Quantum information and atomic networks

Erika Andersson

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Preface

This thesis is based on work mostly done at the Royal Institute of Technology in Stockholm, where I had the privilege to work with Prof. Stig Stenholm at the section of Laser physics and quantum optics. It has been a pleasure to work out the factors of $\hbar$, π and $1/2$ in his inspiring ideas. Many thanks go to my collegues and collaborators here in Stockholm and elsewhere. Many of you are also my best personal friends outside work.

Without my family, my mother, father, sister and brother, I certainly would not be here at all. Your visits have been welcome breaks, giving me good reasons to increase my share of art museums and concerts.

I have certainly learned a lot during these years, and not only physics. All you happy Lindy Hoppers, Jens and Oskar in particular, have made my life swing. "When in doubt, twist!" applies also outside the dance floor.

Finally I want to thank Patrik, for everything. You are the one I will always trust.

Stockholm, November 2000

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Abstract

Recent advance in cooling and controlling the motion of neutral atoms allows for the fabrication of *microtraps*, where the atoms are confined to regions comparable to their de Broglie wavelength. This opens up the possibility of studying quantum phenomena arising in such environments.

The microtraps may be fabricated on a surface, in the form of channels guiding the atoms, and a beam splitter could be obtained by coupling two channels. What differentiates the system from an optical coupler is the particle-particle interaction. The effects of interactions on the performance of such a beam splitter for atoms are investigated using a simple analytic model verified by numerical calculations.

Microtrap channels could eventually be connected to networks to be used for the manipulation of quantum information and quantum computation. Potential applications of microtraps are further investigated through toy models of simple devices. The goal is here to find mechanisms to effect logical operations on atomic qubits. The performance of these devices is again investigated through analytical considerations and detailed numerical calculations.

The prospects of quantum computation and communication leads us to considering a scheme where the information carried by a qubit is transferred to be shared in such a way that it may be symmetrically accessed by two observers.

Finally, generalised measurement strategies for atomic qubits are investigated. The information may here be carried by different atomic internal states, or by the ground states of different channels in a network of microtraps. Coupling the system to auxiliary states allows for a measurement on the combined system, corresponding to a probability operator measure strategy (POM) on the original system.

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List of publications

This thesis consists of an introductory part, followed by seven scientific publications:

I *Quantum statistics of atoms in microtraps*
E. Andersson, M. T. Fontenelle, and S. Stenholm, Physical Review A **59**, 3841 (1999).

II *Realising a quantum logic gate using microtraps*
E. Andersson and S. Stenholm, accepted for publication in Optics Communications.

III *Quantum logic with channeled atomic particles*
E. Andersson and S. Stenholm, accepted for publication in Journal of Modern Optics.

IV *Bell-state analyzer with channeled atomic particles*
E. Andersson and S. M. Barnett, Physical Review A **62**, 052311 (2000).

V *Shared access to quantum information*
S. Stenholm and E. Andersson, *Physical Review A* **62**, 044301 (2000).

VI *Generalised measurements of atomic qubits*
S. Franke-Arnold, E. Andersson, S. M. Barnett, and S. Stenholm, submitted for publication.

VII *Generalised measurements on atoms in microtraps*
E. Andersson, submitted for publication.

1 Introduction

Quantum phenomena are very exciting, in my opinion because they are so utterly different from the everyday life we experience. It might well be that all philosophical consequences of this are not yet fully appreciated. At the very heart of quantum mechanics lies the superposition principle. A system, which can be found in one of two states, may have any linear combination of these states. Multi-particle superposition states may be entangled, that is, correlated so that the two-particle wave function cannot be written as a product of one-particle wave functions. Entanglement is utilised in many spectacular applications like quantum cryptography, teleportation and quantum computation.

Using a beam splitter one can perform simple and beautiful experiments to demonstrate the effects of entanglement [1, 2]. Classically, if two particles are incident on a 50/50 beam splitter, one expects that, in half of the cases, they exit through different output ports, and in half of the cases they exit together through one of the output ports. In complete contrast to this, if two photons are incident on the beam splitter simultaneously, one through each input port, quantum mechanics will predict that if the spatial part of the wave function is symmetric, the two photons will always exit through the same output port. If the spatial part of the wave function is antisymmetric, they will always exit through different output ports. Since the symmetry of the bosonic wave function applies to the overall state, both these possibilities may be realised if the polarisation part of the two-photon wave function is symmetric respectively antisymmetric [2]. The quantum mechanical prediction is indeed found to hold in complete contrast to the intuitive classical statistics.

Advances in cooling and trapping neutral atoms may allow us to reach the regime where the atomic de Broglie wave length is of the same dimensions as the trapping structures. If, for example, these traps can be used to construct a microscopic beam splitter for atoms, and if we are able to control the input states on the single-atom level, experiments similar to the beam splitter experiment outlined above could be performed with atoms. The traps can be made very small, and might be connected to form microdevices and networks allowing us to store and manipulate quantum information and perform calculations.

In this thesis, I investigate some of the potential uses of such microtraps. In chapter 2, the different experimental realisations currently available are reviewed. After that, in chapter 3, I proceed to consider the physical mechanisms which are of interest when designing microdevices, such as coupling between channels and particle-particle interaction. It was already mentioned that the microtraps could ultimately be used for the processing of quantum information. In chapter 4, I give a brief review of quantum computation and develop toy models of simple devices which could be used to implement conditional logic with atoms in microtraps. In chapter 5, a scheme to share quantum information is discussed. Chapter 6 deals with generalised measurements, so-called probability operator measure strategies, and how these could be realised on atoms or ions. Finally, in chapter 7, some conclusions and an outlook are offered.

Contribution of the author

My supervisor, Prof, Stig Stenholm, has provided ideas, discussions and encouragment all through the thesis work. In Papers I, II, and III, the main part of the numerical and analytical calculations is my work. The paraxial approximation was suggested by my supervisor, and the calculation in two spatial dimensions in Paper I was carried out by Dr. Márcia Fontenelle. The spark initiating Paper IV was a visit by Prof. Steve Barnett, after which I carried out the calculations and and wrote the paper. In Paper V, I finished and perfected the calculations initiated by my supervisor. In Paper VI, I have carried out the analytical work connected to the Rydberg atom scheme, including the effects of decay, and done about half the writing. Paper VII is my own work, but I am of course indebted to Prof. Stig Stenholm, Prof. Steve Barnett and Dr. Sonja Franke-Arnold for discussions.

Short introduction to the papers

Paper I

We consider the possibility of manipulating quantum information using linear microtraps acting as single-mode wave guides for atoms. A beam splitter for material particles acts as an example. The effect of particle interactions on the output correlations is investigated numerically and explained through a simple analytic model.

Paper II

An atomic analogue of the electronic Coulomb blockade is investigated using numerical calculations. A particle is trapped in a quantum dot-like structure by a suitable laser pulse; the next particle trying to pass through will feel the presence of the trapped particle and (ideally) be reflected. Thus, where the second particle is directed depends on the presence or absence of the first particle.

Paper III

The possibility to realise a quantum logic gate with three coupled channels is considered. The presence of a particle can act as a control qubit steering a target particle from one course into another. We discuss the quantum dynamics of the system using a simple analytic model and verify these predictions by wave packet calculations.

Paper IV

We suggest some basic devices which might be built using neutral-atom microtrap networks. The case of a controlled-NOT gate and a Bell-state analyser are considered.

Paper V

We consider the transfer of a qubit from one observer to another in such a way that it can be symmetrically accessed by both observers. Since it is not possible to clone quantum states, only one of them is allowed to actually read the information carried in the qubit, but which one of them, the observers may decide later.

Paper VI

Probability operator measure (POM) strategies are a generalisation of von Neumann measurements. We suggest ways to experimentally realise such measurements on atoms or ions, with techniques presently available. The qubit states are different internal states of atoms or ions.

Paper VII

It is outlined how to perform generalised meaurements on atoms in a network of microtraps. The information is here carried by the spatial degree of freedom, i.e. in which microtrap guide the atom is found.

2 Microtraps

If a particle is confined to a region of the order of the de Broglie wave length, quantum phenomena will appear. With the advent and subsequent improvement of cooling and trapping techniques [3], it has become possible to reach this regime with atoms. When temperature dives into the sub-milliKelvin range, the atomic de Broglie wave length typically increases to the order of 100 nm, which is well within the reach of modern nanofabrication techniques. Lithographic traps for neutral atoms were suggested by Weinsten and Libbrecht [4], and today there is a lot of experimental activity in the field, as will be reviewed below. This motivates us to investigate the possible uses of such microtraps for atoms.

In this thesis, we will refer to a *microstructure* as a particle environment where the dimensions of the structures are of the order of the de Broglie wave length. Semiconductor heterostructures and metallic nanostructures are examples of existing man-made microstructures for electrons. When the dimensions of the systems diminish, quantum effects such as Coulomb blockade [5] are observed. Electrons have the advantage of a small mass, which implies a large de Broglie wave length; they possess charge, and are thus easy to manipulate with electric fields. This of course also means that the electrons are more sensitive to stray fields, which is a disadvantage for interferometric applications.

In analogy with semiconductor heterostructures for electrons, a possibility which immediately comes to mind is to utilise neutral-atom microstructures as integrated atom optics devices. The microtraps can be made very extended and thin, and such atomic quantum wires could be connected to form networks, where the particles would travel, interacting with the potential of the device and with other particles. The real challenge is to achieve coherent transport of single atomic particles. This would allow many exciting applications, since concepts without classical counterparts like superposition states and entanglement are the keys to the occurrence of quantum phenomena. Ultimately, microdevices could be used to manipulate quantum information and perform quantum computational tasks. Experimentally, we are still far from reaching this goal, but components such as beam splitters [6, 7] and an "atomic conveyor belt" [8] have already been realised. In these experiments, a thermal atomic cloud propagates through the device.

Most tests of quantum mechanics have been made using photons, see for example [1, 2]. Photons are rewarding to work with, since they are fairly easy to generate, for example, the entangled states needed to test Bell inequalities can be obtained by parametric down-conversion [9]. Photons do not interact, and propagate undisturbed over long distances; their detection is well established, even if there are limits on efficiency and the ideal photon detector still exists only in the mind of a theorist. Massive particles, on the other hand, have a different energy dispersion relation. They can be well localised, brought to rest and they interact with each other. These aspects make it of interest to perform conceptually similar experiments also with material particles, for example with atoms.

Interference and diffraction has of course been observed with material particles. A spectacular example is the interference of large fullerene C_{60} molecules [10]. Tests of quantum correlations have also been performed using massive particles, with a beamsplitter for electrons made of GaAs-GaAlAs semiconductor heterostructures [11]. It is difficult to launch single electrons into well controlled input states of the conduction band of a semiconductor; in this experiment, streams of electrons were directed onto the beamsplitter, and the shot noise of the output streams was seen to be suppressed in accordance with fermionic quantum statistics. Dephasing in electron interference has also been studied, using an interferometer built of semiconductor heterostructures [12]. Here, which-path information was obtained via a quantum point contact. Proposals to create and detect entanglement of single electrons in nanostructures have also been made [13]. With atoms, on the other hand, it might be easier, although not trivial, to control the input states. Microtraps could be used to perform fundamental tests of quantum mechanics with atomic particles, in analogy with the numerous experiments with photons. This is certainly less futuristic than imagining a complete neutral-atom microtrap quantum computer.

2.1 Building surface mounted atom optics

The possibility to manipulate neutral atoms and explore their wave character gave birth to the field of atom optics [14]. A variety of techniques to cool, trap and control the motion of neutral atoms are now available. Microscopic, planar magnetic traps were suggested by Weinstein and Libbrecht [4]. Integrating the traps on a surface, “integrated atom optics”, means robustness and scalability [15, 16, 17]. Below we will briefly go through some proposals for and realisations of microtraps for neutral atoms.

2.2 Magnetic guides

An atom with a magnetic moment $\vec{\mu}$ in a magnetic field $\mathbf{B}$ will experience a potential

$$V_{mag} = -\vec{\mu} \cdot \mathbf{B}. \tag{1}$$

All particles having a permanent magnetic dipole moment can be trapped making use of this. With high field gradients, it is possible to achieve tight confinement, and to create very small magnetic traps for neutral atoms [18].

Traps may be realised with current-carrying wires in different geometries, with or without additional bias fields [19, 20]. The magnetic field of a straight wire carrying a current I is given by

$$\mathbf{B} = \frac{\mu_0}{2\pi} \frac{I}{r} \hat{e}_\phi, \tag{2}$$

where $\hat{e}_\phi$ is the circular unit vector in cylindrical coordinates. An atom with its magnetic moment parallel to $\mathbf{B}$ can be trapped by this attractive “Kepler guide” $1/r$ potential. If an additional homogeneous magnetic field is applied perpendicular to the wire, this magnetic field will cancel that of the wire along a line parallel to the wire. Outside this line, the magnetic field increases in all directions and forms a potential tube, “side guide”, where atoms with $\vec{\mu} \cdot \mathbf{B} < 0$, “weak-field seekers”, can be trapped.

Current-carrying wires may also be fabricated onto a chip [21, 22, 23, 24, 25] in various configurations to achieve micrometer-size magnetic atom guides. Two parallel wires carrying opposite currents, plus an external bias field, will create a potential tube above the wires, where the magnetic field increases in all directions. Using four wires, no external bias field is needed. Wave guides for light, allowing the manipulation of the trapped atoms by lasers, could also be fabricated onto the surface. Typically, the dimensions of the structures is of the order of 100 μm. The state-of-the-art is that trapping frequencies of a few kHz, meaning ground state sizes of roughly 100 nm, can be reached.

As mentioned in the introduction, beam splitters [6, 7] and an "atomic conveyor belt" [8] have been realised using such integrated magnetic guides. By adding a bias field to a magnetic mirror, a corrugated magnetic reflector can be created [26]. Rotating the bias field makes the corrugation move, taking the trapped atoms with it. Current work aims at the observation of interference at the output of an interferometer consisting of two beam splitters [27].

Another possibility is the quadrupole guide obtained by four current-carrying wires embedded in a silica fibre [28]. The atoms are guided in a fifth channel at the centre of the fibre. At the end of the guide the four wires spread out to form a magnetic funnel into which the atoms are released from a magneto-optical trap. With such a guide, propagation over a few centimetres of a cloud of atoms with a radius of approximately 100 μm has been achieved.

2.3 Mirrors and charged structures

Another way to realise a microstructure guiding the motion of neutral atoms is to combine atom mirrors with charged or current-carrying nanostructures [15]. The confining potential is obtained as a combination of the repulsive mirror potential and an attractive potential due to electrically charged structures. These structures may be fabricated on top of the mirror surface using existing nanofabrication techniques. The mirror would prevent the atoms from hitting the structures or the surface, which are typically much hotter than the cold atomic sample; an atom hitting the surface will immediately be lost. By changing the electric fields on different electrodes, one may change the guiding potential.

An atom mirror can be obtained using a blue-detuned evanescent light field [29, 30, 31]. This evanescent wave is created by total internal reflection of a laser beam at an interface between a dielectric and vacuum. Let $\hat{z}$ be perpendicular to the surface. Due to the optical dipole force, the atom experiences a repulsive, exponentially decaying mirror potential

$$U_m = U_0 e^{-kz} \tag{3}$$

proportional to the intensity of the laser beam used, and inversely proportional to the detuning. The scattering rate from the mirror is inversely proportional to the square of the detuning; thus it can be reduced by increasing the detuning, at the expense of higher intensities required to maintain the same mirror potential height. Another possibility is to use a magnetic mirror [32, 33]. An alternating magnetic "floppy disk" pattern with the magnetisation

$$\mathbf{M} = M_0 \cos(kx)\hat{x}, \tag{4}$$

where $\hat{x}$ is parallel to the surface, will, somewhat simplifying the matter, create a magnetic field outside the surface, with the magnitude

$$B = B_0 e^{-kz}, \tag{5}$$

with $\hat{z}$ perpendicular to the surface. Provided the field changes slowly enough in space, the magnetic moment of an atom will follow the field adiabatically, and an atom with its magnetic moment $\vec{\mu}$ antiparallel to $\mathbf{B}$ will experience a repulsive potential μB, which then is exponentially decaying exactly as in the evanescent wave case.

The actual confinement is created by a strong, inhomogeneous electric field. The potential energy of an atom with electric polarisability α in an electric field $\mathbf{E}$ is given by

$$U_{pol} = -\frac{1}{2}\alpha|\mathbf{E}(\mathbf{r})|^2, \tag{6}$$

which, for an atom in its ground state, always will be attractive. The electric field can be created for example by a thin wire with an attractive potential proportional to $1/\rho^2$ (cylindrical coordinates), or a point charge, with a potential proportional to $1/r^4$ (spherical coordnates), resulting in a tube-like or dot-like confinement, respectively. It is possible to adjust the parameters so that quasi single-mode propagation is obtained, or such that there is just one single bound state of the dot. Hence the terms *neutral atom quantum wires* and *neutral atom quantum dots* have been coined [15]. The size of the ground state in such a guide could be of the order of 0.1 μm or even smaller, and the potential depth is a few neV.

2.4 Optical guides

Atoms have also been guided in hollow optical fibres with optical dipole forces confining them to the centre of the fibre [34, 35, 36, 37, 38]. The principle is the same as for the evanescent wave mirror; the light confining the atoms is guided by the same fibre. The diametre of the fibre core can be as small as a few μm, and the diametre of the ground state wave function even less, in principle allowing single mode propagation. One problem is how to avoid heating the atoms through intensity fluctuations and spontaneous emission. It is also not trivial to fuse the fibres together to achieve a beam splitter without backscattering. Hollow laser modes may be used in a similar way for the guidance of cold atoms [39]. Compared to the experiments with hollow optical fibres, the width of the guiding core is larger, but for the same potential height, the width of the potential barrier trapping the atoms is larger, too, leading to longer guiding distances.

Optical lattices [40] also provide a means to confine and manipulate neutral atoms. The interference of multiple laser beams creates periodic light shift potentials, where atoms can be trapped at the potential minima. Just as for evanescent wave mirrors, the optical dissipation may be reduced in a far-detuned optical lattice.

2.5 Loading and detection

In a typical experiment, atoms are trapped and cooled in a magneto-optical trap (MOT), and subsequently transferred to the guiding potential by switching on the guiding fields, and switching off the MOT. Atomic species employed are ^{85}Rb, ^{87}Rb, ^{7}Li and ^{133}Cs. The mirror-MOT technique employs a reflecting layer on top of the atom chip [21]; in this way, one is able to realise all six laser fields needed for the trapping potential close to the surface. With a nested series of magnetic traps, the atomic cloud may be compressed further until finally transferred to the actual guide.

What has been experimentally achieved is transport over a few centimetres of 10^6 to 10^8 atoms in thermal clouds. Trapping frequencies of a few kHz, implying ground state sizes of 100 nm and below, can be reached. This is the regime required for quantum information applications. For the same purpose, it would be of interest to achieve single-mode propagation, and to be able to control just a few atoms accurately. The loading of Bose-condensed atoms into integrated surface traps would solve the single-mode issue; steps in this direction are taken [41]. A MOT may also be used to trap just a few atoms [42, 43, 44], and these might be transferred into a magnetic surface trap; loading into an optical dipole trap with 100% efficiency [45] has been achieved. Single atoms could be detected by ionisation.

Coherent transport of the atoms also requires precise control of the trapping fields and currents. C. Henkel and S. Pötting have given analytical estimates for the heating and decoherence of a cold atom cloud in low-dimensional waveguides [46]. They estimate the scattering rates to be quite large, but that decoherence rates may be reduced by working with smaller structures and lowering their temperature.

3 Modeling microtrap devices for material particles

We imagine a typical microstructure device to consist of a network of channels where the particles propagate, interacting with each other, possibly manipulated by laser beams. The beam splitter and the six-port device considered in Papers I and III bear strong resemblance to multiport couplers as we know them from optics. The crucial difference is that optical multiport devices are linear; material particles, in contrast to photons, interact with each other and this will render the corresponding atomic devices essentially non-linear. This is promising, since non-linear features is precisely what the implementation of conditional logic requires. Below we will consider some of the aspects of the device operation: coupling between the channels and the effect of particle-particle interaction. Finally we will mention wave packet calculations with the split-operator method, which we have used to perform numerical simulations of the devices considered in Papers I, II, and III.

3.1 Coupling mechanisms

In optics, a beam splitter couples two input modes into two output modes according to

$$\begin{bmatrix} a^{\dagger}_{out} \\ b^{\dagger}_{out} \end{bmatrix} = \begin{bmatrix} t^* & r^* \\ r^* & t^* \end{bmatrix} \begin{bmatrix} a^{\dagger}_{in} \\ b^{\dagger}_{in} \end{bmatrix}, \tag{7}$$

where $a^{\dagger}$ and $b^{\dagger}$ are creation operators referring to the input and output modes of the beam splitter; see figure 1. The transmission and reflection coefficients obey the relations

$$|t|^2 + |r|^2 = 1, rt^* + tr^* = 0; \tag{8}$$

see for example Ref. [47].

In analogy with an optical coupler, a beam splitter for atoms might be obtained by letting two atomic quantum wires approach each other so that the particles are allowed to tunnel between the guides. In Paper I, a beam splitter is modeled according to this, with two harmonic valleys approaching each other in one direction, thus creating a coupling between the guides as in figure 2.

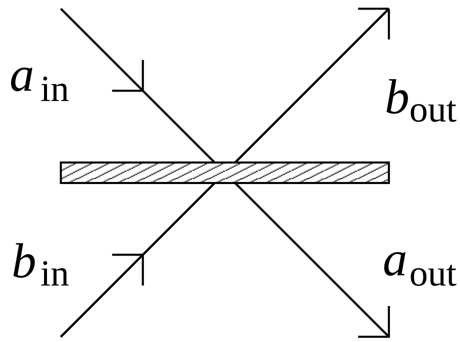

Figure 1: Schematic drawing of a beam splitter. The incoming modes $a_{\rm in}, b_{\rm in}$ are piloted into the outgoing modes $a_{\rm out}, b_{\rm out}$ according to the beam splitter relations.

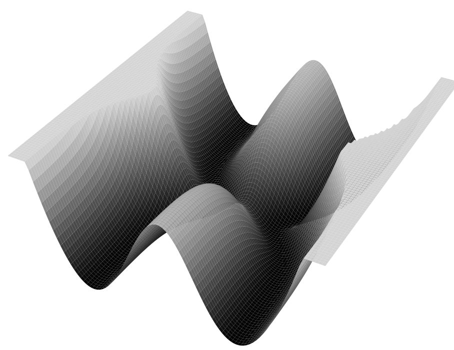

Figure 2: Atomic beam splitter potential. A beam splitter for atoms could be obtained by coupling two guides, in analogy with an optical coupler. The particle is allowed to tunnel between the guides where they approach each other.

A wave packet in a one-dimensional double-well potential $U(x)$ will tunnel across the barrier with a rate

$$T \sim \exp\left[-\int\sqrt{\frac{2m}{\hbar^2}(U(x)-E)}dx\right], \tag{9}$$

where the integral should be evaluated over the potential barrier separating the two wells. By making this potential barrier higher or lower, we can control the tunneling between the two channels.

The coupling can also be understood in terms of the eigenfunctions of the double well potential. The ground state is symmetric, ψ_S, with energy E_S, and the first excited state antisymmetric, ψ_A, with energy E_A. We define the tunneling frequency 2Ω according to

$$\begin{aligned} E_A &= \overline{E} + \hbar\Omega \\ E_S &= \overline{E} - \hbar\Omega. \end{aligned} \tag{10}$$

The time evolution of any initial state can easily be found using the eigenstates. We are interested in where the particle is localised; thus we form the states

$$\begin{aligned} \varphi_L &= \frac{1}{\sqrt{2}}(\psi_S + \psi_A) \\ \varphi_R &= \frac{1}{\sqrt{2}}(\psi_S - \psi_A), \end{aligned} \tag{11}$$

where the subscripts L (R) denote left (right) localization. Starting from φ_L at time $t = 0$, we obtain

$$\begin{aligned} \Psi(t) &= \exp(-iHt/\hbar)\,\varphi_L \\ &= \frac{1}{\sqrt{2}}\exp\left(-i\overline{E}t/\hbar\right)\left(e^{i\Omega t}\psi_S + e^{-i\Omega t}\psi_A\right) \\ &= \exp\left(-i\overline{E}t/\hbar\right)\left(\cos\Omega t\,\varphi_L + i\sin\Omega t\,\varphi_R\right), \end{aligned} \tag{12}$$

which shows that the particle is flipping back and forth between the two wells. For example, if $\Omega t = \pi/4$, the two coupled wells act as a 50/50 beam splitter. Experimentally, oscillations between the two localized states in a double-well potential in a far off-resonant optical lattice have been observed with cesium atoms [48].

This description is motivated as such, if the particles are trapped in any type of “quantum dots”, and the dots are moved closer to each other by changing the currents through the nanostructures. It is also easy to imagine that two coupled channels, where the particles move forward at a steady pace, will behave in the same fashion. A wave packet tunnels when the channels approach each other and is split between the guides. By varying the distance between the channels, the length of the coupling region, that is, the coupling time, and the height of the potential barrier between the guides, one is able to tune the device to perform e.g. 50/50 beam splitting. Using integrated magnetic traps, this kind of beam splitter has been realised experimentally [7]. Due to the exponential dependence in Eq. (9), the transmission and reflexion coefficients of such a beam splitter are extremely sensitive to changes in the guide potential.

In reality, the tunneling frequency Ω will obviously depend on the position along the channels, since the energy levels depend on the potential, and ωt is replaced by $\int \Omega dt$. This, however, changes nothing in the qualitative description. It turns out, as shown in Paper I, that one can describe the behaviour of the two-channel system quite accurately using the simplified treatment above by defining a constant effective Ω and a constant effective coupling time τ. Multiport devices, like the six-port coupler considered in Paper III, may be treated within the same approximation. The real advantage of the scheme becomes apparent when we turn to consider the effects of interactions, which may not directly be incorporated into a matrix like the beam splitter matrix in Eq. (7). This will be the subject of the following paragraph.

3.2 Particle-particle interaction

We need a simple way to estimate the effects of particle-particle interaction, since this interaction will be essential for the performance of the microdevices we consider. In order to do this, we assume the potential to be a multi-well system with the particles occupying different wells. They are allowed to tunnel between the wells or channels as above — this occurs when two channels approach each other — and they feel each other's presence through interactions. Sometimes it is sufficient to consider only the on-site interaction energy of two particles occupying the same channel to achieve a sufficient level of accuracy in the description;

in some cases the interaction between nearest neighbours is also taken into account, as in Paper III, in the description of a device which consists of three coupled channels.

Two particles which are brought close to each other by the channels of the device will perturb each other's energy levels due to their mutual interaction. If we assume the interaction to be weak, that is, the interaction energy to be much less than the level spacing of the modes in the guide, then the shape of the particle wave functions will essentially not change due to the interaction, and the only result will be a phase shift for the two-particle state. This approximation is reasonable since neutral atoms interact very weakly, with scattering lengths usually of the order of a few nm. If the energy shift due to the interaction is denoted by ΔE_{int}, the perturbative expression for this additional phase shift is

$$\Delta\phi_{int} \approx \frac{1}{\hbar}\int dt \Delta E_{int}, \qquad (13)$$

where ΔE_{int} obviously depends on how close the two particles are at each time instant. The fact that $\Delta\phi_{int}$ is large if two particles are found close to each other gives us a way of implementing conditional logic. By allowing atoms trapped in optical lattices or magnetic microtraps to collide and interact conditioned on their internal states, this effect has been proposed to be used for the creation of atom-atom entanglement [49, 50, 51, 52].

In Paper I, we verified that the effect of particle interactions on a beam splitter such as the one considered in Sec. 3.1, is to change the effective coupling constant of the device. A device which acts as a 50/50 beam splitter for a single particle, will no longer perform this action when two particles are incident simultaneously, one through each input. This can easily be understood in the language of Sec. 3.1, since the effect of the interaction is to change the energy levels of the two-particle system, so that the effective coupling Ω is changed. This effect occurs when ΔE_{int} is of the same order of magnitude as the splitting between the symmetric and antisymmetric eigenfunctions of the two wave guides ΔE. One can, however, adjust the potential so that the splitting is 50/50 for two incident particles (in which case the action will obviously not be 50/50 splitting for a single particle).

3.3 Wave packet calculations

When one has a specific configuration in mind, it is motivated to perform detailed numerical investigations of the system. Since the operation of the devices considered depends crucially on the actual motion of the particles, we need a method which takes dynamics into account. The method employed in the work of this thesis, one of the standard ways to simulate the Schrödinger equation, is wave packet calculations with the split-operator method [53]. The wave function (single- or many-particle) is a wave packet in real space. The numerical approach requires both space and time to be discretised. In the split-operator approximation, the time-evolution of the system is divided into small but finite steps Δt, and within each step the time-evolution operator is approximated according to

$$\exp\left[-i(\hat{T}+\hat{U})\Delta t/\hbar\right] \approx \exp\left[-i\hat{T}\Delta t/\hbar\right]\exp\left[-i\hat{U}\Delta t/\hbar\right]. \tag{14}$$

Here $\hat{T}$ denotes the kinetic energy operator, and $\hat{U}$ incorporates both external and interaction potentials. The corrections to (14) are, according to the Baker-Hausdorff formula, determined by the commutator

$$\left[\hat{T},\hat{U}\right]\frac{\Delta t^2}{2\hbar^2}, \tag{15}$$

whose expectation value should be kept sufficiently small to achieve numerical accuracy. Equation (14) enables us to calculate the action of the potential part

$$\exp\left[-i\hat{U}\Delta t/\hbar\right] \tag{16}$$

in a straightforward manner, since $\hat{U}$ is diagonal in position space. The action of the kinetic part

$$\exp\left[-i\hat{T}\Delta t/\hbar\right] \tag{17}$$

is obtained by Fourier transforming the wave function to k-space, whereby the kinetic energy operator $\hat{T}$ becomes diagonal. After obtaining the action of $\hat{T}$ on the wave function, this is transformed back to x-space and the whole procedure repeated. The numerical feasibility of the scheme relies on the Fast Fourier Transform (FFT) algorithm for performing the transforms between x- and k-space, requiring $\mathcal{O}(N \log_2 N)$ steps for N grid points in contrast to $\mathcal{O}(N^2)$ steps for a "normal" discrete Fourier transform.

FFT or not, the problem rapidly becomes intractable when the number of degrees of freedom increases. One has to eliminate as many dimensions as possible exploiting symmetries and simplifying the problem. In Papers I and III we make use of a paraxial approximation. The time evolution of the wave function involves steady propagation through the two-dimensional time independent potential in one — say the z — direction, with more involved dynamics along the other direction. It appears natural to simplify the problem by replacing the steady motion by a time-dependent Hamiltonian, thus reducing the dimensionality of the problem. This approximation can be extended to many-particle wave functions, as has been done in Papers I and III.

4 Quantum devices with channeled atomic particles

4.1 Quantum computation and information processing

Classical computers encode information in macroscopic two-level systems, bits, and manipulate the information using Boolean logic. In the 80's, it was realised by Feynman [54] and Deutsch [55], that information processing based on the laws of quantum mechanics is not constrained by the same rules as classical information processing. The factorisation algorithm discovered by Shor [56] in 1994 initiated an increased interest in quantum computing, since the security of presently used cryptographic protocols relies on our inability to factorise large numbers in a manageable time [57]. There is also a practical need to consider the influence of quantumness on calculations, whether we care about quantum algorithms or not. We are all familiar with the incredibly fast development of electronics and of computers in particular. If the dimensions of the hardware continue to diminish, at some point not too many years ahead, quantum phenomena will start to have effects in any classical computer. Unless we try to harness these effects, they will appear as noise.

A quantum bit, or qubit for short, can, in addition to the classical states 0 and 1, be found in any superposition state of these; this gives rise to quantum parallellism. When measuring the state of the qubit it is of course possible to get only one answer. By entangling an input qubit with another qubit, we can, however, achieve results which are not possible to obtain using the rules of classical computation. For example, we are able to determine whether a function $f : \{0,1\} \rightarrow \{0,1\}$ is constant or varying by computing the value of the function only once [58].

A register is a collection of qubits; quantum computation is the unitary manipulation of these. As a result, quantum computation is necessarily reversible, in contrast to classical computation (the final readout process may involve irreversibility). We need to be able to effect single qubit gates, that is, any unitary transform of one qubit, and two-qubit gates, that is, unitary evolution of a qubit conditioned on the state of another qubit. This is in principle enough to realise any computation. We need not be able to effect arbitrary two-qubit gates, since it can be shown that certain gates, such as the controlled-NOT gate and the phase gate, together with single-qubit operations, are sufficient for the implementation of any other quantum gate [59]. There are several good rewiev articles on the subject; see for example [60, 61].

Up to this time, a wide range of suggestions for how to realise a quantum computer has been considered. Almost every field of physics has its own candidate, with the exception of high-energy physics. The proposal by Cirac and Zoller utilises laser-cooled ions in traps [62, 63]. Polarised photons are suggested in [64]; solid state candidates are qubits realised as bound states of electrons in quantum dots [65] and Josephson-junction qubits [66, 67]. Other systems are electrodynamic cavities [68, 69, 70] and NMR [71, 72]. We will not embark on a discussion of the advantages and drawbacks of all these proposals; let us just note that a crucial point is how to realise the qubit-qubit coupling, while minimising the interaction with the environment, since it is essential that coherence is preserved during the calculation [73, 74, 75]. Also, registers with sufficiently many qubits have to be prepared and the final state read out.

Neutral atoms have the advantage that they are weakly coupled to the environment. Cold controlled collisions between atoms trapped in optical lattices or magnetic microtraps have been suggested to achieve entanglement and obtain quantum logic gates [49, 50, 51, 52]. In this context, it is discussed to use single atoms, but also ensembles of atoms. The two qubit basis states are taken to be different internal atomic states, and the atoms are made to collide by shifting the optical lattices or the magnetic microtraps. The atom-atom interaction seems to be strong enough to realise two-qubit gates. A somewhat different approach is taken in [76], where it is proposed to shift two atoms to the same optical lattice well, and then to induce electric dipoles with an additional laser. This would enable the implementation of a quantum controlled-NOT gate.

In the Introduction, it was mentioned how neutral-atom microtraps could be used for tests of quantum correlations and for constructing devices to manipulate quantum information and perform computations. The particles would travel through a network of microtraps; the information could here be encoded in the spatial degree of freedom. The qubit basis states would be the ground state modes of different channels. The guiding potentials need not be static. It is possible to change the guiding potentials by varying the charge of or the current through the confining structures (depending on the particular realisation), thus altering for example the transmission and reflexion coefficients of a beam splitter acting as a coupler, as described in the previous chapter on modelling microtrap devices. It is also possible to integrate wave guides for light with the microtraps, in order to manipulate the atoms with lasers. Combining linear microtraps and optical lattices might also prove useful. Problems which were touched upon in the chapter on microtraps, and which have to be solved in an actual experiment, is the launching of input particles into the ground state mode of the channels, and the readout, that is, detection of single atoms, at the end of a computation. The guiding potentials also have to be well controlled in order to preserve quantum coherence during calculation.

A single-qubit gate could be obtained by coupling two channels to form a beam splitter, with transmission and reflexion coefficients tuned by the potential barrier between the channels and by the length of the interaction region. In addition to this, one needs to manipulate the phase of a single atom in a particular channel. This may be achieved by introducing path differences or by locally changing the potential, so that the ground state energy is altered. To realise conditional logic some kind of nonlinearity in the device is needed. With atomic particles, it is very natural to use particle-particle interactions as this non-linear ingredient. As explained above, a phase gate or a controlled-NOT gate together with single-qubit operations is sufficient for the implementation of any quantum computational algorithm. In Papers II, III and IV, we have considered different possibilities for realising a controlled-NOT gate with channeled atomic particles. These schemes will be briefly described below.

4.2 Atomic analogue of Coulomb blockade

Imagine a conducting island connected to leads through tunnel junctions. Due to the finite charging energy $e^2/2C$, where C is the capacitance of the island, it does not pay to transfer electrons onto the island for arbitrarily small bias voltage; a finite voltage is needed in order to obtain a current. This phenomenon is referred to as Coulomb blockade [5]. A related experiment has been proposed for neutral atoms [22]. According to this work, transport through a channel with two constrictions would

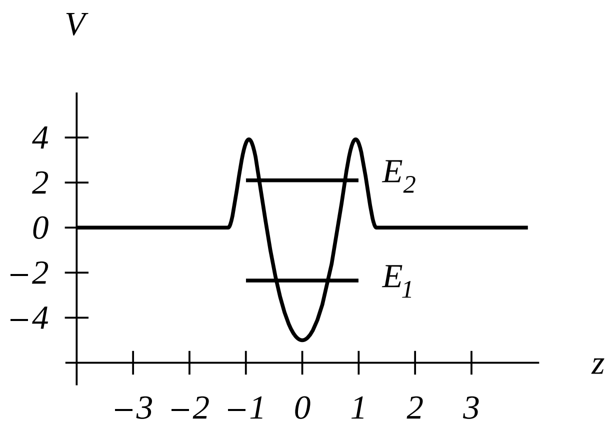

Figure 3: The gate potential $V(z)$. The particles are propagating in the z direction in the matter wave guide, confined in the x and y directions. The energies E_1 of the bound ground state and E_2 of the quasibound tunneling state are indicated with horizontal lines.

show similar quantum features.

To investigate the potential use of a related system, in Paper II, we have considered a wave guide with an additional double barrier structure along the direction of propagation as shown in figure 3. The idea is to take advantage of the particle interaction to obtain gated operation. The well has one bound and one quasibound state. Incoming particles with the correct energy E_2 may tunnel through the structure resonantly if the wave packet is narrow enough in energy; off-resonant components are reflected. If a resonant laser pulse is applied when a particle is tunneling, this may be trapped at the lower level. The next particle arriving at the barriers will feel the presence of the trapped particle through interaction. The interaction energy will ideally shift the second wave packet off resonance, and it will be reflected. Thus the presence of the first "control" particle determines where the second "target" particle is directed. This scenario is schematically outlined in figure 4. The laser pulse acts as a clock pulse, ready to trap a control qubit particle if this is present.

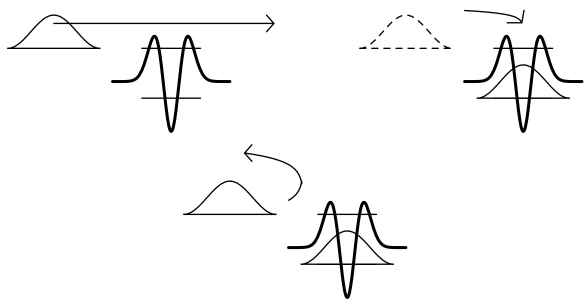

Figure 4: Schematic pictures of the gate operation. A wave packet with the correct energy will tunnel through the double barrier (upper left). Applying a suitable laser pulse, a particle can be trapped in the ground state (upper right). A second particle trying to pass will then be reflected (lower centre).

Our numerical simulations indicate that this device could in principle operate as a quantum controlled-NOT gate. Neutral atoms, however, interact very weakly. To obtain an energy shift sufficiently large, one would have to localise the atoms to a region of a few nanometers. The optimisation of the laser pulse in order to achieve efficient trapping is also not a trivial task. In a possible experiment, the question of how to recycle the control particle has to be solved, since quantum gates should be reversible. Whether this or similar devices can be built or used in spite of these difficulties is still an open question.

4.3 Six-port device for conditional logic

In Paper III, we consider a device consisting of three coupled channels. The target qubit particle enters in the upper channel 1, and if the control qubit is zero, i.e. the control particle is not present in the middle guide 2, it will tunnel and exit in the lower guide 3 as shown in figure 5. Without particle-particle interaction this six-port device would be linear, and if the control particle is incident in the middle channel, the two particles would exit in channels 2 and 3 as shown in figure 6. When interaction is present, this will change the effective coupling constant of the device,

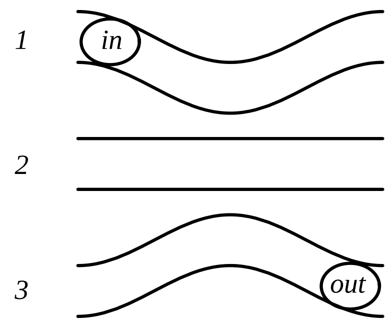

Figure 5: A single particle incident in channel 1 will exit in channel 3.

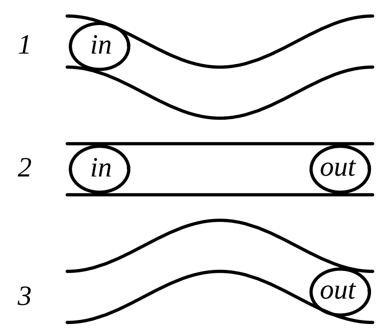

Figure 6: Two particles incident in channels 1 and 2 will exit in channels 2 and 3 supposing no particle-particle interaction is present.

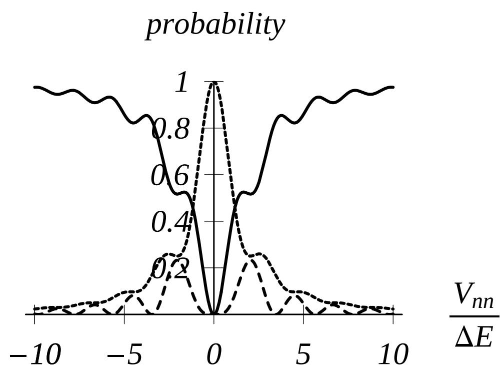

Figure 7: The probabilities, as functions of an effective interaction parameter $V_{nn}/\Delta E$ (further details are found in Paper III), for two particles, incident in inputs 1 and 2, to emerge at different output combinations. The full line denotes the probability to emerge in channels 1 and 2, the dotted line shows the probability to emerge in channels 2 and 3, and the dashed line shows the probability to emerge in channels 1 and 3. The probabilitites to find both particles in the same channel is zero in this approximation. We see that the desired behavior is approached asymptotically for large V_{nn}. Thus, in this limit, the controlled-NOT gate operation is obtained.

in analogy with the case of a beam splitter, as we have seen in Sec. 3.1. The purpose is now that this would force the target and control particles to exit in channels 1 and 2 instead, making the target particle appear in different output channels conditioned on the presence of the control particle. This would allow for the implementation of a controlled-NOT gate.

In order to estimate the effect of the interaction on the performance of the device, we have modeled it as a three-well system with particles occupying the different wells, feeling an on-site and a nearest-neighbour interaction. It is straightforward to solve for the energy eigenvalues and eigenfunctions, and to obtain the time evolution of a general input state. We find that in the limit of large interaction strength, the desired

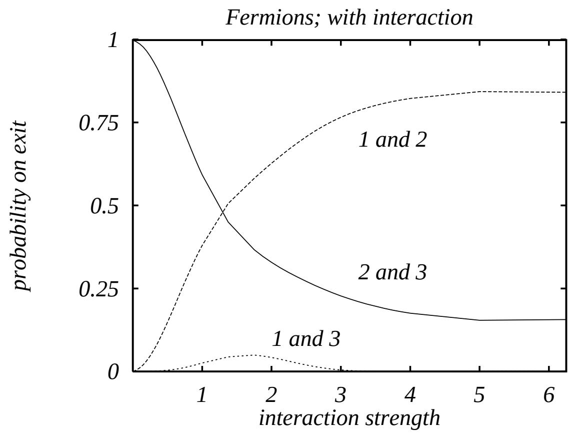

Figure 8: The probabilities, as functions of interaction strength, that two fermions, incident in channels 1 and 2, will exit at different combinations of output ports. Ideally, the probability for appearance in channels 1 and 2 should approach unity. The interaction strength is shown in units of $V_{nn}/\Delta E$ as in figure 7.

behaviour is indeed approached, and, when a control qubit particle is present, the target qubit particle is predicted to exit in channel 1 instead of channel 3, as shown in figure 7.

Numerical simulations verify this behaviour to a certain extent. Figure 8 shows the probabilities for two particles, here fermions, to exit in different combinations of channels, as functions of interaction strength. The deviations at large interaction strengths arise from the fact that the simple analytical model assumes single-mode propagation, and when the interaction becomes comparable to the mode spacing in the guides, higher-lying states are excited and the numerical results deviate from the predicted behaviour. The gate operation could be improved by increasing the coupling time, thus relaxing the need of large interaction, at the expense of more time-consuming numerics. The principle of the gate operation is, however, already illustrated.

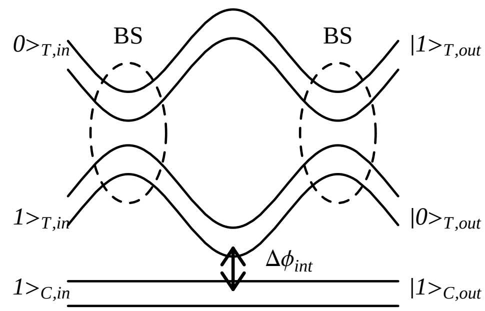

Figure 9: Schematic view of a controlled-NOT gate realised using an interferometer and a phase gate. The target qubit is incident in a superposition of the interferometer input modes. The control qubit is able to flip the target qubit as explained in the text.

4.4 Controlled-NOT gate and Bell-state analyser

Paper IV proposes some more devices which could be obtained by combining neutral-atom microtraps. A Mach-Zehnder interferometer can obviously, as in optics, be obtained using two beam splitters. A phase gate,

$$\begin{array}{lcl} |0\rangle|0\rangle & \rightarrow & |0\rangle|0\rangle \\ |0\rangle|1\rangle & \rightarrow & |0\rangle|1\rangle \\ |1\rangle|0\rangle & \rightarrow & |1\rangle|0\rangle \\ |1\rangle|1\rangle & \rightarrow & -|1\rangle|1\rangle, \end{array} \tag{18}$$

proposed for neutral atoms in [49, 50, 51, 52], together with single-qubit operations, is sufficient to realise a controlled-NOT gate. Combining an interferometer with a third channel and a phase gate as in figure 9 is a way to realise this. The target qubit is incident in the upper two channels, corresponding to $|0\rangle_T$ and $|1\rangle_T$. The two encircled regions are 50/50 beam splitters. The control qubit is incident in the third channel, if it is in state $|1\rangle_C$, and in some other channel (not shown in the picture) if it is in state $|0\rangle_C$. Of course, also the control qubit may be found in a superposition state of $|0\rangle_C$ and $|1\rangle_C$.

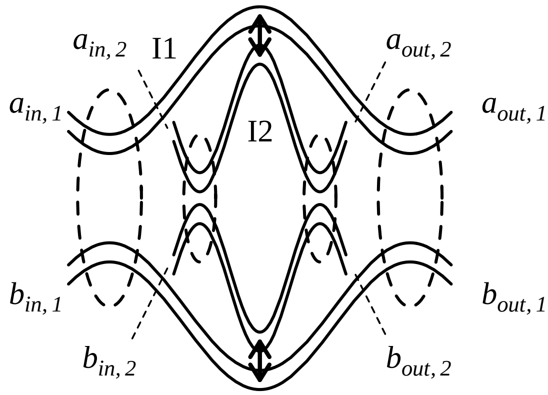

Figure 10: Bell-state analyser consisting of two coupled interferometers.

The double arrow indicates a phase gate, resulting in a phase shift of π for the two-atom wave function if the control qubit is set. If the control qubit is zero, there is no phase shift, and nothing happens with the target qubit. If the control qubit is set, the induced phase shift will result in the the target qubit being flipped on exit:

$$\begin{aligned} |0\rangle_C|0\rangle_T &\rightarrow |0\rangle_C|0\rangle_T \\ |0\rangle_C|1\rangle_T &\rightarrow |0\rangle_C|1\rangle_T \\ |1\rangle_C|0\rangle_T &\rightarrow |1\rangle_C|1\rangle_T \\ |1\rangle_C|1\rangle_T &\rightarrow |1\rangle_C|0\rangle_T. \end{aligned} \tag{19}$$

Since the whole process is a unitary time-evolution, superposition states will also evolve correctly according to these rules.

A Bell-state analyser is a component needed to take full advantage of applications such as quantum teleportation [77] and quantum dense coding [78]. In principle, if we are able to perform a controlled-NOT or phase gate operation, and, in addition to this, linear one-qubit operations, we are also able to conduct Bell measurements.

As described in Paper IV, one way to achieve this is to combine two interferometers I1 and I2 as in figure 10. The two interferometers should be imagined to lie on top of each other. The four possible Bell states we will want to distinguish with this device are

$$
\begin{aligned}
|\Psi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(a_1^{\dagger}b_2^{\dagger} \pm b_1^{\dagger}a_2^{\dagger}\right)|0\rangle, \\
|\Phi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(a_1^{\dagger}a_2^{\dagger} \pm b_1^{\dagger}b_2^{\dagger}\right)|0\rangle,
\end{aligned} \tag{20}
$$

where $a_1^{\dagger}$ and $b_1^{\dagger}$ are creation operators referring to the two modes of interferometer 1 in figure 10, and $a_2^{\dagger}$ and $b_2^{\dagger}$ are creation operators for the modes of interferometer 2, and $|0\rangle$ denotes the vacuum state. If it is not physically possible to have channels crossing each other, which is probably the case for an "atom chip", channels can be made to effectively cross using a beam splitter coupling as in Eq. (12), with Ωt equal to π. The double arrows indicate phase gates, this time, however, not inducing a phase factor of π resulting in a factor of -1 multiplying the wave function, but a phase factor of $\pi/2$.

It is straightforward to check that Bell states will be disentangled according to

$$
\begin{aligned}
|\Psi^{-}\rangle &\rightarrow e^{i\pi/4}a_1^{\dagger}b_2^{\dagger}|0\rangle \\
|\Psi^{+}\rangle &\rightarrow e^{i3\pi/4}b_1^{\dagger}a_2^{\dagger}|0\rangle \\
|\Phi^{-}\rangle &\rightarrow e^{-i\pi/4}a_1^{\dagger}a_2^{\dagger}|0\rangle \\
|\Phi^{+}\rangle &\rightarrow e^{-i3\pi/4}b_1^{\dagger}b_2^{\dagger}|0\rangle.
\end{aligned} \tag{21}
$$

5 Shared access to quantum information

Quantum mechanics does not permit us to make perfect copies of a quantum state; this is the no-cloning theorem [79]. By teleportation, we can, however, transfer a quantum state to another observer, without knowing the state ourselves [77]. Teleporting to more than one location so that only one of the receivers can reconstruct the state [80] is related to quantum copying [81]. A message can also be split in several parts so that no subset of parts is enough to read the message, but the entire set is [82, 83].

In Paper V, we consider the situation where a qubit is potentially transferred from one observer to another, and can be symmetrically accessed by either of the two observers. Since it is not possible to clone quantum states, as already mentioned, only one of the observers can read the information carried in the qubit. The transfer may however be carried out at an earlier instant, and the decision of whether the state has to be read by the reciever or returned to sender can be made later. The process succeeds only with 50% probability, but even in the case of a failure, the quantum information is (ideally) not lost and may be used in further operations.

We want to transfer a qubit of information from one system to another. The Hilbert spaces of each system are four-dimensional, $\mathcal{H}_1 = \{|1\rangle, |2\rangle, |3\rangle, |4\rangle\}$ and $\mathcal{H}_2 = \{|\alpha\rangle, |\beta\rangle, |\gamma\rangle, |\delta\rangle\}$ respectively. The first two components of each Hilbert space will carry the information, and the other two will serve as a scratch pad. In the end, the observers will have access to one of the subsystems each. The first observer wants to transfer the state

$$| \varphi_1^0\rangle = a \mid 1\rangle + b \mid 2\rangle, \tag{22}$$

where the information is carried in the numbers a and b. The initial state of the system is taken to be

$$| \psi_0\rangle =| \varphi_1^0\rangle \mid \varphi_2^0\rangle, \tag{23}$$

where the state of the second system is chosen as

$$| \varphi_2^0\rangle = \frac{1}{\sqrt{2}} \left(| \alpha\rangle + \mid \beta\rangle\right). \tag{24}$$

After the use of two unitary transformations, the final state of the system is found to be of the form

$$\begin{aligned} \mid \Psi_{fin}\rangle = \;& \tfrac{1}{2}\left[(a \mid 1\rangle + b \mid 2\rangle) \otimes (\mid \gamma\rangle + \mid \delta\rangle)\right] + \\ & \tfrac{1}{2}\left[(\mid 3\rangle + \mid 4\rangle) \otimes (a \mid \alpha\rangle + b \mid \beta\rangle)\right]. \end{aligned} \tag{25}$$

The transforms have to be performad jointly on the two qubits, either by having the qubits at the same location initially, or by making use of teleportation [77, 84]. The unitary transformations have produced an entangled state where the original qubit is associated with the scratch pad state in the second two components of system 2, and the scratch pad state in the second two components of system 1 is associated with the original quantum qubit in the first two components of system 2. The second qubit will now be sent or teleported back to the second observer, making the qubit information $\{a, b\}$ accessible to each one of them.

If the first observer makes a projective measurement onto the scratch pad state, and the result is positive, he ascertains that the second oberver is in possession of the original qubit state, and vice versa. The scheme will of course be succesful in only 50 % of the cases. If detection is assumed to be ideally nondestructive, a negative answer implies that the information is found in the space complementary to the scratch pad, and thus the information is not lost, only in the wrong place, and further information processing may take place. The process may be viewed as "teleportation with a delayed choice". Because of the 50% success probability, and the fact that the two observers should trust each other to agree on who should have the qubit in the end, the protocol does of course not solve the question of two-party quantum bit commitment. Bit commitment is a process when one party, say Alice, sends a bit of her choice to another party, say Bob. The bit is contained in a "locked box", and only later Alice sends Bob the key. Quantum bit commitment protocols fail because there is no way to prepare a qubit which is both unmodifiable by Alice and unobservable by Bob [85, 86].

6 Generalised measurements on atoms

If we want to learn something about a quantum system, we have to observe it. Standard textbooks on quantum mechanics usually describe measurements within the von Neumann formalism [87]. An ideal von Neumann measurement is a strange creature. It is a projection onto the orthogonal eigenstates of some observable. The measurement result is one of the eigenvalues, and the wave function is said to "collapse" to the corresponding eigenfunction. All subsequent measurements of the same observable will yield the same result. One usually considers the situation when identical measurements are performed on an ensemble of systems, and outcomes are discussed in terms of probabilities.

Technical advances have now made it possible to make measurements on single quantum systems, to perform measurements that perturb the state of the system only minimally, so-called quantum non-demolition (QND) measurements, and to make repetitive measurements on the same quantum system. The quantum theory of measurement is no longer an esoteric subject of little relevance to "real" physcis. von Neumann measurements are clearly not sufficient to decribe all realistic measurement situations. A von Neumann measurement cannot, for example, directly describe a simultaneous measurement of non-commuting observables, such as the position and momentum of a particle, due to the fact that these cannot be described in the same basis of eigenstates [88, 89].

Given a quantum system, and told to measure its state, we are not always able to learn much unless we have some knowledge about the preparation procedure. Consider for example a photon. It is obviously impossible to tell with certainty what its polarisation is — or was before the measurement — unless we know in which basis to measure it. Quantum cryptography protocols [90] rely on this fact. Now assume that the photon is prepared in one of two non-orthogonal polarisation states. Since the states are non-orthogonal, it is obviously impossible to tell with certainty what the polarisation is, even if we know which two possibilities to choose from. We can of course minimise the error probability, still describing our actions with projective von Neumann measurement with two outcomes.

It is, however, possible to perform a more general kind of measurement with three outcomes, two of them corresponding to either one of the polarisation directions, telling us for certainty in which state the system was prepared. The price we have to pay is that there always will be a finite probability for the third outcome — that the measurement yields no information at all [91, 92]. For an explicit description of this measurement strategy, the reader is referred to Paper VI.

This kind of measurement, as well as the simultaneous probing of non-commuting observables, and other situations, can be described using so-called probability operator measure (POM) strategies [93, 94]. This formalism does, however, not tell us how to perform such measurements on a given physical system. Some experiments on photons, where one wants to distinguish non-orthogonal and linearly dependent states, have been performed [95, 96, 97, 98]. It is of interest to be able to perform generalised measurements also on other types of systems, especially when considering quantum information applications. Papers VI and VII deal with performing such measurements on atomic qubits. An experimental realisation when the qubit states are different internal states of ions or atoms is suggested in Paper VI. In Paper VII, I outline how to implement generalised measurements on atomic particles trapped in microtraps, where the information is encoded in the spatial degree of freedom. Before briefly describing these techniques, a sketch of the general features of POM measurements is presented in the next section.

6.1 Probability operator measures

Probability operator measure (POM), also referred to as positive operator-valued measure (POVM) strategies [93, 94] provide a generalisation of von Neumann measurements. One important difference between von Neumann measurements and these generalised measurements is that the number of outcomes may exceed the number of available preparations or the dimensionality of the Hilbert space of the system to be measured.

It is instructive to picture a generalised measurement to be carried out by coupling the system to be measured to an auxiliary system, and then performing a von Neumann measurement on the combined system. Let ρ_i be the density matrix of the initial system prepared in state i, and ρ^{aux} that of the auxiliary system. The different outcomes, labeled by j, of the measurement on the combined system correspond to orthogonal projectors $\hat{\Pi}_j^{comb}$, acting in the Hilbert space of the combined system.

The probability to obtain result j after preparation i is

$$\begin{aligned} P_{ji} &= Tr[\hat{\Pi}_j^{comb}(\rho_i \otimes \rho^{aux})] \\ &= \sum_{mr,ns} (\hat{\Pi}_j^{comb})_{mr,ns}(\rho_i)_{nm}(\rho^{aux})_{sr}. \end{aligned} \tag{26}$$

If we now define

$$(\hat{\Pi}_j)_{mn} = \sum_{rs} (\hat{\Pi}_j^{comb})_{mr,ns}(\rho^{aux})_{sr}, \tag{27}$$

then the probability P_{ji} to obtain the result labeled by j is given by

$$P_{ji} = \mathrm{Tr}(\hat{\Pi}_j \hat{\rho}_i), \tag{28}$$

where each possible outcome j of the generalised measurement is associated with a Hermitian operator $\hat{\Pi}_j$ acting on the original system. For a von Neumann measurement, the operators $\hat{\Pi}_j$ are the projectors onto the orthonormal eigenstates of the observable to be measured, and there is no need for an auxiliary system. In general, $\hat{\Pi}_j$ are neither orthogonal nor normalized, but the following conditions will always hold:

- All the eigenvalues of the operators $\hat{\Pi}_j$ are positive or zero.
- $\hat{\Pi}_j$ form a decomposition of the identity operator: $\sum_j \hat{\Pi}_j = \hat{\mathbf{1}}$.

The first condition is necessary due to the fact that we are dealing with probabilities, and the second makes sure that we always get a result, even if this result paradoxically enough might provide no information at all. Having obtained a result, it is possible to infer something about the state of the system before the measurement. If the prior probability for preparation i is p_i, the prior probability for obtaining result j will be

$$q_j = \sum_i P_{ji} p_i. \tag{29}$$

If we apply Bayes' theorem, we can calculate the posterior probability Q_{ij} for preparation i:

$$Q_{ij} = P_{ji} p_i / q_j. \tag{30}$$

When faced with a measurement situation, the question one obviously asks is “What POM strategy is the optimal one?”. The answer depends on what we mean by optimal. If we wish to minimise the overall error probability, this is a case of linear optimisation. This is, however, not the only possible definition of "optimal", and in general the question of the best strategy is difficult and lies open.

6.2 Realisation using auxiliary states

Neumark's theorem [99] implies that any POM measurement can be viewed as a von Neumann measurement in an extended Hilbert space [93, 94]. Loosely speaking, in order to have N different outcomes when the dimensionality of the system to be probed is M, where $M \leq N$, we need to map the original system onto N orthogonal states of an extended system. This can be achieved by finding $N - M$ auxiliary states and redistributing the population among all the states, coupling the original and auxiliary degrees of freedom in such a way that the final population probabilities of the basis states of the extended system will correspond to the N different measurement results.

The coupling to the auxiliary states can be thought of as a "premeasurement superoperator". For simplicity, we will here restrict ourselves to POM strategies consisting of rank one matrices. Let us consider a measurement strategy on an M-dimensional system, with the N elements

$$\hat{\Pi}_j = |\Psi_j\rangle\langle\Psi_j|, \quad j = 1, ..., N, \tag{31}$$

where

$$|\Psi_j\rangle = \sum_{i=1}^{M} a_{ji}|i\rangle, \tag{32}$$

and $|i\rangle$, $i = 1, ..., M$, are the orthogonal basis states of the initial system, on which the measurement should be carried out. Let $|i\rangle$, where $i = M+1, ..., N$, be the auxiliary states. It is always possible to find extended states

$$|\Psi'_j\rangle = |\Psi_j\rangle + \sum_{i=M+1}^{N} b_{ji}|i\rangle \tag{33}$$

so that $|\Psi'_j\rangle$ are orthonormal and span the whole extended Hilbert space.

We can construct a unitary transformation

$$\hat{U} = \sum_{j=1}^{N} |j\rangle\langle\Psi'_j|, \tag{34}$$

which, applied to an initial state $|\phi\rangle$ in the Hilbert space of the original system, yields

$$\hat{U}|\phi\rangle = \sum_{j=1}^{N} |j\rangle\langle\Psi'_j|\phi\rangle = \sum_{j=1}^{N} |j\rangle\langle\Psi_j|\phi\rangle, \tag{35}$$

meaning that the probability to find the system in state $|j\rangle$ is exactly $P_j = Tr(\hat{\Pi}_j\hat{\rho})$, with $\hat{\Pi}_j$ given by $|\Psi_j\rangle\langle\Psi_j|$. Thus, in order to effect the desired POM measurement we only need to apply this "premeasurement superoperator" $\hat{U}$ and then to measure the final population in the basis states. This is a useful key to actual physical realisations, and has been used both in Papers VI and VII.

6.3 Measurements on atoms or ions

Given a system on which we want to perform a POM measurement, we have to realise three main points: First, enough auxiliary states have to be found, so that there is one state per required measurement outcome. Secondly, we have to be able to couple the states to realise the transform $\hat{U}$ of Eq. (34). Finally, we have to be able to test for population in the basis states used. When the qubits are carried by the polarisation states of photons [95, 96, 97, 98], $\hat{U}$ is accomplished as a series of rotations of the polarisation, and by phase shifts. Polarising beam splitters are used to separate and recombine different polarisation states. The auxiliary states are unused input ports of beam splitters.

In Paper VI, we suggest how to realise generalised measurements when the states used for encoding the information are different internal states of atoms or ions. We have only considered the case when the system to be measured is a single atom or ion, not yet joint measurements on several particles. For ions in a trap, the qubit basis states may be coupled to other internal auxiliary states using Raman pulses. The final read-out may then be carried out via a fluorescence measurement. The detection efficiency is very high, virtually 100% [100].

Another possibility is to use different atomic Rydberg states as the basis states. Atoms in circular Rydberg states can be used to achieve atom-photon and atom-atom entanglement in experiments where atoms pass through cavities, interacting with the cavity field and undergoing precisely controlled transitions between different Rydberg states; see for example [69]. State-selective detection is obtained using ionisation of the atom. The same existing techniques could easily be used to effect POM measurements on atoms.

6.4 Measurements on atoms in microtraps

In Paper VII, it is outlined how POM measurements could be performed on neutral atoms in microtraps, when the qubit basis states are the ground state modes of different guides in a network. The auxiliary states would simply be additional channels. The atom whose state is to be measured is incident in channels 1 to M only, in an appropriate superposition of these modes, and the auxiliary input ports $M+1$ to N are unused.

The operator $\hat{U}$ in Eq. (34) can be realized by coupling the channels two at a time, using beam splitters and phase shifts. As shown in [101], any unitary transform can be obtained using two-by-two beam splitters and phase shifts. A beam splitter is obtained as described in section 3.1, by letting two guides approach, thus allowing particles to penetrate from one guide to the other. Phase shifts are obtained using path differences or additional electric or magnetic potentials. The final measurement is performed by detecting at which output channel the atom is emerging.

The experimental challenges remain the same as the ones mentioned when discussing other neutral-atom devices built from microstructures. Precise control of the trapping potentials is essential. A beam splitter based on tunneling will work correctly only for guided atoms with a certain transverse energy, making single-mode guides necessary. It is also desirable to be able to release and detect single atoms.

7 Conclusions

In this thesis, I have considered some potential applications of microtraps for neutral atoms. In Paper I, the influence of particle-particle interaction is investigated through a model of a beam splitter for atoms, obtained by coupling two atom guides in analogy with an optical coupler. Such a beam splitter could be used to perform tests of quantum mechanics with massive particles, and as a building block of more complex devices.

We then aim at using the particle-particle interaction to effect logical operations, where the time evolution of a target qubit particle is conditioned on the state of another control qubit particle. Paper II deals with an atomic analogue of Coulomb blockade, and in Paper III a six-port device is studied. In both these "toy model" devices, the presence of one particle will act as a control qubit, steering another target particle into different output states, thus effecting a two-qubit gate. Paper IV considers designs for a controlled-NOT gate and a Bell-state analyser.

In Paper V, we consider a scheme where a state is manipulated so that two observers have symmetric access to the information in a qubit. Only one of them may actually read the information, but which one of them the observers may decide later. The scheme is not fail-safe, but also when not succesful, the information residing in the qubit is not lost.

Finally, in Papers VI and VII, we take up the subject of generalised measurements. Ways to experimentally realise such measurements on atomic qubits are considered, both when the information is carried by different internal states of atoms or ions, and when the qubit basis states are the ground state modes of different guides in a network of microtraps for neutral atoms.

Whether microtrap devices similar to those envisaged in this thesis can be built, and eventually be used to manipulate quantum information and perform computations, depends on to which extent it is possible to launch single atoms into the microtraps and retain their coherence throughout the operation of the device. Neutral atoms interact very weakly; this is an advantage when it comes to avoiding decoherence due to coupling to the environment, but in order to achieve sufficient atom-atom coupling, the atoms have to be brought very close to each other, possibly confined to regions of the size of only a few nanometres. Several potential realisations of the "hardware" are currently being developed. The future may bring with it for example the launching of Bose condensed atoms into microtraps. It will be exciting to follow the experimental progress.

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