A Binary Power Control Scheme for D2D Communications

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***Abstract*—Binary power control (BPC) is known to maximize the capacity of a two-cell interference limited system and performs near optimally for larger systems. However, when device-to-device (D2D) communication underlaying the cellular layer is supported, an objective function that considers the power consumption is more suitable. We find that BPC remains optimal for D2D communications when the weight of the overall power consumption in the utility function is bounded. Building on this insight, we propose a simple near-optimal extended BPC scheme and compare its performance with a recently proposed utility optimal iterative scheme using a realistic multicell simulator. Our results indicate that a near optimal D2D performance can be achieved without lengthy iterations or complex signaling mechanisms.**

I. Introduction

Binary power control (BPC) is known to maximize the capacity of systems that can be clustered into adjacent groups of two cells [1] and well approximate the capacity obtained by the optimal power allocation in systems consisting of a large number of cells [2]. The near optimality of the ON-OFF type of PC makes it particularly attractive in systems where centralized control is viable.

For cellular network controlled device-to-device (D2D) communication, it has been pointed out that rate maximization alone is not an appropriate objective function because of two reasons. First, the underlaying D2D layer must not cause excessive interference to the cellular layer, let alone allocating zero power to a cellular user equipment (CUE) that could be the result of a BPC approach. Secondly, users may value power saving even at the expense of some rate degradation as compared with the rate that could be achieved by a rate maximizing scheme [3]. For example, for low-rate machine type communications, power efficiency is more important than rate [4].

Although the above power allocation schemes are able to balance between overall power consumption and sum rate, they typically require a prohibitive number of iterations. Therefore, it is highly desirable to employ BPC for network controlled D2D communications, especially in situations in which the number of multiplexed D2D pairs and CUEs on a single resource block (RB) is low. From a control plane and signaling perspective, BPC can be a viable PC scheme, since the base station (BS) is a natural point of centralized control as long as the number of D2D pairs per RB is low [5].

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However, it is not known whether BPC can be utility optimal or under which conditions BPC can maximize a utility function that explicitly takes into account the consumed power. This paper establishes the conditions under which BPC is utility optimal and, based on this insight, proposes a practical BPC scheme that well approximates the performance of the iteration based utility optimal schemes previously proposed in the literature.

II. System Model and Problem Formulation

A. System Model

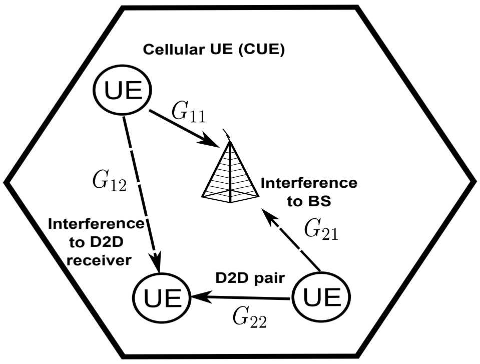

Figure 1. A D2D pair communicating in uplink cellular spectrum causes interference to the cellular BS and suffers interference from CUEs. We assume that there may be at most one CUE and one D2D pair using the same cellular resource block simultaneously.

For ease of presentation, we consider a single cell system supporting network assisted D2D communications (Figure 1). Figure 1 corresponds to the two cell system of [1] in terms of the achievable rate. Let G_{ii} denote the effective path gain between transmitter i and its receiver, and $G_{ij} > 0$ denote the interfering path gain between transmitter i and the receiver associated with transmitter j. Let P_N denote the noise power and P_i be the transmission power of transmitter i.

The signal to interference-plus-noise ratio (SINR) of the transmitting CUE measured at the BS and the SINR of the D2D pair are:

$$\gamma_1(\mathbf{p}) = \frac{P_1 G_{11}}{P_N + P_2 G_{21}}, \quad \gamma_2(\mathbf{p}) = \frac{P_2 G_{22}}{P_N + P_1 G_{12}}, \quad (1)$$

where $\mathbf{p} = [P_1 \ P_2]^T$ is the power allocation vector.

From the SINR equations (1), the maximum achievable rates are given by: $R_i(\mathbf{p}) = W \log_2 \Big(1 + \gamma_i(\mathbf{p})\Big), \quad i = 1, 2.$

B. Problem Formulation

The utility maximization problem for the system of Figure 1 is defined as follows:

$$\underset{\mathbf{p},\mathbf{s}}{\text{maximize}} \sum_{i=1}^{2} \log(s_i) - \omega \sum_{i=1}^{2} P_i \quad (2a)$$

$$\text{subject to } s_i \le R_i(\mathbf{p}), i = 1, 2, \tag{2b}$$
$$\mathbf{s} \succeq 0, \mathbf{p} \in \Omega, \tag{2c}$$

where $\Omega = \{P_1, P_2 | (P_1, P_2) \le P_{\text{max}}\}$, P_{max} is a predefined maximum transmit power level, $\mathbf{s} = [s_1\ s_2]^T$, where s_i is an optimization variable that must fulfill constraints (2b) and (2c) at power allocation $\mathbf{p}$, and ω is a predefined constant in units of $1/W$ that weights the cost of investing a unit power into the system. Note that $\mathbf{s}$ can be considered the achieved transmission rate of User i [6].

The reason for taking the logarithm of the capacities in the utility function of (2) – as proposed by several prior works [6], and specifically for D2D communications in [3] –, is that for most applications increasing the capacity yields diminishing returns from the point of view of the application, such as video or voice quality or end-to-end latency.

To solve this optimization task, previous works have proposed iterative schemes [3], [6] that are hardly feasible in practice. We are therefore interested in applying the BPC scheme as a practical solution approach to Problem 2.

III. A Solution Approach Based on BPC

In the case of (2b) being tight – which can be assumed in a system with a well designed modulation and coding scheme –, we have: $s_i = W \log_2 \Big(1 + \gamma_i(\mathbf{p})\Big)$. We use this to reformulate problem (2):

$$\underset{\mathbf{p}}{\text{maximize}} \sum_{i=1}^{2} \log \left(W \log_2 \Big(1 + \gamma_i(\mathbf{p})\Big) \right) - \omega \sum_{i=1}^{2} P_i \tag{3a}$$
$$\text{subject to } \mathbf{p} \in \Omega, \tag{3b}$$

where we can rewrite the objective function of problem (3) as:

$$O(P_1, P_2) = \log \Big[W^2 \log_2 (1 + \gamma_1(\mathbf{p})) \log_2 (1 + \gamma_2(\mathbf{p})) \Big] - \omega(P_1 + P_2). \tag{4}$$

To find the optimal solutions $(P_1^\star, P_2^\star)$ for problem (3), we need to prove the following lemma, where the main difference with respect to the lemma in [2] is the presence of ω.

Lemma 1. *The optimal transmit power vector has at least one component that is equal to P_{max} provided that $\exists \alpha \in \mathbb{R}, \alpha > 1$ is a scaling factor and*

$$\omega < \frac{\log \left[\frac{\log_2(1+\gamma_1(\alpha\mathbf{p})) \log_2(1+\gamma_2(\alpha\mathbf{p}))}{\log_2(1+\gamma_1(\mathbf{p})) \log_2(1+\gamma_2(\mathbf{p}))} \right]}{(\alpha - 1)(P_1 + P_2)}. \tag{5}$$

Proof. See Appendix I. ■

From Lemma 1, the optimal power allocation is found between the alternatives:

1) **Critical points on the boundaries of Ω:** Either $P_2 = P_{\text{max}}$ or $P_1 = P_{\text{max}}$, which means P_1 or P_2 corresponding to $\frac{\partial O(P_1, P_{\text{max}})}{\partial P_1} = 0$ or $\frac{\partial O(P_{\text{max}}, P_2)}{\partial P_2} = 0$, respectively.
2) **Corner points of Ω:** $(P_{\text{max}}, 0)$, or $(0, P_{\text{max}})$, or $(P_{\text{max}}, P_{\text{max}})$.

Dropping the usage of $W/\log(2)$ and utilizing that the logarithm is a monotonically increasing function, we look for the critical points on the boundary by defining:

$$J(P_1, P_2) \triangleq \log (1 + \gamma_1) + \log (1 + \gamma_2) - \omega(P_1 + P_2),$$
$$= \log \Big[(1 + \gamma_1)(1 + \gamma_2) \Big] - \omega(P_1 + P_2),$$

and finding the critical points of $J(P_1, P_2)$. This leads to the following result.

Result 2. *If the following inequality on P_1 holds, the optimal power allocation $(P_1^\star, P_2^\star)$ lies in the set of corner points.*

$$TP_1^2 + VP_1 + X \ge -(NP_1^4 + QP_1^3), \tag{6}$$

where N, Q, T, V and X are defined by:

$$N = -G_{11}^2 G_{12}^4, \tag{7a}$$
$$Q = -4G_{11}^2 G_{12} G_{22}^2 P_N, \tag{7b}$$
$$T = G_{11} G_{12}^2 \Big[- 2G_{11} P_N (2P_N + P_{max} G_{22}) + 4G_{12} G_{22} P_{max}(P_N + P_{max} G_{21}) \Big], \tag{7c}$$
$$V = -2G_{11}^2 G_{12} P_N (2P_N + P_{max} G_{22})(P_N + P_{max} G_{22}) + 2G_{12}^2 P_{max}(P_N + P_{max} G_{21}) \Big(G_{12} G_{22} (P_N + P_{max} G_{21}) + G_{11}(P_N + G_{22} P_N + P_{max} G_{22}) \Big), \tag{7d}$$
$$X = G_{12}^2 G_{22} P_{max}(P_N + P_{max} G_{21})^2 (2P_N + P_{max} G_{22}) - G_{11}^2 P_N^2 (P_N + P_{max} G_{22})^2. \tag{7e}$$

Proof. See Appendix II. ■

Remark. *Result 2 applies to the case of two users sharing the same time-frequency resource, that is when any time-frequency resource is used by at most two users simultaneously. Therefore, the condition of (8) in Result 2 must be fulfilled by the single User 1 for all values of P_1 within the interval $[0, P_{max}]$.*

Depending on the value of ω and the channel conditions, the optimal solution of problem (3) does not lie in the critical points of the boundaries of Ω; it is within the corner points of the constraint set Ω. Therefore, for D2D communications BPC is optimal for problem (2) when there is a single D2D pair reusing a cellular resource along with a CUE.

IV. BPC for Practical D2D Scenarios

In practice, a CUE-D2D scheduling algorithm selects the particular CUE and D2D pair for resource sharing and assigns a suitable RB to them. Although the BPC is optimal for a single CUE and D2D pair sharing a RB when inequality (6) holds, a practical power control algorithm must be able to handle higher load situations in which there are multiple D2D pairs reusing cellular resources or (6) does not hold. As we will show in the numerical section, in practice, (6) typically holds for a large area of the cell.

Therefore, we propose a power control algorithm suitable in scenarios in which multiple D2D pairs share a RB with a single CUE. Note that if the system has 1 CUE and 1 D2D pair, the algorithm performs exactly equal to the BPC proposed above. We denote by Q the number of available resources, I the total number of users in the system, I_q the number of users associated with resource q and $\mathcal{S}$ the set of users using minimum transmitting power. The algorithm is performed by the BS and requires full channel knowledge to evaluate the objective function in the possible corners. In Algorithm 1, the BS sets the initial power $\mathbf{p}_0$ as P_{max}, evaluates the

Algorithm 1 BPC for multiple D2D pairs

Input: $\mathbf{p}_0 = P_{\max},\ O_0 = O(\mathbf{p}_0),\ l = 0,\ \mathcal{S} = \emptyset$
for $q = 1$ **to** Q **do**
  for $i = 1$ **to** I_q **do**
    $l = l + 1$
    $\mathbf{p}_l = \mathbf{p}_{l-1},\ O_l = O(\mathbf{p}_l)$
    $i^\star = \arg\max_{i \notin \mathcal{S}} O\Big((P_l)_i = 0, (\mathbf{p}_l)_{j \neq i}\Big)$
    if $(O_l)_{i^\star} > O_{l-1}$ **then**
      $(P_l)_{i^\star} = 0$
      $\mathcal{S} = \mathcal{S} + \{i^\star\}$
      $O_l = O(\mathbf{p}_l)$
    end if
  end for
end for
Output: $\mathbf{p}^\star = \mathbf{p}_l$

objective function $O(\mathbf{p}_0)$ at $\mathbf{p}_0$ and initializes the set $\mathcal{S}$ empty. Subsequently, Algorithm 1 loops over the Q resources and the I_q users assigned to resource q. At each I_q loop, the algorithm finds the user $i^\star$ that optimizes the objective function by using minimum power, sets its power to minimum and adds it to the set $\mathcal{S}$ only if such user exists. The output of the algorithm is the transmitting powers. The algorithm performs Q iterations for each I_q^2, and the total complexity is $\mathcal{O}\Big(Q(I_1^2 + \cdots + I_q^2)\Big)$, thus a faster solution than exhaustive search.

In practice BPC can be employed on the time scale of large scale fading, similarly to the time scale used for power control in 3GPP Long Term Evolution (LTE) systems. On this time scale, D2D users can use similar reference signals as the user specific reference signals or the demodulation reference signals (DMRS) of LTE to acquire channel state information either at the base station or at the user equipment and report such channel state information to the serving base station.

V. Numerical Experiments and Discussion

We first consider the single-cell scenario showed in Figure 1, with one CUE and one D2D pair per RB. Subsequently, we study the multi-cell scenario with multiple CUEs and D2D users. For simplicity and to gain insights, we initially assume that each CUE and D2D pair uses a single uplink RB, but later we consider the case where multiple D2D pairs and one CUE reuses the RB. The most important system parameters are summarized in Table I.

Table I
SIMULATION PARAMETERS

Parameter	Value
Number of BSs	1/7
Cell radius	500 m
Number of CUEs per cell	1/2
Number of D2D pairs per cell	1/4
Number of RBs	1/6 RBs
Path Loss coefficient	3.5
Shadowing standard deviation	8 dB
User max transmit power	24 dBm
Values of ω	0.1/1

Figure 2 shows the behaviour of $O(P_1, P_2)$ when $\omega = 0.1$ and $P_1, P_2 \in [-24, 24]$dBm. Note that as the powers increase, the utility also increases and the point that maximizes $O(P_1, P_2)$ is at or close to the corner points.

Next, we consider BPC in a practical case with multiple D2D users in a multicell environment using multiple values

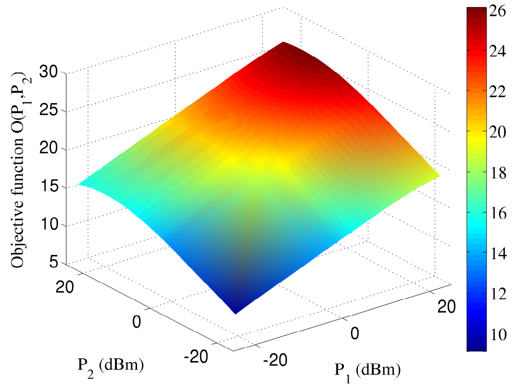

Figure 2. Objective function $O(P_1, P_2)$ with fixed ω and varying powers. We clearly see that as the powers increase the utility tends to the corner point $(P_{\max}, P_{\max})$.

of ω. Figure 3 compares the algorithm proposed in section IV that uses BPC-ω and the algorithm that achieves the optimal solution of problem 2 proposed in [3], denoted by UM-ω. Note that UM reaches a higher utility than BPC, which is expected since it is the optimal solution, but the difference is still acceptable given the difference in the complexity of the algorithms.

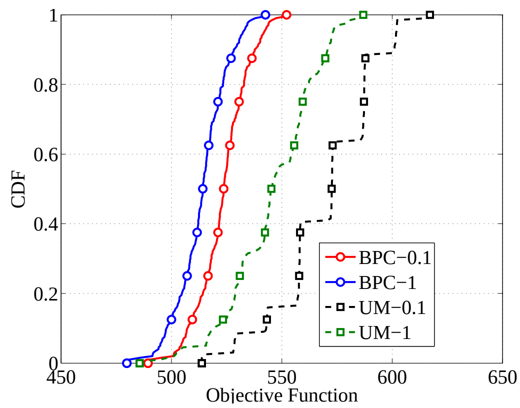

Figure 3. Comparison between the objective function $O(\mathbf{P})$ reached by BPC and utility maximizing power control when multiple D2D users share the resource. We clearly see that BPC has a lower utility but the difference is acceptable given the difference in the complexity between both.

VI. Conclusion

In this paper we asked the question whether BPC is applicable in network-assisted D2D scenarios, in which power consumption of the devices must be taken into account.

We proposed to build on a previously proposed utility optimization approach, in which the rate-power consumption trade-off can be flexibly controlled by the single parameter ω.

We found that under certain channel conditions, BPC can be utility optimal and that the utility maximizing transmit powers are found in the corner points of the power constraint set. The numerical results show that for higher values of ω and multiple D2D users sharing the resource, the optimal transmit powers tend to deviate from the corner points.

Since from a system design perspective, BPC is simpler than iterative algorithms, and can achieve near utility optimal performance, it is a viable alternative as a practical D2D power control algorithm in the following sense:

- BPC does not require iterations;
- BPC is able to balance between spectral and power efficiency in a flexible fashion;
- BPC handles multiple D2D pairs with acceptable performance loss, as our multicell simulations show.

VII. Appendix I

Assuming a scaling factor $\alpha \in \mathbb{R}, \alpha > 1$ and $(P_1, P_2) \in \Omega$:

$$O(\alpha P_1, \alpha P_2) = \log\left[W^2 \log_2\left(1+\gamma_1(\alpha \mathbf{p})\right)\log_2\left(1+\gamma_2(\alpha \mathbf{p})\right)\right] - \omega\alpha(P_1+P_2), \quad (8)$$

where

$$\gamma_1(\alpha \mathbf{p}) = \frac{P_1 G_{11}}{\frac{P_N}{\alpha} + P_2 G_{21}}, \quad \gamma_2(\alpha \mathbf{p}) = \frac{P_2 G_{22}}{\frac{P_N}{\alpha} + P_1 G_{12}}. \quad (9)$$

For $O(\alpha P_1, \alpha P_2) > O(P_1, P_2)$, it is necessary that (5) is fulfilled. Hence, we can increase the objective function $O(P_1, P_2)$ by increasing all components of $\mathbf{p}$ by a factor α, until one component hits the boundary $P_{\max}$. Therefore, the solution of problem (3) has at least one component equal to $P_{\max}$, i.e., the solution to problem (3) sets P_1 or P_2 or both equal to $P_{\max}$.

Appendix II

We aim to prove that the optimal power allocation $(P_1^\star, P_2^\star)$ lies in the set of corner points and not in the critical points. Therefore, we now differentiate $J(P_1, P_{\max})$ with respect to P_1 and we find

$$\frac{\partial J}{\partial P_1} = \frac{AP_1^3 + BP_1^2 + CP_1 + D}{E(P_1)}, \quad (10)$$

where

$$A = -\omega G_{11} G_{12}^2, \quad (11a)$$

$$B = G_{11}G_{12}^2 - \omega\Big[G_{12}^2(P_N + P_{\max}G_{21}) + G_{11}G_{12}(2P_N + P_{\max}G_{22})\Big], \quad (11b)$$

$$C = 2G_{11}G_{12}P_N - \omega\Big[(P_N + P_{\max}G_{22})(G_{11}P_N + G_{12}(P_N + P_{\max}G_{21})) + G_{12}P_N(P_N + P_{\max}G_{21})\Big], \quad (11c)$$

$$D = G_{11}P_N(P_N + P_{\max}G_{22}) - G_{12}G_{22}P_{\max}(P_N + P_{\max}G_{21}) - \omega P_N(P_N + P_{\max}G_{21})(P_N + P_{\max}G_{22}), \quad (11d)$$

$$E(P_1) = (P_N + P_{\max}G_{21} + P_1G_{11})(P_N + P_1G_{12} + P_{\max}G_{22})(P_N + P_1G_{12}). \quad (11e)$$

Since $E(P_1)$ in (10) is always positive, then $\frac{\partial J}{\partial P_1} = 0$ can be found as the solution to $AP_1^3 + BP_1^2 + CP_1 + D$. To find the solution of this cubic equation, we used the MATLAB Symbolic Toolbox, yielding

$$P_1 = F - \frac{H}{F} - \frac{L}{3G_{11}G_{12}^2\omega}, \quad (12)$$

where

$$F = \left\{\left[H^3 + \left(\frac{L^3}{27G_{11}^3G_{12}^6\omega^3} + K - I\right)^2\right]^{\frac{1}{2}} - \frac{L^3}{27G_{11}^3G_{12}^6\omega^3} - K + I\right\}^{\frac{1}{3}}, \quad (13a)$$

$$H = \frac{M}{3G_{11}G_{12}^2\omega} - \frac{L^2}{9G_{11}^2G_{12}^4\omega^2}, \quad I = \frac{LM}{6G_{11}^2G_{12}^4\omega^2}, \quad (13b)$$

$$K = \Big[G_{22}P_{\max}(G_{12}G_{21}P_{\max} + P_N(G_{12} - G_{11})) + \omega P_N(P_N^2 + P_NP_{\max}(G_{21} + G_{22}) + G_{21}G_{22}P_{\max}^2)\Big] \Big/ \Big[2G_{11}G_{12}^2\omega\Big], \quad (13c)$$

$$L = -G_{11}G_{12}^2 + \omega G_{12}\Big(P_N(G_{12} + 2G_{11}) + P_{\max}(G_{12}G_{21} + G_{11}G_{22})\Big), \quad (13d)$$

$$M = -2G_{11}G_{12}P_N + \omega\Big(P_N^2(G_{11} + 2G_{12}) + P_NP_{\max}(2G_{12}G_{21} + G_{22}(G_{11} + G_{12})) + G_{12}G_{21}G_{22}P_{\max}^2\Big). \quad (13e)$$

Since we want a real and non-negative $P_1 \in [0, P_{\max}]$, this can only occur if

$$F > \frac{H}{F} + \frac{L}{3G_{11}G_{12}^2\omega}. \quad (14)$$

Define as P_1^{rn} the value of P_1 that solves equation (12) when the inequality (14) holds. However, it is necessary to check whether P_1^{rn} corresponds to a maximum or minimum, because the objective function is not convex, which allows it to have local optima that maximize or minimize it. Thus, we calculate the second derivative:

$$\frac{\partial^2 J}{\partial P_1^2} = \frac{NP_1^4 + QP_1^3 + TP_1^2 + VP_1 + X}{Z(P_1)}, \quad (15)$$

where N, Q, T, V and X were defined in (7) and $Z(P_1)$ is defined as:

$$Z(P_1) = (P_N + P_1G_{12} + P_{\max}G_{22})^2(P_N + P_1G_{11} + P_{\max}G_{21})^2(P_N + P_1G_{12})^2. \quad (16)$$

Since $Z(P_1)$ is non-negative, the concavity of $J(P_1, P_{\max})$ depends on the numerator. Since N and $Q < 0$, if inequality (6) holds, $J(P_1, P_{\max}) \geq 0$ and then convex, which means that the critical point P_1^{rn} is a minimum point for $J(P_1, P_{\max})$, and due to symmetry, the above analysis also hold for P_2. However, if inequality (6) does not hold, then $J(P_1, P_{\max}) \leq 0$ and then concave, which means that the critical point P_1^{rn} is a maximum point for $J(P_1, P_{\max})$. Therefore, when inequality (6) holds, the critical points on the boundaries of Ω do not maximize our function and we conclude that $(P_1^\star, P_2^\star)$ is found in the set of corner points: $\Delta\Omega = \{(P_{\max}, 0), (0, P_{\max}), (P_{\max}, P_{\max})\}$.

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