Objective evaluation of vehicle handling during winter conditions

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Preface

The following report documents the work conducted for the Master thesis project *Objective evaluation of vehicle handling during winter conditions*, a part of the curriculum for the Master of Science in Vehicle Engineering at KTH Royal Institute of Technology, Stockholm, Sweden. The work has been conducted at the Vehicle Dynamics Attribute Development section at Volvo Car Corporation, Göteborg, Sweden.

Some information used in the proceedings of the project are proprietary to Volvo Car Corporation and hence cannot be published in a public report. This report attempts to give the reader information about the procedures used, results obtained and the conclusions drawn using publicly available information. The figures in this report have been de-classified by removing the scale on their axes, as well as representing time in a relative scale. However, figures and tables containing complete information have been included in a separate Appendix C and D that shall not be published.

We would like to thank Volvo Car Corporation for making this thesis possible. We are grateful for the help given and knowledge shared by the people at the Vehicle Dynamics Attribute Development section and Vehicle Dynamics CAE section, at Volvo. Special thanks to our supervisors; Mikael Nybacka, Assistant Professor at KTH Royal Institute of Technology and Gaspar Gil Gómez, Industrial PhD student at Volvo Car Corporation, for your guidance and support during the thesis. Finally, we thank our family and friends for their encouragement and support.

Göteborg the 17th of June 2015

Alexander Lönnergård

Mohit Hemant Asher
### List of symbols

\[ \begin{align*}
\alpha & \quad \text{Lateral slip angle} \\
\alpha_{12} & \quad \text{Lateral slip angle, front axle} \\
\alpha_{34} & \quad \text{Lateral slip angle, rear axle} \\
\alpha_{\text{UKF}} & \quad \text{Spread of sigma points in UKF} \\
a_x & \quad \text{Longitudinal acceleration} \\
a_y & \quad \text{Lateral acceleration} \\
a_z & \quad \text{Vertical acceleration} \\
\beta & \quad \text{Body slip angle} \\
\beta_{\text{UKF}} & \quad \text{Distribution of the data of states in UKF} \\
b & \quad \text{Distance from CG to rear axle} \\
B & \quad \text{Stiffness factor, Magic Formula tyre model} \\
C & \quad \text{Shape factor, Magic Formula tyre model} \\
C_\alpha & \quad \text{Cornering stiffness} \\
C_{12} & \quad \text{Cornering stiffness, front axle} \\
C_{34} & \quad \text{Cornering stiffness, rear axle} \\
D & \quad \text{Peak value, Magic Formula tyre model} \\
\delta & \quad \text{Average steering angle on the front axle} \\
E & \quad \text{Curvature factor, Magic Formula tyre model} \\
f & \quad \text{Distance from CG to front axle} \\
F & \quad \text{Function value in ANOVA} \\
F_y & \quad \text{Lateral tyre force} \\
F_{y,\text{dyn}} & \quad \text{The dynamic lateral tyre force described with relaxation length} \\
F_{y,12} & \quad \text{Lateral tyre force, front axle} \\
F_{y,34} & \quad \text{Lateral tyre force, rear axle} \\
\dot{F}_{y,12}(t) & \quad \text{Lateral tyre force time derivative as function of time, front axle} \\
\dot{F}_{y,34}(t) & \quad \text{Lateral tyre force time derivative as function of time, rear axle} \\
F_z & \quad \text{Normal load} \\
F_{z,12} & \quad \text{Normal load, front axle} \\
F_{z,34} & \quad \text{Normal load, rear axle} \\
F_{z,\text{nom}} & \quad \text{Nominal normal load} \\
F_{z,\text{nom},12} & \quad \text{Nominal normal load, front axle} \\
F_{z,\text{nom},34} & \quad \text{Nominal normal load, rear axle} \\
\gamma & \quad \text{Camber angle} \\
g & \quad \text{Gravity constant, 9.81} \\
G_f & \quad \text{Gradient of function } f \\
h & \quad \text{Height of centre of gravity} \\
H & \quad \text{Mathematical Hessian} \\
I & \quad \text{Moment of inertia} \\
I_{zz} & \quad \text{Moment of inertia around vertical axis} \\
\kappa_{\text{UKF}} & \quad \text{Scaling factor in UKF} \\
K & \quad \text{Slip stiffness } (B \cdot C \cdot D), \text{ Magic Formula tyre model} \\
\lambda & \quad \text{Weight distribution on the front axle} \\
\lambda_{Cy} & \quad \text{MF: Scale factor of } F_y \text{ shape factor} \\
\lambda_{Ey} & \quad \text{MF: Scale factor of } F_y \text{ curvature factor}
\end{align*} \]
\( \lambda_{F_{x0}} \) MF: Scale factor of nominal (rated) load
\( \lambda_{Fy} \) MF: Scale factor of inclination for \( F_y \)
\( \lambda_{Hy} \) MF: Scale factor of \( F_y \) horizontal shift
\( \lambda_{Ky} \) MF: Scale factor of \( F_y \) cornering stiffness
\( \lambda_{\mu y} \) MF: Scale factor of \( F_y \) peak friction coefficient
\( \lambda_{Vy} \) MF: Scale factor of \( F_y \) vertical shift
\( L \) Wheel base
\( \mu \) Friction coefficient
\( m \) Vehicle mass
\( M \) Mean value
\( MS \) Mean square sum
\( \nu \) Number of degrees of freedom in ANOVA
\( \psi \) Yaw angle
\( \dot{\psi} \) Yaw rate
\( \ddot{\psi} \) Yaw acceleration

\( p \)-value Significance level
\( p_{Cy1} \) MF: Shape factor \( C_{fy} \) for lateral forces
\( p_{Dy1} \) MF: Lateral friction \( \mu_y \)
\( p_{Dy2} \) MF: Variation of friction \( \mu_y \) with load
\( p_{Dy3} \) MF: Variation of friction \( \mu_y \) with squared inclination
\( p_{Ey1} \) MF: Lateral curvature \( E_{fy} \) at \( F_{z,\text{nom}} \)
\( p_{Ey2} \) MF: Variation of curvature \( E_{fy} \) with load
\( p_{Ey3} \) MF: Inclination dependency of curvature \( E_{fy} \)
\( p_{Ey4} \) MF: Variation of curvature \( E_{fy} \) with inclination
\( p_{Ky1} \) MF: Maximum value of stiffness \( K_{fy}/F_{z,\text{nom}} \)
\( p_{Ky2} \) MF: Load at which \( K_{fy} \) reaches maximum value
\( p_{Ky3} \) MF: Variation of \( K_{fy}/F_{z,\text{nom}} \) with inclination
\( p_{Hy1} \) MF: Horizontal shift \( S_{Hy} \) at \( F_{z,\text{nom}} \)
\( p_{Hy2} \) MF: Variation of shift \( S_{Hy} \) with load
\( p_{Hy3} \) MF: Variation of shift \( S_{Hy} \) with inclination
\( p_{Ty1} \) MF: Peak value of relaxation length for lateral direction
\( p_{Ty2} \) MF: Shape factor for lateral relaxation length
\( p_{Vy1} \) MF: Vertical shift in \( S_{Vy}/F_z \) at \( F_{z,\text{nom}} \)
\( p_{Vy2} \) MF: Variation of shift \( S_{Vy}/F_{z} \) with load
\( p_{Vy3} \) MF: Variation of shift \( S_{Vy}/F_{z} \) with inclination
\( p_{Vy4} \) MF: Variation of shift \( S_{Vy}/F_{z} \) with inclination and load
\( Q_{UKF} \) Noise covariance matrix for process data in UKF
\( R_{UKF} \) Noise covariance matrix for measurement data in UKF
\( R \) Curve radius
\( \sigma_\alpha \) Tyre relaxation length
\( S \) Set of constraints for optimization
\( S_{H} \) Horizontal shift, Magic Formula tyre model
\( S_{V} \) Vertical shift, Magic Formula tyre model
\( SS \) Square sum
\( \tau \) Time constant first order low-pass filter
\( t \) Time
$t_0$  Time when the throttle is released in TRIT manoeuvre
$t_n$  Time 1 s after the throttle was released in TRIT manoeuvre
$t_w$  Track width
$v_1$  Lateral tyre deflection
$v_x$  Longitudinal velocity
$v_y$  Lateral velocity
$v_{sy}$  Lateral velocity, contact patch
$v_\xi$  Velocity vector with added noise
$x$  Longitudinal position, driving direction
$X$  MF: Input variable
$y$  Lateral position
$Y$  MF: Output variable
$\zeta_0$  MF: Reduction factor vertical shift of the lateral force = 0
$\zeta_2$  MF: Peak side force reduction factor
$\zeta_3$  MF: Cornering stiffness reduction factor
$\zeta_4$  MF: Reduction factor vertical shift of the lateral force
$z$  Vertical position
List of abbreviations

ANOVA  Analysis of Variance
ARB    Anti-roll bar
BOS    Beginning of Steer
CAE    Computer-Aided Engineering
CG     Centre of Gravity
COS    Completion of Steer
CR     Constant Radius
CRT    VI-CarRealTime
EKF    Extended Kalman Filter
ESC    Electronic Stability Control System
FR     Frequency Response
GM     Grand Mean
HSS    High g Swept Steer
ISO    International Organization for Standards
K&C    Kinematics and Compliance
LF     Left Front
LR     Left Rear
MF     Magic Formula tyre model
MLA    Maximum Lateral Acceleration
MOI    Moment of Inertia
NHTSA  National Highway Traffic Safety Administration
Ref    Reference vehicle (Standard configuration)
RF     Right Front
RR     Right Rear
SLT    Straight Line steady-state Test
Sroll  Suspension roll angle
SSCG   Body slip angle at the centre of gravity
SSF    Body slip angle on the front axle
SSR    Body slip angle on the rear axle
SWA    Steering Wheel Angle
SWD    Sine with Dwell
TNO    Netherlands Organisation for Applied Scientific Research
TP     Throttle Position
TRIT   Throttle Release in Turn
Troll  Total roll angle
Trollr Total roll rate
UKF    Unscented Kalman Filter
Veh 1  Vehicle configuration 1, no anti-roll bar in the rear
Veh 2  Vehicle configuration 2, no anti-roll bar in the front
Veh 3  Vehicle configuration 3, standard vehicle (not the exact same vehicle as Ref)
Abstract

Vehicle handling evaluation is a crucial part of the vehicle development process. The evaluation can be done in two ways, subjectively; by expert test drivers or objectively; by performing repeatable standard manoeuvres usually by steering robots. Subjective testing is resource intensive as prototypes need to be built. Objective testing is less so, as it can be performed in a virtual environment in conjunction with physical testing. In an effort to reduce resources and time used in vehicle development, manufacturers are looking to objective testing to assess vehicle behaviour.

Vehicle handling testing in winter strongly relies on subjective testing. This thesis aims to investigate into the usage of objective test strategy to assess vehicle handling behaviour in winter conditions. Manoeuvres and metrics are defined for summer conditions, but not for winter. Hence the goal was to define new or modified metrics and manoeuvres custom to winter testing.

Data from an objective winter test was obtained and analysed. The manoeuvres used were constant radius (CR), frequency response (FR), sine with dwell (SWD) and throttle release in turn (TRIT). The manoeuvres were compared to public standards from the International Organization for Standards (ISO) and National Highway Traffic Safety Administration (NHTSA) as well as the vehicle manufacturer standards.

The data from a reference vehicle is compared to that from three configuration vehicles, one without anti-roll bar in the front, one without rear anti-roll bar and a standard. The difference in the signals between reference and configuration vehicles is compared to the spread in data of the reference vehicle to determine the signal-to-noise ratio in the manoeuvres. The spread of reference data is analysed to determine the distribution and to differentiate between the two test days. To replicate vehicle behaviour in simulation, winter tyre models using brush and Magic Formula model equations were investigated. These were used in a bicycle and a VI-CarRealTime model. The performance of these are checked and compared. The bicycle model is used in an unscented Kalman filter, to investigate potential improvements in signal processing. The metrics obtained from the study of standards are checked for robustness in winter conditions by analysis of variance (ANOVA) methods. The procedure of selection of metrics from the ANOVA results is explained. Further, the manoeuvres are modified virtually in VI-CarRealTime, from the results of a sensitivity analysis. The difference in metrics between reference and configuration vehicles is maximized.

The final results of the thesis were; a test plan with modified manoeuvres and a set of robust metrics. Also containing important information to aid in the execution of the tests. The conclusions drawn were that the noise in winter testing is high, but the difference between vehicles is statistically significant for some robust metrics. The metrics related to yaw rate were in general more robust. Open-loop throttle and steering control in manoeuvres should be avoided as far as possible. A bicycle model is not complex enough to represent vehicle behaviour at high slip angles. Performance increase of a UKF is not justified as to the effort involved.
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1 Introduction

In this chapter the background, the problem formulation and the expected outcomes of the thesis are presented, as well as the limitations and the scope.

1.1 Background

Vehicle winter testing for manufacturers all over the world has in the last years been concentrated to the north of Sweden. The area consists of large lakes and open spaces that freeze and get covered in snow during the winter. This makes it possible to have large proving grounds which is required in order to test vehicles at the limit. Also the winter season is long, making it possible to test during a longer period compared to other parts in the world. During short time the temperature change can be large and it snows regularly making it easier for developers to test vehicles for different kinds of winter surfaces which is important to see the change in vehicle behaviour. Extremely low temperatures are reached, giving manufacturers a chance to see how vehicle components work in extreme conditions. But still the winter test season is short comparing to summer making it harder for developers to test when wanted and detailed planning is required. Then a detailed test plan is needed together with manoeuvres that do not need to be pre-tested but are robust and can be used when needed [1].

During the phases of the vehicle development, physical testing is performed to assess vehicle behaviour and driving feel. During the early phases these tests require a number of prototypes, a time consuming and expensive procedure. In an effort to reduce the time and cost of development, computer-aided engineering (CAE) tools can be used in an early stage requiring less physical testing needed. This gives the advantage of being able to test multiple configurations by simulating them. It is envisioned that, the CAE tools can be used to anticipate vehicle behaviour and wanted driving feeling to ensure that the vehicle is designed in a preferred way [2, 3].

The CAE tools and physical testing with steering robot give objective values of the vehicle behaviour, whereas test drivers subjectively assess the vehicle [2, 3]. Currently, the objective values are clearly defined for summer testing, though the correlation between the objective metrics and subjective assessment is being researched. Whereas in the case of testing during winter conditions, subjective assessments are strongly relied on [4]. To be able to use the CAE tools for assessing driving feel in winter conditions, correlations between the subjective assessment and objective metrics have to be clearly defined. To this end, objective metrics and manoeuvres that show the difference in vehicle behaviour are needed. The objective metrics need to be robust to be obtainable through simulation with CAE tools. To achieve this, an improved winter tyre model needs to be implemented. This thesis focuses on understanding the vehicle behaviour and definition of manoeuvres and metrics for vehicle handling during winter conditions.


1.2 Problem formulation

Evaluation of vehicle behaviour strongly relies on subjective assessments of expert drivers. In an effort to reduce the resources used during testing, CAE is being used to a larger extent. In case of summer testing, the vehicle manoeuvres and objective metrics are clearly defined. This is not the case for winter testing where objective testing is usually not performed at all. Therefore no manoeuvres or objective metrics are specifically defined for this kind of surface. Different characteristics of the vehicle behaviour could be of interest in winter, hence new or modified manoeuvres and metrics need to be developed. The objectives are not directly applicable for both conditions and the added difficulty of changing surface conditions during winter. This results in a low signal-to-noise ratio. The lead time and the short winter test season exacerbate this problem. Solving the problem would be a stepping stone to correlating subjective assessment to the objective metrics.

1.3 Scope

The thesis is based on data from winter tests performed in the north of Sweden in 2014. The tests consisted of objective testing using a steering robot that controlled steering wheel angle and throttle position input. The vehicle used during the expedition is in the C size segment. The manoeuvres performed were constant radius (CR), frequency response (FR), sine with dwell (SWD) and throttle release in turn (TRIT). The tests were performed with one reference vehicle (Ref) with standard settings which was driven simultaneously to every test run. Another vehicle but the same model, set up to be the same as the reference vehicle, was configured in three ways,

1. No anti-roll bar in the rear (Veh 1)
2. No anti-roll bar in the front (Veh 2),
3. Standard configuration (Veh 3).

The tests were performed during two days without any electronic assistance systems e.g. ESC. For every test run the reference vehicle was run simultaneously to each test with configured vehicle to have a reference that could represent the surface.

The thesis is limited to the following,

- The manoeuvres researched are limited to the standard manoeuvres that are specified by the vehicle manufacturer as well as those approved by the International Organization for Standards and National Highway Traffic Safety Administration.
- The tyre model would be restricted to the tested tyres. The behaviour of these might not be representative of all tyres in low friction conditions.
- The study is limited to a particular class of vehicle. Hence the tools developed might not be valid for other classes.
- The initial analysis is limited to vehicle configurations with changes to the front and rear anti-roll bar.
- Data of only four manoeuvres (CR, FR, SWD and TRIT).
- The vehicle dynamics are mainly considered in lateral direction.
- Designing manoeuvres will only include virtual testing, no physical testing.
1.4 Expected outcomes

This study aims to develop objective metrics that are robust, repeatable and representative of the vehicle behaviour in winter season. To obtain such metrics research into manoeuvres will be conducted, to modify these for winter conditions. In order to utilize CAE tools, a simple winter tyre model would be developed that can replicate winter conditions. Further the simulation is used to validate if the metrics can be obtained. Using the acquired knowledge, an improved test plan would be proposed. The metrics would be analysed to check the signal-to-noise ratio. This would also give an understanding on the requirement of a reference vehicle during winter testing. The expected outcomes of the thesis are:

- Modified/new metrics for winter testing.
- Modified manoeuvres for winter testing, primarily sine with dwell.
- Investigate a Kalman filter to filter measurement data.
- Tool for on-site evaluation of metrics.
- Simple winter tyre model for Kalman filter, simulation program and to implement in driving simulator.
- Propose improved winter test plan with virtual pre-testing.
2 Literature study

In this chapter the theory around procedures in vehicle physical testing, statistical tools and softwares used in the thesis is presented. Giving the reader basic knowledge for further understanding of the thesis.

2.1 Physical testing

2.1.1 Subjective testing

Vehicle manufacturers in an effort to attract customers as well as establish a brand identity, are focused on the subjective aspects of the behaviour of the vehicle. It is considered a vital process in the development procedure to tune the vehicle to obtain specific desired qualities [2].

Subjective testing is that, which is performed by trained test drivers to evaluate the subjective feel of the vehicle. This type of testing hence, requires physical prototypes to be built and tested. Due to these reasons, the cost and time required for the development is increased drastically [2].

2.1.2 Objective testing

Objective testing is performed to objectively quantify the performance of the vehicle. The results of this are measured performance criteria specified prior to testing. The human influence is kept as low as possible and the tests are performed with the use of steering robots. These steering robots control the input to the vehicle within a fine tolerance. Hence these tests are repeatable, giving the opportunity to directly compare one vehicle to another. The main advantage with these type of tests is the possibility of conducting virtual tests by means of simulation. Hence reducing the dependence on physical prototypes to a large extent [3].

Vehicle manufacturers have specific tests to assess particular behaviour of vehicles. These are individual to the particular manufacturer. However, there are standardized tests specified by international and national organizations as well. These are especially used for the purposes of legislation and government policy, e.g. safety regulations [3].

2.1.3 K&C testing

A K&C or Kinematics and Compliance test rig measures the quasi-static kinematic characteristics due to suspension and steering system geometries and compliances due to suspension springs, anti-roll bars, elastomeric bushings and component deformations by applying forces and moments. As the purpose is to measure quasi-static parameters, the rate of displacement in the test is kept low. This reduces the influence of dampers
and inertia of the components [5, 6].

There are different types of K&C rigs available. A simple model is the single axle rig where one axle is measured at a time. A more advanced K&C rig includes all four wheels, i.e. full vehicle testing, which can measure each axle individually. For full vehicle test rigs, see Figure 2.1, the pads that the wheels stand on can either be fixed in the ground plane (x, y) and vertical bounce, roll, pitch motion are executed with a moving centre table or that the pads also move in vertical z-direction without a centre table. When having a centre table the vehicle is fixed to the table via clamps. This makes it possible to have 3 degrees of freedom for each wheel pad (x, y, steering angle) and 6 degrees of freedom for the centre table (x, y, z, roll, pitch, yaw) [6–8].

Some test rigs are also equipped with centre of gravity (CG) and moment of inertia (MOI) measurement possibilities. Here the centre table clamps hold the vehicle and the table is moved. The vehicle is lifted up so the wheels do not touch the wheel pads. Oscillatory motion is applied to the vehicle in pitch, roll and yaw to get MOI around all coordinate axes [5, 6].

The principal parameters that can be measured are suspension rates and hysteresis, bump-steer, roll-steer, roll stiffness distribution, longitudinal and lateral compliance steer, and steering system characteristics. Knowledge of these parameter values and characteristics is essential for thorough understanding and making it possible to model a vehicle in terms of ride, steering and handling. The data extracted is a great base to model a full vehicle [5, 6].


2.2 Standards

When performing physical vehicle testing, manoeuvres need to be defined in a certain way to be able to compare different vehicles and vehicle configurations. The manoeuvres are performed to either subjectively or objectively analyse vehicle behaviour. Objective metrics are calculated values that need to be done in the same way to be valid [9,10].

Objective metrics are related to the particular behaviour of the vehicle that is required to be measured. Hence, to bring out the required behaviour, the vehicle is subjected to a range of manoeuvres. These are required to be standardised to have a repeatable input to the vehicle. Standardisation also allows the possibility of performing the same manoeuvres for different vehicles and vehicle configurations [9,10].

Vehicle manufacturers have their own standard manoeuvres, which are not available to the public. However, there are some standards defined by international and national organizations, such as the International Organization for Standards (ISO) and the American organization, National Highway Traffic Safety Administration (NHTSA). They provide a common platform with which vehicles from different manufacturers can be tested.

The manoeuvres; constant radius (CR) [11], frequency response (FR) [12], and throttle release in turn (TRIT) [13] are defined by the ISO as standards while sine with dwell (SWD) [14] is a NHTSA standard. To be able to find the linear lateral acceleration range and the lateral acceleration limit for a vehicle high g swept steer is used (HSS) [11], which also is presented below. The manoeuvres are related to specific metrics that can be measured from them and the standards define the manoeuvre as well as the metrics that can be measured. Proposed metrics are also mentioned for the ISO and NHTSA standards.

2.2.1 Constant radius manoeuvre

For CR standard the following definitions are included; turning direction, i.e. left or right, longitudinal speed increase and the radius of the circle \( R \).

For CR the standard metrics are; steering wheel angle (SWA) gradient, path curvature gradient, body slip angle (SSCG) gradient, roll angle (Troll) gradient, steering wheel torque (SWT) gradient and steering wheel/side slip angle gradient [11]. The manoeuvre procedure and metrics for the CR manoeuvre are presented in more detail in Table 2.1 and 2.2 respectively. For the vehicle manufacturer manoeuvre and metric standards, see Table D.3 and D.4 in Appendix D.

2.2.2 Frequency response manoeuvre

For FR standard the following definitions are included; the initial longitudinal speed, the initial yaw rate, range of steering frequency and steering wheel amplitude.
Table 2.1: Manoeuvre procedure, International Organization for Standardization: constant radius [11]. The lateral acceleration is noted as $a_y$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turning direction</td>
<td>Left and right</td>
</tr>
<tr>
<td>Speed increase</td>
<td>maximum $a_y$: 0.1 m/s$^3$ to 0.2 m/s$^3$</td>
</tr>
<tr>
<td>Radius</td>
<td>100 m, lower 40 m, minimum 30 m ± 0.5 m</td>
</tr>
</tbody>
</table>

Table 2.2: Metrics, International Organization for Standardization: constant radius [11].

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Plot</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steering wheel angle gradient</td>
<td>SWA/$a_y$</td>
<td>–</td>
</tr>
<tr>
<td>Path curvature gradient</td>
<td>$(1/R)/a_y$</td>
<td>–</td>
</tr>
<tr>
<td>Side slip angle gradient</td>
<td>SSCG/$a_y$</td>
<td>–</td>
</tr>
<tr>
<td>Roll angle gradient</td>
<td>Troll/$a_y$</td>
<td>–</td>
</tr>
<tr>
<td>Steering wheel torque gradient</td>
<td>SWT/$a_y$</td>
<td>–</td>
</tr>
<tr>
<td>Steering wheel angle/Side slip angle gradient</td>
<td>SWA/SSCG</td>
<td>–</td>
</tr>
</tbody>
</table>

For FR the standard metrics are; lateral acceleration ($a_y$) gain, yaw velocity ($\dot{\psi}$) gain, phase angle and phase angle time [12]. The manoeuvre procedure and the metrics for the FR manoeuvre are presented in Table 2.3 and 2.4 respectively. For the vehicle manufacturer manoeuvre and metric standards, see Table D.5 and D.6 in Appendix D.

Table 2.3: Manoeuvre procedure, International Organization for Standardization: frequency response [12].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>100 km/h ± 2 km/h (20 km/h increments)</td>
</tr>
<tr>
<td>Yaw rate at start</td>
<td>0 °/s ± 0.5 °/s</td>
</tr>
<tr>
<td>Frequency range</td>
<td>0.2 – 2 Hz</td>
</tr>
<tr>
<td>SWA</td>
<td>Steady-state cornering SWA at 4 m/s$^2$</td>
</tr>
</tbody>
</table>

2.2.3 Sine with dwell manoeuvre

For SWD standard the following definitions are included; sine frequency, initial turning direction, i.e. left or right, pause time period (dwell), nominal SWA, if equal steering wheel angle is used for both turning directions, steering wheel angle increments (how the steering wheel angle should increase for multiple tests), maximum steering wheel angle and initial speed.

For SWD the standard metrics are; maximum yaw rate ratio I (1 s after completion of steer (COS)), maximum yaw rate ratio II (1.75 s after COS) and minimum lateral displacement (1.07 s after beginning of steer (BOS)) [14]. The manoeuvre procedure
Table 2.4: Metrics, International Organization for Standardization: frequency response [12].

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Plot</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral acceleration gain</td>
<td>$a_y$/SWA</td>
<td>–</td>
</tr>
<tr>
<td>Phase angle time</td>
<td>$a_y$/SWA</td>
<td>SWA as input, $a_y$ as output</td>
</tr>
<tr>
<td>Yaw velocity gain</td>
<td>$\dot{\psi}$/SWA</td>
<td>–</td>
</tr>
<tr>
<td>Phase angle time</td>
<td>$\dot{\psi}$/SWA</td>
<td>SWA as input, $\dot{\psi}$ as output</td>
</tr>
</tbody>
</table>

and the metrics for the SWD manoeuvre are presented in Table 2.5 and 2.6 respectively. For the vehicle manufacturer manoeuvre and metric standards, see Table D.7 and D.8 in Appendix D.

Table 2.5: Manoeuvre procedure, National Highway Traffic Safety Administration: sine with dwell [14].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>7 Hz</td>
</tr>
<tr>
<td>Initial turning direction</td>
<td>Left and right</td>
</tr>
<tr>
<td>Pause time period (dwell)</td>
<td>0.5 s</td>
</tr>
<tr>
<td>Nominal SWA</td>
<td>$1.5 \cdot$ SWA at 0.3 g</td>
</tr>
<tr>
<td>SWA increment</td>
<td>$0.5 \cdot$ SWA at 0.3 g</td>
</tr>
<tr>
<td>Maximum SWA</td>
<td>$6.5 \cdot$ SWA at 0.3 g or 270°</td>
</tr>
<tr>
<td>Speed</td>
<td>$50 \text{ mph} = 80.47 \text{ km/h}$</td>
</tr>
</tbody>
</table>

Table 2.6: Metrics, National Highway Traffic Safety Administration: sine with dwell [14].

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Plot</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum yaw rate ratio I</td>
<td>$t/\dot{\psi}$</td>
<td>1 s after COS</td>
</tr>
<tr>
<td>Maximum yaw rate ratio II</td>
<td>$t/\dot{\psi}$</td>
<td>1.75 s after COS</td>
</tr>
<tr>
<td>Minimum lateral displacement</td>
<td>$t/y$</td>
<td>1.07 s after BOS</td>
</tr>
</tbody>
</table>

2.2.4 Throttle release in turn manoeuvre

For TRIT standard the following definitions are included; two alternative methods are mentioned in the standard, constant radius or constant speed, turning direction, steady-state SWA, SWA deviation, recording time before throttle off, recording time after throttle off and the steady-state time interval.
Table 2.7: Manoeuvre procedure, International Organization for Standardization: throttle release in turn [13].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turning direction</td>
<td>Left and right</td>
</tr>
<tr>
<td>Constant radius method</td>
<td>100 $m$, lower 40 $m$, minimum 30 $m \pm 2$ $m$</td>
</tr>
<tr>
<td>Steady-state SWA</td>
<td>SWA at 4 $m/s^2$</td>
</tr>
<tr>
<td>Constant speed method</td>
<td>100 $km/h \pm 1$ $km/h$ ($\pm 20$ $km/h$ increments)</td>
</tr>
<tr>
<td>SWA deviation</td>
<td>$\pm 3%$ from steady-state value</td>
</tr>
<tr>
<td>Recording time before throttle off</td>
<td>at least 1.3 s (extended 0.2 – 1 s depend. filter)</td>
</tr>
<tr>
<td>Recording time after throttle off</td>
<td>at least 2.0 s (extended 0.2 – 1 s depend. filter)</td>
</tr>
<tr>
<td>Steady-state time interval</td>
<td>0.3 to 1.3 s before throttle off</td>
</tr>
</tbody>
</table>

Table 2.8: Metrics, International Organization for Standardization: throttle release in turn [13]. The throttle position is noted as TP.

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Plot</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average $a_x$</td>
<td>$t/a_x$</td>
<td>during $t_0$ to $t_n$</td>
</tr>
<tr>
<td>Ratio yaw rate</td>
<td>$t/\dot{\psi}$</td>
<td>to $\dot{\psi}_{ref}$ at $t_n$</td>
</tr>
<tr>
<td>Ratio maximum yaw rate</td>
<td>$t/\dot{\psi}$</td>
<td>to $\dot{\psi}<em>{ref}$ at $t</em>{\dot{\psi}_{max}}$</td>
</tr>
<tr>
<td>Difference yaw rate and reference yaw rate</td>
<td>$t/\dot{\psi}$</td>
<td>at $t_n$</td>
</tr>
<tr>
<td>Maximum difference yaw rate and reference yaw rate</td>
<td>TP/$\dot{\psi}$</td>
<td>at throttle off</td>
</tr>
<tr>
<td>Yaw acceleration</td>
<td>$t/\dot{\psi}$</td>
<td>at $t_n$</td>
</tr>
<tr>
<td>Ratio $a_y$ to reference $a_y$</td>
<td>$t/a_y$</td>
<td>at $t_n$</td>
</tr>
<tr>
<td>Maximum SSCG</td>
<td>$t/$SSCG</td>
<td>–</td>
</tr>
<tr>
<td>Difference between SSCG and steady-state SSCG</td>
<td>$t/$SSCG</td>
<td>at $t_n$</td>
</tr>
<tr>
<td>Difference between yaw rate and calculated yaw rate</td>
<td>$t/\dot{\psi}$</td>
<td>at $t_n$</td>
</tr>
<tr>
<td>Path deviation</td>
<td>$t/y$</td>
<td>at $t_n$</td>
</tr>
</tbody>
</table>

Definition of time constants in TRIT manoeuvre; $t_0$ is the time when the throttle release is executed, $t_n$ is the time when one second is gone after throttle release, $t_n = t_0 + 1$ s. Finally, $t_{\dot{\psi}_{max}}$ is the time of the manoeuvre when the yaw rate is at its maximum ($\dot{\psi}_{max}$).

For TRIT the standard metrics are; average longitudinal acceleration during $t_0$ to $t_n$, ratio yaw rate ($\dot{\psi}$) at $t_n$ to reference yaw rate ($\dot{\psi}_{ref}$) at $t_n$, ratio $\dot{\psi}_{max}$ to $\dot{\psi}_{ref}$ at $t_{\dot{\psi}_{max}}$, difference between $\dot{\psi}$ at $t_n$ and $\dot{\psi}_{ref}$ at $t_n$, maximum difference $\dot{\psi}$ and $\dot{\psi}_{max}$ at throttle off, yaw acceleration at $t_n$, ratio lateral acceleration at $t_n$ to reference lateral acceleration at $t_n$, maximum side slip during observation period, difference between side slip at $t_n$ and steady-state side slip, difference between $\dot{\psi}$ at $t_n$ and calculated $\dot{\psi}$ at $t_n$ and path deviation at $t_n$ [13]. The manoeuvre procedure and the metrics for the TRIT manoeuvre are presented in Table 2.7 and 2.8 respectively. For the vehicle manufacturer manoeuvre and metric standards, see Table D.9 and D.10 in Appendix D.
2.2.5 High g swept steer manoeuvre

The HSS manoeuvre is similar to the steady-state cornering method; constant speed/increasing steering wheel angle [11]. The HSS has a higher SWA amplitude to get both linear, non-linear range and the limit of the lateral acceleration. The manoeuvre procedure and the metrics for the HSS manoeuvre are presented in Table 2.9 and 2.10 respectively.

Table 2.9: Manoeuvre procedure, International Organization for Standardization: high g swept steer [11].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turning direction</td>
<td>Left and right</td>
</tr>
<tr>
<td>Speed</td>
<td>100 km/h (20 km/h increments)</td>
</tr>
<tr>
<td>SWA rate</td>
<td>1 °/3s</td>
</tr>
</tbody>
</table>

Table 2.10: Metrics, International Organization for Standardization: high g swept steer [11].

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Plot</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum $a_y$</td>
<td>$t/a_y$</td>
<td>--</td>
</tr>
<tr>
<td>Linear $a_y$ range</td>
<td>SWA/$a_y$</td>
<td>--</td>
</tr>
</tbody>
</table>

2.3 Bicycle model

The bicycle model (one track model) is one of the most simple models for simulating vehicle behaviour in lateral direction [15]. It provides a very simplified method to estimate the behaviour and calculate the vital information of a car during cornering. To explain the model, the assumptions made are described.

- The front and rear axles are represented by one tyre, hence giving it the name bicycle model.
- The CG height of the vehicle is assumed to be zero.
- Hence, the lateral and longitudinal load transfers are assumed to be zero.
- Roll and pitch motions along with motions in the z-direction are neglected.
- Small angles are assumed for the simplification of calculations.

The model is now free to move in the $xy$-plane, see Figure 2.2, and the basic equations from equilibrium around the CG are described in equations (2.1), (2.2) and (2.3).
\[ \uparrow: m(\dot{v}_x - \dot{\psi}v_y) = -F_{y,12}\sin(\delta) \]  
\[ \rightarrow: m(\dot{v}_y + \dot{\psi}v_x) = F_{y,34} + F_{y,12}\cos(\delta) \]  
\[ I_{zz}\ddot{\psi} = fF_{y,12}\cos(\delta) - bF_{y,34} \]

Where,

- \( m \) is the total vehicle mass
- \( \dot{\psi} \) is the yaw rate
- \( v_y \) is the lateral velocity
- \( v_x \) is the longitudinal velocity
- \( f \) is the distance from CG to the front axle
- \( b \) is the distance from CG to the rear axle
- \( F_{y,12} \) is the lateral force on the front axle
- \( F_{y,34} \) is the lateral force on the rear axle
- \( I_{zz} \) is the moment of inertia around the \( z \)-axis
- \( \delta \) is the average steering angle on the front axle

The lateral forces \( F_{y,12} \) and \( F_{y,34} \) are described as functions of the lateral slip angle, equations (2.4) and (2.5). The particular function used is dependent on the tyre model implemented.

\[ F_{y,12} = f(\alpha_{12}) \]  
\[ F_{y,34} = f(\alpha_{34}) \]

Where,

- \( \alpha_{12} \) is the lateral slip angle on the front axle
- \( \alpha_{34} \) is the lateral slip angle on the rear axle
The slip angles are described as, the angles between the vectors of the resultant and longitudinal velocities of the axles, equations (2.6) and (2.7).

\[
\alpha_{12} = \arctan \left( \frac{v_y + \dot{\psi}_f v_x}{v_x} \right) - \delta \quad (2.6)
\]

\[
\alpha_{34} = \arctan \left( \frac{v_y - \dot{\psi}_b v_x}{v_x} \right) 
\]

To solve for state estimation \(v_x, v_y\) and \(\dot{\psi}\), the equations need to be written in state-space form describing the derivative of the wanted state, equations (2.8), (2.9) and (2.10).

\[
\dot{v}_x = -\frac{F_{y,12} \sin(\delta)}{m} + \dot{\psi}v_y 
\]

\[
\dot{v}_y = \frac{F_{y,34} + F_{y,12} \cos(\delta)}{m} - \dot{\psi}v_x
\]

\[
\ddot{\psi} = \frac{fF_{y,12} \cos(\delta) - bF_{y,34}}{I_{zz}}
\]

### 2.4 Tyre models

#### 2.4.1 General

All the forces that propel the vehicle are transmitted through the four tyres. Hence understanding the conditions that affect the behaviour and performance of the tyre is of prime importance in vehicle dynamic studies. This is done with the use of mathematical equations describing the behaviour of the tyre. These set of equations are called a tyre model. These models are based on the functions of the tyre that are important for the particular study that it is used for.

In the book, "Tire and Vehicle Dynamics" [16], H. Pacejka distinguishes the primary tasks of the tyre and the equally important secondary effects. The primary tasks are listed as,

- Load carrying
- Cushioning
- Braking or driving
- Cornering

These primary tasks have the requirement to transmit forces in the \(x, y\) and \(z\)-directions. The secondary effects are crucial to the development of these primary forces. He further makes distinction between quasi steady-state and vibratory behaviour and, between symmetric and anti-symmetric factors.

Several types of mathematical models have been developed, to represent these primary tasks of the tyre. These models vary in scope and complexity depending on the intended
use. On one extreme, there are mathematical tyre models that describe measured tyre characteristics through interpolation and curve fitting schemes, empirical models. On the other extreme, models describe the physical aspects of a tyre with regard to its construction, usually made using finite element methods.

The behaviour of the tyre during dry, summer conditions is different than during winter conditions. It is known that build-up of snow can affect the high slip behaviour of tyres, giving a curve of different shape as that in summer. Also the difference between a summer tyre and winter tyre is of high significance. A winter tyre has a different softer compound. In addition, there are studded tyres and unstudded tyres. Modelling of winter tyres is not a mature area, the concentration of research has been into summer tyres. There are papers describing research in winter tyres like [17] into tread patterns and how forces are changing due to this and different snow depth, but no published study into vehicle handling with winter tyres was found.

2.4.2 Brush model

In this model, the physical conditions at the contact patch are described by simple mathematical equations. The tyre is described as a row of elastic bristles, giving it its name, see Figure 2.3. These bristles come in contact with the road and move from leading to trailing edge of the contact patch. If the tyre has side or longitudinal slip, the bristles develop deflection, generating lateral or longitudinal forces [18].

![Bristles of a Brush tyre model](image)

*Figure 2.3: Bristles of a Brush tyre model [18].*

The brush model is similar to a linear tyre model but with the added saturation in the high slip region, see Figure 2.4a and 2.4b.
Literature study

(a) Deflection of tyre for increasing slip angle A to D [16].

(b) Force and aligning moment generated as function of increasing slip angle [16].

Figure 2.4: Brush model lateral force and aligning moment characteristics.

The lateral force \( F_y \) is expressed in equation (2.11).

\[
F_y = -C_\alpha \cdot \tan(\alpha) \cdot f(\lambda)
\]  

(2.11)

With,

\[
f(\lambda) = \begin{cases} 
\lambda(2 - \lambda) & \lambda \leq 1 \\
1 & \lambda > 1 
\end{cases}
\]  

(2.12)

\[
\lambda = \frac{F_z \mu}{2C_\alpha |\tan(\alpha)|}
\]  

(2.13)

Where,

\( C_\alpha \) is the cornering stiffness
\( \alpha \) is the slip angle
\( F_z \) is the normal load and
\( \mu \) is the friction coefficient

2.4.3 Magic Formula tyre model

The Magic Formula tyre model (MF) is a semi-empirical model used to calculate the steady-state tyre forces and moment characteristics. The model in its basic form is a curve fit to describe these forces and moments that are measured with physical testing. This model is widely used in vehicle dynamic studies as it can represent, quite accurately, most important aspects of tyre behaviour related to this field. The model is also computationally efficient, when compared to finite element method models. The general form of the model for given values of vertical load and camber angles is given in equation (2.14). The curve fitting factors are shown in Figure 2.5.

\[
y = D \sin[C \arctan Bx - E(Bx - \arctan Bx)]
\]  

(2.14)

\[
Y(X) = y(x) + S_Y
\]  

(2.15)

\[
x = X + S_H
\]  

(2.16)
Where,

- $Y$ is the output variable; $F_x$ or $F_y$
- $X$ is the input variable; $\alpha$ or $\kappa$
- $B$ is the stiffness factor
- $C$ is the shape factor
- $D$ is the peak value
- $E$ is the curvature factor
- $S_H$ is the horizontal shift
- $S_V$ is the vertical shift

As the tyre does not operate at one particular normal load and camber case, the constants of the equation are made to change based on these values. Hence, the Magic Formula is a set of curves describing the behaviour of the tyre in a range of normal loads and camber angles.

The formulae for the constants has been updated over the years, to include different effects, such as inflation pressure, as well as to improve the prediction of the model. One of the earlier models that has been developed by MSC Software according to "Tire and Vehicle Dynamics", 2002, is the PAC2002 model [19]. The model is used to calculate pure tyre forces as well as combined operation of the tyre. The pure lateral force equations are listed in equations (2.17) to (2.19).

\[
F_{y0} = D_y \sin[C_y \arctan B_y \alpha_y - E_y (B_y \alpha_y - \arctan(B_y \alpha_y))] + S_{Vy} \tag{2.17}
\]
\[
\alpha_y = \alpha + S_{Hy} \tag{2.18}
\]
\[
\gamma_y = \gamma \cdot \lambda_{\gamma y} \tag{2.19}
\]

The coefficients of the Magic Formula are dependent on the normal load and the camber angle of the tyre, equations (2.20) to (2.28). For further details about parameters refer
list of symbols and [16].

\[ C_y = p_{Cy1} \cdot \lambda_{Cy} \quad (2.20) \]
\[ D_y = \mu_y \cdot F_z \cdot \zeta_2 \quad (2.21) \]
\[ \mu_y = (p_{Dy1} + p_{Dy2} \cdot df_z) \cdot (1 - p_{Dy3} \cdot \gamma^2) \cdot \lambda_{\mu y} \quad (2.22) \]
\[ E_y = (p_{Ey1} + p_{Ey2} \cdot df_z) \cdot \{1 - (p_{Ey3} + p_{Ey4} \cdot \gamma_y) \cdot \text{sgn}(\alpha_y)\} \cdot \lambda_{Ey} \quad (2.23) \]
\[ K_{y0} = p_{Ky1} \cdot F_{z0} \cdot \sin \left(2 \arctan \left\{ \frac{F_z}{p_{Ky2} \cdot F_{z0} \cdot \lambda_{Fz0}} \right\}\right) \cdot \lambda_{Fz0} \cdot \lambda_{Ky} \quad (2.24) \]
\[ K_y = K_{y0} \cdot (1 - p_{Ky3} \cdot |\gamma_y|) \cdot \zeta_3 \quad (2.25) \]
\[ B_y = \frac{K_y}{C_y \cdot D_y} \quad (2.26) \]
\[ S_{Hy} = (p_{Hy1} + p_{Hy2} \cdot df_z) \cdot \lambda_{Hy} + p_{Hy3} \cdot \gamma_y \cdot \zeta_0 + \zeta_4 - 1 \quad (2.27) \]
\[ S_{Vy} = F_z \cdot \{p_{Vy1} + p_{Vy2} \cdot df_z\} \cdot \lambda_{Vy} + (p_{Vy3} + p_{Vy4} \cdot df_z) \cdot \gamma_y \cdot \lambda_{\mu y} \cdot \zeta_4 \quad (2.28) \]

### 2.4.4 Tyre transient behaviour

The slip angle of a tyre is related to the deflection at the contact patch. As the tyre has a mass and inertia, the deflection will take a certain amount of time to reach the steady-state value. Hence, there is a time delay from the instant the force acts on the tyre until it generates a deflection in the tyre, see Figure 2.6. This is especially noticeable in transient manoeuvres. Magnitude of the time lag is dependent on the relaxation length [16].

![Figure 2.6: Response of dynamic lateral force (b), to step input of slip angle (a) [18.]](image)

There are different methods to model this transient behaviour of the tyre. One of the simplest methods is to model the delay as a first order low-pass filter, with a time constant equal to the time lag [20]. The filter is applied to the approximated lateral
force, to give the dynamic lateral force, as given in equation (2.29) [21].

\[
F_{y,\text{dyn}} = \frac{F_y}{1 + s\tau}
\]

(2.29)

where,

\[ F_{y,\text{dyn}} \] is the dynamic lateral force that the tyre produces
\[ F_y \] is the steady-state lateral force that is calculated from the corresponding tyre model
\[ \sigma_\alpha \] is the relaxation length of the tyre
\[ v_x \] is the longitudinal speed of the contact patch of the tyre

Another approach, is to use the Stretched String model [16]. Here, the tyre belt is modelled as a stretched string and the deflection of this is calculated. In Figure 2.7, \( \sigma_\alpha \) is the relaxation length and \( v_1 \) is the tyre deflection in lateral direction. The Stretched String model in lateral direction is expressed in equation (2.31).

\[
\sigma_\alpha \frac{dv_1}{dt} + |v_x| v_1 = \sigma_\alpha v_{sy}
\]

(2.31)

where,

\[ v_1 \] is the tyre deflection
\[ \sigma_\alpha \] is the relaxation length
\[ v_x \] is the longitudinal speed
\[ v_{sy} \] is the lateral velocity in the tyre contact patch

The time lagged lateral slip angle is then calculated, equation (2.32), and is used to determine the lateral force generated by the tyre.

\[
\alpha' = \arctan \left( \frac{v_1}{\sigma_\alpha} \right)
\]

(2.32)
2.5 Kalman filter

2.5.1 General

Kalman filters are widely used for signal processing in the vehicle industry. Especially for state estimation, e.g. for active safety implementation [22]. The model based Kalman filter combines measurement data with that calculated from the model. To achieve a signal with less noise without losing information from the signal. The Kalman filter can be used in real time applications to estimate the needed states without time lag. In applications Kalman filters can be used instead of e.g. digital low-pass filters [22,23].

There are a number of different Kalman filters, e.g. Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). The main difference between these filters is that the EKF uses linearised model equations and the UKF uses non-linear equations [24,25].

The filtering procedure consists of two phases; predict and update phase. Simply stated the procedure starts with the predict phase in which the model is used to calculate and predict the states. This is done by (a); determining sigma points, similar to sample points that are inside the boundaries of the model and measurement noise covariance for a state. They are then (b); propagated through the non-linear model equations restricted by the boundaries of covariance for each point. The sigma points are hence transformed. Next is the update phase (c) and measurement data is added to the points and the mean value of the these finally gives a value close to the true one, see Figure 2.8.

Two matrices, $Q_{UKF}$ and $R_{UKF}$, are used to define covariance of the noise in the filtered signal and to tune the filter. The $Q_{UKF}$ matrix represents the covariance of the processed noise, i.e. the model and $R_{UKF}$ represents the covariance of the measurement noise. The $R_{UKF}$ matrix can be determined with measurement data but the $Q_{UKF}$ matrix is obtained by tuning. There are algorithms to determine the $Q_{UKF}$ matrix but it is not very straight forward to determine. The higher the number of states that have to be filtered, the more advanced it becomes [26].

2.5.2 Algorithm

The procedure for using the UKF and performing the calculations in each step is well known. The algorithm can be extended by, e.g. dual estimation but the basic algorithm
for the UKF is stated below [24]. The weighting factors for calculating states are
presented in equations (2.33) to (2.36).

\[ W_0^{(m)} = \frac{\lambda_{UKF}}{n_{UKF} + \lambda_{UKF}} \]  
(2.33)

\[ W_0^{(c)} = \frac{\lambda_{UKF}}{n_{UKF} + \lambda_{UKF}} + (1 - \alpha_{UKF}^2 + \beta_{UKF}) \]  
(2.34)

\[ W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n_{UKF} + \lambda_{UKF})} \]  
(2.35)

With,

\[ \lambda_{UKF} = \alpha_{UKF}^2(n_{UKF} + \kappa_{UKF}) - n_{UKF} \]  
(2.36)

Where,
\( \alpha_{UKF} \) is the spread of the sigma points
\( \kappa_{UKF} \) is a scaling factor
\( \beta_{UKF} \) determines the distribution of the states
\( n_{UKF} \) is the number of states

Standard values for \( \alpha_{UKF} = 0.001 \), \( \kappa_{UKF} = 0 \) and \( \beta_{UKF} = 2 \) for Gaussian distribution which is the most common approximation of state estimation data distribution.

The initial values for starting the state estimation iteration are calculated as in equations (2.37) to (2.40).

\[ \hat{x}_0 = E[x_0] \]  
(2.37)

\[ P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \]  
(2.38)

\[ \hat{x}_{0a} = E[x^a] = \begin{bmatrix} \hat{x}_0^T & 0 & 0 \end{bmatrix}^T \]  
(2.39)

\[ P_{0a} = E[(x_{0a}^a - \hat{x}_{0a}^a)(x_{0a}^a - \hat{x}_{0a}^a)^T] = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & R_{UKF} & 0 \\ 0 & 0 & Q_{UKF} \end{bmatrix} \]  
(2.40)

Where,
\( \hat{x}_0 \) is the vector with starting values for the states
\( P_0 \) is the error covariance matrix
\( R_{UKF} \) is the measurement noise covariance matrix
\( Q_{UKF} \) is the processed (model) noise covariance matrix

Next step is to calculate the sigma points required. The first time update (the prediction
phase) i.e. the model based states are expressed in equations (2.41) to (2.48).

\[
\chi_{k-1}^a = [\hat{x}_{k-1}^a \ \hat{x}_{k-1}^v \pm \sqrt{(n_{UKF} + \lambda_{UKF})P_{k-1}^a}]
\] (2.41)

\[
\chi_{k|k-1}^a = F[\chi_{k}^x, \chi_{k}^n]
\] (2.42)

\[
\hat{x}_k = \sum_{i=0}^{2n_{UKF}} W_i^{(m)} \chi_{i,k|k-1}^a
\] (2.43)

\[
P_k^- = \sum_{i=0}^{2n_{UKF}} W_i^{(c)} \chi_{i,k|k-1}^a - \hat{x}_k^- \chi_{i,k|k-1}^a - \hat{x}_k^-]^T
\] (2.44)

\[
Y_{k|k-1} = H[\chi_{k|k-1}^x, \chi_{k|k-1}^n]
\] (2.45)

\[
\hat{y}_k^- = \sum_{i=0}^{2n_{UKF}} W_i^{(m)} Y_{i,k|k-1}
\] (2.46)

With,

\[
\chi^a = [(\chi^x)^T (\chi^v)^T (\chi^n)^T]^T
\] (2.47)

\[
x^a = [x^T v^T n^T]^T
\] (2.48)

for \(k \in 1, ..., \infty\).

Where,

\(\chi_{-1}^a\) are the sigma points

\(\chi_{k-1}^a\) are the time updated sigma points by iteration using a solver, e.g. Runge Kutta.

\(P_k^-\) is the predicted estimation of covariance

\(\hat{y}_k^-\) is the measurement residual

The following is the update phase, where the model based states (sigma points) are updated with measurement data. This is done in equations (2.49) to (2.53).

\[
P_{\hat{y}_k\hat{y}_k} = \sum_{i=0}^{2n_{UKF}} W_i^{(c)} [Y_{i,k|k-1} - \hat{y}_k^-] [Y_{i,k|k-1} - \hat{y}_k^-]^T
\] (2.49)

\[
P_{x_ky_k} = \sum_{i=0}^{2n_{UKF}} W_i^{(c)} [\chi_{i,k|k-1} - \hat{x}_k^-] [Y_{i,k|k-1} - \hat{y}_k^-]^T
\] (2.50)

\[
K = P_{x_ky_k} P_{\hat{y}_k\hat{y}_k}^{-1}
\] (2.51)

\[
\hat{x}_k = \hat{x}_k^- + K (y_k - \hat{y}_k^-)
\] (2.52)

\[
P_k = P_k^- - KP_{\hat{y}_k\hat{y}_k} K^T
\] (2.53)

for \(k \in 1, ..., \infty\).

Where,

\(K\) is the optimal Kalman gain matrix

\(\hat{x}_k\) are the updated states (filtered states for the specific point \(k\))

\(P_k\) is the updated covariance matrix

For further details in UKF filtering, see [24].


2.6 Softwares

2.6.1 Sympathy for data

Sympathy for Data is described as a framework for automation of data analysis. The programming is done in Python and Sympathy adds a visual layer for better transparency [27].

A program is created by using nodes structured into a flow, as it is called, to indicate the sequence of steps performed, see Figure 2.9. The nodes are blocks containing python scripts. The nodes have input and output and can also have a GUI to customize settings. Sympathy has its own basic library of nodes but the user can create own nodes and libraries as well.

![Figure 2.9: Flow in Sympathy for data [27].](image)

The advantage of Sympathy for Data is the ability to run batches of data in the same flow at the same time. This makes repeated data processing easier and more transparent. It also has the option of making plots with the data available.

2.6.2 VI-CarRealTime

VI-CarRealTime is a vehicle simulation platform that is based on look-up tables. This reduces the computation time for the simulations, to the extent that it is possible to use this in real time simulation platforms. It uses python as a programming language and is compatible with Adams/Car and Matlab/Simulink [28].

Open-loop manoeuvres can be simulated with in-built manoeuvre files but also new manoeuvres can be designed by the user. This is done in an add-on environment called VI-EventBuilder and measurement data like steering wheel angle and longitudinal velocity can be used as input for manoeuvres [29].

K&C data from either Adams/Car model or measurement data from a test rig can be imported and a vehicle model can be built from the data using the K&C import interface. The program creates look-up tables from the data imported to populate the tables for suspension and steering characteristics. Damper data, though, must be imported separately [29].
CarRealTime creates simulation files that later on can be co-simulated with Matlab/Simulink. This makes it possible to, for instance, run optimization routines in Matlab. The CarRealTime-Simulink block can have multiple inputs and outputs to have the possibility of performing signal processing in Matlab but also to modify inputs during the simulation. Also, activation flags for subsystems can be set on or off, e.g. to deactivate the front or rear anti-roll bar [29].

2.7 Optimization

Mathematical optimization is the process of finding the extremum of a function, by systematically choosing input values.

2.7.1 Gradient based optimization

Gradient based optimization methods, use the first and second derivatives of the function to iteratively find the extremum of the function. The classic example of this is Newton’s method. This is a second order method, using the gradient as well as the Hessian of a function to find the maximum or minimum. The equation is based on a Taylor’s series expansion of the function. An initial point is required for this type of algorithm.

\[ \text{initial: } x_0 \in \mathbb{R}^n \]
\[ \text{iterate: } x_{t+1} = x_t - H^{-1} G_f \]

Where,
- \( \mathbb{R} \) is the set of all real numbers
- \( G_f \) is the gradient of the function \( f \); \( \nabla f(x_t) \)
- \( H \) is the Hessian of the function \( f \); \( \nabla^2 f(x_t) \)

The advantage of these algorithms is that they are computationally efficient and hence faster. Though, the algorithm is susceptible to local minima. The selection of initial point also determines the efficiency as well as effectiveness of the algorithm.

One of the basic in-built optimization algorithms in Matlab is \texttt{fmincon}. It is used for finding the minimum of a constrained non-linear multi-variable function [30]. It is a gradient based method and hence has the same advantages and pitfalls.

2.7.2 Genetic algorithms

Genetic algorithms are based on Darwin’s theory of evolution. The solutions of problems, processed by these set of algorithms, are evolved from generation to generation [31].

The algorithm uses the inputs as the DNA of an organism and calculates the fitness of the organism that is produced. The algorithm is started with a set of solutions called population. It then repeatedly modifies a population of individual solutions, called a generation. The algorithm randomly selects individuals from the current population
and uses them as parents to produce the children for the next generation. The objective is to evolve the organism by means of reproduction, mutation or elitism to obtain an organism that has the highest fitness and hence the optimal solution.

Hence, it can be said that the process of selection of parents to make the new population is the most important process in the algorithm. This is done in a variety of ways. The parents are selected based on their fitness, where individuals with high fitness are used in a crossover process where they interchange their DNA to produce offsprings. Mutation is the process in which the DNA of the offspring are modified in a random manner. The individuals with the highest fitness in a population are kept the same from one generation to the next, this is called elitism.

The performance of the genetic algorithm is dependent on the processes used in coding of the DNA of individuals, the selection of parent, the crossover method and the process of mutation. There are many different methods that can be used for these processes, based on the problem to be solved. In most cases, general thumb rules can be used and modified to improve the performance of the algorithm [32].

2.7.3 Multi-objective optimization

A multi-objective optimization problem can be mathematically represented as,

$$\min \{f_1(x), f_2(x), ..., f_n(x)\}$$

$$x \in S$$

where, $n > 1$ and $S$ is the set of constrains.

In multi-objective optimization, the concept of scalar optimality does not apply, as there are multiple functions. Here a concept called Pareto optimality is used. A set of inputs to the functions is said to be Pareto optimal if all other vectors of inputs have a higher value for at least one of the objective functions $f_i$, with $i = 1, ..., n$ or have the

![Figure 2.10: Pareto fronts in multi-objective optimization [33].](image-url)
same value for all the objective functions. The efficient set is the set of all Pareto optimal points for a particular set of objective functions. This efficient set when displayed is called the Pareto front. The Pareto front can also give an idea of the trade-offs to be considered when optimizing multiple objectives. Pareto curves cannot be computed efficiently in many cases. Thus, approximation methods are frequently used [33].

A multi-objective problem can be solved as a single objective one, by combining its objectives into one single unique objective function. In this method, the weighted sum of all the objectives is used as the new single objective. This approach is generally called the weighted sum method. There is not an a-priori correspondence between a weight vector and the solution. It must also be noted that weighting coefficients do not necessarily correspond to the relative importance of the objectives. Therefore, the decision maker has to choose the weight combination to reproduce a representative part of the Pareto curve. The disadvantage of this method is that non-convex parts of the Pareto curve cannot be reached [33], see Figure 2.10a and 2.10b.

Genetic algorithms can also be used to solve multi-objective optimization as well. The Matlab function gamultiobj finds a local Pareto front for the set of objectives. It is also possible to display the Pareto front that is calculated for the functions. The genetic algorithm works in fundamentally the same way as for the single objective problem. Though, here the elite population is present on the Pareto front of each generation [32].

2.8 Analysis of variance (ANOVA)

ANOVA or analysis of variance is a technique for comparing sample means. It gives the possibility of determining if the data samples could be from the same distribution. The analysis can be of interest e.g. when performing a process with different treatments as a changing variable. Then it can be seen from ANOVA if the effect of different treatments is statistically significant or if the variation is due to noise [34].

\[
M_A = \frac{\sum_{i=1}^{n} A_i}{n} \quad (2.58)
\]
\[
M_B = \frac{\sum_{i=1}^{n} B_i}{n} \quad (2.59)
\]
\[
M_C = \frac{\sum_{i=1}^{n} C_i}{n} \quad (2.60)
\]
\[
GM = \frac{\sum_{i=1}^{n} A_i + \sum_{i=1}^{n} B_i + \sum_{i=1}^{n} C_i}{n + n + n} \quad (2.61)
\]

The analysis is performed by first finding the mean value for each series \((M)\) and the grand mean \((GM)\), the mean value of all data for all treatments, equations (2.58) to (2.61). For example if there are three treatments \((A, B, C)\), for each treatment there is a series of samples 1 to \(n\), series \(A\), series \(B\) and series \(C\), see Table 2.11. The series represent column vectors in the data matrix. Then the difference is determined for
Table 2.11: Example: Sample series, Analysis of variance (ANOVA).

<table>
<thead>
<tr>
<th>Run</th>
<th>Treatment A</th>
<th>Treatment B</th>
<th>Treatment C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A_1$</td>
<td>$B_1$</td>
<td>$C_1$</td>
</tr>
<tr>
<td>2</td>
<td>$A_2$</td>
<td>$B_2$</td>
<td>$C_2$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$n$</td>
<td>$A_n$</td>
<td>$B_n$</td>
<td>$C_n$</td>
</tr>
</tbody>
</table>

Each series by comparing the mean value with the grand mean called $T_{ANOVA}$-matrix or treatments matrix. Then each sample is compared to the grand mean called $D_{ANOVA}$-matrix and the measurement noise or error called $R_{ANOVA}$-matrix is determined by taking away the difference between the value of each sample series and the mean of the series, e.g. $A$, $B$ or $C$. Where $D_{ANOVA}$, $T_{ANOVA}$ and $R_{ANOVA}$ are related to each other as $D_{ANOVA} = T_{ANOVA} + R_{ANOVA}$, see Table 2.12 and Table 2.13.

Table 2.12: Example: $T_{ANOVA}$-matrix.

<table>
<thead>
<tr>
<th>Treatment A</th>
<th>Treatment B</th>
<th>Treatment C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_A$-GM</td>
<td>$M_B$-GM</td>
<td>$M_C$-GM</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$M_A$-GM</td>
<td>$M_B$-GM</td>
<td>$M_C$-GM</td>
</tr>
</tbody>
</table>

Table 2.13: Example: $R_{ANOVA}$-matrix.

<table>
<thead>
<tr>
<th>Treatment A</th>
<th>Treatment B</th>
<th>Treatment C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$-$M_A$</td>
<td>$B_1$-$M_B$</td>
<td>$C_1$-$M_C$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$A_n$-$M_A$</td>
<td>$B_n$-$M_B$</td>
<td>$C_n$-$M_C$</td>
</tr>
</tbody>
</table>

The null hypothesis is stated as, *if the treatment has no effect*. The common significance level is 5% (confidence level is 95%) and the distribution to use is the $F$-distribution. Hence, if the treatment is found to be statistically significant, it provides the opportunity to discard the null hypothesis.

Parameters to be determined for the ANOVA analysis are; number of degrees of freedom $\nu$, total sum of squares $SS$ and mean squares $MS$, these parameters are calculated for the treatment and the error respectively. Thus the observed function value $F$ and significance value or $p$-value for $F$-distribution are determined. The parameters are
determined as,

\[ \nu = n - 1 \]  \hspace{1cm} (2.62)

\[ SS = (a)^2 + (b)^2 + ... + (z)^2 \]  \hspace{1cm} (2.63)

\[ MS = \frac{SS}{\nu} \]  \hspace{1cm} (2.64)

\[ F = \frac{MS_{\text{treatment}}}{MS_{\text{error}}} \]  \hspace{1cm} (2.65)

where, \( n \) is the number of treatments and \( a, b, ..., z \) are elements in the \( T_{\text{ANOVA}} \)- or \( R_{\text{ANOVA}} \)-matrix depending on if it is \( SS_{\text{treatment}} \) or \( SS_{\text{error}} \) that is calculated.

The \( p \)-value is based on the \( F \)-distribution, refer [34], with function value \( F \), the degrees of freedom for treatments \( \nu_{\text{treatment}} \) and the degrees of freedom for the error \( \nu_{\text{error}} \) as inputs. If the \( p \)-value is less than the significance level the null hypothesis can be discarded; if greater than the significance level it cannot be discarded but it is neither possible to say if it is true or not.

Matlab has the one-way ANOVA analysis function `anova1` where the measurement data is input as a matrix [35]. The function calculates all the needed parameters in a table including the \( p \)-value.

2.9 Design of experiments

When performing experiments the factors can be divided into two categories;

- controllable → possible to observe and change
- observable → possible to measure but not to change

Hence when testing, the immediate strategy is to vary one factor at a time. This method is time consuming, due to the large number of experiments required, as well as losing the information about the effects of the interaction of factors. Therefore, testing strategies have been developed to deal with these potential pitfalls. Taguchi methods, also called factorial testing is one such method for multi-variable testing and analysis [15].

2.9.1 Factorial design

Factorial design involves designing and performing of tests with different factor combinations in a structured way. Thus, reducing the time and resources required. The results of these tests can be used to determine the effect of the factors as well as the combination of the factors on the tests [15,34].

The most common method is to use the two-level factorial design, meaning that each factor is tested for two values often referred as a low (−) and a high value (+). For a two-level full factorial design the number of tests required are \( 2^n \). Where, \( n \) is the number of factors for individual factor isolation. For a more general case, the number of tests required are \( p^n \) where \( p \) is the number of levels.
Fractional factorial design is used, when time and resources are limited. Here the effect of the combination of factors can be mixed with the effect of the individual factors. For fractional factorial design the number of tests needed is $2^{n-1}$. The fractional analysis can be done by reducing with every half e.g. half, quarter and eighth part of a full factorial design. If having four factors ($A$, $B$, $C$ and $D$) and having half fractional design the effect of the combination of two factors can be isolated from higher combinations. The disadvantage compared to the full factorial design is that it is not possible to isolate the effect of the combination of two factors. For example, if $AB$ seems to have an large affect it is not possible to say if $AB$ or $CD$ is the driving factor for that effect [15,34].

The resolution of the factorial design is controlled with the number of factors and the number of treatments (runs). There a namely three resolution levels, III, IV and V where V is the full factorial. The higher the resolution, the more information can be extracted from the analysis. The full factorial design with all factors isolated is the highest resolution [15,34].

### 2.9.2 Sensitivity analysis

To get the separate effect of each factor when having multiple factors the test matrix should be orthogonal, i.e. that the product of each vector is zero. If not zero there could be linear combinations of vectors that are not unique which can give misleading conclusions.

<table>
<thead>
<tr>
<th>Run</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$R_1$</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>$R_2$</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>$R_3$</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>$R_4$</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>$R_5$</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>$R_6$</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>$R_7$</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>$R_8$</td>
</tr>
</tbody>
</table>

For a two-level and three factor design ($A$, $B$ and $C$) the factorial design matrix could look like Table 2.14. The combined effect of two or more factors can be investigated by multiplying the corresponding element of each vector with the other ($A_i \cdot B_i$), where $i$ is the run number, see equation (2.66). With the results of the tests, the effect $c$, of each factor can be calculated by multiplying the corresponding element of the result vector with that of each sign vector, divided by the number of changes made to that factor,
see equations (2.67), (2.68) and (2.69) [15,34].

\[
\begin{bmatrix}
- \\
+ \\
- \\
+ \\
- \\
+ \\
- \\
+ \\
\end{bmatrix}
\begin{bmatrix}
- \\
- \\
+ \\
+ \\
- \\
- \\
+ \\
+ \\
\end{bmatrix}
\begin{bmatrix}
+ \\
+ \\
- \\
- \\
+ \\
+ \\
- \\
+ \\
\end{bmatrix}
\]

\( \begin{bmatrix}
- \\
+ \\
- \\
+ \\
- \\
+ \\
- \\
+ \\
\end{bmatrix}
\begin{bmatrix}
- \\
- \\
+ \\
+ \\
- \\
- \\
+ \\
+ \\
\end{bmatrix}
\begin{bmatrix}
+ \\
+ \\
- \\
- \\
+ \\
+ \\
- \\
+ \\
\end{bmatrix}
\]  

(2.66)

\[c_A = \frac{-R_1 + R_2 - R_3 + R_4 - R_5 + R_6 - R_7 + R_8}{4} \]  

(2.67)

\[c_B = \frac{-R_1 - R_2 + R_3 + R_4 - R_5 - R_6 + R_7 + R_8}{4} \]  

(2.68)

\[c_C = \frac{-R_1 - R_2 - R_3 - R_4 + R_5 + R_6 + R_7 + R_8}{4} \]  

(2.69)

### 2.9.3 Pareto chart

The analysis of the results from the sensitivity analysis can be done with the use of a Pareto chart. The Pareto chart displays the absolute value of the effects of all factors as well as combination of factors in descending order, see Figure 2.11 for example with treatments A, B and C. Hence it gives a visual tool to easily determine which factor has the most effect on the results of the experiment [15,34].

![Figure 2.11: Example: Pareto chart.](image)
3 Methodology

In this chapter the processes of the work performed in the thesis are presented. An explanation of the study into the standards of manoeuvres and metrics is given. Along with, detailed information of the processing of data obtained from winter objective testing. The processes used to obtain simulation models are described as well as, the implementation of an Unscented Kalman filter. Statistical tools used to analyse and modify the manoeuvres are listed, along with the calculation tools to determine metrics from winter data is given.

3.1 Manoeuvres and metrics standards

To obtain manoeuvres and metrics specific for winter conditions, was a goal of this thesis. Hence, the standards for manoeuvres and their metrics were researched. As there were no defined manoeuvres or metrics existing for winter conditions, at the time of writing this thesis, standards for summer were checked. The standards defined by the ISO, NHTSA and a vehicle manufacturer were compared. The differences with respect to the definition of the manoeuvres and the metrics that were calculated, were documented. The CR, FR, SWD and TRIT manoeuvres were focused on.

The above comparison was used as a benchmark and hence the manoeuvres performed in a previous winter expedition were analysed. The manoeuvres used had been modified based on considerations of limitations of space, safety as well as theories that were being tested. The data recorded at the winter testing was used to not only compare the input to the vehicle, but also the output from the vehicle. This was done in order to understand the limitations and differences of objective testing during winter. The important signals compared were, SWA and speed as inputs, and lateral acceleration and $x$ and $y$-displacements as outputs.

3.2 Comparison of vehicle configurations

Table 3.1: Combinations of comparisons between vehicles considering configuration, time and vehicle differences.

<table>
<thead>
<tr>
<th>Time</th>
<th>Config &amp; Vehicle</th>
<th>Config &amp; Time</th>
<th>Vehicle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref 1 v/s Ref 2</td>
<td>Ref 1 v/s Veh 1</td>
<td>Veh 1 v/s Veh 2</td>
<td>-</td>
</tr>
<tr>
<td>Ref 1 v/s Ref 3</td>
<td>Ref 2 v/s Veh 2</td>
<td>Veh 1 v/s Veh 3</td>
<td>-</td>
</tr>
<tr>
<td>Ref 2 v/s Ref 3</td>
<td>Ref 3 v/s Veh 3</td>
<td>Veh 2 v/s Veh 3</td>
<td>Ref 3 v/s Veh 3</td>
</tr>
</tbody>
</table>

Table B.1 in Appendix B shows the sequence of the tests that were run. It can be noticed that the differences between the reference (Ref) and configuration vehicles (Veh
1, Veh 2 and Veh 3) were more than just the configuration change in the ARB (anti-roll bar). The changes that can be noticed are those of time, ARB configuration and vehicle components tolerance (Ref 3 v/s Veh 3). Table 3.1 shows the corresponding changes to be taken into account when comparing two vehicles. To understand the difference between the vehicle behaviour on ice and on asphalt in summer, the data from tests conducted using the same vehicle with the same manoeuvres was analysed as well. The configuration Veh 1 was without ARB in the rear, Veh 2 was without ARB in the front and Veh 3 was a standard vehicle (same configuration as reference).

![Graphs showing yaw rate, lateral acceleration, body slip angle, and cross plot SWA v/s lateral acceleration](image)

**Figure 3.1**: Constant radius manoeuvre.

The parameters used for the comparison were, lateral acceleration $a_y$, yaw rate $\dot{\psi}$ and body slip angle $\beta$. The $a_y$ and $\dot{\psi}$ are important as they are measured by the IMU (Inertial Measurement Unit) and do not have to be calculated as in the case of some other parameters. The body slip angle on the other hand is calculated by the IMU and
hence is susceptible to errors, especially at low speed, see Figure B.1 in Appendix B.

Initially, the data was compared visually for the different combinations. This was done to observe trends and locate regions of interest in the data and was performed by overlaying the plots of two vehicles, see Figure 3.1 to 3.6. For complete figures, see Figure D.2 to D.11 in Appendix D. The SWD manoeuvre was performed with different SWA inputs in the previous winter expedition. In this study only the SWA with most change in vehicle behaviour without spinning out was considered. For the FR manoeuvre, the plots were compared both in the time and the frequency domain, see Figure 3.3 and 3.4 respectively. For complete figures, see Figure D.6 and D.7 in Appendix D. This was done with Bode plots of the signals using the SWA signal as the input and the measured data as the output of the vehicle system. Hence, the gain, phase and the coherence of the system could be compared visually.

The other factor that could be checked from these plots, was the variation between the multiple runs that were performed with each vehicle. Hence determining if the differences due to configurations was larger than the differences due to the change in surface conditions, hence a rough estimate about the signal-to-noise ratio. Later in this thesis a statistical method, ANOVA, will be used to investigate the signal-to-noise even further.

![Diagram](image-url)
Figure 3.3: Bode plot of yaw rate, frequency response manoeuvre.

Figure 3.4: Bode plot of lateral acceleration, frequency response manoeuvre.


3.3 Spread of reference vehicle

For the winter expedition, a reference vehicle was run along with the vehicle configuration. Hence there were ten test sets performed with the reference, named Ref 1 to 10. Though, for the CR manoeuvre, only 6 sets were performed to optimize the time and resources used in the testing. The purpose of having a reference vehicle was to get a measure on the changing surface conditions.

Therefore when plotting the data for all the runs of the reference vehicle, it was possible to see the spread of data that is caused by varying surface conditions. This also
gives the possibility to compare the magnitude of spread in the reference vehicle with the magnitude of change in vehicle configuration. Hence giving an estimate about the signal-to-noise ratio.

The analysis of the spread was initially performed visually by plotting the data from the references in the same graph. The signals of yaw rate, lateral acceleration and body slip angle were used for the comparison. The next step was to determine a region of interest where the spread in the behaviour at the different runs could be noticed. A measure of the spread was chosen, in this case the slopes of the signals in the region of interest. These were then analysed statistically, by checking the distribution of the measure. The data was checked for having both normal and lognormal distributions. The reasoning for checking the lognormal distribution was the assumption that, there would be a higher probability of having areas with lower grip than those with higher grip. This would cause a skewness in the data and hence fit a lognormal distribution.

![Graphs showing the spread in reference vehicle data for constant radius manoeuvre.](image)

### Figure 3.7: Spread in reference vehicle data for constant radius manoeuvre.

#### 3.3.1 Constant radius manoeuvre

In the CR manoeuvre, the variation from reference to configuration vehicle can be seen distinctly in the plots of the body slip angle and the cross plot of SWA and lateral acceleration, see Figure 3.1c and 3.1d. Hence, the spread of the reference vehicle was investigated for these signals, to check the magnitude in spread, see Figure 3.7a and 3.7b. For complete figures, see Figure D.12a and D.12b in Appendix D.

#### 3.3.2 Frequency response manoeuvre

In the FR manoeuvre, the spread of data was checked in the frequency domain. This was done as, the noise is lower and as the metrics for this manoeuvre are largely in the frequency domain. The spread in data for the reference vehicle was checked for the yaw
rate and the lateral acceleration, see Figure 3.8. The bode plot of the lateral acceleration similarly showed no spread. For complete figures, see Figure D.13 in Appendix D.

![Figure 3.8: Spread of yaw rate in reference data in frequency domain.](image)

### 3.3.3 Sine with dwell manoeuvre

In the SWD manoeuvre, the region of interest was the yaw rate after the negative SWA peak. Hence the yaw rate was plotted for all the reference vehicles, see Figure 3.9a. For complete figure, see Figure D.14a in Appendix D. The slope was differing for the references and hence a linear regression was made to obtain the slope in that section, see Figure 3.9b. The slope was then used to check for the different distribution. The SWD manoeuvre was performed with different SWA inputs in the previous winter expedition. In this study only the SWA with most change in vehicle behaviour without spinning out was considered.

The body slip angle was also differing for the reference vehicles. The factor that was compared, was the peak-to-peak amplitude of the body slip angle, see Figure 3.10 (Figure D.14b in Appendix D). The peak-to-peak amplitude was used rather than the absolute one, as the measurement of body slip angle was susceptible to offsets. Using peak-to-peak amplitude reduces the error due to these offsets.

### 3.3.4 Throttle release in turn manoeuvre

The yaw rate and the body slip angles were the two signals where the differences between the vehicles could be seen. Hence the spread of the yaw rate and the body slip angle were plotted, see Figure 3.11a and 3.12 (Figure D.17a and D.17b in Appendix D).

In the plot of the yaw rate, the slope after the throttle release event was differing and hence a linear regression was made to check for the distribution of the slope, see Figure
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(a) Spread in yaw rate.

(b) Linear regression for yaw rate slope.

Figure 3.9: Spread of yaw rate in reference vehicle data for sine with dwell manoeuvre.

Figure 3.10: Spread of body slip angle in reference data for sine with dwell manoeuvre.

3.11b. In the case of the body slip angle, a variation could be seen for the reference vehicles, though it was not possible to represent the difference with a quantitative measure.
Figure 3.11: Spread of yaw rate in reference vehicle data for throttle release in turn manoeuvre.

Figure 3.12: Spread of reference vehicle data in body slip angle in throttle release in turn manoeuvre.

3.4 Simulation

3.4.1 Bicycle model

A vehicle model was intended to be used in the Kalman filter as well as to check if the model could be used to simulate the tests that were conducted in winter on a low friction surface. A simple model was chosen to assess the performance, as well as for speed in the Kalman filtering process. A single track or bicycle model was chosen. The model was built in Matlab and the states were solved using a Runge-Kutta integration
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algorithm, **ode45**. This required the equation of the model (2.1), (2.2) and (2.3) to be expressed in the state-space form, see equations (2.9) and (2.10).

This simplified model was then modified to suit the particular needs and requirements. These additions were,

- non-linear tyre model
- longitudinal load transfer
- tyre transient behaviour.

The reasoning for the inclusion of these phenomena were as follows. A non-linear tyre model was chosen as the lateral accelerations that were to be investigated were beyond the linear range of the tyre and vehicle on that surface. The two tyre models that were used were the,

- Brush model
- Magic Formula model

The longitudinal load transfer was included for the TRIT manoeuvre. On the assumption that the transient behaviour was appearing due to the longitudinal load transfer when releasing the throttle and engine braking was excited. Hence the normal loads on the front, \( F_{z,12} \) and rear, \( F_{z,34} \) axles were calculated with equation (3.2) and (3.1) respectively.

\[
F_{z,34} = \frac{f \cdot m \cdot g + h \cdot m \cdot a_x}{f + b} \tag{3.1}
\]

\[
F_{z,12} = m \cdot g - F_{z,34} \tag{3.2}
\]

Where,

- \( h \) is the vertical height of the vehicle above the ground in static condition
- \( a_x \) is the longitudinal acceleration of the vehicle

Tyre transient behaviour was chosen to better represent the response in the FR and SWD manoeuvres. The implementation of this was performed using the first order low pass filter to the calculated lateral force. Equation (2.29) is modified to obtain the equation for the dynamic lateral force in the time domain. The equations (3.3) and (3.4) for the front and rear dynamic lateral force are obtained in the state-space form and can then be added to the equation of the bicycle model. Here \( \sigma_{12} \) and \( \sigma_{34} \) are kept constant.

\[
\dot{F}_{y,12}(t) = (F_{y,12} - F_{y,12}(t)) \cdot \frac{v_x}{\sigma_{12}} \tag{3.3}
\]

\[
\dot{F}_{y,34}(t) = (F_{y,34} - F_{y,34}(t)) \cdot \frac{v_x}{\sigma_{34}} \tag{3.4}
\]

Where,

- \( F_{y,12} \) is the calculated lateral force on front axle
- \( F_{y,34} \) is the calculated lateral force on rear axle
- \( \dot{F}_{y,12}(t) \) is the time derivative of the force on the front axle at time \( t \)
- \( \dot{F}_{y,34}(t) \) is the time derivative of the force on the rear axle at time \( t \)
- \( \sigma_{12} \) is the relaxation length on the front axle
- \( \sigma_{34} \) is the relaxation length on the rear axle
3.4.2 VI-CarRealTime

As mentioned, VI-CarRealTime (CRT) uses look-up tables to perform the necessary calculations for the vehicle dynamic simulations. Hence the characteristics of the suspension and the steering as well as the properties of the vehicle had to be input in the form of tables to the software. In order to obtain a model of a real vehicle, the compliances of the sub-systems had to be input as well. The K&C importer, allows the calculation of the data points necessary to populate the suspension and steering look-up tables. There are some options to control the level of accuracy required. Direct, cross and single compliance tests can be input. Compliance at a single load case and upto a maximum of three load cases can be input with this tool. However, as K&C tests are quasi-static, the characteristics of the dampers had to be input separately. The data that can be imported was as follows,

- **Kinematics**
  - Vertical wheel travel for front and rear axle respectively.
  - Steering for three height levels (three load cases).

- **Compliance**
  - Lateral, in phase and anti-phase for three height levels (three load cases).
  - Longitudinal braking, in phase and anti-phase, single and combined for three height levels (three load cases).
  - Aligning torque, in phase and anti-phase, single and combined, three height levels (three load cases).
  - Steering for three height levels (three load cases).

- **Anti-roll**
  - opposite travel roll, with and without any anti-roll bars.

The vehicle measured in the K&C test-rig, was the configuration vehicle that had been used in the previous winter expedition. In order to obtain a fairly accurate model, with a limitation of time, the direct and cross compliance at three load cases were measured. The three load cases were represented by the jounce levels, i.e. 0 $mm$ representing the static height, $\pm 60 \, mm$ representing the higher and lower load cases. The vehicle was measured in four configurations, standard configuration (Ref/Veh 3), without anti-roll bar in the rear (Veh 1) and without anti-roll bar in the front (Veh 2) and without both anti-roll bars. The vehicle was also measured with different tyres, a 16 in summer tyre and a 17 in unstudded winter tyre. However due to limitation of time, limited tests were performed with the winter tyre in order to check the effect of tyre on the measurements. Hence the data with the summer tyre was used to build the final model.

After the creation of the model, data for the damper had to be manually imported. The damping force as a function of displacement speed and the curve of the motion ratio as a function of wheel displacement were obtained from data sheets of the dampers as well as an Adams/Car model. The CG z-location as well as inertias that had been measured for the vehicle in the K&C rig. Along with the calculated x and y-locations
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of the CG for the vehicle loaded with all fluids and a driver as well as a passenger, see equations (3.5) to (3.8). These were manually input to the software.

\[ \frac{b}{L} \cdot m = m_{LF} + m_{RF} \]  
\[ f = L - b \]  
\[ CG_x = \frac{L}{2} - f \]  
\[ CG_y = \frac{t_w}{2} - \left( t_w - \frac{m_{LF} + m_{LR}}{m} \cdot t_w \right) \]

Where,
- \( m_{LF} \) is the left front corner weight
- \( m_{LR} \) is the left rear corner weight
- \( m_{RF} \) is the right front corner weight
- \( t_w \) is the track width

Simulations in VI-CarRealTime was used for mainly two purposes,

1. Find behaviour for existing data (high spread in data).
2. Modify/design new manoeuvres.

1. Due to high spread in measurement data in previous winter tests, simulation was used to see how the vehicle should behave and also if the difference between inputs of speed and SWA would affect the behaviour or if the change only was due to changes in surface. If the simulation of different runs were very close to each other or replicated, it would be an evidence of that the changes in surface would matter a lot in reality. On the other hand if there would be differences in runs, it would be concluded that relative small changes in SWA and speed matters as well. This was done by getting the SWA input and the speed map from real measurements and have them as input to the vehicle model in CRT. This was done both for Ref, Veh 1 and Veh 2 to be able to see if a difference was noticeable in simulation. From the results of this analysis it would be easier to understand how the manoeuvres would be modified.

2. To modify the manoeuvres with the consideration of space required to perform them. Hence, the test tracks were modelled/built with VI-Road and used for simulation. To run the open-loop manoeuvres, new python scripts were designed with VI-EventBuilder (a program within CarRealTime). The inputs to the simulation were steering wheel angle profile and speed profile. The simulation procedure included the steps,

- Check traction – was the manoeuvre feasible.
- Check if the manoeuvre fitted the test track space.
- Get metrics in Sympathy for data.
- Check if the steady-state time period was long enough to get steady-state metrics.
- See if difference between Ref, Veh 1 and Veh 2 was noticeable. If not the driver inputs were changed.
3.5 Winter tyre modelling

For simulation of the manoeuvres that were performed in the previous winter expedition, tyre models that could be used with the vehicle models available had to be made. As there were two vehicle models, the tyre models that were developed could be split into two categories.

- Axle tyre models - for use with the bicycle model.
- Tyre models - for use with VI-CarRealTime model.

In all cases, only the pure lateral force generation of the tyre was developed. This was due to the fact that, the manoeuvres that were studied are largely lateral in nature.

3.5.1 Brush model with axle characteristics

The lateral force of the axles was modelled initially using the brush tyre model equations (2.11), (2.12) and (2.13). The slip angles were calculated from the bicycle model equations (2.6) and (2.7) for the front and rear axles respectively. These were used to calculate the corresponding axle lateral force $F_{y}$. This was then used to calculate the dynamic lateral force $F_{y,dyn}$ using equations (3.3) and (3.4) for the front and rear axles respectively.

To obtain the lateral stiffness and the friction coefficients, $\mu_{12}$ and $\mu_{34}$, the above model was optimized to fit the measurement data. The signals of yaw rate and lateral acceleration were chosen to calculate the error to minimize, making this a multi-objective optimization. As the Pareto front was not known, genetic optimization method was chosen. In this way, the shape of the Pareto front could also be checked. The measurement data was chosen from one run of the CR manoeuvre. This was done to have a more steady-state initially and not introduce a large amount of error due to the simple relaxation behaviour. The run was selected on the criterion that the fluctuations of signal at the end were lower. Due to the time taken by the Runge-Kutta solver, the overall optimization time was substantially long.

The performance of the optimized model was checked against the CR, FR and SWD manoeuvres. The TRIT manoeuvre was not considered because it had high spread in measurement data and different behaviours for the same vehicle was found, see section 3.3.4. The criteria were the normalized mean squared error of the lateral acceleration and yaw rate from measurement and simulation.

Due to the bad performance in the SWD manoeuvre, the optimization scope was expanded to include all manoeuvres. Hence optimization of the model against all runs of the CR, FR and SWD manoeuvres was attempted. This was done to check if it was possible to obtain an average model that could represent all the characteristics from the different manoeuvres moderately well. Due to the large scope of the optimization, genetic algorithms would be extremely inefficient. Hence, as the Pareto front of the previous optimization was convex, a weighted mean method was attempted. Equal weighting was given to all error values. The results of the optimization were checked by
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comparing the performance of the new model to that of the previous one, see section 4.3.1.

3.5.2 Magic Formula model with axle characteristics

The lateral force was also modelled using the Magic Formula equations. The equations used were obtained by simplifying the PAC2002 set of equations, (2.20) to (2.28). The major simplification that was made is the removal of camber and tyre asymmetry terms. As the model is of the axle and not an individual tyre. The other simplifications were to the change in the parameters with variation of normal load, see equations (3.9) to (3.15). The slip angle is negative in the equation for the lateral force (3.15), due to the sign convention followed in the bicycle model.

\[
\frac{df_z}{z} = \frac{F_z - F_{z,\text{nom}}}{F_{z,\text{nom}}} \\
\mu_y = \mu + d\mu \cdot df_z \\
D_y = \mu_y \cdot F_z \\
C_y = c_1 \\
E_y = e_1 \\
B_y = \frac{C_\alpha}{C_y \cdot D_y} \\
F_y = D_y \cdot \sin\{C_y \cdot \arctan(B_y \cdot (-\alpha) - E_y \cdot (B_y) \cdot (-\alpha) - \arctan(B_y \cdot (-\alpha))}\}
\]

Where,
- \(F_z\) is the normal load on one axle
- \(F_{z,\text{nom}}\) is the nominal (static) normal load on one axle
- \(\mu\) is the friction coefficient
- \(C_\alpha\) is the cornering stiffness on one axle
- \(c_1\) is a constant to be fitted
- \(e_1\) is a constant to be fitted
- \(\alpha\) is the lateral slip angle

To obtain the values of the constants of this model, a similar method was used as in the previous section 3.5.1. Multi-objective optimization against the yaw rate and the lateral acceleration was performed. The model was optimized to fit the measurement data from one run of the CR manoeuvre. It must be noted that the model was optimized against the same run as in the previous section, see section 3.5.1. The performance of this model was then compared to that of the brush models from the previous section to assess the qualities of the two models for modelling winter tyres.

3.5.3 Magic Formula model for individual wheel

For use with VI-CarRealTime, the lateral force developed by a single tyre had to be developed. Hence one equation was required to represent all four tyres of the vehicle model. There was also the intent to check the possibility of obtaining a tyre model from vehicle measurement data. The first step in this was to obtain a Magic Formula tyre
model for another tyre of same size but different generation. This was done sequentially.

First, the simplest form of the Magic Formula was used, see equation (2.14), where the coefficients of the formula were kept constant. The constants were then optimized to fit the measurement data for the lowest load and no camber case. The optimization was performed using genetic algorithms in conjunction with gradient based optimization \texttt{fmincon}. This was done to check the slope of the function around the local minimum obtained by the genetic algorithm. The performance of the resulting fit was checked for the other load and camber cases.

As it was noticed that the camber caused many changes in the tyre characteristic curves, the effects of camber were introduced in the Magic Formula. First, the previous model was used to obtain the coefficients for the fits to the measurement data for the lowest load case with a value of positive and negative camber respectively. The equations for the coefficients were then solved to obtain values of the constants linked to camber, see equations (3.16) to (3.22). The values obtained from this were hence used as the constrains in the optimization of the Magic Formula with the effect of camber. The optimization of these constants gave a tyre model that could represent the lateral force curves of the tyre at a given load and different camber values.

\[
\alpha_y = \alpha + S_{Hy} \tag{3.16}
\]

\[
C_y = p_{Cy1} \tag{3.17}
\]

\[
D_y = D \cdot (1 - p_{Dy3} \gamma^2) \cdot F_z \tag{3.18}
\]

\[
B_y = B \cdot (1 - p_{Ky3} |\gamma|) \tag{3.19}
\]

\[
E_y = p_{Ey1} \cdot (1 - (p_{Ey3} + p_{Ey4} \gamma) \text{sgn}(\alpha_y)) \tag{3.20}
\]

\[
S_{Hy} = S_H + p_{Hy3} \cdot \gamma \tag{3.21}
\]

\[
S_{Vy} = (S_V + p_{Vy3} \cdot \gamma) \cdot F_z \tag{3.22}
\]

The next step was to introduce the dependence on normal load. A similar procedure was used to obtain the ranges of the constants for the load dependence. The equations for the Magic Formula that were used are (3.23) to (3.25). It can be noticed that the tyre lateral stiffness is kept constant. As it was not possible to solve the equations for the constants. This is addressed in the next step. The model was optimized to obtain the values of the constants to be able to represent the load and camber cases that were tested.

\[
D_y = (p_{Dy1} + p_{Dy2} \cdot df_z) \cdot (1 - p_{Dy3} \gamma^2) \cdot F_z \tag{3.23}
\]

\[
S_{Hy} = p_{Hy1} + p_{Hy2} \cdot df_z + p_{Hy3} \cdot \gamma - 1 \tag{3.24}
\]

\[
S_{Vy} = (p_{Vy1} + p_{Vy2} \cdot df_z + p_{Vy3} \cdot \gamma) \cdot F_z \tag{3.25}
\]

The coefficients for the lateral stiffness of the tyre were used and optimized as well. Though the results obtained were not satisfactory. The previous model had better performance and hence was used further.
3.5.4 Co-simulation

A tyre model was obtained for a studded winter tyre from the previous section, see section 3.5.3. As the tyre used for the modelling was not the same tyre as used during the winter expedition, it was decided to use the measurement data to tweak the available tyre model to fit the measurement data.

The procedure selected for this was to use co-simulation of VI-CarRealTime in Matlab/Simulink with a gradient based optimization routine, fmincon, to modify the constants of the tyre model. Due to the complexity of the model, the simulation time was greater hence resulting in the optimization routine being extremely time consuming. Due to these and other considerations, this was not attempted.

3.6 Unscented Kalman filter

3.6.1 Implementation

An Unscented Kalman filter was selected to investigate potential improvements in signal processing over a fixed frequency low-pass filter. The aim was to implement the UKF in the metrics calculation tool.

Fixed frequency low-pass filters can cause loss of data in the signals, such as reduction in amplitude and time lags. This is as the cut-off frequency is fixed and can cause over or under filtering when the signal frequency is changing, such as in transient vehicle manoeuvres. Hence, the advantage of a UKF, of having model based filtration, are particularly noticeable in highly transient manoeuvres.

There are many types of Kalman filters, see section 2.5. From these, the Unscented Kalman filter was chosen over the Extended Kalman filter, as the EKF needs linearised equations. Hence, the vehicle model would have been simplified and would not have been able to replicate behaviour in the required transient manoeuvres, reducing the accuracy of the model predictions.

The algorithm was implemented in Matlab by using basic functions by Simo Särkkä for validation [36].

3.6.2 Model

The model used and implemented in the UKF was the non-linear bicycle model with brush model, see section 3.5.1. The solver to calculate the states with the bicycle model
was a fourth order Runge Kutta, see equations (3.26) to (3.34).

\[ x_1 = x \quad (3.26) \]
\[ dx_1 = f_x(x_1) \cdot dt \quad (3.27) \]
\[ x_2 = x + 0.5 \cdot dx_1 \quad (3.28) \]
\[ dx_2 = f_x(x_2) \cdot dt \quad (3.29) \]
\[ x_3 = x + 0.5 \cdot dx_2 \quad (3.30) \]
\[ dx_3 = f_x(x_3) \cdot dt \quad (3.31) \]
\[ x_4 = x + dx_3 \quad (3.32) \]
\[ dx_4 = f_x(x_4) \cdot dt \quad (3.33) \]
\[ x_{\text{update}} = x + \frac{1}{6} \cdot (dx_1 + 2dx_2 + 2dx_3 + dx_4) \quad (3.34) \]

Where,
- \( x \) is the vector with states
- \( f_x \) is the model equation
- \( dt \) is the discrete time step

### 3.6.3 Tuning

The UKF needed tuning of the noise covariance matrices \( Q_{\text{UKF}} \) and \( R_{\text{UKF}} \) to achieve the best possible signal. The filter was tuned to previous winter measured data. To get the steady-state and the transient behaviour, data for CR and FR was used. The measurement noise covariance was given by the manufacturer of the IMU that was used during the tests. Also a standstill test of the IMU was performed during about 30 minutes. From the standstill test the actual noise could be calculated and then the covariance was calculated from that data. The noise is the deviation from the mean or zero in the case of standstill depending on the inclination of the position. The test was not performed on a surface that was completely flat so the noise of the mean was determined. From the tuning it was seen that the manufacturer data was a bit optimistic probably because when they were determined the conditions were optimal; indoors, no wind, totally flat etc.

According to the manufacturer of the IMU the error was 0.01 m/s\(^2\) and that the noise covariance then was 0.01\(^2\) m/s\(^2\). When used at the track with different disturbance it was proposed to be a bit higher, 0.03\(^2\) m/s\(^2\). From the standstill test the noise was found by taking away the mean to be sure of having noise around zero and then the covariance was calculated for the time series. The resulting noise covariance were,

- \( v_x = 1.0868 \cdot 10^{-5} \)
- \( v_y = 1.5362 \cdot 10^{-5} \)
- \( \dot{\psi} = 1.1655 \cdot 10^{-5} \)

Tuning for these values made the signal similar to the measurement data i.e. no noise reduction, because of the low noise covariance. If more reliability was put on the model, the matrices in the algorithm got singular, i.e. it was not possible to filter the signal.
Another approach was needed. Instead the manufacturer noise covariance values were used and tuned.

The processed noise covariance was not straight forward to determine so \( Q_{UKF} \) was determined by tuning the UKF with measurement data. It was tuned to make the filtered signal have as little noise as possible, but still not deviate from the real signal. The tuning of \( Q_{UKF} \) and \( R_{UKF} \) was performed for all three vehicle configurations, Ref, Veh 1 and Veh 2. The noise covariance of the measured data was also needed to be tuned to get less noise of the filtered signal without any offset from the measured data. The resulting \( Q_{UKF} \) and \( R_{UKF} \) matrices after tuning are presented in equations (3.35) and (3.36).

\[
Q_{UKF} = \begin{bmatrix}
0.05 & 0 & 0 & 0 & 0 \\
0 & 0.01 & 0 & 0 & 0 \\
0 & 0 & 0.001650 & 0 & 0 \\
0 & 0 & 0 & 0.001650 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\] (3.35) \hspace{1cm}
\[
R_{UKF} = \begin{bmatrix}
0.0971^2 & 0 & 0 & 0 & 0 \\
0 & 0.0971^2 & 0 & 0 & 0 \\
0 & 0 & 0.0971^2 & 0 & 0 \\
0 & 0 & 0 & 10^{12} & 0 \\
0 & 0 & 0 & 0 & 10^{12}
\end{bmatrix}
\] (3.36)

The covariance for the calculated forces with relaxation length was put to zero in the \( Q_{UKF} \) matrix and \( 10^{12} \) in the \( R_{UKF} \) matrix. This because only the calculations are trusted so then the noise was set to zero and no measurements exist so then they were put to a large number that was tuned.

3.6.4 Comparison with low-pass filter

In order to see if the UKF could perform better than the existing filters in the metrics calculations, the filter was compared to a high order Butterworth filter for different cut-off frequencies, 8 and 10 Hz. The state-of-the-art method of signal filtering at the time was Butterworth low-pass filter with 10 Hz cut-off frequency. The two filters were compared for steady-state and transient manoeuvre data, CR and FR, from previous winter tests. The signal amplitude, the noise of the signal and if the filtered signal was following the real one with no or small offset were compared.

3.7 Statistical analysis

3.7.1 Analysis of variance (ANOVA)

Analysis of variance (ANOVA) was used to determine the robustness of metrics. The null hypothesis formulated was that, \textit{the configurations are the same}. The variance in the summer metrics were analysed to see if they would obtain a \( p \)-value less than 5\%, i.e. \( p < 0.05 \). This would then mean that, the metric is able to discard the null hypothesis and can differentiate between the configurations. If the \( p \)-value is higher, then the spread is too great and the configuration change is not statistically significant.
Meaning that the metric cannot discard the null hypothesis. Thus, it is not able to
differentiate between the configurations.

ANOVA was also performed for the reference vehicles, i.e Ref 1, 2 and 3. The null
hypothesis was the same in this case as well, that the configurations are the same. But
here, a high \( p \)-value was required so that a metric does not discard the null hypothesis.
If the \( p \)-value is low, the metric is not robust and is affected by noise from changing
surface conditions. Finally also an ANOVA analysis was performed between Ref 3 and
Veh 3 which were two different vehicle but the same standard configuration. This was
done to see if there was a statistical difference between the same configuration but dif-
ferent vehicles. This was not done for SWD because the runs for Veh 3 from previous
winter test had shorter manoeuvre time, so the metrics could not be calculated. The
data used for ANOVA was; measurement data from previous winter testing and mea-
surement data for the same vehicle and manoeuvres but in summer from 2015.

Four different outcomes were possible from ANOVA, see Table 3.2. The wanted outcome
was high change for vehicle configurations (\( p_{\text{Veh}} < 0.05 \)) and high noise for references
(\( p_{\text{Ref}} >> 0.05 \)). In that case it is clear that there is a change in vehicle behaviour due
to configuration and that the reference vehicles are the same. In the extreme case of
having change between references and noise in vehicle configurations there could be
no conclusion, it would seem extremely strange to have change in the same vehicle
but not in different vehicle configurations. The third case was to have noise in both
reference and vehicle configurations. In that case the results would be so influenced
by the noise that nothing could be said more than that noise is changing more than the
difference in configuration. The fourth outcome was to have change in both references
and vehicle configurations. In order to see if the change was largest between references
or vehicle configurations, the \( p \)-values could be compared and the the lowest \( p \)-value
would probably have the largest impact on the change.

Table 3.2: Possible outcomes from analysis of variance (ANOVA) of references and vehicle
configurations (Config). The arrow pointing up ↑ indicates high \( p \)-value and the arrow pointing
down ↓ indicates low \( p \)-value.

<table>
<thead>
<tr>
<th>Ref/Veh</th>
<th>↑ ( p )-value (Noise)</th>
<th>↓ ( p )-value (Change)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref/Ref</td>
<td>↓ ( p )-value (Noise)</td>
<td>Higher noise than</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Config changes</td>
</tr>
<tr>
<td>↑ ( p )-value (Noise)</td>
<td></td>
<td>Low surface change</td>
</tr>
<tr>
<td></td>
<td></td>
<td>and High config change</td>
</tr>
<tr>
<td>↓ ( p )-value (Change)</td>
<td></td>
<td>Change in surface?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>or Change in config?</td>
</tr>
</tbody>
</table>

The calculation of these statistical values was performed using the inbuilt Matlab
\texttt{anova1} function, to calculate the one-way \( p \)-value.
### 3.7.2 Sensitivity analysis

One of the aims of the thesis was to have robust metrics. It was known that the signal to noise ratio in winter is very low. One method to increase this was to reduce the noise in the measurement, another approach was to increase the signal strength. With respect to the thesis, this refers to the difference between the reference and configuration vehicles. A larger difference gives the possibility that despite the noise, the measurements from these different vehicles will not overlap. To achieve this, the manoeuvres were modified to amplify the differences between the vehicles.

A factorial design procedure was chosen along with a sensitivity analysis to study the effects of multiple factors on the differences between metrics from the reference and configuration vehicles. The effect of the combination of factors was also a point to investigate. This was performed for the FR, TRIT and SWD manoeuvres. The CR manoeuvre was not considered because of only one parameter to change, the speed profile, and simulation time was long. The metrics for these manoeuvres were then calculated using the corresponding Sympathy flow for the Ref, Veh 1 and Veh 2. For the analysis the standard metrics were used. The differences between the metrics from Ref and Veh 1, equation (3.37), as well as between Ref and Veh 2, equation (3.38), were calculated and used in the sensitivity analysis. The aim was to increase the magnitude of the difference for these two combinations.

\[
\Delta_1 = |M_{Veh1} - M_{Ref}| \tag{3.37}
\]

\[
\Delta_2 = |M_{Veh2} - M_{Ref}| \tag{3.38}
\]

Where,
- \( \Delta \) is the absolute difference in metric
- \( M \) is a metric calculated from the Sympathy flow

For the FR manoeuvre, the factors to be investigated were the longitudinal speed, \( v_x \), that the manoeuvre was conducted at and the maximum SWA amplitude of the sinusoidal wave. The two-level, full factorial design matrix was chosen, see Table 3.3. Hence, by a series of pre-tests, the values of the two levels were decided upon, see Table D.28 in Appendix D. As the difference between the reference and configuration vehicles was noticed to be small from the previous winter expedition, a higher speed and higher SWA amplitude were chosen for investigation.

<table>
<thead>
<tr>
<th>Run</th>
<th>( v_x )</th>
<th>SWA</th>
<th>( v_x ) · SWA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

In the SWD manoeuvre, pre-tests were made to determine the effect of the parameters and hence, the factors to be modified were the first SWA peak \( \text{SWA}_1 \), the second SWA
peak $\text{SWA}_2$, the dwell time in the second SWA peak $t_{\text{dwell},2}$ and the longitudinal speed, $v_x$. The levels of the factors were chosen from series of pre-tests that narrowed down the range to the values shown in Table D.29 in Appendix D. As the number of factors were high, a full factorial matrix would require too many tests, $2^4 \cdot 3 = 48$, as each run for the three vehicle combinations. Hence a partial factorial matrix was decided, see Table 3.4.

**Table 3.4: Factorial design matrix: sine with dwell.**

<table>
<thead>
<tr>
<th>Run</th>
<th>SWA$_1$</th>
<th>SWA$_2$</th>
<th>$t_{\text{dwell},2}$</th>
<th>$v_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

For the TRIT manoeuvre, the factors that could be modified were the longitudinal speed, $v_x$, SWA amplitude and the gear. The gear was modified to determine the average longitudinal deceleration after the throttle release event. Here as well a two-level, full factorial design matrix was selected, see Table 3.5. The levels were decided to investigate a higher speed, a lower SWA amplitude and two different gears representing high and low amounts of engine braking, see Table D.30 in Appendix D.

**Table 3.5: Factorial design matrix: throttle release in turn.**

<table>
<thead>
<tr>
<th>Run</th>
<th>$v_x$</th>
<th>SWA</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

The tests were performed and the metrics calculated for all the vehicle combinations. The absolute value of the difference between each metric for the reference and vehicle combinations was calculated. The result of this was used in the sensitivity analysis and displayed in a Pareto chart. This gave the factor which had the largest effect on the difference in the metrics.
Post the analysis, the manoeuvre was simulated again for Ref, Veh 1 and Veh 2. The required space and if the vehicle was spinning out, was checked. A noise analysis was performed to check robustness against velocity change of the manoeuvre (noticed in the previous winter test). This was done by getting the noise from multiple tests from previous winter tests and adding that noise to the speed for the manoeuvre. Then simulating the runs with speed with noise $v_\xi$ and the robustness of the manoeuvre could be determined. The noise was determined by taking away the mean value from the data vector. Then the noise was added to the specific speed for each manoeuvre, see equations (3.39) and (3.40).

$$\xi = V - \text{mean}(V)$$  \hspace{1cm} (3.39)

$$v_\xi = v_{\text{spec}} + \xi$$ \hspace{1cm} (3.40)

Where,
- $\xi$ is the noise
- $V$ is the vector with data
- $v_{\text{spec}}$ is the specific speed for a certain manoeuvre

### 3.8 Modification of manoeuvres

The winter manoeuvres that were performed were different from the standards specified. Hence, to check and modify the manoeuvres to suit the conditions in winter, tools such as the sensitivity analysis were used. To facilitate this, the parameters of the manoeuvre had to be specified and checked so as to consider the limitations due to the conditions. The modified manoeuvres were simulated in CRT. In order to reduce time and effort for implementation of multiple manoeuvres, a Matlab script was made to modify the parameters of the manoeuvres as well as record these in .dcd files to be used in CRT.

The basic modifications considered for all manoeuvres were,

- The range of lateral acceleration for linear behaviour.
- The lateral acceleration limit.
- Steering wheel angle amplitude.
- The longitudinal speed the manoeuvre was executed with.
- The space required for the manoeuvre.

The modification of manoeuvres is described below for each manoeuvre.

#### 3.8.1 Constant radius manoeuvre

The parameters that define the manoeuvre are as follows,

- Method - Path following v/s Steering wheel angle input.
- Speed profile
- Radius

The method and the radius were decided from limitations of the track, in this case path following method was chosen. Hence the speed profile, including the initial speed as well as rate of increase of speed, was modified. The aim was to have a smooth, gradual
increase of speed to maintain close to quasi-static conditions at all times. This would also reduce the harsh control of the acceleration robot reducing the longitudinal slip that can occur on the wheels. The initial speed was to be chosen to reduce the time required for the manoeuvre as well as obtain the information of the linear region of the vehicle, see Figure 3.13 (Figure D.28 in Appendix D).

![Design of constant radius manoeuvre speed profile input.](image)

**Figure 3.13: Design of constant radius manoeuvre speed profile input.**

### 3.8.2 Frequency response manoeuvre

The parameters for the frequency response manoeuvre are,

- SWA amplitude
- SWA frequency range
- Rate of change of frequency
- Speed
- Gear
- Space required

The SWA input is the main factor to be changed in this manoeuvre. The frequency range was chosen to be similar to that mentioned in the standards as this covers the range that is of interest in vehicle handling studies. The rate of change of frequency was determined by fixing the time taken for the manoeuvre. The speed and SWA were determined by the sensitivity analysis with the added consideration of the space taken by the manoeuvre. Figure 3.14 shows the designed input for the FR manoeuvre (Figure D.29 in Appendix D).
3.8.3 Sine with dwell manoeuvre

The parameters that are used to define this manoeuvre are,

- SWA amplitude – positive and negative.
• Frequency of the sine wave.
• Dwell in the peaks.
• Speed
• Gear

Here as well, the SWA input is of prime importance. The frequency of the sine waves determine the rate of change of SWA between the peaks. The standard mentions that this parameter is closely related to the energy that can be built-up in the system [14]. This was selected to be the same as that of the standards. The speed, the SWA amplitudes and the dwell times were selected based on the sensitivity analysis. The Matlab program gave the option to modify the SWA amplitudes, which could be set different from each other, and the dwell times, see Figure 3.15 (Figure D.30 in Appendix D).

3.8.4 Throttle release in turn manoeuvre

![Figure 3.16: Design of throttle release in turn steering wheel angle input.](image)

The parameters that are used to define the TRIT manoeuvre are,

• SWA amplitude
• Speed
• Gear – longitudinal acceleration
• Throttle position
• Steady-state portion
Methodology

In this manoeuvre, the throttle release time and the recording before and after is of prime importance. The SWA amplitude was selected from analysing the maximum capabilities of the vehicle, see Figure 3.16 (Figure D.31 in Appendix D). The gear was selected to obtain deceleration for a reaction from the vehicle. Hence a lower gear to increase the braking due to engine inertia. The manoeuvre was designed so that the vehicle stabilized before and after the release event.

3.8.5 High g swept steer manoeuvre

A HSS manoeuvre was also designed, to be able to find linear range of the lateral acceleration v/s SWA. The linear range of the lateral acceleration was required for definition of metrics but also to determine suitable steering wheel angle inputs for new winter manoeuvres. The SWA input was designed in CRT to reduce the required space for the manoeuvre but also avoiding too high lateral acceleration rate.

3.9 Metrics calculation

3.9.1 Calculation tool

The metrics calculation tool used was Sympathy for Data. Flows specific for each manoeuvre were present. Though, these flows were made for data obtained from summer testing. The data files were pointed to by a system of .log files, the data was read into Sympathy and was processed by the nodes in the sequence that they were connected. The original flows were modified to suit the particular needs for metrics custom to winter conditions.

The first step was to configure the flows to read the data obtained from objective testing, which were in .txt files. Originally, the flows read .sydata which is one of the default formats of the program. Due to the lack of space and to prevent duplicity of data, the flow was modified to read the .txt files. The advantage of this is that Matlab also can read .txt files. As VI-CarRealTime exports data in another format, i.e. .csv, hence another flow was required to calculate metrics from simulation data.

The next step was to determine the linear lateral range in winter conditions for the calculation of metrics. A virtual HSS test was performed in VI-CarRealTime to this end. The linear range was needed in order to determine different linear gradients and to determine the SWA amplitudes for corresponding lateral acceleration values.

The corresponding ranges of lateral acceleration were modified in the flows. The data was then input and processed step-wise, to check for calculation errors. One issue was the short duration of the steady-state time in the TRIT manoeuvre. This caused some metrics to have a large spread and unreasonable values. Another issue noticed, was the cut-off frequency in the flow for the FR manoeuvre. This was set to a very low value and could cause aliasing. Hence the value of this was increased to 10 Hz.

The next step was to calculate the metrics for the data from the winter expedition, as well as simulated runs in CRT. The robustness of the metrics was analysed as described
Methodology

in the following section.

Guidance for metrics calculation tool for the vehicle manufacturer is presented in section C.3 in Appendix C.

3.9.2 Selection of metrics

In order to determine robust metrics for winter conditions. The metrics from ISO, NHTSA and manufacturer standards were analysed. This was done using ANOVA, see section 2.8 and 3.7.1.

In order to have robust metrics the $p$-value should be high for references to $not$ have change and low for vehicle configurations to have change between configurations. The significance level is normally 5%, see section 2.8, but metrics with $p$-values below 20% were considered.

First the summer metrics were checked. If a difference was not noticed in summer data then it would probably not be seen in winter due to more noise. On the other hand, difference in winter maybe not can be seen in summer. This was also considered. Then the $p$-values for reference data from previous winter test were considered to get the signal-to-noise ratio for the metrics. In other words checking the robustness of the metric. Next the vehicle configuration $p$-values were checked to see if a difference in configurations could be seen in winter. The last step was to check if the metrics were calculable for simulation data and also if the change between Ref, Veh 1 and Veh 2 was seen. This was done for absolute and relative values. The relative metric values were calculated as in equation (3.41) and (3.42).

$$\Delta_{\text{rel, Veh1}} = M_{\text{Veh1}} - M_{\text{Ref}}$$  \hspace{1cm} (3.41)

$$\Delta_{\text{rel, Veh2}} = M_{\text{Veh2}} - M_{\text{Ref}}$$  \hspace{1cm} (3.42)

Where,

- $\Delta_{\text{rel}}$ is the relative change in metric between vehicle configuration and Ref
- $M$ is the metric value

When the ANOVA analysis and the relative changes in metrics were calculated the metrics were ranked in three levels; ROBUST, POSSIBLE and NOISY. The metrics rated as ROBUST had appropriate $p$-values and change in the metric for the simulated manoeuvres was observed. Some ROBUST metrics were also stared (⋆) which meant that they were performing very good, i.e. lower than 5% significance level and showed large difference in simulation metrics. If the metric was rated POSSIBLE one of the ANOVA analysis showed poor $p$-values but still a change in simulation data was seen. Finally the rating NOISY indicated that multiple $p$-values were low and the metric did not perform good. Some of the metrics rated as NOISY were also related to e.g. time and turning direction. These metrics were used to calculate other metrics and did not give any information about handling.

Defining new metrics were not considered as a large number of metrics from public and manufacturer standards were available and could be reused.
4 Results

In this chapter the results achieved by performing the methodology in the thesis are presented. Thus, giving the reader a detailed description of the outcomes of the procedures executed in the project.

4.1 Comparison of vehicle configurations

As mentioned, the comparisons between two vehicles can be with regard to multiple factors. In the following section, the graphs of only two combinations are shown.

The combinations chosen were, Veh 1 v/s Veh 2 and Ref 3 v/s Veh 3. These combinations were selected as, Veh 1 and Veh 2 are the configuration vehicles, with no ARB rear and front respectively. Hence, their behaviour was assumed to be the most different. The reference combination, Ref 3 v/s Veh 3 as seen in Table 3.1, have only the parameter of being different vehicles (vehicle components tolerance). Hence, these should be the same in theory. The comparison was done by visually comparing the signals of yaw rate $\dot{\psi}$, lateral acceleration $a_y$ and body slip angle $\beta$ between the combinations, for all the manoeuvres.

4.1.1 Constant radius manoeuvre

In the CR manoeuvre, as mentioned, the body slip angle and the cross plot of SWA and lateral acceleration were compared, see Figure 4.1 and 4.2. In the Figure 4.1a, it can
be seen that the body slip angle for the Ref 3 and Veh 3 are very similar. For complete figures, see Figure D.3 and D.4 in Appendix D. Both signals have the downward trend close to the 80% completion mark, though with differing slopes. Whereas, for the Veh 1, it can be seen that the trend in the same region is to first increase and then decrease. For the Veh 2, the trend is to remain constant, ending in fluctuations. As seen in Figure 3.7a for the spread in reference vehicle, the trend does not vary much.

![Comparison of cross plot SWA/\(\alpha_y\) for constant radius manoeuvre.](image)

### 4.1.2 Frequency response manoeuvre

In the FR manoeuvre, since the time domain plots are quite noisy, the comparison was performed in the frequency domain. This is also advantageous as the metrics for this manoeuvre are calculated in this domain. The bode plots of the yaw rate and the lateral acceleration were made. For complete figures, see Figure D.6 and D.7 in Appendix D. From these plots, almost no noticeable difference is present. It must be noted that the plots are the comparison between the Veh 1 and Veh 2, which are the most different. It was the same case with the other comparisons as well. The signals in the frequency domain were too similar to make any differentiation.

### 4.1.3 Sine with dwell manoeuvre

In the SWD manoeuvre, the yaw rate showed differences between vehicles, see Figure 4.3 (Figure D.8 in Appendix D). For the comparison between Ref 3 and Veh 3, the signals seem to be quite close. It was the same for the comparison between the Veh 1 and Veh 2, though there was an outlier for Veh 1. Whereas, for the comparison between Ref 2 and Veh 2 a difference can be noticed, see Figure 3.5a. It was noted that in some cases, the outliers made it difficult to estimate the trend. Because of the few tests conducted. Thus, making it hard to determine a difference between vehicles.
Results

The body slip angle was compared as well in the SWD manoeuvre. As can be seen in Figure 4.4 (Figure D.9 in Appendix D), the differences between Ref 3 and Veh 3 are quite small, apart from the outlier. It is the same case for the comparison of Veh 1 and Veh 2. As seen from the spread of references, see Figure 3.10, the magnitude of spread in the reference vehicle is as large as the difference in reference and vehicle configuration.
4.1.4 Throttle release in turn manoeuvre

In the TRIT manoeuvre, the signals of yaw rate and the body slip angle were used for comparisons, see Figure 4.5 and 4.6 (Figure D.10 and D.11 in Appendix D). In the case of the yaw rate, the signal was noisy, but the general slope was compared. For both pairs of comparisons, the slope was quite similar.

The body slip for the TRIT manoeuvre had quite different behaviour. Though as in the case of Veh 1 and Veh 2, the behaviour differs for the same vehicle.

Figure 4.5: Comparison of yaw rate for throttle release in turn manoeuvre.

Figure 4.6: Comparison of body slip angle for throttle release in turn manoeuvre.
4.2 Spread of reference vehicle

The spread of yaw rate $\dot{\psi}$, lateral acceleration $a_y$ and body slip angle $\beta$ between Ref 1 to Ref 10 are presented for each manoeuvre, see section 3.3. For CR there were only Ref 1 to Ref 6.

4.2.1 Constant radius manoeuvre

The signals of $\dot{\psi}$, $a_y$ and $\beta$ were compared to check for the spread in the reference vehicle. It was seen that there was minimal spread, apart from the fluctuations at the end of the run, refer section 3.3.1.

4.2.2 Frequency response manoeuvre

The spread in FR manoeuvre for $\dot{\psi}$, $a_y$ and $\beta$ was not noticeable in the frequency domain. In the time domain, some differences were noticed in the low frequency region, but these were mainly due to differences in SWA input, refer section 3.3.2 in methodology.

4.2.3 Sine with dwell manoeuvre

In the SWD manoeuvre, the yaw rate slope was compared for the spread in the reference vehicle, see section 3.3.3. The distribution of the slope was analysed to check for normal distribution, see Figure 4.7a. As this did not fit well, a lognormal distribution was checked, see Figure 4.7b. It can be seen that the data fits this distribution better. Hence, this distribution was used to analyse the spread in the data.

![Figure 4.7: Probability check curves for yaw rate data for sine with dwell.](a) Normal probability for yaw rate. (b) Lognormal probability for yaw rate.)

The lognormal probability density of the data was plotted, see Figure 4.8a (Figure D.15a in Appendix D). It can be noticed that the data is present in the low probability regions
as well. It was noticed from Figure 3.9b, that the data from the two days of testing had differing spread. Hence the probability function was plotted for the two data from the two days separately, see Figure 4.8b (Figure D.15b in Appendix D). Resulting in the data being more concentrated in the higher probability regions.

![Graphs](image)

(a) Lognormal distribution for yaw rate for all runs on both days.  
(b) Lognormal distribution of yaw rate, separated for the two test days.

**Figure 4.8: Distribution curves for yaw rate data for sine with dwell.**

![Graphs](image)

(a) Normal probability of body slip angle.  
(b) Lognormal probability of body slip angle.

**Figure 4.9: Probability check curves of body slip angle data for sine with dwell.**

A similar method was used for the spread in peak-to-peak value of body slip angle. The normal and lognormal distributions were checked, see Figure 4.9a and 4.9b. It can be noticed that the lognormal distribution fits the data better. Thus, in this case as well, the probability density function was plotted first for all the data together, see Figure
Results

4.10a and then for the two days separately, Figure 4.10b (Figure D.16a and D.16b in Appendix D). It can be noticed that the data in day 1 was more spread than for day 2. This was also noticed from the time domain plots of the yaw rate and the body slip angle.

![Graphs showing distribution curves of body slip angle for all runs and separated for the two days.](image1)

Figure 4.10: Distribution curves of body slip angle for sine with dwell.

4.2.4 Throttle release in turn manoeuvre

![Graphs showing normal and lognormal probability of yaw rate.](image2)

Figure 4.11: Probability check curves of yaw rate data for the throttle release in turn.

In the TRIT manoeuvre, the slope of the yaw rate was checked for the spread in the reference vehicle. The data was checked for its distribution as done in the previous
section, see Figure 4.11a and 4.11b. It can be noticed that the data fits the normal distribution better. Hence this was used to analyse the spread.

The normal probability density of the data was plotted for the data of the two days together, Figure 4.12a and separately, Figure 4.12b (Figure D.18a and D.18 in Appendix D). It can be noticed that in the case where the data is separated, the concentration of data points is on the high probability regions.

![Normal distribution of yaw rate for the two test days.](image1)

![Normal distribution of yaw rate, separated for the two test days.](image2)

*Figure 4.12: Distribution curves of yaw rate for throttle release in turn.*

![Spread of throttle position for the two test days.](image3)

![Spread of longitudinal acceleration for the two test days.](image4)

*Figure 4.13: Difference in longitudinal characteristics for throttle release in turn.*

Finally, in Figure 4.13 the spread in throttle position and the longitudinal acceleration
are presented. It can be noticed that the throttle position signals are different between Day 1 and 2. Hence, the longitudinal accelerations were slightly different as well, see Figure 4.13b (Figure D.19a and D.19b in Appendix D).

4.3 Winter tyre modelling

4.3.1 Brush model for axle characteristics

The brush model was optimized initially with one run of the CR manoeuvre. Hence the performance of the model was checked against the measured values of yaw rate $\dot{\psi}$ and the lateral acceleration $a_y$. The performance of the model in the CR manoeuvre was checked, see Figure 4.14a and 4.14b. It can be noticed that the fit in the linear region is quite good, but with the fluctuations, the model is not able to represent the real data.

![Comparison of yaw rate.](image1)

![Comparison of lateral acceleration.](image2)

*Figure 4.14: Performance of brush model in constant radius manoeuvre.*
The performance of the model was also checked against the FR manoeuvre, see Figure 4.15a and 4.15b. It can be noticed that the amplitude of the response of the model increases with the decrease in frequency up to a certain point. This behaviour cannot be seen in the measurement data. The phase difference of the simulation and the model change with frequency as well. In the low frequency region, the model is able to better represent the behaviour of the real vehicle.

The model was used to simulate the SWD manoeuvre, as well. Here it was noticed that it had some instability, see Figure 4.16a and 4.16b. The model was not able to follow the behaviour of the real vehicle. Due to the instability, the model was then optimized with multiple manoeuvres to obtain an average model. Hence the performance of the current model was recorded, by listing the mean squared error of the yaw rate and lateral acceleration for all runs in CR, FR and SWD manoeuvres, see Table B.2, B.3 and B.4 respectively in Appendix B.
Post the optimization, the performance of the model was checked to see the improvements. For the CR and FR manoeuvres, the fit of the new model was not much different from the previous. Whereas, in the case of the SWD manoeuvre, there were drastic changes. The model was not unstable, though was not able to follow the behaviour of the real vehicle, see Figure 4.17a and 4.17b. The performance of the new model was recorded in the same manner, by listing the mean squared error of the yaw rate and the lateral acceleration, see Table B.2, B.3 and B.4. It was noticed that, for the CR and FR manoeuvres, the model is slightly compromised and the performance is better for the SWD manoeuvre. The final values of the constants used in the brush model are given in Table D.11 in Appendix D.

![Comparison of yaw rate and lateral acceleration](image)

**Figure 4.17:** Performance of brush model after second optimization in sine with dwell.

### 4.3.2 Magic Formula model for axle characteristics

The Magic Formula model given in equations (3.9) to (3.15) were used to model the behaviour of the real vehicle. As mentioned, the model was optimized to the yaw rate and lateral acceleration of a run in the CR manoeuvre. The performance of this model was hence checked. The final constants used are given in Table D.12 in Appendix D.

In the case of the CR manoeuvre, the model was able to represent the behaviour of the real vehicle, see Figure 4.18a and 4.18b. As can be noticed, the model follows the curve in the linear region quite well but is not able to follow in the fluctuations.
The FR manoeuvre was also used to assess the performance of the Magic tyre formula model, see Figure 4.19a and 4.19b. It can be seen that the behaviour of the model is similar to that seen with the brush model. The amplitude increase is seen here as well.
Results

The performance of the model was checked with the SWD manoeuvre, see Figure 4.20a and 4.20b. It can be noticed that a similar case of instability is present in this case as well.

(a) Comparison of yaw rate.

(b) Comparison of lateral acceleration.

*Figure 4.20: Performance of Magic Formula in sine with dwell manoeuvre.*

4.3.3 Magic Formula tyre model

The performance of the Magic Formula tyre model was checked with keeping the coefficients constant as described earlier. The model was able to fit the lowest load and zero camber case. On checking with the higher loads, the model had quite good performance. Though when the lowest load with positive and negative camber cases were checked, the model did not fit the measurement data.

Hence, the coefficients of camber were introduced, as seen in equations (3.16) to (3.22). The range of the constants were calculated as mentioned and then used in an optimization routine. The results were then checked. It could be seen that the lowest load cases fit for all three cambers. But the camber cases for the higher loads did not.

Therefore, the coefficients for normal load were introduced, as given in equations (3.23) to (3.25). It can be noticed that the lateral stiffness variation with load was not modelled, as the calculation for the range of the constants was not possible. The constants were optimized and hence the performance of this model was checked. It can be seen that the model fits the cases of normal load and camber quite well. Except in some regions, where there was noise in the measurement data.

The constants for the variation of lateral stiffness were then introduced as well and were optimized and tuned. The performance of this model was compromised as compared to that of the previous model. If the model fit for one load case, the fit for the other load cases was not good. Hence the previous model was used further.
At this point, the TNO tyre model was obtained as well. Hence the performance of this tyre model was compared to a professionally fit tyre model. It could be noticed that the tyre models were almost the same, see Figure 4.21. The TNO model was used further in the simulation with VI-CarRealTime as it also had the longitudinal effects and it was in the standard format for use with the software.

![Graphs showing comparison of fit for custom and TNO supplied tyre models](image)

Figure 4.21: Comparison of fit for custom tyre model and TNO supplied tyre model for stud-ded winter tyre on ice.

### 4.4 Unscented Kalman filter

The UKF was compared to a Butterworth high order low-pass for 8 Hz and 10 Hz. In Figure 4.22 the UKF v/s low-pass 8 Hz is presented and in Figure 4.23 the UKF v/s low-pass 10 Hz is presented. For complete figures, see Figure D.20 and D.21 in Appendix D. In both the figures the green signal is the real, the red is the UKF and finally the blue is the low-pass filter. The figures are presenting a transient FR manoeuvre in
the high frequency range.

It can be seen from comparing Figure 4.22 and 4.23, that the low-pass filter performs as well as the UKF for the higher cut-off frequency.

Figure 4.22: UKF v/s Butterworth high order low-pass filter with cut-off frequency 8 Hz.

Figure 4.23: UKF v/s Butterworth high order low-pass filter with cut-off frequency 10 Hz.
4.5 VI-CarRealTime simulation

4.5.1 VI-CarRealTime v/s bicycle model

To validate and compare which model that suited the measurement data from previous winter test the best, the CRT model and bicycle model were compared. This was done for reference data of yaw rate and lateral acceleration. The tyre model of the CRT model was the one fitted for the same tyre but a newer generation. From the comparison it was seen that the CRT model performed the better. The CR manoeuvre is presented in Figure 4.24 which can be compared to the bicycle model in Figure 4.14a and 4.14b (Figure D.22 in Appendix D). For the FR manoeuvre the CRT model is presented in Figure 4.25 and the bicycle model in Figure 4.15a and 4.15b (Figure D.23 in Appendix D). Finally the SWD manoeuvre is presented in Figure 4.26 for the CRT model and in Figure 4.17a and 4.17b for the bicycle model (Figure D.24 in Appendix D).

Figure 4.24: Constant radius: real v/s simulated data for VI-CarRealTime model.
Figure 4.25: Frequency response: real v/s simulated data for VI-CarRealTime model.

Figure 4.26: Sine with dwell: real v/s simulated data for VI-CarRealTime model.

4.5.2 Frequency response manoeuvre

The sensitivity to four different speed noise profiles for the vehicle configurations models, Veh 1 and Veh 2, were checked for yaw rate in FR manoeuvre in CRT, see Figure 4.27 (Figure D.25 in Appendix D). It can be observed that the speed noise does not affect the response.
4.5.3 Sine with dwell manoeuvre

Figure 4.27: Frequency response: speed following manoeuvre with four different speed noise profiles.

Figure 4.28: Sine with dwell: speed following manoeuvre with four speed noise profiles for vehicle configurations; Veh 1 and Veh 2.
The SWD manoeuvre was checked for sensitivity by running the vehicle configurations Veh 1 and Veh 2 for the speed mapping from a run performed in previous winter test. Also the same vehicle configurations were plotted with no throttle and with constant throttle application. The yaw rate was checked. The simulation was done for speed following, no throttle and constant throttle, see Figure 4.28 and 4.29 (Figure D.26 and D.27 in Appendix D). It was observed that in the case of no throttle, the differences between the vehicles was very small and could not be noticed.

### 4.6 Modification of manoeuvres

From the sensitivity analysis, the parameter most affecting the difference between the metrics of the reference vehicle and the configuration vehicle were obtained. These results, along with the the parameters identified in section 3.7.2, were simulated in CRT and used to propose modified manoeuvres. Apart from the four available manoeuvres, two other, i.e. HSS and straight line steady-state test (SLT), were proposed as well. These would be auxiliary manoeuvres, the metrics from these would be used to perform the main manoeuvres. A complete test plan for the vehicle manufacturer is presented in section C.2 in Appendix C. Also an estimate of required space for the modified v/s previous manoeuvres can be found in Figure C.1 in Appendix C.

#### 4.6.1 Constant radius manoeuvre

For the constant radius manoeuvre, the parameters that could be changed were quite few. Due to track limitations, path following method was chosen. Though, to reduce the time required for the manoeuvre, the initial speed was increased. This does not
affect the calculation of the metrics as well. Another advantage with this was that the manoeuvre could be performed with a higher gear which could reduce the throttle variation required to maintain the desired speed. The speed rate was intended to be reduced, to achieve a smoother increase in the longitudinal speed and hence less harsh control by the acceleration robot. But, the increase in time due to this was very high and hence the speed rate was kept unchanged. In Table 4.1 a comparison is made between the manoeuvre from the previous winter test and the modified manoeuvre (Table C.1 in Appendix C).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Previous</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>Path following, increasing speed</td>
<td>same as previous</td>
</tr>
<tr>
<td>Initial speed</td>
<td>$v_{x, in}$</td>
<td>$v_{in, mod} &gt; v_{x, in}$</td>
</tr>
<tr>
<td>Speed rate</td>
<td>Linearly increasing</td>
<td>same as previous</td>
</tr>
<tr>
<td>Gear</td>
<td>Unknown</td>
<td>Highest possible gear</td>
</tr>
</tbody>
</table>

4.6.2 Frequency response manoeuvre

For the frequency response manoeuvre, the parameters that were checked in the sensitivity analysis were, the speed and the SWA amplitude. From this, it was observed that the speed of the manoeuvre was the parameter that influenced most metrics to the largest extent, for both Veh 1 and Veh 2. This was followed by the combination of speed and SWA amplitude.

The effect of speed was then investigated and it was observed that increasing the speed increased the difference in the metrics. The combination on the other hand did not have a consensus with the metrics.

From the other parameters identified, the rate of change of frequency was kept unchanged, as it would increase the time required for the manoeuvre drastically. The longitudinal space required by the manoeuvre was largely dependant on the speed. Hence the increase in speed was limited to the space available on the track. Hence a compromise was made between the two parameters taking into account the specific size of the track that was available to the vehicle manufacturer. In Table 4.2 a comparison is made between the manoeuvre from the previous winter test and the modified manoeuvre (Table C.2 in Appendix C).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Previous</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>$v_x$</td>
<td>$v_{mod} = v_x + 10 km/h$</td>
</tr>
<tr>
<td>SWA amplitude</td>
<td>SWA</td>
<td>same as previous</td>
</tr>
<tr>
<td>Frequency rate</td>
<td>Fixed</td>
<td>same as previous</td>
</tr>
<tr>
<td>Frequency range</td>
<td>Standard</td>
<td>same as previous</td>
</tr>
</tbody>
</table>
4.6.3 Sine with dwell manoeuvre

As mentioned, for the sine with dwell manoeuvre, the parameters checked in the sensitivity analysis were the two SWA amplitudes, the second dwell and the speed of the manoeuvre. In this case, the second SWA amplitude was the parameter that affected the most number of metrics to the largest extent. This was followed by the speed.

The effect of the two factors was then investigated. For the second SWA amplitude, the consensus was that a higher amplitude gave larger differences for both Veh 1 and Veh 2. In the case of speed though, some metrics preferred a higher speed, where some preferred a lower speed.

From the other parameters, the throttle position and gear were important factors. To reduce the longitudinal slip in the tyres for the duration of the manoeuvre, it was decided to perform the manoeuvre in the highest gear possible. For the throttle position, a constant throttle was chosen, as this had shown to increase the difference between reference and configuration vehicle in simulation, see section 4.5.3. The SLT manoeuvre was hence chosen to determine the steady-state throttle position for a particular gear and speed. This throttle position was chosen to be used in the SWD manoeuvre. In Table 4.3 a comparison is made between the manoeuvre from the previous winter test and the modified manoeuvre (Table C.3 in Appendix C).

\begin{table}[h]
\centering
\caption{Manoeuvre design, previous v/s modified: sine with dwell.}
\begin{tabular}{l|l|l}
\hline
Parameter & Previous & Modified \\
\hline
Method & Speed following & Constant throttle \\
Speed & \(v_x\) & same as previous \\
Gear & Unknown & Highest possible gear \\
SWA amplitude steps & 8 & 4 \\
Frequency & Varying & Standard \\
First dwell time & \(t_{\text{dwell},1}\) & \(t_{\text{dwell},1,\text{mod}} < t_{\text{dwell},1}\) \\
Second dwell time & \(t_{\text{dwell},2}\) & \(t_{\text{dwell},2,\text{mod}} > t_{\text{dwell},2}\) \\
\hline
\end{tabular}
\end{table}

4.6.4 Throttle release in turn manoeuvre

For the TRIT manoeuvre, the parameters that were checked in the sensitivity analysis were, the SWA amplitude, the speed and the gear representing the longitudinal deceleration after the release event. From this it was observed that the speed was the factor that affected most metrics to the largest extent. It must be mentioned that this was the case for approximately 85% of the metrics for both Veh 1 and Veh 2. The other metrics were affected by the SWA amplitude and only a couple by the gear used.

On investigating further the effect of the speed, it was found that a higher speed favoured a larger difference between the reference and configuration vehicles for a large number of metrics. For the SWA amplitude, a higher amplitude resulted in a larger difference.
From the other parameters, the space required was an important factor due to track limitations. The longitudinal space available was larger than the lateral space available. Hence increasing the speed, but lowering the SWA amplitude would help in this regard. In Table 4.4 a comparison is made between the manoeuvre from the previous winter test and the modified manoeuvre (Table C.4 in Appendix C).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Previous</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>Constant SWA</td>
<td>same as previous</td>
</tr>
<tr>
<td>Speed</td>
<td>$v_x$</td>
<td>$v_{\text{mod}} &gt; v_x$</td>
</tr>
<tr>
<td>SWA amplitude</td>
<td>SWA</td>
<td>$\text{SWA}_{\text{mod}} &lt; \text{SWA}$</td>
</tr>
<tr>
<td>Recording time before throttle off</td>
<td>$t_b$</td>
<td>$t_{b,\text{mod}} &gt; t_b$</td>
</tr>
<tr>
<td>Steady-state time interval</td>
<td>$t_s$</td>
<td>$t_{s,\text{mod}} &gt; t_s$</td>
</tr>
<tr>
<td>Gear</td>
<td>Unknown</td>
<td>Low as possible</td>
</tr>
</tbody>
</table>

### 4.6.5 High g swept steer manoeuvre

The HSS is an auxiliary manoeuvre, to be used in order to define the inputs for the other manoeuvres, so as to scale the lateral acceleration to the same level.

In this manoeuvre, constant speed is maintained and the SWA input is increased at a rate so as to not exceed a specified rate of increase of lateral acceleration. The speed is determined as the speed to be used in the main manoeuvre (Table C.5 in Appendix C).

### 4.6.6 Straight line steady-state test

The SLT is used to determine the throttle position for a corresponding speed. This is used to determine the throttle position to be used in the SWD manoeuvre (Table C.6 in Appendix C).

### 4.7 Selection of metrics

As mentioned ANOVA was performed for the data from the vehicle configurations, the reference vehicles, between Ref 3 and Veh 3, and for the summer tests in 2015. These results are used to rate the metrics into three levels: Robust, Possible, and Noisy. Representing the performance of the metrics in ANOVA, as well as simulation data of the modified manoeuvres.

Robust was stated when all tests agree that the metric is robust and show difference in vehicle configurations. Possible was stated when one or two procedures did not agree and finally Noisy was when none of the tests showed robustness or difference. The selected metrics are presented for each manoeuvre in the following sections.
4.7.1 Constant radius manoeuvre

The ANOVA analysis was done for the three vehicle configurations for the previous winter test with resulting $p$-value in Table A.1 in Appendix A. For the references in Table A.2 and between Ref 3 and Veh 3 in Table A.3 in Appendix A. In Table A.4 in Appendix A the $p$-values for vehicle configurations from the summer test in 2015 are presented. For ANOVA tables with vehicle configurations the metrics marked with a star (*) have a $p$-value that discard the null hypothesis. Only public standard metrics are presented. For ANOVA tables for vehicle manufacturer metrics, see Table D.13 to D.16 in Appendix D. The metrics are hence rated and divided into the three mentioned levels, see Table 4.5. The rated vehicle manufacturer metrics are presented in Table C.8 in Appendix C.

Table 4.5: Metrics defined for winter conditions: constant radius. Body slip angle at CG is presented as $\beta_{CG}$.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Metrics</th>
<th>Plot</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robust</td>
<td>Steering wheel angle gradient</td>
<td>$\text{SWA}/a_y$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Roll angle gradient</td>
<td>$\text{Troll}/a_y$</td>
<td>–</td>
</tr>
<tr>
<td>Possible</td>
<td>Steering wheel torque gradient</td>
<td>$\text{SWT}/a_y$</td>
<td>–</td>
</tr>
<tr>
<td>Noisy</td>
<td>Path curvature gradient</td>
<td>$(1/R)/a_y$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Side slip angle gradient</td>
<td>$\beta_{CG}/a_y$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Steering wheel angle/Side slip angle gradient</td>
<td>$\text{SWA}/\beta_{CG}$</td>
<td>–</td>
</tr>
</tbody>
</table>

4.7.2 Frequency response manoeuvre

The ANOVA analysis was done for the three vehicle configurations for the previous winter test with resulting $p$-value in Table A.5 in Appendix A. For the references in Table A.6 and between Ref 3 and Veh 3 in Table A.7 in Appendix A. In Table A.8 in Appendix A the $p$-values for vehicle configurations from the summer test in 2015 are presented. For ANOVA tables with vehicle configurations the metrics marked with a star (*) have a $p$-value that discard the null hypothesis. Only public standard metrics are presented. For ANOVA tables for vehicle manufacturer metrics, see Table D.17 to D.20 in Appendix D. The metrics are hence rated and divided into the three mentioned levels, see Table 4.6. It can be mentioned that for the metrics from the vehicle manufacturer, the ones defined by yaw rate were observed to be more robust. The rated vehicle manufacturer metrics are presented in Table C.9 in Appendix C.

4.7.3 Sine with dwell manoeuvre

The ANOVA analysis was done for the three vehicle configurations for the previous winter test with resulting $p$-value in Table A.9 in Appendix A. For the references in Table A.10 in Appendix A. In Table A.11 in Appendix A the $p$-values for vehicle configurations from the summer test in 2015 are presented. For ANOVA tables with vehicle configurations the metrics marked with a star (*) have a $p$-value that discard the null
Table 4.6: Metrics defined for winter conditions: frequency response. Yaw rate is presented as $\dot{\psi}$.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Metrics</th>
<th>Plot</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROBUST</td>
<td>Phase angle time</td>
<td>$\dot{\psi}$/SWA</td>
<td>SWA as input, $\dot{\psi}$ as output</td>
</tr>
<tr>
<td>POSSIBLE</td>
<td>Phase angle time</td>
<td>$a_y$/SWA</td>
<td>SWA as input, $a_y$ as output</td>
</tr>
<tr>
<td></td>
<td>Lateral acceleration gain</td>
<td>$a_y$/SWA</td>
<td>–</td>
</tr>
<tr>
<td>NOISY</td>
<td>Yaw velocity gain</td>
<td>$\dot{\psi}$/SWA</td>
<td>–</td>
</tr>
</tbody>
</table>

hypothesis. Only public standard metrics are presented. For ANOVA tables for vehicle manufacturer metrics, see Table D.21 to D.23 in Appendix D. The metrics are hence rated and divided into the three mentioned levels, see Table 4.7. It can be noticed that no public metrics were robust enough, but multiple manufacturer metrics were. They were related to yaw rate, lateral acceleration and body slip angle. The rated vehicle manufacturer metrics are presented in Table C.10 in Appendix C.

Table 4.7: Metrics defined for winter conditions: sine with dwell.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Metrics</th>
<th>Plot</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROBUST</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>POSSIBLE</td>
<td>Maximum yaw rate ratio I</td>
<td>$t/\dot{\psi}$</td>
<td>1 s after COS</td>
</tr>
<tr>
<td></td>
<td>Maximum yaw rate ratio II</td>
<td>$t/\dot{\psi}$</td>
<td>1.75 s after COS</td>
</tr>
<tr>
<td>NOISY</td>
<td>Minimum lateral displacement</td>
<td>$t/y$</td>
<td>1.07 s after BOS</td>
</tr>
</tbody>
</table>

4.7.4 Throttle release in turn manoeuvre

The ANOVA analysis was done for the three vehicle configurations for the previous winter test with resulting p-value in Table A.12 in Appendix A. For the references in Table A.13 and between Ref 3 and Veh 3 in Table A.14 in Appendix A. In Table A.15 in Appendix A the p-values for vehicle configurations from the summer test in 2015 are presented. For ANOVA tables with vehicle configurations the metrics marked with a star (*) have a p-value that discard the null hypothesis. Only public standard metrics are presented. For ANOVA tables for vehicle manufacturer metrics, see Table D.24 to D.27 in Appendix D. The metrics are hence rated and divided into the three mentioned levels, see Table 4.8. The rated vehicle manufacturer metrics are presented in Table C.11 in Appendix C.
Table 4.8: Metrics defined for winter conditions: throttle release in turn. Yaw rate is presented as $\dot{\psi}$, body slip angle as $\beta$ and throttle position as TP.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Metrics</th>
<th>Plot</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROBUST</td>
<td>Maximum difference $\dot{\psi}$ and $\dot{\psi}_{\text{reference}}$</td>
<td>TP/\dot{\psi}</td>
<td>at throttle off</td>
</tr>
<tr>
<td></td>
<td>Ratio maximum $\dot{\psi}$</td>
<td>t/\dot{\psi}</td>
<td>to $\dot{\psi}<em>{\text{ref}}$ at $\dot{\psi}</em>{\text{max}}$</td>
</tr>
<tr>
<td>POSSIBLE</td>
<td>Maximum $\beta_{CG}$</td>
<td>t/\beta_{CG}</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Average $a_x$</td>
<td>t/a_x</td>
<td>during $t_0$ to $t_n$</td>
</tr>
<tr>
<td></td>
<td>Difference $\beta_{CG}$ and $\beta_{CG,\text{steady-state}}$</td>
<td>t/\beta_{CG}</td>
<td>at $t_n$</td>
</tr>
<tr>
<td></td>
<td>Difference $\dot{\psi}$ and $\dot{\psi}_{\text{reference}}$</td>
<td>t/\dot{\psi}</td>
<td>at $t_n$</td>
</tr>
<tr>
<td></td>
<td>Difference $\dot{\psi}$ and $\dot{\psi}_{\text{calculated}}$</td>
<td>t/\dot{\psi}</td>
<td>at $t_n$</td>
</tr>
<tr>
<td></td>
<td>Ratio $\dot{\psi}$</td>
<td>t/\dot{\psi}</td>
<td>to $\dot{\psi}_{\text{ref}}$ at $t_n$</td>
</tr>
<tr>
<td>NOISY</td>
<td>Ratio $a_y$ to reference $a_y$</td>
<td>t/a_y</td>
<td>at $t_n$</td>
</tr>
<tr>
<td></td>
<td>Yaw acceleration</td>
<td>t/\dot{\psi}</td>
<td>at $t_n$</td>
</tr>
<tr>
<td></td>
<td>Path deviation</td>
<td>t/y</td>
<td>at $t_n$</td>
</tr>
</tbody>
</table>
5 Discussion

In this chapter the results, the interpretations and the suggestions made are discussed.

5.1 Analysis of data

Initially the winter test data was analysed visually to determine trends, regions of interest and to get an idea of the noise that was present in the data. First, the comparison was made between vehicles to compare them. On noticing the variation in data for each vehicle, the spread of the reference vehicle was plotted and analysed. Hence a clearer picture was obtained about the signal-to-noise ratio in the data. The following sections discuss the results obtained for each manoeuvre from these analyses.

5.1.1 Constant radius manoeuvre

It was observed that the spread in the body slip angle data is minimal for this manoeuvre and at the same time, a difference can be noticed between the reference and configuration vehicle. Hence a fair estimate could be drawn about the steady-state behaviour of a vehicle from this manoeuvre.

One interesting observation, was that Veh 1 and Veh 2 had similar trends though in theory they should be opposite to each other.

The observation that the acceleration and steering robot fluctuations showed that the manoeuvre was not pure open-loop in nature. Increasing the amount of noise in the data. This also gave an indication of surface conditions. It was theorized that running on the same path with studded winter tyres polished the ice lowering the grip available. Which could explain the increase in fluctuations for the tests performed later in the day as compared to those performed earlier. Hence the repeatability of the tests was decreased.

5.1.2 Frequency response manoeuvre

The time domain data for the FR manoeuvre did not show any difference between the configurations. The data, when analysed in the frequency domain showed no difference as well. As the noise in the time domain was larger, further analysis was performed in the frequency domain.

The spread of data for the reference vehicle was checked in the frequency domain and here as well no difference could be observed. This gave an indication that the manoeuvre was not pushing the vehicles and hence no difference was being noticed.
5.1.3 Sine with dwell manoeuvre

The comparison of data from reference and configuration vehicles showed that different behaviours could be noticed. Though, the presence of outliers made it difficult to areas general trends. Hence the spread of data for reference vehicle was analysed. This showed that the magnitude of the spread was as large as that of the difference between vehicles. The analysis of the spread of data clearly showed the difference between the two test days. As well as the spread that can occur on a particular day. The cause of the differing spreads is open to speculation at the moment. But it seems likely that the temperature and cloud cover are major factors.

Another point to note is that many amplitude levels were tested, to determine a narrower range of interest. It was also noticed that the lower amplitude levels did not show much difference and the very high levels were unstable. Reducing the number of amplitude levels would increase efficiency of testing.

5.1.4 Throttle release in turn manoeuvre

From the comparison of the data between reference and vehicle configuration, it is quite difficult to determine or quantify differences due to the noise. In the case of body slip angle, there were three distinctly different behaviours that could be noticed. But, these were present for almost all vehicles. On analysing the behaviour for the reference vehicles, no distinguishing factor could be observed.

The statistical analysis of the slope of the yaw rate showed some interesting results. Showing clearly the difference between the two test days also with regard to the difference in the spread. Hence a deeper analysis into the cause of differing spread would be required.

5.2 Winter tyre modelling

To be able to simulate the behaviour of the vehicle in winter conditions, a model representing the behaviour of the tyre was required. This was attempted in two ways for two separate models, i.e. a simple bicycle model and a complex VI-CarRealTime one.

5.2.1 Axle model

An axle model was created for use in the bicycle model that would be used in the UKF as well as to check its performance. From the results that were obtained, it clearly shows that the simplified model is not able to represent vehicle behaviour in all cases. Hence to get a model that can represent both limit as well as linear behaviour, optimizing with all manoeuvres gives better results. Obtaining a more average model.

The SWD manoeuvre is quite transient as well as saturates the tyre to a large extent. The brush model is not able to represent this behaviour. A more complex Magic Formula model might be able to represent this better as it gives more control of the curve
It was also noticed that the tyre relaxation proved to be very effective at reducing the error between the simulation and the measurement. However there is still scope of improvement by using more complex models.

### 5.2.2 Tyre model

The winter tyre model that was fit using test rig data is noticed to be similar to the professionally fitted tyre model. However, in the case of the custom fit model, tyre hysteresis was not considered. Though the value seems to be small.

During the fitting process, the presence of local minima was noticed, where different combinations of the parameters resulted in similar curves.

### 5.3 Unscented Kalman filter

The Butterworth low-pass filter performed really well when having a cut-off frequency between 8 to 10 Hz. This because the frequency response manoeuvre had a steering wheel angle frequency range close to half the cut-off frequency. Meaning that if lower cut-off frequency was used, amplitude and frequency was lost in the higher SWA frequency range, also time lags were introduced. A cut-off frequency over 10 Hz did not remove enough noise. Generally the highest frequency in a signal doubled is the minimum as cut-off frequency. From this result it was decided to not use the UKF in the metrics calculations tool. Another disadvantage for the UKF was the required calculation time. The low-pass filter did not need as long time. Another disadvantage for the UKF is that for every vehicle analysed a custom model is required. This demands time and resources. The low-pass filter can be adapted to any signal. In order to use the UKF probably a more advanced model is needed, e.g. a two-track model. The disadvantage with the two-track model would probably be that the calculation time would increase even more.

### 5.4 K&C measurements

The K&C tests performed to obtain a model in VI-CarRealTime were quite different from the standard measurements performed. Hence some issues were noticed during the procedure. However, with prior knowledge of what tests are to be performed, most issues can be avoided. The K&C measurements were performed for the vehicle with both a summer tyre and an unstudded winter tyre on a larger rim. Due to a limit on the time, the full set of tests was not performed with the winter tyre. On comparing the K&C data a difference could be noticed between the two measurements. But on simulating the resulting models, there was a very small difference. This could also be as the lateral accelerations in the manoeuvres are low, i.e. 0.4 g. With higher forces, the differences may become larger.
5.5 VI-CarRealTime

The importation of the K&C data to obtain a complete model required a substantial investment of time and effort. It also depends largely on the accurate and correct measurement of the vehicle. The software used for importation was not without issues and post creation of the model, manual tweaking was required.

However, the created final model worked very well with the supplied tyre model and was able to represent the vehicle behaviour quite well. Though the tyre model was for a newer generation of the tyre used during testing.

5.5.1 Frequency response manoeuvre

One observation of the response of the model in the FR manoeuvre, was the variation of the amplitude with frequency was visible in the time domain plots. Which could not be seen in the measurement data. It could also be seen from simulation that the FR manoeuvre was robust to speed noise.

5.5.2 Sine with dwell manoeuvre

In the SWD manoeuvre on simulating the test runs, spread in data was observed. From this observation the conclusion was drawn that the speed and throttle control noise affected the results to a large extent. On simulating with no throttle and a constant throttle, a larger difference between reference and configuration vehicle was noticed for constant throttle.

5.6 Metrics

It was noticed that for most manoeuvres, the yaw rate was relatively robust. This was followed by the lateral acceleration and then the body slip angle. This gives an idea of the signals which are most important and which can be used to differentiate vehicles.

Due to a lack of time and availability of a large number of metrics, investigation into definition of new metrics was not carried out. However, using similar metrics from summer entails that analyses and tuning using these metrics could be used for winter as well and could be advantageous and reduce the tools needed to be developed.

5.7 Winter test plan

The development of objective testing methods would have to continue beyond the scope of this thesis. Hence a major goal was to provide suggestions for future winter expeditions. A large portion of this was to modify the manoeuvres to suit the conditions prevalent in winter, in general, and on the test track to be used, in particular. The other aspect is that the manoeuvre should make the difference between vehicles larger, to increase the signal-to-noise ratio. Apart from the definition of the manoeuvres, some auxiliary information was also collected that could aid in the successful execution of the tests. These are summed up in the following sections.
5.7.1 Constant radius manoeuvre

The CR manoeuvre was performed with path following making it not a pure open-loop manoeuvre. The fluctuations observed in the steering and acceleration robots are due to this. Hence it was proposed to perform this in an open-loop method such as constant speed/variable SWA. However, due to the large amount of space required, this would not have been possible to perform. Hence the suggestions were made to use a higher initial speed and hence a higher gear to damp the fluctuations on the driven wheels.

Another aspect of the fluctuations is the degradation of the surface by running over the same path. To reduce this as well as not require repeated surface preparations, the suggestion was made to change the centre of the circular path to be followed, see Figure 5.1. The problem could be that the vehicle behaviour is changed when crossing old paths. Another option could be to wear out the surface before the actual testing by running multiple pre-laps, e.g. 20 times. This to have a surface that cannot change as much.

![Figure 5.1: Overlapping run paths for constant radius manoeuvre by changing circle centre of the steering robot path. The black circle is the original path.](image)

5.7.2 Frequency response manoeuvre

It was noticed that in the FR manoeuvre, differences between vehicles could not be noticed. On further investigation it was noticed that the manoeuvre might have been too mild. The sensitivity analysis gave results to increase the differences between vehicles.

The speed and hence longitudinal space required was an important parameter as the track available was not completely straight. Due to this a manual input has to be given over the signal to the steering robot. If given too rapidly it could cause variation in behaviour of the vehicle.

5.7.3 Sine with dwell manoeuvre

The SWD manoeuvre was sensitive to noise from surface and throttle application. Hence the suggestion to perform it with a constant throttle at a high gear. The number of amplitude levels was reduced as well. Thus a larger number of tests can be run in the same amount of time. With which a better statistical result can be obtained.
The manoeuvre performed in the previous winter expedition had been modified from the standard by increasing the first dwell time and reducing the second dwell time. But on analysis of the manoeuvre, it was found that tending towards the standard in this regard increased the difference between vehicles.

5.7.4 Throttle release in turn manoeuvre

The cause of the different throttle positions between the two test days was not confirmed. The two possible causes seem to be either a different gear was used or the calibration of the acceleration robot was not correct. Hence it is suggested to check the gear used as well as the calibration of the robot before a set of runs.

The other observation was that the SWA amplitude used corresponded to larger than 60% of the maximum lateral acceleration. It was theorized that the effect of the high SWA input eclipses the reaction due to the release of the throttle. Hence it was suggested, despite the result from the sensitivity analysis, to reduce the SWA amplitude.

5.8 Sources of error

Sources of error that can be identified for the methodology processes in the thesis are presented below.

- Vehicle modelling
  - Vehicle mass and distribution.
  - Vehicle inertia.
  - Vehicle powertrain – e.g. gear ratio for deceleration during TRIT.
  - Not enough K&C data – only three load cases.
  - Data for a newer generation of tyre than that was used during the previous expedition was implemented.

- Tyre modelling
  - Fitting only yaw rate and lateral acceleration for the tyre model.
  - MF fit is not made for all load cases.
  - Not all constants are used for MF fit.
  - Genetic optimization algorithm was used but is still not a 100% secure model for finding the global minimum.

- Physical testing – steering robot
  - Vehicle mass and distribution.
  - Change in track surface - temperature, preparation and from driving multiple tests on the same track.
  - Tyre wear/tyre pressure during testing.
  - Gear or throttle position calibration that was used during TRIT. It was noticed that the throttle position was different between two test days for the manoeuvre. Hence, the calibration of the throttle position was not done or that a different gear was used on the two days.
  - Different steering robots were used for the two test vehicles.
In e.g. the FR manoeuvre two different operators were driving, triggering the robot with different initial vehicle direction and manual input was hence different.

• Physical testing – K&C rig measurements
  
  – Vehicle CG height – Corner weights were achieved by adding weight (metal plates) in different places, but the CG will not be correct. It would be higher in reality due to the mass distribution of a person in vertical direction resulted in different longitudinal deceleration.
  – Longitudinal acceleration compliance test was performed with straps to attach the wheel to the pad which will not be correct due to added forces. Instead braking compliance was used for both acceleration and braking in the vehicle model in CRT.
6 Future work

In this chapter recommendations for further analyses of the results and the methodology in the thesis are presented.

This thesis was an investigation into techniques that could be used to objectively assess vehicle handling in winter conditions. It is envisioned that the methods and results in this thesis are a base from which further improvements can be achieved. The following are areas which could be investigated into further:

- This thesis was limited to three configurations, analysis of the other configurations tested could be performed.
- The bicycle model with brush tyre model was too simple to represent the vehicle behaviour. Two distinct paths are present when introducing more complex parameters. The usage of which can be depending on the particular requirements and available information.
  - A complex tyre model with simple vehicle model. For example an optimized Magic Formula model with more parameters on a bicycle model.
  - A complex vehicle model with simple tyre model. For example, a two track vehicle model with brush tyre model.
- An algorithm could be used for the UKF to tune the noise covariance matrices, obtaining better precision.
- A complete K&C measurement for the vehicle, measuring the individual compliances for each wheel.
- Implementation of an accurate power steering model in VI-CarRealTime to simulate SWT.
- To check the feasibility of the modified manoeuvres with physical testing.
- Investigation of usage of reference vehicle to scale metrics of configuration vehicles. In order to achieve possibility of direct comparison between vehicles.
- To check spread of data obtained from physical testing with modified manoeuvres, to assess performance as compared to previous manoeuvres.
- To check the p-values from ANOVA of metrics obtained from modified manoeuvres. To check improvement of robustness in metrics.
- Implementation of a GUI system in the metrics calculation tool to increase ease of use.
- Tests including manoeuvres to assess steering metrics in winter conditions.
7 Conclusions

In this chapter the conclusions drawn, from the results obtained in the thesis, and the final thoughts on the topic are described.

The main goals of the thesis were to obtain new or modified metrics and manoeuvres, along with a winter tyre model, a tool for on-site evaluation and an improved test plan.

From the initial analysis of the data, the main issue with performing objective winter was apparent. The change in surface conditions causes a large spread in the data, whose magnitude is as large as the difference between vehicles. Due to the changing conditions, comparing vehicles tested at different times is harder still. A reference vehicle is required to determine the differences in surface. However no method is available to scale vehicle behaviour.

To replicate vehicle behaviour in simulation, a simple bicycle model with a brush tyre model is too simple. This is true at high slip angles in manoeuvres such as the sine with dwell. However, a full vehicle model obtained from K&C data with a professionally fit advanced tyre model can be used to replicate real vehicle behaviour in winter conditions.

For signal processing, the resources required to implement an unscented Kalman filter outweigh its potential benefits. A high-order Butterworth low-pass filter with appropriate cut-off frequency performs as well as the UKF. This is especially true in the case that the measured data is of high quality with low errors.

Some summer metrics are robust and could be used to objectively measure vehicle behaviour in winter conditions, with some modifications to the limits and ranges they are calculated at. In general yaw rate metrics were observed to be more robust to surface changes, while also being able to differentiate vehicle behaviour.

The manoeuvres to be used in winter testing had to be modified to increase the differences between the reference and configuration vehicles. The manoeuvres themselves should be performed as open-loop as far as possible, excessive control from the steering and the acceleration robots introduce significant amount of error into the tests. The biggest limitation to this, was the physical space available on the track to perform different manoeuvres safely.

Hence with the recommended modifications, it is believed that the signal-to-noise ratio in the metrics can be increased along with the statistical significance of the differences between the vehicles. Thus giving the opportunity to be able to differentiate between the vehicles. It is also hoped that improvements in the simulation will allow more use of CAE tools in the development of vehicles, in winter as well as summer conditions.
References


References


Appendix

This appendix includes extra data related to manoeuvres and metrics.

A.1 Selection of metrics

A.1.1 Constant radius manoeuvre

Table A.1: Vehicle configurations Analysis of variance (ANOVA) analysis previous winter test: constant radius. Stared $p$-values (*) are values that discard the null hypothesis.

<table>
<thead>
<tr>
<th>Metric</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steering wheel torque gradient 0.2 $g$</td>
<td>0.7682</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.3 $g$</td>
<td>0.8265</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.1 $g$</td>
<td>0.6881</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.05 $g$</td>
<td>0.4408</td>
</tr>
<tr>
<td>Steering wheel angle gradient from 0.05 to 0.14 $g$</td>
<td>0.9852</td>
</tr>
<tr>
<td>Roll angle gradient from 0.05 to 0.14 $g$</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table A.2: References Analysis of variance (ANOVA) analysis previous winter test: constant radius.

<table>
<thead>
<tr>
<th>Metric</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steering wheel torque gradient 0.2 $g$</td>
<td>0.8378</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.3 $g$</td>
<td>0.8058</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.1 $g$</td>
<td>0.8766</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.05 $g$</td>
<td>0.9145</td>
</tr>
<tr>
<td>Steering wheel torque gradient at max $a_y$</td>
<td>0.4026</td>
</tr>
<tr>
<td>Steering wheel angle gradient from 0.05 to 0.14 $g$</td>
<td>0.9852</td>
</tr>
<tr>
<td>Roll angle gradient from 0.05 to 0.14 $g$</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
### Appendix

#### Table A.3: Ref 3 vs Veh 3 Analysis of variance (ANOVA) analysis previous winter test: constant radius.

<table>
<thead>
<tr>
<th>Metric</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steering wheel torque gradient 0.2 g</td>
<td>0.0516</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.3 g</td>
<td>0.0637</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.1 g</td>
<td>0.0364</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.05 g</td>
<td>0.0339</td>
</tr>
<tr>
<td>Steering wheel torque gradient at max $a_y$</td>
<td>0.3739</td>
</tr>
<tr>
<td>Steering wheel angle gradient from 0.05 to 0.14 g</td>
<td>0.0052</td>
</tr>
<tr>
<td>Roll angle gradient from 0.05 to 0.14 g</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

#### Table A.4: Vehicle configurations Analysis of variance (ANOVA) analysis for summer test in 2015: constant radius. Stared $p$-values (⋆) are values that discard the null hypothesis.

<table>
<thead>
<tr>
<th>Metric</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steering wheel torque gradient 0.2 g</td>
<td>0.1509</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.3 g</td>
<td>0.1401</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.1 g</td>
<td>0.3115</td>
</tr>
<tr>
<td>Steering wheel torque gradient 0.05 g</td>
<td>0.2893</td>
</tr>
<tr>
<td>Steering wheel torque gradient at max $a_y$</td>
<td>0.4874</td>
</tr>
<tr>
<td>Steering wheel angle gradient from 0.05 to 0.14 g</td>
<td>0.1614</td>
</tr>
<tr>
<td>Roll angle gradient from 0.05 to 0.14 g⋆</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

#### A.1.2 Frequency response manoeuvre

#### Table A.5: Vehicle configurations Analysis of variance (ANOVA) analysis previous winter test: frequency response. Stared $p$-values (⋆) are values that discard the null hypothesis.

<table>
<thead>
<tr>
<th>Metric</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral acceleration gain at 3 $dB$ frequency</td>
<td>0.7208</td>
</tr>
<tr>
<td>Yaw velocity gain⋆</td>
<td>0.0360</td>
</tr>
<tr>
<td>SWA/$a_y$ phase angle time if no crossings</td>
<td>0.6256</td>
</tr>
<tr>
<td>SWA/$a_y$ phase angle time crossing</td>
<td>0.5900</td>
</tr>
<tr>
<td>SWA/Yaw rate phase angle time if no crossings</td>
<td>0.3487</td>
</tr>
<tr>
<td>SWA/Yaw rate phase angle time crossing⋆</td>
<td>0.0387</td>
</tr>
</tbody>
</table>

A.2
Table A.6: References Analysis of variance (ANOVA) analysis previous winter test: frequency response.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral acceleration gain at 3 dB frequency</td>
<td>0.0340</td>
</tr>
<tr>
<td>Yaw velocity gain</td>
<td>0.3707</td>
</tr>
<tr>
<td>SWA/$a_y$ phase angle time if no crossings</td>
<td>0.1723</td>
</tr>
<tr>
<td>SWA/$a_y$ phase angle time crossing</td>
<td>0.4360</td>
</tr>
<tr>
<td>SWA/Yaw rate phase angle time if no crossings</td>
<td>0.8056</td>
</tr>
<tr>
<td>SWA/Yaw rate phase angle time crossing</td>
<td>0.1578</td>
</tr>
</tbody>
</table>

Table A.7: Ref 3 vs Veh 3 Analysis of variance (ANOVA) analysis previous winter test: frequency response.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral acceleration gain at 3 dB frequency</td>
<td>0.6172</td>
</tr>
<tr>
<td>Yaw velocity gain</td>
<td>0.2237</td>
</tr>
<tr>
<td>SWA/$a_y$ phase angle time if no crossings</td>
<td>0.8459</td>
</tr>
<tr>
<td>SWA/$a_y$ phase angle time crossing</td>
<td>0.7083</td>
</tr>
<tr>
<td>SWA/Yaw rate phase angle time if no crossings</td>
<td>0.9515</td>
</tr>
<tr>
<td>SWA/Yaw rate phase angle time crossing</td>
<td>0.4202</td>
</tr>
</tbody>
</table>

Table A.8: Vehicle configurations Analysis of variance (ANOVA) analysis for summer test in 2015: frequency response.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral acceleration gain at 3 dB frequency</td>
<td>0.1102</td>
</tr>
<tr>
<td>Yaw velocity gain</td>
<td>0.3148</td>
</tr>
<tr>
<td>SWA/$a_y$ phase angle time if no crossings</td>
<td>0.1562</td>
</tr>
<tr>
<td>SWA/$a_y$ phase angle time crossing</td>
<td>0.0770</td>
</tr>
<tr>
<td>SWA/Yaw rate phase angle time if no crossings</td>
<td>0.1195</td>
</tr>
<tr>
<td>SWA/Yaw rate phase angle time crossing</td>
<td>0.0728</td>
</tr>
</tbody>
</table>


**A.1.3 Sine with dwell manoeuvre**

Table A.9: Vehicle configurations Analysis of variance (ANOVA) analysis previous winter test: sine with dwell.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw rate ratio 1 s after COS (I)</td>
<td>0.6278</td>
</tr>
<tr>
<td>Yaw rate ratio 1.75 s after COS (II)</td>
<td>0.1912</td>
</tr>
<tr>
<td>Lateral displacement 1.07 s after BOS</td>
<td>NaN</td>
</tr>
<tr>
<td>Lateral displacement at COS</td>
<td>NaN</td>
</tr>
</tbody>
</table>

Table A.10: References Analysis of variance (ANOVA) analysis previous winter test: Sine with dwell.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw rate ratio 1 s after COS (I)</td>
<td>0.4554</td>
</tr>
<tr>
<td>Yaw rate ratio 1.75 s after COS (II)</td>
<td>0.5884</td>
</tr>
<tr>
<td>Lateral displacement 1.07 s after BOS</td>
<td>NaN</td>
</tr>
<tr>
<td>Lateral displacement at COS</td>
<td>NaN</td>
</tr>
</tbody>
</table>

Table A.11: Vehicle configurations Analysis of variance (ANOVA) analysis for summer test in 2015: sine with dwell. Stared p-values (⋆) are values that discard the null hypothesis.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw rate ratio 1 s after COS (I)</td>
<td>0.0001</td>
</tr>
<tr>
<td>Yaw rate ratio 1.75 s after COS (II)</td>
<td>0.0021</td>
</tr>
<tr>
<td>Lateral displacement 1.07 s after BOS</td>
<td>NaN</td>
</tr>
<tr>
<td>Lateral displacement at COS</td>
<td>NaN</td>
</tr>
</tbody>
</table>

**A.1.4 Throttle release in turn manoeuvre**

Table A.12: Vehicle configurations Analysis of variance (ANOVA) analysis previous winter test: throttle release in turn.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum SSCG during observation period</td>
<td>0.6346</td>
</tr>
<tr>
<td>Ratio maximum yaw rate</td>
<td>0.1065</td>
</tr>
<tr>
<td>Average $a_x$</td>
<td>0.7552</td>
</tr>
<tr>
<td>Difference between SSCG at $t_n$ and steady state SSCG</td>
<td>0.4962</td>
</tr>
<tr>
<td>Difference yaw rate and reference yaw rate</td>
<td>0.2941</td>
</tr>
</tbody>
</table>
Table A.13: References Analysis of variance (ANOVA) analysis previous winter test: throttle release in turn.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum SSCG during observation period</td>
<td>0.5921</td>
</tr>
<tr>
<td>Ratio maximum yaw rate</td>
<td>0.6357</td>
</tr>
<tr>
<td>Average $a_x$</td>
<td>0.3569</td>
</tr>
<tr>
<td>Difference between SSCG at $t_n$ and steady state SSCG</td>
<td>0.5131</td>
</tr>
<tr>
<td>Difference yaw rate and reference yaw rate</td>
<td>0.3729</td>
</tr>
</tbody>
</table>

Table A.14: Ref 3 vs Veh 3 Analysis of variance (ANOVA) analysis previous winter test: throttle release in turn.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum SSCG during observation period</td>
<td>0.4953</td>
</tr>
<tr>
<td>Ratio maximum yaw rate</td>
<td>0.3271</td>
</tr>
<tr>
<td>Average $a_x$</td>
<td>0.0002</td>
</tr>
<tr>
<td>Difference between SSCG at $t_n$ and steady state SSCG</td>
<td>0.2599</td>
</tr>
<tr>
<td>Difference yaw rate and reference yaw rate</td>
<td>0.1632</td>
</tr>
</tbody>
</table>

Table A.15: Vehicle configurations Analysis of variance (ANOVA) analysis for summer test in 2015: throttle release in turn. Stared $p$-values (⋆) are values that discard the null hypothesis.

<table>
<thead>
<tr>
<th>Metric</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum SSCG during observation period⋆</td>
<td>0.0244</td>
</tr>
<tr>
<td>Ratio maximum yaw rate</td>
<td>0.1667</td>
</tr>
<tr>
<td>Average $a_x$⋆</td>
<td>0.0000</td>
</tr>
<tr>
<td>Difference between SSCG at $t_n$ and steady state SSCG</td>
<td>0.1197</td>
</tr>
<tr>
<td>Difference yaw rate and reference yaw rate⋆</td>
<td>0.0166</td>
</tr>
</tbody>
</table>
This appendix includes extra data related to tools and algorithms used in the thesis.

![Diagram showing error of inertial measurement unit (IMU) in body slip angle estimation](image)

**Figure B.1:** Error of inertial measurement unit (IMU) in body slip angle estimation, as a function of vehicle speed.

**Table B.1:** Test sequence, previous winter test, where \( i = 1, 2, 3, ... \)

<table>
<thead>
<tr>
<th>Manoeuvre sequence</th>
<th>FR</th>
<th>SWD</th>
<th>TRIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration</td>
<td>Ref ( i )</td>
<td>Ref ( i )</td>
<td>Ref ( i )</td>
</tr>
<tr>
<td></td>
<td>Veh ( i )</td>
<td>Veh ( i )</td>
<td>Veh ( i )</td>
</tr>
</tbody>
</table>

The test sequence in Table B.1 was repeated for each reference and vehicle combination; Ref 1 v/s Veh 2, Ref 2 v/s Veh 2 and Ref 3 v/s Veh 3, with a break between each set of testing.
Table B.2: Performance of brush and Magic Formula axle models for the constant radius manoeuvre.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Run</th>
<th>Brush model</th>
<th>Optimized brush model</th>
<th>Magic Formula model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\dot{\psi}$</td>
<td>$a_y$</td>
<td>$\dot{\psi}$</td>
</tr>
<tr>
<td>Ref 1</td>
<td>Run 1</td>
<td>0.066</td>
<td>0.130</td>
<td>0.073</td>
</tr>
<tr>
<td></td>
<td>Run 2</td>
<td>12.205</td>
<td>0.346</td>
<td>17.309</td>
</tr>
<tr>
<td></td>
<td>Run 3</td>
<td>0.102</td>
<td>0.381</td>
<td>0.105</td>
</tr>
<tr>
<td>Ref 2</td>
<td>Run 1</td>
<td>0.062</td>
<td>0.420</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td>Run 2</td>
<td>0.067</td>
<td>0.111</td>
<td>0.072</td>
</tr>
<tr>
<td></td>
<td>Run 3</td>
<td>0.089</td>
<td>0.181</td>
<td>0.109</td>
</tr>
<tr>
<td>Ref 3</td>
<td>Run 1</td>
<td>0.085</td>
<td>0.275</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td>Run 2</td>
<td>2.013</td>
<td>0.180</td>
<td>2.010</td>
</tr>
<tr>
<td></td>
<td>Run 3</td>
<td>0.095</td>
<td>0.124</td>
<td>0.105</td>
</tr>
</tbody>
</table>
Table B.3: Performance of brush and Magic Formula axle models for the frequency response manoeuvre.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Run</th>
<th>Brush model</th>
<th>Optimized brush model</th>
<th>Magic Formula model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\dot{\psi}$</td>
<td>$a_y$</td>
<td>$\dot{\psi}$</td>
</tr>
<tr>
<td></td>
<td>Run 2</td>
<td>36.739</td>
<td>7.860</td>
<td>36.454</td>
</tr>
<tr>
<td></td>
<td>Run 3</td>
<td>48.188</td>
<td>114.553</td>
<td>46.686</td>
</tr>
<tr>
<td>Ref 2</td>
<td>Run 1</td>
<td>88.326</td>
<td>17.328</td>
<td>72.240</td>
</tr>
<tr>
<td></td>
<td>Run 2</td>
<td>33.703</td>
<td>27.507</td>
<td>33.232</td>
</tr>
<tr>
<td></td>
<td>Run 4</td>
<td>5.244</td>
<td>198.939</td>
<td>5.196</td>
</tr>
<tr>
<td>Ref 3</td>
<td>Run 1</td>
<td>44.980</td>
<td>32.910</td>
<td>42.130</td>
</tr>
<tr>
<td></td>
<td>Run 3</td>
<td>70.672</td>
<td>1518.2</td>
<td>70.986</td>
</tr>
<tr>
<td></td>
<td>Run 4</td>
<td>245.662</td>
<td>76.668</td>
<td>221.033</td>
</tr>
</tbody>
</table>
Table B.4: Performance of brush and Magic Formula axle models for the sine with dwell manoeuvre.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Run</th>
<th>Brush model</th>
<th>Optimized brush model</th>
<th>Magic Formula model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\dot{\psi}$</td>
<td>$a_y$</td>
<td>$\dot{\psi}$</td>
</tr>
<tr>
<td>Ref 1</td>
<td>Run 1</td>
<td>1773.1</td>
<td>527.4</td>
<td>34.406</td>
</tr>
<tr>
<td></td>
<td>Run 2</td>
<td>934.6</td>
<td>180.6</td>
<td>18.324</td>
</tr>
<tr>
<td></td>
<td>Run 3</td>
<td>11583</td>
<td>156.7</td>
<td>16.818</td>
</tr>
<tr>
<td></td>
<td>Run 4</td>
<td>127.1</td>
<td>188.5</td>
<td>7.862</td>
</tr>
<tr>
<td>Ref 2</td>
<td>Run 1</td>
<td>724.5</td>
<td>679.4</td>
<td>2.154</td>
</tr>
<tr>
<td></td>
<td>Run 2</td>
<td>7555.2</td>
<td>220.0</td>
<td>3.0154</td>
</tr>
<tr>
<td></td>
<td>Run 3</td>
<td>9445.5</td>
<td>156.9</td>
<td>2.291</td>
</tr>
<tr>
<td></td>
<td>Run 4</td>
<td>249.6</td>
<td>49.8</td>
<td>11.601</td>
</tr>
<tr>
<td>Ref 3</td>
<td>Run 1</td>
<td>197.0</td>
<td>124.</td>
<td>2.257</td>
</tr>
<tr>
<td></td>
<td>Run 2</td>
<td>497.2</td>
<td>790.1</td>
<td>2.034</td>
</tr>
<tr>
<td></td>
<td>Run 3</td>
<td>2479.4</td>
<td>305.0</td>
<td>8.161</td>
</tr>
<tr>
<td></td>
<td>Run 4</td>
<td>882.9</td>
<td>471.3</td>
<td>8.3664</td>
</tr>
</tbody>
</table>