GEOMETRY, MECHANICS AND TRANSMISSIVITY
OF ROCK FRACTURES

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FOREWORD

This research work was accomplished at the Division of Engineering Geology, Royal Institute of Technology, KTH, Stockholm, during autumn 1996-spring 2001. A three months period in 1999 was spent at the Department of Civil, Environmental, and Architectural Engineering, University of Colorado at Boulder (CU), CO, USA.

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ABSTRACT

This Thesis work investigates methods and tools for characterising, testing and modelling the behaviour of rock fractures. Using a 3D-laser-scanning technique, the topography of the surfaces and their position with respect to one another are measured. From the fracture topography, fracture roughness, angularity and aperture are quantified; the major features used for characterisation. The standard deviations for the asperity heights, surface slopes and aperture are determined. These statistical parameters usually increase/decrease according to power laws of the sampling size, and sometimes reach a sill beyond which they become constant. Also the number of contact spots with a certain area decreases according to a power-law function of the area. These power-law relations reveal the self-affine fractal nature of roughness and aperture. Roughness is “persistent” while aperture varies between “persistent” and “anti-persistent”, probably depending on the degree of match of the fracture walls.

The fractal models for roughness, aperture and contact area are used to develop a constitutive model, based on contact mechanics, for describing the fracture normal and shear deformability. The experimental testing results of normal deformability are simulated well by the model whereas fracture shear deformability is not as well modelled. The model predicts well fracture dilation but is too stiff compared to rock samples. A mathematical description of the aperture pattern during shearing is also formulated. The mean value and covariance of the aperture in shearing is calculated and verifies reported observations.

The aperture map of samples is inserted in a numerical program for flow calculation. The “integral transform method” is used for solving the Reynolds’ equation; it transforms the fracture transmissivity pattern into a frequency-based function. This closely resembles the power laws that describe fractals. This function can be described directly from the fractal properties of the aperture, with noticeable economy of input data. The modelling also affirms that the density of the aperture data greatly affects the calculated flow.

KEYWORDS: anisotropy, angularity, anti-persistence, aperture, channelling, closure, contact area, contact mechanics, cross-correlation, deformability, dilation, flow, fractals, fracture, joint, laboratory, modelling, normal loading, persistence, power spectra, roughness, shearing, stationarity, tortuosity, transmissivity, 3D-laser scanning, variogram.
SOMMARIO

Questa Tesi di ricerca presenta lo studio di alcuni metodi e strumenti per la caratterizzazione, l’esecuzione di prove di laboratorio e la modellazione del comportamento delle fratture in roccia. La topografia delle superfici della frattura e la loro posizione reciproca sono digitalizzate con una tecnica di misurazione laser tridimensionale. La scabrezza, l’angolosità e l’apertura sono calcolate sulla base della topografia della frattura e costituiscono le principali proprietà impiegate per la caratterizzazione. La deviazione standard dell’apertura, dell’altezza e pendenza delle asperità delle superfici della frattura sono funzioni della potenza crescente/decrescente della dimensione del campione. In qualche caso, la deviazione standard raggiunge una soglia oltre la quale diventa costante. Anche il numero delle superfici di contatto tra le pareti della frattura segue una legge di potenza dell’area della superficie del contatto. Le leggi di potenza riscontrate per scabrezza, angolosità, apertura e superfici di contatto rivelano la natura frattale della geometria della frattura. In particolare, la scabrezza risulta essere “persistente” mentre l’apertura può variare tra “persistente” e “anti-persistente”, probabilmente in funzione del grado di accoppiamento tra le superfici della frattura.

Il modello frattale della scabrezza e delle superfici di contatto può essere utilizzato per sviluppare un modello costitutivo che simula il comportamento deformativo normale e di taglio della frattura. I risultati sperimentali confermano la validità del modello di comportamento normale della frattura. Pur calcolando la dilatanza, il modello di comportamento di taglio non dà altrettanto buoni risultati per quel che riguarda la rigidezza al taglio della frattura. Il modello simula anche l’evoluzione dell’apertura durante il taglio: il valore medio e la correlazione dell’apertura sono calcolati in funzione dello spostamento di taglio e verificano le osservazioni riportate in letteratura.

La mappa dell’apertura può essere inserita in un programma per il calcolo numerico del flusso d’acqua nella frattura. Il programma utilizza un metodo di “trasformazione integrale” per la risoluzione dell’equazione di Reynolds: trasforma la conduttività idraulica della frattura in una funzione della frequenza spaziale. Questa funzione ha una stretta rassomiglianza con le leggi di potenza frattali. Perciò, la funzione potrebbe essere descritta direttamente attraverso le proprietà frattali dell’apertura, con un notevole risparmio in termini di dati richiesti per la caratterizzazione idraulica della frattura. Inoltre, la modellazione indica l’importanza della densità dei valori di apertura sul flusso calcolato.
SAMMANFATTNING


Ett beräkningsprogram har utvecklats som använder sig av variationen hos sprickvidden för att beräkna flödet i sprickan. Programmet löser Reynolds ekvation genom att transformera sprickans transmissivitet till en frekvensbaserad funktion som liknar de fraktala potessambanden. Denna funktion kan beskrivas med hjälp av de fraktala egenskaperna hos sprickvidden, med anmärkningsvärt få indata. Modelleringsresultaten visar att datatätheten för sprickvidden har stor betydelse för det beräknade flödet genom sprickan.
PREFACE

This Doctoral Thesis consists of an overview of the research work on fracture geometry, mechanics and transmissivity that was published in the papers collected in Appendice:


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1. INTRODUCTION

Fractures often determine the response of rock masses in association to engineering projects since they often mark the weakest part of the rock mass on a whole. Fractures often govern the stability of slopes and underground excavations, determine the possibility and degree for extraction of water, hydrocarbons and geothermal fluids from reservoirs, as well as controls the flow of groundwater through the rock mass. A major part of the recent research in the field of rock mechanics has been devoted to study of fractures in the rock mass and their importance for a successful, environmentally safe, final disposal site for radioactive waste and spent nuclear fuel.

In hard rocks the fracture network conducts 80 to 90 per cent of the groundwater flow; the remaining part takes place in the matrix. Transport of pollutants through the rock mass, escaping from the repositories, primarily would be along fractures. The stability of excavations, at the near field, is also almost completely dependent upon the rock fractures as well as on the integrity of the installed supports and of the engineered barriers for waste isolation.

To make a proper evaluation of a potential site it is often necessary, to simultaneously consider the mechanical, hydraulic, thermal and chemical processes that occur in the rock mass; using a sophisticated model for rock and fracture behaviour. This requires quantitative information concerning the rock fracture geometry, which in addition to the knowledge of the physical properties of the rock material, can be used in the models to understand the behaviour of the rock mass on a whole.

Stability and safety assessment of the underground works needs the knowledge of the behaviour of the rock mass, which in turn depends on the behaviour of the single fractures. The fractures, on the other hand, often present geometrical and hydro-mechanical properties that change with the spatial position, the scale of interest and the applied constraints. The limited accessibility of the rock mass often hinders the direct study of the fractures forcing the engineers to extrapolate properties from those of the borehole samples to those of fractures in real scale. One way of getting around this problem is by gathering large numbers of samples, measuring and testing them in order to obtain an experimental result that mirrors the variability at the site. Another important aspect is the ability of measuring quantitative properties that can then directly be employed in the calculations. For this reason, the properties have to be reasonably easy to measure and own a clear physical meaning.

This research work proposes a random field model for fracture geometry description. The different entries from the authors’ previous publications are combined in the effort of formulating a general model for rock fracture geometry (Papers A, B and C). The definition of extended stationarity is introduced to consider the variation of the statistics of roughness, angularity and aperture with changing scale (Wang et al., 1988). Fracture surfaces and aperture are interpreted as probabilistic fractal object below well-defined thresholds of the sampling size. These stationarity thresholds are recognised based on experimental observation and from the data available in the literature. The fractal model describes the major feature of roughness, angularity and aperture:

a) scale dependency, by means the Hurst exponent $H$ or the fractal dimension $D$;

b) persistence or anti-persistence, which give a measure of the geometrical complexity of the distribution ($H$ larger, equal or lower than 0.5);

c) magnitude of the statistics at a certain scale, to be used for comparing different samples.
Beyond the stationarity threshold, the statistics of aperture and roughness seems to stabilise and become constant irrespective of the sample size and aperture and roughness cease to be fractal.

The correlation between the roughness of the two surfaces of the rock fracture is also investigated. It is found that this relationship is mainly determined by the statistical properties of the aperture, e.g. variance and variogram. It also allows for a formulation of the aperture statistics during shearing of the rock fracture. The formulation explicitly gives the aperture variance with non-restrictive assumptions. It also verifies the basic statistical relations for the fracture “at rest” (no shearing). The model gives qualitatively the same results as reported from experimental (Yeo et al., 1998) and numerical works (Power & Durham, 1997, Borri Brunetto et al., 1998).

The parameters obtained from the geometrical characterisation of the samples are input into the constitutive model for fracture normal and shear deformability based on contact mechanics also developed in this research work (Papers C and D). The constitutive model is then applied for simulating the results of some laboratory tests of normal loading and shearing; the experimental results are also presented (Paper E).

The final section of this thesis presents an attempt of integrating the fractal description of the fracture aperture into a model that calculate the flow and pressure distribution in a fracture (Paper F). This shows the possibility of describing the fracture transmissivity through a limited number of parameters correlated to the aperture properties. Having a few parameters to describe the transmissivity pattern of the fracture would open great opportunities of applying the model to complicated fracture networks.
2. SAMPLE DESCRIPTION

Two kinds of samples were analysed for geometrical characterisation: i) fractures in rock blocks (size about 10×20 cm; Papers A-D and F) (Figure 1); ii) fractures in borehole cores (diameter 51 and 61 mm; Paper E) (Figure 2). The types of rocks analysed were: gneiss (block; Stockholm), diorite and granite (cores; Åspö, southern Sweden) and schist (block; Offerdal, central Sweden). The block-sized samples were characterised geometrically only, while the core samples were also tested in the laboratory under normal and shear loading.

Figure 1. Fracture in a block of Stockholm Gneiss.

Figure 2. Core samples of diorite and granite from borehole KG0048A01 from the prototype repository site at Åspö HRL.
3. GEOMETRY

Rock fracture morphology has two main geometrical features: roughness and aperture. Roughness and, at larger scales, waviness concern the characteristics of the single fracture wall, while aperture is related to the juxtaposition of the two walls of the fracture. Initially, roughness was inferred by eye inspection of a specimen and comparison with a set of reference profiles, which where chosen from typical geometries and shear strength characteristics (Barton & Choubay, 1977). Further developments dealt with the topography of the fracture surface, trying to relate the statistics of the asperity height with the empirical Joint Roughness Coefficient (JRC) by Barton (Tse & Cruden, 1979). The fact that the statistics of the asperity height of a rock fracture vary with sample size brought to the conclusion that the traditional methods for roughness characterisation were not suitable. Instead, scientists started applying the power spectrum technique (Sayles & Thomas, 1978) and fractal theory (Brown & Scholz, 1985, Power & Tullis, 1991) to characterise rough fracture surfaces. The new approaches could explicitly handle the scaling properties of the surface statistics, and treat the surface roughness as a non-stationary random process.

From the very early investigations of the geometry of the joints, it was noticed that aperture derives from some degree of mismatch of the walls of rock fractures, in other words, from the non-correlation between the two fracture surfaces. Brown et al. (1986) investigated the non-correlated component of the roughness of the fracture walls by using the composite topography of the rock walls with respect to a common coordinate reference system. It was found that the composed topography differed from the topography of the fracture surfaces in the slope of the power spectrum and on the fact that the low frequencies (large wavelength components) were missing. However, other authors concentrated on the aperture distribution only, by paying attention to the aperture frequency distribution (Gale, 1990) and aperture spatial correlation (Gentier, 1990, Hakami et al., 1995). Already Brown et al. (1986) found that aperture showed a linear power spectrum, in a certain wavelength range, but it was Cox & Wang (1993) who had the idea that aperture, like roughness, could be treated as a fractal. They applied a slit-island technique for the determination of the fractal dimension of aperture, which, however, led to different results depending on the chosen cut-offs. Power & Tullis (1992) derived the relationship between the variance of the aperture, the variance of the asperity height and the cross-covariance between the surfaces of a rock fracture. They also suggested the applicability of this relationship for different scales of observation.

3.1. Theory

If the fracture surface is divided into a regular grid of squares on its in-plane projection, each square sub-sample can be interpolated by a planar facet. The distance between the points on the surface and the planar facet is defined as the “reduced asperity height”, $ah$. The mean plane of the sample is assumed to be the reference co-ordinate plane, and the slope of the facet and its orientation are defined with respect to one reference axis. This pair of angles is called 3D-co-latitude ($\alpha, \theta$) according to the definition by Riss et al. (1995). Moreover, the 2D or directional co-latitude is given by the apparent slope of the facets along a given direction. The apparent slope defines the surface “angularity”. This can be positive or negative, therefore it is possible to distinguish between the forward and backward angularity feature of a surface for a given direction.
Figure 3. Square planar facets of size $h$ interpolate the points measured on the fracture surface. The distance between the points and the facets defines the reduced asperity height, $a_h$. The angles of the 3D-co-latitude, $(\alpha, \theta)$ are also illustrated.

If the position of the upper surface is known with respect to the lower surface, then the geometry of the fracture aperture can be determined. The map of the distance between the point on one surface and those on the other surface of the fracture can be used for a rough investigation of the aperture entity and pattern.

A slightly more sophisticated technique for aperture determination consists of measuring the distance between the walls of the fracture, and not between single points. This is obtained by selecting groups of three neighbouring points on one wall of the fracture to define a triangular planar surface. The points of the other wall that project perpendicularly inside the triangle are selected and the distances from the points to the triangular surface define as the aperture (Figure 4). This technique allows for an aperture determination that is independent of the selected co-ordinate reference system. Owing to the accuracy of the measurements, the contact between the walls of the joint might be assumed every time the aperture determination gives a value lower than 50 $\mu$m.
Figure 4. Aperture determination: reference surface interpolated by triangles and facing points on the other surface of the fracture.

3.1.1. Model for fracture roughness

The variance of the reduced asperity height \( \sigma_{ah}^2 \) (i.e. the square of the standard deviation), calculated for sub-samples of different sizes, shows to be a power law of the sampling size \( h \) as follows:

\[
\sigma_{ah}^2 = G_{ah} h^{2H}
\]

where \( H \) is the Hurst exponent and \( G_{ah} \) is a proportionality constant. The same kind of relationship applies to the variance of the slopes of the asperity surface \( \sigma_{slope}^2 \):

\[
\sigma_{slopes}^2 = G_{slope} h^{2H-2}
\]

and to the variogram of the asperity height \( \gamma_{ah}(h) \) that quantifies the correlation between points located at a distance \( h \) on the surface (see definition in Paper B):

\[
\gamma_{ah}(h) = G_{slope} h^{2H}
\]

where \( G_{slope} \) is a proportionality constant. (When defining the variogram, \( h \) can also be seen as a delay distance \( \delta \) between the points on the surface.) Self-affine fractal objects (Mandelbrot, 1977) present the same kind of power-laws of the statistics of the roughness, thus self-affinity is assumed for the asperity heights (out-of-plane component of the surface topography) while isotropy and self-similarity is assumed for the position co-ordinates (in-plane component). Self-affine fractal objects have a complicated geometry at all scales, and the magnification of some details appears to be the same as the whole object only if different scale factors are used in different directions. On the other hand, for self-similar fractal objects, the scale factor is the same in all directions.

A simple relation links the Hurst exponent, \( H \), of a self-affine fractal surface to its fractal dimension \( D \) as:

\[
D = 3 - H
\]
To express the correlation between points on the same fracture surface, the definition of auto-covariance has to be modified. In fact, the auto-covariance of a process exists only if the variogram of the process is bounded, which means that the process is stationary to some degree. In other cases, one has to refer to the extended definition of stationarity given in Paper B. Since all samples of the same size statistically should have the same standard deviation and variogram, it is therefore possible to assume that the definition of auto-covariance $\text{cov}(ah,ah')$ (see definition in Paper B) is valid for such conditions:

$$\text{cov}(ah,ah') = \text{cov}_{ah}(\delta) = \sigma_{ah}^2 - \frac{1}{2} \gamma_{ah}(\delta)$$

The auto-covariance becomes zero when the delay distance equals the correlation length $\delta_c$ of the asperity surface (Poon et al., 1992):

$$\delta_c = \left( \frac{2G_{ah}}{G_{\text{slope}}} \right)^{1/2\eta} h$$

and consequently, Eq. (5) is valid for delay distances shorter than the correlation length $\delta_c$ (Figure 5). The correlation length continues to be linearly related to the sample size until the roughness stationarity threshold length $\Theta$ is reached.

![Figure 5. Scheme of the auto-covariance and variogram of the surface asperity height for the Stockholm Gneiss. For a sample size of 10 mm, the correlation length is 1.75 mm. However, according to Eq. (6), the correlation length is scale dependent until the stationarity threshold of the roughness is reached.](image)

In the literature and as shown by the results reported in Papers A, B and E, the Hurst exponent of the roughness of natural surfaces is generally equal to or larger than 0.5. According to the theory of Brownian motion (Mandelbrot, 1982), the fracture surface can then be defined as a persistent random variable. The persistence of a random variable determines that it will likely have increments of the same sign when neighbouring points are considered. In other words, if the asperity height increases at a certain point, it is probably increasing at close points, and vice versa.

Odling (1994) found a negative correlation between the JRC (Barton & Choubey, 1977) and fractal dimension and thus a positive correlation between JRC and the Hurst exponent, while the correlation was markedly positive between the amplitude of the asperity heights and JRC. The same observation was done for the Hurst exponent of the analysed samples (Paper E). This leads to the conclusion that rougher profiles seem to have a smaller fractal dimension, and a larger Hurst exponent. Moreover, to define the amplitude of a fractal
object, the sample size has to be defined. In other words, to compare roughness profiles and surfaces in terms of amplitude of the asperities, one always has to consider the scale of the comparison (Barton, 1990). This explains the fact that both the Hurst exponent and the fractal dimension do not give information about amplitude of the roughness, but only about its geometrical scaling.

3.1.2. Model for fracture aperture

The same self-affine fractal model shown for the fracture surface also applies to the fracture aperture $a$. With slightly different notation it can be written:

$$\sigma_a^2 = G_a h^{2H_a}$$  \quad (7)

$$\gamma_a(h) = G_m h^{2H_a}$$  \quad (8)

$$\delta_{ac} = \left(\frac{2G_a}{G_m}\right)^{\frac{1}{2H_a}} h$$  \quad (9)

where $h$ is the sampling size; $H_a$ is the Hurst exponent of the aperture; $G_a$ is a proportionality constant for the aperture variance $\sigma_a^2$; $G_m$ is a proportionality constant for the aperture variogram $\gamma_a(h)$; $\delta_{ac}$ is the correlation length of the aperture. As previously observed about the roughness, an aperture stationarity threshold, $\Theta_a$, can also be defined, when the mean value and variance of the aperture reach a sill.

In contrast to observations about the roughness, for which $H>0.5$, the aperture seems to exhibit a Hurst exponent which ranges between 0 and 1 (Papers C and E). In Paper E, it was observed that samples with surfaces extensively in contact (>10% of the nominal surface) exhibit an Hurst exponent of the aperture larger than 0.5; on the other hand, samples with small total contact area have an Hurst exponent less than 0.5. These observations agree with the fact that small Hurst exponents correspond to low correlated or anti-persistent aperture and thus small contact areas, and vice versa. An increase of the correlation and persistence is often associated to fractures that experienced shearing and mismatching (Chen et al., 2000).

Cox et al. (1993) obtained values of the Hurst exponent of the aperture in the range of 0.60 to 0.70. However, by admission of the authors, the variogram method should have predicted much higher fractal dimensions than the “slit-island” method, to which they gave more credit, and thus much lower Hurst exponents. (It is still an open question whether the “slit-island” method is suitable for determining the fractal dimension of self-affine surfaces.)

Brown et al. (1986) observed how the profiles of the asperity heights are different from the profiles of the “composite topography” of the joint, which is the sum of the asperity heights of the walls of the fracture when computed from the same reference system (Figure 6). From the plots, it was also evident that the complexity of the aperture distribution was higher than that of the roughness distribution, which means that the aperture profiles are sometimes coarser than the roughness profiles.

It could be concluded that the aperture of matching fractures could be assimilated to an anti-persistent self-affine fractal, for which increasing trends are most probably followed by decreasing trends, and vice versa. As observed for the roughness, the Hurst exponent of the aperture should be regarded as a descriptor of the complexity of the fracture void distribution. Furthermore, it gives a measure of the scale or sample size dependency of the aperture statistics. In the extreme case of a non-fractal random aperture, the Hurst
exponent degenerates to zero, and the ordered fractal structure of increasing aperture variance with increasing sample size is lost. This is also what happens when the stationarity threshold for the aperture is reached.

Several authors focused on the stationarity of the aperture distribution and on the correlation length (e.g. Gale, 1990, Gentier, 1990, Hakami & Larsson, 1996). The approach illustrated here, instead, seems to be more suitable to describe the small-scale features of the aperture, where stationarity has not been reached. The Hurst exponent could then be the input parameter for hydro-mechanical constitutive models where the contact area and the micro-scale flow pattern are described.

Figure 6. Surface profiles and composite topographical profile for a natural fracture in diorite are shown above. Below, the power spectra of the same surface profile (a) and of the composite topography (b) (spectrum (b) was shifted one decade to the right for clarity) (from Brown et al., 1986).
3.1.3. Correlation roughness-aperture for a matched fracture

Power & Tullis (1992) calculated the cross-covariance between the facing surfaces of a fracture in rock and concluded that it was related to the variance of the asperity height and of the aperture, at a certain scale:

\[
\text{cov}(ah_1, ah_2) = \sigma_{ah}^2 - \frac{\sigma_a^2}{2}
\] (10)

where \( ah_1 \) and \( ah_2 \) are the asperity heights of the facing fracture walls; \( \sigma_{ah}^2 \) is the variance of the asperity height (assumed coincident for both fracture surfaces); \( \sigma_a^2 \) is the variance of the fracture aperture. Equation (10) expresses the correlation between the fracture surfaces. The correlation increases as the variance of the asperity heights, with the assumption that the aperture had reached stationarity. For very large samples, the cross-covariance between the two surfaces will equal the variance of the surface asperity height, meaning that the two sides of the fracture tend to coincide. The same equation applies for small scales, where the reduction by half of the aperture variance is not negligible, and gives rise to a lower degree of correlation. The relationship in Eq. (10) points out the fact that the aperture can be considered as the non-correlated component of the fracture geometry, and becomes less important for the correlation of the surfaces of the fracture when larger samples are considered and aperture reaches stationarity. Finally, the cross-covariance in Eq. (10) also becomes constant when roughness reaches stationarity.

The same relationship applied to the cross-covariance between the asperity height and the aperture give the following (Paper B):

\[
\text{cov}(ah_1, a) = -\frac{\sigma_a^2}{2}
\] (11)

where it is stated that the correlation between aperture, \( a \), and asperity height, \( ah_1 \), only depends on the aperture variance. This relation becomes constant as soon as the aperture reaches stationarity. Equations (10) and (11) are written for a zero delay distance \( \delta \) between the points at which the correlation is computed.

3.1.4. Correlation roughness-aperture for a sheared fracture

Shearing is equivalent to the geometrical process of shifting the upper surface with respect to the lower surface along a dilation slope \( S \). All damage produced on the roughness asperities are hereby neglected. For a shear displacement \( \nu \), the mean value of the aperture, \( \mu_{av} \) that was equal to \( \mu_a \) for the fracture “at rest”, becomes:

\[
\mu_{av} = \mu_a + S \nu
\] (12)

while the variance of the aperture \( \sigma_{av}^2 \) changes as follows:

\[
\sigma_{av}^2 = \sigma_a^2 + \gamma_{slope}(\nu) - 2 \text{cov}(\Delta ah_1^\nu, a)
\] (13)

where \( \Delta ah_1^\nu \) is the increment of the asperity height for an interval \( \nu \). The variance of the aperture increases with shearing by a term equal to the variogram of the asperity height calculated for a distance equal to the shearing displacement \( \nu \). The term deduced is of importance for small shearing displacements, but becomes constant when the delay distance reaches of the aperture stationarity threshold.

The spatial correlation of the aperture can be inferred along the shearing direction as:
\[
\text{cov}(a_v, a_v^{(\delta,0)}) = \sigma_v^2 + \frac{1}{2} \left[ \gamma_{\text{slope}}(v + \delta) + \gamma_{\text{slope}}(v - \delta) \right] - \gamma_{\text{slope}}(\delta) - \\
- \text{cov}(\Delta h_1^{(v+\delta)}, a) - \text{cov}(\Delta h_1^{(v-\delta)}, a)
\]

(14)

and perpendicularly to the shearing direction (delay distance \( \xi \)) as:

\[
\text{cov}(a_v, a_v^{(0,\xi)}) = \sigma_v^2 + \gamma_{\text{slope}}(\mu) - \gamma_{\text{slope}}(\xi) - 2 \text{cov}(\Delta h_1^{\mu}, a)
\]

(15)

where the additional term \( \mu \) is a working variable defined by:

\[
\mu = \sqrt{v^2 + \xi^2}
\]

(16)

By comparing Eq. (13) with Eqs. (14) and (15), one recognises a term equivalent to the variogram of the aperture during shearing. This term changes from the semi-variogram of the aperture “at rest”, to the variogram of the asperity height, when the shearing displacement is large. Assuming that the aperture distribution is still fractal during shearing, its shape varies between two fractal shapes: a first one with the Hurst exponent of the aperture “at rest” and a second one with the same Hurst exponent as the roughness. If the aperture “at rest” is observed to have a smaller Hurst exponent than the roughness, then the aperture becomes more persistent during shearing because the Hurst exponent increases; in addition, the aperture correlation length increases. This process will stop as soon as the shear displacement becomes as large as the correlation length of the surface roughness.

These changes of the aperture geometry were also observed by Power & Durham (1997), results from numerical shearing simulations of a digital image of a real rock joint. In their simulations, the upper surface of the fracture was rigidly translated with respect to the lower surface, and the evolution of the aperture was observed. The aperture and pattern of contact area changed. In particular, it was found that the size of the contact spots (zones with zero aperture) increased while their frequency diminished. This is in agreement with the results obtained here (Papers B and E). The growth of the contact spots is justified by the increase of the aperture correlation length; the reduction of the number of contact spots is due to the increase of persistence and Hurst exponent of the aperture distribution. Borri Brunetto et al. (1998) observed the same behaviour. They carried out a numerical simulation of shearing of the digital image of a perfectly mated sandstone fracture. A threshold was observed for the shear displacement beyond which there was no more correlation between the fracture surfaces. This could coincide with the correlation length for the roughness for that particular sample.

3.1.5. Contact area

The characteristics of the aperture of the fracture govern the extension and distribution of the contacts between the facing surfaces. By interpreting the fracture aperture as a fractal, the contact area between the facing surfaces can also be treated as a fractal: thus, the fractal dimension of the contact area can be derived directly from that of the aperture distribution. In fact, the contact area can be seen as a cut-off point, a zero level of the fracture aperture. In other words, the contact areas can be seen as “lakes” on a “landscape” (Mandelbrot, 1977). By knowing the fractal dimension of the contours of the contact areas (“lakes”), which depend on the fractal dimension of the aperture distribution (“landscape”), it is possible to evaluate the number \( N \) of contacts having an area, \( S \), larger than a reference area \( s \). For an isotropic and self-similar distribution, the following relation holds:
\[ N(S > s) = \left( \frac{s}{s_l} \right)^{(1-D_a)/2} = N_{tot} \left( \frac{s}{s_m} \right)^{(1-D_a)/2} \]  

(17)

and \( s_l \) and \( s_m \) are the largest and smallest measured contact areas, respectively; \( D_a \) is the fractal dimension of the aperture distribution; \( N_{tot} \) is the total number of contact areas experimentally measured. The cumulative frequency distribution (Pr) for the contact areas can be written as (Mandelbrot, 1982):

\[ \text{Pr}(S > s) = C_a \ s^{(1-D_a)/2} \]  

(18)

and \( C \) is the proportionality constant:

\[ C_a = \left( \frac{s_l}{N_{tot}} \right)^{(1-D_a)/2} = \left( \frac{s_m}{N_{tot}} \right)^{(1-D_a)/2} \]  

(19)

The frequency distribution of the contact areas can be obtained from the cumulative frequency distribution in Eq. (18), and it has the following form:

\[ f(s) = -C_a \left( \frac{1-D_a}{2} \right) s^{-(1+D_a)/2} \]  

(20)

By knowing the frequency distribution of the contact areas, it is possible to evaluate the total contact area as a function of the maximum recorded contact \( s_l \) (Majumdar & Bhushan, 1990):

\[ A_c = N_{tot} \int_0^{s_l} f(s) sd s = -C_a \left( \frac{1-D_a}{2} \right) \int_0^{s_l} s^{-(1+D_a)/2} sd s \]  

(21)

and substituting and integrating, it results:

\[ A_c = \frac{D_a - 1}{3-D_a} s_l \]  

(22)

From Eq. (22) it follows that, for a given largest contact area \( s_l \), the total contact area will increase by increasing the fractal dimension of the aperture. Equation (22) also represents the area of an infinite series of small islands, which cumulative contour length, on the other hand, would not converge (Mandelbrot, 1977). According to this interpretation, the fractal dimension of the contact areas becomes the measure of the fragmentation of the distribution of the contact spots. From Eq. (22), it also follows that, if both the total area of all the contacts and the area of the largest contact is known, then it should be possible to determine the fractal dimension of the contours of the contacts and of the aperture. As contact area is a sub-set of the aperture, the fractal model for contact area would apply until the stationarity threshold of the aperture was reached.

3.2. Measuring technique

Topographical measurements of the fracture surfaces were carried out by means of a 3D-laser-scanning system. A full description is given in Paper A and Lanaro et al. (1998). The system is composed of a laser sensor, an electronic control unit (ECU), a Co-ordinate Measuring Machine (CMM) and a computer. A laser source projects a linear light beam on the surface of the sample. Two Charged Coupled Devices (CCD) cameras record the image of the beam. By a stereographic triangulation technique, the co-ordinates of 600
points on the object along the laser beam are measured with an accuracy of ±50 µm. The user can program the path of the scanner over the sample. Moreover, the sensor is inclined with respect to a vertical axis, so that vertical surfaces can also be measured.

The topographical data collected by the scanner were exported to a computer program for reverse engineering (Surfacer, Imageware, 1998). The program contains an editor in which standard and user-defined functions can be combined to form routines for analysis. These were used to translate and rotate data, interpolate data with planes, calculate distances and angles, evaluate statistics of asperity height and aperture, etc. A rendered plot of the topography of one fracture surface is given in Figure 7.

Figure 7. 3D perspective view of the topography of the Stockholm Gneiss visualised by Surfacer (Imageware, 1998).

Papers A contains the description of the technique developed for matching the walls of a rock fracture sample so that the in-between void space can be studied. This technique simply consists of gluing some reference spheres onto the rock blocks when the fracture is closed (Figure 8, a). Next, the spheres are scanned. Then, the sample is opened and the surfaces are scanned individually together with their reference spheres (Figure 8, b). By means of rigid translations, the topography of the surfaces with the respective spheres is relocated to its original position with high accuracy. Due to the fact that the relocation uses a large number of points measured on calibrated objects, the accuracy of the relocation is higher than that of the scanning of one surface.
3.3. Experimental results

3.3.1. Roughness and angularity

The surface of the sample is divided into a grid with squared cells to measure the scaling of roughness and angularity according to the theory described in Sec. 3.1.1. Sub-samples of 2, 5, 10, 20, 40 or 60 mm were analysed.

Scale dependency of the asperity height

The frequency distribution of the reduced asperity height is shown in Figure 9 and 10 for the Stockholm Gneiss for different sampling sizes. The results show quite wide range of graph shapes, all of which were obtained from the same sampling size. The reported frequency distributions have a zero mean, this is because the asperity height is reduced by the planar trends.

Skewness indicates the symmetry of the statistical distribution around the mean; kurtosis characterises the peakedness or flatness of a distribution; a positive peakedness indicates a relatively narrow distribution, while for the Gaussian distribution the value is 3. In this case, skewness varies between positive and negative without any defined pattern and kurtosis ranges between −0.50 and 7.78, and is often smaller than the unit.
The relocation technique also allows one to compare the statistical distribution of the asperity height of the upper and lower surface of the fracture. In this particular case, very often the two distributions are almost coincidental.

Figure 9. Frequency distributions of the reduced asperity height for the Stockholm Gneiss sample. The figure shows results for square sampling sizes of 2, 5 and 10 mm. The solid line represents the frequency distribution for the lower surface, and the dashed line represents the upper surface.
As observed in Figures 9 and 10, the standard deviation of the frequency distribution of the asperity height $\sigma_{ah}$ depends on the sample size. When plotting the standard deviation of the asperity height versus the sampling size on a log-log diagram, the graphs are shown to be linear inside a certain range of sample sizes (Figures 11 and 12). In fact, the Offerdal Schist reaches a sill for sampling sizes of about 20 mm, while the graphs of both the Stockholm Gneiss (Paper A) and of the diorite from Åspö HRL (Paper E; Figure 12) are linear across the whole range of sampling sizes. Moreover, the standard deviation of the asperity height is characterised by a frequency distribution that appears to be Gaussian-shaped (Figure 11). The standard deviation of the reduced asperity height for the sample of the Offerdal Schist reaches a sill. This means that the statistics of the asperity height have
become constant independently of the chosen sample size. The Stockholm Gneiss (Paper A) and the samples of diorite from Åspö HRL (Figure 12) do not show this property; no sill is reached on the log-log plot of the standard deviation of the asperity height versus sampling size. All sub-samples in the range of sizes analysed here give different results in term of statistics of the asperity height. In other words, none of them is representative for roughness: larger samples would be characterised by larger standard deviations.

**OFFERDAL SCHIST**

![Figure 11. Plot of the standard deviation of the reduced asperity heights, $\sigma_{ah}$, versus the sampling size, $h$, for the sample of the Offerdal Schist.](image)

![Figure 12. Plot of the standard deviation of the reduced asperity heights, $\sigma_{ah}$, versus the sampling size, $h$, for the sample of the Stockholm Gneiss.](image)

**Scale dependency of the asperity slope**

The slopes of the planar facets interpolating the sub-samples of the fracture surfaces were also analysed. The apparent slopes are reported for the Offerdal Schist and the Stockholm Gneiss in Figures 13 and 14, respectively.
The asperity slopes for the sample of Stockholm Gneiss, on the other hand, seem to be less sensitive to the sampling size. The frequency distributions of the slopes are quite Gaussian-shaped for the sampling size of 2 mm. Due to the smaller number of data available and scale dependency for the larger sampling size of 10 mm, the distributions appear to be more irregular and peaked. The range of slope angles reduces for larger samples.

The standard deviation of the distribution of the slopes can also be calculated and plotted as a function of the sub-sample size on which it was determined. The plots are linear for small sample sizes; then, the slope drastically reduces when the sub-sample size approaches the sample size. This is because the fitting plane of the sub-sample always tends to coincide with the mean plane of the surface.
Figure 14. Frequency distributions of the apparent slopes of the planar facets interpolating the surface of the sample of Stockholm Gneiss, for sampling size of 2 and 10 mm. The apparent slopes are evaluated along different direction on the fracture plane.

Figure 15. Plot of the variance of the slope $\sigma^2_{\text{slope}}$ versus the sampling size $h$ for the samples of Offerdal Schist and Stockholm Gneiss.
Figure 16. Plot of the standard deviation of the slope $\sigma_{\text{slope}}$ versus the sampling size $h$ for the core sample KA3579G-9.43 from Åspö HRL.

Power spectra

Fourier analyses were performed on the surface topography of the two walls of the fracture samples from blocks. An overview of the theoretical background of this technique is presented in Paper A. The surface topography is given in term of spatial coordinates of the measured points, where the reference plane is the global interpolation plane of the fracture. The Fourier analysis returns the power spectral density function of the spatial frequency along two orthogonal axes. In both cases the perspectives of the power spectral function are almost conical, with the co-ordinate vertical axis as the symmetry axis (Figure 17).

Figure 17. Perspective of the power density of the asperity height as a function of the spatial frequency for the Stockholm Gneiss sample.

The cross-sections of the power density functions along the co-ordinate planes appear to be roughly linear on log-log diagrams. The related interpolations by exponential functions of the frequency for the Stockholm Gneiss are given in Figure 18.
Figure 18. Coordinate cross-sections of the power spectral density function for the sample of Stockholm Gneiss (cf. Figure 17).

Anisotropy

The frequency distributions of the apparent slopes of the surface for the Stockholm Gneiss vary depending on the considered direction on the plane of the fracture (Figure 14). While the overall shape of the distributions is almost the same in all cases, the mean slope is not zero as would be expected for an isotropic surface, but ranges from about $-5^\circ$ for an orientation of $0^\circ$ to about $3^\circ$ for an orientation of $90^\circ$, respectively.

Anisotropy is revealed also by plotting the maximum apparent slopes along different directions in the fracture plane on a rose diagram as shown in Figure 19. In this case, the influence of very steep singular asperities is determinant, since the values in the graphs are not averaged by any statistical operators. This kind of diagram could help in understanding the results from the shearing tests of a fracture by quantifying the relationship between the slope of very steep asperities and the measured dilation of the sample.

Figure 19. Rose diagram of the maximum and minimum apparent asperity slopes for the Stockholm Gneiss sample. Values of the angle for different sampling sizes are shown.
The cross-sections of the power spectral density functions also reveal anisotropy (Figure 18). In fact, the Stockholm Gneiss shows different behaviour depending on the analysed direction (Paper A).

3.3.2. Aperture

The void distribution was evaluated for samples for very low normal loads due to self-weight (1-2.5 kPa). The grey scale maps the aperture can be directly obtained by plotting the distances between the point of the upper and of the lower surfaces of the fracture (Paper A and E; Figures 20 and 21). Thanks to the high density of the measured data on the surface topography, the maps are of valuable help in visualising the distribution of the fracture aperture.

For the Stockholm Gneiss, different sampling sizes have been analysed for the purpose of aperture investigation. With the same technique adopted for the roughness analysis, the sample was split into two square areas of 60×60 mm. Each of the areas was then divided

Figure 20. Grey-scale map of the aperture distribution of the fracture sample of Offerdal Schist.

Figure 21. Grey-scale map of the aperture distribution of the core sample KG0021A01–40.74 from Åspö HRL (core diameter 51 mm).
into sub-samples of 10, 20, 30 or 40 mm side length. For each sub-sample, the statistical analysis of the data gives the frequency distribution and standard deviation of the aperture. The collected data for the Stockholm Gneiss show that the frequency distribution of the aperture always exhibits a fairly Gaussian shape, except for the sub-samples where the mean value of the aperture is of the same order of magnitude as the accuracy of the scanner (50 µm). In this case, the frequency of the aperture turns into a lognormal distribution. The Gaussian shape of the aperture distribution is much better defined than the frequency distribution of the asperity height. Some of the sub-samples show slightly asymmetrical distributions, with a not negligible tie towards the higher apertures. Typical frequency distributions of the aperture for three different sampling sizes are shown in Figure 22. The mean aperture value varies between 0.055 mm and 0.204 mm depending on the particular position of the selected area on the sample. If the most frequent value of the mean aperture is taken for each window size, then that value appears to be almost the same independent of the scale of observation (Paper F). On the other hand, the range of the aperture means increases for smaller sampling sizes.

Figure 22. Frequency distributions of the aperture for the sample of Stockholm Gneiss. The graphs show results for sub-samples of 2, 10 and 30 mm side.
Scale dependency of the aperture

While the range of the mean values of the aperture for different sampling sizes reduces by increasing the window size, the aperture standard deviation increases. The log-log diagram of the standard deviation of the joint aperture versus the sampling size (Figure 23) appears to be an almost linear, i.e. a power law, for sampling sizes smaller than 30 mm. The fractal dimension derives from the exponent of the power law and was about 2.711 ($H_a=0.289$). Beyond that limit, the standard deviation tends to stabilise. Notice that the larger the size of the sub-samples, the fewer the data of standard deviations available.

As shown in Figure 23, the aperture standard deviation reaches a sill for sampling size larger than 30 mm. The aperture distribution can be considered to be statistically stationary for samples larger than the threshold of 30 mm, thus complete statistical information could then be obtained from those samples. For sampling sizes less than 30 mm the diagram shows a power-law relation indicating that the aperture has a fractal behaviour. The correlation length for the aperture of the sample of Stockholm Gneiss obtained according to Eq. (9) is 6.7 mm in value.

Contact area

The contact areas of the samples were identified from the plots of the aperture under a threshold of 50 µm, in two different ways: i) by eye recognition and counting on the aperture maps; ii) by image-analysis technique from the plots of the aperture maps. With both techniques, the number of contacts with a given area was determined; their cumulative-frequency distributions are similar to those in Figures 24 and 25. With both techniques, a power law of the cumulative frequency distribution was observed, as in Eq. (18). From here, the fractal dimension of the aperture could be determined ranging between 3 and 2 (Papers D and E).

It is evident (Figure 25) that the fractal model for the contact area does not hold for the whole range of contact areas, but it seems to lose validity for the largest areas. This could suggest that the aperture of the sample had reached stationarity, and very unlikely larger areas than about 50 mm² would be observed for this or another sample from the same fracture.
Figure 24. Cumulative frequency distribution of the contact areas for the sample of Stockholm Gneiss (aperture threshold = 50 µm).

Figure 25. Cumulative frequency distribution of the contact areas for sample KA3579A-9.43 from Åspö HRL (aperture threshold = 50 µm).
4. MECHANICS

The geometry of the fracture determines both points where the forces between the surfaces are exchanged, and the direction of those forces. Thus, for modelling the mechanical behaviour of a fracture, detailed information about the contact areas and the slopes of the surfaces at the contacts is required. Such information is not easy to obtain; it requires special equipment and techniques for identifying the contacts. This is why most of the proposed constitutive models in the literature have either: i) to rely on data gathered from the results of experiments of normal and shear loading; ii) to make hypothesis about the fracture geometry; iii) to directly use the fracture geometry. For example, the models by Amadei & Saeb (1990), Grasselli & Egger (2000) assume constitutive equations empirically based on experimental results. Wang and Kwasniewski (2000) formalised their model for fracture normal and shear deformability on contact mechanics. They assumed a simplified fracture geometry that only considers roughness, ignoring aperture and contact area. Myer (2000) assumed the fracture to be a collection of elliptical cracks interacting with each other at rock bridges that simulate the contact spots. This model poses problems in characterising the equivalent cracks.

From the conceptualisation of the fracture geometry proposed in this Thesis, many of the simplifications necessary in earlier models can be avoided. This leads to constitutive equations that purely contain geometrical variables or mechanical parameters concerning the rock at the contacts (Papers D and E). Contact mechanics seems to suite very well the fractal theory for the contact areas. For this reason, in the following sections contact mechanics is used for modelling the fracture behaviour during normal and shear loading. The theoretical results are then validated against experimental tests.

4.1. Theory

4.1.1. Contact behaviour of a fractal fracture in normal loading

Hertzian mechanics assumes an ideal contact between two spheres or between a sphere and a plane. Based on the stress distribution and deformation at the contacts, a mathematical expression of the normal load can be obtained for elastic and elasto-plastic conditions. When the normal deformation or closure $\delta$ at the contact is small with respect to the radius of the sphere, the area of the contact can be approximated by:

$$s = \pi R \delta$$  \hspace{1cm} (23)

and $R$ is the equivalent asperity radius. According to Hertzian theory in elastic regime, at the contact between a sphere of radius $R$ and a planar surface there is a force $P_e$ that is a function of the normal displacement $\delta$ (Bushan, 1999):

$$P_e = \frac{4}{3} E' \sqrt{R} \delta^{3/2}$$  \hspace{1cm} (24)

The equivalent deformation modulus $E'$ can be determined from respectively the deformation moduli $E_1$ and $E_2$, and Poisson’s ratios, $\nu_1$, $\nu_2$, of the touching surfaces, as follows:

$$E' = \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)^{-1}$$  \hspace{1cm} (25)
From Eqs (23) and (24), the relation between contact force, displacement and contact area can be written as:

\[ P_e = \frac{4}{3} E' \frac{\delta}{\pi} \]  

(26)

When the contact undergoes plastic yielding, the mean contact pressure becomes a function of the surface hardness \( H \) (Williams, 1994):

\[ p_p = \frac{1}{K} H \]  

(27)

where \( K \) is a proportionality constant ranging between 0.5 and 2 (Majumdar, 1991). The plastic yielding pressure can be taken as the yield strength of the rock material. The force at certain contact can then be written as:

\[ P_p = p_p s \]  

(28)

Thus, the contacts in elastic regime still follow the law in Eq. (26), while those in plastic regime are governed by Eq. (28). The critical deformation \( \delta_c \) at which plasticity occurs for a particular contact has been shown to be (Greenwood, & Williamson, 1966):

\[ \delta_c = \left( \frac{\pi H}{2E'} \right)^2 R \]  

(29)

If a multitude of contacts is considered, for a certain deformation level \( \delta \), there might be some contacts under plasticity conditions. By recalling Eq. (23), it is possible to obtain the size \( s_p \) of the largest area under plastic conditions for a certain deformation level \( \delta \):

\[ s_p = \left( \frac{4E'\delta}{3\sqrt{\pi} p_p} \right)^2 \]  

(30)

Because small contact areas are related to small radii of curvature, from Eq. (29) it would follow that small contacts undergo plasticity before large contacts, during normal loading of a rock fracture (Majumdar, 1991). This is in contrast with Greenwood & Williamson’s model (1966).

If the largest and smallest contact areas are \( s_l \) and \( s_s \), respectively, by integrating the normal load in Eq. (26) acting on the single contact spot to all contact spots having the frequency distribution in Eq. (20), the total elastic normal load \( PE \) can be obtained:

\[ PE = N \int_{s_s}^{s_l} P_e f(s) ds = \frac{4}{3} \frac{E'}{\sqrt{\pi}} \frac{D_a - 1}{2 - D_a} s_i^{1-D_a} s_j^{2-D_a} \delta \left[ s_i^{2} \delta_j^{2} - s_j^{2} \delta_i^{2} \right] \]  

(31)

Eq. (31) applies for a particular fracture closure \( \delta \), when one knows the entity of the smallest and largest contact areas at that particular deformation level. However, it is quite improbable that these values are available for each deformation level during fracture deformation. It is more realistic that the map of the contact spots is measured at another known closure level \( \delta_0 \). As shown in Eq. (23), the changes in the contact area from \( s_0 \) to \( s \) during loading and closure from \( \delta_0 \) to \( \delta \), is:

\[ \frac{s}{s_0} = \frac{\delta}{\delta_0} \]  

(32)
Moreover, we assume that the frequency of a certain area \( s_0 \) stays the same when, due to deformation, it expands to \( s \) according to Eq. (32). The frequency distribution of the contact area for the closure \( \delta \) can be inferred from the frequency distribution it had at the reference closure \( \delta_0 \) (see Eq. (20)):  

\[
f'(s) = f(s_0) = -C_a \left( \frac{1-D_a}{2} \right) \left( \frac{\delta}{\delta_0} \right)^{-(1+D_a)/2} \]  

(33)

Thus, when inferring the normal load \( PE \) at a closure \( \delta \) for which the contact area distribution has not been measured, Eq. (31) is modified as follow:  

\[
PE = N_{tot} \int_{s_s}^{s} P_p f'(s^*) ds^* = \frac{4}{3} \sqrt{\frac{E^*}{\pi}} \frac{D_a}{2 - D_a} \frac{1-D_a}{s_{10}} \left[ \frac{2-D_a}{s_{0}^{1/2}} - \frac{2-D_a}{s_{0}^{1/2}} \right] \left( \frac{\delta}{\delta_0} \right)^{3/2} \]  

(34)

This coincides with Eq. (31) when the closure has its initial value. Equation (34) allows one to infer the normal behaviour of a fracture under elastic deformation. The size of the sample governs upon the size of the largest contact area. The size of the smallest contact area, should be on the order of the mineral crystals in the rock. In practice, this value varies depending on the accuracy of the measuring system.

When the smallest contact spot undergoes plastic deformation, the normal pressure will level out and becomes constant. An integration of the normal load over all the plasticised contact spots \( (s < s_p) \), gives:  

\[
PP = N_{tot} \int_{s_s}^{s_p} P_p f(s) ds = -p_p s_{l} \frac{1-D_a}{2} \frac{1-D_a}{s_{p}} \left[ s_{p}^{3-D_a/2} - s_{s}^{3-D_a/2} \right] \]  

(35)

Since the contact areas evolve during normal loading, Equation (35) can be rewritten as:  

\[
PP = N_{tot} \int_{s_s}^{s_p} P_p f'(s^*) ds^* = p_p s_{l0} \left[ \frac{1-D_a}{2} \frac{D_a}{3-D_a} \right] s_{0}^{1-D_a/2} \left[ s_{p}^{3-D_a/2} - s_{s0}^{3-D_a/2} \right] \left( \frac{\delta}{\delta_0} \right)^{2} \]  

(36)

It is very likely that elastic and plastic deformation coexist in the same fracture, thus, the total normal load acting on the contact spots can be easily obtained by the summation of Eqs. (31) and (35). For a closure \( \delta \) that is different than \( \delta_0 \), the normal load is:  

\[
PTOT = PE + PP =
\]

\[
= \frac{4}{3} \sqrt{\frac{E^*}{\pi}} \frac{D_a}{2 - D_a} \frac{1-D_a}{s_{10}} \left[ \frac{2-D_a}{s_{0}^{1/2}} - \frac{2-D_a}{s_{0}^{1/2}} \right] \left( \frac{\delta}{\delta_0} \right)^{1+D_a/2} +
\]

\[
+ p_p s_{l0} \left[ \frac{1-D_a}{2} \frac{D_a}{3-D_a} \right] s_{p} \left[ \frac{3-D_a}{2} - s_{s0} \frac{3-D_a}{2} \right] \left( \frac{\delta}{\delta_0} \right)^{2} \]  

(37)

4.1.2. Contact behaviour of a fractal fracture in shearing

The solution for a Hertzian contact between two spheres is described in the literature (Bushan, 1999). The solution postulates the occurrence of slip in a peripheral annular area of the contact. The solution holds for a model where slip occurs. However, the values of the tensions due to shearing in the annular area would be unrealistically high for a model
where slip does not take place. The relative displacement $\gamma$ between the walls of the fracture due to shearing can be expressed by:

$$
\gamma = \frac{3\mu P}{16 \sqrt{\frac{s}{\pi}}} \left[ \frac{2-v_1}{G_1} + \frac{2-v_2}{G_2} \right] \left[ 1 - \left( 1 - \frac{Q}{\mu P} \right)^{\frac{2}{3}} \right] 
$$

(38)

From this equation, the shearing force $Q$ can be obtained as a function of the shear displacement $\gamma$ and the normal load $P$. It follows that:

$$
Q = \mu P \left[ \left( 1 - \left( 1 - \frac{Q}{\mu P} \right)^{\frac{2}{3}} \right) \right]
$$

(39)

$\mu$ is the frictional coefficient of the contact and $G'$ the equivalent shear deformation modulus defined as:

$$
G' = \left( \frac{2-v_1}{G_1} + \frac{2-v_2}{G_2} \right)^{-1}
$$

(40)

The shear force applies to the area $s$ under a normal load $P$, which is also a function of the contact area. If plastic deformation is neglected, then:

$$
Q_s = \frac{\mu E \delta}{3} \sqrt{\frac{s}{\pi}} \left[ 1 - \left( 1 - \frac{4G'}{\mu E \delta} \right)^{\frac{3}{2}} \right]
$$

(41)

By knowing the frequency distribution of the contact areas, Eq. (41) can be integrated over the whole fracture. By assuming a frequency distribution of the form in Eq. (20), the elastic response of the fracture to shear becomes $QE$:

$$
QE = \int_{s_{min}}^{s_{max}} N_{me} Q_s(s) f(s) ds =
$$

$$
= -\mu \left[ 1 - \left( 1 - \frac{4G'}{\mu E \delta} \right)^{\frac{3}{2}} \right] P E
$$

(42)

If plasticity due to normal loading is considered, a constant normal pressure is assumed at the contact area. Then Eq. (39) would not be easy to integrate. However, a simplification can be made in order to overcome this inconvenience; the expression between square brackets can be substituted by its polynomial approximation.
\[
\left(1 - \frac{16\sqrt{\frac{s}{\pi}} G' \gamma}{3\mu P}\right)^{3/2} = a_3 \left(\frac{16\sqrt{\frac{s}{\pi}} G' \gamma}{3\mu P}\right)^3 + a_2 \left(\frac{16\sqrt{\frac{s}{\pi}} G' \gamma}{3\mu P}\right)^2 + a_1 \left(\frac{16\sqrt{\frac{s}{\pi}} G' \gamma}{3\mu P}\right) + a_0
\]

(43)

And the dimensional coefficients of the approximation are: \(a_0 = 1.0099\), \(a_1 = -0.8433\), \(a_2 = 0.4973\) and \(a_3 = -0.6189\). By introducing this approximation in Eq. (39), it becomes:

\[
Q_s \approx \mu P \left\{-a_3 \left(\frac{16\sqrt{\frac{s}{\pi}} G' \gamma}{3\mu P}\right)^3 - a_2 \left(\frac{16\sqrt{\frac{s}{\pi}} G' \gamma}{3\mu P}\right)^2 - a_1 \left(\frac{16\sqrt{\frac{s}{\pi}} G' \gamma}{3\mu P}\right)\right\} 
\]

(44)

To count for all the contact areas of the sample, Eq. (44) has to be integrated according to the frequency distribution in Eq. (20) to obtain the total shear force \(QTOT\):

\[
QTOT = QP + QE = N_{tot} \left\{ \int_{s_p}^{s} Q_p f(s) ds + \int_{s_p}^{s} Q_e f(s) ds \right\} 
\]

(45)

and because the plasticity at the contacts, the normal load can be expressed as the product of the plasticity pressure \(p_p\) and the contact area \(s (s < s_p)\):

\[
QP = -\mu p_p s_{p}^{D_p/2} \left\{ \frac{1 - D_a}{2} a_3 \left(\frac{16G' \gamma}{3\mu p_p \sqrt{\pi}}\right)^3 \frac{2}{D_a} \left(s_{p}^{1-D_a/2} - s_{p}^{1-D_a/2}\right) + \right.
\]

\[
- a_2 \left(\frac{16G' \gamma}{3\mu p_p \sqrt{\pi}}\right)^2 \frac{2}{1-D_a} \left(s_{p}^{1-D_a/2} - s_{p}^{1-D_a/2}\right) + \right.
\]

\[
+ a_1 \left(\frac{16G' \gamma}{3\mu p_p \sqrt{\pi}}\right) \frac{2}{2-D_a} \left(s_{p}^{2-D_a/2} - s_{p}^{2-D_a/2}\right) \right\} 
\]

(46)

The stick-slip condition in Eq. (38) applies only if the shearing force does not exceed the frictional force \(\mu P\) acting on a contact of area \(s\). This condition results into an inequality of the kind:

\[
16\sqrt{\frac{s}{\pi}} G' \gamma < 3\mu P
\]

(47)

Two possible expressions of the normal load \(P\) acting on one contact area are available: the elastic normal load in Eq. (26) and the plastic normal load in Eq. (28). When substituting the normal load into Eq. (47), the following condition results:
In elastic conditions, Eq. (48) states that slipping occurs when the ratio between normal and shear displacement exceeds a certain limit that depends only on the mechanical properties of the rock fracture, and not on the contact area $s$. If plasticity is induced by normal loading, then the areas undergoing slipping are smaller than a parabolic function of the shear displacement as:

$$s_{\text{slip}} = \left( \frac{16G'\gamma}{3\sqrt{\pi}\mu_p} \right)^2 < s$$

(49)

If slipping occurs, the tangential force $Q$ equals the surface friction $\mu P$. By integrating the tangential force for all the areas under slipping, we obtain:

$$QS\text{LIPE} = N_{\text{tot}} \int_{s_{\text{slip}}}^{s_m} \mu P f(s) ds = \frac{4}{3} \mu \frac{1-D_a}{2-D_a} E' \delta s_j^{\frac{D_a-1}{2}} \left( s_{\text{slip}}^{2-D_a} - s_s^{2-D_a} \right)$$

(50)

in elastic conditions, and:

$$QS\text{LIPE} = N_{\text{tot}} \int_{s_{\text{slip}}}^{s_m} \mu P f(s) ds = -\mu P_s s_j^{\frac{D_a-1}{2}} \frac{1-D_a}{3-D_a} \left( s_{\text{slip}}^{3-D_a} - s_s^{3-D_a} \right)$$

(51)

in plastic conditions. Equations (50) and (51) are substituted into Eq. (45) if the conditions in Eqs. (47) and (48) are satisfied.

### 4.1.3. Dilation

The mean plane of a fracture sample of size $h$ will eventually be inclined with respect to the mean plane of the whole fracture due to waviness and large scale roughness. This inclination can also be defined as the asperity slope for the asperity base $h$. For an isotropic fractal, the geometrical properties should be directionally invariant, thus the asperity slope should have zero mean. For a normal distribution of the asperity slopes, forward and backward slopes can be represented by the positive and negative branches of the distribution. The maximum and minimum expected values of the slope would then be related to the standard deviation of the asperity slope (Barton, 1990), when assuming a certain confidence level. A possible choice would be to define that the maximum asperity slope $S$ with base $h$ is equal to the standard deviation of the slope distribution multiplied by a factor $\alpha$ (Paper C):

$$S(h) = \alpha \sqrt{G_{\text{slope}}} h^{H-1}$$

(52)

where $H$ is the Hurst exponent of the roughness. In this way, the maximum asperity slope would be governed by a power law of the asperity base $h$. Anisotropy of roughness causes the frequency distribution to be shifted towards positive or negative values, and this could be easily taken into consideration by adding a constant positive or negative angle to Eq. (52). On the other hand, an asperity base $h$ that is too short would lead to an unrealistic value for the slope, nearing infinity.

For small asperity slopes, the tangent of the slope angle $S$ can be replaced by the angle itself (in radiant). During shearing, the elementary dilation increment due to pure kinematic mechanism will be $S(\nu) dv$. Integrating for a shear dislocation of $A$, gives:
\[ d = \int_{0}^{\hat{\Lambda}} S(\nu) d\nu = \alpha \sqrt{G_{\text{slope}} \frac{\Lambda_{\text{crit}}}{H}} \quad (53) \]

In Sec. 4.1.2, the contact areas between the facing surfaces of the samples have, for hypothetical reasons, been considered parallel to the mean plane of the fracture, or in other words, flat. This is usually not the case for rock fractures because their asperities are inclined by a certain angle with respect to the mean plane. By considering all contact spots with the same inclination, the slope of the asperity does not significantly change the previously calculated integrals. If we consider the asperity slope, the resultant of the tractions and pressures on the contact areas are inclined with respect to the mean fracture plane. A reference rotation has to be considered, which gives the resultant forces parallel \(FT\) and orthogonal \(FN\) to the mean fracture plane. By introducing a matrix notation, we can write:

\[
\begin{bmatrix}
FN \\
FT
\end{bmatrix} =
\begin{bmatrix}
\cos S(\nu) & -\sin S(\nu) \\
\sin S(\nu) & \cos S(\nu)
\end{bmatrix}
\begin{bmatrix}
PTOT \\
QTOT
\end{bmatrix}
\]

(54)

where the angle \(S\) is the asperity slope for a certain shear displacement \(\nu\) as defined in Eq. (52). An analogous matrix relation holds for the local and the global displacements:

\[
\begin{bmatrix}
du \\
dv
\end{bmatrix} =
\begin{bmatrix}
\cos S(\nu) & -\sin S(\nu) \\
\sin S(\nu) & \cos S(\nu)
\end{bmatrix}
\begin{bmatrix}
d\delta \\
d\gamma
\end{bmatrix}
\]

(55)

4.2. Laboratory tests

4.2.1. Normal loading

Normal load testing was carried out on fracture samples using a standard MTS servo-hydraulic stiff machine, at the Laboratory of the Department of Structural Engineering, KTH. The tests were carried out to determine the normal stiffness of the fractures for rock mechanics characterisation. Samples were taken from the rock mass located at the prototype repository site at Aspö HRL (Lanaro, 2001a). The technical specifications of the laboratory equipment are listed in Table 1. Two different types of gauges were used for recording deformations during testing: extensometers of type 632.11C-20 and 632.12C-20.

The samples from borehole cores were prepared according to two procedures depending on the inclination of the fractures with respect to the core axis. The first set of samples did not require any special preparation since the fractures were nearly perpendicular to the core axis. The cores were simply checked for planarity of the end surfaces and taped so that the two halves of the fracture would hold together during manipulation and installation of the displacement gauges (Figure 26).
Table 1. Technical specifications for the servo-hydraulic loading machine MTS 311.21s.

<table>
<thead>
<tr>
<th></th>
<th>MTS 311.21s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>MTS 311.21s</td>
</tr>
<tr>
<td>Force</td>
<td>500 kN in compression/extension</td>
</tr>
<tr>
<td>Maximum plate distance excursion</td>
<td>150 mm</td>
</tr>
<tr>
<td>Maximum sample size</td>
<td>2280 mm/609 mm height/width</td>
</tr>
<tr>
<td>Weight</td>
<td>20 KN</td>
</tr>
<tr>
<td>Stiffness</td>
<td>$1 \times 10^9$ N/m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accessories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic holder</td>
</tr>
<tr>
<td>Spherically connected compression plates</td>
</tr>
<tr>
<td>Different extensometers</td>
</tr>
<tr>
<td>50 kN load gauge</td>
</tr>
</tbody>
</table>

The procedure for testing the samples with inclined fractures was a bit more complicated than for non-inclined fractures. The setup used was the same for both the shear and normal load tests. The cores were cut off parallel to the fracture plane, inclined to the core, at a distance of about three centimetres from the fracture. Then, the two halves of the sample are taped together. Steel rings were needed to hold the fracture blocks in the shear box machine. Quick-hardening concrete (EMACO® T 926) was poured into the first ring and the sample pressed into the ring so that the fracture plane remained outside the concrete.
After a few hours, the concrete inside the first ring was hard. The fracture sample moulded into the ring was turned up side down and pressed into the second ring filled with fresh concrete. Some mutters are used as spacers between the rings, so that the fracture is positioned outside the concrete. The mutters are removed before normal loading.

Figure 27. Setup for normal load testing of a sample with fracture inclined with respect to the core axis.

Results

The samples of rock fracture and intact rock were tested according to a given loading sequence in three steps:

1\(^{\text{st}}\) loading cycle: from about 0.5 MPa to 5 MPa; three loading-unloading cycles between the same stress levels;

2\(^{\text{nd}}\) loading cycle: from about 0.5 MPa to 10 MPa; three loading-unloading cycles between the same stress levels;

3\(^{\text{rd}}\) loading cycle: from about 0.5 MPa to 15 MPa; three loading-unloading cycles between the same stress levels;

The loading speed was 0.5 MPa/s for all tests.

The normal stress acting on each sample was back-calculated based on the area of the elliptical or circular fracture surface and from the total load applied by the loading machine. For each of the fracture samples, a sample of the intact rock taken from the same core, fracture free, was also tested using the same conditions. The deformation measured in the laboratory consisted of a contribution from the fracture deformation and one from intact rock deformation; the latter contribution has to be removed for determination of the fracture stiffness.

Fracture at a right angle with respect to the core axis exhibit the typical load versus deformation curve reported in the literature (see Figure 28). All deformation curves start
from a normal stress level of about 0.5 MPa, a value that was imposed to avoid instability of the testing equipment.

All deformation curves for the fracture inclined with respect to core axis are presented in Figure 29. The graphs range from typical asymptotic curves to almost linear curves.

Figure 28. Normal stress versus closure of fracture samples with fracture at a right angle with respect to the core axis (Set 4).

Figure 29. Normal stress versus closure of samples with fracture inclined with respect to core axis (Set 1).
Remarks

Due to the fracture geometry, some of the samples needed a special preparation before normal loading. This implied the use of steel rings and cement mortar. Although the sample preparation was done very well in advance with respect to testing (about 20 hours before) and with quickly hardening cement, some anomalies of the sample behaviour occurred. This can be explained by plastic deformation of the mortar developed at high normal stress. Furthermore, slippage between the contact rock-mortar and steel-mortar cannot be excluded. Therefore, it was decided to reduce the maximum normal stress from 30 MPa to 15 MPa.

For the samples prepared with steel rings, the normal deformations between the rings are measured. The deformation of the fracture was obtained from that of the whole sample by deducing the deformation of the intact rock sample. This was obtained by testing intact rock samples moulded in steel rings. For the core samples with no steel-ring preparation, only deformations of the intact rock and of the fracture are recorded.

Some minor deviations between the results obtained for borehole cores and for setups with steel rings and mortar were also observed. The curves of normal stress versus closure of the two sets of samples diverged for normal stresses larger than about 8 MPa. This could be explained by the initiation of the plastic deformation.

The fractures perpendicular to the core axis were taken from a sub-horizontal fracture set, while the inclined fractures belonged to a sub-vertical fracture set from the prototype repository at Åspö HRL. Aperture measurements on the samples gave the mean and the mean standard deviation of 0.208 mm and 0.152 mm for the samples with fractures perpendicular to the core axis, and 0.777 mm and 0.762 mm for the inclined fractures, respectively. Thus, the samples with inclined fractures have an aperture that in average is 3.7 times larger than the aperture of the samples with perpendicular fractures. This geometrical property greatly affects the normal stiffness of the fractures in the way that large mean and standard deviation of the aperture indicates smaller total contact area between the surfaces of the fracture. Small total contact area of the fractures produces lower normal stiffness.

Some difference between perpendicular and inclined fractures can also be due to the sample size. In fact, the samples from the sub-vertical fracture set were taken from cores of 51 mm in diameter, while the sub-horizontal fractures were collected from cores of 61 mm diameter.

4.2.2. Shearing

Direct shear testing was also carried out at the Geo-technological Laboratory, KTH, for the characterisation of the fracture samples. All fracture samples were moulded into steel rings with cement mortar as described in Sec 4.2.1 for the samples with fracture inclined with respect to the core axis.

The aim of the test campaign was to determine the shear stiffness of fracture samples at four levels of normal stress: 0.5, 1, 5, 10 MPa (Lanaro, 2001b). Shear and normal displacements during testing were measured together with shear and normal load. The shear stiffness was obtained for a shear stress of about half the peak shear strength and also for the peak shear strength. Moreover, the peak and residual shear strength envelopes were determined for the two fracture sets considered. An unloading-reloading cycle was performed after the samples had reached the peak-shear strength for the purpose of measuring the reloading shear stiffness. The evaluation of the correlation between the geometrical and mechanical properties of joints was the second objective of the test campaign (Paper E).
Tests were performed with a machine designed in 1964, upgraded in 1973 (Thurner) and automated with a computerised acquisition system at the beginning of the ’90. The machine can accommodate samples with diameter of 150 mm and a free height of maximum 50 mm for shear testing and 150 mm for normal testing, respectively. The rings containing the upper and lower part of the samples are placed on a steel table. Pivots at the top and at the bottom of the sample guarantee a centred position of the normal load. The normal load is applied by means of a system of leverages with a counterbalancing beam. During testing the beam has to be in horizontal position. Thus, the position of the beam should be adjusted depending upon the vertical deformation of the sample. This is done by screwing downward the beam by using two wheels. The horizontal shear force is applied at constant speed by a combination of two electrical engines for back and forward movements. The horizontal force is applied to the middle height of the upper ring. A jaw system inhibits rotations of the upper half of the sample. Forces, vertical and horizontal, and deformations, vertical and horizontal, are directly recorded by electrical transducers connected to a personal computer. All transducers have an accuracy of less than 1% (2% being the maximum allowed by ISRM’s standards (1981)) and are chosen for having an optimal bridge and minimum sensitivity to moisture. Each reading is recorded with four digits, except for time that is recorded with seven digits. A view of the testing equipment and the acquisition unit is given in Figure 30.

The tests were carried out according to “Suggested method for laboratory determination of the direct shear strength of rock fractures” (ISRM, 1981). Tests were conducted with a shearing speed of 0.002 mm/sec. The samples of rock fracture were tested under the following normal load stresses:

- 0.5 MPa or 1 MPa until a shear displacement of about 2 mm occurred, with an unloading-loading cycle at a shear displacement of about 0.25 mm;
- 5 MPa until a shear displacement of about 3 mm occurred, with an unloading-loading cycle at a shear displacement of about 0.5 mm;
- 10 MPa until a shear displacement of about 4 mm occurred, with an unloading-loading cycle at a shear displacement of about 1 mm.

Figure 30. Shear machine and acquisition system.
Figure 31. Gauge readings for sample KG0040A01 – 27.16 during direct shearing: a) shear stress versus shear displacement; b) dilation versus shear displacement.

Results

Figure 31 shows typical results of shear stress versus shear displacement and for dilation versus shear displacement. For the tested samples, the dilation versus the shear displacement plots almost linearly on a double logarithmic diagram (Figure 32).
Figure 32. Dilation versus shear displacement in double logarithmic diagrams for some core fracture samples.

Remarks
Although the simplicity of the testing machine, some interesting properties of the fractures during shearing could be observed. By performing a cycle of unloading-reloading of the sample when this had reached the shear peak strength, it could be observed that: i) no shear displacements were recorded during unloading; ii) the fracture exhibited a stiff behaviour during reloading. The stiffness calculated during reloading was 30-100% larger
than the initial stiffness. This also suggests that the shear behaviour can be elastic if the shear load is smaller than the largest shear load experienced by the fracture.

It was also observed that the dilation curves for the tested samples almost systematically follow a power-law of the shear displacement (see Figure 33). This is in agreement with the theoretical formulation in Eq. (53).

4.3. Modelling

4.3.1. Normal loading

The model for describing the fracture normal behaviour proposed in Sec. 4.1.1 is a closed form solution. There is no need of finite or boundary element formulations for determining the fracture behaviour. Computer programs that simulate the rock mass behaviour would certainly benefit from such model for considering the behaviour of the single fractures. Moreover, this research work develops a methodology for gathering the parameters involved in the simulation.

The model fits quite well the experimental results of normal loading on real fractures (Papers D and E). Some of the parameters have to be adjusted to optimise this fitting. However, as demonstrated in Paper E, the adjusted parameters are quite close to the ones from fracture mechanical characterisation (Dahlström 1998). Some examples of simulation of the fracture normal loading response are shown in Figures 33 and 34.

![Figure 33. Modelling of the normal load behaviour of the samples from Åspö HRL with fracture at a right angle with the core axis.](image-url)
Figure 34. Modelling of the normal load behaviour of the samples from Äspö HRL with fracture inclined with respect to the core axis.
4.3.2. Shearing

The model for describing the fracture shear behaviour needs some more efforts for overcoming the limits that are: i) the evolution of the contact area during shearing is not completely understood; ii) the dilation angle is a function of the normal load applied to the fracture and of the hardness and strength of the rock at the contacts; these aspects are also the topic of many recent studies (e.g. Plesha, 1995; DeJong et al., 2000).

Some hints about the variation of the pattern of the contact spots can be obtained by simulating the evolution of the aperture during shearing (Papers B and C). For a shear displacement \( \nu \), the mean value of the aperture, \( \mu_\nu \), and the variance of the aperture, \( \sigma_\nu^2 \), can be predict by Eqs (12) and (13). By comparing Eq. (13) and Eq. (14), one recognises a term equivalent to the variogram of the aperture during shearing. This term changes from the semi-variogram of the initial aperture, to the variogram of the roughness, for a large shear displacement. If the aperture distribution is fractal during shearing, then its equivalent Hurst exponent \( H^* \) would vary between that of the initial aperture and that of the roughness. In Figure 35, the evolution of the equivalent Hurst exponent during shearing is given for the fracture in Paper C.

![Figure 35](image_url)

**Figure 35.** Evolution of the equivalent Hurst exponent \( H^* \) of the aperture during shearing.

These changes of the aperture geometry were also observed by Power & Durham (1997) as result of numerical shearing simulations of the digital image of a real rock joint. In their simulations, the upper surface of the fracture was rigidly translated with respect to the lower surface, and the evolution of the aperture was observed. The aperture and pattern of contact area was observed to change as well. In particular, it was found that the size of the contact spots increased while their number diminished during shearing. This is in agreement with the results obtained in Paper B. The growth of the contact spots is justified by the increase of the aperture correlation length; the reduction of the number of contact spots is due to the increase of persistence and Hurst exponent of the aperture distribution. Borri Brunetto et al. (1998) observed the same behaviour by carrying out numerical simulations of shearing of the digital image of a perfectly mated sandstone fracture. A threshold was observed for the shear displacement beyond which there was no more correlation between the fracture surfaces. This could coincide with the correlation length for the roughness for that particular sample.

The covariance function in Eq. (14) becomes zero for a certain delay interval \( \delta_a^* \). This distance is also defined as correlation distance of the aperture during shearing. As for the
Hurst exponent and fractal dimension of the aperture, also this correlation length evolves during shearing as shown in Figure 36. Its behaviour can be approximated very well by a power law of the kind:

\[
\delta_a^n = \delta_a + C_\delta \nu^\rho 
\]

(56)

where \(\delta_a\) is the correlation length of the initial aperture distribution, while \(C_\delta\) and \(\rho\) are two constants determined by fitting the numerical results.

**Figure 36.** Numerical simulation of the evolution of the correlation distance of the aperture during shearing and power law interpolation proposed in Eq. (56).

Concerning dilation, the experimental results quite clearly show the validity of the power law in Eq. (53). From the analysis in Paper E, it results that the parameters involved change with the level of the normal load, and present a certain spreading for the samples of the same fracture set. However, it seems possible to use the same equation not only for predict the dilation for low normal loads, but also for high normal loads. It is not clear yet how the parameters are affected by asperity damage, normal load and moisture.
5. FLOW

The ability of modelling the hydraulic behaviour of natural rock fractures has been recognised as one of the most difficult challenges for inferring and guaranteeing the functionality and safety of underground excavations and waste disposals.

Fractures in rock mainly determine rock mass behaviour for fluid and gas flow and contaminant transport. It is then important to understand the physics of the flow in a single fracture to be able to simulate the behaviour of complicated fracture networks in the rock mass. The need of modelling fracture networks implies that the geometrical description of the fracture has to be kept as simple as possible due to the amount of topological parameters already involved in the analyses.

Many recent studies have addressed the problem of flow determination inside rock fractures of known aperture distribution. In most cases (e.g. Hakami & Larsson, 1996; Capasso et al., 1999; Pyrak-Nolte & Morris, 2000), the aperture distribution along the whole fracture sample had to be specified. In other words, irrespective of the mathematical algorithm for flow calculation, a huge amount of data consisting in thousands of aperture values and their locations has to be input in the analyses. This is of course feasible when considering just one rock fracture, but it makes these methods unpractical when several fractures in a fracture network have to be considered.

On the other hand, commercial software often considers very simplified fracture models for calculating the hydraulic response of a rock mass. In most cases, the Darcy’s law and parallel plate model are applied to each fracture or portion of fracture in the network (MAFIC, Miller et al., 1995; NAPSAC, AEA Technology, 1998). Some other analogue models that simulate fractures with pipes suffer the problem of getting reasonable fracture parameters (Nordqvist et al., 1995).

Amadei & Illangasekare (1992) proposed an analytical technique for calculate the flow in a single fracture called “integral transform method”. The technique solves the Reynolds’ equations for flow by means of Fourier transforms of the fracture transmissivity distribution. It calculates the hydraulic head distribution on the fracture plane and the flow rate. This technique also allows for the determination of the flow of viscoplastic materials (e.g. grouting) flowing and clogging through a fracture (Amadei, 2000; Amadei & Savage, 2000). However, the technique has never been tested on aperture distributions of real fractures, and its analytical results have not been compared to experimental laboratory or field tests.

Although this technique also needs the fracture to be described by the aperture map, the Fourier transform method suggests a new possibility of investigating the transmissivity of rock fractures. The idea is to describe the fracture transmissivity pattern by directly giving the Fourier transform. The transforms characterise the magnitude, spatial correlation and variability of the fracture transmissivity of a fracture. For natural fractures, the transform appears to be a characteristic function that could be defined by means of a limited number of parameters (Paper F). A few parameters that describe the complicated flow pattern in the fracture are an evident advantage in numerically modelling of large fracture networks.

5.1. Theory

The conservation of the mass contained inside an elementary volume of fluid delimited by the fracture walls, is given by:
\[-\left[\frac{\partial (av_x)}{\partial x} + \frac{\partial (av_y)}{\partial y}\right] = \left(\frac{1}{K_a} + a\beta\right) \frac{\partial p}{\partial t} + \sum_{i=1}^{N} Q_i \delta \left(x-x_i\right) \delta(y-y_i)\]  \hspace{1cm} (57)

where \(a\) is the aperture; \(v_x\) and \(v_y\) are the fluid velocity components; \(K_a\) is the fracture wall stiffness; \(\beta\) is the fluid compressibility; \(Q_i\) is the volume rate of flow at the source or sink position \(i\); and \(\delta\) is the Delta Dirac function. In laminar flow, the velocity components can be defined according to the Darcy’s law:

\[v_x = -K \frac{\partial h}{\partial x}, \quad v_y = -K \frac{\partial h}{\partial y}\]  \hspace{1cm} (58)

where \(K\) is the hydraulic conductivity of the fracture. By introducing Eqs. (58) into (57), it gives the governing equation for flow in a rock fracture. In laminar flow conditions, Louis (1969) proposed the following empirical expression for the transmissivity:

\[T = \frac{K}{a} = g \frac{a^3}{12\nu} \frac{1}{1 + 8.85 \left(\frac{k}{2b}\right)^{1.5}}\]  \hspace{1cm} (59)

where \(g\) is the gravity acceleration; \(\nu\) is the kinematic viscosity of the fluid; \(k\) is a “roughness” height. The coefficient \(\xi\) is 0 for \(k/2b \leq 0.033\) and is 1 for \(k/2b > 0.033\) when the flow ceases to be parallel plate flow.

Eq. (57) can be solved by using finite Fourier transforms of the fracture transmissivity and of the hydraulic head inside a rectangular fracture with two opposite impervious edges parallel to the overall direction of flow \(x\) (Amadei & Illangasekare, 1992; Amadei et al., 1994). By assuming the hydraulic head to be constant along the open ends of the fracture, its distribution can be expressed by the sum of a linear and a residual function as:

\[h(x,y) = \tilde{h}(x,y) + \frac{\lambda - \alpha}{L} x + \alpha\]  \hspace{1cm} (60)

where \(\lambda\) and \(\alpha\) are the given hydraulic heads at the end of the fracture of length \(L\) and width \(H\); \(\tilde{h}\) is the residual of the actual hydraulic head with respect to the linear approximation. The finite Fourier transforms of the residual hydraulic head and transmissivity are respectively:

\[\Delta(n,m) = \sum_{j=1}^{J} \sum_{i=1}^{S} \tilde{h}(x_j, y_i) \sin \left(\frac{n\pi x_j}{L}\right) \cos \left(\frac{m\pi y_i}{H}\right) \Delta x \Delta y\]  \hspace{1cm} (61)

\[\Gamma(n,m) = \sum_{j=1}^{J} \sum_{i=1}^{S} T(x_j, y_i) \sin \left(\frac{n\pi x_j}{L}\right) \cos \left(\frac{m\pi y_i}{H}\right) \Delta x \Delta y\]  \hspace{1cm} (62)

\(J\) and \(S\) are the number of values of \(\tilde{h}\) and \(T\) considered along \(x\) and \(y\) direction, respectively; \(\Delta x = L/J\) and \(\Delta y = H/S\); \(n\) and \(m\) are indexes of each term of the Fourier transform, with \(n, m \leq N\), the number terms of the truncated Fourier series. The convergence and accuracy of the solution depend on the truncation number \(N\) of the finite Fourier transforms.

By using a “\(\cos x-sin y\)” Fourier transform of the residual hydraulic head \(\tilde{h}\), the boundary condition of the problem are ensured and Eq. (57) results into a linear system of
equations in the unknown $\bar{h}$. This system can be solved by using the Gauss-Siedel method, so that all the coefficient of the transform of the residual hydraulic head $h$ can be calculated. The hydraulic head inside the fracture is obtained as:

$$h(x,y) = \frac{2}{LH} \sum_{n=1}^{N} \Delta(n,0) \sin \frac{n\pi x}{L} + \sum_{n=1}^{N} \sum_{m=1}^{N} \Delta(n,m) \sin \frac{n\pi x}{L} \cos \frac{m\pi y}{H} + \frac{\lambda - \alpha}{L} x + \alpha$$

(63)

5.2. Transmissivity and fracture aperture

The absolute value of the “cosx-siny” Fourier transform of the fracture transmissivity seems to have a power-law shape when plotted as a function of the frequency (Figure 37):

$$G_T(s) = G_{T0}s^{GEXP}$$

(64)

Power laws are also common shapes of the transform of the geometry of fractal objects. As it was demonstrated in this research, aperture appears to be governed by fractal laws, thus its Fourier transform $G$ always has a power-law expression. For an isotropic case:

$$G(q,s) = G_0 \left[ q^2 + s^2 \right]^{\alpha_{1/2}}$$

(65)

where $q$ and $s$ are frequency values; $G_0$ is a dimensional proportionality coefficient. The power exponent $\alpha$ is related to the fractal dimension $D_a$ of the aperture according to a polynomial that depends on the degree of the transform. It is then straightforward that there should be a relation between the power exponent of the transform of the transmissivity and the fractal dimension of the aperture, of which the transmissivity is a function (Eq. (59)).

For the two square areas of 60×60 mm from the fracture of Stockholm Gneiss, it was observed that the transform of the transmissivity had almost the same power-law parameters; and that the transmissivity average value should only affect the first term of the transmissivity transform (for zero frequency). Having parameters that describe the transmissivity pattern (like the mean transmissivity, power-law exponent and dimensional coefficient) allows one to quantitatively characterise the hydraulic properties of the fracture. Comparison and classification of different samples would then be possible for site characterisation.
Figure 37. Double logarithmic diagrams of the absolute “cosx-cosy” Fourier transform of the transmissivity versus spatial frequency for two areas of the fracture of Stockholm Gneiss along two perpendicular directions x and y.

5.3. Remarks

The results of flow obtained with the “integral transform method” show to greatly depend on the choice of the aperture data density and roughness parameter. Reducing the density of the data changes the statistics of the aperture because aperture is sample-size dependent. For a given aperture map, also the roughness parameter should be determined from the comparison of numerical and laboratory results for one and the same fracture.

Flow through rock fractures is very sensitive to normal load and shearing (e.g. Zimmerman et al., 1990; Chen et al., 2000): the shear dilation significantly enhances the permeability of the fracture while the opposite effect has normal loading. Thus, for determining fracture transmissivity, instead of the roughness parameter by Louis, it would be more correct to consider an “aperture variation coefficient”. For low normal loads, the roughness of the fracture basically does not change during shearing, although the fracture experienced closure or dilation. The dependency of the aperture from the normal load or the shear displacement (Papers B and E) could be used to determine the aperture maps for these loading conditions. The effect of loading on the fracture flow would then explicitly considered.

By characterising the transmissivity by means of its Fourier transform, the scale dependency of the aperture would also be directly considered. The calculation of the transform requires a large amount of aperture data as the sample increases in size. However, this would not change the number of parameters needed for the power-law model of the transmissivity.

The illustrated analytical method for fracture flow would be suitable for modelling the hydraulic behaviour of fractures when these are parts of a three-dimensional fracture network. Moreover, the actual formulation could be modified so that closed fractures and intersections between fractures could be considered.
6. DISCUSSION

This study shows that two geometrical features can conceptually describe the topography of a rock fracture surface: roughness and angularity. Roughness depends on the asperity height, while angularity is governed by the slope of the planar facets interpolating the surface. The statistics of these quantities (e.g. variance) are functions of the sampling scale, and for this reason are well described by a power-law or self-affine fractal model (Mandelbrot, 1977). The variation of the statistics is sometimes bounded by stationarity thresholds beyond which the fractal model ceases to apply (Paper A). This kind of characterisation should be performed on samples larger than the roughness stationarity threshold as that defines the minimum representative size of the samples.

Based on the experimental results, an analogous model was formulated for the fracture aperture. The standard deviation of the aperture was observed to increase with increase of the sample size, but seemed to reach a sill defined as the aperture stationarity threshold (Paper B). Furthermore, the analysis of the contact spots showed that their contours have a fractal shape. Because the contact spots are areas where the aperture is zero, the same geometrical rules that apply to contact spots should apply to aperture. This was calculated in Paper D: the fractal dimension obtained from the statistics of the aperture and from the contact area analysis for the same sample was roughly the same (maximum difference 20%).

Aperture and roughness are more or less correlated depending on the degree of mismatch between the walls of the fracture. The correlation between the aperture and the asperity height was calculated and seems to depend only on the aperture characteristics (Paper B). It was also observed that a large difference in asperity height between two points is likely to be associated with a large variation of the aperture. In turn, this means that aperture should be larger at the peaks and throats of the surfaces of the fracture, and that contacts would be mainly located on the slopes of the surfaces.

The experimental results presented here along with available data in the literature suggest that, while the fracture roughness behaves as a persistent self-affine fractal (0.5<H<1), the aperture can be characterised as a persistent or anti-persistent self-affine fractal (0<H<1). Anti-persistence of the aperture is expected in relation with mated fractures while persistence is related with unmated fractures that have experienced some degree of shearing. This fact has a great effect on the spatial structure of the roughness and aperture distributions. As the Hurst exponent diminishes, the fractal dimension increases and the complexity of the fractals grows. An increase of geometrical complexity of the aperture distribution reduces the fracture transmissivity of fluids; produces tortuosity and channelling in the fracture flow; determines the pathways and deposition patterns of substances transported by flow; influences the conditions for capillary bubble trapping and gas accumulation in the fracture (Jarsjö & Destouni, 1998).

The formulated geometrical model for describing the fracture morphology (Paper B) allows the simulation of shearing of the fracture: it can predict the mean value and covariance of the aperture distribution (Paper C). Aperture mean value and variance depend not only on the characteristics of the initial aperture, but also on the surface properties of the fracture walls (Wang et al., 1988), and in particular on the variogram of the roughness and surface slope at the contacts. According to the geometrical model, the aperture maintains its fractal nature during shearing and its Hurst exponent evolves. The aperture covariance is mainly affected by the variogram of the roughness, therefore, it seems reasonable that aperture behaviour changes from that of the initial aperture to that of the roughness. Consequently, the Hurst exponent of the aperture should vary between these two extremes and the correlation length would increase from that of the initial
aperture to that of the roughness. A behaviour similar to that predicted theoretically was observed in the laboratory by Yeo et al. (1998). They tested epoxy-resin replicas of a sandstone sample and measured the increase of the mean and variance of the aperture during shearing.

The methods used for investigating roughness, angularity and aperture of natural rock joints could be easily extended to fractures with strong anisotropy. In such case, directional parameters should be computed from the fracture geometry. For instance, rectangular sampling schemes could be used instead of the square ones adopted here. Moreover, directional instead of omnidirectional variograms may be used. The probability distribution of the contact areas should be modelled with a formulation suitable for self-affine and anisotropic fractal objects. Results presented by Lanaro et al. (1998) showed how stereographical projections and directional frequency distributions of the surface slopes could highlight anisotropy of the surface angularity. Furthermore, forward and backward directions could also be distinguished. In Paper A, the power spectral technique was applied for the evaluation of the fractal dimension of roughness. This technique captured the anisotropy of the fracture surface by identifying two different values of the fractal dimension along the two orthogonal directions of investigation.

The geometrical properties discussed above are directly applicable to the formulated constitutive model for fracture normal and shear deformability (Papers D and E). To complete the necessary data set some mechanical properties of the rock surface must be specified so that the contact spots can be characterised.

The model for simulating the normal deformability of the fracture presents some features that makes it compatible with the geometrical description of the fractures. By implementing the fractal theory for describing the contact areas, the fracture behaviour becomes scale dependent. The largest contact area considered in the model depends on the size of the sample, if no stationarity threshold is encountered. The assumption that some of the contacts might undergo yielding allows the simulation of monotonic loading. However, the contacts can behave elastically during unloading and reloading up to the previous yielding. Cycles of loading and unloading/reloading can then be simulated. The extension of the contact area also changes as a function of the imposed normal load in a realistic way. All these features make the model particularly suitable for simulating experimental results (Paper E).

The model for normal deformability substantially differs from the other developed so far. In fact, the idealisation of the fracture geometry is very effective and proved experimentally sound. Thus, there is not a need for a large amount of input data specifying the exact position of each contact (unlike the approach developed by Borri-Brunetto et al., 1998). On the other hand, it does not consider the mutual interaction of one contact with its neighbouring contact. Roughness is not considered as a key parameter for normal deformability like in other models, e.g. Wang, 1994; Wang & Kwansiewski, 2000. Instead, the contact area represents the fracture geometry. Moreover, no mechanical tests on the fractures are necessary for providing input mechanical parameters as often is needed for other models (e.g. Bandis et al., 1983). The model predicts yielding of certain contacts, unlike in Brown & Scholz model (1986), that can behave elastically during reloading.

The model for fracture shear deformability, based on the geometry of the contact areas, presents the same advantages as does the model for normal deformability. In this case, however, the influence of roughness on the kinematics of the blocks becomes determinant. Thus, the equivalent slope of the contact areas needs to be specified as a function of the shear displacement. In Paper C such a relation is presented based on the fractal model for angularity and is verified against experimental results in Paper E. Furthermore, the same relation seems to properly describe dilation at different levels of normal loading.
The advantages of the proposed model for fracture shear deformability are: i) the model is completely theoretical; ii) it could simulate deformability, peak-shear and residual-shear conditions; iii) it can model fracture dilation; iv) it is suitable for either normal stress or normal stiffness constrains in loading. However, the present model formulation is not totally satisfactory because: i) it does not account for damage of the surface asperities; ii) it is not able to realistically predict the peak shear displacement; and iii) it requires the knowledge of the variation of the contact areas and related fractal parameters during shearing.

Another application of the fractal approach to aperture is for modelling fracture flow. The fractal model for aperture also describes, other than the statistics, the spatial distribution of the aperture. Thus, there is potential use for quantifying channelling and tortuosity of the flow, although this cannot be done directly with the present knowledge. The “integral transform method” (Paper F) overcomes the problem by calculating the Fourier transform of the transmissivity of the fracture. In this way, the transmissivity values do not directly enter the Reynolds’ equation and the number of parameters used in the calculation markedly decreases. Because the transform spans over a wide range of frequencies, it depends on the density of the aperture data provided as input and also on the size of the sample. This raises the question of what suitable data density should be used for numerical modelling of fracture flow. In fact, reducing the density of the data performs a sampling of the aperture values and, since aperture is scale dependent, modifies its statistics by smoothing the distribution. This is reflected by the varying model results for flow obtained for the same sample. A fractal definition of the transform of the fracture transmissivity could be applied and the transform could be calculated on a high-density aperture map, thus approximated by a power-law or fractal function, and finally be inputted into the Reynolds’ equation. In this way, a large aperture data set would not weight down the calculation but only improve the resolution of the transform of the transmissivity. Additional research should be done on the mathematics of the relation between fractal aperture and transform of the transmissivity. Moreover, laboratory results of fluid flow through rock fractures should be carried out to validate the “integral transform method” and refine a technique for choosing a suitable density for the aperture map.
7. CONCLUSIONS

The 3D-laser-scanning technique has been successful in many applications where a high density and accuracy of the geometrical data is required. This is obtained thanks to the fast scanning procedure and the large size of the samples that can be measured. The research work presented in this thesis shows that 3D-laser-scanning technique is then very suitable for measuring rock fracture morphology. Here, dense data measured on large surfaces are needed for determining roughness and angularity of the fracture. Furthermore, the accuracy of the measurements allows development of a new technique of relocation of the topography of the surfaces once the fracture is opened. In this way, the aperture of the fracture can be measured and analysed in relation to the morphology of the surfaces.

Roughness and aperture seem to be two independent properties of the fracture morphology that originate either at the time of formation of the fracture or later. Their independence reduces when the fracture is sheared because roughness usually effects larger scales than aperture, and the role of roughness overtakes that of aperture.

This focuses the attention on the variation of roughness and aperture in relation to the sample size. For this purpose, sub-samples of fractures, with a size ranging from a few millimetres to decimetres, were investigated. The slope of planar facets interpolating the sub-sample surface with respect to the sample mean plane, and the surface heights with respect to the planar facets, were chosen as descriptors of roughness. A similar approach was taken for the aperture determination that was defined as the shortest distance between the fracture surfaces at each measured point. By analysing the multitude of data, it was observed that the standard deviation of the three parameters was not constant when increasing the size of the sub-samples: for aperture and asperity height the standard deviation increases, while for the asperity slope it decreases. The variation of the standard deviations was proved to follow power laws of the sampling size, which are a typical property of probabilistic fractal objects. The Hurst exponent could describe the scaling properties of roughness and aperture. Roughness showed to be persistent (Hurst exponent ≥ 0.5), but aperture could span from persistent to anti-persistent (0 < Hurst exponent < 1). This observation is of major importance because both persistence and anti-persistence of a fractal object are related to its geometrical complexity. The Hurst exponent, besides representing scaling, could be used for classifying tortuosity and channelling of the fluid flow in a fracture. Moreover, by interpreting aperture as a fractal object, the probability distribution of the contact area between the facing surfaces of a fracture can be mathematically described.

The relocation technique also allowed for measuring the cross-correlation between the asperity height and aperture. The cross-correlation was also evaluated theoretically; it depends on the aperture variance and variogram.

Based on the fractal models for roughness and aperture, and on the formulation of the cross-correlation between asperity height and aperture, a model for describing the aperture during shearing was set up. This model is based on the assumption that the dilation angle is known and that no damage or penetration of the fracture surfaces will occur during shearing. The model gives an increasing mean value and variance of the aperture for an increasing shear displacement. It is interesting to note that as the shearing displacement increases, the aperture becomes more and more dependent on the variogram of the asperity height of the fracture. Consequently, the Hurst exponent of the aperture seems to evolve from its initial value (“at rest”) to the Hurst exponent of the surface of the fracture. It also implies that the aperture distribution changes during shearing, and becomes more persistent. Thus, it can be concluded that an aperture pattern with low Hurst exponent...
corresponds to mated fractures, and \textit{vice versa}. This has a great effect on the flow and frictional properties of the fracture.

The developed theory contains all the elements for establishing a constitutive model for hydro-mechanical behaviour of natural rock fractures. In fact, the formulation does not consider a particular rock material or type of fracture, and its damage sensibility and strength can be characterised separately. From the frequency distribution of the contact areas, a model for describing the normal loading behaviour of the measured fractures was set up. This model is based on the assumption that each contact spot responds to normal loading in the same manner as the contact between two spheres. Elastic and plastic deformations are considered, so that the elasto-plastic model simulates loading while the elastic model simulates unloading-reloading of the fracture. The experimental results carried out on the samples could then be compared to the predictions made by the model; these are in very good agreement.

The same idea of integrating the resistant force for each contact spot was used for determining the shear behaviour of the fracture. This attempt was not completely successful for a number of reasons: i) the contact area of the fracture varies during shearing as does its fractal dimension; a more detailed geometrical analysis of the contact area during shearing is needed; ii) a criterion for degradation, bulking and seating of the asperities needs to be implemented into the model. On the other hand, the kinematics of the fracture dilation was well predicted by a simple power-law relation based on the scale dependency of the asperity slope. In comparison with the experimental results, it was also observed that the response depends on the level of normal loads that inhibits dilation.

The description of the fracture aperture as a fractal object opens up a series of new applications. One of those is the characterisation of the fracture geometry with respect to fluid flow. If one wants to avoid describing aperture either by giving average statistics or the whole map of values, then the Fourier transform of the fracture transmissivity could be used. The transforms of the geometry of fractal objects usually behave as power laws. Thus, just a few parameters would be necessary for describing the whole pattern of the fracture transmissivity. This gives major advantages, requires less data for numerical simulations of flow and takes into account the spatial variability of the aperture in the fracture plane.
8. FURTHER RESEARCH

The theoretical model proposed for the description of rock fracture morphology provides a mathematical explanation for most of the geometrical features measured on rock fracture specimens. However, there still are some questions about the general validity of the model primarily due to the limited number of samples and their relatively small size. In the author’s opinion, several geometrical tests should be performed to verify:

i) that roughness is systematically persistent;

ii) that aperture can be whether persistent or anti-persistent, depending on the fracture mismatching and degree of shearing;

iii) that the Hurst exponents do not vary with scale for aperture and roughness in their fractal range;

iv) that Eqs (13) and (15) which describe the covariance between the aperture and the variogram of asperity height, are general and robust.

In the geometrical model, the fractal parameters are assumed to be constant. However, there might be “nested” structures with multiple levels of stationarity at different scales. This should be investigated by testing large fracture specimens.

The formulation of the aperture evolution during shearing, which has been qualitatively proved based on numerical data from the literature, should be given experimental and quantitative confirmation. This could be achieved by applying the relocation technique to the fracture samples during shear testing so that the relative position of the fracture walls could be inferred from the position of reference spheres. These spheres are always accessible during testing, including when the fracture void is injected with mortar or resins. Moreover, the technique allows a direct investigation of the surface damage during shearing. The surfaces could be scanned before and after shearing so that all changes of the morphology can be detected and related to the original shape of the surfaces.

More effort should be devoted to the characterisation of the fracture contact spots. Particular attention should be paid to the evaluation of the asperity slopes of the fracture surfaces at the contacts in order to clarify the mechanism of dilation during shearing. Furthermore, it would be very interesting to study if there is any change in the total area of the contact spots as a function of the sample size. In fact, the fractality of the aperture should be mirrored by the fractality of the contact area, which implies scale dependency. The evolution of the contact area during shearing has been one of the major objects of this study; the constitutive model for fracture behaviour proposed in this thesis requires the knowledge the contact area for inferring both the normal and shear behaviour of the fracture.

The model developed for rock fracture geometry can be used to generate artificial fractures in the real rock masse. Monte-Carlo simulations can then be performed on different fracture patterns and geometries, used to predict possible responses.

A rock mass usually contains several fracture sets that intersect each other. There is a need to understand the effects of the geometry of the fracture intersections on the rock mass behaviour. The relocation technique developed in this research work could be used to study the geometry of rock joint intersections in three dimensions.

The comparison of the constitutive model with the experimental behaviour of the fractures gives the opportunity of highlighting some of the limits that exist in the theory. With respect to the model of fracture normal deformability, the main aspects that require some improvements are:
i) the relation between the contact area of the single spot and the global fracture closure (see Eq. (23)). This assumes linear elasticity and an approximation of the geometry for large radii of curvature compared to the normal displacement at the contact;

ii) the plasticity criterion in Eq. (28): a more realistic criterion than that of perfect plasticity could be assumed.

These improvements would allow the following up of the behaviour of the fracture at higher levels of normal stress, and probably would be able to reach the asymptote of the closure often shown by the experimental tests.

More research effort should be dedicated for refining the model for fracture shear behaviour. Besides a more accurate study of the pattern changes of the contact area during shearing, a criterion for implementing asperity damage, gauge formation and thickening of the fracture surfaces should be used. In fact, the actual formulation does not consider the damaging effects due to the normal load but only its value and related deformations.

Also the new approach for integrating the geometrical characterisation of the fracture with a tool for calculating fluid flow and fluid pressure distribution needs some further development as:

i) further numerical analysis should be performed to check the likelihood of the formulation for describing the fracture transmissivity by means of power-law parameters.

ii) a theoretical study should be carried out with respect to the relation between the “cosx-sinx” Fourier transform of the fracture transmissivity and the fractal properties of the aperture distribution.

iii) the numerical code and the power-law formulation should be validated against experimental results.

These improvements would result in a model that covers almost all hydro-mechanical aspects related to the behaviour of natural fractures in rock. The fewer number of parameters required by the model would make it very suitable for modelling large fracture networks with limited computational effort.
9. REFERENCES


