# Review of the possibilities of magnetic energy storage using permanent magnets in Halbach configurations

Dr. Daniel Månsson, Elektroteknisk teori och konstruktion,
Skolan för Elektro- och systemteknik,
Kungliga Tekniska högskolan, manssond@kth.se

***Abstract – Here we review the possibilities, advantages and disadvantages of using magnetic material to form structures with one-sided fluxes (i.e., "Halbach arrays") for purpose of energy storage. These can form highly non-linear magnetic springs which can be widely adapted for specific cases, but an optimal parameter point exist for energy storage.***

## 1. INTRODUCTION – REASONS FOR ENERGY STORAGE

The climate is slowly but surely changing. Several methods to combat this is discussed and implemented. The more realistic and beneficial approach to mitigate this crises is to reduce the energy demand of society (with sustained rate of improvement of standard of living around the globe). The predominantly used energy carrier today is electricity without which our modern civilization would perish. Thus, it is logic to look at improving the carrier system, i.e., the power grid. The concept of a "smart grid" has been introduced and (at the time of writing) started to be implemented at several smaller networks around the globe. A smart grid incorporates (to a much larger extent than most current power grids):

1) Information and communication technology, i.e., ICT via "smart appliances" in homes (only active when there exist a surplus of cheap and "clean" energy) or for, e.g., extensive real-time measurements of voltage and current in the grid.

2) Distributed renewable power production via, e.g., wind, solar or geothermal power production and several spatially distributed positions in the grid supplementing existing sources.

3) Energy storage, which is required as the renewable power production is inherently stochastic over time and location and peak production hours might, thus, not coincide with peak power requirements.

Thus, this inevitable need for storing the energy produced in an intermediary state in wait for the demand to increase is, from many perspectives, an interesting scientific topic. The stored energy can be in the form of chemical energy in batteries, potential energy of elevated water in a basin (i.e., pumped hydro), electromagnetic energy in the charge of a capacitor, kinetic energy in the momentum of a flywheel etc.

Here, we will discuss a more novel approach, in which magnetic material is used in a special magnetization scheme, to form a magnetic spring, in which energy is stored mechanically when the configuration is compressed. Barring, e.g., mechanical losses, the energy can then later be utilized when they are again released and repel each other. As we shall see, if such a configuration uses two normal bar magnets this is not an optimal usage of magnetic material. Also, we will assume that modern NdFeB magnets are used as they can give strong magnetic field (e.g., 1.3 T for N42 graded NdFeB).

## 2. MAGNETIC SPRINGS – NONLINEAR VIA HALBACH CONFIGURATIONS

The particular magnetization scheme is known as a "Halbach array" was first described by J.C. Mallison [1] but later rediscovered and further explained by K. Halbach [2]. The particular magnetization leads to a structure having (ideally) a one sided flux (i.e., the magnetic field is concentrated to the enhancing side and is ideally zero on the opposing, cancellation side of the structure, see left figure in Fig. 1). (This constitutes however not a magnetic monopole as the field lines forms closed loops.) They were originally envisioned to be used for more accurate reading and writing information on magnetic tapes [1] but are today used in, e.g., so called "wigglers", magnetic levitation of high-speed trains, magnetic bearings or in brushless motors. Halbach arrays were first described as a continuous change in the magnetization direction along one axis of the structure [1] but were later discretized to a structure of finite number of elements [2].



To investigate the use of Halbach arrays as an energy storage media we will first have to know how they act as magnetic springs and, thus, we have to investigate the restorative force created when two such opposing magnetic structures are pushed together (see right figure in Fig. 1). We will study this by following the path of $\boldsymbol{M} \to \boldsymbol{B} \to \bar{\bar{T}} \to \boldsymbol{F}$ , i.e., the magnetizations of the structures will give us the magnetic flux density between (and around) them from which we can calculate Maxwell's stress tensor around one of the structures and then we can get the force on this structure.

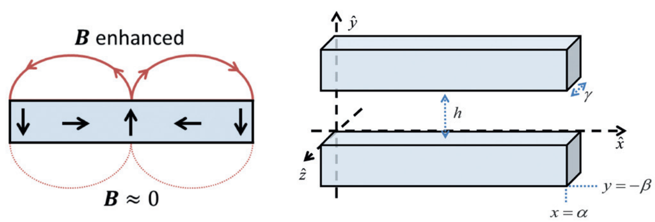


Figure 1. A single Halbach array with rotating direction of magnetization is shown alongside two Halbach arrays with the dimensions (α, β, γ).

The Magnetization of the upper and lower Halbach array is given by ("+" for upper and "-" for lower) $\boldsymbol{M} = M_0\ (\hat{\boldsymbol{x}} sin(kx) \pm \hat{\boldsymbol{y}} cos\ (kx))$. Here, $\kappa = 2\pi/\lambda$ is the wavenumber and $\lambda$ is the wavelength of the magnetization in the material (i.e., the length between consequent maxima in the magnetic field). Taking one array at a time, we can get the magnetic flux density from $B = \mu_0\ (\boldsymbol{M}+\boldsymbol{H})$, but since $\nabla \cdot \boldsymbol{B} = 0$ and also $\nabla \times \boldsymbol{H} = 0$ (as we have no free currents in the magnetic material) we get $\nabla \cdot \boldsymbol{H} = -\nabla \cdot \boldsymbol{M}$ and as we can now described the magnetic field with a scalar potential alone [3], i.e., $\boldsymbol{H} = -\nabla\phi$ , we have $\nabla \cdot \boldsymbol{M} = \nabla \cdot \nabla\, \phi = \nabla^2\, \phi$ which can be solved for the scalar potential [4]. Using the expression for $\phi$ we can then acquire the expressions for the magnetic field around the array but for our purposes we are here only interested in the magnetic field in the gap between the arrays as this is the only field that will contribute directly to the behavior of the magnetic spring. Thus, the magnetic field below the upper array (i.e., on the enhancing side, $\mathbf{H}_1$) and above the lower array, $\mathbf{H}_2$, can be calculated. However, as the coordinate system has y = 0 at the top of the lower array we shift the variable for the upper array (i.e., simply change "y" to "y – $(h+\beta)$"):

$$\left.\begin{array}{c} \boldsymbol{H}_1 = M_0\left(1-e^{k\beta}\right)e^{k(y-(h+\beta))} \\ \left(\hat{\boldsymbol{x}}sin(kx) - \hat{\boldsymbol{y}}\cos(kx)\right) \\ \boldsymbol{H}_2 = -M_0\left(1-e^{-k\beta}\right)e^{-ky} \\ \left(\hat{\boldsymbol{x}}sin(kx) + \hat{\boldsymbol{y}}\cos(kx)\right) \end{array}\right\}$$

Finally, the total magnetic field in the gap between the two arrays is given by $\boldsymbol{H}_{tot} = \boldsymbol{H}_1 + \boldsymbol{H}_2$,

$$\begin{array}{c} \boldsymbol{H}_{tot} = M_0 sin(kx) \\ \left[\left(1-e^{k\beta}\right)e^{k(y-(h+\beta))} - \left(1-e^{-k\beta}\right)e^{-ky}\right]\hat{\boldsymbol{x}} \\ - M_0 cos(kx) \\ \left[\left(1-e^{k\beta}\right)e^{k(y-(h+\beta))} + \left(1-e^{-k\beta}\right)e^{-ky}\right]\hat{\boldsymbol{y}} \end{array}$$

B

We assume that the arrays are pressed together "slow enough" that we can assume that no electric fields are induced (i.e., $\nabla \times E = -d\boldsymbol{B}/dt \approx 0$, thus, Maxwell's stress tensor is given simply by [3] $\mathbb{T}_{ij} = \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$. The force on the lower array (which is, of course, equal in magnitude but opposite in direction for the other array) is given by the closed surface integral (taken over a surface so $\mathbf{F} = \oiint \mathbb{T}\cdot d\boldsymbol{a}$. As most of the magnetic field is located in the gap and only a small, as we shall see, negligible portion is on the external edges of the arrays it is sufficient to calculate this surface integral on the top of the lower array. In addition, for storage purposes we only concern our self with the vertical component of the force, i.e., $F_y$ and as our magnetic field is independent upon z, several of the tensor components are zero (see [4] for details) the potentially complicated expression becomes

$$\boldsymbol{F} = \oiint \mathbb{T}\cdot d\boldsymbol{a} \approx \iint_{top} T_{yy,top} da_{top} = \gamma \int_{x=0}^{\alpha} T_{yy,top} dx.$$

Remarkably, we can reduce the complicated surface integral into a line integral on the top surface alone. The tensor element becomes $\mathbb{T}_{yy} = \frac{1}{\mu_0}\left(B_yB_y - \frac{1}{2}\left(B_x^2 + B_y^2\right)\right)$ and if we integrated this along the line (where y = 0) we get the restorative force, i.e., $F_y = \boldsymbol{F}\cdot\hat{\boldsymbol{y}}$, for the magnetic spring (see [4] and [5] for details):

$$F_y = \gamma \frac{{B_r}^2}{2\mu_0\mu_r^2}\left(\frac{\alpha c_1}{2} + \frac{\lambda}{8\pi}c_2 \sin\left(\frac{4\pi\alpha}{\lambda}\right)\right)$$

$$\begin{cases} c_1 = \theta - \psi \\ c_2 = \theta + \psi \end{cases} \text{and where}$$

$$\begin{cases} \theta = \left(\left(1-e^{k\beta}\right)e^{k(-(h+\beta))} + \left(1-e^{-k\beta}\right)\right)^2 \\ \psi = \left(\left(1-e^{k\beta}\right)e^{k(-(h+\beta))} - \left(1-e^{-k\beta}\right)\right)^2 \end{cases}$$

Here we also used that $\boldsymbol{B} = \mu_0\boldsymbol{H}$ in the gap and the magnetic remanence, often given by manufacturers, $B_r = M_0\mu_0\mu_r$. Thus, we finally have an expression we can use to analyze the situation. Below, in Fig. 2, Halbach arrays with dimensions α = 5 cm, β = 1 cm and γ = 1 cm and NdFeB material with $B_r$ = 1.3 T and $\mu_r$ = 1.05 is used in the dual configuration (shown in Fig. 1).

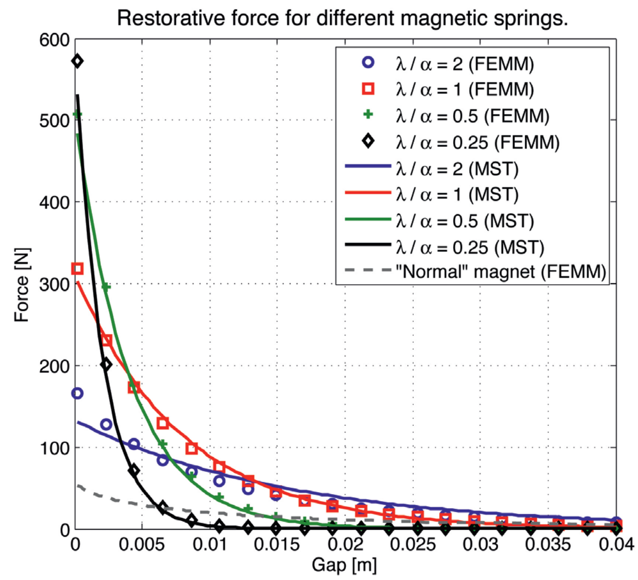


Figure 2. The restorative force created for two opposing Halbach arrays is highly non-linear and much larger than if two "normal" magnets had been used. Adopted from [5] (reproduced with courtesy of The Electromagnetics Academy).

The analytical derived restorative forces (denoted *MST*) are compared to numerical simulations using the FEMM software [6] and as can be seen the agreements justifies our approximations above. This type of magnetic spring is far from the simple linear variant often described by *Hooke's law* (i.e., $F_y = -k\dot{y}$ against the direction of compression) which can be seen when varying the relative wavelength (in comparison to the width α). In addition, varying the dimension β, will also greatly change the shape of the curves in Fig. 2 (see [5]). Thus, a highly non-constant, non-linear and parameter dependent spring stiffness is formed, i.e., $k = k(\lambda/\alpha, \beta, \gamma, h)$, and significant freedom of design exist to adapt to different applications.

## 3. ENERGY STORAGE –ADVANTAGES AND DISADVANTAGES

Compressing a mechanical spring will store potential energy in the configuration and compressing the two Halbach arrays will store potential energy in the magnetic field between the two arrays. If we integrate the restorative force over the distance of compression (assuming that the Halbach arrays originally have some separation distance $h \approx h'$ and are then compressed together ($h \approx 0$)) this gives us the energy spent during the compression. Assuming no deformations take place, the energy stored is the negative of this, i.e., $W = -\int_{h=h'}^{0} F_y$. This integral can best be handled via numerical integration in e.g. *Matlab*. If we divide the energy with the volume of the devices (i.e., $2\alpha\beta\gamma$) we get the energy density of the storage device and this is shown in Fig. 3.

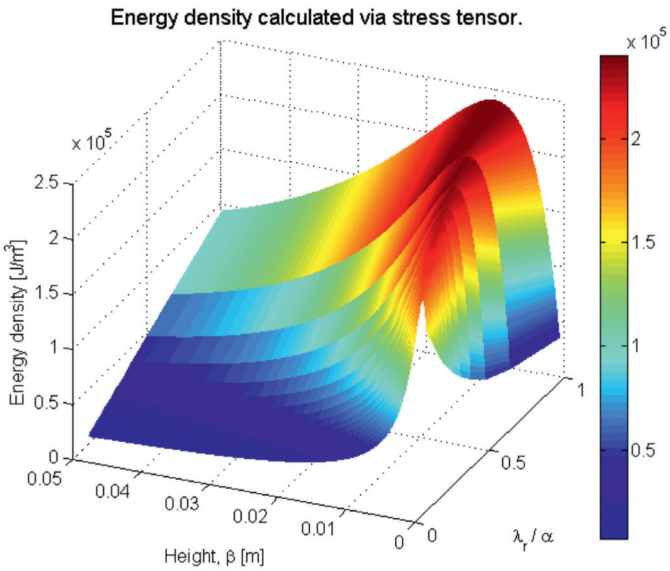


Figure 3. The optimum in the energy density as a function of dimensions and wavelength is easily seen. Adopted from [4] (reproduced with courtesy of The Electromagnetics Academy)

As can be seen there is a large dependency on the dimensions and the wavelength and there exist an optimal point of almost 250 kJ/m$^3$. This is comparable with some other commercial energy storage systems (e.g., compressed air energy storage). This optimum is important as the cost of the material increases linearly with any of the dimension, e.g., β, but the energy density will not. Thus, it is erroneous to simply increase any of the dimensions (i.e., larger magnets) to optimize the stored energy. For given λ, after the maximum point has been reached, it is much more cost effective to use additional material to form additional dual configurations [7]. Still, as the cost of vital rare earth materials (e.g., Neodymium) are today very high, and no direct replacement for these exists, the commercial realization of such an energy storage system is, thus, currently not realistic. Importantly however, this is the only principal disadvantage of the energy storage system and might be amended through the ongoing future material research.

The advantages of storing energy with this method is that, due to the nature of the medium, the stored energy doesn't, for normal conditions, degrade measurably even over very long periods of time (as the remanence of the magnets don't decrease over time). In addition, the charge and discharge times can, compared to most other energy storage systems, be very short without introducing intrinsic losses.

However, when the magnets are compressed quickly, currents are induced, both in the magnets themselves, but also in surrounding conducting structures (via $\nabla \text{x} \text{E} = -d\boldsymbol{B}/dt$). However, it was seen [7] that even if the surrounding structure is a solid conducting tube of copper and the magnets are compressed with a speed of 50 m/s, the resulting opposing force on the moving magnets (due to *Lenz law*) is negligible (much less than 1% compared to the maximum restorative forces in Fig. 2).

Also, during compression, the magnetic field inside the material is increased. This could potentially increase the temperature and if it is increased above the working temperature (here, 80°C) the magnets will start to lose their magnetization. In actuality, it is however only at the very corners of the structures where the magnetic field increases considerably. The power lost as heat is given by [8, pp. 221, Eq. (5.169)] and if the compression then happens at 50 m/s this still only produces 1.2 J lost as heat. The temperature increase is then given by $\Delta T = 1.2/(\text{mC}) \approx 0{,}3°C$ (with $\text{m} \approx 7.4 \cdot 10^{-3}\ kg$ and $C \approx 0{,}5\ kJ\ /kgC°$ ). Considering likely modes of normal operation and available cooling methods, this is deemed manageable (see [7] for details).

An additional issue, yet to be mentioned, is internal shearing forces. A continuous magnetization, as given by $\mathbf{M} = M_0\left(\hat{\boldsymbol{x}}sin(kx) \pm \hat{\boldsymbol{y}}cos\,(kx)\right)$ can't be made today and instead pieces of "normal" magnets are put together to form a discrete Halbach array. By increasing the number of such segments, and making them slimmer (and decreasing the difference in magnetization direction between two subsequent segments) we can approximate a continuous Halbach array quite well [7]. Each individual segment in a single array will feel a force (from the other segments in that array) to dislodge it. Fortunately the force on each segment is lower than the force required to damage the material and it is also lower than required to break the bond formed by modern adhesives [7]. In addition, the internal forces decrease with increased discretization of the Halbach array.

## CONCLUSION

A dual Halbach array configuration can be used to store potential mechanical energy in the magnetic field. This can be done fast and, normally, over a long period of time without significant obstacles besides the current financial cost of quality magnetic material.


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