Magnet Losses in Inverter-fed High-speed PM Machines

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Magnet Losses in Inverter-Fed High-Speed PM Machines

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Abstract

This master thesis deals with the estimation of magnet losses in a Permanent Magnet (PM) motor inserted in a nut-runner. This type of machine has interesting features such as being slot-less and running at a very high speed (30000 rpm). An extensive literature review was performed in order to investigate the state of the art in estimation of the losses in magnets of a PM machine. Analytical models to calculate the no-load back-emf and the magnetic flux density in the air-gap due to the currents in the stator are presented first. Furthermore, several of the analytical models for calculating losses in magnets described in the literature were tested and adapted to the case of a slot-less machine with a parallel-magnetized ring. Then, a numerical estimation of the losses with finite element method (FEM) 2D was carried out. In addition, a detailed investigation of the effect of simulation settings (e.g., mesh size, time-step, remanent magnetic flux density in the magnet, superposition of the losses, etc.) was performed. Finally, calculation of losses with 3D FEM are also included in order to compare the calculated losses with both analytical and FEM 2D results. The estimation of the losses includes the variation of these with frequency for a range of frequencies between 10 and 100 kHz.

Keywords
Eddy currents, FEM 2D, FEM 3D, magnet losses, nut-runner, PM motor, slot-less winding
Sammanfattning

Detta examensarbete handlar om uppskattningen av magnetförluster i en permanentmagnet motor (PM) införd i en mutterdragare. Denna typ av maskin har intressanta funktioner, som att den är *slot-less* och att den körs i en hög hastighet (30000 rpm). En omfattande litteraturstudie utfördes för att kunna uppskatta förluster i magneterna på bästa sätt. Först presenteras analytiska modeller för att beräkna den elektromotoriska kraften (EMK) och den magnetiska flödestätheten i luftgapet som uppkommer på grund av strömmarna i statorn. Dessutom har flera av de analytiska modellerna för beräkning av förlusterna som beskrivits i litteraturen testats och anpassats till en *slot-less* maskin med en parallelmagnetiserad ring. En numerisk uppskattning av förlusterna har sedan utförts med hjälp av finita elementmetoden (FEM) 2D. Därtill har en detaljerad undersökning genomförts hur olika parameterinställningar påverka utfallet. De FEM parametrar som har undersökt har bland annat bestått av beräkningsnätets storlek, tidssteg, remanens flödestätheten i magneten och om *superposition* av förlusterna gäller. Till sist har beräkningar för förluster med 3D FEM utförts och jämförts med resultaten för både de analytiska och FEM 2D resultaten. Uppskattning av förluster innefattar variationen av dessa med ett frekvensområde mellan 10 och 100 kHz.

Nyckelorden

*Virvelströmmar, FEM 2D, FEM 3D, förluster i magnet, mutterdragare, permanentmagnetiserad motor, luftlindad lindning*
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Chapter 1

Introduction

1.1 Background

The development of new magnetic materials along with the improvement and spread of power converters have allowed that Permanent Magnet Synchronous Machines (PMSMs) become available for a series of applications. These applications are as varied as for example; power tools, fans, water pumps, through propulsion of electrical vehicles (cars, ships, etc.) and even wind power generation. All of this due to characteristics such as small size, high efficiency and reliability. During the design of PMSMs the estimation of performance such as losses is of prime importance in order to fulfil the increasingly strict efficiency requirements. Generally, the most representative losses in an electrical machine are the copper and iron losses. However, in an inverter-fed machine, as the frequency increases, iron and magnet losses may start to be dominant. The main reason is the high amount of harmonics that are fed by the inverter to the machine due to the non-sinusoidal characteristic of the feeding source. Given the sealed characteristic of the machines, cooling the losses of the rotor is challenging. Additionally, the presence of other mechanical losses (e.g., air friction losses, bearing losses, etc.), makes that the heat generated in the rotor has a poor dissipating path. Consequently, the risk of demagnetization of the magnets in the rotor may be high.

1.2 Thesis scope

This thesis is focused on the study of eddy currents and the consequent losses generated in the magnets of a high-speed, inverter-fed PM machine. Both analytical models and FEM simulations are used for this study. The main goal of this thesis is an attempt to enhance the interpretation of the losses appearing in the machine as a result of the harmonics fed by the pulse wide modulation (PWM) in the inverter. A parallel study
of the losses in windings is being carried out in order to have a complete picture of the phenomenon of the harmonic losses.

1.3 Thesis outline

In order to predict the value of magnet losses, a literature review was performed. This is presented in Chapter 2. Chapter 3 is devoted to the description of the machine and the analytical models selected to determine the no-load back-emf $E_0$, magnetic flux density in the air-gap due to the current in the windings $B_{\delta w}$ and the magnet losses. The influence of the mesh and time-step while running FEM 2D simulations is presented in Chapter 4, along with some preliminary results of the magnet losses. Chapter 5 introduces FEM 3D simulations with preliminary results for no-load back-emf $E_0$, magnetic flux density in the air-gap due to the currents in the windings $B_{\delta w}$ and magnet losses. The analysis of the results obtained by analytical models, FEM 2D and FEM 3D simulations is presented in Chapter 6. Lastly, in Chapter 7 conclusions are drawn and future work is proposed.
Chapter 2

Literature review

2.1 Introduction

Losses in magnet is a currently hot topic in the research about PMSMs, with many recent publications. A number of articles have been reviewed and this chapter focuses on the state of the art in the study of losses in permanent magnets due to eddy currents induced by both time (PWM commutation) and space harmonics (slot effect, non-sinusoidal stator magneto-motive force (mmf) distribution). Two main groups were identified in the study of losses in the magnets: models accounting for the reaction field of the eddy currents in the magnet and models neglecting this phenomenon. At the end of this chapter, a summary table is presented with the articles which are considered as most relevant.

2.2 Eddy currents

The mechanisms governing the eddy currents in a magnet are the same as for the eddy currents in electrical sheets or solid conductors. A time varying magnetic flux density, in this case generated by the mmf of the currents in the stator windings $B_{\delta w}$, penetrates the surface of the magnet. This incident magnetic flux density originates eddy currents as illustrated in figure 2.1. It is appropriate to clarify that eddy currents are originated only by time and space harmonics in the stator mmf. That is, the fundamental in space of the mmf at synchronous frequency is seen by the magnets in the rotor as a DC field, consequently, does not create eddy currents. A method to counteract these losses, similar to the lamination technique implemented in a stator core, is the implementation of circumferential and axial segmentations of the magnets [1], [2], [3], [4].
CHAPTER 2. LITERATURE REVIEW

Figure 2.1: Eddy currents [5].

The major consequence of these eddy currents is the heating generated by Joule effect, as the magnets are made of a material with a high conductivity. Additionally, two important phenomena by which the calculation of the losses in the magnet can be affected are:

- Reaction field of eddy currents $B_{eddy}$.

- Skin effect.

The reaction field of eddy currents $B_{eddy}$ is generated by the eddy currents themselves, since these are varying in time as well [6]. This $B_{eddy}$ opposes to the external magnetic field $B_{δw}$ that is inducing the eddy currents. Consequently, the value of $B_{δw}$ is reduced. In addition, $B_{δw}$ causes the displacement of the current inside the conductor (figure 2.2), being forced to flow close to the conductor’s surface. Hence, the effective area of the conductor is reduced with increasing frequency. This is the definition of the skin effect [7].

Figure 2.2: Skin effect in a long straight round single conductor [6].
2.3 Literature study on losses in magnets

Although a broad variety of literature may be found, most of the articles related with the study of the eddy current losses in magnets are focused on how to deal with the reduction of these losses. The three main techniques are; segmenting of the magnets both circumferentially and axially and performing skewing of the magnets in the rotor [8]. Other references focused on the measurement of the losses in the magnets [9] and accounting for the permeance variation due to the presence of slots in the stator and the effect of the load [10]. In the literature, when calculating losses in magnets two main groups of models are found:

- Models neglecting the reaction field of eddy currents.
- Models accounting for eddy current reaction field.

Additionally, it was decided to present the articles distributed in three different sub-groups as follows:

1. Models accounting for both time and space harmonics.
2. Models accounting only for time harmonics.
3. Models accounting only for space harmonics.

Note that when several articles are in a same sub-group additional criteria is applied. Firstly, the priority is given to the articles which models presented are validated by experimental measurements. Secondly, the similarities between the machine studied in the article and the one analysed in this report are considered. Otherwise, the articles are presented in chronological order, that is, most recent first. Additionally, the articles in the first two sub-groups are considered to be more relevant for this study.

2.3.1 Models neglecting the reaction field of eddy currents

Models accounting for both time and space harmonics

Zhu, Schofield & Howe. The study of losses in the magnets starts with the estimation of the magnetic field distribution in the rotor region. The analytical prediction of the magnetic field distribution $B$ in the air-gap assumes a magneto-static field model [11]. This model is applied to a slotted machine with a retaining sleeve over the magnets as shown in figure 2.3 and accounts for magnets with both parallel and radial magnetization, as well as, time and space harmonics. This analytical model makes the following assumptions:
• The end-effects are neglected, that is, it is assumed that the eddy currents only flow in the axial direction.

• The retaining sleeve is assumed to be non-magnetic.

• The magnets and the sleeve are assumed to be homogeneous and having constant and isotropic permeabilities and conductivities.

• A 2D permeance function is introduced for accounting the slotting effect of the stator.

![Diagram of an internal rotor machine]

Figure 2.3: Model of an internal rotor machine [12].

Ishak, Zhu and Howe. Adopting similar assumptions, an analysis applicable to machines with a fractional number of slots per pole $q$ is presented in [13]. This 2D problem considers the loss contribution from space harmonics introducing the effect of time harmonics. The model is also applicable to machines fed with higher number of phases and neglects the effect of eddy currents in the magnetic flux density in the air-gap. The implemented model is presented in figure 2.4 and does not include a retaining sleeve over the rotor, differing from the case presented in [11].
Wang, Atallah, Chin, Arshad & Lendenmann. The study reported in [3] considers both space and time harmonics. Figure 2.5 shows the machine with radial segmentation under investigation. Additionally, the study points that there is a limit in the number of segments at which the reduction of the losses is no longer effective. Furthermore, it comes to the conclusion that the losses in the segments are not equally distributed, something to be aware of when considering the risk of demagnetization. The study makes the following assumptions:

- The iron and stator cores are of a material with infinite permeability.
- The analysis assumes a slot-less machine with the stator currents modelled as current sheets.

Wu, Zhu, Staton, Popescu & Hawkins. A very detailed study, accounting for the effect of slots in the stator and load is presented in [10]. It accounts for the influence of the interaction of harmonics on the losses. Consequently, time and space harmonics are considered as well as parallel and radial magnetization of the magnets. The machine geometry is presented in figure 2.6

- The iron materials have infinite permeability.
- The end-effects are negligible.
- The magnet has linear properties.
- The slots have simplified geometry (figure 2.6).
- The laminations in the rotor and stator have zero conductivity.
• The current density in the conductor area is assumed to have an uniform distribution.

• The permeability of the gaps between magnets are equal to the permeability of magnets.

• The magnets are perfectly insulated.

Models accounting only for time harmonics

Polinder & Hoeijmakers. A machine for a gas-turbine generator is studied in [14]. The model is developed for calculation of the losses in magnets with circumferential segmentation as in figure 2.7b. The losses in the magnets are modelled as resistances introduced to the electrical equivalent circuit of the machine. This study accounts for time harmonics. In addition to the assumptions made in previous documents the following are also adopted:

• The magnetic flux density $B$ is assumed to be constant over the magnet width.

• The current density has only a component in the $z$-direction (figure 2.7a).
Figure 2.7: (a) Cross-section of a magnet segment, (b) Cross-section of a two pole PM machine [14].

Huang, Bettayeb, Kaczmarek & Vannier. Accounting only for time harmonics, a study considering both axial and radial segmentation is reported in [15]. The geometry to which the model is applied is shown in figure 2.8. This study gives some recommendations about when the skin effect should be considered or not. Furthermore, it describes that segmentation may be inefficient. This can only be found out with a model where the skin effect is taken into account in the calculation of the losses.

Figure 2.8: Cross-section of one pole magnet and one segmentation [15].
Models accounting only for space harmonics

Pyrhönen, Jussila, Alexandrova, Rafajdus & Nerg. Figure 2.10 shows the geometry studied in [16]. This study is focused on the calculation of the losses in the magnet due to the permeance variations in the stator and the harmonics due to the distribution of the windings. Hence, time harmonics are not considered. The assumptions described in the article are as follows:

- $B$ can be calculated based on Carter’s classical theory.
- The magnet material is linear and well conducting.
- The eddy currents in the magnet follow the surface impedance with a phase shift of 45°.
- The eddy currents may be easily traceable together with their resistance considering the skin depth in the magnet material.
- The rotor yoke material is assumed as non-conducting or laminated without affecting the behaviour of the flux.

Aslan, Semail & Legranger. The study presented in [17] accounts only for space harmonics in machines with concentrated windings. As described in figure 2.9, it is focused on the interaction of the wavelengths of space harmonics in the mmf with the magnets and the resulting losses. It assumes the following:

- The losses resulting from the effect of slots in the stator are neglected.
- The losses in the magnet correspond to the summation of losses caused by each parasitic harmonic in the mmf.
- The variation of $B$ with the magnet thickness and axial length is neglected.
Atallah, Howe, Mellor & Stone. Another model considering the magnets circumferentially segmented is presented in [18]. The analysis is applied to 3 and 6-phase machines. Additionally, it assumes that the current is uniformly distributed around the stator as shown in figures 2.11a and 2.11b, neglecting the effect of the slots in the stator. However, it considers only the losses due to space harmonics for designs in which the number of poles in the fundamental stator mmf is lower than the number of poles in the rotor, that is, the torque is obtained as a result of the interaction between a harmonic in the stator mmf of higher index and the magnet. The accuracy of this study is claimed to be reduced as the speed of the machine increases and this model is applicable only to machines that have at least one magnet segment per pole in the rotor. The assumptions for this model are:

- The stator and rotor are infinitely permeable.
- The magnets are assumed with high resistivity and low recoil permeability.
- The stator winding is represented as an equivalent current sheet (i.e., slotting effect neglected).
Figure 2.11: (a) OuterRotor machine, (b) Inner-rotor machine [18].

2.3.2 Models accounting for eddy current reaction field

Models accounting for both time and space harmonics

Zhu, Schofield & Howe. A method for calculating the losses in a rotor of a machine with a retaining sleeve is presented in [19]. The analysis is applied to the same type of machine as in [11] in figure 2.3. It brought some improvements in the estimation of the losses at the cost of increasing the complexity of the expressions used to determine the value of the losses. In addition, this study accounts for both time and space harmonics. Important assumptions are:

- The windings of the stator are represented as current sheets placed in the slots openings, figure 2.3.
- The variation of the permeance due to the presence of slots is omitted.
- The materials of both the magnets and the retaining sleeve are assumed to be homogeneous and isotropic.

In [12] a model based on [19] is presented. The model accounts for several types of windings (overlapping and non-overlapping) and presents expressions for calculation of the magnetic flux density B and magnetic field strength H in three main regions as described in figure 2.3. The effect that the stator slots have in the magnetic flux density distribution is still neglected. The assumptions are the same as for [11] and [19], considering both time and space harmonics as well as either AC or DC machines with inner or outer rotor. Additionally, the experimental set-up for the measurements in an actual machine is described in the article.
Markovic & Perriard. Accounting for both time and space harmonics, a study focusing on slot-less machines, as in figure 2.12, is presented in [20]. The authors claim that it may be applicable to several different configurations. In this study the following assumptions are adopted:

- The material of the shaft is assumed linear and conductive.
- The stator iron is assumed as non-conducting linear and with infinite permeability.
- The operating point in the $BH$ curve is assumed to be far from saturation.
- It is assumed that the remanent magnetic flux density of the magnet $B_r$ does not have any influence in the induced eddy currents.
- The influence of the end-effects is disregarded.

![Figure 2.12: PM motor configuration [20].](image)

Qazalbash, Sharkh, Irenji, Wills & Abusara. A study on PM generators connected to non-controlled rectifiers is presented in [21]. Figures 2.13a and 2.13b shows the geometry of the machine and the model implemented. This study accounts for the influence of the stator slots and the space harmonics in the stator mmf in the distribution of the magnetic flux density $B$. In addition, the study is applied with arc-shaped magnets with parallel magnetization accounting for space and time harmonics. It is also applied to a rotor with a non-conductive, non-magnetic sleeve. Some additional assumptions are:
• The magnet is assumed to be a conductive region with zero magnetization.
• The end-effect is neglected.
• The model assumes the use of a ring magnet.
• The machine has negligible saturation.

Figure 2.13: A quarter of the model of the PM machine, and cylindrical slot-less model of a PM machine [21].

Dubas & Rahideh. A study applied to inset PM synchronous machines is presented in [22]. Additionally, this article makes a description of the studies performed before by other authors since 1995. It accounts for both time and space harmonics. A well detailed list of assumptions is:

• The end-effects are disregarded.
• The effect of the slots in the stator is neglected.
• The remanent magnetic flux density in the magnet $B_r$ is neglected.
• Any saturation effect in the rotor and stator iron is null.
• There is no air-spaces between magnets and iron inter-poles, figure 2.14.
• The permeability and conductivity of the magnets are assumed constant.
• The induced eddy current density in the magnets circulates in $z$-direction.
• The faces of the magnets are radially shaped (figure 2.14).
Martin, Zaïm, Tounzi & Bernard. An improved study is presented in [23]. A new method based on a magneto-dynamic problem in a conductive ring using Cartesian coordinates is introduced. Accounting only for circumferential segmentation, it considers the effect of the slots in the stator. Figure 2.15 shows the geometry and main dimensions implemented in the model. Some general assumptions are:

- The conductive ring is assumed to be homogeneous, linear and isotropic.
- Infinite iron permeability.
- The length of the ring is much higher than pole pitch.
- The stator current is assumed as an equivalent current density sheet.
Mirzaei, Binder, Funieru & Susic. The study reported in [2] accounts for both time and space harmonics and considers axial and circumferential segmentation of the magnets. This study goes further and makes a comparison of the calculated losses when accounting for the reaction field of the eddy currents. The geometry proposed for this study is shown in figure 2.16. The main assumptions are:

- The normal component of $B$ is only studied.
- The normal component of $J$ is neglected.
- Infinite stator and rotor iron permeability.

2.3.3 Comparative table of the literature review

The main purpose of this section is to summarize the different studies reviewed so far in the calculation of losses in magnets of PM machines. Table 2.1 is composed by six main fields: authors and year of publication, type of study, the type of machine to which the study was applied, the level of complexity and accuracy and the type of validation made by the authors. Hence, instead of evaluating the quality of the article, this section is expected to give the reader a brief overview about each study presented in this chapter. Note that the criteria complexity and accuracy levels are a personal judgement based only on the results and expressions presented in the articles.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Focus of Study</th>
<th>Type of machine</th>
<th>Compl. Level</th>
<th>Accura. Level</th>
<th>Validation</th>
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Table 2.1: References summary (continues).
<table>
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<th>Reference</th>
<th>Focus of Study</th>
<th>Type of machine</th>
<th>Compl. Level</th>
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<th>Validation</th>
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Table 2.1: References summary (continues).
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<tr>
<th>Reference</th>
<th>Focus of Study</th>
<th>Type of machine</th>
<th>Compl. Level</th>
<th>Accura. Level</th>
<th>Validation</th>
</tr>
</thead>
</table>

Table 2.1: References summary.
2.4 Summary and conclusions

In this chapter a description of the eddy current phenomenon and its consequences were presented as a motivation for this study. In addition, a classification of the literature available in the calculation of the losses in a magnet was performed and summarized in table 2.1. As mentioned in previous sections, the priority was given to the models accounting for time and space harmonics, accounting for the reaction field of eddy currents and with similarities with the machine analysed in this project. Hence, the models reported in [20] and [21] were selected and a reproduction of these was attempted. However, the results were not satisfactory due to either lack of information or misinterpretation of the expressions described. A second selection was focused in one model neglecting the reaction field of eddy currents and a second model accounting for this effect. Thus, the models presented in [15] and [12] are further described in Chapter 3. Additionally, some preliminary conclusions are:

- There are certainly few articles reporting both experimental and FEM 3D simulations validations. Furthermore, only two of the studies; [20] and [21], are applied to models of machines with similar characteristics to the machine studied in this project (i.e., slot-less with a magnetized ring).

- In general, several common assumptions were identified in all articles reviewed for the calculation of losses in magnets. Among these, the neglect of the end-effects is the most common since all models are developed as a 2 dimensional problem. FEM simulations will allow to verify the impact of such assumption.
Chapter 3

Description of the analytical model

3.1 Introduction

This chapter may be divided in three major parts, as follows. Firstly, the machine studied in this project is introduced, that is, main dimensions and working characteristics. Secondly, the analytical expressions for calculating the no-load back-emf $E_0$ and the magnetic flux density in the air-gap due to the currents in the windings $B_{\delta w}$. Lastly, the description of two analytical models [15] and [12] used for the calculation of losses in the magnets reviewed in Chapter 2.

3.2 Description of the machine

3.2.1 Main dimensions

The main dimensions of the slot-less machine analysed in this study are described in table 3.1 and figure 3.3. Note that the analytical calculations and FEM 2D and 3D simulations are performed on a machine with a length equal to a single magnet segment $l$. 
### Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active length $L_a$ [mm]</td>
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<tr>
<td>Maximum speed $n_{max}$ [rpm]</td>
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</tr>
<tr>
<td>Air-gap length $l_g$ [mm]</td>
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<td>Number of winding turns per phase $N_c$</td>
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<tr>
<td>Magnet axial length $l$ [mm]</td>
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<tr>
<td>Magnet thickness $h$ [mm]</td>
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</tr>
<tr>
<td>Magnet radius $R_m$ [mm]</td>
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</tr>
<tr>
<td>Shaft diameter $R_s$ [mm]</td>
<td>2.5</td>
</tr>
<tr>
<td>Inner stator diameter $R_s$ [mm]</td>
<td>23.2</td>
</tr>
<tr>
<td>Outer stator diameter [mm]</td>
<td>31</td>
</tr>
<tr>
<td>Number of poles $p$</td>
<td>2</td>
</tr>
<tr>
<td>Peak phase current $I$ [A]</td>
<td>1</td>
</tr>
<tr>
<td>Fundamental frequency $f_0$ [Hz]</td>
<td>500</td>
</tr>
</tbody>
</table>

Winding type: distributed $q = 1$

Table 3.1: Main machine dimensions.

#### 3.2.2 Working characteristics

The machine analysed in this project is applied to the power drive of a nut-runner. Figure 3.1 illustrates the working cycle of this type of machine [24]. This working cycle is divided in two stages. The first, in which the machine is running at maximum speed with zero torque for reaching the tightening point. The second stage at which the machine reaches nominal speed as the torque is increased to reach the correct value of
torque applied to the nut. This study is focused on the first operational stage, since it is this region in which the company has more interest.

Given the PWM technique implemented in the converter, the running at no-load may involve relatively high losses due to the appearance of harmonics at 1 time and 2 times the switching frequency $f_{sw}$. Figure 3.2 shows the harmonic distribution of the inverter output voltage at no-load.

![Figure 3.2: FFT of PWM voltage [24].](image)

For the coming analyses, it is assumed that the switching frequency $f_{sw}$ is 10 kHz. Additionally, the fundamental frequency $f_0$ is calculated as:

$$f_0 = \frac{n_{max}}{60} \quad (3.1)$$

With $n_{max}$ the maximum speed equal to 30000 rpm. For this study one harmonic from each group is selected. Thus, the harmonics indexes $n_1$ and $n_2$ appearing at $f_{sw}$ and $2f_{sw}$ respectively, are given by:

$$n_1 = \frac{f_{sw}}{f_0}$$

$$n_2 = \frac{2f_{sw}}{f_0} \quad (3.2)$$

### 3.3 Analytical calculation of the no-load back-emf

For a slot-less machine with a magnet ring with parallel magnetization $M$ as described in figure 3.3, the no-load back-emf per turn per phase per pole $\dot{E}_{turn}$ can be calculated with the expression [25]:
\[ \dot{E}_{\text{turn}} = \frac{\dot{B}_{\text{sm}} L_a \pi f}{p(R_{w2}^2 - R_{w1}^2)} \sin \kappa \cdot \frac{\kappa}{C_e} \] (3.3)

Where,

- \( L_a \) corresponds to the axial length of the machine.
- \( \kappa \) is the angular width of winding sections (figure 3.3).
- \( p \) is the number of poles.
- \( R_{w1} \) and \( R_{w2} \) are the internal and external radii of the copper in the windings as in figure 3.3.

The peak value of magnetic flux density at the stator surface due to the magnet flux \( \dot{B}_{\text{sm}} \) can be obtained with the expression:

\[ \dot{B}_{\text{sm}} = \mu_0 H_{cB} \left( \frac{R_m^2 - R_r^2}{R_s^2 - R_r^2} \right) \] (3.4)

Where \( H_{cB} \) is the coercive strength of magnet material, found in the manufacturer data-sheet [26]. The constant for no-load induced voltage \( C_e \) is given by the geometrical dimensions of the machine and can be found in [25].

### 3.4 Analytical calculation of B in the air-gap

The analytical calculation of the magnetic flux density in the air-gap of a slot-less machine due to the stator currents \( B_{\delta w} \) is presented in [27]. According to figure 3.4, the radial component of the magnetic flux density in region II \( B_{rII} \) is given by the
CHAPTER 3. DESCRIPTION OF THE ANALYTICAL MODEL  

expression:

\[ B_{rII}(r, \theta) = \sum_{m=1,2,3,...} \mu_0 J_m R_c \left[ -\frac{(mp+2)}{2} \left( \frac{R_s}{R_c} \right)^{2mp} + 2 \left( \frac{R_s}{R_c} \right)^{(mp+2)} \right] \left[ 1 - \left( \frac{R_s}{R_c} \right)^{2mp} \right] \left[ \left( \frac{R_c}{R_r} \right)^{(mp+1)} \left( \frac{r}{R_r} \right)^{(mp-1)} \left[ \left( \frac{R_c}{r} \right)^{(mp+1)} \right] \cos(mp\theta) \right] \]  

(3.5)

Where,

- \( m \) is the space harmonic index.
- \( r \) is the radius at which \( B \) is calculated.
- \( R_c \) is the inner radius of the winding.

![Slot-less machine geometry for calculating \( B_{\delta w} \), [27].](image)

Note that there is no difference between \( R_c \) and \( R_{w1} \) in figures 3.4 and 3.3, respectively. The reason for using different notations is to keep the original expressions used by each author avoiding misleading the reader. The current density \( J_m \) is given by:

\[ J_m = \frac{4N_c I_p}{\pi(R_s^2 - R_c^2)} K_{wm} \]  

(3.6)

With \( K_{wm} \) as the winding factor, which in turn is given by:

\[ K_{wm} = \sin \left( \frac{m\pi}{2} \right) \frac{\sin \left( \frac{m\pi}{6} \right)}{\left( \frac{m\pi}{6} \right)} \]  

(3.7)

Similarly, the angular component (or tangential component) of the magnetic flux den-
sity in region II $\hat{B}_{\theta II}$ can be calculated with the following expression:

$$B_{\theta II}(r, \theta) = - \sum_{m=1,2,3,...} \mu_0 J_m R_c \left[ \frac{-\mu_0 J_m R_c \left( \frac{R_s}{R_c} \right)^{2mp} + 2 \left( \frac{R_c}{R_s} \right)^{(mp+2)} + \left( \frac{mp-2}{2} \right)}{1 - \left( \frac{R_s}{R_c} \right)^{2mp}} \right]$$

$$\cdot \left[ \left( \frac{R_c}{R_r} \right)^{(mp+1)} \left( \frac{r}{R_r} \right)^{(mp-1)} - \left( \frac{R_c}{r} \right)^{(mp+1)} \right] \sin(mp\theta)$$

(3.8)

Note that in the original article, the space harmonic index is defined as $n$. In this report $m$ is used instead to make coming formulations consistent with the ones described previously.

### 3.5 Analytical calculation of magnet losses

#### 3.5.1 Model neglecting the reaction effect of eddy currents

Huang, Bettayeb, Kaczmarek & Vannier

As described in [15] the calculation of the losses in the magnet when the skin effect is disregarded (low frequencies) is given by the following expression:

$$P_m = \frac{V_m \hat{B}_{rII}^2 \omega_h^2}{16 \rho_m} \cdot \frac{w^2 l^2}{l^2 + w^2}$$

(3.9)

With,

- $V_m$ as the volume of the magnet (assuming that this is of a rectangular section).
- $w$ is the radial span of the magnet (figure 2.8).
- $\hat{B}_{rII}$ is the peak magnetic flux density in the air-gap due to the mmf of the stator current, calculated in section 3.4.
- $\omega_h$ is the electrical angular frequency of the applied harmonic current.
- $\rho_m$ is the resistivity of the magnet.

The main assumptions adopted for the calculation of the losses by this model are:

- The skin effect is neglected.
- $B$ is homogeneous over the magnet width $w$.
- The width $w$ is much smaller than the magnet length $l$, this way neglecting end effects.
In this model the magnet ring is assumed to have a rectangular section as in figure 2.8 with a total width $w$ of $(R_r + R_m)/2$. In addition, this study presents an alternative for the calculation of losses for higher frequencies. Taking [28] as reference, the power losses per area exposed to a field $H$ can be calculated as:

$$\frac{P}{S} = \frac{1}{2} H_{\tan}^2 R_s$$  \hspace{1cm} (3.10)

With $H_{\tan}$ as the peak tangential incident magnetic field, $S$ being the tangential surface given by:

$$S = 2h(l + w)$$  \hspace{1cm} (3.11)

And $R_s$ is the surface impedance:

$$R_s = \frac{1}{\delta_m \sigma_m}$$  \hspace{1cm} (3.12)

Where $\delta_m$, the skin depth, is given by:

$$\delta_m = \sqrt{\frac{2}{\omega_h \sigma_m \mu_m \mu_0}}$$  \hspace{1cm} (3.13)

And $\sigma_m$ is the conductivity of the magnet. The criterion for selecting either of the two methods is based on how large the skin depth is in comparison to the magnet dimensions $w$, $h$ and $l$.

### 3.5.2 Model accounting for the reaction effect of the eddy currents

Zhu, Schofield & Howe

This model presented in [12] defines the losses in the region III (figure 2.3) as:
\[ P_{III} = 2\alpha_p \pi L_a R_s^2 \omega_r \mu_0 \mu_m \mu_{sl}^2 \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \frac{(n + m)}{m} j_{nm}^2 \]  

(3.14)

\[
\cdot \text{Re}\left\{ \frac{C_9}{K} \left[ \left( \frac{\tau_m R_r}{m} Y_{m-1}(\tau_m R_r) - Y_m(\tau_m R_r) \right) J_m(\tau_m R_m) \right.ight.
\]

\[ - \left( \frac{\tau_m R_r}{m} J_{m-1}(\tau_m R_r) - J_m(\tau_m R_r) \right) Y_m(\tau_m R_m) \left. \right] \frac{C_9^*}{K^*} \left[ \left( \frac{\tau_m R_r}{m} Y_{m-1}(\tau_m R_r) - Y_m(\tau_m R_r) \right) \right.
\]

\[ \cdot \left( \frac{\tau_m R_r}{m} J_{m-1}(\tau_m R_r) - J_m(\tau_m R_r) \right) \left. \right] - \left( \frac{\tau_m R_r}{m} Y_{m-1}(\tau_m R_r) - Y_m(\tau_m R_r) \right) \left. \right] \right\}^* \}

Where, the sub-indexes \( m \) and \( n \) correspond to space and time harmonics, respectively. The functions \( J_m \) and \( Y_m \) are Bessel functions of first and second kind of \( m \) order, respectively. And the harmonic amplitude of the equivalent current sheet distribution:

\[ J_{nm} = \frac{3N_c I K_{dpv} K_{sow}}{\pi R_s} \]  

(3.15)

Where, \( K_{sow} \) and \( K_{dpv} \) the slot opening and winding factors defined in [29]. Other parameters such as \( C_9, K, \tau_m, \tau_{sl} \) and \( \mu_{sl} \) are described and derived in the article itself. Therefore, for sake of simplicity and to prevent any misinterpretation, their descriptions are omitted in this report but the readers are encouraged to review them in each article. The main assumptions adopted in this model are:

- The end-effects are neglected, that is, it is assumed that the eddy currents only flow in the axial direction.
- The retaining sleeve is assumed to be non-magnetic.
- The magnets and the sleeve are assumed to be homogeneous and having constant and isotropic permeabilities and conductivities.
- The windings of the stator are represented as current sheets placed in the slots openings, figure 2.3.
- The variation of the permeance due to the presence of slots is omitted.

Note that for the calculation of losses in the magnet with this model, the coefficient \( K_{sow} \) is assumed to be equal to 1, due to the absence of slots. Additionally, the thickness
of the retaining sleeve in figure 2.3 is set to zero and the corresponding permeability 
\( \mu_{sl} = 1 \) and conductivity \( \sigma_{sl} \approx 0 \).

3.6 Summary and conclusions

In this chapter a description of the machine analysed in this project was carried out. A switching frequency for both analytical and FEM calculations was selected. In addition, a description of the analytical models selected for the calculation of the magnet losses was performed. The models introduced in section 3.5.1 are denominated as Huang\(_a\) and Huang\(_b\) respectively and the results are presented in Chapter 6. Similarly, the model presented in section 3.5.2 is denominated as Zhu and the results are also presented in Chapter 6. Additionally, as presented in Chapter 2 there are differences between the geometries considered for selected magnet loss models and the actual machine. Therefore, it was required to adapt the expressions to the actual machine characteristics.
Chapter 4

FEM 2D simulations

4.1 Introduction

This chapter is focused on simulations by 2D FEM method. The FEM software selected is FLUX™ v12 from Cedrat. All simulations were performed in the Transient Magnetic module. The results for $E_0$, magnetic flux density in the air-gap due to the fundamental of the stator mmf $B_{5w(1)}$ and magnet losses are reported. Furthermore, for the calculation of the losses in the magnet, some important considerations are studied:

- Influence of the mesh density.
- Effect of the time-step in the results.
- Application of the principle of superposition.
- Influence of the remanent magnetic flux density of the magnet $B_r$ on the losses.
- Calculation of losses simulating the rotor fixed with zero speed.

An analysis of the losses variation with frequency is presented as well. Therefore, a range of frequencies for $n_1$ was selected from 10 to 100 kHz. Even though the highest switching frequency is around 50 kHz the selection of this range allows to verify the behaviour of the losses with frequency. The methods and selection of the various parameters serve as reference for the FEM 3D simulations.

4.2 No-load back-emf

This section describes the simulation performed for calculating $E_0$ of the machine. In order to determine the voltage induced in the windings, it is necessary to define an electrical circuit (figure 4.2) associated to the geometry shown in figure 4.1. The main parameters adopted for these simulations are presented in table 4.1. The magnet was
modelled as a *Linear magnet described by the $B_r$ module* and a relative permeability $\mu_m$ with parallel magnetization. The shaft was modelled as *Air or vacuum region*. The stator was modelled as *Magnetic non-conductive region* and as *FLU_M270_35A*. The resistors were simulated with a very high value of resistance in order to emulate an open circuit during no load. Regarding to the boundary conditions, the geometry was surrounded by an infinite box set as *Air or vacuum region*. The simulation was run varying the position of the rotor every 5° and the *aided mesh* was implemented.

![2D geometry simulated and magnetization arrows.](image1)

*Figure 4.1: 2D geometry simulated and magnetization arrows.*

![Equivalent circuit for calculating $E_0$ in 2D simulations.](image2)

*Figure 4.2: Equivalent circuit for calculating $E_0$ in 2D simulations.*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanent magnetic flux density $B_m$ [T]</td>
<td>1.12</td>
</tr>
<tr>
<td>Current $I$ [A]</td>
<td>0</td>
</tr>
<tr>
<td>Relative permeability $\mu_m$</td>
<td>1.05</td>
</tr>
<tr>
<td>Resistance $[\Omega]$</td>
<td>$1 \times 10^5$</td>
</tr>
</tbody>
</table>

*Table 4.1: Parameters definition for back-emf simulations.*

The obtained values of $E_0$ in each phase are presented in figure 4.3. The peak value is equal to $E_0=8.49$ V.
As explained previously in this document, the values of $E_0$ are calculated for a single magnet segment. That is, for the complete machine it would be necessary to multiply this value by 14 (i.e., the number of magnet segments). It is important to point out the sinusoidal nature of $E_0$ due to the parallel magnetization of the magnet in combination with the slot-less winding.

### 4.3 B in the air-gap due to stator currents

In order to obtain $B_{\delta w(1)}$ an initial current was applied as follows:

\[
\begin{align*}
I_a &= I \cos(\omega t) \\
I_b &= I \cos \left(\omega t - \frac{2\pi}{3}\right) \\
I_c &= I \cos \left(\omega t - \frac{4\pi}{3}\right)
\end{align*}
\]

Where $\omega$ is the fundamental angular frequency, given as $2\pi f_0$. Then, a path was drawn in the geometry in order to obtain the normal component of $B_{\delta w(1)}$ represented by the white dotted contour in figure 4.4. The magnet was modelled as a Magnetic non-conductive region with the definition of the parameters described in table 4.2. The remaining regions were kept as in section 4.2.
Figure 4.4: Path definition for plotting of $B_{\delta w}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanent magnetic flux density $B_m$ [T]</td>
<td>-</td>
</tr>
<tr>
<td>Current $I$ [A]</td>
<td>1</td>
</tr>
<tr>
<td>Relative permeability $\mu_r$</td>
<td>1.05</td>
</tr>
<tr>
<td>Resistance [Ω]</td>
<td>$1 \times 10^5$</td>
</tr>
</tbody>
</table>

Table 4.2: Parameters definition for calculation of $B_{\delta w(1)}$.

Figure 4.5 shows the results of $B_{\delta w(1)}$ with a maximum value of $7.5 \times 10^{-3}$ T. Note that the wave-shape is not sinusoidal since the winding is not sinusoidally distributed ($q = 1$).

Figure 4.5: FEM 2D normal component of $B_{\delta w(1)}$. 
Applying FFT to $B_{\delta w(1)}$ the harmonic spectrum shown in figure 4.6 was obtained. This spectrum is in line with theory; only odd harmonics are present. In addition, in a Y-connected synchronous machine, if the space harmonics indexes $m$ are multiple of 3, the voltages in each phase have same angle. Consequently, there will be no third harmonics either [30].

4.4 Magnet losses

4.4.1 Mesh size validations

FLUX$^\text{TM}$ software gives the option of activating an aided mesh in its user interface. In addition to this type of mesh, two more meshes were implemented with different sizes of elements. They are defined as follows:

- **Mesh 1**: Aided mesh.
- **Mesh 2**: Coarser mesh.
- **Mesh 3**: Finer mesh.

This selection of mesh size was focused on the regions which are believed to be more critical. For all three meshes, the smallest mesh elements were located within the magnet region and the air-gap. Furthermore, the size of the elements in these regions was selected to be lower than the skin depth of the magnet $\delta_m$ at 10 kHz. Additionally, it was decided to link the time-step for each mesh to the size of the elements.

**Mesh 1**

For this type of mesh, the aided mesh option was activated. Figure 4.7 shows the distribution of the elements in each region. The selection of the time-step in this case, was based on the suggestions from the tutorials of FLUX$^\text{TM}$. With a number of steps $n_{\text{steps}}=140$ the time step was selected as:
\[ t_{\text{step}_1} = \frac{T_0}{n_{\text{step}}} \]  

(4.2)

With \( T_0 \) as the period of the fundamental frequency \( f_0 = 500 \text{ Hz} \).

Mesh 2

For this specific case, the term "coarser mesh" refers to a mesh of lower quality when compared with the "aided mesh". Some parameters defining the mesh in the software were set up manually and the aided mesh was disabled. The software offers several options for defining the size of the elements required. Among them, the number of elements that a mesh line should have (Arithmetic). This alternative was selected and the definition of the mesh lines is presented in figure 4.8.

The number of elements chosen for every mesh line is introduced in table 4.3. A rough
calculation of the size of the elements inside the magnet can be done with the perimeter described by the magnet radius $R_m$ and the number of segments $n_{\text{seg}}$ defined for this line:

$$size_2 = \frac{2\pi R_m}{n_{\text{segment}}}$$

(4.3)

This yields $size_2 = 1.31$ mm which is lower than the skin depth by a factor of 3 approximately. Note that the skin depth for frequencies of 10 and 20 kHz are 5.8 and 4.1 mm respectively (according to equation 3.13). The geometry meshed with Mesh 2 is shown in figure 4.9. The definition of the time-step was achieved by introducing $size_2$ in equation 3.13. An equivalent frequency $f_{\text{step2}} = 197.11$ kHz was obtained. Consequently, the time-step for Mesh 2 was $t_{\text{step2}} = 5.07 \times 10^{-6}$ s.

![Figure 4.9: Mesh density of Mesh 2.](image1)

![Figure 4.10: Mesh density of Mesh 3.](image2)

<table>
<thead>
<tr>
<th>Mesh line</th>
<th>Mesh 2</th>
<th>Mesh 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft</td>
<td>18</td>
<td>32</td>
</tr>
<tr>
<td>Magnet</td>
<td>54</td>
<td>96</td>
</tr>
<tr>
<td>In_winding</td>
<td>54</td>
<td>96</td>
</tr>
<tr>
<td>Out_winding</td>
<td>18</td>
<td>36</td>
</tr>
<tr>
<td>Stator</td>
<td>18</td>
<td>32</td>
</tr>
</tbody>
</table>

Table 4.3: Definition of line regions for 2D simulations.

**Mesh 3**

The definition "finer mesh" is applied to a mesh in which the elements are finer in comparison to those of the "aided mesh". The number of elements for each mesh
line is also described in table 1 and figure 4.10 shows the distribution of the elements in Mesh 3. For the calculation of the time-step, similar procedure was followed as for Mesh 2. The equivalent frequency for the calculated segment size $size_3$ is $f_{step3} = 1404.40$ kHz and the time-step $t_{step3} = 7.12 \times 10^{-7}$ s.

### 4.4.2 Simulations results

The main goal of this mesh density study was calculating the losses in the magnet. As it was described in Chapter 3, only time harmonics $n_1$ and $n_2$ were considered. Hence, the currents applied in the simulations were defined as:

\[
I_a = I \cos(n_1\omega t) + I \cos(n_2\omega t)
\]
\[
I_b = I \cos\left(n_1\left[\omega t - \frac{2\pi}{3}\right]\right) + I \cos\left(n_2\left[\omega t - \frac{2\pi}{3}\right]\right)
\]
\[
I_c = I \cos\left(n_1\left[\omega t - \frac{4\pi}{3}\right]\right) + I \cos\left(n_2\left[\omega t - \frac{4\pi}{3}\right]\right)
\]

(4.4)

Additionally, figure 4.11 shows the circuit defined for the computation of the magnet losses in FEM 2D simulations.

![Figure 4.11: Equivalent circuit for calculating magnet losses in 2D simulations.](image)

Two sets of simulations were performed. First, each one of the three meshes with their corresponding time-step. The calculated values of losses in the magnet are shown in figure 4.12 and presented in table 4.5.
Table 4.4: Parameters definition for calculation of losses in the magnet.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanent magnetic flux density $B_m$ [T]</td>
<td>0</td>
</tr>
<tr>
<td>Current $I$ [A]</td>
<td>1</td>
</tr>
<tr>
<td>Relative permeability $\mu_r$</td>
<td>1.05</td>
</tr>
<tr>
<td>Resistivity $\rho_m$ [$\Omega m$]</td>
<td>$1.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>Resistances [$\Omega$]</td>
<td>$1 \times 10^5$</td>
</tr>
<tr>
<td>Harmonic index $n_1$</td>
<td>20</td>
</tr>
<tr>
<td>Harmonic index $n_2$</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 4.5: Magnet losses and simulation times for the three mesh types.

<table>
<thead>
<tr>
<th>Mesh type</th>
<th>Magnet losses [W]</th>
<th>Simulation time [s]</th>
<th>Losses deviation [%]</th>
<th>Time deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh 1</td>
<td>1.61</td>
<td>73</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mesh 2</td>
<td>2.28</td>
<td>63</td>
<td>41.61</td>
<td>-13.70</td>
</tr>
<tr>
<td>Mesh 3</td>
<td>2.75</td>
<td>920</td>
<td>70.81</td>
<td>1160.27</td>
</tr>
</tbody>
</table>

Table 4.5 describes the deviations of magnet losses and simulation times for the different meshes and time-steps, taking as reference the aided mesh or Mesh 1. Note that positive values of deviation indicates that the values to which the reference is contrasted to are larger and vice-versa. The second set, was run keeping the same value of time-
step for the three mesh types. The results are shown in figure 4.13. The results are summarized in table 4.6. In this case, the time-step corresponding to the Mesh 2 was chosen.

![Figure 4.13: Magnet losses for the three mesh densities and same time-step.](image)

Table 3 describes the deviation in losses and simulation time taking Mesh 1 as reference.

<table>
<thead>
<tr>
<th>Mesh type</th>
<th>Magnet losses [W]</th>
<th>Simulation time [s]</th>
<th>Losses deviation [%]</th>
<th>Time deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh 1</td>
<td>1.61</td>
<td>73</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mesh 2</td>
<td>1.61</td>
<td>43</td>
<td>0</td>
<td>-41.10</td>
</tr>
<tr>
<td>Mesh 3</td>
<td>1.61</td>
<td>91</td>
<td>0</td>
<td>24.66</td>
</tr>
</tbody>
</table>

Table 4.6: Magnet loss and simulation times for three mesh types and same time-step.

The selection of the time-step showed to have a more significant incidence than the size of the mesh elements in the calculated values of magnet losses. According to figure 4.13 and table 4.6, simulations with same time-step and different mesh densities yield identical values of losses. On the other hand, as shown in figure 4.12 and table 4.5, different time-steps gave different values of losses. In addition, the reduction of the time-step and size of mesh elements resulted in the increment of the simulation time. Hence, the definition of the time-step was studied further in the next section of this chapter.
4.4.3 Time-step validation

As shown in section 4.4.2, the losses were not varying with the size of the mesh elements for the three tested meshes. However, there was a dependence with the time-step, the frequency and the skin depth. Therefore, it was necessary to investigate if the calculation of the losses is affected by changes in the value of the time-step. Thus, the time-step is formulated as a function of the frequency, as follows:

\[ t_{\text{step}} = \frac{1}{f_{\text{sw}} k_{\text{time}}} \]  

(4.5)

Where \( k_{\text{time}} \) is the number of samples per period. The factor \( k_{\text{time}} \) was taken from 5 to 50 in steps of 5. It is important to remark that this analysis was applied to the finer mesh (i.e., Mesh 3). The variation of the losses with the number of points per period \( k_{\text{time}} \) is shown in figure 4.14. As it may be seen, the lower \( k_{\text{time}} \), the lower the calculated value of losses and as the number of sampling points increased, the results became more stable converging to a value.

![Figure 4.14: Magnet losses for different \( k_{\text{time}} \).](image)

The wave shape of the currents applied for every number of samples was analysed as well and the plots are shown in figure 4.15.
Figure 4.15 gives an indication of the required \( k_{\text{time}} \) for a good resolution of the results. For example, the current in dotted line corresponding to a \( k_{\text{time}} = 5 \) illustrates that this is not accurately represented. On the other hand, the current presented in dashed line corresponding to \( k_{\text{time}} = 25 \) shows a more realistic trend. Therefore, the results obtained with \( k_{\text{time}} \leq 25 \) might not be sufficiently reliable. In contrast, the current shown in continues line corresponding to \( k_{\text{time}} = 50 \) showed that the current applied has a smooth and more realistic behaviour, which is accomplished with a number of samples \( k_{\text{time}} \geq 30 \).

<table>
<thead>
<tr>
<th>( k_{\text{time}} )</th>
<th>Magnet losses [W]</th>
<th>Simulation time [s]</th>
<th>Losses deviation [%]</th>
<th>Time deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.37</td>
<td>64</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>1.90</td>
<td>122</td>
<td>38.69</td>
<td>90.63</td>
</tr>
<tr>
<td>15</td>
<td>2.14</td>
<td>200</td>
<td>12.63</td>
<td>212.50</td>
</tr>
<tr>
<td>20</td>
<td>2.29</td>
<td>249</td>
<td>7.01</td>
<td>289.06</td>
</tr>
<tr>
<td>25</td>
<td>2.38</td>
<td>318</td>
<td>3.93</td>
<td>396.88</td>
</tr>
<tr>
<td>30</td>
<td>2.45</td>
<td>398</td>
<td>2.94</td>
<td>521.88</td>
</tr>
<tr>
<td>35</td>
<td>2.50</td>
<td>456</td>
<td>2.04</td>
<td>612.50</td>
</tr>
<tr>
<td>40</td>
<td>2.54</td>
<td>503</td>
<td>1.60</td>
<td>685.94</td>
</tr>
<tr>
<td>45</td>
<td>2.57</td>
<td>574</td>
<td>1.18</td>
<td>796.88</td>
</tr>
<tr>
<td>50</td>
<td>2.59</td>
<td>660</td>
<td>0.78</td>
<td>931.25</td>
</tr>
</tbody>
</table>

Table 4.7: Variation of losses and simulation time with number of steps \( k_{\text{time}} \).
Table 4.7 shows the variation of the losses with the number of samples $k_{time}$. Additionally, the deviation in losses of each result with respect to the previous one is presented in column 4. The deviation of simulation time between each value of $k_{time}$ and the initial value of $k_{time} = 5$ is presented in column 5. The trend is that the deviation in losses between a new value and the calculated previously is reducing as the number of samples is increased. Then, it is required to define at which level of deviation is considered that a reliable value has been reached. If it is assumed that the solution is reached when a variation of 1% is achieved, according to table 4.7, a number of sampling points $k_{time} = 50$ is required. It is also appropriate to mention that the simulation time is increased as the factor $k_{time}$ increases. Hence, the selection of this factor is a compromise between time and accuracy.

### 4.4.4 Superposition of the losses

The main focus of this section was to verify if the principle of superposition might be applied to the simulations for different harmonic indexes. This was performed keeping the rotor speed at a value of 30000 rpm, and varying the harmonic indexes $n_1$ and $n_2$ accordingly in order to obtain a variation of the switching frequency $f_{sw}$ from 10 to 100 kHz in steps of 1 kHz. Note that for these simulations values of $n_1$ multiples of three are not considered (i.e., $n_1 = 21, 24, 27, \ldots$). Three different cases were studied:

- **Case 1**: Losses only due to the harmonic index $n_1$ and a phase shift of $\pi/3$.
- **Case 2**: Losses only due to the harmonic index $n_2$ and a phase shift of $\pi/2$.
- **Case 3**: Total losses due to both $n_1$ and $n_2$.

Thus, the variation between the calculated losses in *Case 3* and the summation of the losses calculated in *Case 1* and *Case 2* should be negligible.

#### Case 1

The currents applied in this simulation are described as follows:

\[
\begin{align*}
I_a &= I \cos \left( n_1 \left[ \omega t - \frac{\pi}{3} \right] \right) \\
I_b &= I \cos \left( n_1 \left[ \omega t - \frac{2\pi}{3} - \frac{\pi}{3} \right] \right) \\
I_c &= I \cos \left( n_1 \left[ \omega t - \frac{4\pi}{3} - \frac{\pi}{3} \right] \right)
\end{align*}
\]  

(4.6)

The results are presented in figure 4.16.
Case 2

Similar procedure is applied for Case 2. The currents applied in this simulation are:

\[
I_a = I \cos \left( n_2 \left[ \omega t - \frac{\pi}{2} \right] \right)
\]
\[
I_b = I \cos \left( n_2 \left[ \omega t - \frac{2\pi}{3} - \frac{\pi}{2} \right] \right)
\]
\[
I_c = I \cos \left( n_2 \left[ \omega t - \frac{4\pi}{3} - \frac{\pi}{2} \right] \right)
\]  
(4.7)

The results are presented in figure 4.16.

Case 3

The total losses with the incidence of both harmonic indexes \( n_1 \) and \( n_2 \) were calculated performing simulations with the currents described in section 4.4.2. The results are presented in figure 4.17.

Results

Figure 4.16 shows the losses calculated in Case 1 (blue plot, inverted triangles) and Case 2 (black plot, diamonds) versus frequency. This distinction is necessary, since for the calculation of the losses for case 1 a range of frequency from 10 to 100 kHz was defined. On the other hand, for case 2, the calculation of the losses was performed for frequencies from 20 to 200 kHz, since \( n_2 = 2n_1 \).

![Magnet losses vs frequency for n1 and n2 separately.](image)

Figure 4.16: Magnet losses vs frequency for \( n_1 \) and \( n_2 \) separately.

Then, a comparison of the losses calculated in case 3 and the summation of cases 1 and 2 is presented in figure 4.17. As it can be noticed, the deviation between both
results is minimum which is confirmed with figure 4.18. These deviations are calculated having Case 3 as reference and are fluctuating within -1 and 1% . This leads to the conclusion that is possible to apply superposition principle and calculate the losses for one harmonic at a time and them sum up for each current component in the spectrum.

![Figure 4.17: Total magnet losses vs. frequency.](image1)

![Figure 4.18: Loss deviation vs. frequency.](image2)

### 4.4.5 Effect of remanent magnetic flux density on losses

These simulations were intended to evaluate the effect that $B_r$ could have in the calculation of the magnet losses. Hence, it was decided to evaluate the losses in two different scenarios:
• **Magnet OFF**: Setting the magnet region as a solid conducting region ($B_r=0$ T).

• **Magnet ON**: Setting the magnet as a *Linear magnet described by the $B_r$ module*. That is, setting the value of $B_r=1.12$ T.

In addition to these two scenarios, it was decided to evaluate the effect at extreme frequencies, that is, 10 and 100 kHz. Note that these simulations were performed with the complete harmonic current as defined for *Case 3* and in section 4.4.2.

**Magnet OFF**

Since two frequencies were selected, it was decided to change the mesh density accordingly. This was carried out by initially calculating the equivalent time-step for running the simulations. As described in section 4.4.3, $k_{time}=50$ was adopted. With this value and the two frequencies it was possible to determine the value required of time-step, and the size of the elements in the mesh, as described in section 4.4.1. For a frequency of $f_{sw} \cdot k_{time} = 500$ kHz the skin depth is 0.80 mm. On the other hand, for a frequency of 5000 kHz a skin depth of 0.25 mm was calculated. The results are presented in section 4.4.5 in figures 4.20a and 4.22a for the distribution of the losses in the magnet. Additionally, figures 4.19a and 4.21a show the distribution of the current inside the magnet and table 4.8 summarizes the results obtained.

**Magnet ON**

For the simulations performed in this scenario, the same mesh size as function of the selected frequencies was selected. However, as explained above, the material of the magnet region was selected as *Linear magnet described by the $B_r$ module*. Similarly, the results for the losses in the magnet are presented in figures 4.20b and 4.22b. The current density distribution in figures 4.19b and 4.21b and the mean values are also shown in table 4.8.

**Results**

The distribution of the current density $J$ over the magnet region is presented by pairs in order to allow a better comparison between both cases when the magnet is OFF (left picture) and ON (right picture). For a frequency of 10 kHz, the instantaneous values at $t = 5 \times 10^{-4}$ s are presented in figures 4.19a and 4.19b. There are no significant variations in the distribution of the current density between the two cases. The positive value of the current (represented in yellow) and the negative value of the current (shown in purple) indicates the direction in which this is flowing. However, there is an unusual behaviour of the current density distribution since a symmetrical distribution would
be expected. This undulating behaviour may be explained as a consequence of the reaction field of eddy currents $B_{eddy}$. Nevertheless, something important to point out is that the current seems to be flowing in the total area of the magnet region.

![Figure 4.19: Current distribution at 10 kHz ($t = 5 \times 10^{-4}$ s); (a) magnet OFF, (b) magnet ON.](image)

![Figure 4.20: Loss distribution at 10 kHz ($t = 5 \times 10^{-4}$ s); (a) magnet OFF, (b) magnet ON.](image)

The instantaneous distribution of the losses at 10 kHz and $t = 5 \times 10^{-4}$ s shows no variation when the magnet is OFF and ON either according to figures 4.20a and 4.20b. The value of skin depth at 10 kHz ($\delta_m = 5.8$ mm) can be verified in figure 4.20 as the area where the losses start appearing covers the region between the inner and the outer diameters of the magnet. A similar undulating distribution was obtained
which is expected as the losses are proportional to the square of the current. Figures 4.21a and 4.21b show the current density distribution at a frequency of 100 kHz and $t = 1 \times 10^{-4}$ s. The undulating trend is still present, but a region inside the magnet can be identified in which the current density is equal to zero. This confirms that at 100 kHz the skin-effect is much more pronounced.

![Figure 4.21: Current distribution at 100 kHz ($t = 1 \times 10^{-4}$ s); (a) magnet OFF, (b) magnet ON.](image)

Figures 4.22a and 4.22b illustrates the losses at frequency of 100 kHz. At this frequency, the skin depth ($\delta_m = 1.8$ mm) can be verified in figure 4.22 as the area where the losses are high covers approximately 1/3 of the region between the inner and the

![Figure 4.22: Loss distribution at 100 kHz ($t = 1 \times 10^{-4}$ s); (a) magnet OFF, (b) magnet ON.](image)
outer diameters of the magnet. This corroborates how the skin effect becomes more noticeable as the frequency increases with a large region with approximately zero losses. Figures 4.23 and 4.24 show the calculated losses vs. time when the magnet is ON and OFF for frequencies of 10 and 100 kHz respectively. It is clear that there are not significant variations in either of the cases.

Figure 4.23: Magnet losses at 10 kHz, with magnet ON and OFF.

Figure 4.24: Magnet losses at 100 kHz, with magnet ON and OFF.
<table>
<thead>
<tr>
<th>Frequency [kHz]</th>
<th>Magnet losses (OFF) [W]</th>
<th>Magnet losses (ON) [W]</th>
<th>Losses deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.60</td>
<td>2.59</td>
<td>-0.38</td>
</tr>
<tr>
<td>100</td>
<td>10.09</td>
<td>10.08</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

Table 4.8: Magnet losses for both magnet OFF and ON.

There is not noticeable variation in the calculated value of losses for both magnet OFF and magnet ON and this is shown in the low deviation (less than 1 %) with the losses when the magnet is set as a solid conductor (magnet OFF) as reference.

### 4.4.6 Simulation at zero speed in the rotor

An alternative for running the simulations was setting the speed of the rotor \( n_{\text{max}} \) as zero. Consequently, the only parameter varying with time would be the applied current. However, it was necessary to determine the direction of rotation of the harmonic indexes \( n_1 \) and \( n_2 \) in order to define the relative speed of rotation. The procedure to verify if the harmonic component of the magnetic field is rotating forward or backward is described as follows; if the current applied to phase \( b \) has a phase shift of \( 2\pi/3 \), the product of this phase shift and the time harmonic index \( n_1 = 20 \) results in:

\[
n_1 \cdot \frac{2\pi}{3} = \frac{40\pi}{3}
\]

Then it is required to express this number as a summation of one angle equivalent to a whole revolution \( (2\pi) \) and the phase shift \( 2\pi/3 \). For instance, the former equation can be expressed in two different ways. The first one:

\[
n_1 \cdot \frac{2\pi}{3} = \frac{40\pi}{3} = \frac{38\pi}{3} + \frac{2\pi}{3}
\]

Since 38 is not divisible by 3 then this expression can be discarded. The second one:

\[
n_1 \cdot \frac{2\pi}{3} = \frac{40\pi}{3} = \frac{42\pi}{3} - \frac{2\pi}{3}
\]

In this second expression 42 is indeed divisible by 3 and the negative sign in \(-2\pi/3\) is an indication that the harmonic index \( n_1 \) is rotating in the same direction as the fundamental (forward) and for the definition of the applied current, it is necessary to subtract the fundamental frequency to \( n_1 \). Similar procedure is followed for \( n_2 \), which yields that this harmonic index is rotating in opposite direction to the fundamental (backward) and in this case, the fundamental frequency needs to be added. Again, it was decided to perform this analysis at the extreme frequencies (i.e., 10 and 100 kHz).
Figure 4.25: Magnet losses at 10 kHz, zero speed and 30000 rpm.

Figure 4.26: Magnet losses at 100 kHz, zero speed and 30000 rpm.

<table>
<thead>
<tr>
<th>Simulation type</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>Magnet losses [W]</th>
<th>Simulation time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 kHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_{rotor} = 30000$</td>
<td>20</td>
<td>40</td>
<td>2.59</td>
<td>244</td>
</tr>
<tr>
<td>$n_{rotor} = 0$</td>
<td>19</td>
<td>41</td>
<td>2.59</td>
<td>238</td>
</tr>
<tr>
<td>100 kHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_{rotor} = 30000$</td>
<td>200</td>
<td>400</td>
<td>10.38</td>
<td>594</td>
</tr>
<tr>
<td>$n_{rotor} = 0$</td>
<td>199</td>
<td>401</td>
<td>10.39</td>
<td>627</td>
</tr>
</tbody>
</table>

Table 4.9: Magnet losses at different speeds for two different frequencies.

From the results shown in table 4.9 variations in the calculated losses are negligible.
However, the simulation time is increased for a frequency of 100 kHz and \( n_{\text{rotor}} = 0 \). It was also identified for this case that the initial transient lasts for a longer time. For these simulations the magnet was modelled as a solid conductor (\textit{Magnet OFF}).

4.5 Summary and conclusions

In this chapter preliminary results of \( E_0 \) and \( B_{\delta_w(1)} \) were obtained for the validation of the machine. In addition, the various parameters when simulating magnet losses were introduced and verified. That is, mesh density, time-step, frequency dependence of the losses, superposition principle, effect of remanent magnetic flux density on losses and simulation at zero speed in the rotor. Consequently, some preliminary conclusions are:

- There was no significant effect on the calculated losses when varying the density of the mesh. On the other hand, the time-step selected had an important incidence on the calculated values. However, both mesh density and time-step showed to have direct incidence in the simulation times.

- The possibility of performing superposition when calculating the losses was verified. This means that there is no interaction between harmonics from group 1 (harmonics at 1 time \( f_{sw}, n_1 \)) and group 2 (harmonics at 2 times \( f_{sw}, n_2 \)). This is valid for the specific case of a slot-less machine.

- There is no effect of \( B_r \) in the calculated values of magnet losses.

- The calculation of the losses with the rotor with zero speed yielded similar results. However, the simulation times were not reduced.
Chapter 5

FEM 3D Simulations

5.1 Introduction

In this chapter, the results obtained by FEM 3D simulation method for $E_0$, $B_{sw(1)}$ and losses in the magnet are presented. In order to make a fair comparison with the results from both analytical models and FEM 2D simulations, same dimensions, mesh parameters and time-step were adopted. As for FEM 2D simulations, the analysis in this chapter was performed for a single magnet element. Additionally, for FEM 3D simulations. Two options were tested:

- Simulations with meshed coils.
- Simulations with non-meshed coils.

5.2 Machine validations

5.2.1 No-load back-emf

Meshed coils

The geometry for this type of simulation is presented in figure 5.1. The volumes corresponding to the windings (i.e., yellow, red and magenta regions) were modelled as coil conductors with the same specifications as for FEM 2D simulations. The infinite box was set as Air or vacuum region. Additionally, the setting of the parameters is identical as shown in table 4.1. The results of $E_0$ for each phase are shown in figure 5.2.
As expected, due to the parallel magnetization of the magnet, the wave-shape of $E_0$ is extremely sinusoidal. However, the value of $E_0$ calculated for this case was 2.87 V, significantly lower when compared with FEM 2D results in section 4.2.

**Non-meshed coils**

Three non-meshed coils were created and a coil conductor was assigned to each one. Consequently, the volumes corresponding to each winding were set as *Air or vacuum regions*. The confection of each coil winding was done by means of coordinates and the shape selected for each coil was simplified as it is illustrated in figures 5.3a and 5.3b. A slight variation in the calculated value of $E_0$ among the three phases is expected, since all three coils should be identical. However, this variation is considered negligible. Similar set of parameters were implemented for simulations with non-meshed coils. The resulting $E_0$ is presented in figure 5.4.
Figure 5.3: Non-meshed coils definition for calculation of $E_0$; (a) isometry, (b) lateral view.

Figure 5.4: Back-emf 3D simulations with non-meshed coils.

Again, the wave shape of $E_0$ in this case is sinusoidal, but in comparison to the option with meshed coils, the peak value is much similar to the FEM 2D results with a value of 9.28 V.

5.2.2 B in the air-gap

Meshed coils

With same parameters definition as for FEM 2D simulations in table 4.2, the magnetic flux density in the air-gap due to the fundamental of the mmf $B_{\delta w(1)}$ was calculated. Two paths were defined in order to obtain the distribution of $B$; at the middle region of the magnet domain (path 1 in figure 5.5a, yellow contour) and at one of the corners
of the magnet (path 2 in figure 5.5a, yellow contour) right in the middle of the air-gap (figure 5.5b white contour). The results for both paths are shown in figure 5.6:

![Figure 5.5: Definition of paths for plotting B in the air-gap; (a) isometry, (b) top view.](image)

Figure 5.5: Definition of paths for plotting B in the air-gap; (a) isometry, (b) top view.

![Figure 5.6: Normal component of B_{δw(1)} with meshed coils.](image)

Figure 5.6: Normal component of $B_{δw(1)}$ with meshed coils.

The wave-shape of $B_{δw(1)}$ has a similar distribution as for the obtained in FEM 2D. However, as shown previously with $E_0$ results, the maximum value of $B_{δw(1)}$ with a value of $3.31 \times 10^{-3}$ T is not in the same range. In addition, the value of $B_{δw(1)}$ in Path 2 was $2.61 \times 10^{-3}$ T (around 21.15 % reduction). This could be a result of the fringing flux effect. This phenomenon appears when an air-gap is present in the path of a magnetic flux density $B$ [31]. Furthermore, its effect is more noticeable with increasing frequency. However, the incidence of this phenomenon in the magnet losses is not studied further.
The harmonic spectrum of $B$ in path 1 is illustrated in figure 5.7. A very similar distribution of harmonic order and amplitude is found as in section 4.3.

**Non-meshed coils**

The normal component of $B_{\delta w}(1)$ was calculated following the same procedure and adopting similar parameters as for meshed coils and FEM 2D simulations. The simplified shape of the coils in figures 5.3a and 5.3b was kept and the length of the coils increased in order to avoid any incidence of the end effects in the calculations, that is, the length of the coils was adopted as $L_a = 65$ mm. The results for paths 1 and 2 are shown in figure 5.8.

A maximum value of $7.35 \times 10^{-3}$ T was obtained for $B_{\delta w}(1)$ in Path 1. In contrast, the value in Path 2 was of $6.98 \times 10^{-3}$ T. That is, a deviation of 5.03 %. The harmonic spectrum of plot in path 1 is illustrated in figure 5.9 and the harmonic content follows the same distribution as for FEM 2D simulations and FEM 3D with meshed coils.
The simulations in FEM 3D with non-meshed coils yielded more satisfactory results when compared to FEM 2D simulations. Therefore, in the following all FEM 3D simulations are using non-meshed coils.

5.3 Calculation of the losses in the magnet

The simulation settings presented in Chapter 4 applies to the calculation of the losses in FEM 3D. That is, same time-step factor and a finer mesh were selected in order to guarantee that the values calculated are representative and that a fair comparison can be done with FEM 2D simulation results. The calculation of losses in the magnet was performed at frequencies of 10, 25, 40, 55, 70, 85 and 100 kHz and the results are shown in figure 5.10.

The trend of the losses with frequency differs from FEM 2D simulations. In this case, the nature of the losses is almost linear, in contrast to the logarithmic behaviour of
the losses obtained with FEM 2D simulations (figure 4.17). In order to evaluate the behaviour of the eddy currents in the magnet, the current density $J$ was plotted over the surface of the magnet for a frequency of 10 kHz and $t = 5.2 \times 10^{-5}$ s as shown in figure 5.11. Note that yellow colour and larger arrows indicate maximum values. At first glance the magnet could be divided into two regions in terms of the eddy current flow indicating the two incident values of $B_{sw}$. In addition, some hot-spots of critical regions in which these currents are the highest are identified as the boundaries between the shaft and magnet regions at the top and bottom faces of the magnet. This could be due to the curvature described by these regions in which the currents are forced to vary their trajectory.

![Image of current density distribution over the magnet at 10 kHz](image)

Figure 5.11: Current density distribution over the magnet at 10 kHz ($t = 5.2 \times 10^{-5}$ s).

![Image of current density distribution at 10 kHz](image)

Figure 5.12: Current density distribution at 10 kHz ($t = 5.2 \times 10^{-5}$ s); (a) $xz$ sub-region, (b) $xz$ plane.
For performing a better analysis of the behaviour of the eddy currents inside the magnet, two 2D sub-regions were defined inside the magnet volume. One parallel to \(xz\) axis and a second one parallel to \(yz\) axis. The eddy current plots for grid \(xz\) are shown in figures 5.12a and 5.12b. As predicted in the literature, the eddy currents are forced to flow in the periphery of the magnet. Regarding the plots on the \(yz\) sub-region in figures 5.13a and 5.13b, the currents induced in both regions find a common returning path. The plane \(xy\) is a transition region in which currents change from the top region to the bottom region, thus closing the loop.

![Figure 5.13: Current density distribution at 10 kHz \((t = 5.2 \times 10^{-5} \text{ s})\); (a) \(yz\) sub-region, (b) \(xz\) plane.](image)

![Figure 5.14: Loss distribution at 10 kHz \((t = 5.2 \times 10^{-5} \text{ s})\); (a) over the magnet, (b) \(xy\) plane.](image)
The distribution of the losses is also plotted and shown in figures 5.14a for the complete magnet volume, 5.14b for the $xy$ plane, 5.15a for the $xz$ plane and 5.15b with the $yz$ plane, respectively. The behaviour of the losses is in line with the behaviour of the eddy currents, showing the highest values at the surface of the top and bottom regions and with hot-spots in the boundary between the shaft and magnet volumes. Similar analysis may be applied for a frequency of 100 kHz. The results are presented in appendix A at the end of this document. An identical behaviour was identified, but with different magnitudes of eddy currents and losses.

![Figures 5.15](image)

Figure 5.15: Loss distribution at 10 kHz ($t = 5.2 \times 10^{-5}$ s); (a) $xz$ plane, (b) $yz$ plane.

### 5.4 Summary and conclusions

This chapter presented the results of $E_0$, $B_{\delta w(1)}$ and magnet losses calculated with FEM 3D simulations. Two approaches were verified; simulations with meshed coils and with non-meshed coils. Meshed coils $E_0$ and $B_{\delta w(1)}$ calculations showed a large deviation when compared to FEM 2D results. Consequently, the calculation of the losses was performed only with non-meshed coils. In addition, a different trend of the losses with frequency was identified when comparing to FEM 2D results. This could be anticipated since the FEM 3D model no longer neglects the end-effects.
Chapter 6

Analysis of results

6.1 Introduction

This chapter summarizes the results obtained from analytical calculations and FEM 2D and 3D simulations. The main objective is to contrast, validate and, to some extent, analyse the reasons of the deviations of the results among models and simulations. The analysis of $E_0$, $B_{\delta w(1)}$ includes analytical calculations, FEM 2D and 3D with both meshed and non-meshed coils. The analysis for losses in the magnet excludes the meshed coils.

6.2 No-load back-emf

The results of $E_0$ are shown in table 6.1. The terms 3D FEM_M and 3D FEM_NM in rows 3 and 4, refer to simulations in 3D with meshed and non-meshed coils respectively. The deviation in percentage presented in column 3 is calculated taking the analytical calculation as reference. Positive values in the deviation mean that the reference value is lower than the contrasted value. On the other hand, negative values represent higher value of the reference when compared. As can be seen, there is a good agreement between the value calculated analytically and the results from FEM 2D simulations. For the case of FEM 3D simulations with meshed coils, as it was presented in Chapter 5 the results are far from the analytical values confirming that this type of simulation may not be adequate. Regarding FEM 3D simulations with non-meshed coils, the deviation is larger and the values of $E_0$ higher than the analytical calculations due probably to the effect of end windings.
Table 6.1: $E_0$ results for different methods.

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>$E_0$ [V]</th>
<th>Deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>8.41</td>
<td>-</td>
</tr>
<tr>
<td>2D FEM</td>
<td>8.49</td>
<td>0.95</td>
</tr>
<tr>
<td>3D FEM_M</td>
<td>2.87</td>
<td>-65.87</td>
</tr>
<tr>
<td>3D FEM_NM</td>
<td>9.28</td>
<td>10.34</td>
</tr>
</tbody>
</table>

In order to give an insight of the difference among values, figure 6.1 shows the wave-shapes of $E_0$ calculated in FEM simulations when varying the position of the rotor equivalent to time variation. It is important to point out the smooth sinusoidal behaviour as a result of the magnetization of the magnet ring.

![Figure 6.1: $E_0$ vs. rotor position comparison.](image)

6.3 B in the air-gap

Similar to $E_0$ results, the peak values of the magnetic flux density $B_{5u(1)}$ are shown in table 6.2. The wave-shapes shown in figure 6.2 are similar to some extent. However, the calculated value from 3D simulations with meshed coils shows the largest deviation in column 3 of table 6.2. Again, as it was corroborated for the FEM 2D simulations, this is an indication that selecting this type of simulations could be a source of error.
Table 6.2: $B_{\delta w(1)}$ for different calculations.

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>$B_{\delta w(1)}$ [T]</th>
<th>Deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>$5.50 \times 10^{-3}$</td>
<td>-</td>
</tr>
<tr>
<td>2D FEM</td>
<td>$7.50 \times 10^{-3}$</td>
<td>36.36</td>
</tr>
<tr>
<td>3D FEM_M</td>
<td>$3.31 \times 10^{-3}$</td>
<td>-39.82</td>
</tr>
<tr>
<td>3D FEM_NM</td>
<td>$7.35 \times 10^{-3}$</td>
<td>33.64</td>
</tr>
</tbody>
</table>

The analytical values of $B_{\delta w(1)}$ are much lower when compared to the values obtained with FEM 2D and 3D with non-meshed coils for the same Path in the air-gap. This deviation could be a result of setting the permeability of the magnet as $\mu_r = 1.05$ in both FEM 2D and 3D simulations which could yield higher values of $B_{\delta w}$.

6.4 Magnet losses

6.4.1 Analytical calculations

The magnet losses calculated with the analytical models described in Chapter 3 are shown in table 6.3. The calculation was done applying superposition of the results. The losses were calculated for harmonic indexes $n_1$ and $n_2$ separately and later summed up. A plot with the results is presented in figure 6.3. Note that a logarithmic scale was implemented in $y$ axis in order to facilitate the analysis.
The losses calculated with Model Huang_a [15] show the largest deviation compared to the losses estimated by FEM 2D simulations (dashed line in figure 6.3). As the frequency increases, the results start deviating dramatically resulting in the overestimation of the losses. This was expected, since expression 3.9 shows how losses depend on the square of the frequency. On the other hand, the losses calculated with Model Huang_b [28] shows a similar behaviour to the losses obtained by FEM 2D method but with much lower values, resulting in an underestimation of the losses. This is due mainly to the low values calculated for $B_{\delta u}$. Model Zhu [12], expected to give more reliable results given its level of complexity, yielded the best results of all three analytical models evaluated. It shows a similar trend when compared with FEM
2D simulations, and with the lowest deviation of the three models evaluated. Table 6.4 shows the deviation of each analytical model compared to FEM 2D results.

<table>
<thead>
<tr>
<th>Freq. [kHz]</th>
<th>Loss deviation Model Huang_a [%]</th>
<th>Loss deviation Model Huang_b [%]</th>
<th>Loss deviation Model Zhu [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-85.71</td>
<td>-52.90</td>
<td>29.73</td>
</tr>
<tr>
<td>25</td>
<td>-53.68</td>
<td>-61.63</td>
<td>8.95</td>
</tr>
<tr>
<td>40</td>
<td>-6.42</td>
<td>-61.66</td>
<td>7.36</td>
</tr>
<tr>
<td>55</td>
<td>51.61</td>
<td>-61.53</td>
<td>8.04</td>
</tr>
<tr>
<td>70</td>
<td>117.32</td>
<td>-61.57</td>
<td>8.19</td>
</tr>
<tr>
<td>85</td>
<td>190.74</td>
<td>-61.57</td>
<td>8.29</td>
</tr>
<tr>
<td>100</td>
<td>269.00</td>
<td>-61.80</td>
<td>8.00</td>
</tr>
</tbody>
</table>

Table 6.4: Magnet losses deviations of analytical models.

### 6.4.2 FEM simulations

Table 6.5 shows the result of losses for FEM 2D and 3D simulations. In addition, for each frequency the deviation is calculated taking FEM 2D simulation results as reference.

<table>
<thead>
<tr>
<th>Freq. [kHz]</th>
<th>Loss 2D simulation [W]</th>
<th>Loss 3D simulation [W]</th>
<th>Deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.59</td>
<td>0.44</td>
<td>-83.01</td>
</tr>
<tr>
<td>25</td>
<td>5.03</td>
<td>2.22</td>
<td>-55.86</td>
</tr>
<tr>
<td>40</td>
<td>6.39</td>
<td>4.99</td>
<td>-21.91</td>
</tr>
<tr>
<td>55</td>
<td>7.46</td>
<td>8.26</td>
<td>10.72</td>
</tr>
<tr>
<td>70</td>
<td>8.43</td>
<td>11.16</td>
<td>32.38</td>
</tr>
<tr>
<td>85</td>
<td>9.29</td>
<td>13.91</td>
<td>49.73</td>
</tr>
<tr>
<td>100</td>
<td>10.13</td>
<td>16.96</td>
<td>67.42</td>
</tr>
</tbody>
</table>

Table 6.5: Magnet losses vs. frequency FEM 2D and 3D.

The results of magnet losses obtained by FEM 3D simulations are presented together with the FEM 2D results in figure 6.4.
As can be observed there is a large deviation between both FEM methods at both low and high frequency. The simulations in FEM 2D assume that eddy currents are flowing only in one single direction in the magnet, as shown in Chapter 4. On the other hand, the model for FEM 3D simulations, considers the total volume of the magnet, and consequently, the currents are describing a totally different path as shown in Chapter 5. This is an indication of the main differences in the calculated losses. Next section, investigates this problem more in depth.

### 6.5 Investigation of B in the air-gap vs frequency

This section of the report is oriented to investigate the causes of the large deviation between the losses calculated in FEM 2D and 3D simulations. As it was explained, the losses vary with the square of the incident $B_{\delta w}$ in the air-gap. In addition, as reported in the literature, the effect of the reaction field created by eddy currents could be the source of such deviations. Consequently, the investigation of the effect of frequency in $B_{\delta w}$ is the first step to have a better understanding about the distribution of the losses in both types of simulations. In that sense, four sets of frequencies were selected; 10, 40, 70 and 100 kHz. The plots for $B_{\delta w}$ in the air-gap for FEM 2D simulations are presented in figure 6.5.
As the frequency increases the magnetic flux density in the air-gap $B_{δw}$ is reducing as shown in figure 6.5. This is due to what was defined in the literature review as the reaction effect of the eddy currents in $B_{δw}$ [19]. The circulation of the eddy currents in the magnet generates a magnetic field density of reaction $B_{eddy}$ that opposes to $B_{δw}$. This reduction is significant in FEM 2D simulations, in which $B_{δw}$ is reduced by around 50%. Similar procedure was followed in order to calculate $B_{δw}$ in the air-gap for FEM 3D simulations. The results are presented in figures 6.6 and 6.7. As pointed out in previous sections, the normal component of $B_{δw}$ was plotted for both Path 1 and 2.

Figure 6.5: FEM 2D variation of $B_{δw}$ with frequency.

Figure 6.6: FEM 3D variation of $B_{δw}$ with frequency at 10 and 40 kHz.
For $B_{\delta w}$ in Path 1 the reduction is around 30%. On the other hand, for Path 2 the reduction is around 5%. This behaviour is unusual since the values of $B_{\delta w}$ in Path 2 are expected to be lower than for Path 1, as it was shown in Chapter 5. However, there is an explanation and that is the skin effect itself. At lower frequencies, the magnetic flux density lines inside the magnet due to the eddy currents $B_{\text{eddy}}$ are more evenly distributed, as can be seen in figure 6.8a. But, as the frequency starts increasing and the current starts being forced to flow towards the surface of the magnet, $B_{\text{eddy}}$ lines start deforming creating a "void" zone inside the magnet and resulting in an uneven distribution of $B_{\delta w}$. Figure 6.9 shows the distribution of $B_{\delta w}$ when a path is created in the air-gap in the axial direction covering the entire length $l$ of the magnet (green line in figures 6.8aa and 6.8ab).

Figure 6.7: FEM 3D variation of $B_{\delta w}$ with frequency at 70 and 100 kHz.

Figure 6.8: $B$ distribution at; (a) 10 kHz, (b) 100 kHz.
This also indicates the source of the differences in losses. As mentioned before, the reduction of $B_{\delta w}$ in the air-gap for FEM 2D simulations was around 50%, and for FEM 3D in the worst case, 30%. Having lower values of $B_{\delta w}$ in 2D simulations, implies that the resulting losses are going to be lower as the losses are dependent of the square of $B_{\delta w}$. In addition, the reason for which the reaction field of eddy currents is more significant in FEM 2D simulations, is that in this type of simulations the impedance seen by the eddy currents is lower. Consequently, the magnitude of the eddy currents is higher which in turn yields a higher $B_{\text{eddy}}$ and lower $B_{\delta w}$ as a result.

6.6 Magnet losses with double the magnet length

This investigation was extended to the case in which the magnet length was affected by a factor of 2. The motivation for this selection, is to show that for a given value of length of magnet $l$, the deviation between the losses calculated with both FEM 2D and 3D should start reducing. The reduction of the deviation between FEM 2D and FEM 3D simulations is around 27% and 12% at lower and higher frequencies respectively (tables 6.6 and 6.5). In addition, the losses for a magnet length of $l$ and $2l$ are shown in figure 6.10. There is not much to mention about the results from FEM 2D simulations, since they are simply the same values calculated in Chapter 4 multiplied by 2. However, in figure 6.10 two frequencies at which the calculated losses in both FEM 2D and 3D simulations are similar. These frequencies are at around 48 kHz for $l$ and 30 kHz for $2l$. This could be an indication that as the length of the magnet increases, the curve representing the losses calculated by FEM 2D starts approaching
to the curve representing FEM 3D results. It is expected for this to happen at a value in which \( l \gg w \) at which the end-effect of the returning loop for the current in FEM 3D simulations start being insignificant. Nevertheless, this would require more verifications through simulations and it is proposed as a future work.

![Graph showing magnet losses comparison between FEM 2D and 3D simulations](image)

Figure 6.10: Magnet losses comparison between FEM 2D and 3D simulations 2\( l \).

<table>
<thead>
<tr>
<th>Freq. [kHz]</th>
<th>Loss 2D simulation [W]</th>
<th>Loss 3D simulation [W]</th>
<th>Deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5.18</td>
<td>2.22</td>
<td>-57.14</td>
</tr>
<tr>
<td>25</td>
<td>10.06</td>
<td>9.12</td>
<td>-9.34</td>
</tr>
<tr>
<td>40</td>
<td>12.78</td>
<td>14.63</td>
<td>14.48</td>
</tr>
<tr>
<td>55</td>
<td>14.92</td>
<td>20.71</td>
<td>38.81</td>
</tr>
<tr>
<td>70</td>
<td>16.86</td>
<td>26.15</td>
<td>55.10</td>
</tr>
<tr>
<td>85</td>
<td>18.58</td>
<td>28.04</td>
<td>50.92</td>
</tr>
<tr>
<td>100</td>
<td>20.26</td>
<td>31.40</td>
<td>54.99</td>
</tr>
</tbody>
</table>

Table 6.6: Magnet losses FEM 2D and 3D 2\( l \).

### 6.7 Summary and conclusions

This chapter included the analysis of the results of the analytical models presented in Chapter 3. Additionally, the results of FEM 2D and 3D simulations from Chapters 4 and 5 respectively, were contrasted. The validation of \( E_0 \) gave satisfactory results with very low deviations when compared with the analytical model. However, the validation
of $B_{\delta w}$ yielded a high deviation, which could be a result of setting the permeability of the magnet in both FEM 2D and 3D simulations as $\mu_r = 1.05$. A major conclusion of this chapter is that an analytical model for calculating the losses in magnets was found fitting the results obtained with FEM 2D simulations. However, the calculation of the losses neglecting the end-effects (analytical models and FEM 2D simulations) might not be sufficient for the machine under study with a length of the magnet lower than the magnet width ($l < w$).
Chapter 7

Conclusions and future work

7.1 Conclusions

Losses in magnets is a trendy topic and a matter of concern due to the risks that these represent in PM machines. This can be seen in the amount of literature available at the start of the literature review. However, few articles developed a model for calculating the losses in a machine with similar characteristics to the one analysed in this report. In general, the models are developed for slotted machines and with magnet segments with a length much larger than the width $l \gg w$. In addition, all models neglect the end-effects, which as it was shown from simulation results, has a strong incidence in the calculated value of losses, at least, when the length of the magnet is lower than its width ($l < w$). The summary of the conclusions about this regard is presented as follows:

- There are certainly few articles reporting both experimental and FEM 3D simulations validations. Furthermore, only two of the studies; [20] and [21], are applied to models of machines with similar characteristics to the machine studied in this project (i.e., slot-less with a magnetized ring).

- In general, several common assumptions were identified in all articles reviewed for the calculation of losses in magnets. Among these, the neglect of the end-effects is the most common since all models are developed as a 2 dimensional problem. FEM simulations allowed to verify the impact of such assumption.

- As presented in Chapter 2 there are differences between the selected models for calculating magnet losses and the actual machine. However, it was required to adapt the expressions to the actual machine dimensions.

In this thesis a study of the different parameters to be accounted for when running FEM simulations was carried out. It was found that the time-step values have a more signif-
significant effect than the mesh density when calculating losses in magnets. Furthermore, some hints are given regarding the selection of the time-step when simulating magnet losses in a machine. Some additional conclusions, regarding FEM 2D simulations are:

- There was no significant effect on the calculated losses when varying the density of the mesh. On the other hand, the time-step selected had an important incidence on the calculated values. However, both mesh density and time-step showed to have direct incidence in the simulation times.

- The possibility of performing superposition when calculating the losses was verified. This means that there is no interaction between harmonics from group 1 (harmonics at 1 time \( f_{sw}, n_1 \)) and group 2 (harmonics at 2 times \( f_{sw}, n_2 \)). This is valid for the specific case of a slot-less machine.

- There is no effect of \( B_r \) in the calculated values of magnet losses.

- The calculation of the losses with the rotor with zero speed yielded similar results. However, the simulation times were not reduced.

Two analytical models found in the literature review were selected and tested. Additionally, in order to validate FEM 2D simulations, FEM 3D simulations were run. However, different results were obtained. Two important conclusions are:

- An analytical model for calculating the losses in magnets was found fitting the results obtained with FEM 2D simulations. However, the calculation of the losses neglecting the end-effects (analytical models and FEM 2D simulations) might not be sufficient for the machine under study with a length of the magnet lower than the magnet width \( (l < w) \).

- A different trend of the losses with frequency was identified when comparing to FEM 2D results. This could be anticipated since the FEM 3D model no longer neglects the end-effects.

### 7.2 Future work

The validity of either FEM 2D or 3D simulations needs to be confirmed. This can only be done through experimental measurements in the actual machine. In this sense, an overview about the methods implemented for performing measurements is presented in Appendix B of this document.

As it is reported in the literature review, it would be interesting to evaluate to what point the segmentation that is currently implemented in the machine is well exploited.
This could allow to verify the distribution of the losses in each segment. Therefore, for performing these verifications, simulations of the entire machine in FEM 3D are required.

It is necessary to evaluate the behaviour of the losses when varying the length of the magnet segments, accounting for the impact of changing the impedance. This is related to the fact that in this study the model implemented in FEM 2D and 3D is fed by a current source which has a constant value. However, as the length of the magnet segments is changed, the equivalent impedance of the machine is changed.

While doing measurements, the machine is fed by harmonic voltage sources. This means that, in another study, the amplitudes of the current harmonics will need to be derived accounting for the variation of the machine impedance with frequency introduced by the eddy currents in the magnets.
References


Appendix A

FEM 3D current and loss distributions in the magnet at 100 kHz

Figure A.1: Current density distribution over the magnet at 100 kHz ($t = 5.6 \times 10^{-6}$ s).
Figure A.2: Current density distribution at 100 kHz \((t = 5.6 \times 10^{-6} \text{ s})\); (a) \(xz\) sub-region, (b) \(xz\) plane.

Figure A.3: Current density distribution at 100 kHz \((t = 5.6 \times 10^{-6} \text{ s})\); (a) \(yz\) sub-region, (b) \(xz\) plane.
Figure A.4: Loss distribution at 100 kHz \((t = 5.6 \times 10^{-6} \text{ s})\); (a) over the magnet, (b) \(xy\) plane.

Figure A.5: Loss distribution at 100 kHz \((t = 5.6 \times 10^{-6} \text{ s})\); (a) \(xz\) plane, (b) \(yz\) plane.
Appendix B

Measurements overview

B.1 Introduction

This chapter could be seen as an extension of the literature review and is focused on the measurement methods of the losses in a PM machine. Given the importance of measurements in the validation of the analytical models and FEM simulations presented throughout, it was found that only a few number of the articles reviewed were validated by experimental measurements in an actual machine. Furthermore, a description of the measurement set-up is done only in [12]. The measurement of the losses in the magnets of a PM machine is a cumbersome task, since it is difficult to differentiate the losses that appear in the machine (i.e., iron losses, copper losses, mechanical losses, magnet losses, etc.).

B.2 Measurements

B.2.1 Method reported by Zhu, Schofield & Howe

The measurements performed in [12] for validating the analytical model were implemented with the thermometric method. According to this method, the specific power loss $Q$ in W/m$^3$ is given by the expression:

\[
Q = C_p \rho \left( \frac{dT}{dt} \right)_{t=0} \quad \text{(B.1)}
\]

With $C_p$ as the specific heat of magnets and $\rho$ the density of the material under analysis. The disposition of such thermistor is described in B.1a. The losses are calculated as:

\[
P = MC_p \left( \frac{dT}{dt} \right)_{t=0} \quad \text{(B.2)}
\]
Where $M$ is the total mass and $dT$ the initial rate of rise of temperature determined by the measurements. The thermistor used for measurements was of precision negative temperature coefficient type and placed on the surface of one of the magnets. As shown in figure B.1b, each pole is composed by two skewed PM in order to reduce the incidence of the cogging torque. In this specific case, the motor was equipped with oil forced cooling and the test was performed keeping the temperature constant (around ±1 °C) with the help of a heat exchanger.

![Figure B.1: Position of the thermistor on the PM; (a) rotor of the machine, (b) circumferentially displaced magnet segments, [12].](image)

B.2.2 Method reported by Malloy, Martinez-Botas & Lamperth

An additional experimental method for the measurement of losses in the magnet is reported in this work. As presented in [12] the losses are determined by the temperature gradient in the surface of the rotor and solving equation B.1. The next step requires to determine the magnet temperature, which is accomplished by measuring the voltage constant of the machine $k_e$ in V·s/rad, with the expression:

$$k_e(T_m) = \frac{e(T_m)}{\omega} \quad (B.3)$$

With,

- $e$ as the fundamental back-emf.
- $\omega$ the angular speed.
- $T_m$ the magnet temperature in °C.

In order to determine the relationship of temperature of the magnet and induced back-emf, the temperature on magnet surface was measured with the use of a thermocouple.
at the same time that the constant $k_e$ was measured. In order to validate this method, several values of temperature were applied to the magnets with the help of an oven. The results for this experimental set-up are shown in figure B.2a.

Figure B.2: (a) voltage constant and magnet temp. rise ratio, (b) single phase equivalent circuit for a PM machine at 50°C. [9]

The main goal of this set of measurements is the calculation of the coefficient $\alpha$, which will allow to determine the magnet temperature rise $\theta_m$, with the expression:

$$\theta_m = \alpha(k_{e1} - k_{e2}) \quad \text{(B.4)}$$

Each voltage constant of the machine is determined with the equivalent circuit shown in figure B.2b and with the expression:

$$k_{e_n} = \frac{\sqrt{v^2 - (\omega L_s i)^2} - iR}{\omega} \quad \text{(B.5)}$$