likely to be confused with the phasor notation [13]. Only the important equations and transformations of space vector theory that are often used in this work, will be briefly explained here. Detailed information about space vector theory can be found in [15, 16].

2.2.1 Space vector representation

Unlike phasor notation [13], the space vector representation allows the user to describe a variable frequency three-phase system without any zero sequence components. As an example, three-phase voltages \( u_a(t), u_b(t) \) and \( u_c(t) \) can be defined such that

\[
\begin{align*}
  u_a(t) + u_b(t) + u_c(t) &= 0, \quad \forall t \\
  u_a(t) &= U_a \cos(\theta_a(t) + \phi_a) \\
  u_b(t) &= U_b \cos\left(\theta_b(t) - \frac{2\pi}{3} + \phi_b\right) \\
  u_c(t) &= U_c \cos\left(\theta_c(t) - \frac{4\pi}{3} + \phi_c\right)
\end{align*}
\]  

(2.1)

and

\[
\begin{align*}
  \theta_a(t) &= \int_0^t \Phi_a(\tau) d\tau. \\
  \theta_b(t) &= \int_0^t \Phi_b(\tau) d\tau. \\
  \theta_c(t) &= \int_0^t \Phi_c(\tau) d\tau
\end{align*}
\]  

(2.3)

With this definition of \( \theta(t) \), it is assured that the angular frequency of the three-phase system \( \Theta(t) \) is a time varying quantity and at a given instant all three voltages have the same instantaneous angular frequency.

Since the three components add to zero, it becomes possible to describe them as a system with two degree of freedom, which can be given by the complex valued space vector

\[
\begin{align*}
  \mathbf{u}_s(t) &= \frac{2}{3} K \left[ u_a(t) + e^{j \frac{2\pi}{3}} u_b(t) + e^{j \frac{4\pi}{3}} u_c(t) \right]. \\
  K &= \text{scaling constant that can be arbitrarily fixed (methods of selecting } K \text{ in a meaningful manner will be discussed later). It is understood that } \mathbf{u}_s(t) \text{ is a space vector rotating at the}
\end{align*}
\]  

(2.4)
sensorless control of PMSMs for high-speed…

In usual notation $u_s(t)$ then has the real and imaginary components described as

$$u_s(t) = u_a(t) + j u_b(t)$$

This α-β coordinate system, which is stationary with respect to the rotating voltage space vector $u_s(t)$ at stator frequency, is said to be stator coordinates and the three-phase to two-phase transformation is described by the linear matrix transformation

$$\begin{bmatrix} u_a(t) \\ u_b(t) \end{bmatrix} = K \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} u_a(t) \\ u_b(t) \\ u_c(t) \end{bmatrix} K^T.$$  \hspace{2cm} (2.6)

The inverse transformation is not straightforward as the transformation matrix is not symmetric. However, by augmenting (2.6) with (2.1) it can be easily shown that the inverse transformation (two-phase to three-phase transformation) is

$$\begin{bmatrix} u_a(t) \\ u_b(t) \\ u_c(t) \end{bmatrix} = K^{-1} \begin{bmatrix} 1 & 0 \\ -\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ -\frac{1}{2} & -\frac{2}{2} \end{bmatrix} \begin{bmatrix} u_a(t) \\ u_b(t) \end{bmatrix}.$$  \hspace{2cm} (2.7)

Fixing the directions of the α-β reference frame with respect to the magnetic axes of the three-phase stator windings is usually done by establishing the positive direction of the α-real axis of the two-phase system to be the same as the positive direction of the magnetic axis of phase A. This is known to be a stator oriented coordinate system and is shown in Figure 2.1.

**Figure 2.1:** Location of magnetic axes

**Figure 2.2:** α-β and d-q coordinate systems
2.2.2 Synchronous coordinates

Due to the rotating nature of the electrical quantities in $\alpha-\beta$ coordinate system, the current, voltage and flux linkage signals in the stator frame will have alternating characteristics. The modeling, analysis and controller design of three-phase AC machines can further be simplified by transforming the quantities onto a coordinate system that is rotating in synchronism with the fundamental stator frequency. Such a set of axes is called a synchronous coordinate system. The corresponding transformation to obtain the same space vector in synchronous coordinates is done by applying the vector rotation

$$u(t) = e^{-j\theta} u_s(t). \quad (2.8)$$

Here again, the complex space vector $u(t)$ has a real and an imaginary component, usually known as $d$-$q$ components respectively. They are described as

$$u(t) = u_d(t) + ju_q(t). \quad (2.9)$$

This particular vector rotation and the inverse of it can also be expressed in the form of linear matrix transformations

$$\begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} u_a(t) \\ u_b(t) \end{bmatrix} T_{SR} \quad (2.10)$$

and

$$\begin{bmatrix} u_a(t) \\ u_b(t) \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix} T_{SR} \quad (2.11)$$

Graphical illustration of the two axis systems is given in Figure 2.2.

2.2.3 Selection of scaling constant

As mentioned earlier, the selected value for scaling constant $K$ may change from one user to the other. For the space vector description in (2.4), three scaling conventions that can be useful in various three-phase applications will presented here [14]. If $U_a(t) = U_b(t) = U_c(t) = U$ and $\phi_a = \phi_b = \phi_c = \phi$ are substituted in (2.2), and assumed that the angular frequency to be quasi-steady at $\omega$, the corresponding voltage space vectors in stator and synchronous coordinates will be

$$u_s(t) = KUe^{j(\omega t + \phi)}. \quad (2.12)$$

(i) Peak value scaling ($K = 1$)

When $K$ is set to $1$, the magnitude of the space vector is equal to the peak value of the corresponding phase quantities. This is called as peak value scaling and is specially useful in implementing digital control schemes. The reason is, with peak value scaling, the peak values of the two-phase quantities in the discrete system will be the same as the peak values of the corresponding three-phase quantities. This will enable the user to obtain the optimum use of the dynamic ranges of A/D and D/A converters and will also make the comparison of digital and analog quantities easier.
(ii) **RMS value scaling (\( K = I/\sqrt{2} \))**

When this scaling is used, the RMS value of the three-phase quantities will be the same as the magnitude of the corresponding space vector. This is somewhat similar to our choice of the magnitude of phasors discussed in the Section 2.3.1.

(iii) **Power invariant transformation (\( K = \sqrt{3/2} \))**

If a symmetrical three-phase system is described from the equivalent per-phase voltage and current phasors (having phase angles of \( \phi_u \) and \( \phi_i \) respectively)

\[
\hat{U} = \frac{U}{\sqrt{2}} e^{j\phi_u} \quad \text{and} \quad \hat{I} = \frac{I}{\sqrt{2}} e^{j\phi_i},
\]

the expression for the average power dissipation in the three-phase system will be

\[
P = \frac{3}{2} UI \cos(\phi_u - \phi_i).
\]

(2.13)

If the corresponding space vector description of the same system is

\[
u_s = K U e^{j(\omega t + \phi_u)} \quad \text{and} \quad i_s = K I e^{j(\omega t + \phi_i)},
\]

the instantaneous power dissipation will be

\[
P = \text{Re}(\mathbf{u}_s \cdot \mathbf{i}_s^*) = K^2 UI \cos(\phi_u - \phi_i).
\]

(2.14)

This will give the same expression as in (2.13), when \( K = \sqrt{3/2} \). Since the expressions will appear to be the same as they look like in phasor notation, this scaling is believed to be analytically friendly. However, in this application \( K=1 \) (peak value scaling) will be used due to the several advantages for digital control applications.

### 2.3 Dynamic model of PMSM

With this background of auxiliary tools used in the three-phase system description, it is now appropriate to look at the modeling of the PMSM for the variable speed operation. This is also referred to as dynamic modeling of PMSM. The conventional approach of modeling of a general PMSM can be found in [7] and [17]. The particular motors considered in this work are surface mounted PMSMs. A more compact and simplified modeling procedure is presented in Appendix B, which can only be applied for PMSMs with negligible saliency (for surface mounted PMSMs). Basic assumptions, that form the fundamental platform for all types of analysis, are mentioned below [7].

1. All soft magnetic media are assumed to have linear magnetic properties (magnetic saturation in the iron is not considered).
2. The permanent magnet material is assumed to have linear, temperature independent demagnetization curve.
3. Windings and the magnet are assumed to produce radially directed and sinusoidally distributed flux density in the air gap.
4. Hysteresis and eddy current losses are neglected.
5. Resistances and inductances are assumed to be independent of temperature and frequency. Thus skin effects are neglected.
It is assumed that the stator winding is Y-connected. Thus, zero sequence currents do not exist.

Even though the simplifications obtained through these assumptions are essential for the validity of the model, it will be necessary to take into account at least some of these non-ideal-natures of the machine in the process of implementing proper control strategies. One such example is the temperature and frequency dependence of machine parameters. As dealt with later in this part of the thesis, this behavior can be critical for the accurate performance of sensorless control algorithms.

In the coming sections, the general PMSM machine model equations derived from first principles will be presented [7, 17]. All the equations here are derived in normalized (per-unit) quantities including time (see Appendix A for details).

2.3.1 Typical inductance variation

In the three-phase stator of a PMSM, the magnetic axes of the three windings are 120° electrical degrees apart from each other as shown in Figure 2.3 for a two-pole machine.

Understanding the variation of self-inductance of one stator winding is essential for the inductance calculation that can be found in many textbooks [17]. Typical variation of self-inductance versus the angular position of the permanent magnet rotor flux (angle $\theta$ in Figure 2.3) is shown in Figure 2.4.

This means that the self-inductance of each phase varies with a sinusoidal amplitude $L_g$, about an average value ($L_{aa}=L_{aa}+L_{al}$), where $L_{al}$ is the leakage inductance, which is assumed to be a constant. Minimum of the sinusoidal variation occurs, when the permanent magnet rotor flux is in-line with the magnetic axis of the particular phase. Obviously, the maximum occurs, when permanent magnet rotor flux is perpendicular with the magnetic axis. Computation of the self-inductance of each phase and mutual inductance between the phases is required to find the corresponding flux linkages [7, 17]. Once the flux linkages are known, the voltage equations for the machine in three-phase can be written.

2.3.2 Voltage equations of three-phase system

Voltage equations for three-phases can be written in compact matrix form as
\[
\mathbf{u} = \mathbf{R} \mathbf{I} + \mathbf{L} \frac{d\mathbf{I}}{dt} + \mathbf{\omega} \frac{d\mathbf{L}}{d\theta} + \mathbf{\omega} \frac{d\mathbf{\theta}}{d\theta},
\]

where

\[
\mathbf{u} = [u_a \ u_b \ u_c]^T, \quad \mathbf{L} = [l_{aa} \ l_{bb} \ l_{cc}]^T, \quad \mathbf{\theta} = \begin{bmatrix} \psi_m \cos(\theta) \\ \psi_m \cos\left(\theta - \frac{2\pi}{3}\right) \\ \psi_m \cos\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}^T.
\]

are column vectors and

\[
\mathbf{L} = \begin{bmatrix}
    l_{aa} + l_1 - l_g \cos(2\theta) & -\frac{1}{2} l_{aa} - l_g \cos\left(2\theta - \frac{2\pi}{3}\right) & -\frac{1}{2} l_{aa} - l_g \cos\left(2\theta + \frac{2\pi}{3}\right) \\
    -\frac{1}{2} l_{aa} - l_g \cos\left(2\theta + \frac{2\pi}{3}\right) & l_{aa} + l_1 - l_g \cos\left(2\theta + \frac{2\pi}{3}\right) & -\frac{1}{2} l_{aa} - l_g \cos(2\theta) \\
    -\frac{1}{2} l_{aa} - l_g \cos(2\theta) & -\frac{1}{2} l_{aa} - l_g \cos\left(2\theta + \frac{2\pi}{3}\right) & l_{aa} + l_1 - l_g \cos\left(2\theta + \frac{2\pi}{3}\right)
\end{bmatrix}
\]

\[
\mathbf{R} = \begin{bmatrix}
    R_s & 0 & 0 \\
    0 & R_s & 0 \\
    0 & 0 & R_s
\end{bmatrix}
\]

are symmetric matrices given by machine parameters.

### 2.3.3 Transformation to two-axis stator coordinates

The transformation matrices \(T_{32}\) and \(T_{23}\) described in (2.6) and (2.7) respectively are now used to transform the voltage equation (2.15) into two-axis stator coordinates. Transformed equation after introducing space vector quantities is given by

\[
\mathbf{u}_s = \mathbf{R}_s^T \mathbf{i}_s + \mathbf{L}_s^T \frac{d\mathbf{i}_s}{dt} + \mathbf{\omega} \frac{d\mathbf{L}_s^T}{d\theta} \mathbf{i}_s + \mathbf{\omega} \frac{d\mathbf{\theta}_m}{d\theta}.
\]

The transformed inductance and resistance matrices are

\[
\mathbf{L}_s^T = T_{32} \mathbf{L} T_{23} = \begin{bmatrix}
    \frac{3}{2} l_{aa} + l_1 - \frac{3}{2} l_g \cos(2\theta) & -\frac{3}{2} l_g \sin(2\theta) \\
    -\frac{3}{2} l_g \sin(2\theta) & \frac{3}{2} l_{aa} + l_1 + \frac{3}{2} l_g \cos(2\theta)
\end{bmatrix}
\]

\[
\mathbf{R}_s^T = T_{32} \mathbf{R} T_{23} = \begin{bmatrix}
    R_s & 0 \\
    0 & R_s
\end{bmatrix}
\]

Transformed permanent magnet flux linkage can be expressed as

\[
\mathbf{\theta}_m^s = T_{32} \mathbf{\theta}_m = \begin{bmatrix} \psi_m \cos(\theta) \\ \psi_m \cos\left(\theta - \frac{2\pi}{3}\right) \\ \psi_m \cos\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}.
\]

The next step is to write voltage equations in synchronous coordinates by doing the axis transformation discussed in Section 2.2.2.

### 2.3.4 Voltage equation in synchronous coordinates

The next step is to transform the voltage equation (2.16) into synchronous coordinates. Transformation matrices \(T_{SR}\) and \(T_{RS}\) described in (2.10) and (2.11) respectively are used for the purpose. Corresponding transformation results-in
\[ u = \begin{bmatrix} R_d & 0 \\ 0 & R_q \end{bmatrix} i_d + \begin{bmatrix} L_{d} & 0 \\ 0 & L_q \end{bmatrix} \frac{di}{dt} + \omega \begin{bmatrix} 0 & -L_q \\ L_d & 0 \end{bmatrix} i_d + \omega \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \phi_m \] (2.19)

where

\[ \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \left( L_{aa} - L_g \right) + L_l & 0 \\ 0 & \frac{3}{2} \left( L_{aa} + L_g \right) + L_l \end{bmatrix}. \] (2.20)

Equation (2.19) can now be rewritten as

\[ u_d = R_d i_d + L_d \frac{di_d}{dt} - \omega L_q i_q \]
\[ u_q = R_q i_q + L_q \frac{di_q}{dt} + \omega L_q i_d + \omega \psi_m. \] (2.21)

### 2.3.5 Torque expression

To obtain the expression for the instantaneous torque of the PMSM, the basic relationship of electromechanical energy conversion is used [16]. This is based on the principle that electrical energy absorbed from the supply must be equal to the sum of the energy stored in the magnetic field, the mechanical energy exerted by the rotor movement (work done by the mechanical torque) and the losses in the system. With peak value scaling \((K=1)\), the torque developed can be derived as

\[ T = \frac{3}{2} n_p \left[ \phi_m i_q - (L_q - L_d) \dot{i}_d i_q \right], \] (2.22)

where \(n_p\) is the pole pair number. With this, all the dynamical equations required to fully describe the PMSM are now available.

### 2.3.6 Complete dynamic model of PMSM

It is now possible to present the complete dynamic model of the PMSM using (2.21), (2.22) and the equation for the mechanical dynamics of the rotor. This can be described as

\[ \frac{di_d}{dt} = -\frac{R_d}{L_d} i_d + \frac{\omega L_q}{L_d} i_q + \frac{1}{L_d} u_d \]
\[ \frac{di_q}{dt} = -\frac{R_q}{L_q} i_q - \frac{\omega L_d}{L_q} i_d - \frac{\omega \psi_m}{L_q} + \frac{1}{L_q} u_q \]
\[ T = \frac{3}{2} n_p \left[ \phi_m i_q - (L_q - L_d) \dot{i}_d i_q \right] \]
\[ \frac{d\omega}{dt} = \frac{1}{J} \left( T - T_l - B\omega \right). \] (2.23)

The model is shown in block-diagram form in Figure 2.5.
This model can be considered as a general PMSM model valid for both salient (buried magnet type) and non-salient (surface mounted type). In fact, Equation (2.23) is simulated (in continuous time) as the PMSM machine model in this work. Even though it is a surface mounted PMSM (i.e. $L_d = L_q = L_s$), a saliency of 5% is assumed to comply with the experimental measurements made on the inductance (see Chapter 9).

2.4 Vector control of PMSMs - Basics

Quick torque response is a keyword in modern industrial drive systems such as servo drives. The already built up air gap flux (due to permanent magnets) in PMSMs, makes it a better contender to induction machine in this respect. The reason for this is that a faster torque response can be obtained by simply achieving a fast enough current control (time constant of current dynamics is usually much smaller than that of flux dynamics).

With the aid of transformation to synchronous coordinates, instead of dealing with AC voltage and current signals, one has the advantage of handling DC signals at the controller design stage. One big advantage this offers is the easy tuning of the controllers. This is in contrast with the case, when the controllers are to be designed for AC signals. Then, since the supply frequency varies, fixing the bandwidth of controllers to perform equally well over a wide frequency range, can be a difficult problem.

By inspection of (2.22) it can be seen that the torque produced by a non-salient PMSM mainly depends on $q$-axis stator current. On the other hand, (2.22) also indicates that in a salient pole PMSM, a $d$-axis current of opposite sign (which should not be too high to demagnetize the permanent magnets) to $q$-axis stator current can actively contribute to the torque production. In both types of machines, field-weakening operation can be achieved by

![Figure 2.5: PMSM model in block-diagram form](image_url)
changing the $d$-axis current. Otherwise, commonly used technique is to keep the $d$-axis current at zero and control the $q$-axis current according to the torque requirement.

This explanation and also the model equations of the PMSM in synchronous coordinates shows that the AC PMSM motor can be made to behave like a separately excited DC motor by the use of feedback current control [14]. This method, first recorded in [18], is known as the vector control. The aim of this section is to introduce the basic elements required for the implementation of a vector control scheme. The modules involved are clearly shown in Figure 2.6 and their basic functionality is described afterwards.

**Figure 2.6: Basic vector control structure**

- **Current sensors:** Values of all three-phase currents are required for the coordinate transformations discussed before. Yet, due to the symmetry of the three-phase system, it is common practice to measure only two line currents and estimate the third as the negative sum of the other two.

- **3-2 phase transformation:** In this block, the mathematical operation in (2.6) is performed on the measured current values, to obtain the corresponding currents in stator coordinates (i.e. $i_a$ and $i_b$). This can be implemented in analog form using hardware electronics or in digital form, if digital control is used.

- **$\alpha\beta$ to $d-q$ Transformation:** In this block, the mathematical operation in (2.10) is performed on the currents in stator coordinates. This results in the currents in synchronous coordinates (i.e. $i_d$ and $i_q$). This block requires an extra input, which is the rotor position information that is to be used as the transformation angle in $\sin(\theta)$ and $\cos(\theta)$ terms.

- **Current controller:** In this block, a current controller is implemented. The task is to control the $d$-axis and $q$-axis currents according to the need of flux and torque respectively. In addition to the current information, this block requires the rotor position information also. The reason is the dependency of current dynamics on rotor speed according to (2.23). Current controller can also be implemented both in analog or digital form. Methods of current control are discussed in detail in Chapter 4.
• **d-q to α-β Transformation:** The command voltages from the controller are transformed back to stator coordinates in this block (2.11). Here again, the rotor position information is required as an extra input.

• **2-3 Phase transformation:** This is the last step in transforming the command voltages from the controller back to command signals in the three-phases. The mathematical operation in (2.7) is performed in this block in analog or digital form.

• **PWM generation:** Three modulated pulse patterns corresponding to the three command voltages that define the current requirements in each phase are generated in this block. The outputs of this are used to switch the current elements in the inverter. Here again, analog or digital methods can be used for PWM generation. More information on PWM generation methods used in this work can be found in Chapter 3.

• **Inverter:** This is the power module or the actuator of the system. Three switched-voltage waveforms, of which the averages are sinusoidal, are applied to the motor by the inverter. Application of switched voltages introduces harmonics into the three-phase system. Though the inverter is assumed to be an ideal device for most of the analytical purposes, it has several non-ideal characteristics. How these non-idealities are dealt with is discussed in Chapter 8.

Once the basic vector controller in Figure 2.6 has been implemented, any other controller can be cascaded around it to obtain the required motion of the PMSM. For example, if a speed controller has to be implemented, the measured speed information from the machine can be used as a feedback to a suitable controller structure, which will decide the \( q \)-axis current demand at a given instant. This will be the input to the \( q \)-axis current reference in the vector controller. The typical speed controller implementation of a PMSM is depicted in Figure 2.7.

![Figure 2.7: Speed controller cascaded to the basic vector controller](image)

Problems associated with output saturations of cascaded controllers like this are theoretically dealt with in Chapter 6. With this basic knowledge on PMSM modeling and control, a description about the PMSMs test rig used in this work will be presented in the next chapter.
3. Experimental PMSM drive set-up

This chapter gives first hand information on the PMSM drive set-up used to implement all digital motion control methods for testing. The prime objective of this work is to verify the performance of the suggested sensorless control strategy together with other motion control techniques for high-speed applications. Therefore, it was decided to choose another high-speed drive system already in use in industry, which employs a different sensorless control strategy as the reference drive. The particular drive chosen was an angle grinder application that operates at a rated speed of 30,000 rpm (see Table 3.1 for detailed specifications). Two back-to-back connected PMSM machines with same specifications are used in the test rig, one as the drive motor and the other as the load machine. This chapter will give basic details of the PMSM machines, inverters, PWM generation, current and voltage sensors, rotor position sensors (incremental encoders), brake chopper for load side inverter and the digital signal processing systems.

3.1 Specifications of the test drive and the reference drive

![Back to back connected PMSMs](image)

Figure 3.1: Back to back connected PMSMs

Working at rotational speeds as high as 30,000 rpm needs specially designed motors and power electronic devices etc. Therefore it was decided to employ PMSMs designed for lower supply voltage and run the machines at 2-3 pu speed (2-3 times higher than rated) in order to reach the vicinity of the speed range of interest. The next approach was to correlate the results obtained from this system to the performance of the application of interest. In Figure 3.1 shown is the view of the two PMSMs, connected back-to-back. Table 3.1 shows the specifications of both drives (drive chosen and the reference drive). In addition to the parameters mentioned above, inertia of the shaft coupling used to connect the two PMSMs is 0.000869 kgm².

3.2 IGBT inverter

Two inverters were employed, one on the motor side and the other one on load side. The load machine runs in generating mode and hence feeds power into the DC link of the corresponding inverter. In order to limit the DC link voltage, a brake chopper circuit was
designed. This is discussed in Section 3.7. The two inverters, which are of same type, have the following specifications.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PMSMs in test rig</th>
<th>Reference Drive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated voltage / VAC</td>
<td>230</td>
<td>230</td>
</tr>
<tr>
<td>Induced emf / V_{rms,l-l} /1000 rpm</td>
<td>42</td>
<td>6</td>
</tr>
<tr>
<td>Rated current / A</td>
<td>16.3</td>
<td>8</td>
</tr>
<tr>
<td>Rated torque / Nm</td>
<td>10.5</td>
<td>0.8</td>
</tr>
<tr>
<td>Rated speed / rpm</td>
<td>3000</td>
<td>30000</td>
</tr>
<tr>
<td>Number of poles</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Rotor inertia / kgm^2</td>
<td>0.00167</td>
<td>2.7x10^{-5}</td>
</tr>
<tr>
<td>Stator resistance / \Omega_{l-l}</td>
<td>0.33</td>
<td>1.2</td>
</tr>
<tr>
<td>Inductance / mH_{l-l}</td>
<td>3.2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3.1: Specifications of the test drive and the reference drive

Inverter specifications for the motor side and load side:

- **Input supply**: 3 phase, 400V_{l-l}
- **Average DC-link voltage**: 575 V
- **Output power**: 15 kW
- **Maximum output current**: 100 A
- **Maximum switching frequency**: 15 kHz

3.3 Digital Signal Processor systems

For the purposes of controlling the motor and the load, two Digital Signal Processor (DSP) systems were employed. The advantage of using a digital control system for the load is that it gives the user the possibility of testing the drive under different loading conditions and load patterns. The two DSP systems are briefly described here.

3.3.1. DSP system for the motor

As this was the system, on which sensorless control strategies were implemented, higher demands had to be put on the DSP system. A modular type system from dSPACE™ was used for the purpose. In fact, this was the same system used for the AMB application described in Part 1 (Chapter 2) of the thesis was upgraded for this purpose.

**Input/output requirements:**

The motor side requires the following I/O functions.

(a) **Sampling of three line currents**: Even if two line currents are sufficient due to the symmetrical nature of the load, it is important to have the third line current also sampled for the evaluation purposes. This requires 3 A/D converter inputs.

(b) **DC link voltage sampling**: Since DC link voltage is used as a measured signal in the control algorithms, this will require another A/D converter input.

(c) **Position encoder**: The incremental encoder signal has to be read into the DSP in order to obtain the position information. The dSPACE™ provides a circuit board, which can drive five incremental encoders in parallel and count the output pulse streams from these encoders.
(d) **PWM generation**: According to the PWM generation method used here, one D/A channel is required for the square wave output, which generates the triangular wave. To send the three PWM reference signals three more A/D converters are required.

Apart from the incremental encoder board, which was added to the previous system, the rest of the circuit modules are the same as used in the AMB application. Specifications of the incremental encoder board DS3001 can be found in [30]. Specifications of other circuit modules can be found in Chapter 2 of Part 1 of this thesis. The software environment is based on program development in Matlab\textsuperscript{TM}/SIMULINK\textsuperscript{TM}/RTI (Real-Time Interface) platform [19, 20, 21]. As in the AMB application, real time parameter tuning is done through the COCKPIT\textsuperscript{TM} [22] and on-line data acquisition is done using TRACE\textsuperscript{TM} [23]. The hardware arrangement of the DSP system is schematically shown in Figure 3.2.

![Hardware arrangement of the DSP system (Motor side)](image)

**Figure 3.2**: Hardware arrangement of the DSP system (Motor side)

### 3.3.2. DSP system for the load machine

The load has fewer demands on the DSP system. A single board DSP system from dSPACE\textsuperscript{TM} was used for the purpose.

**Input output requirements:**

The load side needs the following I/O functions.

(a) **Sampling of two line currents**: Two line currents are sufficient due to the symmetrical nature of the load. This requires 2 A/D converter inputs.

(b) **Position encoder**: The incremental encoder signal has to be read into the DSP in order to obtain the position information. One of the two onboard incremental encoder interfaces on the dSPACE controller board was used for the purpose.

(c) **PWM generation**: Since there is an onboard slave processor, which can generate PWM waveforms, the Digital I/O was used to get the generated PWM signals out to the inverter.

Specifications of the I/O channels of the single board DSP system DS1102 can be found in [31]. A list of available I/O channels is as follows.

1. Two 16-bit A/D channels
2. Two 12-bit A/D channels
3. Four 12-bit D/A channels
4. 2 incremental encoder interface channels
5. Digital I/O based on the TMS320P14 DSP micro-controller (offers six PWM circuits)
The schematic block diagram of the controller board is shown in Figure 3.3.

**Figure 3.3: Hardware arrangement of the DSP system for load machine**

The software development platform is the same as the other DSP system.

### 3.4 PWM generation and synchronous sampling

In inverter fed machine control applications, current sampling can be a problem due to the switched nature of the line voltages. The most common technique that is used is the so-called synchronous sampling. This method allows the design engineer to do away with current filtering, which can cause phase delays in the sampled currents. Synchronous sampling is tightly linked with the PWM generation and the concept is easily understood by referring to Figure 3.4. Different methods were used to accomplish the synchronous sampling on the motor side and on the load side. This was due to the different capabilities of the DSP systems on the two sides.

**Figure 3.4: Concept of synchronous sampling**

#### 3.4.1 PWM generation on the motor side

On the motor side, triangular PWM generation is used. To obtain the triangular wave shape, a bipolar square wave output from an A/D channel of the DSP system was integrated using an
analog integrator. The schematic block diagram of the method is shown in Figure 3.5. The method of course is not the best way of generating the PWM, as there are more accurate internal timer based digital PWM generation schemes. Yet, this solution had to be used to suit the particular DSP system capabilities.

![Block Diagram of PWM Generation](image)

**Figure 3.5: PWM generation on the motor side**

The frequency of the square wave in this case is half the base sampling frequency of the DSP. This means that the ADCs take one sample in, at every rising and falling edge of the bipolar square wave (in other words, it is at the same instants as the top and bottom peaks of the triangular wave occurs). These exactly are the suitable instances for synchronous sampling.

### 3.4.2 PWM generation on the load side

All available features of the Digital Signal Processing (DSP) system for the load side were discussed in Section 3.3.2. In this section, the method of PWM generation on the load side will be briefly explained. This method can be called as digital PWM, as the signals are generated in the DSP system and sent out. The reader may refer to Figure 3.3 also for more details of the hardware arrangement of the DSP system. In addition to the master DSP (TMS320C31), in this system there is a slave micro-controller (TMS320P14), which works as a Digital I/O sub-system. This system can be customised according to the application and has the ability to execute an application specific DSP program in parallel to the master DSP.

The Digital I/O sub-system has a compare sub-system consisting of six compare registers and six action registers controlling the output pins. The compare registers continuously compare their values with the counter registers of the two timers of TMS320P14. If a match is found the contents of the action registers determine, which kind of action takes place on the output pins. This slave processor arrangement saves computation time of the master DSP, while the PWM generation is also more accurate than the method used on the motor side.

Implementing synchronous sampling of the line current with this PWM generation needs some external logic circuitry. In this DSP system, PWM generation in all six channels have been synchronised to one internal clock in order to achieve higher speed. Thus, it can be observed that the rising edges of all channels occur at the same instance. In addition, since the PWM signal generation is carried out not from the master processor but from the slave micro controller, the pattern is not at all synchronised with the sampling operation of the currents. A method to shift the PWM pulses to the middle of each PWM period has been suggested in [27]. To do this, two PWM channels are required for the correct PWM pulse generation of each phase. Once the reference values for PWM generation are calculated from the vector controller, each of them are fed to a signal diversion module shown in Figure 3.6 (a). The
module will determine the amount of modulation required in the two channels dedicated for that particular phase, so that the correct PWM pulse will be generated by the inverter at the centre of the period. Those two new reference values are fed to the corresponding inputs of the PWM generation module in the SIMULINK™ controller block model. When the two channels are XORed, the resulting output will produce a pulse at the centre of its PWM period. The next problem is how to synchronise the sampling with PWM pattern. This is done by placing the ADC block model inside a subsystem triggered by a hardware interrupt block, provided with the dSPACE™ block library for SIMULINK™ [28]. The external trigger signal can be generated by connecting any of the 6 PWM channel outputs (all of them have their rising edges at the beginning of the PWM period) to pin 6 of the output connector of DS1102. The function of pin 6 is to read user interrupts to the master processor of the DSP system. Thus, the control algorithm is executed as an interrupt service routine triggered by the rising edge of the unshifted PWM pulses. Important waveforms corresponding to the PWM generation of one phase is shown in Figure 3.6 (b).

![Diagram](a) Signal diversion module  (b) Waveforms at key locations

**Figure 3.6**: PWM generation on the load side

### 3.5 Current and voltage sensors

In the set-up, Hall effect current sensors are used to sample the line currents and voltage sensor using the same Hall effect phenomena is employed to sample the DC link voltage. Current measuring circuit is based on LEM LA 100-P current transducer and the voltage measuring circuit uses a LEM LV 25-P voltage transducer. Details of the design of current and voltage measuring circuits and experimental verification of the sensitivities can be found in Appendix C. Only the experimentally verified sensitivities of the measuring circuits are given here.

- Sensitivity of current measurement = 0.22 V/A
- Attention of the voltage measuring circuit = 1.175%

### 3.6 Incremental encoder for position sensing

Both PMSMs are equipped with similar position encoders (Litton G71 S). On the motor, the purpose of having a position sensor is to enable comparison of the measured position with the estimated rotor position, when it is run sensorless. Another advantage is to switch over from sensorless control to sensored control under emergency conditions. Each encoder is TTL compatible and produces 2048 pulses per revolution. Specifications of the position encoders used can be found in Appendix C.
3.7 Brake chopper – means of protection during machine braking

In actual operation of an electrical machine, there are situations, where the speed has to be brought down from a higher value to a lower value or to standstill. These situations can basically be classified into two operating conditions.

(a) Normal operation

Under normal operating conditions, the machine speed will have to be reduced, reversed or brought to zero. Depending on the speed and current controller performance, speed reduction and machine stoppage may cause negative $q$-axis currents. On the other hand $q$-axis current must be reversed for speed reversal. If the speed controller is kept active during the deceleration and if the $q$-axis current is not limited to positive values, active braking of the machine is achieved. However, whether it is possible to use active braking depends upon the capabilities of the inverter. During the active braking, the inverter recovers kinetic energy from the machine. This kinetic energy obviously is converted back to electrical energy and must be stored in the DC-link capacitance. This may increase the DC-link voltage above safety margins unless suitable methods are used to limit it. Thus, active braking is only possible, if the drive inverter has a suitable mechanism to dissipate the excess energy recovered during the process.

(b) Emergency shut down

Under emergency conditions machine speed has to be brought to standstill by stopping the voltage supply. This is usually done by two methods.

(I). One method is to stop the IGBT switching by disconnecting the PWM signal input. When this is done, the machine is still kept connected to the inverter and the freewheeling diodes of the IGBTs will act as a three-phase diode rectifier to rectify the three-phase back emf from the machine to the DC link for a very short period. As the speed drops, the back emf will also drop so that the rectified three-phase voltage will gradually become lower than the DC-link voltage. This will automatically stop the rectification by the freewheeling diodes. Deceleration of the machine beyond this instant totally depends on the mechanical dynamics of the drive and the user will no longer have any electrical control of the drive. However, this method of emergency shut down can be safer for the power modules of the inverter.

(II). The other method is to disconnect the three-phase supply after the inverter stage. This will totally isolate the drive. Kinetic energy of the rotor can only be dissipated as friction and windage losses in the drive just as it is in the last part of the first method.

Thus, during braking of the motor, or during stoppage of PWM input signals for IGBT switching, kinetic energy in the rotor is converted to electrical energy as back emf and pumped back to the DC-link through the inverter stage. This obviously causes the DC-link voltage to increase. The energy transfer must therefore be limited in order to save the DC-link capacitance. One method, which is energy efficient – though complex – is to feed it back to the AC mains. A simpler – less efficient – method is to dissipate the excess energy in the DC-link as heat in a high power resistor supplied from a DC chopper circuit. This type of a chopper is called a brake chopper and was employed in this application as a safety measure.

The brake chopper circuit designed here is based on MITSUBSHI™ hybrid IGBT driving IC M57959L. Details of design, input signal selection and performance of the brake chopper circuit are given in Appendix C. By testing, it was found that the actual DC-link voltages to switch ON and OFF the brake chopper signal are 669V and 663V respectively. The difference is due to the hysteresis generated from the protection logic circuits in the inverter hardware.
3.8 Grounding, Earthing and Shielding – never works without it!

Avoiding corrupted signals is essential for any electronic circuit to function properly. Especially in a digital control system, where a lot of different types of signals are sampled and converted into discrete signals, one has to be very careful about the purity of the signals being sampled. Electromagnetic compatibility (EMC) is a key word in this area. It has become so important that since the beginning of 1992, all electrical and electronic equipment marketed within the European Union have to meet approval and application requirements as laid down by the general EMC directive [24].

The significant part of the EMC problems existing in a variable speed AC drive is due to the radiation from powerful high frequency sources – namely, the inverter switching. The power cable that carries the switched voltage signals tends to radiate high frequency glitches [25]. The incremental encoder signal cable can pick up these glitches, causing errors in pulse counting or unnecessary initialization of pulse counting. The same phenomenon can affect current and voltage sampling too.

Proper grounding, earthing and shielding can always be helpful in this kind of situations. Grounding creates a short, impedance-free path for the interference current to flowing direct from the chassis of the electronics apparatus back to the interference source. On the other hand, a functional earth makes sure the proper functioning of circuits and systems by defining the voltage reference. The zero logic level of a computer is one such example. The protective earth is used to earth all exposed parts of a system.

The best points to ground and the most suitable way to shield can change from one application to the other. This makes it difficult to establish a standard method of grounding, earthing and shielding common to all applications. Yet, a lot of literature can be found on this topic suggesting some general rules. Two such rules on shielding, which are immediately related to this problem, are mentioned below [26].

Rule 1: An electrostatic shield enclosure, to be effective, should be connected to the zero-signal reference potential of any circuitry contained within the shield.

Rule 2: The shield conductor should be connected to the zero-signal reference potential at the signal-earth connections.

Figure 3.7 shows the method of shielding and grounding for the PMSM drive set-up of interest.

Some important comments on selecting the incremental encoder cable must also be made at this stage. Since differential signals are sent from each phase of such an encoder, twisted pairs must be used for carrying the two signals of each phase. Thus, a twisted pair cable with outer shield is recommended for incremental encoder signal. The D-sub connectors at each end of the cable must also be shielded type and the cable shielded must be galvanically connected to the connector shield.

Some very important information on Electromagnetic Emission (EMI) related to power cable shielding of modern PWM AC drives can be found in [29].
3.9 Basic characterization of the system

The experimental set-up explained so far is an integration of a number of electronic circuit modules. It is of course essential to test each module to be sure of their proper operation. Some basic tests are also required as an evaluation of the system performance. In addition, to develop simulation models of the control strategies, a lot of information from the real-time prototype is required. Some of the measurements done in order to characterize the system are presented in Appendix C. All the per-unit quantities of interest for the PMSM under consideration will be computed in this section. Definitions of all the base quantities for per-unit calculations can be found in Appendix A. As the first step however, the base values for the fundamental electrical parameters must be carefully established giving emphasis to the meaningfulness of their choices. The fundamental base quantities for this application are voltage, current, electrical angular frequency and time. Corresponding base values are selected as follows.

(a) **Voltage** \( (U_{\text{base}}) \): The peak value of the per-phase induced emf of the machine at rated speed is taken as the base value of voltage. According to the manufacturer information mentioned earlier in this chapter (induced line to line rms emf at 1000rpm is 42 V), this results in

\[
U_{\text{base}} = \frac{\sqrt{2}U_{3000\text{rpm}, l-l}}{\sqrt{3}} = \frac{\sqrt{2} \times 126}{\sqrt{3}} = 103 \text{ V}.
\]  

(b) **Current** \( (I_{\text{base}}) \): The peak value of the rated current \( (16.3 \text{ A} \text{ rms}) \) of the machine is used here.

\[
I_{\text{base}} = \sqrt{2} \times 16.3 = 23.1 \text{ A}
\]  

(c) **Electrical angular speed** \( (\omega_{\text{base}}) \): Electrical angular frequency of the supply voltage, when the machine is rotating at rated synchronous speed \( (3000 \text{ rpm}) \) is taken as the base value for electrical angular frequency.
\[ \omega_{base} = \frac{p}{2}, \quad \omega_{mech} = \frac{6}{2} \times \frac{2\pi \times 3000}{60} = 942 \text{ rad} / \text{s} \]  \hspace{1cm} (3.3)

(d) Time (\(t_{base}\)): The inverse of the base value for electrical angular frequency is chosen to be the base value for time.

\[ t_{base} = \frac{1}{\omega_{base}} = \frac{1}{942} = 0.00106 \text{s} / \text{rad} \]  \hspace{1cm} (3.4)

With the establishment of these fundamental base values, it is possible now to calculate all the other base values for this drive defined in Chapter 2. The Table 3.2 shows the corresponding quantities.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux</td>
<td>0.109 Wb</td>
</tr>
<tr>
<td>Impedance</td>
<td>4.5 ( \Omega )</td>
</tr>
<tr>
<td>Inductance</td>
<td>4.8 mH</td>
</tr>
<tr>
<td>Mechanical angular speed</td>
<td>314 rad/s</td>
</tr>
<tr>
<td>Power</td>
<td>3569 W</td>
</tr>
<tr>
<td>Torque</td>
<td>11.4 Nm</td>
</tr>
<tr>
<td>Inertia</td>
<td>0.385 \times 10^{-4} kgm^2</td>
</tr>
<tr>
<td>Damping constant</td>
<td>0.0363 Nm/(rad/s)</td>
</tr>
</tbody>
</table>

**Table 3.2: Base values for the drive**

Another machine parameter that can be estimated from the available data is the permanent magnet flux. This is done from the induced line to line voltage information (at 1000rpm induced line to line voltage is 42 V rms). This gives the relationship

\[ 100\pi \times \psi_{m} = \frac{\sqrt{2} \times 42}{\sqrt{3}} \]

\[ \psi_{m} = 0.109 \text{ Wb}. \]

This value is important to find the corresponding per-unit permanent magnet flux. All important per-unit quantities for the drive can now be calculated using the manufacturer information (see Table 3.1) and the base quantities given in Table 3.2. For basic simulations and controller designs, per-unit values calculated from the manufacturer data are used. The per-unit values of interest are shown in Table 3.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Actual value</th>
<th>Per-unit value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_d)</td>
<td>1.6 mH</td>
<td>0.33</td>
</tr>
<tr>
<td>(L_q) (Assumed to be 5% higher than (L_d))</td>
<td>1.68 mH</td>
<td>0.35</td>
</tr>
<tr>
<td>(R_s)</td>
<td>0.1663 ( \Omega )</td>
<td>0.037</td>
</tr>
<tr>
<td>(\psi_{m})</td>
<td>0.109 Wb</td>
<td>1</td>
</tr>
<tr>
<td>Total inertia (J)</td>
<td>4.209 \times 10^{-3} kgm^2</td>
<td>109.3</td>
</tr>
<tr>
<td>Damping*</td>
<td>0.00456 Nm/(rad/s)</td>
<td>0.1256</td>
</tr>
</tbody>
</table>

**Table 3.3: Per-unit values for the drive**

\* The damping constant has been determined experimentally, as there is no information on damping constant available from the manufacturer. A simple test carried out to estimate this quantity is described in Appendix C.
4. Current control implementation

Several current control methods for three-phase inverter fed applications, have been suggested in the literature. In the beginning of this chapter, a brief review of current control methods for rotating machines will be presented. Emphasis will be given to reported work on synchronous frame current control methods, since this is today the widely used technique for PMSMs. This will be followed by some suggestions on the implementation of a current controller with fast dynamic response, as applied in this work.

4.1 Review of current control methods

In variable speed drive applications, obtaining fast dynamic responses of flux and torque is an essential requirement. To achieve this objective some form of a current controller has to be employed that will select the correct voltage vector such that the motor current will follow the desired current commands. In terms of the vector controller discussed in Chapter 2, this means achieving fast dynamics of $d$-axis and $q$-axis currents respectively. The basic requirements for such a current controller can be summarized as follows [32].

- Ideal reference tracking over a wide output frequency range (no phase and amplitude errors).
- High dynamic response of the system.
- Constant switching frequency to guarantee safe operation of converter semiconductor power devices.
- Low harmonic content.
- Good DC link voltage utilization.

To achieve these objectives, there are several different methods that have been reported in the past. Some of the basic strategies will be discussed in the next section.

4.1.1 Basic strategies of current control

There are several well-known current control methods for inverter fed motor drives [32]. Implementation aspects, merits, demerits and comparisons of these methods have been well documented and can be easily found in the form of textbook [33] or journal publication [32]. The most common current control methods are hysteresis and bang-bang current controllers [14, 34], hysteresis control with delta modulator [33], stator-frame PI control with separate PWM generation [14] and synchronous-frame current control [33, 14]. The methods differ from each other in terms of factors such as, complexity, location (i.e. in which frame the controller is implemented), etc.

Synchronous-frame current control is simply implementing the controllers for $d$ and $q$-axis currents in the vector controller shown in Figure 2.6 of Chapter 2. Recent advancements in digital signal processor and microcontroller technology have made it possible to sample the line currents in the three-phase system and compute the corresponding currents in the synchronous reference frame, which is otherwise a difficult computation to be implemented using analog electronics. As also mentioned in Chapter 2, all current and voltage signals of interest become DC quantities in this frame. This makes it easier to design and tune current controllers that perform equally well over the whole operating frequency range of the AC drive. Some disadvantages are the computational overhead due to the coordinate transformation (see Equations (2.10) and (2.11)), presence of cross-
coupling terms (see Equation (2.21)) and the need of sufficiently accurate machine parameter estimates to implement axis decoupling (will be discussed in Section 4.2.1). Even at the cost of these drawbacks, synchronous frame current control opens up a wide range of advanced controller design possibilities in addition to basic PI controllers. Thus, it is the most commonly used current control method today. Some of the advanced synchronous frame current control methods will be briefed in the next section.

4.1.2 Advances in synchronous frame current control

The standard PI controller still seems to dominate in this area of application. Not surprisingly, a lot of work can be found in the literature on advancements of standard PI controllers. Some of the literature also suggests controllers that have different structures. However, the prime objective of all these studies is to improve the transient response to the fastest possible, overcoming the maximum applicable voltage constraint in the inverter. Another task is to maintain the steady state current demand under disturbance conditions (measurement noise and torque disturbances for example). The second requirement is satisfactorily fulfilled by sufficient integral action together with synchronous sampling of current signals and suitable filtering of the sampled data if necessary. Some of the obstacles in achieving the expected fast dynamics and known methods to overcome these obstacles will be summarized below.

The adverse effect of cross coupling (see (2.21)) is that, any transient change in one of the currents will affect the value of the other current. An alternative method to widely used decoupling is to design more advanced multivariable controllers. Design of such a multivariable state feedback controller with integral action can be found in [35]. Another major problem in sampled data systems is the inherent delay of measurement information due to the zero order hold nature of sampling. To overcome this problem, the authors of [36] suggest a deadbeat current controller based on the discrete machine model. An open loop prediction method is used in [36] to obtain the motor currents at the end of each sampling period. The maximum voltage constraint of the inverter will be dealt with from a different point of view in Chapter 6 of this part of the thesis. Some previous work dealing with this problem can be found in [37, 38, 39]. The main idea in all these works can be summarized as follows. The $d$-axis command voltage is slightly changed during the transient period (i.e. to make a slight reduction of flux). This gives room for the $q$-axis command voltage during the transient period to be increased, thereby increasing the torque producing $q$-axis current, while still keeping the magnitude of the command voltage within the maximum voltage constraint.

In this section some of the advancements of synchronous frame current control from the very basic PI control structure were briefly presented. Some other methods that should be mentioned are the introduction of a feed forward term [35], non-linear controllers [32, 40] and knowledge based methods [32]. Frequency response characteristics of synchronous frame current control have been studied both theoretically and experimentally in [41]. Literature can also be found on dynamic analysis of synchronous frame current control [106] and operation in the field weakening range [107].

4.2 Synchronous frame current control applied to PMSMs

In this section, main issues that must be considered, when applying the synchronous frame current control method to PMSMs will be presented. The dynamical equations of the $d$-axis and $q$-axis currents in (2.21) can be rewritten as
The cross coupling between the two axes results in a complex dynamical system. A simpler and widely used method to remove this complexity is to incorporate the knowledge on the machine parameters and sampled currents to compute the cross coupling effect on each axis and reduce it accordingly to obtain a decoupled system. This is explained in the next section.

4.2.1 Axis decoupling

Axis decoupling will be explained here using a classical control system approach. Equation (4.1) can be expressed in complex space vector form as

\[
\begin{align*}
\frac{d i_d}{dt} &= -\frac{R_d}{L_d} i_d + \frac{\omega L_q}{L_d} i_q + \frac{1}{L_d} u_d, \\
\frac{d i_q}{dt} &= -\frac{R_q}{L_q} i_q - \frac{\omega L_d}{L_q} i_d - \frac{\omega \Psi_m}{L_q} + \frac{1}{L_q} u_q.
\end{align*}
\]  

(4.1)

If accurate knowledge on \(L_d, L_q, \Psi_m\) and speed \(\omega\) is available, instead of the system input \(u_s\), a new input can be defined such that

\[
u_s = u'_s + j\omega L_i_s + E.
\]  

(4.3)

This gives rise to an inner decoupling loop shown in Figure 4.1. Since the motor speed and hence the back emf also, have much slower dynamics compared to that of currents, the term \(E\) in (4.2) can also be treated as a disturbance to the control system. If treated in this manner, \(E\) can be excluded from the new input computation in (4.3). The closed loop current controller implementation is shown in Figure 4.1. The controller structure with axis decoupling is shown in Figure 4.2.

4.2.2 System after axis decoupling

With the inner decoupling loop, the effective process seen by the new input can be given as
This could be written in matrix transfer function form as

\[
\begin{bmatrix}
    i_d \\
    i_q
\end{bmatrix} = G_p(s) \begin{bmatrix}
    u'_d \\
    u'_q
\end{bmatrix},
\]

where

\[
G_p^{-1}(s) = \begin{bmatrix}
    L_q s + R_s & 0 \\
    0 & L_q s + R_s
\end{bmatrix}.
\]

It is clear from (4.4) that the decoupled system gives rise to two parallel first order systems in the d and q-axis, which are much easier to deal with from a control engineer’s point of view. Basic design criterions for the controller of the first order system is to achieve sufficiently fast transient performance and the tracking capability of current reference with zero steady state error. In fact, zero steady state tracking error in d-q frame implies zero amplitude and phase error in each phase of the three-phase system.

Even though the controller design now seems to be fairly straightforward, obtaining fast enough closed loop response can be a challenging task, in real implementation. This design problem is dealt with in the coming sections of this chapter. Presenting the design suggestions and introducing the practical difficulties in obtaining the expected performance from these designs will be covered later. Advanced digital control methods that can be used to overcome some of the practical difficulties will also be discussed.

4.3 Basic Internal Model Control – an easy way to get started

As mentioned earlier, when a decoupled current control method is used, the control problem reduces to designing a controller of a first order system (a first order system that is essentially open loop stable in this case). This imposes lesser demands on the controllers from the closed loop stability point of view. However, the controller design must be capable of achieving sufficiently fast transient response as well as steady state reference tracking capability. A well-designed PI controller is sufficient for ordinary performance. Yet, as a beginning to a thorough discussion on this issue, it is of interest to investigate the possibilities of easy design and tuning of the controllers, which could offer better performance. The basic Internal Model Control (IMC) design suggested in [6, 104] is presented here as a possible candidate and will be discussed in detail.

4.3.1 Fundamentals of IMC

The problem encountered here is to design controllers for two de-coupled (independent) currents, of which the dynamics are described by (4.5) in first order matrix transfer function form. Design of basic IMC controller for this system is described here. The underlying principle of the IMC controller can be easily understood by referring to Figure 4.3 (a) and (b). Figure 4.3 (a) shows the typical IMC structure and Figure 4.3 (b) shows the same structure re-drawn as the classical control
Current control implementation

system structure. By referring to Figure 4.3 (a), one can easily understand the fact that the output \( u(t) \) of the IMC controller \( G_c(s) \), drives the actual plant \( G_p(s) \) and also an internal model \( \hat{G}_p(s) \) and the error between the actual plant output and the internal model output is fed back to the controller. Therefore the overall controller transfer function is

\[
F(s) = \left[ 1 - G_c(s)\hat{G}_p(s) \right]^{-1} G_c(s).
\] (4.7)

Then, if perfect modeling is assumed, that is

\[
\hat{G}_p(s) = G_p(s),
\] (4.8)

resulting closed loop transfer function will be

\[
G_{cl}(s) = G_p(s)G_c(s).
\] (4.9)

Since it is already known that \( G_p(s) \) is open loop stable, \( G_c(s) \) can be selected such that

\[
G_c(s) = \frac{\alpha}{s+\alpha} G_p^{-1}(s).
\] (4.10)

This choice of \( G_c(s) \) will give low-pass filtering of bandwidth \( \alpha \) to the feedback error quantity. The resulting controller transfer function can be written as

\[
F(s) = \frac{\alpha}{s} G_p^{-1}(s),
\] (4.11)

which contains the integral action required to eliminate the steady state error. The overall closed loop transfer function will be

\[
G_c(s) = \frac{\alpha}{s+\alpha} I = \begin{bmatrix} \alpha & 0 \\ \frac{\alpha}{s+\alpha} & 0 \\ 0 & \frac{\alpha}{s+\alpha} \end{bmatrix},
\] (4.12)

which has the first order characteristics. So, this IMC controller design will result in a closed loop current control bandwidth of \( \alpha \).

The IMC design presented here is a very basic approach for a simple first order system. One immediate requirement in a design for a higher order process is the modification of the low-pass filter
term used in the controller transfer function in (4.10). If the process model is of \( n^{th} \) order \((n > 1)\),
then the order of the filter must be at least \( n \) or higher in order for the controller transfer function to be causal. A general design procedure for an \( n^{th} \) order process can be found in [6, 42].

4.3.2 Isn’t that a PI controller?

The matrix controller transfer function obtained from the IMC design is \( F(s) \) given by (4.11). It is now interesting to investigate the resulting controller on each axis. Controller transfer functions of each axis can be written as

\[
\begin{align*}
F_d(s) &= \frac{\alpha}{s} \left( L_d s + R_s \right) = \alpha L_d + \frac{\alpha R_s}{s} \\
F_q(s) &= \frac{\alpha}{s} \left( L_q s + R_s \right) = \alpha L_q + \frac{\alpha R_s}{s},
\end{align*}
\]  

(4.13)

It is clear from (4.13) that the final outcome is two PI controllers, of which the proportional part is proportional to the inductance value, while the integral part is proportional to the resistance term. In fact, having an explicit integral part in the controller structure is an important positive feature in this design due to two reasons. One is obviously the ability of the integral controller to eliminate the steady state error of the currents. The other reason is that the implementation of anti-windup compensation schemes under output saturating conditions becomes easier, when there is an explicit integrator in the controller structure.

In the machine control area, it is a common practice to design PI current controllers in such a way that the pole of the current dynamics is cancelled by the zero of the PI controller. Therefore, one could conclude that the simplest form of IMC design for a first order system gives similar results as this standard design. However, the IMC design presented so far is the very basic approach for the simple first order model. The method can be very effective and also robust, when applied to more sophisticated process models [42]. Advanced IMC design strategies will be considered later on in this chapter. For these designs, the process model for the current dynamics will be modified to represent some other practical issues in sampled data systems.

4.3.3 Some comments on IMC

It is of importance to make some preliminary comments on IMC approach at this stage. Only the important points will be mentioned here.

- Since a pole zero cancellation is involved, the method is valid only for open loop stable processes, when applied in the simple way as presented here.
- It was shown that the resulting controller parameters depend on the process model parameters. These are the resistance and inductance of each axis in this particular application. This demands accurate estimations of model parameters for IMC design of any process.
- Model mismatch can degrade the expected performance of the closed loop system.
- Stator resistance and inductance of an electrical machine tend to vary considerably with factors such as, supply frequency, temperature, current level in the stator-winding etc. This can always affect the closed loop performance of the current dynamics, if not compensated properly. The issue of parameter estimation of a PMSM at various operating frequencies will be addressed in Chapter 9.
The next step is to implement an IMC design at simulation and experimental levels in order to further investigate its performance.

### 4.3.4 Implementation of IMC in simulation level

In the simulation stage, it is straightforward to work with per-unit machine parameters. Normalized time will also be used. This means that one has to convert the sampling time that is going to be used in the digital control implementation also into a normalized value. As an example, for a sampling time of 125μs (corresponding to 8kHz sampling frequency), the time step used in the digital controller simulations is 0.11775rad (based on Equation (3.4), $T_{sn} = 125μs \times 942rad/s$). The controller matrix transfer function according to (4.11) will be

$$ F(s) = \frac{\alpha}{s} \begin{bmatrix} L_d s + R_s & 0 \\ 0 & L_q s + R_s \end{bmatrix}. \quad (4.14) $$

From continuous-time controller description in (4.14), it is now possible to obtain the discrete-time controllers for $d$-axis and $q$-axis with the per-unit machine parameters for the prototype PMSM ($L_d = 0.33$, $L_q = 0.35$, $R_s = 0.037$). Discrete equivalent of the proportional part is straightforward, while there are several methods to approximate the continuous-time integral part in discrete-time [43, 51]. Three widely used methods are trapezoidal integration, forward-rectangular integration and backward-rectangular integration. The discretized controllers for each axis with forward-rectangular integration rule can be given in discrete-time operator form as

$$ u_d(k) = \alpha L_d + \frac{\alpha R_s T_{snq}^{-1}}{1 - q^{-1}} e_d(k) \quad (4.15) $$

and

$$ u_q(k) = \alpha L_q + \frac{\alpha R_s T_{snq}^{-1}}{1 - q^{-1}} e_q(k). \quad (4.16) $$

Forward-rectangular integration rule is preferred here due to the simpler structure than that of the trapezoidal integration rule [51].

It must be noted here that the error quantities driving the controllers, $e_d(k)$ and $e_q(k)$ are defined in terms of the reference and measured currents by

$$ e_d(k) = i_{dref}(k) - i_d(k-1) $$

$$ e_q(k) = i_{qref}(k) - i_q(k-1). \quad (4.17) $$

The measurement at $k^{th}$ instant of the current is not available for the $k^{th}$ control signal computation in this type of a sampled data system. The effect of this will be discussed in detail later in this chapter. Step response simulations for the $d$-axis with $\alpha = 4 \text{ pu}$, which corresponds to a real-time closed loop bandwidth of 600Hz is shown in Figure 4.4.
The next step is to check the controller performance in real-time.

### 4.3.5 Real-time implementation

Using correct scaling between the quantities in the real system and control algorithm is very important. Preserving the normalized parameters in the controller is recommended in order to keep the dissimilarities between the controller design and implementation to a minimum. Specially, in the RTI/SIMULINK™ programming environment, this approach becomes very effective in terms of programming time. Actual to per-unit conversion of the sampled currents is done just after the ADC stage and the inverse transformation of the control signal outputs to the PWM stage is done just before the DAC stage of the DSP system. Even though these conversions are straightforward and appear as pure scaling factors, normalized time to real-time conversion of a control algorithm is rather complicated.

Transforming a controller designed in per-unit time scale into real-time has to be thoroughly understood. Else, serious mismatch between the design and real implementation may occur, resulting in oscillations and instabilities. This can be looked upon in many ways. According to (4.14), continuous normalized time equation for the \(d\)-axis current controller can be written as

\[
\frac{\alpha L_d}{R_c} e_d (t_n) + \alpha R_c \int e_d (t_n) dt_n,
\]

where \(t_n\) denotes the normalized time (per-unit time). The substitution \(t_n = t / t_{\text{base}} = t \omega_{\text{base}}\) can now be made and this yields a continuous real-time description of the controller given by

\[
u_d (t) = \frac{\alpha L_d}{R_c} e_d (t) + \alpha R_c \omega_{\text{base}} \int e_d (t) dt .
\]

The discrete-time operator description of the controller in real-time with forward-rectangular integration rule is

\[
u_d (k) = \left[ \frac{\alpha L_d + \alpha R_c T_s \omega_{\text{base}}}{1 - q^{-1}} \right] e_d (k).
\]
\[ u_d(k) = \left[ \alpha L_d + \alpha R_s T_s q^{-1} \right] e_d(k). \tag{4.21} \]

Now the difference between (4.20) and (4.21) is the \( \omega_{\text{base}} \) factor coming in the integral gain. Since \( T_s \omega_{\text{base}} = T_{sn} \), what effectively has happened in (4.20) is that the computation of the integral part with the normalized integration time step \( T_{sn} \), even if the sampling time of the actual implementation is \( T_s \).

Implementation of Equation (4.20) as the controller in real-time ideally should give the performance similar to that of simulation results in Figure 4.4. However, experimental results show that the controller performance is well below the expected. There are two major reasons for this behavior. One is the non-ideal nature of the inverter. Main non-idealities are the switching dead-time, resistive voltage drop of inverter switching elements and the DC-link voltage drop. These problems will be discussed in detail in Chapter 8. Another problem is the dependency of the integral part on the resistance term \( (R_s) \). If there is going to be a mismatch in the effective \( d \)-axis resistance in the stator and the estimated value used in the controller, there will be performance degradation. It is possible to experimentally verify that the lack of integral action still remaining even after compensating for the inverter non-idealities can be eliminated by increasing the integral gain used in the controller by a factor of 2 - 3. This gives some indication about the uncertainty in the machine parameters under transient and varying frequency conditions. Improvement of performance with the compensated inverter and increased integral part is illustrated in the first experimental result of the basic IMC implementation shown in Figure 4.5. As in the simulation results, step-response of the \( d \)-axis with \( \alpha = 4 \text{ pu} \), which corresponds to a real time closed loop bandwidth of 600Hz, is shown in Figure 4.6 with the dead-time compensation and doubled resistance term \( (R_s) \).
This behavior clearly motivates the need for more investigation on inverter non-idealities and machine parameter estimation under variable frequency conditions. In fact, the reason to bring up these two factors at this point is to let the reader aware of two very important practical problems that may arise, when attempting to tune a certain control implementation to achieve the expected performance. Both these issues will be taken up in the following chapters.

However, even after fine tuning of the controller, it can be seen from Figure 4.6 that for larger steps, the step-response deviates from the expected first order behavior, causing overshoots. This is due to the delayed nature of the current measurement illustrated in (4.17). Next section explains it clearly.

![Figure 4.6: Step responses of d-axis current with $\alpha = 4 \text{ pu}$ - the step inputs applied are 0.2, 0.3, 0.5 and 0.7 pu.](image)

### 4.4 Inherent delay in sampled data systems

Implementation of a digital control system essentially requires some form of an interface between the digital computing device (a micro-controller, computer or a DSP) and the continuous-time process. As far as the line currents and phase voltages are concerned in this application, ADCs are used for sampling the line currents (measurement inputs) and DACs are used to drive the PWM generation circuit (control signal outputs). If it is assumed that the sampling time is much larger than the computation time of the control algorithm, the obvious way to do this operation can be given in the form of a flow chart shown in Figure 4.7 (a). Corresponding timing diagram is shown in Figure 4.7 (b).

By looking at the timing-diagram, it can be understood that there is a delay between the instant of sampling the measurement and sending the control signal output, which is computed based on that measurement. This delay comprises the A/D conversion time, control signal computation time and D/A conversion time (this is referred to as total computation time). Under these circumstances, the control signal output sent at a certain instant has always been computed based on measurements that are delayed by some amount of time. In applications such as computer controlled chemical processes, where the sampling time is of the order of milliseconds (or even higher) against a total computation time of several microseconds, this delay can be neglected. When looking at the delay in this manner, it is convenient to call it as “computation delay”, as was done here. Modeling and compensating the computation delay, which is a fraction of sampling time has been tackled in previous contributions [45, 46].
Current control implementation

Figure 4.7: I/O function handling in a digital control system (a) Flow chart (b) Timing-diagram

However, the situation changes as the total computation time gets closer to the sampling time. This is the case for digital control applications with high sampling frequencies of the order of kilohertz. With the development of the digital control area, signal-processing devices such as DSPs appeared, which can operate at very high sampling frequencies for closed loop control applications. Yet, depending on the ratio (computation time / sampling time), the I/O function handling method has to be changed for better performance. If the above ratio is much closer to one (it is closer to zero for applications with slower dynamics such as chemical processes), the modified flow chart and the timing diagram shown in Figure 4.8 (a) and (b) are considered to be showing the more appropriate way of handling associated I/O functions. This actually is the method used in the DSP system for the PMSM application in this thesis.

Figure 4.8: Modified I/O functions handling (a) Flow chart (b) Timing-diagram
According to this timing diagram, it is clear that the measurement used for control signal computed and sent out at a certain instant is exactly one sampling time delayed.

In digital control applications with this type of I/O function handling, the resulting delay is not interpreted as a computation delay by the control engineers. Instead it is referred to as “sampling delay” with the idea that there is a lack of up-to-date measurement information for control signal computation at a given instant [36]. This again has been a known problem over a long period of time and has been tackled in several previous contributions from a theoretical point of view [44].

It has been identified that this sampling delay can also be a significant problem for fast transient performance of current dynamics. The consequences of this delay are the overshoots of the $d$ and $q$-axis current step-responses as illustrated in Figure 4.9 for the basic IMC implementation. Figure 4.9 shows a deteriorated step response for a set-point change of 0.7 pu of the closed loop basic IMC current controller, when the bandwidth is raised to $\alpha = 6$ pu (corresponding to 900 kHz). The dead-time compensation and doubled $R_s$ term in the integral gain are still used.

![Figure 4.9: Deteriorated step response for a set point change of 0.7 pu](image)

One straightforward way to deal with this problem is to use the designed controller as it is and try to make compensations for the delayed measurement information. Some previous work can also be found on compensating techniques that can improve the closed loop performance [36]. A predictive observer method based on the discrete machine model will be presented in the next section.

### 4.5 Predictive observer for sampling delay compensation

When all I/O function handling is done at the end of a sampling period as explained in the previous section, control signal computation is done using delayed current information. This is better elaborated by means of the sequences of measured current and sent out voltage signals shown in Figure 4.8 (b). If the machine model is accurately known, some additional computations can be done in the processor, before the execution of the control algorithm in order to obtain a predicted value of the current that will be existing in the system, when the next control signal is sent out. Then the control signal can be computed based on this predicted current. This will be further explained next.
The predictive observer method is based on the simple idea that the control signal computation can be improved, if it is done based on the actual current value that will be in the machine, when the particular voltage is sent out through the DAC. In other words, instead of computing $u(k+1)$ based on $i(k)$, the aim here is to compute $u(k+1)$ using a predicted value of $i(k+1)$, called as $\hat{i}(k+1)$. For this purpose the control voltage information sent out at $k^{th}$ sampling instant, $u(k)$ and the current existed in the system at that instant $i(k)$, are available for the design engineer. Corresponding flow chart and the signal sequence are shown in Figure 4.10 (a) and (b) as before. The design problem can now be explained as finding out a suitable method for the block that does the prediction shown in the flow chart in Figure 4.10 (a). In other words the design must come up with an appropriate method that will produce the predicted current sequence from the measured current sequence shown in the Figure 4.10 (b). The observer will be the predictive mechanism in this application. The best possible predictive observer structure will be derived step by step with clarifications on the disadvantages of its intermediate stages. The first step is to introduce the discrete time model for motor current dynamics.

**Figure 4.10: Typical predictive mechanism for current prediction**

4.5.1 Discrete machine model in state-space form

For the purpose of current prediction, one obviously needs a mathematical model of the motor current dynamics. Given the fact that accurate machine parameter values are available, the $dq$ frame mathematical model in Equation (4.1) can be used for the purpose. However, the continuous-time model for current dynamics in (4.1) has to be converted into a discrete-time model for it to be implemented in a DSP. As it has been described in [44], there are several methods for the purpose. The method used here is the matched z-parameter model. From (4.1), continuous-time current dynamics in state space form can be derived as
The cross-coupled nature or the non-linearity in (4.22) can be eliminated by making the assumption that the speed dependent cross-coupling terms can be treated as disturbance to the linear state space system description. The modified state-space model can be given as

\[
\begin{bmatrix}
\frac{d i_d}{dt} \\
\frac{d i_q}{dt} \\
\end{bmatrix} = \begin{bmatrix}
\frac{-R}{L_d} & 0 \\
0 & \frac{-R}{L_q}
\end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} & 0 \\
0 & \frac{1}{L_q}
\end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix} + \begin{bmatrix} \frac{\omega L_q}{L_d} \\
\frac{-\omega L_q}{L_q}
\end{bmatrix} \begin{bmatrix} 0 \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{\omega L}{L_q} \\
0
\end{bmatrix} \begin{bmatrix} \omega \psi_m \\ 0 \end{bmatrix}.
\]

(4.23)

This yields the new state space description can be given as

\[
\frac{d i}{dt} = Ai + Bu + D_e i + E_e \omega
\]

(4.24)

\[
y = Ci,
\]

where

\[
A = \begin{bmatrix}
\frac{-R}{L_d} & 0 \\
0 & \frac{-R}{L_q}
\end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L_d} & 0 \\
0 & \frac{1}{L_q}
\end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_e = \begin{bmatrix} 0 & \frac{\omega L_q}{L_d} \\
\frac{-\omega L_q}{L_q} & 0
\end{bmatrix}, \quad E_e = \begin{bmatrix} 0 \\
\frac{\omega L}{L_q}
\end{bmatrix}.
\]

The following linear state space description extracted from (4.24) given by

\[
\begin{bmatrix}
\frac{d i_d}{dt} \\
\frac{d i_q}{dt} \\
\end{bmatrix} = \begin{bmatrix}
\frac{-R}{L_d} & 0 \\
0 & \frac{-R}{L_q}
\end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} & 0 \\
0 & \frac{1}{L_q}
\end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix} + \begin{bmatrix} 0 \\ 0
\end{bmatrix} \begin{bmatrix} \omega \psi_m \\ 0 \end{bmatrix}.
\]

(4.25)

can now be discretized with Zero-Order-Hold sampling [47] to obtain the discrete state space machine model. With the assumption that \(\omega(k)\) and \(i(k)\) are held constant during the sampling period, the complete discrete machine model with transformed \(D_e\) and \(E_e\) will be

\[
i(k+1) = \hat{O} i(k) + \hat{A} u(k) + D \hat{t}(k) + E \omega(k)
\]

\[
y(k) = C i(k).
\]

(4.26)

where
The resulting complete discrete-time state space model is graphically depicted in Figure 4.11.

\[ \tilde{\Phi} = e^{A T_m} = \begin{bmatrix} e^{\frac{R_s T_m}{T_d}} & 0 \\ 0 & e^{\frac{R_s T_m}{T_q}} \end{bmatrix}, \quad \tilde{\Delta} = \int_0^{T_m} e^{A s} d\Sigma = \begin{bmatrix} 1 - e^{\frac{R_s T_m}{T_d}} & 0 \\ 0 & 1 - e^{\frac{R_s T_m}{T_q}} \end{bmatrix} \]

\[ D = T_m \int_0^1 e^{A s} d\Sigma D_c = \begin{bmatrix} 0 & \alpha L_d \left(1 - \exp\left(-\frac{R_s T_m}{T_d}\right)\right) \\ \alpha L_d \left(1 - \exp\left(-\frac{R_s T_m}{T_q}\right)\right) & R_s \end{bmatrix}, \quad E = T_m \int_0^1 e^{A s} d\Sigma E_c = \begin{bmatrix} 0 & \Psi_m \left(1 - \exp\left(-\frac{R_s T_m}{T_d}\right)\right) \\ R_s & \Psi_m \left(1 - \exp\left(-\frac{R_s T_m}{T_q}\right)\right) \end{bmatrix} \]

\[ \dot{u}(k) = \begin{bmatrix} \Gamma^{\top} \\ \Theta \end{bmatrix}, \quad \dot{i}(k) = \begin{bmatrix} \Phi^{\top} \\ \Phi \end{bmatrix} \]

**Figure 4.11: Discrete-time state space machine model**

At this point, the effect due to the inherent sampling delay of sampled data systems must be included into the actual machine model. This occurs, when closed loop control is implemented based on the sampled current information as it was explained earlier in this chapter. This would be the case, if \( u(k) \) is the control signal computed at time instant \( k \), based on the current measurement \( i(k) \). However, the computed \( u(k) \) is input to the to the process after one sample time delay. Thus, the actual process model equation will be

\[ i(k+1) = \tilde{\Phi} i(k) + \tilde{\Delta} u(k-1) + D i(k) + E \omega(k). \]  

(4.27)

This clearly shows that the current at \((k+1)^{th}\) instant \( i(k+1) \), is excited by \( u(k-1) \), the computation of which is based on \( i(k-2) \). Whereas, it would have been preferred to have \( i(k+1) \) excited by \( u(k) \), the computation of which is based on \( i(k) \). Thus, there is a need for a one step ahead predictive observer in parallel to the actual plant. The predictive observer can now be developed based on the same discrete machine model. This procedure will be discussed in the following sections.

4.5.2 State-space closed loop observer – proportional error feedback

The machine model structure shown in Figure 4.11 can now be implemented in the DSP in parallel to the actual machine and driven from the same current controller output signal \( u(k) \). Then the model will produce a reconstruction of the actual current output from the machine. Corresponding state space equations for this simple observer can be given as
\[ \dot{i}(k+1) = \hat{O} \dot{i}(k) + \hat{A}u(k-1) + D\dot{i}(k) + E\omega(k) \]  \hspace{1cm} (4.28)

with \( \dot{i}(k) \) denoting the estimated quantities of the corresponding currents. \( u(k-1) \) is the control signal based on \( \dot{i}(k) \). The output of the predictor \( \dot{i}(k+1) \) can now be used to compute \( u(k) \). In fact the disturbance terms can also be implemented as shown in (4.28) to obtain a system closest to the real situation. However, for the analysis and design, only the linear part in (4.28) must be considered in order to enable the use of linear system design theory. The open loop observer implementation in (4.28) is possible in this case, since the open loop current dynamics is stable. However, the reconstruction can be improved by introducing the difference between the measured and predicted outputs, \( y(k) - \hat{C}\dot{i}(k) \) as a proportional feedback [47] and is always recommended in automatic control. The closed loop observer is described by

\[ \dot{i}(k+1) = \hat{O} \dot{i}(k) + \hat{A}u(k-1) + K_p \left[ y(k) - \hat{C}\dot{i}(k) \right] + D\dot{i}(k) + E\omega(k) \]

\[ K_p = \begin{bmatrix} K_{pd} & 0 \\ 0 & K_{pq} \end{bmatrix} \]

The new observer structure is shown in Figure 4.12.

\[ \begin{align*}
\dot{i}(k+1) &= \hat{O} \dot{i}(k) + \hat{A}u(k-1) + K_p \left[ y(k) - \hat{C}\dot{i}(k) \right] + D\dot{i}(k) + E\omega(k) \\
K_p &= \begin{bmatrix} K_{pd} & 0 \\ 0 & K_{pq} \end{bmatrix}
\end{align*} \]  \hspace{1cm} (4.29)

\[ \begin{align*}
\dot{i}(k+1) &= \hat{O} \dot{i}(k) + \hat{A}u(k-1) + K_p \left[ y(k) - \hat{C}\dot{i}(k) \right] + D\dot{i}(k) + E\omega(k) \\
K_p &= \begin{bmatrix} K_{pd} & 0 \\ 0 & K_{pq} \end{bmatrix}
\end{align*} \]  \hspace{1cm} (4.30)

The user introduced proportional feedback gain matrix has to be designed to achieve desired performance. Subtracting (4.29) from (4.27) gives

\[ \tilde{i}(k+1) = [\hat{O} - K_pC]\dot{i}(k), \]  \hspace{1cm} (4.30)

where \( \tilde{i}(k) = i(k) - \dot{i}(k) \) and the disturbance terms have been neglected here as mentioned. Then purpose of the design can be looked upon as stabilizing the prediction error dynamics. Prediction error dynamics has to be much faster than the dynamics of the current, in order to obtain accurate predictions from the observer. This automatically gives out the design criterion to determine \( K_p \). A pole placement design can be done for this problem based on the characteristic equation for the system,
Simulated performance of the closed loop predictive observer with proportional error feedback, when the prediction error dynamics is forced to be approximately a decade faster than the actual current dynamics is shown in Figure 4.13. Also shown in the same figure is the performance, when there is a mismatch in the initial conditions (less by 5%) of the observer and actual process.

The results show that the performance of the predictor is still not satisfactory, particularly when there is a mismatch between the initial conditions of the actual machine and the implemented observer model. How to overcome this problem is discussed in the next section.

Figure 4.13: Performance of closed loop observer with proportional error feedback – with perfect model matching and with 5% difference in the initial conditions

4.5.3 Observer with proportional and integral error feedback

To eliminate the steady state prediction error, one can introduce an integral feedback of the prediction error in addition to the proportional part suggested in the earlier section [48]. The modified predictive observer equation is

\[
\hat{i}(k + 1) = \hat{\mathbf{O}} \hat{i}(k) + \hat{\mathbf{A}} \mathbf{u}(k - 1) + K_p \left[ y(k) - C \hat{\mathbf{i}}(k) \right] + K_i \sum_{n=0}^{k} \left[ y(n) - C \hat{\mathbf{i}}(n) \right] + D \mathbf{i}(k) + E \omega(k)
\]

\[
K_i = \begin{bmatrix}
K_{pd} & 0 \\
0 & K_{iq}
\end{bmatrix}
\]

(4.32)

Following the same approach as in the previous section (subtracting (4.32) from (4.27)) gives

\[
\tilde{i}(k + 1) = \left[ \hat{\mathbf{O}} - K_p C \right] \tilde{i}(k) - K_i C \sum_{n=0}^{k} \tilde{i}(n).
\]

(4.33)

Equation (4.33) can now be further simplified by introducing an additional state \( e(k) \) representing the integral state and augmenting it with the system matrix to give

\[
\begin{bmatrix}
\tilde{i}(k + 1) \\
e(k + 1)
\end{bmatrix} = \begin{bmatrix}
\hat{\mathbf{O}} - K_p C & -I \\
K_i C & I
\end{bmatrix} \begin{bmatrix}
\tilde{i}(k) \\
\mathbf{e}(k)
\end{bmatrix}.
\]

(4.34)

Fixing the values of the proportional and integral gain matrices, \( K_p \) and \( K_i \) can now be done based
on the new characteristic equation for prediction error dynamics given by
\[
\det \begin{bmatrix} \dot{\mathbf{O}} - K_p \mathbf{C} & -\mathbf{I} \\ \mathbf{K}_c \mathbf{C} & \mathbf{I} \end{bmatrix} = 0.
\] (4.35)

The new observer structure is shown in Figure 4.14. As in the previous case, performance of the closed loop predictive observer with proportional and integral error feedback, when the prediction error dynamics is forced to be approximately a decade faster than the actual current dynamics is shown in Figure 4.15. Also shown in the same figure is the performance, when there is a mismatch in the initial conditions (less by 5%) of the observer and actual process.

The results show that the predictor with proportional and integral error feedback is capable of
removing the mismatch in the initial conditions in a satisfactory manner. Under these circumstances, the recommended closed loop observer structure for the current prediction is the last configuration with proportional and integral error feedback. The new current controller structure with the predictive observer and its stability considerations will be discussed in the next section.

### 4.5.4 Using the current prediction and stability

The aim of employing a current predictor as mentioned before is to obtain a prediction of currents at the \((k)^{th}\) instant \(\hat{i}(k)\), to be fed into the current controller. When this predicted information is used as the feedback information, current controller structure differs from the previous one and the new structure is shown in Figure 4.16.

When the predictor is designed as explained in the previous section and the predicted current is used as the feedback information for the current controller, it is obvious that the closed loop system changes from the previous one. The new system consists of the actual process, predictor and the controller. Therefore an analysis must be done to verify the closed loop stability of the new complete system. More information on this can be found in [47].

![Figure 4.16: New current controller structure using predicted current as feedback](image)

### 4.5.5 Implementation in real-time

Unlike in the case of IMC current controller, the observer does not interact with the external real-time system directly. It is in fact implemented in per-unit quantities inside the control algorithm. However, a discrete-time integration takes place with a time step of \(T_s\) in the state space implementation. The discretization of the continuous time machine model according to Equation (4.26) needs the sampling time information. At this point one has to use the real sampling time in all appropriate places. All machine parameters must be substituted in per-unit form in order to minimize the scaling problems. The implementation structure is the same as shown in Figure 4.16.

### 4.5.6 Experimental results

Two types of results will be presented here. First only the performance of the current prediction will be verified without using the predicted current information for the closed loop control purpose. Later some step response results and prediction error plots will be presented with the current control loop closed using the predicted current information.
(a) Performance of the current prediction

Performance of the predictor with proportional error feedback (see Section 4.5.2) and proportional and integral error feedback (see Section 4.5.3) will be presented here. The results from the real implementation of the closed loop observer with proportional error feedback are shown in Figure 4.17.

![Graph showing performance of the predictor with proportional error feedback](image)

**Figure 4.17: Performance of observer with proportional error feedback**

This result shows that the performance during the transient condition is still not satisfactory. Then the integral feedback was introduced and as before real-time implementation results for the closed loop observer with proportional and integral error feedback are shown in Figure 4.18.

![Graph showing performance of the observer with proportional and integral error feedback](image)

**Figure 4.18: Performance observer with proportional and integral error feedback**

This shows that the introduction of the integral error feedback has dramatically reduced the prediction error even during the transient stage. Thus the argument on proportional-integral error feedback observer design is very well justified. Now it is possible to check the closed loop performance of this observer.
(b) Performance of the predictor based closed loop current control

After the verification of the prediction capability, it is now possible to close the current control loop with the predicted current information and observe the closed loop performance.

Figures 4.19 and 4.20 show the performance of closed loop observers with different error feedback for closed loop control – experimental.

As was mentioned earlier, open loop observer is not suitable for this prediction. The closed loop observer with proportional error feedback is first tested for the same \( d \)-current step. The results are shown in Figure 4.19.

The same test was performed with the predictive observer with proportional and integral error feedback. The results are shown in Figure 4.20. It can be seen here that there is a slight improvement in the estimation convergence of the estimation error with the introduction of the integral term.
Then the more interesting thing to check is whether it is possible to use higher closed loop bandwidths with the same basic IMC based current controller so that it can be compared with the basic IMC performance at higher current controller bandwidth shown in Figure 4.9. This is shown in Figure 4.21, where a closed loop bandwidth of $\alpha = 4 \text{ pu}$ (corresponds to 600 Hz) is used together with de-tuned (doubled) integral gain and dead time compensation as before.

The results show that the unfavorable current overshoot is successfully reduced by the use of predicted current information for the closed loop feedback control. Thus the aim of the predictive observer design is successfully achieved.

### 4.5.7 Importance of observer approach

The most important feature in this approach is that it gives an easy-to-understand overall controller structure that can be fine-tuned using the physical understanding of the system performance. As an example, if the amount of noise in the sampled currents is high, the current predictor sensitivity (bandwidth) must be lowered. By this its sensitivity to measurement noise can be lowered resulting in better control signals. Even though there will be several modules in the overall controller structure with this approach, one advantage is the fact that the design of each module is simpler and fine-tuning becomes straightforward. In contrast, it will be seen that the compact advanced IMC design discussed in the following section results in a more complex structure, of which fine tuning is more difficult.

### 4.6 Advanced IMC design

When designing a discrete-time controller for a continuous plant, one can have several approaches. A continuous-time controller design can be done with the continuous-time model of the plant. The
resulting controller is then discretized. When this method is used, the designer has the advantage of using the wide spectrum of existing continuous-time controller design techniques. However, even if a certain controller is stable in the continuous-time, its discrete equivalent can be unstable, especially when the design is complex.

The basic IMC design presented in Section 4.3 is a good example for this complete continuous time approach. The sampling delay problem encountered can also be pointed out as an example for disadvantages of this method. The reason is that, the zero order hold nature of the control signal output from the discrete-time controller and the inherent sampling delay that was explained in Section 4.4 were not taken into account in the design procedure.

The predictive observer in the previous section can be used as a remedy, if the controller design procedure is still preferred to be in the continuous domain. However, this situation motivates the need for investigating the possibilities of other design approaches. Another widely used method is to obtain a discrete-time model of the continuous-time process and do the design in the discrete domain. There are several methods to obtain the discrete-time equivalent of a continuous-time process [44]. The more common approach is the matched z-transformed model (like the state space discrete motor model discussed in the previous section). After the discretization, it is much easier to represent any additional time delays appearing in the complete digital control implementation like the sampling delay appeared in this control scheme. Previous work can be found both on analyzing systems with such delay [45] and also designing controllers for systems of this nature [46]. This work will follow on the possibilities of extending the concepts in basic IMC design to a complete discrete-domain design that can compensate for the sampling delay problem. In contrast to the observer method, this approach will result in a compact controller. In other words, this can be looked upon as merging the current predictor and controller into one unit.

4.6.1 Generalized discrete-time IMC design

The underlying principle in doing the IMC design in discrete domain for a process with arbitrary input time delay will be presented here. Analogous to Figure 4.3 (a) and (b), the corresponding discrete-time IMC structure can be graphically depicted as in Figure 4.22 (a) and (b).

![Diagram](https://via.placeholder.com/150)

(a) Typical IMC structure  (b) Classical control system IMC structure

**Figure 4.22: Block diagrams for general discrete IMC controller design**

The original process model \( G_p(q^{-1}) \) is again considered to be an open loop stable non-minimum phase (no right hand plane zeros in the transfer function), with an arbitrary time delay \( d \) (where \( d \) is an integer) at the input. The process and the time delay together is treated as the plant \( G_q(q^{-1}) \) for which the discrete IMC has to be designed. The derivation of the controller is analogous to that was done in Section 4.3. The controller transfer function is
Then, if perfect modeling is assumed, that is
\[
\hat{G}_p(q^{-1}) = G_p(q^{-1}),
\]
the resulting transfer function will be,
\[
G_{cl}(q^{-1}) = \frac{G_p(q^{-1})G_c(q^{-1})}{1 - \hat{G}_p(q^{-1})G_c(q^{-1}) + G_p(q^{-1})G_c(q^{-1})} = G_p(q^{-1})G_c(q^{-1}).
\]

Since it has already been assumed that \(G_p(q^{-1})\) is open loop stable, \(G_c(q^{-1})\) can be selected such that
\[
G_c(q^{-1}) = \left[\frac{H(q^{-1})}{G_p(q^{-1})}\right]^{-1}.
\]
This choice of \(G_c(q^{-1})\) will result in a closed loop transfer function derived from (4.38) given by
\[
G_{cl}(q^{-1}) = \frac{H(q^{-1})}{G_p(q^{-1})}q^{-d}.
\]

From this it is clear that the time delay \(d\) still exists between the set-point and the output as in the case of predictive observer design. Since the plant dynamics has now been cancelled, the user has the freedom to decide the polynomials \(H(q^{-1})\) and \(P(q^{-1})\) depending on the desired closed loop response. It must also be noted here that the choice of the polynomials must satisfy the additional condition
\[
\frac{H(1)}{P(1)} = 1
\]

to have unity steady state gain of closed loop system. With this choice of \(G_c(q^{-1})\), it is now interesting to look at the resulting final IMC controller \(F(q^{-1})\) from (4.36) given by
As was mentioned in the case of basic IMC design, it is preferred to have explicit integral action in the controller to eliminate the steady state tracking error of the controlled variable. From (4.42), it can be shown that

\[ P(q^{-1})H(q^{-1})^{-d} = 0. \]  

(4.43)

With the choice of \( H(q^{-1}) \) and \( P(q^{-1}) \) polynomials according to (4.41), it is clear that \( F(1) \rightarrow \infty \). This means that the controller transfer function has integral action. In fact since

\[ P(q^{-1})H(q^{-1})^{-d} \bigg|_{q=1} = 0, \]  

(4.44)

\( q = 1 \) is a factor of (4.44), which means that there is an explicit integrator included in the controller transfer function.

The fact that the controller even for the generalized design contains integral action is a very interesting feature in IMC design. As the next step, application of this general derivation for current control purpose will be discussed.

4.6.2 Delay Compensating IMC (DC-IMC) for current control

In this section, derivation of the discrete-time IMC design procedure will be done using the generalized approach discussed earlier. The aim here is to incorporate the time delay at the process input appearing due to I/O function handling method of the DSP system explained in Section 4.4, in the design procedure. As in the basic IMC design, it is possible to use matrix transfer function notation for this derivation also. However, for easy readability, the complete derivation will be done only for the \( d-axis \). The notation used will be the same as in the general derivation with an extra suffix \( d \) to denote \( d-axis \). The process model without input delay for the \( d-axis \) is
Sensorless control of PMSMs for high-speed…

\[
G^*_{pd}\left(q^{-1}\right) = \frac{1}{\frac{R}{R_s}} \left[ 1 - \exp\left( -\frac{R_s}{L_d} T_s \right) \right]^{-1} \frac{1}{1 - \exp\left( -\frac{R_s}{L_d} T_s \right) q^{-1}}.
\]  

(4.45)

Then the plant model with the delay included will be (with \(d = 1\))

\[
G_{pd}\left(q^{-1}\right) = \frac{1}{\frac{R}{R_d}} \left[ 1 - \exp\left( -\frac{R_s}{L_d} T_s \right) \right]^{q-2} \frac{1}{1 - \exp\left( -\frac{R_s}{L_d} T_s \right) q^{-1}}.
\]  

(4.46)

If the controller transfer function is selected according to (4.39) with a first order low-pass filter of bandwidth \(\alpha\) defined by the \(H(q^{-1})\) and \(P(q^{-1})\) polynomials, the result will be

\[
G_{cd}\left(q^{-1}\right) = \left[ 1 - \exp\left( -\alpha \frac{\Theta}{\theta_n} \right) q^{-1} \right]^{-1} \frac{R_d}{1 - \exp\left( -\frac{R_s}{L_d} T_s \right) q^{-1}}.
\]  

(4.47)

The final IMC controller can be derived according to (4.42) as

\[
F_{pd}\left(q^{-1}\right) = \left[ 1 - \exp\left( -\alpha \frac{\Theta}{\theta_n} \right) q^{-1} \right]^{-1} \frac{R_s}{1 - \exp\left( -\frac{R_s}{L_d} T_s \right) q^{-1}}.
\]  

(4.48)

It can be seen from (4.48) that the denominator polynomial vanishes when \(q^{-1} = 1\), making \((q^{-1}-1)\) one of its factors. The denominator polynomial can now be factorized to obtain the explicit integrator and the simplified IMC controller can be given as

\[
F_{pd}\left(q^{-1}\right) = \frac{R_s}{1 - \exp\left( -\frac{R_s}{L_d} T_s \right) q^{-1}} \times \left[ 1 - \exp\left( -\frac{R_s}{L_d} T_s \right) q^{-1} \right]^{-1}.
\]  

(4.49)

The discrete IMC controller for the \(q\)-axis can easily be derived from (4.49) just by changing inductance value accordingly. This controller was implemented in simulation level and the results will be presented in the next section.

4.6.3 Design and simulation results
4.6.4 Real-time implementation

Normalized time to real-time conversion of the controller is again the main difficulty in the real-time implementation. As explained in Section 4.3.5, this is easily done by making the substitution, $T_{sn}=T_s \times \omega_{base}$ in (4.49) to give correctly scaled real-time controller transfer function

$$F_d(q^{-1}) = \frac{R_s}{1 - \exp \left(-\frac{\alpha T_{sn}}{L_d} \frac{\omega_{base}}{T_s} \right)} \times \left[ 1 - \exp \left(-\frac{R}{L_d} \frac{T_s}{T_{sn}} \frac{\omega_{base}}{s} \right) \right]^{-1}.$$

Here again experimental results with the new DC-IMC implementation gives some indication on the uncertainty in the machine parameters under transient and varying frequency conditions. Improvement of performance with the compensated inverter and increased integral part is illustrated in the first experimental result of the DC-IMC implementation shown in Figure 4.24 with $\alpha = 4 \text{ pu}$. 

Since the derivation was done in discrete domain, implementation is straightforward unlike in the basic IMC design. For easy comparison between the methods, the same actual sampling time of $125\mu s$ (corresponding to $8kHz$ sampling frequency), which gives a normalized time step of $0.11775\text{rad}$, is used in the digital controller simulations. Step-response simulations for the $d$-axis with $\alpha = 4 \text{ pu}$, which corresponds to a real time closed loop bandwidth of $600 \text{ Hz}$ is shown in Figure 4.23. The delay of two sampling periods in the beginning is due to the zero-order-hold nature of the discrete time system and the input delay to to I/O function handling. The controller performance in real-time will be tested in the next step.
As in the simulation results, step-response of the \( d \)-axis with \( \alpha = 4 \) pu, which corresponds to a real time closed loop bandwidth of 600 Hz, is shown in Figure 4.25 with the dead-time compensation and doubled resistance term \( (R_s) \).

**Figure 4.25:** Step responses of \( d \)-axis current for step inputs of 0.2, 0.3, 0.5 and 0.7 pu.

The results presented so far are meant for easy comparison with basic IMC discussed in the beginning of this chapter. Some more results with much higher current controller bandwidth must be presented here to justify its sampling delay handling capability. In Figure 4.26 the same set of step response tests are shown with \( \alpha = 10 \) pu (corresponding to 900 Hz), which may be compared with the results shown in Figure 4.9, which was presented at the start of the discussion on the sampling delay problem. Here again it must be noted that these results are with an inverter compensated (for inverter non-idealities) to behave like an ideal inverter together with a doubled integral part from the typical design.
Current control implementation

Figure 4.26: Step responses of d-axis current - the step inputs applied are 0.2, 0.3, 0.5 and 0.7 pu.

From these results it can be seen that the performance deterioration observed with basic IMC at the same bandwidth is no longer existing with DC-IMC. Thus, the new design has been able to meet the expected delay-compensating capability.

4.7 Concluding remarks

As illustrated also, all controller and observer design methods discussed in this chapter are dependent on the machine parameters. Therefore, accurate machine parameters are required for better performance of the control strategies. The stator resistance and inductance may vary considerably with the variation of the supply voltage frequency, temperature and also the current level in the machine (the amount of penetration of the stator flux into the rotor iron depends on the current level). Due to this phenomena it is an extremely difficult task to establish the best set of machine parameters for the full speed range of an electrical machine. As a consequence a controller de-tuning may occur depending on the operating point of the machine. Machine parameter estimation will be discussed in Chapter 9 of this work.

Another important point to mention here is the relatively low stator inductance and especially resistance values of a PMSM compared to an induction machine. Due to this, the RLC network formed together with the DC-link capacitance and stator winding, when the machine is inverter driven, may tend to cause heavy current oscillations at some operating frequencies. In addition, it was seen in the case of basic IMC design, that the amount of integral action that is determined by the stator resistance term may not be enough to bring the currents to the desired steady state value faster. This means that some amount of de-tuning from exact IMC design has to be done for a PMSM current control to be acceptable.

When it comes to fine tuning of a controller, another comment must be made here on the complexity of controller structure. In the so-called modular type controller structure, fine-tuning is more straightforward. Since it is possible to explicitly identify the proportional and integral controllers, the implementation engineer always has the advantage of having a better physical insight into the fine tuning process. The same applies to the fine-tuning of the observer to perform satisfactorily under noisy measurement conditions (the faster the closed loop observer dynamics, the higher the noise content in the predictions). On the contrary, in the more compact discrete IMC design or similar approaches, offers poor physical insight for the implementation engineer to use in the fine tuning process. However, this approach may have advantages like, compactness and lesser execution time.
etc.

Another positive remark towards the observer approach is that with this method, predicted current information will be explicitly available inside the control algorithm to be used for any purposes other than current control. One such example is the rotor speed and position estimation algorithms that will be discussed in Chapter 7. It will be shown that for these estimations, the $d$-axis and $q$-axis currents are required. It is an advantage to have these predicted current information explicitly for the estimation algorithm to produce better rotor speed and position estimations. With the DC-IMC, this possibility will no longer be there due to the compactness of the method.

Throughout this design procedure, it was assumed that there are no non-linearities in the control loop and asymptotic stability was the design criterion. However, in a typical variable speed AC drive, there are saturation type non-linearities like the maximum voltage limit of the inverter and also some non-idealities in the inverter. Yet, this kind of designing the controller for the linear system and separately treating the non-linearities is an accepted method in many control engineering applications [49]. These non-linear issues will be discussed in detail in the following chapters.
5. Speed controller design

After the complete vector controller has been implemented successfully, additional control loops can be included around it in cascade to have either a speed controlled (speed servo) or angular position controlled (position servo) drive. This was mentioned in Chapter 2 also (see Figure 2.7). Unless flux-weakening operation is used, the electromagnetic torque depends only on the \( q \)-axis current in synchronous coordinates. This is the situation for the surface mounted PMSMs considered here. Thus, all these outer loops must be built up around the \( q \)-axis current controller. The discussion in this chapter will be limited only to speed controller design, as this work deals with high-speed applications.

5.1 Basic requirements for speed controller design

Improving the transient response is one feature expected from a properly designed speed controller. The mechanical time constant \( (J/B) \) for the prototype drive is about 0.9 s, which is not satisfactory for a high-speed application. Therefore, a good speed controller designed for this drive must give rise to a closed loop speed response that is several times faster than the open loop plant.

Another aspect is tracking the steady state speed set-point accurately during the operation. To achieve zero steady state error situation, it is well known that the controller must have integral action.

Canceling the influence of unknown load torque variations is another feature expected. Especially in applications like an angle grinder, this kind of unknown load torque variation must be tackled by the speed controller, in order to maintain the operating speed at set value. Fast recovery of speed, after a sudden change of load situation is an essential feature.

However, the speed controller design can be considered as an easier task, when compared to the current controller design, which is a Multi Input Multi Output (MIMO) problem. As it was explained in Chapter 4, the vector controller decouples the flux producing current \( (i_d) \) and the torque producing current \( (i_q) \). This enables the speed controller design to be treated as a Single Input Single Output (SISO) problem (when flux-weakening is not used). What is then left is to decide a suitable controller structure that can fulfill the above basic requirements and design its parameters. For this purpose, a suitable model for the speed or the mechanical dynamics of the motor has to be established. This will be discussed in detail in the next section.

5.2 Formulation of design problem

The aim here is to establish a suitable model for the mechanical dynamics of the motor. This has to start from the torque expression and the differential equation for speed in the PMSM model of Equation (2.23). From these two equations with zero \( d \)-axis current, speed dynamics can be given as

\[
\frac{d\omega}{dt} = \frac{1}{J} \left( \frac{3}{2} n \psi \omega i_q - T_i - B\omega \right). \tag{5.1}
\]

Since the speed control loop is added around the \( q \)-axis current control loop, the closed loop current dynamics must also be considered for the model to be accurate. If the basic IMC design is
assumed with a closed loop current control bandwidth of $\alpha$, dynamical equation for $q$-axis current is given by

$$\frac{di_q}{dt} = \alpha i_{q_{ref}} - \alpha i_q.$$  \hspace{1cm} (5.2)

Taking the Laplace transforms of (5.1) and (5.2) and combining the two together, results in

$$\omega(s) = \frac{3}{2} \frac{n_p \Psi_m}{(Js + B)} \cdot \frac{\alpha}{(s + \alpha)} i_{q_{ref}}(s) - \frac{1}{(Js + B)} T_i(s).$$ \hspace{1cm} (5.3)

This continuous-time model for the PMSM speed dynamics is graphically depicted in Figure 5.1.

![Figure 5.1: Continuous-time model for the PMSM speed dynamics](image)

With this arrangement, it is now clear that the speed controller has to produce the $q$-axis current reference, which in other words is the torque demand for a particular set-point of speed, as its output. It is now interesting to see if it is possible to make any simplifications of this second order model. The immediate possibility is to neglect the current dynamics (plant electrical pole), by assuming the fact that closed loop current dynamics is much faster than the mechanical dynamics of the motor. This assumption is a very common one in motor applications. Then the pole at $(s+\alpha)$ in (5.3) can be neglected to give the first order system

$$\omega(s) = \frac{3}{2} \frac{n_p \Psi_m}{(Js + B)} i_{q_{ref}}(s) - \frac{1}{(Js + B)} T_i(s).$$ \hspace{1cm} (5.4)

To justify this model reduction, one could compare the pole locations or the corresponding time constants of the electrical and mechanical dynamics. As shown in Figure 5.2, the mechanical pole is at $s = -1.15 \times 10^{-3}$ (equivalent to 109.3/0.1256 pu - 924 ms - time constant), while the electrical pole is at $s = -2$ (equivalent to 0.5 pu - 0.53 ms - time constant).

It can be seen that the mechanical pole ($s = -1.15 \times 10^{-3}$) and the electrical pole ($s = -2$) are more than three decades apart from each other. Thus the assumption of neglecting the electrical pole for the speed controller design is very well justified. However, trying to bring the speed controller bandwidth unnecessarily closer to the current controller bandwidth can also cause oscillation problems. In fact, any such attempt will nullify the validity of the assumption made here and the current controller dynamics may tend to affect the speed dynamics under these circumstances. Possible speed controller structures will be introduced in the next section.
5.3 Overview of existing speed controllers

Current control methods for PMSMs were discussed in detail in Chapter 4. The aim of this chapter is to look into the overall control system, which essentially consists of an inner current control loop and an outer speed control loop in cascade. It is therefore important to investigate also into the existing speed control methods for motor drives at this stage. A detailed survey of speed controllers for permanent magnet synchronous machines can be found in [50]. A brief overview will be presented here.

5.3.1 Speed control structures – a classification

Speed controller structures used in motor drive applications in general can be classified into three major categories.

1. Loops without a reference feed-forward term

In this type of structures, one does not find a feed-forward term proportional to the speed reference in the controller. Only the speed error (difference between the speed set-point and the measured speed) is used as the controller input. This category can further be sub-divided into two. This subdivision is done based on the way the disturbances are treated by the controller.

1.1 Loops without disturbance feed-forward term

Typical examples for this type of controllers are the Proportional Integral (PI) and Integral Proportional (IP) controllers. Both structures can sometimes be extended with a derivative term to give PID and IPD controllers respectively. These are the most common speed controllers suggested by many authors and the basic PI and IP structures are shown in Figure 5.3 (a) and (b) respectively.

![Figure 5.3: PI and IP controllers](image)
The difference between the two structures can be illustrated by obtaining the closed loop transfer functions for each case starting from the controller equations in the s-domain. The controller s-domain equation for the PI controller is given by

\[ i_{qref}(s) = \frac{K_p s + K_I}{s} [\omega_{ref}(s) - \omega(s)]. \] (5.5)

Then the closed loop system description will be (with reduced first order model (5.4) for mechanical dynamics)

\[ \omega(s) = \frac{3}{2} n_p \Psi_m (K_p s + K_I) \frac{\omega_{ref}(s)}{s(Js + B) + \frac{3}{2} n_p \Psi_m (K_p s + K_I)}. \] (5.6)

If one examines (5.6) carefully, the possibility of canceling plant mechanical pole from the controller zero will be revealed. Since the pole cancelled is a stable one (a pole in the left half plane), it is a perfectly safe design method from control theory point of view. In fact, the choice of \( K_I/K_P = B/J \) and fixing \( K_P \) to give desired closed loop pole is the design strategy. This again is very similar to basic IMC design presented for current controller design in Chapter 4. However, this way of looking at the problem is the typical approach from machine control point of view and is more familiar to machine control engineers. The interesting feature with this design is that the closed loop system becomes first order, when perfect pole zero cancellation is achieved. However, the presence of model errors can cause imperfect pole-zero cancellation resulting in an un-cancelled zero in the closed loop system. Thus, overshoots can occur in the speed response.

Similar to this analysis, the IP controller is defined from the equation

\[ i_{qref}(s) = \frac{K_I}{s} [\omega_{ref}(s) - \omega(s)] - K_P \omega(s) \]

\[ = \frac{K_I}{s} \omega_{ref}(s) - \frac{K_P s + K_I}{s} \omega(s). \] (5.7)

Then the closed loop system description will be (again with reduced first order model for mechanical dynamics)

\[ \omega(s) = \frac{3}{2} n_p \Psi_m K_I \frac{\omega_{ref}(s)}{s(Js + B) + \frac{3}{2} n_p \Psi_m (K_p s + K_I)}. \] (5.8)

In (5.8) it can be seen that there is no possibility of a pole zero cancellation unlike in (5.6). This result in a second order closed loop system.

In general a pole placement design can be done to determine \( K_P \) and \( K_I \) for both these controller structures [43, 47, 51]. Their performance is further discussed in [50].

1.2 Loops with disturbance feed-forward terms:

In situations, where the information about the disturbances occurring in the process is available as measurements (an example is the torque measurement of a drive system), a feed-forward term based on disturbance information can be associated in the controller structure. This could result in a better regulation performance. The controller structure is shown in Figure 5.4.
As before, a PI or IP controller can be employed as the feedback controller together with this feed-forward term. If PI control is assumed as an example, the typical controller equation for a motor drive having a load torque disturbance feed-forward term will be

\[ i_{qref}(s) = \frac{K_p s + K_I}{s} [\omega_{ref}(s) - \omega(s)] + G_p(s)T_d(s). \]  

(5.9)

Thus, in addition to the PI controller design, a suitable criterion has to be used to determine the disturbance (torque in this case) feed-forward component of the controller output.

2. Loops with a reference-proportional feed-forward term:

A proportional-reference feed-forward term such as the one shown in Figure 5.5, sets a zero in the closed loop reference to output transfer function, independent of the pole placement of the rest of the controller. This is the first step towards having separate tuning for regulation and tracking.

Here again, the controller can be designed using a suitable pole placement to either PI or IP controller structure. As an illustration, complete controller equation with a PI controller and the feed-forward term can be given as

\[ i_{qref}(s) = \frac{K_p s + K_I}{s} [\omega_{ref}(s) - \omega(s)] + K_F \omega_{ref}(s) \]

\[ = \left[ \left( K_p + K_F \right) + \frac{K_F}{s} \right] \omega_{ref}(s) - \frac{K_p s + K_I}{s} \omega(s). \]

(5.10)
where $K_F$ is the reference-proportional feed-forward gain. This yields the closed loop system given by

$$\omega(s) = \frac{3}{2} n_p \psi_m \left[ (K_P + K_F) s + K_I \right] \omega_{ref}(s).$$

(5.11)

The equation (5.11) shows the additional zero set by the feed-forward term in the closed loop system dynamics.

3. RST Controllers:

This is a typical polynomial controller with a structure shown in Figure 5.6. A pole placement design can be done to find the polynomials $R$, $S$ and $T$, according to the demanded performance of the closed loop system.

$$i_{qref}(s) = \frac{T(s)}{R(s)} \omega_{ref}(s) - \frac{S(s)}{R(s)} \omega(s),$$

(5.12)

where $R(s)$, $S(s)$ and $T(s)$ are polynomials of $s$. Polynomial controller design methods can be found in [47]. The design and tuning aspects of all these controller structures are described in detail in [50].

In the next section, design of the speed controller for this application will be discussed.

5.4 Design and implementation of the speed controller

The design of the speed controller for this application is done in exactly the same way as basic IMC design for the current controller discussed in Chapter 4. There is only a slight difference in this approach with respect to PI controller design explained in the previous section. In the PI controller design, only the plant mechanical pole is cancelled from the controller zero. On the contrary, in the basic IMC approach the complete plant transfer function is cancelled by the choice of the controller transfer function and the closed loop bandwidth is fixed from the pre-filtering term (see Section 4.3.1). This design will further establish the link between the conventional PI controller design and basic IMC design.
5.4.1 Basic IMC design for the speed controller

The design procedure will be the same as in Section 4.3.1. The controller structure is shown in Figure 5.7. With neglecting the faster plant electrical pole, the transfer function for motor speed dynamics, $G_M(s)$, is given by

$$G_M(s) = \frac{3}{2} n_p \frac{\Psi_m}{(Js + B)} = \frac{K_M}{(Js + B)}. \quad (5.13)$$

The complete IMC controller transfer function is

$$F_M(s) = \left[1 - G_{CM}(s) \bar{G}_M(s)\right]^{-1} G_{CM}(s). \quad (5.14)$$

If perfect modeling is assumed, that is

$$\bar{G}_M(s) = G_M(s), \quad (5.15)$$

resulting closed loop transfer function will be

$$G_{clm}(s) = G_M(s) G_{CM}(s). \quad (5.16)$$

Since it is already known that $G_M(s)$ is open loop stable, $G_{CM}(s)$ can be selected such that

$$G_{CM}(s) = \frac{\alpha_s}{s + \alpha_s} G_M^{-1}(s). \quad (5.17)$$

As before, this choice of $G_{CM}(s)$ will give low-pass filtering of bandwidth $\alpha_s$ to the feedback error quantity. The resulting controller transfer function can be written as

$$F_M(s) = \frac{\alpha_s}{s} G_M^{-1}(s)$$

$$= \left(2 \cdot \frac{J \alpha_s}{3 n_p \Psi_m}\right) + \left(2 \cdot \frac{B \alpha_s}{3 n_p \Psi_m}\right) s^{-1}, \quad (5.18)$$

which is a PI controller. Corresponding closed loop speed dynamics is given by
\[ \omega(s) = \frac{\alpha_s}{(s + \alpha_s)} \omega_{ref}(s). \]  

(5.19)

Implementation issues will be treated in the next section.

### 5.4.2 Implementation in simulation level

The discrete-time implementation of the continuous-time PI controller in (5.18) is done similar to the discrete-time implementation of the current controllers discussed in Chapter 4. With forward-rectangular integration rule, the discrete-time controller equation will be

\[ i_{q \text{ref}}(k) = \left[ \frac{2}{3} \cdot \frac{J \alpha_s}{n_p \Psi_m} \right] + \left[ \frac{2}{3} \cdot \frac{B \alpha_s}{n_p \Psi_m} \right] T_s q^{-1} \left[ \omega_{ref}(k) - \omega(k) \right]. \]  

(5.20)

Shown in Figure 5.8 is a step response simulation of speed with a closed loop speed controller bandwidth of \( \alpha_s = 0.05 \).

![Figure 5.8: Step response of speed (\( J = 109.3, B = 0.1256, n_p = 3, \Psi_m = 1 \) and \( \alpha_s = 0.005 \) in per-unit)](image)

### 5.4.3 Implementation in real-time

Here again the problem is the discrete implementation of a controller designed in continuous domain. The proportional and integral parts of the controller can be implemented in discrete real-time in exactly the same way as described in Chapter 4. The conversion of the integration real-time step to normalized time step must be done here also. The resulting discrete real-time controller equation is

\[ i_{q \text{ref}}(k) = \left[ \frac{2}{3} \cdot \frac{J \alpha_s}{n_p \Psi_m} \right] + \left[ \frac{2}{3} \cdot \frac{B \alpha_s}{n_p \Psi_m} \right] T_s q^{-1} \left[ \omega_{ref}(k) - \omega(k) \right]. \]  

(5.21)

A step response of speed with a closed loop bandwidth of \( \alpha_s = 0.05 \) pu is shown in Figure 5.9. In fact for easy comparison the same simulated response in Figure 5.8 has also been plotted in Figure 5.9 with the time scale converted (according to the definitions used it is easily seen that Real-time = Per-unit time \( \times \alpha_{\text{base}} \)).
The difference between the real-time response and the simulated response shows the poor quality of the machine parameter estimation. In fact, the response with de-tuned integral gain illustrates how the expected performance can be obtained. According to (5.18), it can be seen that the integral part depends on the damping, while the proportional part depends on the inertia of the drive. Thus, de-tuned (increased) integral part means mainly a discrepancy in the estimate of damping – hence insufficient integral action, when it comes to IMC controller design. This clearly shows again as in current control design, that the basic IMC approach is a good way of reaching the ultimate tuning values of the controller for a simple process faster.

### 5.4.4 Some comments

Some of the important points on machine parameters used in this design must be mentioned here. This could be of importance for the effective utilization of the design method. The inertia value used in the design must be the total equivalent inertia of all the rotating parts driven by the motor. As an example, if the motor drives a load coupled through a gear arrangement, the inertia of the load must be translated to an equivalent quantity on the rotor shaft using the gear ratio [52].

Similarly the damping constant \( B \) must also be the equivalent damping constant of all the rotating parts connected to the rotor. This will make it difficult for the designer to analytically find the suitable value for \( B \). An experimental method similar to the one described in Appendix C can be used for the purpose. Thus, some fine-tuning will be necessary in order to obtain the best performance from the controller.

Like in the case of current controller design, possible non-linearities that could exist in the control loop were not taken into this design procedure and it was the asymptotic stability criterion that was used. However, since it is preferred to keep the steady state current of the machine around the rated current, some saturation type non-linearities can be assumed at the speed controller output. This is in addition to non-linearities mentioned in Chapter 4. As mentioned earlier, these non-linear issues will be discussed in the next chapter.

### 5.5 Advanced speed controller design

The PI controller structure suggested in the previous section is capable of removing the steady state error due to a step like input as the speed reference. This controller performs quite satisfactorily as a speed controller for several industrial applications and in fact it is the most widely used speed controller in drive applications. However, there can be many industrial motion control applications, which are more demanding in terms of the speed controller performance.
One such famous example is the optimized motion of a train from one station to the other. A common method used is to accelerate the train up to the top speed and continue at maximum speed until the train gets closer to the next stop by a certain distance and then decelerate. This gives rise to the following speed profile shown in Figure 5.10.

![Speed profile for the motion of a train](image)

*Figure 5.10: Speed profile for the motion of a train*

Since the area under the speed profile gives the total distance traveled, some form of a cost function can be used to find out the amount of acceleration and deceleration required and corresponding time intervals ($T_1$, $T_2$, $T_3$ in this case) to give expected optimum performance. For this to be successful, the speed controllers used in the traction drives must be capable of tracking not only the constant speed set points, but also ramp like set-point changes without a steady state error. Thus “ramp tracking capability” becomes a key word in some motion control applications.

An ordinary PI controller is not capable of tracking ramp-like set-point changes with zero steady state error. Characteristics of a controller capable of doing this will be explained next.

### 5.5.1 Ramp tracking controller design

Basic requirements of a controller capable of tracking speed ramps without steady state error can be analytically found. This in fact is a standard result in automatic control theory [43]. It can be easily shown that for a first order process such as motor transfer function to track input ramps, the controller transfer function $F_M(s)$ must at least have double poles at the origin. This basic condition leads to defining the general form of the controller transfer function. However, similar to PI and IP controllers discussed earlier, two controller structures with double integrators can be suggested for ramp tracking feature. The two structures are shown in Figure 5.11 (a) and (b). Both these designs will be done and implemented to illustrate the difference between them in terms of performance.

The controller transfer function for the structure in Figure 5.11 (a) (Ramp Tracking Controller 1) can be given as

$$F_M(s) = K_P + \frac{K_i}{s} + \frac{K_2}{s^2}.$$  \hfill (5.22)

Corresponding closed loop transfer function is
This shows that there will be two zeros in the closed loop transfer function. The required closed loop performance can be achieved by doing a suitable pole placement design for the characteristic equation

\[
s^3 + \left( \frac{B + K_M K_P}{J} \right) s^2 + \frac{K_M K_1}{J} s + \frac{K_M K_2}{J} = 0,
\]

which will give the values of the controller parameters \( K_P, K_i, \) and \( K_2 [43] \). Like in the previous design, it is recommended to do this design also with per-unit quantities and translate later to real-time, when it come to real-time implementation.

![Figure 5.11: Two possible ramp tracking controller structures](image)

The controller equation for the structure in Figure 5.11 (b) (Ramp Tracking Controller 2) is

\[
i_{qref}(s) = \left( \frac{K_1}{s} + \frac{K_2}{s^2} \right) \omega_{ref}(s) - \omega(s) - K_P \omega(s)
\]

\[
i_{qref}(s) = \left( \frac{K_1}{s} + \frac{K_2}{s^2} \right) \omega_{ref}(s) - \left( K_P + \frac{K_1}{s} + \frac{K_2}{s^2} \right) \omega(s).
\]

Corresponding closed loop transfer function is

\[
G_{clm}(s) = \frac{K_M \left( \frac{K_1}{J} + \frac{K_2}{J} \right)}{s^3 + \left( \frac{B + K_M K_P}{J} \right) s^2 + \frac{K_M K_1}{J} s + \frac{K_M K_2}{J}}.
\]

This shows that there will be only one zero in the closed loop transfer function for this controller configuration, which is analogous to IP controller structure. However, the characteristic equation is the same as for the previous one.

In the next section under implementation it will be illustrated that for the same pole placement, the two controllers show different performance in terms of overshoot etc., while having good ramp tracking capability. In fact, this is an interesting feature in the two structures.
5.5.2 Implementation in simulation level

As before, continuous time controllers can be implemented in discrete-time using the forward-
rectangular integration rule to convert integrators. The discrete implementation corresponding to the
controller given in Equation (5.26) (Ramp Tracking Controller 1) is

\[
q_{\text{ref}}(k) = K_p + \frac{K_1 T_{sn} q^{-1}}{1-q^{-1}} + \frac{K_2 T_{sn} q^{-2}}{(1-q^{-1})^2} \left[ \omega_{\text{ref}}(k) - \omega(k) \right].
\] (5.27)

The discrete-time controller equation corresponding to the Ramp Tracking Controller 2 (structure in
(5.29)) is

\[
q_{\text{ref}}(k) = \left[ \frac{K_1 T_{sn} q^{-1}}{1-q^{-1}} + \frac{K_2 T_{sn} q^{-2}}{(1-q^{-1})^2} \right] \omega_{\text{ref}}(k) - \left[ K_p + \frac{K_1 T_{sn} q^{-1}}{1-q^{-1}} + \frac{K_2 T_{sn} q^{-2}}{(1-q^{-1})^2} \right] \omega(k). \] (5.28)

The speed responses (for step and ramp inputs) with both these controller structures, for the same
pole placement are shown in Figure 5.12.

![Figure 5.12: Step and ramp response for the two Ramp Tracking Controller structures (poles are placed at –0.009, -0.00225 and –0.045 pu)](image)

As before this implementation in simulation level was done with per-unit parameters and time. Real-
time implementation will be discussed in the next section.

5.5.3 Real-time implementation

Once again, when the per-unit to real-time conversion is done, the time step correction has to be
made. This is done by replacing \( T_{sn} \) in (5.31) and (5.32) from \( T_b \times \omega_{\text{base}} \). Real-time controller
equation corresponding to (5.31) is
\[ i_{qref}(k) = \left[ K_p + \frac{K_1 T_s \alpha_{base} q^{-1}}{1 - q^{-1}} + \frac{K_2 T_s^2 \alpha_{base}^2 q^{-2}}{(1 - q^{-1})^2} \right] \left[ \omega_{ref}(k) - \omega(k) \right] \]  

and that corresponding to (5.32) can be given as

\[ i_{qref}(k) = \left[ \frac{K_1 T_s \alpha_{base} q^{-1}}{1 - q^{-1}} + \frac{K_2 T_s^2 \alpha_{base}^2 q^{-2}}{(1 - q^{-1})^2} \right] i_{qref}(k) - \left[ \frac{K_1 T_s \alpha_{base} q^{-1}}{1 - q^{-1}} + \frac{K_2 T_s^2 \alpha_{base}^2 q^{-2}}{(1 - q^{-1})^2} \right] \omega(k) \]  

Experimental results (for step and ramp inputs) for the two controller implementations with the pole placement similar to the one in the simulation results presented in the previous section are shown in Figure 5.13.

![Figure 5.13: Step and ramp response – experimental](image)

(poles are placed at –0.009, -0.00225 and –0.045 pu)

It must be noted here that this particular pole placement is such that the q-axis current is not raised above the rated motor current (this was limited to 0.5 pu due to the safety of the test rig) during the transient period with both input types (step and ramp). The transient performance can be further improved by making a pole placement to give larger closed loop bandwidth. However, this must be done after implementing a suitable controller anti-windup scheme to remove the adverse effects of current limiting at the speed controller output. As an illustration, shown in Figure 5.14 are the step and ramp responses of the two controllers with a faster pole placement after implementing an anti-windup scheme. Then it can be seen that for the step input both controllers tend to saturate during the transient period. Controller output saturation and the resulting integrator windup problem will be the focus of Chapter 6. Only the experimental results to justify the performance of the ramp tracking controllers are presented here.
5.6 Some general comments on speed controllers

Like in the current controllers, the sampling delay in the speed measurement exists, if a speed sensor is used for the measurement purpose. However, this delay does not become pronounced as the speed dynamics is much slower than the current dynamics. In other words the ratio (mechanical time constant / sampling time) is much higher than the ratio (electrical time constant / sampling time). Thus, the need for any sampling delay compensation does not arise in this case.

If an estimation method is used to obtain the speed information (sensorless operation), depending on the method used, there is a possibility that the speed controller can be provided with up-to-date speed information.

Speed controller design based on mechanical parameters of the machine, as it has been done in this work may not give the best performance at once. The reason again is the possible deviation of the mechanical machine parameters depending on different loading condition, temperature etc. Yet, this approach will be very useful in order to reach best tuning values for the speed controller easily. In that respect, the method will be more useful for the industry applications.

Figure 5.14: Step and ramp response – experimental
(poles are placed at –0.05, -0.0125 and –0.25 pu)

Steady state current in the motor during the operation has to be limited to the rated current due to the thermal limitations of the machine. This can be done by simply checking the current and activating a safety switch to shut down the system, when over-current occurs. This is the more conventional way of achieving over-current protection and it is still in use. However, the number of emergency shut downs can be reduced and the extra costs for restarting can be saved by incorporating the current limitation into the speed controller. These issues will be discussed in detail in the next chapter.
6. Saturation phenomena in AC drive control

A typical current and speed control loops of an electrical drive forms a cascaded system with the faster current control loop being the inner loop and the slower speed control loop being the outer. Even though both these processes were assumed to be perfectly linear during the controller design, it may not be so in actual practice. There are some limitations in a variable speed AC drive system that can be translated into the closed loop cascaded control structure as saturation type non-linearities. Controllers with output saturations can come across windup problem, which was addressed in Chapter 4 of Part 1 of the thesis. Special techniques must then be employed to eliminate these controller windup problems in order to achieve expected performance. Having a good anti-windup scheme in the control structure increases its reliability and robustness. This enables the designer to force the system to be at extreme conditions (maximum realizable control signals can be used without reaching instabilities) during transients to achieve the fastest possible response out of it. In this chapter, such limitations in an AC drive and how they can be translated into the control loop will first be revealed. Suitable anti-windup schemes will then be presented for the cascaded system together with some novel ideas on how to exploit the cascaded control structure with anti-windup compensation to achieve improved performance. To suit the application of interest in this work, the methods developed and experimental results will be for a surface mounted PMSM. However, some hints on how the concepts can be extended for any variable speed AC drive will also be mentioned towards the end of the chapter.

6.1 Cascaded nature of machine control structure and limitations

Clearly identifying the controllers that are in cascade in the case of an AC drive is the first task in this approach. This will be followed by investigating the possible limitations in a drive system and how they can be translated into the closed loop controller structure.

6.1.1 Cascaded AC drive control structure and its advantages

In the previous chapter on speed controller design, it was explained that the speed response can be described as a function of \( q \)-axis reference current (see Equation (5.4)). Thus, the speed controller output can simply be the \( q \)-axis reference current that can produce the required torque at a given instant. It was also mentioned in Chapter 5 that this system forms a typical series cascaded control structure with \( q \)-axis current controller being the inner (secondary) loop, while the speed controller being the outer (primary) loop. This is common to any AC drive controlled in synchronous reference frame (provided that flux-weakening is not used). In addition, if the position control is also required another loop surrounding the current and speed loops must be built, making the number of controllers in cascade equal to three.

Advantages of cascaded control have been generally discussed in [53]. Some of the principal advantages of the cascade control in the case of AC drive control will be mentioned below.

- Disturbances arising in the inner current loop are corrected by the inner current controller, often before they can influence the outer speed loop.

- The steady state error in the \( q \)-current (thus the phase and amplitude errors in the three line currents in the three-phase system) is reduced by the inner loop. This improves the dynamics of the outer speed loop.
• Gain variations in the inner loop of the structure are overcome within its own loop.
• The inner current loop permits an exact manipulation of the \( q \)-axis current by the outer speed controller.

With this explanation, the need for much faster current dynamics with respect to the speed controller dynamics is again emphasized. The next step is to identify the possible limits in the control system.

6.1.2 Typical limitations and their translation to drive control structure

Any actuator has its own limitations. Thus actuator saturation is present in all systems [54]. The actuator in a typical AC drive application is usually a PWM inverter. The maximum output sinusoidal voltage that can be produced by the inverter is the saturation limit in this case. This is further elaborated below.

(a) Voltage limitation

Maximum average sinusoidal output voltage that can be produced in each phase by the inverter is the output voltage corresponding to full modulation from the PWM scheme. This is proportional to the DC-link voltage associated and the exact relationship depends on the type of PWM generation scheme employed. Even though this voltage limitation exists in the inverter stage in real life, this can be translated into the current controller structure by taking into account the signal transitions taking place in between the control algorithm and the inverter output. The advantage of this translation is that the designer then has the ability to implement suitable measures to eliminate the adverse effects of this limitation at the current controller stage.

In space vector form, this limitation becomes a limit on the maximum magnitude of the voltage space vector that can be produced (see Equation (2.12)). With suitable choice of scaling factors and axis transformations, the voltage limit can be translated to the synchronous reference frame located just outside the current controller [37, 38]. The reason for this is the fact that current controller determines the components of the output voltage vector \((u_d, u_q)\) in synchronous reference frame. The corresponding general limitation is

\[
\sqrt{u_d^2 + u_q^2} \leq U_{MAX}.
\] (6.1)

If this limitation is not taken into account in the actual implementation, current controller can easily get windup during the fast transients. In fact, if one needs to take into account this voltage limitation, the best possibility is to assume a saturation type non-linearity just outside the current controller, which limits the voltage command according to (6.1). Then suitable anti-windup compensation schemes can be employed. This actually is a general description of the voltage limit. There can be further simplifications of the expression for limitation depending on the particular AC drive and the control strategy used.

(b) Current limitation

During the steady state operation of the machine, the three-line currents in the motor must be kept within the rated value. Excess current can heat-up the machine and one common practice in industrial drives is to use temperature sensing of the stator winding to activate a thermal protection circuitry to switch off the drive under over heated conditions. However, this can also be treated from controller design point of view by properly translating this “user introduced safety limitation” into the controller structure.
As in the case of voltage limit, the current limitation on the three lines (stator currents) can be interpreted as a limit on the maximum magnitude of the stator current space vector. This again can be transformed into \( d-q \) frame with the correct scaling factor and axis transformations. The two components of the stator current space vector in synchronous reference frame can then be limited accordingly by limiting their reference values \((i_{dref}, i_{qref})\). The corresponding inequality will be

\[
\sqrt{i_{dref}^2 + i_{qref}^2} \leq I_{MAX}.
\]  (6.2)

Since \( i_d \) is the flux producing current, there will not be any outer loops around the \( d \)-axis current control loop. Therefore, the limit on \( d \)-axis is straightforward. However, if the machine is operated as a veritable speed drive as an example, there will be an outer speed control loop as explained earlier around the \( q \)-axis current controller. Thus the limitation that was imposed on the \( q \)-axis current reference will actually be a saturation type non-linearity outside the speed controller. This translation enables the designers to treat the “user introduced safety limitation” from the control theory point of view and perhaps avoid the number of thermal shut downs of the drive [55].

### 6.1.3 Limitations as applied to surface mounted PMSMs

Since this work has the special focus on surface mounted PMSMs; some more simplifications that can be done on the limitation inequalities based on this machine type will be illustrated here.

A surface mounted PMSM is usually controlled with zero \( d \)-axis current as the air-gap flux is produced from the permanent magnets. When used as a variable speed drive, as explained before, the speed controller determines the \( q \)-axis reference current of the motor. Since the \( d \)-axis reference current is forced to zero, for near-perfect axis decoupling, the \( d \)-axis current controller output \( u_d \) will be in the vicinity of zero. This means that the above general inequalities (6.1) and (6.2) can be simplified such that

\[
|u_q| \leq U_{MAX}
\]

\[
|i_{qref}| \leq I_{MAX}.
\]  (6.3)

These simplifications remove the multivariable nature of the saturation problem. The resulting controller structure is graphically depicted in Figure 6.1.

Some comment must be made on the best location to place the voltage saturation. It is known that the axis decoupling is done as an inner loop to the current controller. This means that as shown in Figure 4.2, the decoupling terms are added to the current controller outputs of each axis. The best place to assume the voltage limitation as shown in Figure 6.1 is after this summing point of decoupling term to the \( q \)-axis. By doing so, it is possible to limit the final \( q \)-axis command voltage output. Then whatever the anti-windup scheme used, it will take care of the resultant saturation effect from \( q \)-axis current controller output and the decoupling terms. A brief overview on how these two limitations are handled from controller design point of view will be presented in the following section.
6.2 How this has been treated before?

Before presenting the strategy used to overcome these two saturation problems, an investigation of previous contributions in this area will be presented. Achieving fast closed loop current control dynamics has been the goal of many researchers in the past. As an obvious outcome of this attempt, most of the previous contributions have identified voltage limitation of the inverter as an obstacle for fast current control. Thus, there is an abundance of literature on motor control applications that successfully treat the saturation due to voltage limitation of the inverter.

6.2.1 Previous work on voltage limitation

Some previous work on the saturation of current controller due to the voltage limitation will be briefly here.

An optimal control method for general three-phase PWM converters has been suggested in [37] and [38]. The basic concept there is to find the optimal control voltage for tracking the reference current with minimum time under the voltage constraint (limit) of the inverter.

The reference voltage vector ($u_d + ju_q$) is assumed to be limited to a hexagon in [36]. The hexagon is the result of six different maximum voltage vectors that can be generated from an inverter. A current predictor has been implemented in this work, which uses a look-up table based on measured data to find the actual voltage applied to the machine for a given reference voltage vector. In order to avoid saturation, corresponding boundary values of the voltage limiting hexagon are assumed as applied voltages in the machine during large signal transient. The method as it is can be called an empirical technique rather than an approach from the control theory point of view.

A fast current control method for PMSMs has been proposed in [39] that exploits the cross coupling between the $d$-axis and $q$-axis currents. In this application a reference modification part is incorporated with the generally used synchronous frame PI controller for the fast transient response. The $d$-axis current is kept at zero as usual for a PMSM and all limitations can then be assumed to act along $q$-axis control loop as it was shown. Incorporating the voltage limitation has been done by defining a maximum for the $q$-axis current gradient based on the voltage limitation.

6.2.2 Previous work on current limitation

Unlike voltage limitation, attempts to treat the current limitation as a saturation type non-linearity at the speed controller output is rarely found. As mentioned before, one reason could be the thermal over current protection usually provided in an industrial drive as a safety for the stator winding.
Another point that could be of interest here is the relatively shorter transient time of speed compared to the temperature dynamics of the stator winding. Due to this slow temperature dynamics, application of a higher current than the rated may not be that harmful to the stator in many applications. Thus, assuming such a limitation for the speed controller design can be considered as less significant.

However, a new anti-windup PI controller is proposed in [55] to improve the speed controller performance of variable speed motor drives. In this work, rated current limitation has been translated into a $q$-current limitation outside the speed controller. Then an integrator anti-windup scheme has been proposed for the integrator of the speed controller.

One common feature in all these work is that the saturation effect has been treated as a problem local to current controller or speed controller. Assuming both limitations at the same time and trying to treat it globally (considering the cascaded structure) has to the knowledge of the author, never been attempted before. In that respect, approach in this work will be novel and is a clear contribution to the machine control area.

6.3 Available solutions for saturation problem

Actuator saturation in closed loop control systems has been a known problem over the years [54] and a discussion on this implementation problem can be found in many of the standard textbooks on control engineering [47]. Some of the basic anti-windup schemes reported so far were introduced under the Active Magnetic Bearing application (see Chapter 4 of Part 1) in this thesis. The area of anti-windup compensator design is so wide that it is easy to find new methods that show improved performance [56]. Work dedicated to performance of control systems with power amplifier saturation and their stability, which is a part of the saturation problem in this application can also be found in the literature [57, 58].

6.3.1 Saturation of multivariable systems

The general saturation problem in an AC drive, when controlled in synchronous coordinates is a multivariable problem. It was explained earlier also that the voltage saturation affects the output voltage vector from the current controller, which is the vector sum of $d$-axis and $q$-axis voltage commands. Therefore, previous work on control of multivariable systems with output constraints is also of interest at this stage.

Digital implementation of PID controllers for a class of multiple input multiple output systems with saturating actuators has been presented in [59]. Different possibilities of implementing integrator anti-windup schemes for two-input two-output PI controllers etc. have been explained in this work. In [60], a general theory to address the anti-windup and bumpless transfer problem for multivariable systems has been presented. A reformulation of the conditioning technique (which will be discussed in detail later) for the saturation problem of constrained multivariable systems can be found in [61].

The immediate interest in this work is to treat the two saturation type non-linearities present in a typical PMSM drive as a series cascaded control structure. This motivates the need for a better investigation of available anti-windup compensation methods for cascaded control systems. This is done in the next section.
6.4 Saturation of series-cascaded systems

It is possible to find several methods to remove the saturation problem of series-cascaded controllers with output saturation. A brief overview of a couple of them will be presented here.

6.4.1 Local anti-reset windup

One straightforward method is to treat the two saturations located outside each controller in series cascade locally and implement anti-windup schemes for each controller independently. This has been presented in [49], which has a structure shown in Figure 6.2.

\[
\begin{align*}
&\text{PI}_1 & & \text{NL}_1 & & \text{PI}_2 & & \text{NL}_2 \\
&w_1 & & u_1 & & u_2 = w_2 & & u_2 = w_1 & & u_1 & & u_1^r \\
&y_1 & & y_1 & & y_2 & & y_2 & & + & & +
\end{align*}
\]

**Figure 6.2:** Anti-reset windup for controllers in series cascade - local implementation

The equations for each controller, when both are of type PI with anti-reset windup can be given as

\[
e_i(k) = w_i(k) - y_i(k)
\]

\[
u_i(k) = K_{pi}e_i(k) + I_i(k)
\]

\[
u_{i}^r(k) = nl(u_i(k))
\]

\[
I_i(k + 1) = I_i(k) + K_{pi}e_i(k) + K_{awci} [u_{i}^r(k) - u_i(k)], \quad i = 1, 2.
\]

where \(e_i\) - control errors, \(w_i\) – set-points, \(y_i\) - controlled variables (measurements), \(u_i\) - desired control outputs, \(u_{i}^r\) - realizable control outputs, \(I_i\) - integral states and \(K_{pi}, K_{i}, K_{awci}\) are proportional, integral and anti-windup compensator gains respectively.

One disadvantage in this method is the possibility of primary (outer) controller windup due to the saturation at the secondary (inner) controller output. If this happens, there is no mechanism in this method to remove the windup in the primary controller. This drawback has been addressed somewhat successfully in the next method presented here.

6.4.2 Anti-reset windup with modified tracking signal

When a series cascaded controller structure is in operation, control goal is to bring the controlled variable from inner loop (\(y_i\)) equal to the realizable control output of the outer loop (\(u_{i}^r\)). If the transient of \(y_i\) is going to be affected due to saturation at the inner loop controller output, as mentioned earlier, it can also cause the outer loop controller to saturate in the long run. Therefore, saturation information occurring at the inner loop can be indirectly transferred to the outer loop by using \(y_i\) in place of \(u_{i}^r\) in the anti-reset algorithm described in (6.4). This method is described in [53]. The modified cascaded structure is shown in Figure 6.3.
Corresponding control algorithm can be given as

\[ e_i(k) = w_i(k) - y_i(k) \]
\[ u_i(k) = K_{pi} e_i(k) + I_i(k) \]
\[ u_i^r(k) = nl(u_i(k)) \]  

\[ I_1(k+1) = I_1(k) + K_{II} e_1(k) + K_{aw1} [u_1^r(k) - u_1(k)] \]
\[ I_2(k+1) = I_2(k) + K_{II} e_2(k) + K_{aw2} [y_1(k) - u_2(k)] \]

However, one drawback of this method is that the removal of local windup (outer controller windup due to outer loop saturation) may now get delayed or rather will depend on the response time of the inner control loop.

### 6.5 The conditioning technique for controllers with output constraints

The conditioning technique, which is described in this section, is capable of using the saturation information of both, inner and outer loops into account for the anti-windup mechanism in the outer control loop. This method is considered to be the most reliable method presented so far for the saturation problem in series-cascaded control systems with output constraints. The method can be applied for multiple input multiple output (MIMO) systems with constraints. This has been presented by Hanus et. al. in [62]. Mismatch between the desired control variables \( u \) and the actual (or realizable) ones \( u_r \) yields an inconsistency of the controller state variables \( v \). The restoration of the controller consistency is done by the conditioning technique. This forces the controller to come back to the normal mode of operation as soon as it can do so. If the controller inconsistency occurred due to the application of reference inputs \( w \), a set of auxiliary reference inputs called realizable reference inputs \( (w_r) \) are calculated from this method. Those realizable reference inputs are such that, if \( w_r \) had been applied to the controller instead of \( w \), the control variables would have been equal to \( u_r \), (controllers will be just at saturation). This is the basic idea of the conditioning technique. The theoretical derivation of the method will be done in the following sections.

#### 6.5.1 Basic theory for a general controller

This derivation of the conditioning technique for controllers with output saturation is true for any MIMO system as mentioned before. The classical unconditioned control loop with non-linear saturation at the output is shown in transfer function form and state space form in Figure 6.4.
The following are the basic variable definitions involved.

- \( w \) - reference variables
- \( y \) - output variables
- \( u \) - desired controlling variables
- \( v \) - desired controller state variables
- \( w' \) - realizable reference variables
- \( u' \) - realizable (actual) controlling variables
- \( v' \) - realizable controller state variables

The most general form of discrete-time state variable representation for the controller shown in Figure 6.4 can be given as

\[
\begin{align*}
\dot{v}(t+1) &= f(v(t), w(t), y(t), t) \\
u(t) &= g(v(t), w(t), y(t), t),
\end{align*}
\]

(6.6)

where \( f \) and \( g \) are functions of \( v(t), w(t), y(t) \) and \( t \). Due to the output saturation shown in Figure 6.4, there can be a mismatch between \( u \) and \( u' \). This makes inconsistency in control state variables \( v \). This can be avoided and the state consistency can be restored by calculating the so-called auxiliary reference inputs \( w' \) and applying them to the controller. These \( w' \)s are such that, if \( w' \) had been applied to the controller in place of \( w \), the control variables \( u \) would have been \( u' \). If it is assumed that \( w' \) can be calculated, the resulting \( v' \) states found with these new inputs will be necessarily consistent. Hence,

\[
\begin{align*}
\dot{v}'(t+1) &= f\left(v'(t), w'(t), y(t), t \right) \\
u'(t) &= g\left(v'(t), w'(t), y(t), t \right).
\end{align*}
\]

(6.7)

With these new reference variables \( w' \), the consistency of the state variables \( v' \) is restored. Since the internal states are now consistent, the computation of \( u \) can be done using the consistent states \( v' \) instead of \( v \). This results in
In this general description, the realizable reference variables $w'_r$ are implicitly defined by the equations (6.7) and (6.8). Explicit form of $w'_r$ can be obtained for the special case, where (6.7) is linear in $w$. Therefore, if the descriptions for $u'_r$ in (6.7) and $u$ in (6.8) linear in $w'$ and $w$ respectively, are

$$u'_r(t) = c[v'(t), y(t), t] + D_f[v'(t), y(t), t]w'(t)$$

(6.9)

and

$$u(t) = c[v'(t), y(t), t] + D_f[v'(t), y(t), t]w(t).$$

(6.10)

Here again $c$ is a function of $v(t), y(t)$ and $t$, while $D_f$ is a functional matrix of appropriate dimensions. The difference of (6.9) and (6.10) will explicitly define the realizable references $w'(t)$ from the equation

$$u'(t) - u(t) = D_f[v'(t), y(t), t][w'(t) - w(t)].$$

(6.11)

Solving of (6.11) for $w'(t)$ demands that $D_f$ be of full rank. When the dimensions of $w(t)$ and $u(t)$ are equal, there will be a unique solution for $w'(t)$ from (6.11) given by

$$w'(t) = w(t) + D_f^{-1}[v'(t-1), y(t-1), t][u'(t-1) - u(t-1)].$$

(6.12)

Hence the corresponding conditioned controller will be

$$w'(t-1) = w(t-1) + D_f^{-1}[v'(t-1), y(t-1), t-1][u'(t-1) - u(t-1)].$$

$$v'(t) = f[v'(t-1), w'(t-1), y(t-1), t-1]$$

$$u(t) = c[v'(t), y(t), t] + D[v'(t), y(t), t]w(t).$$

(6.13)

This is a somewhat general form of the conditioned controller. The equations can further be simplified for the case, where the system under consideration is of Linear Time Invariant (LTI). This is explained in the next section.

### 6.5.2 Conditioning technique for LTI controllers

For LTI controllers, functions $f$ and $g$ become linear in $v(t), w(t)$ and $y(t)$, while they will be time invariant, hence independent of $t$. Thus, (6.6) can be rewritten as

$$v(t+1) = Av(t) + Bw(t) - Ey(t)$$

$$u(t) = Cv(t) + Dw(t) - Fy(t).$$

(6.14)

where $A, B, C, D, E$ and $F$ are matrices of appropriate dimensions. Then the corresponding conditioned controller analogous to (6.13) is

$$w'(t-1) = w(t-1) + D_f^{-1}[u'(t-1) - u(t-1)].$$

$$v'(t) = A v'(t-1) + B w'(t-1) - E y(t-1)$$

$$u(t) = C [v'(t) + D w(t) - F y(t)].$$

(6.15)

For this to be realizable, the matrix $D$ must be of full rank. Equation (6.14) in transfer function form is given by
Saturation phenomena in AC drive control

\[ u(t) = T(q)w(t) - S(q)y(t) \]  \hspace{1cm} (6.16)

where \( T(q) \) and \( S(q) \) are transfer functions defined in terms of unit delay operator \( q \). The relationship between \( T(q) \), \( S(q) \) and \( A, B, C, D, E, F \) can be easily derived by describing (6.14) in operator form as

\[ v(t) = (Iq - A)^{-1}[Bw(t) - Ey(t)] \]  \hspace{1cm} (6.17)

which yields

\[ u(t) = [C(Iq - A)^{-1}B + D]w(t) - [C(Iq - A)^{-1}E + F]y(t). \]  \hspace{1cm} (6.18)

From (6.18) it can be seen that the matrix \( D \) is the asymptotic value of \( T(q) \) as \( q \) tends to infinity. This is elaborated by

\[ \lim_{q \to \infty} T(q) = \lim_{q \to \infty} \left[ \frac{CB}{Iq - A} + D \right] = D. \]  \hspace{1cm} (6.19)

The conditioned controller in (6.15) can now be written as

\[ u(t) = [C(Iq - A)^{-1}B]w'(t) + Dw(t) - S(q)y(t) \]  \hspace{1cm} (6.20)

or,

\[ w'(t-1) = w(t-1) + D^{-1} \{ u'(t-1) - u(t-1) \} \]

\[ u(t) = [T(q) - D]w'(t) - S(q)y(t) + Dw(t). \]  \hspace{1cm} (6.21)

This is the operator description of the general conditioned controller. The conditioned discrete-time controller described in (6.21) is shown in Figure 6.5.

![Conditioned discrete-time controller](image)

**Figure 6.5: Conditioned discrete-time controller**

The structure shown here is the most general form and in some instances it may be possible to obtain more simple controller structures depending on the original controller transfer function. This will be illustrated, when the method is applied to two cascaded PI controllers later in this chapter. In [62], another form of the same conditioning technique, referred to as self-conditioned structure, can be found. The next step is to extend this result to the series cascaded controller structure. This is done in the next section.

6.5.3 Extension to series-cascaded control systems
Two controllers in typical series cascade are shown in Figure 6.6. This in fact is analogous to cascaded current and speed controllers in a variable speed AC drive described earlier in this chapter and depicted in Figure 6.1.

![Series-cascaded controllers](image)

The controller description here is similar to the standard description in (6.16) and can be given as

\[ u_i(t) = T_i(q)w_i(t) - S_i(q)y_i(t) \quad i = 1,2. \]  \tag{6.22}

The suffix \( i \) here denotes whether it is inner (\( i = 1 \)) or outer (\( i = 2 \)) loop in the cascade. Series cascaded nature leads to the additional relationship

\[ w_i(t) = u_i(t) \]  \tag{6.23}

Application of the conditioning technique to the inner loop is straightforward and is analogous to (6.19) and (6.12). This is given by

\[ w_i(t) = w_i(t) + D_i^{-1}\{u_i(t) - u_i(t)\} \tag{6.24} \]

and

\[ D_i = \lim_{q \to \infty} T_i(q) \tag{6.25} \]

Conditioning of the outer loop becomes a complex task as this must be done considering the non-linearities in both inner and outer loops. The realizable reference input for the outer loop \( w_2^*(t) \) now means that, if \( w_2^*(t) \) is applied as inputs, the final outputs from the series cascade \( u_i(t) \) would be equal to \( u_i^*(t) \). This means that the computation of \( w_2^*(t) \) needs the conditioned equations corresponding to both controllers written as

\[ v_i(t) = A_i v_i(t-1) + B_i w_i(t-1) - E_i y_i(t-1) \]
\[ u_i(t) = C_i v_i(t) + D_i w_i(t) - F_i y_i(t) \tag{6.26} \]
\[ u_i^*(t) = \text{NL}_i\{u_i(t)\} \]

where

\[ T_i(q) = C_i (Iq - A_i)^{-1} B_i + D_i \]
\[ S_i(q) = C_i (Iq - A_i)^{-1} E_i + F_i, \quad i = 1,2. \]  \tag{6.27}

The realizable references of the inner loop \( w_i^*(t) \) are found from (6.24). Due to the cascaded nature of the controllers, they become the realizable controls of the outer loop, which can be different from the limited control variables \( u_i^*(t) \). From (6.26), it can now be deduced that
From (6.26) and (6.27) it can be written that
\[
\begin{align*}
\dot{u}_2(t) - w'_2(t) &= D_2(t)(w_2(t) - w'_2(t)) \\
w'_2(t) &= w_2(t) + D_2^{-1}(t)\left[w'_2(t) - u_2(t)\right]
\end{align*}
\]
(6.29)

Substituting for \(w'_2(t)\) yields
\[
w'_2(t) = w_2(t) + D_2^{-1}\left[w_2(t) - w'_2(t)\right] + D_1^{-1}\left[u'_1(t) - u_2(t)\right]
\]
(6.30)

Since \(w'_2(t)\) can now be replaced according to (6.23), the final expression for \(w'_2(t)\) will be
\[
w'_2(t) = w_2(t) + D_2^{-1}\left[u'_2(t) - u_2(t)\right] + D_1^{-1}\left[u'_1(t) - u_2(t)\right]
\]
(6.31)

Conditioned-cascaded structure is shown in Figure 6.7.

**Figure 6.7:** Conditioned-cascaded structure

The equations (6.24) and (6.31) describes how the realizable references are computed for the series-cascaded general controller structure. In the next section, how this can be applied to two PI controllers in a series-cascaded PMSM drive control and its stability will be discussed.

### 6.6 Application of conditioning technique for PMSM control

The conditioning technique for series cascaded control systems derived in the previous section can now be applied to the PMSM speed controller structure shown in Figure 6.1. In this section, the conditioned controllers for some of the current and speed controllers discussed in Chapter 4 and 5 will be derived. Two PI controllers in cascade will be given priority, as this is the most common controller structure in the motor control area. The issue of closed loop stability of the conditioned controller, when applied for a PMSM drive will also be addressed.

#### 6.6.1 Conditioned PI controllers in cascade

The two PI controllers in cascade can be written similar to the form in (6.22) as
The corresponding conditioning constants $D_i$ (for the SISO case matrices $D_i$ reduce to constants) will then be

$$D_i = \lim_{q \to 0} \begin{bmatrix} K_p + \frac{K_n}{q-1} \end{bmatrix} = K_{p_i}, \quad i = 1, 2. \quad (6.33)$$

Equations for the two unconditioned PI controllers in state space form can be written as (this is easily deduced from (6.26) by substituting $A_i = 1, B_i = 1, C_i = K_{pi}$ and $D_i = F_i = K_{pi}$)

$$v_i(t) = v_i(t) + K_n \left[ w_i(t) - y_i(t) \right]$$

$$u_i(t) = v_i(t) + K_{p_i} \left[ w(t) - y_i(t) \right], \quad i = 1, 2. \quad (6.34)$$

The equations (6.24) and (6.31) can now be used to compute the realizable references for the inner and outer control loops as

$$w_i'(t) = w_i(t) + \frac{1}{K_{p_i}} \left\{ u_i'(t) - u_i(t) \right\} \quad (6.35)$$

and

$$w_2'(t) = w_2(t) + \frac{1}{K_{p_2}} \left\{ u_2'(t) - u_2(t) \right\} + \frac{1}{K_{p_1}} \left\{ u_1'(t) - u_1(t) \right\} \quad (6.36)$$

It is advisable to always start off from Equation (6.21), whenever the conditioned form for a certain controller has to be derived. Simpler implementation structures than what was shown in Figure 6.5 can of course be obtained by manipulating the terms after applying the technique. Following this approach the operator description of the cascaded PI controllers can be given as,

1. **Inner PI controller**

$$w_i'(t) = w_i(t) + \frac{1}{K_{p_i}} \left\{ u_i'(t) - u_i(t) \right\}$$

$$u_i(t) = \left[ K_{p_i} + \frac{K_{i1}}{q-1} - K_{p_i} \right] w_i'(t) - \left[ K_{p_i} + \frac{K_{i1}}{q-1} \right] y_i(t) + K_{p_1} w_i(t)$$

$$= \frac{K_{i1}}{q-1} \left\{ w_i'(t) - y_i(t) \right\} + K_{p_1} \left\{ w_i(t) - y_i(t) \right\} \quad (6.37)$$

2. **Outer PI controller**

$$w_2'(t) = w_2(t) + \frac{1}{K_{p_2}} \left\{ u_2'(t) - u_2(t) \right\} + \frac{1}{K_{p_1}} \left\{ u_1'(t) - u_1(t) \right\}$$

$$u_2(t) = \left[ K_{p_2} + \frac{K_{i2}}{q-1} - K_{p_2} \right] w_2'(t) - \left[ K_{p_2} + \frac{K_{i2}}{q-1} \right] y_2(t) + K_{p_2} w_2(t)$$

$$= \frac{K_{i2}}{q-1} \left\{ w_2'(t) - y_2(t) \right\} + K_{p_2} \left\{ w_2(t) - y_2(t) \right\} \quad (6.38)$$

The resulting conditioned cascaded controller structure is shown in Figure 6.8.
One important point to consider here is the placement of the voltage limit in the cascaded structure with respect to the summing point of the decoupling and back emf terms. If the machine parameters have been estimated with reasonable accuracy, it can be assumed that the decoupling is accomplished fairly well. However, there is still a slight possibility of cross coupling effects coming into action during transients. Such effects may cause additional saturation at the voltage limit. Conditioning the \textit{q-axis} current controller is best done, if it can take into account all possible signals that can cause voltage saturation. Thus, the best location for the voltage saturation in the control structure is after the summing point of the decoupling and back emf terms as shown in Figure 6.8.

![Conditioned-cascaded PI controller structure](image)

\textit{Figure 6.8: Conditioned-cascaded PI controller structure}

A slightly different form of the same implementation must also be mentioned here. The main difference here is that it eliminates the need to compute the realizable reference. Instead this implementation becomes very similar to the tracking algorithm used in the AMB application. The only difference being the fact that the anti-windup gain is fixed (derived from the conditioning technique) in this case, while in the tracking algorithm the anti-windup gain is treated as a tuning parameter. This is done by substituting for \( w'_i(t) \) in the expressions for \( u'_i(t) \) in Equations (6.37) and (6.38). This would result in the conditioned PI controllers to be described as

1. **Inner PI controller**

   \[
   u_i(t) = \frac{K_{i1}}{q-1} \left( (w_i(t) - y_i(t)) + \frac{1}{K_{p1}} (u'_i(t) - u_i(t)) \right) + K_{p1} \left( w_i(t) - y_i(t) \right) \]  
   \hspace{10cm} (6.39)

2. **Outer PI controller**

   \[
   u_i(t) = \frac{K_{i2}}{q-1} \left( (w_i(t) - y_i(t)) + \frac{1}{K_{p2}} (u'_i(t) - u_i(t)) + \frac{1}{K_{p1}} (u'_i(t) - u_i(t)) \right) + K_{p2} \left( w_i(t) - y_i(t) \right) \]  
   \hspace{10cm} (6.40)

Thus the realizable reference computation is vanished from the control algorithm and the modified structure is shown in Figure 6.9.
6.6.2 BIBO stability in PMSM application

The classical Bounded Input Bounded Output (BIBO) stability for the conditioning technique can be established by argument even for this series cascaded structure. This is possible since the two processes in cascade (namely, the $q$-axis current dynamics and mechanical dynamics of the rotor) are open loop stable. It was earlier established the fact that both these dynamics can be approximated to be stable first order processes. Complete closed loop cascaded control system with the anti-windup mechanism (conditioning technique in this case) is shown in Figure 6.10.

A stable first order system will always give a bounded output for a bounded input, which is called as BIBO stability. In this case therefore, both the current and speed dynamics are BIBO stable. When the speed and current controllers are designed considering the system without saturation type non-linearities in the control loop, the closed loop system also becomes BIBO stable. Therefore, by the design itself the stability of the system, when none of the saturation effects are active, is guaranteed. The presence of saturation at the speed and current controller outputs causes a discontinuity of the signal flow from the controllers to the process, whenever the signals reach saturation levels. This causes the closed loop of the system to break.

Any saturation occurring in the speed controller will send the bounded current $u'_2(t)$ through the motor corresponding to which there is a bounded speed value. In the same way, saturation at the current controller output will give rise to a certain bounded voltage $u'_1(t)$ be applied to the motor that would result in another bounded speed.

As mentioned before, any saturation terminates the continuity of the closed loop. Thus, the boundedness of the outputs of the controllers under saturating conditions are not guaranteed unless
there are suitable anti-windup mechanisms. Infact, the purpose of anti-windup mechanisms used here is to make the controller transfer functions stable so that their outputs will be bounded to $u_i'(t), \quad (i = 1,2)$, whenever saturation occurs at the output of any of the controllers.

With this argument, it is clear that under saturation conditions the two controller outputs and the outputs of the two processes of interest ($q$-axis current dynamics and motor speed dynamics) will have bounded output signal values with the conditioning technique. Hence the stability is guaranteed. It must be noted here that a more general proof of stability of the conditioning technique applied to cascaded control systems using a different approach can be found in [62].

The proof of BIBO stability in this section is correct from the argument point of view. However, it may give a better physical insight into the stability problem, if it is possible to present some information regarding the maximum set-point change of speed that can be made on top of a given operating speed under a certain load. This way of looking into the problem is important, when one thinks about how the load torque may affect the stability. The Liyapunov theory approach [64] similar to the one suggested in [63] can be useful in that respect. However, this analysis is not presented in this work.

6.6.3 Simulation and experimental results

In fact due to the time scale difference between the closed loop responses of current and speed control, current saturation may not be reached for a step change in speed reference with normal tuning values. However, to illustrate the fact that the cascaded conditioned control system is capable of handling this situation, a simulation result is shown in Figure 6.11. The current controller bandwidth is de-tuned (made unnecessarily fast) and the voltage limit is contracted here to force the current controller into saturation during the step change of speed.

![Figure 6.11: Step change of speed that causes both controllers to saturate – simulation](Speed controller is PI with $|I_{\text{MAX}}| = 0.5 \text{ pu}, |U_{\text{MAX}}| = 1.0 \text{ pu}, \alpha = 10 \text{ pu}, \alpha_r = 0.5 \text{ pu})
The same de-tuning can be done in the experimental set-up also to illustrate the real-time capability of the conditioning technique. This is shown in Figure 6.12. However, it must be noted here that under normal tuning parameters of the speed and current controllers, this case can not be easily implemented. One major problem against this is the noise in the current measurements in a typical inverter-fed motor drive.

It can be seen from the results that the voltage saturation occurs only for just one sampling instant during the step response. However, due to the de-tuning of the current controller closed loop current response has become unnecessarily fast and it shows sensitivity to noise in the sampled current. This is visible in the $q$-axis voltage plot and in fact some of the noise have been cut off from the voltage limitation. This again is a good proof for the proper operation of the cascaded conditioning technique.

These results show that dealing with the current saturation in cascaded form is not that significant especially for the machine start-up due to the time scale difference between the two control loops in cascade. This might raise doubts about the usefulness of cascaded conditioning technique in electrical machine control among the reader. Yet, there are many other hazardous conditions that can arise in a drive system, where sudden current controller saturation occurs (rotor getting stuck in case of an angle grinder application is one example). In addition, some methods of making use of the cascaded nature that will very well justify this approach, will also be suggested later in the chapter.

Before completing this section on application of the conditioning technique, some control configurations other than conventional PI controllers will be investigated. They are DC-IMC and ramp tracking controllers suggested for current and speed control in Chapter 4 and 5 respectively.

6.6.4 Conditioned DC-IMC
Controller transfer function for DC-IMC in time-normalized form is given in (4.49) and the same in real-time form is given in (4.50) respectively. For this illustration of deriving the conditioned DC-IMC, only the time-normalized controller transfer function is used. The real-time version can be easily derived by performing the scaling operation explained in Chapter 4 and 5. Corresponding controller equation in the form of (6.16) will be

\[ u(t) = F_d(q)w(t) - F_d(q)y(t), \]  

(6.41)

where \( F_d(q) \) is according to (4.49). Then the conditioning gain can be computed from the limit

\[ D_{DCIMC} = \lim_{q \to \infty} T(q) = \lim_{q \to \infty} F_d(q^{-1}) = \frac{R_d \left[ 1 - \exp\left(-\alpha T_{sn}\right) \right]}{1 - \exp\left(\frac{R_d}{L_d T_{sn}}\right)}. \]  

(6.42)

Thus the corresponding conditioned DC-IMC controller equation according to (6.21) is

\[ w'([t] - 1) = w(t - 1) + D_{DCIMC}^{-1} \times \{ u'([t] - 1) - u(t - 1) \} \]

\[ u(t) = \{ F_d(q) - D_{DCIMC} \} w'(t) - F_d(q)y(t) + D_{DCIMC}w(t) \]

\[ = F_d(q)\{ w'(t) - y(t) \} + D_{DCIMC}\{ w(t) - w'(t) \} \]  

(6.43)

The structure can be depicted as shown in Figure 6.13.

**Figure 6.13:** Conditioned DC-IMC controller structure

### 6.6.5 Conditioned ramp tracking controllers

Two ramp tracking controller configurations were discussed in Chapter 5 for speed control purposes. Being the outer loop, conditioning the speed controller always needs properly scaled saturation information from the inner current controller according to (6.31). However, information fed back from the saturation of inner current controller and its scaling (\( D_{i}^{-1} \)) is determined by the current controller structure. What is changing with the speed controller structure is the scaling factor for the conditioning of outer loop (\( D_{i}^{-1} \)). The way in which this is derived will only be illustrated here and providing the correct saturation from the inner control loop then is fairly straightforward. As before, the time-normalized controller equations ((5.31) and (5.32)) will be considered.

1. **Ramp tracking controller 1**

As in the case of DC-IMC, controller equation (5.31) in the form of (6.16) is
Then the conditioning gain can be computed from the limit

\[
D_2 = \lim_{q \to \infty} T(q) = \lim_{q \to \infty} \left[ K_p + \frac{K_1 T_{in} q^{-1} + K_2 T_{in}^2 q^{-2}}{1 - q^{-1}} \right] = K_p
\]  \hspace{1cm} (6.45)

According to (6.21) the conditioned controller equation can now be written as

\[
u(t) = \left[ K_p + \frac{K_1 T_{in} q^{-1} + K_2 T_{in}^2 q^{-2}}{1 - q^{-1}} \right] \omega(t) - \left[ K_p + \frac{K_1 T_{in} q^{-1} + K_2 T_{in}^2 q^{-2}}{1 - q^{-1}} \right] y(t) + K_p \omega(t)
\]

\[
u(t) = \left[ K_p + \frac{K_1 T_{in} q^{-1} + K_2 T_{in}^2 q^{-2}}{1 - q^{-1}} \right] \left\{ \omega(t) - y(t) \right\} + K_p \left\{ \omega(t) - y(t) \right\}
\]

\hspace{1cm} (6.46)

The corresponding structure of the “Conditioned ramp tracking controller 1” is shown in Figure 6.14.

![Conditioned ramp tracking controller-1](image)

**Figure 6.14: Conditioned ramp tracking controller-1**

2. Ramp tracking controller 2

As in the case of first ramp tracking controller, the Equation (5.32) in the form of (6.16) is

\[
u(t) = \left[ K_p + \frac{K_1 T_{in} q^{-1} + K_2 T_{in}^2 q^{-2}}{1 - q^{-1}} \right] \omega(t) - \left[ K_p + \frac{K_1 T_{in} q^{-1} + K_2 T_{in}^2 q^{-2}}{1 - q^{-1}} \right] y(t).
\]

\hspace{1cm} (6.47)

Then, if the conditioning gain is computed from the direct limit, as before

\[
D_2 = \lim_{q \to \infty} T(q) = \frac{K_1 T_{in} q^{-1} + K_2 T_{in}^2 q^{-2}}{1 - q^{-1}} = 0.
\]

\hspace{1cm} (6.48)

The conditioning gain becomes zero making it impossible to use the method for this ramp tracking controller configuration. However, this problem has been addressed in respect to conditioning technique by previous researchers and the method of applying the conditioning technique to
controllers with input delays can be found in [65]. The excess time delay in \( T(q) \) transfer function that causes the anti-windup gain to vanish is taken out as a factor and a new filtered set-point variable is defined as 

\[
\omega_f(t) = \omega(t-1). \tag{6.49}
\]

This results in the new controller equation given by

\[
u(t) = \left[ \frac{K_p T_{sn}}{1 - q^{-1}} + \frac{K_2 T_{sn}^2 q^{-1}}{1 - q^{-1}} \right] \omega_f(t) - \left[ K_p + \frac{K_p T_{sn} q^{-1}}{1 - q^{-1}} + \frac{K_2 T_{sn}^2 q^{-2}}{(1 - q^{-1})^2} \right] y(t). \tag{6.50}\]

The new controller description gives rise to a non-zero conditioning gain found as

\[
D_z = \lim_{q \rightarrow \infty} T(q) = \left[ \frac{K_p T_{sn}}{1 - q^{-1}} + \frac{K_2 T_{sn}^2 q^{-1}}{1 - q^{-1}} \right] = K_p T_{sn}. \tag{6.51}\]

Now it is possible to write the conditioned controller equation using (6.21) in combination with (6.54). This can be described as

\[
u(t) = \left[ \frac{K_p T_{sn}}{1 - q^{-1}} + \frac{K_2 T_{sn}^2 q^{-1}}{1 - q^{-1}} - K_1 T_{sn} \right] \omega_f(t) - \left[ K_p + \frac{K_p T_{sn} q^{-1}}{1 - q^{-1}} + \frac{K_2 T_{sn}^2 q^{-2}}{(1 - q^{-1})^2} \right] y(t) + K_1 T_{sn} \omega_f(t) \tag{6.52}\]

The corresponding structure of the “Conditioned ramp tracking controller 2” is shown in Figure 6.15.

![Figure 6.15: Conditioned ramp tracking controller-1](image)

The basic derivation of the cascaded conditioning technique and its application to some controller configurations that were discussed in this thesis have now been completed with sufficient verifications of the methods with both simulation and experimental results. In the following section some novel concepts will be presented on how this powerful cascaded controller structure can be used to improve the performance and reliability of an electrical drive. These concepts are novel to the machine control area and hence can be highlighted as clear contributions resulting from this work. In
fact, these methods will enable the control engineers to look at the saturation type non-linearities present in the machine control loop as positive features instead of sighting them as problematic negative effects. Hence, the section has been named as “Exploiting the cascaded nature”.

6.7 Exploiting the cascaded nature – some new concepts

As mentioned earlier too, employing a good anti-windup scheme increases the reliability and robustness of the drive. This is the case with the cascaded-conditioned controller also. With this, the speed controller can be made even faster in dynamics. In this section, some new concepts on making use of this extra reliability added to the drive from the anti-windup mechanism will be presented.

6.7.1 Relaxed current saturation for higher acceleration

With this cascaded control structure, it is now clear that the designer can force the machine to accelerate under rated current during speed transients. It was earlier mentioned that the temperature dynamics of the stator winding is much slower than the closed loop speed controller dynamics. Due to this reason, it is in fact possible to apply a torque producing current larger than the rated current during a short speed transient. This is absolutely harmless to the stator winding, if it is done with total control over it. The cascaded controller structure with anti-windup enables this. Since the current limitation of the drive is done by the saturation limit assumed at the speed controller output, relaxing the current limits during a speed transient will automatically force a higher current into the motor. The overall stability of the drive will still be preserved from the anti-windup mechanism, since any voltage saturation resulting from this relaxation can again be properly eliminated. During the steady state operation, current limit must be kept at rated current value as a safety measure for the stator winding.

However, to avoid steep changes in the motor torque, it is recommended to vary the current limit smoothly back to the rated current, after relaxation during the transient. This can be done by using some form of a time varying function to define the current limit. To illustrate the idea, one suitable variation for $\pm I_{MAX}(t)$ can be given as

$$|I_{MAX}| = \left|I_r + k_R \omega_e(t)\right|$$  \hspace{1cm} (6.53) when

$$|\omega_e(t)| \geq \delta$$, where

$$\omega_e(t) = \omega_{ref}(t) - \omega_{mes}(t)$$  \hspace{1cm} (6.54)

and $\delta$ is a small positive number. The amount of relaxation is determined by the gain term $k_R$. The value of $k_R$ must be determined based on the maximum possible set-point change in speed that may occur during the machine operation.
The design criterion that must be used here is the maximum current (above the rated) that can be allowed to flow through the machine during the largest possible transient. Improvement of the speed response, when time varying current limit relaxation is done can be seen in Figure 6.16.

However, having a good understanding on the $q$-axis current variation with speed is important, when doing this relaxation. If the back emf generated from the machine is positive and sufficiently large, the $q$-axis current can only increase relatively slowly, but can drop faster. On the other hand, if the back emf is negative, $q$-current rises faster and drops slowly [39]. This can be easily understood by inspecting the dynamical equation of $q$-axis current in Equation (2.23) in Chapter 2. Due to this reason, unnecessary relaxation of the current limit, while the machine is picking up from low speeds will demand a higher rate of change of current from the power modules of the inverter.

This explanation shows how the concept of current relaxation can be used to define the specifications for the power electronic actuator (inverter). Hence, these ideas might be useful in designing a complete drive system that includes the machine and the drive inverter. One example is the design of integral motors that have the inverter drive fabricated in the motor housing itself [10]. Especially in this kind of applications and also in general, optimized drive component design to make the overall drive system cost effective is of high demand in the industry. When control algorithms such as the one suggested here are capable of achieving good dynamical performance with higher degree of reliability, the designer can use it to reduce the safety margins for the inverter design. This will reduce the overall cost for the drive considerably. These ideas can be further strengthened by continued research in this area.

### 6.7.2 On-line emulation of DC-link voltage fluctuations

The voltage limitation assumed at the current controller output is actually a transformation of the maximum voltage applicable from the inverter used in the drive. It was explained that this limit is directly related to the DC-link voltage of the inverter. A straightforward way is to estimate the

---

**Figure 6.16:** Step change in reference speed with current limitation constant & relaxed

$k = 0.5, \delta = 0.05, q$-current limit at steady state: 0.5 pu, speed step 1pu
nominal DC-link voltage and define the voltage limit at the current controller output based on that estimation. This is briefly explained below.

**Translation of voltage limit on to \(d'q'\) frame:**

![Diagram of voltage limit translation](image)

*Figure 6.17: Translation of voltage limit on to \(d'q'\) frame*

When peak value scaling is assumed there is no magnitude change occurring in the coordinate transformation stages. The scaling at the output stage of the current controller must be such that 1 pu \(d\)-axis voltage must give rise to an output voltage of 103 V from the inverter leg of interest. All the scale factors associated in this case with respect to the test rig used here are shown in Figure 6.17. The scale factor used at the current controller output stage and the location of the translated saturation is also shown.

However, in industrial environments, DC-link voltage of the inverter can fluctuate as a result of voltage fluctuations in the three-phase supply. Under this kind of situation, it is clear that the constant limit corresponding to nominal DC-link voltage does not exactly model the actual constraint existing in the inverter. This can deteriorate the performance of cascaded control structure with conditioning technique.

Consequences of this imperfect transformation of the saturation are illustrated in Figure 6.18. This is the case, when the actual DC-link voltage fluctuate (drops for a shorter period) considerably so that the inverter can not produce the demanded current from the inverter. The results shown is obtained by emulating a DC-link fluctuation. This was done by defining a second voltage limit after the limit used for the anti-windup mechanism and contracting it according to the fluctuation that must be emulated.
Saturation phenomena in AC drive control

Figure 6.18: DC-link voltage fluctuation without on-line voltage limit estimation – emulating voltage limit dropped from ±2.86pu to ±0.8pu.

It can be seen here that the cascaded system can handle this voltage fluctuation. Yet the voltage limit just outside the current controller used to represent the real voltage limit outside seems to saturate. There seem to be a sudden spike in the q-current that exceeds the expected 0.5 pu limit. This may be due to imperfections in the axis decoupling. However, this degradation of performance can be eliminated and the control structure can be made more robust by sampling the DC-link voltage and estimating the saturation limit at the current controller output on-line. In this drive with peak value scaling for axis transformation, on-line estimation of the voltage limit in per-unit is done according to the equation

\[ U_{\text{MAX}} = \frac{U_{\text{DCm}}}{2} \times \frac{1}{U_{\text{base}}}, \]  

where \( U_{\text{DCm}} \) is the measured DC-link voltage. This will exactly translate the effective voltage constraint in the inverter stage to the current controller output. However, if the DC-link voltage drop is significant, the demanded q-axis current will not be produced. Thus operating speed will drop to the “realizable speed” corresponding to present DC-link voltage. Yet the cascaded anti-windup mechanism will restore the operating speed as the DC-link voltage recovers. This is shown in Figure 6.19.

6.7.3 PMSM as a voltage or current controlled drive

The initial aim of the controller design considering output saturations was to come up with a reliable control strategy for variable speed PMSM applications. As a result of treating the control system as a cascaded one and solving the saturation problems in a global manner, new possibilities have emerged to achieve improved performance. Two such possibilities were presented in Section 6.7.1 and 6.7.2. Since the global stability of the cascaded control system is guaranteed, some indirect methods of varying the machine speed without changing its set-point will now be suggested. The methods suggested here are not recommended for speed changes that demand good dynamic
performance during transient. However, the concepts can be used for situations, where overall system safety and undisrupted operation take the priority over good dynamic performance (such as faulty conditions of power supply etc.).

Figure 6.19: DC-link voltage fluctuation with on-line voltage limit estimation - voltage limit dropped from $\pm 2.86\text{pu}$ to $\pm 0.8\text{pu}$. The speed drops to realizable speed during the drop and recovers as the DC-link voltage rises to the normal value

(a) Constant voltage operation below set-speed

This fixed input voltage operation can be used to reduce the machine speed below a certain operating point set by the speed reference. Depending on the load torque $T_{l0}$, the speed controller will then determine the $q$-current required and thereby the $q$-axis command voltage ($u_{q0}$) by the current controller. The so-called “Constant voltage operation below set-speed” can now be performed for any $q$-axis voltage below $u_{q0}$ by shrinking the voltage limit of the cascaded system below that value ($u_{q0}$) according to the requirement. This will actually be a situation similar to DC-link fluctuation discussed before. The difference is that the shrinking of voltage limit is purposely done in this case, whereas under a DC-link voltage fluctuation the shrinking occurs as a result of system fault.

Consequence of this voltage limit shrinking again will be the inadequacy of $q$-current to overcome the load torque $T_{l0}$ at the initial speed reference. Therefore, the operating speed will drop to the corresponding realizable speed determined by the conditioning technique. So the new operating speed corresponding to any voltage limit setting below $u_{q0}$ will always be determined by the conditioning technique and will actually be the speed, where torque balance occurs. However, since the $q$-axis command voltage will always be in saturation, the machine will be supplied by a known voltage determined by the voltage limit, hence “Constant voltage operation below set-speed”. This operation is illustrated in Figure 6.20.

In fact, this method can be incorporated to force all the motor drives in a production line to a “pause mode” under faulty conditions in power supply. This will be advantageous than making an
emergency shut down of the drives both in terms of restarting time and cost. The normal operation can be restored, once the fault has been cleared, simply by relaxing the voltage limitation back to its normal value.

![Graph of voltage and speed over time](image)

**Figure 6.20:** Fixed input voltage operation below set speed - voltage limit is dropped from 2.86 pu to 0.8, 0.7 and 0.6 pu in steps

(b) **Constant current operation below set-speed**

As before, after the machine is brought up to a desired operating point from the speed reference setting, the current limit can now be reduced to obtain “constant current” operation. If the \( q \)-current corresponding to initial speed setting is \( i_{q0} \), this operation will be possible for current limit settings below that value. Again the torque production will drop due to this shrinking of current limitation and the speed will also drop to the realizable speed. Since it is the current controller that is saturated during this operating mode, a constant current determined by the current limit will flow through the machine resulting the so called “constant current” operation. This operation is illustrated in Figure 6.21.
Figure 6.21: Fixed input voltage operation

This again is useful for a series of drive systems in a production facility, controlled by a central control computer [67]. If there is a temporary drop in the supply power to the overall system, the whole production process can be slowed down by controlling the power delivered to each drive in the network. This kind of regulation can be achieved by appropriately contracting the current saturation of each drive based on a pre-defined criterion.

6.8 Some comments on conditioning technique

Some possible drawbacks of the conditioning technique and also some methods that can be used to obtain improved performance from it will also be briefly explained here. Being a well recognized integrator anti-windup method, a lot of other research work investigating the conditioning technique further can easily be found from the literature [65, 66]. One major drawback of the conditioning technique and one method to improve the performance further will be mentioned here.

6.8.1 Inherent “Short Sightedness”

Earlier in this chapter, it was mentioned that any controller with a suitable anti-windup scheme can be tuned to give faster closed loop system response. This tuning can be done such that the control output reaches upper saturation limit for a positive set-point change. If a conditioned controller is further tuned so that the control signal drops to the lower saturation from the upper, before settling down to the steady state value, one could observe this “Short Sightedness” property. This type of behavior of the control signal is usually called double saturation in control engineering. This shortcoming is explained in detail in [65] and [66] and said to an inherent property of the conditioning technique. This property must be taken into account, when a conditioned controller is fine tuned to obtain faster response.
6.8.2 Improvements through generalized conditioning technique

As a solution to the above-mentioned short sightedness, the authors of [65] and [66] have suggested a simple modification for the realizable reference computation given in Equation (6.21). This has been named as introducing cautiousness to the computation, so that the change in the modified set-point at the controller de-saturation is made smoother. The modified computation scheme is given by

\[
\begin{align*}
    &w'(t-1) = w(t-1) + \frac{1}{D+\rho} \times \left[ u'(t-1) - u(t-1) \right] \\
    &u(t) = [T(q) - D]w'(t) - S(q)y(t) + Dw(t),
\end{align*}
\]

(6.56)

where \( \rho \) is a tuning parameter matrix of the same dimensions as \( D \) in the general case. Those two factors are very important for any design using conditioning technique as the anti-windup scheme.

6.9 Hints on extending for a general AC drive

When it comes to a general AC drive the main difference that may occur from the cascaded structure discussed before will be the possible saturation of the \( d \)-axis current controller output. When non-zero \( d \)-currents are used (in induction machines \( d \)-current is used to produce air gap flux), the voltage limitation acts on the resultant voltage vector according to the inequality in (6.1). This yields the unsaturated operating region shown in Figure 6.22.

![Figure 6.22: General operating region for the voltage vector](image)

Now the cascaded structure becomes complex with voltage saturation of inner loop being a multivariable problem. Some methods similar to the one suggested in [39] that uses cross coupling between the axes may also be useful to eliminate the saturation effects from the inner current loop. However, treating the system as a cascade and deriving a global mechanism can also be efficient as the conditioning technique is valid for multi-input multi-output systems.

Further research is required in this area to investigate these possibilities, which might yield control strategies that can be novel contributions to the machine control area and the area of controller design with output saturations.
7. Sensorless control strategy

A basic definition for sensorless control of AC drives was established in the introduction to the thesis. The idea is to eliminate the need for a shaft mounted sensor for speed and rotor position measurement. This is done by employing an estimation method that provides rotor position information. Accurate rotor position information is essential for the proper operation of a PMSM (this is analytically shown in Appendix B). Methods on how to control the motor current and speed, when accurate rotor position and speed information is available were presented in Chapter 4 and 5 respectively. The aim of this chapter is to present a rotor position and speed estimation strategy that is capable of producing the accurate information required for the current and speed controllers. This enables the removal of the shaft-mounted sensor from the machine resulting in sensorless operation.

7.1 An overview of sensorless control methods

The back emf of a PMSM is sinusoidal in contrast to that of a brushless DC motor [7]. This enables the user to obtain constant torque with very low ripple, provided the machine is fed with sinusoidal currents. For this to be accomplished, rotor position information is continuously required. This is the main difference in PMSM control, compared to brushless DC machine. In brushless DC machines, the three stator windings are commutated sequentially every 60° with the back emf wave from to permanent magnets on the rotor [68]. This drastically reduces the resolution of the rotor position information required in case of a brushless DC motor. As mentioned before, this is not the case for a PMSM operation. This demands accurate and reliable estimation techniques of rotor position and speed, when it comes to sensorless operation of a PMSM. Since the main interest in this work is the sensorless operation of PMSMs, reported sensorless control methods for this type of machines will be classified in the beginning.

7.1.1 Classification of available methods

Rotor position estimation methods reported so far can be classified into five different categories. Each of these will be briefly explained here with some suitable references for application examples.

(a) Position estimation based on the terminal voltage and current measurement

The method is based on field orientation. Stator voltage and current signals are used to construct a flux linkage position signal, through which the phase angle of the stator current can be controlled. Since the induced emf is the rate of change of flux linkage, an analog integration can be implemented using the voltage and current measurements that would give the stator flux linkage vector. The additional information required is the stator resistance per phase so that one can reduce the resistive voltage drop from the measured terminal voltage to obtain the line-to-line stator flux linkage. The method is clearly explained in [68]. Two application examples of this method can be found in [69] and [70].

Some disadvantages of the method must be mentioned here. Being an inverter fed machine, input voltage of a PMSM is necessarily (if not line-start type) a switched voltage with a lot of high frequency switching noise in it. To measure and use this information in an algorithm, a lot of filtering (analog in many cases) has to be done as shown in [68]. In addition to this, the use of stator resistance term in the calculation can induce errors as the resistance may change considerably with the operating temperature.
(b) Position information based on the hypothetical rotor position

This method is based on the idea that any difference between the actual rotor position and a hypothetical rotor position must reflect as a deviation between the detected and estimated states of the system. Applied voltage to the motor in this case is determined by the controller based on a hypothetical rotor position, which may not necessarily be the same as actual rotor position. The ideal applied voltage is calculated using the instantaneous voltage equation of the motor and the measured currents. Now the error between these voltages can be linearly related to the error between actual and hypothetical rotor positions. Self-synchronisation is possible by reducing this angle difference to zero. This can be done by the following procedure.

1. Measure the three-phase voltages and convert them to the estimated synchronous reference frame using hypothetical rotor position. These are the actual $d$-$q$ components of the applied voltage.

2. Use the measured currents and the motor model to determine the hypothetical $d$-$q$ components of the applied voltage.

3. The difference between the $d$-axis actual and hypothetical voltage components is now proportional to the error between the actual and hypothetical rotor positions. This can be eliminated by making the angle error zero, which is practically achieved by changing the rotor speed.

The method again is described in [68]. This has successfully been applied in [71]. The fact that a complete machine model is used in the sensorless control algorithm implies several possibilities of erroneous estimations. The main reasons are machine parameter variations due to frequency and temperature. The need for terminal voltage measurement can also cause problems similar to those mentioned under the first method.

(c) Sensorless operation based on Kalman filtering

Since the model of the machine is known to a certain degree of accuracy, Kalman filter approach is also feasible. As well known, it is an optimal state estimation method. The measured voltages and currents can be transformed into stationery reference frame ($\alpha$-$\beta$) and using the state equations of the machine together with a Kalman filter, unknown states (rotor position and speed) of the system can be estimated. The estimation is constantly corrected by an additional term based on the measurements. The method is computationally heavy and parameter sensitive. An application example can be found in [72].

(d) Position estimation based on state observers

Estimation methods based on state observers can be found very often in the literature. Just like in the case of Kalman filter approach, a machine model based on the estimated parameters is employed in this case also. In many cases it is the PMSM machine model described in Equation (2.23) in state space form that is used [73, 74]. Since the machine parameters can vary with frequency, temperature and ageing, some algorithms use on-line parameter estimation schemes to update the machine parameter values of the observer model [75]. Due to the cross-coupling of the machine model in synchronous reference frame, observer structure obviously becomes non-linear and the design becomes complex. However, since some states of the system like $d$-axis and $q$-axis currents are measurable, the observer order can be reduced to obtain simpler structures such as in [76].
However, most of the methods are very much machine parameter sensitive as the observer is based on the complete machine model. They could also be computationally heavy.

(e) Estimation based on inductance variation of the machine (injection methods)

Most of the above methods are based on the fact that the back emf information due to the rotor movement is available. In fact the back emf is the driving force or the term that carries the rotor position information in many of these methods. Thus they need some initial rotor movement for the algorithms to operate. Hence most of them fail at near zero speeds and are not suitable for position servo applications. This has motivated some researchers to look for other physical quantities that may vary with the geometrical location of the rotor relative to the stator winding. Sensing the inductance variation of the stator is one such method that can also work at zero speed. In many of these methods, some high frequency carrier signal or any other form is injected into the stator winding together with the stator currents [77].

This type of techniques can be specially applied for PMSMs with buried magnets as their winding inductance vary as the rotor rotates in the air gap. This is due to the difference in reactance along $d$-axis and $q$-axis of this type of rotors. Therefore, the variation of self-inductance with the rotor position can be determined by injecting a variable frequency sinusoidal signal into one winding of the machine and measuring the terminal voltage and current. These methods are also referred to as injection methods.

The well known INdirect Flux detection by On-line Reactance Measurement (INFORM) method suggested in [78] and further developed in [79] can also be put into this category. One drawback of such methods is the difficulty of using them with relatively low switching frequencies of the inverters, because of the inability to generate high frequency injection signal. However, injection methods are superior to methods depending on back emf in the near zero speed range and at standstill.

In addition to these standard methods, some artificial intelligent techniques such as fuzzy logic and neural network based controllers can also be found in recent literature [80]. Whatever the method used, it should be clear from the description in this section that there are some basic practical problems that must be successfully dealt with. These problems will be summarised in the following section.

7.1.2 Practical problems in estimation methods - a summary

It is now possible to summarise all practical difficulties that may arise against a successful implementation of an estimation strategy. Some of these were pointed out in the previous section, when the estimation methods were classified.

Obtaining input line voltages to the motor as an information is always difficult because of the switched nature of the waveforms. Even if digital control is essentially used here, analog pre-filtering is required in order to remove the high frequency switching noise and extract the fundamental sinusoidal wave out of it. Any estimation algorithm requiring input voltages will have this problem in the implementation stage.

Methods using an exact motor model (observer and Kalman filter methods) will always have higher computation overhead and sometimes will also require matrix inverse computations [79]. This will put higher demands on the digital control system used.
Machine parameter dependency of the estimation accuracy is the biggest problem in these kind of sensorless control applications. Machine parameters such as $d$-$q$ inductance and resistance vary considerably with stator temperature and the supply frequency. In addition, parameters like rotor inertia and damping will always depend on the load driven by the motor. This can be a significant problem for observer methods incorporating the equation of the mechanical dynamics of the machine.

Under these circumstances, any estimation algorithm having the following properties is preferable for the purpose of sensorless control.

- Algorithm does not require input line voltage information to the motor
- Less computational overhead
- Very low machine parameter dependency

The estimation algorithm presented in this part of the thesis becomes special in this respect.

### 7.2 Voltage error based estimation

In many observer-based methods, it is the error between the sampled and reconstructed currents that is used for feedback stabilisation of the observer. In the method presented in this work, it is the $d$-axis voltage error, which is used as the driving signal of the adaptive observer mechanism. This basic method was first reported in [6]. In this work, first the adaptive observer will be further investigated for its non-linear stability and performance. Then some methods to improve the performance will also be suggested. Later in the chapter, a new algorithm based on the same concept will be presented with the capability of tracking speed ramps (“Ramp tracking algorithm”). The non-linear stability and performance of the new algorithm will also be investigated. In the first part of the next section, the formulation of the adaptive observer based on the $d$-axis voltage error will be presented.

#### 7.2.1 Basics of observer formulation - a practical insight

A more theoretical approach to the adaptive observer formulation can be found in [6]. A somewhat practical approach that will give a better physical insight into the rotor position estimation problem will be presented here. The whole concept is based on the fact that there exists two synchronous frames in this type of a sensorless machine control application. One is the actual synchronous frame found in the machine (this will be called as actual $dq$ frame). The other is the hypothetical synchronous frame found from the sensorless control algorithm (this will be called as estimated $dq$ frame - $d'q'$). Whether these two coincide with each other depends on the availability of the actual rotor position. If direct rotor position sensing is not used, then the control scheme is said to be sensorless as explained earlier. The whole sensorless control problem can then be looked upon as an attempt of bringing these two axes systems to coincide with each other. The situation, when the two systems are not coinciding (in fact, the estimated frame $d'q'$ is assumed to be lagging in this case) is graphically depicted in Figure 7.1.

#### 7.2.2 Sign convention
Before moving into the derivation of the adaptive observer, it is important to establish the sign convention used in this derivation. If \( a_i \) is any physical time varying machine parameter of interest, the following conventions will be used in the derivations.

\[
\begin{align*}
  a_i^0 & \quad \text{a true machine parameter} \\
  a_i & \quad \text{a fixed model parameter} \\
  \hat{a}_i & \quad \text{an estimated machine parameter} \\
  a_i & \quad \text{estimation error of a machine parameter defined as } a_i = a_i^0 - \hat{a}_i
\end{align*}
\]

The following example will be helpful for the better understanding of the convention.

**Example:**

Actual rotor speed at a given instance is denoted as \( \omega^0 \). Accordingly, the estimated speed of the rotor is \( \omega \) and the speed estimation error will be \( \omega = \omega^0 - \hat{\omega} \). The basics of the adaptive observer formulation will be presented in the next section.

---

**Figure 7.1** Actual and estimated dq frames

### 7.2.3 Adaptive observer formulation

A better theoretical approach in formulating and justifying this way of treating the speed and position estimation problem can be found in [6]. The observer formulation presented here will have much more insight into what really is happening to the interaction between the hypothetical synchronous frame incorporated by the sensorless control algorithm and the actual synchronous frame that can be located in the real machine as was mentioned in the beginning.

Since both, the speed and rotor position measurements are required for the speed control of PMSM, any sensorless control method must be capable of giving out accurate estimates of those two quantities. The rotor position is the integral of rotor speed. This makes the estimation a difficult task with non-linearities coming into the estimation algorithm. The basic estimation problem for the rotor speed and position can be formulated as

\[
\begin{align*}
  \dot{\omega}(t) & = \gamma_1(t) \Phi_1(t) \kappa(t) \\
  \dot{\phi}(t) & = \omega(t) + \gamma_2(t) \Phi_2(t) \kappa(t)
\end{align*}
\]

(7.1)
where $\Phi_1(t)$ and $\Phi_2(t)$ are the negative gradients of the error, $\varepsilon(t)$ that drives this adaptation mechanism. In the equations, $\gamma_1$ and $\gamma_2$ are two adaptation gains.

The problem now is to select an error quantity that can be measured from the machine, which can drive this adaptation mechanism. It must be such that the error should converge to zero as the estimated synchronous reference frame coincides with the actual synchronous reference frame. The so-called physical insight into this synchronising problem becomes important at this stage.

Since the voltage is the rate of change of flux linkage, when a PMSM is rotated with a speed of $\omega^0$, a voltage of the magnitude $\omega^0 L_d \Psi_m$ ($\Psi_m$ is the flux due to permanent magnets) will be induced along the actual $q$-axis due to permanent magnet flux. If the machine is run sensorless with an angle error between the actual $dq$ frame in the real machine and the estimated $d'q'$ frame in the control processor, this back emf term will appear as two components induced along $d'$-axis and $q'$-axis. In Figure 7.1 this scenario is clearly illustrated together with the sign convention (i.e. The rotational direction of the rotor is anticlockwise. The estimated $d'q'$ frame is lagging the actual $dq$ frame, thereby giving a negative induced voltage along the $d$-axis). In fact, this happens since the coordinate transformation in the control algorithm is done using an erroneous angle information as the rotor position.

This lead to an error voltage vector (due to wrongly oriented back emf with respect to estimated $d'q'$ frame) reflected in the estimated $d'q'$ frame, when the machine is run with erroneous angle estimation [6]. By referring to Figure 7.1, this can be expressed as

$$
\varepsilon = 
\begin{bmatrix}
\varepsilon_d \\
\varepsilon_q
\end{bmatrix} = L_d \Psi_m
\begin{bmatrix}
-\omega^0 \sin \tilde{\Theta} \\
\omega^0 \cos \tilde{\Theta} - \omega
\end{bmatrix}.
$$

This error vector, which will be referred to as “voltage error vector” in this thesis must be investigated to understand its behaviour with the angle estimation error $\tilde{\Theta}$. The $d$-axis voltage error ($\varepsilon_d$) converges to zero as $\tilde{\Theta}$ reaches zero (this means that the estimation reaches the actual rotor position). Since $\sin \tilde{\Theta}$ can be approximated by $\tilde{\Theta}$ for very small angles, $\varepsilon_d$ will not lose the information regarding rotor position estimation error and its sign (since $\sin \tilde{\Theta}$ changes sign with the zero crossing) as the two axes systems coincide. In contrast, $\cos \tilde{\Theta}$ will converge to 1 and is insensitive to zero crossing of $\tilde{\Theta}$. Even though $\varepsilon_q$ will not lose information regarding rotor position estimation error as the two axis systems coincide, the sensitivity of the angle error will be very low. The next step is to utilize the error voltage vector as the actuating signal for the adaptive observer in an efficient way.

The adaptive mechanism used here is also a gradient search method [81]. Unlike in Model Reference Adaptive Systems (MRAS), the appropriate error quantity and the direction vectors for the gradient search procedure in this case are chosen by inspection [82]. This is done by taking into account the understanding available on the variation of the error quantity as explained before. In the error vector above, the first element converges to zero as $\tilde{\Theta}$ goes to zero and it carries the rotor position estimation error information (including the sign) that must be extracted. On the other hand, the second element will also converge to $0$ as $\tilde{\Theta}$ goes to zero, while being insensitive to sign (being a cosine term it will not carry the information about the sign of the angle error). In addition it has very low sensitivity to the position estimation error information due to this cosine variation. Therefore, a row gradient vector can be constructed as
Sensorless control strategy

\( \Phi_1 = \Phi_2 = [-1 \ 0] \) \hspace{1cm} (7.3)

which then lead to

\[ \Phi_1 \dot{\xi}(t) = \Phi_2 \dot{\xi}(t) = \omega^0 L_d \Psi_m \sin \tilde{\theta}. \] \hspace{1cm} (7.4)

This would result in the non-linear observer for the speed and position estimation described as

\[ \dot{\omega} = \gamma_1 \omega^0 L_d \Psi_m \sin \tilde{\theta}, \]

\[ \dot{\theta} = \dot{\omega} + \gamma_2 \omega^0 L_d \Psi_m \sin \tilde{\theta}. \] \hspace{1cm} (7.5)

Determination of the adaptation gains \( \gamma_1 \) and \( \gamma_2 \) must be done in such a way that the non-linear observer is stable. This requires following of proper design procedure. The term \( \tilde{\theta} \) here is not a directly known quantity. However, a term proportional to \( \sin \tilde{\theta} \) can be computed from the available information, which will be explained later. Thus, for the analysis and design of the observer (7.5) will be used.

7.2.4 Design considerations

With the adaptive mechanism discussed above, \( \dot{\omega} \) and \( \dot{\theta} \) are expected to converge to \( \omega^0 \) and \( \theta^0 \) respectively. A linearization process can be done here by making the substitution \( \tilde{a}_i = a_i^0 - \dot{a}_i \), which leads to

\[ \omega^0 - \tilde{\omega} = \gamma_1 \omega^0 L_d \Psi_m \sin \tilde{\theta}, \]

\[ \theta^0 - \dot{\theta} = \dot{\omega} + \gamma_2 \omega^0 L_d \Psi_m \sin \tilde{\theta}. \] \hspace{1cm} (7.6)

At this point, the assumption that \( \omega^0 \) has a quasi steady time variation compared to the estimation error dynamics is made (this in fact is a standard assumption made in adaptive control). Thus \( \omega^0 \) is treated as a constant and with the additional relationship \( \theta^0 = \omega^0 \) the observer can be rewritten in terms of speed and position estimation errors described as

\[ \dot{\tilde{\omega}} = -\gamma_1 \omega^0 L_d \Psi_m \sin \tilde{\theta}, \]

\[ \dot{\tilde{\theta}} = \dot{\omega} - \gamma_2 \omega^0 L_d \Psi_m \sin \tilde{\theta}. \] \hspace{1cm} (7.7)

With the approximation \( \sin \tilde{\theta} = \tilde{\theta} \) as \( \tilde{\theta} \to 0 \) and the substitutions

\[ \gamma_1' = \gamma_1 \omega^0 L_d \Psi_m, \]

\[ \gamma_2' = \gamma_2 \omega^0 L_d \Psi_m, \] \hspace{1cm} (7.8)

a linearized observer can be obtained given by

\[ \begin{bmatrix} \dot{\tilde{\omega}} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} 0 & -\gamma_1' \\ 1 & -\gamma_2' \end{bmatrix} \begin{bmatrix} \tilde{\omega} \\ \tilde{\theta} \end{bmatrix}. \] \hspace{1cm} (7.9)

The characteristic polynomial of this system given by

\[ p(\lambda) = \begin{vmatrix} \lambda & \gamma_1' \\ -1 & \lambda + \gamma_2' \end{vmatrix} = \lambda^2 + \gamma_2' \lambda + \gamma_1' \] \hspace{1cm} (7.10)
appears to give rise to an asymptotically stable system, if \( \{y_1', y_2'\} \neq 0 \). The adaptation gains can now be selected such that this linearised observer is stable. A simple pole placement design can be done to fix the adaptation gains \( \{y_1', y_2'\} \), based on the characteristic equation obtained earlier [43]. Double poles at \( s = -\rho \) is a reasonable design [6]. In fact, this pole placement will result in an observer of closed loop bandwidth \( \rho \), which would lead to

\[
\begin{align*}
\gamma_1' &= \rho^2 \rightarrow \gamma_1 = \frac{\rho^2}{\omega^0 L_d \Psi_m} \\
\gamma_2' &= 2\rho \rightarrow \gamma_2 = \frac{2\rho}{\omega^0 L_d \Psi_m}.
\end{align*}
\]  

(7.11)

This way of linearised design is a valid method for many non-linear applications. However, the validity or the effective region of stability for this design must be investigated using non-linear analytical tools. This will be done later in the chapter.

By inspecting (7.11) it can be seen that the rotor speed information is required in the adaptation gain calculation. Since there is no direct measurement available, one has to use the estimated speed or some filtered version of it in place of \( \omega^0 \). Since the speed term is located in the denominator of the adaptation gains, this method will not perform well in the low speed region (cause very high adaptation gain values). Yet it is possible to use a different method (“Start-up technique”) to obtain the initial rotor movement and later hand over the control of the PMSM to the adaptation algorithm provided that it is activated with a suitable set of initial conditions. This will be discussed later.

### 7.2.5 Filtered speed for adaptation gain calculation

As mentioned before, adaptation gain calculation needs the rotor speed information \( \omega^0 \) according to (7.11). A first order low-pass filter can be used to filter the estimated speed output from the observer and the filtered speed can be used in place of \( \omega^0 \) in the adaptation gain calculation [6]. If the filtered speed is denoted as \( \omega_f \), the \( s \) domain expression for the filter can be given as

\[
\omega_f(s) = \frac{k_f \alpha_s}{s + k_f \alpha_s} \omega(s).
\]  

(7.12)

The term \( k_f \alpha_s \) is the bandwidth. The filter factor \( k_f \) can be couple of times (3 to 4) the closed loop speed controller bandwidth \( \alpha_s \). The need for this sort of filtering is due to the possibility of noise being present in the estimated speed. Since the sampled line currents are going to be used in \textit{d-axis} voltage error calculation, as it will be shown later in the chapter, the observer outputs (\( \hat{\omega} \) and \( \hat{\Theta} \)) can be noisy.

### 7.2.6 Basic observer implementation

Now it is possible to present the complete observer equations in continuous domain in the form of differential equations. This can be done by substituting the corresponding values in (7.1) and augmenting the filter equation into the system. Thus, the complete adaptive observer will be
\[ \dot{\theta} = -\frac{p^2}{\omega_j \hat{L}_d \hat{\Psi}_m} e_d \]
\[ \dot{\hat{\theta}} = \hat{\omega} - \frac{2p}{\omega_j \hat{L}_d \hat{\Psi}_m} e_d \]  
\[ \omega_y = k_j \alpha_y (\hat{\omega} - \omega_j) \]  

Since the machine parameter values that must be used here are estimated quantities, they are denoted as \( \hat{L}_d \) and \( \hat{\Psi}_m \). Discrete-time implementation of this will be discussed under the implementation issues later in the chapter.

### 7.2.7 d-axis voltage error calculation

The immediate question that may arise is the method of computation of \( d\)-axis voltage error \( e_d \). This is done based on the assumption that the inverter is ideal and the command voltages determined by the current controller outputs of the vector controller are exactly applied in the machine as expected. Deriving the equation for the computation of \( e_d \) can be done starting from the voltage equation for \( d\)-axis motor current found in (2.21). The basic idea of this computation is to check whether \( d\)-axis voltage equation holds in the estimated \( d'q' \) frame. However, some simplifications are also done to reduce the computation effort. The \( d\)-axis voltage equation written in \( d'q' \) frame will be

\[ u'_d = \hat{R}_d i'_d + \hat{L}_d \frac{di'_d}{dt} - \hat{\omega}\hat{L}_q i'_q, \]  

where \( a'_i \) \( (i = d, q) \) refers to a quantity in estimated \( d'q' \) frame and \( \hat{R}_d, \hat{L}_d \) and \( \hat{L}_q \) here are estimated machine parameters. If there is no error between the estimated and actual rotor positions (7.14) must be valid even in the estimated frame, when computed from the sampled line currents and estimated machine parameters. When there is an angle error and the two axis systems are not coinciding, the back emf term, which is aligned with \( q'\)-axis under ideal conditions will be resolved on to both \( d'\) and \( q'\) axes as shown in Figure 7.1. This will in fact make an imbalance between the left and right hand sides of (7.14) and the deficit will be equal in magnitude to the back emf component now oriented along negative \( d'\)-axis as shown in Figure 7.1. Thus defining \( e_d \) as

\[ e_d = -\left( u'_d - \hat{R}_d i'_d - \hat{L}_d \frac{di'_d}{dt} + \hat{\omega}\hat{L}_q i'_q \right) \]  

is the most appropriate way of \( e_d \) computation. However, since the current controller assures fast closed loop current dynamics, the sampled currents may be assumed quasi steady and the derivative term can be neglected. In fact attempting to use the derivative term may cause serious problems due to noise in the sampled line currents. This results in a more simplified form of \( e_d \) computation given by

\[ e_d = -\left( u'_d - \hat{R}_d i'_d + \hat{\omega}\hat{L}_q i'_q \right). \]  

Inspecting this equation, it can be seen that the speed information is also required. In a discrete-time implementation, the speed estimation computed in the previous computation cycle is used for this calculation. This will also be discussed under implementation issues. Yet another simplification that
can be done is to use the reference values of the $d'$-axis and $q'$-axis currents, again assuming fast current control. This assumption will reduce the effect of noise also on the estimation, as the noise in the sampled line currents will not be directly affecting the $\varepsilon_d$ computation any longer, when this assumption is used. The modified equation will be

$$
\varepsilon_d = -\left( u'_d - \hat{R}_i i'_d + \hat{\omega} L_q i'_{qref} \right). 
$$

One must thoroughly keep in mind the fact that the inverter, which is not an ideal device, is assumed to be ideal in this computation as was mentioned earlier also. Effect of non-ideal inverter can be a significant problem for the accuracy of the speed and rotor position estimation and how to compensate for the non-ideal behaviour of the inverter will be the topic of the next chapter.

### 7.2.8 Parameter dependency, computation overhead and operating speed range

By inspecting Equations (7.13) together with (7.16) or (7.17), one can get information regarding the machine parameter values required for the implementation of the adaptive observer. They are $\psi_m$, $R_s$, $L_d$ and $L_q$. All these quantities must be accurately estimated from the machine using suitable tests and manufacturer’s data. However, these parameters (mostly $R_s$) can vary considerably with temperature and frequency. Compensating for the temperature variations can be comparatively easier than frequency dependent variation. The main reason is the complicated nature of the variation of machine parameters with frequency. Some work on this aspect will be presented in Chapter 9 of the thesis.

As for the computation overhead, the whole estimation algorithm only requires the implementation of Equation (7.16) or (7.17) together with Equation (7.13). Thus the method has less demand on computation effort compared to other observer-based methods that usually incorporate a complete PMSM model [73, 75, 76].

Some comments on the performance of the algorithm over the full operating speed range of interest will now be given. Experimental implementation has revealed that the algorithm does not produce accurate information below 10% (approximately) of the base speed. In addition, it totally fails and becomes unstable at zero speed or standstill.

#### (a) Mathematical explanation

This can be mathematically explained with the help of Equation (7.11), which describes the adaptation gain computation. As was mentioned earlier also, the $\omega^2$ term in the denominator of the equations for adaptation gain computations will cause them to reach very high values at low speeds and in fact they will become infinity at zero speed. Therefore, at low speeds the observer will be more sensitive to noise content in the computed $\varepsilon_d$.

#### (b) Physical explanation

An explanation that gives a better physical insight into this phenomenon will now be given. It is now clear that the rotor position information coming into this algorithm is based on the resolved component of back emf of the PMSM on to the $d'$-axis, when there is an estimation error in rotor position. At low speeds, this back emf component is low and still the sampled line currents may have the same amount of measurement noise. Thus, the signal-to-noise ratio (SNR) is lower at low speeds. At higher speeds for the same amount of estimation error, the back emf component will be larger resulting in a higher SNR. This explains the problem physically. The more important factor
here is the possibility of the algorithm to perform better and better as the operating speed increases (SNR goes up). This makes the algorithm a promising candidate for high-speed applications.

Another important point is the speed reversal of the machine with this algorithm. The algorithm becomes unstable and cannot therefore handle the zero crossing of the speed. Therefore, the algorithm must be disabled as the speed drops closer to 10% of the base speed and the machine must be decelerated open loop (slowly rotated q-axis current may be used) afterwards. Once the machine is forced to cross the zero speed and it is accelerated up to about 10% of the base speed in the opposite direction, the algorithm can be re-started with a different set of initial conditions corresponding to its new rotational direction. Careful inspection of the design will reveal the fact that the pole placement design of the observer will be still valid (stable) as long as the algorithm is activated with the correct set of initial conditions for the speed and rotor position with the same sign.

7.3 Non-linear stability analysis

The observer design so far was based on the linearised design done earlier, which guarantees the asymptotic stability. However, the start-up and also the speed reversal of the machine running with this sensorless algorithm need it to be activated with a suitable set of initial conditions for the estimated rotor speed and position. This raises a critical question about the capability of the algorithm to converge to the actual rotor speed and position starting from the user defined set of initial conditions. That also addresses the degree of uncertainty that the user can have on the actual rotor speed and position by the time the algorithm is activated. This situation highly motivates the need of carrying out a complete non-linear analysis on the already designed non-linear observer in order to get a better insight into the converging capabilities. Main focus of this section will be the non-linear analysis.

7.3.1 Liapunov stability

The stability of this non-linear system described by (7.7) (with the substitution in (7.8)) can be proved by considering the following Liapunov function [6, 105].

\[
V(\omega, \theta) = \frac{1}{2} \omega^2 + \gamma_1 \sin \eta \eta + \frac{1}{2} \omega^2 + \gamma_1 \left(1 - \cos \theta \right) \quad (7.18)
\]

At the fixed points \{\omega, \theta\} = \{0, 2n\pi\}, \ n = 0, \pm 1, \pm 2,... of the non-linear system, \ V(0, 2n\pi) = 0 and, \ V(\omega, \theta) = 0, \forall \{\omega \neq 0, \theta \neq 2n\pi\}.

In addition, the time derivative of \ V will be

\[
\dot{V} = \frac{\partial V}{\partial \omega} \dot{\omega} + \frac{\partial V}{\partial \theta} \dot{\theta} = \omega \dot{\theta} + \gamma_1 \sin \theta \dot{\theta} = \omega (-\gamma_1 \sin \theta \dot{\theta}) + \gamma_1 \sin \theta (\omega - \gamma_1 \sin \theta) \quad (7.19)
\]

So the Liapunov stability criterion is satisfied by \ V provided that \ \{\gamma_1, \gamma_2\} \neq 0. Since the linearised design guarantees this condition, it can be concluded that the non-linear observer is stable at the required fixed points, when it is designed using the linearised system. However, the exact fixed point...
to which the solution will reach once the algorithm is activated with a certain set of initial conditions is not yet known. Further analysis is required for this purpose.

### 7.3.2 Fixed points and eigen vectors

With the double poles placed at \( \lambda = -\rho \) the non-linear observer for the the speed and position estimation errors that can be derived from (7.7) will be of the form

\[
\tilde{\omega} = -\rho \sin \tilde{\theta} \\
\tilde{\theta} = \tilde{\omega} - 2\rho \sin \tilde{\theta}.
\]  

(7.20)

With the form, \( \dot{x} = f(x) \) and the Taylor series expansion around the equilibrium point \( x^* \), found as the solution for \( f(x) = 0 \) will give the linearized version of the system as, \( \Delta \dot{x} = F(x^*) \Delta x \), where

\[
F(x^*) = \left[ \frac{\partial f(x)}{\partial x} \right]_{x=x^*}.
\]

(7.21)

Corresponding to the fixed points \( \{\tilde{\omega}, \tilde{\theta}\} = \{(2m+1)\pi, 0, 2m\pi\}, m = 0, \pm 1, \pm 2, \ldots \), the system in Equation (7.20) will result in

\[
F(\tilde{\omega}^{(n)}, \tilde{\theta}^{(n)}) = \begin{bmatrix}
0 & -\rho^2 \cos \tilde{\theta} \\
1 & -2\rho \cos \tilde{\theta}
\end{bmatrix}_{(2m+1)\pi} = \begin{bmatrix}
0 & -\rho^2 \\
1 & 2\rho
\end{bmatrix}
\]

and

\[
det(\lambda - F(\tilde{\omega}^{(n)}, \tilde{\theta}^{(n)})) = \begin{vmatrix}
\lambda & -\rho^2 \\
-1 & \lambda - 2\rho
\end{vmatrix} = \lambda^2 - 2\rho \lambda - \rho^2 
\]

(7.22)

\[
\lambda = \rho(1 \pm \sqrt{2})
\]

and

\[
F(\tilde{\omega}^{(2)}, \tilde{\theta}^{(2)}) = \begin{bmatrix}
0 & -\rho^2 \cos \tilde{\theta} \\
1 & -2\rho \cos \tilde{\theta}
\end{bmatrix}_{0, 2m\pi} = \begin{bmatrix}
0 & -\rho^2 \\
1 & -2\rho
\end{bmatrix}
\]

\[
det(\lambda - F(\tilde{\omega}^{(2)}, \tilde{\theta}^{(2)})) = \begin{vmatrix}
\lambda & \rho^2 \\
-1 & \lambda + 2\rho
\end{vmatrix} = \lambda^2 + 2\rho \lambda + \rho^2 
\]

(7.23)

\[
\lambda = -\rho, -\rho.
\]

Since the eigen values are of opposite signs, the fixed point in (7.22) is a saddle point and the corresponding eigen vectors \( \{v_1, v_2\} \) are found to be

\[
(\lambda_1, J - F(\tilde{\omega}^{(n)}, \tilde{\theta}^{(n)}))\begin{bmatrix}
\tilde{\omega} \\
\tilde{\theta}
\end{bmatrix} = \begin{bmatrix}
\rho(1 + \sqrt{2}) & -\rho^2 \\
-1 & \rho(1 + \sqrt{2}) - 2\rho
\end{bmatrix}\begin{bmatrix}
\tilde{\omega} \\
\tilde{\theta}
\end{bmatrix} = 0
\]

(7.24)

\[
v_1 = \begin{bmatrix}
\frac{\rho(\sqrt{2} - 1)}{\sqrt{\rho^2(\sqrt{2} - 1)^2 + 1}} & \frac{1}{\sqrt{\rho^2(\sqrt{2} - 1)^2 + 1}}
\end{bmatrix}
\]

and since \( v_1^T \cdot v_2^* = 0 \) (orthogonality condition),
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\[
\mathbf{v}_2^{*1} = \begin{bmatrix} -1 & \frac{\rho(\sqrt{2}-1)}{\sqrt{\rho^2(\sqrt{2}-1)^2+1}} \\ \sqrt{\rho^2(\sqrt{2}-1)^2+1} & -\frac{1}{\sqrt{\rho^2+1}} \end{bmatrix}^T.
\] (7.25)

Then the stable manifold in the vicinity of \( \{\hat{\omega},\hat{\theta}\} = \{0,(2m+1)\pi\} \) will be tangent to \( \mathbf{v}_2^{*1} \) (eigen vector corresponding to negative eigen value) and the unstable manifold will be tangent to \( \mathbf{v}_1^{*1} \). In contrast, the fixed point given by (7.23) is a sink, of which the eigen vectors, \( \{\mathbf{v}_1^{*2},\mathbf{v}_2^{*2}\} \) are

\[
(\lambda_2 I - F(\hat{\omega}_2,\hat{\theta}_2))\begin{bmatrix} \hat{\omega} \\ \hat{\theta} \end{bmatrix} = \begin{bmatrix} -\rho & \rho^2 \\ -1 & -\rho + 2\rho \end{bmatrix} \begin{bmatrix} \hat{\omega} \\ \hat{\theta} \end{bmatrix} = 0
\]

\[
\mathbf{v}_2^{*2} = \begin{bmatrix} \frac{\rho}{\sqrt{\rho^2+1}} \\ \frac{1}{\sqrt{\rho^2+1}} \end{bmatrix}^T
\] (7.26)

and

\[
\mathbf{v}_2^{*2} = \begin{bmatrix} -1 & \frac{\rho}{\sqrt{\rho^2+1}} \\ \sqrt{\rho^2+1} & \frac{1}{\sqrt{\rho^2+1}} \end{bmatrix}^T
\] (7.27)

This enables drawing of the phase portrait, which can be used to study the non-linear flows of the system.

7.3.3 Phase portrait

Now by numerically solving the set of differential equations in (7.20), phase portrait of this non-linear system can be drawn, which will give an idea on the convergence of the algorithm starting from a certain set of initial conditions. This is shown in Figure 7.2.

\textbf{Figure 7.2: Phase portrait of the non-linear observer}
With this analysis it is clear that for properly selected adaptation gains, the algorithm will definitely converge to correct speed but not the correct rotor position (if \( \tilde{\theta} = 2m\pi, \ m = \pm 1, \pm 2, \ldots \)), the estimated angle will either be lagging or leading by an integer multiple of an electrical cycle. Even though the machine can ideally operate with this kind of a situation in steady state (since a correct estimation of speed is available with the synchronisation to the rotor position), the following unfavourable situations can occur in the drive system.

1. Since the applied voltage vector is not synchronously rotated with the rotor, the electromagnetic torque produced will drop, forcing the speed controller to increase the \( q \)-axis current. This could result in saturation of the inverter causing instabilities.

2. Unsynchronised rotation of the voltage vector will cause high torque ripples and generate jerks in the machine. Unfavourable acoustic noise could also result in.

3. Since the algorithm is not valid for zero rotor speed the algorithm has to be activated after the rotor has started to rotate and any mismatch in the initial conditions of the estimated speed and position can cause instabilities right at the beginning.

This means that convergence of the algorithm to any stable point other than \( \{\tilde{\omega}, \tilde{\theta}\} = \{0, 0\} \) is not advisable. Therefore, the safest situation as far as the overall stability of the drive system is concerned is to activate the adaptation algorithm from a set of initial conditions for the estimated speed and position (they are related to \( \{\tilde{\omega}, \tilde{\theta}\} \)) so that the estimation errors will converge to \( \{\tilde{\omega}, \tilde{\theta}\} = \{0, 0\} \) (i.e.: the sink at the origin).

These reasons motivate further analysis of the non-linear system to find out the basin of attraction to each sink located at \( \{\tilde{\omega}, \tilde{\theta}\} = \{0, 2m\pi\}, \ m = 0, \pm 1, \pm 2, \ldots \) This will give the information on the maximum allowable speed and position estimation error before the machine control is taken over by the adaptive algorithm. This is discussed in the next section.

### 7.3.4 Basin of attraction

The basin of attraction of a sink \( \hat{x} \) is the union of all solution curves that converges to \( \hat{x} \). If the stability boundary is considered, obviously there can not be any other equilibrium points that are sinks on it. Therefore, the stability boundary is the union of the stable manifolds of the unstable equilibrium points on the stability boundary. The Liapunov function approach can be used to estimate the basin of attraction. The same Liapunov function used to prove the stability of this algorithm earlier can be used with the designed values for the adaptation gains (Equation (7.18) with \( \gamma = \rho^2 \)). This yields

\[
V = \frac{1}{2} \tilde{\omega}^2 + \rho^2 (1 - \cos \tilde{\theta}). \tag{7.28}
\]

This is graphically shown in Figure 7.3 and since

\[
\text{grad}(V) = \begin{bmatrix} \tilde{\omega} & \rho^2 \sin \tilde{\theta} \end{bmatrix}
\]

the gradient system defined as

\[
\dot{x} = -\text{grad}(V) \tag{7.30}
\]
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has fixed points at \( \chi^* = \{ \tilde{\omega}, \tilde{\Theta} \} = \{ 0, (2m+1)\pi \}, \{ 0, 2m\pi \}, \ m = 0, \pm 1, \pm 2, \ldots \).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7_3.png}
\caption{Liapunov function surface}
\end{figure}

The Hessian matrix of \( V \) evaluated at \( \chi^* \), is

\[
H(\chi^*) = \left[ \frac{\partial^2 V}{\partial x^2} \right]_{\chi^*} = \left[ \frac{\partial (\text{grad}(V))}{\partial x} \right]_{\chi^*} = \begin{bmatrix} 1 & 0 \\ 0 & \rho^2 \cos \tilde{\Theta} \end{bmatrix}_{\chi^*}.
\]  

(7.31)

At \( \{ \tilde{\omega}, \tilde{\Theta} \} = \{ 0, (2m+1)\pi \}, \)

\[
H(0, (2m+1)\pi) = \begin{bmatrix} 1 & 0 \\ 0 & -\rho^2 \end{bmatrix}
\]  

(7.32)

and at \( \{ \tilde{\omega}, \tilde{\Theta} \} = \{ 0, 2m\pi \}, \)

\[
H(0, 2m\pi) = \begin{bmatrix} 1 & 0 \\ 0 & \rho^2 \end{bmatrix}
\]  

(7.33)

Since (7.32) is indefinite, \( \{ \tilde{\omega}, \tilde{\Theta} \} = \{ 0, (2m+1)\pi \} \) is a saddle point of the gradient system, while the positive definite (7.33) corresponds to a sink. According to previous argument about the stability boundary, the points \( \{ \tilde{\omega}, \tilde{\Theta} \} = \{ 0, (2m+1)\pi \} \) must be on the stability boundaries of corresponding sinks located at \( \{ \tilde{\omega}, \tilde{\Theta} \} = \{ 0, 2m\pi \}. \) If a level curve \( c \) on the Liapunov surface \( V \) is defined such that

\[
V(\tilde{\omega}, \tilde{\Theta}) = c
\]  

(7.34)

\( \text{grad}(V) \) will always be the outward normal to curve \( c \) at all point on \( c \). Then

\[
\mathbf{\hat{c}} = \text{grad}(V)^\top \mathbf{\hat{c}} = |\text{grad}(V)| \times \mathbf{\hat{c}} \times \cos \varphi < 0,
\]  

(7.35)
since \( W'(x) < 0 \). This gives rises to the constraint \( \pi/2 < \varphi \leq \pi \), which follows that the derivative \( \& \) always directs to the interior of the level curve \( c \) making it impossible for the solution to escape from it. This is depicted in Figure 7.4.

\[
V(x) = c
\]

\[
\theta \quad \text{grad}(v) \rightarrow \mathbf{x}^*
\]

**Figure 7.4:** Graphical interpretation of the basin of attraction

At the limiting case such curves passing through \( \{\ddot{\omega}, \ddot{\varphi}\} = \{0, (2m+1)\pi\} \) can be found giving rise the relationship

\[
V(0, (2m+1)\pi) = \left[ \frac{1}{2} \ddot{\omega}^2 + \rho^2 (l - \cos \ddot{\varphi}) \right]_{(0, (2m+1)\pi)} = 2\rho^2 = c. \tag{7.36}
\]

Then the basin of attraction of each sink is given by the curve described by

\[
\frac{1}{2} \ddot{\omega}^2 + \rho^2 (l - \cos \ddot{\varphi}) = 2\rho^2. \tag{7.37}
\]

**Figure 7.5:** Basin of attraction superimposed on phase portrait

Figure 7.5 shows the basins of attraction superimposed on the phase portrait. This gives a better physical insight into the problem of using unfavourable initial conditions in the adaptation algorithm. It clearly shows that any initial condition within the basin of attraction of the stable point \( \{\ddot{\omega}, \ddot{\varphi}\} = \{0, 0\} \)
by default converges to it. Some initial conditions outside the basin of attraction can deviate to \( \{\overline{\omega}, \overline{\theta}\} = \{0.2\pi\} \), if they are located to the left of the stable manifold of \( \{\overline{\omega}, \overline{\theta}\} = \{0, \pi\} \). With this analysis now there is a clear idea about the allowable estimation errors in the initial state so that the algorithm definitely converges to \( \{\overline{\omega}, \overline{\theta}\} = \{0, 0\} \).

In the next section implementation issues of the proposed adaptation algorithm will be discussed.

### 7.4 Implementation issues

Under this section, the discussion will be started by presenting a graphical representation of the sensorless control structure that will show the exact locations, where to obtain the input signals for the adaptive observer. Per-unit implementation of the observer in simulation level and the real-time implementation will be addressed next.

#### 7.4.1 Complete sensorless control structure

It was earlier mentioned that the adaptive observer implementation needs either Equations (7.13) and (7.16) or Equations (7.13) and (7.17). The information required are the \( d \)-axis command voltage, sampled \( d' \)-axis current and sampled \( q' \)-axis current (will be reference currents, if (7.17) is used. The two outputs from the adaptive observer, rotor speed and position will be used in the speed and current controllers and also in the co-ordinate transformations. To be more precise, speed information is required for the purpose of axis decoupling in the current controller and also in the speed controller. The rotor position information is mainly required for the co-ordinate transformations. The two possibilities of implementation are depicted in Figure 7.6.

![Diagram](image)

**Figure 7.6:** Two implementation possibilities: (a) Reference current based and (b) Measured current based estimations

Discrete-time implementation in the simulation level will be discussed next. For simplicity, the implementation issues of one of the combinations (Equations (7.13) and (7.17)) will be discussed hereafter. Repeating the same procedures for the remaining combination of equations (Equations (7.13) and (7.16)) is straightforward.

#### 7.4.2 Simulation model in per-unit
The discrete-time version of the Equations (7.13) and (7.17) has to be obtained for the implementation purposes. The forward Euler integration rule is used here to accomplish the integration. Discrete set of equations can be given as follows.

(a) **Voltage error computation**

The computation of $\varepsilon_{eq}$ at the beginning of $k^{th}$ sampling period is done according to

$$
\varepsilon_{eq}(k) = -(u'_d(k-1) - \hat{R}_s i'_{dref}(k-1) + \omega(k-1)\hat{L}_q i'_{qref}(k-1))
$$

(7.38)

All values of $u'_d$, $i'_{dref}$, $i'_{qref}$ and $\omega$ must be taken from the previous computation cycle in order to avoid algebraic loops being formed. In the case of Equations (7.13) and (7.16), the values of $i'_d$ and $i'_q$ used will be computed from the sampled line currents and hence will also be delayed by one sampling period.

(b) **Adaptive observer**

The discrete-time observer with forward Euler integration can be given as

$$
\hat{\omega}(k) = \hat{\omega}(k-1) - \frac{\rho^2}{\omega_j (k-1) \hat{L}_d \hat{\Psi}_m} T_s \varepsilon_d (k)
$$

$$
\hat{\theta}(k) = \hat{\theta}(k-1) + T_s \omega_j (k-1) - \frac{2\rho}{\omega_j (k-1) \hat{L}_d \hat{\Psi}_m} T_s \varepsilon_d (k)
$$

(7.39)

$$
\omega_j (k) = \omega_j (k-1) + k_j \alpha_j T_s \omega (k-1) - \omega_j (k-1)
$$

The changes required in terms of scaling, when it comes to real-time implementation will be discussed next.

**7.4.3 Real-time implementation**

In fact, there will not be any modification required in the $d$-axis voltage error computation described in (7.38) in the real-time implementation. However, in the adaptive observer equations the type of scaling that was done in the current and speed controller implementations has to be done. The modified real-time adaptive observer equations are given by

$$
\hat{\omega}(k) = \hat{\omega}(k-1) - \frac{\rho^2}{\omega_j (k-1) \hat{L}_d \hat{\Psi}_m} T_s \omega_{base} \varepsilon_d (k)
$$

$$
\hat{\theta}(k) = \hat{\theta}(k-1) + T_s \omega_{base} \hat{\omega}(k-1) - \frac{2\rho}{\omega_j (k-1) \hat{L}_d \hat{\Psi}_m} T_s \omega_{base} \varepsilon_d (k)
$$

(7.40)

$$
\omega_j (k) = \omega_j (k-1) + k_j \alpha_j T_s \omega_{base} \omega (k-1) - \omega_j (k-1)
$$

Verification of the adaptive observer performance in the simulation and experimental level will be done in the next section.

**7.5 Simulation and Experimental verification**

Performance of the algorithm can be tested in many ways. The discussion here will be started by presenting simple step-response results obtained from both simulations and experiments. Later the
discussion will progress towards the behaviour of speed and position estimation errors, their convergence properties and the start-up technique of the machine from standstill with the algorithm.

### 7.5.1 Step response tests

The response of speed for a step change in set-point, when the machine is run with the sensorless control algorithm was obtained both by simulation and from the real-time implementation. The simulated results are shown in Figure 7.7 (a). The machine speed is first brought to a value above zero (0.3 pu) using the start-up technique that will be explained later in this section. For comparison purposes, the calculated speed from the simulated machine model has also been plotted on the same graph. The same size of step input was applied to the real system with same controller and estimator tuning values. The result is shown in Figure 7.7 (b). In this figure the actual position measured from the incremental encoder has also been plotted for comparison purpose.

\[(a) \text{ Simulated step response (0.3 – 1.5 pu step applied at 1500 per-unit time)}\]

\[(b) \text{ Experimental step response (0.3 – 1.5 pu step applied at 0 s)}\]

**Figure 7.7:** Step response test results of sensorless drive

\((\alpha = 6, \rho = 0.15, \alpha_s = 0.02 \text{ pu})\)

Even though the step response test result is a standard way of evaluating the system response, the errors in the position and speed estimations during this speed transient give a better physical insight into the performance of the adaptive observer algorithm. This is shown in the next section.

### 7.5.2 Angle and speed estimation error

In this section estimation errors of rotor position and speed obtained from simulation and real-time tests will be presented.

1. **Rotor position estimation error**
Rotor position estimation error from the simulation $\tilde{\Theta}_{\text{sim}}$ is computed according to
\[\tilde{\Theta}_{\text{sim}} = \Theta_{\text{MOD}}^0 - \hat{\Theta}_{\text{ESTS}}, \tag{7.41}\]
where $\Theta_{\text{MOD}}^0$ is the rotor position computed from the simulated machine model and $\hat{\Theta}_{\text{ESTS}}$ is the estimated rotor position obtained from the simulated adaptive observer. The simulated rotor position estimation error variation during the step response test explained in the previous section is shown in Figure 7.8 (a). The same error for the experimental system $\tilde{\Theta}_{\text{ex}}$ is calculated from
\[\tilde{\Theta}_{\text{ex}} = \Theta_{\text{MES}}^0 - \hat{\Theta}_{\text{EST}}, \tag{7.42}\]
where $\Theta_{\text{MES}}^0$ is the rotor position measured from the incremental encoder fitted in the machine and $\hat{\Theta}_{\text{EST}}$ is the estimated rotor position obtained from the implemented adaptive observer. The experimental rotor position estimation error variation during the step response test explained in the previous section is shown in Figure 7.8 (b).

![Figure 7.8: Position estimation error during step change](image)

(a) Simulated rotor position estimation error
(0.3 – 1.5 pu step applied at 1500 per-unit time)

(b) Experimental rotor position estimation error
(0.3 – 1.5 pu step applied at 0 s)

Sudden discontinuities visible in the experimental rotor position estimation error is due to quantisation problem occurring, when the measured and estimated rotor positions are limited to principle values of angle $-\pi \leq \Theta_{\text{MES}}^0, \hat{\Theta}_{\text{EST}} \leq \pi$ in this case.

2. **Speed estimation error**
Similar to computation of rotor position estimation error, speed estimation error $\omega_{sim}$ for the simulations is defined by

$$\omega_{sim} = \omega_{0,MOD} - \omega_{ESTS}, \quad (7.43)$$

where $\omega_{0,MOD}$ is the speed computed from the simulated machine model and $\omega_{ESTS}$ is the estimated speed obtained from the simulated adaptive observer. The simulated speed estimation error variation during the step response test explained in the previous section is shown in Figure 7.9 (a). The same error for the experimental system $\omega_{ex}$ is calculated from

$$\omega_{ex} = \omega_{0,MES} - \omega_{EST}, \quad (7.44)$$

where $\omega_{0,MES}$ is the rotor position measured from the incremental encoder fitted in the machine and $\omega_{EST}$ is the estimated rotor position obtained from the implemented adaptive observer. The experimental rotor position estimation error variation during the step response test explained in the previous section is shown in Figure 7.9 (b).

![Graph of simulated speed estimation error](a) Simulated speed estimation error (0.3 – 1.5 pu step applied at 1500 per-unit time)

![Graph of experimental speed estimation error](b) Experimental speed estimation error (0.3 – 1.5 pu step applied at 0 s)

Figure 7.9: Speed estimation error during step change ($\alpha = 6$, $\rho = 0.15$, $\alpha_c = 0.02$ pu)

The next performance that must be verified is the convergence capability (or the attractivity) of the adaptive observer starting from a certain set of initial conditions for speed and rotor position. This will be presented in the next section.

### 7.5.3 Verification of the region of attraction

From the simulation and experimental results presented in this section, the non-linear analysis that was conducted in order to find out the basin of attraction for the algorithm can be verified. For this
test, the machine speed is first brought up to 0.5 pu using the speed sensor signal. Then the control is switched over to sensorless algorithm. Normally the best sensor-to-sensorless transfer is performed by setting the initial conditions of the adaptive observer to 0.5 pu speed and 0 rad rotor position and making the change over, when the measured rotor position is very closer to 0 rad. This can be called as somewhat smooth sensor-to-sensorless transfer. For the verification of the basin of attraction, different sets of initial conditions are used including the one that has the best possible combination. From the simulations, it is possible to obtain noise free estimation errors so that they can be plotted on \(\hat{\Theta}_{\text{sim}} Vs \hat{\Theta}_{\text{sim}}\) plane to give a better insight into the attractivity region. Such a plot is shown in Figure 7.10.

![Figure 7.10: Verification of attractivity region by simulation](image)

However, the quantization noise problem observed in the experimental rotor position estimation error makes it difficult to present results as a \(\Theta_{\text{ex}} Vs \Theta_{\text{ex}}\) plot. Therefore, the time variations of the estimation error quantities during the convergence process will be presented. For simplicity, either the initial condition for speed or the initial condition for rotor position will be deviated one at a time. Figure 7.11 (a) shows the convergence of estimation errors, when the initial condition of the speed estimation is deviated from operating speed 0.5 pu. The initial condition for the rotor position estimation is kept at zero, which is the change over angle from sensor-sensorless operation. Figure 7.11 (b) shows the convergence of estimation errors, when the initial condition of the rotor position estimation is deviated from zero. In this case initial condition for the speed estimation is kept at 0.5 pu.

The results show the robustness of the algorithm for the errors in the initial conditions. This in fact is the main motivating factor in presenting the completely sensorless start-up technique “Kick-start” in the next section.

### 7.5.4 Start-up method – Kick-start

The method is based on the ability of the algorithm to converge to the correct rotor position and speed from a reasonably large deviation in initial conditions. It was earlier mentioned that the algorithm totally fails at zero speed. Thus, some initial rotor movement in the preferred rotational direction must be initiated before enabling the adaptive mechanism. This can be done by first aligning
the rotor along the estimated $d'$-axis of the control system and then applying a sufficiently large $q$-current pulse. At the falling edge of the $q$-current pulse, the adaptive mechanism is enabled.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig7_11.png}
\caption{(a) Initial condition for speed estimation is deviated from 0.5 (to 0.2, 0.3, 0.4, 0.6, 0.7 and 0.8 pu respectively)}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig7_11.png}
\caption{(b) Initial condition for rotor position estimation is deviated from 0 (to -2.0, -1.5, -1, -0.5, 0.5, 1, 1.5 and 2.0 rad respectively)}
\end{figure}

**Figure 7.11**: Verification of attractivity region by experiments

Application of the $d$-current is done with zero co-ordinate transformation angle. Then the positive $q$-current pulse initiates the rotor movement in the positive direction or in other words “kicks the rotor to start”. Hence the method is named as “Kick-start”. At this point the design engineer must have a rough estimate of the rotor position and the speed at the end of the $q$-current pulse. In fact, these estimates can be used as the initial conditions for the adaptive algorithm. Figure 7.12 shows all-important signal variations experimentally obtained during a kick-start process.
The application of the $d$-current pulse was manually done in this particular case and that is the reason for such long pulse duration. It is the same reason that has made a long delay between the $d$-current pulse and the $q$-current pulse. Procedures after the application of $q$-current pulse are automated. Application of the $d$-current pulse and the following $q$-current pulse that must start at the falling edge of the $d$-current pulse can also be easily linked together to create a fully automated “Kick-start” procedure.

![Graph of experimental results of “Kick-start” procedure](image)

**Figure 7.12: Experimental results of “Kick-start” procedure**

After this basic performance evaluations of the algorithm, it is now interesting to look at the ways and means of improving the performance of the adaptive observer. Some approaches made with this regard and new results obtained through these approaches will be presented in the following sections.

### 7.6 Bandwidth improvement through periodic disturbance cancellation

In this section, some attempts to improve the overall performance of the drive system using the sensorless control algorithm will be described and results will be presented. The approach here is based upon the fact that faster closed loop speed response can be obtained by increasing the closed loop speed controller bandwidth $\alpha$, as much as possible. To do this, when running sensorless the speed controller requires faster speed estimate. This demands the closed loop observer bandwidth $\rho$ to be increased, which is limited by the closed loop current controller bandwidth $\alpha$. The reason is in the cascaded system; the inner most current controller, the adaptive observer and the outer most speed controller must hold the inequality $\alpha > \rho > \alpha_1$ in terms of closed loop bandwidths for proper implementation.

While presenting the basic sensorless control strategy, Harnefors [6] suggests some limitations for the bandwidths of the control loops in the algorithm. The criterion that has been presented is to keep the estimator bandwidth $\rho$, 10 times slower than the current controller bandwidth $\alpha$. It was also mentioned that instabilities occurred in the algorithm as $\rho$ was made faster or larger than $\alpha/10$ and emphasised the importance of having a faster estimator, which enables the achievement of a faster speed controller. Apart from that, the presence of space harmonics in the sampled line currents was pointed out as the possible reason for the instability. The research in this area that will be presented
in the following sections was based on these observations and the discussion in the following is based on the phenomenon that were observed, when attempting to bring the bandwidths of the current controller and the observer close together.

7.6.1 Harmonic components present in the system

At this point it is worthwhile to have an overview on the type of harmonic components in the sampled line currents. At the same time it is important to understand how they appear in the transformed $d'q'$ frame. This is because, since the control strategy is implemented in estimated $d'q'$ frame, any attempt to cancel these disturbances must also be done based on the disturbance information present in the $d'q'$ frame. Figure 7.13 (a) and (b) shows the Discrete Fourier Transforms (DFTs) of positive sequence components of the sampled line currents, while sensorless control algorithm is ON and OFF respectively. Figure 7.13 (c) and (d) shows the DFTs of negative sequence components of the sampled line currents, while sensorless control algorithm is ON and OFF respectively. The speed reference for the machine used was 0.3 pu, which corresponds to 900 rpm and a fundamental frequency of 45 Hz. Corresponding bandwidth values in the normalised frame were $\alpha=1$ and $p=0.5$. The frequency axes of the DFT plots have been adjusted accordingly.

These observations give some important information on the system behaviour, when the estimation algorithm is active. In addition to the higher order harmonics of integer order, one could notice the presence of sub-harmonics of 1/3$^{rd}$ (at 15 Hz) and 2/3$^{rd}$ (at 30 Hz) 4/3$^{rd}$ (at 60 Hz) and 5/3$^{rd}$ (at 75 Hz) in the line currents (both positive and negative sequence components).

![DFT plots](image)

**Fig. 7.13:** DFTs positive and negative sequence components in line currents

Mapping of these positive and negative sequence sub-harmonic components on to $d$-$q$ frame take place according to
\[
\omega_{dq} = \omega_{sh} - \omega,
\]  

(7.45)

where \(\omega_{dq}\) is the resulting frequency in \(d-q\) frame and \(\omega_{sh}\) is the sub-harmonic frequency (with sequence denoted by sign) and synchronous reference frequency respectively. Then Table 7.1 shows how the positive and negative sequence sub-harmonic current components are mapped on to \(d'-q'\) frame according to (7.45). Only the important sub-harmonics are shown.

<table>
<thead>
<tr>
<th>Positive sequence</th>
<th>Negative sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_{dh}) [Hz]</td>
<td>(\omega_{d0}) [Hz]</td>
</tr>
<tr>
<td>+15</td>
<td>-30</td>
</tr>
<tr>
<td>+30</td>
<td>-15</td>
</tr>
<tr>
<td>+60</td>
<td>+15</td>
</tr>
<tr>
<td>+75</td>
<td>+30</td>
</tr>
<tr>
<td>+105</td>
<td>+60</td>
</tr>
<tr>
<td>+120</td>
<td>+75</td>
</tr>
</tbody>
</table>

Table 7.1: Mapping of sub-harmonics

This shows that some combinations of sub-harmonics in the stationary frame appear at the same frequency and depending on the phase difference between the harmonics, sub-harmonics of larger amplitudes in the \(d'-q'\) frame could be observed. To justify this argument, the DFT of \(d\)-axis voltage, which is the input source of these sub-harmonics into the estimator stage, is shown in Figure 7.14 (a). Figure 7.14 (b) shows the DFT of the same signal with sensorless controller switched OFF.

![DFTs of d-axis voltage](image)

(a) Sensorless ON  (b) Sensorless OFF

Figure 7.14: DFTs of \(d\)-axis voltage

This shows a clear increment of 15Hz, 45Hz, 60Hz and 75Hz sub-harmonics, when the estimator algorithm is active. Another important observation one could make is that the rate of increase of specially 1/3\(^{rd}\) and 2/3\(^{rd}\) sub-harmonics in the line currents is much higher as the estimator bandwidth \(p\) is increased (note that these two sub-harmonics contribute to generate 1/3\(^{rd}\), 2/3\(^{rd}\), 4/3\(^{rd}\) and 5/3\(^{rd}\) sub-harmonics in the \(d-q\) frame).

### 8.6.2 Effect of harmonics on the sensorless control algorithm
At this point it is important to pay attention to the control algorithm implemented in normalised $d^\prime q^\prime$ frame. This gives an understanding about where these sub-harmonics are going to be located in the normalised frequency axis. It is now clear that at 0.3 pu speed reference, they appear at the locations 0.1 (15 Hz), 0.2 (30 Hz), 0.4 (60 Hz) and, 0.5 (75 Hz) in the normalised frequency axis of the $d^\prime q^\prime$ frame. This means that they fall well within the bandwidth of the estimator ($\rho = 0.5$), making it unstable as $\rho$ is increased.

7.6.3 Periodic disturbance cancellation

With the background about the periodic disturbances in the system explained in the previous section, application of a periodic disturbance rejection or cancellation technique to get rid of these sub-harmonics will be presented here. In fact this method was successfully used in the AMB application to cancel periodic disturbances appearing in the position sensor signals due to whirling of the unbalance rotor shaft during rotation. The historical background, basic theory and some suggestions for improved performance of this Adaptive Periodic Disturbance Cancellation (APDC) technique were discussed in detail in Chapter 6 of the Part 1 of this thesis. Hence, a description about the location, where the APDC technique was implemented in the control algorithm will be presented here.

Since this is a non-linear algorithm with some cross coupling terms, it is not straightforward to decide the exact location where the cancellation of the periodic disturbance must be done. One way of arguing is to treat this periodic disturbance as though it is coming into the slower closed loop system (consisting of the estimator and speed controller) through $u_d$, and try to cancel the disturbance in $u_d$ just before sending it into the estimator (see Figure 7.6 (a)).

The argument against is that since there is a faster inner current control loop, any added signal in the slower outer loop can be reflected in the inner loop faster and the corresponding effect can once again be fed back to the estimator. If one thinks in that manner it could be better to try and cancel the disturbance in $i_d$ and $i_q$ inside the faster current control loop. These two methods were tested both at simulation level and real-time (see Figure 7.6 (b)).

7.6.4 Simulation and experiment results

The DFT observations obtained with sensorless ON and sensorless OFF show that the evolution of the $4/3^{rd}$ and $5/3^{rd}$ sub-harmonics in the $d^\prime q^\prime$ frame is significant, when the estimation algorithm is
active. Therefore, in simulations the APDC technique suggested was used to cancel the periodic disturbances of the order $4/3^{rd}$ and $5/3^{rd}$ of the speed reference. Cancellation technique has shown successful result [83] at both locations suggested in the previous section. The effect of cancellation technique on estimated speed and $d$-axis voltage is presented in Figure 7.15 (a) and (b) as an illustration.

For the simulations, the periodic disturbance generated in the motor was modelled as two sinusoidal disturbances in $d$ and $q$-axis currents, which are $90^0$ out of phase. For this particular simulation, the disturbance frequency is $4/3^{rd}$ of the reference speed (0.3 pu) and the APDC was used for $d$-axis voltage. The time axis of the plots is in per unit.

The method was implemented to cancel the $1/3^{rd}$ sub-harmonic in the $d$-axis voltage signal in the experimental set-up and in Figure 7.16 DFT plots of $d$-axis voltage with and without cancellation are shown [84]. It can be seen that the APDC has been able to reduce the unwanted sub-harmonic ($1/3^{rd}$ at 15 Hz in this case) by about 15 dB, which is considerable.

![Figure 7.16: Cancellation of 15 Hz sub-harmonic in real system](image)

### 7.6.5 Some comments

Even though the method seems to perform satisfactorily, the computational overhead on the DSP due to this becomes very high. In addition since parallel structures are required to cancel each sub-harmonic the APDC structure required for completely removing all unwanted disturbances becomes a complex one. Another drawback is the variable frequency nature of the drive system. This means that the APDC tuned frequency (frequency of the generated sine and cosine signals within the algorithm) must also vary with the rotational speed. Due to all these practical problems, improving the drive performance by simply trying to increase the closed loop bandwidth towards the current controller was not considered as a productive effort. Instead, several other improvements that can be made in the current and speed control, such as accounting for saturation problems in the controllers, compensating for the non-idealities of the inverter etc. were attempted to obtain better overall performance. In the next section, the capability of the algorithm to track speed ramps, which is an important feature for a drive system used in motion control will be investigated.

### 7.7 Ramp tracking capability of the adaptation algorithm
In high-speed applications fast acceleration and deceleration is considered as an essential feature. This gives rise the need of an analysis that would verify the ramp tracking capability of this adaptation algorithm. If a constant acceleration situation is considered, it can be derived from (7.7) (with the substitution in (7.8)) that
\[ \omega^0 = k_r \]
\[ \omega = \gamma' \sin \theta \rightarrow \omega = k_r - \gamma' \sin \theta = k_r - \gamma' \theta \]
\[ \hat{\omega} = \hat{\omega} + \gamma' \sin \theta \rightarrow \hat{\omega} = \omega - \gamma' \sin \theta = \omega - \gamma' \theta. \]

Steady state values of \( \hat{\omega} \) and \( \tilde{\theta} \) can now be found by substituting \( \{\hat{\omega}, \tilde{\theta}\} = \{0, 0\} \) in the above equations, yielding
\[ \tilde{\theta} = \frac{k_r}{\gamma'}, \]
\[ \tilde{\omega} = \frac{k_r \gamma'}{\gamma'}. \]

This shows that both, the position error and speed error will not be absolutely forced to zero by this algorithm and the only thing possible is to try and select the magnitudes of \( \gamma', \gamma'' \) such that the corresponding errors are minimized [6]. In Figure 7.17 (a) the actual and estimated speed variations during a ramp tracking is shown. Figure 7.17 (b) shows the position estimation error during this process.

![Figure 7.17: Ramp tracking capability of the adaptation algorithm](image)

This situation motivates the need of investigating the possibilities of improving the adaptation algorithm to give better ramp tracking capability. New suggestions on the basic adaptive observer discussed so far will be presented in the following section.

### 7.8 Ramp tracking adaptive observer

The solution suggested here is to increase the order of the observer by one with the introduction of an integral state into the speed adaptation. Since the angle estimation is by default associated with the integral of estimated speed, introduction of such an integral term to the angle estimation proves to be meaningless. The integral state, which is supposed to take away the steady state error of speed estimation is selected as the integral of \( d' \)-axis voltage error in this case. As far as the author of this
thesis is aware of, this modification has never been reported before and hence is a new contribution for this way of using voltage error in speed and position estimation algorithms.

With this newly introduced integral state, the modified non-linear observer for the speed and position estimation in its basic form will be

\[
\begin{align*}
\dot{\hat{\theta}}(t) &= \gamma_1(t) \Phi_1(t) \varepsilon(t) + e(t) \\
\dot{\varepsilon}(t) &= \gamma_1(t) \Phi_1(t) \varepsilon(t) \\
\theta^\ast(t) &= \dot{\omega}(t) + \gamma_2(t) \Phi_2(t) \varepsilon(t),
\end{align*}
\]

(7.48)

where \( e \) is the new integral state. The choice of the error quantity that drives the adaptation mechanism is exactly the same as before and hence the direction vectors in this case are selected to be

\[
\Phi_1 = \Phi_2 = \Phi_3 = [-1 \ 0],
\]

(7.49)

which then lead to

\[
\begin{align*}
\Phi_1 \varepsilon(t) &= \Phi_2 \varepsilon(t) = \Phi_3 \varepsilon(t) = \omega^0 L_d \Psi_m \sin \tilde{\theta}.
\end{align*}
\]

(7.50)

The linearised observer design of the new observer will be discussed in the next section.

### 7.8.1 Design considerations

Since the formulation of the new adaptive observer is similar to the earlier one, the discussion here will start straight away from the design considerations. With the introduction of the additional integral state, the new non-linear observer for the speed and position estimation will be

\[
\begin{align*}
\dot{\hat{\theta}} &= \gamma_1 \omega^0 L_d \Psi_m \sin \tilde{\theta} + e \\
\dot{\varepsilon} &= \gamma_1 \omega^0 L_d \Psi_m \sin \tilde{\theta} \\
\theta^\ast &= \dot{\omega} + \gamma_2 \omega^0 L_d \Psi_m \sin \tilde{\theta}.
\end{align*}
\]

(7.51)

As before, the substitution \( \tilde{a}_i = a^0_i - \hat{a}_i \) leads to

\[
\begin{align*}
\omega^0 - \dot{\theta} &= \gamma_1 \omega^0 L_d \Psi_m \sin \tilde{\theta} + e \\
\dot{\varepsilon} &= \gamma_1 \omega^0 L_d \Psi_m \sin \tilde{\theta} \\
\theta^\ast - \tilde{\theta} &= \omega^0 - \tilde{\omega} + \gamma_2 \omega^0 L_d \Psi_m \sin \tilde{\theta}.
\end{align*}
\]

(7.52)

With the approximation \( \sin \tilde{\theta} = \tilde{\theta} \) as \( \tilde{\theta} \to 0 \) and the substitutions

\[
\begin{align*}
\gamma'_1 &= \gamma_1 \omega^0 L_d \Psi_m \\
\gamma'_2 &= \gamma_2 \omega^0 L_d \Psi_m \\
\gamma'_3 &= \gamma_3 \omega^0 L_d \Psi_m,
\end{align*}
\]

(7.53)

the above system yields the following linearized observer

\[
\begin{bmatrix}
\dot{\hat{\theta}} \\
\dot{\varepsilon} \\
\theta^\ast
\end{bmatrix} =
\begin{bmatrix}
0 & -1 & -\gamma'_1 & \tilde{\theta} \\
0 & 0 & \gamma'_2 & e \\
1 & 0 & -\gamma'_3 & \tilde{\theta}
\end{bmatrix}
\]

(7.54)
As before, the characteristic polynomial of this system is given by

\[ p(\lambda) = \begin{vmatrix} \lambda & 1 & \gamma'_i \\ 0 & \lambda & -\gamma'_i \\ -1 & 0 & \lambda + \gamma'_i \end{vmatrix} = \lambda^3 + \gamma'_i \lambda^2 + \gamma'_i \lambda + \gamma'_i, \]  

(7.55)

which is asymptotically stable, if \( \gamma'_1, \gamma'_2, \gamma'_3 \) satisfies the Routh-Hurwitz criterion [43]. This raises the following conditions.

1. \( \gamma'_1, \gamma'_2, \gamma'_3 \) must be of the same sign
2. Hurwitz determinant 1, \( D_1 = \gamma'_2 > 0 \)
3. Hurwitz determinant 2, \( D_2 = \begin{vmatrix} \gamma'_2 & \gamma'_1 \\ 1 & \gamma'_1 \end{vmatrix} = \gamma'_2 \gamma'_1 - \gamma'_1 > 0 \)

As before the same pole placement design can be done to fix the adaptation gains \( \{\gamma'_1, \gamma'_2, \gamma'_3\} \), based on the characteristic equation obtained earlier. Having triple poles at \( s = -\rho \) to give a closed loop bandwidth of \( \rho \) as before would lead to

\[ \gamma'_1 = 3\rho^2 \rightarrow \gamma_1 = \frac{3\rho^2}{\omega^0 L_d \Psi_m} \]
\[ \gamma'_2 = \rho^3 \rightarrow \gamma_i = \frac{\rho^3}{\omega^0 L_d \Psi_m} \]
\[ \gamma'_2 = 3\rho \rightarrow \gamma_2 = \frac{3\rho}{\omega^0 L_d \Psi_m} \]  

(7.56)

However, poor performance in the low speed region will still remain even with this modified algorithm. Yet as mentioned earlier, it is possible to use a different method to obtain the initial rotor movement and later handover the control of the PMSM to the adaptation algorithm. Most of the practical points mentioned in respect to the previous algorithm such as the use of a filtered speed term for adaptation gain computation, \textit{d-axis} voltage error calculation method, parameter dependency, computation overhead and information on operating speed range apply in the same way to this new algorithm also.

### 7.8.2 Basic observer implementation

Now it is possible to present the complete observer equations in continuous domain in the form of differential equations. This can be done by substituting the corresponding values in (7.51) and augmenting the filter equation (7.12) into the system as before. Thus, the complete ramp tracking adaptive observer will be
\begin{equation}
\dot{\theta} = -\frac{3\rho^2}{\omega_j L_d \hat{\Psi}_m} \epsilon_d + \epsilon
\end{equation}

\begin{equation}
\dot{\epsilon} = -\frac{\rho^3}{\omega_j L_d \hat{\Psi}_m} \epsilon_d
\end{equation}

\begin{equation}
\dot{\theta}^e = \tilde{\omega} - \frac{3\rho}{\omega_j L_d \hat{\Psi}_m} \epsilon_d
\end{equation}

\begin{equation}
\omega_j = k_j \alpha_j (\tilde{\omega} - \omega_j).
\end{equation}

As in the earlier case, since the machine parameter values that must be used here are estimated quantities, they are denoted as \( \hat{L}_d \) and \( \hat{\Psi}_m \). Discrete-time implementation of this will be discussed under the implementation issues later in the chapter.

### 7.9 Non-linear stability analysis of the new algorithm

Due to the same reasoning that was made in the beginning of Section 7.3, a complete non-linear analysis of this new algorithm must also be carried out. Yet, the graphical interpretation of the results of this analysis may not give an elaborate physical insight into the convergence process of the algorithm due to two reasons. They are,

1. It is difficult to find a globally valid Liapunov function for this system. Since order has gone up by one to a third order system, the Liapunov analysis becomes a tuff task. The reason is inability to find a globally valid Liapunov candidate for this complex non-linear system description. All standard methods suggested in textbooks [85, 86] seem to fail (or do not give rise to a straightforward Liapunov candidate) in this particular case.

2. The additional integral state makes it difficult to comment on the convergence of the algorithm due to the three dimensional nature.

Eventhough it is still possible to find a locally valid Liapunov candidate by means of numerical proofs, the particular approach will not be presented here due to the above reasoning.

#### 7.9.1 Fixed points and eigen vectors

The analysis is the same as for the previous case except for the fact that the observer is of third order due to the introduction of the integral state.

With tripple poles placed at \( s = -\rho \) the non-linear observer for the the speed and position estimation errors will be of the form

\begin{equation}
\dot{\theta}^e = -e - 3\rho^2 \sin \tilde{\theta}
\end{equation}

\begin{equation}
\dot{\epsilon} = \rho^3 \sin \tilde{\theta}
\end{equation}

\begin{equation}
\dot{\theta}^e = \tilde{\omega} - 3\rho \sin \tilde{\theta}.
\end{equation}

As before, this will lead to the linearized version of the system \( \Delta \dot{x} = F_n(x^*) \Delta x \), where corresponding to the fixed points \( \{ \tilde{\omega}, e, \tilde{\theta} \} = \{0,0,(2m+1)\pi\}, \{0,0,2m\pi\} \) \( m = 0, \pm 1, \pm 2, \ldots \), the system in equation (7.66) will result in
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\[ F_n(\tilde{\omega}^n, e^{*1}, \tilde{\theta}^n) = \begin{bmatrix} 0 & -1 & -3\rho^2 \cos \tilde{\theta} \\ 0 & 0 & \rho^3 \cos \tilde{\theta} \\ 1 & 0 & -3\rho \cos \tilde{\theta} \end{bmatrix} \frac{1}{0,0,(2m+1)\pi} \begin{bmatrix} 0 & -1 & 3\rho^2 \\ 0 & 0 & -\rho^3 \\ 1 & 0 & 3\rho \end{bmatrix} \ (7.59) \]

\[ \det(J - F(\tilde{\omega}^n, e^{*1}, \tilde{\theta}^n)) = \begin{vmatrix} \lambda & 1 & -3\rho^2 \\ 0 & \lambda & \rho^3 \\ -1 & 0 & \lambda - 3\rho \end{vmatrix} = \lambda^3 - 3\rho \lambda^2 - 3\rho^2 \lambda - \rho^3 \]

and

\[ F_n(\tilde{\omega}^{*2}, e^{*2}, \tilde{\theta}^{*2}) = \begin{bmatrix} 0 & -1 & -3\rho^2 \cos \tilde{\theta} \\ 0 & 0 & \rho^3 \cos \tilde{\theta} \\ 1 & 0 & -3\rho \cos \tilde{\theta} \end{bmatrix} \frac{1}{0,0,2m\pi} \begin{bmatrix} 0 & -1 & -3\rho^2 \\ 0 & 0 & \rho^3 \\ 1 & 0 & -3\rho \end{bmatrix} \]

\[ \det(J - F(\tilde{\omega}^{*2}, e^{*2}, \tilde{\theta}^{*2})) = \begin{vmatrix} \lambda & 1 & 3\rho^2 \\ 0 & \lambda & -\rho^3 \\ -1 & 0 & \lambda + 3\rho \end{vmatrix} = \lambda^3 + 3\rho \lambda^2 + 3\rho^2 \lambda + \rho^3 \ (7.60) \]

\[ \lambda = -\rho, -\rho, -\rho. \]

Since the coefficients of \( \lambda \) in the characteristic polynomial in (7.59) is of different signs (this is because the value of \( \rho \) is selected to be positive), its roots must be of different signs, i.e. the eigenvalues are also of opposite signs. Therefore, the fixed point in (7.59) is a saddle point and the corresponding eigenvectors \( \{v_1, v_2, v_3\} \) can be found by following the previous approach.

Similarly, the fixed point given by (7.60) is a sink, as it’s eigenvalues are all negative. The corresponding eigenvectors \( \{v_1^{*2}, v_2^{*2}, v_3^{*2}\} \) can also be found. At this point the tedious analytical derivation of eigenvectors will not be performed.

7.9.2 Phase portrait

Figure 7.18: Phase portrait of the new observer
Corresponding phase portrait in 3-dimensional frame is shown in Figure 7.18 (a). This obviously is a complex one to be compared with the phase portrait of the previous algorithm. In Figure 7.18 (b), the same projected on to $e = 0$ plane is shown.

Implementation issues of the new algorithm will be discussed in the next section.

### 7.10 Implementation issues

The sensorless control structure for the new algorithm is exactly the same as before and is in fact similar to Figure 7.7. Voltage error computation is also done in the same way and can be referred to Equation (7.16) or (7.17). Only the discrete-time implementation of the new adaptive observer will be described here.

#### 7.10.1 Simulation model in per-unit

The discrete-time observer with forward Euler integration can be given as

$$
\dot{e}(k) = e(k-1) - \frac{\rho}{\omega_f(k-1)\hat{L}_d\hat{\Psi}_m} - T_s\hat{e}_d(k) + T_s e(k-1)
$$

$$
\theta(k) = \theta(k-1) + T_s \dot{e}(k-1) - \frac{3\rho}{\omega_f(k-1)\hat{L}_d\hat{\Psi}_m} - T_s \hat{e}_d(k)
$$

$$
\omega_f(k) = \omega_f(k-1) + k_f \alpha T_s (\dot{\theta}(k-1) - \omega_f(k-1))
$$

The changes required in terms of scaling, when it comes to real-time implementation will be discussed next.

#### 7.10.2 Real-time implementation

In the new adaptive observer equations also the scaling has to be done as before. The modified real-time adaptive observer equations are given by

$$
\dot{\hat{e}}(k) = \hat{e}(k-1) - \frac{3\rho^2}{\omega_f(k-1)\hat{L}_d\hat{\Psi}_m} - T_s \hat{\omega}_b \hat{e}_d(k) + T_s \hat{\omega}_b e(k-1)
$$

$$
\theta(k) = \theta(k-1) + T_s \hat{\omega}_b \dot{\hat{e}}(k-1) - \frac{3\rho}{\omega_f(k-1)\hat{L}_d\hat{\Psi}_m} - T_s \hat{\omega}_b \hat{e}_d(k)
$$

$$
\omega_f(k) = \omega_f(k-1) + k_f \alpha T_s \hat{\omega}_b (\dot{\theta}(k-1) - \omega_f(k-1))
$$

Verification of the adaptive observer performance in the simulation and experimental level will be done in the next section.

### 7.11 Simulation and experimental verification
As in the previous case performance of the algorithm can be tested in many ways. The discussion here will be started by presenting simple step response results obtained from both simulations and experiments. Results similar to the ones that were presented in the case of first adaptive observer will be presented here as well. However, “Kick start” will not be performed, as its implementation is straightforward in this case also.

7.11.1 Step response tests

Even though the observer is capable of tracking speed ramps, first its step response will also be studied. The simulated results are shown in Figure 7.19 (a). The machine speed is first brought to a value above zero (0.3 pu) using the start-up technique that will be explained later in this section. For comparison purposes, the calculated speed from the simulated machine model has also been plotted on the same graph. The same size of step input was applied to the real system with the same controller and estimator tuning values. The result is shown in Figure 7.19 (b). In this figure the actual position measured from the incremental encoder has also been plotted for comparison purpose.

(a) Simulated step response (0.3 – 1.5 pu step applied at 1000 per-unit time)

(b) Experimental step response (0.3 – 1.5 pu step applied at 0 s)

Figure 7.19: Step response test results of sensorless drive

\[ \alpha = 6, \ \rho = 0.15, \ \alpha_s = 0.02 \ \text{pu} \]

Ramp tracking capability will be presented later in the chapter.

7.11.2 Angle and speed estimation error

The focus here is the estimation errors of rotor position and speed estimation error variations during the above step response.

(a) Rotor position estimation error

In this section estimation errors of rotor position and speed obtained from simulation and real-time tests will be presented. Rotor position estimation error for the simulations and experiments are
computed according to Equations (7.41) and (7.42) as before. The simulated and experimental rotor position estimation errors are shown in Figure 7.20 (a) and (b) respectively.

(b) Speed estimation error

Speed estimation errors from simulation and experiments are computed from the Equation (7.43) and (7.44). The simulated and experimental speed estimation errors are shown in Figure 7.21 (a) and (b) respectively.

The region of attraction will be verified in the next section.
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7.11.3 Verification of the region of attraction

From the analysis it was not possible to find a clear region of attraction for this third order adaptive observer. Thus, simulated results on the region of attraction is very important, in particular, when implementing special techniques such as “Kick-start” etc. As in the previous case the machine speed is first brought up to 0.5 pu using the speed sensor signal. Then the control is switched over to sensorless algorithm. Being a system of three states, presenting results plotted on $\omega_{\text{sim}} V s \ \Theta_{\text{sim}}$ plane becomes somewhat meaningless for this case even at the simulation level. However, simulated results are given in $\omega_{\text{sim}} V s \ \Theta_{\text{sim}}$ plane for easy comparison with the previous algorithm. They are shown in Figure 7.22.
As before, the quantization noise problem observed in the experimental rotor position estimation error makes it difficult to present results as a $\tilde{\omega}_r$ Vs $\tilde{\theta}_r$ plot. Therefore, the time variations of the estimation error quantities during the convergence process will be presented.

Figure 7.23 (a) shows the convergence of estimation errors, when the initial condition of the speed estimation is deviated from operating speed 0.5 pu. The initial condition for the rotor position estimation is kept at zero, which is the changeover angle from sensor-sensorless operation. Figure 7.23 (b) shows the convergence of estimation errors, when the initial condition of the rotor position estimation is deviated from zero. In this case initial condition for the speed estimation is kept at 0.5 pu.

(b) Initial condition for rotor position estimation is deviated from 0 (to -2.0, -1.5, -1, -0.5, 0.5, 1, 1.5 and 2.0 rad respectively)

Figure 7.23: Verification of attractivity region by experiments
7.12 Verification of ramp tracking capability

Since the new algorithm was meant to be used for tracking speed ramps, its tracking capability has to be first tested analytically before moving onto simulations and experiments.

7.12.1 Analytical test on ramp tracking capability

As before, when the constant acceleration situation is considered

\[\dot{\omega} = \gamma' \sin \tilde{\theta} + \int \gamma' \sin \tilde{\theta} dt \rightarrow \dot{\omega} = k_r - \gamma' \sin \tilde{\theta} - \int \gamma' \sin \tilde{\theta} dt \approx k_r - \gamma' \tilde{\theta} - \int \gamma' \tilde{\theta} dt \quad (7.63)\]

\[\dot{\tilde{\theta}} = \omega + \gamma' \sin \tilde{\theta} \rightarrow \dot{\tilde{\theta}} = \dot{\omega} - \gamma' \sin \tilde{\theta} = \dot{\omega} - \gamma' \tilde{\theta}.\]

By putting \(\{\theta, \dot{\theta}\} = \{0,0\}\) in the above equations

\[0 = k_r - \gamma' \tilde{\theta} - \int \gamma' \tilde{\theta} dt\]

\[0 = \dot{\omega} - \gamma' \tilde{\theta}. \quad (7.64)\]

The Laplace transforms of the above two equations yields

\[\tilde{\theta}(s) = \frac{k_r s}{\gamma' s + \gamma'} \quad (7.65)\]

\[\dot{\omega}(s) = \frac{\gamma' k_r s}{\gamma' s + \gamma'}.\]

The steady state values of \(\dot{\omega}\) and \(\tilde{\theta}\) can now prove to be zero (by substituting \(s = 0\)) from the above derivation. This means that the modified algorithm is capable of bringing the steady state estimation errors of speed and position down to zero, while tracking a speed ramp.

7.12.2 Verification by simulation and experiments

![Figure 7.24: Simulated ramp tracking capability of the modified adaptation algorithm](image)

In fact, to make use of the ramp tracking capability of this new algorithm, a speed controller with similar capabilities must also be used. For this purpose any structure out of the two ramp tracking speed controllers (Equation (5.29) or (5.30)) suggested in Chapter 5 can be incorporated. The simulation and experimental results shown here were obtained with ramp tracking controller 1 together with ramp tracking adaptation algorithm for speed and position estimation. Estimation error
variations are also shown in the respective figures. Figure 7.24 shows the simulated ramp tracking process.

![Graph showing simulated ramp tracking process](image)

**Figure 7.24:** Simulated ramp tracking process

The experimental ramp tracking is shown in Figure 7.25. From these results it can be seen that the new observer with the additional integral state of the $d$-axis voltage error has better tracking capability with the added ability to track speed ramps. This ability as it was pointed out in the chapter on speed controller design, is a very important feature in digital motion control applications. In fact, even for the angle grinder application, it is advisable to ramp-up the speed reference so that the top speed is reached in a known period of time. This applies also for the deceleration as well and ramping down the set-point is considered to be better and safer for applications of this nature.

7.13 A summary

The first voltage error based non-linear observer in [6] was further investigated and a modified observer was suggested in this chapter. Before winding up this chapter, important issues that were brought up will be summarised.

With the explanations that were given during the observer formulation, it is now clear that both algorithms investigated here are suitable for the sensorless control of PMSMs at high-speed range. In fact, the accuracy of speed and position estimations will improve, as the operating speed goes up due to the higher SNR (see Section 7.2.8). Thus, it can be concluded here that the two algorithms are suitable for sensorless control of PMSMs in the high-speed range.

The non-linear analysis of the observer led the way to present the start-up technique “Kick-start”, when the PMSM is controlled sensorless. In fact, the robustness shown by both algorithms is sufficient to start the PMSM with this method, as the algorithms are not capable of providing rotor position information at standstill. Thus, the “Kick-start” method is a good candidate for the start-up problem especially for applications such as the angle grinder that do not have to overcome the load torque from the beginning. If the machine is loaded at standstill (like in traction applications), then it
will not be possible to do the initial alignment of the rotor with the $d$-current pulse. Hence, the “Kick-start” method will not be applicable.

The speed reversal and stopping the machine were not experimentally illustrated here. Yet, the information provided here are sufficient to derive suitable methods to handle those two situations. For the speed reversal, disabling the algorithm during the zero crossing can be incorporated (see Section 7.2.8). Stopping the machine, when operated with the sensorless control algorithm must also be done carefully. One easy solution is to decelerate the machine down to 10% base speed and disable the algorithm, while applying a $d$-current pulse at the same time.

The modified algorithm with ramp tracking capability proved to produce more accurate speed and position estimates. Main reason is the capability of the newly introduced integral state to eliminate the steady state error in speed estimation. In fact, the ramp tracking capability will also be very useful in applying the sensorless control strategy to PMSMs that are used as actuators for motion control systems.

During the derivation of the adaptation algorithms it was mentioned that the inverter non-idealities and machine parameter variations could cause the estimations to deteriorate. How these non-idealities are compensated will be the focus of the next chapter. Machine parameter estimation will be discussed in Chapter 9.
8. Inverter is not an ideal device

For analytical and modeling purposes in vector control theory, the inverter is assumed to be an ideal device as mentioned in previous chapters. This leads to the assumption that the command voltages sent out from the vector controller are actually applied to the motor as expected. Due to the typical characteristics of electronic components used in inverters, they deviate (or are sometimes forced to deviate by the design engineers) from the ideal behavior. This causes an error between the command voltage from the vector controller and the actual applied voltage in the motor at a given instant. This leads to distortions of the expected average sinusoidal voltage waveforms from the inverter output. Previously published work on this issue can also be found in the literature [87, 88, 89, 90]. The aim of this chapter is to describe more dominant non-idealities of an inverter and present some compensation methods together with experimental results.

8.1 Impact of a non-ideal actuator

The above mentioned error can cause a performance degradation in any inverter driven motor drive. Yet, the effect can be less in the case of variable speed operation with speed sensors, because of the feedback information. However, the dynamical response of the current controllers may also degrade as it was illustrated in Chapter 4 under current controller design. In fact non-idealities of the inverters have been identified as one of the main reasons for instabilities and oscillations in variable speed AC drive systems. The effect of non-idealities can be more devastating in the case of sensorless control of PMSM. According to (7.16) or (7.17) in Chapter 7, in this particular sensorless control application, it is also assumed that the command voltage from the current controller output is the actual applied voltage in the machine. The presence of non-idealities then causes erroneous estimation of speed and rotor position by the adaptation algorithm.

Being a switching device with nonlinear characteristics, modeling of the inverter other than a linear gain (this can be called as the time average model with ideal approximation) is obviously a difficult task. Therefore, one good way of handling this problem is to apply possible compensation methods so that the overall performance of the inverter is brought as close as possible to the ideal behavior.

Switching dead-time, resistive voltage drop in switching components and DC-link voltage fluctuations are considered to be the most dominant non-idealities of an inverter. They will be discussed in detail in the following sections.

8.2 Switching dead-time

Inverter switching dead-time also known as blanking time is a non-ideality introduced by the designers as a safety measure. This can be explained with the aid of Figure 8.1, which shows one inverter pole. Figure 8.1 (a) shows an ideal inverter pole, while Figure 8.1 (b) shows an actual inverter pole.

In an ideal inverter, the upper and lower switching elements (considered to be ideal switches) are assumed to be changing their state at the same time instant. In practice, when power electronic components like thyristers or IGBTs are used as the switching elements, one must always take into consideration the finite time taken by such elements for switching on (turn-on time) and switching off (turn-off time) [87, 52].
It is assumed that $Q_1$ is OFF and $Q_2$ is ON at a certain instance and it is required to change their states. If the switch-on signal to $Q_1$ and switch-off signal to $Q_2$, are applied at the same time instant, due to turn-on and turn-off time delays, there can be a period, where both switches are conducting. This can cause direct short circuit of the DC-link through the two switches in a particular inverter pole.

The usual technique to avoid this hazardous situation is to introduce a time delay $T_d$ between the switch-off signal to the conducting switch and switch-on signal to the non-conducting switch (switch-on is always delayed to switch-off). This is known as the inverter switching dead-time and the effect of this user-introduced delay is analyzed in this section.

### 8.2.1 Impact of dead-time at a glance

At this stage, it is interesting to see the mathematical impact of dead-time on the expected average sinusoidal fundamental voltage from an inverter. When supplying inductive loads, the presence of freewheeling diodes, bypassing the switches as shown in Figure 8.1 (b) makes this analysis a complex task. It is known that the turn-on delay $T_{d-on}$ and the turn-off delay $T_{d-off}$ of a semiconductor switch depends on the current it delivers to the load. Therefore, the effective dead-time $T_d$, when the dead-time introduced by the controller is $T_{dc}$, can be written as

$$T_{d-on} = f_{d-on}(I_{SA})$$

$$T_{d-off} = f_{d-off}(I_{SA})$$

$$T_d = T_{dc} + T_{d-on} - T_{d-off}. \quad (8.1)$$

Figure 8.2 (a), (b) and (c), show the actual conducting and non-conducting periods of $Q_1$ and $Q_2$, with respect to the sign of the current flow [88, 90]. Each situation is considered separately.

**a) $I_A > 0$**

This explanation refers to Figure 8.2 (a). During effective dead-time, $Q_1$ and $Q_2$ are non-conducting. A positive $I_A$ can only flow through $D_2$. Therefore, the node $A$ will continue to be tied to $-U_{DC}/2$ until $Q_1$ starts conducting. When $Q_1$ starts conducting, since the node $A$ is connected to $+U_{DC}/2$, the positive current will rise.

At the end of the positive pulse, it is now required to switch-off $Q_1$. After the switch-off signal is applied to $Q_1$, the positive current will still continue to increase until $Q_1$ is completely off. Once $Q_1$ becomes non-conducting, positive current will start flowing through $D_2$ again and the node $A$ will be effectively connected to $-U_{DC}/2$. 
Inverter is not an ideal device

Now the actual conducting time due to the applied ideal switch-on signal is given by

\[ T_{a\text{-}on} = T_{i\text{-}on} + T_{d\text{-off}} - T_{dc} - T_{d\text{-off}} = T_{i\text{-}on} - T_d. \]  

(8.2)

(b) \( I_A < 0 \)

Now, referring to Figure 8.2 (b), when \( Q_1 \) and \( Q_2 \) are non-conducting, a negative \( I_A \) can only flow through \( D_1 \). Therefore, \(-U_{DC}/2 \) will only appear at node \( A \) until \( Q_2 \) is effectively switched-off. Current will start rising towards zero just as \( Q_1 \) starts conducting.

When the switch-off signal to \( Q_1 \) is applied, \( D_1 \) will still keep conducting even after the switch-off of \( Q_1 \). The potential at \( A \) will change from \(+U_{DC}/2 \) to \(-U_{DC}/2 \), soon after \( Q_2 \) starts conducting.

The actual conducting time due to the applied ideal switch-on signal in this case is given by

\[ T_{a\text{-}on} = T_{i\text{-}on} + T_{dc} + T_{d\text{-off}} - T_{d\text{-off}} = T_{i\text{-}on} + T_d. \]  

(8.3)

(c) \( I_A \) changes sign during the pulse

This explanation refers to Figure 8.2 (c). Here it is assumed that \( I_A < 0 \) (much closer to 0 V so that \( I_A \) can cross over to positive region during switch-on pulse to \( Q_1 \)), when switch-on pulse to \( Q_1 \) is applied. As explained under the condition \( I_A < 0 \), a negative \( I_A \) can only flow through \( D_1 \). Therefore, \(-U_{DC}/2 \) will only appear at node \( A \) until \( Q_2 \) is effectively switched-off. Current will start rising towards zero just as \( Q_1 \) starts conducting. In this case current will cross over to the positive side before the switch-on period of \( Q_1 \) is over.
By the time the switch-off signal to $Q_i$ is applied, $I_A$ will have a positive increasing value. After the switch-off signal is applied to $Q_i$, the positive current will still continue to increase until $Q_i$ is completely off (this is quite similar to switch-off process under $I_A > 0$ explained earlier). When $Q_i$ becomes non-conducting, positive current will start flowing through $D_2$ again and the node $A$ will be effectively connected to $-U_{DC}/2$.

From Figure 8.2 (c) it can be seen that apart from a time shift of $T_{d-off}$ of the positive pulse, the dead-time does not change the pulse width in this case. The following equation can be derived for the actual on time in this case.

$$T_{a-on} = T_{i-on}$$ \hspace{1cm} (8.4)

This shows that applying dead-time compensation in actual practice has to be totally dependent on the sign of the current in each phase. Since the dead-time has no bad effect, when the phase current is in the vicinity of zero, special care has to be taken on deciding when to apply the particular compensation scheme.

With this background knowledge on the behavior of switching dead-time, the focus will now move on to mathematical description of dead-time.

### 8.2.2 Mathematical insight into the effect of dead-time

The aim here is to mathematically describe the effect of dead-time explained above. This will give an idea of how the output voltage in each phase is affected by the presence of switching dead-time. Equation (8.2) and (8.3) can now be written in compact form to give the actual on time of the upper switch of the inverter leg as

$$T_{a-on} = T_{i-on} - T_d \, \text{sgn}(I_A).$$ \hspace{1cm} (8.5)

The same information can be presented in duty cycle form general to the three phases with the following approach [48].

---

![Figure 8.3: Duty cycle form of dead-time effect](image-url)
According to Figure 8.3, the expression for duty ratio \( d \) is

\[
d = \frac{T_{a\_on}}{T_{a\_on} + T_{a\_off}} = T_{1\_on} / T_s. \tag{8.6}
\]

Dividing (8.5) by \( T_{a\_on} + T_{a\_off} \) will lead to a general expression for duty ratio of all three-phases given by

\[
d_k = d_{i,k} - \frac{T_d}{T_s} \text{sgn}(I_A) = d_{i,k} - \Delta d_k, \quad k = A, B, C.
\tag{8.7}
\]

The duty ratio error due to dead-time in each phase is given by the term \( \Delta d_k \), \( k = A, B, C \) in (8.7). Average phase voltage in each phase \( V_{kO} , k = A, B, C \), over a switching period can now be derived as

\[
V_{kO} = \left[ \frac{T_{a\_on} - T_{a\_off}}{T_{a\_on} + T_{a\_off}} \right] \frac{U_{DC}}{2} - \Delta d_k U_{DC}.
\tag{8.8}
\]

Substituting for \( d_k \) from (8.7) leads to

\[
V_{kO} = (2d_{i,k} - 1) \frac{U_{DC}}{2} - \Delta d_k U_{DC}
\tag{8.9}
\]

The description of the effect of dead-time in phase voltage quantities as in (8.10) makes it possible to define a corresponding space vector representation [88, 48]. This is described in the next section.

### 8.2.3 Space vector representation of dead-time

With peak value scaling, the expression of error voltage space vector due to the dead-time effect can be written as

\[
\Delta V_s(t) = \frac{2}{3} \left[ \Delta V_{A0}(t) + a \cdot \Delta V_{B0}(t) + a^2 \cdot \Delta V_{C0}(t) \right]. \tag{8.11}
\]

where \( a = e^{j2\pi/3} \).

This can be further simplified as
\[
\Delta V_s(t) = \frac{2}{3} \left[ \frac{T_d}{T_s} U_{DC} \text{sgn}(I_A) + a \frac{T_d}{T_s} U_{DC} \text{sgn}(I_B) + a^2 \frac{T_d}{T_s} U_{DC} \text{sgn}(I_C) \right] \\
= \frac{2}{3} \frac{T_d}{T_s} U_{DC} \left[ \text{sgn}(I_A) + a \text{sgn}(I_B) + a^2 \text{sgn}(I_C) \right] \\
= \frac{4}{3} \frac{T_d}{T_s} U_{DC} \text{sgn}(I_S),
\]

where \( \text{sgn}(I_S) \) is defined as a sign vector of unit magnitude given by
\[
\text{sgn}(I_S) = \frac{\text{sgn}(I_A) + a \text{sgn}(I_B) + a^2 \text{sgn}(I_C)}{\text{sgn}(I_A) + a \text{sgn}(I_B) + a^2 \text{sgn}(I_C)}.
\]

The actual stator voltage space vector (distorted due to the effect of dead-time) applied to the machine by the inverter is given by,
\[
V_s(t) = V_a(t) - \Delta V_s(t).
\]

The symmetry of the three-phase system demands the fact that two of the phase currents must be of the same sign and opposite to the third at a given instant. As can be seen from (8.13) this gives rise to six different distortion-space-vectors of the same magnitude.

The sector of stator coordinate plane within which each distortion-space-vector becomes active as the stator current vector rotates in stator frame is graphically depicted in Figure 8.4 [91]. The distortion that would take place in the applied stator voltage space vector, when the stator current space vector is lying in the sector \((I_A > 0, I_B < 0, I_C < 0)\) is also shown in the same figure.

By observing (8.12) one could come to the conclusion that the distortion due to dead-time for a given inverter becomes more pronounced as the switching frequency of the inverter increases. In addition, observation of Figure 8.4 reveals the fact that the distortion effect can be more pronounced, if the injected voltage magnitude (inverter output voltage) is smaller.

### 8.3 On-state voltage drops in switching components

The switching elements used in an inverter, namely power transistors and freewheeling diodes have finite on-state voltages across them, when they are in conducting mode. These
can have two basic components. One component is the voltage required to bring the device to the conducting mode (the cut-in voltage of a diode for example). There can be another current dependent voltage drop across the device as well. The aim of this section is to have a mathematical insight into the adverse effects of these voltage drops.

8.3.1 On-state voltage drops in detail

It is convenient in this case also to consider one inverter pole for the analysis. The approach here is to replace the actual transistors and diodes in the pole from ideal components as follows.

- An actual transistor is replaced from an ideal switch, a resistor and an ideal voltage source equal to on-state voltage drop of the transistor.
- An actual diode is replaced by an ideal diode, a resistor and an ideal voltage source equal to on-state voltage drop of the diode.

The idealized inverter pole is shown in Figure 8.5.

![Figure 8.5: Idealized inverter pole representing resistive voltage drops](image)

With this way of representing the switching components, it is possible to express the on-state voltage drops across them mathematically. The on-state voltage drop across a transistor can be given as

\[ V_T = V_{T_{on}} + R_T I_A. \] (8.15)

For a diode the corresponding expression will be

\[ V_D = V_{D_{on}} + R_D I_A. \] (8.16)

It is understood that a positive current can only flow through \( Q_1 \) or \( D_2 \). Similarly a negative current can only flow through \( Q_2 \) or \( D_1 \). This makes the corresponding distortion current-dependent. This is graphically shown in Figure 8.6. Each case will be separately considered here.

(a) \( I_A > 0 \)

During the conduction period of \( Q_1 \), the positive current will flow through it and the output voltage will be equal to \( (U_{DC}/2-V_T) \). When \( Q_1 \) is in non-conducting mode, positive current will flow through \( D_2 \) and the corresponding output voltage will be \( -(U_{DC}/2-V_D) \).
(b) $I_A < 0$

During the conduction period of $Q_2$, the negative current will flow through it and the output voltage will be equal to $(U_{DC}/2 + V_T)$. When $Q_2$ is in non-conducting mode, negative current will flow through $D_1$ and the corresponding voltage will be $(U_{DC}/2 + V_D)$.

8.3.2 Mathematical description of on-state voltage drops

The approach will be similar to the one that was followed in the analysis of dead-time distortion. The average pole voltage over one switching period can be found in this case also.

(a) $I_k > 0$

For positive phase current in a certain phase, the average pole voltage over one switching period is

$$V_{kO} = \frac{\left( \frac{U_{DC}}{2} - V_{T,k} \right) T_{on} - \left( \frac{U_{DC}}{2} + V_{D,k} \right) T_{off}}{T_{on} + T_{off}}$$

$$= \frac{\left( T_{on} - T_{off} \right) \frac{U_{DC}}{2}}{T_{on} + T_{off}} - \frac{V_{T,k} T_{on} + V_{D,k} T_{off}}{T_{on} + T_{off}}$$

$$= \frac{(2d_k - 1) U_{DC}}{2} - \frac{V_{T,k} T_{on} + V_{D,k} T_{off}}{T_{on} + T_{off}}$$

$$= V_{i,kO} - \Delta V_{kO}, \quad k = A, B, C.$$  \hspace{1cm} (8.17)

(b) $I_k < 0$

For negative phase current in a certain phase, the average pole voltage over one switching period is
Inverter is not an ideal device

Now (8.17) and (8.18) can be combined and written as

\[ V_{ko} = V_{kxo} - \Delta V_{ko} \operatorname{sgn}(I_k), \quad k = A, B, C. \quad (8.19) \]

It is now clear that (8.19) is somewhat analogous to the combination of (8.9) and (8.10) in the case of dead-time distortion. However, the duty cycle dependency of \( \Delta V_{ko} \) in this case (this can be seen from (8.17) and (8.18)) makes this a complex problem. This complexity can be reduced by making the assumption that \( V_{T,k} \) and \( V_{D,k} \) are equal (say \( V_{R,k} \)). Then a simplified form of phase voltage distortion due to on-state voltage drop can be written as

\[ V_{ko} = V_{kxo} - V_{R,k} \operatorname{sgn}(I_k), \quad k = A, B, C, \quad (8.20) \]

where

\[ V_{R,k} = V_{T,k} = V_{D,k} = V_T + R_L I_k, \quad k = A, B, C. \quad (8.21) \]

Since this distortion is also analogous to the distortion due to dead-time, one can continue to represent it in space vector form also. This is not done here, as the approach is exactly the same as in Section 8.2.3. However, unlike in dead-time distortion, this contains a term proportional to the amplitude of the current space vector.

### 8.4 DC-link voltage fluctuations

All equations that were considered in this chapter so far assumed the fact that the DC-link voltage is constant (constant \( U_{DC} \)). For this to be true, the DC-link voltage has to be infinitely stiff. However, in practice this requires a large DC-link capacitance, which can be bulky and expensive [48]. On the other hand, a large DC-link capacitance may cause much higher harmonic rejection into the AC mains [90]. This means that the DC-link voltage ripple is inevitable in practice. The ripple frequency and amplitude may depend on the particular rectifier configuration used and the loading condition [89, 48]. Voltage fluctuations in the supply three-phase voltage can also contribute to DC-link voltage fluctuations.

As a consequence of DC-link voltage ripple, input voltage of the machine might be influenced causing some low frequency distortions in phase currents [48]. The ripple is most pronounced during large torque reversals of the machine and it can ultimately cause instabilities of the whole drive system.

Since the ripple occurs about a certain nominal DC level (\( U_{DC,n} \)) of the bus voltage, the corresponding effect of the fluctuation on each phase voltage can be expressed as a scaling of the expected average voltage over one modulation cycle. This can be given as
The non-idealities discussed in the three previous sections are the more pronounced inverter non-idealities present.

### 8.5 A simple model for a real inverter

Mathematical interpretation of the inverter non-idealities discussed so far leads to build a simple per-phase model for the time-average behavior of an inverter. The model can then be used to analyze the harmonic distortions that take place in the output voltage waveforms due to each of the non-idealities. Model building will be done in this section. Similar work can be found in [90]. In fact, what is meant by “per-phase model” here is that the mathematical operations shown in the figures presented in this section must be carried out in parallel in all three phases to obtain the real three-phase inverter model. Thus the quantities shown in the figures are three-phase voltage and current vectors.

#### 8.5.1 Models for each non-idealty

First models for each non-idealty will be introduced and later in the next section they will be integrated together to make the complete inverter model.

(a) **Dead-time model**

The dead-time model is simply the implementation of (8.9) and (8.10). This is graphically shown in Figure 8.7 (a). The Equation (8.10) needs the PWM switching period $T_s$, effective dead-time $T_{d}$, DC-link voltage $U_{DC}$ and the sign of the current in each phase for its implementation.

![Dead-time model](image)

(b) **Model for on-state voltage drop**

![Model for on-state voltage drop](image)

(c) **DC-link voltage fluctuation model**

![DC-link voltage fluctuation model](image)

*Figure 8.7: Models for inverter non-idealities*
(b) Model for on-state voltage drop in components

The model for the on-state voltage drops in the switching elements has the same structure as dead-time model. It is the implementation of (8.20) and (8.21). This is graphically shown in Figure 8.7 (b). In addition to the quantities required for the dead-time model, one needs the instantaneous phase current, device on-state voltage drop \( V_g \) and device on-state resistance \( R_c \) for this implementation.

(c) DC-link voltage fluctuation model

Here Equation (8.22), which is more straightforward than the other two will be implemented. Figure 8.7 (c) shows the linear gain that models the DC-link voltage fluctuation.

8.5.2 Complete inverter model

The three models are put together to obtain the complete per-phase actual inverter model. The dead-time model and the model for the on-state voltage drops can be cascaded together, while the DC-link voltage fluctuation model has to be inserted between the inverter output command and the three phase load as shown in Figure 8.8.

![Complete actual inverter model](image)

Figure 8.8: Complete actual inverter model

In order to do any simulation studies with this model, it is understood that one needs the basic information that was mentioned above from the real inverter of interest. Therefore, the practical use of the model will be dealt with in Section 8.7 on simulation and experimental validation.

8.6 Implementation of compensation methods

Explanations and analysis so far reveals the fact that these non-idealities can considerably degrade the performance of the drive, if not compensated properly. Compensation schemes for all three non-idealities discussed here can be found in previous literature also [87, 88, 89, 90]. Compensation schemes for each non-ideality will be taken up separately.
8.6.1 Compensation for switching dead-time

Compensation methods reported so far for the compensation of switching dead-time can basically be divided into two categories.

(a) Feedback compensation methods

The approach here is to measure the distortion that takes place in a particular pulse width due to dead-time, using a combinatorial logic arrangement, and to appropriately compensate for it in the following pulse. Details of this method can be found in [87] and [48]. This can be used, where PWM generation is done digitally and the design engineer is free to manipulate the duty ratio from the internal timers.

This is not the case for the motor drive used in this application, where analog triangular carrier wave PWM generation is used. For such applications, feed-forward methods have been suggested in [48]. Two feed-forward compensation methods will be proposed and tested here.

(b) Feed-forward compensation methods

In a PWM generation scheme, where the digital control system generates analog reference levels for PWM patterns, the user always has the ability to make small changes in the amount of modulation in each phase by adding a desired quantity to the reference levels. It is always possible to have prior knowledge on the PWM switching period $T_s$, effective dead-time $T_d$, and DC-link voltage $U_{DC}$. As the sampled line current information is available to the control algorithm, the sign of the current can also be decided. This makes it possible to work out the feed-forward term that must be added to the reference levels of each phase as the compensation term for dead-time distortion. This is a phase-wise compensation scheme as it is done in the stator frame for three-phases. The scheme is graphically depicted in Figure 8.9.

![Figure 8.9: Phase wise dead-time compensation](image)

In Section 8.2.3, it was shown that dead-time distortion could be expressed in space vector form. This makes it is possible to do the compensation in either two-axis stator frame or in synchronous reference frame. In this approach, the two axis current information has to be used to determine the present location (the sector in which the current vector lies) of the stator current vector. Compensating vector can then be selected as the opposite of the effective distortion vector at that instant. The effective distortion vector at a given instant will always be one out of the six possibilities discussed in Section 8.2.3. Feed-forward compensation implemented in stator coordinates ($a\beta$ frame) is shown in Figure 8.10.

In the phase wise implementation, it can be seen that the compensation of each phase is de-activated, when the particular current crosses zero. The same disabling must be done in stator frame implementation also. Careful inspection of the three line current waveforms reveal the fact that zero crossing of any of the three currents occurs, when the position of the stator
current space vector passes the boundaries of the sectors (they are of $\pi/6 \text{ rad}$ angular width) shown in Figure 8.4. Thus, the most suitable thing to do is to disable the compensation (to have a dead zone) as the stator current vector passes these boundaries.

Both these methods will be implemented and the results will be presented in Section 8.7.

8.6.2 Compensation for on-state voltage drops

The structure for the compensation of on-state voltage drops is almost the same as that for the switching dead-time compensation. The main difference is that the computation of the compensation terms needs the line current information. The effects of these on-state voltage drops however, is negligible compared to the effect coming from the presence of dead-time specially for relatively low current applications (for low currents, the on-state resistive voltage drop component will be lower) as is the case in this application example. Thus, separate compensation was not implemented for resistive voltage drop in this work. To further strengthen this argument, some more information from the manufacturer’s data of the IGBTs used in the inverter incorporated will be presented in the next section.

8.6.3 Compensation for DC-link voltage fluctuations

DC-link voltage fluctuation model shown in Figure 8.9 reveals that it is an attenuation by the ratio between measured instantaneous DC-link voltage and the nominal DC-link voltage. As a feed forward compensation method, multiplication of the reference voltage inputs to the inverter from the reciprocal of the above ratio can be done. For this purpose, the instantaneous DC-link voltage must be sampled into the system. The scheme is shown in Figure 8.11.
Simulation and experimental validation of these methods will be done in the following section.

### 8.7 Simulation and experimental validation

Some of the basic analysis that can be done in the simulation level on inverter non-idealities will be presented here. This will be followed by experimental results from the implementation of feed-forward compensation methods discussed in the previous section. Basic measurements taken from the inverter employed will be presented as the first step.

#### 8.7.1 Basic measurements from the inverter

As pointed out earlier, in order to simulate and implement compensation methods, some basic quantities has to be measured from the inverter used in a particular application.

**(a) Dead-time**

If the dead-time information for the inverter is not given by the manufacturer, it can be determined by doing some basic measurements. This is done by observing the PWM pulse applied and the resulting output voltage pulse from the inverter, on the oscilloscope. This is done for three different constant current levels (namely, positive, negative and zero). The resulting waveforms are shown in Figure 8.12 (a), (b) and (c) respectively.

![Figure 8.12: Dead-time measurement](image_url)
When delivering a positive output current, with $T_{i_{-on}} = 104.6 \, \mu\text{s}$ and $T_{a_{-on}} = 100.8 \, \mu\text{s}$, the following equation can be obtained from (8.2) as

$$100.8 = 104.6 + T_{d_{-off}} - T_{dc} - T_{d_{-on}}$$

$$T_d = T_{dc} + T_{d_{-on}} - T_{d_{-off}} = 3.8 \, \mu\text{s}. \quad (8.23)$$

Similarly, for the negative current condition (8.3) with $T_{i_{-on}} = 93.8 \, \mu\text{s}$ and $T_{a_{-on}} = 97.8$ yields

$$97.8 = 93.8 + T_{dc} + T_{d_{-on}} - T_{d_{-off}}$$

$$T_d = T_{dc} + T_{d_{-on}} - T_{d_{-off}} = 4.0 \, \mu\text{s}. \quad (8.24)$$

When the output current is zero, (8.4) results in

$$T_{a_{-on}} = T_{i_{-on}} = 100 \, \mu\text{s}. \quad (8.25)$$

This result confirms the analytically predicted behavior of the dead-time distortion. An effective dead-time value of $4 \, \mu\text{s}$ will be used for simulation and actual implementation of compensation techniques in this work.

(b) Determination of on-state parameters for switching devices

Measurement of on-state parameters from the IGBT modules in an inverter is a difficult task. The best possible way to obtain these values is to approximate them from the characteristic curves provided in the data sheets from the manufacturer. Forward current against Forward-voltage characteristics for the IGBT module (TOSHIBA MG100Q2YS40) used in this inverter is shown in Figure 8.13.

![Forward-current Vs forward-voltage characteristics](image)

**Figure 8.13:** Forward-current Vs forward-voltage characteristics

From the characteristics it can be approximated that,

- On-state constant voltage drop ($V_g$) = $0.91 \, \text{V}$
- Device on-state resistance ($R_c$) = $64 \, \text{m}\Omega$
It was mentioned in Chapter 3 that, the rated current of the motor supplied by this inverter is 16.3A. This means that the operating point of the IGBT will not reach the ohmic region of the forward-current Vs forward-voltage characteristics. Consequently, the on-state constant voltage drop $V_g$ will be the dominant component. Hence, for compensation purposes in this application, a negligible device on-state resistance $R_c$ is assumed. In fact, it can be seen that on-state constant voltage drop is also of negligible importance. Thus, the argument presented in the previous section regarding the non-implementation of a separate compensation scheme for the on-state voltage drop compensation in this low current application is very well justified.

### 8.7.2 Harmonic distortion due to dead-time – by simulation

Total harmonic distortion of the inverter output voltage due to dead-time can be analytically found [48]. As it is out side the scope of this work, the analytical approach will not be presented here. Yet, in order to illustrate the functionality of the dead-time model suggested in the Section 8.5.1, some simulation results will be presented here. The distortion occurring in the fundamental 50Hz sinusoidal voltage (when supplying an inductive load) for the measured dead-time is shown in Figure 8.14.

![Figure 8.14: Dead-time distortion](image)

Frequency domain analysis of this will give information on the total harmonic distortion. To illustrate this, the Discrete Fourier Transform (DFT) of the above distorted voltage signal is shown in Figure 8.15.

![Figure 8.15: Dead-time distortion in frequency domain](image)

Since the on-state voltage drops and DC-link voltage fluctuation are load dependent non-idealities, simulation studies will not be done on those two. Yet, the models developed for these non-idealities in Section 8.5 can be used for simulation studies for pre defined loading conditions and DC-link ripple patterns.
8.7.3 Experimental results of dead-time compensation

The effect of dead-time compensation on the step response of \(d\)-axis current was illustrated already in Chapter 4 under the discussion on basic IMC current controllers (see Figure 4.5 of Chapter 4). Comparing the DFTs of uncompensated and compensated output currents is another method of performance evaluation in this case. The effect of the compensation method must then be visible as a reduction of harmonic content (\(5^{th}\) and \(7^{th}\) harmonics as an example).

(a) Phase-wise dead-time compensation

Phase-wise dead-time compensation scheme presented in the previous section was implemented and Figure 8.16 shows the DFTs of one line current with and without activation of the compensation scheme. The improvement achieved by the method from the reductions visible in the \(3^{rd}\) and \(5^{th}\) harmonics.

For this experiment the machine was run at 0.33 pu speed (almost closer to 50 Hz supply frequency) and remarkable improvement can be seen from the reductions of \(3^{rd}\) and \(5^{th}\) harmonic contents.

![Figure 8.16: Phase-wise dead-time compensation](image)

(b) Stator frame dead-time compensation

The stator frame dead-time compensation was also implemented and results similar to Phase-wise implementation were obtained. Frequency domain results as before (DFT of a line current with and without activation of the compensation scheme) are shown in Figure 8.17.

![Figure 8.17: Stator frame dead-time compensation](image)

The reductions visible in \(3^{rd}\) and \(5^{th}\) harmonics show that the stator frame compensation is also successful. Compensation for DC-link voltage fluctuations will be the topic of the next section.
8.7.4 Compensation for DC-link voltage fluctuations

The scheme discussed in the previous section was successfully implemented. In fact, to be able to observe a remarkable impact from the method, the machine must be sufficiently loaded and subjected to load torque fluctuations. Similar impact can be seen, when the machine is forced to a faster accelerate from rest. This also draws higher torque producing current from the DC-link capacitance causing the DC-link voltage to drop. This type of step change in speed with and without the DC-link voltage compensation is shown in Figure 8.18 as a demonstration.

![Figure 8.18: Compensation for DC-link voltage fluctuations](image)

By the successful implementation of these compensation methods the validity of the ideal inverter assumption that was made in the adaptive observer design can be increased. Apart from that the compensation in fact boosts the dynamic response of the current controllers also as was demonstrated earlier. Machine parameter estimation, which is another important task in achieving better performance of all control strategies on electrical machines is the focus of the next chapter.
9. Machine Parameter Estimation

The performance of most of the control designs discussed in this part of the thesis depends on the actual machine parameters. In the case of current controller design, the IMC controllers and predictive observer method need the accurate values of $R_s$, $L_d$ and $L_q$. The speed controller design does not need those values, if only the mechanical dynamics are considered for the design, as it was done in Chapter 5. However, the knowledge about the damping and the total inertia of the rotating parts will be required to obtain approximate controller parameters that will lead to the final tuning values faster. The adaptive observer for speed and position estimation on the other hand needs accurate values of $R_s$, $L_d$ and $L_q$. Mechanical parameters of the plant can be determined by analytical methods as well as experimental methods. In fact, the total inertia of the rotating parts in this experimental set-up was analytically found using manufacturer data and measurements, whereas, the damping was found as discussed in Appendix C, by an experimental method. The focus in this chapter is the estimation of plant electrical parameters, namely, $R_s$, $L_d$ and $L_q$ under different operating conditions. Towards the end of the chapter, some concepts on how to use the characterised variations will be presented with experimental results to justify them.

9.1 Overview of the estimation problem

Since it affects even the tuning of the current controller, not having accurate information regarding the plant electrical parameters (namely, $R_s$, $L_d$ and $L_q$) can cause performance deterioration in the closed loop current control and speed estimation. Measuring those quantities, when the machine is at standstill is fairly straightforward. The main challenge is estimating the machine parameters during the rotating condition. In fact, there are two variations that are of interest.

- Machine parameter variation with temperature
- Machine parameter variation with operating frequency or varying speed

Both these variations are of utmost importance. However, the issue of machine parameter variation with temperature is not addressed in this part of the thesis as the experimental set-up is not equipped with suitable temperature sensors to measure the temperature of the stator winding. Yet, several contributions of temperature dependent machine parameter variation can be easily found [92, 93, 94, 95]. The focus in this chapter will be on the frequency-dependent machine parameter variation.

9.1.1 Machine parameter variation with frequency

One can identify two aspects of characterising the frequency-dependent machine parameter variation.

(a) Parameter variation within the operating frequency range

Characterising this variation will give the information regarding the effective machine parameters of the real machine at each and every steady state operating points within the operating frequency range. This information is important for better performance of the speed and position estimation algorithms, as one has to use the effective machine parameters at each operating point in order to obtain best possible estimates from the algorithms.

(b) Parameter variation above the operating range of frequencies
When a step change of \( d \)-axis or \( q \)-axis current is made, it excites the high frequency modes of the process electrical dynamics. Thus, as far as the transient behaviour of the motor currents is concerned, what matters are the machine parameter values at high frequencies. Therefore, all current controller designs could require the machine parameter values at high frequencies (in the vicinity of desired closed loop bandwidth). This motivates the need to study the high frequency dependency of electrical machine parameters.

When approaching to the problem from the digital control theory point of view, several system identification techniques can be applied. Among them, the Maximum Likelihood approach [96, 97] and the Recursive Estimation [98] seem to be widely used. In some sensorless control methods, a machine model is incorporated in the speed and rotor position estimation algorithm. Even in the adaptive observer approach presented here, machine parameters are essentially used, though a direct machine model is not involved in the estimation strategy. The model based approach is further improved in some contributions by developing complex models for the machine, taking parameter variations and loss components into account [99, 100].

The direct measurement from the machine using some methods at different supply frequencies is a straightforward way of characterising the machine parameter variation with frequency. However, the fact that a PMSM is supplied by an inverter (in many cases) with a switched voltage waveform makes it difficult to establish reliable measuring strategies. This is somewhat easier, if the machine is operated in the generating mode or if the machine is fed with continuous sinusoidal line voltages. A novel general method of determining \( d \) and \( q \)-axis reactance of PMSMs without rotor position information has been suggested in [101]. The approach presented in this work is a combination of improved machine modelling and direct measurement from the machine.

### 9.2 Physical insight into frequency dependent parameter variation

Having a better physical understanding about how the machine parameters are likely to change with varying supply frequency is important. This can also be useful in developing improved circuit models that take into account frequency dependent machine parameter variations of the PMSM. This can be done to a certain extent by looking into the physical arrangement of one machine pole of a surface mounted PMSM in \( d \) and \( q \) axes as shown in Figure 9.1.

![Figure 9.1: Physical arrangement of one machine pole](image-url)
Since the permeability of permanent magnets are assumed to the same as that of air, the magnetic
circuit along the $d$-axis has less iron. This makes the reactance along $d$-axis lower than the
reactance of $q$-axis. Hence, $d$-axis inductance $L_d$ will be lower than the $q$-axis inductance $L_q$.

In the same way, some predictions can be made on the expected variation of resistance along the $d$
and $q$-axis conducting paths with varying supply frequency. In fact, these resistance variations refer
to the active losses occurring along each path, when a current of a certain frequency is flowing.
When looked upon in this manner, the effective $d$ and $q$-axis resistances ($R_d$ and $R_q$ ($R_d$, $R_q$) respectively) at a given supply frequency can be defined in terms of the active power losses along
each conducting path ($P_{ad}$ and $P_{aq}$). Since the $d$-axis has got a longer iron path due to the
permanent magnets, the effective resistance $R_d$ along $d$-axis will be slightly higher than $R_q$. However,
this approach needs some modifications to the basic PMSM circuit model as the active losses in the
machine have two components.

1. Copper loss in the stator windings
2. Core loss in each magnetic circuit

The copper loss in the stator will be due to the constant resistance ($R_s$) in the stator winding as long
as the supply frequencies of interest are well below the frequency range that causes significant skin
effect in the stator conductors. The core loss has two components: hysteresis loss and eddy-current
loss. The eddy current loss is proportional to the square of the supply frequency, while hysteresis
loss is proportional to it [13]. However, these two losses can be approximately represented by
assuming a constant resistance ($R_c$) (thus neglecting the hysteresis losses) in parallel to the machine
internal voltage as shown in the electrical circuit for phase A in Figure 9.2 [99].

![Figure 9.2: Modified circuit model for phase A](image)

This new per-phase circuit can now be used to derive a more accurate $dq$ frame model for PMSM
as in [99]. This way of modelling the PMSM can lead to a more complex model in $dq$ frame, which
may not be Linear Time Invariant (LTI) due to the frequency dependent nature of the resulting
machine parameters. This can be avoided by using the already known time scale difference between
the speed and current dynamics in the control system. This will be further explained at the end of this
chapter under the concluding remarks. The measurement strategy used in this work leads to have a
simpler approach, which will be explained in the next section.

### 9.3 Estimation strategy in this work – Locked rotor measurements

The aim in this work is to come up with a simple machine parameter estimation strategy that can be
used to estimate the $d$ and $q$-axis machine parameters ($R_d$, $R_q$, $L_d$ and $L_q$) effective at a given
operating frequency assuming a per-phase model similar to that shown in Figure 9.2. The methods
will be verified with experimental results. The parameter estimation method will be briefly introduced here before moving into the detailed analysis.

9.3.1 Brief description of locked rotor measurements

The d-axis of the rotor is aligned with the resultant magnetic axis of phases A and B and locked. Aligning the rotor in this manner can be easily and accurately done by injecting a sufficiently large current into phase A and taking out from phase B. Accurate rotor position measurement is not required for this purpose. A variable frequency sinusoidal voltage source (single-phase) is used to supply the stator windings of the machine, connected in different combinations. Using a Dynamic Signal Analyser, it is now possible to measure the power spectrums of the input current and voltage and also the magnitude and phase of the cross-spectrum (i.e.: voltage/current). Using these information it is possible to compute the resistive and inductive components in the conducting path formed by the particular winding combination at different frequencies. If the relationship between the winding combinations and $R_d$, $R_q$, $L_d$ and $L_q$ is known, the machine parameter values can easily be computed at each supply frequency. Details of the winding connections and the relationships of interest will be presented later in the next section.

9.3.2 Theory behind stationary measurement method

The method is especially suitable for machines that do not have access to the internal star point of the star-connected three-phase stator windings. Since the measurements are done under stationery condition, the back emf term shown in Figure 9.2 vanishes resulting a passive RL circuit ($R_s$ and $L_l$ will be in series with the parallel combination of $R_c$ and $L_{ma}$). The arrangement of the three windings in terms of their magnetic axes and the alignment of the permanent magnet rotor, when a current is injected from phase A and taken out from phase B, with the phase C open circuit is shown in Figure 9.3.

The rotor position shown in Figure 9.3 is the locked rotor orientation maintained throughout the tests. Two sets of measurements are required with two different combinations of stator winding connections to compute all the machine parameters ($R_d$, $R_q$, $L_d$ and $L_q$). The two methods of connection are shown in Figure 9.4 (a) and (b).

![Figure 9.3: Rotor orientation for stationery measurements](image)

![Figure 9.4: Different stator winding for parameter estimation](image)
The possible estimations from each connection method will be explained separately.

(a) Injection into phase A and taken out from phase B (Connection (a))

When this current injection is done, the magnetic flux produced will be in the direction of the resultant magnetic flux direction of the phases A and B (see Figure 9.3 and 9.4 (a)). Thus, the flux path will be along the permanent magnet flux direction with this way of rotor alignment. This means that the d-axis machine parameters can be estimated from the measurements taken from this test. The equivalent impedance \( z_{ed} (\omega) \) – is the resultant impedence of the passive network formed by \( R_s \) and \( L_l \) in series with the parallel combination of \( R_c \) and \( L_{aa} \) – of this winding connection can be given as

\[
\begin{align*}
  z_{ed} (\omega) &= 2R_s + 2R_s(\omega) + j2\omega[L_r + L_s(\omega)],
\end{align*}
\]

where

\[
\begin{align*}
  R_s(\omega) &= R_s^2 \frac{\omega^2 L_{aa}^2}{R_c^2 + \omega^2 L_{aa}^2},
\end{align*}
\]

and

\[
\begin{align*}
  L_s(\omega) &= R_c^2 \frac{\omega^2 L_{aa}^2}{R_c^2 + \omega^2 L_{aa}^2}.
\end{align*}
\]

This shows that the equivalent circuit reduces to a simple RL series network across inputs A and B. The simple analysis below will give the relationship of the resistance and inductance estimates made from the test to \( R_{d} \) and \( L_{d} \).

With this configuration of winding connection the phase currents in A and B have the relationship

\[
\begin{align*}
  i_b &= -i_a.
\end{align*}
\]

The expressions for flux linkages produced in each phase, when (9.4) is also taken into account will be

\[
\begin{align*}
  \psi_a &= (L_r + L_i) i_a - \frac{1}{2} L_r(-i_a)
  &= \left( \frac{3}{2} L_r + L_i \right) i_a, \quad \text{(9.5)}
\end{align*}
\]

and

\[
\begin{align*}
  \psi_b &= -\frac{1}{2} L_r(i_a) + (L_r + L_i)(-i_a)
  &= -\left( \frac{3}{2} L_r + L_i \right) i_a, \quad \text{(9.6)}
\end{align*}
\]

Thus the total flux linkage produced by this winding configuration will be

\[
\begin{align*}
  \psi_a - \psi_b &= 2\left( \frac{3}{2} L_r + L_i \right) i_a, \quad \text{(9.7)}
\end{align*}
\]

which result in the expression for the line-to-line inductance between phase A and phase B to be given by
\[ \Psi_{ab} = L_{ab}i_a = \Psi_a - \Psi_b = 2\left(\frac{3}{2}L_r + L_l\right)i_a \]

\[ L_{ab} = 2\left(\frac{3}{2}L_r + L_l\right) \]  

(9.8)

Analogous to Equation (2.20) the \textit{d-axis} inductance can now be computed as

\[ L_d = \frac{1}{2}L_{ab} = \left(\frac{3}{2}L_r + L_l\right). \]  

(9.9)

Therefore, the inductance estimated by supplying this winding configuration would be equal to twice the \textit{d-axis} inductance. When the power spectrum magnitude and phase information is available, it is possible to compute the active losses along the conducting path. With this particular configuration, it will simply be the active losses in the \textit{d}-direction of the rotor; hence the so-called \textit{d-axis} resistance can be computed from this active loss component. According to Equation (2.19) of Chapter 2, the \textit{d-axis} resistance is simply the resistance per phase of the stator winding, when its flux vector is directing along the \textit{d-axis} of the rotor (which is usually taken as \( R_s \), if frequency dependency is not taken into account). Since two phases are in series here, the estimated resistance equivalent to loss component \([2R_s + 2R_s(\omega)]\) gives an estimate of twice the \textit{d-axis} resistance. The \textit{d-axis} resistance and inductance estimates from the test can therefore be computed from the equations

\[ R_d = \frac{\text{Estimated resistive component}}{2}, \]  

(9.10)

and

\[ L_d = \frac{\text{Estimated inductive component}(L_{ab})}{2}. \]  

(9.11)

Once the experimental variations have been obtained, a curve fitting can be made to verify the theoretical validity of the results. This can be done according to the functions

\[ R_d(\omega) = R_s + R_s(\omega) = R_s + \frac{k_1^{rd} \omega^2}{1 + k_2^{rd} \omega^2} \]  

(9.12)

and

\[ L_d(\omega) = \frac{3}{2}L_r(\omega) + L_l = \frac{k_1^{ld}}{1 + k_2^{ld} \omega^2} + L_l \]  

(9.13)

respectively \((k_1^{rd}, k_2^{rd}, k_1^{ld}, k_2^{ld}\) are constants). However, since an accurate estimate of \( L_l \) is not available, that term in (9.13) will have to be neglected for the curve fit.

\textbf{(b) Injection into phase C and taken out from phases AB in parallel (Connection (b))}

When this current injection is done, the magnetic flux produced will be in the direction of the magnetic flux of the phase C (see Figure 9.3 and 9.4 (b)). The flux produced by phases A and B perpendicular to that direction will oppose each other and hence cancel out. Thus, the flux path will be along the direction perpendicular to permanent magnet flux direction. This means that the \textit{q-axis} machine parameters can be estimated from the measurements taken from this test.
The equivalent impedance \( z_{eq}(\omega) \) – is the resultant impedance of the passive network formed by phase C having modified circuit similar to Figure 9.2 series with the parallel combination of phase A and B also having modified circuits. Voltage sources representing the back emfs in all three phases vanish, since the rotor is stationary.) of this winding connection referring to Figure 9.4 (b) can be given as 

\[
\frac{3}{2}R_s + \frac{3}{2}R_r(\omega) + j\frac{3}{2}\omega[L_I + L_q(\omega)].
\]

(9.14)

This shows that the corresponding equivalent circuit again reduces to a simple RL series network across inputs C and AB. The simple analysis below will give the relationship of the resistance and inductance estimates made from the test to \( R_q \) and \( L_q \). With this configuration of winding connection the phase currents in A and BC have the relationship

\[
i_a = i_b = -\frac{1}{2}i_c.
\]

(9.15)

The expressions for flux linkages produced in each phase in usual notation, when (9.15) is also taken into account will be

\[
\psi_c = -\frac{1}{2}L_c\left(-\frac{i_c}{2}\right) - \frac{1}{2}L_r\left(-\frac{i_c}{2}\right) + (L_c + L_q)i_c
\]

\[
= \left(\frac{3}{2}L_c + L_q\right)i_c,
\]

(9.16)

and

\[
\psi_b = -\frac{1}{2}L_c\left(i_c\right) + (L_c + L_q)\left(-\frac{i_c}{2}\right) - \frac{1}{2}L_r\left(-\frac{i_c}{2}\right)
\]

\[
= -\left(\frac{3}{4}L_c + \frac{L_q}{2}\right)i_c.
\]

(9.17)

Thus, the total flux linkage produced by the windings C and B will be

\[
\psi_c - \psi_b = \frac{3}{2}\left(\frac{3}{2}L_c + L_q\right)i_c,
\]

(9.18)

which result in the expression for the line-to line inductance between phase A and phase B to be given this time by

\[
\psi_{cb} = L_{cb}i_c = \psi_c - \psi_b = \frac{3}{2}\left(\frac{3}{2}L_c + L_q\right)i_c
\]

\[
L_{cb} = \frac{3}{2}\left(\frac{3}{2}L_c + L_q\right)
\]

(9.19)

Analogous to Equation (2.20) the \( q\)-axis inductance can now be computed as

\[
L_q = \frac{2}{3}L_{cb} = \left(\frac{3}{2}L_c + L_q\right).
\]

(9.20)

Therefore, the inductance estimated by supplying this winding configuration would be equal to three-half of the \( q\)-axis inductance. When the power spectrum magnitude and phase information is
available, it is possible to compute the active losses along the conducting path as it was explained earlier. With this particular configuration, it will simply be the active losses (factored) in the \( q \)-direction of the rotor. According to Equation (2.19) of Chapter 2, the \( q \)-axis resistance will simply be the resistance per phase of the stator winding, when it is producing the flux vector along \( q \)-axis direction. Since phase C is in series with the two phases A and B in parallel, the loss component gives an estimate of three-half the \( q \)-axis resistance. The \( q \)-axis resistance and inductance estimates from the test can therefore be computed from the equations

\[
R_q = \frac{2}{3} \text{(Estimated resistive component)} \tag{9.21}
\]

and

\[
L_q = \frac{2}{3} \text{(Estimated inductive component)}. \tag{9.22}
\]

Curve fitting similar to \( d \)-axis parameters can be performed here also and the corresponding functions for \( R_q(\omega) \) and \( L_q(\omega) \) will be of similar structure as described by (9.12) and (9.13) respectively.

### 9.3.3 Method of computation from the measurements

The method of computation from the measurements will be shown here to give a better insight into the estimation procedure.

#### (a) Computation of \( d \)-axis parameters

If it is assumed that following are the data recorded at a certain frequency \( \omega \ \text{rad/s} \) with winding connection (a) in Figure 9.4.

- Power spectrum magnitude of voltage = \( V_d \ \text{V} \)
- Power spectrum magnitude of current = \( I_d \ \text{A} \)
- Cross spectrum magnitude = \( Z_d V_{rms}^2 \)
- Cross spectrum phase = \( \Phi \ \text{rad} \)

\( R_s \) is available as manufacturer data and can also be measured easily by an ohmmeter.

Equating the active power dissipation in the network results in

\[
R_d(\omega) = R_s + R_q(\omega) = \frac{V_d I_d \cos(\Phi)}{2 I_d^2}. \tag{9.23}
\]

Equating the reactive power dissipation in the network results in

\[
L_q(\omega) = \frac{3}{2} L_r(\omega) + L_s = \frac{V_d I_d \sin(\Phi)}{2 \omega I_d^2}. \tag{9.24}
\]

#### (b) Computation of \( q \)-axis parameters

If it is assumed that following are the data recorded at a certain frequency \( \omega \ \text{rad/s} \) with winding connection (b) in Figure 9.4.

- Power spectrum magnitude of voltage = \( V_q \ \text{V} \)
Power spectrum magnitude of current $= I_q \, A$

Cross spectrum magnitude $= Z_q V_{\text{rms}}$

Cross spectrum phase $= \Phi \text{ rad}$

Equating the active power dissipation in the network results in

$$R_q(\omega) = R_s + R_e(\omega) = \frac{2}{3} \times \frac{V_q I_q \cos(\Phi)}{I_q^2}.$$  \hfill (9.25)

Equating the reactive power dissipation in the network results in

$$L_q(\omega) = \frac{3}{2} L_r(\omega) + L_i = \frac{2}{3} \frac{V_q I_q \sin(\Phi)}{\omega l_q^2}.$$  \hfill (9.26)

### 9.3.4 Computed parameter variations

Figure 9.5 shows the $d$ and $q$-axis effective resistance variations ($R_q(\omega), R_e(\omega)$) with frequency, while Figure 9.6 shows the variations of the two inductances ($L_q(\omega), L_i(\omega)$) with frequency in per-unit.

![Figure 9.5: Resistance variations](image)

![Figure 9.6: Inductance variations](image)
Some suggestions on making use of these variations to improve the performance of control strategies in this work will be presented in the following section.

9.4 Making use of the estimated parameter variations

In the beginning of this chapter it was mentioned that there are two frequency regions of interest, when it comes to the study of frequency-dependent machine parameter variations. The two regions are

- Operating frequency range
- Frequencies above the operating range

Since the machine was operated within the speed range 0-3 pu, the operating frequency range in this case was from 0 – 450 Hz. As the measurements were taken up to 2 kHz, a reasonable coverage of both these frequency ranges has been made. In this section, some concepts will be presented on making use of the obtained parameter variations. The operating frequency range will be considered first.

9.4.1 Use of parameter variation within the operating frequency range

This machine parameter variation refers to effective resistance and inductance values of the machine in the steady state operation at a particular supply frequency. Thus, this is of interest for the sensorless control algorithms presented in Chapter 7. It can be seen from the Equations (7.16) and (7.17) of Chapter 7 that the $d$-axis voltage error computation incorporates the estimated values of $R_s$ and $L_q$. In fact, since it is the $d$-axis voltage equation, which is used here, it must now be modified to

$$
\varepsilon_d = -\left(u_d' - \hat{R}_d(\hat{\omega})y_d' + \hat{\omega}\hat{L}_q(\hat{\omega})y_q'\right)
$$

and

$$
\varepsilon_d = -\left(u_d' - \hat{R}_d(\hat{\omega})y_{d,ref}' + \hat{\omega}\hat{L}_q(\hat{\omega})y_{q,ref}'\right)
$$

according to the new way of considering different $d$ and $q$-axis resistance values discussed in this chapter. It can now be seen that both $R_d$ and $L_q$ play an important role in the error voltage computation. Thus, during a considerably large speed change, the frequency dependency of the two parameters can affect the accuracy of computed $\varepsilon_d$. Obviously this affects the estimated speed and rotor position also. Since the frequency dependent variation of the two parameters are now available from the measurements, one remedy is to change the two parameters with the operating frequency of the machine. This will be illustrated here.

(a) Use of frequency dependent $d$-axis resistance term for $\varepsilon_d$ computation

A curve, which obeys Equation (9.12), can be fitted to the $R_d$ variation at 3 A current level shown in Figure 9.5 as mentioned before. An optimization routine based on least square approximation method was used in this case [102]. This is shown in Figure 9.7 and the corresponding equation for the $R_d$ variation in per-unit will be

$$
R_d(\omega) = R_0 + R_1(\omega) = 0.037 + \frac{(0.005525)\omega^2}{1 + (0.001283)\omega^2}
$$

(9.29)
Figure 9.7: Curve fitting for the variation of $R_d$

The Equation (9.29) can now be used to vary the term $R_d$ on-line in the equations for $\varepsilon_d$ computation ((9.27) or (9.28)). This was implemented in real-time and the position and speed estimation errors for the same step response test (0.3 pu to 1.5 pu) with on-line $R_d$ updating mechanism is shown in Figure 9.8. For these experiments, the filtered estimated speed itself was used as the frequency value ($\omega$) in (9.29).

(a) Estimation errors with on-line $d$-axis resistance updating

(b) On-line variation of resistance term during step change

Figure 9.8: Improved performance with on-line resistance term variation
These variations may be compared with the corresponding variations for the same step response initially shown in Figure 7.8 and 7.9 of Chapter 7. A significant improvement in the position estimation error variation (steady state error has reduced) can be observed.

It must be mentioned here that the resistance variation used was not the best possible. It was obtained based on the measurements done at a supply current level of 3 A. One must consider the current level of the machine at each operating frequency also and fit a suitable curve to obtain the best $R_d$ variation. However, obtaining the $R_d$ variation at several current levels was not possible in this case due to limitations of the sinusoidal voltage supply used for the measurements.

(b) Effect of frequency dependent $q$-axis inductance term for $e_d$ computation

In the same way, the $L_q$ term of the $e_d$ computation can also be characterized from a fitted curve according to (9.13). With the assumption that the leakage induction term is negligible, the corresponding equation that characterises the $L_q$ variation at 3 A can be given as

$$L_q(\omega) = \frac{0.3494}{1 + (8.061e^{-\omega})\omega^2}.$$  (9.30)

From (9.30) it can be seen that the variation can be neglected and hence no further study on the effect of frequency dependent variation of $L_q$ was done in this work. It must also be noted here about the rotor oscillations that occurred, when the measurement on the $q$-axis was carried out. This is due to the torque pulsations caused by the stationary flux wave created by the sinusoidal injected current. These oscillations can cause errors in the $L_q$ estimations.

9.4.2 Use of parameter variation above the operating frequency range

It was mentioned and also experimentally verified in Chapter 4 that a value higher than the measured stator resistance has to be used in the basic IMC design to achieve sufficient integral action in the current controller. In fact this was the same with advanced DC-IMC design. This observation can now be explained with the frequency dependent $d$ and $q$-axis resistance variations.

The best approximation for the effective machine parameter values at a selected closed loop bandwidth $\alpha$ pu can be taken as the machine parameter values corresponding to the frequency $\alpha$ pu, which can be selected from the frequency dependent variations.

Since the insufficient integral action was the basic problem encountered, a criterion can be proposed to select the suitable resistance value that must be used in place of integral gain of the basic IMC and DC-IMC designs. This must be based on the frequency dependent variation of the $d$ and $q$-axis resistance. If the variation shown in Figure 9.7 and described in the Equation (9.29) is taken as an illustration, the $d$-axis resistance at 4 pu frequency is found to be 0.1236 pu. This is approximately 2.38 times the DC resistance ($R_s=0.037$ pu) of $R_d$. In Chapter 4 it was noticed that $R_d$ must be approximately doubled from the value of $R_s$ to obtain reasonably good step response characteristics. Based on these observations, the criterion for resistance term selection can now be expressed as below.

**Criterion for resistance term selection in IMC designs:**

“If the desired closed loop bandwidth of the current controller is $\alpha$ pu, the corresponding $R_d$ value (if the design is for $d$-axis) to be used in IMC and DC-IMC controllers can be computed from (9.24) as
\[ R_d(\alpha) = R_s + R_v(\alpha) = 0.037 + \frac{(0.005525)\alpha^2}{1 + (0.001283)\alpha^2}. \] (9.31)

This is verified for the \textit{d-axis} with \( \alpha = 4 \) pu \((R_d(4) = 0.1236 = 2.38 \times R_s \text{ pu})\) in Figure 9.9 for the basic IMC design described by (4.20) of Chapter 4. This criterion may be used as a “rule of thumb” for fine-tuning of the current controllers designed based on the IMC principle provided that the frequency dependant \( d \) and \( q \)-axis resistance variations are available to the designer.

\textbf{Figure 9.9:} Verification of the design criterion for current controllers

In the same way \textit{q-axis} current controller design must be done based on the fitted curve for frequency variation of \( R_q \). Since there is no significant variation of the inductance terms on both axes, its effect on the proportional gain can be neglected and the manufacturer data can straight away be used in place of the inductance term in the IMC and DC-IMC controller equations.

\textbf{9.5 Concluding remarks on estimation method}

As it was mentioned earlier, this way of modelling the core loss leads to frequency varying machine parameters. This can cause serious analytical problems to the dynamic modelling of the PMSM. The time varying nature of the machine parameters results in a system that is not LTI. It must be noted here that what has been done in [99] is the steady state analysis of the new machine model in \( dq \) frame. The dynamic modelling of the new machine structure in \( dq \) frame can be an interesting topic for further research. With a quasi-steady approximation, any speed change may be assumed to take place by following a sequence of operating points that are quasi-stable in terms of the \( d \) and \( q \)-axis currents. Thus, simple replacement of the resistance terms in the original error voltage calculations with the newly estimated effective resistance term is very well justified.

In the same way, the design criterion for the current controller is also in total agreement with LTI theory. In this case the argument is that the system model for the current controller design must contain machine parameter values effective at a frequency corresponding to the expected closed loop bandwidth of the current dynamics.

Another important assumption was the constant stator resistance term \( R_s \). For this to be true, the skin effects must be negligible within the operating frequency range of the machine. The diameter of the stator windings for the machine used in the test rig was not available from the manufacturer data. If that information is available, it is recommended to check whether the assumption of constant \( R_s \) over the whole operating frequency range is really valid based on empirical equations that can be found in the literature [103].
10. Correlation of results, conclusions and future work

In this chapter, the performance of the PMSM drive with all the control strategies will first be verified using some experimental results. This will be followed by some conclusions and the main contributions of this work will also be highlighted. Suggestions for future research based on the results obtained in this work will be made towards the end of the chapter.

10.1 Performance of the complete control strategy

Successful performance of a sensorless control drive strategy is always an integration of many digital motion control techniques. A reliable rotor position and speed estimation strategy, fast current control, suitable speed control with a reliable controller anti-windup mechanism and compensation techniques for the inverter non-idealities are essential features of such a control scheme. All these issues have been addressed in this work and each method suggested was verified both by simulation and experimental results. Before winding up this part of the thesis, some results must be presented on the performance of the sensorless control drive system, with all these control strategies. These results will be useful in predicting or estimating what can be expected from these control strategies, when used with a drive having similar specifications as the reference drive (see Section 3.1 of Chapter 3).

(a) Start-up and acceleration using first algorithm in [6]  
(step change of speed set-point is made with the Kick-start)

(b) Acceleration from 0.2 pu to 2 pu using ramp tracking algorithm

Figure 10.1: Start-up and acceleration up to 2 pu speed

Figure 10.1 (a) shows the start-up of the PMSM drive with Kick-start and acceleration up to 2 pu (6000 rpm) speed using the first algorithm. Acceleration from 0.2 pu up to 2.0 pu speed using the ramp tracking algorithm is shown in Figure 10.1 (b).
Figure 10.2 shows the important variations of several variables, when a load torque of approximately 20% of the rated torque is applied, while the machine is rotated at 1 pu speed with the ramp tracking sensorless control algorithm. Estimated speed, measured speed, speed and rotor position estimation errors and corresponding $q$-axis current variation have been plotted.

10.2 Conclusions and contributions

This work has produced a complete sensorless control drive strategy for surface mounted PMSMs working at high-speed range. All digital motion control strategies suggested in this work have been experimentally verified and the practical difficulties associated with their implementation have also been theoretically dealt with. The investigation on the sensorless control strategy first suggested in [6] shows promising results to confirm its usefulness in the high-speed range. The new algorithm with added ramp tracking capability also proves to be applicable in the high-speed range. In fact, the second algorithm is a better contender for applications such as angle grinders. This is because the machine can be accelerated up to the rated speed in a specified time by ramping up the speed set value and vice versa. The start-up method suggested based on the robustness of the two algorithms is also suitable for applications similar to an angle grinder, where the machine is not loaded until it is accelerated up to the rated speed.
The other auxiliary techniques suggested within this part of the thesis contribute to achieve the final goal of well performing sensorless control drive. One of the two methods suggested for current control (i.e. predictive observer based method and DC-IMC) with compensation for sampling delay can be incorporated to achieve a high closed loop current control bandwidth. The conditioning technique based anti-windup mechanism improves the reliability of the drive and also helps to obtain faster closed loop performance for speed. In fact, the method opens up several new possibilities to improve the drive performance. More importantly, the concepts presented can be extended for other type of AC drives also. Compensation schemes for inverter non-idealities have also improved the performance. The methods suggested to incorporate the machine parameter variations into the sensorless control scheme has also improved the quality of the speed and rotor position estimations. This approach has also produced a new criterion to tune the current controllers of a PMSM drive.

10.2.1 Contributions of this work

The main contributions of this work according to the knowledge of this author will be highlighted within this section.

(a) DC-IMC for sampling delay compensation in current control

The advanced current controller suggested based on internal model control principle is believed to be a new contribution that has not been applied in machine control area. The general derivation, which shows that explicit integral action can be achieved from the controller of any order is also an important result.

(b) Conditioning technique based integrator anti-windup mechanism

Treating the speed and current control loops as a series cascaded system in tackling the controller windup problem is clearly a new approach in machine control area. The method has improved the reliability of the drive significantly. The concepts suggested based on the preliminary implementation are also novel. “Relaxed current saturation” helps to improve the transient performance, while the method of “On-line emulation of DC-link voltage fluctuations” improves the reliability. The “Constant current operation below set-speed” and “Constant voltage operation below set-speed” can be used to improve the safety of the drive system under fault conditions.

(c) Investigation of position and speed estimation algorithm suggested in [6]

Practical implementation of the sensorless control algorithm proposed by Harnefors [6] and investigation of its performance under practical constraints can be mentioned as a major contribution resulting from this part of the thesis. Investigation on the region of attraction, which led the way to the start-up technique, can also be highlighted under this.

(d) New rotor position and speed estimation algorithm

The modified algorithm for speed and position with the capability of tracking speed ramps is a clear contribution to the sensorless control area and can be very useful in several digital motion control applications.

(e) Start-up method – Kick-start

The “Kick-start” method suggested for the two algorithms is also a new addition to this voltage error based estimation method that has not been reported in [6].
Correlation of results, conclusions and future work

(f) Incorporating frequency dependent machine parameter variation

The frequency dependent stator resistance variation in the \(d\)-axis voltage error computation suggested in Chapter 9 is also a new addition to the estimation algorithm. The criterion suggested for current controller tuning can also be highlighted as a new approach suggested based on the implementation issues come across during this work.

10.3 Future work

The work done in this part of the thesis has raised several important issues that could lead to promising results after continued research. They will be mentioned in this section.

(a) Use of predicted currents in the estimation algorithm

One step-ahead prediction method suggested in Chapter 4 for sampling delay compensation could be incorporated to improve the estimation accuracy of the rotor position and speed estimation algorithm. If Equation (7.17) is used for the \(d\)-axis voltage error computation, it will be possible to use these predicted current values instead of measured currents.

(b) Conditioning technique based integrator anti-windup scheme

The stability of the cascaded integrator anti-windup method suggested in Chapter 6 must be further investigated. A Liyapunov approach can be helpful as was mentioned in Section 6.6.2. Apart for that, generalization of this method for any AC drive in variable speed operation is also a significant contribution to the area. In fact, the method will be somewhat complex, when applied to an induction motor in flux weakening operation. This is because both \(d\) and \(q\)-axis currents are changed during this type operation and hence the corresponding saturation must be defined as in Equation (6.1) in Chapter 6.

(c) Future work on estimation algorithms

One important point to raise here is the investigation of the possibility to incorporate \(q\)-axis voltage error also into the estimation algorithm. The speed reversal of the machine, when operated from the sensorless control algorithm will also be an interesting area to investigate. Improvements to the “Kick-start” method must also be investigated. This is necessary since the procedure suggested here can fail, if the machine is loaded initially.

(d) Machine parameter estimations

Further research is essential to make concrete conclusions on the suggestions made regarding incorporating the frequency dependent machine parameter variations. One point to mention is the recording the parameter variation at different current levels to obtain a better understanding on the behavior of equivalent stator resistance (as defined in Chapter 9) variation with frequency. The concepts presented on current controller tuning can also be studied further by following this approach. However, the accuracy of the measurements taken from these locked rotor tests must be verified by conducting some form of an experiment to estimate the machine parameters under rotating conditions. One possible approach can be as explained in [101].
Reference


Sensorless control of PMSMs for high-speed…


Appendix A: Per-unit notation

In this report, for all modeling and analytical purposes, the fundamental base values are taken as,

- **Voltage**: $U_{base}$
- **Current**: $I_{base}$
- **Electrical angular frequency**: $\omega_{base}$
- **Time**: $t_{base}$

In this Appendix, only the basic derivations that are important for the modeling and analytical purposes in this work will be presented.

### A.1 Derivation of other base values

Base values for all the other quantities involved can be derived, by using their mathematical relationships with the fundamental base values. They are shown in Table A.1.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Equation</th>
<th>Equation Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux</td>
<td>$\Psi_{base} = U_{base}/\omega_{base}$</td>
<td>A.1</td>
</tr>
<tr>
<td>Impedance</td>
<td>$Z_{base} = U_{base}/I_{base}$</td>
<td>A.2</td>
</tr>
<tr>
<td>Inductance</td>
<td>$L_{base} = Z_{base}/\omega_{base}$</td>
<td>A.3</td>
</tr>
<tr>
<td>Mechanical angular speed</td>
<td>$\omega_{base,mech} = 2.\omega_{base}/p$</td>
<td>A.4</td>
</tr>
<tr>
<td>Power</td>
<td>$S_{base} = K.U_{base}.I_{base}$</td>
<td>A.5</td>
</tr>
<tr>
<td>Torque</td>
<td>$T_{base} = S_{base}/\omega_{base,mech}$</td>
<td>A.6</td>
</tr>
<tr>
<td>Time</td>
<td>$t_{base} = 1/\omega_{base}$</td>
<td>A.7</td>
</tr>
<tr>
<td>Inertia</td>
<td>$J_{base} = T_{base}/\omega_{base}.\omega_{base,mech}$</td>
<td>A.8</td>
</tr>
<tr>
<td>Damping constant</td>
<td>$B_{base} = T_{base}/\omega_{base,mech}$</td>
<td>A.9</td>
</tr>
</tbody>
</table>

**Table A.1: Definitions of base values**

### A.2 Why the time is normalised?

It is not a common practice to normalise the time. Normalising the time according to the way that has been suggested gives some advantages. It is illustrated by means of an example here.
The simple differential equation describing the \textit{RL} series circuit shown in Figure A.1.

\[ L \frac{di}{dt} + Ri = u \] \hspace{1cm} (A.10)

If all the other quantities except time are normalised in this equation, the result will be

\[ L_n \cdot L_{\text{base}} \frac{d(i_n \cdot i_{\text{base}})}{dt} + R_n \cdot Z_{\text{base}}(i_n \cdot i_{\text{base}}) = u_n \cdot U_{\text{base}} \]

\[ L_n \cdot \frac{Z_{\text{base}}}{\omega_{\text{base}}} \frac{d(i_n \cdot i_{\text{base}})}{dt} + R_n \cdot Z_{\text{base}}(i_n \cdot i_{\text{base}}) = u_n \cdot U_{\text{base}} \] \hspace{1cm} (A.11)

\[ \frac{L_n}{\omega_{\text{base}}} \frac{d}{dt} + R_n \cdot i_n = u_n. \]

In contrast, if the time is also normalised it is easy to show that

\[ L_n \frac{di_n}{dt} + R_n \cdot i_n = u_n. \] \hspace{1cm} (A.12)

Unlike equation (A.11), the equation with normalised time, (A.12) appears exactly the same as (A.10). This means that the time normalised per-unit equations appear to be the same as their real valued equations. This is an advantage from the analytical point of view as the possibility of making errors by accidentally omitting certain factors is minimum in this way of normalising.
Appendix B: Dynamic modeling of non-salient PMSMs

For surface mounted PMSMs with negligible saliency, some assumptions can be made that will eliminate the need of involving the three-phase voltage equations that is usually done in the conventional approach. The important assumption made here is assuming that the stator inductance does not change as the rotor rotates in the air gap. This will remove the angle dependency of the inductance matrix in (2.15). Figure B.1 (a), shows the basic principle of PMSM. Corresponding dynamic equivalent circuit is shown in Figure B.1 (b).

![Figure B.1: PMSM – basic principle and equivalent circuit](image)

B.1 PMSM in stator coordinates

In space vector form, the input voltage to the stator must hold the following relationship in stator coordinates given by

\[ u_s^* - R_s i_s^* - \frac{d\theta_s^*}{dt} = 0 \]  

(B.1)

The stator flux linkage has two components described by

\[ \theta_s^* = L_s i_s^* + \theta_m \]  

(B.2)

Since \( \Psi_m \) is the flux in the air gap oriented in the direct axis (\( d \)-axis) due to permanent magnets on the rotor, it can be expressed in the stator reference frame as

\[ \theta_m = \varphi_m e^{j\psi} \]  

(B.3)

\( \Psi_m \) is the magnitude of the permanent magnet flux linkage. This yields in

\[ \frac{d\theta_s^*}{dt} = L_s \frac{di_s^*}{dt} + \frac{d}{dt} \left( \varphi_m e^{j\psi} \right) \]  

(B.4)

Since \( \theta_s^* = \omega_s \), (B.1) can be rewritten as

\[ u_s^* - R_s i_s^* - \left( L_s \frac{di_s^*}{dt} + j\omega_s \varphi_m e^{j\psi} \right) = 0. \]  

(B.5)
This results in the following complex differential equation describing the PMSM in stationary reference frame.

\[ L_s \frac{di_s}{dt} = u_s' - R_s i_s' - j\omega_s \psi_m e^{j\alpha}, \]  

(B.6)

where, \( j\omega_s \psi_m e^{j\alpha} \) is the back-emf term and the equation describes the dynamic equivalent circuit depicted in Figure B.1 (b).

### B.2 Into synchronous coordinates

To transform the equation into synchronous coordinates, the vector rotation is now applied with an angle \( \theta \).

\[ L_s \frac{d[e^{j\alpha}i_s]}{dt} = e^{j\alpha}u_s' - R_e e^{j\alpha}i_s' - j\omega_s \psi_m e^{j\alpha}, \]

\[ L_s \frac{di_s}{dt} + L_s i_s \frac{d[j\theta]}{dt} = u_s - R_s i_s - j\omega_s \psi_m e^{j(\alpha + \theta)} \]  

(B.7)

Since the synchronous coordinate system is assumed to rotate in synchronism with the stator voltage vector

\[ \frac{d[j\theta]}{dt} = j\omega, \]  

(B.8)

and if the angle error between the applied stator voltage vector and the actual rotor position is defined as \( \widetilde{\theta} = \theta - \theta \), the result will be

\[ L_s \frac{di_s}{dt} = u_s - j\omega L_s i_s - R_s i_s - j\omega \psi_m e^{j\widetilde{\theta}}. \]  

(B.9)

This means that, the back-emf will be constant only if \( \widetilde{\theta} \) is constant.

### B.3 Torque equation

The electromagnetic torque produced in PMSM with peak value scaling can be expressed as

\[ \tau_c = \frac{3}{2} n_p \text{Im}\{i_s' \psi_m \} \]  

(B.10)

From (B.2) it can be easily shown that the stator flux linkage in synchronous coordinates is

\[ \psi_s = L_s i_s + \psi_m e^{j\alpha}. \]  

(B.11)

This results in

\[ \tau_c = \frac{3}{2} n_p \text{Im}\{L_s (i_{sd} - j i_{sq}) + \psi_m (\cos \widetilde{\theta} - j \sin \widetilde{\theta})\} (i_{sd} + j i_{sq}) \]

\[ = \frac{3}{2} n_p \psi_m (i_{sq} \cos \widetilde{\theta} - i_{sd} \sin \widetilde{\theta}) \]  

(B.12)

Equations (B.9) and (B.12) demand the fact that \( \widetilde{\theta} \) be constant, if the back-emf and the electromagnetic torque produced to be constant respectively. Time varying \( \widetilde{\theta} \) can cause
torque ripples according to (B.12). This means that in order to get the best performance from the PMSM, the stator voltage vector must always be applied in orientation with the \textit{d-axis} of the rotor. Mathematically this implies that \(^{\hat{\theta}} = \theta - \phi = 0\).

Unlike in the case of a synchronous generator, where the stator voltage is a result of the rotation – hence synchronized with the rotor by default – PMSM operation (motoring mode) demands the information about the rotor position. This also implies that the PMSM has to be inverter fed in order to align the applied stator voltage vector as accurately as possible with the \textit{d-axis} of the rotor. Usual practice is to employ a resolver or a position encoder to sense the rotor position and use that information for the control purposes. In sensorless control, some form of an algorithm is used to estimate the rotor position and speed. Inputs to such an algorithm are the sampled line currents and sometimes the line voltages or DC link voltage of the inverter.

\textbf{B.4 Two axis model of PMSM}

With \(^{\hat{\theta}} = \theta - \phi = 0\) decoupled (B.9) and (B.12) can be used to obtain the PMSM machine model. This is given by

\[
\begin{align*}
\frac{d}{dt} i_d &= -\frac{R}{L} i_d + \frac{\omega L}{L_s} i_q \frac{1}{L_s} u_d \\
\frac{d}{dt} i_q &= -\frac{R}{L} i_q - \frac{\omega L_d}{L_s} i_d - \frac{\omega \psi_m}{L_s} + \frac{1}{L_s} u_q \\
T &= \frac{3}{2} n_p [\phi_m i_q]
\end{align*}
\]  

\[ (B.13) \]

\[
\frac{d\omega}{dt} = \frac{1}{J} (T - T_i - B\omega).
\]

The same can be obtained by substituting \(L_d = L_q = L_s\) in (2.23). Yet the above approach serves a useful purpose, because it is relatively easy to derive. Another advantage is this procedure can be used to mathematically show the importance of accurate rotor position information for the proper operation of a PMSM.
Appendix C: PMSM test rig – Design and characterization

All important design information about the PMSM test rig and its characterization will be presented in this Appendix.

C.1 Design and characterization of current and voltage sensor circuits

The Hall effect current and voltage sensors have the specifications given in Table C.1.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary nominal r.m.s. current</td>
<td>100A</td>
<td>Nominal primary current</td>
<td>10mA</td>
</tr>
<tr>
<td>Primary current measuring range</td>
<td>0±150A</td>
<td>Primary internal resistance (+70°C)</td>
<td>250Ω</td>
</tr>
<tr>
<td>Secondary nominal r.m.s. current</td>
<td>50mA</td>
<td>Secondary internal resistance</td>
<td>110Ω</td>
</tr>
<tr>
<td>Conversion ratio</td>
<td>1:2000</td>
<td>Turns ratio</td>
<td>2500:1000</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>±12…15V</td>
<td>Nominal output current</td>
<td>25mA</td>
</tr>
<tr>
<td>Design temperature</td>
<td>70°C</td>
<td>Supply voltage</td>
<td>±12…15V</td>
</tr>
<tr>
<td>Maximum measuring resistance</td>
<td>110Ω</td>
<td>Design temperature</td>
<td>70°C</td>
</tr>
</tbody>
</table>

*Table C.1: Specifications of the current and voltage sensors*

C.1.1 Design of current sensor

Rated r.m.s. current of the machine = 16.3A

Peak current = 23A

Peak design current = 10% of Peak current = 25 A

This means since the primary nominal current of the Hall sensor is 100 A, four turns of the conductor can be passed through the Hall sensor for improved sensitivity.

Then the secondary current for 25 A peak design current in the conductor = 50mA.

Input range of the A/D converter used for current sampling = ± 10V

If the precision external measuring resistance $R_m$ is selected to be = 110Ω

Output voltage range from the sensor corresponding to peak design current = ± 5.5V.

This is well within the range of the A/D converter (±10 V). Figure C.1 shows the complete current sensing circuit for one phase.

C.1.2 Design of the voltage sensor

Specifications of Hall effect voltage sensor are as given in Table C.1.

(a) Design of primary resistance

The purpose of the voltage sensor is sampling the DC-link voltage of the drive inverter to be used as one measurement information in the control algorithms. Therefore, during the design one has to accompany for possible DC-link voltage rises.
Making allowances for those fluctuations, maximum measuring voltage was selected to be $700\text{V}$.

Since the nominal primary current is $10\text{mA}$ and the nominal primary resistance is $250\text{\,\Omega}$, the primary resistance is calculated to be $\frac{700}{10\times10^{-3}} - 250 = 69.75\times10^3\text{\,\Omega}$.

Power dissipation in $R_1$ is 
\[\left(10\times10^{-3}\right)^2 \times 69.75 \times 10^3 = 6.975\text{W}\]

So to avoid overheating, the maximum power that can be dissipated in $R_1$ is selected to be $\leq 25\text{W}$.

A bank of 2 W metal film precision resistors of equivalent resistance $69.75\times10^3\text{\,\Omega}$ was used as $R_1$.

---

**Figure C.1**: Current sensing circuit

**Figure C.2**: Voltage sensing circuit

(b) Design of measuring resistance

Nominal primary current $= 10\text{mA}$

Corresponding secondary current $= \frac{2500\times10\text{mA}}{1000} = 25\text{mA}$

Input range of the A/D converter used for voltage sampling is $\pm 10\text{V}$.

So, if the measuring resistance $R_m = 330\Omega$.

The output range from sensor is $\pm 8.25\text{V}$.

This again is well within the range of the A/D converter ($\pm 10\text{V}$). Figure C.2 shows the complete voltage sensing circuit.

C.1.3 characterization of the sensors

A balanced resistive load was supplied with the inverter and different current values of the first phase of the three-phase system was recorded together with corresponding sampled voltage values from the sensor output. External current measurement was done using both, a clip-on ammeter and an oscilloscope current probe. Corresponding variation is plotted in Figure C.3. It follows that the sensitivity of the current sensor circuits is $0.22\,\text{V/A}$. The voltage sensor behavior was also tested around its designed measuring voltage. Measured and
sampled values of some DC link voltage settings were recorded. External measurement was done using an isolated voltage probe. Corresponding variation is plotted in Figure C.4. From the variation it can be found that the voltage sensor causes an attenuation of 1.175%.

Figure C.3: Current sensor characteristics  Figure C.4: Voltage sensor characteristics

C.2 Specifications of incremental encoders

Electrical and mechanical specifications of the position encoders used are given in Table C.2.

<table>
<thead>
<tr>
<th>Description</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage</td>
<td>5V ± 5%</td>
</tr>
<tr>
<td>Current requirement</td>
<td>max. 180mA (without load)</td>
</tr>
<tr>
<td>Output frequency range</td>
<td>0..400kHz</td>
</tr>
<tr>
<td>Hence, maximum operating speed</td>
<td>$180 \times 10^5/(\text{increments/revolution})$ r.p.m.</td>
</tr>
<tr>
<td>Weight</td>
<td>250g</td>
</tr>
<tr>
<td>Starting torque (at 25°C)</td>
<td>0.005 Nm max.</td>
</tr>
<tr>
<td>Maximum operating speed</td>
<td>12000 r.p.m.</td>
</tr>
<tr>
<td>Rotor inertia</td>
<td>$3.5 \times 10^{-6} \text{kgm}^2$</td>
</tr>
<tr>
<td>No. of increments per revolution</td>
<td>2048</td>
</tr>
</tbody>
</table>

Table C.2: Specifications of the position encoders

C.3 Brake chopper design

Design calculations for the brake chopper employed in the test rig will be presented here. The chopper circuit used here is based on MITSUBSHI hybrid IGBT driving IC M57959L. The complete circuit diagram is shown in Figure C.5.
C.3.1 Input signal selection for the brake chopper:

In order to activate the brake chopper circuit, there has to be some sort of a sensing system of the DC-link voltage. This must provide an alarm signal, when the DC-link voltage exceeds the safety limit. The particular alarm can then be used as the activating signal to the brake chopper. The protective logic circuit of the inverter shown in Figure C.6 generates alarm signals for over voltage and under voltage of the DC-link, which goes further to stop IGBT switching. One needs to reset the protective logic circuit in order to come back to the normal operation. In between those voltage limits, the same circuit generates the activating signal for a brake chopper. The simple analysis following will give the particular DC-link voltage levels at which each alarm signal is generated.

![Figure C.5: Driver circuit](image1)

![Figure C.6: Protective logic circuit](image2)

The reference voltages for the three comparators can be calculated from the simple voltage divider arrangement seen, when looking into section plane AA. They are

\[
V_{U\_ref} = \left[15 \times 10^3 / (1 + 5.6 + 15) \times 10^3 \right] \times 6.2V = 4.3V
\]

\[
V_{B\_ref} = \left[(5.6 + 15) \times 10^3 / (1 + 5.6 + 15) \times 10^3 \right] \times 6.2V = 5.91V
\]

\[
V_{O\_ref} = 6.2V
\]

If the hysteresis created by the OPAMP comparator stage that generate input signal for the brake chopper is neglected, corresponding DC-link voltages to activate each alarm signal can be easily found as below.

\[
V_U = \left[(470 + 150 + 5.6) \times 10^3 / 5.6 \times 10^3 \right] \times 4.3V = 480V
\]

\[
V_B = \left[(470 + 150 + 5.6) \times 10^3 / 5.6 \times 10^3 \right] \times 5.91V = 660V
\]

\[
V_O = \left[(470 + 150 + 5.6) \times 10^3 / 5.6 \times 10^3 \right] \times 6.2V = 692.6V
\]

By testing, it was found that the actual DC-link voltages to switch ON and OFF the brake chopper signal are 669V and 663V respectively. The difference is due to the hysteresis generated from the comparator.

C.4 Experiment to determine damping constant

Since two rotors are connected with a coupling, the inertia and the damping of the combination becomes much different from the unconnected machine. Furthermore, information on the damping of the motor itself is rarely available. A simple test, which can be
carried out to experimentally estimate the damping (it is possible to estimate the rotor inertia analytically) of the combined mechanical system is described here.

The test is to let the machine run at 1 pu steady state speed and disconnect the supply voltage instantly. Then the machine freely decelerates down to zero speed. The machine is thus electrically isolated, which means that the electromagnetic torque opposing the rotation of the machine becomes negligible. The deceleration of the motor under these conditions is affected by the following opposing torques.

- Torque due to viscous damping: this would be the most dominant one.
- Electromagnetic torque due to eddy currents: the eddy currents in the rotor will have a certain interaction with the permanent magnet flux. This will create a certain braking torque.
- Resistance due to air: the air resistance from the motion of the rotor will always be there.

It is difficult to model the torque due to eddy current effect and the air resistance. If it is assumed that the resistive torque is completely due to viscous damping, it can be assumed to be approximately proportional to speed. The Laplace transformed equation of the mechanical dynamics of the motor according to (2.23) can be given as

\[
\omega(s) = \frac{1}{(Js + B)} (T(s) - T_i(s)).
\]  

(C.1)

When the load torque \( T_l \) is zero, using the final value theorem, steady state torque for 1 pu speed can be shown to be equal to \( B \) pu. The time response of speed against electromagnetic torque under this particular condition can be found from the analysis below. It is assumed that \( B \) pu steady state torque is removed at \( t=0 \), while the machine is running at 1 pu speed. This means that the initial conditions are

\[
\omega(0) = 1 \text{ and } \omega^{(1)}(0) = \frac{d\omega}{dt}_{t=0} = 0.
\]  

(C.2)

Taking the Laplace transforms of both sides of the differential equation

\[
\frac{d\omega}{dt} + \frac{B}{J} \omega = 0
\]  

(C.3)

results in

\[
s\omega(s) - \omega(0) + \frac{B}{J} \omega(s) = 0
\]

\[
\omega(s) = \frac{1}{s + \frac{B}{J}}.
\]  

(C.4)

The speed response for a shut down of torque from \( B \) pu steady state down to zero is given by

\[
\omega(t) = e^{-\frac{B}{J}}. \quad t \geq 0
\]  

(C.5)

The ratio \( J/B \) is known as the motor time constant. This can be estimated from the deceleration data shown in Figure C.7. One straightforward method is to find the time taken for the speed to drop down to 63% of full speed and take that value as an approximation for the time constant. The other method is to use the data and do a curve fit using an optimization
method to estimate the time constant. The Figure C.7 shows that the time constant obtained by curve fitting is the same as the value obtained by the direct approach. For easy interpretation of data, the time scale has been converted into per-unit in this case.

Figure C.7: Free deceleration and fitted curve

The per-unit rotor time constant obtained from the optimization process is 0.001149 pu. Since it is possible to estimate the rotor inertia analytically, this leads to a per-unit damping calculated below.

\[ J = 109.3 pu \]

\[ \frac{B}{f} = 0.001149 pu \]

\[ B = 0.1256 pu \] \hspace{1cm} (C.6)

The deviation of the rotor dynamics from the analytically expected behavior can be due to the neglected forces on the rotor mentioned in the beginning of this discussion. Yet the damping value obtained can be used as a good approximation for the simulation and control design purposes. On the other hand, going for more complex mechanical models of the machine will require complex controller structures as well.