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Methods for Experimental Estimation of Anelastic Material Properties

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Doctoral Thesis

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To my beloved wife, Sara,
and our children, Benjamin and Elias

Preface

The work presented in this thesis is the result from research conducted at the structural dynamics research group, Department of Structures and Materials, Aeronautics Division, Swedish Defence Agency FOI (former Aeronautical Research Institute of Sweden, FFA). The author gratefully acknowledge the financial support provided mainly by the Swedish Defence Material Administration (FMV Flyg).

Thanks to Krister Dovstam (co-author in paper A) for enlightening discussions on modal methods in the field of material damping and structural dynamics. Thanks also to my former colleague Dr. Stella Papadia-Einarsson for valuable discussions during the course of this work, in particular the support during the preparation of paper B. Many tanks to Dr. Adam Zdunek, Niklas Sehlstedt, and Ulf Tengzelius for discussions and valuable comments in the end of this work and thanks also to my former colleague Lennart Nystedt for help with the measurements. Special thanks also to all my colleagues at the Department of Structures and Materials, for creating the best scientific working environment.

Finally, I would like to thank my supervisor Prof. Peter Göransson for encouragement and support.

Stockholm in November 2001

Mats Dalenbring

CONTENTS

ABSTRACT

Reduction and passive control of structural vibrations through addition energy dissipating materials is fundamental in vibroacoustic design of modern transport applications. The present work proposes improved methods for estimation of three-dimensional constitutive anelastic (damping) parameters for such materials, in particular those exhibiting isotropic and transversely isotropic material symmetry. The constitutive relationship used throughout this work is based on a linear viscoelastic model, with connection to thermodynamics. For a class of isotropic materials, a general experimental material damping estimation methodology based on a combined modal vibration model and neural net (NN) technique is proposed. In addition a simplified damping approximation method is proposed, based on a least-squares estimation technique. A combined experimental and numerical investigation is presented for an Aluminium-Plexiglas plate structure using material parameters estimated for each individual part. From excellent agreement between experiments and numerical vibration response simulations it is demonstrated that the proposed methodology yield very accurate results in the whole frequency interval of estimation. The estimation strategy was slightly modified for application to transversely isotropic materials and successfully demonstrated on an Aluminium-Plexiglas laminate structure. For this latter case homogeneous elastic and anelastic (damping) properties were used. Finally, the estimation technique is validated experimentally, with good result, on two for practical application interesting composite structures, a Carbon fibre-epoxy laminate and a three-layer Aluminium laminate structure with an embedded constrained viscoelastic layer.

KEYWORDS: Constitutive material damping modelling, isotropy, transverse isotropy, homogeneous materials, linear viscoelasticity, experimental damping estimation, composite laminate structures, finite element displacement modes, modal analysis, structural vibration modelling

THESIS

PART I Overview of field of research

PART II Appended papers

Paper A: K. Dovstam and M. Dalenbring (1997) Damping Function Estimation Based on Modal Receptance Models and Neural Nets. *Research Journal Computational Mechanics* 19, 271-286.

Paper B: Dalenbring, M. (1999): Damping Function Estimation Based on Measured Vibration Frequency Responses and Finite-Element Displacement Modes. *Mechanical Systems and Signal Processing* 13(4), 547-569.

Paper C: Dalenbring, M. (2001). Validation of Estimated Anelastic Isotropic Material Properties and Vibration Response Prediction. Submitted for publication in *The Journal of Sound and Vibration.*

Paper D: Dalenbring, M. (2001). An Explicit Formulation of a Three-dimensional Material Damping Model with Transverse Isotropy. Accepted for publication in *International Journal Solids Structures*.

Paper E: Dalenbring, M. (2001). Experimental Material Damping Estimation for Transversely Isotropic Laminate Structures. Recommended for publication in *International Journal Solids Structures*.

PARTITION OF WORK BETWEEN AUTHORS

Paper A: Krister Dovstam did the planning of the first phase in this project, involving mainly the adopted theoretical basis for the modal vibration response model. Development of estimation methodology was done jointly by the authors. Mats Dalenbring did all practical finite-element, neural-network implementations and numerical simulations. Krister Dovstam did the final report writing in collaboration with Mats Dalenbring.

1. BACKGROUND

Designs of modern transportation systems must satisfy objectives concerned with cost, for example fuel consumption, and weight, to allow for increased payload capacity. In addition developing contemporary manufacturing processes, e.g., high-speed milling, further make it possible to produce large sections of a construction in one piece.

Altogether these requirements, in combination with the technology developed to meet them, drive the introduction of new structural concepts which in turn pose new challenges for noise and vibration design. One example of this is the structural damping, caused by e.g. friction in joints between structural parts, which in an assembled structure may be reduced substantially if the number of joints is decreased. This typically leads to a higher level of structural vibration, which is in conflict with increasing demands on passenger comfort and vibration safety margins for sensitive equipment. Furthermore there is a trend in modern engineering applications, especially in the aerospace and automotive industries, to exploit new combinations of different materials in the form of composite structures. To cope with these challenges, there is a need for computationally efficient material models with the capability to accurately represent the dynamic behaviour.

A perfectly elastic material is a theoretical idealisation. In practice, the chemical constitution and imperfections in the material typically introduce anelasticity and dissipation of vibration energy or damping. For crystalline materials, e.g., metallic materials, a very low level of internal material damping may be observed. Traditional engineering damping models are usually based on a constant damping loss factor model, proportional to the strain (hysteretic damping) or alternatively proportional to the strain rate (viscous damping). In the case of light damping a simpler material structural damping (loss) factor model, estimated at each resonance from measured vibration frequency response functions, may be justified.

For a large class of real materials including, polymers, rubber and viscoelastic layers[1] damping is substantially higher. The constitutive, material specific, relationship between stresses and strains, including damping, is then usually modelled in the frequency domain by using dynamic, complex frequency dependent, material moduli. A natural consequence of a more detailed constitutive description of the physical reality is the increased complexity of the material model involved. One argument presented against the use of such models is that most refined models in the literature have a major drawback in requiring a large number of

[1] used in constrained layer damping treatments

parameters to accurately describe observed material behaviour. Thus there seems to be a common need for a candidate material model parameterisations which is simple both in estimation and operation in order to be successful in applications. A successful application of a given technique depend also on the number of moduli (degree of material symmetry) and important types of vibration deformation occurring in the structure under investigation.

From a practical point of view, it is only possible to measure structural deformations which, in general, are three-dimensional and non-homogeneous. To separate different damping mechanisms the ideal situation would be to have a number of independent experimental tests with homogeneous deformation. An alternative approach would be to estimate material damping indirectly from measured structural vibrations (frequency response functions) on a test piece of the material. However, this requires a toolbox containing the following three components: a general three-dimensional constitutive law, an accurate vibration model of the test piece and suitable estimation procedures. The work presented in this thesis constitutes a proposal for such a toolbox.

2. THEORY AND APPLICATIONS

2.1. Overview of Previous Related Work

In the literature, there exists an extensive amount of theoretical works about linear three-dimensional constitutive modelling of material. Various aspects of constitutive modelling in general structures are available in the form of numerous state-of-the-art references and modern textbooks, e.g. Chandra *et al.* (1999), Gibson (1990) and Finegan and Gibson (1999), Gibson (1994) and Sun and Lu (1995). However, in practical applications the material damping identification/estimation procedures are usually based on purely experimental, cf. Bert (1973), Ewins (1986), Woodhouse (1998) and Balmes (1997), analytical or two-dimensional constant loss-factor models using rather simple damping models, Abrahamsson *et al.* (1996), Lin and Ewins (1994), Imregun *et al.* (1995a, 1995b) and Gelin and Ghouati (1995). To the knowledge of the author there are only few works published about material parameter estimation techniques based on more general three-dimensional constitutive models e.g., Ohkami *et al.* (1997, 1998, 1999). However, it is important to account for the three-dimensional state of deformation in the structure, depending on the geometry, even at rather low frequencies, cf. Hwang and Gibson (1991). These three-dimensional effects are especially important near boundaries and joints.

2.1.1. Linear Viscoelastic Materials

The classic viscoelasticity starts with Boltzmann (1876), Zener (1948) and thermodynamic viscoelastic models by Biot (1955, 1956). The linear three-dimensional time domain relationship between stresses $\sigma_{ij}(\mathbf{x},t)$ and strains $\varepsilon_{kl}(\mathbf{x},t)$ in a standard linear viscoelastic solid is given by a convolution integral, a Boltzmann superposition integral, as

$$\sigma_{ij}(\mathbf{x},t) = C_{ijkl}(\mathbf{x},0)\varepsilon_{kl}(\mathbf{x},t) - \int_0^t \frac{dC_{ijkl}(\mathbf{x},t-\tau)}{d\tau}\varepsilon_{kl}(\mathbf{x},\tau)d\tau\,, \tag{1}$$

where $C_{ijkl}(\mathbf{x},t)$ is the material stiffness tensor components, including dissipation of vibration energy, defined by the integral term. Restrictions imposed by thermodynamics and fading memory are discussed in the literature, e.g., Biot (1955,1956), Christensen (1979), Enelund (1996), Lesieutre and Bianchini (1995) and Dovstam (1995, 2000a). In the general case the material stiffness tensor is temperature dependent, but here and in the following isothermal conditions are assumed, which means that temperature rise due to dissipation of vibration energy is neglected. The use of linear models is common practice in the field of vibroacoustics, cf. Fahy (1985) and Norton (1989). This is acceptable due to the fact that material non-linearities are localised mainly to areas where stress concentrations are present, i.e., near abrupt changes in geometry and joints between connected structures.

In vibroacoustic applications involving stationary vibrations it is convenient to use the standard complex modulus approach. Frequency domain properties are then defined by applying the elastic-viscoelastic correspondence principle at a constitutive level, with each component of the unrelaxed stiffness tensor $C_{ijkl}(\mathbf{x},t)$ directly related to the fully relaxed elastic material stiffness and material specific dissipation functions, see e.g. Flügge (1967), Sun *et al.* (1995) or Ohayon and Soize (1998).

Formally, the Laplace transformed complex constitutive relationship between stress and strain is given in the frequency domain as

$$\sigma_{ij}(\mathbf{x},s) \;= \left[C^e_{ijkl}(\mathbf{x}) + C^a_{ijkl}(\mathbf{x},s)\right]\varepsilon_{kl}(\mathbf{x},s), \tag{2}$$

where $C^e_{ijkl}(\mathbf{x})$ are the relaxed elastic components of the constitutive stiffness tensor and $C^a_{ijkl}(\mathbf{x},s)$ the augmentation ($s = i2\pi f$ and f is the current frequency of vibration). The augmentations of the relaxed elasticities, cf. Dovstam (1995, 2000a), determined by complex frequency dependent damping functions, are treated as constitutive, material specific parameters in this work.

The tensor $C^{a}_{ijkl}(\mathbf{x},s)$ may be parameterised in several alternative ways. From a list of more recent publications covering both isotropic and anisotropic materials one may mention Lesieutre *et al.* (1990, 1995) on thermodynamic models using hidden variables, Bagley and Torvik (1983) and Enelund (1996) and Gaul (1999) on fractional models and Golla and Hughes (1985) on minioscillator models.

In applications to anisotropic composite laminate structures it is natural to utilise also the concept of effective macro-mechanical models. These are defined as homogeneous and generally anisotropic representative volumes of the composite structure with the interaction between structural details included at a less refined constitutive level. In analogy to the isotropic case the elastic, static, material parameters of the structure need to be known in order to estimate the anelastic material damping parameters. Methods for extraction of elastic material parameters of laminates are usually based on two-dimensional plate and shell theories in connection with higher-order deformation theory, cf. Reddy and Phan (1985) and recently Khdeir and Reddy (1999), Carrera (2000) and Kant and Swaminathan (2000). Alternative, three-dimensional, effective anisotropic material models start with Postma (1955) and Rytov (1956), in the field of geophysics, proposing effective solutions for wave propagation in infinite periodically layered medium of two isotropic materials. By assuming a state of deformation in the laminate structure, effective elastic material moduli for general orthotropic material symmetry may be given in a closed-form "rule of mixture" as geometrically weighted averages of the constituent parts of the laminate structure, cf. Christensen (1979). Similar methods, for extraction of three-dimensional effective elastic material moduli, have been developed by Chou and Carleone (1972), Pagano (1969, 1970, 1974), Sun *et al.* (1988, 1996) and recently by Chen and Tsai (1996) and Whitcomb and Noh (2000), by using the first-order approximation that interlaminar shear stresses are constant through the laminate, cf. Pipes and Pagano (1970) and Hyer (1998). To the author's knowledge there are very few works published on three-dimensional modelling of damped composite plates, which also was confirmed in the review paper by Chandra *et al.* (1999).

2.1.2. Frequency Domain Vibration Response Models

Frequency domain response models for simulation of damped vibration in structures are extensively described in a number of modern textbooks, e.g. Sun and Lu (1995), Ohayon and Soize (1998) or Gibson (1994). The focus in this thesis is on three-dimensional models capable of handling general geometries and boundary conditions. In this context, it is natural to use the finite-element method.

There are two major methods available for calculation of structure vibration frequency response. The first approach, referenced to as the direct (frequency by frequency) finite-element method (FEM), is defined as straightforward discretisation of the complex frequency domain weak formulation of the boundary value problem. The second, alternative approach is based on modal series expansion of the vibration field by using a complete three-dimensional displacement modal basis. The modal basis is defined as solutions to a corresponding elastic eigenvalue problem of the test specimen, cf. Ohayon and Soize (1998), approximated using FEM. The associated modal coefficient spectra are approximated by projection of the vibration field onto the basis and do generally depend on the elastic properties, excitation (modal forces) and the material damping, given in terms of modal loss factors, cf. Nashif *et al.* (1985) and Sun and Lu (1995).

In built-up structures the damping model is often based on the so-called strain energy method, introduced by Ungar and Kerwin (1962). A finite-element application of the strain energy method was first introduced by Johnson and Kienholz (1982), where the modal loss factors are conveniently calculated using partial modal strain energies approximated by using standard finite-element matrix projections, cf. also Nashif *et al.* (1985) and Sun and Lu (1995). In the following a few recent works are mentioned, Hwang and Gibson (1991), Saravanos and Chamis (1991), Barrett (1992), Rikards *et al.* (1993, 1994), Saravanos (1994), Saravanos and Pereira (1995), Koo and Lee (1995), Yarlagadda and Lesieutre (1995), Korjakin *et al.* (1998) and Maher *et al.* (1999). However, most of these works are based on one- or two-dimensional models and constant loss factor models. A general three-dimensional modal vibration response model was defined by Dovstam (1997, 1998a, 1998b, 2000b), and applied to isotropic materials with the modal loss factors determined by two frequency dependent material damping functions, each one associated with the two Lame´s moduli, and corresponding partial modal strain energies.

Another important aspect is the existence of modal coupling, in laminates, due to geometry, orientation, stacking sequence, constituent layer properties and the vibration mode of interest. Topics such as coupling and three-dimensional effects (interlaminar stresses) on damping of laminates have been investigated by Hwang and Gibson (1991, 1992a, 1992b) and Hwang *et al.* (1992c) by performing three-dimensional analyses based on the strain energy method and layer-wise finite-element analysis on typical laminate structures. Further, there exists another type of modal coupling, present also in homogeneous structures, due to the damping itself. The last effect is taken care of by using an unconditionally convergent three-dimensional modal vibration model, based on separate series expansions of the displacement and the stress fields, proposed by Dovstam (1998b, 2000b).

2.1.3. Estimation Procedures

Material damping properties may be estimated indirectly by minimising the difference between the vibration model and the corresponding measured vibration frequency response functions, on a suitable test of the structure. The separation of elastic and anelastic parts of each complex dynamic material modulus is essential in order to estimate correctly the constitutive properties of the material. In order to succeed, all deformations related to a studied parameter, reflected in the vibration model, have to be excited also in the experimental test set-up. Finally, a robust estimation procedure is needed.

There are many different optimisation methods available in the literature, but yet no single preferred solution. The optimal candidate material damping function estimation method should be robust and insensitive to noise in measured data and computationally efficient. In standard textbooks, Press *et al.* (1992), there exist traditional so-called gradient optimisation methods. In applications of gradient methods one of the key issues is to find the balance between spending computer power on improving search direction, using higher-order terms (using the Hessian matrix) and step length control or using simpler methods, numerical differentiation and a larger number of iterations. Another problem is the difficulty shared by many gradient optimisation methods, that the algorithm may get trapped in a local minimum. Gradient methods are local in the sense that the search direction, steepest decent, is optimal in the neighbourhood of the current point. These methods require starting values close to the solution, in order to find the global minima of the quadratic distance between the model and measured vibration response. In order to avoid some of the shortcomings of traditional methods, especially the global search performance, several relatively new and alternative methods as, e.g., genetic algorithms, simulated annealing and Artificial-Neural-Networks have been proposed in the literature, cf. Goldberg (1989) and Haykin (1994). The Neural-Network technique have been successfully applied, to a wide range of problems in the literature, cf. Haykin (1994), Rahim (1994), Sjöberg (1994), as a practical and efficient way to construct a non-linear input-output mapping of general nature. It has further been shown, cf. Haykin (1994), that the neural-network technique fulfil the universal approximation theorem, which states that a multi-layer network with back-propagation learning can approximate any continuous function to any accuracy, provided that the hidden layer is large enough. The main disadvantage of the standard Neural-Network/back-propagation algorithm is that it is a first-order gradient technique, slow in convergence, performing a zigzag walk toward the optimal solution, cf. Haykin (1994) and Tang and Kwan (1993).

2.2. Present Work

The focus of the present thesis is to develop a general (three-dimensional) material damping estimation methodology, applicable to a wide class of important engineering materials, based on integration of suitable vibration models, measurements and proper estimation tools. The approach taken here uses some of the basic concepts of traditional experimental modal analysis, cf. Ewins (1986), modified to take into account the internal material damping behaviour. The main contribution of the present work is improved methods for estimation of three-dimensional constitutive damping parameters for materials with isotropic and transversely isotropic material symmetry. The starting point is a three-dimensional linear material model incorporated in a modal vibration response model derived by Dovstam (1997, 2000b). Provided that we know accurately the geometry and the elastic material properties from static measurement, damping properties may be extracted through the application of a modal vibration response model and proper estimation (model optimisation) tools. The modal vibration model is based on finite-element approximations of structural (elastic) properties of the test structure and on laboratory measurements, at controlled temperature, moisture, excitation and boundary conditions.

2.2.1. Estimation of Anelastic Material Properties

In paper A, a material damping estimation methodology is proposed, based on construction and training of a Neural-Network (NN) and the back-propagation algorithm, and applied to a Plexiglas test structure. The Neural-Network/back-propagation algorithm was chosen since it is possible to explicitly differentiate the modal vibration model with respect to homogeneous isotropic material damping parameters.

An alternative to the Neural-Network approach was developed (paper B) on the basis of the following arguments. From the results of estimation on the Plexiglas test structure, in paper A, one may observe that the mean value[2] of modal partial strain energy, i.e., the strain energy associated with the shear modulus to the total modal strain energy, is about 90 percent. The partial modal strain energy is a combination of both the geometric characteristics, with a relatively small thickness compared to the global dimensions, and the elastic material properties of the test structure. This observation motivates the introduction of an approximate modal damping function, neglecting the structural damping contribution associated with Lame´s modulus, providing a simplified unique relationship between the modal (structural) damping and the isotropic material damping and a reduced number of parameters to be estimated (paper B). A similar approximative approach is adopted in paper E for the transversely isotropic case. Note here also that, for nearly incompressible materials, Zdunek (1992), as for example rubber-isolators, this assumption is almost exactly fulfilled, as the deformation associated with volumetric change is very small, due to the high bulk modulus or Lame´s modulus.

The approximative least-squares estimation technique developed in paper B is based on the above-mentioned assumption and a modal vibration response model. The modal (Fourier) coefficient spectra in the response model, are extracted, fitted in standard least-square sense, cf. Strang (1986), by a projection of a sub-set of the entire vibration displacement field onto a sub-set of the modal basis, cf. Dovstam (1998a). The modal damping function is determined at each damped resonance from given (FE computed) modal data and the estimated modal coefficient. Finally, real material damping function amplitudes are estimated by using the incremental least-square damping approximation method, proposed in paper B for isotropic materials and in paper E for the case of transversely isotropic material symmetry.

The latter material symmetry is valid for a class of balanced fibre composite laminate structures and stacked isotropic materials including constrained viscoelastic damping treatment layers. The transversely isotropic vibration model is first tested numerically, in paper D, on a three-layer Aluminium-Plexiglas laminate structure by comparing a homogeneous model of the structure, using estimated homogeneous material parameters, with a finite-element model with detailed modelling of the constituents.

Paper E presents an approximate method for three-dimensional transversely isotropic material damping estimation by straightforward generalisation of the experimental estimation methodology proposed for isotropic materials in paper B. In this case the complexity in modelling and extraction of material parameters increase compared to isotropic materials. Effects of damping are included, by proper assumptions of the state of deformation in the

[2] over 140 number of elastic modes

continuum and the use of five constitutive damping functions associated with each independent elastic moduli. The distribution of elastic strain-energies on each independent material moduli are calculated and dominant damping contributions are investigated in order to simplify the description of the homogenised material models.

Apart from the mathematical description of the physical reality, the accuracy and robustness in estimation will of course depend on uncertainties in measurement and model parameters, cf. Ziaei-Rad and Imregun (1996), Hjelmstad (1996) and Beatty and Chewning (1979). In the following, paper B and C, some systematic sources of uncertainty are listed and briefly discussed. It is here assumed that the proposed finite-element models of the test structures are sufficiently accurate for our purpose with respect to element discretisation errors and possible element locking, cf. Reddy (1997) and Côté and Charron (2001). The uncertainties of random character are assumed to be small (averaged out) in the measurement process and not addressed in the thesis. Air damping in the surrounding media and boundary damping from the suspension of the test structures are further assumed negligible or inherent in the damping estimation process of the structures, cf. De Visscher *et al.* (1997). Two important sources of uncertainties are related to the uncertainty in measurement position localisation and distribution and direction of the excitation force, Olbrecht *et al.* (1996), Lee and Chou (1996) and Dalenbring and Einarsson (1999). These uncertainties may to some extent be studied from reciprocal measurements, i.e. interchange of the response and excitation position. Other important sources of uncertainties are due to the manufacture process, i.e., varying geometry, mass distribution and elastic material properties of the structure. The effects of uncertainty in elastic moduli on extracted damping from dynamic response of highly damped systems are numerically investigated in Einarsson and Dalenbring (2000) or Dalenbring (2000), by performing Monte Carlo simulations, cf. Press *et al*. (1992) and Kleiber and Hien (1992). From the result in Dalenbring (2000) it may be concluded that the estimation method, proposed in paper B, is robust with respect to uncertainty (typical scatter) in elastic data. The validity of extracted material parameters is also strictly limited to the frequency interval of estimation.

2.2.2. Application to Experimental Test Structures

The estimation technique has been applied to a number of different structures, all tested in laboratory condition. The measurements were performed on each test structure hanging in long thin threads, in order to simulate a stress-free boundary condition. The dynamic behaviour of the structures were measured in terms of forced vibrations, frequency response functions[3] (FRFs), at a chosen number of points distributed over the surface of each plate. Each structure was excited with a random input signal, by an electrodynamic shaker. The input force spectrum component, normal to the structure surface, was measured by means of a Piezo-electric force transducer. The velocity of the structure was measured using a non-contact Laser-Doppler-Vibrometer (LDV) transducer; cf. detailed descriptions in paper A, paper B, paper D and Stanbridge and Ewins (1995).

In the first part, papers A and B, a mixed numerical-experimental material damping estimation approach is proposed for isotropic materials and applied to a rectangular test structure made from Plexiglas[4]. The Plexiglas material is chosen based on its highly damped material behaviour in combination with a high degree of material simplicity, with approximately isotropic material symmetry on a macroscopic scale, cf. Muzeau and Perez (1993), Goble and Wolff (1993), Kral *et al.* (1998), Ferry (1980) and Eklind and Maurer (1996). Estimated Plexiglas material damping properties are validated, on a structure with different non-rectangular geometry, in paper B. The proposed isotropic damping estimation methodology is finally validated experimentally by comparing measured and predicted structural vibrations (in paper C) for a combined Plexiglas-Aluminium structure in a set-up exhibiting a completely different set of boundary conditions compared to the free conditions used in estimation of material parameters for Plexiglas.

The results from the tests in papers A, B, C and Figure 1, show a very good overall agreement between measured vibration responses and model predictions. Thus it is shown that the proposed estimation methodology is straightforward, robust and accurate. The results do also verify the hypothesis concerning the estimated[5] material (constitutive) parameter independence of geometry, excitation and boundary conditions.

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[3] defined as quotient spectra between displacement $\tilde{\mathbf{u}}(\mathbf{x},s)$ (at point $\mathbf{x}$) and excitation $\tilde{\mathbf{F}}(\mathbf{x}_e,s)$, at position $\mathbf{x}_e$.

[4] PMMA, Polymethyl Metacrylate

[5] in the frequency interval 40-500 Hz

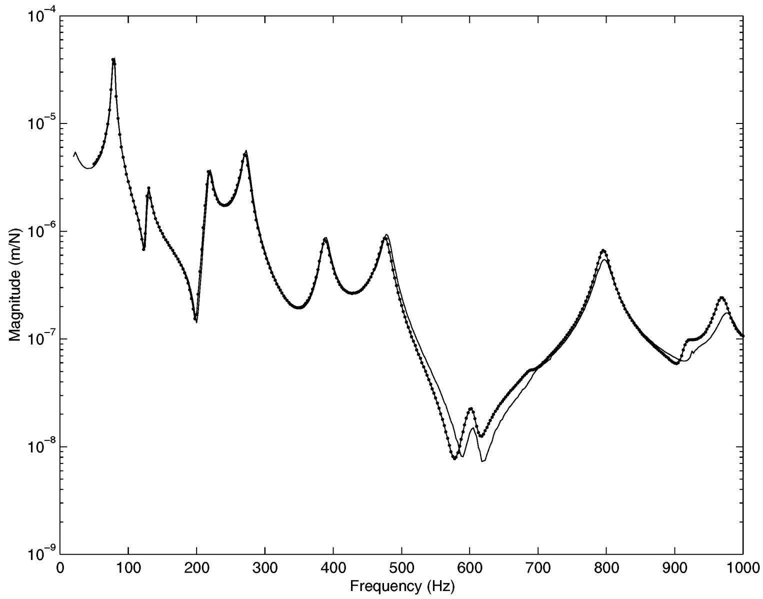

Figure 1. Typical FE calculated (dotted-solid curve) transfer receptance FRF (response R_{33} normal to the plane of the plate in point 29 and normal excitation at position 287) for the two-layer Al-Plexiglas laminate plate (with detailed modelling of the constituents) and corresponding measurements (solid curve), in the frequency interval 40-1000 Hz (paper C).

Secondly, the proposed method is applied to two different transversely isotropic experimental laminate composite structures, with symmetric lay-ups in order to avoid geometric coupling effects. The first experimental structure in paper E is a 32-layer carbon fibre-epoxy[6] laminate structure with a relatively low level of material damping. The second test structure (paper E) is a three-layer Aluminium laminate with an embedded viscoelastic layer[7], having extremely high material damping. In this case the material model of the test structures are based on a constitutive modelling using homogenised elastic material parameters augmented, from the start, with homogeneous material laminate damping properties. The proposed estimation methodology is in this case inherently approximate as

[6] prepreg Siba Geigy HTA/6376
[7] of the type 3M112P05

the model is based on three-dimensional dynamic homogenisation of the laminate, a first order constitutive approximation of the macro-mechanics.

The results from the experimental test structures in paper E, using estimated material damping properties on the same structures but different excitation points, show that the agreement between measured vibration responses and homogeneous model predictions is not as good as for isotropic materials but still good, Figures 2 and 3. Thus it may once more be concluded that the proposed method in paper E provides a straightforward, robust and accurate estimation methodology.

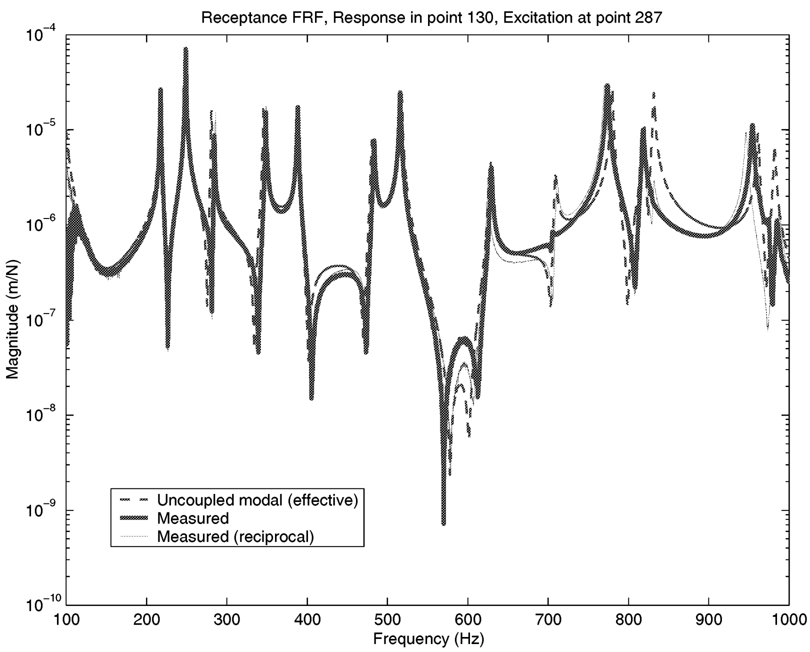

Figure 2. Typical calculated (dashed curve) transfer receptance FRF (response R_{33} normal to the plane of the plate in point 130 and normal excitation at position 287) for the carbon fibre-epoxy composite laminate plate (paper E) and corresponding measurements (solid curves) in the frequency interval 100-1000 Hz.

3. SUMMARY AND FUTURE WORK

In the following, the scope of the work is summarised and the originality of the present work is pointed out together with future extensions.

- Proposition of a new general (three-dimensional) isotropic material damping estimation procedure is given in paper A. The estimation is based on, measured vibration frequency response functions and a modal response model, finite-element approximations and Artificial-Neural-Networks. The technique requires no prior assumptions about the distribution of the damping between the two isotropic moduli.
- Implementations are made within the framework of a general purpose, finite element code.
- Proposition of a new simplified material damping estimation procedure, for homogeneous isotropic plate structures, applied to a Plexiglas test structure, is presented in paper B.
- Experimental validation of the estimation methodology with respect to the material (constitutive) parameter independence of geometry and boundary conditions is presented in paper C, using two composite Aluminium-Plexiglas structures.
- An explicit formulation of a viscoelastic modal vibration model, suitable for estimation of transversely isotropic material properties, is proposed in paper D.
- Proposition of a three-dimensional material damping estimation procedure and validation of estimated approximate homogenised three-dimensional transversely isotropic viscoelastic material properties for two experimental composite laminate structures are presented in paper E.

A natural extension of the present thesis work is further development of procedures applicable in connection with general three-dimensional homogenisation including damping, derivations of explicit damping functions and estimation tools for material with general anisotropy. Future work may also involve investigations of the sensitivity of the estimation method to uncertainties in the model parameters, in order to construct optimal test and estimation conditions, and improved methods to find bounds on estimated material parameters.

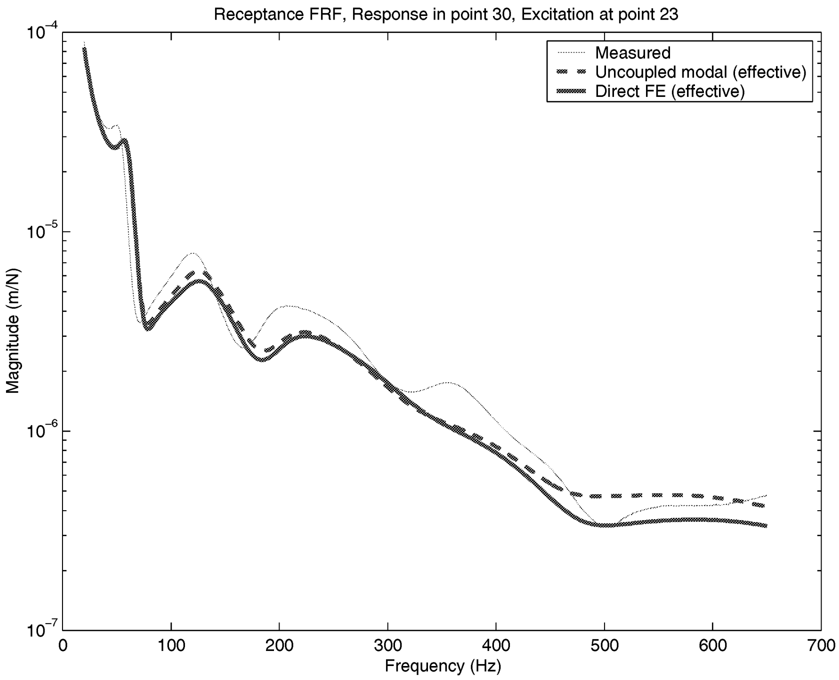

Figure 3. Typical measured (thin solid curve) transfer receptance FRF (response R_{33} normal to the plane of the plate at point 30 and normal excitation at position 23) for the Aluminium-visco-Aluminium laminate plate, corresponding effective uncoupled modal response model (thick dashed curve) and direct FE calculated (thick solid curve) vibration response, in the frequency interval 20-650 Hz (paper D).

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