Design and Evaluation
of a
Compact 15 kW PM Integral Motor

Peter Thelin

Royal Institute of Technology
Department of Electrical Engineering
Electrical Machines and Power Electronics
Stockholm 2002
Design and Evaluation
of a
Compact 15 kW PM Integral Motor

Peter Thelin

Royal Institute of Technology
Department of Electrical Engineering
Electrical Machines and Power Electronics

Stockholm 2002

Submitted to the School of Computer Science, Electrical Engineering and Engineering Physics, Royal Institute of Technology KTH, in partial fulfilment of the requirements for the degree of Doctor of Technology.
Abstract

This thesis deals with the integral motor of tomorrow, and particularly with a variable speed, sensorless permanent magnet synchronous motor with an integrated converter. The rated power is 15 kW at 1500 r/min. The outer dimensions are approximately the same as for the equivalent standard induction motor.

Control strategies for pumps and fans, i.e. suitable loads for variable speed motors, are briefly described. The huge energy savings that can be made by reducing the speed instead of throttling/choking the flow are pointed out. Compared to installing an induction motor with a separate converter, a PM integral motor will probably pay-off in less than a year.

A totally analytical expression for calculating the airgap flux density of permanent magnet motors with buried magnets is derived. The analytical expression includes axial leakage, and iron saturation of the most narrow part of the magnetic circuit of the machine.

A computer program for optimization of PM motors with buried magnets has been developed. It was used to design the manufactured prototype PM integral motor, and the parameters are investigated with analytical and/or FEM calculations. The optimization program is also used to suggest near-optimum pole numbers for desired powers (4-37 kW) and speeds (750-3000 r/min) of inverter-fed PM motors. Results show that compact buried PM motors should have relatively large airgaps and high NdFeB-magnet masses to improve the efficiency. Ferrite magnets are unsuitable.

Measurements on the manufactured PM motor, the novel concept of stator integrated filter coils, and the complete PM integral motor are presented. Special attention was given to temperature and overall efficiency measurements.

The rotor cage losses were investigated by time-stepping FEM. Four short circuit fault conditions were also examined in order to evaluate the risks of demagnetization of the buried magnets.

Keywords: integral motor, integrated motor drive, PM motor, PMSM, compact motor, filter coil, airgap flux density, axial leakage
Acknowledgements

I would like to thank my project leader Dr. Juliette Soulard for her good suggestions and for her support. She always took her time to answer when I came with my - not so short - “five-minute questions”. Thanks to my supervisor Prof. Chandur Sadarangani for guidance, encouragement, patience and help during this work, and for providing such a good atmosphere at our Division. Thanks also to my former project leader Prof. Hans-Peter Nee for his enthusiasm, for encouraging me to commence PhD studies, and for giving me good advice whenever I asked for it.

Many thanks to the skilful people who manufactured the PM integral motor prototype: Lic. Tech. (honorary) Sven Karlsson, Mr. Karl-Erik Abrahamsson and their staff in the workshop at ABB Corporate Research (heat-sinks and PM motor), and Mr. Ulf Karlsson, Mr. Thord Nilson, Lic. Tech. Lars Lindberg and their colleagues at Inmotion Technologies (converter and control circuits), Lic. Tech. Anders Lindberg, also at Inmotion Technologies, deserves credit for measuring the temperatures at different loads on the complete PM integral motor.

I would like to thank Dr. Jörgen Engström, with whom I have been sharing office spaces for about six years, for his companionship and all the discussions we have had on electrical machines etc. He is also remembered for all his playful initiatives like go-cart competitions, the slotcar track, and the force-feedback wheel.

I am very grateful to my “personal” system administrator (Unix/Linux) Lic. Tech. Karsten Kretschmar who always took his time to help me out when I needed it. I would also like to thank him for all the fun we have had at our Division, at the different conference sites etc.

Other persons who have offered me help with different computer related obstacles should not be forgotten. Thanks to: Mr. Thomas Korssell, Dr. Anders Lundgren, Dr. Erik Thunberg, Lic. Tech. Fredrik Carlsson, Dr. Rémy Kolessar, and Mr. Peter Lönn.

I would like to thank Dr. Eckart Nipp, Dr. Thomas Bäckström, and Lic. Tech. Peter Kjellqvist for all the interesting discussions on electrical machines, and other subjects, which we have had during our years at the Division.
Many, many thanks to Lic. Tech. Mats Leksell for discussions on electrical machines, control theory, the enigmas of the Universe, and for the answers he has given to most of my obvious/peculiar questions. His motto is always good to keep in mind: “Still confused, but on a higher level!”.

Thanks to Mr. Jan Timmerman for rebuilding the calorimetric measurement equipment, and for keeping me company during many long evenings down in the “dungeons” observing the Termos.

Thanks to Prof. Lennart Harnefors for introducing me to the food and beverages of Broncos Bar.

Thanks to Mr. Jan-Olov Brännvall and Mr. Yngve Eriksson for several skilful mechanical arrangements in the Electrical Machines Laboratory.

Thanks to Lic. Tech. Louis Lefevre for helping me to get started with MEGA, to Dr. Roger Hill-Cottingham at University of Bath for prompt answers to my questions about MEGA, and to Dr. Sanath Alahakoon for advise on using Simulink.

The members of the Electrical Machines and Power Electronics division are acknowledged for their good friendship, support, and all the funny leisure activities we have had together in the name of Roebels SK.

Also the “administrative” personal should be remembered for their good work; thank you Mr. Göte Bergh, Mrs. Eva Pettersson and Mrs. Astrid Myhrman.

I am also grateful to the following persons - in alphabetical order - who have contributed to my work in different ways:
Mr. Per-Olof Berg, Lic. Tech. Hans-Olof Dahlberg, Mr. Gert Hallgren, Mr. Tapio Haring, Mr. Michael Henze, Mr. Gösta Jansson, Mr. Mikael Lagerberg, Mr. Bo Malmros, Mr. Jürgen Mökander, Mr. Edward Paro, and Mr. Bengt Rydholm.

Thanks to the following persons, who I believe have and are doing an excellent job in teaching: Prof. Stefan Östlund (in the courses Electricity and Electrical Machines at KTH), Mr. Hans Mogensen (my high-school teacher in Electrical Machines), Mr. Dan Ögren (my high-school teacher in Electricity), Mr. Bengt Ivansson (my science and mathematics teacher in secondary school), and Mrs. Karin Jonsson (my teacher in primary
Thanks to Dr. Lars Jonsson and his colleagues at the Division of Electromagnetic Theory, for discussions on analytical calculations of magnetic reluctance.

The PMD-programme at the Competence Centre in Electric Power Engineering, where ABB Corporate Research, ABB Motors, Höganäs AB, Inmotion Technologies, ITT Flygt, Sura Magnets and the Swedish National Energy Administration (STEM) participate, are gratefully acknowledged for the financial support of the work.

Many, many warm hugs to my parents, Birgit and Lars Thelin. They have always supported me and provided for me in the best of ways. Tack för att ni finns, Pappa och Mamma!

Finally, I would like to send bunches of roses to my wife Carolina Thelin for her love, relaxing neck massages, culinary abilities, and for coping with my odd working hours. Hopefully we can spend more time together in Idet from now on! :o)

Stockholm, February 2002

Peter Thelin
Contents

1 Introduction ......................................................................................... 15
  1.1 Major driving forces for permanent magnet motors and integrated motor drives ......................................................... 15
  1.2 Outline of the thesis and publications ........................................ 18

2 Advantages of PM integral motors ................................................. 21
  2.1 Control strategies for pumps and fans ....................................... 21
     2.1.1 Pumps ........................................................................... 21
     2.1.2 Fans ............................................................................. 28
  2.2 Conventional integral motors ...................................................... 32
  2.3 Economical comparisons ............................................................ 37
     2.3.1 PM integral motor vs a converter-fed induction motor 37
     2.3.2 Magnet cost versus pay-off time and monetary saving 44
  2.4 Conclusions ............................................................................... 47

3 Accurate modelling of the airgap flux density of buried PMSM:s ................................................................. 49
  3.1 Analytical calculation of the airgap flux density of PM synchronous motors with buried magnets ............... 50
     3.1.1 Introduction .................................................................. 50
     3.1.2 Design principle ......................................................... 51
     3.1.3 Derivation of an expression for the airgap flux density 52
     3.1.4 Conclusion ................................................................... 59
  3.2 An analytical expression for the airgap flux density including iron saturation and axial leakage............. 60
     3.2.1 Modelling of axial leakages ......................................... 61
     3.2.2 Iron saturation and the final analytical expression ....... 66
  3.3 Iterative compensation for the magnetic saturation of stator and rotor teeth and yokes ............................. 71
  3.4 Conclusions ............................................................................... 74

4 Flux densities of the accurate models compared to FEM and measurements ................................................... 75
  4.1 Comparisons between analytical and FEM calculated axial leakage reluctances of the rotor ....................... 75
     4.1.1 Calculating axial leakage reluctance using 2D-FEM... 76
     4.1.2 Conclusions ............................................................... 81
4.2 Comparisons between analytical-, iterative-, FEM-calculated, and “measured” flux density .......................... 82
  4.2.1 Iterative and analytical calculations for Motors A-E.... 82
  4.2.2 Results of the iterative and analytical calculations...... 86
  4.2.3 Analysis of the results................................................... 89
4.3 FEM investigations of the no-load voltage of PM synchronous motors......................................................... 90
  4.3.1 Methods for calculating the induced no-load voltage... 91
  4.3.2 The vector magnetic potential method ....................... 92
  4.3.3 Calculations and comparisons of the induced no-load voltages for Motors A-E.......................................... 94
  4.3.4 Conclusions................................................................. 97
4.4 3D-FEM calculation of the influence of axial leakage flux for Motor A.......................................................... 98
4.5 Conclusions..................................................................... 100
5 Optimization of buried PMSM:s......................................... 101
  5.1 Optimization program.................................................... 101
   5.1.1 General layout of the computer program............... 101
   5.1.2 Description of the different parameters................. 103
   5.1.3 Calculation of losses................................................. 111
   5.1.4 Calculation of copper temperature....................... 116
   5.1.5 Efficiency versus speed.......................................... 116
  5.2 Choice of pole number for inverter-fed PMSM:s.......... 117
   5.2.1 Introduction............................................................. 118
   5.2.2 Computer program.................................................. 119
   5.2.3 Results..................................................................... 121
   5.2.4 Comments on the results........................................ 122
   5.2.5 Conclusion............................................................... 125
  5.3 Using Ferrite magnets instead of NdFeB magnets in the optimization of an 8 pole motor ................................. 125
  5.4 Conclusions.................................................................. 126
6 Prototype PM integral motor design ............................... 127
  6.1 Project description and specifications.......................... 127
   6.1.1 Background to the project........................................ 127
   6.1.2 Equivalent standard induction motor..................... 128
   6.1.3 Specifications for the PM integral motor.................. 129
6.2 Optimization ................................................................. 130
6.2.1 PM motor parameters and first results ...................... 130
6.2.2 Fine-tuned parameters of the chosen 8 pole motor .... 134
6.2.3 Fine-tuned parameters of a 12 pole motor ............... 137
6.2.4 Fine-tuned parameters of an 8 pole motor design opti-
mized with a more accurate flux density model .......... 137
6.3 Analytical and FEM calculations of the optimized 8 pole
motor design ................................................................. 138
6.3.1 FEM calculations of the airgap flux densities .......... 138
6.3.2 Number of winding turns per stator slot .............. 143
6.3.3 Calculation of fundamental d- and q-inductances .... 149
6.3.4 Saliency ratio ....................................................... 152
6.3.5 Field weakening region ....................................... 153
6.3.6 Torque characteristics ........................................ 155
6.3.7 Mechanical strength ........................................... 158
6.3.8 Converter circuit ................................................ 159
6.3.9 Corner coils - A new integral motor stator design .... 160
6.4 Investigation of dummy heat sinks and airflows ......... 168
6.5 Prototype design changes and prototype
manufacturing .................................................................. 170
6.5.1 Changes in the PM motor design ......................... 170
6.5.2 Manufacturing of the prototype motor ............... 175
6.6 Conclusions ............................................................ 178
7 Measurements ............................................................. 179
7.1 Measurements on the prototype PM motor .............. 179
7.1.1 Airflows and temperatures of real heat-sink #1 .... 179
7.1.2 Airflows and temperatures of real heat-sink #2 .... 184
7.1.3 Torque measurements .......................................... 186
7.1.4 Induced stator voltages ........................................ 188
7.1.5 Bearing voltage ................................................... 190
7.1.6 Measurement of the stator winding resistance .... 191
7.1.7 Measurements of d- and q-inductances .......... 191
7.1.8 Stator winding temperature ................................ 197
7.1.9 Corner coils ...................................................... 199
7.1.10 Instruments ....................................................... 202
7.2 Measurements on the complete PM integral motor .... 203
7.2.1 Temperature measurements ............................... 203
7.2.2 Line current with and without line-filter and DC-link inductance ............................................................. 205
7.2.3 Efficiency measurements ............................................. 207
7.3 Conclusions ................................................................. 218

8 Time-stepping FEM investigations of rotor cage losses and fault conditions ................................................. 219
8.1 Rotor cage losses ........................................................... 219
8.2 Fault conditions ............................................................ 232
8.3 Conclusions ................................................................. 242

9 Conclusions and Future work ......................................... 243
9.1 Conclusions ................................................................. 243
9.2 Future work ............................................................... 245

References ........................................................................ 247

List of symbols .................................................................... 253

Appendix A ....................................................................... 263
Appendix B ....................................................................... 271
1 Introduction

This chapter explains the increased interest for permanent magnet machines and the integration of motor and converter to one unit. A brief background to the PM integral motor project is given. Finally, the outline of this thesis is presented and international publications by the author are listed.

1.1 Major driving forces for permanent magnet motors and integrated motor drives

About 135 years ago, the scientist Werner von Siemens presented one of his discoveries\(^1\) which he had entitled “Über die Umwandlung von Arbeitskraft in elektrischen Strom ohne permanente Magnete” [23]. Freely translated this title reads: “On conversion of mechanical power into electrical power without permanent magnets”. The last two decades, the research trend for alternating-current rotating electrical machines has been the opposite. The increased interest and use of permanent magnet materials have several reasons. The rather new mixture of rare-earth metals, such as Neodymium (Nd) and Boron (B) in combination with the not so rare iron (Fe) resulted in permanent magnets with both high remanent flux density and high coercive magnetic field intensity, compared to the earlier ferrite magnets [35] [62]. The NdFeB-magnets are therefore said to be high energy density magnets. Further, the environmental concern is growing worldwide. We have finally realized that we have to take care of our planet Earth. One of the big issues during this new century is to reduce the emissions of carbon dioxide (CO\(_2\)). This gas, which is one of the so-called Green House gases, is believed to be the main contributor to global warming. The reduction of CO\(_2\) gas can be done in mainly two ways; changing from fossil based energy conversion to alternative renewable energy supplies and by energy savings. One way of saving energy is to decrease the use of energy. This is not a probable outcome, since the world’s consumption of energy has been increasing the last century. When the emerging countries raise their standard of living, the demand for energy will also grow rapidly. The second alternative is to use the existing energy in a better way, that is to reduce the losses when energy is

\(^1\) What Werner von Siemens had discovered was that a shunt DC generator can, under certain conditions, self-excite due to the remanence of the iron [23].
transported between supply and load, and when energy is transformed from one form into another. Here the NdFeB-magnets play an important role. A large part of the world’s energy consumption is used for rotating electrical machines, usually induction motors. The operating principle of the induction machine requires magnetizing currents in the stator windings and currents in the rotor cage. Both of these currents cause heat losses. These losses are avoided if permanent magnets are used in the rotor. This may imply a reduction of the total losses of the machine by about 50% [33].

The development of power electronics, which has been going on since the 1960’s, has also made enormous progress. Nowadays non-expensive, reliable, low-loss diodes, and especially electronic valves such as insulated gate bipolar transistors (IGBT:s) and metal oxide semiconductor field effect transistors (MOSFET:s) are off-the-shelf products. This has opened up for new solutions when it comes to the way an electrical machine can be supplied and run. The normal (old fashion) way of running a rotating machine is to choose a suitable pole number of the machine and/or to change the gear/chain/belt transmission ratio to obtain the wanted maximum speed of the load, and connect the machine directly to the mains. Normally the load, which e.g. may be a pump or a fan, is then throttled with a valve or choked with a barrier to obtain the required fluid or gas flow. This measure causes unnecessary losses at the valve/barrier and reduces the efficiency of the pump or the fan drastically. The remedy is to remove the valve/barrier and reduce the speed instead. By doing so, the optimum efficiency of the pump or the fan may be maintained and the losses of the valve/barrier are eliminated. To reduce the speed of the machine, a voltage source of variable output voltage and variable output frequency is required. Such a voltage and frequency converter can be built by using power electronic devices. Normally the alternating three-phase voltages are rectified into a constant - or slightly pulsating - voltage in the intermediate link of the converter. The intermediate link voltage is then inverted, i.e. “cut up” into short pulses, and applied to the three motor terminals in a special pattern. By changing the pulse pattern, different fundamental voltage levels and frequencies can be obtained. This method is called pulse width modulation (PWM). A system, consisting of a motor and a converter, goes under the name variable speed drive (VSD) or adjustable speed drive (ASD). Inverter-fed motors are found e.g. in chemical, paper, wood and steel industries. The load is often a pump, fan, compressor, mixer or conveyor [53]. The converter cabinets are sometimes placed in the vicinity of the motors, but placements in separate
Introduction

rooms are also common.

The next natural step in the evolution of electrical machines and power electronics was to integrate them both into one single “package”. These motors are frequently referred to as integral motors or integrated motor drives because of the integrated converter circuit. The name Integral Motor® first appeared as a marketing name of such a drive from ABB Motors, and is the name that has been used in this thesis. The name Integrated Motor Drive® (IMD®) is the marketing name for these motors from TB Wood’s. Other manufacturers on the market, which sell integrated adjustable/variable speed drives are e.g. SEW Eurodrive, Danfoss Drives and Controls / Brook Hansen, and Siemens. More manufacturers are given in Chapter 2. The benefits of integral motors are e.g. improved efficiency of the load due to the possibility of controlling the speed, less EMC problems due to the containment of inverter and cables connecting the inverter and the motor, easier and cheaper installation and commissioning etc.

Common characteristics for many of the integral motors on the market are “large” outer dimensions and “low” output powers. The integration of the converter circuit is often - basically - done by mounting an adapted converter cabinet on a conventional motor. This implies a fairly large axial/height/width extension of the motor. This extension can be both a negative eye-catcher, and a real space problem for some customers, e.g. original equipment manufacturers (OEM) who want to fit the motor into another product. The output power of many integrated drives are normally kept at low values - e.g. below 1,5 kW or below 7,5 kW - due to thermal considerations.

It has for some time been the opinion of our research group that the time has come to make a “real” integration of the converter circuit and the motor, to obtain a compact integral motor with high efficiency. Therefore this project was launched in 1996. The project is a pilot project (name: KIM) in the Permanent Magnet Drives programme (PMD-programme), which is within the Competence Centre in Electric Power Engineering at the Royal Institute of Technology (KTH). The purpose has been to show that it is possible to develop a variable speed 15 kW, 1500 r/min permanent magnet (PM) integral motor that is both compact, see Fig. 1.1, and worth its price. The idea is that both motor and converter should be cheap to manufacture, and that the installation and use should be as easy as with a standard induction motor.
1.2 Outline of the thesis and publications

In this thesis the analysis and verification of a 15 kW, 1500 r/min permanent magnet (PM) integral motor, with the same outer dimensions as the equivalent standard induction motor, but with the possibility of operating with speed control and at a higher efficiency is presented. The design of the PM motor is also described in this thesis. The control and power electronics for the PM integral motor are not treated in this thesis. They have been investigated by others in the research group: Prof. Lennart Harnefors did his PhD on control of induction and PM motors. The sensorless algorithms implemented in the PM integral motor are based on his work [30]. Lic. Tech. Karsten Kretschmar has shown that a very small intermediate link polypropylene capacitor is sufficient for this application [40] [41]. To improve the curve forms of the line-side input currents of the PM integral motor, he has also performed research on different types of converters. The most promising candidate seems to be the so-called Vienna-rectifier with tolerance band control [42] [43]. The PM motor was built at ABB Corporate Research, Sweden. The power electronics and control circuits were developed, manufactured and programmed at Inmotion Technologies, Sweden. Both companies are members of the Competence Centre in Electric Power Engineering at the Royal Institute of Technology (KTH).

The outline of the thesis is as follows:

Chapter 2 gives general information about pumps and fans, since they are the most suitable loads for a PM integral motor. The advantage of using speed control instead of throttle/barrier control is pointed out. Some
conventional induction integral motors are also described. The financial benefits of using a PM integral motor instead of an induction motor with converter are shown.

Chapter 3 presents an analytical expression for the airgap flux density of permanent magnet motors with buried magnets. The analytical expression includes axial leakage flux and magnetic saturation. To simplify the analytical expression, only the most saturated part of the iron circuit is considered. Further, an analytical-iterative calculation method for the airgap flux density of permanent magnet motors with buried magnets - including tooth and yoke saturations, and axial leakage flux - is derived. Accurate values of the PM motor airgap flux density are required for use in the optimizations in Chapter 5.

Chapter 4 compares the results from using the models in Chapter 3 to FEM calculated values and to values obtained from measurements.

Chapter 5 gives a description of the computer program that has been developed to optimize the design of PM motors with buried magnets. The result of the computer program is a set of design parameters that define a PM motor with the lowest losses according to the used loss models. Based on the optimization program, pole numbers for inverter-fed PM synchronous motors for different powers and speeds are suggested.

Chapter 6 deals with the design of the PM motor prototype. The PM motor design is made with the use of the computer program described in Chapter 5. The design parameters are checked and/or determined with analytical and/or FEM calculations. The concept of stator integrated filter coils is introduced. The design changes of the prototype motor, which were done during the manufacturing process, are given. The most important effects of the changes are also presented.

Chapter 7 presents measurements made on the manufactured PM motor, the stator integrated filter coils and the real heat-sink. Measurements made on the complete PM integral motor prototype are also presented. The different temperatures (stator winding, filter coil, heat-sink, DC-link capacitor etc.) are shown when the PM integral motor is loaded with different torques at different speeds. Extensive measurements of the thermal steady state efficiencies of the PM integral motor, the PM motor, and the converter for different torques and speeds are presented. To improve the accuracy, a calorimetric measurement method was also used.
Chapter 8 investigates the high-frequency losses in the rotor cage by using time-stepping FEM calculations. These losses were neglected in the optimization program in Chapter 5. Four short-circuit fault conditions for the PM motor are also studied with the time-stepping FEM software.

Chapter 9 contains conclusions made from the presented work. An outlook to the future, regarding subjects suitable for further investigations, is also made.

Parts of this work have been published in the proceedings of international conferences:

- Analytical Calculation of the Airgap Flux Density of PM-Motors with Buried Magnets, P. Thelin and H.-P. Nee, ICEM'98, [75].
- Suggestions Regarding the Pole-Number of Inverter-Fed PM-Synchronous Motors with Buried Magnets, P. Thelin and H.-P. Nee, PEVD'98, [76].
- Calculation of the Airgap Flux Density of PM Synchronous Motors with Buried Magnets including Axial Leakage and Teeth Saturation, P. Thelin and H.-P. Nee, EMD'99, [77].
- Development and Efficiency Measurements of a Compact 15 kW 1500 r/min Integral Permanent Magnet Synchronous Motor, P. Thelin and H.-P. Nee, IAS 2000, [80].
- Comparison between Different Ways to Calculate the Induced No-Load Voltage of PM Synchronous Motors using Finite Element Methods, P. Thelin, J. Soulard, H.-P. Nee, and C. Sadarangani, PEDS’01, [81].

Other publications of the author of this thesis are:

2 Advantages of PM integral motors

Pumps and fans (blowers) are particularly suitable loads for integral motors equipped with shaft-mounted cooling fans. There are mainly two reasons for this: pumps and fans have a quadratic torque dependence on speed which implies that high torques are only required at speeds where the integral motor is well-cooled, and large amounts of electrical energy can be saved by applying speed control instead of throttle or barrier control. To make the reader more familiar with these kinds of loads, this chapter shows typical pump and fan characteristics. A brief description of different control methods for pumps and fans is also presented. To show the industrial state-of-the-art in this area, some general information about commercially existing induction integral motors on the market are given. One motive for using permanent magnet integral motors is financial. Therefore, in the end of this chapter some economical models are derived and used to estimate pay-off times and monetary savings that can be made by installing a permanent magnet integral motor instead of an induction motor with a separate converter.

2.1 Control strategies for pumps and fans

This section will give some general information about pumps and fans. Typical pump and fan characteristics are shown in diagrams, and different control strategies are described.

2.1.1 Pumps

Displacement pumps and Centrifugal pumps
A pump is used for transport of liquid material, e.g. water to households [66], or different kinds of sewage water. For transportation purposes water can be mixed with some solid content. The solid content can e.g. be pulp, iron, sand, clay or mud. Some industrial production require water, e.g. breweries, while others, e.g. a nuclear power plant, uses water in the cooling system to transport energy.

The two most common pumps are the displacement pump and the centrifugal pump, where the centrifugal pump is by far the most used pump on the market [66]. One type of displacement pump has a construction which is similar to an internal combustion engine, regarding the piston and the
cylinder. Another type of displacement pump consists of a flexible hose which is bent in a U-turn around a rotating wheel equipped with a few teeth. The flexible hose is compressed at several points by the teeth of the wheel. The fluid, which is captured between two compressed points of the hose, moves with the turning of the wheel. These two designs imply a constant displacement, i.e. movement of a constant volume of fluid, per rotational turn of the shaft of the pump. Basically the centrifugal pump consists of a rotating disc equipped with “shovels”, where the liquid is made to enter at the centre of the disc and is then pushed radially outwards by the centrifugal forces. Due to these two different principles of operation the displacement pump and the centrifugal pump show different behaviours to a change of speed \( n \), regarding volumetric flow rate \( Q \), head \( H \), required shaft torque \( T \), and required power \( P \). Table 2.1 summarizes these proportionalities for the two types of pumps [66].

Table 2.1 Affinity rules for a displacement pump and a centrifugal pump [66].

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Displacement pump</th>
<th>Centrifugal pump</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow ( Q )</td>
<td>(- n)</td>
<td>(- n)</td>
</tr>
<tr>
<td>Head ( H )</td>
<td>Constant</td>
<td>(- n^2)</td>
</tr>
<tr>
<td>Torque ( T )</td>
<td>Constant</td>
<td>(- n^2)</td>
</tr>
<tr>
<td>Power ( P )</td>
<td>(- n)</td>
<td>(- n^3)</td>
</tr>
</tbody>
</table>

A pump is characterized by its head-flow curve (HQ-curve), see Fig. 2.1. The head \( H \) is equivalent to the height the pump can lift water of a certain flow \( Q \) through a friction-less pipeline in the gravity field of the earth. Normally a pipeline has friction so the available height will be reduced. The head, which normally is given in metres of water column, is a measure of the pressure given by the pump. For an electrical engineer it can be advantageous to compare the head with the voltage of an electric source, and the volumetric flow rate with the current. The “electric source equivalent” of the pump will have a non-linear internal resistance.
Advantages of PM integral motors

Fig. 2.1 Typical HQ-curves for different speeds of a centrifugal pump [66]. Two different pipeline characteristics are also shown.

The head which is required to get a particular flow through a pipeline can be sub-divided into a static head and a dynamic head [22]. The static head is equivalent to the height difference between input and output of a closed pipeline, while the dynamic head is dependent on the friction of the pipeline to liquid flow. The system connected to the pump can therefore be represented by either; Pipeline I without a static head or Pipeline II with a static head, see Fig. 2.1. Pipeline I has an almost quadratic dependence of head on the flow rate, while Pipeline II shows a more flat dependence. For an electrical engineer, the Pipeline I characteristic can be seen as a non-linear resistance. The Pipeline II characteristic is then equivalent to a non-linear resistance in series with a zener diode. A difference between the “electric load equivalent” and the pump is that the static head of the system will consume some power even when the flow is zero.

The mechanical input power to the pump can be calculated [66] as

\[ P_{in} = \frac{HQ\delta a_g}{\eta_{pump}} \]  

(2.1)

where \( \delta \) is the mass density of the liquid, \( a_g \) is the acceleration due to gravity, and \( \eta_{pump} \) is the efficiency of the pump. The efficiency of a pump is normally optimum near the pump’s rated point of operation. An example of typical equi-efficiency lines of a centrifugal pump is shown in Fig. 2.2.
Sometimes the volumetric flow rate where the pump is to be used is well known. In other cases it has to be based on coarse measurements or estimations. A “tool” for choosing the size of the pump(s) is to make a duration graph [66]. The duration graph is based on a (measured) flow versus time diagram or a frequency-of-flow versus flow diagram. The duration graph shows the different sizes of flows and for how long time they appear. If the measurements are unreliable it might be better to use only the maximum, minimum and average flow and rely on statistical methods. One can then assume a binomial distribution to be a good approximation, when drawing the duration graph [66].

**Methods of controlling the volumetric flow rate**

The required volumetric flow rate can be controlled in different ways. Basically the different methods are divided into two groups: discontinuous control and continuous control. To this day the discontinuous control methods are the most common methods for controlling the volumetric flow rate [66].

The discontinuous control methods are

![Diagram showing typical equi-efficiency lines for a centrifugal pump](image-url)
Advantages of PM integral motors

- to use a 2-speed motor, or sometimes even a 3-speed motor
- to use a single speed motor and the on/off method
- to connect pumps (equipped with single-speed motors) in series or in parallel, in combination with the on/off method.

By using a 2- or a 3-speed motor, two or three different liquid flows can be obtained, see Fig. 2.1. If sometimes a zero flow is required, the number of possible flows increase to three or four, respectively. Multi-speed motors are normally induction motors with different number of poles. This implies that the achievable speeds - for a 50 Hz supply - will be slightly below 3000 r/min, 1500 r/min, 1000 r/min, 750 r/min etc. If these speeds do not coincide with the required speed for a wanted liquid flow, the required speed can then be obtained by connecting the pump to the motor via a chain or a belt drive. This means that probably only one of the liquid flows will be produced at an optimum efficiency of the pump. The multi-speed motor can also be combined with the on/off method, see paragraph below.

\[ \text{Fig. 2.3 } \quad \text{A single-speed motor driving a pump connected to a sump and a pipeline with a valve.} \]

If a single-speed motor is used, the pump, motor and any chain or belt drive can be chosen so the pump operates at an optimum efficiency for a flow that is higher than the required flow. To obtain the required flow, or a lower flow, the motor is stopped for a while now and then. This is called the on/off method or the start/stop method [66]. To be able to run in this mode, the pump has to be connected to or put into a special “input buffer”, see Fig. 2.3. This “buffer” consists of a certain volume where the surplus flow can accumulate. This input buffer is called sump, sink, drain or - for a submersible pump - pump pit. The volume of the sump is decided
both by the number of starts and stops of the motor, and by the space available for the sump. A smaller sump increases the number of starts and stops, and vice versa. This method can have high efficiency, but the disadvantages are the large size of the sump and extra maintenance costs due to the high number of starts and stops. On the other hand, the large number of starts and stops may not necessarily be only disadvantageous, since the pump has a chance to “clean itself” from clogging material and long fibrous material every time it start and stops [22].

*Fig. 2.4 Series (left) and parallel (right) connection of pumps.*

Series connection of two, or several pumps, is used when the required head is higher than what can be obtained for one single pump or when the mechanical dimensions of one single pump would be too large, see Fig. 2.4. The head $H'$ of the “new” resulting HQ-curve is obtained by adding the heads $H_1$, $H_2$, ... of the series connected pumps for each value of flow $Q$ [66].

Parallel connection of two or several pumps are normally used when large flows are required, see Fig. 2.4. The flow $Q'$ of the “new” resulting HQ-curve is obtained by adding the flows $Q_1$, $Q_2$, ... of the parallel connected pumps for each value of head $H$ [66]. The on/off method - applied to only one of the pumps - is the most common method for controlling the flow from several parallel connected pumps. The pump that is chosen for start and stop may be moved (electrically) among the pumps to reduce the wear and tear due to the repetitive starts and stops. When parallel connection of several pumps are used, the number and size of pumps are not ob-
Advantages of PM integral motors

Previous. One should e.g. consider minimum energy consumption, volume of the sump, number of starts/stops, and the redundancy that is required for the system [66].

The continuous methods are

• valve throttling
• variable speed control

Valve throttling implies that a valve is inserted in the pipeline of the system connected to the pump [66], see Fig. 2.3. The valves changes the pipeline characteristic of the system, i.e. the valve increases the head of the system. Assume that the required flow of a system reduces from 100% to 50%, see Fig. 2.1. This implies that the operating point is moved from point 0 to point 2, i.e. the head of the system increases from 100% to 112%. At the same time the efficiency of the pump is reduced from around 80% to 55%, see Fig. 2.2. According to Equation (2.1) this means that the input power to the pump, which now delivers only half the flow, has only reduced by 19%. Throttling is the control method that has by far the lowest efficiency, and should be avoided!

Variable speed control means that the speed of the motor, which is driving the pump, is changed. This implies a change of the HQ-curve of the pump instead of a change of the pipeline characteristic, see Fig. 2.1. An advantage of this method is that the pump almost maintains its efficiency, see the equi-efficiency lines of Fig. 2.2. Let us again assume that the required flow of a system reduces from 100% to 50%, see Fig. 2.1. This implies that the operating point now is moved from point 0 to point 2’ by a reduction of the speed. The required head of the system has now decreased from 100% to 40%, if we assume that we have the characteristic of Pipeline I (no static head). At the same time the efficiency of the pump is only reduced to 76%, see Fig. 2.2. According to Equation (2.1) this means that the input power to the pump, which now delivers only half the flow, has reduced by 79%! If we had the characteristic of Pipeline II (with static head) instead, the input power reduction of the pump is 61%. This example shows that a tremendous energy saving can be made by changing a system from throttle control to variable speed control. The lower the static head of the system, the larger the saving.
Variable speed control can also be used instead of the on/off method in a multi-pump system, but the energy savings are not as pronounced as in the case of valve throttling [22]. If several pumps are used, normally only one pump is running with variable speed. Sometimes the best overall efficiency is then obtained if this single variable speed pump is run at a constant speed (at a system efficiency optimum) with the on/off method. It is also possible to run all of the pumps in a multi-pump system with variable speed, but the initial cost of this solution will be very high compared to the small gain. Multi-pumps, which are run with the on/off method, often require a smaller sump volume than variable speed pumps. For a single variable speed pump - with sophisticated control - the sump volume can be slightly smaller for the variable speed method than for the on/off method, though the difference is often negligible. An advantage of a variable speed pump is that a smooth change in the flow, which is required sometimes, can be achieved. A disadvantage is that such pumps are more sensitive to clogging. This is a result of the reduced speed of the impeller vanes and of the liquid, causing long fibrous material to get stuck on the vanes and large quantities of sediment to deposit in the sump. The life of a variable speed motor and pump can be expected to be longer - due to smoother operation and lower speed - than an on/off controlled motor and pump with many starts and stops. On the other hand, the run-time of the variable speed system is longer than for the on/off controlled system, making the estimation of the life more difficult.

2.1.2 Fans

Axial and Radial fans
A fan (blower) transports gas, normally air, while a compressor is designed to provide energy to tools etc. There are mainly two types of fans; axial and radial. The axial fan has a propeller with blades through which the air is transported in the axial direction, similar to the propeller(s) of an aeroplane. Large axial fans (MW-range) can have blades with adjustable angle to improve the efficiency. Advantages of an axial fan are small size and direction of air flow. A radial fan consists of a rotating disc with “shovels”. The air goes axially into the centre of the disc and is pushed outwards radially by centrifugal forces. Sometimes the radial fan is connected to the motor via a belt or a chain drive. The following descriptions in this sub-section are made for a radial fan but most of them apply to an axial fan as well [66].
Fan characteristic
The data of a fan is normally given as a ΔpQ-curve in a diagram. The diagram shows the pressure increase over the fan Δp versus the gas flow Q for different speeds n at 20 °C and an air pressure of one atmosphere, see Fig. 2.5. The required input power of the fan is also shown separately in the same diagram. Some installation system lines, here numbered 2 to 5, are also shown in Fig. 2.5. Again, as in the case of pumps, it can be advantageous for an electrical engineer to compare the fan with an electric source.

*Fig. 2.5* Typical fan characteristic for a medium sized radial fan with backwardly bent shovels [66]. Some installation system lines, here numbered 2 to 5, are also shown.
A fan installation can e.g. consist of a fan with input and/or output ducts, flow ducts (channels), damper, filter, heater (or cooler) and a moisture regulator. Common for these components is that the pressure drop over them increases with the square of the air flow. Such a system is called a \textit{constant flow installation}, and the pressure drop curve will have the following shape:

\[ \Delta p = C Q^2 \]  

(2.2)

where \( C \) is a constant for the installation. Some systems require the flow to vary, e.g. the ventilation during day time and night time of an office. Such a system is called a \textit{variable flow installation}. Normally this system is designed with a minimum pressure in the ducts. The minimum pressure can be maintained by feedback control of the signal from a pressure sensor mounted in the outlet duct [66]. The pressure drop curve for a variable flow installation has the following shape:

\[ \Delta p = \Delta p_0 + C Q^2 \]  

(2.3)

where \( \Delta p_0 \) is the required minimum pressure in the outlet duct.

Table 2.2 presents how the flow, the pressure increase and the required input power vary with speed for a fan in a constant flow installation.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\textbf{Quantity} & \textbf{Radial fan} \\
\hline
Flow \( Q \) & \( -n \) \\
\hline
Increase of pressure \( \Delta p \) & \( -n^2 \) \\
\hline
Torque \( T \) & \( -n^2 \) \\
\hline
Power \( P \) & \( -n^3 \) \\
\hline
\end{tabular}
\caption{Affinity rules for a radial fan in a constant flow installation [66].}
\end{table}
Advantages of PM integral motors

The mechanical input power to the fan can be calculated as

\[ P_{in} = \frac{\Delta p \cdot Q}{\eta_{fan}} \]  (2.4)

where \( \eta_{fan} \) is the efficiency of the fan.

The fan diagrams are normally given for a gas density of 1.2 kg/m\(^3\), i.e. the mass density of air at a temperature of 20 °C and a pressure of one atmosphere. One atmosphere equals 1.013 bar = 101.3 kPa = 760 mm Hg. If the air temperature, pressure and/or humidity differ from these values, the density of air will change. Table 2.1 shows how a change in mass density \( \delta \) affects the quantities of the fan. Both the fan characteristic and the installation system line have to be recalculated for the new density of the gas, see e.g. [66].

Table 2.3  Affinity rules for a radial fan when the mass density of the transported gas changes [66].

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Radial fan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow ( Q )</td>
<td>Constant</td>
</tr>
<tr>
<td>Increase of pressure ( \Delta p )</td>
<td>( - \delta )</td>
</tr>
<tr>
<td>Torque ( T )</td>
<td>( - \delta )</td>
</tr>
<tr>
<td>Power ( P )</td>
<td>( - \delta )</td>
</tr>
<tr>
<td>Efficiency ( \eta_{fan} )</td>
<td>Constant</td>
</tr>
</tbody>
</table>

Methods of controlling the flow
There are different methods for controlling the flow rate from a fan, and they are listed below [66]:

- Introduce a barrier in the flow path
- Change the speed of the fan
- Insert a sliding damper at the gas inlet of the fan
- Control the angle of the blades (only axial fans)
The first two methods are the most commonly known, and were also discussed for pumps in the former sub-section. The introduction of a barrier in the flow path changes the installation line of the system, while the reduction of speed changes the characteristic of the fan instead:

**Example:** Assume a fan is operating at its rated point of operation; 1800 Pa and 11 m$^3$/s, see point X in Fig. 2.5. From point Y in Fig. 2.5 we can also see that the fan requires an input power of 29 kW. Using Equation (2.4) we can calculate the efficiency of the fan to 68%. If the flow is to be reduced to 6.5 m$^3$/s by inserting a barrier, the pressure at the point of operation would increase to 2600 Pa, see point A in Fig. 2.5. The input power to the fan is only reduced to 24 kW, see point B in Fig. 2.5. The efficiency of the fan is now 70%. If speed control is used instead of a barrier, the new point of operation would be 550 Pa and 6.5 m$^3$/s, see point A’ in Fig. 2.5. The required input power is now only 5.8 kW (point B’), which is 18 kW less than with the barrier! The efficiency of the fan in this operating point is 62%. This example shows that an enormous energy saving can be made by using variable speed control instead of choking the flow with a barrier.

The curves of Fig. 2.5 are for a radial fan with shovels that are bent backwards, so called B-shovels. Fans with forwardly bent shovels (F-shovels) have different characteristics and the energy savings might be smaller [66].

For some applications the insertion of a sliding damper on the fan inlet side can, together with a well chosen 2-speed motor, give same energy savings as a variable speed solution but to a lower cost [66]. Controlling the blade angle is only possible for axial fans due to construction difficulties.

### 2.2 Conventional integral motors

**Induction integral motors**

Induction motors were investigated in the laboratories before the 1990’s, and the company *Grundfos* in Denmark combined an induction motor and an inverter for their pumps already in 1991 [6]. However, the company *Franz Morat KG* in Germany was probably one of the first to produce integral motors commercially for industrial use in 1993 [6]. In 1996 the production discontinued since the market was not ready and the integral
motors did not fit into the main profile of the company. Nowadays many different integral motors are available on the market. Numerous manufacturers have them on their sales program, and research on the integration of power electronics and motor to one package is very much in focus. The research area covers both induction integral motors, as the early paper [27], and the paper [21], as well as permanent magnet integral motors e.g. in [4] and [13]. The integral motors of these papers had output powers below 2.2 kW. The induction integral motors that are, or have been, commercially available normally have an output power below 7.5 kW (i.e. about 10 hp). This is not the absolute limit but around this power the thermal problems start increasing, according to some manufacturers [7]. Rockwell Automation claimed that thermal problems arise already at 3.7 kW (5 hp), while Siemens saw a possible increase to 15 kW before the end of 2001 [6]. One exception is VEM Motors, which already offers integral motors up to 22 kW [6]. Their smaller integral motors are equipped with drives from Danfoss in Denmark, while the larger sizes use drives from Emotron in Sweden [6]. Higher powers call for more complex designs regarding e.g. the heat sink(s). Also the amount of copper and the iron quality of the motor, and the amount of silicon in the converter have to be increased. This leads to more expensive products. Another limitation is that above some (unspecified) power, the converter - equipped with the components of today - becomes larger than the motor and the idea of an integral motor is no longer obvious. Some manufacturers of induction integral motors that are - or have been - commercially available, and some data about the integral motors, are shown in Table 2.4 below [7].

Placement of the converter
Most manufacturers have chosen to place the converter on top of the motor, some of them also allow it to be placed on the left or the right side of the motor [7]. A top placement conserves the footprint of the motor but increases the height. A few manufacturers go for a placement of the converter at the non-drive end. This placement extends the axial length of the motor but keeps the height and reduces the risk of harmful vibrations [7]. These manufacturers also claimed that the converter circuit is better protected from heat, especially the rising heat of the motor after it is turned off. The placement of the converter circuit is also indicated in Table 2.4.
Table 2.4 Some manufacturers of induction integral motors that are, or have been, commercially available, and some data of their motors [7]. Single phase voltages are valid for lower motor powers. Maximum speed in brackets.

<table>
<thead>
<tr>
<th>Company (and its URL)</th>
<th>Product name</th>
<th>Output powers [kW]</th>
<th>Speed range [r/min]</th>
<th>Input voltage [Vrms]</th>
<th>Conv. placement</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABB (<a href="http://www.abb.com/motors&amp;drives">www.abb.com/motors&amp;drives</a>)</td>
<td>Comp-AC™</td>
<td>0.55-2.2</td>
<td>[Output freq: 0-250 Hz]</td>
<td>3~: 380-480 or 500</td>
<td>Top</td>
</tr>
<tr>
<td>ABB (<a href="http://www.abb.com/motors&amp;drives">www.abb.com/motors&amp;drives</a>)</td>
<td>Integral Motor®</td>
<td>0.75-7.5</td>
<td>300-1500 (3000 or 6000)</td>
<td>1~: 240 3~: 380-460</td>
<td>Axial</td>
</tr>
<tr>
<td>Baldor Electric (<a href="http://www.baldor.com">www.baldor.com</a>)</td>
<td>SmartMotor®</td>
<td>0.75-7.5</td>
<td>180-1800 (3600)</td>
<td>1~: 230 3~: 460</td>
<td>Top</td>
</tr>
<tr>
<td>Bonfiglioli Group (<a href="http://www.bonfiglioli.com">www.bonfiglioli.com</a>)</td>
<td>LMS Series</td>
<td>0.37-4</td>
<td>[Output freq: 0-100 Hz]</td>
<td>3~: 400-500</td>
<td>Top</td>
</tr>
<tr>
<td>Carpanelli Motori (<a href="http://www.carpanelli.it">www.carpanelli.it</a>)</td>
<td>MII Series</td>
<td>0.75-4</td>
<td>?</td>
<td>1~: 220 3~: 380-440</td>
<td>Top</td>
</tr>
<tr>
<td>Danfoss (<a href="http://www.danfoss.com">www.danfoss.com</a>)</td>
<td>VLT® DriveMotor FCM 300</td>
<td>0.55-7.5</td>
<td>-6000</td>
<td>3~: 380-480</td>
<td>Top or Sides</td>
</tr>
<tr>
<td>Danfoss Bauer (<a href="http://www.danfoss.com">www.danfoss.com</a>)</td>
<td>EtaSolution</td>
<td>0.12-7.5</td>
<td>?</td>
<td>3~: 380-480</td>
<td>Top or Sides</td>
</tr>
<tr>
<td>Franklin Electric (<a href="http://www.fele.com">www.fele.com</a>)</td>
<td>IMDS</td>
<td>0.25-0.75</td>
<td>-3450 (4800)</td>
<td>1~: 115 or 230</td>
<td>Top</td>
</tr>
<tr>
<td>Grundfos (<a href="http://www.us.grundfos.com">www.us.grundfos.com</a>)</td>
<td>MLE Motor</td>
<td>0.37-7.5</td>
<td>?</td>
<td>1~: 230 3~: 330 or 460</td>
<td>Top</td>
</tr>
<tr>
<td>Hanning Elektro-Werke (<a href="http://www.hanning.de">www.hanning.de</a>)</td>
<td>Varicon</td>
<td>0.55-1.5</td>
<td>0-1400 or 0-2800 (2250 or 4500)</td>
<td>1~: 230 3~: 400</td>
<td>Axial</td>
</tr>
<tr>
<td>Invensys Brook Crompton (<a href="http://www.brook">www.brook</a> crompton.com)</td>
<td>VSM ‘W’ Series</td>
<td>0.55-7.5</td>
<td>?</td>
<td>3~: 380-480</td>
<td>Top or Sides</td>
</tr>
<tr>
<td>Kebco (<a href="http://www.kebco.com">www.kebco.com</a>)</td>
<td>Combidrive</td>
<td>0.75-2.2</td>
<td>?</td>
<td>1~: 230 3~: 460</td>
<td>Top</td>
</tr>
<tr>
<td>Lenz (<a href="http://www.lenze.com">www.lenze.com</a>)</td>
<td>8200 moteck</td>
<td>0.12-2.2</td>
<td>?</td>
<td>1~: 230 3~: 320-550</td>
<td>Top or Sides</td>
</tr>
<tr>
<td>Leroy-Somer (<a href="http://www.leroy-somer.com">www.leroy-somer.com</a>)</td>
<td>Varmeca</td>
<td>0.25-7.5</td>
<td>?</td>
<td>1~: 230 3~: 400 or 460</td>
<td>Top</td>
</tr>
</tbody>
</table>
Advantages of PM integral motors

Integral motors also exist in the electric vehicle industry, but they are normally water cooled. Water cooling reduces the thermal problems, which are strongly associated with compact integral motors.

Costs and predicted market growth

To make a fair comparison, the cost of an integral motor must be compared to the cost of a separate motor and a separate converter including the total installation cost [8]. Siemens claimed that the cost of their integral motor was 80%-97% (large to small) of the cost of their separate motor and converter plus enclosure, cables and filter, according to [8]. Baldor claimed the cost of their integral motor to be “roughly the same” or just slightly higher than a separate alternative [8]. ABB postulated that their integral motor was “fairly comparable versus a capable drive and a variable-speed motor”, or even more cost-effective when compared to a full-featured AC drive and motor [8]. MagneTek stated that their integral motor in average, i.e. across all versions, costed 25% less than separate

<table>
<thead>
<tr>
<th>Company (and its URL)</th>
<th>Product name</th>
<th>Output powers [kW]</th>
<th>Speed range [r/min]</th>
<th>Input voltage [V rms]</th>
<th>Conv. placement</th>
</tr>
</thead>
<tbody>
<tr>
<td>MagneTek Drives (<a href="http://www.magnetek.com">www.magnetek.com</a>)</td>
<td>IntelliPac 100</td>
<td>0.25-0.75</td>
<td>[Output freq: 0.1-120 Hz]</td>
<td>1~: 230</td>
<td>Top</td>
</tr>
<tr>
<td>Mannesmann Dematic (<a href="http://www.dematic-us.com">www.dematic-us.com</a>)</td>
<td>Indrive</td>
<td>0.22-3.6</td>
<td>-1400 or -2900</td>
<td>3~: 380-500</td>
<td>Sides</td>
</tr>
<tr>
<td>Rockwell Automation (<a href="http://www.rockwellautomation.com">www.rockwellautomation.com</a>)</td>
<td>13291 VSM 500</td>
<td>0.75-3.7</td>
<td>?</td>
<td>1~: 115 1~ or 3~: 230 3~: 460</td>
<td>Top</td>
</tr>
<tr>
<td>SEW-Eurodrive (<a href="http://www.seweurodrive.com">www.seweurodrive.com</a>)</td>
<td>Movimot®</td>
<td>0.37-1.5</td>
<td>[Output freq: 2-100 Hz]</td>
<td>3~: 380-500</td>
<td>Top</td>
</tr>
<tr>
<td>Spang Power Electronics (<a href="http://www.spangpower.com">www.spangpower.com</a>)</td>
<td>SPE100</td>
<td>0.19-1.5</td>
<td>-1450 or -1750</td>
<td>1~: 115 or 230</td>
<td>Top</td>
</tr>
<tr>
<td>VEM Motors (<a href="http://www.vem-group.com">www.vem-group.com</a>)</td>
<td>Compact Drive</td>
<td>0.55-22</td>
<td>0-3000 or 0-6000</td>
<td>3~: 380-480</td>
<td>Top</td>
</tr>
<tr>
<td>WEG Electric (<a href="http://www.weglectric.com">www.weglectric.com</a>)</td>
<td>MotorDrive MDW-01</td>
<td>0.37-3.7</td>
<td>?</td>
<td>1~ or 3~: 230 3~: 380-480</td>
<td>Top</td>
</tr>
</tbody>
</table>
alternatives when cables and installations are included [8]. Also Rockwell Automation believed their integral motors to be less costly than separate products, at least above the lower powers [8].

There are several surveys on the integral motor market. In the Californian Motion Tech Trends’ survey, from the first quarter of the year 2000, North America is said to still be in the early adoption stage while Europe is in the growth stage of market development [6]. The sales of integrated AC induction motor drives above 1 hp (0.74 kW) in North America was 1.6 M$ (2600 units) in 1999, and is expected to grow to 16.5 M$ (12000 units) in 2005 [6], [7]. The british Frost & Sullivan’s analysis from the end of 1999 showed a European market value for integrated drives of 46.4 M$ (40000 units) in 1999, [6], [7]. The top three European market areas in 1999 were Germany, Italy and France [6]. Further, Frost & Sullivan expects the European market to grow to 195 M$ (219 000 units) by 2006 [6], [7]. Another report in 1999, from the british Intex Management Services, predicts the European market plus the U.S. market for integrated drives to be 500 M$ by 2005.

Advantages and disadvantages of integral motors

Here follows a short list, pointing out advantages and disadvantages of integral motors. Some of the pros are:

• Variable/adjustable speed, which may increase the efficiency of the load. Increased efficiency leads to reduced energy costs.
• Easy installation, leading to reduced installation costs.
• Easy commissioning, leading to reduced commissioning costs.
• No space for a converter cabinet is required.
• Reduced EMC problems, both radiated and line-carried, due to the containment of the inverter and the cable from the inverter to the motor.
• Reduced stock inventory, since one integral motor can replace several induction motors with different pole numbers for a certain torque.

Some of the cons of integral motors:

• Novel motor concept scares potential buyers.
• Slightly more expensive to buy than a motor with a separate converter.
• The converter circuit is exposed to the same environment as the motor, regarding vibrations and heat.
Advantages of PM integral motors

2.3 Economical comparisons

One of the strongest arguments for using integral motors is financial. By reducing the speed of the motor, the throttle/barrier in a pump/fan system can be removed. This can imply enormous energy savings, see Section 2.1. That is not only a benefit of integral motors, but of all adjustable/variable speed drives, whether the converter is integrated with the motor or not. On the other hand, an integral motor may be easier to install and commission than a motor with a separate converter. Most commercially available integral motors contain an induction motor. The operation of an induction motor requires magnetizing currents in the stator windings and currents in the rotor bars. These currents create losses. By using a permanent magnet (PM) synchronous motor instead of an induction motor, the integral motor can be made smaller and will have higher efficiency. To highlight this fact, this section contains an example which will show the required efficiencies versus wanted pay-off times and the monetary saving that can be made due to reduced losses etc. when a PM motor is used. Some assumptions will also be made to investigate how the magnet mass influences the pay-off time of a PM integral motor.

2.3.1 A PM integral motor vs a converter-fed induction motor

A PM integral motor will of course be more expensive to buy than solely a standard induction motor. Neither will the cost of a PM integral motor, compared to an induction motor with a separate converter, be in favour of the integral motor, due to the cost of permanent magnets, more difficult manufacturing process and (at least for now) lower production volumes and relatively novel technology. On the other hand, the installation cost and occupied space will be lesser for an integral motor than for an induction motor with a separate converter. Another advantage, due to the integration, can be reduced material consumption, e.g. in converter cabinets, special cables etc.

To be able to make fair comparisons between different drive systems, some economical models for calculating the required efficiency have to be derived and thereafter the possible monetary saving can be calculated. First the present value of an estimated cost in the future is required:

A cost today \( C_0 \) will in the future, due to the inflation \( i_k \) from year \( k - 1 \) to year \( k \) (where \( k = 1, 2, \ldots, n \)), be

\[
C_k = C_0 (1 + i_k)^{k-1}
\]
in $n$ years. This future cost $C_n'$ will appear today, due to the interest rate $r_k$ year $k$ (where $k = 1, 2, \ldots, n$), as a cost at the present value of

$$C_n = C_0 \cdot (1 + i_1) \cdot (1 + i_2) \cdot \ldots \cdot (1 + i_n)$$  \hspace{1cm} (2.5)$$

Let us assume that the price of a standard induction/asynchronous motor is given as

$$E_a = M_a + T_a + V_a$$  \hspace{1cm} (2.7)$$

where $M_a$ is the material cost, $T_a$ is the manufacturing cost and $V_a$ is the sum of profit, sales & administration costs (S&A), and overhead costs (OH).

Let us also assume that the price of a permanent magnet integral motor is given as

$$E_i = M_i + T_i + P_i + K_i + V_i$$  \hspace{1cm} (2.8)$$

where $M_i$ is the material cost (except permanent magnets and converter), $T_i$ is the manufacturing cost, $P_i$ is the cost of permanent magnets, $K_i$ is the cost of the converter and $V_i$ is the sum of profit, sales & administration costs (S&A), and overhead costs (OH).

We can also define:

- $p_0$, $p_1$, $\ldots$, $p_m$ as the mean value of the electrical energy price year 0, 1, $\ldots$, $m$
- $W_1, W_2, \ldots, W_m$ as the shaft energy consumption year 1, 2, $\ldots$, $m$
- $r_1$, $r_2$, $\ldots$, $r_m$ as the interest rate year 1, 2, $\ldots$, $m$
- $i_1$, $i_2$, $\ldots$, $i_m$ as the inflation from year 0 to 1, 1 to 2, $\ldots$, $m - 1$ to $m$
- $\eta_a$ as the average efficiency of the induction motor
- $\eta_i$ as the average efficiency of the integral motor
Advantages of PM integral motors

The cost of the energy loss year $k$ is then

$$C_k = W_{k,\text{loss}} \cdot p_k = \frac{W_k}{\eta} \cdot (1 - \eta) \cdot p_k = p_k \cdot W_k \cdot \left(\frac{1}{\eta} - 1\right) \quad (2.9)$$

Let us also, for simplicity, introduce the factor

$$N_k = \{\text{Compare to Eq. (2.6)}\} = \frac{(1 + i_1) \cdot (1 + i_2) \cdot \ldots \cdot (1 + i_k)}{(1 + r_1) \cdot (1 + r_2) \cdot \ldots \cdot (1 + r_k)} \quad (2.10)$$

If both the induction motor and the PM integral motor are bought, installed and started on the 1:st of January year 1, and - for simplicity - the electrical energy bill is to be paid the last of December each year, the two following expressions for the life-cycle cost (except installation, maintenance and recycling etc.) of each motor can, by the use of equation (2.6)-(2.10), be stated

$$S_a = E_a + p_0 W_1 \left(\frac{1}{\eta_a} - 1\right) N_1 + p_0 W_2 \left(\frac{1}{\eta_a} - 1\right) N_2 + \ldots + p_0 W_n \left(\frac{1}{\eta_a} - 1\right) N_n \quad (2.11)$$

$$S_i = E_i + p_0 W_1 \left(\frac{1}{\eta_i} - 1\right) N_1 + p_0 W_2 \left(\frac{1}{\eta_i} - 1\right) N_2 + \ldots + p_0 W_n \left(\frac{1}{\eta_i} - 1\right) N_n \quad (2.12)$$

where the substitution

$$\frac{p_k}{(1 + r_1)(1 + r_2)\ldots(1 + r_k)} = \frac{p_0(1 + i_1)(1 + i_2)\ldots(1 + i_k)}{(1 + r_1)(1 + r_2)\ldots(1 + r_k)} = p_0 N_k \quad (2.13)$$

has been used.

Setting the two life-cycle costs of Equations (2.11) and (2.12) equal and solving for the efficiency of the PM integral motor $\eta_i(n)$ (i.e. the required efficiency of a integral motor for a pay-off time of $n$ years) gives

$$\eta_i(n) = \frac{1}{\eta_a} - \frac{1}{\frac{E_i - E_a}{p_0 W_1 \left(\frac{1 + i_1}{1 + r_1}\right) + \ldots + p_0 W_n \left(\frac{(1 + i_1)(1 + i_2)(1 + i_n)}{(1 + r_1)(1 + r_2)\ldots(1 + r_n)}\right)}} \quad (2.14)$$

The present value of the monetary saving that is made the first year after
pay-off, i.e. year $n + 1$, is then given as

$$B''_{n+1} = \left( \frac{W_{n+1}}{\eta_a} \cdot (1 - \eta_a) - \frac{W_{n+1}}{\eta_i(n)} \cdot (1 - \eta_i(n)) \right)p_0N_{n+1} = \ldots = (2.15)$$

$$= p_0W_{n+1}\left( \frac{1}{\eta_a} - \frac{1}{\eta_i(n)} \right)(\frac{1 + i_1}{1 + r_1} \cdot \frac{1 + i_2}{1 + r_2} \cdot \ldots \cdot \frac{1 + i_{n+1}}{1 + r_{n+1}})$$

The equations given above are most easily handled in a computer program. In the following examples, the software MATLAB has been used, both for calculations and for visualizing the results. It has been shown earlier that a permanent magnet integral motor cannot “compete” with a conventional induction motor which is running at its rated load [82]. On the other hand, the permanent magnet integral motor will pay-off in less than two years if it replaces an induction motor feeding a pump with a throttled valve [82]. In the latter case the efficiency of the pump was supposed to be increased from 50% to 70% [82], but without decreasing the so-called head of the system. A decrease of the head reduces the required input power of the system, see sub-section 2.1.1. The following first example deals with a permanent magnet integral motor “versus” a converter-fed induction motor. The second example will illustrate how the magnet cost can influence the pay-off time of a PM integral motor.

Example 1
An economically interesting comparison is the choice between buying and installing a PM integral motor and a converter-fed induction motor. Assume that a 15 kW standard induction motor will be run at its rated operating point: 98 Nm at 1460 r/min with an efficiency of 90% [31]. The induction motor is fed from a separate converter with an assumed constant efficiency of 97%. There is no point in running the induction motor at rated speed with a converter, but it serves as an example in this comparison. Normally the efficiency of the induction motor is reduced when it is fed from a converter. This normally implies a de-rating of the motor power. These two factors have been neglected in this simple analysis. The required efficiency of the PM integral motor (for different profits etc.) versus the wanted pay-off time is investigated in this example.

Assume that the total price of the induction motor, including the converter and installation costs, is $E_a,\text{conv}=24500$ SEK. (Induction motor: 7500
Advantages of PM integral motors

SEK i.e. $M_a = 1950$ SEK, $T_a = 2550$ SEK and $V_a = 3000$ SEK. Converter:
8000 SEK. Installation costs: 9000 SEK, include work, material and ini-
tial start-up procedure.) Profit, sales & administration costs, and over-
head costs of the induction motor are together only 12% of the total price.

Assume that the total price of a PM integral motor, including installation
costs, is $E = 24750-29550$ SEK. (PM integral motor: 15450 SEK, i.e.
$M_i = 1950$ SEK, $T_i = 4000$ SEK, $P_i = 1500$ SEK, and $K_i = 8000$ SEK. Prof-
it, sales & administration costs, and overhead costs: $V_i = 4800-9600$ SEK.
Installation costs: 4500 SEK, include work and material. Initial start-up
procedure is not required for an integral motor.) Profit, sales & adminis-
tration costs, and overhead costs of the PM integral motor are together
19%-32% of the total price.

Let the maximum studied pay-off time be $n = 20$ years.

The future values of the following quantities are impossible to estimate
accurately, which is why they are assumed to be the same from year to
year:

- The price of electrical energy is set to $p_0 = 0.45$ SEK/kWh. Including
  network fees but without VAT. (Mean price for one small industry in
  Sweden 1998. From one electrical energy supplier.)
- The shaft energy consumption each year is set to
  $W = W_1 = W_2 = \cdots = W_{21} = 15 \cdot 24 \cdot 7 \cdot 51 = 128520$ kWh (i.e.
rated load during 51 weeks out of 52 a year, one week off for mainte-
nance etc.)
- The interest rate is set to $r = r_1 = r_2 = \cdots = r_{21} = 15\%$ (which is
  the internal interest rate of the company investing in a PM integral
  motor).
- The inflation is set to $i = i_1 = i_2 = \cdots = i_{21} = 2.5\%$ (Mean value of
  electrical energy price inflation for industries in Sweden 1991-1997.)

The results of the calculations are shown in Fig. 2.6, Fig. 2.7 and Fig. 2.8.

Fig. 2.6 shows the required efficiency of the PM integral motor versus the
desired pay-off time for different profits etc. As we can see, e.g. a pay off-
time of 2 years and a profit etc. of 7200 SEK, would require an efficiency
of 89.4% for the PM integral motor.
Fig. 2.6 Required efficiency of the PM integral motor versus the desired pay-off time for different profits etc., if the PM integral motor is installed instead of a converter-fed induction motor.

Fig. 2.7 Present value of the monetary saving that can be made the first year after pay-off versus pay-off time for three different profits etc., if the PM integral motor is installed instead of a converter-fed induction motor.
Fig. 2.7 shows the present value of the monetary saving that can be made the first year after the pay-off time is over. E.g. a pay-off time of 2 years and a profit of 7200 SEK give rise to a saving of 1113 SEK (present value) the first year after pay-off. From Fig. 2.7 it is easy to believe that a higher monetary saving is obtained if a higher profit is chosen. This is not true. The reason for this “illusion” is that a higher profit requires a higher efficiency for a certain pay-off time. Therefore it is better to see the monetary saving versus PM integral motor efficiency, which is shown in Fig. 2.8. The “4800 SEK”- and “7200 SEK”-curves would, of course, extend further and always above the “9600 SEK”-curve if the efficiency of the PM integral motor for these profits etc. is increased.

Conclusions

The conclusion of Example 1 is that a PM integral motor can compete with a converter-fed induction motor, though the PM integral motor will probably be more expensive to buy. The reasons for this are that an integral motor with a PM rotor can be designed to have a higher efficiency and is cheaper to install than an induction motor with a converter.
2.3.2 Magnet cost versus pay-off time and monetary saving

One of the arguments against permanent magnet motors is the cost of the permanent magnets. A common figure, when it comes to the price of permanent magnets, is about 1000 SEK/kg for NdFeB-magnets. This price is valid for rectangular shaped magnets. Curve shaped magnets or magnets with other shapes have a higher price. This price of 1000 SEK/kg will probably drop in the (near) future. The price can also vary from one manufacturer to another. In one case that is familiar to the author, there was a difference in price by a factor 6 between two different magnet manufacturers. The cost of magnets can e.g. be several thousands of SEK for a PM motor with a torque of 100 Nm. A PM integral motor is probably not as sensitive to the magnet price as a line-start PM motor. This is due to the relatively high cost of the converter, which is included in the price of a PM integral motor. The cost of magnets will therefore be a relatively smaller expense for a PM integral motor.

A PM motor requires a certain amount of magnets to operate with high efficiency. The efficiency increases with the amount of magnets, inside a reasonable range. One of the reasons for the increased efficiency is the possibility of changing the airgap length. With increased airgap length the same torque-producing airgap flux density can be retained by increasing the magnet mass, see Section 3.1. The increased airgap will, on the other hand, decrease the flux from the armature reaction and thereby decrease the iron losses in the machine, see Section 3.2.3. An increased airgap will also reduce the rotor surface load and no-load stray losses, but increase the axial leakage flux from the rotor.

Higher efficiency will probably decrease the pay-off time, though the PM integral motor becomes slightly more expensive. The size of the monetary saving that can be made, when the pay-off time is over, will also be larger with an increased efficiency.

In the former example in sub-section 2.3.1 the total cost of NdFeB-magnets was set to 1500 SEK. It would be interesting to see how a change in the amount of magnets, and thereby a change in efficiency, effects the pay-off time. Such an example would give a hint regarding the choice between a slightly cheaper PM integral motor with slightly lower efficiency and a slightly more expensive PM integral motor with slightly higher efficiency.
Example 2
In this example the PM integral motor is compared to a converter-fed induction motor, while the magnet mass of the PM integral motor is varied. The total price of the induction motor, converter and installation is set to 24500 SEK, see Example 1. Profit, sales & administration costs, and overhead costs of the induction motor are only 12% of the total price. The same run-time, price and percentage assumptions as in example 1 are made, except for the cost of magnets. Let us assume that - at least inside a small interval - an increase in magnet mass by 0.5 kg would increase the efficiency of the PM motor by 0.5 percentage units, see sub-section 6.2.1. This assumed relationship is shown in Table 2.5. The profit, sales & administration costs, and overhead costs of the integral motor are set to 9600 SEK. The total price of the PM integral motor is then 29050-31050 SEK. Profit, sales & administration costs, and overhead costs are 31-33% of the total PM integral motor price, and therefore a reasonable profit can be made. The induction motor is assumed to have a constant efficiency of 90% and its converter a constant efficiency of 97%.

Table 2.5 Assumed relationship between magnet mass and PM motor efficiency.

<table>
<thead>
<tr>
<th>Magnet mass [kg]</th>
<th>Magnet cost [SEK]</th>
<th>PM motor efficiency [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>92</td>
</tr>
<tr>
<td>1.5</td>
<td>1500</td>
<td>92.5</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>93</td>
</tr>
<tr>
<td>2.5</td>
<td>2500</td>
<td>93.5</td>
</tr>
<tr>
<td>3</td>
<td>3000</td>
<td>94</td>
</tr>
</tbody>
</table>

Fig. 2.9 and Fig. 2.10 show the results of the calculations. The curves in Fig. 2.9 have been aborted the year before the required efficiency is higher than the one which is achievable according to Table 2.5.

As we can see from Fig. 2.9 the shortest pay-off time is achieved when we use 3 kg of magnets. For curiosity it can be mentioned that the magnet prices of Table 2.5 had to be raised by a factor 3.1 before the five different magnet masses gave the same pay-off time. This pay-off time was 8 years. Using the magnet prices of Table 2.5, the profit, S&A, and OH were also reduced from 9600 SEK to 7200 SEK and 4800 SEK. This reduces the shortest pay-off time to 2 and 1 year, respectively. The shortest
pay-off times were then achieved for all magnet masses.

![Graph showing required efficiency of the PM integral motor versus desired pay-off time for different magnet masses.](image)

**Fig. 2.9** Required efficiency of the PM integral motor versus desired pay-off time for different magnet masses. Profit etc. was set to 9600 SEK.

![Graph showing present value of the monetary saving that is made the first year after pay-off, versus the pay-off time for different magnet masses.](image)

**Fig. 2.10** Present value of the monetary saving that is made the first year after pay-off, versus the pay-off time for different magnet masses. Profit etc. was set to 9600 SEK.
Conclusion
It seems as if the shortest pay-off time, with a low profit, is not very de-
pendent on the magnet mass. With a reasonable profit, the shortest pay-
off time is achieved with a high magnet mass. The pay-off time with a
reasonable profit is of course longer than with a low profit. Example 2 in-
dicates that we should not let the high price of NdFeB-magnets (approx-
imately 1000 SEK/kg) prevent us from using a high magnet mass, at least
not for a PM integral motor where we want to make a reasonable profit.

2.4 Conclusions

This chapter has given some general information about pumps and fans.
It was also pointed out that large energy savings can be obtained by using
speed control instead of throttle/barrier control. Numerous induction in-
tegral motors on the market were presented. Numerical examples have
shown the benefits of a PM integral motor instead of an induction motor
with converter, and the use of a high mass of permanent magnet material.

In the following chapter models for accurate calculations of the airgap
flux density of PM motors with buried magnets will be presented. The
models include axial leakage flux of the rotor and magnetic saturation.
3 Accurate modelling of the airgap flux density of buried PMSM:s

In this chapter analytical models for accurate calculations of the airgap flux density of permanent magnet synchronous motors (PMSM:s) with buried magnets are derived. Both axial leakage flux and iron saturations are taken into consideration. The airgap flux density is required both for analytical torque and iron-loss calculations as well as an analytical calculation of the induced no-load voltage of the machine. Accurate analytical models are a necessity for time-effective optimizations of a machine. In the following chapter these models are compared to FEM calculations and to values based on measurements of the induced no-load voltages.

The first section of this chapter presents a magnetic model for analytical calculation of the airgap flux density of unsaturated PMSM:s with buried magnets. The first section is mainly based on a paper presented by the author at the ICEM’98-conference [75]. It was found that there was still a difference between the analytical and the FEM calculated values of the airgap flux densities. This is probably due to iron saturation phenomena. A disagreement between the time-stepping FEM calculated induced no-load voltage and the measured induced no-load voltage was also observed. The axial leakage flux is believed to play an important role in this latter case. To try to overcome these disagreements, the analytical flux density model of the first section was developed further.

Section two presents a newly derived, so far unpublished, totally analytical expression for the airgap flux density. The analytical expression includes axial leakage, and iron saturation of the most saturated iron part of the machine. Approximate analytical expressions for the axial leakage reluctance are derived. This section is partly based on a paper presented by the author at the PEVD2000-conference [79].

The third section of this chapter gives a brief description of how to compensate for iron saturations in stator and rotor teeth and yokes in an iterative manner. A fictitious extra airgap is introduced in the magnetic model from the first section. This section is mainly based on a paper presented by the author at the EMD’99-conference [77].
3.1 Analytical calculation of the airgap flux density of PM synchronous motors with buried magnets

Analytical calculations of the airgap flux density of permanent magnet (PM) motors with buried magnets are not so common in literature. In [69] an analytical expression is given, that takes flux concentration into consideration. Papers [84] [36] present analytical models which also include saturation of the iron bridges. In this section an analytical expression - which takes flux concentration, internal airgaps, flux barriers and saturated iron bridges into account - has been derived [75].

3.1.1 Introduction

Permanent magnet synchronous motors (PMSM:s) with buried magnets have been considered in a wide range of drives including both variable-speed drives [51] and mains-connected (i.e. line-start) drives [12] [33]. A buried magnet design has two main advantages compared to surface mounted [70] and inset [59] magnet designs:

1. Flux concentration can be achieved [34], enabling high airgap flux densities. This is especially interesting for low-speed drives, where the iron losses in the stator are not as important as at high speeds.
2. The rotor can be made with a squirrel cage, which is used as a starter and damper winding for mains-connected motors. For variable-speed drives the cage can be used as a means to keep the rotor together mechanically.

The three main drawbacks are:

1. Increased q-axis inductance
2. Additional high-frequency losses if the rotor is equipped with a cage
3. Loss of magnet flux through iron bridges holding the magnets in position

The increased q-axis inductance is, however, not necessarily a drawback. If the reluctance torque is used, a higher utilization of the motor can be achieved. Moreover, the harmonic losses caused by inverter-supply are
decreased by an increased inductance. If, however, a conventional controller using only q-axis current for torque production is employed, the q-axis inductance can be reduced by means of a radial flux barrier across each pole [74]. Doing so, the iron losses of the motor can be decreased and the power factor can be increased. The latter implies lower ratings of the inverter.

This section deals with the topic of analytical calculation of the airgap flux density of motors with buried magnets. If analytical optimization methods are used in the design of a new motor series, analytical expressions for various quantities are necessary. One of the most important quantities is the airgap flux density. The main problem when calculating the airgap flux density is the representation of the highly saturated iron bridges. Depending on the centrifugal forces, the thickness of the bridges can be chosen from approximately 0.7 mm [33] to 2 mm. The thicker the bridges, the more magnet flux is lost as leakage through the bridges. Due to the non-linear magnetic properties of the iron bridges, analytical prediction of the airgap flux density becomes inaccurate unless a good representation of the iron is used. In this section an attempt is made to achieve satisfactory accuracy from analytical calculation of the airgap flux density using simple analytical expressions.

3.1.2 Design principle

From experience it has been found that buried magnet rotors can be assembled comparatively easy. The magnets are just inserted into punched slots in the laminated rotor iron. No bandaging is required for instance. Other advantages are that the magnets are protected from physical damage and demagnetizing currents. Burying the magnets admits different magnet configurations [34]. The magnets can e.g. be placed close to the rotor surface or in V-shape (i.e. with flux concentration), see Fig. 3.1.

The two rotors depicted in Fig. 3.1 are equipped with a cast aluminium squirrel cage for mechanical stability. The cage has only two bars per pole. To reduce the iron losses from the q-flux, each pole in the V-shaped alternative is equipped with an air-filled slot in the radial direction.

A disadvantage with buried magnets is the presence of iron bridges between magnets, and between magnets and airgap etc. The iron bridges will “short-circuit” some of the magnet-flux and therefore reduce the air-
Fig. 3.1 Examples of 8-pole PM motor designs with a thin squirrel cage and one buried magnet per pole (left), and buried magnets in V-shape.

gap flux density. Though flux barriers, if such are present, are designed to have high reluctance there will be some leakage flux through them, as well. To be able to insert the magnets into the slots, the slots have to be made slightly larger than the magnets. This will also reduce the airgap flux density. To calculate analytically the airgap flux density in motors with unsaturated stators and permanent magnets buried in the rotor, one needs to introduce the effects of the iron bridges, flux barriers and slot tolerances in the model.

3.1.3 Derivation of an expression for the airgap flux density

Magnetic model and definition of parameters
By looking at one pole of a simple rotor geometry (see Fig. 3.2), we can use the theory of “Equivalent Magnetic Circuits” (see e.g. [69]) to calculate the flux density in the airgap. The derivation is performed in a number of steps. First the permanent magnets, internal airgaps and iron bridges are replaced by a magnetic Thévenin-equivalent. Thereafter the remaining magnetic circuit is connected to the Thévenin-equivalent. Finally, with the use of the achieved equivalent magnetic circuit, the airgap flux density can be calculated. MMF-drops (magneto motive force) in the rotor- and stator-iron have been neglected. On the other hand, the iron bridges are assumed to be fully saturated. The airgap flux density is as-
sumed to be rectangular (see Fig. 3.3), and the effect of stator slotting is taken into account by the use of the Carter factor.

Fig. 3.2  Definition of parameters for one rotor pole.

Fig. 3.3  Rectangular airgap flux density and corresponding fundamental airgap flux density.
Design and Evaluation of a Compact 15 kW PM Integral Motor

Representation of Permanent Magnets

Fig. 3.4 Demagnetization curves in the second quadrant for a NdFeB-magnet. The load line of a linear magnetic circuit is also shown.

The second quadrant of the demagnetization curve of a NdFeB-magnet (Neodymium-Iron-Boron) can be approximated by a straight line [33], see Fig. 3.4. This implies that we can regard the permanent magnet as a constant MMF-source in series with a constant internal reluctance. The magnitude and the reluctance of the MMF-source can be found as:

\[ f_m = H_c l_m = \frac{B_r}{\mu_r \mu_0} l_m \quad (3.1) \]

and

\[ R_m = \frac{l_m}{\mu_r \mu_0 w_m L} \quad (3.2) \]

respectively, where \( H_c \) is the coercive magnetic field intensity of the magnet, \( l_m \) is the thickness of the magnet, \( B_r \) is the remanent flux density of the magnet, \( \mu_r \) is the relative permeability of the magnet, \( \mu_0 \) is the permeability of free space, \( w_m \) is the width of the magnet and \( L \) is the axial length of the magnet, which is equal to the rotor length. \( H_D, B_D \) denote
Accurate modelling of the airgap flux density of buried PMSM:s

a critical point (at magnet temperature $T$) where an irreversible demagnetization of the magnet can take place, and $H_m$, $B_m$ denote a possible operating point.

If the rotor has more than one magnet per pole, e.g. due to flux concentration, Equation (3.1) and (3.2) will still be valid if all the magnets have the same thickness (which normally is the case):

$$l_m = l_{m1} = l_{m2} = \ldots$$

(3.3)

In this case the magnet width $w_m$ will be the sum of the (different) magnet widths under one pole:

$$w_m = w_{m1} + w_{m2} + \ldots$$

(3.4)

**Internal airgap**

Due to the tolerances required for inserting the magnets into the slots an extra airgap will be added to the magnetic circuit, see Fig. 3.2. The reluctance of the extra airgap can be expressed as:

$$\mathcal{R}_i = \frac{I_i}{\mu_0 w_m L}$$

(3.5)

where $\mu_0$ is the permeability of free space and $w_m$ is the width of the magnet (see also equation (3.3) and (3.4)). $L$ is the axial length of the magnet, which is equal to the rotor length. The thickness of the internal airgap is defined as:

$$l_i = l_{slot} - l_m$$

(3.6)

where $l_{slot}$ is the thickness of the magnet slot and $l_m$ is the thickness of the magnet.

**Iron bridges**

The iron bridges, required to keep the rotor together mechanically, will “short-circuit” some of the flux from the magnets. Depending on the thickness and number of iron bridges the “lost” flux can be quite significant. A sufficiently good model is achieved if the iron bridges are regarded as constant flux sources (or better: sinks) which have to be completely saturated before any “useful” flux can cross the airgap. The constant flux
required to saturate the iron bridges under one pole can be found as:

\[ \Phi_{\text{sat}} = B_{\text{sat}} w_{Fe} k_f L \]  

(3.7)

where \( B_{\text{sat}} \) is the flux density in saturated iron and \( k_f \) is the stacking factor for the iron lamination. \( L \) is the axial length of the magnet, which is equal to the rotor length. \( w_{Fe} \) is the sum of the (different) iron bridge widths under one pole:

\[ w_{Fe} = w_{Fe1} + w_{Fe2} + \ldots \]  

(3.8)

**A Thévenin-equivalent for magnet, internal airgap and iron bridges**

![Thévenin-equivalent diagram](image)

**Fig. 3.5 A magnetic Thévenin-equivalent for magnet, internal airgap and iron bridges.**

Now a magnetic Thévenin-equivalent for the permanent magnet, internal airgap and iron bridges can be established. The MMF and internal reluctance of the Thévenin-equivalent can be found as:

\[ F_{\text{Th}} = F_m - (R_m + R_i) \Phi_{\text{sat}} \]  

(3.9)

and

\[ \mathcal{R}_{Th} = R_m + R_i \]  

(3.10)

respectively, where \( F_m \) is the MMF of the magnet, \( R_m \) is the internal reluctance of the magnet, \( R_i \) is the reluctance of the internal airgap and \( \Phi_{\text{sat}} \) is the flux required to saturate the iron bridges.
Saturated iron bridges
When the iron bridges are completely saturated they can be regarded as air instead of iron. The resulting reluctance can be found by connecting all saturated iron bridges under one pole in parallel:

\[
\mathcal{R}_{Fe} = \mathcal{R}_{Fe1} \parallel \mathcal{R}_{Fe2} \parallel \cdots = \frac{1}{\mu_0 L} \cdot \frac{1}{\frac{w_{Fe1}}{I_{Fe1}} + \frac{w_{Fe2}}{I_{Fe2}} + \cdots} \quad (3.11)
\]

where \( w_{Fe1}, w_{Fe2}, \ldots \) are the widths of the saturated iron bridges, and \( I_{Fe1}, I_{Fe2}, \ldots \) are the thicknesses of the saturated iron bridges.

Flux barriers
Flux barriers can be composed of rotor bars, air-filled slots, the air-filled space between the magnets etc. If there are any flux barriers in the rotor they are made of a non-magnetic material. Therefore the barriers can be regarded as air. The resulting reluctance of the barriers can be found by connecting all flux barriers under one pole in parallel:

\[
\mathcal{R}_b = \mathcal{R}_{b1} \parallel \mathcal{R}_{b2} \parallel \cdots = \frac{1}{\mu_0 L} \cdot \frac{1}{\frac{w_{b1}}{I_{b1}} + \frac{w_{b2}}{I_{b2}} + \cdots} \quad (3.12)
\]

where \( w_{b1}, w_{b2}, \ldots \) are the widths of the flux barriers, and \( I_{b1}, I_{b2}, \ldots \) are the thicknesses of the flux barriers.

Non-rectangular flux-barriers might be approximated with their average width and average length, or - even better - the correct equivalent width to length quotient can be derived.

Airgap
The reluctance of the airgap is simply:

\[
\mathcal{R}_g = \frac{k_c g}{\mu_0 w_g L} \quad (3.13)
\]

where \( g \) is the airgap length (thickness) and \( k_c \) is the Carter factor (which takes the increased reluctance, due to slotting, into account). \( w_g \) is the true circumferential pole width on the rotor surface (see Fig. 3.2) and can be expressed as:
Design and Evaluation of a Compact 15 kW PM Integral Motor

\[ w_g = r \cdot \frac{2 \alpha \cdot \pi}{180^\circ} \cdot \frac{2}{p} \]  
(3.14)

where \( r \) is the rotor radius, \( \alpha \) is the electrical angle (in degrees) of half the true pole width on the rotor surface (see Fig. 3.2 and Fig. 3.3) and \( p \) is the number of poles. The Carter factor can be found as [32]:

\[ k_c = \frac{\lambda}{\lambda - \gamma_c g} \]  
(3.15)

where \( g \) is the airgap. \( \gamma_c \) is defined as [32]:

\[ \gamma_c = \frac{4}{\pi} \cdot \left( \frac{u_{slo}}{2g} \cdot \text{atan}\left( \frac{u_{slo}}{2g} \right) - \frac{1}{2} \ln\left(1 + \left( \frac{u_{slo}}{2g} \right)^2 \right) \right) \]  
(3.16)

and

\[ \lambda = \frac{2\pi \cdot (r + g)}{Q} \]  
(3.17)

is the slot pitch. \( u_{slo} \) is the stator slot opening and \( Q \) is the number of stator slots.

**Airgap flux density**

![Magnetic Thévenin-equivalent](image)

*Fig. 3.6 The magnetic Thévenin-equivalent loaded with the reluctance of the airgap, the flux barriers and the saturated iron bridges.*

The rectangular airgap flux density \( B_g \) (see Fig. 3.3) can now be found
by connecting the reluctances of equation (3.11), (3.12) and (3.13) to the Thévenin-equivalent represented by equation (3.9) and (3.10), see Fig. 3.6. Solving for the airgap flux $\Phi_g$ and dividing by the true rotor pole area gives:

$$B_g = \frac{\Phi_g}{w_g L} = \frac{1}{w_g L} \cdot \frac{I_{Th}}{R_{Th} + R_{Fe} / R_b + R_b / R_g + R_{Fe} / R_b}$$

(3.18)

Inserting equation (3.9)-(3.13) into equation (3.18) and simplifying yield:

$$B_g = \frac{B_r - B_{sat} k_f w_{Fe} w_m (1 + \mu r l_m)}{(\frac{w_w}{w_m} + \frac{w'}{T} \cdot \frac{k_c g}{w_m} (1 + \mu r l_m) + \mu r l_m)}$$

(3.19)

where

$$\frac{w'}{T} = \frac{w_{Fe1}}{l_{Fe1}} + \frac{w_{Fe2}}{l_{Fe2}} + \ldots + \frac{w_{b1}}{l_{b1}} + \frac{w_{b2}}{l_{b2}} + \ldots$$

(3.20)

Using Fourier analysis, the peak value of the fundamental airgap flux density (see Fig. 3.3) can be found as:

$$\hat{B}_{(1)g} = \frac{4}{\pi} B_g \sin(\alpha)$$

(3.21)

where $\alpha$ is the electrical angle of half the true pole width on the rotor surface.

### 3.1.4 Conclusion

In this section, an analytical expression for the airgap flux density in unsaturated PM motors with buried magnets and iron bridges has been presented. The analytical expression also takes magnet-mounting tolerances and flux-barriers into account.

However, when the magnetic circuit is saturated the corresponding ampere-turn drops must be taken into account. For machines with relatively large airgaps and relatively short axial rotor lengths, the axial leakage...
flux must also be regarded. This implies that the magnetic model needs to be improved. Therefore, axial leakage flux and magnetic saturation will be taken into consideration in the next section.

3.2 An analytical expression for the airgap flux density including iron saturation and axial leakage

To find a totally analytical expression for the airgap flux density, which takes axial leakage and iron saturation into account, we can start off with Equation (3.18). An axial reluctance $R_a$ can be inserted into the magnetic model in parallel to the already paralleled reluctances of the flux barriers $R_b$, the saturated iron bridges $R_{Fe}$ and the airgap $R_g$, see Fig. 3.7. A new non-linear reluctance $R_{nar}$, representing the most narrow - i.e. most saturated - iron part of the magnetic circuit, is also inserted in series with the airgap reluctance $R_g$, see Fig. 3.7. The resulting equation is

$$B_g = \frac{\Phi_g}{w_g L} =$$

$$= \frac{1}{w_g L} \frac{\mathcal{F}_{Th}}{R_{Th} + R_{Fe} / / R_b / / R_a / / (R_g + R_{nar})} \cdot \frac{R_{Fe} / / R_b / / R_a}{(R_g + R_{nar}) + R_{Fe} / / R_b / / R_a}$$

where the parameters were defined in Section 3.1. The importance of
identifying the area which saturates first in a complicated magnetic circuit was pointed out already in [11]. The following two sub-sections present the derivation of the axial leakage reluctance $R_a$ and modelling of iron saturation as well as the final expression for the airgap flux density.

### 3.2.1 Modelling of axial leakages

The axial leakage flux of the rotor - see Fig. 3.8 - is often neglected in radial flux machines. This is due to the fact that the influence from the axial leakage flux is normally very small, therefore calculations are based on a cross-section of the machine. The same assumption is usually made when performing FEM calculations. Another reason for doing this assumption is that 2D-FEM is more common, cheaper, faster and easier to use than 3D-FEM.

For PM machines with relatively large airgaps and relatively short axial rotor lengths, the axial leakage flux has a larger influence on the radial torque-producing flux. To estimate the influence of axial leakage flux, some analytical expressions for the axial leakage reluctance will be presented in this section. This section is mainly based on a paper\(^1\) presented by the author at the PEVD2000-conference [79]. A practical application of this model was tried in [45].

Analytical expressions for the axial reluctance

Fig. 3.9 Typical field lines of the axial leakage flux (from 2D-FEM). The picture shows an axial cross-section view of the upper half of the stator, the rotor and the shaft for a radial flux machine.

To get a view of the appearance of the axial leakage flux, a 2D-FEM calculation on a simplified geometry was made. Fig. 3.9 shows a typical magnetic field line plot of the axial leakage flux in an axial cross-section view of the upper half of the stator, the rotor and the shaft of a radial flux machine with buried magnets. The two buried magnets, which originally were placed in V-shape, have been replaced by one single buried magnet at the average height of the V, i.e. some simplifications were made (see also Fig. 3.10):

- Magnets in V-shape have been replaced with one magnet at half the height of the V.
- Magnets in U-shape have been replaced with one magnet at the bottom of the U.
- The influence of the “cross-saturation” (from the radial flux) on the axial leakage flux has been neglected.
- The iron material of the rotor and the stator is set to have a very high permeability.
- A cross-section of the rotor is regarded to have an infinite depth, i.e. a rotor with a given radius but an “infinite circumference”.

The last simplification will result in a lower calculated axial reluctance than the real axial reluctance. To compensate for this, the axial reluctanc-
Accurate modelling of the airgap flux density of buried PMSM:s

es are calculated with a non-magnetic shaft. (This was also tried in an axi-symmetric FEM calculation, but no real significant change of the calculated radial flux was observed.)

![Diagram](image)

**Fig. 3.10 Simplifications for calculating the axial leakage reluctance of one rotor pole.**

As can be seen from Fig. 3.9, the main part of the axial leakage flux is concentrated to the vicinity of the magnet, though some penetrates the stator iron. The part of the flux that penetrates the stator iron will partly link with the stator winding, thereby slightly contributing to the induced voltage and the torque production. Due to these two reasons it should be sufficiently accurate to consider only the leakage flux close to the magnet. Therefore, the stator core and the rotor shaft are omitted in the following. Furthermore, due to symmetry, it is enough to use only half of the rotor, see Fig. 3.10.

**Estimating the permeances of probable flux paths**

The analytical calculation of the axial reluctance is not easy, or as [63] so vividly expresses it:

*The precise mathematical calculation of the permeance of flux paths through air, except in a few special cases, is a practical impossibility. This is because the flux does not usually confine itself to any particular path which has a simple mathematical law.*
[63] suggests the heuristic method of “estimating the permeances of probable flux paths”. With this method, applied to the simplified rotor geometry of Fig. 3.10, the field lines are divided into two regions; one at a distance from the magnet where the field lines follow circular paths, and one close to the magnet where the field lines follow a path with a mean length. See Fig. 3.11. Compare also to Fig. 3.9.

This results in the following approximate expression for the total axial reluctance per pole of the two rotor sides [63]:

$$\mathcal{R}_{a,\text{prob}} = 0.5 \cdot \frac{1}{\Lambda_{\text{circular}} + \Lambda_{\text{mean}}} = \frac{0.5}{\frac{\mu_0 w_m}{\pi} \ln\left(1 + \frac{2h}{l_{\text{slot}}}ight) + 0.26 \mu_0 w_m}$$

(3.23)

where $\Lambda_{\text{circular}}$ and $\Lambda_{\text{mean}}$ are the permeances of the circular path and the mean path, respectively. In [63], it is mentioned that functions of complex variables (i.e. the method of conformal mapping, see e.g. [26]) may also be used. The result of this is that the factor 0.26 in Equation (3.23) reduces to 0.24 and 0.22 for a thick and a thin sample, respectively [63]. From this, one can conclude that the approximate method is accurate enough for this application.

The model mentioned above is less valid if the magnet slot is displaced from the centre in the vertical direction (in Fig. 3.11). Further, the model above neglects the leakage flux that will appear outside the circular paths. Therefore, the axial reluctance obtained from this model will be an overestimation. To decrease the magnitude of the axial reluctance another model is also suggested in the following.
Accurate modelling of the airgap flux density of buried PMSM:s

The shortest path model
Another approach, which shows quite good agreement with the FEM calculated values (see Chapter 4), is to assume that the flux goes the shortest path from the “north pole” to the “south pole”, see Fig. 3.12.

![Fig. 3.12 Derivation of axial reluctance for the left rotor side with a magnet slot displaced in the vertical direction.](image)

As this model compensates for the absence of some flux paths which are not considered in the probable flux path model, it gives a good estimate of the total axial reluctance. This reluctance can be derived in the following manner:

The differential permeance of the air path from the centre of the magnet slot up to the height \( y \) in Fig. 3.12 is given by

\[
\frac{\text{d} \Lambda}{\mu_0} \cdot \frac{w_m \text{dy}}{l_{\text{slot}}/2 + y} = \frac{w_m \text{dy}}{l_{\text{slot}}/2 + y} \tag{3.24}
\]

where \( w_m \) is the magnet width and \( l_{\text{slot}} \) is the thickness of the magnet slot. The total permeance of the upper air path is then given by integrating over the height \( h_1 \):

\[
\Lambda_1 = \int_0^{h_1} \mu_0 \cdot \frac{w_m \text{dy}}{l_{\text{slot}}/2 + y} = \mu_0 w_m \ln\left(1 + \frac{2h_1}{l_{\text{slot}}/2}\right) \tag{3.25}
\]

The permeance of the lower air path is found in the same manner:

\[
\Lambda_2 = \int_0^{h_2} \mu_0 \cdot \frac{w_m \text{dy}}{l_{\text{slot}}/2 + y} = \mu_0 w_m \ln\left(1 + \frac{2h_2}{l_{\text{slot}}/2}\right) \tag{3.26}
\]
The total axial reluctance per pole of the two rotor sides is then given as

\[ R_{\text{a,short}} = 0.5 \cdot \left( \frac{1}{\Lambda_1} + \frac{1}{\Lambda_2} \right) = \]
\[ = \frac{1}{2\mu_0 w_m} \left( \frac{1}{\ln(1 + \frac{2h_1}{l_{\text{slot}}})} + \frac{1}{\ln(1 + \frac{2h_2}{l_{\text{slot}}})} \right) \]  
(3.27)

**Tangential leakage flux**

The tangential leakage flux, i.e. the inter-pole leakage through the airgap or through airgap - stator-tooth-tip - airgap, has not been considered. This is mainly due to the fact that the tangential leakage flux

- influences the shape of the flux in the airgap only in the q-axis direction where the fundamental component of the flux is less weighted in the Fourier analysis
- partly depends on the rotor position
- is also present in the results of the FEM calculations with which the analytical calculations are compared

**Conclusions**

The behaviour of the axial leakage flux of a radial flux machine has been discussed. Two different analytical expressions for the axial leakage reluctance of Equation (3.22) have been presented, and they will be tested in Section 4.1.

**3.2.2 Iron saturation and the final analytical expression**

**Modelling of iron saturation**

The reluctance \( R_{\text{nar}} \) of the most narrow - i.e. most saturated - iron part in Equation (3.22) can be written as

\[ R_{\text{nar}} = R_{\text{nar}}(B_{\text{nar}}) = \frac{l_{\text{nar}}}{\mu_0 w_{\text{nar}} L} \cdot \frac{1}{\mu_{r,Fe}(B_{\text{nar}})} \]  
(3.28)

where \( B_{\text{nar}} \) is the flux density in the most narrow part. The inverse of the relative permeability of the iron material \( 1/\mu_{r,Fe}(B_{\text{nar}}) \) is obtained for each value of \( B \) from the BH-curve data by using:
The relevant part of the curve of the inverse relative permeability can normally be approximated with the following equation:

\[
\frac{1}{\mu_{r,Fe}(B_{nar})} = \frac{1}{\mu_{r,Fe}(B)} = \mu_0 \cdot \frac{H}{B}
\] (3.29)

where the coefficients \(a\) and \(b\) are found from a curvefit. It was found that it is important to get a good curve-fitting around the “knee” of the curve. Substituting \(B_{nar}\) with \(cB_g\) yields

\[
\frac{1}{\mu_{r,Fe}(B_{nar})} = \frac{a}{b - B_{nar}}
\] (3.30)

Now, the most saturated iron part of the magnetic flux path must be identified. This can be done by looking at the smallest total width of the main flux path. In this analysis, it has been assumed that saturation will occur first in the rotor teeth (subscript \(tr\)) or the stator teeth (subscript \(ts\)) or the rotor yoke (subscript \(yr\)) or the stator yoke (subscript \(ys\)). This saturation will therefore be the dominating one. Introduce the most narrow flux path width according to

\[
w_{nar} = \min(\gamma_{tr}w_{tr}, \gamma_{ts}w_{ts}, k_r w_{yr}, 2w_{ys})
\] (3.32)

The quotient \(l_{nar}/w_{nar}\) of the same narrow path, and the factor \(c\) are given by Table 3.1. The other parameters are defined below.

If it would happen that two - or more - of the smallest total widths of Equation (3.32) would be (almost) exactly equal, the sum of the two - or more - corresponding lengths can simply be used in the quotient \(l_{nar}/w_{nar}\) in Table 3.1.
Table 3.1 Expressions for the different parameters.

<table>
<thead>
<tr>
<th>If $w_{nar} = \gamma_r w_{tr}$</th>
<th>$\gamma_s w_{ts}$</th>
<th>$k_r w_{yr}$</th>
<th>$2w_{ys}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>set $l_{nar} = \frac{l_{tr}}{w_{nar}}$</td>
<td>$\frac{l_{tr}}{\gamma_r w_{tr}}$</td>
<td>$\frac{l_{ts}}{\gamma_s w_{ts}}$</td>
<td>$\frac{l_{yr}}{k_r w_{yr}}$</td>
</tr>
<tr>
<td>and set $c = \frac{w_{g}}{\gamma_r w_{tr} k_f}$</td>
<td>$\frac{w_{g}}{\gamma_s w_{ts} k_f}$</td>
<td>$\frac{w_{g}}{k_r w_{yr} k_f}$</td>
<td>$\frac{w_{g}}{2w_{ys} k_f}$</td>
</tr>
</tbody>
</table>

$w_{tr}$ and $w_{ts}$ are the widths of a rotor and a stator tooth, respectively. $l_{tr}$ and $l_{ts}$ are the lengths of a rotor and a stator tooth, respectively. $k_f$ is the stacking factor for the iron lamination, $w_{ys}$ is the thickness of the stator yoke (i.e. back), and $l_{ys}$ is the length (or the sum of the lengths) of the part(s) in the stator yoke that are subjected to one half of the total pole flux, see Fig. 3.1. $w_{yr}$ is the width of the most narrow part of the rotor yoke (i.e. the part that will carry most flux per width), and $l_{yr}$ is the length of the same narrow part, see Fig. 3.13. Further, we have:

$$k_r = \begin{cases} 1 & \text{if most narrow part carries the total pole-flux} \\ 2 & \text{if most narrow part carries half of the total pole-flux} \end{cases}$$

(3.33)

With an airgap flux according to Fig. 3.3, only a certain number of stator teeth $\gamma_s$ and rotor teeth $\gamma_r$ will conduct the magnet flux. The number of active stator teeth (i.e. teeth which are conducting the flux), is then found as

$$\gamma_s = Q_s \cdot \frac{w_{g}}{2\pi r}$$

(3.34)

where $Q_s$ is the number of stator teeth, $r$ is the rotor radius, and $w_{g}$ is the true circumferential pole width on the rotor surface given by Equation (3.14), see also Fig. 3.2. In this analysis, $\gamma_s$ is allowed to be a positive real number. It may be more correct to choose the closest integer instead. The number of active rotor teeth $\gamma_r$ is given by the rotor design. $\gamma_r$ is normally a positive integer.
Analytical expression

Solving Equation (3.22) with respect to $B_g$, by using the software Maple, gives the two roots

$$B_g = \frac{-(\alpha c + \Gamma) \pm \sqrt{((\alpha c + \Gamma)^2 + 4\alpha \beta b)}}{2\beta}$$

(3.35)

where the positive sign in front of the square-root gives the correct solution. A negative $B_g$ indicates that the iron bridges are too thick. The peak value of the fundamental airgap flux density is given by Equation (3.21).

Note that Equation (3.35) is a totally analytical expression for the airgap flux density, which takes into account the saturated iron bridges, flux barriers, internal airgaps, axial leakage flux, and the iron saturation of the stator or the rotor teeth or yoke.

It is also possible to obtain an analytical solution if the most and the second most narrow parts - i.e. two different versions of Equation (3.31); one with a $c_1$ and one with a $c_2$ - are inserted in Equation (3.22), but the analytical expression becomes extensively large. No analytical solutions exist if more than two narrow parts of this kind are used in Eq. (3.22).
The substituted parameters $\alpha$, $\beta$ and $\Gamma$ are given below:

$$\alpha = (B_r l_m w_m - B_{sat} k_f l_m w_{Fe}) \cdot \frac{I'}{w'}$$  \hspace{1cm} (3.36)

$$\beta = -c \cdot \left( I_m \cdot \left( k_c g + w_g \cdot \frac{I'}{w'} \right) + \mu_r k_c g w_m \cdot \frac{I'}{w'} \right)$$  \hspace{1cm} (3.37)

$$\Gamma = l_m \cdot \left( b k_c g + w_g \cdot \left( a \cdot \frac{l_{nar}}{w_{nar}} + b \cdot \frac{I'}{w'} \right) \right) +$$

$$+ \mu_r w_m \cdot \frac{I'}{w'} \cdot \left( b k_c g + a w_g \cdot \frac{l_{nar}}{w_{nar}} \right)$$  \hspace{1cm} (3.38)

where

$$l_m' = l_m + \mu_r l_i$$  \hspace{1cm} (3.39)

and

$$\frac{l'}{w'} = \frac{1}{\frac{w_{Fe1}}{l_{Fe1}} + \frac{w_{Fe2}}{l_{Fe2}} + \ldots + \frac{w_{b_1}}{l_{b_1}} + \frac{w_{b_2}}{l_{b_2}} + \ldots + \frac{1}{\mu_0 L k_a}}$$  \hspace{1cm} (3.40)

$$= \frac{1}{\frac{w_{Fe1}}{l_{Fe1}} + \frac{w_{Fe2}}{l_{Fe2}} + \ldots + \frac{w_{b_1}}{l_{b_1}} + \frac{w_{b_2}}{l_{b_2}} + \ldots + \frac{w_m}{L} k_a}$$

where the axial leakage factor is

$$k_a = 2 \cdot \frac{\ln \left( 1 + \frac{2h_1}{l_{slot}} \right) \cdot \ln \left( 1 + \frac{2h_2}{l_{slot}} \right)}{\ln \left( 1 + \frac{2h_1}{l_{slot}} \right) + \ln \left( 1 + \frac{2h_2}{l_{slot}} \right)}$$  \hspace{1cm} (3.41)

if Equation (3.27) is used for $\Re_a$. The two heights $h_1$ and $h_2$ are defined in Fig. 3.12. The other geometrical parameters are defined in Fig. 3.2.
There are also two alternative ways to use Equation (3.31). The first alternative is to make two - or even several - curvefits to different parts of the inverse relative permeability curve given by Equation (3.29), and use them one by one in Equation (3.35). The second alternative is to choose a very small value (e.g. \(10^{-9}\) T) for \(a\) in Equation (3.31). This implies that the reluctance of the most narrow part of the magnetic circuit will be negligible until the flux density value in this part is very close to the chosen value of \(b\). Very close to the flux density value \(b\), the reluctance will rapidly grow to be very large.

**Conclusion**

A totally analytical expression for the airgap flux density in a PM machine with buried magnets has been derived. The expression includes saturated iron bridges, flux barriers, internal airgaps, and magnetic saturation of the most narrow iron part of the machine.

In the next section an attempt is made to include iron saturations of the stator and rotor teeth and yokes at the same time in an analytical-iterative manner.

### 3.3 Iterative compensation for the magnetic saturation of stator and rotor teeth and yokes

![Fig. 3.14](image)

*Fig. 3.14 The magnetic Thévenin-equivalent loaded with the reluctances of the airgap plus the reluctances of stator and rotor teeth and yokes, the flux barriers, the saturated iron bridges, and the two axial reluctances, which are pointing in and out of the paper.*
This section gives a brief summary of the proposed analytical-iterative method for determining the airgap flux density and is partly based on a paper\(^1\) presented by the author at the *EMD’99-conference* [77]. The method is an approximation since the true field-plot is unknown and quite large assumptions are made.

**Fictitious extra airgap**

When the flux density increases, the teeth and yokes of the stator and the rotor start to saturate. These saturations give rise to MMF-drops, which represent the increased reluctances of the teeth and yokes. This is mainly why Equation (3.19) only holds for unsaturated machines. This phenomenon can be compensated for by introducing a fictitious extra airgap \(g_e\) in the magnetic circuit of sub-section 3.1.3, see Fig. 3.14. The extra airgap represents the increased reluctances of the stator and the rotor teeth and yokes. This extra airgap is inserted into Equation (3.19) at the places where the airgap term \(k_c g\) exists, i.e.

\[
k_{e_w} = k_c g + g_e = k_c g + \mu_0 R_{tot} w_L \quad (3.42)
\]

**Axial leakage**

The axial leakage of the rotor is also taken into account in this model, see Fig. 3.14. This is done by using Equation (3.40), which includes the axial leakage factor \(k_a\), in Equation (3.19).

**Iterative calculation procedure**

When the fictitious extra airgap is introduced, the airgap flux density will be reduced. A reduced airgap flux density implies lower saturation levels of the iron. Therefore, a new value of the fictitious extra airgap has to be calculated and used in Equation (3.19). This iterative procedure is repeated until the value of the airgap flux density is almost constant. This iterative procedure is best described by the flow-chart in Fig. 3.15.

To be able to calculate the reluctances of the teeth and yokes, an analytical expression for the relation between the flux density and the magnetic field intensity of the iron material is needed. Since an iterative calculation procedure already is required, a better representation of the magnetic iron saturation than in sub-section 3.2.2 can be used. A modified and simpli-

---

Accurate modelling of the airgap flux density of buried PMSM:s

The fied Langevin-expression, more accurate for higher magnetic field intensities, was used [58] [15]. This an-hysteretic function describes a relation between the flux density and the magnetic field intensity, and can be seen as an average magnetization curve of the material. The magnetic field intensity of this Langevin-expression cannot be written as an explicit function of the flux density. Instead, a numerical approach had to be used. A common and simple method for numerical solutions of equations is the Newton-Raphson method, see e.g. [62]. The calculations and expressions mentioned above are described in detail in [77] and [79].

![Flow-chart describing the iterative calculation procedure for determining the airgap flux density, when iron saturations of teeth and yokes are compensated for by introducing a fictious extra airgap in the magnetic circuit.](image-url)

**Fig. 3.15** Flow-chart describing the iterative calculation procedure for determining the airgap flux density, when iron saturations of teeth and yokes are compensated for by introducing a fictious extra airgap in the magnetic circuit.
The iterative calculation procedure for the extra airgap (not for the Newton-Raphson method) turned out to have a bad convergence for a machine which was heavily saturated. Often the extra airgap length, and the flux density, alternated between a high and a low value. To avoid this phenomenon it was found that it was better to set the new extra airgap to the old value plus 10% (or less) of the difference of the new and the old value. In this way a better - but slow - convergence was obtained.

Conclusions
An analytical-iterative calculation procedure for the airgap flux density was briefly described. The calculation procedure is complex but includes saturations of stator and rotor teeth and yokes.

3.4 Conclusions
This chapter has presented three models for calculation of the airgap flux density of PM synchronous motors with buried magnets. The first model is used for analytical calculations of unsaturated machines and includes internal airgaps, flux barriers and saturated iron bridges. The second model results in a totally analytical expression for the airgap flux density of saturated PM motors with buried magnets. The totally analytical expression includes internal airgaps, flux barriers, saturated iron bridges, axial leakage flux and saturation of the most narrow part of the stator or rotor teeth or yokes. The third model gives the airgap flux density through analytical-iterative calculations. It includes internal airgaps, flux barriers, saturated iron bridges, axial leakage flux and saturations of the stator and rotor teeth and yokes.

The accuracy of these models will be compared to FEM calculations and to values based on measurements in the following chapter.
4 Flux densities of the accurate models compared to FEM and measurements

To check the validity of the models derived in the former chapter, the present chapter has been devoted to comparisons between analytical values, FEM calculated values and values based on measurements. Five manufactured PM motor prototypes have been used. Four of these motors are line-start motors, while one motor is inverter-fed.

The first section of this chapter compares values from the two analytical equations for axial reluctance from Section 3.2.1 with values obtained from 2D-FEM calculations.

The second section makes comparisons between FEM calculations, values based on measurements, and the results from the different analytical models in Chapter 3.

The third section presents some different methods to calculate the induced no-load voltage with FEM. The “vector magnetic potential” method, using FEM, is explained.

Section four contains a 3D-FEM calculation of the influence of axial leakage flux for one of the prototype motors.

4.1 Comparisons between analytical and FEM calculated axial leakage reluctances of the rotor

To compare the axial leakage reluctance from Equations (3.23) and (3.27) with FEM calculations, five different - but typical - magnet configurations will be studied. In this comparison, the FEM calculated values are regarded as the correct values, although they are based on 2D-FEM calculations.
4.1.1 Calculating axial leakage reluctance using 2D-FEM

Fig. 4.1 Typical field lines of the axial leakage flux, from 2D-FEM. The picture shows an axial cross-section view of the upper half of the stator, the rotor and the shaft for a radial flux machine.

A typical magnetic field line plot of the axial leakage flux in an axial cross-section view of the upper half of the stator, the rotor and the shaft of a radial flux machine with buried magnets is shown in Fig. 4.1. The two magnets, which originally were placed in V-shape, have been replaced by one single magnet at the average height of the V, i.e. the same simplifications as described in Section 3.2.1 were made in the FEM calculations.

The axial reluctance of one pole for the two sides of the rotor “seen” by the magnet in Fig. 4.1 can be found as

\[
\mathcal{R}_a = \frac{f_m}{\Phi_a} - \mathcal{R}_{mi} = \frac{H_c l_m}{(w_{m1} + w_{m2} + \ldots) \Phi_a} - \mathcal{R}_{mi} \tag{4.1}
\]

where the reluctance of the magnet plus the internal airgap surrounding the magnet is given from Equations (3.2) and (3.5) as

\[
\mathcal{R}_{mi} = \mathcal{R}_m + \mathcal{R}_i = \frac{l_m}{\mu_r \mu_0 w_m L} + \frac{l_i}{\mu_0 w_m L} = \frac{l_m + \mu_r l_i}{\mu_0 \mu_0 w_m L} \tag{4.2}
\]

\(f_m\) is the MMF of the magnet, \(\Phi_a\) is the total axial flux through the magnets of one pole, \(H_c\) is the coercive magnetic field intensity of the magnet.
at operating temperature, $l_m$ is the thickness of the magnet and $w_{m1}$, $w_{m2}$, ... are the widths of each magnet. $l_i$ is the thickness of the internal airgap surrounding the magnet, $L$ is the axial rotor length and $\mu_r$ is the relative permeability of the magnet. $\phi_a$ is the axial flux (through the magnets) per unit magnet width, see Fig. 4.1. $\phi_a$ is obtained from a 2D-FEM calculation.

By using Equations (4.1) and (4.2) above and the parameters of the machine depicted in Fig. 4.1, the obtained value of the axial reluctance of one pole for the two sides of the rotor is

$$R_a = \frac{4224}{(0,04 + 0,04) \cdot 12,57 \cdot 10^{-3}} - 431,5 \cdot 10^3 = 3,77 \text{ MH}^{-1}$$

by using a FEM calculation. (H$^{-1}$=A/Wb=A/Vs) To be able to compare the analytical equations of Section 3.2.1 with FEM calculations, five different - but typical - magnet configurations are tried. In the following five cases the rotor shaft and the stator iron are omitted, according to the assumptions made in Section 3.2.1. Due to symmetry, only half of the rotor is used in the FEM calculations.

**Case 1. Centred magnet**

*Fig. 4.2* Axial field lines from the left half of the simplified rotor with a magnet which is centred in the vertical direction.
Again the total axial reluctance per pole for both sides of the rotor is calculated by the use of FEM and equation (4.1), and the result is

$$R_a = 4.03 \text{ MH}^{-1}$$

(4.4)

As can be seen from Equation (4.4) the axial reluctance is now higher (+7%) but still in the same range, compared to the value of Equation (4.3).

**Case 2. Displaced magnet**

It is also interesting to see how a vertical displacement of the magnet affects the axial reluctance, see Fig. 4.3.

![Fig. 4.3 Axial field lines from the left half of the simplified rotor when the magnet is displaced in the vertical direction.](image)

The axial reluctance is again calculated with FEM and Equation (4.1):

$$R_a = 4.36 \text{ MH}^{-1}$$

(4.5)

This measure increases the axial reluctance (+8%), compared to Case 1.

**Case 3. Magnet close to edge**

Fig. 4.4 shows the field lines when the magnet is moved to a new position, even closer to the edge. The axial reluctance is again calculated with FEM and Equation (4.1):

$$R_a = 5.69 \text{ MH}^{-1}$$

(4.6)
The axial reluctance has increased further (+41%), compared to Case 1.

*Fig. 4.4* Axial field lines from the left half of the simplified rotor when the magnet is close to the edge.

**Case 4. Thicker magnet**

It is also interesting to see how a thicker magnet affects the axial reluctance. Fig. 4.5 shows the field lines when the thickness of the magnet slot is 10 mm instead of 5 mm.

*Fig. 4.5* Axial field lines from the left half of the simplified rotor with a thicker magnet slot.

The axial reluctance is again calculated with FEM and Equation (4.1):

\[ \Re_a = 4.91 \text{ MH}^{-1} \]  

(4.7)

This measure increased the axial reluctance (+22%), compared to Case 1.
**Case 5. Thinner magnet**

Fig. 4.6 shows the field lines when the thickness of the magnet slot is reduced to 2.5 mm.

![Field lines with thinner magnet slot](image)

**Fig. 4.6** Axial field lines from the left half of the simplified rotor with a thinner magnet slot.

The axial reluctance is again calculated with FEM and equation (4.1):

\[
\mathcal{R}_a = 3.40 \text{ MH}^{-1}
\]  

(4.8)

This measure reduces the axial reluctance (-16%), compared to Case 1.

By calculating the axial reluctance with Equations (3.23) and (3.27), respectively, a comparison with the FEM calculated values of Equations (4.4)-(4.8) can be made. In this comparison, the FEM calculated values are regarded as the correct values. In Equation (3.23), the smallest value of \( h_1 \) and \( h_2 \) was used for \( h \). The results of the calculations are summarized in Table 4.1. For an easier comparison, the values are also shown in the diagram of Fig. 4.7.

From Fig. 4.7 it can be seen that \( \mathcal{R}_{a,\text{short}} \) seems to show slightly better agreement with \( \mathcal{R}_{a,\text{FEM}} \). The suggestion is therefore to use \( \mathcal{R}_a \) according to Equation (3.27). Another observation that can be made is that the magnitude of the axial reluctance does not change very much, when the magnet placement and the thickness of the magnet slot are altered within reasonable ranges. This implies, as will be deduced in Section 4.3, that the influence of the axial leakage flux will mostly depend on the ratio between the airgap length and the axial rotor length.
Table 4.1  FEM and analytically calculated values of the axial reluctance for magnet configuration Cases 1-5.

<table>
<thead>
<tr>
<th>Case #</th>
<th>$\Re_{a,FEM}$ [MH$^{-1}$]</th>
<th>$\Re_{a,prob}$ [MH$^{-1}$]</th>
<th>$\Re_{a,short}$ [MH$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.03</td>
<td>4.87</td>
<td>4.16</td>
</tr>
<tr>
<td>2</td>
<td>4.36</td>
<td>6.22</td>
<td>4.71</td>
</tr>
<tr>
<td>3</td>
<td>5.69</td>
<td>10.35</td>
<td>8.84</td>
</tr>
<tr>
<td>4</td>
<td>4.91</td>
<td>6.22</td>
<td>5.87</td>
</tr>
<tr>
<td>5</td>
<td>3.40</td>
<td>4.01</td>
<td>3.23</td>
</tr>
</tbody>
</table>

4.1.2 Conclusions

The axial leakage flux of the rotor have been investigated by means of 2D-FEM calculations. Neither of the two suggested models of Section 3.2.1 show perfect agreement to FEM. The “shortest path” model, i.e. Equation (3.27), shows satisfactory agreement with 2D-FEM for a thin and centred magnet, and is therefore recommended.
4.2 Comparisons between analytical-, iterative-, FEM-calculated, and “measured” flux density

To check the validity of Equations (3.19), (3.35), and (3.42) in combination with (3.19), five PM motor designs have been examined. The analytical calculations and the analytical-iterative calculations are compared to FEM calculations. The 2D-FEM calculations were performed with the software ACE\(^1\). All five PM motors have been manufactured so experimental values (calculated from the induced no-load voltage) of the airgap flux densities are also available.

4.2.1 Iterative and analytical calculations for Motors A-E

Motor A, which has 8 poles and is inverter-fed, has approximately the geometry shown in Fig. 3.1 (right-hand side), see sub-section 3.1.2. The geometrical parameters of Motor A were identified by using Fig. 3.2, Fig. 3.12, and Fig. 3.13, and are given below:

- Number of stator slots \(Q=48\)
- Number of poles \(p=8\)
- Rotor radius \(r=81\) mm
- Airgap \(g=2,9\) mm
- Slot-opening at airgap \(u_{slot}=3\) mm
- Flux density in saturated iron bridges \(B_{sat}=1,85\) T
- Stacking factor for iron lamination \(k_{fill,Fe}=0,94\)
- Iron bridges between airgap and rotor-bars:
  \(w_{Fe1}=w_{Fe2}=1\) mm and \(l_{Fe1}=l_{Fe2}=5\) mm
- Saturated iron bridges between rotor bars and magnets:
  \(w_{Fe3}=w_{Fe4}=1\) mm and \(l_{Fe3}=l_{Fe4}=5\) mm
- Flux barrier (i.e. rotor-bar in q-direction): \(w_{b1}=w_{b2}=5\) mm and approximated with average length \(l_{b1}=l_{b2}=6,25\) mm
- Flux barrier between saturated iron bridge and magnet in the magnet slot: Approximated with average width \(w_{b3}=w_{b4}=1,05\) mm and

---

\(^{1}\) FEM program from ABB Corporate Research.
Flux densities of the accurate models compared to FEM and measurements

\( l_{b3} = l_{b4} = 5 \text{ mm} \)

- Flux barrier between the two magnets in the magnet slot: Approximated with average width \( w_{b5} = 6.3 \text{ mm} \) and \( l_{b5} = 5 \text{ mm} \)

- Thickness of magnet slot \( l_{\text{slot}} = 5 \text{ mm} \)

- Thickness of magnet \( l_m = 4.8 \text{ mm} \)

- Width of magnet \( w_{m1} = w_{m2} = 40 \text{ mm} \)

- The NdFeB-magnet is assumed to have a remanent flux density \( B_r = 1.22 \text{ T} \) (at approx. \( 20^\circ \text{C} \)) and \( \mu_r = 1.044 \)

- The electrical angle for half the true pole width on the rotor-surface \( \alpha = 75 \text{ degrees} \)

- Using equation (3.14) gives \( w_g = 53.0 \text{ mm} \)

- Equations (3.15)-(3.17) gives \( k_c = 1.045 \)

- Height above equivalent magnet: \( h_1 = 24.75 \text{ mm} \)

- Height below equivalent magnet: \( h_2 = 24.75 \text{ mm} \)

- Length of a stator tooth: \( l_{ts} = 31.4 \text{ mm} \)

- Width of a stator tooth: \( w_{ts} = 5.93 \text{ mm} \)

- Length of a rotor tooth: \( l_{tr} = 5 \text{ mm} \)

- Width of a “rotor tooth”: approximated with \( w_{tr} = w_g / 2 = 26.5 \text{ mm} \)

- Number of active stator teeth: \( \gamma_s = 5 \), by using Equation (3.34)

- Number of active “rotor teeth”: \( \gamma_r = 2 \), given by rotor design

- Length of active stator yoke: \( l_{ys} = 9.16 \text{ mm} \)

- Width of stator yoke: \( w_{ys} = 11.7 \text{ mm} \)

- Length of active rotor yoke: approximated with minimum length: \( l_{yr} = 1.81 \text{ mm} \)

- Width of active rotor yoke: approximated with \( w_{yr} = 74 \text{ mm} \)

- Rotor flux split up on two paths: \( k_r = 2 \)

The axial leakage flux of Motor A is shown in Fig. 4.1.

Motors B-E have geometries which are somewhat similar to the one in Fig. 3.1 (right-hand side) in sub-section 3.1.2, but the rotor cages have deeper bars and a higher number of bars since these motors are line-start
motors. Motors B–E have 4, 6, 16 and 4 poles, respectively. The results of analytical calculation with Equation (3.19) are shown in Table 4.2.

**Analytical and analytical-iterative calculations**

Analytical calculations with Equation (3.19) were performed and the results are shown in Table 4.2.

The analytical-iterative process of calculating the airgap flux density, using a fictitious extra airgap, was employed. A flow-chart of the iterative process is shown in Fig. 3.15. The calculations are described more thoroughly in [77]. The results of these calculations are shown in Table 4.2.

**Totally analytical calculation**

The totally analytical calculation of the airgap flux density of Equation (3.35) requires the inverse of the relative permeability versus the flux density $1/\mu_{r,Fe}(B_{nar})$ of the used iron material.

Motor A is equipped with an iron quality having iron losses approximately equal to the iron losses of the iron quality CK27 from [52]. Assuming similar magnetic properties for the two materials, $1/\mu_{r,Fe}(B_{nar})$ can be calculated from the BH-curve of CK27 by using Equation (3.29), and it is shown as the solid curve in Fig. 4.8.

![Fig. 4.8 The inverse of the relative permeability versus the flux density for the iron material CK27 (solid), and the positive valued curve-fitted equation (dashed).]
The curve of Fig. 4.8 can be approximated with Equation (3.30) with a satisfactory result. This was done by visually fitting Equation (3.30) to the curve of Fig. 4.8 by adjusting the two parameters \( a \) and \( b \). To get a good fitting for the lower values and around the “knee” of the curve, the parameters were chosen to \( a = 0.0009 \) T and \( b = 1.95 \) T. A curve-fitting command (\textit{lsqcurvefit}) in \textit{Matlab} was also tried, but it is believed that an ocular examination by the author was preferable when it came to suppressing the curve for the lower values and finding a suitable shape around the knee. The positive valued curve of the curve-fitted equation is also shown in Fig. 4.8 (dashed curve). The curve-fitted equation has also a (false) negative valued curve, not shown in Fig. 4.8, which gives rise to the false root of Equation (3.35). The result from Equation (3.35) is presented in Table 4.2.

Motors B-E have an iron quality named Scotsil 530-50\(^1\), thickness 0.50 mm. The results of the visual curve-fitting for the inverse of the relative permeability of this material are \( a = 0.0003 \) T and \( b = 2.1 \) T. The parameters were chosen to suppress the curve for lower values, and to get a good fitting around the “knee” of the curve. The positive valued curve of the curve-fitted equation and the calculated inverse relative permeability are plotted in Fig. 4.9. The results from Equation (3.35) are presented in Table 4.2.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig49.png}
\caption{The inverse of the relative permeability versus the flux density for the iron material Scotsil 530-50 (solid), and the positive valued curve-fitted equation (dashed).}
\end{figure}

\(^1\) From Sankey Laminations Ltd.
4.2.2 Results of the iterative and analytical calculations

The results of the analytical, analytical-iterative, and totally analytical calculations are shown in Table 4.2. The FEM-calculated values, the values calculated from the measured induced no-load voltages, the highest flux density level in the rotor and stator teeth, and in the rotor and stator yokes (from FEM calculations), and the assumed saturated flux density level of the iron are also shown in Table 4.2. The experimental values are calculated by the use of the induced no-load voltages in combination with Equation (4.9). Note that the slot leakage flux is neglected in the calculations of the experimental flux density values. To compensate for this, the experimental values have also been corrected. The correction factors were obtained as the ratios between voltages calculated from airgap flux densities and voltages calculated from vector magnetic potentials. These voltages are found in Table 4.4. To get a better overview, the values of Motors A, B and D are also shown in Fig. 4.10 and Fig. 4.11.

Table 4.2  Flux densities of the five examined Motors A-E.

<table>
<thead>
<tr>
<th>Motor</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{(1)g, 2D-FEM} )</td>
<td>0.88</td>
<td>0.64</td>
<td>0.82</td>
<td>1.11</td>
<td>0.79</td>
</tr>
<tr>
<td>( B_{(1)g, neglect} )</td>
<td>1.14</td>
<td>0.83</td>
<td>1.02</td>
<td>1.83</td>
<td>0.93</td>
</tr>
<tr>
<td>( B_{(1)g} )</td>
<td>0.92</td>
<td>0.64</td>
<td>0.79</td>
<td>1.44</td>
<td>0.84</td>
</tr>
<tr>
<td>( B_{(1)g, axi FEM} )</td>
<td>0.88</td>
<td>0.63</td>
<td>0.78</td>
<td>1.42</td>
<td>0.83</td>
</tr>
<tr>
<td>( B_{(1)g, axi} )</td>
<td>0.88</td>
<td>0.63</td>
<td>0.78</td>
<td>1.42</td>
<td>0.83</td>
</tr>
<tr>
<td>( B_{(1)g, axi, comp, analyt} )</td>
<td>0.87</td>
<td>0.63</td>
<td>0.78</td>
<td>1.25</td>
<td>0.83</td>
</tr>
<tr>
<td>( B_{(1)g, comp, iter} )</td>
<td>0.85</td>
<td>0.63</td>
<td>0.78</td>
<td>1.25</td>
<td>0.83</td>
</tr>
<tr>
<td>( B_{(1)g, axi, comp, iter} )</td>
<td>0.84</td>
<td>0.63</td>
<td>0.78</td>
<td>1.24</td>
<td>0.82</td>
</tr>
<tr>
<td>( B_{(1)g, esperi} )</td>
<td>0.79</td>
<td>0.60</td>
<td>0.77</td>
<td>1.07</td>
<td>0.73</td>
</tr>
<tr>
<td>( B_{(1)g, esperi, slot} )</td>
<td><strong>0.80</strong></td>
<td><strong>0.60</strong></td>
<td><strong>0.77</strong></td>
<td><strong>1.11</strong></td>
<td><strong>0.73</strong></td>
</tr>
<tr>
<td>( B_{max, rot. tooth} )</td>
<td>(1.4)</td>
<td>1.7</td>
<td>1.8</td>
<td>2</td>
<td>1.8</td>
</tr>
<tr>
<td>( B_{max, stat. tooth} )</td>
<td>1.3</td>
<td>1.2</td>
<td>1.6</td>
<td>1.8</td>
<td>1.5</td>
</tr>
<tr>
<td>( B_{max, rot. yoke} )</td>
<td>0.7</td>
<td>0.7</td>
<td>1</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>( B_{max, stat. yoke} )</td>
<td>1.6</td>
<td>1.3</td>
<td>1.3</td>
<td>1.2</td>
<td>1.6</td>
</tr>
<tr>
<td>( B_{sat} )</td>
<td>1.85</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Flux densities of the accurate models compared to FEM and measurements

The different subscripts of the peak value of the fundamental airgap flux density from the magnets of Table 4.2 are explained below:

- \( \hat{B}_{g, 2D-FEM}^{(1)} \): 2D-FEM calculated value in the middle of the airgap.
- \( \hat{B}_{g, \text{neglect}}^{(1)} \): Analytical value from Eq. (3.19), neglecting iron bridges, flux barriers and internal airgaps (i.e. \( w_{Fe} = 0 \), \( w'/l' = 0 \) and \( l_i = 0 \)).
- \( \hat{B}_{g}^{(1)} \): Analytical value from Eq. (3.19) without axial leakage (\( \Re_a = \infty \)) and without iron saturation (\( g_e = 0 \)).
- \( \hat{B}_{g, axiFEM}^{(1)} \): Value from Eq. (3.19) in combination with Eq. (3.40), taking axial leakage flux into account with 2D-FEM (\( \Re_a \ll \infty \)). Without iron saturation (\( g_e = 0 \)).
- \( \hat{B}_{g, axi}^{(1)} \): Analytical value from Eq. (3.19) in combination with Eq. (3.40), taking axial leakage flux into account with Eq. (3.27) (\( \Re_a \ll \infty \)). Without iron saturation (\( g_e = 0 \)).
- \( \hat{B}_{g, axi, \text{comp, analyt}}^{(1)} \): Totally analytical value from Eq. (3.35), taking axial leakage flux into account with Eq. (3.41) (\( k_a > 0 \)). Includes analytical calculation of the saturation of the most narrow part of the teeth or the yokes.
- \( \hat{B}_{g, \text{comp, iter}}^{(1)} \): Analytical value from Eq. (3.19) in combination with Eq. (3.42), iteratively compensated for saturations of teeth and yokes (\( g_e > 0 \)). Without axial leakage (\( k_a = 0 \)).
- \( \hat{B}_{g, axi, \text{comp, iter}}^{(1)} \): Analytical value from Eq. (3.19) in combination with Eq. (3.42), taking axial leakage flux into account with Eq. (3.40) and (3.41) (\( k_a > 0 \)). Iteratively compensated for saturations of teeth and yokes (\( g_e > 0 \)).
- \( \hat{B}_{g, \text{experi}}^{(1)} \): Value calculated from measurements by the use of Eq. (4.9).
- \( \hat{B}_{g, \text{experi,slot}}^{(1)} \): Value calculated from measurements by the use of Eq. (4.9), and compensated for slot leakage by multiplying with the ratio of “airgap flux voltage” to “vector magnetic potential voltage” of Table 4.4.
Fig. 4.10 Comparison among FEM, iterative, analytical and experimental values for Motors A and B, respectively.
Flux densities of the accurate models compared to FEM and measurements

4.2.3 Analysis of the results

From Table 4.2, Fig. 4.10 and Fig. 4.11 it can be seen that there are differences between the FEM calculated and experimentally determined values. Some of this discrepancy can be due to axial leakage, as in the case of Motor A which has a relatively large airgap and a relatively short rotor length. When the axial leakage of Motor A is taken into account, the airgap flux density reduces by around 4%. One can also see that the analytically calculated axial reluctances have the same influence on the result as the FEM calculated axial reluctances for all the five motors.

Furthermore, the analytical-iterative compensation for iron saturation has reduced the airgap flux density values for the motors which were saturated.

The totally analytical calculations have been successful, showing good agreement with the analytical-iterative method for four of the motors. This indicates that it may often be enough to take only the most saturated iron part into account.

A general source of errors in the analytical values is also the assumption
that the airgap flux density has a quasi-square shape. In cases where for instance a rotor tooth is heavily saturated, the flux may move to an adjacent tooth (e.g. from the edge to the centre). In this way the rectangular shaped airgap flux density may become slightly more sinusoidal and the analytical model does not apply perfectly. However, the fundamental component may not have to be influenced to the same degree as the total flux since the region close to the peak of the fundamental component is weighted more heavily than other regions when a Fourier analysis is performed on the waveshape.

The overall conclusion is that the totally analytical equation is preferable since it is analytical, includes iron saturation and does not require iterative calculations. The strength of this equation is that it provides the user with a more realistic answer, since it “automatically” chokes the airgap flux if iron saturation occurs.

4.3 FEM investigations of the no-load voltage of PM synchronous motors

The accurate calculation of the induced no-load voltage of permanent magnet synchronous motors (PMSM) is not an easy task. It is nevertheless important since the machine behaviour is related to this voltage. Purely analytical calculations are not always sufficiently correct, since they seldom account for the effects of e.g. leakage flux and iron saturation. Also the use of Finite Element Methods (FEM) can give erroneous values sometimes. For instance, if the induced voltage of the stator winding is calculated analytically from the FEM-calculated value of the airgap flux density, the obtained voltage will be an over-estimation since a part of the circumferential inter-pole leakage flux in the airgap and the stator slot leakage flux have been neglected. In this section, three different calculation methods are shown, and compared to each other and to measured values of manufactured prototype machines. Two of the three methods do not require a FEM software package that can perform time-stepping. Time-stepping can be both time-consuming and cost-expensive. This section is partly based on a paper\(^1\) presented by the author at \textit{Peds’01} [81].

---

4.3.1 Methods for calculating the induced no-load voltage

**Combined FEM and analytical calculation**
An easy way of calculating the RMS-value of the fundamental component of the induced no-load voltage of a stator *winding* of a rotating machine is to use the following equation

\[
E_{\text{(1)wind}} = \sqrt{2} qn_s \hat{B}_{(1)g} r L \omega_s \cdot \frac{k_{(1)w}}{c}
\]

(4.9)

from Chapter 6. This method requires only the static FEM-calculation of the airgap flux density. Alternatively the airgap flux density can be calculated without a FEM program if a satisfactory good analytical model exists, see e.g. Section 3.2. The major drawback of using Equation (4.9) is that the leakage flux across the stator slots is not taken into account, not even when the FEM-calculated value of the airgap flux density is used.

**Time-stepping calculation with FEM**
The strength of a time-stepping FEM software is, of course, that it yields the best estimation of the induced no-load voltage of the stator winding since it takes into consideration the rotating behaviour of the machine. The drawbacks of time-stepping are that such softwares normally are slightly more expensive to purchase, and setting up the problem and solving is much more time-consuming.

**Vector magnetic potential calculation using FEM**
To get the accuracy of the time-stepping FEM-calculation, which also takes the leakage flux of the stator slots into account, the averaged magnetic vector potentials of the stator slots from static FEM-calculation can be used if the number of turns per slot is sufficiently high. Only a few static FEM calculations are required. This method was used in e.g. [3].

If the number of stator slots per pole is sufficiently high, it may even be enough with one single static FEM-calculation to get a pretty accurate value of the fundamental component of the induced voltage. [9] suggests a similar method, using only one single FEM-calculation, but that method requires access to the stiffness matrix of the FEM program. The following sub-section will give a brief description of how the “vector magnetic potential method” is used.
4.3.2 The vector magnetic potential method

From Maxwell’s Equations [14] we have

$$\nabla \cdot \mathbf{B} = 0 \quad \text{(No isolated magnetic charge)} \quad (4.10)$$

which implies that $\mathbf{B}$ can be expressed as the curl of another vector field $\mathbf{A}$, i.e.

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (4.11)$$

where overlined letters indicate vector quantities. The vector magnetic potential $\mathbf{A}$ of a certain point is therefore a purely mathematical quantity which expresses the total amount of flux per unit length circulating around that point.

The instantaneous flux $\Phi$ through a coil of the winding is given by integrating the flux density $\mathbf{B}$ over the coil area $S$ which is bounded by the contour $C$ [14]:

$$\Phi = \oint_{S} \mathbf{B} \cdot d\mathbf{s} = \oint_{S} (\nabla \times \mathbf{A}) \cdot d\mathbf{s} \quad (4.12)$$

By using Stoke’s theorem [14], Equation (4.12) can be transformed into

$$\Phi = \oint_{C} (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_{C} \mathbf{A} \cdot d\mathbf{l} \quad (4.13)$$

Equation (4.13) can be interpreted in the following way: The flux through a coil of the stator winding is given by integrating the vector magnetic potential along one coil of the winding. Neglecting the end-windings of the machine and looking at a cross-section of the machine geometry, we can derive

$$\Phi = \oint_{C} \mathbf{A} \cdot d\mathbf{l} = L \cdot (A_{\text{slot, forward}} + A_{\text{slot, back}}) = L \cdot (A_{\text{left}} - A_{\text{right}}) \quad (4.14)$$

where $L$ is the axial length of the stator, $A_{\text{left}}$ and $A_{\text{right}}$ are the cross-section averaged vector magnetic potentials of the stator slots containing the coil. Note that the vector magnetic potential will have the same sign
Flux densities of the accurate models compared to FEM and measurements

when travelling along the contour (i.e. the coil), but opposite signs in a cross-section view of the machine.

Furthermore, the instantaneous flux linkage of the winding is found as

\[ \Psi_{wind} = \frac{p}{2} \cdot \frac{1}{c} \cdot \Psi_{coil} = \frac{p}{2} \cdot \frac{1}{c} \cdot N\Phi \] (4.15)

where \( p \) is the number of poles, \( c \) is the number of parallel circuits in the winding, \( N \) is the number of turns of a coil and \( \Phi \) is the instantaneous flux through one coil of the winding and is given by Equation (4.14). The flux linkages for 90 electrical degrees (one quarter of a period) are required. The remaining flux linkage values are obtained by mirroring of the first quarter.

There are now two slightly different ways to calculate the induced no-load voltage of the winding. The instantaneous value of the induced no-load voltage is given by

\[ e = \frac{d\Psi_{wind}}{dt} \] (4.16)

The RMS-value of the fundamental component of the induced no-load voltage is found from a Fourier analysis (\( \mathcal{F} \)) [65] (e.g. with Matlab) of one period of the waveform of the voltage:

\[ E_{(1),e} = \frac{1}{\sqrt{2}} \cdot \mathcal{F}(e_{0-T})|_{(1)} \] (4.17)

A possible difficulty of using Equation (4.17) is that Equation (4.16) contains the derivative of the flux. The derivative will be determined numerically, and numerical derivatives are known to sometimes give large errors.

The second alternative is to first perform the Fourier analysis of one period of the waveform of the flux linkage from Equation (4.15) to find the fundamental component of the flux linkage. The result is then multiplied with the synchronous speed \( \omega_s \). That is, the RMS-value of the fundamental component of the induced no-load voltage of the winding is given by
Design and Evaluation of a Compact 15 kW PM Integral Motor

\[ E_{(1)}, \psi = \frac{1}{\sqrt{2}} \cdot \omega_s \cdot \mathcal{J}(\Psi_{0,q}) \] 

Equation (4.18) is believed to give a more accurate result than Equation (4.17).

4.3.3 Calculations and comparisons of the induced no-load voltages for Motors A-E

To compare the different methods of calculating the induced no-load voltage, five different manufactured PM synchronous motors (Motors A-E) have been used. These are the same motors as were used in Section 4.2.

The measured no-load voltages, when running the machines as generators (or, as in one case; as a motor) are given in Table 4.4.

The time-stepping FEM calculations were performed with MEGA\(^1\). These results are shown in both Table 4.3 and Table 4.4.

The combined analytical and FEM calculations were performed by using Equation (4.9) and the airgap flux density values \( \bar{B}_{(1)}, 2D\text{-FEM} \) of Table 4.2. The results are presented in Table 4.4.

Vector magnetic potential calculations were performed for Motors A-E. The calculations for Motors A-C and E were done with a mechanical rotational angle of 2° and three static FEM calculations. For the two 4-pole machines, with 36 stator slots each, the obtained voltage waveforms will then consist of 88 points. 88 points per period is more than enough to estimate the fundamental component of the induced voltage accurately. Motor D, which needed two poles to obtain the required stator symmetry and had a more complicated stator winding, was subjected to six static FEM calculations. The results are shown in Table 4.3. Also the vector magnetic potential calculations without any rotation of the rotor were performed for Motors A-E. These results are also shown in Table 4.3. A more detailed description of the “vector magnetic potential method”, applied to Motor A, is shown in Appendix A.

---

1. FEM program from University of Bath.
Flux densities of the accurate models compared to FEM and measurements

For the vector magnetic potential calculations which are performed without any rotation of the rotor, the number of points in the waveforms are linked to the number of stator slots per pole. Therefore, this quotient is given in Table 4.3.

By inserting Equation (3.40) into Equation (3.19) it is seen that the geometry dependent axial leakage factor \( k_a \) is multiplied by the quotient between airgap length \( g \) and axial rotor length \( L \), as it enters Equation (3.19). This quotient can be seen as an indication of how sensitive the machine is for axial leakage flux. Therefore the airgap lengths, the axial rotor lengths and the ratios of airgap length to axial rotor length of the five motors are also given in Table 4.4.

Table 4.3  The RMS-values of the fundamental component of the calculated induced no-load phase voltages at a magnet temperature of 20
\( ^\circ \)C for the five motors.

<table>
<thead>
<tr>
<th>Motor</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stator slots per pole</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>4.5</td>
<td>9</td>
</tr>
<tr>
<td>Voltage from time-stepping ([V_{RMS}])</td>
<td>202</td>
<td>172</td>
<td>193</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Voltage from vector magnetic potentials using Eq. (4.17) ([V_{RMS}]) (Number of static FEM-calc. &amp; Number of points in Fourier Analysis)</td>
<td>202 (3&amp;60)</td>
<td>174 (3&amp;88)</td>
<td>194 (3&amp;60)</td>
<td>246 (6&amp;88)</td>
<td>203 (3&amp;88)</td>
</tr>
<tr>
<td>Voltage from vector magnetic potentials using Eq. (4.18) ([V_{RMS}]) (No. of FEM-calc. &amp; Pts in FA)</td>
<td>202 (3&amp;60)</td>
<td>174 (3&amp;88)</td>
<td>194 (3&amp;60)</td>
<td>246 (6&amp;88)</td>
<td>203 (3&amp;88)</td>
</tr>
<tr>
<td>Voltage from vector magnetic potentials using Eq. (4.17) ([V_{RMS}]) (One FEM-calc. &amp; Pts in FA)</td>
<td>200 (1&amp;12)</td>
<td>174 (1&amp;18)</td>
<td>193 (1&amp;12)</td>
<td>235 (1&amp;10)</td>
<td>202 (1&amp;18)</td>
</tr>
<tr>
<td>Voltage from vector magnetic potentials using Eq. (4.18) ([V_{RMS}]) (One FEM-calc. &amp; Pts in FA)</td>
<td>202 (1&amp;12)</td>
<td>175 (1&amp;18)</td>
<td>196 (1&amp;12)</td>
<td>243 (1&amp;10)</td>
<td>203 (1&amp;18)</td>
</tr>
</tbody>
</table>

---: Not performed
Table 4.4  The RMS-values of the fundamental component of the measured and calculated induced no-load phase voltages. (20°C.)

<table>
<thead>
<tr>
<th>Motor</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of poles</td>
<td>8</td>
<td>4</td>
<td>6</td>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>Airgap length [mm]</td>
<td>2.9</td>
<td>0.68</td>
<td>0.38</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>Rotor length [mm]</td>
<td>110</td>
<td>115</td>
<td>115</td>
<td>170</td>
<td>148</td>
</tr>
<tr>
<td>Airgap length to axial rotor length ratio [10⁻³]</td>
<td>26</td>
<td>5.9</td>
<td>3.3</td>
<td>2.9</td>
<td>6.8</td>
</tr>
<tr>
<td>Measured voltage [V_RMS]</td>
<td>184</td>
<td>162</td>
<td>184</td>
<td>245</td>
<td>188</td>
</tr>
<tr>
<td>Voltage from time-stepping [V_RMS]</td>
<td>202</td>
<td>172</td>
<td>193</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Voltage from vector magnetic potentials using Eq. (4.18) [V_RMS] (3, 3, 3, 6 &amp; 3 FEM-calc.)</td>
<td>202</td>
<td>174</td>
<td>194</td>
<td>246</td>
<td>203</td>
</tr>
<tr>
<td>Voltage from airgap flux density [V_RMS]</td>
<td>205</td>
<td>174</td>
<td>195</td>
<td>255</td>
<td>203</td>
</tr>
</tbody>
</table>

1: 245 V (true RMS) at no-load motor operation with minimum current, i.e. without frequency analysis.
---: Not performed.

Fig. 4.12 Relative comparison among measured and calculated values of the induced no-load voltages of the five Motors A-E.
Analysis of the two Tables
From Table 4.3 it can be seen that all four methods, which use the vector magnetic potentials, give almost similar results and are in good agreement with the values obtained by time-stepping, at least for Motors A, B and C. When a high number of points are used in the Fourier analysis there is no difference in the results between the two different ways of calculating the induced voltage. When the number of points is reduced, the calculation based on a Fourier analysis of the flux linkage seems to be preferable.

From Table 4.4 and Fig. 4.12 it can be seen that using the airgap flux density for calculating the induced voltage has been successful - when compared to the vector magnetic potential method - for Motors B, C and E. Motor A has a relatively long airgap (2.9 mm) and it is believed to be the reason for the slightly higher voltage based on the airgap flux density, since that method neglects circumferential leakage flux in the airgap. Motor D has high flux density values in the stator teeth which lead to stator slot leakage. Stator slot leakage is also neglected when using the airgap flux density to calculate the induced voltage, and is a possible reason to this high value for Motor D.

4.3.4 Conclusions
The overall conclusion that can be made from the present section is that using the easiest method, i.e. the airgap flux density, is accurate enough for unsaturated machines with relatively small airgaps. Otherwise the vector magnetic potential method or time-stepping simulations have to be employed. For the vector magnetic potential method, it seems as if one single static FEM calculation can be enough, though the accuracy can be improved by carrying out static FEM calculations for three rotor positions.

The relatively large differences between measured and calculated voltage values that still exist are probably due to other effects; axial leakage flux (compare the ratios of airgap length to axial rotor length), manufacturing tolerances of the used materials and inaccuracies in the measurements. The influence of the axial leakage flux was estimated with 2D-FEM calculations in Section 4.2. To investigate the axial leakage further, a 3D-FEM calculation of its influence on Motor A has been done in the following section.
4.4 3D-FEM calculation of the influence of axial leakage flux for Motor A

In Section 4.3 results of time-stepping transient 2D-FEM calculations of the induced no-load voltages were presented. These calculations could still not present results in perfect agreement with measurements. As only 2D analysis was used, the disagreement can be due to something in the third dimension, i.e. in the axial direction of the motors. Earlier in this chapter, the influence of the axial leakage flux of the rotor has been estimated with a simplified 2D model. From Fig. 4.12 and Table 4.4 one can conclude that machines with a large value of the quotient airgap length to axial rotor length seem to show larger differences between values from measurements and time-stepping 2D-FEM. Motor A, which has the largest value of this quotient among the five motors, shows a reduction of the airgap flux density of 4.3% (0.88T/0.92T, see Table 4.2.) when the axial leakage flux is taken into account. However, the obtained induced no-load voltage is still 5% larger than the measured voltage. This might imply that the simplified 2D model for the axial leakage flux underestimates its influence. This possibility was investigated with the 3D-FEM program FLUX3D\(^1\) in collaboration with Dr. Jörgen Engström [25].

---

1. FEM program from Cedrat.
A simplified geometry of one pole of Motor A was used in the static 3D-FEM calculations, see Fig. 4.13. The simplified geometry had a smooth, i.e. slotless, stator surface and the two internal airgaps surrounding the magnets were omitted by increasing the magnet thickness from 4.8 mm to 5 mm. The relative permeability of the magnets was set to 1.05 and a BH-curve of the iron material CK27 was used.

The first two 3D-FEM calculations were performed with a remanent flux density of the magnet equal to unity. This gave an airgap flux from the rotor of 1.71 mVs when no axial leakage flux was allowed. When axial leakage flux could propagate freely, the airgap flux was decreased to 1.58 mVs. This yields a reduction of the airgap flux of 7.6%. This is larger than the 4.3% that was predicted by the simplified 2D model.

One must bear in mind that it is the fundamental component of the airgap flux density that produces the fundamental component of the induced voltage. Therefore two new 3D-FEM calculations were performed, again with and without the possibility for the axial flux to appear. A more correct remanent flux density of $4.8 \text{ mm} / 5 \text{ mm} \cdot 1.22 \text{ T} = 1.17 \text{ T}$ was introduced to compensate for the thicker magnets that were used in the 3D calculations. The result was an airgap flux density which had a fundamental component with a peak value of 0.904 T when no axial leakage was allowed, see Fig. 4.14. This is higher than the value of 0.88 T which was obtained in the 2D-FEM calculations earlier in this thesis, but may be explained by the lack of stator slotting.

![Airgap flux densities with and without axial leakage flux.](image)

*Fig. 4.14* Airgap flux densities with and without axial leakage flux.
When axial leakage flux was allowed the peak value of the fundamental component of the airgap flux density decreased to 0.838 T in the middle of the rotor. At the edges of the rotor the peak values were 0.828 T, see Fig. 4.14.

The reduced airgap flux density is remarkably constant along the rotor, which is not the case in a slotless machine with surface mounted magnets [25]. This is probably due to the fact that the iron above the magnets, in a buried magnet design, redistributes the flux along the rotor when a part of the flux is “drained” through the air at the two end-sides of the rotor. Assuming that the airgap flux density drops linearly along the rotor, the average value of the two flux densities 0.833 T is to be used. This implies that the fundamental component of the airgap flux density reduces by 7.9% due to axial leakage. When this reduction is applied to the voltage obtained from the time-stepping 2D-FEM calculation for Motor A, it decreases to 186 V. The measured voltage of Motor A was 184 V, a difference of only 1.1%.

One can see from the 3D calculations above that the axial leakage flux can play an even more important role than what was predicted by the 2D models. This study shows that for motors with a large quotient airgap length to axial rotor length, a 3D-FEM calculation may be required to accurately predict the induced no-load voltage.

### 4.5 Conclusions

This chapter has compared the three airgap flux density models from Chapter 3 to each other, to 2D-FEM calculations and to values calculated from measurements. The agreement is satisfactory when iron saturation and axial leakage are included. Axial leakage should be included for machines with high values of the quotient airgap length to axial rotor length. A 3D-FEM calculation showed that the 2D models for axial leakage might underestimate its influence. Different ways to calculate the induced no-load voltage have also been discussed.

The following chapter will describe an optimization program which has been developed. It is used to find near-optimum parameters for PM motors. The copper and iron loss models etc. that are used in the program are presented. One of the airgap flux density models from Chapter 3 is also used.
5  Optimization of buried PMSM:s

The first section of this chapter presents an optimization procedure for permanent magnet synchronous motors with buried magnets. The second section gives some general results based on the optimization program.

5.1 Optimization program

An electrical machine can be described as a complex system of parameters, effecting each other. Changing one parameter to improve something, normally changes something else in a negative direction. It is therefore not possible to optimize the design of an electrical machine by optimizing one thing at a time. A solution to this can be to create a model of the electrical machine and use it to find a set of parameters that give the machine the desired properties. This section gives an overview and a description of the optimizing computer program that has been developed in order to find parameters that enables a construction of a buried PM motor with high efficiency.

5.1.1 General layout of the computer program

The development of the computer program started with a diploma work presented in [74], and has continued during this work. The computer program is written in Matlab-code, but for faster execution a Matlab-to-C software compiler package was employed. Fig. 5.1 presents a flow-chart of the optimization program. The aim of the optimization program is to find the set of parameters (rotor radii, current density etc.) - for each pole number - that gives the PM motor highest efficiency for a desired torque and speed.

The computer program does not use any optimization method, speaking in terms of optimization theory. Instead the results of all parameter values are tried. This can be done since the “area” that has to be “scanned” is (quite) limited, due to machine and physical restrictions. Also the step-size of the parameters effect the computational time. By choosing an appropriate step-size for all parameters, the calculation time can be kept to a reasonable length. Even if the “optimum” exists inside a step, this optimum is too narrow to be used, provided that the step-size is chosen reasonably small. An advantage of this method is that one can be sure that
the found optimum is a global optimum. On the other hand, as the searched “area” is probably quite “smooth” anyway, the use of a real optimization method would probably not lead to problems but this question has not been further explored in this work. Another difficulty might be to state the objective function (required by an optimization method), since the program contains an iterative calculation process.

Fig. 5.1  Flow-chart describing the optimization program.
To reduce complexity of the computer program, the following assumptions were made:

- Time-harmonic copper losses generated by the inverter are neglected.
- Time-harmonic iron losses generated by the inverter are neglected.
- Losses due to space-harmonic effects are disregarded.
- Current and field displacement due to eddy currents are neglected.
- Magnetic saturation is neglected, except in the iron bridges which are assumed to be totally saturated.
- The MMF-drop in iron is neglected since the flux densities are kept lower than certain values.

High frequency losses of the rotor cage are investigated in Chapter 8. The following subsections will define constants, limitations, the different parameters, and methods of calculating flux densities and losses etc. The description-order is done in accordance with the appearance in the flow-chart of Fig. 5.1.

### 5.1.2 Description of the different parameters

#### Constants

The following values are constant throughout the program:

- Relative permeability of magnet
- Outer radius of stator core
- Shaft radius
- Width and thickness of iron bridges to be saturated in rotor
- Thickness of rotor bars
- Flux density level when magnetic saturation occurs in iron
- Thickness of magnet slots
- Thickness of magnets
- Width of slot openings
- Radial length of slot openings
- Required torque
- Copper fill factor
- Angle between magnet flux and stator current vector
- Ambient temperature
- Temperature rise of cooling air due to converter losses
- Temperature difference from stator copper to stator iron
- Thermal resistance from stator copper to ambient
- Number of stator slots (constant for each pole number)
Design and Evaluation of a Compact 15 kW PM Integral Motor

- Stacking factor for iron lamination
- Density of iron

Limitations
The following limitations have been set:

- Minimum radial length between shaft and magnets
- Minimum thickness of stator yoke
- Maximum average temperature of copper in stator winding
- Maximum flux density in stator teeth
- Maximum flux density in stator yoke

Magnet width
Depending on shaft radius, rotor radius, number of poles, magnet position, magnet thickness etc. different magnet widths are possible. The maximum total magnet width of a pole is derived with geometrical calculations in the beginning of the program. The total magnet widths $w_m$ versus pole number and rotor radius are saved in a matrix for later use in the program. See also the paragraph Magnet positions below.

Pole-numbers
The pole number $p$ is varied from the lowest number of poles $p_{min}$ - e.g. 2 - to the highest $p_{max}$ - e.g. 16 -, with a step of 2 or 4.

Rotor length
The axial rotor length $L$ is varied from a minimum value (i.e. start value) up to a maximum value $L_{r,max}$. The maximum value is given as

$$L_{r,max} = L_{max} - L_{endwind,tot} = L_{max} - k_{endwind} \pi r \cdot \frac{2}{p} \tag{5.1}$$

where $L_{max}$ is the maximum available axial length for the stator core plus the two end windings. $r$ is the rotor radius, $p$ is the number of poles and $k_{endwind}$ is a factor that takes into account the axial length of the two end windings. This factor is approximately in the range of 0,9-1,1 for 2, 4 and maybe also for 6 pole machines [54], [67]. For higher pole numbers the value is higher.

Airgap length
The airgap length is varied from the minimum length - e.g. 0,5 mm - up to the maximum length - e.g. 5 mm -, with a step of e.g. 0,2 mm.
**Optimization of buried PMSMs**

---

**Current density**

The current density $J_{Cu}$ in the copper of the stator winding is varied from the smallest value - e.g. 0.5 A/mm$^2$ - up to the highest - e.g. 6 A/mm$^2$ -, with a step of e.g. 0.5 A/mm$^2$.

The current density in combination with the current loading (i.e. the linear current density) and the copper fill factor will give the required slot area and slot depth.

**Current loading**

The RMS value of the fundamental current loading $K_{(1)s}$ is found from the torque expression given by [72]

$$T = 2\pi r^2 L B_{(1)gm} K_{(1)s} \sin(\beta) \quad (5.2)$$

where $r$ is the rotor radius, $L$ is the axial rotor length, $B_{(1)gm}$ is the RMS value of the fundamental airgap flux density from the magnets, and $\beta$ is the electrical angle between the magnet flux and the stator current vector. Equation (5.2) does not take reluctance torque into account. For a motor with magnetic saliency, Equation (5.2) will only be valid if the d-current is equal to zero. This is equivalent to the angle $\beta$ being 90 degrees.

The total RMS value of the magneto motive force (MMF) of each slot is then given as

$$M_s = n_s I_s = \frac{K_{(1)s}}{k_{(1)w}} \cdot \frac{2\pi (r + g)}{Q} \quad (5.3)$$

where $n_s$ is the number of stator winding turns per stator slot ($n_s$ is *not* decided by the program), $r$ is the rotor radius, $g$ is the airgap length and $Q$ is the number of stator slots. $k_{(1)w}$ is the winding factor for the fundamental, given by the following general expression for a three phase winding ($\nu = 1$ for the fundamental) [66]

$$k_{(u)w} = k_{(u)d} \cdot k_{(u)p} \cdot k_{(u)s} = \frac{\sin\left(\frac{\nu \pi}{6}\right)}{q \sin\left(\frac{\nu \pi}{6q}\right)} \cdot \sin\left(\frac{\nu_{sp} \pi}{6q}\right) \cdot \frac{\sin\left(\frac{\nu_{sp} \pi}{2\tau_p}\right)}{\nu_{sp} \pi \frac{2\tau_p}{\nu_{sp}}} \quad (5.4)$$
where subscript $d$, $p$, and $s$ denote distribution, pitch and skew factor, respectively. $\nu$ is the space harmonic order number, $q$ is the number of slots per pole and phase, $y_{sp}$ is the pitch (in number of slots) of a short-pitched coil, $\rho_s$ is the peripheral length of the skew and $\tau_p$ is the peripheral length of a pole-pitch.

**Yoke thickness, copper area, slot area and slot depth**

The stator yoke thickness is given by the minimum allowable yoke thickness $d_{y,\text{min}}$ or the thickness required to keep the flux density of the yoke below the limit value, i.e.

$$d_y = \max \left( d_{y,\text{min}}, \frac{2}{p} \frac{\hat{B}_{(1)g,ms}(r+g)}{\hat{B}_{(1)y,max} k_f} \right)$$  \hspace{1cm} (5.5)

where $\hat{B}_{(1)g,ms}$ is the airgap flux density due to magnets and stator currents, $\hat{B}_{(1)y,max}$ is the maximum allowable flux density in the stator yoke/back, $k_f$ is the stacking factor for the stator iron lamination.

The required copper area of a slot is found as

$$A_{Cu,\text{slot}} = \frac{M_s}{J_{Cu}}$$  \hspace{1cm} (5.6)

while the required slot area is given as

$$A_{\text{slot}} = \frac{A_{Cu,\text{slot}}}{k_{f,Cu}} + A_{\text{slot-opening}}$$  \hspace{1cm} (5.7)

where $k_{f,Cu}$ is the copper fill factor. $A_{\text{slot-opening}}$ is the extra area required due to the semi-closed slot opening. This extra slot-opening area is added since the slot area is used to calculate the required slot depth. The slot-opening area is given by

$$A_{\text{slot-opening}} = \gamma \cdot \frac{\pi (r + g + d_{\text{slot-opening}})^2 - \pi (r + g)^2}{Q}$$  \hspace{1cm} (5.8)

where $d_{\text{slot-opening}}$ is the radial length of the slot opening. The total slot depth, with each tooth having parallel sides, is approximately given as:
The length that is not required for the slot depth is added to the thickness of the stator yoke, i.e.

\[ d_s = -(r + g)\gamma + \sqrt{((r + g)\gamma)^2 + \frac{Q\text{slot}A}{\pi}} \] \hspace{1cm} (5.9)

The length that is not required for the slot depth is added to the thickness of the stator yoke, i.e.

\[ d_y = r_{so} - d_s - g - r \] \hspace{1cm} (5.10)

where \( r_{so} \) is the outer radius of the stator. If the required slot depth is larger than the available depth, the design is not possible to realize.

**Pole width**

The electrical angle of half the true pole width on the rotor surface \( \alpha \) is varied from a small value - e.g. 60 degrees - up to a high value - e.g. 90 degrees - , with a step of e.g. 5 degrees. Fig. 5.2 defines the electrical angle of half the true pole width on the rotor surface. For a real machine with *surface mounted* or *inset mounted* magnets the true pole width is determined by the magnet width. In a real machine with *buried* magnets the true pole width can be changed by increasing the widths of the rotor bars and/or by introducing air-filled slots beside the rotor bars.

![Fig. 5.2 Definition of the electrical angle \( \alpha \) of half the true pole width on the rotor surface, here shown for a 4 pole rotor.](image)

A value that is supposed to minimize cogging \([71]\) is also checked:

\[ \alpha_{cog,min} = \frac{90^\circ}{Q} \cdot pk < 90^\circ \] \hspace{1cm} (5.11)
where $Q$ is the number of stator slots, $p$ is the number of poles and $k$ is a positive integer. The highest angle value closest to $90^\circ$ was used. Some extra degrees may have to be added to this angle value, to account for tangential leakage flux in the airgap. This has been neglected in the computer program. The size of the tangential leakage flux is dependent on the airgap length. If $f_s$ is the fundamental electrical stator frequency, the cogging torque has a fundamental mechanical frequency of

$$f_{cog} = \frac{Q f_s \cdot 2}{p}$$

(5.12)

More details about cogging, and ways to reduce it, are found e.g. in [10].

**Slot-pitch ratio**

The slot to slot-pitch ratio $\gamma$ is varied from a small value - e.g. 0.3 - to a high value - e.g. 0.65 - , with a step of e.g. 0.05. It is defined as

$$\gamma = \frac{\alpha_{slot}}{\alpha_{slot-pitch}}$$

(5.13)

Fig. 5.3 illustrates the two angles. The angles are measured at the airgap. Semi-closed slot openings do not effect the definition of the angles. Each tooth has parallel sides, while the slots have a more triangular shape.

*Fig. 5.3 Definition of angles for the slot to slot-pitch ratio. (Principal sketch with reduced number of teeth.)*
Magnet positions
Thirteen different magnet positions can be tried out. Only buried magnets are considered. Position #1 to #7 are not tried for pole number 2.

Position #1 to #7
Position #1 start with two magnets in V-shape, to achieve flux concentration. The four following positions (#2 to #5) are achieved by folding the two magnets upwards/outwards, until they are aligned. See Fig. 5.4. The total magnet width is given by the geometrical calculation mentioned in subsection Magnet width above. Position #6 and #7 are found by reducing the magnet width of position #5 to 90% and 80%, respectively. The magnet slot is kept at the same width as for magnet position #5, see Fig. 5.4. Each pole is also equipped with a radial air-filled slot, to reduce the flux from the armature reaction [74].

Fig. 5.4  Magnet position #1 to #7.

Position #8 to #10
Position #8 to #10 contain small magnet pieces, mounted quite close to the rotor surface. The magnets are situated inside/below the bars of the rotor cage, see Fig. 5.5. As many magnet pieces as possible are used. The width of each magnet piece is larger than 15 mm and smaller than 25 mm. For position #8 the full space of the magnet slot is used. For position #9 and #10 the magnet width is reduced to 90% and 80% of full magnet width, respectively. The size of the magnet slots remain the same.
Position #11 to #13
Position #11 to #13 are quite similar to position #8 to #10, but here as few magnet pieces as possible are used. The magnets pieces are again placed close to the rotor surface, but still inside/below the bars of the rotor cage. Each magnet piece is not allowed to be wider than 50 mm. See Fig. 5.6. For position #11 the full space of the magnet slot is used. For position #12 and #13 the magnet width is reduced to 90% and 80% of full magnet width, respectively. The size of the magnet slots remain the same.
Optimization of buried PMSMs

Rotor radius
The rotor radius is varied from a small value up to a large value. The smallest value should at least be bigger than the sum of shaft radius, rotor cage and magnet thickness. The maximum value should not be larger than

\[ r_{\text{max}} = r_{so} - d_{y, \text{min}} - g \]  

(5.14)

where \( r_{so} \) is the stator outer radius, \( d_{y, \text{min}} \) is the minimum allowable thickness of the stator yoke and \( g \) is the thickness of the airgap.

Copper temperature
The average copper temperature is set to a start value and it is assumed that there is no thermal steady-state present. The different loss terms with this copper temperature are calculated. These losses result in a new average copper temperature which is again, together with a small extra temperature step \( T_{Cu, \text{step}} = 1 \) °C, used to calculate the losses. If the average copper temperature is higher than maximum allowable, e.g. 145 °C for insulation class F, the temperature loop is terminated. Thermal steady-state is assumed to be reached if the difference between two consecutive copper temperatures is smaller than two times the small temperature step, i.e. \(< 2T_{Cu, \text{step}}\). If the losses are the smallest so far in the calculation procedure, the current set of parameters is saved. The temperature loop is then terminated.

5.1.3 Calculation of losses
This section contains a description of the calculation of the different loss terms.

Copper losses
The fundamental copper losses are treated like normal ohmic losses:

\[ P_{Cu} = Q A_{Cu, \text{slot}} J_{Cu}^2 L + \sigma \cdot \frac{2\pi (r + g + d_s/2)}{p} \]  

(5.15)

where \( Q \) is the number of stator slots, \( A_{Cu, \text{slot}} \) is the copper area in one slot and is given by equation (5.6), \( J_{Cu} \) is the current density in copper, \( L \) is the length of the stator, \( r \) is the rotor radius, \( g \) is the airgap length, \( d_s \) is the depth of the slot, \( p \) is the number of poles and \( \sigma \) is the ratio between the true length of the end winding and the average coil pitch. Ac-
according to [70] $\sigma=1.6$ is a normal value. $\rho_{Cu}$ is the resistivity of copper at the calculated working temperature $T_{Cu}$, and is given by

$$\rho_{Cu} = 1.72 \cdot 10^{-8} (1 + (T_{Cu} - 20) \cdot 3.9 \cdot 10^{-3}) \quad [\Omega m^2/m] \quad (5.16)$$

where the copper temperature must be given in degrees Centigrade.

**Iron losses**

The iron losses consist of two parts; the hysteresis and eddy-current losses. According to [74] the fundamental iron loss density versus the flux density can be written as

$$P_{(1)Fe}(\hat{B}_{(1)}) = k_h \hat{B}_{(1)}^{v_h} f_s + k_e \hat{B}_{(1)}^{v_e} f_s^2 \quad [W/kg] \quad (5.17)$$

where $\hat{B}_{(1)}$ is the peak value of the fundamental flux density in the iron and $f_s$ is the electrical stator frequency. The values of the coefficients and exponents in equation (5.17) were derived from measurements made by [2] on the material DK70 (lamination thickness 0.5 mm), and they can be found in Table 5.1. DK70 is a standard iron-quality with a low content of silicon. The values found in Table 5.1 are derived from measurements with sinusoidal excitation voltage.

**Table 5.1 Coefficients and exponents for Equation (5.17).**

<table>
<thead>
<tr>
<th>$\hat{B}_{(1)}$ [T]</th>
<th>$k_h$</th>
<th>$v_h$</th>
<th>$k_e$</th>
<th>$v_e$</th>
<th>$f_s$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-0.1</td>
<td>0.113</td>
<td>2.33</td>
<td>3.538</td>
<td>1.6</td>
<td>0-10500</td>
</tr>
<tr>
<td>0.1-0.2</td>
<td>0.113</td>
<td>2.33</td>
<td>2.963</td>
<td>1.6</td>
<td>0-5000</td>
</tr>
<tr>
<td>0.2-0.4</td>
<td>0.0723</td>
<td>2.06</td>
<td>1.193</td>
<td>1.7</td>
<td>50-2400</td>
</tr>
<tr>
<td>0.4-0.8</td>
<td>0.0433</td>
<td>1.50</td>
<td>6.680</td>
<td>1.8</td>
<td>50-1200</td>
</tr>
<tr>
<td>0.8-1.2</td>
<td>0.0442</td>
<td>1.58</td>
<td>3.887</td>
<td>1.9</td>
<td>50-800</td>
</tr>
<tr>
<td>1.2-1.5</td>
<td>0.0434</td>
<td>1.67</td>
<td>4.052</td>
<td>1.9</td>
<td>50-600</td>
</tr>
</tbody>
</table>

The iron losses from the fundamental flux density in the stator teeth and
the stator yoke are then given as

\[ P_{(1)Fe,ms} = \left( p_{(1)Fe}(\hat{B}_{(1)t,ms})V_t + p_{(1)Fe}(\hat{B}_{(1)y,ms})V_y \right) k_f \delta_{Fe} \]  

(5.18)

where \( p_{(1)Fe}(\hat{B}_{(1)}) \) is the iron loss density given by Equation (5.17), \( k_f \) is the stacking factor for the stator iron lamination and \( \delta_{Fe} \) is the iron density. The volume of the stator teeth - where each tooth has parallel sides - is approximately given as

\[ V_t = 2\pi(r + g)(1 - \gamma) d_s L \]  

(5.19)

where \( \gamma \) is the slot to slot-pitch ratio, \( d_s \) is the slot depth and \( L \) is the axial length of the stator core. The volume of the stator yoke is given by

\[ V_y = \pi(r_s^2 - (r + g + d_s)^2) L \]  

(5.20)

where \( d_y \) is the thickness of the stator yoke. \( \hat{B}_{(1)t,ms} \) is the peak value of the fundamental flux density in a tooth due to magnets and stator currents, given from integration of the airgap flux density over one slot-pitch [74]

\[ \hat{B}_{(1)t,ms} = \frac{4}{p} \cdot \frac{\hat{B}_{(1)y,ms}}{k_f} \cdot \frac{r + g}{w_t} \sin\left( \frac{p\pi}{2Q} \right) \]  

(5.21)

where \( w_t \) is the width of a stator tooth. The width of a stator tooth is found as

\[ w_t = (1 - \gamma) \frac{2\pi(r + g)}{Q} \]  

(5.22)

while the width of the stator slot at the airgap, neglecting semi-closed slots, is given by

\[ w_s = \gamma \frac{2\pi(r + g)}{Q} \]  

(5.23)

For machines with “many” stator slots and “few” poles Equation (5.21) can, by using equation (5.22), be rewritten as
Design and Evaluation of a Compact 15 kW PM Integral Motor

\[ \hat{B}_{(1)t, ms} = \begin{cases} \text{High Q, low } p \\ \sin x = x \text{ for small } x \end{cases} = \frac{1}{k_f} \cdot \frac{\hat{B}_{(1)g, ms}}{(1 - \gamma)} \]  

(5.24)

\( \hat{B}_{(1)y, ms} \) is the peak value of the fundamental flux density in the stator yoke due to magnets and stator currents, given from integration of the air-gap flux density over one pole-pitch [74]

\[ \hat{B}_{(1)y, ms} = \frac{2}{p} \cdot \frac{\hat{B}_{(1)g, ms}}{k_f} \cdot \frac{r + g}{d_y} \]  

(5.25)

where \( d_y \) is the thickness of the stator yoke.

The fundamental airgap flux density \( \hat{B}_{(1)g, ms} \) is the vector sum of the fundamental airgap flux density from the magnet \( \hat{B}_{(1)g, m} \) and the fundamental airgap flux density from the armature reaction \( \hat{B}_{(1)g, r} \):

\[ \hat{B}_{(1)g, ms} = \sqrt{(\hat{B}_{(1)g, m} + \hat{B}_{(1)g, r} \cos(\beta))^2 + (\hat{B}_{(1)g, r} \sin(\beta))^2} \]  

(5.26)

where \( \beta \) is the angle between magnet flux and stator current vector.

The flux produced by the magnets decrease when the calculated operating temperature of the magnets increase. The remanent flux density versus magnet temperature is given by equation (5.27), which was achieved by a second-order curvefit of the data for the NdFeB-magnet VACODYM 400HR from the product catalogue [62]:

\[ B_r = -3.84 \cdot 10^{-6} T_{mag}^2 - 7.09 \cdot 10^{-3} T_{mag} + 1.17 \]  

[T]  

(5.27)

The temperature of the magnets \( T_{mag} \) is assumed to be the same as the rotor temperature. The rotor temperature is assumed to be equal to the temperature of the air in the airgap. The airgap temperature is approximately the same as the stator iron temperature, which is assumed to be 10 °C below the temperature of the copper winding [74], i.e.

\[ T_{mag} = T_{Cu} - \Delta T_{Cu-Fe} = T_{Cu} - 10 \]  

[°C]  

(5.28)

where \( T_{Cu} \) must be given in degrees Centigrade.

The airgap flux density from the magnets \( \hat{B}_{(1)g, m} \) is calculated with an
early version of Equation (3.19). A correction factor of $k_{corr,m} = 0.92$ was introduced to reduce the analytically calculated airgap flux density to about the same level as the one calculated using a FEM program.

The airgap flux density from the stator current, i.e. the armature reaction flux, was calculated with the following Equation [28]

$$
B_{g(1),s} = 1.35\mu_0 \frac{M_s q k_{(1)w}}{k_{c} g} \tag{5.29}
$$

where $M_s$ is the total RMS value for the MMF of each slot, given by Equation (5.3). $q$ is the number of slots per pole and phase. $k_{(1)w}$ is the winding factor for the fundamental, given by Equation (5.4). $k_c$ is the Carter factor, given by Equation (3.15). $g$ is the length of the airgap. It is assumed that the stator current is purely a q-current. If the armature reaction flux was higher than that required to saturate the iron bridges in the q-direction of the rotor, a reduction of the MMF and an increase of the effective airgap length were introduced to equation (5.29). A correction factor of $k_{corr,s} = 0.78$ was used together with Equation (5.29) to achieve the same analytical result from Equation (5.26) as from FEM calculations.

**Stray load losses from the end windings**

The end windings are mainly surrounded by air. The stray load losses from the end windings are difficult to predict. According to [5] the end winding inductance seems to be more than inversely proportional to the pole number $p$. This might imply that also the stray load losses from the end windings decrease with the pole number, since they are a result of the flux which is derived from the product of the inductance and the current. To somehow take the stray load losses from the end windings into consideration they were set to

$$
P_{end, stray} = 5\% \cdot P_{(1)Fe, ms}(\frac{2}{p})^{1.5} \tag{5.30}
$$

where $P_{(1)Fe, ms}$ is the total fundamental iron losses from magnets and stator current, given by Equation (5.18).

**Saving sets of parameters**

The set of parameters - for each pole number - that has the lowest sum of the losses described above, is saved.
5.1.4 Calculation of copper temperature

The (average) copper temperature is calculated as the sum of the temperature of the ambient air $T_{amb}$, the temperature rise of the air through the converter heat-sink $\Delta T_{conv}$ and the temperature rise of the winding $\Delta T_{Cu}$. This is expressed as

$$ T_{Cu} = T_{amb} + \Delta T_{conv} + \Delta T_{Cu} \quad (5.31) $$

The temperature rise of the copper winding is estimated as [66]

$$ \Delta T_{Cu} = k_{cs} \frac{P_{Cu}}{N} (P_{Cu} + P_{Cu,r} + P_{Fe}) $$

$$ = 0 \text{ for PM motor} \quad (5.32) $$

where $k_{cs}$ is an empirical value of the thermal resistance, $P_{Cu}$ is the stator copper loss, $P_{Cu,r}$ is the rotor copper loss and $P_{Fe}$ is the iron loss. A PM machine can be assumed to have zero rotor copper loss, since the rotor runs at synchronous speed.

5.1.5 Efficiency versus speed

The estimated overall efficiency of the PM integral motor is plotted versus speed. The plot is done for all existing pole numbers and the saved set of parameters for each pole number is used to calculate the efficiency. Finally also the saved sets of parameters, for each pole number, are listed.

To estimate the overall efficiency of the PM integral motor, the efficiency of the converter has been included. Also fan, windage and bearing-friction losses have been taken into account. Time and space harmonic losses are neglected.

Efficiency of the converter

The converter is assumed to have a constant efficiency, set to [57]

$$ \eta_{conv} = 97\% \quad (5.33) $$
Fan, windage and bearing-friction losses
Since the speed of the PM integral motor is variable, a shaft mounted fan may not give sufficient cooling. In the early stage of the PM integral motor design it was therefore assumed that a separate fan should be used [74]. The fan was assumed to have a constant output power of 50 W and an efficiency of 60%, i.e.

\[ P_{\text{fan}} = \frac{50}{0.60} = 83 \text{ W} \]  

(5.34)

The windage losses on the rotor surface are fairly small. They have been set to 5 W at 1500 r/min and proportional to the third power of the speed [74]:

\[ P_{\text{windage}} = 5 \cdot \left( \frac{n}{1500} \right)^3 \text{ [W]} \]  

(5.35)

where \( n \) is the speed in r/min.

The bearing losses (including losses in the seals) have been set to 50 W at 1500 r/min, and directly proportional to the speed [74]:

\[ P_{\text{bear}} = 50 \cdot \frac{n}{1500} \text{ [W]} \]  

(5.36)

where \( n \) is the speed in r/min.

5.2 Choice of pole number for inverter-fed PMSM:s
When designing an inverter-fed PM motor one is quite free to choose a number of poles which utilizes the machine optimally. This chapter gives a suggestion regarding the pole number for a desired power and speed of the motor, and is mainly based on a paper\(^1\) presented by the author at the PEVD’98 conference [76].

5.2.1 Introduction

Permanent magnet synchronous motors (PMSM) with buried magnets are often considered for variable-speed drives, [51]. Since the variable-speed drive requires an inverter one is quite free to choose the number of poles in the motor. This is possible since, for a given mechanical speed, the inverter frequency can be raised when the number of poles increases. For induction motor (IM) drives the choice of pole number is a compromise between inverter size and motor size, [55]. Increasing the pole number of an IM drive implies higher magnetizing current but also reduced size of the motor. As the PMSM normally does not need any magnetizing current supplied by the inverter, the pole number can be increased without the undesired effect of decreased power factor. The freedom of choosing the pole number is thus significantly higher for a PMSM than for an IM. An optimization of the efficiency with certain volume constraints and with the pole number as the main variable is consequently an interesting and highly relevant task.

As in the case of the IM an increased pole number leads to a smaller motor and lower copper losses. On the other hand, the increased number of poles requires a higher stator frequency which is why the iron losses increase. A common rule of thumb is to choose a high pole number for low-speed motors and vice versa, but there are no sharp border-lines between the different areas. This chapter deals with the topic of choosing the pole number when designing a permanent magnet synchronous motor with buried magnets. The design with buried magnets, mentioned by e.g. [34], was chosen because it was found in previous studies that the efficiency of motors operated at moderate speeds was related to the airgap flux density and the airgap length. Larger airgaps require higher levels of magnetic excitation, which is why flux concentration is needed. As flux concentration can be achieved with buried magnets, this design was chosen. In this way a considerable freedom in choosing airgap flux densities was obtained.

The computer program described in Section 5.1 is used to determine near-optimum parameters for different motors. The interpretation of the results finally lead to suggestions regarding the pole number for a desired power and speed of the motor. Some other relevant motor parameters are also given. The following sections will present loss models, constants, limitations and results.
5.2.2 Computer program

In the computer program (see Section 5.1) different rotor radii, air-gap lengths, magnet widths, slot-tooth ratios, slot-depths, back-thicknesses, current densities etc. are tried out. When running the program one obtains, for each number of poles, a set of parameters of the desired motor with the highest efficiency (according to the used models of iron and copper losses) at a certain chosen speed.

Copper and iron losses
The copper and iron losses are calculated with the use of the equations given in Section 5.1.3. Only fundamental losses have been considered.

Fan, windage and bearing-friction losses
Since the fan, windage and bearing-friction losses are independent of the number of poles, at a certain mechanical speed, they were not considered in the analysis.

Stray losses
The stray losses are difficult to predict, and therefore they have been disregarded in the analysis.

Motor geometry
The chosen motor geometry has buried permanent magnets. The permanent magnets are buried in V-shape inside the rotor to enable flux concentration. The rotor is equipped with a cast aluminium squirrel-cage for mechanical stability. The cage has only two bars per pole. To reduce the iron losses from the armature reaction, each pole is equipped with an air-filled slot in the radial direction.

Constants
The following quantities have constant values:

- Angle between rotor flux and stator current vector: 90°, i.e. only current in q-direction
- Relative permeability of magnet: 1.05
- Magnet density: 7500 kg/m³
- Iron density: 7750 kg/m³
- Stacking factor for iron lamination: 0.94
- Copper fill factor: 0.44
- Thickness of saturated iron bridges: 1 mm
- Flux density in saturated iron bridges: 2.4 T
- Slot opening at airgap: 3 mm
- Ambient temperature: 40 °C

**Constants linked to the pole number**

The following quantities are determined by the pole number:

- The number of stator slots is set to 36 for 4, 6 and 12 pole motors, while 8 and 16 pole motors have 48. (To avoid fractional pitch winding.)
- The electrical angle of half the pole width on the rotor surface is set to 82.5°, 75°, 75°, 60° and 60° for pole number 4, 6, 8, 12 and 16, respectively. These values will probably minimize cogging.

**Motor-size dependent constants**

The following quantities have constant values for a certain motor-size:

- Stator core outer radius: According to a standard induction motor with equal speed and power rating as the considered PMSM.
- Shaft radius: According to a standard induction motor with equal speed and power rating.
- Rotor length: 80% of the rotor length of a standard induction motor with equal speed and power rating.
- Thickness of rotor bars: 5 mm for motor-sizes equal to or smaller than 15 kW, 1500 r/min. Increasing linearly with larger frame-size.
- Torque: According to a standard induction motor with equal power at its rated speed.
- Magnet thickness: 4.8 mm for a motor-size of 15 kW, 1500 r/min, and increasing approximately linearly with increasing frame-size.

**Parameter ranges**

The parameters are allowed to vary within reasonable ranges:

- Number of poles: 4, 6, 8, 12 and 16.
- Rotor radius: From the rotor shaft radius plus 30 mm up to the stator core outer radius minus 25 mm, with a step of 1 mm.
- Ratio of slot angle to slot-pitch angle: 0.3-0.7 with a step of 0.05.
- Current density: 2-7 A/mm², step 0.5 A/mm².
- Airgap: 1-7.5 mm, step 0.25 mm.
• The total width of the magnets under one pole is varied in five steps. From maximum flux concentration with two magnets in V-shape per pole down to one magnet per pole. I.e. only magnet position #1 to #5 is used, see Section 5.1.2.

Limitations
The following limitations have been made:

• Peak value of fundamental flux density in stator teeth from magnets and armature reaction: <1.6 T
• Peak value of fundamental flux density in stator back from magnets and armature reaction: <1.6 T
• Thickness of stator back: ≥ 10 mm
• Distance between rotor shaft and magnets: ≥ 6 mm
• Average temperature of copper: ≤ 145 °C

5.2.3 Results
The different parameters were allowed to attain all values within the specified ranges. In this way several thousands of designs were analysed for each desired rated power and speed.

Motors in the power-range 4 kW to 37 kW and speeds between 750 r/min and 3000 r/min have been examined. The shaft and stator outer radius were always set according to a standard induction motor with equal speed and power rating. The active length of the stator was set to 80% of the corresponding induction motor, since a PM-motor is expected to provide 15% to 30% more torque than an induction motor [68]. For each desired power and speed, the pole number of the motor having the very highest efficiency was chosen.

Fig. 5.7 shows the pole number results of the examined motors, summarized in a diagram.

Pole-numbers \( p \), magnet masses \( m_{NdFeB} \), airgap lengths \( g \), peak values of fundamental airgap flux densities from the magnets \( B_{g,m} \), current densities \( J_{Cu} \) and corresponding efficiencies \( \eta_{(1)CuFe} \) (based on the fundamental iron and copper losses) are shown in Table 5.2.
5.2.4 Comments on the results

Choosing a better iron-quality (i.e. higher content of silicon) would probably result in a higher suggested pole number in some cases. This is especially interesting as the speed is increased.

Since the slot openings are set to 3 mm, the 8 and 16 pole motors (which both are equipped with 48 stator slots instead of 36) require a larger stator inner radius to "exist". This might disqualify some 8 pole motors in favour of the 12 pole motor, at least for small motor-sizes.

It is important to note that time-harmonic losses due to the inverter supply are disregarded in the analysis. As these losses increase with the pole number, the optimal pole number is sometimes lower than the numbers in Table 5.2. In some cases where for instance an 8 pole design has only a slightly lower efficiency than a 12 pole design (according to the simpli-
The 8 pole design is most certainly better if all effects are considered. Some airgap lengths in Table 5.2 should probably be slightly reduced if axial leakage is considered, see sub-section 6.2.4.

Table 5.2 Relevant motor data for different powers and speeds.

<table>
<thead>
<tr>
<th></th>
<th>750</th>
<th>1000</th>
<th>1500</th>
<th>3000</th>
<th>t/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 kW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(p)</td>
<td>8</td>
<td>12</td>
<td>12</td>
<td>6</td>
<td>#</td>
</tr>
<tr>
<td>(m_{NdFeB})</td>
<td>2.39</td>
<td>2.34</td>
<td>1.45</td>
<td>0.56</td>
<td>kg</td>
</tr>
<tr>
<td>(g)</td>
<td>2.5</td>
<td>2.5</td>
<td>2</td>
<td>1.75</td>
<td>mm</td>
</tr>
<tr>
<td>(\hat{B}_{(1)s,m})</td>
<td>0.73</td>
<td>0.84</td>
<td>0.84</td>
<td>0.61</td>
<td>T</td>
</tr>
<tr>
<td>(J_{Cu})</td>
<td>2</td>
<td>3.5</td>
<td>5</td>
<td>3</td>
<td>A/mm²</td>
</tr>
<tr>
<td>(\eta_{(1)CuFe})</td>
<td>95.8</td>
<td>95.2</td>
<td>94.9</td>
<td>95.8</td>
<td>%</td>
</tr>
<tr>
<td>5.5 kW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(p)</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>6</td>
<td>#</td>
</tr>
<tr>
<td>(m_{NdFeB})</td>
<td>4.44</td>
<td>2.59</td>
<td>1.72</td>
<td>1.72</td>
<td>kg</td>
</tr>
<tr>
<td>(g)</td>
<td>3.5</td>
<td>2.25</td>
<td>2.25</td>
<td>2.75</td>
<td>mm</td>
</tr>
<tr>
<td>(\hat{B}_{(1)s,m})</td>
<td>0.81</td>
<td>0.84</td>
<td>0.83</td>
<td>0.53</td>
<td>T</td>
</tr>
<tr>
<td>(J_{Cu})</td>
<td>2.5</td>
<td>4.5</td>
<td>4</td>
<td>2.5</td>
<td>A/mm²</td>
</tr>
<tr>
<td>(\eta_{(1)CuFe})</td>
<td>95.8</td>
<td>94.9</td>
<td>95.5</td>
<td>96.4</td>
<td>%</td>
</tr>
<tr>
<td>7.5 kW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(p)</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>6</td>
<td>#</td>
</tr>
<tr>
<td>(m_{NdFeB})</td>
<td>6.17</td>
<td>4.03</td>
<td>2.34</td>
<td>0.88</td>
<td>kg</td>
</tr>
<tr>
<td>(g)</td>
<td>3.75</td>
<td>3.5</td>
<td>2.25</td>
<td>2</td>
<td>mm</td>
</tr>
<tr>
<td>(\hat{B}_{(1)s,m})</td>
<td>0.77</td>
<td>0.81</td>
<td>0.83</td>
<td>0.62</td>
<td>T</td>
</tr>
<tr>
<td>(J_{Cu})</td>
<td>2.5</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>A/mm²</td>
</tr>
<tr>
<td>(\eta_{(1)CuFe})</td>
<td>96.0</td>
<td>96.0</td>
<td>95.7</td>
<td>96.4</td>
<td>%</td>
</tr>
<tr>
<td>11 kW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(p)</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>6</td>
<td>#</td>
</tr>
<tr>
<td>(m_{NdFeB})</td>
<td>8.56</td>
<td>4.97</td>
<td>3.36</td>
<td>1.49</td>
<td>kg</td>
</tr>
<tr>
<td>(g)</td>
<td>4.5</td>
<td>3.25</td>
<td>3.75</td>
<td>3.75</td>
<td>mm</td>
</tr>
<tr>
<td>(\hat{B}_{(1)s,m})</td>
<td>0.76</td>
<td>0.84</td>
<td>0.76</td>
<td>0.58</td>
<td>T</td>
</tr>
<tr>
<td>(J_{Cu})</td>
<td>2.5</td>
<td>3</td>
<td>3.5</td>
<td>2.5</td>
<td>A/mm²</td>
</tr>
<tr>
<td>(\eta_{(1)CuFe})</td>
<td>96.3</td>
<td>96.2</td>
<td>96.3</td>
<td>97.0</td>
<td>%</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>1000</td>
<td>1500</td>
<td>3000</td>
<td>t/min</td>
</tr>
<tr>
<td>------------------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>15 kW</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p )</td>
<td>12</td>
<td>8</td>
<td>12</td>
<td>6</td>
<td>#</td>
</tr>
<tr>
<td>( m_{NdFeB} )</td>
<td>9.73</td>
<td>4.58</td>
<td>3.27</td>
<td>1.78</td>
<td>kg</td>
</tr>
<tr>
<td>( g )</td>
<td>4.5</td>
<td>3</td>
<td>3.75</td>
<td>3.5</td>
<td>mm</td>
</tr>
<tr>
<td>( \hat{B}_{(1)B,m} )</td>
<td>0.83</td>
<td>0.79</td>
<td>0.78</td>
<td>0.60</td>
<td>T</td>
</tr>
<tr>
<td>( I_{Cu} )</td>
<td>2.5</td>
<td>2.5</td>
<td>3.5</td>
<td>2.5</td>
<td>A/mm²</td>
</tr>
<tr>
<td>( \eta_{(1)CuFe} )</td>
<td>96.6</td>
<td>96.6</td>
<td>96.5</td>
<td>97.1</td>
<td>%</td>
</tr>
<tr>
<td><strong>18.5 kW</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p )</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>6</td>
<td>#</td>
</tr>
<tr>
<td>( m_{NdFeB} )</td>
<td>7.53</td>
<td>6.28</td>
<td>3.88</td>
<td>2.26</td>
<td>kg</td>
</tr>
<tr>
<td>( g )</td>
<td>3.75</td>
<td>3.75</td>
<td>3</td>
<td>3.5</td>
<td>mm</td>
</tr>
<tr>
<td>( \hat{B}_{(1)B,m} )</td>
<td>0.80</td>
<td>0.80</td>
<td>0.79</td>
<td>0.61</td>
<td>T</td>
</tr>
<tr>
<td>( I_{Cu} )</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
<td>3</td>
<td>A/mm²</td>
</tr>
<tr>
<td>( \eta_{(1)CuFe} )</td>
<td>97.0</td>
<td>96.9</td>
<td>96.9</td>
<td>97.2</td>
<td>%</td>
</tr>
<tr>
<td><strong>22 kW</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p )</td>
<td>12</td>
<td>8</td>
<td>8</td>
<td>6</td>
<td>#</td>
</tr>
<tr>
<td>( m_{NdFeB} )</td>
<td>9.77</td>
<td>6.83</td>
<td>4.70</td>
<td>2.71</td>
<td>kg</td>
</tr>
<tr>
<td>( g )</td>
<td>4</td>
<td>3.75</td>
<td>3</td>
<td>4.75</td>
<td>mm</td>
</tr>
<tr>
<td>( \hat{B}_{(1)B,m} )</td>
<td>0.84</td>
<td>0.80</td>
<td>0.79</td>
<td>0.56</td>
<td>T</td>
</tr>
<tr>
<td>( I_{Cu} )</td>
<td>2.5</td>
<td>2.5</td>
<td>3</td>
<td>2.5</td>
<td>A/mm²</td>
</tr>
<tr>
<td>( \eta_{(1)CuFe} )</td>
<td>97.0</td>
<td>96.9</td>
<td>97.0</td>
<td>97.3</td>
<td>%</td>
</tr>
<tr>
<td><strong>30 kW</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p )</td>
<td>12</td>
<td>12</td>
<td>8</td>
<td>6</td>
<td>#</td>
</tr>
<tr>
<td>( m_{NdFeB} )</td>
<td>13.4</td>
<td>9.51</td>
<td>6.18</td>
<td>3.57</td>
<td>kg</td>
</tr>
<tr>
<td>( g )</td>
<td>4.75</td>
<td>4.75</td>
<td>3.75</td>
<td>5.25</td>
<td>mm</td>
</tr>
<tr>
<td>( \hat{B}_{(1)B,m} )</td>
<td>0.84</td>
<td>0.75</td>
<td>0.79</td>
<td>0.57</td>
<td>T</td>
</tr>
<tr>
<td>( I_{Cu} )</td>
<td>2.5</td>
<td>2.5</td>
<td>3</td>
<td>2.5</td>
<td>A/mm²</td>
</tr>
<tr>
<td>( \eta_{(1)CuFe} )</td>
<td>97.1</td>
<td>97.2</td>
<td>97.2</td>
<td>97.6</td>
<td>%</td>
</tr>
<tr>
<td><strong>37 kW</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p )</td>
<td>-</td>
<td>8</td>
<td>8</td>
<td>6</td>
<td>#</td>
</tr>
<tr>
<td>( m_{NdFeB} )</td>
<td>-</td>
<td>10.2</td>
<td>6.26</td>
<td>4.43</td>
<td>kg</td>
</tr>
<tr>
<td>( g )</td>
<td>-</td>
<td>4.25</td>
<td>4.75</td>
<td>5.5</td>
<td>mm</td>
</tr>
<tr>
<td>( \hat{B}_{(1)B,m} )</td>
<td>-</td>
<td>0.79</td>
<td>0.71</td>
<td>0.55</td>
<td>T</td>
</tr>
<tr>
<td>( I_{Cu} )</td>
<td>-</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>A/mm²</td>
</tr>
<tr>
<td>( \eta_{(1)CuFe} )</td>
<td>-</td>
<td>97.3</td>
<td>97.3</td>
<td>97.7</td>
<td>%</td>
</tr>
</tbody>
</table>
5.2.5 Conclusion

In this section, some suggestions regarding the choice of pole number of inverter-fed PMSM:s with buried magnets are given. Two loss-models, one for iron and one for copper, were used to find the set of parameters giving the highest efficiency at a certain torque and speed. For each power and speed the near-optimum number of poles was plotted in a diagram. The diagram verifies the commonly known rule of thumb; higher speed - lower number of poles and vice versa, but gives also some ideas regarding the border-lines between the different pole numbers.

5.3 Using Ferrite magnets instead of NdFeB magnets in the optimization of an 8 pole motor

Using Ferrite (Fe) magnets, instead of NdFeB magnets, may reduce the total cost of the PM motor. Ferrites have lower remanent flux density than NdFeB:s. Reduced flux density will cause a decrease in efficiency, prolong the pay-off time and decrease future monetary savings. An estimation of the reduction in efficiency for an 8 pole 15 kW 1500 r/min PM motor, when using Ferrites instead of NdFeB:s, is made in this section. (An 8 pole design with NdFeB:s has an efficiency of 96%.)

Replacing the NdFeB:s with Ferrites

To estimate the efficiency that can be obtained in a PM motor with Ferrite magnets, the data of the NdFeB magnets in the optimization program were replaced by the numbers corresponding to a Strontium Ferrite (FeSr) magnet. The chosen Ferrite had a remanent flux density of \( B_{r,FeSr} = 0.4 \, \text{T} \) at 20 °C [35] and the relative permeability could be calculated to \( \mu_{r,FeSr} = B_{r,FeSr}/\mu_0[H_{c,B=0} = 1.06 \) by prolonging the linear part of the demagnetization curve to the x-axis \( (H_{c,B=0} = -300 \, \text{kA/m}) \). The “critical knee” of the curve (see Fig. 3.4) is situated in the second quadrant, at around +0.09 T and -240 kA/m [35]. The Ferrite magnet was assumed to have a temperature dependence similar to the NdFeB:s.

Results

The optimization program was run again for an 8 pole 15 kW 1500 r/min motor, and the result was that not a single possible design was found! Therefore, the rotor length had to be increased from 144 mm - as in the
Design and Evaluation of a Compact 15 kW PM Integral Motor

case where NdFeB magnets are used - to 212 mm until an existing motor design was found by the program. The now obtained coarse design had e.g. an airgap length of 0,9 mm, an airgap flux density of 0,36 T (peak), a current density $6 \text{ A/mm}^2$ (RMS), required 6,8 kg of FeSr magnet (maximum flux concentration in V-shape), and had a poor efficiency of 92%.

At first it may be a little surprising that a rotor length of 212 mm is required to find a possible design, especially since the equivalent standard induction motor already operates with a rotor length of 180 mm. What one must bare in mind is that the induction motor and the PM synchronous motor have two different operating principles. The torque production of the PM synchronous motor in the optimization program is based on the product of stator current and magnet flux, and no reluctance torque is used. This implies that an infinitely long rotor would be required as the remanent flux density of the magnet approaches zero.

**Conclusions**

The results from the optimization program show that it is **not possible** to design an 8 pole 15 kW 1500 r/min PM motor with a rotor length of 144 mm (i.e. 80% of the induction motor), if it is equipped with Ferrite magnets instead of NdFeB magnets. To find an existing motor design, the rotor length had to be increased to 212 mm. The estimated efficiency of the PM motor was then **reduced** from 96% (NdFeB) to 92% (FeSr).

### 5.4 Conclusions

This chapter has presented an optimization program for buried PMSM:s. The loss models used by the program are also presented. The optimization program was used to suggest suitable pole numbers of inverter-fed PMSM:s for different powers and speeds. The commonly known rule of thumb, i.e. higher speed - lower number of poles, was verified. It is seen that the airgaps of the PM motors are relatively large. They would probably be slightly reduced if axial leakage is considered. It was also shown that NdFeB-magnets are required to find a compact 8 pole 15 kW 1500 r/min buried PM motor design, i.e. Ferrite magnets are not possible to use.

The following chapter shows the development and design of the PM motor for a compact 15 kW 1500 r/min PM integral motor prototype. The near-optimum parameters are obtained by the use of the optimization program in the present chapter.
6 Prototype PM integral motor design

6.1 Project description and specifications

6.1.1 Background to the project

The purpose of the project\(^1\) has been to develop a variable speed 15 kW, 1500 r/min permanent magnet (PM) integral motor that is both compact, see Fig. 6.1, and worth its price. A paper\(^2\) about the development etc. was presented by the author at the *IAS 2000 Annual Meeting* [80].

The idea is that both motor and converter should be cheap to manufacture, and installation and use should be as easy as with a standard induction motor.

![Fig. 6.1 A standard induction motor and the proposed compact PM integral motor.](image)

In the short-term perspective the goal is to let the PM integral motor replace simple speed-controlled induction motor drive systems, preferably pump and fan systems with long run-times. Since the efficiency of the PM integral motor should be higher than the induction motor drive system, the PM integral motor is expected to pay-off within a couple of years, see Section 2.3. One can also expect a longer life of a PM integral motor since a PM synchronous motor has only minor rotor losses, compared to an induction motor. The reduced rotor losses will reduce the temperature of the shaft bearings, and thereby increase their life. For induction motors

---

1. The project is a pilot project (name: KIM) in the Permanent Magnet Drives programme (PMD-programme), which is within the Competence Centre in Electric Power Engineering at the Royal Institute of Technology (KTH).
the bearing temperature can be a limiting factor. Also the stator tempera-
ture is expected to be lower, due to the increased efficiency. A decreased
stator winding temperature will lead to an increased life of the winding.

In the long-term perspective the goal is to let the integral motor replace
the standard induction motor in applications where higher efficiency is
required. The gain will probably be largest for high pole numbers, since
the displacement power factor ($\cos \phi$) reduces with increasing pole
number for induction motors [31].

An integral motor is also able to replace a series of standard induction
motors which are designed for a certain torque but for different speeds.
This could reduce the variety of standard induction motors by a factor 4-
5. This would lead to e.g. reduced stocking costs for the customers. For
the manufacturers a reduced number of production lines are required.

6.1.2 Equivalent standard induction motor

The choice of an equivalent standard induction motor fell upon a 4 pole,
totally enclosed, squirrel cage, three-phase, aluminium frame induction
motor from ABB Motors. The motor (type MBT 160L) has the following
relevant data [31]:

- Rated power: 15 kW
- Rated speed: 1460 r/min
- Rated torque: 98 Nm
- Rated voltage and frequency: 380-420 V, 50 Hz
- Rated current: 29 A (at 415 V)
- Efficiency and Displacement power factor ($\cos \phi$):
  - 89%  0.87  @ 5/4 load
  - 90%  0.86  @ 4/4 load
  - 90%  0.82  @ 3/4 load
  - 88,5% 0.75  @ 2/4 load
  - 82,5% 0.52  @ 1/4 load
- Starting current to rated current ratio: 8,5
- Starting torque to rated torque ratio: 3,5
- Maximum torque to rated torque ratio: 3,8
- IEC frame size: 160 (i.e. the shaft height is 160 mm)
- Total length including shaft: 602,5 mm
- Axial length of the stator core plus the two end windings: 294 mm
6.1.3 Specifications for the PM integral motor

Discussions with experts in the field of electrical machines led to the following goal and specifications for the development of the PM integral motor.

**Most important goal**
The PM integral motor shall have higher efficiency than the equivalent induction motor drive system, and (if possible) the same outer dimensions as a standard induction motor with equivalent power and speed ratings.

**Specifications**

- Maximum continuous output power (i.e. shaft power) shall be 15 kW.
- The speed shall be adjustable within \( \pm 1500 \text{ r/min} \).
- Maximum continuous shaft torque shall be 98 Nm.
- Maximum intermittent torque, i.e. maximum torque for a shorter period of time (e.g. 1 min), should be \( 1.2 \cdot 98 \text{ Nm} = 118 \text{ Nm} \).
- A shaft-mounted radial fan should be used.
- The shaft height should be 160 mm.
- The size and placement of the mounting holes of the PM integral motor shall agree with standards.
- The axial length of the stator core plus the axial length of the end windings should be 194 mm. This is 100 mm shorter than in the 15 kW standard induction motor. The main part of the converter circuit is assumed to fit axially inside these 100 mm, see Fig. 6.1. Parts that do not fit (e.g. speed-control circuits, line-filter etc.) should be placed inside an increased (wider and/or higher) terminal box.
- The PM motor should have a Y-connected stator winding, to avoid the induced circulating currents which may occur in a D-connected winding.
- The permanent magnets should be buried inside the rotor (i.e. interior mounted) [74]. See also sub-section 3.1.1.
- The integral motor should be built for a three-phase supply with a line-to-line voltage of \( 400 \text{ V}^{+6\% \cdot -10\%} \cdot 50 \text{ Hz} \).
- The converter should primarily consist of a standard converter solution, which might be modified. Also new converter designs might be possible alternatives.
• The integral motor should be controlled without a shaft sensor.
• The integral motor should be built for pump and fan applications.
• The integral motor must fulfil the current (and in the near future) EMC directives.
• Insulated bearings should maybe be considered, to avoid bearing currents.

A shaft torque of 98 Nm at 1500 r/min implies a maximum continuous output power of 15.4 kW. A short-time torque of 118 Nm implies an increase of the converter output power by 1.2 times as well, since the converter cannot operate at over-load for many seconds. The induced circulating currents which may occur in a D-connected winding are due to a magnet flux containing the 3rd space harmonic and its multiples.

Further discussions with members of the working group of the project led to the following suggestions to achieve a compact integral motor:

• The converter of the integral motor should be equipped with only a small intermediate link capacitor.
• The filter coils of the converter circuit could perhaps be integrated with the stator core.

6.2 Optimization

The computer program described in Section 5.1 is used to determine near-optimum parameters for the prototype PM integral motor. First the pole number and parameters are decided, using “coarse” step-lengths, then the parameters are fine-tuned with a smaller step-length.

6.2.1 PM motor parameters and first results

Constants
The following quantities have constant values:

• Stator core outer radius: 127 mm
• Shaft radius: 27 mm
• Rotor length: 110 mm
• Thickness of rotor bars: 5 mm
• Torque: 98+1=99 Nm, where 1 Nm has been added to compensate for
Prototype PM integral motor design

- Different friction losses in the motor
  - Magnet thickness: 4.8 mm. The magnet thickness is constant since it turned out that the airgap flux density varies much more with the magnet width than with the magnet thickness, at least for buried magnets and the dimensions considered here. See also Equation (3.19).
  - Angle between rotor flux and stator current vector: $90^\circ$, i.e. only current in q-direction
  - Relative permeability of magnet: 1.05
  - Magnet density: 7500 kg/m$^3$
  - Iron density: 7750 kg/m$^3$
  - Stacking factor for iron lamination: 0.94
  - Copper fill factor: 0.60 (chosen high to obtain a compact motor)
  - Thickness of saturated iron bridges: 1 mm
  - Flux density in saturated iron bridges: 2.4 T
  - Slot opening at airgap: 3 mm
  - Total radial thickness of semi-closed slot opening: 2 mm
  - Thermal resistance, stator to ambient: $1.5 \cdot 1.3 \cdot 0.076 = 0.148 \, ^\circ C/W$
  - Temperature rise of the airflow through the converter heat-sink: 5 °C
  - Ambient temperature: 40 °C

A rotor length of 110 mm is very short, but necessary to be sure that both the PM motor and the converter fit inside the length of the standard induction motor. The available axial length of stator core and two end windings is 294 mm in the standard induction motor. About 100 mm is expected to be required for the converter. This leaves 194 mm for the PM stator plus two end windings. According to [68] a PM machine may be expected to provide 15% to 30% more torque than an induction motor. This might imply that the rotor can be made 13% to 23% shorter for the same torque. The length of the rotor of the standard induction motor is 180 mm. The proposed PM rotor of 110 mm is 39% shorter than the rotor of the induction motor.

The thermal resistance from stator to ambient has been increased with a factor of 1.5 and a factor of 1.3. The factor 1.5 is due to the decreased axial length of the stator housing, in the early motor design. This length was supposed to be decreased to about 2/3 of the length of the standard induction motor stator housing. The thermal resistance $R_{th}$ is inversely proportional to the area [37] of the stator housing, i.e.
The factor 1.3 is due to the decreased airflow to the motor, because of the converter heat-sink. This has been taken into account by using Equation (7.2) for the heat transfer coefficient:

\[
R_{th, integral} = \frac{1}{\alpha_{integral}} R_{th, standard} = \frac{0.6}{v_{integral}} R_{th, standard} = \{ q - v \} = \left( \frac{q_{standard}}{q_{integral}} \right)^{0.6} R_{th, standard} = \left( \frac{157}{101} \right)^{0.6} R_{th, standard} = 1.3 R_{th, standard}
\]

where the two airflows have been taken from the measurements in Section 6.4.

**Constants linked to the pole number**

The following quantities are determined by the pole number:

- The number of stator slots is set to 36 for 2, 4, 6 and 12 pole motors, while 8 and 16 pole motors have 48. (To avoid fractional pitch winding.)
- The electrical angle of half the true pole width on the rotor surface is set to 85°, 80°, 75°, 75°, 60° and 60° for pole number 2, 4, 6, 8, 12 and 16, respectively. These values will probably minimize cogging.

**Parameter ranges**

The parameters are allowed to vary within reasonable ranges:

- Number of poles: 2, 4, 6, 8, 12 and 16.
- Rotor radius: 32 mm to 117 mm minus airgap length, step 1 mm.
- Ratio of slot angle to slot-pitch angle: 0.3-0.6 with a step of 0.05.
- Current density: 2-6 A/mm², step 0.5 A/mm².
- Airgap: 0.5-3.3 mm, step 0.2 mm.
- The total width of the magnets under one pole is varied in 13 steps. From maximum flux concentration with two magnets in V-shape per
pole down to one magnet per pole, and also with many small magnet pieces close to the rotor surface. I.e. magnet positions #1 to #13 are used, see Section 5.1.

**Limitations**
The following limitations have been made:

- Peak value of fundamental flux density in stator teeth from magnets and armature reaction: <1.6 T
- Peak value of fundamental flux density in stator back from magnets and armature reaction: <1.6 T
- Thickness of stator back: ≥ 10 mm
- Distance between rotor shaft and magnets: ≥ 5 mm
- Average temperature of copper: ≤ 145 °C

**Results**
The optimization program required about 1 hour of computational CPU time on a Hewlett Packard work station, model 715/100. The results of the optimization program are shown in Fig. 6.2. Over one hundred thousand geometries have been tried by the optimization program. Not a single 2, 4 or 6 pole design are within the specified limits. 46,6924 and 1610 possible designs for 8, 12 and 16 poles respectively, were found by the program. The set of parameters, for each pole number, that minimize the losses at 1460 r/min was saved by the program.

It can be seen in Fig. 6.2 that the 12 pole motor has the highest efficiency, in this simplified analysis. This is in agreement with the suggestions given in Section 5.2, though the axial rotor lengths in that section was only decreased by 20%. The rotor length in this section is decreased by 39%, compared to an equivalent standard induction motor. It is important to note that time-harmonic losses due to the inverter supply are disregarded in the analysis. As these losses increase with the pole number, the optimal pole number is sometimes lower. In some cases where for instance an 8 pole design has only a slightly lower efficiency than a 12-pole design (according to the simplified analysis), the 8 pole design is most certainly better if all effects are considered.

Since the goal also was to have a higher efficiency than the equivalent standard induction motor - at 98 Nm, 1460 r/min - the 8 pole design was chosen, due to the reasoning above.
Fig. 6.2 Efficiency versus speed for different pole numbers. Coarse optimization results when the loss is minimized (left), and when the product of losses and magnet mass is minimized (right).

For comparison the coarse design parameters that minimize the product of magnet mass and power loss (at 1460 r/min) were also saved by the program. These results are also shown in Fig. 6.2. For example the magnet mass of the 8 pole motor has now decreased by, 0,83 kg and the efficiency at 1460 r/min has decreased by, 0,67 percentage units.

6.2.2 Fine-tuned parameters of the chosen 8 pole motor

It was decided that the integral motor should have an 8 pole design, see Results in Section 6.2.1. The coarse parameters of the 8 pole motor can now be fine-tuned by decreasing the parameter ranges and step sizes:

Parameter ranges for fine-tuning

- Number of poles: 8
- Rotor radius: 32 mm to 117 mm minus airgap length, step 1 mm.
- Ratio of slot angle to slot-pitch angle: 0,4-0,5 with a step of, 0,01.
- Current density: 3,2-4,2 A/mm², step 0,1 A/mm².
- Airgap: 2,3-3,3 mm, step 0,1 mm.
- The total width of the magnets under one pole is varied in 13 steps: From maximum flux concentration with two magnets in V-shape per pole down to one magnet per pole, and also with many small magnet pieces close to the rotor surface. I.e. magnet positions #1 to #13 are used, see Section 5.1.
Results
The results of the fine-tuning of the parameters with the optimization program is shown in Table 6.1 below.

Table 6.1 Fine-tuned design parameters of the 8 pole motor, a 12 pole motor, and an 8 pole motor (8') using a more accurate airgap flux density model. Search criteria: Minimized losses at 1460 r/min. Iron losses and temperatures are given at 1500 r/min. The four quantity values marked with a * were pre-determined.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Fine-tuned 8 pole</th>
<th>Fine-tuned 12 pole</th>
<th>Fine-tuned 8' pole</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stator slots $Q$</td>
<td>48$^\circ$</td>
<td>36$^\circ$</td>
<td>48$^\circ$</td>
<td>---</td>
</tr>
<tr>
<td>Rotor radius $r$</td>
<td>81</td>
<td>74</td>
<td>83</td>
<td>mm</td>
</tr>
<tr>
<td>Rotor length $L$</td>
<td>110$^\circ$</td>
<td>110$^\circ$</td>
<td>110$^\circ$</td>
<td>mm</td>
</tr>
<tr>
<td>Yoke thickness $d_y$</td>
<td>11.7</td>
<td>15.4</td>
<td>12.0</td>
<td>mm</td>
</tr>
<tr>
<td>Slot depth $d_s$</td>
<td>31.4</td>
<td>35.3</td>
<td>29.7</td>
<td>mm</td>
</tr>
<tr>
<td>Slot width $w_s$ (at airgap, without semi-closure)</td>
<td>5.05</td>
<td>5.19</td>
<td>4.99</td>
<td>mm</td>
</tr>
<tr>
<td>Tooth width $w_t$</td>
<td>5.93</td>
<td>8.12</td>
<td>6.10</td>
<td>mm</td>
</tr>
<tr>
<td>Airgap $g$</td>
<td>2.9</td>
<td>2.3</td>
<td>2.3</td>
<td>mm</td>
</tr>
<tr>
<td>Slot to slot-pitch ratio $\gamma$</td>
<td>0.46</td>
<td>0.39</td>
<td>0.45</td>
<td>---</td>
</tr>
<tr>
<td>True pole angle $\alpha$</td>
<td>75$^\circ$</td>
<td>60$^\circ$</td>
<td>75$^\circ$</td>
<td>el. deg.</td>
</tr>
<tr>
<td>Current density $J_{Cu}$</td>
<td>3.5</td>
<td>3.0</td>
<td>3.8</td>
<td>A/mm$^2$ (RMS)</td>
</tr>
<tr>
<td>Fundamental current loading $K_{1s}$</td>
<td>39.41</td>
<td>38.06</td>
<td>38.87</td>
<td>kA/m (RMS)</td>
</tr>
<tr>
<td>Fundamental winding factor $k_{1w}$</td>
<td>0.9659</td>
<td>1</td>
<td>0.9659</td>
<td>---</td>
</tr>
<tr>
<td>Magneto motive force per slot $M_s$</td>
<td>448</td>
<td>507</td>
<td>449</td>
<td>A/l= A (RMS)</td>
</tr>
<tr>
<td>Total copper area per slot $A_{Cu,slot}$</td>
<td>128.0</td>
<td>168.9</td>
<td>118.3</td>
<td>mm$^2$</td>
</tr>
<tr>
<td>Slot area $A_{slot}$</td>
<td>223.6</td>
<td>292.1</td>
<td>207.3</td>
<td>mm$^2$</td>
</tr>
<tr>
<td>Carter’s factor (stator) $k_c$</td>
<td>1.045</td>
<td>1.046</td>
<td>1.055</td>
<td>---</td>
</tr>
<tr>
<td>Remanent flux density of magnet at calculated magnet temperature $B_r$</td>
<td>1.045</td>
<td>1.078</td>
<td>1.034</td>
<td>T</td>
</tr>
<tr>
<td>Airgap flux density from magnet $B_{1(1)r,m}$</td>
<td>0.730</td>
<td>0.914</td>
<td>0.716</td>
<td>T</td>
</tr>
<tr>
<td>Airgap flux density from stator current $B_{1(1)r,s}$</td>
<td>0.331</td>
<td>0.278</td>
<td>0.393</td>
<td>T</td>
</tr>
</tbody>
</table>
Most of the fine-tuned parameters for the 8 pole motor of Table 6.1 were used to manufacture the 8 pole prototype motor. During manufacturing some parameter values had to be changed. The parameter changes and the effects of the changes are described and estimated in Section 6.5.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Fine-tuned 8 pole</th>
<th>Fine-tuned 12 pole</th>
<th>Fine-tuned 8' pole</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airgap flux density from magnets and stator current $B_{(1)g,ms}$</td>
<td>0.802</td>
<td>0.956</td>
<td>0.817</td>
<td>T</td>
</tr>
<tr>
<td>Flux density in stator teeth $B_{(1)l,ms}$</td>
<td>1.56</td>
<td>1.59</td>
<td>1.56</td>
<td>T</td>
</tr>
<tr>
<td>Flux density in stator yoke $B_{(1)y,ms}$</td>
<td>1.53</td>
<td>0.84</td>
<td>1.55</td>
<td>T</td>
</tr>
<tr>
<td>Copper losses $P_{Cu}$</td>
<td>425</td>
<td>232</td>
<td>473</td>
<td>W</td>
</tr>
<tr>
<td>Eddy-current losses in stator teeth $P_{(1)te,ms}$</td>
<td>44.7</td>
<td>116</td>
<td>43.8</td>
<td>W</td>
</tr>
<tr>
<td>Hysteresis losses in stator teeth $P_{(1)h,ms}$</td>
<td>65.5</td>
<td>117</td>
<td>64.2</td>
<td>W</td>
</tr>
<tr>
<td>Eddy-current losses in stator yoke $P_{(1)ye,ms}$</td>
<td>42.8</td>
<td>34.6</td>
<td>44.7</td>
<td>W</td>
</tr>
<tr>
<td>Hysteresis losses in stator yoke $P_{(1)yh,ms}$</td>
<td>63.1</td>
<td>46.5</td>
<td>65.7</td>
<td>W</td>
</tr>
<tr>
<td>Stray load losses from end windings $P_{stray, end}$</td>
<td>1.4</td>
<td>1.1</td>
<td>1.4</td>
<td>W</td>
</tr>
<tr>
<td>Total iron losses $P_{(1)Fe,ms}$</td>
<td>217</td>
<td>315</td>
<td>220</td>
<td>W</td>
</tr>
<tr>
<td>Iron loss density $P_{(1)Fe,ms}$</td>
<td>15.2</td>
<td>18</td>
<td>15.4</td>
<td>W/kg</td>
</tr>
<tr>
<td>Total iron + copper losses $P_{(1)CuFe}$</td>
<td>642</td>
<td>547</td>
<td>693</td>
<td>W</td>
</tr>
<tr>
<td>Total losses (PM motor) $P_{loss}$</td>
<td>781</td>
<td>685</td>
<td>831</td>
<td>W</td>
</tr>
<tr>
<td>Magnet position</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>#</td>
</tr>
<tr>
<td>Total magnet width (per pole) $w_m$</td>
<td>79.9</td>
<td>61.7</td>
<td>84.1</td>
<td>mm</td>
</tr>
<tr>
<td>Magnet thickness $l_m$</td>
<td>4.8&quot;</td>
<td>4.8&quot;</td>
<td>4.8&quot;</td>
<td>mm</td>
</tr>
<tr>
<td>Magnet mass $m_{NdFeB}$</td>
<td>2.53</td>
<td>2.93</td>
<td>2.66</td>
<td>kg</td>
</tr>
<tr>
<td>Copper mass $m_{Cu}$</td>
<td>12.9</td>
<td>10.2</td>
<td>11.9</td>
<td>kg</td>
</tr>
<tr>
<td>Bulk iron mass $m_{Fe}$</td>
<td>51.7</td>
<td>51.7</td>
<td>51.7</td>
<td>kg</td>
</tr>
<tr>
<td>Laser cut or punched iron mass $m'_{Fe}$</td>
<td>25.6</td>
<td>25.5</td>
<td>26.3</td>
<td>kg</td>
</tr>
<tr>
<td>Total efficiency @ 1460 r/min $\eta_{1460}$</td>
<td>92.25</td>
<td>92.84</td>
<td>91.96</td>
<td>%</td>
</tr>
<tr>
<td>Total efficiency @ 1500 r/min $\eta_{1500}$</td>
<td>92.32</td>
<td>92.87</td>
<td>92.03</td>
<td>%</td>
</tr>
<tr>
<td>Copper temperature $T_{Cu}$</td>
<td>122</td>
<td>97</td>
<td>129</td>
<td>°C</td>
</tr>
<tr>
<td>Magnet temperature $T_{mag}$</td>
<td>112</td>
<td>87</td>
<td>119</td>
<td>°C</td>
</tr>
</tbody>
</table>
6.2.3 Fine-tuned parameters of a 12 pole motor

It was decided that the PM integral motor should have an 8 pole design, see the paragraph Results in Section 6.2.1. It can still be of interest to see how the near-optimum parameters of a 12 pole motor differs from the near-optimum parameters of the chosen 8 pole motor. The coarse parameters of the 12 pole motor (whose efficiency curve is shown in Fig. 6.2) have been fine-tuned with the optimization program in a similar way as for the 8 pole motor. The results are shown in Table 6.1.

From Table 6.1, one can conclude that the 12 pole motor design has more iron losses (+45%) but less copper losses (-45%), resulting in lower total losses (-12%), compared to the 8 pole design. The 12 pole has also much lower copper and magnet temperatures. All this together gives the 12 pole PM integral motor design 0,59 percentage units (0,64%) better efficiency than the 8 pole alternative at 1460 r/min. It could certainly be interesting to build a 12 pole PM integral motor prototype as well. One must keep in mind, though, that the high frequency losses are neglected in the analysis and the obtained results may therefore favour the 12 pole motor.

6.2.4 Fine-tuned parameters of an 8 pole motor design optimized with a more accurate flux density model

The optimization program, which was described in Chapter 5 and used for the prototype motor design in the present chapter, contained an early version of Equation (3.19) for calculation of the airgap flux density, as mentioned in sub-section 5.1.3. Improved models for calculation of the airgap flux density were later derived and are presented in Chapter 3. To investigate how the totally analytical flux density expression of Equation (3.35) would influence the 8 pole PM motor design, it was implemented in the optimization program. Equation (3.35) includes axial leakage and iron saturation of the most narrow part of the machine. The flux density correction factor of \( k_{corr,m} = 0.92 \), see sub-section 5.1.3, was removed from the optimization program. Instead, a new axial leakage reluctance correction factor of \( k_{corr,axi} = 1.81 \) was introduced to increase the axial leakage factor \( k_a \) of Equation (3.41). The correction factor is introduced since both the 2D-FEM and the analytically calculated axial leakage reluctances seem to be underestimated, see Section 4.4. The factor 1.81 was obtained by increasing the factor \( k_a \) of Equation (3.41) for Motor A in sub-section 4.2.2 until the airgap flux density \( \hat{B}_{(1)g,axi} \) was decreased by
7.6%, compared to the airgap flux density value $\hat{B}_{(1)g}$ which does not include axial leakage. (1.58mVs/1.71mVs=92.4%, see Section 4.4.)

The improved optimization program was run and the coarse parameters of the “improved” 8 pole motor were then fine-tuned in a similar way as in sub-section 6.2.2. The results are shown in Table 6.1. The “improved” 8 pole motor is denoted 8’.

From Table 6.1, one can see that the largest relative change in the geometry is that the airgap now is reduced by 21%, compared to the earlier 8 pole motor design in sub-section 6.2.2. A reduction of the airgap is not surprising since the axial leakage flux of the rotor, which was not taken into account in sub-section 6.2.2, increases with an increasing airgap (for a constant rotor length). The airgap, which has decreased from 2.9 mm to 2.3 mm, is still relatively large. This also implies that the airgaps of the designs in Table 5.2 would probably be reduced if axial leakage is taken into account. The 8’ pole PM integral motor design has 0.29 percentage units (0.3%) lower efficiency than the 8 pole alternative at 1460 r/min.

6.3 Analytical and FEM calculations of the optimized 8 pole motor design

6.3.1 FEM calculations of the airgap flux densities

It is important that the analytical values of the airgap flux densities used in the optimization agree with FEM calculations. The airgap flux densities due to magnets and stator currents at the calculated magnet temperature 112 °C (see Table 6.1) have been examined with the FEM software ACE\textsuperscript{1}. The magnets were modelled as single-turn coils carrying the current $I_m = H_i l_m = (B_r/\mu_0) l_m = 3801$ A [33], where the values are found in Table 6.1. The mesh used in the FEM calculations had about 10 000 elements. The mesh is a kind of web with many, many small triangular elements, adaptively made and used by the FEM program. The mesh had been refined successively until hardly no change in the flux densities was observed. The motor analysed in Fig. 6.3 to Fig. 6.9 was designed according to the parameters from the fine-tuned optimization, found in Table 6.1.

\textsuperscript{1} FEM program from ABB Corporate Research.
Airgap flux density due to the magnets

Fig. 6.3 shows a field plot of one pole of the motor with the magnet flux only, at 112 °C. Fig. 6.4 shows the corresponding Fourier analysis of the airgap flux. As can be seen from Fig. 6.4 a value of $\hat{B}_{(1)g,m} = 0.75$ T was obtained from the FEM analysis, which is quite close to the value of 0.73 T obtained in the optimization model, see Table 6.1.

Fig. 6.3  Field lines and flux densities with only magnets at 112 °C.

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Amplitude [T]</th>
<th>Percentage</th>
<th>Phase [el. deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.91</td>
<td>100</td>
<td>180</td>
</tr>
<tr>
<td>2,1</td>
<td>0.61</td>
<td>68.5%</td>
<td>-180</td>
</tr>
<tr>
<td>2,2</td>
<td>0.33</td>
<td>32%</td>
<td>90</td>
</tr>
</tbody>
</table>

Fig. 6.4  Fourier analysis of the airgap flux density due to the magnets at 112 °C. The columns indicate (left to right): space harmonic order, peak value of flux density in T, relative magnitude with respect to the fundamental, and the phase angle in el. deg.
Airgap flux density due to stator currents

Fig. 6.5 shows a field plot of one pole of the motor with only the flux from the stator currents. Fig. 6.6 shows the corresponding Fourier analysis of the airgap flux. As can be seen from Fig. 6.6 a value of $B_{(1)g,s} = 0.44$ T was obtained from the FEM analysis. The value of 0.33 T that was used in the optimization (see Table 6.1) is much smaller than the FEM calculated value of 0.44 T. This is not so strange, since the iron bridges were not saturated by the magnet flux in this FEM calculation.

![Field lines and flux densities with only stator currents.](image)

**Fig. 6.5**  Field lines and flux densities with only stator currents.

![Fourier analysis of the airgap flux density due to the stator currents. For explanation, see Fig. 6.4.](image)

**Fig. 6.6**  Fourier analysis of the airgap flux density due to the stator currents. For explanation, see Fig. 6.4.
Airgap flux density due to magnets and stator currents

Fig. 6.7 shows a field plot of one pole of the motor with magnets and stator currents, at 112 °C. Fig. 6.8 shows the corresponding Fourier analysis of the airgap flux. As can be seen from Fig. 6.8 a value of $\hat{B}_{(1)g,ms} = 0.81$ T was obtained from the FEM analysis, which is quite close to the value of 0.80 T obtained in the optimization model, see Table 6.1. It can also be seen that the flux density levels in stator teeth and yoke are equal to or below the design limit-value of 1.6 T. The design limit-value was chosen as low as 1.6 T to avoid magnetic saturation.

![Fig. 6.7](image)

**Fig. 6.7** Field lines and flux densities with magnets and stator currents at 112 °C.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Amplitude</th>
<th>Phase</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.003</td>
<td>151.2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.004</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.004</td>
<td>151.2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.004</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.004</td>
<td>151.2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.004</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.004</td>
<td>151.2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.004</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.004</td>
<td>151.2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.004</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 6.8](image)

**Fig. 6.8** Fourier analysis of the airgap flux density due to magnets and stator currents at 112 °C. For explanation, see Fig. 6.4.
Demagnetizing effect under steady-state condition

If the stator current is too high, the armature reaction flux can be so large that the flux density of the magnet decreases below the flux density level of the critical knee of the demagnetization curve of the magnet. The critical knee is the point on the demagnetizing curve where the B-H relation no longer is linear. If that happens, the magnet is irreversibly demagnetized, and has therefore lost some - or all - of its magnetic strength. This demagnetizing current can be due to different faults, e.g. a short-circuit. The critical knee of the used NdFeB-magnet is situated below the level of zero flux density, at least up to a magnet temperature of 120 °C [62]. The calculated magnet temperature is 112 °C. To examine how the magnets are effected by demagnetizing current, a current of as much as 10 times the rated current (i.e. \(10I_n=300\) A) was applied in the negative d-direction. The magnets were also present. A field line plot from the static FEM calculation is shown in Fig. 6.9, where the field lines are represented by arrows. The lengths of the arrows are a measure of the flux density. It can be seen the flux density is low in the magnets, but still above zero. Only the magnet part closest to the rotor shaft has a reversed direction of the flux, i.e. a flux density below zero. In reality, the short-circuit rotor cage

![Field lines and magnets](image)

**Fig. 6.9** Resulting field lines, represented by arrows, from the magnets and 10 times the rated current in negative d-axis direction. The lengths of the arrows are a measure of the flux density.
will shield off stator current transients and the magnets will be even better protected than what is shown in Fig. 6.9. This has been investigated by using time-stepping fixed-speed FEM calculations, see Section 8.2.

**Conclusion**

The conclusion is that the flux densities used in the optimization agrees well with FEM calculations. The magnets are well protected from demagnetizing stator currents.

### 6.3.2 Number of winding turns per stator slot

It is remarkable to note that so far in the design procedure, the number of winding turns per stator slot have not been decided. The number of turns is decided from the achievable output voltage from the inverter and the desired modulation index. According to [83] the DC-voltage from an ideal \( p_{\text{pulse}} \)-pulse diode bridge rectifier with continuous current is given by

\[
 U_d = \hat{U}_{(1)}{_{\text{l-l,mains}}} \frac{p_{\text{pulse}}}{\pi} \sin \left( \frac{\pi}{p_{\text{pulse}}} \right) = \\
\]

\[
= \{6\text{-pulse diode bridge}\} = \\
= \sqrt{2} \cdot 400 \cdot \frac{6}{\pi} \sin \left( \frac{\pi}{6} \right) = 540 \text{ V}
\]

where \( \hat{U}_{(1)}{_{\text{l-l,mains}}} \) is the peak-value of the fundamental line-to-line voltage on the mains-side. For an ideal three-phase inverter the RMS-value of the maximum fundamental line-to-line voltage depends on the modulation method used. The times for when to switch the valves of the inverter are obtained by comparing three reference waves - one for each phase - with a single triangular shaped wave. When a reference wave is larger than the triangular wave, the corresponding valves are switched on, and vice versa. The triangular wave has a high frequency, e.g 4 kHz, and this frequency is called switching frequency. If the reference waves are square waves, the maximum fundamental output voltage from the inverter is obtained. The disadvantage is that a lot of time harmonics, e.g. 5, 7, 11 and 13, are obtained as well. The output line-to-line voltage has a quasi-square shape and the RMS-value of the fundamental line-to-line volt-
age is obtained as [83]:

$$U_{(1)l-l,\text{quasi-square},\text{max}} = 0.780U_d$$  \hspace{1cm} (6.4)$$

By changing the square shaped reference waves to sinusoids, the harmonic content of the output voltage is reduced. The disadvantage is that also the fundamental voltage is reduced. The output line-to-line voltage is then a pulse width modulated (PWM) pattern, and the RMS-value of the fundamental line-to-line voltage is obtained as [83]:

$$U_{(1)l-l,\text{sine-PWM},\text{max}} = 0.612U_d$$  \hspace{1cm} (6.5)$$

Since the 3rd harmonic voltage and its multiples (9, 15, 21 etc.) are only visible in the phase potential, not in the phase and line-to-line voltage, they can be added to the sinusoidal reference waves. The sinusoidal reference waves are then turned into quasi-sinusoidal reference waves, and are therefore called quasines. With quasine shaped reference waves, some extra fundamental voltage can be obtained from the inverter, compared to sine shaped reference waves. The output line-to-line voltage is again a pulse width modulated (PWM) pattern, and the RMS-value of the fundamental line-to-line voltage is obtained as [83]:

$$U_{(1)l-l,\text{quasine-PWM},\text{max}} = 1.155 U_{(1)l-l,\text{sine-PWM},\text{max}} = 0.707U_d$$  \hspace{1cm} (6.6)$$

Since the proposed PM machine will be Y-connected, the induced phase voltage will be equal to the voltage induced in a winding. The RMS-value of the fundamental voltage induced in a winding is given by the following equation [53]:

$$E_{(1)\text{ph}} = \sqrt{2} qn_s \hat{B}_{(1)g} r L \omega_s \frac{k_{(1)w}}{c}$$  \hspace{1cm} (6.7)$$

where

$$\omega_s = \omega_{\text{mech}} \frac{p}{2}$$  \hspace{1cm} (6.8)$$

and $q$ is the number of stator slots per pole per phase. $n_s$ is the number of winding turns per stator slot, $\hat{B}_{(1)g}$ is the airgap flux density, $r$ is the rotor radius (or the radius where the airgap flux density has been calcu-
lated), \( L \) is the axial rotor length, \( k_{(1)w} \) is the winding factor for the fundamental and \( c \) is the number of parallel-connected coils in a winding. \( \omega_{\text{mech}} \) is the angular rotation frequency of the shaft and \( p \) is the number of poles.

The available voltage from the inverter will be reduced due to voltage drops over the filter coils, rectifier diodes and the inverter IGBT:s. A coarse estimation of the different (fundamental) voltage drops are made as follows:

- \( U_{(1)X, \text{filter}} = 2\pi f_{\text{mains}} L_{\text{filter}} I_{(1), \text{mains}} = 2 \text{ V} \)
- \( U_{(1)R, \text{filter}} = R_{\text{filter}} I_{(1), \text{mains}} = 0,2 \text{ V} \ « U_{(1)X, \text{filter}} \)
- \( U_{\text{diode}} = 0,8-1,6 \text{ V} \)
- \( U_{\text{IGBT}} = 2,5-3 \text{ V} \)

where \( f_{\text{mains}} \) is the fundamental frequency of the mains current, \( L_{\text{filter}} \) and \( R_{\text{filter}} \) are the inductance and the resistance of the line filter, respectively, see sub-section 6.3.9. The RMS-value of the fundamental of the quasi-square mains-side current at rated load can be found as

\[
I_{(1), \text{mains}} = \frac{P_{\text{shaft}}}{\sqrt{3} U_{(1)l-L, \text{mains}} \eta_i} = \frac{15,4 \cdot 10^3}{\sqrt{3} \cdot 400 \cdot 0,923} = 24,1 \text{ A} \quad (6.9)
\]

where the total efficiency of the integral motor \( \eta_i \) is found in Table 6.1, Section 6.2.2.

As can be seen from the list above, the different voltage drops are negligible compared to the mains voltage and the DC-link voltage. By using Equations (6.3) and (6.6) the RMS-value of the available fundamental phase voltage from the inverter, neglecting voltage drops, is:

\[
U_{(1)\text{ph,quasine-PWM}} = m \cdot \frac{1}{\sqrt{3}} 0,707 U_d \quad \text{for} \quad 0 < m < 1 \quad (6.10)
\]

where \( m \) is the modulation index for the inverter. \( m \) is defined as the ratio between the amplitude of the reference wave and the amplitude of the triangular shaped wave. If \( m \) is between 0 and 1 the output voltage of the inverter is directly proportional to \( m \), and the harmonic content of the
Fig. 6.10  Phasor-diagram of a loaded PM-machine with d-current equal to zero. (Motor references.)

voltage is low. For values of \( m \) higher than 1, the output voltage is no longer directly proportional to \( m \), and the harmonic content of the voltage increases. The choice of the maximum modulation index \( m \), inside the range 0 to 1, is a trade-off between a better utilization of the inverter and a higher risk of time harmonics in the output voltage.

According to the phasor diagram in Fig. 6.10 the RMS-value of the required fundamental phase voltage of the PM motor can, considering stator resistance and leakage inductance, be expressed as

\[
U_{\text{ph,PM}}(1) = \sqrt{\left( E_{\text{ph}}(1) + R_s I_q \right)^2 + \left( U_{\text{phX}}(1) + \omega_s L_{\text{leak}} I_q \right)^2} \tag{6.11}
\]

where the inductive voltage drop has been split up into two components, since \( U_{\text{phX}}(1) \) was based on FEM calculations while \( \omega_s L_{\text{leak}} I_q \) was calculated analytically.

Equation (6.11) can be rewritten to achieve an expression for the PM motor voltage containing the number of turns per slot \( n_s \). Let the induced voltage at the angular frequency \( \omega_s \) be given by Equation (6.7) with the FEM calculated value \( B_{(1)g} = \hat{B}_{(1)g,m} \) at calculated magnet temperature:

\[
E_{\text{ph}}(1) = k_E \cdot n_s \tag{6.12}
\]
The flux density due to the stator current is in quadrature with the flux density from the magnets, and can be found as

\[ \hat{B}_{1g,s} = \sqrt{\hat{B}_{1g,ms}^2 - \hat{B}_{1g,m}^2} \]  

(6.13)

The flux density in Equation (6.13) gives rise to an inductive phase voltage of (according to Equation (6.7))

\[ U_{(1)phX} = k_U \cdot n_s \]  

(6.14)

The leakage inductance, versus the squared number of turns per slot, is given by the following approximate expression [70]:

\[ L_{leak} = L_{slot, leak} + L_{end, leak} = n_s^2 \cdot \frac{12 \mu_0 (p^2 L (d_s / w_s + 1,5) + 4r)}{\pi Q} \left( \frac{1}{R_{leak}} \right) \]  

(6.15)

where \( p \) is the number of poles, \( L \) is the axial stator length, \( Q \) is the number of stator slots, \( d_s \) is the depth of the stator slot and \( w_s \) is the width of the stator slot. \( R_{leak} \) is the magnetic leakage reluctance of slots and end windings.

The \( q \)-current \( I_q \) is given by the expression

\[ I_q = \frac{M_s}{n_s} \]  

(6.16)

where \( M_s \) is the RMS-value of the stator MMF per slot.

The phase-resistance of the PM motor at the calculated stator copper temperature is

\[ R_s = \frac{P_{Cu}}{3I_q^2} = \frac{P_{Cu}}{3M_s^2} = \frac{n_s^2}{k_R} \cdot \frac{1}{M_s} \]  

(6.17)

Equation (6.11) can now be rewritten as
The required modulation index is now given as

\[ m = \frac{U_{(1)ph,PM}}{U_{(1)ph,quasine-PWM,max}} \]  

(6.19)

Using Equations (6.3) and (6.6), and the ratio 1/\(\sqrt[3]{3}\) results in the voltage

\[ U_{(1)ph,quasine-PWM,max} = 220 \text{ V} \]  

Table 6.2 was established to show the different modulation indices versus number of winding turns per stator slot for different temperatures when the motor is run with rated torque at 1500 r/min (i.e. \(\omega_s = 2\pi 100 \text{ rad/s}\)). This operating point was chosen since it requires a higher modulation index than operation at no-load, see Equation (6.18). The numerical data were taken from Fig. 6.4 and Table 6.1. It is important to adapt the airgap flux density \(B_{(1)g,m}\) and the copper losses \(P_{Cu}\) to the used temperatures.

**Table 6.2** Modulation index versus number of winding turns per stator slot for two different temperatures.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Operation</th>
<th>(n_s=14) t</th>
<th>(n_s=15) t</th>
<th>(n_s=16) t</th>
<th>(n_s=17) t</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_{Cu}=20^\circ C)</td>
<td>98 Nm 1500 r/min</td>
<td>m=…</td>
<td>0.94</td>
<td>1.01</td>
<td>1.08</td>
</tr>
<tr>
<td>(T_{mag}=20^\circ C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T_{Cu}=122^\circ C)</td>
<td>98 Nm 1500 r/min</td>
<td>0.88</td>
<td>0.94</td>
<td>1.00</td>
<td>1.06</td>
</tr>
<tr>
<td>(T_{mag}=112^\circ C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
After studying Table 6.2, the choice of number of turns per slot fell upon

\[ n_s = 15 \ t \]  \hspace{1cm} (6.20)

since it implies a good utilization of the converter over the whole temperature range, and the safety margin to \( m = 1 \) is sufficient at thermal steady-state. Rated torque with cold motor might require slight over-modulation for a while, but the motor temperature will soon increase in that case.

### 6.3.3 Calculation of fundamental d- and q-inductances

The fundamental d- and q-inductances at calculated magnet temperature \( T_{\text{magnet}} = 112 \ ^\circ C \) were calculated with FEM, using the software ACE\(^1\). The magnets were modelled as single-turn coils carrying the current \( I_m = H_c I_m = (B_c/(\mu_0)) I_m = 3801 \ A \) [33], where the values are found in Table 6.1. In the following analysis the magnet flux - and therefore the induced voltage - has been assumed to be constant, despite of the iron saturation effects due the stator current. There are two reasons for this; it is complicated to take this fact into consideration, and the same assumption is made during the measurements on the prototype machine (see sub-section 7.1.7).

### Stator slot and end winding leakage inductance

The stator slot and end winding leakage inductance must be taken into consideration, especially since both airgap length and slot depth are relatively large. According to [70] an approximate value for this inductance is given by Equation (6.15):

\[ L_{\text{leak}} = L_{\text{slot, leak}} + L_{\text{end, leak}} = \ldots = 1.22 \ \text{mH} + 0.35 \ \text{mH} = 1.57 \ \text{mH} \]  \hspace{1cm} (6.21)

---

1. FEM program from ABB Corporate Research.
d- and q-inductances

First the peak value of the fundamental airgap flux density with only magnets inserted was calculated. This flux density was $B_{(1)g, m} = 0.752$ T, resulting (with the use of Equation (6.7)) in an induced fundamental phase-voltage of $E_{(1)ph} = 176$ V. By applying also stator current from 5 A to 40 A in positive and negative d-direction, new flux densities in the airgap $B_{(1)g, ms}$ were obtained. (Rated stator current is $I_n = M_s / n_s = 448 / 15 = 29.9$ A.) With the use of Equation (6.7) the corresponding fundamental phase-voltages $U_{(1)ph}$ can be calculated. The fundamental magnetizing d-inductances (due to positive and negative d-currents) at calculated magnet temperature (112 °C) can now be calculated as:

$$L_{nd} = \frac{X_{nd}}{\omega_s} = \frac{U_{(1)ph} - E_{(1)ph}}{\omega_s I_d}$$  \hspace{1cm} (6.22)

where $\omega_s = 2 \pi 100$ rad/s. The fundamental d-inductances at the calculated magnet temperature 112 °C are found by adding the leakage inductance of Equation (6.21) to the magnetizing d-inductance:

$$L_d = L_{nd} + L_{leak}$$  \hspace{1cm} (6.23)

The d-inductance versus d-current is plotted in Fig. 6.11. The d-inductance can be approximated (using the software Curvefit) with a first order equation (i.e. a straight line) for positive d-currents:

\[ \text{Fig. 6.11 Calculated d-inductance versus d-current (left), and calculated q-inductance versus q-current (right). Magnet temperature 112 °C.} \]
Prototype PM integral motor design

\[ L_{d+,1st} = 4.90 - 0.0309 I_d \text{ [mH]} \quad \text{for} \quad 0 < I_d < 40 \text{ A} \]  \hspace{1cm} (6.24) 

while the d-inductance for negative d-currents is almost constant and can be represented by the mean value:

\[ L_{d-} = 4.95 \text{ [mH]} \quad \text{for} \quad -40 \text{ A} < I_d < 0 \]  \hspace{1cm} (6.25) 

The magnetizing q-inductance was calculated in a similar way as the d-inductance. Again the peak value of the fundamental airgap flux density from only the magnets is \( \tilde{B}_{(1)g.m} = 0.750 \text{ T} \) and the corresponding induced voltage is \( E_{(1)ph} = 175 \text{ V} \). Inserting current from 5 A to 40 A in q-direction results in different peak values of the fundamental airgap flux density \( \tilde{B}_{(1)g.ms} \). This leads - with the use of Equation (6.7) - to different phase voltages \( U_{(1)ph} \). Since the magnet flux and the armature reaction flux are at an electrical 90 degree angle, the magnetizing q-inductance at calculated magnet temperature \( (112 \text{ °C}) \) can be found as:

\[ L_{mq} = \frac{X_{mq}}{\omega_s} = \frac{\sqrt{U_{(1)ph}^2 - E_{(1)ph}^2}}{\omega_s I_q} \]  \hspace{1cm} (6.26) 

where \( \omega_s = 2\pi 100 \text{ rad/s} \). The q-inductance is found as the sum of the magnetizing q-inductance and the leakage inductance according to:

\[ L_q = L_{mq} + L_{leak} \]  \hspace{1cm} (6.27) 

where \( L_{leak} \) is given by Equation (6.21). Since the q-axis direction does not contain any magnets, the q-inductance does not depend on the sign of the q-current. The calculated values of the q-inductance are therefore valid for both positive and negative q-currents.

Fig. 6.11 shows a plot of the q-inductance versus the current in q-direction. The q-inductance can be approximated with a fourth order equation (using the software Curvefit):

\[ L_{q,4th} = 3.83 \cdot 10^{-7} I_q^4 - 1.34 \cdot 10^{-3} I_q^2 + 6.35 \text{ [mH]} \quad \text{for} \quad -40 < I_q < 40 \text{ A} \]  \hspace{1cm} (6.28)
6.3.4 Saliency ratio

The magnetic saliency ratio describes the relation between the inductances in the two different directions of the rotor in a machine. There are different definitions for the saliency ratio, but here the following definition is used:

\[
\xi = \frac{L_q - L_d}{L_d}
\]  

(6.29)

where \(L_d\) and \(L_q\) are the d- and q-inductances, respectively. The saliency ratio is of great importance for the control of the machine. The saliency ratio is required when finding the initial rotor position during the start-up procedure [30], [73]. A saliency larger than zero is in theory enough, but due to noisy signals etc. about 5%-10\% is required in practice. The saliency ratios of the PM machine, for different values of current, are obtained by inserting the d- and q-inductances from Fig. 6.11 into Equation (6.29). The results are presented as graphs in Fig. 6.12. Any cross-coupling between d- and q-direction has been neglected. As can be seen from Fig. 6.12, the saliency is more than 10\% for zero to rated current in any direction. Rated stator current is \(I_n = M / n_s = 448/15 = 29.9\) A.

Fig. 6.12 Calculated saliency ratio versus d- and q-current at a magnet temperature of 112 °C. Cross-coupling between d- and q-direction is neglected.
The compact integral motor described in this thesis has not been designed to operate in the field weakening region since pump and fan loads have a quadratic torque curve. However, it can be of interest to investigate which speed that could be reached if field weakening is employed and this has been done in the following sub-section.

### 6.3.5 Field weakening region

If a motor is run with a reduced magnetic flux, i.e. lower than the rated flux, it is said to be operating in the field weakening region. One reason for reducing the flux is that higher speeds can be reached with the same inverter. For an induction machine the field weakening region is easily entered by increasing the frequency above the base frequency at a constant magnitude of the voltage. This reduces the flux - and thereby also the available torque - of the induction machine, since the flux is proportional to the ratio between voltage and stator frequency. The lower part of the field weakening region is called the constant power region. This name arises from the fact that the power of the induction motor can be kept constant in this region, since the speed increases as much as the torque drops. For a permanent magnet machine, which has a constant flux from the magnets, to enter the field weakening region a current in the negative d-direction must be applied. This negative d-current gives rise to a flux which counteracts the flux through the stator windings from the magnets.

![Phasor diagrams](image)

*Fig. 6.13 Phasor diagrams showing the PM motor operating at rated load and base-speed (left), and at no-load with maximum speed (right). (Motor references.)*
The base-speed of the PM integral motor is 1500 r/min. The maximum speed for continuous operation in field weakening is found when the rated motor current is applied in the negative $d$-direction. From the left and right phasor diagrams of Fig. 6.13 the following two equations for the rated terminal voltage of the motor can be stated:

$$ U_n = \sqrt{(E_n + R_s I_{q,n})^2 + (\omega_n L_q I_{q,n})^2} \quad (6.30) $$

and

$$ U_n = \sqrt{\left(\frac{\omega_{\text{max}}}{\omega_n} \cdot E_n + \omega_{\text{max}} I_d L_d\right)^2 + (R_s I_d)^2} \quad (6.31) $$

By setting Equations (6.30) and (6.31) equal and solving for $\omega_{\text{max}}$, the following expression for the maximum speed of the compact PM integral motor is found:

$$ n_{\text{max}} = \frac{60}{2\pi} \cdot \frac{2}{p} \cdot \omega_{\text{max}} = \frac{60}{2\pi} \cdot \frac{2}{p} \cdot \omega_n \cdot \sqrt{(E_n + R_s I_{q,n})^2 + (\omega_n L_q I_{q,n})^2 - (R_s I_d)^2} \quad (6.32) $$

where $p = 8$, $\omega_n = 2\pi 100$ rad/s, $I_d = -I_n = -30$ A, $I_{q,n} = I_n = 30$ A, $L_d = 4.95$ mH, $L_q = 5.46$ mH, $R_s = 0.16$ Ω at 122 °C and $E_n = 175.5$ V at 112 °C.

This implies that the field weakening range for the PM integral motor is about 2.5 times its base-speed. This is a pretty high value, especially since the PM motor was not designed for field weakening operation. The reason for this high value is the relatively high leakage inductance, see Equation (6.21). The leakage inductance acts in two ways; first it increases the required voltage from the inverter in normal operation, secondly it gives rise to a counter-acting voltage drop in field weakening operation. If the leakage inductance is subtracted from the $d$- and $q$-inductances in Equation (6.32), the field weakening range is reduced to 2610 r/min, i.e. 1.7 times the base-speed. To improve the field weakening range even further with the same inverter, the $d$-inductance has to be raised. The $d$-
inductance can be increased by using thinner and wider magnets.

Note that the mechanical properties of the rotor at speeds above 1800 r/min have not been investigated. The iron losses and their thermal effect have not been examined either.

Expressions and limitations of field weakening operation are e.g. given in [38] and [60].

6.3.6 Torque characteristics

Cogging, magnet and total torque at rated load

The cogging torque, magnet torque and total torque at rated load (98+1 Nm, $T_{max}$=112 °C) versus the rotor position were calculated with FEM software ACE\(^1\). The magnets were modelled as single-turn coils carrying the current $I_m = H \cdot I_m = (B_y / (\mu_y \mu_0)) I_m = 3801$ A [33], where the values are found in Table 6.1. The stator was rotated from 0 mechanical degrees to 15 mechanical degrees, with a step of $\Phi_{mech} = 1.5$ mechanical degrees. Also the stator currents (the “stator current sheet”) were changed to move simultaneously with the rotor movement. The step-wise movement of stator and currents were made manually, i.e. each angle required two new FEM calculations (one for cogging torque and one for total torque). At the mechanical angle 0 degrees the stator current in phase R had a peak value and was spatially placed opposite a rotor magnet, i.e. the torque was - for all angles - calculated with the d-current equal to zero. The reason for rotating the stator, instead of the rotor, is that the stator has a simpler geometry and therefore requires less work.

The cogging torque and the total torque versus the mechanical rotation angle are shown in Fig. 6.14. The cogging torque is only about 2% (peak-to-peak) of the mean total torque. By subtracting the cogging torque from the total torque, the magnet torque - i.e. torque due to interaction between magnet flux and stator current without cogging - was also obtained. The average total torque of 96.9 Nm (from FEM) and the expected average torque of 99 Nm are also shown in Fig. 6.14 as a solid straight line and a dashed straight line, respectively.

\(^{1}\) FEM program from ABB Corporate Research.

---

1. FEM program from ABB Corporate Research.
Design and Evaluation of a Compact 15 kW PM Integral Motor

Fig. 6.14 FEM calculated cogging torque, magnet torque and total torque at a magnet temperature of 112°C. The averaged total torque (from FEM) is shown as a solid straight line. The expected average torque is shown as a dashed straight line.

Torque versus load angle and stator current
Since the designed PM-motor has saliency, i.e. the d-inductance is not equal to the q-inductance, the saliency will contribute to the torque production if there is a current in d-direction. This torque contribution is called reluctance torque. Depending on the sign of the d-current and the size of the q- to d-inductance ratio, the reluctance torque will either increase or the decrease magnet torque. The integral motor is designed for being run with the d-current equal to zero, i.e. the torque will be produced only by the interaction between magnet flux and stator current, while the reluctance torque is set to zero. If one wishes to take advantage of the extra torque produced by the reluctance torque, it can still be of interest to see how the total torque depends on the load angle and the stator current. According to [28], the torque of a synchronous machine with saliency, neglecting all losses, can be expressed as
Prototype PM integral motor design

\[ T = \frac{3}{\omega_{mech}} \cdot \left( \frac{E_{ph} U_{ph}}{X_d} \cdot \sin(\delta) + \frac{U_{ph}^2}{2} \cdot \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \cdot \sin(2\delta) \right) \quad (6.33) \]

where \( \delta \) is the electrical load angle, i.e. the angle between \( U_{ph} \) and \( E_{ph} \) (see Fig. 6.15).

\[ Fig. 6.15 \quad \text{Phasor diagram for a salient synchronous machine. (Motor references.)} \]

Equation (6.33) can be rewritten [24] as

\[ T = \frac{3}{\omega} \cdot \frac{P}{2} \left( \frac{E_{ph} I}{\omega_q} \cdot \sin(\beta) + \frac{I^2}{2} \cdot (L_d - L_q) \cdot \sin(2\beta) \right) \quad (6.34) \]

where \( \beta \) is the angle between the magnet flux phasor \( \Phi_m \) (i.e. d-direction) and the stator current phasor \( I \), see Fig. 6.15.

Only in the latter of the two torque equations, the magnet torque and the reluctance torque are clearly separable.

Both total torque and magnet torque versus stator current and load angle, at calculated operating temperature \( T_{mag} = 112 \degree C \), are plotted in Fig.
6.16. The plots were obtained by the use of Equations (6.34), (6.24), (6.25) and (6.28). The induced phase-voltage is set to $E_{ph} = E_{(1)ph} = 175$ V, see sub-section 6.3.3. As can be seen from Fig. 6.16, the reluctance torque has hardly no effect at all on the total torque.

![Fig. 6.16 Total torque (solid) and magnet torque (dashed) versus stator current (5-30 A) and load angle $\beta$ at a calculated magnet temperature of 112 $^\circ$C.](image)

### 6.3.7 Mechanical strength

**Critical speed**

The fundamental frequency of the critical speed (or eigen frequency or resonance frequency) of the rotor is a function of rotor radius and rotor length [72]:

$$f_{\text{critical}} = \frac{r}{L^2}$$  \hspace{1cm} (6.35)

where $r$ is the rotor radius and $L$ is the axial length of the rotor. Since the PM integral motor has almost the same rotor radius as the standard induction motor but a shorter rotor length, the critical speed will be higher than for the standard induction motor. It will therefore not be investigated further.
Early PM rotor design
Mechanical 2D-FEM calculations on an early PM rotor design with buried magnets were done by ITT Flygt. The calculations were done with both a centrifugal force and a torque.

The applied force was equal to the “centrifugal force” at 1500 r/min. Neither magnets, nor aluminium bars were inserted in the rotor. The radial deformation and the mechanical stress levels are so low (< 16 MPa) that no problems can be expected.

A torque of 99 Nm applied to the rotor showed mechanical stress levels of up to 110 MPa, which is close to acceptable, especially considering iron material fatigue. A recommendation was to increase the iron bridges between the magnets of different poles from 2 mm to 3 mm. The iron bridges between the magnets were increased from 2 mm to 4 mm, since the widths of these iron bridges hardly effects the magnet widths at all. This change was done early in the optimization program.

Final PM rotor design
Mechanical FEM calculations on the final rotor design have been done by ABB Corporate Research. The FEM calculations were done in 2D with the FEM software ACE/LUCAS\(^1\). The calculations were performed by applying a force equal to the “centrifugal force” at 1800 r/min. The E-modulus of the magnet material was set to a very low value, i.e. the magnets do not contribute to the mechanical strength. The mechanical strength of the aluminium bars have not been included either. The results of the FEM calculations are

- All mechanical stress levels are low, i.e. less than 10 MPa.
- The radial deformation at the circumference of the rotor is negligible, i.e. less than 1 \(\mu\)m.

6.3.8 Converter circuit
The PM integral motor will be equipped with a 20 kVA converter, integrated with the PM motor. The converter will be controlled sensorless by a digital signal processor (DSP). The sensorless control algorithms are derived in [30]. The converter consists - basically - of a 6-pulse diode

\(^1\) FEM program from ABB Corporate Research.
bridge rectifier, an intermediate link (DC-link) and a 3-phase IGBT inverter, see Fig. 6.17. The converter is developed and built by Inmotion Technologies (former Atlas Copco Controls). The converter is a new design based on conventional power electronics, custom-built for this application. The DC-link has a very small capacitor, investigated in [40] [41].

Fig. 6.17 Basic layout of the converter circuit for the PM integral motor.

6.3.9 Corner coils - A new integral motor stator design

The converter circuit of the PM integral motor has a line filter and an intermediate-link filter. These filters consist of capacitors and inductances, see Fig. 6.17. The line filter inductances act as commutating inductances and decrease the time-derivative of the line current when the diode bridge commutates. Thereby the time-harmonic content of the line current is reduced. The inductance of the intermediate-link is a smoothing inductance, which decreases the pulsations of the DC-current in the intermediate-link. These four inductances, which are heavy magnetic iron circuits, are also quite volume consuming. To gather together all heavy magnetic devices to the same space in the machine, and to reduce the size of the converter, the idea of integrating the inductances with the stator core came up, see sub-section 6.1.3. This integration would also be a step forward in using the iron lamination more effectively. Later it was found out that a patent on this matter already existed [16]. The present sub-section has been presented in a paper at the EPE’99-conference [78].

Corner coil design

The first thought was, since there are four inductances in the converter circuit, to place one inductance in each “corner” of the motor. See Fig. 6.18. After observing a real induction machine it was soon realized that this placement of the inductances would interfere with the mounting holes at the feet of the machine. There are two solutions to this problem:

1. Reduce the axial length of the two lower inductances, i.e. the two inductances that are closest to the feet of the machine.
2. Do not use the two lower “corners”, i.e. the two “corners” that are closest to the feet of the machine.

The first solution would result in different values of inductance between the upper and lower “corners”, something which is not preferable. It would also require two different geometries of the stator iron lamination sheets. Two different lamination geometries would complicate the iron punching procedure as well as the stacking of the stator core. These drawbacks are considered to be so large that solution number 2, i.e. to not use the lower corners, is preferable. Solution number 2 implies that all four filter coils (i.e. inductances) have to be placed in the two upper corners. The filter coils can then be placed either “back-to-back” or “side-by-side”, see Fig. 6.19.
After some consideration it was realized that the “side-by-side” placement is better than the “back-to-back” ditto, due to four reasons:

1. Less iron material is wasted.
2. Higher inductance value is achieved.
3. Only one stator iron lamination geometry is required.
4. The winding procedure of the filter coils is easier.

The corner coils (i.e. the filter coils in the corners) have been designed according to the following important criteria:

- High inductance.
- Linear inductance.
- Low magnetic coupling to the magnetic circuits of stator and rotor.
- Low mutual inductance between the filter coils.
- Allow heat transport from stator winding and iron to the outside of the stator core yoke.

The suggested solution can be seen in Fig. 6.19 (right) and in Fig. 6.21. The coils of the suggested solution have their “iron yoke legs” pointing in the radial and not the tangential direction. This is done to minimize the stator flux through the coils, since the stator flux mostly travels in tangential direction.
Fig. 6.20 Simulated line current of the PM integral motor with a shaft power of 15 kW and a 10 μF DC-link capacitor [40].

The current going through the corner coils of the line filter will have somewhat of a quasi-square shape [40], [41], see Fig. 6.20. The current through the corner coil of the DC-link will be almost constant with the same magnitude as the “peak value” (or magnitude) of the current through the line filter coils. The corner coil current is approximately given as the “peak value” (or magnitude) of the quasi-square current and can be found as [83]:

\[
I_{cc} = I_{\text{quasi-square}} = \frac{I_{(1)\text{mains}}}{0.780} = \frac{24.1}{0.780} = 31 \text{ A}
\]  

(6.36)

where \(I_{(1)\text{mains}}\) is the RMS value of the fundamental current on the mains-side, given by Equation (6.9). The flux path of the filter coil contains an airgap. The airgap is used in the design to adjust the flux density level in the iron. By keeping the flux density level (far) below the magnetic saturation knee in the BH-curve of the iron material, the inductance is kept approximately linear. The iron saturation normally starts at 1.5-1.7 T, of course depending on the used iron quality. Setting the flux density level of iron in a corner coil as low as \(B_{\text{iron,cc}} = 1.14 \text{ T}\) almost insures that magnetic saturation does not appear. The flux density is deliberately chosen very low since the quasi-square current can have quite large pulsations, depending on the size of DC-link inductance and DC-link capac-
ittance. The airgap length $l_\delta$ of a corner coil, required to keep the flux density of the corner coil airgap down to the level $B_\delta = k_f B_{\text{iron,cc}}$, can be found by applying Ampère’s circuital law on the magnetic circuit:

$$\frac{B_\delta l_\delta}{\mu_0} = N_{cc} \hat{I}_{cc} \Rightarrow l_\delta = \frac{\mu_0 N_{cc} \hat{I}_{cc}}{B_\delta} = \frac{\mu_0 N_{cc} \hat{I}_{cc}}{k_f B_{\text{iron,cc}}}$$

(6.37)

where the MMF-drop of the iron is neglected since the flux density is chosen so low.

The inductance of a corner coil is then given by

$$L_{cc} = \frac{\Psi_{cc}}{\hat{I}_{cc}} = \frac{N_{cc} B_\delta l_\delta}{\hat{I}_{cc}} = \frac{N_{cc} B_{\text{iron,cc}} k_f 2 w_{cc} L}{\hat{I}_{cc}}$$

(6.38)

Allowing a DC-link corner coil current density of about $J_{Cu,cc} = 5$ A/mm², the required copper area is:

$$A_{Cu,cc} = \frac{\hat{I}_{cc}}{J_{Cu,cc}} = 6 \text{ mm}^2$$

(6.39)

A 6 mm² copper wire with insulation has an outer diameter of about 5 mm. Here a trade-off between number of corner coil turns and width of the magnetic flux path has to be done. Three different reasonable possibilities are investigated, see Fig. 6.21. The flux path widths are set to 5 mm, 6 mm and 7 mm, allowing for 8, 6 and 3 conductors, respectively.

Fig. 6.21 Three different reasonable possibilities for a corner coil.
By using Equation (6.37) the required corner coil airgap lengths for these three cases are obtained. Corresponding inductances can be found by the use of Equation (6.38). Table 6.3 summarizes the results.

Table 6.3  Flux path widths, number of turns, airgap lengths and inductances of the three corner coils shown in Fig. 6.21.

<table>
<thead>
<tr>
<th>Flux path ( w_{cc} ) [mm]</th>
<th>Number of turns ( N_{cc} )</th>
<th>Airgap ( l_{g} ) [mm]</th>
<th>Inductance ( L_{cc} ) [mH]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>8</td>
<td>0.29</td>
<td>0.30</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.22</td>
<td>0.27</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>0.11</td>
<td>0.16</td>
</tr>
</tbody>
</table>

The higher inductance, the better it is for the line filter. On the other hand the stator core will (in some way) be shrink fitted into a stator housing. This requires a certain width of the iron bridges (i.e. flux paths), to withstand the mechanical pressure. A good trade-off between inductance and flux path width in this case seems to be to choose a flux path of 6 mm.

To check the analytical calculation, a FEM calculation was performed with the FEM software ACE. The calculation was done on a geometry with 6 mm flux paths and 6 turns, using a current of 31 A. This results in an inductance of 0.24 mH, which is quite close to the analytical value of 0.27 mH.

Copper losses in the corner coils
The corner coils will give rise to additional copper losses (Ohmic) in the machine. The copper losses are given by the total RMS value of the current in each corner coil. The RMS value of the current in the DC-link coil is equal to the “peak” value of the quasi-square current, i.e.

\[
I_{cc,4} = \hat{I}_{cc} = 31 \text{ A}
\]  

while the RMS value of the line filter coil currents is given by:

\[
I_{cc,1-3} = \sqrt{\frac{1}{T} \int_{0}^{T} i^2 dt} = \sqrt{\frac{1}{T} \left( \int_{0}^{T/3} 0^2 dt + \int_{T/3}^{T} \hat{I}_{cc}^2 dt \right)} = \hat{I}_{cc} \sqrt{\frac{2}{3}} = 25.3 \text{ A}
\]  

(6.41)
The corner coils have been numbered from 1 to 4, in clockwise direction seen from the non-drive end, see Fig. 6.22.

The resistance of a corner coil at 112 °C can be estimated to:

$$R_{cc,112°C} = \rho_{Cu,112°C} \cdot \frac{l_{Cu,cc}}{A_{Cu,cc}} = 7.0 \text{ mΩ / corner coil} \quad (6.42)$$

The power losses at the calculated temperature of 112 °C are then 4.5 W per line filter coil and 6.7 W in the DC-link coil. This results in a total corner coil copper loss of 20 W, which is almost negligible compared to the stator winding copper losses of 425 W.

Iron losses in the corner coils

The DC-link coil is assumed to carry a quite constant flux and has therefore almost no iron losses. A very rough estimation of the size of the fundamental iron losses of the three line filter coils is found as:

$$P_{Fe,cc1-3} = 2 \cdot p_{(1)Fe}(B_{iron,cc}) \cdot w_{cc} \cdot l_{cc,path} \cdot L \cdot k \cdot \delta_{Fe} =$$

$$= 2 \cdot 3.57 \cdot 0.006 \cdot 0.088 \cdot 0.11 \cdot 0.94 \cdot 7750 = 3 \text{ W (per line filter coil)} \quad (6.43)$$

where a 50 Hz sinusoidal flux variation with a peak value of $B_{iron,cc}$ was assumed. $p_{(1)Fe}(B_{iron,cc})$ is the iron loss density given by Equation (3.38) and $l_{cc,path}$ is the mean length of the magnetic flux path of a corner coil. The result is a total line filter corner coil iron loss of about 9 W, which is almost negligible compared to the stator iron losses of 217 W.

Influence on airgap flux density and magnetic coupling between coils

It was checked with a FEM calculation that the introduction of the corner coils does not effect the airgap flux density. Further FEM calculations were done to examine the magnetic coupling between the different corner coils (i.e. mutual inductance), and between a corner coil and the magnetic circuit of the motor. The left-hand side picture of Fig. 6.22 shows the field lines from a 6 turn corner coil carrying the current 31 A. No magnets or stator currents are applied. The flux linkage with other corner coils, and with the magnetic circuit of the motor is negligible.

In Fig. 6.22 (right-hand side picture) the stator has been rotated to a position where one of the rotor poles is opposing the middle iron leg of
corner coil number 3. This alignment is assumed to give a “high” rotor flux through the corner coil. The flux through corner coil number 3, here denoted peak flux, is $\Phi_{cc, 3} = 3.3 \, \mu$Vs. Surprisingly the flux through corner coil number 4 is larger: $\Phi_{cc, 4} = 5.2 \, \mu$Vs. This indicates that the maximum flux through a corner coil is not obtained when a pole is opposing a coil. The flux through corner coil number 4 would result in an induced corner coil voltage of (assuming sinusoidal flux variation):

$$\dot{e}_{cc, 4} = N_{cc} \frac{d\Phi_{cc, 4}}{dt} \bigg|_{max} = N_{cc} \omega_s \Phi_{cc, 4} = 20 \, mV \quad (6.44)$$

where $N_{cc} = 6 \, t$ and $\omega_s = 2\pi 100 \, \text{rad/s}$. A corner coil voltage with a peak value of 20 mV (or in that range) is considered to be negligible compared to the peak value of the fundamental phase voltage of the mains, which is 325 V.

**Conclusion**

The conclusion is that the introduction of stator integrated corner coils can be made, without interfering with the rest of the magnetic circuit of the stator.
6.4 Investigation of dummy heat sinks and airflows

Introduction
As a step in the development of the compact 15 kW permanent magnet synchronous integral motor some preliminary converter heat-sink designs etc. were investigated. Temperatures and corresponding airflows were measured. The cooling airflow of the proposed integral motor will go through the converter heat-sink and in to the centre of the fan. From the fan the airflow will continue out over the sides of the stator housing, see Fig. 6.23.

Fig. 6.23 The cooling airflow of the proposed integral motor.

The airflow to the motor will be both reduced and pre-heated, due to the converter heat-sink. The measurements were performed on two dummy heat-sinks mounted at the rear (i.e., non-drive end) of a standard induction motor. Dummy heat-sink #1 and #2 have been manufactured by ABB Corporate Research and KTH, respectively. This section gives a very brief summary of the tests, highlights the most interesting results and gives some conclusions. More details about the tests are found in [82].

Dummy heat sinks

Fig. 6.24 Dummy heat-sink #1 (left) and #2 (right). The number of fins has deliberately been reduced in the drawings.
Two different dummy heat-sinks have been tested in the laboratory. They have different shape and different way of operation, see Fig. 6.24. The first heat-sink has horizontal fins while the power electronics should be mounted vertically. The second heat-sink has vertical fins and a horizontal space for the power electronics. The two dummy heat-sinks were mounted - one at a time - at the rear (non-drive end) of a standard 15 kW induction motor. The induction motor was running at 1460 r/min, no-load. To simulate the losses of the power electronics, three resistors were mounted on the heat-sink where the power electronics were supposed to be. The three resistors were loaded with 200 W each (i.e., a total power loss of 600 W) to simulate a converter of 20 kVA with an efficiency of 97%. Outside the three resistors was a box filled with thermal insulation.

Dummy heat-sink #1 (see Fig. 6.24) was tested in a various number of constellations. The best combination seemed to consist of a fan designed for 1000 r/min, an air inlet to the motor with a diameter of 174 mm (a reduction of the original inlet area by 30%) and a distance of 6,5 mm between heat-sink and air inlet of the induction machine. This reduces the airflow measured for the standard induction motor (with the 1000 r/min fan) from approximately 157 l/s (0,157 m$^3$/s) to approximately 101 l/s. An airflow of 101 l/s (pre-heated 5° C) will probably still be sufficient for the PM motor since the PM motor itself will have lower losses than the standard induction motor. The active heat-sink fin area with this combination is estimated to 0,30 m$^2$. The power electronics can operate safely up to a heat-sink temperature of approximately 70 °C [57]. Assuming an ambient temperature of 40 °C the allowable temperature rise of the heat-sink is 30 °C. The measured temperature rise of the heat-sink (on the power electronics side) was approximately 31 °C, which is acceptable.

The second dummy heat-sink (see Fig. 6.24) showed much higher temperature rises than dummy heat-sink #1. These rises were probably due to smaller heat-sink area and lower level of airflow. The active heat-sink fin area was estimated to 0,25 m$^2$ and the airflow to 64 l/s. Since not much could be altered to improve dummy heat-sink #2, it was rejected.

**Conclusions on dummy heat-sinks**

A heat-sink similar to dummy heat-sink #1 and an air inlet hole with a diameter of about 174 mm should be sufficient for the PM integral motor. With a small distance (6,5 mm) between the air inlet and the heat-sink the airflow could be increased (for the PM-motor) without reducing the cooling capability of the heat-sink significantly.
6.5 Prototype design changes and prototype manufacturing

6.5.1 Changes in the PM motor design

During the manufacturing process it was realized that some things had to be changed in the PM motor design. The changes, and the effects of the changes are described and examined in this section.

Iron quality
The iron quality used in the optimization was DK70 (0.50 mm) from Surahammars Bruks AB, see [52]. The FEM calculations were done with a magnetization curve for DK66. These iron qualities were not used in the manufactured prototype. Instead, an iron quality with losses similar to the losses of iron quality CK27 was used. CK27 has lower iron losses than DK70. It contains a higher level of silicon than DK70, which increases the resistivity and thereby reduces the eddy current losses. A higher level of silicon lowers the saturation level of the material. FEM calculations were done to examine the influence of the change of iron quality on the airgap flux densities. A magnetization curve for CK27 was used, assuming that an iron quality with losses similar to CK27 would have approximately the same magnetic properties. The surprising results from the FEM calculations is that the airgap flux densities are almost unchanged. This is probably due to two reasons:

- The iron bridges in the rotor are more easily saturated.
- The flux density levels of the rest of the motor are still below the “saturation knee” in the magnetisation curve.

These two reasons counter-act each other. The number of ampere-turns that are consumed, due to a lower magnetisation curve of the iron material in the rotor and stator magnetic circuits, are compensated for by the more easily saturated iron bridges. The PM motor design with buried magnet seems to be “self-adjusting”, and therefore not so sensitive to the iron quality, at least not in this particular case.

Conclusion
Changing iron quality from DK70 to something with losses similar to CK27 will not effect the airgap flux densities.
Remanent flux density

The computer program that was used for the PM motor optimization did not take axial leakage flux into account. During the time of manufacturing the axial leakage flux was estimated with 2D-FEM calculations. These calculations showed that the radial (torque producing) flux reduces by about 4% [82]! The axial leakage flux has therefore been further examined in Chapter 3. The high reduction of 4% is probably due to two reasons:

- The relatively large air gap of 2.9 mm
- The relatively short axial rotor length of 110 mm

To compensate for the axial leakage flux, the quality of the NdFeB magnets was changed from Vacodym 400HR to Vacodym 396HR. 396HR has a remanent flux density of 1.22 T, which is about 5% more than 400HR [62]. The maximum allowable continuous temperature of 396HR is 160 °C, i.e. 20 °C lower than Vacodym 400HR. Fig. 6.25 shows a Fourier analysis of the airgap flux density with 396HR magnets at 20 °C. The analysis is made at 20 °C for later comparisons with measurements (subsection 7.1.4).

![Fourier analysis of the airgap flux density due to the magnets Vacodym 396HR at 20 °C. The columns indicate (left to right): the space harmonic order, peak value of flux density in T, relative magnitude with respect to the fundamental component, and the phase angle in electrical degrees.](image)
Copper temperature
Some changes that will effect the copper temperature were made. They are described here, and their effects are investigated.

Copper fill factor
The copper fill factor was set to \( k_{f,Cu} = 0.60 \) during the optimization procedure, see sub-section 6.2.1. This turned out to be a very optimistic value. The value was defined as the ratio of pure copper area to total area of a stator slot with sharp corners. In reality the corners of the slots have a certain radius, and the winding is not occupying the slot all the way out to the airgap. There must for example be space for slot insulation and slot wedges. The result was that when the coil was inserted in the slot with a “normal” level of filling (that would admit automatic machine winding) the copper fill factor was reduced. The designed copper area of a slot was set to \( A_{Cu,slot} = 128 \text{ mm}^2 \), see Table 6.1. The manufactured winding consists of the following conductors:

- 2 parallel wires with a diameter of 0.95 mm
- 4 parallel wires with a diameter of 0.90 mm
- 4 parallel wires with a diameter of 0.85 mm

For a winding with 15 turns per slot, the copper area per slot is \( A'_{Cu,slot} = 93.5 \text{ mm}^2 \). The “manufactured” copper fill factor is then given as

\[
k'_{f,Cu} = k_{f,Cu} \frac{A'_{Cu,slot}}{A_{Cu,slot}} = 0.44
\]

This results in an increased current density in the copper of the stator winding. The current density increases from 3.5 A/mm\(^2\) to 4.8 A/mm\(^2\), which will lead to higher copper losses.

The designed winding had, after compensation for the reduced copper fill factor, a weight of

\[
m_{Cu,comp} = m_{Cu} \frac{A'_{Cu,slot}}{A_{Cu,slot}} = 9.42 \text{ kg}
\]

where \( m_{Cu} \) is the copper mass from Table 6.1. One coil of the manufactured three-phase stator winding had a weight of approximately 0.85 kg.
Prototype PM integral motor design

which results in a total copper weight of about \( m'_{Cu} = 10.2 \text{ kg} \). The increased copper weight of the manufactured winding can be due to slightly longer end windings than expected.

**Iron losses**

Since the iron quality has been changed from DK70 to an iron quality with losses similar to the losses of iron quality CK27, the iron losses will be lower. The loss reduction is due to the higher resistivity of CK27, which leads to reduced eddy-current losses. The hysteresis losses can be assumed to be approximately unchanged. The eddy-current loss is proportional to the induced voltage squared over the resistance. The eddy-current loss reduction can be estimated to

\[
k_{eddy} = \frac{U_{induced}^2}{R_{DK70}} = \frac{\rho_{DK70}}{\rho_{CK27}} = \frac{25}{54} = 0.46 \quad (6.47)
\]

where \( \rho_{DK70}=25 \ \mu\Omega\text{cm} \) and \( \rho_{CK27}=54 \ \mu\Omega\text{cm} \) are the resistivities of the two iron qualities, respectively [52].

Also the density of the iron material is reduced from 7750 kg/m\(^3\) (DK70) to 7610 kg/m\(^3\) (CK27) [52].

**Stator housing area**

The stator housing area of the manufactured prototype motor has not been reduced to 2/3, as was assumed during the optimizations (see sub-section 6.2.1). Instead the area of the stator housing is approximately unchanged. This implies that the temperature increase factor which was given by Equation (6.1) can be reduced from 1.5 to 1.

**Cooling airflow and temperature rise**

The measured airflow of the manufactured prototype motor is approximately 80 l/s, see sub-section 7.1.2. This is lower than the value of 101 l/s that was used during the optimizations, see sub-section 6.2.1. This implies that the temperature factor, which was given by Equation (6.2), should be increased from a value of 1.3 to 1.5. This increase is found by using Equation (6.2):
Also the temperature rise of the airflow through the converter heat-sink has increased from $\Delta T_{\text{conv}} = \Delta T_9 = 5 \, ^\circ\text{C}$ (see sub-section 6.4) to $\Delta T_{\text{conv}} = \Delta T_8 = 9 \, ^\circ\text{C}$ (see sub-section 7.1.2).

Re-run of the fine-tuning optimization program

The fine-tuning optimization program was re-run with the old parameters but with the new copper fill factor, the eddy-current reduction factor, the two temperature increase factors and the increased temperature of the cooling air described above. The most important changes are listed here:

- The copper temperature increased from 122 $^\circ\text{C}$ to 124 $^\circ\text{C}$.
- The copper losses increased from 425 W to 587 W.
- The iron losses decreased from 217 W to 167 W.
- The total losses increased from 781 W to 892 W.
- The efficiency at 1500 r/min decreased from 92.32% to 91.68%.

The copper temperature is almost unchanged. It is obvious that the operating temperature will be very dependent on the thermal resistances inside the motor, and from the motor to the surrounding air.

Radial air-filled slot

The triangular-shaped rotor bar and the radial air-filled triangular-shaped slot in each rotor pole were replaced with two rectangular slots. The outer slot contains an aluminium bar with dimension 20 mm by 5 mm, which is a part of the rotor cage. The inner slot contains a stainless steel bar instead of air, for increased mechanical strength. These changes do not effect the airgap flux density from the magnets, nor the airgap flux from the stator currents.

Increased number of turns in corner coils

During the winding procedure of the corner coils, it was realized that there was space for one extra turn in each corner coil. Since the iron flux density level was set as low as 1.14 T, it was decided that the number of turns should be increased from 6 to 7. This increase has the following impact on the analytically calculated corner coil inductance:

\[
R_{\text{th,integral}} = \left( \frac{157}{80} \right)^{0.6} R_{\text{th,standard}} = 1.5 R_{\text{th,standard}}
\]  

(6.48)
The flux density in the iron of a corner coil increases to

\[ B'_{iron,cc} = B_{iron,cc} \cdot \frac{N'_{cc}}{N_{cc}} = 1.14 \cdot \frac{7}{6} = 1.33 \, \text{T} \quad (6.50) \]

which is still quite low.

Resistance and copper loss of the corner coils increase by a factor of \(7/6\), as well. Since the total copper loss of the four corner coils was as low as 20 W (see sub-section 6.3.9), the increase to 23 W is negligible.

The sum of the iron losses of the three line filter coils were in the range of 9 W, see sub-section 6.3.9. With the increased flux density of the corner coils \(B'_{iron,cc} = 1.33 \, \text{T}\), the iron losses increase to about 12 W, calculated by the use of Equation (6.43). The corner coil iron losses are still negligible.

### 6.5.2 Manufacturing of the prototype motor

The prototype motor was manufactured at ABB Corporate Research while the winding of the stator was done by Sjödins AB. The iron laminations were laser-cut by LASAB. The converter and control circuits were built by Inmotion Technologies (former Atlas Copco Controls). Below some pictures from the manufacturing process are shown:

**Rotor**

The rotor lamination was laser-cut, and stacked on the shaft, see Fig. 6.26. The magnets were inserted and glued in the magnet slots of the rotor. Each pole required 4 magnet pieces with dimensions 4.8 mm by 55 mm by 40 mm. The rotor bars and end plates are made of aluminium. The first idea was to die-cast the rotor bars and the end plates, but there was no suitable mould available. The rotor bars were then supposed to be welded (by Mig-Mag or Tig-Tag technique etc.) to the end plates. This was not possible, due to the strong magnetic field from the rotor magnets! A string could be made, but the quality was not acceptable. The final solution was to make a connection with screws, visible in Fig. 6.26 (right).
Fig. 6.26 Stacking of the rotor lamination (left), and the complete PM rotor (right).

**Stator and housing**

The stator was stacked, welded and wound, see Fig. 6.27. The corner coil windings were also inserted, and can partly be seen in Fig. 6.29. The stator core was then placed in a specially designed housing, see Fig. 6.28. The housing consists of the lower half of a standard induction motor housing. The upper half of the standard housing was removed and replaced by two vertical side-walls and a top lid. The lid is tightened to the walls with screws. The white colour on the stator core in Fig. 6.28 is silicon grease, which is used to improve the heat transfer from the stator core to the stator housing.

Fig. 6.27 Stator stacking with the help of a manufactured jig (left). Windings of the stator (right).
Fig. 6.28  Open view of the PM integral motor. The converter heat-sink is removed and the air inlet hole to the fan is visible.

Fig. 6.29  Close-up of the rear end winding and the shaft-mounted fan. The corner coil connections are also visible.
The complete PM integral motor
Fig. 6.30 shows a side view of the complete PM integral motor. In Fig. 6.31 the prototype integral motor is, for volume comparison, placed close to a standard induction motor with the same power and speed ratings.

Fig. 6.30 The compact integral motor prototype, painted in cobalt-blue.

Fig. 6.31 A standard 15 kW induction motor (left) and the compact 15 kW PM integral motor (right), seen from above.

6.6 Conclusions

This chapter has presented the compact PM integral motor project. Near-optimum PM motor design parameters were given and verified with FEM calculations. The next chapter will present measurements on the machine.
7 Measurements

In this chapter measurements on the manufactured PM motor, and on the complete PM integral motor and its converter are presented.

7.1 Measurements on the prototype PM motor

Some different measurements have been performed on the manufactured prototype PM motor, and they are presented in this section. The measurements are performed to verify the calculations, show the possible operating range of the PM motor and of the heat-sink, and give values required for the sensorless control of the motor.

7.1.1 Airflows and temperatures of real heat-sink #1

The first real heat-sink was manufactured by ABB Corporate Research and was based on the same idea as dummy heat-sink #1, with the power electronics placed vertically, see Section 6.4. The difference is that this real heat-sink has radially placed cooling fins inside a circular domain, see Fig. 7.1. Radially placed fins should improve the airflow, but it also causes an area reduction of the active fins by about 24\%, i.e. a reduction to 0.23 m\(^2\). An extra, horizontal, small heat-sink has also been added beneath the terminal box to provide air to the upper fins of the big heat-sink, see Fig. 7.1. This small extra heat-sink is the roof of the converter box.
All the time during the airflow and temperature measurements the fan was run at 1460 r/min (rotating counter-clockwise, seen from the non-drive end). During the temperature measurements the three resistors were again attached to the rear of the heat-sink (see Section 6.4), simulating power electronic losses of about 600 W.

**Airflow measurements**

The airflow was again measured with the *motor-in-a-box* principle described in [82]. The airflow of the manufactured prototype motor was approximately 77 l/s. (Air speed $v_i = 2.4$ m/s, radius $r_{io} = 75$ mm and radius $r_o = 125$ mm.) The “1000 r/min fan” was used.

To increase the airflow different measures were tried:

- “Shaping” the air outlets on the left, right and upper side of the motor (to give a smoother path for the airflow). This, in fact, slightly reduced the airflow to approximately 74 l/s.
- Removing the cover of the upper air outlet of the motor could only increase the airflow to approximately 80 l/s.
- Instead, removing the heat-sink gave an airflow of approx. 82 l/s.
- If both the upper air outlet cover of the motor and the heat-sink were removed, the airflow increased to approximately 95 l/s.

This implies that the largest airflow limitations are neither the air outlets, nor the heat-sink. Probably the air inlet hole (diameter 140 mm) to the fan is the limiting factor, see Fig. 7.1. Another possibility is that the flow of the fan is too small since the blades had been shortened to fit into the stator.

To examine the influence of the air inlet hole area, the diameter of the hole was reduced from 140 mm to 90 mm, since it was easier to decrease than to increase the diameter of the existing hole. This is equivalent to a reduction of the area of the air inlet hole by 59%. The airflow reduced from 95 l/s to 64 l/s, a reduction by 33%.

There is a small (axial) distance of about 9 mm between the fan and the air inlet plate (i.e., the plate with the air inlet hole). Reducing this distance to about 2 mm had no noticeable effect on the airflow.

**Temperature measurements of real heat-sink #1**

Ten thermo-couples were used to register the temperatures of the heat-sink, PM-motor, ambient, influent and effluent air etc. The placement of
Measurements

Fig. 7.2 Placement of the 10 thermo-couples on the real heat-sink #1.

the thermo-couples can be found in Fig. 7.2. For temperatures referring to the motor, the motor is seen from the non-drive end. The resistors (supplying the losses) were mounted on the heat-sink inside the sealed box for the power electronics, but they were not embedded in thermal insulation.

Table 7.1 Temperatures and airflow of real heat-sink #1 with an air inlet diameter of 140 mm (@9 mm distance), and 160 mm (@ zero distance). Ambient temperature was 26 °C, and 24-25 °C.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>140 mm, n=1460 r/min</td>
<td>r_{in}=75 mm</td>
<td>v_f=2.4 m/s</td>
<td>q_{air}=77 l/s</td>
<td></td>
<td>80</td>
<td>618</td>
<td>56</td>
<td>66</td>
<td>44</td>
</tr>
<tr>
<td>ΔT=0-80 min</td>
<td>ΔT=T_{x},T_{10}</td>
<td>30</td>
<td>40</td>
<td>18</td>
<td>33</td>
<td>49</td>
<td>8</td>
<td>1</td>
<td>7</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>160 mm, n=1460 r/min</td>
<td>80</td>
<td>618</td>
<td>52</td>
<td>62</td>
<td>42</td>
<td>54</td>
<td>74</td>
<td>33</td>
<td>26</td>
<td>32</td>
<td>35</td>
</tr>
<tr>
<td>ΔT=0-80 min</td>
<td>ΔT=T_{x},T_{10}</td>
<td>27</td>
<td>37</td>
<td>17</td>
<td>29</td>
<td>49</td>
<td>8</td>
<td>1</td>
<td>7</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

With real heat-sink #1 the temperature rise of the heat-sink at thermal steady-state (i.e., after about 80 min) was between ΔT_1=30 °C (at top) to ΔT_2=40 °C (at bottom), see Table 7.1. The highest temperature rise was ΔT_5=49 °C, inside the sealed power electronics box.
Temperatures with turbulence-boosters
Small flat plastic bars were attached to the sharpened outside edges of the radial fins of the heat-sink. The plastic bars were meant to increase the airflow turbulence, since a turbulent airflow has better cooling capability, compared to a laminar airflow [37]. This proved to be true, and the temperature rises were reduced by 1 to 3 °C.

Temperatures with different air inlet diameters and space distances
The diameter of the air inlet hole was increased from 140 mm to 160 mm. This is equivalent to an air inlet hole area increase of 31%. This change hardly affected the temperatures, but a significant increase of airflow could be sensed.

There is an axial distance of about 9 mm between the air inlet plate of the fan and the heat-sink. By filling this empty space, more of the airflow is forced to travel through the heat-sink. On the other hand, the total airflow is reduced. By varying the air inlet diameter the amount of airflow is changed. Without any empty space (i.e., zero distance) between air inlet plate and heat-sink, air inlet diameters 120, 140 and 160 mm were used. The largest diameter gave the best results (i.e., the lowest temperature rises) and they can be found in Table 7.1. The temperature rises of the heat-sink were about 3 °C lower than without the filling.

Ambient temperature: 36 °C

<table>
<thead>
<tr>
<th>Temperature</th>
<th>64</th>
<th>62</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>81 81 81 78 78</td>
<td>77 73 75 75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

▲ +1500 r/min

▲ -1500 r/min

Fig. 7.3 Steady-state temperatures of real heat-sink #1 at 621 W, +/-1500 r/min. Ambient temperature was 36 °C.

Here also the rear lid of the power electronics box was removed, and the steady-state temperatures in a few more spots were registered with a hand-held temperature probe (see Fig. 7.3). The ambient temperature was 36 °C. The speed of the fan was 1500 r/min during these measurements,
Measurements

and both directions of rotation were tried. The power loss in the resistors was 621 W. It is worth noting that the upper part of the heat-sink seems to have better cooling than the lower part. At the left side, close to the resistor, a temperature rise of 45 °C was measured, which is more than what is allowed for the power electronics. From Fig. 7.3 it can also be seen that there is a temperature rise of about 16 °C from heat-sink to the “backs” of the resistors. These high temperatures of the resistor “backs” can be the explanation to why the air temperature $T_s = 75$ °C ($\Delta T_s = 49$ °C) inside the power electronics box is so high. Embedding the resistors in thermal insulation lowered the temperature rise inside the power electronics box to $\Delta T_s = 21$ °C, while the other temperatures were almost unchanged.

Choking the small heat-sink with a piece of tape (while the air inlet diameter was 120 mm) caused an extra temperature rise of the small heat-sink of approx. 6 °C, but hardly had any effect on the other temperatures.

Comments and suggestion

The temperature rises of real heat-sink #1 are probably slightly too high, i.e. more than 30 °C. It seems like the performed changes above can only effect the temperatures with a couple of degrees Centigrade. Since a significant increase of the airflow could be sensed when the air inlet diameter was increased to 160 mm, this is probably an easy and preferable thing to do. Increased airflow results in a better cooling of the PM-motor.

According to [37] the rate at which energy transfer (i.e., the power loss) by convection takes place can be stated as:

$$P = \alpha_{\text{heat}} A_{\text{surface}} (T_{\text{surface}} - T_{\text{fluid}})$$  \hspace{1cm} (7.1)

where $\alpha_{\text{heat}}$ is the heat transfer coefficient, $A_{\text{surface}}$ is the area of the heated surface, $T_{\text{surface}}$ is the temperature of the heated surface and $T_{\text{fluid}}$ is the temperature of the cooling fluid. A typical value for the convection heat transfer coefficient for gases at forced convection (i.e., with a fan) might be in the range of 25 to 250 W/m$^2$K, [37]. The following approximate expression for the heat transfer coefficient is given by [70]:

$$\alpha_{\text{heat}} = 20 v_{\text{fluid}}^{0.6}$$  \hspace{1cm} (7.2)

where $v_{\text{fluid}}$ is the velocity of the cooling medium. Looking at the com-
bination of Equations (7.1) and (7.2), it can be seen that the heat-sink area is probably more important than the velocity of the cooling medium (here: the airflow). The suggestion is therefore to manufacture a new (real) heat-sink with more cooling area.

7.1.2 Airflows and temperatures of real heat-sink #2

Fig. 7.4 Real heat-sink #2.

To improve the cooling capability, a new heat-sink was manufactured by ABB Corporate Research, see Fig. 7.4. Real heat-sink #2 is based on the same idea as real heat-sink #1 (see Fig. 7.1), but the axial length of the fins has been increased from 39 mm to 59 mm. Extra fins have also been added outside “the circular fin domain”, compare Fig. 7.1 and Fig. 7.4. Also the air inlet plate has been mounted on the fins of the heat-sink, to increase the cooling area. The active fin cooling area of real heat-sink #2 can be estimated to be at least 50% larger than for real heat-sink #1, i.e. at least 0.34 m². The diameter of the hole in the air inlet plate is 160 mm, and the fins have sharp corners to improve the turbulence of the airflow.

Airflow measurements

The airflow was again measured with the motor-in-a-box principle described in [82]. The airflow of the manufactured prototype motor, equipped with the real heat-sink #2, was approximately 80 l/s at 1500 r/min. (Air speed \( v_l = 2.5 \) m/s, radius \( r_{io} = 75 \) mm and radius \( r_o = 125 \) mm.) The “1000 r/min fan” was used.

Temperature measurements of real heat-sink #2

The temperature measurements were done at 5 different speeds. The results are presented in Table 7.2. The placement of the thermo-couples is
shown in Fig. 7.2, but with some changes: $T_5$ is now the lid of the converter box, $T_6$ is the ambient temperature, and $T_{10}$ was out of order. For temperatures referring to the motor, the motor is seen from the non-drive end. The temperature of the top heat-sink of the PM machine ($T_9$) is high because the PM machine was loaded with the rated current of 30 A when the speed was 1500 r/min. In Table 7.2 it can be seen that decreasing the speed from 1500 r/min to 500 r/min increases the temperatures with about 20 °C. The temperatures at some extra spots on the big heat-sink were also registered at 1500 r/min. The measurements were done with a handheld temperature probe, and the results are presented in Fig. 7.5.

**Table 7.2** Temperatures and airflows of real heat-sink #2. Ambient temperature was 34–35 °C.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>n=1500 r/min</td>
<td>$t_w=75$ mm</td>
<td>$\nu=2,5$ m/s</td>
<td>$q_{air}=80$ l/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>621</td>
<td>20</td>
<td>20</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>420</td>
<td>56</td>
<td>63</td>
<td>47</td>
<td>58</td>
<td>48</td>
<td>34</td>
<td>35</td>
<td>43</td>
<td>61</td>
<td></td>
</tr>
<tr>
<td>n=1250 r/min</td>
<td>$t_w=80$ mm</td>
<td>$\nu=2,1$ m/s</td>
<td>$q_{air}=70$ l/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>530</td>
<td>621</td>
<td>60</td>
<td>66</td>
<td>50</td>
<td>62</td>
<td>51</td>
<td>35</td>
<td>36</td>
<td>44</td>
<td>61</td>
</tr>
<tr>
<td>n=1000 r/min</td>
<td>$t_w=90$ mm</td>
<td>$\nu=1,6$ m/s</td>
<td>$q_{air}=59$ l/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>610</td>
<td>621</td>
<td>63</td>
<td>70</td>
<td>53</td>
<td>66</td>
<td>53</td>
<td>35</td>
<td>36</td>
<td>46</td>
<td>60</td>
</tr>
<tr>
<td>n=750 r/min</td>
<td>$t_w=95$ mm</td>
<td>$\nu=1,2$ m/s</td>
<td>$q_{air}=46$ l/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>650</td>
<td>619</td>
<td>68</td>
<td>76</td>
<td>57</td>
<td>71</td>
<td>57</td>
<td>35</td>
<td>36</td>
<td>48</td>
<td>59</td>
</tr>
<tr>
<td>n=500 r/min</td>
<td>$t_w=95$ mm</td>
<td>$\nu=0,75$ m/s</td>
<td>$q_{air}=29$ l/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>730</td>
<td>619</td>
<td>76</td>
<td>85</td>
<td>63</td>
<td>79</td>
<td>62</td>
<td>35</td>
<td>36</td>
<td>52</td>
<td>58</td>
</tr>
<tr>
<td>$\Delta t=420$ min</td>
<td>$\Delta T=T_{1500}$, $T_{1500}=...$ at 1500 r/min</td>
<td>22</td>
<td>29</td>
<td>13</td>
<td>24</td>
<td>14</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>27</td>
</tr>
</tbody>
</table>

**Fig. 7.5** Steady-state temperatures of real heat-sink #2 at 621 W of losses, 1500 r/min. Ambient temperature was 34 °C.
Conclusion
Real heat-sink #2 has decreased the highest temperature rises by about 10 °C, compared to real heat-sink #1. The temperature rises are now quite moderate, and the real heat-sink #2 should be sufficient for cooling the power electronics at 1500 r/min.

7.1.3 Torque measurements

Cogging torque
The peak cogging torque was measured with a rod and a weight. The rod had a length of 2.29 m, and the weight had a mass of 99.8 g. The midpoint of the horizontal rod was attached to the motor shaft, see Fig. 7.6.

![Fig. 7.6 Measurement set-up for the cogging torque.](image)

By carefully moving the weight outwards on the rod, and noting the length where the rod “tilts over”, the peak cogging torque was found. To compensate for any imbalances etc., the measurement was performed on both the left and the right part of the rod and the mean value of the length was used. Table 7.3 presents the results. Here it should also be mentioned that depending on the direction of the rotation of the rotor, before the measurement, two different values of the peak cogging torque could be found. If the rotor, before the measurement, is rotated in the same direction as it will rotate due to the weight, a higher torque value is achieved, and vice versa. This phenomenon is probably a result of hysteresis effects, i.e. slight remanent flux, in the teeth. Since the rotor, during normal operation, does not change direction of rotation very often the higher torque value is more correct to use.

<table>
<thead>
<tr>
<th>Direction of rotation</th>
<th>$l_{left}$ [cm]</th>
<th>$l_{right}$ [cm]</th>
<th>$l_{mean}$ [cm]</th>
<th>$T_{cogging}$ [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>“reverse and forward”</td>
<td>92</td>
<td>76</td>
<td>84</td>
<td>(0.82)</td>
</tr>
<tr>
<td>“only forward”</td>
<td>109</td>
<td>99</td>
<td>104</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 7.3 Measured rod lengths and peak cogging torque at 20 °C.
The measured value of the peak cogging torque of 1.0 Nm (at 20 °C) shows good agreement with the FEM calculated value of 1 Nm (at calculated magnet temperature of 112 °C, see Fig. 6.14). The two different magnet temperatures used effect the torque value slightly, but the agreement is still satisfactory.

**Torque versus load angle and stator current**

The torque versus load angle and stator current was measured by applying DC-current through two of the phase windings of the machine. A bar with a length of 3 meters was attached to the rotor shaft, see Fig. 7.7.

![Fig. 7.7 Measurement set-up for measuring torque versus load angle and current.](image)

The bar was balanced with a counter-weight of 5 kg. It was equipped with 5 bolts placed at an equi-distant length of 0.5 m from the rotor shaft. Also a phasor was attached to the rotor shaft to show the actual load angle $\beta$ in mechanical degrees.

![Fig. 7.8 Measured torque (solid) and calculated torque (dashed) versus load angle and current at 20 °C.](image)
A DC-current corresponding to the momentarily value of the AC-current was applied in positive direction through phase R and in negative direction of phase S. This moment in time ($\omega t = 11\pi/6$) was chosen because it results in a better power loss distribution between the three phase windings than e.g. the instant $\omega t = 0$. By measuring the length of the bar and the force required to achieve a desired load angle $\beta$ for a desired value of current, the torques could be calculated. Measured torque versus load angle for different values of stator current is shown in Fig. 7.8. Fig. 7.8 does also contain the calculated torque at 20 $^\circ$C, according to Equations (6.34), (6.24), (6.25) and (6.28). The induced phase voltage in Equation (6.34) at 20 $^\circ$C is set to 194 V, given by Equation (6.12).

### 7.1.4 Induced stator voltages

The induced no-load voltage of the PM machine was measured at 20 $^\circ$C. The PM machine was driven at 1500 r/min by a DC motor. The following paragraphs present the results.

**Induced phase and line-to-line voltage**

The phase voltage wave-form of phase R to the isolated neutral point N of the machine was measured with an oscilloscope, and is plotted in Fig. 7.9. Table 7.4 shows measured and calculated fundamental and harmonic content of the wave-form. The harmonic content was measured with a frequency analyser. The calculated voltages are found by using Equation (6.7), and the flux density values from Fig. 6.25 in combination with an axial reduction factor calculated as $\hat{B}_{(1)g, axi}/\hat{B}_{(1)g} = 0.96$ using the values of Motor A in Table 4.2, sub-section 4.2.2. The agreement between calculated and measured voltage is satisfactory for the fundamental component and time harmonic number 3, 5 and 7. The measured higher harmonics are lower than calculated. The phase voltage contains a pronounced 3:rd harmonic. This 3:rd harmonic and its multiples will not be seen in the line-to-line voltage.

The line-to-line voltage wave-form of phase R to phase S is plotted in Fig. 7.9. Table 7.4 shows the measured and calculated fundamental and harmonic content of the wave-form. The calculated voltages are found in the same way as for the phase voltages. The agreement between calculated and measured voltage is satisfactory for the fundamental component and time harmonic number 5 and 7. The measured higher harmonics are lower than calculated.
The RMS-value of the induced fundamental line-to-line voltage, at a magnet temperature of 20 °C, versus speed was also measured. The relationship is, as expected, linear.

Table 7.4 Measured and calculated fundamental and harmonic content of the induced phase voltage and the induced line-to-line voltage at a magnet temperature of 20 °C, 1500 r/min, no-load.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>184</td>
<td>196</td>
<td>315</td>
<td>340</td>
</tr>
<tr>
<td>3</td>
<td>299</td>
<td>34.7</td>
<td>39</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>500</td>
<td>4.55</td>
<td>5.3</td>
<td>8.21</td>
<td>9.2</td>
</tr>
<tr>
<td>7</td>
<td>700</td>
<td>1.20</td>
<td>1.2</td>
<td>1.93</td>
<td>2.0</td>
</tr>
<tr>
<td>9</td>
<td>899</td>
<td>1.04</td>
<td>2.9</td>
<td>0.12</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>1099</td>
<td>4.82</td>
<td>12</td>
<td>8.96</td>
<td>21</td>
</tr>
<tr>
<td>13</td>
<td>1298</td>
<td>0.63</td>
<td>3.2</td>
<td>1.01</td>
<td>5.6</td>
</tr>
<tr>
<td>15</td>
<td>1498</td>
<td>0.51</td>
<td>-</td>
<td>0.10</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>1698</td>
<td>0.13</td>
<td>-</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>19</td>
<td>1898</td>
<td>0.05</td>
<td>-</td>
<td>0.073</td>
<td>-</td>
</tr>
<tr>
<td>21</td>
<td>2098</td>
<td>0.02</td>
<td>1.1</td>
<td>0.019</td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td>2297</td>
<td>0.15</td>
<td>4.9</td>
<td>0.33</td>
<td>8.4</td>
</tr>
<tr>
<td>25</td>
<td>2496</td>
<td>0.19</td>
<td>3.4</td>
<td>0.26</td>
<td>5.9</td>
</tr>
</tbody>
</table>
7.1.5 Bearing voltage

Bearing currents are currents passing from one bearing ring, through the rolling elements and to the other ring. Bearing currents can destroy the raceways and rolling elements of the bearing since the temperatures of the arcs can be very high (a phenomenon comparable to welding). The voltage over the bearing, causing the current, is due to the sharp voltage spikes from the inverter or due to unsymmetry in the electric and/or magnetic circuit of the motor. Since the designed PM machine - at least geometrically - has a highly unsymmetric stator magnetic circuit, bearing voltages can be expected. To examine the presence of bearing voltages a small ball-bearing ball was mounted against the rear shaft end with a piece of brass sheet, see Fig. 7.10. The purpose of the bearing ball is to act as a conductor, to enable measurements of the voltage between the inner and the outer rings of the shaft bearings. The bearing voltage is not a measure of the bearing current but should at least give some indication if a problem should be anticipated. N.B: The bearings are not insulated during the test, i.e. the bearings might be conducting bearing currents and the observed voltage is the remaining voltage drop across the bearings. An example of the measured voltage is plotted in Fig. 7.10. The bearing voltage is extremely weak, and both magnitude and harmonic content change when a load in the 100 kΩ range is removed from the measurement points. This indicates that the bearing voltage is due to capacitive coupling from the stator winding and not due to magnetic unsymmetry in the stator circuit. This assumption is further proven by the fact that no current could be measured when the inner and outer bearing rings were short-circuited with an ampere-meter.
The conclusion is that bearing currents due to unsymmetry in the magnetic or electric circuit will not be a problem. How inverter supply will effect the bearings is not examined here. Bearing currents due to inverter supply have a capacitive coupling from the stator winding to the rotor body. Since the airgap is relatively large (2.9 mm) these currents are not expected to be a major problem.

7.1.6 Measurement of the stator winding resistance

The stator winding DC resistance at 20 °C was measured by applying a DC current of 6 A between phase R and phase S of the Y-connected PM machine. 6 A is 20% of the rated current. By measuring the DC voltage, the winding resistance is found as

\[ R_{s, 20^\circ C} = \frac{1}{2} \cdot \frac{U_{DC}}{I_{DC}} = \frac{1}{2} \cdot \frac{2.01}{6} = 0.168 \, \Omega/winding \]  

which is slightly lower, but the agreement is still satisfactory. The higher measured resistance is probably due to slightly longer end windings than expected, see sub-section 6.5.1.

7.1.7 Measurements of d- and q-inductances

During the inductance measurements the induced voltage has been assumed to be constant (i.e. equal to the induced no-load voltage), though it slightly reduces due to iron saturation effects from the stator currents.

**d-inductance**

The d-inductance was measured when running the PM machine as a motor at no-load. Running under motor-conditions makes it possible to measure the d-inductance with both positive and negative d-currents, see Fig. 7.11.
The PM motor was supplied from a synchronous generator with variable output voltage and frequency. The PM motor was run “open-loop”, i.e. no control method was used. The output line-to-line voltage of the synchronous generator at 50 Hz, no-load is shown in Fig. 7.12. The PM motor was connected via a power-meter and 25 A fast-blow fuses.

The measurements were performed at a stator frequency of 50 Hz, which equals a shaft speed of 750 r/min (and is half of the rated speed of the PM machine), due to three reasons:

1. It was the maximum output frequency of the synchronous generator.
2. The iron-, fan- and friction-losses are significantly lower at lower speed.
3. The rotor was oscillating heavily for stator frequencies of 15 to 30 Hz.
(The rotor oscillation phenomenon, which had a peak around 22 Hz, may be an obstacle to perform this kind of d-inductance measurement on some machines. The oscillations have not been investigated further.)

Before the measurements began the supply voltage was adjusted to minimize the stator current to the PM motor. The current could not be tuned to zero (see row 7 in Table 7.5), due to the losses in the PM motor and the insufficient resolution of the output voltage from the synchronous generator. The d-inductance was measured with both positive and negative values of the d-current by increasing and decreasing the supply voltage.

Fig. 7.12 shows the waveform of the current of the PM motor at $I_s = |I_{d*}| = 30$ A. Due to the internal inductance of the synchronous generator, the supply voltage could not be reduced enough to reach the rated current in negative d-direction $I_s = |I_d| = |-30| A = 30$ A. Table 7.5 shows the measured quantities and the d-inductance for positive and negative d-currents. The calculated d-inductances, from Fig. 6.11, are also shown in Table 7.5. The agreement between calculated and measured d-inductances is good.

**Table 7.5 Measured quantities when running the PM machine as a motor at 750 r/min, no-load. For comparison, the calculated d-inductances are shown as well.**

<table>
<thead>
<tr>
<th>Desired d-current $I_{d*}$ [A]</th>
<th>Obtained stator current $I_s$ [A]</th>
<th>Supply voltage $U_{l-l}$ [V]</th>
<th>Active power $P$ [W]</th>
<th>Reactive power $Q$ [kVAR]</th>
<th>Measured d-reactance $X_{d50Hz}$ [$\Omega$]</th>
<th>Measured d-inductance $L_d$ [mH]</th>
<th>Calculated d-inductance $L_{d}$ [mH]</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>30</td>
<td>231</td>
<td>630</td>
<td>12.06</td>
<td>1.30</td>
<td>4.15</td>
<td>3.95</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>222</td>
<td>470</td>
<td>9.71</td>
<td>1.37</td>
<td>4.37</td>
<td>4.09</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>213</td>
<td>331</td>
<td>7.4</td>
<td>1.43</td>
<td>4.56</td>
<td>4.25</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>202</td>
<td>228</td>
<td>5.27</td>
<td>1.50</td>
<td>4.77</td>
<td>4.43</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>190</td>
<td>150</td>
<td>3.3</td>
<td>1.55</td>
<td>4.94</td>
<td>4.61</td>
</tr>
<tr>
<td>5</td>
<td>5.0</td>
<td>177</td>
<td>101</td>
<td>1.52</td>
<td>1.57</td>
<td>5.0</td>
<td>4.76</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>163</td>
<td>80</td>
<td>0.11</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>-5</td>
<td>5.0</td>
<td>149</td>
<td>86</td>
<td>1.3</td>
<td>1.67</td>
<td>5.33</td>
<td>4.92</td>
</tr>
<tr>
<td>-10</td>
<td>10</td>
<td>134</td>
<td>125</td>
<td>2.3</td>
<td>1.67</td>
<td>5.33</td>
<td>4.93</td>
</tr>
<tr>
<td>-15</td>
<td>15</td>
<td>120</td>
<td>190</td>
<td>3.1</td>
<td>1.66</td>
<td>5.27</td>
<td>4.94</td>
</tr>
<tr>
<td>-20</td>
<td>18.7</td>
<td>109</td>
<td>260</td>
<td>3.5</td>
<td>1.67</td>
<td>5.31</td>
<td>4.95</td>
</tr>
<tr>
<td>-25</td>
<td>23.3</td>
<td>95.8</td>
<td>370</td>
<td>3.8</td>
<td>1.67</td>
<td>5.30</td>
<td>4.95</td>
</tr>
<tr>
<td>-30</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>4.95</td>
</tr>
</tbody>
</table>
The measured d-inductance (see Table 7.5) can be approximated (using the software Curvefit) with a first order equation for positive d-currents, and the mean value for negative d-currents:

\[
L_{d+1st} = 5.25 - 0.0353I_d \quad [\text{mH}] \quad \text{for } 0 < I_d < 30 \text{ A}
\]

\[
L_d = 5.31 \quad [\text{mH}] \quad \text{for } -30 \text{ A} < I_d < 0
\]  

(7.5)

During the tests the current was assumed to be only reactive, and the load angle \(\delta\) equal to zero. The first approximation is acceptable since, in the worst case, the supply line-to-line voltage is only 95.8 V while the active power loss is 370 W at 23.3 A. This is equivalent to a Y-connected resistance of 0.227 \(\Omega\), resulting in a resistive phase voltage drop of 5.3 V. The resistive voltage drop is in quadrature with the inductive voltage, resulting in an inductive voltage of 95.4 V, which is just 0.5% smaller than the supply voltage. The second approximation can be justified by assuming that the total power loss has to be transferred to the rotor, i.e. over the airgap. The required load angle for the worst case (370 W, 95.8 V), neglecting the reluctance torque, is only \(\delta = 2.3\) degrees.

**q-inductance**

The q-inductance measurements were performed when running the PM machine as a generator at 750 r/min driven by a DC motor, feeding a purely resistive load. The terminal voltage was measured for a stator current of 0 to 30 A, see Table 7.6. A general phasor-diagram for this generator-condition can be found in Fig. 7.13. (The phasor-diagram is given with motor-references to make it easier to realize that the machine is working with negative d-current with this kind of load.) The cross-coupling of d-inductance to q-current and of q-inductance to d-current is neglected in the following calculations.
From the phasor diagram in Fig. 7.13 we can state:

\[ I_d = -I \sin((\phi - \delta) - 180^\circ) = I \sin(\phi - \delta) \]  
(7.6)

\[ I_q = -I \cos((\phi - \delta) - 180^\circ) = I \cos(\phi - \delta) \]  
(7.7)

Looking solely at the components in the q-axis direction gives

\[ U \cos \delta = E + X_d I_d + R_s I_q \]  
(7.8)

Inserting Equations (7.6)-(7.7) into Equation (7.8) yields

\[ U \cos \delta = E + X_d I \sin(\phi - \delta) + R_s I \cos(\phi - \delta) \]  
(7.9)

Performing the same exercise for the d-axis direction gives

\[ U \sin \delta = X_q I_q - R_s I_d = X_q I \cos(\phi - \delta) - R_s I \sin(\phi - \delta) \]  
(7.10)

Solving Equation (7.10) for the q-reactance gives

\[ X_q = \frac{U \sin \delta + R_s I_d}{I_q} \]  
(7.11)
Table 7.6  Measured quantities when running the PM machine as a
generator at 750 r/min, feeding a purely resistive load. For
comparison, the calculated q-inductances are given as well.

<table>
<thead>
<tr>
<th>Terminal voltage $U_{ll}$ [V]</th>
<th>Stator current $I_s$ [A]</th>
<th>Active power $P$ [W]</th>
<th>Load angle $\delta$ [deg]</th>
<th>d-current $I_d$ [A]</th>
<th>q-current $I_q$ [A]</th>
<th>Measured q-reactance $X_{50Hz}$ [$\Omega$]</th>
<th>Measured q-inductance $L_q$ [mH]</th>
<th>Calculated q-inductance $L_q$ [mH]</th>
</tr>
</thead>
<tbody>
<tr>
<td>162</td>
<td>0</td>
<td>0</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>159,6</td>
<td>4,99</td>
<td>1375</td>
<td>-5,43</td>
<td>-0,47</td>
<td>-4,97</td>
<td>1,77</td>
<td>5,64</td>
<td>6,32</td>
</tr>
<tr>
<td>155,8</td>
<td>9,71</td>
<td>2618</td>
<td>-10,35</td>
<td>-1,74</td>
<td>-9,55</td>
<td>1,72</td>
<td>5,48</td>
<td>6,23</td>
</tr>
<tr>
<td>150,7</td>
<td>14,58</td>
<td>3803</td>
<td>-15,59</td>
<td>-3,92</td>
<td>-14,0</td>
<td>1,71</td>
<td>5,45</td>
<td>6,10</td>
</tr>
<tr>
<td>143,6</td>
<td>19,78</td>
<td>4912</td>
<td>-21,51</td>
<td>-7,25</td>
<td>-18,4</td>
<td>1,72</td>
<td>5,47</td>
<td>5,94</td>
</tr>
<tr>
<td>134,8</td>
<td>24,89</td>
<td>5800</td>
<td>-27,91</td>
<td>-11,7</td>
<td>-22,0</td>
<td>1,75</td>
<td>5,56</td>
<td>5,79</td>
</tr>
<tr>
<td>141,6</td>
<td>30,04</td>
<td>7360</td>
<td>-33,97</td>
<td>-16,8</td>
<td>-24,9</td>
<td>1,94 ($\text{at 56,7 Hz}$)</td>
<td>5,47 ($\text{at 56,7 Hz}$)</td>
<td>5,67</td>
</tr>
</tbody>
</table>

With the use of voltage (i.e. phase-voltage, $U_{ll}/\sqrt{3}$) and current from Table 7.6, and the value of the induced voltage $E$ ($U_{ll}/\sqrt{3}$ at $I_s$=0), the load angle $\delta$ for each load could be found by trying which $\delta$ (between 0 and -90°) that best satisfies Equation (7.9). Since the PM machine is running as a generator, feeding a purely resistive load, the phase angle is $\phi = 180^\circ$ (instead of 0°, due to motor references). The value of the d-reactance at 50 Hz is set to $X_d = \omega L_d = 1,67 \, \Omega$ (according to Equation (7.5)) since the d-current is negative with this type of load.

When the load angle was known, the d-current, q-current, q-reactance (at 50 Hz) and q-inductance could easily be calculated with the help of Equations (7.6), (7.7) and (7.11). These results are also found in Table 7.6. In the last row of Table 7.6 the frequency was raised to 56,7 Hz, to achieve the desired value of stator current. This also affects the values of induced voltage, and d- and q-reactance, but not the q-inductance. For comparison the calculated inductances, using Equation (6.28), are also shown in Table 7.6. The agreement between calculated and measured inductances is satisfactory, though no current influence can be seen in the measured inductances.

In [56] an analytical expression for the q-inductance, which does not require measurements of the load angle $\delta$, is derived. A no-load and a load test, both in motor operation, give the d- and q-inductances, respectively. Since the PM machine in the present thesis had been run as a generator in the second measurement, this method was not used.
7.1.8 Stator winding temperature

The stator winding temperature was measured implicitly by measuring the DC resistance of the stator winding. The DC resistance was measured by applying a DC current of 6 A through phase R and phase S of the Y-connected PM machine and measuring the DC voltage. The DC current was applied when the PM machine was disconnected and had stopped rotating. The result will be an average temperature of the stator winding, and will therefore not predict hot spot temperatures. The measurements were performed by running the PM machine as a generator, driven by a DC motor at 1500 r/min.

Ambient temperature

To simulate real operating conditions the tests were done in a sealed room where the room temperature reached a final value of 34 °C.

Converter losses

The losses of the converter were simulated by applying a power totalling 620 W in three resistors, mounted on the converter heat sink (see also Section 6.4). This causes a pre-heating of the cooling airflow to the PM machine of about 9 °C.

Corner coil losses

To simulate the copper losses in the corner coils, the corner coils were series connected and fed with a DC current of 27 A. This DC current is an equivalent current, which represents the average copper losses of the four corner coils during normal operation. The equivalent DC current is calculated as

\[
I_{DC_{equiv}} = \frac{1}{4}(3I_{cc1-3} + I_{cc4}) = 27 \text{ A}
\]  

(7.12)

where the currents of the corner coils are given by Equations (6.40) and (6.41). This resulted in a measured power loss of about 38 W.

Temperature tests

The average temperature of the stator winding can be calculated as

\[
T_{Cu} = \frac{1}{\alpha_{Cu}} \cdot \left( \frac{R_x}{R_{s,start}} - 1 \right) + T_{Cu,start}
\]  

(7.13)
where $\alpha_{Cu} = 3.9 \cdot 10^{-3} \, \text{K}^{-1}$ is the resistance temperature coefficient of copper, $R_s$ is the measured resistance of the winding, $R_{s,\text{start}}$ is the resistance of the stator winding when the measurement starts, and $T_{Cu,\text{start}}$ is the temperature of the stator winding at the start of the measurement.

The temperature tests were done by loading the PM machine with a resistive three phase load. During the first tests reduced current were used, but since the stator winding temperature seemed to be moderate, the rated current of 30 A was used. In the final test the PM machine had been run as a generator at rated current for 7.5 hours, delivering 12 kW to the resistive load. When the PM machine was stopped the stator resistance was measured at intervals of 10 seconds, during 1 minute. This gives a decaying temperature curve. The temperature when the PM machine was disconnected can be estimated to be around 100 °C. Table 7.7 shows the stator resistance and the decaying winding temperature. $T_{\text{amb}}$ is the ambient temperature. Since the PM machine was running as a generator, instead of as a motor, the flux in the machine is reduced. Running under motor conditions will therefore cause some more iron losses. The iron losses will increase the stator winding temperature slightly. The winding has a class F insulation, i.e. maximum allowable temperature is 145 °C (not including a safety margin of 10 °C for hot spots). The conclusion is that even with some extra iron losses and an ambient temperature of 40 °C, the winding temperature will be far below the critical 145 °C.

Table 7.7  Stator winding temperature of the PM machine after 7.5 hours of generator operation at 1500 r/min and 30 A (RMS).

<table>
<thead>
<tr>
<th>$t$ [min:s]</th>
<th>$R_s$ [Ω]</th>
<th>$I_{\text{rms}}$ [A]</th>
<th>$P$ [kW]</th>
<th>$T$ [Nm]</th>
<th>$T_{\text{amb}}$ [°C]</th>
<th>$T_{Cu}$ [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.168</td>
<td>---</td>
<td>(14)</td>
<td>(94)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>450:00</td>
<td>---</td>
<td>29.8</td>
<td>12</td>
<td>82</td>
<td>34</td>
<td>---</td>
</tr>
<tr>
<td>450:10</td>
<td>0,218</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>97.8</td>
</tr>
<tr>
<td>450:20</td>
<td>0,217</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>95.9</td>
</tr>
<tr>
<td>450:30</td>
<td>0,217</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>95.2</td>
</tr>
<tr>
<td>450:40</td>
<td>0,216</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>94.6</td>
</tr>
<tr>
<td>450:50</td>
<td>0,216</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>94.0</td>
</tr>
<tr>
<td>451:00</td>
<td>0,215</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>93.4</td>
</tr>
</tbody>
</table>
7.1.9 Corner coils

DC resistance of the corner coils
The DC resistance of the four corner coils at 23 °C was measured by applying a smooth DC current of $I_{DC} = 24$ A through the four series connected coils. This gave rise to a total voltage of $U_{DC} = 1.07$ V. This voltage includes the conductors of 2 times 0.6 m to each coil, as well. The DC resistance of each corner coil is now found as

$$R_{cc, 23°C} = \frac{1}{4} \cdot \frac{U_{DC}}{I_{DC}} = 11.1 \text{ mΩ / corner coil} \quad (7.14)$$

This agrees quite well with the analytical value of 9.6 mΩ per corner coil (including conductors of 2 times 0.6 m), found by using Equation (6.42) with 7 corner coil turns and a temperature of 23 °C.

Induced voltage in the corner coils due to the rotor magnet flux
The induced voltage in the corner coils, when the PM machine is running at no-load with 1500 r/min (driven by a DC motor), was measured with an oscilloscope. The voltage curve-forms of the four corner coils are shown in Fig. 7.14. The temperature of the magnets was 20 °C.

![Fig. 7.14 Induced voltage in the four corner coils. The PM machine was driven by a DC motor at 1500 r/min, no-load.](image)

The induced fundamental voltage in corner coil 1 is only 40 mV (RMS), and in corner coil 2 it is only 24 mV (RMS). This is considered to be neg-
ligible compared to the main’s phase voltage of 230 V (RMS). The calculated induced fundamental voltage, according to Equation (6.44) but with 7 corner coil turns, is in the range of 17 mV (RMS). Fig. 7.14 indicates that the magnetic coupling from the stator to the corner coils, though negligible, is quite complicated.

**Fundamental mutual inductance between corner coils**
The fundamental mutual inductances between the corner coils were measured by applying a sinusoidal current with a fundamental RMS value of $I_{(1)cc1} = 20.3$ A and a frequency of $f = 50$ Hz through corner coil number 1. This gives rise to a voltage in corner coil 1 with a fundamental RMS value of $U_{(1)cc1} = 1.95$ V, and a voltage in corner coil 2 with a fundamental RMS value of $U_{(1)cc2} = 4.8$ mV, see Fig. 7.15. The voltages induced in corner coils 3 and 4 are smaller than 0.2 mV, and therefore negligible. The current and voltages have been measured with a frequency analyser. The mutual inductance between corner coil 1 and 2 can be calculated as

$$L_{(1)cc12} = \frac{1}{2\pi f} \cdot \frac{U_{(1)cc2}}{I_{(1)cc1}} = 0.75 \, \mu H$$  \hspace{1cm} (7.15)

which is about 400 times smaller than the self inductance of a corner coil, see next sub-section. The conclusion is that the mutual inductance is negligible.

**Fig. 7.15 Voltage across corner coil 1 (left) and corner coil 2 (right) with a sinusoidal current through corner coil 1.**

**Fundamental self inductance of the corner coils**
The fundamental self inductance of the four corner coils were measured
in a similar way as the mutual inductance. By applying a sinusoidal current with a frequency of \( f = 50 \text{ Hz} \) through the four series connected coils, the voltage across each coil could be measured. The applied current had a peak value from 5 A up to 35 A. By using the peak value of the current, the saturation level in the iron is kept under control. The currents and voltages have been measured with a frequency analyser. The fundamental self inductance of each corner coil is now given as

\[
L_{(1)cc} = \frac{1}{2\pi f} \sqrt{\left(\frac{U_{(1)cc}}{I_{(1)cc}}\right)^2 - R_{cc}^2}
\]

(7.16)

where \( R_{cc} \) is the corner coil resistance according to Equation (7.14). The results are presented in Table 7.8. The measured inductances decrease slightly with increasing current but the linearity is still satisfactory.

The analytically designed inductance was 0.37 mH, see sub-section 6.5.1. With 7 turns instead of 6, and neglecting magnetic saturation, a FEM calculated inductance of \( 0.24 \times (7/6)^2 = 0.33 \text{ mH} \) is expected. The measured inductances show satisfactory agreement with the expected inductance from the FEM calculation. The inductance differences between the corner coils can be due to slightly different airgap lengths in the magnetic circuits of the corner coils. The designed airgap length was \( l_\delta = 0.22 \text{ mm} \). Measuring the airgap lengths of one iron lamination sheet gave airgaps of 0.20 mm < \( l_\delta < 0.25 \text{ mm} \) and 0.15 mm < \( l_\delta < 0.20 \text{ mm} \) for the two corners, respectively.

**Table 7.8** Measured fundamental inductances of the four corner coils versus the peak value of the sinusoidal corner coil current.

<table>
<thead>
<tr>
<th>Current ( \hat{I}_{(1)cc} ) [A]</th>
<th>Inductance ( L_{(1)cc1} ) [mH]</th>
<th>Inductance ( L_{(1)cc2} ) [mH]</th>
<th>Inductance ( L_{(1)cc3} ) [mH]</th>
<th>Inductance ( L_{(1)cc4} ) [mH]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>0.32</td>
<td>0.34</td>
<td>0.28</td>
<td>0.29</td>
</tr>
<tr>
<td>9.9</td>
<td>0.33</td>
<td>0.34</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>15.0</td>
<td>0.32</td>
<td>0.34</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>19.9</td>
<td>0.32</td>
<td>0.33</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>25.0</td>
<td>0.31</td>
<td>0.33</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>30.0</td>
<td>0.30</td>
<td>0.32</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>34.9</td>
<td>0.30</td>
<td>0.32</td>
<td>0.28</td>
<td>0.28</td>
</tr>
</tbody>
</table>
7.1.10 Instruments

The following instruments were used for the measurements in Section 7.1:

Airflow and heat-sink temperature measurements

- DC-voltage: Digital multi meter HP E2377A (No: 2932J06895)
- DC-current: Analogue instrument Norma (No: V960185)
- Shaft speed: Ono Sokki HT-430
- Air speed: TA 3000
- Temperature: Comark 5000 (10 inputs) and FLUKE 80T-150

Torque, stator voltage, bearing voltage, stator resistance, d- and q-inductances, stator winding temperature and corner coil measurements

- Force meter: 0-5 kg (KTH063, EaEm1020)
- Digital oscilloscope: LeCroy 9304A
- Frequency analyser: Hewlett Packard Dynamic signal analyzer 3562A (KTH212, EaEm708)
- Digital power meter (incl. voltage and current): Yokogawa 2533
- Opto-coupled probe: Nicolet Isobe 3000
- DC-voltage: Analogue instrument Siemens (KTH143, EaEm 030)
- DC-current: Analogue instrument Norma (V960141 & V960143)
- AC-current: Analogue instrument Norma (W880003) and current transformer AEG 50/5A, 20/5A (KTH062, EaEm 422)
- Shaft speed: Ono Sokki HT-430
Measurements

7.2 Measurements on the complete PM integral motor

Fig. 7.16 The converter mounted on the heat-sink of the integral motor.

This section describes the different measurements that has been performed on the manufactured compact PM integral motor.

The converter and control circuits were developed, built and tested by Inmotion Technologies (former Atlas Copco Controls). The converter was mounted on the heat-sink of the PM integral motor, see Fig. 7.16. After some minor fine tuning of the control, the PM integral motor was commissioned and performed according to specifications. The digital signal processor (DSP) and control algorithms had been tested earlier, on another drive system [48].

7.2.1 Temperature measurements

Inmotion Technologies has done some temperature measurements with different torques and speeds. The temperatures were measured at the converter heat-sink, the DC-link capacitor, a corner coil, two spots on the rear end winding as well as at an ambient point. Torque and current of the PM motor were also measured. Thermo-couples were used for the temperature measurements. All temperatures, except ambient and one of the winding spots, are read by the DSP. The DSP can therefore interrupt the running of the motor if the temperatures exceed certain pre-set limits. The DSP, which also is situated inside the converter box, is used for the control of the motor. One of the two winding spot temperatures is read from a hand-held thermometer (spot 1), and is about 15 °C higher than the other spot (spot 2). The measured quantities are presented in the following two paragraphs.
100 Nm and 1500 r/min (i.e. approximately rated torque and speed)
First the motor was loaded with 100 Nm at 1500 r/min which is 2 Nm more than rated torque. The steady-state winding temperature at spot 1 is 90 °C, with an ambient temperature of 29 °C, see Fig. 7.17 and Table 7.9. The earlier measured average temperature of the copper winding was around 100 °C, with an ambient temperature of 34 °C, see Section 7.1.8. The calculated operating average temperature of the winding was 124 °C, with an ambient temperature of 40 °C, see section 8.1.3. It can be noted that the calculated temperature is higher than the real measured temperature. The conclusion is that all measured temperatures with this load are quite moderate.

![Fig. 7.17 Temperatures, torque and PM motor current versus time, when the PM integral motor is loaded with 100 Nm at 1500 r/min.](image)

For a given output power a slight increase of the PM motor current can be observed in Fig. 7.17. This is because the remanent flux density of the magnet is reduced when the temperature of the magnet increases. To keep a constant torque, the control system increases the current.

**Steady-state temperatures for different speeds and torques**
A torque of 100 Nm at 750 r/min and 375 r/min was also tested. The measured steady-state temperatures are found in Table 7.9. All temperatures are still moderate for 750 r/min. All temperatures increased but were still acceptable for the windings and the corner coils at 375 r/min. The temperature rise of the heat-sink is here more than the allowed 30 °C. This is not a problem since the most temperature sensitive part of the power electronics is the DC-link capacitor [57], which is less hot.

Finally, the overload capability was tested with a torque of 120 Nm at 1500 r/min, see Table 7.9. This is equivalent to 22% more than the rated torque of 98 Nm. The heat-sink and capacitor temperatures are moderate. The winding and corner coil temperatures are now quite high, but there is no danger yet.
Table 7.9  Steady state temperatures for different speeds and torques.

<table>
<thead>
<tr>
<th>Load</th>
<th>Ambient °C</th>
<th>Capacitor °C</th>
<th>Heat-sink °C</th>
<th>Corner coil °C</th>
<th>Winding, spot 1 °C</th>
<th>Winding, spot 2 °C</th>
<th>Run-time [h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 Nm 1500 r/min</td>
<td>29</td>
<td>38</td>
<td>46</td>
<td>80</td>
<td>90</td>
<td>---</td>
<td>4.5</td>
</tr>
<tr>
<td>100 Nm 750 r/min</td>
<td>27</td>
<td>42</td>
<td>49</td>
<td>79</td>
<td>96</td>
<td>---</td>
<td>5</td>
</tr>
<tr>
<td>100 Nm 375 r/min</td>
<td>22</td>
<td>46</td>
<td>57</td>
<td>92</td>
<td>112</td>
<td>98</td>
<td>5</td>
</tr>
<tr>
<td>120 Nm 1500 r/min</td>
<td>31</td>
<td>42</td>
<td>55</td>
<td>107</td>
<td>126</td>
<td>110</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Conclusion on temperatures

The conclusion is that the prototype PM integral motor can be loaded continuously with the rated torque of 100 Nm at speeds from 1500 r/min down to 375 r/min, without violating the temperature limits. Also a continuous torque of 120 Nm at 1500 r/min is possible. These loads and speeds are possible with an ambient temperature up to 40 °C.

7.2.2  Line current with and without line-filter and DC-link inductance

The line current of the PM integral motor at 1500 r/min and 100 Nm, with and without the line-filter and the DC-link inductance (see Fig. 6.17), was measured with an oscilloscope. The fundamental frequency of the mains was 50 Hz, and the switching frequency of the inverter was 4 kHz. The converter of the PM integral motor is equipped with a 70 µF DC-link polypropylene capacitor. Due to the sampling frequency of the oscilloscope, which was 10 kHz, frequencies above 5 kHz cannot be detected.

Fig. 7.18 shows a plot of the line current and the corresponding frequency spectrum, without line-filter and DC-link inductance. The two side bands of the switching frequency are visible.

Fig. 7.19 shows a plot of the line current and the corresponding frequency spectrum, with line-filter and DC-link inductances. The two side-bands of the switching frequency, and some other high frequency components, are now eliminated.
Design and Evaluation of a Compact 15 kW PM Integral Motor

Fig. 7.18 Measured line current of the converter of the PM integral motor at 1500 r/min and 100 Nm, without line-filter and DC-link inductance (left). Corresponding frequency spectrum (right). The converter is equipped with a 70 μF DC-link capacitor.

Fig. 7.19 Measured line current of the converter of the PM integral motor at 1500 r/min and 100 Nm, with line-filter and DC-link inductance (left). Corresponding frequency spectrum (right). The converter is equipped with a 70 μF DC-link capacitor.

In [42] [43] investigations have been done on how to improve the waveforms of the converter input currents. This may be necessary to meet possible future legislations in this area. The best choice seems to be to use a so-called Vienna-rectifier with tolerance band control [42] [43]. It has a high efficiency and the corner coil inductance value is sufficiently high.
7.2.3 Efficiency measurements

The efficiencies of the compact PM integral motor, its converter and its PM motor have been measured. The measurements have been performed for five different speeds and four different torques, i.e. for twenty different operating points. Each measurement was made at thermal steady state. Thermal steady state was defined as when the temperature change was less than one degree Centigrade (or Kelvin) per hour. This implies that each measurement lasted for several hours.

The efficiencies are based on the input and output powers of the converter and of the PM motor. Two digital power meters were used, one\(^1\) for the input power of the converter and the other\(^2\) for both the output power of the converter and the input power of the PM motor, see Fig. 7.20. The mechanical output power, which is determined as torque times mechanical angular frequency, was measured using a torque-meter\(^3\) attached between the shafts of the PM integral motor and the load. The load was a DC-machine which was run as a generator feeding a resistive load. The torque-meter was also equipped with a function for determining the speed of the shaft. On top of the input and output powers, all voltages, all currents, two motor temperatures and the ambient temperature were recorded. A table containing measured data is found in Appendix B.

![Fig. 7.20 Measurement setup for determining the efficiencies of the compact PM integral motor and of its converter and PM motor.](image)

The three different measured efficiencies were determined as

1. Yokogawa Digital Power Meter WT1030, modified to 100A(peak)-range
2. Yokogawa Digital Power Meter 2533 DC/AC
3. Ono Sokki Digital Torque Detector SS101 combined with Meter TS-800B
Design and Evaluation of a Compact 15 kW PM Integral Motor

\[
\eta = \frac{P_{\text{out}} \pm |\Delta P_{\text{out}}|}{P_{\text{in}} \mp |\Delta P_{\text{in}}|} \quad (7.17)
\]

where first \(\Delta P_{\text{out}} = \Delta P_{\text{in}} = 0\). \(P_{\text{out}}\) and \(P_{\text{in}}\) are given in Table B1 in Appendix B. By applying the inaccuracies of the different used torque and power meters (\(\Delta P_{\text{out}}, \Delta P_{\text{in}}\)) to Equation (7.17), the upper and lower values of the efficiencies were obtained, see Table B2 in Appendix B. The inaccuracies of the torque meter and the two power meters are given by their respective manuals [20], [17], and [18] combined with [19]. The inaccuracies (in Watts) of each instrument are then calculated as follows:

\[
\Delta P_{TM} = \pm 0.2\% \cdot T_{\text{Full scale}} \cdot \omega_{\text{shaft}} = \pm 0.2 \cdot \omega_{\text{shaft}} \quad (7.18)
\]

\[
\Delta P_{2533} = \begin{cases} 
\pm (0.1\% \cdot P + 0.2\% \cdot 2U_{\text{range}}I_{\text{range}}) & \text{if } 45\text{Hz} < f < 66\text{Hz} \\
\pm (0.2\% \cdot P + 0.4\% \cdot 2U_{\text{range}}I_{\text{range}}) & \text{if } 20\text{Hz} < f \leq 45\text{Hz} \\
\pm (0.2\% \cdot P + 0.4\% \cdot 2U_{\text{range}}I_{\text{range}}) & \text{if } 66\text{Hz} \leq f < 2\text{kHz}
\end{cases} \quad (7.19)
\]

\[
\Delta P_{1030} = \begin{cases} 
\pm (0.3\% \cdot P + 0.5\% \cdot 2U_{\text{range}}I_{\text{range}}) & \text{if } 0.5\text{Hz} \leq f < 45\text{Hz} \\
\pm (0.2\% \cdot P + 0.1\% \cdot 2U_{\text{range}}I_{\text{range}}) & \text{if } 45\text{Hz} \leq f \leq 66\text{Hz} \\
\pm (0.3\% \cdot P + 0.2\% \cdot 2U_{\text{range}}I_{\text{range}}) & \text{if } 66\text{Hz} < f \leq 1\text{kHz}
\end{cases} \quad (7.20)
\]

where \(P\) is the measured three phase power of the corresponding power meter. \(U_{\text{range}}\) and \(I_{\text{range}}\) are the used voltage and current ranges of each measurement, respectively. Note that these are worst case scenarios, the measured values are probably more accurate than that. The internal power loss of the power meter connected to the PM motor is less than 6 W for the rated PM motor current [17], and has therefore been neglected. The speed signal from the torque meter is regarded to have a negligible error.

The following first three diagrams show the efficiencies versus speed for different torques of the PM integral motor, of the converter and of the PM motor, respectively.

The remaining three diagrams show comparisons between the measured efficiencies of the PM integral motor and the listed efficiencies of mains-connected induction motors.
Efficiency of the PM integral motor

The measured steady-state efficiency for 100 Nm and 1500 r/min is 91.1% (+/-0.7 percentage units) with an ambient temperature of 32 °C. The PM integral motor had been running for 6 hours. The winding temperature (spot 1) was 100 °C and the corner coil temperature was 89 °C.

The calculated efficiency was 91.7% at 100 Nm and 1500 r/min, with an ambient temperature of 40 °C and a calculated winding temperature of 124 °C, see sub-section 6.5.1. This agreement must be considered as satisfactory, due to two reasons; the calculated efficiency is inside the accuracy interval of the measured efficiency, and the high-frequency losses - due to inverter supply - were neglected in the calculations.

The highest measured efficiency 91.7% (+/-0.6 percentage units) is obtained for 75 Nm and 1500 r/min, while the lowest efficiency is 81.2% (+/-0.6 percentage units) for 100 Nm and 500 r/min.

The highest measurement inaccuracy was +/-1.3 percentage units, which was obtained for 25 Nm and 750 r/min.

All PM integral motor efficiencies, and corresponding inaccuracies, are found in Table B2 in Appendix B.
Efficiency of the converter

![Graph showing measured efficiency of the converter at thermal steady state versus speed for different torques. The switching frequency was 4 kHz.](image)

The measured steady-state efficiency of the converter, when the PM motor is operating at 100 Nm and 1500 r/min, is 96.9% with an inaccuracy of +/-2.1 percentage units. The converter output power was 16.7 kW in this operating point, see Table B1 in Appendix B.

The expected efficiency was 97%, see sub-section 5.1.5, and the agreement is satisfactory if the inaccuracy of the measurement is disregarded.

The highest measurement inaccuracy was +/-2.6 percentage units. This high inaccuracy was obtained when the PM motor was delivering a torque of 100 Nm at 500 r/min. The converter had then an efficiency of 93.5%.

All converter efficiencies and inaccuracies are found in Table B2 in Appendix B.
Measurements

Efficiency of the PM motor

Fig. 7.23 Measured efficiency of the PM motor at thermal steady state versus speed for different torques. The switching frequency was 4 kHz.

At 100 Nm and 1500 r/min, the measured steady-state efficiency of the PM motor is 94.0% (+1.8/-1.7 percentage units). The ambient temperature was 32 °C, and the PM integral motor had been running for 6 hours.

The calculated efficiency at 100 Nm and 1500 r/min was 0.54 percentage units higher. This calculation was based on an ambient temperature of 40 °C and a calculated winding temperature of 124 °C, see sub-section 6.5.1. Again the agreement is considered to be satisfactory since the calculated efficiency is inside the accuracy interval of the measured efficiency, and the high-frequency losses had been disregarded in the calculations.

The highest inaccuracy is obtained when the PM motor is delivering 25 Nm at 500 r/min. This inaccuracy is +2.5/-2.4 percentage units.

All inaccuracies and efficiencies of the PM motor are found in Table B2 in Appendix B.
Calorimetric measurement of the efficiency of the PM integral motor

As can be seen in the former paragraphs, the accuracies of the measured efficiencies are low. This is especially a problem when measuring high efficiencies. To improve the accuracy of one efficiency measurement, a calorimetric measurement method was used. The calorimetric measurement setup was developed and built in another, earlier project [1]. When this earlier project finished the equipment was dismounted, but has now been rebuilt again. The equipment consists of a 2m x 2m x 2m thermally insulated closed room with a heat-exchanger. By measurements of the cooling fluid mass flow and of the input and output temperatures, the total losses inside the closed room can be calculated. The thermal leakage through the walls of the room are reduced to a minimum by adjusting the inside temperature equal to the outside temperature. A more detailed description of the system is found in [1]. Calorimetric measurements have also been used by e.g. [47]. In [47] it was shown that the relative error of the losses obtained from an input-output power measurement, approaches infinity as the efficiency of the measured object approaches unity.

The calorimetric measurements began by calibrating the calorimetric equipment. A measured electrical power loss of 2492 W (+/- 29 W) was applied inside the closed room and the calorimetrically measured value was 2422 W. This yields an inaccuracy of about 4%, which is not so good. Though one must keep in mind that this relatively high inaccuracy applies to the losses, and would therefore have a smaller impact on an efficiency calculation. To improve the accuracy of the calorimetric measurement, it was decided to make two measurements. In the first measurement the losses of the compact PM integral motor is measured calorimetrically. In the second measurement an electrical power is supplied to resistors inside the closed room. The magnitude of the electrical power is adjusted until almost the equivalent calorimetrically measured power loss, as in the former measurement, is obtained. Performing two measurements, instead of one, is more complicated and more than twice as time-consuming. On the other hand, the accuracy of the result improves tremendously since the repeatabilities of the thermo-couples and the mass flow meter are high.

During the first measurement the compact PM integral motor was run at 1460 r/min and 98 Nm, which is the rated operating point of the equivalent standard induction motor. This speed and torque equals a mechanical output power of \( P_{out} = 14.98 \text{ kW} \). After about 6 hours thermal steady state was reached. The ambient, end-winding and corner coil tempera-
Measurements

Temperatures were 24 °C, 90 °C and 79 °C, respectively. The electrical input power to the PM integral motor was $P_{in} = 16.33$ kW. The calorimetrically measured power loss was 1474 W.

In the second measurement, a calorimetrically measured power loss of 1478 W was - after hours of adjustments - obtained when the electrical power to the resistors inside the closed room was measured to 1520 W. A linear compensation for this small power difference will then yield a power loss of $P_{loss} = 1520 \cdot \frac{1474}{1478} = 1515$ W for the PM integral motor.

The efficiency and efficiency inaccuracy of the PM integral motor can now be calculated in three different ways. Equations (7.18) and (7.20), and the following equations are used

\[
\eta_{in+out} = \frac{P_{out} \pm |\Delta P_{out}|}{P_{in} \mp |\Delta P_{in}|} = 91.8\%^{+0.7\%}_{-0.7\%} \quad (7.21)
\]

\[
\eta_{in+loss} = \frac{P_{in} \pm |\Delta P_{in}| - P_{loss} \pm |\Delta P_{loss}|}{P_{in} \mp |\Delta P_{in}|} = 90.7\%^{+0.1\%}_{-0.1\%} \quad (7.22)
\]

\[
\eta_{out+loss} = \frac{P_{out} \pm |\Delta P_{out}|}{P_{out} \mp |\Delta P_{out}| + P_{loss} \mp |\Delta P_{loss}|} = 90.8\%^{+0.07\%}_{-0.07\%} \quad (7.23)
\]

From the three results above, one can see that the efficiency based on the losses and the output power is most accurate. The reason for this is that the torque meter is more accurate than the digital power meter for this load condition. These three results show the usefulness of a calorimetric measurement method. The input-output efficiency, and the most accurate calorimetrically obtained efficiency - including the efficiency inaccuracies - are plotted as two vertical lines in Fig. 7.21.

This paragraph has shown how a calorimetric measurement method can improve the accuracy of the determined efficiency tremendously. The disadvantages of such a method are that it requires an extraordinary measurement equipment and it is also quite time-consuming.

In the following three paragraphs the efficiencies of the PM integral mo-
Design and Evaluation of a Compact 15 kW PM Integral Motor

tor from Fig. 7.21 are compared to the listed efficiencies of both larger and smaller induction motors. Note that the induction motor efficiencies do not include any converter efficiencies. This implies that if the PM integral motor has higher efficiency than the induction motors without converters, it will most certainly have higher efficiency than induction motors equipped with converters.

Efficiency comparison: PM integral motor vs 3 induction motors, size 160

The listed efficiencies of three different mains-connected induction motors at full, three quarters, half and a quarter of the rated load torque [31], and the measured efficiencies of the PM integral motor from Fig. 7.21 have been plotted in Fig. 7.24. The induction motor efficiencies are given for the synchronous speeds since not all the corresponding speeds are given in [31]. Some data of the three different induction motors [31] are given in Table 7.10. The PM integral motor has a frame size of 160 mm.

As can be seen in Fig. 7.24, the PM integral motor has higher efficiency than the induction motors in all the compared operating points.

**Fig. 7.24** Comparisons among measured efficiencies of the PM integral motor at thermal steady state and the listed efficiencies of three different mains-connected induction motors at full and part load [31].
Table 7.10 Data of three different induction motors used in Fig. 7.24 [31].

<table>
<thead>
<tr>
<th>Motor</th>
<th>Frame size [mm]</th>
<th>Number of poles</th>
<th>Synchronous speed [r/min]</th>
<th>Rated power [kW]</th>
<th>Rated torque [Nm]</th>
<th>Loading (torque)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>160</td>
<td>8</td>
<td>750</td>
<td>7.5</td>
<td>100</td>
<td>100,75,50,25%</td>
</tr>
<tr>
<td>b</td>
<td>160</td>
<td>6</td>
<td>1000</td>
<td>11</td>
<td>110</td>
<td>100,75,50,25%</td>
</tr>
<tr>
<td>c</td>
<td>160</td>
<td>4</td>
<td>1500</td>
<td>15</td>
<td>98</td>
<td>100,75,50,25%</td>
</tr>
</tbody>
</table>

Efficiency comparison: PM integral motor vs 9 larger induction motors

Fig. 7.25 Comparisons among measured efficiencies of the PM integral motor at thermal steady state and the listed efficiencies of nine different larger mains-connected induction motors at part load [31].

If the PM integral motor has a higher efficiency than larger induction motors, which are running at reduced load torques, it may be motivated to replace the induction motors with PM integral motors. The listed efficiencies of nine different larger mains-connected induction motors at reduced torques and synchronous speeds [31], have been plotted in Fig. 7.25 together with the measured efficiencies of the PM integral motor from Fig. 7.21. The induction motor efficiencies are given for the synchronous speeds since the corresponding speeds are not given in [31]. Some data of the nine induction motors [31] are given in Table 7.11.
Fig. 7.25 shows that the PM integral motor has higher efficiency than the larger induction motor in seven of the nine compared operating points.

**Table 7.11 Data of the nine different larger induction motors used in Fig. 7.25 [31].**

<table>
<thead>
<tr>
<th>Motor</th>
<th>Frame size [mm]</th>
<th>Number of poles</th>
<th>Synchronous speed [r/min]</th>
<th>Rated power [kW]</th>
<th>Rated torque [Nm]</th>
<th>Loading (torque)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>180</td>
<td>8</td>
<td>750</td>
<td>11</td>
<td>147</td>
<td>75%</td>
</tr>
<tr>
<td>e</td>
<td>180</td>
<td>6</td>
<td>1000</td>
<td>15</td>
<td>148</td>
<td>75%</td>
</tr>
<tr>
<td>f</td>
<td>180</td>
<td>4</td>
<td>1500</td>
<td>22</td>
<td>143</td>
<td>75%</td>
</tr>
<tr>
<td>g</td>
<td>200</td>
<td>8</td>
<td>750</td>
<td>15</td>
<td>200</td>
<td>50%</td>
</tr>
<tr>
<td>h</td>
<td>200</td>
<td>6</td>
<td>1000</td>
<td>22</td>
<td>216</td>
<td>50%</td>
</tr>
<tr>
<td>i</td>
<td>200</td>
<td>4</td>
<td>1500</td>
<td>30</td>
<td>195</td>
<td>50%</td>
</tr>
<tr>
<td>j</td>
<td>250</td>
<td>8</td>
<td>750</td>
<td>30</td>
<td>392</td>
<td>25%</td>
</tr>
<tr>
<td>k</td>
<td>250</td>
<td>6</td>
<td>1000</td>
<td>37</td>
<td>361</td>
<td>25%</td>
</tr>
<tr>
<td>l</td>
<td>250</td>
<td>4</td>
<td>1500</td>
<td>55</td>
<td>357</td>
<td>25%</td>
</tr>
</tbody>
</table>

**Efficiency comparison: PM integral motor vs 9 smaller induction motors**

**Fig. 7.26 Comparisons among measured efficiencies of the PM integral motor at thermal steady state and the listed efficiencies of nine different smaller mains-connected induction motors at full load [31].**
In some special cases it might be possible, despite the lower frame sizes, to replace smaller induction motors running at rated torque and rated speed with a PM integral motor operating with reduced torque and reduced speed. The listed efficiencies of nine different smaller mains-connected induction motors at rated torques and rated speeds [31], have been plotted in Fig. 7.26 together with the measured efficiencies of the PM integral motor from Fig. 7.21. Some data of the nine induction motors [31] are given in Table 7.12.

In Fig. 7.26 it can be seen that the PM integral motor has higher efficiency than the smaller induction motors in all nine operating points.

**Table 7.12 Data of the nine different smaller induction motors used in Fig. 7.26 [31].**

<table>
<thead>
<tr>
<th>Motor</th>
<th>Frame size [mm]</th>
<th>Number of poles</th>
<th>Rated speed [r/min]</th>
<th>Rated power [kW]</th>
<th>Rated torque [Nm]</th>
<th>Loading (torque)</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>160</td>
<td>8</td>
<td>725</td>
<td>5.5</td>
<td>73</td>
<td>100%</td>
</tr>
<tr>
<td>n</td>
<td>160</td>
<td>6</td>
<td>975</td>
<td>7.5</td>
<td>74</td>
<td>100%</td>
</tr>
<tr>
<td>o</td>
<td>160</td>
<td>4</td>
<td>1460</td>
<td>11</td>
<td>72</td>
<td>100%</td>
</tr>
<tr>
<td>p</td>
<td>160</td>
<td>8</td>
<td>730</td>
<td>4</td>
<td>53</td>
<td>100%</td>
</tr>
<tr>
<td>q</td>
<td>132</td>
<td>6</td>
<td>935</td>
<td>5.5</td>
<td>57</td>
<td>100%</td>
</tr>
<tr>
<td>r</td>
<td>132</td>
<td>4</td>
<td>1440</td>
<td>7.5</td>
<td>50</td>
<td>100%</td>
</tr>
<tr>
<td>s</td>
<td>132</td>
<td>8</td>
<td>705</td>
<td>2.2</td>
<td>30</td>
<td>100%</td>
</tr>
<tr>
<td>t</td>
<td>132</td>
<td>6</td>
<td>950</td>
<td>3</td>
<td>30,5</td>
<td>100%</td>
</tr>
<tr>
<td>u</td>
<td>112</td>
<td>4</td>
<td>1440</td>
<td>4</td>
<td>27</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Conclusions**

In this section it has been shown that the measured efficiency of the manufactured PM integral motor is slightly lower than calculated, but still higher than an equivalent standard induction motor (without converter) at rated torque and rated speed.

The converter of the PM integral motor has an efficiency at rated load which corresponds well to what was promised by the manufacturer.

If the inaccuracies of the efficiency measurements are taken into consideration, some results have quite large errors. To improve this, a calorimetric measurement was performed for the rated operating point. The
calorimetric measurement increases the accuracy of the obtained efficiency. The calorimetrically measured efficiency is slightly lower than the efficiency calculated from measurements of the input and output powers.

In all compared cases the PM integral motor has a higher efficiency than induction motors with different pole numbers but similar frame size, running at full and part load. In most cases the PM integral motor has a higher efficiency than both some larger and some smaller induction motors running at reduced and full load, respectively. Note the all the induction motor efficiencies are given for mains-connected motors, i.e. no converter efficiencies are included in these numbers.

7.3 Conclusions

This chapter has presented measurements on heat-sinks and on the manufactured PM motor, its converter and the complete PM integral motor.

Most of the measured quantities show a satisfactory agreement to the analytically and/or FEM calculated values.

Temperature measurements on the complete PM integral motor indicate that a torque of 100 Nm is possible for speeds between 375 r/min and 1500 r/min. For 1500 r/min, even a torque of 120 Nm is possible.

Efficiency measurements - based on input and output powers - on the PM motor, the converter, and the PM integral motor give approximately the expected results. The inaccuracies of the measurements are quite high in some cases. To improve the accuracy of the measurement, a calorimetric measurement method was employed for the PM integral motor at the rated operating point. This results in a slightly lower efficiency than with the input-output power method, but with a very high accuracy.

Efficiency comparisons between the PM integral motor and different induction motors (without converters) show that the PM integral motor is preferable in most cases.

In the following chapter, the high-frequency losses of the rotor cage are estimated with time-stepping fixed-speed 2D-FEM calculations. Four different fault conditions are also investigated with the FEM software.
8 Time-stepping FEM investigations of rotor cage losses and fault conditions

In the optimization procedure of Chapter 5, the high-frequency losses of the motor - which are difficult to predict - were assumed to be small, and were therefore neglected. To investigate the high-frequency losses of the rotor cage, fixed-speed time-stepping 2D-FEM calculations have been carried out in this chapter with the software \textit{MEGA}\textsuperscript{1}. The high-frequency iron losses of the rotor and of the stator were not included in the FEM simulations.

In case of a short circuit, high mechanically damaging peak torques can appear and there is also a risk of permanently demagnetizing the permanent magnets of the motor. In Chapter 6 a static FEM calculation with demagnetizing current was carried out. In the present chapter, four short-circuit fault conditions have been simulated with fixed-speed time-stepping 2D-FEM calculations. Again \textit{MEGA} is used.

8.1 Rotor cage losses

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig8_1.png}
\caption{The combined geometry and mesh of one pole.}
\end{figure}

The rotor body of the compact PM integral motor is equipped with a

\begin{enumerate}
\item FEM program from University of Bath.
\end{enumerate}
sparse squirrel cage. The rotor cage is used as a means to keep the rotor together mechanically, see Chapter 3. To calculate the losses of the rotor cage the PM integral motor geometry was entered into the FEM program. The combined geometry and mesh is shown in Fig. 8.1. Due to symmetry it is enough to model only one pole. The centre of winding R was placed in front of the north (N) pole of the rotor, see Fig. 8.2. The windings and the rotor cage were connected according to Fig. 8.2. A permanent magnet material with a remanent flux density of 1.22 T and a relative permeability of 1.05 was used. This is equivalent to the magnet that was used in the manufactured motor, see subsection 6.5.1.

![Diagram of stator winding connections and rotor start position](image)

**Fig. 8.2** The stator winding connections and the start position of the rotor in the time-stepping FEM calculations. The field lines from only the magnets, and the number of winding turns are also shown.

The iron material has iron losses similar to CK27 [52] and a modified BH-curve was entered. The BH-curve was modified to take a lamination stacking factor of \( k_f = 0.94 \) into consideration. According to [44] a new equivalent relative permeability of the laminated iron is calculated as:
where \( \mu_{r, Fe} \) is the relative permeability of the non-laminated iron, calculated from the original BH-curve. The conductivity of the rotor aluminium bars was set to \( 3.54 \times 10^7 \) S/m at 20 °C [14], and the two aluminium end-plates of the rotor have been modelled as ideal short circuits. The losses of the rotor cage will therefore be a slight overestimation. Since the geometry consists of only one pole, the results were multiplied by the number of poles, which is 8.

**Static FEM calculation**

First a static calculation of the airgap flux density was carried out. The result is the flux density shown in Fig. 8.3, which has a fundamental component with a peak value of 0.884 T. This is in good agreement with earlier FEM calculations, see subsection 6.5.1. The static torque was also checked. Symmetric three-phase currents, where the current of the second phase (S) equals its peak value of 42.4 A, were applied to the stator windings with the rotor position shown in Fig. 8.2. This implies that the stator current phasor is lagging the q-axis by 30° (el.). Therefore, an analytically calculated torque of \( 0.88T/0.73T \times 99Nm \times \sin(90° - 8/2 \cdot 7.5°) = 103 Nm \) can be expected. The obtained torque from the FEM calculation was 97 Nm, which is satisfactory. It also indicates that the winding connections in the FEM program are done in a correct way.

\[
\mu_{r, Fe, eq} = \mu_{r, Fe}k_f + (1 - k_f)
\]  

(8.1)

**“Turning on” the magnets**

The first measure in the time-stepping FEM calculations is to “turn on” the permanent magnets, while the speed is constant and equal to zero.
This procedure can be performed with a coarse time-step length, e.g. 1 ms. With a rotor cage of aluminium (conductivity: \( \sigma = 3,54 \cdot 10^7 \text{ S/m} \)) the induced voltages have vanished after less than 40 ms. The following five simulations have therefore been restarted at the time 50 ms with a fixed rotor speed equal to 1500 r/min.

**Rotor loss with open cage. Only magnets.**
The time-step length was now reduced to 0.1 ms. The rotor was made to rotate 1500 r/min in the counter-clockwise direction. At first, the rotor cage was open and only the magnets were present. The rotor bars had a conductivity of \( \sigma = 3,54 \cdot 10^7 \text{ S/m} \). The rotor bar losses are only due to eddy current losses in the bars. The simulation was run for 250 ms. The total instantaneous power loss of the rotor bars are shown in Fig. 8.4. The average power loss is found from averaging over the steady state loss values, e.g. between 150 ms and 300 ms:

\[
P_{bars, m, open} = \frac{1}{t_{end} - t_{steady}} \int_{t_{steady}}^{t_{end}} P_b(t) dt = 0.43 \text{ W} \quad (8.2)
\]

which is a very small power loss.

**Fig. 8.4** The total instantaneous power loss of the rotor bars during 7 ms almost at the end of the simulation. Only magnets are present. The rotor cage was open.
Rotors loss with short circuited cage. Only magnets.
The second time-stepping simulation was similar to the former, except that the rotor cage was short circuited. The rotor bar losses now arise from both eddy currents in the bars and circulating bar currents. The simulation was run for 250 ms. The total instantaneous power loss of the rotor bars are shown in Fig. 8.5. The average power loss of the rotor bars at steady state is found as:

\[
P_{\text{bars, m}} = \frac{1}{t_{\text{end}} - t_{\text{steady}}} \int_{t_{\text{steady}}}^{t_{\text{end}}} P_b(t) dt = 0.45 \text{ W}
\]  

(8.3)

which still is a very small power loss.

![Fig. 8.5](image.png)

**Fig. 8.5** The total instantaneous power loss of the rotor bars during 7 ms almost at the end of the simulation. Only magnets are present. The rotor cage was short circuited.

Rotors loss with short circuited cage. Magnets and sinusoidal currents.
As earlier, the time-stepping was restarted at \( t_{\text{restart}} = 50 \text{ ms} \) and with a time-step of 0.1 ms, but this time the stator windings were supplied with sinusoidal three-phase currents. The currents, which were modelled with 100 points per 100 Hz period, are given by the following equations:
Design and Evaluation of a Compact 15 kW PM Integral Motor

\[ i_R = \sqrt{2} \cdot I_n \cos \left( \frac{180^\circ}{\pi} \cdot \omega_s t - \varphi_{\text{restart}} - \varphi_q \right) \]

\[ i_S = \sqrt{2} \cdot I_n \cos \left( \frac{180^\circ}{\pi} \cdot \omega_s t - \varphi_{\text{restart}} - \varphi_q - 240^\circ \right) \]  

\[ i_T = \sqrt{2} \cdot I_n \cos \left( \frac{180^\circ}{\pi} \cdot \omega_s t - \varphi_{\text{restart}} - \varphi_q - 120^\circ \right) \]  

(8.4)

where \( I_n = 30 \) A (RMS) is the rated PM motor current and \( \omega_s = 2\pi 100 \) rad/s is the rated electrical angular frequency of the motor. The phase sequence of currents S and T have been shifted to obtain a current sheet rotating in the counter-clockwise direction. This has been done since the rotor was already set to rotate in this direction, see Fig. 8.2. Further, the electrical restart delay angle is found as

\[ \varphi_{\text{restart}} = \frac{180^\circ}{\pi} \cdot \omega_s t_{\text{restart}} = 1800^\circ = 5 \cdot 360^\circ \Leftrightarrow 0^\circ \]  

(8.5)

and the electrical angle required to get purely q-current with the given rotor position is found from Fig. 8.2:

\[ \varphi_q = \frac{45^\circ_{\text{mech}}}{\frac{p}{2}} \cdot \frac{P}{2} = 90^\circ \]  

(8.6)

Again the simulation was run for 250 ms. Then a second restart, with a time step of 5 \( \mu \)s, was done at 250 ms and run for 50 ms. The total instantaneous power loss of the rotor bars is shown in Fig. 8.6. The average power loss of the rotor bars at steady state is:

\[ P_{\text{bars,ms}} = \frac{1}{t_{\text{end}} - t_{\text{steady}}} \int_{t_{\text{steady}}}^{t_{\text{end}}} P_b(t) dt = 8.0 \text{ W} \]  

(8.7)

which is larger than earlier, but still negligible for a 15 kW motor.
Fig. 8.6 The total instantaneous power loss of the rotor bars during 5 ms almost at the end of the simulation. Magnets and sinusoidal stator currents are present. The rotor cage was short circuited.

Rotor loss with short circuited cage. Magnets, 100 Hz and 4 kHz currents. The compact PM integral motor is not fed with sinusoidal current of 100 Hz, as was investigated in the former paragraph. Instead a pulse width modulated (PWM) voltage is applied to the terminals. The switching frequency of the triangular shaped carrier signal is constant and equal to 4 kHz. According to [50] a PWM modulated voltage source will give rise to many time harmonic voltages of different frequencies. If the ratio between the switching frequency and the fundamental frequency is large (i.e. >21), odd and a multiple of three, the following frequencies and amplitudes (in per cent of the fundamental voltage at full modulation, i.e. m = 1) of the most significant time harmonic voltages are present [50]:

- \( f = f_{sw} \pm 2f_s \) and a relative amplitude of 32%
- \( f = f_{sw} \pm 4f_s \) and a relative amplitude of 1.8%
- \( f = 2f_{sw} \pm f_s \) and a relative amplitude of 18%
- \( f = 2f_{sw} \pm 5f_s \) and a relative amplitude of 3.3%

where \( f_s \) is the frequency of the fundamental component of the voltage and \( f_{sw} \) is the switching frequency. According to [29], it can be shown...
that the values above are valid also if the frequency ratios are not odd and not a multiple of three, as long as the same triangular shaped carrier signal is used for the three phases. On the other hand, quasine shaped reference waves (i.e. third harmonic injection) will change this bulleted list.

The ratio between the switching frequency and the rated frequency of the PM integral motor is $4\text{kHz}/100\text{Hz}=40>21$. The two larger time harmonic voltages with a relative amplitude of 32%, situated around the switching frequency can be expected to give the largest contribution to the losses in the rotor cage. The reasons for this are that they have a high relative amplitude and they will - due to the lower frequency - give rise to larger air-gap fluxes, compared to the time harmonic voltages around twice the switching frequency. Further, the induced voltages due to these larger air-gap fluxes will see a smaller cage reactance, giving rise to higher values of current in the rotor cage. The lower resistance of the rotor cage at around the switching frequency, due to lower skin effect, will not be decreased so much, compared to the value at around twice the switching frequency, that it compensates for the higher rotor cage current.

The time harmonic voltages around the switching frequency can be treated separately. The ones around twice the switching frequency cannot since they interact with each other, e.g. by creating a pulsating rotor flux [54].

To investigate the sensitivity of the rotor cage for high-frequency time harmonics in the supply current, symmetric sinusoidal three-phase currents of 4 kHz was added to the sinusoidal 100 Hz currents used in the former paragraph. The high-frequency current was set to have a magnitude of 1 A (RMS), which is about 3.3% of the magnitude of the fundamental current $I_n = 30$ A (RMS). The phase sequence of the high-frequency current is opposite of the low frequency ditto, and thereby also opposite of the direction of the rotor. This phase sequence is chosen to obtain what is probably a worst case, regarding the losses in the rotor cage. The three stator currents, which this time were modelled with 1000 points per 100 Hz period, are given by the following equations:
where \( i_R \), \( i_S \) and \( i_T \) are the three currents given by Equation (8.4).

\[
\begin{align*}
i_R' &= i_R + \sqrt{2} \cdot \frac{I_n}{30} \cdot \cos\left(\frac{180^\circ}{\pi} \cdot 2\pi 4000 t\right) \\
i_S' &= i_S + \sqrt{2} \cdot \frac{I_n}{30} \cdot \cos\left(\frac{180^\circ}{\pi} \cdot 2\pi 4000 t - 120^\circ\right) \\
i_T' &= i_T + \sqrt{2} \cdot \frac{I_n}{30} \cdot \cos\left(\frac{180^\circ}{\pi} \cdot 2\pi 4000 t - 240^\circ\right)
\end{align*}
\]

\[(8.8)\]

Again the time-stepping simulation was restarted at 50 ms and run for 250 ms. Then a second restart, with a time step of 5 \( \mu \)s, was done at 250 ms and run for 50 ms. The total instantaneous power loss of the rotor bars at the end of the simulation is shown in Fig. 8.7. The average power loss of the rotor bars at steady state is:

\[
P_{bars,ms,4kHz} = \frac{1}{t_{end} - t_{steady}} \int_{t_{steady}}^{t_{end}} P_b(t) dt = 16.5 \text{ W} \quad (8.9)
\]

which is still negligible.
Even if one is assuming that all four time harmonic currents, i.e. the high amplitude side bands around the switching frequency and around twice the switching frequency, each contribute with the equivalent amount of losses, the total bar losses would still not be more than 42 W.

**Rotor loss with short circuited cage. Magnets, 100 Hz and 8 kHz currents.**

Though it is not totally correct - according to the former paragraph - to apply a single three-phase time harmonic voltage of around twice the switching frequency, it is interesting to see the losses it gives rise to. The stator currents given by Equation (8.8) were used again, but the values of 4000 (Hz) were replaced by 8000 (Hz). The currents were modelled with 1000 points per 100 Hz period. Again the time-stepping simulation was restarted at 50 ms and run for 250 ms. Then a second restart, with a time step of 5 μs, was done at 250 ms and run for 50 ms. The total instantaneous power loss of the rotor bars at the end of the simulation is shown in Fig. 8.8.

![Graph](image)

*Fig. 8.8 The total instantaneous power loss of the rotor bars during 5 ms almost at the end of the simulation. Magnets, 100 Hz and 8 kHz stator currents are present. The rotor cage was short circuited.*

The average power loss of the rotor bars at steady state is:
which is higher than with 4 kHz currents, see Equation (8.9).

The magnets and the (purely) sinusoidal 100 Hz currents gave rise to rotor cage losses totalling 8.0 W, while the addition of 4 kHz and 8 kHz currents gave total rotor cage losses of 16.5 W and 21.0 W, respectively. The two high-frequency stator currents, which have equal magnitudes, are forced through the stator windings. They will therefore give rise to rotor currents of equivalent magnitudes. The difference in losses for the 8 kHz current, compared to the 4 kHz current, comes only from increased resistance of the rotor bars due to skin effect [54]. The approximate rotor resistance ratio $k_{skin}$ for the 8 kHz and 4 kHz currents can be found from the following equation:

$$
k_{skin} = \frac{R_{rotor, 8kHz}}{R_{rotor, 4kHz}} = \frac{P_{loss, 8kHz}/I_{rotor, 8kHz}^2}{P_{loss, 4kHz}/I_{rotor, 4kHz}^2}
$$

(8.11)

Since we here have

$$
I_{rotor, 8kHz} = I_{rotor, 4kHz}
$$

(8.12)

Equation (8.11) will boil down to

$$
k_{skin} = \frac{P_{loss, 8kHz}}{P_{loss, 4kHz}} = \frac{21.0 - 8.0}{16.5 - 8.0} = 1.53
$$

(8.13)

**Rotor loss with short circuited cage. Magnets and measured currents.**

Finally, three measured currents of the PM motor prototype were used in the FEM simulation. The three measured currents are shown in Fig. 8.9. The oscilloscope was set to sample with 250 kS/s per channel, i.e. 2500 points per 100 Hz period of the current waveform. The PM integral motor was operating at 1500 r/min delivering a torque of 100 Nm at the time of the measurement.
Fig. 8.9 Three measured PM motor currents. The PM integral motor was operating at 1500 r/min and 100 Nm.

The result of a frequency analysis of the phase current $i_R$ is shown in Fig. 8.10. The magnitudes of the time harmonic currents are below $0.8 \text{ A (RMS)}$, i.e. less than $2.8\%$ of the fundamental current component. The fundamental current component is less than the rated $30 \text{ A}$, since the PM motor was many hours from reaching thermal steady-state at the time of the measurement.
Fig. 8.10 Frequency analysis of the PM motor current in phase R, when the PM integral motor was operating at 1500 r/min and 100 Nm. The fundamental frequency is 100 Hz.

The time-stepping simulation was restarted at 50 ms and run for 250 ms. Then a second restart, with a time step of 5 µs, was done at 250 ms and run for 50 ms. The total instantaneous power loss of the rotor bars is shown in Fig. 8.11. The average power loss of the rotor bars at steady state is:

\[
P_{\text{bars,ms,meas}} = \frac{1}{t_{\text{end}} - t_{\text{steady}}} \int_{t_{\text{steady}}}^{t_{\text{end}}} P_b(t) dt = 21.8 \text{ W} \quad (8.14)
\]

which is negligible for the 15 kW PM integral motor.
Conclusion

The conclusion that can be made is that the (high-frequency) losses of the rotor bars of the manufactured PM integral motor prototype are negligible. This fact justifies the neglecting of these losses that was made in the optimization program in Section 5.1.

8.2 Fault conditions

In case of a short circuit, there is a risk of permanently demagnetizing the permanent magnets of a motor, due to the strong opposing magnetic field from the short circuit current(s). Motors with surface or inset mounted magnets are more sensitive to this than buried designs. The peak torque at a short circuit can also be very high, leading to mechanical failures of the rotor, the shaft coupling or the load. One can identify at least four severe fault conditions for an inverter-fed permanent magnet motor [64] (see Fig. 8.12), and they are listed below:
1. Short circuit between one terminal of the machine and the (normally) isolated neutral point of the machine.
2. Short circuit between two terminals of the machine.
3. Short circuit between all three terminals of the machine.
4. Short circuit in one of the diodes or valves of the inverter, giving rise to a direct current (DC) in the machine even in short circuit steady state.

Fault condition #1 requires an insulation fault of the stator winding, or - if the isolated neutral point of the machine is present in the terminal box - a short circuit in the terminal box of the machine to occur. Fault condition #1 is named 1-phase short circuit in this thesis.

Fault condition #2 and #3 can occur due to a short circuit in the terminal box of the machine or by damage to the cable connecting the machine and the inverter. Fault condition #2 is named 2-phase short circuit in this thesis.

Fault condition #3 can also occur if the three upper or the three lower switches of the inverter are turned on at the same time, or if there is a short circuit of the intermediate link. Fault condition #3 is named 3-phase short circuit in this thesis. Paper [85] uses the name symmetrical short circuit for this fault condition.

Fault condition #4 can occur if one of the six diodes is short circuited. A short circuit can be due to high reverse voltage over a diode, high forward current in a diode or impurities in the manufacturing of a diode [39]. Fault condition #4 can also arise from an erroneous gate signal to a valve. An erroneous gate signal can appear e.g. if the electromagnetic compatibility (EMC) of the control circuit is too low for the environment where it is being operated, or be due to an error in the software code. Fault condition #4 is named 1-phase inverter short circuit in this thesis. Paper [85] uses the name asymmetrical single-phase short circuit for this fault condition.

In all four cases above it is assumed that the switchings of the valves in the output inverter are stopped immediately when the fault occurs. To simplify the simulations, it is also assumed that the motor was running at no-load, driving a friction-less load with an infinite inertia. This implies that the PM machine changes from motor operation to generator operation without any change of speed, and will continue to run at that speed. Due to the friction-less no-load operation, it is also assumed that all three stator currents are equal to zero when the different short circuits appear.
By using the equivalent circuit of the machine in Fig. 8.12, the RMS values of the steady state short circuit currents for three of the four fault conditions are estimated to:

\[ I_{\text{short, 1ph}} = \frac{E}{\sqrt{R_s^2 + \left(\omega_s \cdot \frac{2}{3} \cdot L_s\right)^2}} = \left\{ R_s \ll \omega_s L_s \right\} = \frac{3}{2} \cdot \frac{E}{\omega_s L_s} = 96 \text{ A} \]  

(8.15)

\[ I_{\text{short, 2ph}} = \frac{E \cdot \sqrt{3}}{\sqrt{\left(2R_s\right)^2 + \left(2\omega_s L_s\right)^2}} = \left\{ R_s \ll \omega_s L_s \right\} = \frac{\sqrt{3}}{2} \cdot \frac{E}{\omega_s L_s} = 56 \text{ A} \]  

(8.16)

\[ I_{\text{short, 3ph}} = \frac{E}{\sqrt{R_s^2 + \left(\omega_s L_s\right)^2}} = \left\{ R_s \ll \omega_s L_s \right\} = \frac{E}{\omega_s L_s} = 64 \text{ A} \]  

(8.17)

\[ I_{\text{short, DC}} = \frac{E}{R_s} > \left\{ R_s \ll \omega_s L_s \right\} > \begin{cases} I_{\text{short, 1ph}} \\ I_{\text{short, 2ph}} \\ I_{\text{short, 3ph}} \end{cases} \]  

(8.18)

where \( E = 202 \text{ V}, R_s = 0.168 \Omega, \omega_s = 2\pi \cdot 100 \text{ rad/s} \) and \( L_s = 5 \text{ mH} \).
The inductance per phase of Equation (8.15) has been reduced to two thirds of its normal three-phase value since the three-phase inductance per phase is based on currents in all three phases [46]. It can be seen that the last fault condition probably gives a larger steady state current than the other cases.

To investigate these four fault conditions more thoroughly, time-stepping FEM calculations have been performed. Again the 2D-FEM software MEGA was used. Fig. 8.13 shows how the four fault conditions of Fig. 8.12 were simulated in the FEM program. All simulations started by “turning on” the magnets from 0 to 0.1 s, with zero speed of the rotor and with a coarse time step of 1 ms. The time stepping simulations were then restarted from 50 ms with the rotor rotating at 1500 r/min, and the time step was reduced to 0.1 ms. Then another restart was made for each simulation. At the second restarts the appropriate fault conditions were introduced in the circuit, according to the table in Fig. 8.13. The faults were made to appear at instants in time giving “worst cases”, regarding the amplitudes of the first peak of the short circuit currents. The objectives of the investigation are to see the amplitude of the first current peak, the lowest flux density levels in the magnets, the maximum peak torque after the short circuit, and the maximum peak torque at steady state. The torque calculation of the FEM program is based on Maxwell’s stress.

A permanent magnet can be damaged, i.e. loose all or a part of its remanent flux density, if the magnet is exposed to a high flux in the opposite direction of the magnet’s magnetization. Risk of demagnetization is present if the counter-acting flux lowers the flux density in the magnet to a point that is below the so-called critical knee of the magnet’s BH-curve, see Section 3.1. For an isotropic permanent magnet, i.e. a magnet material which has similar magnetic properties in all directions, the demagnetizing flux in quadrature to the direction of the magnet’s magnetization must also be taken into consideration [49]. Therefore, for an isotropic permanent magnet it is the magnitude and direction of the applied flux that are of importance [49]. Most permanent magnets with a high remanent flux density have been exposed to a magnetic field early in the manufacturing process, long before the real magnetization of the magnets take place. The early magnetic field aligns the domains of the material already during the pressing of the magnetic powder. After the pressing, the magnetic domains are more or less stuck in the direction of the magnet’s magnetization. These magnet materials are therefore anisotropic, i.e. they do not have similar magnetic properties in all directions. This im-
plies that anisotropic magnet materials are not sensitive to demagnetizing fluxes in the quadrature direction of the magnet’s magnetization, at least as long as these applied fluxes are not considerably high [49].

In FEM calculation it is normal that both the magnet flux and a counter-acting flux from the stator currents are present at the same time. The flux densities obtained from the FEM calculations are then the resulting flux densities. For many NdFeB-magnets, the critical knee of the BH-curve (see e.g. Fig. 3.4) is situated in the third quadrant, i.e. below zero flux density, even for higher temperatures [62]. This implies - both for isotropic and anisotropic magnet materials - that the permanent magnet is safe as long as the resulting normal component (i.e. the component parallel to the direction of magnetization) of the flux density in the magnet is above zero. For an isotropic permanent magnet to be safe, the tangential component shall be small compared to the remanent flux density of the magnet, while an anisotropic permanent magnet is safe even for a higher flux density.

![FEM model and extra circuits](image)

**Fig. 8.13** One pole FEM model and extra circuits used for the simulations in MEGA of the four investigated fault conditions.
1-phase short circuit

The 1-phase short circuit of phase R to the isolated neutral point of the Y-connected PM motor winding was introduced at 86.0 ms, i.e. at a moment in time when the no-load phase voltage $u_R$ changed from negative to positive. The maximum transient peak torque of 623 Nm was obtained at 89.9 ms. The maximum peak current of 400 A appears at 90.7 ms, see Fig. 8.14.

The minimum resulting flux densities in the magnets appear at 92.3 ms. The resulting flux density components normal and tangential to a line through the centre of the left and the right magnets at 92.3 ms are shown.

\begin{align*}
\Phi_n & = 0.47 \text{T} \\
\Phi_t & = -0.094 \text{T} \\
\Phi_{n,\text{short}} & = 0.50 \text{T} \\
\Phi_{t,\text{short}} & = 0.065 \text{T}
\end{align*}

Fig. 8.14 Short circuit current at a 1-phase fault.

Fig. 8.15 Flux densities along the centre line of the left magnet (left pic.) and the right magnet (right pic.) at no-load, and 6.3 ms after a 1-phase short circuit. (n: normal, t: tangential)
in Fig. 8.15. It can be seen that there is no risk of demagnetisation, since the resulting normal component (i.e. the component parallel to the direction of magnetization) of the flux density in the magnet is above zero and the tangential component is small.

The maximum peak torque in steady state is 205 Nm. The steady state current is not sinusoidal. Table 8.1 summarizes the results.

2-phase short circuit

![Graph](image)

Fig. 8.16 Short circuit currents at a 2-phase fault.

The 2-phase short circuit of phases R and T was introduced at 85.2 ms, i.e. at a moment in time when the no-load phase-to-phase voltage $u_{RT}$ changed from negative to positive. The maximum transient peak torque of 687 Nm was obtained at 88.9 ms. The maximum peak current of 260 A appears at 89.7 ms, see Fig. 8.16.

The minimum resulting flux densities in the magnets appear at 91.7 ms. The resulting flux density components normal and tangential to a line through the centre of the left and the right magnets at 91.7 ms have similar shapes as the curves of Fig. 8.15, but the normal flux densities are lower. The lowest normal component in the left and the right magnet is 0.44 T and 0.47 T, respectively. The largest tangential components are -0.10 T and 0.068 T. Therefore there is no risk of demagnetisation.

The maximum peak torque in steady state is 163 Nm, and one can see that the steady state current is not sinusoidal. See also Table 8.1.
3-phase short circuit

Fig. 8.17 Short circuit currents at a 3-phase fault.

The 3-phase short circuit was introduced at 87.0 ms, i.e. at a moment in time when the no-load phase voltage $u_R$ changed from negative to positive. The maximum transient peak torque of 494 Nm was obtained at 89.9 ms. The maximum peak current of 270 A in phase R appears at 90.8 ms, see Fig. 8.17.

The minimum resulting flux densities in the magnets appear at 94.1 ms. Again, the shape of the resulting flux density components normal and tangential to a line through the centre of the left and the right magnets at 94.1 ms are similar to the curves of Fig. 8.15. The lowest normal components for the left and right magnets are 0.37 T and 0.39 T, respectively. The largest tangential components are -0.11 T and 0.077 T. Even in this case, the magnets will not be demagnetized.

The maximum peak torque in steady state is 30 Nm. (See also Table 8.1.) The steady state current is almost sinusoidal and has an RMS-value of 65 A, which is in good agreement with the analytical value of Equation (8.17).
1-phase inverter short circuit

Fig. 8.18 Short circuit currents at a 1-phase inverter short circuit.

The 1-phase inverter short circuit was introduced at 86.8 ms, i.e. at a moment in time when the no-load phase-to-phase voltage $u_{RS}$ changed from negative to positive. The maximum transient peak torque of 703 Nm was obtained at 90.6 ms. The maximum peak current of 341 A appears in phase S at 92.1 ms, see Fig. 8.18.

The minimum resulting flux densities in the magnets appear at 94.8 ms. The resulting flux density components normal and tangential to a line through the centre of the left and the right magnets at 94.8 ms are shown

Fig. 8.19 Flux densities in the centre of the left and right magnets at no-load, and 8.0 ms after a 1-phase inverter short circuit. (n: normal, t: tangential)
in Fig. 8.19. It can be seen that there is no risk of demagnetisation, since the resulting normal component of the flux density in the magnet is above zero and the tangential component is small.

The maximum peak torque in steady state is 417 Nm. The direct current of phase S, i.e. the average value, in steady state is 124 A, see Table 8.1.

The results from the FEM simulations of the four different fault conditions are summarized in Table 8.1.

Table 8.1 Results of the FEM simulations for the four fault conditions. The worst numbers are in bold text. (The rated torque of the PM motor is 98 Nm and the rated current is 30 A_{RMS}).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Fault #1 (1-ph)</th>
<th>Fault #2 (2-ph)</th>
<th>Fault #3 (3-ph)</th>
<th>Fault #4 (DC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient peak torque [Nm]</td>
<td>623</td>
<td>687</td>
<td>494</td>
<td>703</td>
</tr>
<tr>
<td>Steady state peak torque [Nm]</td>
<td>205</td>
<td>163</td>
<td>30</td>
<td>417</td>
</tr>
<tr>
<td>Transient current peak [A]</td>
<td>400</td>
<td>260</td>
<td>270</td>
<td>341</td>
</tr>
<tr>
<td>Steady state current peak [A]</td>
<td>183</td>
<td>112</td>
<td>94</td>
<td>160, 264, 196 (R, S, T)</td>
</tr>
<tr>
<td>Steady state current [A_{RMS}]</td>
<td>102</td>
<td>69</td>
<td>65</td>
<td>80, 147, 95 (R, S, T)</td>
</tr>
<tr>
<td>Steady state direct current [A]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>56, 124, 68 (R, S, T)</td>
</tr>
<tr>
<td>Lowest flux density [T] (left magnet, n-comp.)</td>
<td>0,47</td>
<td>0,44</td>
<td>0,37</td>
<td>0,33</td>
</tr>
<tr>
<td>Lowest flux density [T] (right magnet, n-comp.)</td>
<td>0,50</td>
<td>0,47</td>
<td>0,39</td>
<td>0,36</td>
</tr>
</tbody>
</table>

It has not been investigated how a transient torque of 703 Nm would effect the rotor mechanically.

Conclusions
The time-stepping FEM simulations show that the analytically calculated values of the steady state short circuit currents are not correct, except in the three-phase case. The FEM simulations also show that single phase
short circuits are the worst faults:

- A 1-phase short circuit (#1.) gives the highest transient peak current (400 A).
- A 1-phase inverter short circuit (#4.) gives the highest transient peak torque (703 Nm).
- A 1-phase inverter short circuit (#4.) gives the highest steady state peak torque (417 Nm).
- A 1-phase inverter short circuit (#4.) gives the lowest resulting flux density levels in the permanent magnets (here a reduction from 0.67 T to 0.33 T).

In [85] it is even suggested to transfer a 1-phase inverter short circuit into a 3-phase short circuit by turning on the corresponding two upper or lower valves of the inverter. This measure reduces both the risk of demagnetization of the permanent magnets and reduce the braking torque [85].

In our case, the resulting flux density levels in the magnets, in the direction of magnetization, are above zero, implying that the magnets are not demagnetized. The conductivity of the magnets was set to zero in the FEM simulations. In the real PM machine the magnets will be even better protected than what is predicted by the FEM simulations, due to the eddy currents that will flow inside the magnets. On the other hand, these eddy currents may cause severe heating of the magnets, and thereby a loss of the magnetization, if the fault occurs for a long period of time. This has not been further investigated.

### 8.3 Conclusions

For the compact PM integral motor, high-frequency rotor cage losses and short circuit fault conditions have been simulated with fixed-speed time-stepping 2D-FEM calculations. The rotor losses are negligible and the magnets are well protected from demagnetizing currents. A transient torque of about seven times the rated torque can appear if a 1-phase inverter short circuit occurs.

The following chapter concludes the thesis and propose possible future work.
9 Conclusions and future work

9.1 Conclusions

In this thesis a vision of tomorrow’s integral motor has been presented. It has been shown that it is possible manufacture a compact, sensorless, variable speed permanent magnet synchronous integral motor for 15 kW and 1500 r/min. The outer dimensions are approximately the same as for the equivalent standard induction motor.

A brief description of control strategies for pumps and fans (blowers) were given. Pumps and fans are suitable loads for variable/adjustable speed motors, since enormous energy savings can be made by reducing the speed instead of throttling/choking the flow of the pump/fan. Numerous induction integral motors that are, or have been, commercially available were listed. It is shown that the installation and use of a PM integral motor for speed control is advantageous compared to installing an induction motor with a separate converter, and will probably pay-off in less than a year. The present value of the monetary saving that can be made, due to reduced energy costs, can be several thousands of SEK (hundreds of €/$). It was also shown that the high price of NdFeB-magnets is no excuse to use less magnet material, particularly if a reasonable profit on the PM integral motor is required.

A totally analytical expression for the airgap flux density of permanent magnet motors with buried magnets has been derived. The analytical expression includes axial leakage and iron saturation of the most narrow part of the magnetic circuit of the machine. It shows satisfactory agreement with FEM calculations. The axial leakage flux reduces the radial, torque producing flux, and its influence was estimated with an analytical 2D model. A 3D-FEM calculation showed that the 2D model might underestimate the axial leakage. The influence of the axial leakage flux is normally negligible, but for motors with relatively large airgaps and relatively short rotor lengths it is higher and must be taken into consideration. An analytical-iterative calculation method - which includes saturations of all teeth and both yokes - was also tried, but it is slightly too complex for practical implementation and use.

An optimization program for PM motors was developed. It gives the set
of parameters of the desired PM motor having the lowest losses at a certain torque and speed, according to the used loss models. The optimization program was used to suggest near-optimum pole numbers of inverter-fed PM motors, for a desired power and speed. The investigated power and speed ranges are 4 kW to 37 kW and 750 r/min to 3000 r/min, respectively. The suggestions confirm the commonly known rule of thumb: higher speed - lower number of poles and vice versa. It was also shown that inverter-fed PM motors with buried magnets should have relatively large airgaps to obtain high efficiency, at least if only q-current is used for the torque production. A large airgap will reduce the armature reaction flux, and thereby the iron losses. The large airgap will also reduce the load and no-load stray losses of the rotor surface. To retain the airgap flux density from the magnets, a high magnet mass then has to be used. The obtained relatively large airgaps should probably be slightly smaller if axial leakage is taken into consideration. It was also shown that NdFeB-magnets, instead of Ferrites, are required to obtain a compact buried PM motor design.

The optimization program was used to obtain design parameters for the manufactured compact 15 kW 1500 r/min prototype PM motor. The results were compared with 2D-FEM calculations and the agreement was satisfactory. The novel concept of stator integrated filter coils was proposed and investigated. The four coils serve as line and intermediate-link filters, and are integrated in the two upper “corners” of the stator core. The corner coils were designed not to interfere with each other, nor the magnetic circuit of the stator.

Measurements made on the manufactured PM motor agree quite well with the analytical and/or FEM calculated values. Measurements also show that the stator integration of filter coils has been successful and the interference is negligible. Temperature measurements on the stator winding and the (final) converter heat-sink show moderate or acceptable temperature rises. Temperature measurements during load tests on the complete PM integral motor show that it can operate with a torque of 100 Nm at speeds between 375 r/min and 1500 r/min. At 1500 r/min, the torque can even be increased to 120 Nm. The rated torque is 98 Nm. Efficiency measurements on the PM integral motor, its PM motor and converter show good agreement with calculations. A calorimetric measurement method was employed to increase the accuracy for one operating point. For the operating point 98 Nm and 1460 r/min, the PM integral motor has an overall efficiency of 90.8% (+/- 0.07 percentage units).
Finally, the high-frequency losses of the rotor cage - which were neglected in the optimization program - were investigated by the use of fixed-speed time-stepping 2D-FEM calculations. These losses turned out to be around 22 W, which is negligible for a 15 kW motor. The FEM software was also used to investigate four fault conditions of the PM motor: 1-, 2- and 3-phase short circuits, and short circuit of one of the diodes or valves of the inverter. The last fault turned out to be the worst case, with a peak torque of about seven times the rated torque. The buried magnets were well-protected against demagnetization in all the four cases.

9.2 Future work

The first logical continuation of the present work would be to subject the prototype to a long-term test. The long-term test will give an indication to the robustness of the PM integral motor when exposed to an industrial environment (vibration, dust, moisture, voltage sags etc.).

The losses of the PM motor were measured globally. It could be of interest to conduct experiments to split up the losses in their different terms. The model for the iron losses with load would probably need to be further investigated as the analytical expression for the airgap flux density from the stator current did not agree satisfactorily with FEM calculations before correction.

Many assumptions were done to obtain the model of the axial leakage. It could be interesting to use 3D-FEM calculations to help improving the analytical expression.

The motor is started with an open-loop control scheme. At about 10% of nominal speed, sensorless operation takes over. Since the motor has some magnetic saliency, the initial rotor position algorithm could be used during start-up. However, the rotor cage could interfere with this method. Therefore, the possible use of the initial rotor position algorithm could be investigated. Another more general question is: does the rotor cage have a positive or negative impact on the control of the motor?

Many interesting studies could also be conducted on the power electronics part of the PM integral motor:

The PM integral motor has a diode bridge rectifier and a small DC-link
capacitor. Therefore it can hardly cope with any re-generated power, e.g. from braking. Solutions to this can be made in the rectifier circuit, in the intermediate link or in the PM motor control software, and could be examined.

Improving the curve forms of the line currents of the converter is another interesting subject. This could be done in order to meet future possible legislations in this area, something which is already going on in e.g. [43]. The inductance of the corner coils is not so high. Improvements in the design of the corner coils, to increase the inductance, would contribute to the work mentioned above.

The use of other types of converters, e.g. matrix converters, is an interesting subject. The escalating development of silicon carbide semiconductors might open up for novel solutions for the converter circuit. It could therefore be subject for further studies, as well.

With a few improvements and an increased sensitivity to energy savings, the concept of the PM integral motor we developed could surely become a product of tomorrow.
References


[54] H.-P. Nee, Royal Institute of Technology, Sweden. Personal communication.


[64] B. Rydholm, ABB Corporate Research, Sweden. Personal communication.


### List of symbols

#### Lower-case letters

- $a$: Flux density value found from a curvefit etc.
- $a_g$: Acceleration due to gravity. (About 9.81 m/s$^2$ on Earth.)
- $b$: Flux density value found from a curvefit etc.
- $c$: 1.) Number of parallel-connected coils in a winding. 2.) Flux density enhancement factor.
- $d_y$: Thickness of the stator yoke.
- $d_{y, \text{min}}$: Minimum allowable thickness of the stator yoke.
- $e$: Instantaneous value of induced voltage.
- $e_{cc}$: Peak value of induced voltage in a corner coil.
- $f$: Frequency.
- $f_{cog}$: Fundamental mechanical frequency of the cogging torque.
- $f_{\text{critical}}$: First critical speed of the rotor.
- $f_{\text{mains}}$: Fundamental frequency of the mains.
- $f_s$: Fundamental electrical stator frequency.
- $f_{\text{sw}}$: Switching frequency of the converter.
- $i$: 1.) Instantaneous value of current. 2.) Inflation
- $g$: The airgap length of the motor.
- $g_e$: Fictitious extra airgap to compensate for iron saturations.
- $h$: 1.) Height above/below the equivalent magnet. 2.) Factor for slots.
- $h_1$: Height above the equivalent magnet.
- $h_2$: Height below the equivalent magnet.
- $i_1, i_2, \ldots, i_m$: Inflation from year 0 to 1, 1 to 2, ..., m-1 to m.
- $k$: A positive integer.
- $k_a$: Axial leakage factor.
- $k_c$: The Carter factor.
- $k_{\text{corr, axi}}$: Correction factor for $k_a$ from 3D-FEM.
- $k_{\text{corr, m}}$: Correction factor for airgap flux density from the magnets.
- $k_{\text{corr, s}}$: Correction factor for airgap flux density from the stator current.
- $k_{cs}$: Empirical value of the thermal resistance.
- $k_e$: Coefficient for calculating eddy-current losses.
- $k_E$: Induced voltage per winding turn.
- $k_{f, Cu}$: Copper fill factor of the stator winding.
- $k'_{f, Cu}$: Copper fill factor of the manufactured stator winding.
Stacking factor for iron lamination.

Coefficient for calculating hysteresis losses.

Equals two if most narrow part carries half the pole flux, else unity.

Factor for increased resistance due to skin effect.

Resistive voltage drop per winding turn.

Inductive voltage drop per winding turn.

Winding factor for space harmonic number $u$.

Distribution factor for space harmonic number $u$.

Pitch factor for space harmonic number $u$.

Skew factor for space harmonic number $u$.

A positive integer.

The (different) flux barrier lengths under one pole.

Mean length of the flux path of a corner coil.

The airgap length of a corner coil.

Thickens of the internal airgap (in the magnet slot).

Left length of the “cogging-rod”.

Magnet thickness, equal for all magnets under one pole.

Mean value of left $l_{left}$ and right $l_{right}$ rod length.

Sum of $l_{mi}$ and $\mu_{s}l_{i}$.

Length of the most narrow iron part, under one pole.

Right length of the “cogging-rod”.

Thickness of magnet slot.

1.) Modulation index. 2.) A positive integer.

Mass of the copper winding of the stator.

Mass of the manufactured copper winding of the stator.

Mass of the copper winding of the stator, compensated for decreased copper fill factor.

Mass of the bulk iron lamination (square-shaped).

Mass of the cut iron lamination (excluding corner coils).

Mass of the neodymium-iron-boron magnets.

1.) Mechanical shaft speed. 2.) A positive integer.

Number of stator winding turns per slot.

Number of poles.

Mean value of electrical energy price year $k$.

Pulse number of the rectifier diode bridge.

Mean value of electrical energy price year 0, 1, ..., $m$. 
$p_{(1)Fe}(\hat{B}_{(1)})$  Fundamental iron loss density versus the peak value of the fundamental flux density.
$q$  Number of stator slots per pole per phase.
$q_{air}$  Airflow of the air inlet tube.
$q_{integral}$  Airflow to the PM integral motor.
$q_{standard}$  Airflow to the standard induction motor.
$r$  1.) Rotor radius. 2.) Interest rate.
$r_{io}$  Radius inside the air inlet tube where the airspeed starts to drop.
$r_{max}$  Maximum rotor radius.
$r_{o}$  Outer radius of the air inlet tube.
$r_{so}$  Outer radius of stator core.
$r_1, r_2, \ldots, r_m$  Interest rate year 1, 2, ..., m.
$t$  Time.
$u_{cc}$  Instantaneous voltage of a corner coil.
$v_{air}$  Velocity of the airflow in the air inlet tube.
$v_e$  Exponent for calculating eddy-current losses.
$v_{fluid}$  Velocity of the cooling fluid.
$v_h$  Exponent for calculating hysteresis losses.
$v_{integral}$  Velocity of the airflow in the air inlet tube to the integral motor.
$v_{standard}$  Velocity of the airflow in the air inlet tube to the standard motor.
$W_{b1}, W_{b2}, \ldots$  The (different) flux barrier widths under one pole.
$W_{cc}$  Width of the flux path, i.e. the outer iron leg, of a corner coil.
$W_{Fe}$  The sum of the (different) iron bridge widths under one pole.
$W_{Fe1}, W_{Fe2}, \ldots$  The (different) iron bridge widths under one pole.
$W_g$  The true circumferential pole width on the rotor surface.
$W_m$  The sum of the (different) magnet widths under one pole.
$W_{m1}, W_{m2}, \ldots$  The (different) magnet widths under one pole.
$W_{nar}$  Width of the most narrow iron part, under one pole.
$W_s$  Stator slot width at the airgap (neglecting semi-closed slots).
$W_t$  Stator tooth width.
$w' / l'$  Equivalent width to length ratio.
$y_{sp}$  The pitch (in number of slots) of a short-pitched coil.

**Capitals**

$A$  Vector magnetic potential.
$A_{Cu,slot}$  Total copper area per stator slot.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A'_\text{Cu,slot}$</td>
<td>Manufactured total copper area per stator slot.</td>
</tr>
<tr>
<td>$A_\text{slot}$</td>
<td>Total area per stator slot.</td>
</tr>
<tr>
<td>$A_\text{slot-opening}$</td>
<td>Area of slot-opening and semi-closure.</td>
</tr>
<tr>
<td>$A_\text{surface}$</td>
<td>Area of a surface in general.</td>
</tr>
<tr>
<td>$B_g$</td>
<td>The magnitude of the rectangular/quasi-square airgap flux density.</td>
</tr>
<tr>
<td>$B_{\text{iron,cc}}$</td>
<td>Desired flux density of the iron in a corner coil.</td>
</tr>
<tr>
<td>$B'_\text{iron,cc}$</td>
<td>Manufactured flux density of the iron in a corner coil.</td>
</tr>
<tr>
<td>$B^{\prime}_{n+1}$</td>
<td>The present value of the monetary saving that is made the first year after pay-off, i.e. year $n+1$ if the pay-off time was $n$ years.</td>
</tr>
<tr>
<td>$B_r$</td>
<td>Remanent flux density of the magnet at temperature $T_{\text{mag}}$.</td>
</tr>
<tr>
<td>$B_{\text{sat}}$</td>
<td>Assumed saturated flux density level of iron.</td>
</tr>
<tr>
<td>$B_{(1)g,m}$</td>
<td>RMS value of the fundamental airgap flux density from the magnet.</td>
</tr>
<tr>
<td>$B_{(1)g}$</td>
<td>Peak value of the fundamental airgap flux density.</td>
</tr>
<tr>
<td>$B_{(1)g,m}$</td>
<td>Peak value of the fundamental airgap flux density from the magnet.</td>
</tr>
<tr>
<td>$B_{(1)g,ms}$</td>
<td>Peak value of the fundamental airgap flux density from magnet and stator current.</td>
</tr>
<tr>
<td>$B_{(1)s}$</td>
<td>Peak value of the fundamental airgap flux density from the stator current.</td>
</tr>
<tr>
<td>$B_{(1)t,\text{max}}$</td>
<td>Maximum allowable peak value of the fundamental flux density in the stator teeth.</td>
</tr>
<tr>
<td>$B_{(1)t,ms}$</td>
<td>Peak value of the fundamental flux density in the stator teeth, due to magnet and stator current.</td>
</tr>
<tr>
<td>$B_{(1)y,\text{max}}$</td>
<td>Maximum allowable peak value of the fundamental flux density in the stator yoke.</td>
</tr>
<tr>
<td>$B_{(1)y,ms}$</td>
<td>Peak value of the fundamental flux density in the stator yoke, due to magnet and stator current.</td>
</tr>
<tr>
<td>$B_\delta$</td>
<td>Flux density in the airgap of a corner coil.</td>
</tr>
<tr>
<td>$C$</td>
<td>1.) A constant in general. 2.) Capacitance. 3.) Contour of surface.</td>
</tr>
<tr>
<td>$C_k$</td>
<td>The cost year $k$.</td>
</tr>
<tr>
<td>$C_n$</td>
<td>The future cost year $n$.</td>
</tr>
<tr>
<td>$C^{\prime\prime}_n$</td>
<td>The present value of the future cost year $n$.</td>
</tr>
<tr>
<td>$C_0$</td>
<td>The cost today.</td>
</tr>
<tr>
<td>$E, E_{\text{ph}}$</td>
<td>RMS value of induced phase voltage.</td>
</tr>
<tr>
<td>$E_a$</td>
<td>The price of a standard induction motor.</td>
</tr>
<tr>
<td>$E_{a,\text{conv}}$</td>
<td>The price of the standard induction motor plus converter and installation costs.</td>
</tr>
<tr>
<td>$E_i$</td>
<td>The price of an integral motor.</td>
</tr>
<tr>
<td>$E_n$</td>
<td>Rated phase voltage (RMS).</td>
</tr>
<tr>
<td>$E_{(1)ph}$</td>
<td>RMS value of induced fundamental phase voltage.</td>
</tr>
<tr>
<td>$F$</td>
<td>Force.</td>
</tr>
</tbody>
</table>
\( \mathcal{J}_m \) \hspace{1cm} MMF of the magnet.
\( \mathcal{J}_{Th} \) \hspace{1cm} MMF of the magnetic Thévenin-equivalent.
\( \mathcal{J}(x) |_{(n)} \) \hspace{1cm} Fourier analysis of \( x \) with respect to component number \( n \).
\( H \) \hspace{1cm} Head of a pump.
\( H_c \) \hspace{1cm} Coercive magnetic field intensity of the magnet.
\( H_{c,B = 0} \) \hspace{1cm} \( H_c \) obtained by extending the linear part of the curve to \( B = 0 \).
\( I \) \hspace{1cm} 1.) Stator current phasor. 2.) RMS value of a current in general.
\( I_{cc,1}, \ldots, I_{cc,4} \) \hspace{1cm} RMS value of the current through corner coil 1 to 4.
\( I_{cc} \) \hspace{1cm} Peak value of the current in a corner coil.
\( I_d \) \hspace{1cm} Current in d-direction.
\( I_d^* \) \hspace{1cm} Desired current in d-direction.
\( I_{DC} \) \hspace{1cm} DC current.
\( I_{DC, equiv} \) \hspace{1cm} Equivalent DC current, representing the copper losses of the four corner coils.
\( I_m \) \hspace{1cm} Current of a single-turn coil, used to model a magnet in FEM.
\( I_n \) \hspace{1cm} RMS value of rated stator current.
\( I_q \) \hspace{1cm} Current in q-direction.
\( I_{quasi-square} \) \hspace{1cm} Peak value/Magnitude of the quasi-square shaped current on the mains-side of the converter.
\( I_s \) \hspace{1cm} RMS value of the (fundamental) stator current.
\( I_{(1), mains} \) \hspace{1cm} RMS value of the fundamental current on the mains-side of the converter.
\( J_{Cu} \) \hspace{1cm} RMS value of the current density in the winding copper of the stator.
\( J_{Cu, cc} \) \hspace{1cm} RMS value of the current density in the winding copper of a corner coil.
\( K_i \) \hspace{1cm} The cost of the converter.
\( K_{(1)s} \) \hspace{1cm} The current loading, i.e. the RMS value of the fundamental linear current density of the stator.
\( L \) \hspace{1cm} Axial length of rotor or stator.
\( L_{cc} \) \hspace{1cm} Inductance of a corner coil.
\( L'_{cc} \) \hspace{1cm} Manufactured inductance of a corner coil.
\( L_{endwind, tot} \) \hspace{1cm} The axial length of the two end windings.
\( L_{filter} \) \hspace{1cm} Inductance of a filter coil.
\( L_{leak} \) \hspace{1cm} Leakage inductance per winding of the motor. Here equivalent to phase leakage inductance due to Y-connection of the motor.
\( L_{slot, leak} \) \hspace{1cm} Slot leakage inductance per winding of the motor. Here equivalent to phase slot leakage inductance due to Y-connection of the motor.
\( L_{end, leak} \) \hspace{1cm} End leakage inductance per winding of the motor. Here equivalent to phase end leakage inductance due to Y-connection of the motor.
\( L_{max} \) \hspace{1cm} Maximum available axial length for the stator core plus the two end windings.
$L_{md}$ Magnetizing inductance in d-direction.
$L_{mq}$ Magnetizing inductance in q-direction.
$L_{r,max}$ Maximum value of axial rotor length.
$L_{(1)cc}$ Fundamental inductance of a corner coil.
$L_{(1)cc12}$ Fundamental mutual inductance between corner coil 1 and 2.
$M_F$ Material cost of a standard induction motor.
$M_i$ Material cost (except permanent magnets and converter) of a PM integral motor.
$M_S$ RMS value of the MMF per stator slot.
$M_p$ Peak value of the MMF per stator slot.
$N_c$ Number of turns in general.
$N_{cc}$ Number of winding turns of a corner coil.
$N_{k}$ Combined interest rate and inflation factor for year k.
$P$ Power
$P_{bars}$ Average total power loss of the rotor bars at steady-state.
$P_{bear}$ Power losses due to friction of bearings and seals.
$P_{Cu}$ (Fundamental) resistive losses of the stator copper winding.
$P_{Cu,cc1}, \ldots, P_{Cu,cc4}$ Copper losses of corner coil 1 to 4.
$P_{Cu,r}$ Copper losses of the rotor.
$P_{end,stray}$ Stray load losses from the end windings.
$P_{fan}$ Power losses due to the fan.
$P_{Fe}$ Iron losses.
$P_{Fe,cc1-3}$ Iron losses of corner coil 1 to 3.
$P_i$ The cost of permanent magnets.
$P_{in}$ Input power.
$P_{loss}$ 1.) The sum of fundamental copper and iron losses, fan, windage and bearing friction losses. 2.) Power loss in general.
$P_{out}$ Output power.
$P_{shaft}$ Mechanical output power of the shaft of the motor.
$P_{windage}$ Power losses due to air friction of the rotating rotor surfaces.
$P_{(1)CuFe}$ Fundamental copper and iron losses.
$P_{(1)Fe,ms}$ Fundamental iron losses due to magnet and stator current.
$P_{(1)te,ms}$ Fundamental eddy-current losses in the stator teeth, due to magnet and stator current.
$P_{(1)th,ms}$ Fundamental hysteresis losses in the stator teeth, due to magnet and stator current.
$P_{(1)ye,ms}$ Fundamental eddy-current losses in the stator yoke, due to magnet and stator current.
$P_{(1)yh,ms}$ Fundamental hysteresis losses in the stator yoke, due to magnet and stator current.
Flow of a pump or a fan. 2.) Number of stator slots.
Number of rotor slots.
Number of stator slots.
DC resistance of a corner coil.
Resistance of an eddy current loop in the iron material CK27.
Resistance of an eddy current loop in the iron material DK70.
DC resistance of a filter coil.
Axial reluctance, "seen" by the magnets, under one pole.
Resulting reluctance of the flux barriers, under one pole.
Reluctance of flux barrier 1, 2, ... under one pole.
Reluctance of the most narrow iron part, under one pole.
Resulting reluctance of the saturated iron bridges, under one pole.
Reluctance of saturated iron bridge 1, 2, ... under one pole.
Reluctance of the internal extra airgap, surrounding the magnet, under one pole.
Leakage reluctance of slots and end windings.
Internal reluctance of the magnets, under one pole.
Sum of the reluctance of the magnet and the internal airgap surrounding the magnet, under one pole.
Stator winding DC resistance. Here equivalent to phase resistance due to Y-connection of the motor.
Thermal resistance.
Thermal resistance from integral motor stator copper to ambient cooling air.
Thermal resistance from standard induction motor stator copper to cooling air.
Sum of reluctances of stator and rotor teeth and yokes.
Resulting reluctance of the rotor teeth.
Resulting reluctance of the stator teeth.
Internal reluctance of the magnetic Thévenin-equivalent.
Reluctance of the rotor yoke.
Reluctance of the stator yoke.
Life-cycle cost (except installation, maintenance and recycling etc.) of a standard induction motor.
Life-cycle cost (except installation, maintenance and recycling etc.) of an integral motor.
1.) Torque. 2.) Period time.
Manufacturing cost of a standard induction motor.
The ambient temperature.
Cogging torque.
Temperature of the stator copper winding.
\( T_{Cu,\text{start}} \) Temperature of the stator copper winding when the measurement starts.
\( T_{\text{fluid}} \) Temperature of the cooling fluid.
\( T_i \) Manufacturing cost of an integral motor.
\( T_{\text{mag}} \) 1.) Magnet torque. 2.) Temperature of the magnets.
\( T_{\text{magnet}} \) Temperature of the magnets.
\( T_{\text{surface}} \) Temperature of a surface in general.
\( T_{\text{tot}} \) The sum of cogging torque and magnet torque.
\( T_1, \ldots, T_{10} \) Temperatures of measurement points 1 to 10.
\( U \) RMS value of phase voltage.
\( U_d \) DC voltage of the intermediate link.
\( U_{\text{diode}} \) On-stage voltage drop of a rectifier diode.
\( U_{\text{DC}} \) DC voltage.
\( U_{\text{IGBT}} \) On-stage voltage drop of an inverter IGBT.
\( U_{\text{induced}} \) RMS value of voltage induced in iron lamination.
\( U_{\text{(1)cc}} \) RMS value of the fundamental voltage across a corner coil.
\( U_{\text{(1)i-l,\text{mains}}} \) Peak value of the fundamental line-to-line voltage of the mains.
\( U_{\text{(1)i-l,\text{quasi-square,max}}} \) RMS value of the inverter output line-to-line voltage with square-shaped reference waves.
\( U_{\text{(1)i-l,\text{sine-PWM,max}}} \) RMS value of the inverter output line-to-line voltage with sine-shaped reference waves.
\( U_{\text{(1)i-l,\text{quasine-PWM,max}}} \) RMS value of the inverter output line-to-line voltage with quasine-shaped reference waves.
\( U_{\text{(1)ph,PM}} \) RMS value of the total fundamental phase voltage of the PM motor, including resistance and leakage inductance voltage drops.
\( U_{\text{(1)phX}} \) RMS value of the fundamental inductive phase voltage across the magnetizing inductance.
\( U_{\text{(1)R,\text{filter}}} \) RMS value of the fundamental resistive voltage drop of the filter coil.
\( U_{\text{(1)X,\text{filter}}} \) RMS value of the fundamental inductive voltage drop of the filter coil.
\( V_a \) Sum of profit, sales & administration costs, and overhead costs of a standard induction motor.
\( V_i \) Sum of profit, sales & administration costs, and overhead costs of an integral motor.
\( V_t \) The volume of the stator teeth.
\( V_y \) The volume of the stator yoke.
\( W_{k,\text{loss}} \) The loss energy year k.
\( W_1, W_2, \ldots, W_m \) Shaft energy consumption year 1, 2, ..., m.
\( X_d \) Reactance in d-direction.
\( X_q \) Reactance in q-direction.
\( X_{md} \) Magnetizing reactance in d-direction.
\( X_{mq} \) Magnetizing reactance in q-direction.
Lower-case greek letters

\( \alpha \)  
1.) The electrical angle of half the true pole width on the rotor surface. 2.) Substituted parameter.

\( \alpha_{\text{cog.min}} \)  
A value of the angle \( \alpha \) that probably will minimize cogging torque.

\( \alpha_{Cu} \)  
Resistance temperature coefficient of copper.

\( \alpha_{\text{heat}} \)  
Heat transfer coefficient.

\( \alpha_{\text{integral}} \)  
Heat transfer coefficient of the integral motor.

\( \alpha_{\text{rot}} \)  
Mechanical rotation angle of the rotor.

\( \alpha_{\text{standard}} \)  
Heat transfer coefficient of the standard induction motor.

\( \alpha_{\text{slot}} \)  
The angle of the slot at the stator airgap surface, neglecting semi-closed slot-openings.

\( \alpha_{\text{slot-pitch}} \), \( \alpha_{sp} \)  
The angle of the slot-pitch at the stator airgap surface.

\( \beta \)  
1.) Electrical angle between magnet flux and stator current phasor. 2.) Substituted parameter.

\( \gamma \)  
The slot to slot-pitch ratio.

\( \gamma_c \)  
Coefficient used for calculating the Carter factor.

\( \gamma_r \)  
Number of “active” (i.e. flux-conducting) rotor teeth.

\( \gamma_s \)  
Number of “active” (i.e. flux-conducting) stator teeth.

\( \delta \)  
Mass density.

\( \delta_{Fe} \)  
Mass density of the non-laminated iron material.

\( \eta \)  
Efficiency.

\( \eta_a \)  
Efficiency of a standard induction motor.

\( \eta_{\text{conv}} \)  
Efficiency of the converter.

\( \eta_{\text{fan}} \)  
Efficiency of a fan (blower).

\( \eta_i \)  
Efficiency of an integral motor.

\( \eta_{\text{pump}} \)  
Efficiency of a pump.

\( \eta_{1460} \)  
Efficiency of a motor at a shaft speed of 1460 r/min.

\( \eta_{1500} \)  
Efficiency of a motor at a shaft speed of 1500 r/min.

\( \eta_{(1)CuFe} \)  
Efficiency of a motor, based only on fundamental copper and iron losses.

\( \lambda \)  
The slot pitch.

\( \mu_r \)  
Relative permeability of the magnet.

\( \mu_{r,Fe} \)  
Relative permeability of the iron material.

\( \mu_{r,Fe,eq} \)  
Equivalent relative permeability, including lamination fill factor.

\( \mu_0 \)  
Permeability of free space. (\( 4\pi \times 10^{-7} \) H/m)

\( \nu \)  
The space harmonic order number.

\( \xi \)  
Magnetic saliency ratio.

\( \rho_{\text{CK27}} \)  
Resistivity of the iron quality CK27.
\( \rho_{Cu} \) Resistivity of copper.
\( \rho_{DK70} \) Resistivity of the iron quality DK70.
\( \rho_s \) The peripheral length of the skew.
\( \pi \) Circumference to diameter ratio of a circle. \((3.141592654...).\)
\( \sigma \) 1.) The ratio between the true length of the end winding and the average coil pitch. 2.) Conductivity
\( \phi \) Phase angle.
\( \phi_a \) Axial flux (through magnets) per unit magnet width, under one pole.
\( \phi_{el} \) Electrical rotation angle.
\( \phi_{mech} \) Mechanical rotation angle.
\( \omega_{max} \) Maximum electrical angular frequency in field weakening.
\( \omega_j \) Electrical angular frequency of the stator.
\( \omega_{mech} \) Mechanical angular frequency of the rotor shaft.

**Capital greek letters**

\( \Delta p \) Pressure increase or drop.
\( \Delta P_{in} \) Measurement error in the input power.
\( \Delta P_{out} \) Measurement error in the output power.
\( \Delta T_{conv} \) Temperature rise of the air passing through the heat-sink of the converter.
\( \Delta T_{Cu-Fe} \) The temperature drop from stator copper to stator iron.
\( \Delta T_1, \ldots, \Delta T_{10} \) Temperature difference of point 1-10 to the temperature of ambient air.
\( \Gamma \) Substituted parameter.
\( \Lambda \) Magnetic permeance.
\( \phi \) Phase angle of the current, i.e. the angle between voltage and current.
\( \Phi_{cc} \) Flux of a corner coil.
\( \Phi_g \) Airgap flux under one pole.
\( \Phi_m \) Magnet flux phasor.
\( \Phi_{sat} \) Flux required to saturate the iron bridges under one pole.
\( \Psi_{cc} \) Flux linkage of a corner coil.
\( \Psi_{coil} \) Flux linkage of a coil of the stator winding.
\( \Psi_{wind} \) Flux linkage of the stator winding.
Appendix A

Using the vector magnetic potentials of the stator slots to estimate the induced no-load voltage

This appendix gives a suggestion of how to minimize the manual work required for the calculations of the no-load voltage, first in general and then in an example.

If the machine has symmetry in the stator geometry, i.e. all north poles and south poles of the rotor “see” a similar stator geometry at the same instant of time, and if the rotor flux waveform in the airgap is symmetric around the d-axes, it is enough to turn the rotor only half a slot-pitch. Due to this symmetry, the required number of FEM calculations is immediately reduced by a factor two. It is then enough to do the FEM calculations on a geometry consisting of only one single pole. Note that symmetry in the geometry is present even if the left- and right-hand side outer slots are divided in halves. If the symmetries described above are not present, more than one pole of the geometry is required and a total rotation of one full slot-pitch might be needed.

A step-by-step suggestion of how to perform the flux linkage calculations is given as follows:

1. Turn the rotor to a position where a north pole of the rotor, i.e. the d-direction, faces the centre of one of the stator coils
2. Perform a FEM calculation of the field lines, with satisfactory accuracy
3. It is now a good idea to check that the averaged vector magnetic potentials of the stator slots on both sides of the d-axis have values of approximately equivalent magnitudes but with opposite signs
4. Save the first set (index 1) of averaged values of the vector magnetic potentials for all stator slots of the geometry numbered 1 to \( m \) (including any outer left- and right-hand side radial half slots as well), i.e. \( A_{1:1} \) to \( A_{1:m} \) counting in a clock-wise direction.
5. Rotate the rotor counter-clockwise with a mechanical angle (in degrees) of the slot-pitch \( \alpha_{sp} \) divided by e.g. five:

\[
\alpha_{rot} = \frac{1}{5} \cdot \frac{\lambda}{(r + g)} \cdot \frac{180^\circ}{\pi} = \frac{\alpha_{sg}}{5}
\]  

(A.1)
where \( \lambda \) is the slot pitch, \( r \) is the rotor radius and \( g \) is the airgap length. To simplify the Fourier analysis, i.e. to have equi-distant time-steps between the points of the waveform, it is a good idea to choose the rotation angle \( \alpha_{rot} \) in such a way that

\[
k \cdot \alpha_{rot} = \frac{\lambda}{2}
\]

or

\[
(k + 0.5) \cdot \alpha_{rot} = \frac{\lambda}{2}
\]

where \( k \) is a positive integer.

6. Repeat step 4 and 5 (with index 2, 3, ...) for \( k \) times, where \( k \) is given by Equation (A.2).

7. Step 4-6 will result in \( l = k + 1 \) rows, each row containing the vector magnetic potentials of the stator slots of the geometry for that rotor position. This matrix represents the turning of the rotor by half a slot-pitch. If there is symmetry in the stator geometry, the vector magnetic potential values for the continuation of the rotation to a full slot-pitch can be found from these former values. The values are easily obtained since the remaining rotation gives rise to the same values but with an opposite sign, in reverse order and displaced one slot in the counter-clockwise direction. See an example in Fig. A.1. The continuation of this matrix is basically given by performing the following operations:

\[
\begin{align*}
A_{2l,m} &= A_{1,m} & A_{2l-1,m} &= A_{2,m} \\
A_{2l,m-1} &= -A_{1,1} & A_{2l-1,m-1} &= -A_{2,1} \\
A_{2l,m-2} &= -A_{1,2} & A_{2l-1,m-2} &= -A_{2,2} & \ldots (A.3) \\
A_{2l,m-3} &= -A_{1,3} & A_{2l-1,m-3} &= -A_{2,3} \\
\ldots & \ldots & \ldots & \ldots
\end{align*}
\]

Note: If there is no symmetry in the stator geometry after one pole, more than one pole is required and the rotation might have to be continued to a full slot-pitch instead of using the “mirroring” according to (A.3).
8. The flux linkage of the winding, for the rotation angles between 0° and the angle of the slot-pitch \( \alpha_{sp} \), can now be calculated by using Equation (4.15). When using Equation (4.15), the slots which are containing the two sides of the coil have to be identified.

The flux linkage expression, when using a one pole geometry, will have the following shape:

\[
\Psi_{x=1, \ldots, 2}(0^\circ \text{ to } \alpha_{sp}) =
\]

\[
= \frac{p}{2} \cdot \frac{n_s}{c} \cdot L \cdot \left( \frac{A_{x,1}}{h} + A_{x,2} + \ldots \right) - \left( \ldots + A_{x,m-1} + \frac{A_{x,m}}{h} \right)
\]

\[(A.4)\]

where \( n_s \) is the number of winding turns per stator slot, and \( c \) is the number of parallel circuits in the winding. The factor 2 in front of
comes from the symmetric placement of the coil over the pole. (If two poles of the stator geometry are used, the factor 2 have to be replaced by the factor 1.) $h = 1$ if the two outer stator slots are totally inside the geometry and $h = 2$ if they have been split radially in two halves. Note that the only vector magnetic potentials that should be used inside the left-hand side parenthesis in Equation (A.4) are from the slots that contains one side of the coil, and similarly for the right-hand side parenthesis.

The flux linkages of the winding for the following $\alpha_{sp}$ to $2\alpha_{sp}$ mechanical degrees (and so on for $2\alpha_{sp}$ to $3\alpha_{sp}$ etc.) are found by assuming that the coil is moved virtually by one slot in the clockwise direction. Again the slots containing the coil are identified and the flux linkage expression will now be

$$\Psi_{x = 1, \ldots, 2}(\alpha_{sp} \text{ to } 2\alpha_{sp}) =$$

$$= \frac{p}{2} \cdot 2 \cdot \frac{n_s}{c} \cdot L \cdot \left( (A_{x,2} + A_{x,3} + \ldots) - \left( \ldots + \frac{A_{x,m}}{h} \right) \right)$$

(A.5)

This calculation procedure is repeated $i$ times, i.e. until a quarter of an electrical period $T/4$ (or $90^\circ$el.) is within the investigated interval:

$$90^\circ \in \frac{p}{2} \cdot ((i-1)\alpha_{sp} \text{ to } i\alpha_{sp})$$

(A.6)

9. The now obtained set of flux linkage values correspond to electrical angles between $90^\circ$ and $180^\circ$. The flux linkage values from $0^\circ$ to $90^\circ$ are given by “mirroring” the first set of values around the angle $90^\circ$.

10. To obtain a full period, all the flux linkage values from point 9 above are “mirrored” around the x-axis and displaced $180^\circ$.

11. For a sufficiently high number of stator slots and a sufficiently low pole-number it may be enough with one single static FEM calculation of the vector magnetic potentials. Equations (A.4) and (A.5) are still used but only with their first and last positions ($0^\circ$, $\alpha_{sp}$, $2\alpha_{sp}$, $\ldots$), i.e. no turning of the rotor is required.

By using two areas with averaged vector magnetic potentials per stator slot, this method works for a two-layer winding, as well.
As an example, the vector magnetic potential calculations for Motor A are given. Motor A has 8 poles and 2 slots per pole per phase. First, one of the north poles of the rotor was placed in front of the centre of a coil belonging to phase R, see Fig. A.2. A FEM calculation with \( ACE^1 \) of the vector magnetic potentials of the slots was performed and the results are found in the first row of Table A.1.

The vector magnetic potentials were integrated over the slot areas and divided by the slot areas to obtain averaged values. Motor A has a slot-pitch angle of 7.5 mechanical degrees and according to Equations (A.1) and (A.2), a rotation of \( \alpha_{rot} = 1.5 \) mechanical degrees is suitable. The rotor was turned 1.5 mechanical degrees in the counter-clockwise direction, and then another 1.5 degrees. A new FEM calculation was made after each rotation, and the vector magnetic potentials are found in Table A.1, rows 2 and 3, respectively. Rows 4 to 6 are found by “mirroring”, according to Equation (A.3). Totally a number of \( l = k + 1 = 3 \) static FEM calculations were required, and these yield 60 points per period for the flux linkage and voltage waveforms. The results are presented in Table 4.3.

1. FEM program from ABB Corporate Research
Table A.1  Vector magnetic potentials of the stator slots of one pole of Motor A. Arrows show the mirror-procedure for the first to the last row. (Hollow arrow-heads indicate a multiplication by \(-1\).)

<table>
<thead>
<tr>
<th>x</th>
<th>(\alpha_{rot})</th>
<th>(A_{x,1}) [Vs/m]</th>
<th>(A_{x,2}) [Vs/m]</th>
<th>(A_{x,3}) [Vs/m]</th>
<th>(A_{x,4}) [Vs/m]</th>
<th>(A_{x,5}) [Vs/m]</th>
<th>(A_{x,6}) [Vs/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0°</td>
<td>0.0184</td>
<td>0.01139</td>
<td>0.003796</td>
<td>-0.003807</td>
<td>-0.01139</td>
<td>-0.0184</td>
</tr>
<tr>
<td>2</td>
<td>1.5°</td>
<td>0.01729</td>
<td>0.00907</td>
<td>0.002309</td>
<td>-0.005295</td>
<td>-0.01288</td>
<td>-0.01908</td>
</tr>
<tr>
<td>3</td>
<td>3°</td>
<td>0.01594</td>
<td>0.008394</td>
<td>0.009309</td>
<td>-0.006833</td>
<td>-0.01441</td>
<td>-0.01938</td>
</tr>
<tr>
<td>4</td>
<td>(4.5°)</td>
<td>0.01441</td>
<td>0.006833</td>
<td>-0.0007809</td>
<td>-0.008394</td>
<td>-0.01594</td>
<td>-0.01938</td>
</tr>
<tr>
<td>5</td>
<td>(6°)</td>
<td>0.01288</td>
<td>0.005295</td>
<td>-0.002309</td>
<td>-0.009907</td>
<td>-0.01729</td>
<td>-0.01908</td>
</tr>
<tr>
<td>6</td>
<td>(7.5°)</td>
<td>0.01139</td>
<td>0.003807</td>
<td>-0.003796</td>
<td>-0.01139</td>
<td>-0.0184</td>
<td>-0.0184</td>
</tr>
</tbody>
</table>

The three flux linkage expressions for 0 to 22.5 mechanical degrees, i.e. 90 to 180 electrical degrees, are identified by using Equations (A.4), (A.5) etc. and Fig. A.2:

\[
\Psi_{x = 1, \ldots, 6(0^\circ \text{ to } 7.5^\circ)} = \frac{8}{2} \cdot 2 \cdot \frac{15}{1} \cdot 0.11 \cdot (A_{x,1} - A_{x,6}) \quad (A.7)
\]

\[
\Psi_{x = 1, \ldots, 6(7.5^\circ \text{ to } 15^\circ)} = \frac{8}{2} \cdot 2 \cdot \frac{15}{1} \cdot 0.11 \cdot (A_{x,1} + A_{x,2}) \quad (A.8)
\]

\[
\Psi_{x = 1, \ldots, 6(15^\circ \text{ to } 22.5^\circ)} = \frac{8}{2} \cdot 2 \cdot \frac{15}{1} \cdot 0.11 \cdot (A_{x,2} + A_{x,3}) \quad (A.9)
\]

The obtained flux linkage of Motor A is shown in Fig. A.3. Using Equation (4.18) with the synchronous speed \(\omega_s = 2\pi 100\) rad/s gives a fundamental induced no-load voltage with an RMS-value of 202 V.

The waveform of the induced voltage is given from the flux linkage, by the use of Equation (4.16), and is shown in Fig. A.4. A Fourier analysis of the waveform, according to Equation (4.17), gives a fundamental induced no-load voltage with an RMS-value of 202 V.

As mentioned earlier, it is also possible to perform the calculations without the mechanical rotation of the rotor, though with a slightly reduced accuracy. The voltage calculations are then based on the vector magnetic
Fig. A.3  The no-load flux linkage of a winding for Motor A, based on the vector magnetic potentials from three static FEM calculations. Magnets at 20°C.

potential values from one single static FEM calculation, which is equivalent to using only the first (x = 1) and the last (x = 2l) flux linkage values of Table A.1. For Motor A, the flux linkages given by Equations (A.7)-(A.9) with x = 1 and x = 6 and increased to one full electrical period, is shown in Fig. A.5. The points in Fig. A.5 have been joined by straight lines for better visibility. Using Equation (4.18) and the synchronous speed \( \omega_s = 2\pi 100 \text{ rad/s} \) give a fundamental induced no-load voltage with an RMS-value of 202 V. This value can also be found in Table 4.3.
The waveform of the induced voltage is given from the flux linkage, by the use of Equation (4.16). The voltage waveform is shown in Fig. A.6. A Fourier analysis of the waveform (i.e. 12 points), according to Equation (4.17), gives a fundamental induced no-load voltage with an RMS-value of 200 V.

Fig. A.5 The no-load flux linkage of a winding for Motor A, based on the vector magnetic potentials from one single static FEM calculation. The 12 points have been joined by straight lines. Magnets at 20 °C.

Fig. A.6 The induced no-load voltage of a winding for Motor A. Measurement (left) and based on the vector magnetic potentials from one single static FEM calculation (right). The 12 points have been joined by straight lines. Magnets at 20 °C.
Appendix B

Table B1 Measured values of the compact PM integral motor at thermal steady state. Steady state is defined as $dT/dt < 1 ^\circ$C/h.

<table>
<thead>
<tr>
<th>Run-time</th>
<th>T [Nm]</th>
<th>n [r/min]</th>
<th>$U_{1030}$ [V RMS]</th>
<th>$I_{1030}$ [A RMS]</th>
<th>$P_{1030}$ [kW]</th>
<th>$U_{2533}$ [V RMS]</th>
<th>$I_{2533}$ [A RMS]</th>
<th>$P_{2533}$ [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6:00</td>
<td>99.75</td>
<td>1499.5</td>
<td>391.1 k600</td>
<td>26.95 k600</td>
<td>17.19</td>
<td>405.0 k600</td>
<td>30.88 k600</td>
<td>16.66</td>
</tr>
<tr>
<td>4:10</td>
<td>100.0</td>
<td>1249.7</td>
<td>393.4 k600</td>
<td>23.75 k600</td>
<td>14.58</td>
<td>373.7 k600</td>
<td>31.10 k600</td>
<td>14.08</td>
</tr>
<tr>
<td>6:45</td>
<td>99.6</td>
<td>999.6</td>
<td>392.5 k600</td>
<td>20.06 k600</td>
<td>11.79</td>
<td>335.4 k600</td>
<td>30.92 k600</td>
<td>11.31</td>
</tr>
<tr>
<td>6:25</td>
<td>100.1</td>
<td>749.7</td>
<td>397.3 k600</td>
<td>16.24 k600</td>
<td>9.132</td>
<td>293.5 k600</td>
<td>31.14 k600</td>
<td>8.69</td>
</tr>
<tr>
<td>6:00</td>
<td>100.1</td>
<td>500.1</td>
<td>393.7 k600</td>
<td>13.08 k600</td>
<td>6.457</td>
<td>242.9 k600</td>
<td>31.31 k600</td>
<td>6.04</td>
</tr>
<tr>
<td>4:15</td>
<td>75.4</td>
<td>1499.25</td>
<td>395.9 k600</td>
<td>20.70 k600</td>
<td>12.903</td>
<td>401.4 k600</td>
<td>22.872 k600</td>
<td>12.55</td>
</tr>
<tr>
<td>3:30</td>
<td>75.1</td>
<td>1250.1</td>
<td>395.0 k600</td>
<td>18.82 k600</td>
<td>10.798</td>
<td>368.4 k600</td>
<td>22.775 k600</td>
<td>10.48</td>
</tr>
<tr>
<td>4:35</td>
<td>74.9</td>
<td>999.6</td>
<td>391.1 k600</td>
<td>15.84 k600</td>
<td>8.710</td>
<td>327.0 k600</td>
<td>22.598 k600</td>
<td>8.40</td>
</tr>
<tr>
<td>5:20</td>
<td>74.7</td>
<td>750.8</td>
<td>392.0 k600</td>
<td>12.91 k600</td>
<td>6.642</td>
<td>285.2 k600</td>
<td>22.444 k600</td>
<td>6.352</td>
</tr>
<tr>
<td>4:30</td>
<td>74.9</td>
<td>500.2</td>
<td>397.5 k600</td>
<td>10.60 k600</td>
<td>4.610</td>
<td>237.5 k600</td>
<td>22.420 k600</td>
<td>4.336</td>
</tr>
<tr>
<td>4:00</td>
<td>50.75</td>
<td>1499.5</td>
<td>391.9 k600</td>
<td>14.99 k600</td>
<td>8.725</td>
<td>394.4 k600</td>
<td>15.394 k600</td>
<td>8.51</td>
</tr>
<tr>
<td>4:40</td>
<td>50.1</td>
<td>1249.2</td>
<td>395.5 k600</td>
<td>13.80 k600</td>
<td>7.200</td>
<td>363.4 k600</td>
<td>15.080 k600</td>
<td>7.01</td>
</tr>
<tr>
<td>3:50</td>
<td>50.5</td>
<td>999.5</td>
<td>388.1 k600</td>
<td>11.38 k600</td>
<td>5.849</td>
<td>323.6 k600</td>
<td>15.100 k600</td>
<td>5.655</td>
</tr>
<tr>
<td>4:00</td>
<td>49.7</td>
<td>751.4</td>
<td>394.2 k600</td>
<td>9.337 k600</td>
<td>4.386</td>
<td>281.2 k600</td>
<td>14.781 k600</td>
<td>4.202</td>
</tr>
<tr>
<td>4:05</td>
<td>49.8</td>
<td>500.1</td>
<td>395.2 k600</td>
<td>7.234 k600</td>
<td>3.007</td>
<td>231.2 k600</td>
<td>14.712 k600</td>
<td>2.834</td>
</tr>
<tr>
<td>4:15</td>
<td>25.0</td>
<td>1499.3</td>
<td>392.5 k600</td>
<td>9.208 k600</td>
<td>4.442</td>
<td>390.3 k600</td>
<td>7.909 k600</td>
<td>4.331</td>
</tr>
<tr>
<td>5:10</td>
<td>25.2</td>
<td>1249.2</td>
<td>395.6 k600</td>
<td>8.215 k600</td>
<td>3.734</td>
<td>359.6 k600</td>
<td>7.824 k600</td>
<td>3.623</td>
</tr>
<tr>
<td>4:00</td>
<td>25.1</td>
<td>999.4</td>
<td>396.3 k600</td>
<td>7.492 k600</td>
<td>2.990</td>
<td>321.3 k600</td>
<td>7.705 k600</td>
<td>2.884</td>
</tr>
<tr>
<td>4:05</td>
<td>25.4</td>
<td>749.8</td>
<td>397.9 k600</td>
<td>6.234 k600</td>
<td>2.286</td>
<td>278.3 k600</td>
<td>7.695 k600</td>
<td>2.185</td>
</tr>
<tr>
<td>4:10</td>
<td>24.8</td>
<td>500.2</td>
<td>393.1 k600</td>
<td>4.128 k600</td>
<td>1.521</td>
<td>228.0 k600</td>
<td>7.474 k600</td>
<td>1.430</td>
</tr>
</tbody>
</table>

Total run-time = $\Sigma$ run-times = 93 h and 40 min

The table contains (from left to right): Run-time, torque and speed. Line-to-line voltage, current, and active power of the converter. Line-to-line voltage, current, and active power out of the converter into the PM motor. (The voltage- and current-value subscripts are the used voltage and current ranges of the power meters, respectively. Some run-times are shorter than others since some measurements started from higher temperatures.)
Table B2  Measured efficiencies, efficiency inaccuracies and temperatures of the PM integral motor, the converter, and the PM motor at thermal steady state, i.e. $dT/dt<1 \, ^\circ C/h$. (%u: percentage units, Amb: Ambient, End: End winding spot 1, CC: Corner Coil).

<table>
<thead>
<tr>
<th>Torque:</th>
<th>25 Nm</th>
<th>50 Nm</th>
<th>75 Nm</th>
<th>100 Nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed: 500 r/min Overall efficiency:</td>
<td>85,4% ±1,2%u</td>
<td>86,7% ±0,9%u</td>
<td>85,1% ±0,6%u</td>
<td>81,2% ±0,6%u</td>
</tr>
<tr>
<td>Amb/End/CC temp:</td>
<td>26°C / 36°C / 35°C</td>
<td>26°C / 49°C / 45°C</td>
<td>27°C / 74°C / 64°C</td>
<td>28°C / 118°C / 98°C</td>
</tr>
<tr>
<td>Speed: 750 r/min Overall efficiency:</td>
<td>87,2% ±1,3%u</td>
<td>89,2% ±0,8%u</td>
<td>88,4% ±0,7%u</td>
<td>86,1% ±0,6%u</td>
</tr>
<tr>
<td>Speed: 1000 r/min Overall efficiency:</td>
<td>87,9% ±1,2%u</td>
<td>90,4% ±0,9%u</td>
<td>90,0% ±0,7%u</td>
<td>88,4% ±0,5%u</td>
</tr>
<tr>
<td>Amb/End/CC temp:</td>
<td>27°C / 41°C / 40°C</td>
<td>27°C / 51°C / 49°C</td>
<td>28°C / 71°C / 64°C</td>
<td>31°C / 104°C / 90°C</td>
</tr>
<tr>
<td>Speed: 1250 r/min Overall efficiency:</td>
<td>88,3% ±1,2%u</td>
<td>90,9% ±0,9%u</td>
<td>91,0% ±0,6%u</td>
<td>89,8% ±0,7%u</td>
</tr>
<tr>
<td>Amb/End/CC temp:</td>
<td>27°C / 43°C / 42°C</td>
<td>28°C / 53°C / 51°C</td>
<td>31°C / 73°C / 67°C</td>
<td>32°C / 102°C / 89°C</td>
</tr>
<tr>
<td>Speed: 1500 r/min Overall efficiency:</td>
<td>88,4% ±1,1%u</td>
<td>91,3% ±0,8%u</td>
<td>91,7% ±0,6%u</td>
<td>91,1% ±0,7%u</td>
</tr>
<tr>
<td>Amb/End/CC temp:</td>
<td>28°C / 45°C / 44°C</td>
<td>29°C / 56°C / 53°C</td>
<td>31°C / 73°C / 68°C</td>
<td>32°C / 100°C / 89°C</td>
</tr>
<tr>
<td>Speed: 500 r/min Converter efficiency:</td>
<td>94,0% ±2,3%u</td>
<td>94,2% ±2,4%u</td>
<td>94,1% ±1,7%u</td>
<td>93,5% ±2,6%u</td>
</tr>
<tr>
<td>Speed: 750 r/min Converter efficiency:</td>
<td>95,6% ±1,3%u</td>
<td>95,8% ±1,7%u</td>
<td>95,6% ±1,0%u</td>
<td>95,2% ±1,2%u</td>
</tr>
<tr>
<td>Speed: 1000 r/min Converter efficiency:</td>
<td>96,5% ±1,6%u</td>
<td>96,7% ±1,6%u</td>
<td>96,4% ±1,2%u</td>
<td>95,9% ±1,6%u</td>
</tr>
<tr>
<td>Speed: 1250 r/min Converter efficiency:</td>
<td>97,0% ±2,0%u</td>
<td>97,2% ±2,1%u</td>
<td>97,1% ±1,5%u</td>
<td>96,6% ±2,4%u</td>
</tr>
<tr>
<td>Speed: 1500 r/min Converter efficiency:</td>
<td>97,5% ±1,7%u</td>
<td>97,5% ±1,8%u</td>
<td>97,3% ±1,3%u</td>
<td>96,9% ±2,1%u</td>
</tr>
<tr>
<td>Speed: 500 r/min PM motor efficiency:</td>
<td>90,8% ±2,5%u</td>
<td>92,0% ±2,2%u</td>
<td>90,5% ±1,4%u</td>
<td>86,8% ±2,1%u</td>
</tr>
<tr>
<td>Speed: 750 r/min PM motor efficiency:</td>
<td>91,3% ±1,3%u</td>
<td>93,1% ±1,6%u</td>
<td>92,5% ±0,7%u</td>
<td>90,4% ±0,9%u</td>
</tr>
<tr>
<td>Speed: 1000 r/min PM motor efficiency:</td>
<td>91,1% ±1,7%u</td>
<td>93,5% ±1,4%u</td>
<td>93,3% ±1,0%u</td>
<td>92,2% ±1,4%u</td>
</tr>
<tr>
<td>Speed: 1250 r/min PM motor efficiency:</td>
<td>91,0% ±2,1%u</td>
<td>93,5% ±1,9%u</td>
<td>93,8% ±1,3%u</td>
<td>92,9% ±2,0%u</td>
</tr>
<tr>
<td>Speed: 1500 r/min PM motor efficiency:</td>
<td>90,6% ±1,9%u</td>
<td>93,6% ±1,6%u</td>
<td>94,3% ±1,2%u</td>
<td>94,0% ±1,8%u</td>
</tr>
</tbody>
</table>