Validation of simulation tool for C-arm X-ray systems

Source and scatter model

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ABSTRACT

Continuous improvement of image quality is one of the priorities in medical imaging. Therefore, development of a simulation tool allowing to generate realistic images would be of great value to understand better the impact of the components on the image quality metrics and to choose imaging set-ups or new design features to optimize output of existing systems and to prototype new ones and to formalize the link between objective and subjective image quality metrics.

Therefore, the purpose of this project, was to contribute to adaptation and validation of an existing simulator for simulation of C-arm X-ray imaging.

Firstly, the study of the existing simulation tool was performed to choose further development axes.

Afterwards, preliminary estimations of simulation complexity by evaluating the number of photons for a given imaging examination were performed.

Previous studies\(^1\) showed the determining impact of focal spot on imaging performance (reducing the limiting spatial frequency in common examination conditions) of X-ray interventional imaging systems. Therefore, the work focused on the improvements of source model, in particular realistic focal spot was defined and simulations of images with close-to-real sharpness were performed and compared to experimentally acquired images.

Finally, a part of this project was dedicated to scatter study. An experimental set-up and "scatter map" analysis were designed to determine the scatter evolution as function of imaging field-of-view. First simulations were also performed.

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INTRODUCTION

Interventional radiology allows the radiologists to visualize the anatomy of a patient as well as to guide tools during medical procedures such as stent or implant deployment. These procedures are alternatives to invasive surgical interventions and thus offer several advantages for a patient, such as shorter hospitalisation or shorter recovery time.[1]

This is why the improvement of the quality of images generated by interventional radiology devices remains a priority of continuous development. In the search for a more efficient way of assessing the impact and the interplay of the imaging chain components on image quality, Vascular Image Quality team at General Electric Healthcare in Buc defined the need for a simulation tool for C-arm X-ray system.

The use of such simulator aims at optimizing the performance of existing systems and prototyping new designs. Indeed, the simulator would lead to an improved understanding of the interplay of components of the C-arm imaging system and development of clearer link between on the one hand objective image quality metrics, such as limiting spatial resolution, contrast, and on the other hand subjective image quality metrics such as wire visibility in the image.

In order to do so, an existing simulation tool validated for other imaging techniques was selected to be adapted to simulations of X-ray imaging with C-arm system.

First section of this report presents general explanation of X-ray imaging chain, underlying physical principles and the principles of image formation.
Afterwards, the state-of-the-art and functioning of the selected simulator is presented.
Then the contributions of this project to the simulator functioning are presented through methods and obtained results.
1. PRESENTATION OF X-RAY INTERVENTIONAL IMAGING

This section briefly presents the principles of X-ray imaging using digital C-arm imaging system. First, the role of different components is explained as well as their assembly within the imaging chain. Then, an overview of main physical aspects is presented. Finally, the principles of the image generation are described.

For more detailed information on these aspects, the reader can refer to [2], [3].

X-ray interventional imaging is a radiographic imaging technique. In simple words, radiographic imaging consists in exposition of a patient to X-ray photons (also referred to as X-rays, X-ray radiation, X-ray beam) that are attenuated by the crossed matter (bones, fat, fluids, air, etc.) and detected at the detector. Depending on the composition and length of imaged structures, X-ray beam is more or less attenuated which results in different signal intensity on the detector. Furthermore, in interventional imaging, acquired images are displayed continuously during medical intervention. Examples of common interventions are stent or implant deployment during which the surgeon has the possibility to control the navigation of the deployed stent (or implant) through the vessels and to check the correct position. This enables decreased invasiveness of the operation, shorter recovery time for patient and less hospitalisation fees.

1.1. Digital C-arm imaging system for X-ray interventional imaging

One of the most common geometries used for X-ray interventional imaging systems is so called C-arm X-ray system, as shown in Figure 1. The imaging chain is a complex assembly. It’s main components are:

- generator and X-ray tube
- filters
- collimator
- patient table
- anti-scatter grid
- detector
- processing and post-processing units
- monitors.
1.1.1. X-ray tube

X-ray photons are produced in X-ray tube. The essential components of an X-ray tube are a cathode and an anode. The cathode, which is a tungsten filament in our case, is heated by applying a high electrical current. As consequence, electrons from this filament are ejected and accelerated by an acceleration potential, also called tube voltage or beam energy, towards the anode. The anode is a rotating tungsten target contained in a vacuum. The accelerated electrons collide with the target, penetrate and interact with anode material which is accompanied by loss of energy. Some of this energy is converted to X-ray radiation: bremsstrahlung radiation and characteristic radiation. However most of the energy is converted to heat.

Bremsstrahlung radiation results from deceleration of electron when it interacts with a positive nucleus. Low energy bremsstrahlung radiation is partially absorbed already in anode. Characteristic radiation occurs when accelerated electron collides and ejects an electron from anode's atom's shell. The latter one is replaced by an electron from inner shell, accompanied by emission of a photon with energy depending on the binding energy of the considered electron.

The two radiation types form emission spectrum, i.e. the distribution of number of X-ray photons over the corresponding energies.

Since target anode is tilted to reduce the overheating of the thermal focal spot, photons travel through different path lengths before leaving anode. Thus the emergent X-ray photon beam presents anisotropic fluence. This phenomenon is known as "heel effect" and will be further developed later on.

adapted from "Interventional Image Quality Team.pptx", presented on 11th September 2012
1.1.2. Flat filters and collimator

In general, flat filters are thin metal sheets used in order to absorb low energy X-ray photons of the emitted spectrum. Indeed, if not filtered out, low-energy photons would not contribute to image formation but would be absorbed by outer body layers and increase uselessly the radiation dose to patient.

Collimator is highly absorbent blade used to limit the field-of-view (FOV), and thus the irradiated zone of the patient's body, only to the zones of interest where the intervention occurs.

1.1.3. Patient table and patient

Composition and geometry of the crossed tissue of patient's body and of the patient table determine the degree of attenuation of X-ray photons travelling through it. Moreover, they contribute largely to scattered photons generation, discussed later.

Furthermore, in the case of X-ray interventional imaging, the imaging set-up implies a high contribution of scatter radiation to the signal in the final image, since large FOV and thick patients are involved.

1.1.4. Anti-scatter grid

Anti-scatter grid is highly absorbent grid with thin blades parallel to primary photons. It is usually placed at the detector entrance and aims at reducing the scatter radiation and thus reducing the degradation of contrast in the image due to scattered photons, as developed a bit later.

1.1.5. Detector

C-arm systems for X-ray interventional systems present flat digital detectors. In our case, energy-integrating detection with indirect conversion were used (as opposed to photon-counting detection and direct conversion).

The indirect conversion consists in two main steps. First, conversion of X-ray photons to visible photons take place in scintillator layer, composed of Cesium Iodide columns doped with Thalium. Then visible photons are collected at active photodiode surface and converted to electrons. These electrons are read-out over a given period of time and converted to digital signal using an analogue-to-digital convertor in order to obtain the digital intensity signal to be displayed on monitors after some processing and post-processing steps.
1.2. Interaction of X-ray radiation with matter

X-ray photons are high energy electromagnetic waves that are attenuated as they travel through a matter. For X-ray interventional imaging, energy of X-ray photon beam ranges between 10 keV \(^3\) and 140 keV.

The main interactions that occur within this energy range are photoelectric absorption, Compton scattering and Rayleigh scattering.

Photoelectric absorption, or photoelectric effect, is a process during which a photon interacts with an electron of an atom of the crossed material. The energy of the X-ray photon is completely absorbed as it is transferred to the electron which leads to ejection of the electron from the considered electronic shell. The probability of photoelectric effect depends on energy of the incoming photon. A photon can undergo photoelectric absorption due to interaction with an electron if the energy is at least equal to the binding energy of the electron. Below the binding energy the photoelectric interaction is energetically not possible.

A photon can also undergo a diffusion, also called a scatter event.

Compton scattering is an event when an X-ray photon interacts with an electron of crossed absorbent material, but do not lose all its energy. Only a fraction of its kinetic energy is lost serving to accelerate the interacting electron. And the X-ray photon is deviated from its incoming direction. Compton effect is possible if the energy of the X-ray photon is superior to the binding energy of the ejected electron. Furthermore it occurs predominantly with soft tissues.

In general, for a given material, photoelectric effect is predominant for lower energies and Compton effect is predominant for medium and higher energies. Indeed, Compton effect is the dominant interaction process within the clinical energy range used for X-ray interventional imaging.

Rayleigh scattering is an event during which an X-ray photon interacts with a whole atom of crossed material. The interacting X-ray photon does not lose energy and does not eject an electron of the target atom, but it is deviated from its initial direction. This interaction is more probable at lower energies (up to 30 keV, depending on the target material).

Interaction cross-section, \(\sigma\), and linear attenuation coefficient, \(\mu\)

Total interaction cross section, \(\sigma_{\text{tot}}\), expressed in cm\(^2\), expresses the probability that a photon with initial energy \(E\) will interact with a target particle of a given imaged material. It can be expressed as sum of various considered interactions that can occur. The three main interactions within energy range specified above are already mentioned: photoelectric effect, Compton and Rayleigh diffusion. Then the total cross section can be expressed as:

\[\sigma_{\text{tot}} = \sigma_{\text{photo}} + \sigma_{\text{Compton}} + \sigma_{\text{Rayleigh}}\]

\(^3\) keV: kiloelectronvolt: 1 keV = 1,6022.10\(^{-16}\) Joules which corresponds to kinetic energy acquired by an electron which is accelerated by an electrical potential of 1 volt.
\[ \sigma_{\text{tot}}(E) = \sigma_{\text{PE}}(E) + \sigma_{\text{Compton}}(E) + \sigma_{\text{Rayleigh}}(E) \]  
(1)

\( \sigma_{\text{PE}} \): cross section of photoelectric effect, in cm\(^2\)

\( \sigma_{\text{Compton}} \): cross section of Compton diffusion, in cm\(^2\)

\( \sigma_{\text{Rayleigh}} \): cross section of Rayleigh diffusion, in cm\(^2\)

For a given direction, the quality of the X-ray beam, i.e. the intensity and the energy, is altered due to absorption and diffusion (scatter) events. Linear attenuation coefficient, \( \mu \), represents the probability of photons being removed from a monoenergetic beam per unit thickness of crossed material. It is expressed in cm\(^{-1}\). It depends on the density of the crossed material and on the X-ray beam energy.

The link between linear attenuation coefficient and cross section is given by the relation:

\[ \mu(E) = \rho \sigma_{\text{tot}}(E) \]  
(2)

with \( \rho \) the density of element which interacts with X-ray photons per unit volume, expressed in cm\(^3\). For compounds, the attenuation is expressed through weighted sum of normalized linear attenuation coefficients of each of \( k \) components of the compound, according to the relation:

\[ \frac{\mu}{\rho} = \sum_k w_k \left( \frac{\mu}{\rho} \right)_k \]  
(3)

with \( w_k \) and \( \left( \frac{\mu}{\rho} \right)_k \) corresponding respectively to weight fraction of component \( k \) and to mass attenuation coefficient of component \( k \).

If we consider \( I_0 \) the initial X-ray beam intensity, then the intensity in the given direction, after having crossed a distance \( L \) (in cm) of a structure with density \( \rho \) (in cm\(^3\)) and linear attenuation \( \mu \) (in cm\(^{-1}\)), presents an exponential decrease, governed by Beer-Lambert law:

\[ I(E) = I_0 e^{-\mu(E)L} \]  
(4)

1.3. Image generation in radiology and aspects impacting the image quality

Image formation using C-arm X-ray system allows to acquire images by projection of conic X-ray photon beam emitted from the source, travelling through patient and acquired by a flat detector. Image formation is based on X-ray attenuation difference. As X-ray photon beam crosses a medium (patient) composed of various materials, the X-ray beam exiting the medium is attenuated differently in different directions depending on the crossed materials and the path length travelled through the material.

The impact of various components of the imaging chain depends on set-up geometry which depends on the aim of a given imaging intervention. A more exhaustive list of image quality metrics and
components that affect these metrics are presented in Table 1, presented at the end of this section, adapted from [4]. The following paragraphs present briefly some of the aspects and impacts on image.

Non-processed radiographic images acquired by a C-arm system present radial and longitudinal non-uniformity due to system's design. The radial non-uniformity, "dome effect", consists in signal intensity degradation as moving radially away from the detector centre. This is explained by the fact that detector is flat and the cone X-ray beam travels longer distance from source to the extremities of the detector than from source to the centre of the detector, as schematized by Figure 2.

The longitudinal non-uniformity occurs in cathode - anode direction in tube plane, i.e. top - bottom direction in image plane, and is also known as "heel effect". Heel effect is due to the intrinsic anode attenuation: the beam of electrons hits and penetrates the anode, a volume of interactions and photon generation exists near the area where electrons hit the anode. This implies that depending on where a photon is generated and on the direction of the photon trajectory, the considered photon travels different distance in the tungsten anode and therefore undergoes different attenuation before it leaves anode. Moreover, anode target presents an angle in order to spread slightly the electrons colliding with the target anode and thus to decrease the overheating of the thermal focal spot. This target angle also implies that generated photons travel various length within anode before emerging out of it. Consequently, they present various attenuation.

As depicted in Figure 3a, the beam rays emerging towards cathode present lower mean energy but higher fluence. Whereas the beam rays emerging towards anode (having travelled longer distance in the anode) are hardened (i.e. they have higher mean energy) and with lower fluence, since low energy photons of this direction were stopped before exiting the anode, but higher mean energy [5].
Figure 3: Heel effect principle and electron depth penetration model; 3a: Effect on the quality of photon beam; 3b: Detail of anode: \( l_c \), electron penetration depth, \( l_p \), distance travelled by photon before leaving the anode and \( d \): photon path through anode as considered by [6].

Since large field-of-view are often used in interventional imaging, heel effect is more pronounced, because the angle between the photons travelling towards the "bottom" of the detector and those travelling towards the "top" of the detector increases with detector dimensions. This larger angle implies also a higher difference in path lengths (and thus attenuation) before X-ray photons exit the anode. Furthermore, the fact that photons exit anode in multiple directions implies that at anode surface, a finite size zone, as opposed to a single point source, represents the "source" of the X-ray photons emitted towards the detector. This zone is referred to as "thermal focal spot". The projection of the thermal focal spot on a plane parallel to detector plane is referred to as "optical focal spot", i.e. a focal spot as seen from detector.

In addition to intensity and energy variation of X-ray photon beam due to anode target angle and spread of X-ray photons, the size of optical focal spot depends also on the considered position on the detector. As represented in Figure 4, optical focal spot is larger and more blurred in the top of the image and smaller and sharper, at the bottom of the image, respectively corresponding to the cathode and anode side.

Figure 4: Theoretical scheme of anode, FS, cathode-anode axis and FS distortion, for reference
Larger focal spot increases the **blurriness** in images, as depicted in **Figure 5**. Furthermore, for a given finite focal spot size, blurriness in image increases with object magnification, as schematized in **Figure 6**. In fact, with increasing focal spot size and magnification, the zone of penumbra relative to umbra increases, which means higher blurriness.
These considerations are important, since for vascular X-ray interventional imaging, relatively large focal spots are commonly used (for instance, focal spot with nominal value 1 mm is commonly used for some static imaging of stent deployment, as compared to nominal focal spots of 0.6 mm or 0.3 mm, the later used for other examination protocols of interventional imaging with C-arm systems or in mammographic systems).

NOTE: "Nominal" focal spot dimension refers to dimensions as officially declared respecting the measurement of focal spot size as specified by International Electrotechnical Commission (IEC) standard IEC60336. The actual dimensions of a given focal spot are anisotropic. Therefore the effect of focal spot on blurriness/sharpness in image is an anisotropic phenomenon.

The useful information for image generation which allows to distinguish various structures of a given imaged object is carried by photons that follow a given direction from the source till the detector and undergo, or not, attenuation when crossing the imaged object. These X-ray photons are called primary X-ray photons and form the primary beam.

Let’s call C the contrast between two imaged objects composed respectively of materials with attenuation coefficients $\mu_1$ and $\mu_2$, and the corresponding signal intensities on the image $I_1$ and $I_2$. Then the contrast between these two regions equals:

$$C = \frac{|I_1 - I_2|}{I_1 + I_2}$$

(5)

Since the attenuation coefficient is function of energy, it is necessary to select beam energy with regards to the composition of objects that are imaged in order to increase the difference between the attenuation of the object of interest and the background in order to optimize the contrast in the image.

However the photons from scatter radiation can also impact on the detector and contribute to the detected signal. As these scattered photons were deflected from their initial direction after interaction with imaged object, they do not contribute to the correct structural information in the image. Instead, they can be interpreted as contributors to noise.

Indeed, scatter contribution to X-ray images acquired during vascular interventional X-ray imaging is high due to the system configurations used for these types of imaging interventions. In particular, two aspects are favourable for important scattering: large field-of-views and thick patients (up to 60 cm for obese patients who suffer frequently from cardio-vascular complications requiring stent or implant deployment).

As consequence, several ways to reduce scatter radiation can be employed, for instance anti-scatter grids and set-up using larger air-gap (distance between the patient and the detector). The latter increases the probability that scattered photons leave the field-of-view before arriving to the detector. However some examination set-ups do not allow for large air gap. Anti-scatter grid has highly absorbent blades focalised to the source so that primary photons (coming along given directions) can pass and scattered photons are absorbed. However, also some primary photons are attenuated and some scattered photons can still pass if arrive under an angle defined by blades.
Knowing that the scatter radiation is important when we consider set-up configurations used during vascular interventional imaging, we decided to study experimentally the evolution of scatter radiation contribution to the image as function of used field-of-view and validate the simulation accuracy of this aspect. Methods and results for this study are presented in Section 3.

The amount of X-ray photons contributing to image formation is an interesting piece of information also from simulation point of view because it contributes to computation complexity and computation time, which will become clearer considering explanations about the simulator presented in Section 2. Therefore an estimation of X-ray photons for a given interventional radiographic imaging procedure was evaluated, as presented in Appendix A.

Image processing procedure can deal with some of the artefacts and noise contributions to the final image. However, aspects involving image processing and image display were out of scope of this project therefore for details on this part, the reader can refer to [5].

In order to quantify the effect of focal spot on blurriness of the image we decided to evaluate spatial resolution in experimentally acquired images and those obtained by simulation. Selected image quality metric was Modulation Transfer Function (MTF) which expresses how the imaging system is able to transfer frequency content of the imaged objects. MTF was determined using slant edge method [8] and MTF measurements were performed using a validated software tool, referred to as MTF tool [9].

In order to quantify the scatter contribution to images, scatter-to-image ratios (SPR) were determined using "scatter maps", as detailed in Section 3.

The choice of the aspects developed in this project was made with regards to common set-up configurations used during vascular X-ray interventional imaging, with regards to aspects until now missing in the simulator and with regards to previous studies on impact of various components on image quality [4]. As consequence, contributions to source and scatter model were performed as presented in more details in Section 3. Work on source model aimed at simulation of realistic blurriness of simulated images thanks to implementation of a realistic focal spot. As presented in Figure 7, image blurriness is influenced by numerous aspects. It was necessary to select only several of them within the scope of the project. That is why we focused on focal spot effect on blurriness, considering several object magnifications and several positions on detector.

Work on scatter model aimed at experimental measurement and validation of the evolution of scatter radiation to image signal with varying field-of-view.
NOTE: In order to assess the influence of selected aspects, it is essential to be aware of other contributors, in order to be able to discern and analyze the impact. Briefly, in the scope of the anterior studies [4], the most important contributors to image sharpness/blurriness, apart from the focal spot, were:

X-ray Pulse width and object motion[^4^], dose, **frequency content of objects** (contrast of tissues and dimensions of objects), **magnification**, **pixel size** (sampling limit) and CsI needle dimension, noise (visibility effect), acquisition and processing blur.

Following approximations and simplifications were considered:
**Position in the centre of image only**, neglected noise contribution to visibility, X-ray techniques effect on contrast, frequency content of clinical objects/anatomy, human perception.

In the case of X-ray interventional vascular imaging, the impact of these aspects is important, which also justifies the interest in development of more realistic focal spot model for the simulator purposes. The typical configuration of interest where the impact of FS on image blurriness is dominant include:

- Stationary imaging (else motion blur is the dominant contributor to image blurring)
- High frequency content (high density edge variations and small sub-millimetric objects)
- A clinical example is the intracranial small artery stent imaging, when realistic focal spot model should permit better sharpness at high density edges and better correlation with artifacts [5].

[^4^]: vessel motion cca 130mm/s for adult and pediatric patients (Shechter, Resar and McVeigh; 2006)
<table>
<thead>
<tr>
<th>IQ CTQ</th>
<th>Tube and Generator</th>
<th>Table</th>
<th>Patient</th>
<th>Gantry</th>
<th>Grid</th>
<th>Detector</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Spatial resolution</strong></td>
<td>Focal spot size, target angle, Focal spot distribution</td>
<td>Table height</td>
<td>Field of view, anatomy, patient motion</td>
<td>Field of view, SID/SOD</td>
<td>Grid step</td>
<td>Pixel pitch, scintillator thickness, readout &amp; integration time, size</td>
</tr>
<tr>
<td><strong>Contrast resolution</strong></td>
<td>Off focal radiation, Beam quality, peak power, kV &amp; mA, XRay transparency</td>
<td>Anatomy, XRay</td>
<td>SID/SOD</td>
<td>Anti scatter ratio</td>
<td>CF, electronic noise, bit depth</td>
<td></td>
</tr>
<tr>
<td><strong>Temporal resolution</strong></td>
<td>Pulsed mode, Fast kV, grid mode, frame rate</td>
<td>Table speed</td>
<td>Anatomy, patient motion,</td>
<td>Readout &amp; integration time, Size</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Artifacts</strong></td>
<td>Focal spot alignment, thermal drift, blooming, non uniformity</td>
<td>Table &amp; accessories</td>
<td>Patient motion, Size (scatter), presence of metal/bones, Vibrations, Gantry</td>
<td>Focal Offset drift, bad pixels, gain, MTF uniformity, EMI sensitivity, lag, radiation damage</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Image Noise</strong></td>
<td>kV vs mAs capacity, mR/mAs drift, filament aging, short pulses</td>
<td></td>
<td>Vibration, Gantry accuracy, alignment</td>
<td>Focal Offset drift, bad pixels, gain, MTF uniformity, EMI sensitivity, lag, radiation damage</td>
<td></td>
<td>Electronic noise</td>
</tr>
<tr>
<td><strong>Dose performance</strong></td>
<td>Filtration, Beam quality, Position accuracy</td>
<td>Radiological thickness, Anaotmy,</td>
<td>SID/SOD</td>
<td>DQE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Analysis of impact of various imaging chain components on image quality metrics,
IQ CTQ: Image Quality Critical To Quality
2. EXPLANATION OF FUNCTIONING OF CATSIM SIMULATION TOOL

2.1. Catsim

CatSim (Computer Assisted Tomography Simulator) is a simulation tool developed by GE employees for CT projection images [10] and further customized and improved allowing also simulation of mammographic images [3], and interventional images (in progress)). The following points explain our choice to develop simulation tool for X-ray interventional imaging by adapting CatSim:
- customizability: modular approach, callback functions which allow to define new features to improve and enlarge its initial purpose
- simplicity: Matlab language for high level routines
- efficiency: optimized C code for core routines, multiple threads

The following paragraphs aim at explaining the functioning of CatSim, necessary for the user to customize it to its needs and for the reader to understand the work of this project as reported in later chapters concerning source and scatter simulations. The explications are based on [3], [10], [11], [12], [13], [14].

CatSim is composed of a set of exchangeable functions defining a physical phenomenon (projection, detection, etc.) or architectural components (source, phantom, detector). The parameters of the architectural components are user-specified in configuration file which is sourced by the main function. The configuration file contains either numerical values or callbacks for input files (spectrum distribution, phantom geometry and composition) and for other modular functions (defining the type of detector, type of source, etc.)

To increase the efficiency, some functions call for external functions written in C, C++ to perform heavier computation tasks (object projections, scatter), for external databases (cross sections for attenuation coefficients calculations) and libraries.

![Figure 8: Modular principle of Catsim, adapted from [12]](image)
The general flow (in the scope useful for this work) is schematized in Figure 8 and can be summarized in following steps:
- construction of physical components (constructing source and detector, reading phantom, materials and attenuation coefficients according to parameters defined in configuration file)
- ray projection
- scatter analysis (if selected)
- signal detection
(reconstruction, visualisation or dose computation are also possible, but not used within the scope of this project)
The rest of this chapter explains briefly principles for each of modules: source, projector and phantom, detector, scatter.

2.1.1. SOURCE MODEL

Source model can be subdivided into: spectrum, focal spot, heel effect and filters definitions.

- **Spectrum**: is defined by an external .dat file containing photon intensity distribution. Photon intensity distribution expresses the number of photons per surface (in mm²) per current (in mA) per time (in s) attenuated by 750 mm of air - i.e. after having travelled 750 mm through air after emerging from anode - and attenuated by tube filters, for energy bins defined by 0.5 keV step going up to the maximal energy defined by maximal acceleration potential, i.e. beam energy, in configuration file.
Indeed, correction factors (for units of distribution and distance from the anode at which it is considered) can be defined in the configuration files. Corresponding energy bins can be "resampled" to define wider bins. This is an option aiming at simplifying computation complexity, but accuracy has to be validated if changes of sampling are done!
This spectral distribution is considered as "raw" spectrum that is further affected by attenuating factors (governed by Beer Lambert law) due to heel effect, due to encountered flat filters and objects along the projection from a source sample towards detector sample, as explained a bit later.
**Figure 9** represents an example of raw spectrum that can serve as input for a simulation. As explained in Section 1, the spectrum consists of continuous part due to bremsstrahlung radiation and of discrete peaks corresponding to characteristic radiation. As already mentioned above, the spectrum is distribution of number of photons over photon energies, expressed in keV. Photons with low energies where filtered out. Indeed, these photons would only be absorbed by outer layers of patient and contribute to dose to patient and not to the formation of image. The maximal energy corresponds to beam energy (or tube voltage).
- **Focal spot**: is defined by:
  - source-to-isocentre distance (mm) (consequence of initial CT use),
  - dimensions (width and length in mm),
  - number of samples (points defining the area of focal spot)
  - corresponding weight for each sample.

In this way a point source, a uniformly distributed source or a focal spot with realistic length and width dimensions and intensity distribution can be defined.

*Figure 10* schematize the definition of focal spot in the simulator. On the left, the area of thermal focal spot on target anode is presented. The target anode presents an inclination degree $\alpha$ in order to decrease the overheating of anode due to impacting electrons. In the center, the position of focal spot on target anode, its projection in a plane parallel to detector plane and the projection of the conic X-ray beam towards object and detector are presented. On the right, a scheme of oversampled focal spot is represented.

For previous simulations, point or square focal spots were defined, i.e. equal length and width and uniform intensity distribution. Point focal spot was defined by a single point whose coordinates were specified by the source-to-detector distance. Square focal spot was defined by a focal spot size and a number of samples (the same number in length and width direction).

A **focal spot sample** is a point with a given coordinates within the area specified by focal spot size and a given weight, determined by focal spot intensity distribution. Focal spot samples were equidistant for a given direction in case of square focal spot definition. In this case we talk about regular oversampling of focal spot. The coordinates of the first and the last sample in each direction, length and width, are centred at coordinates so that their difference corresponds to focal spot size.

**NOTE:** One of the main contribution of this work is the integration and simulations using realistic focal spot in order to generate images with close to reality sharpness.
- **Heel effect**: is taken into account by a model based on varying intrinsic attenuation of the spectrum depending on the "emerging angle" of photons from anode. Indeed, the distance that photons travel through tungsten material in anode before they emerge depends on the angle of emergence, as explained in *Section 1*.

- **Flat filters**: are defined by their materials and thicknesses and are taken into account by applying the corresponding attenuation factor to the considered input spectrum.
2.1.2. DETECTOR MODEL

The detector is defined by:
- detector type (callback function for "flat detector" is specified in configuration file in our case),
- physical parameters specified in configuration file:
  - number of detector cells (pixels, also called samples), whose coordinates are calculated considering the source-to-detector distance, the total number of detector cells and their size, specified by pixel pitch size
  - pixel pitch size,
  - fill factor (fraction of pixel corresponding to active area of photodiode contributing to the integration of the incoming signal),
  - source-to-detector distance
  - decimation factor (making possible to group a specified number of cells in each direction (row or/and column of detector matrix)
  - offsets with regards to detector centre, in both directions (column or/and row)
For detection purposes further details can be provided among which:
- scintillator material and depth (for absorption calculations)
- detector prefilters' materials and thickness
- coefficient for Lorentzian fit (model for signal transfer degradation due to optical spread in scintillator, explained later)
- addition of gain and electronic noise
In a bit more details, the detection of incoming signal ("incoming spectrum, i.e. incoming photons \( I^{\text{incoming}} \)) can be schematized by a block diagram as represented in Figure 11. \( I^{\text{view}}(E) \) represents the intensity of X-ray photon beam coming to detector pixel \( i \). \( I^{\text{raw}}(E) \) is the number of photons arriving at the detector without attenuation for energy bin \( E \), \( s \) is the focal spot sub-sampling index, \( I_{i,s,o} \) is the intersection length between the line with index \( i-s \) (linking detector pixel \( i \) and focal spot sample \( s \)) and the object with index \( o \), \( \mu_o(E) \) is the linear attenuation coefficient of object \( o \) at energy \( E \), and \( S \) is total number of source samples.

**NOTE:** \( \frac{1}{s} \) stands for focal spot sample weight in case of uniformly distributed intensity of X-ray photons in focal spot, therefore it can be replaced by a more general expression of "weight", i.e. intensity of the considered focal spot sample, \( w_s \).

\( I^{\text{scatter}}(E) \) represents the scatter contribution to the signal and includes signals due to both Compton and Rayleigh scattering.

The main detection steps are explained in following paragraphs:
- **Quantum efficiency (QDE):** represents the percentage of photons arriving to the detector scintillator that will be absorbed by the scintillator. QDE is expressed here as:
\[ QDE(E, i) = \frac{I_0(E) - I(E)}{I_0(E)} = 1 - e^{-\mu_{\text{scint}}(E)L_{\text{scint}}(i)} \]  

with

\( E \): considered energy bin, in keV  
\( i \): considered ray direction  
\( I_0 \): the intensity of photons arriving to the scintillation layer of the detector  
\( I \): the intensity of photons as after attenuation undergone in scintillation layer of the detector  
\( \mu_{\text{scint}}(E) \): linear attenuation coefficient of scintillation layer (Cesium Iodide in our case), in cm\(^{-1} \)  
\( L_{\text{scint}} \): distance travelled by X-ray photon in scintillation layer before being attenuated, in cm

- QDE increases with scintillator thickness  
- QDE decreases with increasing energies (as attenuation coefficient of scintillator decreases [15])

- **Quantum noise** (\( P \)): is the standard deviation of the number of incoming photons to detector. This quantity is stochastic and its distribution follows Poisson law, thus the standard deviation is proportional to root square of number of incoming photons.

- **Optical gain and Collection efficiency** (\( G \)):  
  **Optical gain**: represents the efficiency of X-ray photons conversion to visible photons (scintillator level), i.e. represents a group of multiplication factors accounting for scintillation gain, light reflection, optical collection efficiency, photodiode efficiency, etc..  
  **Collection efficiency** (or electronic gain): represents the efficiency of visible photons conversion to electrons (photodiode level).  
Optical gain is considered as linear function of energy of incident photons and electronic gain is considered as constant. Although these two conversions are not immediately successive in real system, they are represented by a unique factor \( G \) for simulation purpose.

- **Optical spreading** (\( H \)): represents diffusion of photons in the scintillator. Indeed, visible photons generated in scintillator do not follow the same direction as their generating X-ray photons. Therefore there is a dispersion of visible photons that may reach a neighbouring pixel (instead of the pixel "corresponding to the position of X-ray photon impact"). This dispersion is modelled by a Lorentzian decay [16], [17], [18] and denoted as \( MTF_{\text{optical}} \).

**NOTE**: For simulation purposes, a Lorentzian fit have to be done for each time the scintillator layer of the simulated layer changes (i.e. when simulating an imaging system with a different scintillator layer design).

- **Integration and sampling**: Detector resolution is limited by the pixel pitch (pixel size). Pixels are considered as square and all detected photons are counted for final signal in the considered pixel. For
surface integration detection, surface integration over a square pixel surface is done. This is simulated as a multiplication by sine cardinal function in Fourier domain (referred to as $MTF_{aperture}$ later on).

**Sampling** is also performed to allow signal assignment to each detector element.

Fill fraction simulates the fact that the active pixel area (participating to signal detection, i.e. sensible to visible photons) is lower than the actual pixel size (due to electronic part on the pixel).

- **DAS (Data Acquisition System):** Electronic noise and Analogue-to-Digital conversion

**Electronic Noise:** represents noise independent of intensity of detected signal. For simulation, it can be added as Gaussian distribution with zero average and standard deviation specified by user.

**Analogue-to-Digital conversion** (quantization ramp): represents the conversion of electrons to digital units (counts, gray level...).

In our case, the experimental images were acquired using fixed techniques (which is an acquisition mode when a fixed ramp is used). To avoid that the ramp is not well adapted to used exposure parameters, we disabled the corresponding readout functionalities on used system during our manipulation.

**NOTE:** Our simulations were performed without electronic noise and specifying quantization ramp.
2.1.3. PROJECTOR AND PHANTOM

Phantom: is defined in text file whose name is specified in configuration file and sourced during main function run.

In this work, analytical definitions and simple geometries were used, respecting one of the existing phantom definition syntaxes, specifying:

- position with regards to "isocenter"
  (defined in configuration file via source-to-isocenter distance)
- geometry
  (in this work, cylinder defined by diameter and length, or rectangle parallelepiped defined by its three dimensions or slant edge defined as a square with rotated sides, were used)
- material and density
**Projector**: Primary photons contribution to final image is calculated in CatSim by analytical calculations based on [Beer-Lambert](https://en.wikipedia.org/wiki/Beer%E2%80%93Lambert_law) attenuation law and [Ray-tracing algorithm](https://en.wikipedia.org/wiki/Ray_tracing). For each projection ray (i.e. a projection connecting a focal spot sample i with a detector sample j), an energy-dependent attenuating factor is calculated using Beer-Lambert law, taking into account the attenuation coefficient of imaged object and its thickness along the considered ray. Determination of a given thickness along each ray is done by ray-tracing algorithm:
- all intersection points between X-ray beam and intercepted primitive structures of object are calculated
- sum of distances between consecutive points defining a straight line represents the total crossed thickness; the order of priority is considered if needed.

*Figure 12* represents relative positions of focal spot, imaged spherical object and detector as well as the corresponding spectrum at considered levels (spectrum after emerging from anode, after crossing the attenuating object and before coming to detector, after detection), for a considered energy bin E and along a considered direction i (linking a given focal spot sample and a given detector sample).
$I_i^{\text{raw}}(E_k)$ corresponds to the X-ray photon number for an energy bin $E_k$ after attenuation due distance travelled in anode and after attenuation by flat filters if some are used.

$I_i^{\text{incoming}}(E_k)$ corresponds to the number of X-ray photons coming to the detector, taking into account the geometrical factor, the attenuation due to the crossed object and the scatter contribution to the signal.

Mathematical expression of $I_i^{\text{incoming}}(E_k)$ can be formulated as follows:

$$I_i^{\text{incoming}}(E_k) = I_i^{\text{raw}}(E_k) \frac{A \cos(\theta_i)}{|\vec{r}_i|^2} \exp\left(-\sum_o \mu_o(E_k) l_{iso}\right) + I_i^{\text{scatter}}(E_k) \quad (7)$$

with

A pixel surface, in mm

$\theta_i$ the angle between the given ray direction and unit vector orthogonal to detector surface, in degrees $^\circ$

$\vec{r}_i$ the vector defining ray between the considered focal spot sample and detector sample, with norm in mm

$\mu_o(E_k)$ is the linear attenuation coefficient of object $o$ at energy $E_k$, in cm$^{-1}$

$l_{iso}$ is the intersection length between the line with index i-s (linking detector pixel i and focal spot sample s) and the object with index $o$, in cm

$I_i^{\text{scatter}}(E_k)$ the number of photons due to scattered radiation

$I_i^{\text{view}}$ corresponds to the number of electrons obtained by conversion of detected X-ray photons.

Mathematically, it can be expressed as follows:

$$I_i^{\text{view}} = G_{DAS}\left[\sum_k E_k P(QDE(E_k, i) I_i^{\text{incoming}}(E_k))\right] + N(\sigma_{DAS}) \quad (8)$$

with

$G_{DAS}$: is the total detector gain (combining the optical and the electronic ones), i.e. conversion factor of number of X-ray photons to number of detected electrons

$k$ the index of considered energy bin $E_k$

$P(.)$: is Poisson distribution

$QDE(E_k, i)$: is the quantum detective efficiency of the detector

$N(0, \sigma_{DAS})$: is additive electronic noise modelled by Gaussian distribution with zero average and standard deviation $\sigma_{DAS}$
2.1.4. SCATTER MODEL

Several models are available in CatSim. The following explanations are based in particular on [3] and [10].

**Monte Carlo (MC) model**: The MC engine tracks photon trajectory inside the imaged object until the photon energy is either absorbed by the phantom or detected by the X-ray converter. With increasing number of initially shot photons, N, the accumulated signal provides more realistic estimation of the scattered photons distribution in final image.

In general, the signal provided by conventional MC is rather noisy and demands an excessive calculation time to have a smooth estimation of scatter contribution.

**Monte Carlo hybrid model**: Another method combines analytic equations with MC tracking. Instead of detecting the photon energy, the hybrid simulator calculates for each photon interaction (within the voxelized phantom), a probability map of the photon being scattered to every point of the detector. The probability map is analytically calculated for each interaction of each shot photon using Klein-Nishina (for incoherent) and Lorentz-Mie (for coherent) equations.

With increasing number of photons, the contributions are accumulated into an estimation of the scattered photons projections.

When compared to conventional MC, for the same number of initially shot photons, the hybrid simulator provides a much smoother signal.

As consequence, hybrid model presenting time reduction, as compared to conventional Monte Carlo model, was used in this work.

Finally, when dealing with a simulation tool, it is important to recall the main aspects contributing to the computation complexity:

- number of energy bins for spectrum definition
- number of source samples, i.e. focal spot points
- number of detector samples, i.e. detector pixels
- number of phantom samples (voxels)
- phantom complexity (composition, material), rotation...
- number of threads

*Figure 13* depicts the main steps of both Monte Carlo methods. Generation and running the event are performed in the same way. The difference comes at the step of tracking and updating the information about scatter contribution. Conventional Monte Carlo method tracks all photons until they are absorbed and provides number of scattered photons over detector matrix. Hybrid Monte Carlo updates the scatter field for every position at which a given photon could undergo a scattering event.
Considerations for Monte Carlo simulations:
- each voxel of phantom is characterized by densities of each element contained within the considered voxel
- spectrum is read from the given ASCII file and scaled, considering the source-to-detector distance, and attenuated by filters, if defined

For every shot photon, Monte Carlo (MC) models keep track of:
- \( x_i \): location of scatter event
- \( \vec{d}_i \): direction of photon before it undergoes an event
- \( E_i \): energy of photon before it undergoes an event
- \( \tau_i \): type of event (Compton, Rayleigh or photoelectric absorption*)

*photoelectric absorption location is not tracked in case of hybrid MC

Thus a collection, \( \sum_i \), of every photon interaction during its lifetime is tracked, as depicted in Figure 13 in step 1: \( \sum_i = \{ x_i, \vec{d}_i, E_i, \tau_i \} \).

Event loop presents 4 main steps, as schematized also in Figure 13:
1. an x-ray photon with randomly chosen energy and direction is shot; the random number is generated between 0 and M, with M: maximum of cumulated density function obtained from input spectrum
2. the given photon is approximated to the nearest phantom voxel following the initial direction vector
3. for every voxel in the path, probability of the photon being transmitted to the next voxel or interacting is calculated. If interaction occurs, 3 possibilities exist:
   - Compton scatter: photon changes direction and energy
   - Rayleigh scatter: photon changes direction
   - Photoelectric effect: photon is absorbed
Compton and Rayleigh interactions are tracked and registered until photon reaches the last voxel or escapes from the field-of-view. If photoelectric absorption occurs, it is the end of photon lifetime.

4. Hybrid MC: scatter field is updated: for every tracked scatter event, the probability that the considered photon scatters in the direction of a given detector pixel and that it is completely transmitted along this direction.

    Conventional MC: scattered photons arriving to a given detector pixel are registered.
3. CONTRIBUTIONS TO ADAPTATION OF THE SIMULATOR FOR IMAGING WITH A C-ARM X-RAY SYSTEM

3.1. SOURCE MODEL

In this section, two aspects of source model were approached. First part aimed at optimization of the heel effect simulation. Then, the other part focused on focal spot impact on image sharpness. As consequence, the implementation of various focal spot profiles (in particular, the realistic one) and simulations were carried out to evaluate the possibility to generate images with realistic sharpness.

3.1.1 Heel effect

3.1.1.1. Material and methods

State-of-the-art of Heel effect model:
At the beginning of the project, the simulation of Heel effect was based on model which considered a single localization point of the photon production [19]. The anode penetration depth is expressed and simplified to the following formula [20]:

\[
l_p(\alpha, \gamma) = \frac{l_c \cos(\theta)}{\sin(\theta + \gamma) \cos(\alpha)} \rightarrow l_p(\gamma) = l_c \frac{\cos(\theta)}{\sin(\theta + \gamma)}
\]

with

- \(l_p\): distance travelled by X-ray photon inside target anode before emerging from it, in mm
- \(l_c\): depth of penetration of electrons into anode, in mm
- \(\theta\): angle of target anode, in degrees °
- \(\gamma\): angle of emerging X-ray photons in cathode-anode direction along x-axis as represented in Figure 3b, in Section 1, with regards to vertical (source-detector) axis, in degrees °
- \(\alpha\): angle of emerging x-ary photons in the direction of y-axis as represented in Figure 3b, with regards to vertical (source-detector) axis, in degrees °

For a better visualisation of these geometrical considerations, angles \(\theta, \gamma, \alpha\) as well as the lengths \(l_p, l_c\), are represented in Figure 3b.

The variation in the direction perpendicular to cathode-anode axis (along \(\alpha\) angle) was supposed to be negligible.

The anode penetration depth was experimentally fitted, as function of tube voltage\(^5\). As expressed in the following formula, the attenuation factor had been applied to the spectrum distribution as function of the emerging angle from the anode:

\(^5\) anode electron penetration in mm = A.\text{beam energy}\(^B\); with A and B fitted coefficients for a given focal spot size
\[ I(y, E) = I_0(E) e^{-\mu_W(E)l_p(y)} \]  \hspace{1cm} (10)

with

\[ I_0 : \text{the intensity at the point of photon generation, in number of photons} \]
\[ \mu_W : \text{total attenuation coefficient of tungsten, in cm}^{-1} \]
\[ l_p : \text{distance travelled by a photon before it emerges from the anode, in cm} \]
\[ y : \text{angle between the direction of emerging X-ray photon and vertical axis, in cathode-anode direction, in degrees (°)} \]

However the spectrum files could not be generated for the same anode angle as in real systems [6], thus a compensation element was introduced to take into account the angle used for spectrum file generation and the actual anode angle including tube tilt:

\[ I(y, E) = I_s(E) \times \frac{e^{-\mu_W(E)l_p(y)}}{e^{-\mu_W(E)l_p(\text{ref})}} \]  \hspace{1cm} (11)

\[ I_0 : \text{the intensity at the point of photon generation, in number of photons} \]
\[ \mu_W : \text{total attenuation coefficient of tungsten, in cm}^{-1} \]
\[ l_p : \text{distance travelled by a photon before it emerges from the anode, in cm} \]
\[ y : \text{angle between the direction of emerging X-ray photon and vertical axis, in cathode-anode direction, in degrees (°)} \]
\[ \text{ref} : \text{compensation angle, used for correction of attenuation taking into account the difference between the angle at which the spectrum file was generated [REF 3] and the angle of real system: 11,25° for systems with detector 31 cm and 11,25° + 1°tilt for systems with detector 41 cm. Indeed, all target anodes present the same angle, but a different tilt of tube is used to "irrigate" with X-ray beam larger or smaller area on detector. The following Table 2 sums up the tilt values used for different detector dimensions. In our case, detectors 31 cm and 41 cm were used.} \]

<table>
<thead>
<tr>
<th>Detector dimension</th>
<th>20 cm</th>
<th>31 cm</th>
<th>41 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube tilt</td>
<td>-3°</td>
<td>0°</td>
<td>1°</td>
</tr>
</tbody>
</table>

Table 2: Detail of X-ray tube tilt depending on the size of the detector in system

Moreover the provided spectra already included an attenuation due to tungsten, therefore another "compensation" term taking into account this aspect was applied, even though this cannot retrieve the low energy components of the spectrum, lost due to the attenuation.

The effect of attenuation due to anode tungsten is represented in Figure 14, showing a spectrum with and without considering intrinsic attenuation of X-ray beam due to the anode.
Half-value-layer (HVL) was calculated in order to compare it to experimentally measured HVL value. HVL calculation was performed considering the following relations expressing X-ray beam flux and Air Kerma:

\[
\text{flux}_k(i) = \text{Intensity}(i) \cdot e^{(-\mu_{\text{air}}(r-thick(k)) \cdot 0.1 - \mu_{\text{Al}}(k) \times 0.1)} \times \left(\frac{1000}{r}\right)^2
\]  

Air Kerma\_k = \sum_{i}^{E_{\text{max}}} \text{flux}_k(i) \cdot \frac{\mu_{\text{Air}}}{\rho_{\text{Air}}} (i) \cdot E(i) \cdot \text{Unit conversion factor}

Normalized Air Kerma\_k = \frac{\text{Air Kerma}\_k}{\text{max}(\text{Air Kerma}\_k)}

with

\[
\text{thick} = 0.1k \text{ in cm}
\]

\[
k = (0 : 1 : \text{number of slices}) : \text{index for Aluminium thickness layers}
\]

\[i = (0 : 0.5 : E_{\text{max}}) : \text{index for Energy bins defining the input spectrum}
\]

ConvFactor = \[1000 \left(\frac{\text{R}}{\text{kg}}\right) \cdot 1.6022 \cdot 10^{-16} \left(\text{Joules keV}\right) \cdot 114 \cdot 1000 \left(\text{mR g}^{-1}\text{yr}^{-1}\right)
\]

So that final expression for HVL was:

\[
\text{HVL} = \text{interp1}(-\ln(\text{Normalized Dose}), \text{thick}, -\ln(0.5))
\]

with

\[
\text{interp1} : \text{linear interpolation function, applied to determine the Aluminium thickness necessary to reduce the dose to half of the initial value.}
\]

\[
\text{Normalized Air Kerma}\_k : \text{vector composed of normalized doses values for different thicknesses thin}\text{k(k)}
\]

Figure 14: Effect of attenuation by a 0.001 mm thick tungsten filter on spectrum generated at 75 kVp

Discrepancies between the simulated and the real system heel effect in the image up to 10% had been found previously in the measure of the Half-Value-Layer (HVL).
Therefore, in order to optimize the simulation outcome, we attempted to take into account the distribution of the photon production localizations within the anode.

\[
\text{Figure 15: Distribution of photon generation depths}
\]

The realistic distribution of electron penetration depth, represented in Figure 15, was provided from a Monte Carlo simulation in the previously used conditions and the equivalent distance travelled by photons before they emerge from anode was expressed as follows:

\[
l_e = \sum_i PD_i p_i
\]  

with

\[
PD_i : \text{penetration depth, in mm}
\]

\[
p_i : \text{probability that photon generation occurs at penetration depth } i
\]

The following expression was used for tungsten compensation factor:

\[
l_c e^{\frac{\cos(\delta_{\text{ref}}) - \frac{\pi}{180}}{\sin(\delta_{\text{ref}} - \frac{\pi}{180})} \mu_w(E)}
\]  

with

\[
l_c : \text{equivalent distance travelled by photons before they emerge from anode, in cm}
\]

\[
\delta : \text{construction anode angle, i.e. } \delta = 11^\circ
\]

\[
\delta_{\text{ref}} : \text{compensation angle, used for correction of attenuation taking into account the difference between the angle at which the spectrum file was generated [21], [6] and the angle of real system, in degree}
\]

\[
\mu_w(E) : \text{linear attenuation coefficient in tungsten for a given energy bin } E, \text{ in cm}^{-1}
\]
The compensation term for tungsten attenuation was applied before computation of intensity vector for various emerging angles $\gamma$, as schematized in Figure 3b.
Unlike in the previous approach, where intensity was calculated for one fitted electron penetration depth, we computed the intensity as function of emerging angle $\gamma$, applying the corresponding probability factor as follows:

$$Spec_{\gamma}(E) = \sum_i p(i) Spec(E)e^{PD(i) \frac{\delta}{180}\cos\left(\frac{\pi}{180}\right)\frac{\mu_w(E)}{\sin\left((\delta-\gamma+\text{ref})\frac{\pi}{180}\right)}}$$

(18)

with

$p(i)$: probability that photon generation occurs at penetration depth $i$
$Spec(E)$: input number of photons for a given energy bin $E$, as defined in spectrum file
$PD(i)$: penetration depth, in mm
$\delta$: construction anode angle, i.e. $\delta = 11^\circ$
$\gamma$: emerging angle, in degrees
$\text{ref}$: compensation angle, used for correction of attenuation taking into account the difference between the angle at which the spectrum file was generated [6] and the angle of real system, in degree
$\mu_w(E)$: linear attenuation coefficient in tungsten for a given energy bin $E$, in cm$^{-1}$

For $\gamma \in [-\text{limit angle}; \text{step angle}; \text{limit angle}]$ with limit angle = target angle + tube tilt (in CatSim the target angle is specified already taking into account the tilt angle, ie 8,25°, 11,25 or 12,25° respectively for detectors 20, 31 and 41cm.)

### 3.1.1.2. Results

The following Table 3 sums up the previous and the new results (before and after introduction of electron penetration depth distribution, with 11.25° anode angle). We concluded that this new implementation lead to accuracy improvement.

<table>
<thead>
<tr>
<th>HVL measurements (mm of Al)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experimental</strong></td>
<td>Before</td>
</tr>
<tr>
<td>5,15</td>
<td>5,69</td>
</tr>
</tbody>
</table>

Table 3: Results before and after realistic distribution of electron depth penetration
3.2.1.3. Discussion

An aspect explaining the experimental and the simulated discrepancy was the need for several "compensation terms", since:
- used input spectra files were providing the spectra already attenuated by tungsten anode.
- single photon production point had been considered
- distance travelled by the photons in the anode before emerging was considered as the one perpendicular to the entering electron beam direction.

The previous model used in CatSim simulator integrated:
- "attenuation compensation term" for tungsten attenuation
- "angle compensation factor" taking into account the difference between angle at which the spectrum was produced (they had access only to spectra for various energies for 11° anode angle)
- varying intensity distribution depending the emerging angle $\gamma$ (in cathode-anode direction), considering a single photon generation point [22].

However, the "attenuation compensation" step can “recover” the intensity of the existing energetic components (as before anode intrinsic attenuation), but it is not possible to recover the spectral components of the energies which had been filtered out.

The new model also disposed of spectrum files generated for anode angles 11° and 12° which supposed only 0.25° angle compensation parameter (as compared to -2.75° in case of spectra simulated at 11° and anode angle being at 8.25° for detector 20 cm) and thus probably less bias.

Also, estimating the attenuation contribution for various anode penetration depths contributed to slightly lower discrepancy.

This section presented another "brick" added to heel effect simulation. A realistic distribution of photon generation locations was integrated to existing heel effect model. Results with slightly better accuracy compared to experiment were observed.

3.1.2. Focal spot model

Previous studies showed that focal spot (FS) blur has notable effect on the final image blur and thus that FS has an important impact on system MTF degradation [23].

Therefore, a choice to focus on the focal spot simulation was made within the scope of this project. The aim was to evaluate the necessity to define the focal spot in the simulation tool by the realistic profile distribution and to validate the possibility to simulate accurately the effect of focal spot on image sharpness and to optimize the simulation with regards to computation complexity.

In order to quantify the effect of FS on image sharpness, system Modulation Transfer Function (MTF) was selected as the corresponding pertinent image quality metric [7].
3.1.2.1. Material and methods

3.1.2.1.1. Simulation part

In the previous models used in simulations in CatSim, a point source or a square (uniformly distributed with equal width and length dimensions) finite size sampled source had been used, as explained in the Section 2.

The realistic FS size and form were implemented using experimental data of FS measurement acquired at manufacturing level. The length and width profiles were obtained by sampling the images and summing up the distributions respectively in "length" and "width" directions as depicted on the left in [24].

These length and width profiles were provided as linear distributions of X-ray signal intensities with respect to the corresponding positions, deduced from the Real-time focus meter (RFM) measurement at the detector center for techniques specified by International Electrotechnical Commision standards: IEC60336 and IEC60613. Main parameters specified by these standards are reported in Table 4 below. The declared length and width dimensions are those specified at most at FW15%M as schematized on right in Figure 16.

<table>
<thead>
<tr>
<th>Beam energy (kVp)</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube current (mA)</td>
<td>630</td>
</tr>
<tr>
<td>Tolerance limits for FS with nominal size 1mm</td>
<td>1.40mm, 2.00mm</td>
</tr>
</tbody>
</table>

Table 4: Techniques and maximum permissible values specified by IEC60336 standard

---

6 Figure 17 contains schematic profiles, as the real FS images and profiles were supposed confidential
7 Techniques specified for the FS measurements : For FS with nominal size 1mm: Acceleration potential of tube = 75 kV; Tube current = 630 mA;
8 FW15%M : Full width at 15 % of maximum
As mentioned before, for image analysis, system MTF was chosen as the image quality metric to quantify the sharpness of both, the experimentally acquired and simulated images. MTF measures how the signal (contrast) is passed through the system, as a function of spatial frequency. Due to FS asymmetry, the system spatial resolution is anisotropic. Therefore, a 2D system MTF would best permit to evaluate the studied effect of blurriness in final image. Since the most suitable available phantom was a tungsten slant edge, the analysis was decomposed into two (vertical and horizontal) 1D system MTF analysis.

In the following parts, the system MTF refers to 1D horizontal or vertical system MTF, detector MTF was considered symmetric, since it was measured in conditions where the influence of FS is minimal, as detailed in Section 3.2.2.1.4.

**NOTE:** As a matter of fact, detector MTF is the result of scintillating light spread and pixel aperture. Since detector aperture is supposed to be symmetrical (squared pixels), detector MTF is supposed to be symmetrical. Moreover, the isotropic behavior of detector MTF relies on the readout architecture. One can easily imagine readout mode for which pixels are readout per packet so that the readout pattern is not squared. What is important to note is for the specific measurements in this work, detector MTF is isotropic which has been verified experimentally (detailed later in $MTF_{detector}$ validation part).

---

$^9$ Illustration from X_ray_tube.pdf (for illustrative purposes, for confidentiality reasons, for oral presentation, real profile distributions can be shown)
In order to estimate the effect of the overall system on image sharpness, 1D system MTF can be expressed as the result of multiplication of MTF of different components (focal spot, detector, motion, object) contributing to image blurriness (based on linear and shift-invariant theory):

\[
MTF_{system}(f) = MTF_{FS}(f) \cdot MTF_{detector}(f) \cdot MTF_{motion}(f) \cdot MTF_{object}(f)
\] (19)

In our experiments, we chose the set-up so as to be able to consider static objects (i.e. \(MTF_{motion}\) including tube or patient motion equals 1 and has negligible effect on \(MTF_{system}\)). We also aimed at minimizing the \(MTF_{object}\) effect by choosing as thin phantom as possible, however this contribution to final image blurriness was reproduced also in simulation by specifying a realistic slant edge phantom. As consequence, the global system MTF can be simplified as follows:

\[
MTF_{system}(f) = MTF_{FS}(f) \cdot MTF_{detector}(f) \cdot MTF_{object}(f)
\] (20)

A validated MTF computation tool [9] was used to get MTF data for simulated and experimentally acquired images.

Figure 17: Schemes of realistic FS implementation; LEFT: Adaptation and reading of vectors containing intensity distributions and corresponding positions; Initially the dimensions are defined at FW15\%M as specified by IEC standards; CENTER: Vectors with linear distributions are resampled (if specified by user), centred and normalized to sum up to 1 (as they play role of probability density functions) and plotted for visualisation; RIGHT: Calculated complete coordinates \((x,y,z)\) with corresponding weights are saved as fields of "source" structure; each source sample is used for projection computation by ray-tracing algorithm.
Implementation:
One script containing the vectors with average intensity distributions and corresponding positions, for both directions (length and width), was adjusted with the provided FS profiles. The user needs to insert the correct intensity and position vectors, i.e. the correct length to get the desired FS dimensions. Another script containing three full\textsuperscript{10} FS profiles (maximum, average, minimum) was written. This one takes into account the choice of length, width (both specified by the percentage level for FW\%M), and a regular decimation factor specified by the user in configuration file and returns the desired vectors of positions and of corresponding intensity distributions. The decimation factor allows for resampling the profile distributions using fewer sample points. In order to preserve the specific profile shape of the distribution in case of higher decimation, the user can redefine irregularly spaced distribution vector.
\textbf{NOTE:} Automatized construction of irregular distribution is a future step to be implemented. This is essential to allow optimization of computation time and to keep the possibility to introduce computation complexity elsewhere (for instance in the perspective of complex phantom or dynamic phantom imaging, 3D imaging...).

The function allowing the integration of such defined realistic focal spot profile was adjusted and its functioning can be described by the following points:

\textbf{Inputs:}
- Filename containing the source profiles through intensities and corresponding positions
- Target angle of the anode
- FS unit (mm)
- FS width (mm)
- FS length (mm)

\textbf{Outputs:}
Source structure defined by the fields:
- Number of source samples
- Array with (x,y,z) coordinates for each source sample
- Vector of weights (scaled intensity) corresponding to each source sample
- Vectors defining the directions of the X-ray beam (forward, lateral (fan), longitudinal (cone))
- Number of corners and array with (x,y,z) coordinates for each corner

\textbf{Function flow:}
- A script containing the vectors with intensity distribution and corresponding position for length and width directions is read.
- The intensity vectors corresponding to the length and the width are normalized so that each sums up to 1.
- The position vector is shifted in order to center the focal spot center on (x,z) = (0,0).

\textsuperscript{10} containing all sample points defining the FS distributions as provided by the team at manufacturing level, i.e. 512 sample points for each dimension, at 0.0081 mm step
- The coordinates in y-direction (source - detector axis) are computed taking into account the anode effective angle (- length/tan(anode effective angle)).
- Projection to the anode surface plane is performed and the anode effective angle to get the corresponding y coordinates is taken into the account (taking into account the source-to-detector distance).
- Output coordinates are built.
- The output profile and projected view of FS on anode ("electronic FS") are plotted.
- From an overall simulation perspective, the simulation flow is explained in Section 2. The ray-tracing algorithm computes the projections for every combination of source and detector sampling points, taking into account the attenuation due to the length of the given ray crossing the phantom.
- Each projection traced from a given focal spot sample is weighted by the corresponding profile intensity value. 

*Figure 17* schematises the main steps of realistic focal spot implementation.
3.1.2.1.2. Experimental and simulation set-up:

The experimental set-up was designed using parameters which were shown to assess easily the focal spot blurriness (magnification 1.5 (> 1.3), high frequency content phantom). The validation consisted in two steps:

- Detector MTF validation, assuring that:
  - \( MTF_{\text{detector}} \) component does not introduce imprecision to \( MTF_{\text{system}} \) that would be measured.
  - MTF Tool software is used correctly.

- System MTF validation, aiming at:
  - validation of \( MTF_{\text{system}} \) in the center of the detector by confronting the simulated and experimental data.
  - assessing the impact of focal spot on \( MTF_{\text{system}} \) depending on the position in the image and depending on used magnification.
  - assessing the impact of use of a focal spot size and intensity distribution versus a uniformly distributed focal spot and a point focal spot in simulation, with regards to accuracy with experimental results optimizing the simulation.

3.1.2.1.3. Image acquisition for detector MTF validation

An acquisition using a standard Image Quality System Test (IQST) phantom was done. This is a phantom used for Quality Assurance procedures on the field. The region of interest (ROI) of the acquired image for detector MTF is given in Figure 18a, where measurement of \( MTF_{\text{detector}} \) is performed. IQST test for detector MTF being a standard measurement, set-up and techniques were chosen according to the IQST test operating mode, as briefly listed in the Table 5. The standard IQST phantom is embedded in a holder which was placed at the detector level, as schematized by Figure 18b.

<table>
<thead>
<tr>
<th>Set-up parameter</th>
<th>Used value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy (kVp)</td>
<td>80</td>
</tr>
<tr>
<td>Tube current (mA)</td>
<td>40</td>
</tr>
<tr>
<td>Source-to-isocenter distance (mm)</td>
<td>820</td>
</tr>
<tr>
<td>Source-to-detector distance (SDD) (mm)</td>
<td>1000</td>
</tr>
<tr>
<td>Pulse Width (ms)</td>
<td>10</td>
</tr>
<tr>
<td>Spectral filtration (mm of copper sheet)</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5: Parameters for IQST test for detector MTF

Figure 18: Set-up for detector MTF measurement:
18a: Detail of slant edge used of IQST phantom with regions used for MTF calculation;
18b: Scheme of experimental set-up, Source-to-image distance (SID);
Source-to-object distance (SOD)
The phantom slant edge is static and positioned right in front of detector so as to minimize the effect of motion and object thickness and magnification on measured MTF. The computation of the MTF<sub>detector</sub> was performed by following ways:
- in-built processing software of IQST test
- MTF Tool software for MTF calculation (in order to verify the correct use of the MTF tool software which was afterwards also used for system MTF calculation)

Then a decomposition of the experimental MTF<sub>detector</sub> to MTF<sub>aperture</sub>, taking into account the finite size of active photodiode area, expressed as a proportion ("fill factor") of the detector pixel pitch size, and MTF<sub>optical</sub>, taking into account the scatter of the photons in the scintillator needles. The latter one cannot be easily measured but it was calculated as follows:

$$MTF_{optical} = \frac{MTF_{detector}}{MTF_{aperture}}$$

(21)

### 3.1.2.1.4. Image simulation for detector MTF validation

Phantom files and configuration files were created to reproduce the set-up used in the experiment. Techniques and system geometry parameters are specified in the Table 5.

The phantom for the simulation was designed so as to reproduce materials and dimensions of the experimental phantom IQST.

For question of computation complexity, a detector with 512 × 512 pixels (corresponding to the central part of a larger detector) and 0,2 mm pixel pitch was used for simulation of MTF<sub>detector</sub>. MTF<sub>detector</sub> used in the simulation is modeled again as product of two elements, MTF<sub>aperture</sub> and MTF<sub>optical</sub> [3].

The MTF<sub>aperture</sub> is modeled by an analytic expression depending on the pixel pitch and fill factor. The active area of detector pixel is approximated by a square function, thus after Fourier transformation, in frequency domain, MTF<sub>aperture</sub> is expressed as a sine cardinal function:

$$MTF_{aperture} = |\sin (f \cdot \pi (ppitch.\ fillfactor))/(f \cdot \pi (ppitch.\ fillfactor))|$$

(22)

The MTF<sub>optical</sub> is modelled by a Lorentzian fit of the experimental MTF<sub>optical</sub>. The choice of Lorentzian fit is justified by the form of the decay obtained by division of experimental MTF<sub>detector</sub> by MTF<sub>aperture</sub> and by previous studies [3], [16], [17], [18]. Matlab Curve fitting Toolbox was used for Lorentzian fit determination and Table 6 resumes the found fit parameters and the goodness of the fit up to spatial frequency 2,5 lp/mm. Inspired by a previous work [3] on CatSim validation (and for more general form of searched function for decay model), a linear combination of two Lorentzian components
was chosen. These two components take into account the low and the high frequency contributions to the transfer decay\(^\text{12}\).

\[
MTF_{\text{optical, simulated}} = \frac{ca}{a+f^2} + \frac{(1-c)b}{b+f^2}
\]

(23)

with

\(c\) : weight between both Lorentzian components  
\(a, b\) : adjustment parameters for high and low frequency Lorentzian components, in \(\text{mm}^{-2}\)  
\(f\) : frequency vector, in \(\text{mm}^{-1}\)

Determination of simulated \(MTF_{\text{detector}}\) is done using MTF Tool and the result (developed in the part 3.2.2.1.7. Results, below) is compared to experimental \(MTF_{\text{detector}}\).

<table>
<thead>
<tr>
<th>Lorentzian parameters a, b, c</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ((\text{mm}^{-2}))</td>
<td>3.27</td>
</tr>
<tr>
<td>b ((\text{mm}^{-2}))</td>
<td>0.02</td>
</tr>
<tr>
<td>c (unitless)</td>
<td>0.95</td>
</tr>
<tr>
<td><strong>Goodness of the fit (R-square)</strong></td>
<td>0.99</td>
</tr>
</tbody>
</table>

**Table 6: Parameters found for Lorentzian fit for optical MTF of detector**

**NOTE:** "\(MTF_{\text{detector}}\)" is measured on final images (both acquired and simulated). Therefore it might seem inaccurate to call it "\(MTF_{\text{detector}}\)" and not "\(MTF_{\text{system}}\)" since a priori the effect of all components that might influence the transfer of the frequency content of the imaged object are included in the final image. Actually, "\(MTF_{\text{detector}}\)" refers to the measurement of the MTF using the final image which was acquired (and simulated) in conditions limiting the influence of all other components. Since the standard experimental set-up (IQST test) for "\(MTF_{\text{detector}}\)" determination exists, we decided to adapt the same configuration also for simulation validation. This included a static thin absorbent slant edge (to minimize back-scattering and crystalline effect) placed right in front of the detector to have no magnification.

To conclude on this aspect, by \(MTF_{\text{detector}}\) validation we mean that coherent results were obtained between \(MTF_{\text{detector}}\) measured from simulated images and so called \(MTF_{\text{detector}}\) from experimental acquisitions and calculated by the standard test for \(MTF_{\text{detector}}\). Moreover, simulated \(MTF_{\text{detector}}\) was measured from simulated image using focal spot defined by a single point, i.e. as a "point source" and showed coherent result as compared to measurement from experimental image, which was considered as comforting the assessment that the effect of focal spot in the considered set-up could be neglected.

\(^{12}\) For instance, due to respectively back-scattering and crystalline shape of Cesium Iodide scintillators, the fitted parameters show the dominant effect of the high frequency component (crystalline shape) explaining the decay.
3.1.2.1.5. Image acquisition for system MTF validation

Several acquisitions using the standard IQST phantom (without the holder) were done with the set-up design as schematized in Figure 19 and Table 7.

Details of object and detector positions are provided in Table 8. The techniques (beam energy and tube current) were chosen to correspond to those specified by IEC60336 and IEC60613 standards for a given nominal FS size (1mm, in our study). Indeed, the FS profiles depend on voltage and current. Therefore, we decided to execute first our experiments in these conditions, to avoid introducing discrepancy parameters as compared to the simulation (where the profiles acquired at these specific techniques were integrated).

Experimental $MTF_{system}$ were computed for acquired images using MTF Tool in horizontal and vertical directions and confronted to simulated results (developed in part Results, below).

![Figure 19: Experimental set-up for system MTF evaluation](image)

![Figure 20: Detail of slant edge and link to system MTF](image)

| **Detail of acquisition parameters for experimental images for system MTF evaluation** |
|---------------------------------|------------------|
| **Beam energy (kVp)**           | 75               |
| **Tube current (mA)**           | 630 (150 and 380 for evaluation of variability due to the current change) |
| **IQST phantom thickness (Tungsten slant edge for MTF measure) (mm)** | 3.34 (0.127) |
| **Object magnifications**       | 1.27, 1.3, 1.5, 1.57, 1.8 |
| **Phantom positions**           | Centre, (and also up and down for magnification 1.8) (in cathode - anode direction in detector plane) |

*Table 7: Acquisition parameters for experimental imaging of slant edges for system MTF evaluation*
### Table 8: Details on set-up used for experimental acquisitions for system MTF measurement

<table>
<thead>
<tr>
<th>Object magnification</th>
<th>1.8</th>
<th>1.5</th>
<th>1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source-to-detector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>distance (cm)</td>
<td>126</td>
<td>116</td>
<td>102</td>
</tr>
<tr>
<td>Object-to-detector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>distance (cm)</td>
<td>70</td>
<td>77</td>
<td>79</td>
</tr>
<tr>
<td>Source-to-object</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>distance (cm)</td>
<td>56</td>
<td>33</td>
<td>17</td>
</tr>
</tbody>
</table>

### 3.1.2.1.6. Image simulation for system MTF validation

Simulations reproducing the experimental imaging set-up from Figure 19 and Table 7 were performed. This supposed to:
- define IQST-like phantom
- adapt configuration files to the system geometry and techniques as used in the experiment for every magnification and every considered position.

**NOTE:** Due to current computation limitations, only images corresponding to small regions of interest of detector surface (150 per 150 pixels of 0.2 mm) could be simulated, in order to respect the realistic detector resolution limit.

Therefore the adaptation of configuration and phantom files was more "handy"; since it was not manageable to simulate in one run (within one simulated image) several phantoms at positions within the same image. Figure 20 shows the edges used for measurement of horizontal and vertical MTF<sub>system</sub> computations on simulated images were performed using MTF Tool in both horizontal and vertical directions.

MTF<sub>system</sub> is also estimated analytically, approximating FS to a uniform distribution

\[
MTF_{\text{system}}(f) = MTF_{\text{focal spot}}(f) \cdot MTF_{\text{detector}}(f) \tag{24}
\]

with

\[
MTF_{\text{focal spot}}(f) = \left| \frac{\sin[f \cdot \pi \cdot FS \cdot (mag-1)]}{f \cdot \pi \cdot (FS \cdot (mag-1))} \right| \tag{25}
\]

with

- \(f\): frequency vector, in mm\(^{-1}\)
- \(FS\): focal spot dimension (length or width), in mm
- \(mag\): object magnification

in the centre of the detector. This analytic formula gives also useful relationship between FS, magnification and first zero of \(MTF_{\text{focal spot}}\) which represents the limit spatial frequency of the system for given conditions (FS size and magnification) in our study, since set-up conditions were chosen so that the FS was the limiting component for spatial resolution.
In image plane: \[ f_{\text{cut-off}} = \frac{1}{f_s(mag-1)} \] (26a) and in object plane: \[ f_{\text{cut-off}} = \frac{\text{mag}}{f_s(mag-1)} \] (26b)

Horizontal and vertical \( MTF_{system} \) from simulated images were compared with analytical \( MTF_{system} \) and \( MTF_{system} \) computed from acquired images.

Also, simulations were performed for qualitative focal spot shape evolution as a function of the position in the detector plane. Corresponding phantom and configuration files were created to simulate the FS shape effect on the selected representative ROIs, depicted in Figure 20.

3.2.2.2. Results

We addressed the simulated results with the theoretical behavior predicted by analytical model valid at the center of the detector and with the results from experimental acquisitions.

Three experimental acquisitions were performed using the identical set-up conditions and ten simulations were run using identical configuration file (only random noise parameter changed), in order to assess the repeatability of determination of experimental \( MTF_{system} \) and of simulated \( MTF_{system} \).

**NOTE 1:** Graphical representations of modulation transfer functions (MTF) are presented up to the spatial frequency of 2.5 lp/mm which corresponds to the Nyquist frequency due to the size of pixel pitch of 0.2 mm, even though in the set-ups used for this study, focal spot was the component determining that limiting spatial resolution which was therefore inferior to 2.5 lp/mm. It is essential to keep in mind that only the part of the MTF prior to the first minimum has a physical sense, i.e. reflects the capacity of the imaging system to transfer frequency content of the imaged object. The evolution of MTF beyond the first minimum are affected by aliasing, however this choice of representation up to the Nyquist frequency due to detector pixel size permits to have a unique axis frame which allows for a quick visual comparison of MTF evolution up to their first minimum for simulations with different magnifications, focal spot sizes or focal spot types (point focal spot, uniform or realistic FS).

**NOTE 2:** Graphical representations of MTF are presented in image plane. Therefore the relationship between the focal spot size and magnification can be approximated by relation (26a).

3.1.2.2.1. \( MTF_{detector} \) validation

The \( MTF_{detector} \) (computed by MTF Tool) measured in simulated image was compared to the experimental \( MTF_{detector} \) (calculated both by MTF Tool and IQST integrated software) and to analytical expression of \( MTF_{detector} \) (which was implemented to CatSim simulator).

This comparison showed good accuracy up to the limiting resolution of the detector, i.e.

\[ \frac{1}{2 \cdot \text{ppitch}} = 2.5 \ \frac{\text{lp}}{\text{mm}} \]

Figure 21 represents the superposition of \( MTF_{optical} \) calculated from experimental images and \( MTF_{optical} \) found using Lorentzian fit. Figure 22 represents the superposition of \( MTF_{detector} \) as obtained from experimental and simulated images as well as corresponding tolerance limits defined by [25].
Indeed, it is not interesting to reproduce frequency content above Nyquist frequency, since the sampling frequency is governed by pixel pitch, and above Nyquist frequency the signal affected by aliasing. Standard deviation was estimated only from experimental data, experimental $MTF_{detector}$ provided by IQST software and calculated from acquired image using MTF Tool [9]; Figure 23 shows an example of a simulated slant edge image used for MTF measurement. As evoked before, $MTF_{detector}$ validation presented twofold interest:
- exclude detector impact on potential inaccuracy of $MTF_{system}$ simulation
- verify that used methodology for MTF determination provides meaningful results, since $MTF_{detector}$ determination was confronted to already validated measurement methodologies.

![Figure 21: Optical MTF decay fitted by a Lorentzian function](image1)

![Figure 22: Detector MTF validation, simulation and experimental results comparison, the simulation results fit into tolerance limits defined by gage R&R limits defined by detector quality specifications (at 1lp/mm and at 2lp/mm)](image2)

![Figure 23: Example of a simulated slant edge image for 1D MTF evaluation](image3)
3.1.2.2.2. \( MTF_{\text{system}} \) validation

1. Qualitative reproduction of the thermal focal spot as seen from the detector plane confirmed that the definition of the real size and the difference in length and width dimensions are necessary to simulate the optical FS “distortion” which depends on the position on the detector. Figure 24 shows the simulated images of "point object" using a point source (on left), realistic source (in the middle) and an reference scheme adapted from IEC30663 standard on focal spot measurement.

![Figure 24: Simulated images of a "point object" for qualitative assessment of FS size and shape distortion depending on the position in the field of view, LEFT: Point source; CENTER: realistic FS profile were used; two tungsten cylindrical objects of 0.2mm were placed in the centre (with magnification 2 and 1; one identical object was placed vertical, horizontal and diagonal axis 19.1 cm from detector centre, RIGHT: for reference - theoretical evolution of image size and shape distortion depending on the position in the field of view. [IEC60336 standard]](image)

<table>
<thead>
<tr>
<th>Configuration Parameters for Simulations for System MTF Evaluation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy (kVp)</td>
<td>75</td>
</tr>
<tr>
<td>Tube current (mA)</td>
<td>630</td>
</tr>
<tr>
<td>Pulse width (ms)</td>
<td>10</td>
</tr>
<tr>
<td>Number of pixels in final simulated image</td>
<td>150x150</td>
</tr>
<tr>
<td>Source-to-isocenter distance (mm)</td>
<td>820</td>
</tr>
<tr>
<td>Source-to-detector distance (mm)</td>
<td>1200</td>
</tr>
<tr>
<td>Source-to-object distance (mm)</td>
<td>Depending on object magnification</td>
</tr>
<tr>
<td>Object thickness (tungsten sheet thickness) (mm)</td>
<td>3.34 (0.127)</td>
</tr>
<tr>
<td>Spectral filtration</td>
<td>0</td>
</tr>
<tr>
<td>Target angle (°)</td>
<td>12.25</td>
</tr>
<tr>
<td>Focal spot form and size (mm)</td>
<td></td>
</tr>
<tr>
<td>Ulyss profile distribution</td>
<td>FS width, FS length</td>
</tr>
<tr>
<td>Uniform profile distribution</td>
<td>FS width, FS length</td>
</tr>
<tr>
<td>Point source</td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Configuration parameters for simulations for system MTF evaluation
Configuration details for this set of simulations is provided in Table 10.

<table>
<thead>
<tr>
<th>System parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy (kVp)</td>
<td>120</td>
</tr>
<tr>
<td>Tube current (mA)</td>
<td>50</td>
</tr>
<tr>
<td>Spectral filtration (mm of Cu)</td>
<td>0.3</td>
</tr>
<tr>
<td>Focal spot length (mm)</td>
<td>1; 1.88</td>
</tr>
<tr>
<td>Focal spot width (mm)</td>
<td>1; 1.21</td>
</tr>
<tr>
<td>Source-to-isocenter distance (mm)</td>
<td>820</td>
</tr>
<tr>
<td>Source-to-detector distance (mm)</td>
<td>1290</td>
</tr>
<tr>
<td>Pulse Width (ms)</td>
<td>10</td>
</tr>
<tr>
<td>MAG</td>
<td>1.5</td>
</tr>
<tr>
<td>Pixel pitch</td>
<td>0.2</td>
</tr>
<tr>
<td>Source-to-object distance (mm)</td>
<td>800</td>
</tr>
</tbody>
</table>

**Phantom**

| Diameter (mm) | 0.2 |
| Thickness (mm) | 1 |
| Material       | Tungsten (surrounded by air) |
| Position up, down, left, right on detector plane (mm from central axis) | 191 |

**Source types**

- Point source
- Ulyssé (realistic FS profile)

Table 10: Configuration details for simulations for qualitative assessment of FS distortion depending on position in FOV

2. For a set of experiments and simulations (with corresponding experimental set-up and simulated configuration settings, as detailed in Table 7 and Table 9, experimental results presented lower MTFsystem as compared to MTFsystem obtained from simulated images using the FS defined by average realistic FS distributions (horizontal and vertical) and dimensions specified by FW15%M, i.e.: $FS_{width} = 1.88$ mm (in cathode-anode axis where vertical MTFsystem is evaluated; top - bottom direction in the image) and $FS_{length} = 1.22$ mm (in perpendicular direction; left - right direction in the image).

Table 11 presents values of $MTF_{system}$ (in lp/mm) obtained from simulated image using FS dimensions as specified by IEC standard, and for experimental image.

This set of results also allowed to assess qualitatively a coherent behavior for simulation case:

- for a given FS dimensions, image sharpness is more degraded (limiting spatial frequency is lower) with higher object magnification
- for a fixed magnification, image sharpness is more degraded with larger FS dimension (in "vertical" case corresponding to FS width: $FS_{no\,dilatation} = 1.88$ mm and $FS_{dilatation} = 2.22$ mm.
Several possible reasons for these discrepancies between the experimental and simulated results were identified:

- **Slant edge thickness**: object thickness contributes to geometric blurriness in image, but in simulation we designed phantom corresponding to the real phantom thickness, composition and position in the field of view.

- **Tube vibration**: this was not simulated, but this effect should be already taken into account by the dimensions of FS profiles provided by manufacturing team that were implemented to simulator to define FS size and intensity distribution.

- **MTF computation method**: but MTF Tool is validated tool, we performed basic repeatability evaluation.

- **Magnification imprecision**: the same magnifications were used for simulations and experiment: but experimentally we measured the detector-object distance with a meter with an estimated error up to 1 cm on object-to-detector distance (including the error due to detector position indicated by the control panel of C-arm system and also the error due to our measurement of distance between the detector and the object). Details of distance measurements is given in Table 8.

Real system presents some limitations concerning the minimal object-to-detector and source-to-table distances, so it was necessary to find the allowed positions to obtain desired magnifications.

Source-to-detector distance was displayed by control monitor connected to the C-arm system. The detector and the table positions of C-arm system can be moved by a minimal distance of 1 cm. We consider therefore, that the source-to-detector distance as displayed on control monitor presented an error of 0,5 cm. Object-to-detector distance was measured by a meter, with a precision error of 0,5 cm. With these consideration, error propagated to magnification are less than 2 %.

Moreover, the slant edge was fixed at one extremity with a PMMA block, in order to image it without the patient table in the field-of-view. Consequently, the horizontal alignment of slant edge (parallelism to the detector plane) was only evaluated visually, and thus might present another contribution to magnification imprecision and to thickness of the slant edge, because in simulation the parallelism was assured by the horizontal position of the slant edge.

- **Localisation of the object** (some fractions of mm difference due to rounding when defining the phantom position and detector offset (due to conversion of distance from mm to number of columns or rows), some errors could have occur since for each new object position, a new corresponding file with phantom of slant edge and configuration with offset for the detector ROI were specified.

<table>
<thead>
<tr>
<th>MTF system (in lp/mm)</th>
<th>Mag = 1,8</th>
<th>Mag = 1,5</th>
<th>Mag = 1,3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{vertical FS_{no dilatation}} = 1,88 \text{ mm} )</td>
<td>0.66</td>
<td>1.06</td>
<td>1.77</td>
</tr>
<tr>
<td>( v_{vertical FS_{experimental}} )</td>
<td>0.56</td>
<td>0.90</td>
<td>1.50</td>
</tr>
<tr>
<td>( h_{horizontal FS_{no dilatation}} = 1,22 \text{ mm} )</td>
<td>1.02</td>
<td>1.64</td>
<td>2.73</td>
</tr>
</tbody>
</table>

*Table 11: Theoretical cut-off frequencies for given FS size and magnification; FS considered as rectangle function, in detector plane, for reference; as the slant edge in experiment and simulation was not positioned exactly in centre.
- Off-focal radiation contribution: not simulated
Impact of whole focal spot surface (also components of profile distributions with intensities < 15% of max)
- Tube aging

In addition to the average FS profile, we also disposed of minimal and maximal realistic FS profiles. Thus, a set of simulations was done defining the FS (intensity distribution and positions) using also maximal and minimal profile distributions, in order to estimate a range for $MTF_{system}$ within which the experiment should enter.

To assess also the impact of the low intensities in the FS profiles (under the limit of FW15%M), a set of simulations using a level preserving this parts of distributions were performed, too. These new levels for FW%M were determined for each profile: maximum, minimum and average for FS length and FS width, with visual assessment of parts to be conserved). These sets of simulations were done with magnification 1.5. The representations of $MTF_{system}$ resulting from this set of simulations are presented in Figure 25 and Figure 26. Indeed correct evolution of MTF curves was observed, i.e. simulations with minimal profiles presented highest cut-off frequencies than those using average and maximal profile.

---

13 NOTE: minimum and maximum profiles related to the sample on which FS have been measured, which does not imply that another tube could not give results out of these boundaries
Figure 25: Zoom Horizontal MTF: experimental MTF versus simulated results with min, average and max realistic FS profiles, with FW15%M and FW"low%"M (low percent: depending on the considered profile, between 1% and 5%)
Figure 26: Zoom Vertical MTF: experimental MTF versus simulated results with min, average and max realistic FS profiles, with FW15%M and FW"low%"M (low percent: depending on the considered profile, between 1% and 5%)
No important difference between the simulations at FW15%M and FW"low"%M were obtained. However the simulations using minimal, maximal and average profiles showed variations comparable to those that were observed previously between $MTF_{system}$ resulting from the experimental and simulated images. The differences in cut-off frequencies and corresponding $MTF_{system}$ value are listed in Table 12.

Indeed, horizontal $MTF_{system}$ computed using experimental image was included to the "range" defined by maximal and minimal simulated profiles along most of the spectral components up to limiting spatial frequency. Indeed, good accuracy with $MTF_{system}$ from simulated image using the maximal FS length profile was found which evoked the interpretation that in our experiments, the performance of the tube are close to the maximal case found in profile measurements provided for our project. However, in vertical direction the experimental result was still lower than the range found by simulations. Thereafter, we wanted to assess whether a use of a "size-adjustment" or "dilatation" factor permitting a normalization between simulated and experimental results would be possible.

First approach, Ad Hoc, consisted in determining a common dilatation factor for all three magnifications. We chose the "reference" cut-off frequency to be the one calculated from experimental image obtained with 1,3 magnification, since this is the case where FS presents the smallest effect on blurriness (thus on $MTF_{system}$ degradation). A dilatation factor for FS width was calculated (as the factor needed to obtain a FS width corresponding to our "reference" cut-off frequency) and this new adjusted (larger) FS width was implemented to simulation. The curves in Figure 27 shows that by ad hoc fitting of cut-off frequency, the transfer of the frequency content in experimental and simulated images present good

<table>
<thead>
<tr>
<th></th>
<th>Cut-off frequency (lp/mm)</th>
<th>MTF(at cut-off frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizontal</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimal profile</td>
<td>1,90</td>
<td>0</td>
</tr>
<tr>
<td>Average profile</td>
<td>1,76</td>
<td>0,01</td>
</tr>
<tr>
<td>Maximal profile</td>
<td>1,56</td>
<td>0,02</td>
</tr>
<tr>
<td>Experimental</td>
<td>1,57</td>
<td>0,03</td>
</tr>
<tr>
<td><strong>Relative error</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Max-Min)/Max</td>
<td>12,8 %</td>
<td></td>
</tr>
<tr>
<td>(Exp-Average)/Exp</td>
<td>12,1 %</td>
<td></td>
</tr>
<tr>
<td><strong>Vertical</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimal profile</td>
<td>1,07</td>
<td>0,01</td>
</tr>
<tr>
<td>Average profile</td>
<td>1,03</td>
<td>0</td>
</tr>
<tr>
<td>Maximal profile</td>
<td>0,98</td>
<td>0,01</td>
</tr>
<tr>
<td>Experimental</td>
<td>0,89</td>
<td>0,01</td>
</tr>
<tr>
<td><strong>Relative error</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Max-Min)/Max</td>
<td>8,4 %</td>
<td></td>
</tr>
<tr>
<td>(Exp-Average)/Exp</td>
<td>15,7 %</td>
<td></td>
</tr>
</tbody>
</table>

Table 12: Variation of cut-off frequencies for images simulated with max and min FS profiles
accuracy (by visual assessment) in cases of all three re-simulated magnifications, 1,3, 1,5 and 1,8. This result suggested that a "universal" adjustment factor might be possible to determine. However, since in our experimental set-up, the slant edge was not positioned exactly in the center, the other method consisted in adjustment factor determination using the ratio of cut-off frequencies issued from independent experimental and simulated $MTF_{system}$ curves. Thus we proceeded by following steps:

- evolution of optical FS width along "top-bottom" direction was computed from experimental and simulated (with average profile) results, represented in Figure 28
- a fit was performed using "Basic fitting options" in Matlab, represented in Figure 29
- optical FS in image plane was determined for experimental and simulated fitted curves
- corresponding ratio was calculated
- a ratio between cut-off frequencies from experimental $MTF_{system}$ curve and $MTF_{system}$ curve using simulated image (with average profile with magnification 1.5)

The inverse of the ratio of cut-off frequencies and the ratio of optical FS presented good agreement, as presented in Table 13. In other words, optical FS width at the center of the detector was determined from a set of simulations and experiments with various magnifications and positions in cathode-anode axis. A ratio was determined between the experimental and simulated optical FS width. Then a ratio between cut-off frequencies from the experimental and simulated images (for a given magnification and position away from detector) was found to correspond to the inverse of the ratio of optical FS found before.

For a lack of time, a new simulation with adjusted FS width was not performed to assess the accuracy also for frequencies below the cut-off frequency. (But the previous ratio estimation showed that fitting the cut-off frequency allowed to reconstitute the spectral components below with visually good accuracy).

Since the three experimental measurements for various magnifications were performed always with the same interventional imaging system, thus the same tube, no variability (from tube to tube) on $MTF_{system}$ determination using our slant edge method was estimated.

To have a more solid reason to find a normalizing adjustment factor, the same kind of experiments shall be performed on different systems, so that a larger set of experimental data could be gathered and analyzed, to assess the variability of $MTF_{system}$ obtained by slant edge method.

For simulation simplification, a set of simulations with same FS dimensions but uniform profile distribution were performed. This is actually a simple way to model FS and thus obtain a simple analytical expression. In general, such defined FS presented $MTF_{system}$ shifted towards lower spatial frequencies. This can be explained by the fact that such defined acts as "effectively" larger FS, since the weights of all samples (especially the one at the borders) are equally important. Unlike in realistic profiles which present a slight slope, thus lower intensities at borders, as visualized in Figure 30. Furthermore, by applying an adjustment factor, the simulated results presented better adequacy with experimental results when realistic profiles where used, which suggested the importance of taking into

---

The experimental position of slant edge referred to as "at centre" was shifted from the centre towards the top (in the image), but this position was reproduced in the simulation set-up, so should not be a source of discrepancy.
account the realistic intensity distribution of FS in simulations, as put into evidence in Figure 31. A brief evaluation of number of samples was also performed, as it is important to optimize computation time in simulation. 40 by 30 samples (6.0,0081 mm sampling step of profile distribution) took up to 10 minutes, whereas 512 by 512 (0,0081 mm sampling step) up to 5.5 hours, compared to less 1 minute for a simulation using a point FS. We estimated by simulation the number of samples that reproduce the frequency content with better than 5% uncertainty on first zero to be 20. The evolution of $MTF_{system}$ measured in images simulated with realistic (Figure 32) and with uniform (Figure 33) focal spot distribution. A convergence towards an optimal number of samples was observed. Figure 34 shows an example of simulated images, simulated using realistic focal spot with 40 over 28 samples (which was determined as the limit assuring that the characteristic profile shape was preserved when downsampling regularly the initial distribution which contained 512 over 512 samples) on the left and with 5 over 5 samples on the right. "Jerky edge" was observed in case of under-sampled focal spot definition. Figure 35 represents the effect of downsampling of focal spot distributions on its 3D construction in simulator. In this case, the downsampling was performed "by hand), so the sampling was irregular, in order to preserve the profile shape (i.e. the features of intensity distribution).

An analytical estimation was begun by under-sampling the provided FS profiles that were implemented to simulations and by calculating their Fourier transform. Up to factor 12 (12.0,0081mm sampling step), the error on cut-off frequency was less than 5% on first zero. The values of downsampling factor and corresponding cut-off frequencies of measured $MTF_{system}$ are presented in Table 14.
Figure 27: Resulting system MTF curves for set of experimental and simulated images. Corresponding experimental and simulation conditions were designed (imaging system techniques, phantom, magnification);
- results from simulated images with FS dimensions defined by IEC standard, i.e. FW15%M of linear FS profile, presented slower decay of system MTF (green) than in experimental case (red);
- attempt for adjustment by fitting the cut-off frequency allowed to reproduce the spectral components also by simulation (blue)
Figure 28: Evolution of optical FS as function of position on detector in top-bottom direction, deduced from a set of experiments and simulations for various magnifications; red - simulation with FS width = 1.88; magenta - experiment; green - simulation with FS

Figure 29: Fitting of optical focal spot evolution, pixel pitch = 0.2mm
Simulation FS width = 1.88 mm  
Experiment 
Simulation FS width = 2.56 mm  

<table>
<thead>
<tr>
<th>Fitted curve</th>
<th>Y = -0.0013X + 2.40</th>
<th>Y = -0.0019X + 2.99</th>
<th>Y = -0.0018X + 3.22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpolated central optical FS (mm)</td>
<td>1.71</td>
<td>2.02</td>
<td>2.32</td>
</tr>
<tr>
<td>Optical FS at 322 pixel (slant edge measure)</td>
<td>1.97</td>
<td>2.38</td>
<td>=&gt; ratio ( \frac{FS_{\text{sim}}}{FS_{\text{exp}}} = 0.8277 )</td>
</tr>
<tr>
<td>Cut-off frequencies (lp/mm)</td>
<td>0.977 (min profile)</td>
<td>0.8887</td>
<td>freq(<em>{\text{cutoff Exp}})/freq(</em>{\text{sim}}) = 0.9100</td>
</tr>
<tr>
<td></td>
<td>1.025 (average profile)</td>
<td></td>
<td>freq(<em>{\text{cutoff Exp}})/freq(</em>{\text{sim}}) = 0.8670</td>
</tr>
<tr>
<td></td>
<td>1.074 (max profile)</td>
<td></td>
<td>freq(<em>{\text{cutoff Exp}})/freq(</em>{\text{sim}}) = 0.8275</td>
</tr>
</tbody>
</table>

Table 13: Details of fitted optical FS evolution, Y: optical FS (in pixels), X: position on detector in top-bottom direction (in pixels)

Figure 30: Comparison of FS with uniform and realistic (Ulysse) profile distributions for equal dimensions specified by IEC60336 standard standard
Figure 31: Impact of profile distribution (uniform versus realistic) on sharpness degradation in images; simulations with Uniform FS profile presented lower cut-off frequencies than with the realistic FS profile, using the same FS dimensions.

Figure 32: Effect of number of samples for profile definition, in horizontal case the downsampling seems to have less impact, hypothesis is that the MTF is mostly impacted by the two intensity peaks present in this profile and since downsampling preserved the peaks the system MTF does not change much.
Figure 33: Benchmark for assessment of the effect of number of samples in profile distribution in images, uniform FS profiles were used, a convergence was found for number of 20 samples (12.0,0081mm profile sampling rate).

Figure 34: Downsampling within simulations was performed with main features maintained.

Figure 35: Effect of number of samples for profile distributions using a uniform profile (slant edge for vertical
Basic assessment of repeatability of the $MTF_{system}$ from simulated images was done by $MTF_{system, SIM}$ computation using 11 images generated using the same configuration file. The resulting curves are represented in Figure 36.

Basic assessment of repeatability of the $MTF_{system}$ from experimental images was done by $MTF_{system}$ computation using 4 acquired images using identical imaging conditions. The results are represented in Figure 37, with corresponding standard deviations computed at selected spatial frequencies, detailed in Table 15.

<table>
<thead>
<tr>
<th>Downsampling factor</th>
<th>Vertical limiting frequency (lp/mm)</th>
<th>Spatial frequency</th>
<th>Horizontal limiting frequency (lp/mm)</th>
<th>Spatial frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,2</td>
<td>0,96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1,17</td>
<td>0,93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1,17</td>
<td>0,93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>1,07</td>
<td>0,86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 14: Analytical assessment of downsampling affect on MTF of focal spot

<table>
<thead>
<tr>
<th>Spatial frequency (lp/mm)</th>
<th>0,5</th>
<th>1</th>
<th>1,5</th>
<th>1,709</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation Simulation (lp/mm$^{-1}$)</td>
<td>0,0031</td>
<td>0,0022</td>
<td>0,0043</td>
<td>0,0042</td>
<td>0,0044</td>
</tr>
<tr>
<td>Standard Deviation Experiment (lp/mm$^{-1}$)</td>
<td>0,0037</td>
<td>0,0019</td>
<td>0,0046</td>
<td>0,0010</td>
<td>0,0045</td>
</tr>
<tr>
<td>Spatial frequency (lp/mm)</td>
<td>0,1</td>
<td>0,3</td>
<td>0,5</td>
<td>0,7</td>
<td>0,8</td>
</tr>
<tr>
<td>Standard Deviation Experiment (lp/mm$^{-1}$)</td>
<td>0,0015</td>
<td>0,0016</td>
<td>0,0037</td>
<td>0,0032</td>
<td>0,0151</td>
</tr>
</tbody>
</table>

Table 15: Selected values for repeatability assessment of system MTF measures

Figure 36: Repeatability for simulated system MTF

Figure 37: Repeatability for experimental system MTF
In both cases (experimental and simulations) the MTFs were computed with MTF Tool and least-square error estimation was computed to have an idea of variations due to MTF Tool and our manipulation, but a more robust estimation of tolerance limits for accuracy needs to be performed and used when comparing:
- \( MTF_{systemSIM} \) and \( MTF_{system} \) curves obtained with slant edge set-up for various positions in the field-of-view and for various magnifications;
- \( MTF_{systemSIM} \) using realistic focal spot and uniformly distributed focal spot
- \( MTF_{systemSIM} \) using varying number of samples of a given focal spot distribution type (either uniform or realistic)

in order to evaluate the effect of the realistic FS profile implementation and the possibility to reduce the number of FS distribution samples to optimize computation time.

3.1.2.3. Discussion and conclusions

This work integrated the realistic focal spot profile distribution to simulations and proposed several initiatives in order to assess the impact of anisotropic FS on image sharpness.

The importance of difference in definition of FS length and width, completing the previous considerations of square or point source, allowed to reproduce by simulation the anisotropic effect of FS on image sharpness. Focal spot impact on image sharpness degradation as function of position on the detector and used magnification were studied.

The qualitative evolutions seemed correct, however a more solid work needs to be done on variability of the measurements to be able to define a tolerance range for discrepancies between results from simulations and experiments.

For computation time issue, the impact of sampling level was evaluated in order to estimate the optimal sampling that would allow to reproduce correctly the frequency content, but reduce computation time, too.

For further practical reasons, in particular the accessibility of realistic FS profiles, a comparison of simulations with uniform and realistic profile distributions were performed. The realistic profile showed possibility to approach better the experimental results.

3.1.2.4. Perspectives

The following actions can be undertaken in order to complete focal spot validation for the simulation tool:
- definition the degree of precision expected from the simulations in order to have necessary effect on detectability.
- analysis using a Point spread function (PSF) would take into account the anisotropy of the FS effect, instead of decomposing the problem to vertical and horizontal MTFs, however, this option was not chosen as an experimental set up at various positions on detector was not available.
- definitive conclusion on optimal sampling degree permitting to reduce computation time and
  conserving frequency content in the image
- development of automated irregular downsampling, conserving the characteristic features of FS profile
  by using an irregular histogram. In this work either regular downsampling or irregular "manual"
  downsampling were performed.
- complex assessment of reproducibility and repeatability is to be performed to set a relevant solid
  tolerance limits.

3.2. SCATTER STUDY

This sections reports results concerning an experimental design for scatter contribution measurement as
function of varying field-of-view. An analysis method was developed and first simulations performed.

Scattered radiation presents a non-negligible contribution to X-ray images (noise) in vascular
interventional imaging, mainly due to:
- Large patient thickness range (5 – 35 cm) (and up to 60cm for obese patients)
- Large field-of-view (FOV) areas as systems with 31.31cm² and 41.41cm² detectors are used
Therefore, correct simulation of scatter contribution is important to generate accurate synthetic images
allowing to assess image quality metrics correctly in various imaging modes.

NOTE: for system MTF study and validation, the experimental and simulation set-up were designed so
that the approximation of not considering scatter was justified: patient table out of the field, small
strongly absorbant object (slant edge).
Scatter simulation using hybrid Monte Carlo and conventional Monte Carlo models implemented in
CatSim were validated for conditions close to the one used in interventional X-ray imaging [3], i.e. large
detector and air gap.
The aimed contribution was to validate the evolution of scatter behavior with varying field-of-view and
for thicker phantom (at least 20 cm).

A set of experimental measurements evaluating the effect of varying FOV on scatter-to-primary ratio
(SPR) and thus the effect on scatter contribution to image.

3.2.1. Methods, materials

3.2.1.1. Acquisition of images

Experimental set-ups, schematized in Figure 38, were prepared in order to acquire images used for
scatter evolution.
The scatter contribution was studied considering:
- varying FOV using 31.31 cm² and 41.41 cm² detectors
- four 5 cm Poly(methyl methacrylate) (PMMA) blocks (corresponding to 20 cm human body equivalent thickness)
- phantom composed with thin lead polygons (2 mm thick) distributed over detector surface (material chosen for sufficient absorbance)
- fixed techniques
- fixed position
- no anti-scatter grid

More details on imaging set-up is presented in Table 16.

Scatter-to-primary ratio (SPR) was evaluated from the previously acquired images using “scatter maps”. The Matlab script created for SPR estimation computes the scatter maps along the following lines:
- Acquired raw images were read (several functions at the acquisition and image processing level are disabled during the experimental acquisition of images to reduce their effect on the images, as we want a real estimation of the number of counts per pixel) (Table 39, a).
- Edge detection was performed using Sobel filter for segmentation, using “edge()” function from Matlab Image Processing Toolbox (Table 39, b).
- Binary images were created for two types of area (with scatter contribution and with both scatter and primary contribution)
- Regions of interest (corresponding to the lead squares) were filled by “ones” to obtain a mask with “one” values at positions corresponding to only scatter contribution – the logical inverse of this binary image gives the mask with “one” values at positions corresponding to scatter and primary contribution (Table 39, c).
- Averaged acquired images were multiplied with the masks in order to get images with either only scatter contribution (and zero-value elsewhere) or only primary contribution (and zero value elsewhere) and the contributions were estimated as function of position on the image (Table 39, d).

- Contributions of primary beam were interpolated for the ROI defined by lead squares.

- Ratios of "scatter-to-primary" was performed for these ROI positions and 2D and 3D representations plotted (referred to as "scatter and SPR maps"). An example of "scatter map" is represented in Figure 40.

3.2.1.2. Simulation of images

Simulation set-up reproduced the phantom and system characteristics and techniques as used during the acquisition.

It is important to note that using a large phantom (20 cm by 30 cm by 30 cm) and large FOV implies higher computation time (increasing in particular with the number of detector pixels, phantom voxelization).
Hybrid Monte Carlo model was used to run the scatter simulation, with $3.10^5$ photons to be shot to estimate the scatter field. (Previous studies [3] showed that in their conditions, $10^6$ photons gave accurate scatter contribution estimation, as compared to their case, we used larger phantom, but decided to use a higher detector decimation factor and less photons, for first simulations). In order to reduce the computation time, detector cell decimation was used, less pixels for detector definition, with larger pixel size. This permits us to define the FOVs with correct dimensions in one view, using a reduced number of detector elements, thus reducing the computation complexity. Since the scatter is a low frequency phenomena and the simulation provides an estimation of scatter contribution, this operation is justifiable.

### 3.2.2. Results

Scatter depends mainly on three aspects: beam quality (its energy), the thickness of encountered objects and the field-of-view.

For a fixed beam quality and for a given phantom dimensions (fixed techniques and fixed phantom, as performed in our experimental benchmark with varying FOV), the varying parameter is FOV. More FOV increases, more SPR increases too.

Indeed, scatter is considered as isotropic at clinical energies that are used in our experiments (79 kVp). The primary photons contribution is less important on FOV borders (as the phantom covers the whole field of view, the photons intercepting the phantom under a certain angle have a longer distance to travel inside the phantom, then the photons incoming perpendicularly in the centre of the phantom). For a large FOV (40 cm, 32 cm and 30 cm), the difference between the primary contribution at borders of FOV and its contribution at the centre is larger than for small FOV, whereas scatter presents less fluctuation: thus the difference between the mean and the maximal SPR increases for larger FOV as presented in [Figure 41]. The detail of values used for the plots are presented in [Table 17].

For lack of time, simulation validation was not advanced, only first simulations were performed. They seemed to confirm Gaussian shape of scatter contribution to detected signal intensity, as shown in [Figure 42].
3.2.2.1. Experimental acquisitions:

![Graph: SPR as function of field-of-view for detector 30 cm]

**Figure 41: Scatter map: example of evolution of SPR with FOV for Detector 40 and 30**

<table>
<thead>
<tr>
<th>Detector 40</th>
<th>FOV (cm)</th>
<th>40</th>
<th>32</th>
<th>20</th>
<th>16</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean SPR</td>
<td></td>
<td>3.5026</td>
<td>2.7637</td>
<td>1.5875</td>
<td>1.0832</td>
<td>0.7592</td>
</tr>
<tr>
<td>Max SPR</td>
<td></td>
<td>4.3491</td>
<td>3.3548</td>
<td>1.7515</td>
<td>1.246</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Detector 30</th>
<th>FOV (cm)</th>
<th>30</th>
<th>20</th>
<th>16</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean SPR</td>
<td></td>
<td>3.0598</td>
<td>1.6987</td>
<td>1.2758</td>
<td>0.9042</td>
</tr>
<tr>
<td>Max SPR</td>
<td></td>
<td>3.3912</td>
<td>1.8389</td>
<td>1.3231</td>
<td>1.0946</td>
</tr>
</tbody>
</table>

Table 17: Detail of experimental SPR values obtained for various field of view for Detectors 31 and 41
3.2.2.2. Simulation:

In this section, the experimental set-up close to realistic interventional examination was used and an analysis by scatter maps developed. Simulation part seemed to present coherent scatter behaviour but need to be further developed.

Scatter simulations presented difficulties linked to computation time and memory load causing sometimes unpredictable computation interruption.

3.2.3. Discussion

In this section, the experimental set-up close to realistic interventional examination was used and an analysis by scatter maps developed. Simulation part seemed to present coherent scatter behaviour but need to be further developed.

Scatter simulations presented difficulties linked to computation time and memory load causing sometimes unpredictable computation interruption.

3.3. Construction of irregular histogram for simplification of used distributions

This paragraph aims at reporting the suggestions for construction of the irregular histogram for further optimisation of computation performance.

Optimization of the computation time as well as of the memory load are essential, for instance to allow higher complexity where it remains necessary, such as for complex realistic phantoms simulations.

In the current state of the art of the simulator, it would be interesting to apply irregular histogram to reduce:
- the number of energy bins used to define the spectral distribution files. These files are presented as array of number of photons per mm$^2$ per mAs at a given energy bin, previously specified 0,5 keV-stepwise, i.e. 121 – 241 energy bins for the clinically interesting tube voltages ranging between 60 and 120 keV, in case of vascular interventional x-ray imaging.
- the number of source samples in focal spot distribution file

The accuracy of simulations with these simplified distributions will need to be evaluated to conclude on optimal number of samples for these distributions.
4. CONCLUSION

Personal contributions within this Master's thesis project can be listed as follows:

- Estimation of the number of photons for a given examination conditions
- Contribution to heel effect model
- Implementation of realistic focal spot definition
- Simulation and experimental evaluation of image sharpness (also away from detector centre), quantified by system MTF computation
- Design of experimental set-up and analysis of scatter contribution in a typical X-ray interventional exam

Indeed, the realistic definition, and taking into account the difference between FS width and length appear to be crucial in accurate simulation of anisotropic effect of FS on image sharpness. In fact, depending on given imaging set up, FS can be the element which determines the limiting spatial frequency in cathode-anode direction (where FS dimension is larger), whereas in the other direction it might be the pixel size of the detector. Furthermore, it is important to keep in mind that actual FS dimensions can exceed significantly (IEC60336 standards tolerance limits) the declared nominal FS (declared by a single value for both directions, although for FS 1mm nominal, the tolerance limits go up to 1.4mm over 1.9mm), which makes a huge difference in the determination of limiting spatial resolution factor for a given imaging set-up, as illustrated in Figure 43 hereafter. It is essential to be aware of this aspect, to be careful when suggesting components design, deciding on trade-off between FS and pixel size depending on the clinically interesting imaging conditions.

The use of the synthetic images should permit engineers a unique flexibility in evaluation of a wide range of characteristics of the components in the image acquisition chain. Consequently, it will be possible to determine the value of components which are even not developed yet, preventing from the costs of related prototype development. This unique virtual approach should also permit us a better understanding of the most interesting development axes in interventional radiology for imaging with higher quality and efficiency.

Furthermore, this study contributed to promising perspectives for future designs. The possibility to "play" with simulated images to evaluate the influence of the size and form of the focal spot on spatial resolution of the imaging system. This means the possibility to link a system specification with a clinical observer task. New image quality metrics could be possibly defined with regards to the performance of the system which would present an important contribution to possibilities of image quality evaluation.
Figure 43: Limiting spatial resolution as function of magnification: Illustration of the "competition" between FS size and pixel pitch size in limiting spatial resolution of the system, based on [4], curves were plotted using relation (26b) in object plane.
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Last, but not least, thanks to Alice Caplier from Phelma and Massimiliano Colarieti-Tosti and other helpful people from KTH, for their flexibility and thus for making possible for me to work on my degree project and hopefully to present it in unusual period, due to my shifted schedules, as compared to most students, in the frame of the double degree.
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APPENDIX A

SIMULATION COMPLEXITY ASSESSMENT. PRELIMINARY ESTIMATIONS.

Verification of the spectrum by Air Kerma calculation. Primary photons computation.

This section aims at assessing the total number of primary photons involved during imaging procedures. X-ray spectrum is used as input for calculations.

The coherence of the input X-ray spectrum file used for simulations was assessed by comparison of computed Air Kerma with Air Kerma obtained from a real X-ray examination. Air Kerma is defined [26], (ICRU Report 33, 1980) as "the sum of the kinetic energies of all those primary charged particles released by uncharged particles (here photons) per unit mass"

Air Kerma can be mathematically expressed as:

\[
AirKerma = \sum_{i}^{E_{max}} \frac{\mu_{tr,Air}}{\rho_{Air}}(i) \varphi E(i)
\]

with

\(i\): index for energy bin i, as defined in the input spectrum file
\(E_{max}\): beam energy, corresponding to the highest energy of energy bins defined by input spectrum, in joules
\(\frac{\mu_{tr,Air}}{\rho_{Air}}(i)\): mass energy-transfer coefficient of air (for energy bin i), in cm²/g
\(\mu_{tr,Air}(i)\): linear attenuation coefficient for air at energy bin i, in cm⁻¹
\(\rho_{Air}\): density of air, in cm³/g
\(\varphi\): photon fluence, in cm⁻²
\(E(\cdot)\): energy corresponding to energy bin i, in joules

Air Kerma is usually expressed in units of Gray (Gy) corresponding to joules per kilogram. In order to compare the computed Air Kerma to experimental value, we convert its value from Gray to millirontgen. The previous formula was reformulated, a conversion factor for units was introduced and Air Kerma was computed as follows:

\[
AirKerma = \sum_{i}^{E_{max}} \frac{\mu_{tr,Air}}{\rho_{Air}}(i) \rho_{Spec}(i) \rho_{PW} E(i) \text{ConvFactor}
\]

with

\(\frac{\mu_{tr,Air}}{\rho_{Air}}(i)\): mass energy-transfer coefficient of air (for energy bin i), in cm²/g
\(\mu_{tr,Air}(i)\): linear attenuation coefficient for air at energy bin i, in cm⁻¹
\( \rho_{\text{Air}} \): density of air, in cm\(^3\)/g

\( \text{Spec}(i) = \text{Intensity from spectrum file}(i) \).

**Distance correction factor.**

*Intensty from spectrum file*(i): vector of number of photons per second per mm\(^2\) per corresponding energy bin i at 750 mm from source (attenuated by 750 mm of air)

**Distance correction factor** = \( (\frac{750 \text{ mm}}{\text{SSD}})^2 \) : inverse square law correction, taking into account the distance at which the input spectra is given (750 mm) and the real distance at which Air Kerma shall be evaluated. SSD.

**Surface correction factor** = 100 : correction factor to get the photon flux per area in cm\(^2\) for consistency with \( \frac{\mu_{\text{Air}}}{\rho_{\text{Air}}} \) (i) units

\( I \): tube current, in mA

\( PW \): pulse width, in ms

\( E(i) \): \( i \)-th energy bin (for \( i = 0 : 0.5 : E_{\text{max}} \)), in keV

\( \text{ConvFactor} = 1000 \left( \frac{Z}{kg} \right) \times 1.6022 \times 10^{-16} \left( \frac{\text{Joules}}{\text{keV}} \right) \times 114.1 \times 1000 \left( \frac{\text{mR}}{\text{Gy}} \right) \)

The set-up and imaging parameters were set as specified by the set-up conditions by the calibration file [27]. *Figure 44* represents the system set-up and *Table 18* presents main parameters used for calculation.

<table>
<thead>
<tr>
<th>Set-up parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source-to-skin distance (SSD) (mm)</td>
<td>1000</td>
</tr>
<tr>
<td>Dosimeter-to-detector distance (DDD) (mm)</td>
<td>80</td>
</tr>
<tr>
<td>Tube current x Pulse width (mAs)</td>
<td>1</td>
</tr>
<tr>
<td>Tube voltage (kVp) (= ( E_{\text{max}} ))</td>
<td>120</td>
</tr>
</tbody>
</table>

*Table 18: Set-up parameters for Air Kerma determination*

*Figure 44: Scheme of the set up for Air Kerma verification*
<table>
<thead>
<tr>
<th>Experimental mRmAs factor [27]</th>
<th>Simulated mRmAs factor</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>14,1798 mRmAs</td>
<td>14,4734 mRmAs</td>
<td>2 %</td>
</tr>
</tbody>
</table>

Table 19: Air Kerma verification

The resulting estimated value, presented in Table 19: Air Kerma verification was considered as accurate enough for spectrum validation, presenting only 2% relative error, acceptable with regards to other experimental uncertainties of this measurement\(^{15}\).

This validation, using mRmAs calibration value, allows us to assume that it was justifiable to estimate the number of photons by a simple spectrum integration (which corresponds to summation of the number of photons defined by the spectrum file).

Therefore, the number of photons arriving to detector surface without crossing an attenuator was calculated as follows:

\[
\text{Number of photons (airscan)} = \sum_{i}^{\text{Spec}(i)} I \cdot PW
\]

with

\[
\text{Spec}(i) = \text{Intensity from spectrum file}(i) \left(\frac{750}{S\text{ID}}\right)^{2} \text{SurfaceCorrection}
\]

\[
S\text{ID} : \text{source-to-image distance, i.e. source-to-detector distance}
\]

\[
\text{SurfaceCorrection} = 100 \text{: correction factor to get the photon flux per area in cm}^{2} \text{for consistency with } \mu_{\text{tr,Air}}(i) \text{ units}
\]

\[
I: \text{tube current, in mA}
\]

\[
P\text{W}: \text{pulse width, in ms}
\]

Number of photons at different levels (right after emerging from anode and right before arriving at the detector after having crossed the imaged object) were also estimated using inverse square law for distance correction and Beer Lambert attenuation law to take into account the attenuation due to phantom and patient table. "Equivalent dose" Aluminum thickness was considered for this rough estimation, as the real composition of the table is complex. Moreover, patient support attenuation is expressed in terms of aluminum equivalent thickness to be compliant with normative constraints (21 CFR and IEC).

\(^{15}\) Examples of source of uncertainties (here for a dosimeter RadCal 2026): Beam quality: +/- 5%; Pressure and temperature: +/- 10% if not taken into account (corrected with radcal 2026); Exposure rate dependence: <5% up to 5 mGy/s; Leakage: <5e-15 A with +300 VDC bias
\[ \text{Number of photons at anode level} = \text{Number of photons (airscan)} \cdot \left( \frac{S}{D} \right)^2 \]  
and

\[ \text{Number of photons}_{\text{phantom}} = \sum_{i=1}^{E_{\text{max}}} \text{Spec}(i) \cdot I \cdot W \cdot e^{-\mu_{\text{tot,PMMA}}(i) \cdot \text{thick}_{\text{PMMA}}} \cdot e^{-\mu_{\text{tot,Al}}(i) \cdot \text{thick}_{\text{Al}}} \]  

with

\[ \text{Spec}(i) = \text{Intensity from spectrum file}(i) \cdot \left( \frac{750}{S} \right)^2 \cdot \text{SurfaceCorrection} \]  

\[ \mu_{\text{tot,PMMA}}(i) \]: linear attenuation coefficient for PMMA at energy bin i, in cm\(^{-1}\)
\[ \text{thick}_{\text{PMMA}} \]: crossed thickness of PMMA, in cm
\[ \mu_{\text{tot,Al}}(i) \]: linear attenuation coefficient for Aluminium at energy bin i, in cm\(^{-1}\)
\[ \text{thick}_{\text{Al}} \]: crossed thickness of Aluminium, in cm

To have an idea of computation time estimation, a study of a previous intern [29] was considered and their estimations are reported in Table 20.

Set-up conditions: Mo/Mo spectrum at 28 kVp, 7.5 cm thick PMMA phantom\(^{16}\)

<table>
<thead>
<tr>
<th>Number of photons to be generated</th>
<th>Computation time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(^7)</td>
<td>0.25</td>
</tr>
<tr>
<td>10(^8)</td>
<td>1.5</td>
</tr>
<tr>
<td>10(^9)</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 20: Computation time estimation, data reported from [28]

This gives by linear extrapolation an estimation of **150,1 hours** (ie cca 6 days 6 hours 6 min) for 10\(^{10}\) photons, in the conditions as specified above (being close to real conditions for a mammographic examination).

However in vascular interventional imaging, the patients range from 5 - 35 cm, or even up to 60 cm for obese patients, and polyenergetic spectra with higher maximal energy are used, as well as a larger air gap. Therefore, a priori, a higher number of photons for scatter estimation is needed to reach sufficient statistics at the detector entry.

The number of photons for scatter simulation is one of the essential parameters impacting the simulation time, since it involves Monte Carlo computation. It is therefore interesting to have the approximate number of scatter photons contributing to the image formation, for a given imaging

\(^{16}\) The previous intern, Luo, worked with a Xeon processor 6500, 3,2GHz, 16Go RAM; During this project we used a Z420 with 6cores and 16Go RAM
conditions. Our estimations of number of primary at different stages and of scattered photons are presented in Table 21. The details on considered experimental conditions are presented in Table 22. However, the simulation tool presents different models for scatter simulation, and configuration parameters permitting reduction of time for scatter simulation. These parameters include:
- number of shot photons (i.e. count event)
- detector rows and columns decimation\(^{17}\)
- spectrum energy bins decimation
- source decimation (use of point source for scatter estimation)
- phantom voxelization

Indeed, the simulation is done separately for primary and scatter contribution (respectively analytical and hybrid Monte Carlo computations), as long as examined conditions justify it.

<table>
<thead>
<tr>
<th>Tube current (mA)</th>
<th>Voltage (kVp)</th>
<th>Pulse Width (ms)</th>
<th>Tube current (mAs)</th>
<th>SID (cm)</th>
<th>SSD (cm)</th>
<th>SOD (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>118</td>
<td>77</td>
<td>2</td>
<td>0.24</td>
<td>102</td>
<td>66</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 22: Considered examination conditions for estimation of number of scattered photons. In this case, the techniques (tube current, voltage) are determined by system according to the chosen type of the imaging exam.

\(^{17}\) decimation: reduction of initially defined number of samples, for instance, number of detector pixels, number of focal spot samples, number of spectrum energy bins, number of phantom voxels

\(^{18}\) 20 cm thick, 30 by 30 cm large PMMA phantom

\(^{19}\) Determined for similar case (same phantom), more details in the Section 3 on Scatter study

\(^{20}\) Estimation 1: using analytical formulas for Number of photon and Air Kerma calculation as presented in equations (28), independently of CatSim tool
In this section, a mRmAs calibration method permitted to justify that total number of primary photons can be accurately evaluated for a given imaging conditions by integrating the spectrum used as input for our later simulations. Thus, knowing the scatter-to-primary ratio (SPR), an estimation of scatter photons is also possible. This allows to assess the computation complexity for simulations and justifies also the need for "decimation" options aiming at computation optimization.
Choice of MTF as image quality metric to assess image blurriness was based on explanations given by [7]. Slant edge method for MTF determination [8] and MTF tool [9] were used for MTF measurement, using experimentally acquired and simulated images of slant edges.

Most of the analysis in part concerning focal spot simulations were performed using the open-source validated software tool for measurement of MTF. The latter read a .tif image of an slant edge, calculates corresponding Line spread function (LSF), then Edge spread function and finally Modulation transfer function. This tool was approached from an "end-user" perspective, although precautions assuring its correct use were taken: slant edge image with sufficient length (as defined in [9], 150 pixels) were simulated to assure good MTF measurement accuracy. Also, a discussion with one of the authors of the tool, Ann-Katherine Carton, currently working at GEMS Buc, was possible to assure correct selection of options proposed by the tool.