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Performance Analysis of Cooperative ARQ Systems for Wireless Industrial Networks

Martin Serror*, Yulin Hu[†], Christian Dombrowski*, Klaus Wehrle*, James Gross[‡]

*Chair of Communication and Distributed Systems, RWTH Aachen University, Germany

[†]Chair of Theoretical Information Technology, RWTH Aachen University, Germany

[‡]School of Electrical Engineering, KTH Royal Institute of Technology, Sweden

{serror, dombrowski, wehrle}@comsys.rwth-aachen.de, hu@umic.rwth-aachen.de, james.gross@ee.kth.se

Abstract—The proliferation of wireless communications has lead to a high interest to establish this technology in industrial settings. The main arguments in favor of wireless are reduced costs in deployment and maintenance, as well as increased flexibility. In contrast to home and office environments, industrial settings include mission-critical machine-to-machine applications, demanding stringent requirements for reliability and latency in the area of $1 - 10^{-9}$ PDR and 1 ms, respectively. One way to achieve both is cooperative Automatic Repeat reQuest (ARQ), which leverages spatial diversity. This paper presents a wireless multi-user Time Division Multiple Access system with cooperative ARQ for mission-critical communication. We evaluate two design options analytically, using an outage-capacity model, to investigate whether the relaying of messages should be performed centrally at a multi-antenna AP with perfect Channel State Information (CSI) or decentrally at simultaneously transmitting stations with average CSI. Results indicate that both options are able to achieve the targeted communication guarantees when a certain degree of diversity is implemented, showing a stable system performance even with an increasing number of stations.

I. INTRODUCTION

Recently, there is a high interest in establishing wireless communication systems in the domain of Machine-to-Machine Communications (M2M), and thus to benefit from the high flexibility in deployment and operation such systems offer. In industrial automation, for example, wireless communications reduces installation and maintenance costs by replacing Fieldbus systems, which are inherently affected by wear and tear [1]. Moreover, wireless communications allows for more flexible industrial processes, as production entities do no longer need to be connected by a cable in order to communicate to each other. However, the domain of industrial automation is dominated by safety- and mission-critical applications, which require a precise synchronization to function properly, thus relying on communication guarantees. So far, cable-based solutions were preferred over wireless solutions, as the former are able to provide such guarantees.

To successfully introduce wireless communications to the field of industrial automation, such systems must be able to guarantee ultra-high reliability whilst ensuring ultra-low latency. Typical target values are $1 - 10^{-9}$ PDR for reliability and 1 ms for latency [2]. Unfortunately, Commercial Off-The-Shelf (COTS) wireless solutions, e.g., based on the IEEE 802.11 standard, or even standards targeting industrial automa-

tion, such as WirelessHART and ISA100.11a, fail to provide such strong guarantees [3]. The main reasons for the difficulty to realize a reliable, latency-constrained wireless technology, lie in the propagation characteristics of radio signals, susceptible to fading, shadowing and co-channel interference.

It is known that cooperative Automatic Repeat reQuest (ARQ) in single-user scenarios effectively increases the communication reliability by several orders of magnitude [4]. Yet, there is a lack of performance evaluations of systems supporting multi-user transmission frames with cooperative ARQ. In a previous work [5], we investigated a Time Division Multiple Access (TDMA) system incorporating cooperative ARQ, where transmissions and retransmissions are centrally scheduled by the Access Point (AP). Although we showed that multi-user cooperative ARQ can be effectively combined with a strict time limit, further mechanisms need to be considered to achieve the anticipated reliability. Therefore, in this paper, we extend our analysis in [5] by considering two more sophisticated designs and analytically determining their performance:

C-RELAYS A centralized system that implements cooperative ARQ with perfect Channel State Information (CSI) using multiple transmit/receive antennas at the AP. As a result, the AP adapts slot lengths and schedules transmissions either directly between stations or indirectly via its multiple antennas, depending on the observed link qualities.

D-RELAYS Making use of distributed resources, several over-hearing stations, including the AP, simultaneously retransmit messages that failed in the direct transmission, thus forming a virtual antenna array. The AP is assumed to have similar hardware characteristics as the associated stations, i.e., a single antenna. As AP and stations do not acquire instantaneous CSI, slot lengths are fixed, as well as a dedicated retransmission phase within the frame.

The remainder of this paper is structured as follows. In Sec. II, we define the system model and the two design options, C-RELAYS and D-RELAYS. This model is used to derive the respective message outage probabilities. An analytic performance evaluation comparing both options is provided in Sec. III. The paper is concluded in Sec. IV.

II. SYSTEM MODEL AND CONSIDERED PROTOCOLS

The considered system model is based on the one presented in [5]. In the following, we focus on the extensions C-RELAYS

Fig. 1. Example scenario for transmitting a message m_1 from Tx_1 to Rx_1 , illustrating cooperative ARQ in C-RELAYS (a-b) and in D-RELAYS (c-d).

and D-RELAYS, as well as the derivation of their performance.

A. General Medium Access Control Model

To ensure deterministic access to the channel, a centralized TDMA scheme is chosen, in which the AP grants exclusive transmission rights to N associated stations. In both C-RELAYS and D-RELAYS, the AP schedules N transmissions within a frame duration of T_F . In C-RELAYS, the N transmissions are scheduled either using the direct link between Transmitting Station (Tx) and Receiving Station (Rx), or using the indirect one via the AP, depending on the current link conditions. At the end of the frame follows a designated signaling phase of length $T_{csi} = \alpha_{csi} \cdot T_F$ to acquire CSI. The CSI is thus used to flexibly trade bad against good quality links, which can be further strengthened when including multiple antennas at the AP and implementing antenna selection or more elaborate multi-antenna transmission schemes. Figure 1(a-b) shows an example scenario for C-RELAYS, in which a message is transmitted through the indirect link via the AP.

In D-RELAYS, we apply the frame structure of [5], including a Transmission Phase (TP) and a Retransmission Phase (RP). In the TP, N equal time slots of length T_t are reserved for direct transmissions between stations. A fixed-length RP is reserved for retransmissions, which are simultaneously performed by multiple relays that overheard the messages in the TP. We refer to the duration of RP as $T_R = \alpha_{ret} \cdot T_F$, being a fraction of the entire communication duration.

1) *Timing for Distributed Relaying:* In D-RELAYS, the AP has only average CSI. Therefore, it schedules slot lengths $T_{t,i}$ for all $i \in \{1, \dots, N\}$ stations of equal duration as

$$T_{t,i}(N) = \frac{(T_F - T_R)}{N}. \quad (1)$$

Retransmission slots T_r are also of equal length, where the length depends on the number of scheduled retransmissions, k , i.e., $T_r = T_R/k$. The AP specifies the RP length upfront, not

knowing how many transmissions will fail during the TP. This yields the definition of the single link outage probability [5]:

$$\mathbb{P}_{out}(\bar{\gamma}, \tau) \triangleq 1 - \exp\left[-\bar{\gamma}^{-1} \left(2^{\frac{D}{B} \cdot \frac{1}{\tau}} - 1\right)\right], \quad (2)$$

where $\bar{\gamma}$ denotes the avg. Signal to Noise Ratio (SNR), D the datasize, B the bandwidth and τ is a placeholder for $T_{t,i}$ or T_r .

2) *Timing for Centralized Relaying:* In C-RELAYS, transmission slot lengths differ depending on the current channel state, as the AP acquires CSI during the frame period $T_{csi} \neq 0$. For a station i , we denote the duration of its transmission slot by $T_{t,i}^*$ and it must hold that $\sum_{i=1}^N T_{t,i}^* \leq T_F$. Having instantaneous CSI at the AP, a message can be successfully transmitted as long as a certain transmission duration requirement is satisfied. In particular, the minimal duration $T_{i,j}^*$ for station i transmitting a message to station j reliably via a link with SNR $\gamma_{i,j}$ is

$$T_{i,j}^*(\gamma_{i,j}) = \frac{D}{B \log_2[1 + \gamma_{i,j}]} \cdot \quad (3)$$

Exactly determining $T_{t,i}$ is discussed in Sec. II-C.

B. Message Error Probability for Distributed Relaying

To calculate the message error probability, we extend the model provided in [5] to simultaneous relaying by several stations. The adapted message error probability ϵ is then

$$\epsilon = \mathbb{P}_{out}\left(\bar{\gamma}_L, \frac{T_F - T_R}{N}\right)^{M+1} + \mathbb{P}_{ret} \cdot \sum_{k=1}^N \mathbb{P}_{sim}\left(\frac{\bar{\gamma}_L}{M}, \frac{T_R}{k}, M\right) \cdot P(k), \quad (4)$$

where the first addend refers to Eq. (2), describing the probability that the message was neither correctly received by Rx nor by one of the M relays. The second addend represents the case where at least one of the relays overheard the message, but fails to forward it to Rx depending on how many other messages have to be retransmitted in addition, where $P(k)$ denotes the probability that exactly k messages from other stations are also scheduled for retransmission. Thus, the probability of performing a retransmission, $\mathbb{P}_{ret}$, requires an unsuccessful direct transmission and at least one of the M relays to correctly overhear it.

$$\mathbb{P}_{ret} = \mathbb{P}_{out}\left(\bar{\gamma}_L, \frac{T_F - T_R}{N}\right) \cdot \left(1 - \mathbb{P}_{out}\left(\bar{\gamma}_L, \frac{T_F - T_R}{N}\right)^M\right). \quad (5)$$

Further, $\mathbb{P}_{sim}$ denotes the outage probability for simultaneously relaying a message via M relays. According to [6], the sum of the signals from all transmitting relays is Gamma distributed for a homogeneous topology, leading to

$$\mathbb{P}_{sim}(\bar{\gamma}, T_r, M) = 1 - \sum_{m=0}^{M-1} \frac{1}{m!} \cdot \left(\frac{2^{\frac{D}{B T_r}} - 1}{\bar{\gamma}}\right)^m \cdot \exp\left[-\frac{2^{\frac{D}{B T_r}} - 1}{\bar{\gamma}}\right]. \quad (6)$$

C. Message Error Probability for Centralized Relaying

With perfect CSI, the AP schedules a message from station i to station j via the AP, if this is more reliable than using the direct link. This occurs if the minimal direct transmit

duration, $T_{t,i}^D = T_{i,j}^*(\gamma_L)$, is longer than the two-hop duration, $T_{t,i}^R = T_{i,AP}^*(\gamma_L) + T_{AP,j}^*(\gamma_L)$. The slot length $T_{t,i}$ is thus

$$T_{t,i} = \min \{T_{t,i}^D(\gamma_L), T_{t,i}^R(\gamma_L)\} . \quad (7)$$

Due to random channel fading, it is possible that the duration for transmitting N messages violates the frame length, i.e., $\sum_{i=1}^N T_{t,i} > T_F$, leading to outages of one or more messages.

The exact determination of the probability that $n = 1 \dots N$ messages cannot be scheduled requires first the derivation of the Probability Density Function (PDF) of $T_{t,i}$. Based on the Rayleigh fading distribution, the Cumulative Distribution Function (CDF) of $T_{t,i}^D$ is given by

$$\begin{aligned} F_{T_{t,i}^D}(\tau, \bar{\gamma}_L) &= \Pr \{T_{t,i}^D \leq \tau\} = \Pr \left\{ \gamma_L \geq 2^{\frac{D}{B\tau}} - 1 \right\} \\ &= \exp \left[-\bar{\gamma}_L^{-1} \left(2^{\frac{D}{B\tau}} - 1 \right) \right] . \end{aligned} \quad (8)$$

As all links have the same average $\bar{\gamma}_L$, $T_{t,i}^D$ are i.i.d. with PDF

$$f_{T_{t,i}^D}(\tau, \bar{\gamma}_L) = \exp \left[-\bar{\gamma}_L^{-1} \left(2^{\frac{D}{B\tau}} - 1 \right) \right] \cdot \frac{2^{\frac{D}{B\tau}}}{\bar{\gamma}_L} \cdot \frac{D \ln 2}{B\tau^2} . \quad (9)$$

Similarly, as all station-AP and AP-station links have the same average SNR, the single transmission durations $T_{i,AP}^*$ and $T_{AP,j}^*$ have the PDF given by $f_{T_{i,j}^*}(\tau, \bar{\gamma}_L) = f_{T_{t,i}^D}(\tau, \bar{\gamma}_L)$. However, for the compound transmission $T_{t,i}^R$, the CDF is

$$\begin{aligned} F_{T_{t,i}^R}(\tau, \bar{\gamma}_L) &= P \{x + y \leq \tau\} = P \{y \leq \tau - x\} \\ &= f_{T_{i,j}^*}(\tau, \bar{\gamma}_L) * f_{T_{i,j}^*}(\tau, \bar{\gamma}_L) \\ &= \int_0^{\tau} \frac{2^{\frac{D}{Bx}}}{\bar{\gamma}_L} \cdot \frac{D \ln 2}{Bx^2} \cdot e^{-\frac{2^{\frac{D}{B(\tau-x)} + 2^{\frac{D}{Bx}} - 2}}{\bar{\gamma}_L}} dx , \end{aligned} \quad (10)$$

where $*$ is the convolution operation. Then, the PDF of $T_{t,i}^R$ is

$$\begin{aligned} f_{T_{t,i}^R}(\tau, \bar{\gamma}_L) &= \int_0^{\tau} f_{T_{i,j}^*}(x, \bar{\gamma}_L) \cdot f_{T_{i,j}^*}(\tau - x, \bar{\gamma}_L) dx \\ &= \int_0^{\tau} \frac{2^{\frac{D}{Bx} + \frac{D}{B(\tau-x)}}}{\bar{\gamma}_L^2} \cdot \frac{D^2 (\ln 2)^2}{B^2 (\tau - x)^2} \\ &\quad \cdot e^{-\bar{\gamma}_L^{-1} \left(2^{\frac{D}{B(\tau-x)} + 2^{\frac{D}{Bx}} - 2 \right)} dx . \end{aligned} \quad (11)$$

Recall that the final time length determined by the AP is given by Eq. (7). If the minimum $T_{t,i}$ is longer than a certain duration t , then both, $T_{t,i}^D$ and $T_{t,i}^R$, are longer than t . The probability of this is $(1 - F_{T_{t,i}^R}(t, \bar{\gamma}_L)) \cdot (1 - F_{T_{t,i}^D}(t, \bar{\gamma}_L))$. Therefore, the CDF of $T_{t,i}$ is given by Eq. (12).

$$\begin{aligned} F_{T_{t,i}}(t, \bar{\gamma}_L) &= \Pr \{T_{t,i} \leq t\} \\ &= 1 - (1 - F_{T_{t,i}^R}(t, \bar{\gamma}_L)) (1 - F_{T_{t,i}^D}(t, \bar{\gamma}_L)) . \end{aligned} \quad (12)$$

Hence, the PDF of $T_{t,i}$ is

$$\begin{aligned} f_{T_{t,i}}(t, \bar{\gamma}_L) &= \frac{\partial F_{T_{t,i}}(t, \bar{\gamma}_L)}{\partial t} \\ &= F_{T_{t,i}^R}(t, \bar{\gamma}_L) \cdot f_{T_{t,i}^D}(t, \bar{\gamma}_L) + (1 - F_{T_{t,i}^D}(t, \bar{\gamma}_L)) \cdot f_{T_{t,i}^R}(t, \bar{\gamma}_L) . \end{aligned} \quad (13)$$

Considering that all N messages need to be transmitted in a frame, the minimal time for message $i = 1, \dots, N$ is i.i.d., such that $f_{T_{t,1}}(t, \bar{\gamma}_L) = \dots = f_{T_{t,N}}(t, \bar{\gamma}_L)$. Hence, the PDF of $\sum_{i=1}^N T_{t,i}$ is given as the convolution of N Eq. (13) terms as

TABLE I
EVALUATION PARAMETERS.

Symb.	Value	Description
N	5	Transmissions per frame
D	128 bit	Message size
$\bar{\gamma}_L$	15 dB	Avg. SNR of links
B	20 MHz	Transmission bandwidth
T_F	1 ms	Total time for transmissions
T_R	$\alpha_{\text{ret}} \cdot T_F$; $\alpha_{\text{ret}} = 20\%$	Time for retransmissions (D-RELAYS)
T_{csi}	$\alpha_{\text{csi}} \cdot T_F$; $\alpha_{\text{csi}} = 20\%$	Time for acquiring CSI (C-RELAYS)

$$f_{\sum_{i=1}^N T_{t,i}}(t, \bar{\gamma}_L) = f_{T_{t,1}}(t, \bar{\gamma}_L) * \dots * f_{T_{t,N}}(t, \bar{\gamma}_L) . \quad (14)$$

Therefore, the probability of scheduling all N messages in the frame with length T_F is given by $F_{\sum_{i=1}^N T_{t,i}}(T_F, \bar{\gamma}_L)$. In the following, we present probability $F_{\sum_{i=1}^k T_{t,i}}(T_F, \bar{\gamma}_L)$, $k = 1, \dots, N$ as $F_{\sum_{i=1}^k T_{t,i}}$ for short. If n denotes the number of erroneous messages, then the expected value of n is given by

$$\mathbb{E}[n] = \sum_{k=1}^{N-1} \left\{ F_{\sum_{i=1}^k T_{t,i}} - F_{\sum_{i=1}^{k+1} T_{t,i}} \right\} \cdot (N-k) + N \left(1 - F_{\sum_{i=1}^1 T_{t,i}} \right) . \quad (15)$$

The total message error probability ϵ_{csi} is then given by

$$\epsilon_{\text{csi}} = \frac{\mathbb{E}[n]}{N} . \quad (16)$$

If the AP has $J > 1$ antennas, it selects the single best antenna for receiving a message and the single best one for transmitting. Hence, the time length $T_{t,i}^{\text{R-BA}}$ for transmitting from Tx to the selected AP antenna is the minimum over all links from Tx to the different antennas. The CDF is given by

$$F_{T_{t,i}^{\text{R-BA}}}(\tau, \bar{\gamma}_L) = 1 - \left(1 - F_{T_{t,i}^D}(\tau, \bar{\gamma}_L) \right)^J , \quad (17)$$

with the PDF, given by

$$f_{T_{t,i}^{\text{R-BA}}}(\tau, \bar{\gamma}_L) = J \left(1 - F_{T_{t,i}^D}(\tau, \bar{\gamma}_L) \right)^{J-1} f_{T_{t,i}^D}(\tau, \bar{\gamma}_L) . \quad (18)$$

Similar to (10), the distribution of the sum of these two time lengths is the convolution of $f_{T_{t,i}^{\text{R-BA}}}(\tau, \bar{\gamma}_L)$ and itself.

III. ANALYTICAL PERFORMANCE EVALUATION

For the evaluation, we assume a homogeneous topology, described by the observed SNR at the receiver ($\bar{\gamma}_L$). Table I shows the exact parameterization of our evaluation. As a baseline, we additionally introduce DIRECT, deactivating relaying in both options and making use of perfect CSI if available, i.e., $\alpha_{\text{ret}} = 0$ (D-RELAYS) and $\alpha_{\text{csi}} = 20\%$ (C-RELAYS).

A. Impact of Link Quality

We evaluate the relation of link quality and message error probability for C-RELAYS and D-RELAYS, applying different numbers of antennas/relays. As $N = 5$ stations are considered, the maximum number of available relays is four (three over-hearing stations and the AP). The results in Fig. 2 show that with C-RELAYS the error probability reduces, when enabling relaying via the AP. Indeed, with each additional antenna the system gains more potential transmission paths to select from and thus benefits from higher flexibility. For D-RELAYS, we observe a similar behavior, as a higher number of relays decreases the error probability by several orders of magnitude.

Fig. 2. Relation of ϵ to link quality $\tilde{\gamma}_L$ for homogeneous topology. Solid lines represent D-RELAYS, while dashed lines represent C-RELAYS.

Fig. 3. ϵ for a varying number of transmissions per frame N . Solid lines represent D-RELAYS, while dashed lines represent C-RELAYS.

In this scenario, however, the performance gain arises from the increased SNR when simultaneously receiving a signal from various sources. The performance gain of C-RELAYS compared to D-RELAYS increases with each additional antenna/relay by one order of magnitude, as perfect CSI allows for a more efficient use of the available time and links that are temporarily in a deep fade are systematically avoided.

B. Impact of Number of Messages

To get insights on the scalability of both system options, we incrementally increase the number of stations and with it the transmissions per frame N in Fig. 3. While in C-RELAYS the number of antennas is fixed at the AP, in D-RELAYS each new station is potentially an additional relay. When the number of relays corresponds to the number of antennas (cf. 1 Relay/Antenna), we see an almost constant performance gap between D-RELAYS and C-RELAYS. More interestingly, both options show generally low sensitivity to an increasing N .

When using all available relays, D-RELAYS benefits from a larger N , although the transmission slots become smaller. At a certain point ($N = 10$), the system performance gain approaches a saturation level, as has been observed in [7]. For C-RELAYS with four antennas, the message outage probability is hardly influenced by an increasing N .

C. Impact of Frame Overhead

In D-RELAYS, the selection of T_R represents a trade-off between stronger coding for direct transmissions or retransmissions, while in C-RELAYS T_{csi} indicates how the system behaves when a certain frame time is lost for acquiring CSI. The results for varying α_{ret} and α_{csi} , respectively, are depicted in Fig. 4. An optimal α_{ret} strongly depends on the number

Fig. 4. ϵ for varying α_{ret} (D-RELAYS, solid lines) and α_{csi} (C-RELAYS, dashed lines).

of available relays, nevertheless the optimum is relatively flat allowing for a certain flexibility when some parameters are unknown. The error probability in C-RELAYS slowly increases with a higher CSI overhead until $\alpha_{csi} = 80\%$, showing again the strong stability of the system, once a certain reliability is achieved through diversity.

IV. CONCLUSION

This paper investigates a wireless multi-user transmission system and its suitability to allow for ultra-high reliability at low latencies. We analytically investigate the effects of incorporating cooperative ARQ into TDMA on the message error probability by evaluating a centralized system variant with instantaneous CSI and a decentralized one with simultaneous relaying. Our results show that the message outage probability is significantly reduced in both options, when introducing spatial diversity through multiple antennas or accordingly multiple relays. At a given outage probability, the system performances remain comparatively stable even with an increasing number of transmissions. In this, C-RELAYS benefits from instantaneous CSI to exclude transmission paths that are temporarily in a deep fade, even with including (high) costs to acquire CSI in the frame. D-RELAYS makes use of distributed resources as each station in the system may act as an additional relay, thus introducing additional spacial diversity to counter reduced transmission slot lengths in the frame.

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