1. Introduction

Explaining and predicting the investment behavior of firms are central issues in economics. A starting point for the theoretical analysis of these issues is that firms aim at maximizing the net present value of their investments. In simple cases where firms operate in competitive environments, they invest whenever the net present value is positive since rejecting an investment with a positive net present value is not consistent with profit maximization. In cases where a firm can delay an investment without risking loss of the opportunity, the investment criterion will in general not be merely that the investment has a positive net present value. Instead, it may be more profitable to wait to invest at a later stage.

In practice, uncertainty with regard to both revenue from and cost of investment is an important ingredient in the value maximizing investment decision. This is even more the case when investment is irreversible, that is when investment costs cannot be recovered if the investment turns out to generate less profit than expected. This doctoral thesis focuses on irreversible investment in real estate under uncertainty.

The decision to make an irreversible investment when revenues and costs are uncertain bears a strong resemblance to the decision to exercise a perpetual American call option on a financial asset, which is an option that can be exercised any time in the future. The analogy is as follows: (i) The real asset you get when you invest is analogous to the financial asset you get when you exercise a financial call option. (ii) The investment cost you incur when you make the investment is analogous to the exercise price of the financial option. (iii) Finding the optimal time to make an irreversible investment is analogous to finding the optimal time to exercise (kill) a financial call option.

In the same way as the value of a financial option increases with uncertainty about the future value of the underlying financial asset, the value of a “real option” increases with uncertainty about the future value of the investment project. Since increased option value increases the opportunity cost of killing the option, investments will be delayed more the higher the uncertainty is. Consequently, the higher the uncertainty, the more erroneous the traditional net present value rule – invest as soon as the net present value is positive – will be.

The analogy between investment opportunities and financial options has promoted a long strand of research papers that have analyzed the valuation of investment opportunities and firms investment behavior using methods based on option pricing, see for instance Pindyck (1991) for an overview. Most real option pricing models are descendants of the continuous-time valuation model that was originally developed by McDonald and Siegel (1986). This paper was the first to incorporate risk aversion considerations into the timing problem in a tractable way and to demonstrate how uncertainty can act to delay investments. They analyze the optimal timing of irreversible investment projects when the project value and construction costs are assumed to evolve stochastically over time and the investment opportunity is infinitely lived.
The continuous-time approach of McDonald and Siegel has been used to analyze a variety of investment decisions. For instance, Majd and Pindyck (1987) study sequential investment where a firm invests continuously and can stop and restart the project costlessly. Pindyck (1988) analyzes variable capacity choice in the presence of irreversibility and uncertainty. Dixit (1989) shows how costly entry and exit decisions under uncertainty can lead to “hysteresis”, i.e. unprofitable production is not discontinued and production that would be profitable is delayed. Dixit (1991) analyzes the effect of price ceilings on irreversible investments. Ingersoll and Ross (1992) examine the effect on irreversible investment decisions of interest rate uncertainty. Bar-Ilan and Strange (1996) analyze the effect of construction lags on investment thresholds and investment behavior in markets with chronic excess supply.

The ownership of a property, in all stages of development, involves undertaking a variety of investments that can be characterized as real options. For instance, the owner of undeveloped land, that can potentially become income producing, must decide if and when it is optimal to pay the development cost in order to receive the income stream from the developed property. From an option pricing perspective the owning of undeveloped land is equivalent to owning an option on the value of the developed property. The cost of exercising the option is the cost of developing the land, e.g. the cost of erecting buildings on the land.

The option like nature of land development was first identified by Titman (1985). He uses a discrete time model to demonstrate that uncertainty regarding the market price of building units increases the value of vacant land and delays development activity. Capozza and Helsley (1990) use a continuous time real option model to analyze how growth and uncertainty in rents affect land prices and the timing of land conversion. Land prices and rents are decomposed into additive components explaining the spatial structure of land prices and rents. Capozza and Helsley (1994) analyze the risk structure of land markets where agricultural land can be converted into urban land. The model is basically the same as in Capozza and Helsely (1990) but allows for risk aversion. Grenadier (1995) models the option to develop land and shows how the combination of demand uncertainty, production lags and costly adjustment of the vacancy rate can increase the likelihood of overbuilding.

The fact that we can observe sales of undeveloped land at high prices is a very good example of the shortcomings of the traditional net present value rule. Positive values of land implies that buyers of the land expect a profit from developing the land, i.e. buyers expect the net present value of a future investment to be greater than zero. If this were not the case, a profit maximizing firm or person would not pay for the land. This observation reinforces the need for other investment rules than the net present value rule and the necessity of option pricing based techniques for land valuation.

The timing option is not the only option that comes with the ownership of land. Subjected to zoning restrictions, a landowner also has the option to choose the capacity (density) of a development in an optimal way. For instance, if an office building is to be constructed, a profit maximizing developer will choose to construct the number of units (square meters, floors etc.) that maximizes the difference between the value of the erected building and the construction costs. The option to choose the density of a development is of course valuable and will make land more valuable than it would be if the density could not be chosen in an optimal way.
The option to choose the density of a real estate development was first analyzed by Clarke and Reed (1988). They solve the problem using a general cost function with only the constraints that it is increasing and convex. The problem is solved sequentially, where the land owner first decides on the density of the development and thereafter decides on the optimal timing of the development. They show that uncertainty increases the value of the development option and increases the optimal density of the constructions on the property. Williams (1991) makes a similar analysis in which he shows the potential problem with corner solutions in Clarke and Reed if the cost function is a power function. Capozza and Li (1994) solve the same problem as Clarke and Reed and Williams, but makes the assumption that rents are expected to grow linearly rather than the more common assumption that rents grow exponentially.

In addition to the option to choose the timing and density of a real estate development, there may also be an option to choose between several possible land uses, or mixes of different uses. These options will enhance the value of vacant land even further. Geltner, Riddiough and Stojanovic (1996) analyze the option to develop a property into the best of two different land uses. They find that land use choice may add significantly to land value under typical economic circumstances. They also find that development is significantly delayed when the two land uses have similar values, compared to a situation where one of the values dominate. Childs, Riddiough and Triantis (1996) value undeveloped and developed land when rents are uncertain and the property owner can change the mix of two different uses in an optimal way. They show that flexibility with respect to mixing uses contributes significantly to the value of both undeveloped and developed land.

If a property is already developed, the owner is faced with a variety of operating decisions that can be characterized as real options. For instance, in many cases tenant improvements are necessary to lease vacant space. Undertaking the improvements is equal to exercising an option on the present value of the future rents from the leased space and the exercise price is the cost of the improvements. Grenadier (1995) shows how costly adjustment of the vacancy status of a building can lead to “sticky” vacancy rates, i.e. occupied units remain leased even if demand becomes depressed and units remain vacant even if demand has increased substantially. Other operating decisions that can be characterized as real options are for instance periodical maintenance.

Subjected to zoning restrictions, the owner of a developed property also has an option to redevelop or change the use of the property. This option will increase the value of the property above the value in the current use since the value in the alternative use might become high enough in the future to make it worthwhile to pay the cost of changing the use. Capozza and Li (1994) model the decision to redevelop a property where the property owner chooses both the timing and the density of redevelopment in an optimal way. They are not able to solve the problem analytically and no numerical simulations are undertaken. However, they sketch the solution and show that for each value of the rent in the new use there exists a value of the rent in the current use for which the redevelopment is optimally undertaken. They also show that since the rents at which redevelopment occurs are stochastic, the density of redevelopment will also be stochastic. Grenadier (1996) uses a game-theoretic approach to examine the strategic exercise of redevelopment options in a duopoly where demand for the redeveloped properties is assumed to be uncertain. In this
model the property owner who first exercises his redevelopment option, the leader, takes into consideration the rational exercise policy of the follower. The model provides explanations for different market behavior regarding redevelopment activities.

Finally, a property owner has the option to abandon the property in case the income from the property can no longer motivate the costs that are associated with the ownership of the property. This option is modeled by Williams (1991) who finds that the option to abandon a property can affect its optimal development point and that properties that are more costly to maintain are abandoned sooner.
2. Summary of Essays I-III.

Essay I:  Real Estate Valuation under Uncertainty—Application of Option Theory.

Essay I, written in Swedish, is a study of the option to develop vacant land. At the time it was written, 1995, the application of real option theory to the analysis of real estate investments was rather new. Therefore, one aim of the essay was to present the basic theory of real option pricing and draw attention to its usefulness in analyzing real estate investments. Another aim of the essay was to analyze existing models of land development in the literature and discuss the effects on model results of different assumptions regarding uncertainty and production technology. On the basis of this analysis alternative models for land valuation are proposed. Essay I was presented by the author at seminar, as part of the requirements for a Licentiate Degree in Engineering.

The first part of the essay may serve as an introduction to the basic theory of option pricing and to the real options approach to investment under uncertainty. Starting with some introductory examples of option valuation in a discrete time model, the arbitrage valuation technique in the Black and Scholes (1973) continuous time option-pricing model is demonstrated. The similarity between financial options and investment opportunities in real assets is discussed and the basic real-options approach to valuing an infinitely lived investment opportunity is demonstrated. Standard comparative static results for real options are also derived.

The second part of the essay studies the option to develop vacant land. The analysis is focused on the optimal choice of density and timing of a real estate development: a problem that has recently been studied from a real options perspective by Williams (1991) and Capozza and Li (1994). The approaches followed in these papers involve the same production technology but different assumptions about uncertainty. Williams assumes that cash flows from the developed property and construction costs are lognormally distributed, that is cash flows and construction costs are expected to grow exponentially. Most papers in the real option literature model uncertainty in this way. Instead Capozza and Li assume that cash flows from the developed property are normally distributed, that is cash flows are expected to grow linearly. In both models the production technology is assumed to be of Cobb-Douglas type, that is development costs grow according to a power function when the density of the development increases.

The effects on the results of the different assumptions about uncertainty made by Williams and Capozza and Li are analyzed and discussed. The purpose of this analysis was to explain why the valuation problem as modeled by Williams only has an interior solution under very specific circumstances, while as modeled by Capozza and Li it always has an interior solution.

Williams makes the very specific assumption that the undeveloped property produces a cash flow that is perfectly correlated with the cash flow from a unit of the developed property. If this assumption is not fulfilled, corner solutions arise. Development of the property either takes place immediately or is delayed indefinitely; in other words the development decision becomes a “now or never” decision. Capozza and Li on the
other hand, who assume that rents grow linearly, do not have this problem with corner solutions. The problem of corner solutions is mentioned both by Capozza and Li and by Williams. Capozza and Li specifically point out that one advantage with their specification of uncertainty is that an interior solution is guaranteed. However, neither of the papers explains why corner solutions can arise when rents are expected to grow exponentially.

In Essay I, I show that the corner solutions that appear in Williams’ model are related to the constant elasticity of the Cobb-Douglas cost function. When the production technology is Cobb-Douglas and cash flows are linear in scale, a profit maximizing property owner will always choose the density of a development in such a way that the ratio of the value of the developed property to construction costs, the benefit-cost ratio, is constant. That is, the ratio does not depend on the actual level of cash flows and construction costs at the time the development is initiated. If this benefit-cost ratio differ from the benefit-cost ratio that is needed to make it optimal to exercise the development option, a “stalemate” situation appears. Unless the two benefit-costs ratios by pure coincidence are equal, it is impossible to optimize the density of the development and the timing simultaneously. Hence, the solution to the optimization problem will always be one of two corner solutions. If the benefit-cost ratio that follows from maximizing the density of the development is greater than the ratio at which the development option is optimally exercised, the development will be initiated immediately. If it is the other way around, it will be optimal to delay the development indefinitely.

In the Capozza and Li model the conflict between the optimization of the density and the timing of the development never occur. This is because their specification of uncertainty means that the relative uncertainty will decrease when rents increase. When uncertainty decreases the benefit-cost ratio that optimally triggers a development will fall. Therefore development will be optimally triggered when rents have risen to a level at which the benefit-cost ratio that optimally triggers development is low enough to coincide with the benefit-cost ratio that follows from maximizing the density of the development.

Both the model of Williams and that of Capozza and Li have properties that are not fully satisfying. In Williams’ model, the results are critically dependent on the very specific assumption that the undeveloped property produces a cash flow that is perfectly correlated with the cash flow from a unit of the developed property. It may of course be the case that undeveloped properties produce cash flows that have that characteristics, but if the aim is to study the land development problem in general, this should not be a critical model assumption. In the model of Capozza and Li, the solution is driven by the fact that the development is delayed until the problem becomes “deterministic” enough to warrant exercising the development option. This way of modeling uncertainty is controversial and not fully satisfactory if the aim is to study the effect of uncertainty on land development.

In general, it is also difficult empirically to test real option models in which the production technology is of Cobb-Douglas type. This is underlined by Quigg (1993), who tests predictions of a real option pricing model for vacant land. She uses a Cobb-Douglas cost function and finds the model extremely sensitive to different assumptions of the value of the cost scale parameter. In fact, although the valuation
model is derived for an endogenously determined density of developments, she uses the initial values of rents and construction costs to determine the optimal density of buildings.

In Essay I, I model the option to develop vacant land using other specifications of the production technology than the commonly used Cobb-Douglas function. Instead of the Cobb-Douglas power function with constant elasticity, I propose cost functions that increase exponentially when the density of developments is increased. This is a reasonable assumption when land is scarce, for instance in downtown areas where constructions are mainly vertical. As buildings become taller, it becomes increasingly more expensive to add another floor, and at some level the available production technology will not permit an increase in the number of floors without extreme increases in the construction cost. The uncertainty in the models is specified in the same way as in Williams (1991), that is cash flows are assumed to follow a lognormal diffusion and are therefore expected to grow exponentially.

With an exponential cost function, the problem of simultaneously optimizing the density and timing of a land development has an interior solution when rents are expected to grow exponentially, without the restrictive assumptions in Williams (1991) regarding the cash flow from the undeveloped property. The reason is that the profit maximizing ratio of benefit to cost of undertaking the development will not be independent of the level of rents, as is the case when the production technology is of Cobb-Douglas type. Instead, as cash flows grow, the profit maximizing benefit-cost ratio will increase, and at some point this ratio coincides with the ratio at which the development option can be optimally exercised. That is, it is possible to optimize the density and the timing of the development simultaneously.

To conclude, the use of a Cobb-Douglas technology in investment models can be motivated by the fact that its simplicity makes models tractable and analytical solutions are often permitted. However, Essay I shows the need to take care when modeling development decisions in a Cobb-Douglas technology in order to avoid corner solutions. This is especially important in models where analytical solutions are not available and the solutions cannot be easily analyzed if results are peculiar. The problem of corner solutions can be avoided by using cost functions that are not of constant elasticity. This study has been of great help in my continued work with more complex models of land development, where numerical solution methods were necessary, and in which I used exponential cost functions.

**Essay II: The Option to Change the Use of a Property when Future Property Values and Construction Costs are Uncertain.**

In most cases there is more than one possible use of a property. The possibility to change the use is a valuable option that will add to the value of the property in its current use. The value of the option, and when it is optimally exercised, will depend on investors’ beliefs about the future value of the property in the current and the new use as well as the cost of changing from one use to the other.

In Essay II a real option model is used to value the option to change the use of a property and to study the optimal timing of the redevelopment. The model is an
extension of the model in McDonald and Siegel (1986) in which the value of the investment and the investment cost follow correlated geometric Brownian motions. The extension is that the investment cost consists of two different parts, the construction costs and the lost value of the property in the current use, that are assumed to follow different geometric Brownian motions.

In the McDonald and Siegel case the investment problem has an analytical solution. The investment is undertaken when the ratio of benefit to cost of investing hits a certain trigger ratio. However, when the cost of investing consists of two different parts, the optimal timing of the investment is not given by a certain ratio between the benefit and the cost. The ratio will be different for different relative sizes of the two parts of the exercise cost.

The option to redevelop a property has recently been analyzed by Capozza and Li (1994), Child, Riddiough and Triantis (1996) and Grenadier (1996). However, none of these papers take into consideration the full uncertainty of the problem since they assume that construction costs are constant over time. Grenadier also assumes that the value of the property in the current use is constant. Essay II therefore adds to the analysis by allowing both the value in the current and the new use as well as the cost of changing the use to follow geometric Brownian motions.

Numerical solution methods were employed to value the redevelopment option and study the optimal timing of the redevelopment. The numerical simulations show that the optimal timing and the value of the redevelopment option can differ greatly depending on the assumptions regarding the uncertainty of the state variables. Also the relative size of the two different exercise costs affects the optimal timing and the value of the redevelopment option. This is especially the case when uncertainty is high and the correlation between the two costs is low. The reason is that a mix of the two costs has a “diversification” effect on the profit from exercising the option. If the costs are not perfectly correlated, the volatility of the profit is reduced when the costs become more equal in size, which will reduce the value of the redevelopment option. A lower value of the option will in turn reduce the opportunity cost of killing the option and so the redevelopment will be undertaken earlier.

The model in Essay II, in which the full uncertainty of the economic environment is taken into consideration, can contribute to rational explanations of a puzzling phenomenon in real estate markets, namely construction booms in the face of rising vacancies and declining property values. This recurring phenomenon is usually attributed to irrational behavior of market participants. Notable exceptions are Bar-Ilan and Strange (1996) and Grenadier (1996). Bar-Ilan and Strange show that when a development can be abandoned, a profit maximizing investor might choose to develop in a depressed market to avoid being out of the market if conditions improves. Grenadier shows that fear of preemption can lead to an equilibrium in which the competitors in a duopoly simultaneously exercise their redevelopment options following a decrease in demand.

The results in Essay II suggest that the interaction between uncertainty in property values and construction costs can create a situation in which a construction boom in the midst of a market downturn is a realistic scenario. Triggering events may, e.g., be the circumstance that construction costs decrease or that the value in the current use of
the property decreases faster than the value in the new use. The probability for a so-called recession induced construction boom increases when the value of the property in the new use and construction costs are highly correlated but the value of the property in the current use is of poorly correlated with the other two variables.

**Essay III: Optimal Capacity Choice and Overbuilding**

Recurring cycles of overbuilding is a well-known phenomenon in many property markets. A number of recent papers have analyzed overbuilding from a “real options” perspective and have provided rational explanations for this seemingly irrational behavior. As shown by several papers, the option to choose the capacity of a development affects both the value of development options and the optimal timing of developments. However, no study has analyzed the relationship between the optimal capacity choice and the risk of overbuilding.

Essay III uses a real option model to study the optimal choice of capacity for a property development and deliberate overbuilding. The model of the development process is based on Grenadier (1995), but his model is extended in two ways. The first extension is that the property owner is assumed to optimize the timing and the capacity of the development simultaneously. In Grenadier (1995) the property owner only optimizes the timing of the development. The second extension is that the assumption of risk neutrality is relaxed. Investors are assumed to be risk averse and to price market risk.

Grenadier (1995) measures overbuilding as the probability, at the beginning of construction, that the demand will be insufficient to rent a specified fraction of the units at the time the building is completed. Although this measure of overbuilding reflects the investors’ willingness to risk initial vacancies, it does not explicitly capture the link between the optimal choice of capacity of a development and the possible intention to overbuild. A more suitable measure of overbuilding is instead, for the purpose of the study in Essay III, the following: Define as a measure of the intention to overbuild, the difference between the number of rental units the owner of a vacant site optimally chooses to produce and the number of units that is expected (at the beginning of construction) to be leased at the time of completion of the development. Measured as a fraction of the full capacity, this difference can be expressed as the expected vacancy rate at completion.

Numerical simulations with reasonable parameter values show that in some economic environments, the optimal production strategy can be to produce more units than are expected to be leased at completion of a development. That is, an optimal strategy can be to overbuild deliberately. One important reason for this result is the high degree of irreversibility that generally characterizes the choice of capacity for a real estate development. When a development is completed, the cost of expansion is often very high. Consequently, the capacity of a development cannot be adjusted continuously as demand for space increases. The higher the cost of expansion, the more irreversible the initial capacity decision will be. This irreversibility creates an incentive to produce more capacity than is expected to be leased at completion of the development in order to have enough capacity to satisfy a future increase in demand.
The comparative static results show that the incentive to overbuild is particularly strong when the demand for space is expected to increase quickly. Seen as an isolated event, increased demand volatility increases the incentive to overbuild. However, increased volatility will in general also increase the risk premium required by investors, which will deter from overbuilding. The net effect of increased volatility on overbuilding therefore depends on the relative size of these two counteracting forces. The comparative statics result for the construction time shows that an increase in the construction lag can increase or decrease overbuilding depending on the values of the other parameters in the investment model.
REFERENCES


