Master Thesis

Prediction of the machine tool errors under quasi-static load

Developing methodology through the synthesis of bottom-up and top-down modeling approach

Author: Károly Szipka
Supervisor: Dr Andreas Archenti
Co-supervisor: Theodoros Laspas

KTH Royal Institute of Technology
School of Industrial Engineering and Management
Department of Production Engineering
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Abstract

One of the biggest challenges in the manufacturing industry is to increase the understanding of the sources of the errors and their effects on machining systems accuracy.

In this thesis a new robust empirical evaluation method is developed to predict the machine tool errors under quasi-static load including the effect of the variation of stiffness in the workspace, the geometric and the kinematic errors. These errors are described through combined computational models for a more accurate assessment of the machine tool’s capability.

The purpose of this thesis is to establish such methodology through the synthesis of the bottom-up and the top-down modeling approach, which consists the combination of the direct (single axis measurements by laser interferometer) and indirect (multi-axis measurements by loaded double ball-bar) measurement technics. The bottom-up modeling method with the direct measurement was applied to predict the effects of the geometric and kinematic errors in the workspace of a machine tool. The top-down modeling method with the indirect measurement was employed to evaluate the variation of the static stiffness in the workspace of a machine tool.

The thesis presents a case study demonstrating the applicability of the proposed approach. The evaluation technic extended for machine tools with various kinematic structures. The methodology was implemented on a three and a five axis machine tool and the results expose the potential of the approach.
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1. Introduction

Our industrial society has a fundamental interest in the machine tool market which is strongly connected to the manufacturing industry. In the last 20 years the world’s machine tool production and consumption have shown a growing tendency (see Figure 1). However the market in 1985 was estimated around 30 billion US dollars in the beginning of 2015 it reached the 80 billion US dollar level, with representing a more than 260% growth.

Since the time humans developed the ability to make and use tools, a special relation to precision engineering started to grow. The extent of technology has given this relationship a higher significance. Since the end of the eighteen century, scientists has had an increasing interest in precision. At that time exact measurements became the general attribute to the physical sciences. The history of increasing precision has been broadened to include many parts of the human life and activities. This thesis presents precision in its quantitative sense through measurements and instruments. Engineering precision usually refers to increase in accuracy and decrease in tolerances of produced parts. The extended concept also covers the whole design and manufacturing phase including the executed measurements. The philosophy of precision engineering dates back to the early 1930s [2] when this area was first discussed in a broad context. Later the word philosophy may be used to refer to the fundamental and general investigation, which includes a systematic approach and rely on rational arguments. Several precision engineering principles and a wide range of technological
improvements and knowledge have become part of our heritage. In recent years, precision engineering and metrology plays a vital role in the redefinition of the SI system [3]. Without precision manufacturing this work would not been possible.

From the manufacture of automobiles, integrated circuits to the toy industry vast numbers of areas can be differentiated. However the manufacturing of items can vary on a wide range of sizes, and the relative accuracy has to be compared. Precision manufacturing focuses on the creation of artefacts rather than the workpiece however naturally precision machines will produce precision workpiece. One of the most critical element in this approach is to reduce uncertainty at the interfaces between processes and products [3]. A higher degree of understanding how machines work lead to a better control on the errors that affecting their performance. This knowledge can reveal how design can affect the overall accuracy, resolution or repeatability of the machines and has an enormous additional value in the machining systems performance and capability. From the economic point of view the reduced non-productive time, the lower machine maintaining efforts and the higher workpiece quality have a significant cost effect.

Accurate production can be accomplished through a controlled and deterministic manufacturing process and the deep understanding of the structure and the behaviour of the manufacturing machine. The modern manufacturing precision machines can be investigated as an integrated system with its structural components, sensors and control systems. Important effects can be described through the interactions of the machine tool and machining process. Several test methods and international standards have been introduced to strengthen awareness in the evaluation of the machine tool’ capability. In respect to this complexity, to increase the accuracy of machined part dimensions, a higher understanding of the sources of the errors is required. With this understanding the errors can be compensated or avoided by revising the machine tool and eliminating the error sources. These errors can be described through developing computational models which are used to calculate and predict the positional accuracy of the cutting tool relative to the part being machined.

Combining computational models with robust testing methods results a methodology for a more accurate assessment of the machine tool’s capability. This ability can be used for finding preferred machining system in respect to a given specification [4] or it should implement an evaluation aid through analysis and control of the accuracy loss and act [5]. Such an empirical methodology has to fulfil basic
requirements in respect to economical, reliability and complexity aspects to be applicable in an industrial environment.

The accurate knowledge of positioning performance can be reached in two different modeling methodology. In case of the top-down methodology the whole system after an overview, is decomposed into sub-systems. Sometimes with further break down sub systems refined to base elements. During the bottom-up methodology the sub-systems are investigated as individual systems and the whole top-level is built up from these fragments. A third way can be differentiated with combining these two approaches. In this context the synthesis of this two modeling methodology can provide more relevant research information and opportunities to evaluate and validate the output of such a model.
1.1. Purpose and motivation

The purpose of this thesis is to develop a robust methodology for the prediction of the machine tool errors under quasi-static load by satisfying the requirements of the industrial environment. The methodology deals with the geometric errors and includes the deviations which were induced by an arbitrary applied quasi-static force.

The proposed research goal is reached through the synthesis of the top-down and bottom-up modeling method (see Figure 3). The established methodology consists the combination of the direct and indirect measurement technics. During this sort of model building, emphasis has to be given to avoid redundancy and different type of dependencies.

The fundamental aim of the created methodology:

- offer an adaptable solution for machine tools with different kinematic structures
- implementation of different type of measurements
- describe the variation in static stiffness of machine tool in the workspace
- predict machine tool geometric errors for a given toolpath
- predict machine tool errors under quasi-static load

![Figure 3. The research purpose of the thesis](image-url)
In addition a general goal is to describe the quasi-static behaviour of the machining systems by extending the connection between the accuracy level and the sources of the geometric errors with the effect of the variation of the static stiffness.

Laspas developed a methodology for modeling geometric errors of three-axis machine tools [5]. His model evaluates the geometric accuracy and estimates the machined part accuracy by predicting the error motion of the machine tool. He also applied the concept of functional points. The goal with this work is to extend his work to five-axis machine tools considering the geometric errors of rotary axes.
1.2. Scope and delimitations of the research project

The main focus of the thesis is on the prediction of the errors under quasi-static loads. In chapter 1 an introduction can be found with the outline of the thesis including the research purposes and motivations. Chapter 2 is the presentation of the relevant state of art. It contains the short description of the different sources of errors, a short explanation of the most important connecting terms and definitions. The further structure of the thesis follows the logic of the two methodology’s synthesis.

In chapter 3, the bottom-up modeling method was applied to predict the effects of individual axis geometric errors on the volumetric error of a machine tool. First a direct measurement will be implemented which allows measuring one error motion for one machine component at the same time. The aim of this computational model is to compose these direct measurement data and use them as inputs and calculate the deviations from a given toolpath in the workspace with geometric errors.

In chapter 4, the top-down modeling method was applied to predict the variation of the static stiffness in the workspace of a machine tool. First an indirect measurement will be implemented which approve to measure superposed errors of parallel motion of more machine axes the same time. In this case the aim of this computational model is to decompose the indirect measurement data and use them as inputs and calculate the static stiffness at different points of the workspace.

Chapter 5 is about the summary of the results, also it draws the conclusion and contains the future opportunities where the research project can be continued.

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**Figure 4. The scope and the delimitations of the developed model**
Since the detailed presentation of some additional results would detract the attention from the highlighted topic of the thesis the appendix contains outcomes which were carried out during the research project.

Certain criteria were needed in the thesis to separate appointed characteristics and the outline of the research area. In the whole developed methodology certain error parameters were neglected. The core of the thesis subordinated to the quasi-static description and evaluation of machine tool accuracy. On the Figure 4, the scope, the model boarder and the most important delimitations can be seen. However the short- or long-term change in the deep grey coloured parameters can have an effect on the measurement results (and finally on the model output) they were eliminated in the computational model.
2. State of art

2.1. Concept of machining systems and machining tools

The purpose of machining systems is to add value to a product by creating functional surfaces for example. All the design conceptions and parameters are subordinated to this two aspects. The development of machine design process requires structured models which are able to characterise the behaviour of the machine tool. Therefore, we have to start by defining the primary elements of the machine system.

Machining system refers to the elastic structure of the machine tool in its interdependency to the machining process. In general a machine tool includes the following sub-systems: the elastic structure, the drives and the control system, which includes the measuring system [6]. A general approach is introduced on Figure 5, which emphasises the significance of the interdependency between the machine systems’ components and environment. During the design of machine tools system rigidity, the accuracy of the movements and damping capacity are the most important parameters. Many conflicts can be found between these factors which also point to the importance of model building.

Figure 5. A general approach of the interdependency between the machine systems’ components and environment
The function of machine tools and their components is to generate the motions and forces that are necessary to execute the process. As for the interactions, the manufacturing process contains the workpiece, the manufacturing technology and the tool. Include the effect of these interactions can increase the accuracy of a model when analysing and optimising performance and process parameters. However, disturbances such as forces, heat gradients and moments that take effect during the process may influence the behaviour of the machine [7].

2.1.1. Accuracy of machining tools

The most important factors which critically affect all of the manufactured parts’ quality are the accuracy, the repeatability and the resolution. Furthermore, each components quality has a strong relationship with machine cost. The goal is to maximize the production quality and minimalize the machine cost. Consequently engineers optimize the parameters during the machine design process.

In this paper, terms accuracy, repeatability and resolution are used according to the definitions provided by Slocum which are in correspondence with ISO 230 part 2. Firstly he referred accuracy as “the maximum translational or rotational error between any two points in the machine’s work volume”. Repeatability is “the error between numbers of successive attempts to move the axis to the same position”. In case of repeatability we have to define bidirectional repeatability as well, which can be achieved when the point is approached from two different directions. This includes the effect of backlash in leadscrew. And finally resolution is “the larger of the smallest programmable step or the smallest mechanical step the machine can make during point to point motion” [8].

However, several alignments exist, the final part and form accuracy can be determined by five subsystems. These are the accuracy of the machine tools, the process, the workpiece, the programming, the fixture and the tool. The cutting process can affect the machined part’s accuracy in two ways. First the cutting force induces deformations in the machine tool structure which lead to dimensional and form errors. Next the tool wear, which was caused by machining affects the dimensional errors of the machined part. In this thesis the focus will be on the machine tool accuracy including factors from the following groups: positional accuracy and static/quasi-static accuracy accuracy. The accuracy factors are effected by deformations. Deformations in the kinematic structure in the case of positional accuracy. Deformations can be also caused by thermal expansion, static loads or dynamic loads.
2.2. Literature review

The one of the first machine tool modeling approaches were introduced in the early 60’s. The most common methods contained trigonometric equations and different constructions of error matrices. However the technological conditions and the mathematical apparatus have been changed the goal to predict and compensate the machine tools error is the same.

Measurement methods and investigations by constructing models for identifying the effect of geometric or kinematic errors on motion accuracy of various types multi-axis machine tools has been done from several different approach. An analytical quadratic model was introduced by Ferreira and Liu [9] where the coefficients of the model were obtained by touch trigger probe measurement. Slocum [8] and Donmez [10] introduced two methodologies with several similarities. Both of them was carried out with case studies for describing geometric and thermal errors. After the rigid body assumption homogeneous transformation matrices (HTM) were used to predict and compensate the identified errors. Lin and Shen [11] established a model for five-axis machine tool using the matrix summation approach. Ferreira and Kiriden [12] used direct measurements to define the parameters of the n-dimensional polynomials in the David-Hartenberg (D-H) method to predict geometric and thermal errors. Soons, Theuws and Schleken [13] presented a general methodology and an application in two case study which accounts for errors due to inaccuracies in the geometry, finite stiffness, and thermal deformation of the machine's components. Special statistical techniques were applied to the calibration data to obtain an empirical model for each of the errors. Suh, Lee and Jung [14] focused on rotary axes in their work. They established a model for the calibration of the rotary table including a geometric error model and an error compensation algorithm with experimental apparatus for CNC controllers.

The measurement instruments and the most important definitions are specified in ISO 230-1 [15]. For five-axis machine configurations having a tilting rotary table, ISO 230-7 [16] contain the most important evaluation technics. The telescoping double ball bar (DBB) measuring instrument has been applied to identify geometric and kinematic errors on five-axis machine tools in several papers. Mayer J., Mir Y. and Fortin C. [17] introduced a calibration method of a five-axis machine tool for position independent geometric error parameters using a telescopic magnetic ball bar. Ibaraki S. and Knapp W. [18] in their paper includes DBB usage with several other indirect measurement
schemes for the kinematics of three orthogonal linear axes, as well as the five-axis kinematics with two rotary axes.

Ming, Huapeng and Heikki [19] presents a modeling method connected to the static stiffness of a hybrid serial–parallel robot. The stiffness matrix of the basic element in the robot is evaluated using matrix structural analysis. The analytical stiffness model obtains the deformation results of the robot workspace under certain external loads. Z. M. Levina’s paper [20] deals with the results of theoretical and experimental researches of stiffness of joints and slide-ways. A method of calculating the joint deflections is proposed. The comparison of calculated and experimental results is also given. Daisuke [21] established a 3D stiffness model using contact stiffness. The stiffness in each direction is assumed to be determined by the contact stiffness at the interfaces. Ri Pan [22] aim was to achieve polishing controllability as the tool executes a precessive motion trajectory. His model employed the Jacobian stiffness matrix of the tool. The static stiffness of the machine is finally derived taking tool loading into account along its path. To minimize deformation, the control algorithm that relies on a maximum static stiffness strategy is superposed on the rigid body system tool trajectory model.

Archenti, Nicolescu, Casterman, and Hjelm [23] in their paper presents a novel test instrument called Loaded Double Ball Bar (LDBB), which can be used either as an ordinary double ball bar system to the structure or with applying a predefined load. Important data can be collected from the effect of the load on the machine tool structure from the static deflection point of view. The machine tool elastic deflection can be investigated with this new measurement equipment. In a further study [24] the collected data are used to plot diagrams displaying characteristics of machine tool performance such as static stiffness. The data proposed to be used to predict any accuracy deviations, for trend analysis and later for preventive maintenance. The static behaviour is determined also to be used to improve digital models for process simulations and compensation of errors that are caused by deflection.
2.3. Various sources of errors in machine tools

To describe the errors, which occur in a machine tool we have to understand the inputs of the system outline and the interactions between the elements. These sources of errors in the manufacturing process effects the difference between the nominal and the actual positions of the kinematic structure’s elements. According to several references these errors can be grouped in many different ways. One approach decompose them into the contribution of three error sources. First the contouring error, which is the effect of follow-up errors of the axis drives. The second is the quasi-static geometric errors of the machine, which includes link and motion errors and thermal drift. And the third is the dynamic geometric errors, resulting in the machine structure deflection under dynamic load [25]. Slocum divided machine tool errors into systematic errors, which can be measured and random errors, which prediction is difficult. Systematic errors determine accuracy and random errors influence precision [8]. The most common categorization of errors are the following: kinematic and geometric errors, thermo-mechanical errors, loads, dynamic forces and motion control [26] (see Figure 6).

![Figure 6. The categorization of errors which effect the machining system accuracy](image)

Motion control errors are connected to the positioning of the axis. Servo- and ball-screw drives controlled by NC program commands are used to execute this process. However feedback loops can provide information about the actual position the system can induce errors.
Additionally we can note the existence of the error which is caused by material instability. All of the materials change their attributes in long-term depending on their internal alloying structure. Residual stresses lead to instability in the material which can effect the mechanical properties in different interactions.

2.3.1. Geometric and kinematic error

Geometric errors are caused by the unwanted motions of the structural components of the machine, such as of guideway carriages, cross-slides and work-tables. Geometric imperfections and misalignments lead to these error motions. These imperfections can derive for instance from the deviations in surface straightness and roughness of the guideway or the preload of bearing.

Heat generated by the machine tool and the cutting operation causes temperature changes of the machine tool elements. Due to the complex geometry of the machine structure, concentrated heat sources, such as the drive motors and the spindle bearings, create thermal gradients along the machine structure [10]. However, temperature changes and other error sources make difficult to measure geometric errors separately, it is possible to investigate the effects on the kinematic structure of the machine tool. Such measurement methods will be discussed later.

Geometric and kinematic errors have a tight connection. Geometric errors of the structural components can be partially considered as a resulting effect of the geometric errors during the co-ordinate movement of the functional components. As such, motion errors are functions of at least the position of the carrying axis and occur mainly during the execution of interpolation algorithms [5]. Both types of deviations are the result of production, assembly or operation of the components in the machine tool.

2.3.2. Thermal error

Thermal deformation can effect the machine structural loop after heat generation. The source of the heat can be an electric motor (machine spindle), the cutting process itself or different type of friction in bearings. Temperature changes in the environment provides a different type of affect, which also have to be considered.

The total effect on the accuracy of machining system, may be determined by measurement of the geometric and kinematic behaviour [27] and may effect location and component errors of the machines. However the internal and external thermal sources in machine tools result in thermal distortion of the machine’s structural loop often dominate the accuracy besides other error sources [28], but it requires attention
and compensation since most of the structural metals expands in a range from 8 to 15 μm/m/°C. Investigations of the thermal effects have to take place in a temperature controlled environment.

Reaching the thermal equilibrium before operation can prevent significant amount of thermal error. Most machines require a warm-up time, which can be executed by heating elements such as resistance heaters or temperature controlled fluids. During the operation cooling can play an important role, especially in case of the spindle bearings.

2.3.3. Dynamic error

The vibrations of the machine tools can increase the machine tool wear and decrease the surface quality of the workpiece. Dynamic behaviour of the machines can be characterised in three main quantities: stiffness, mass and damping. The interconnectivity here has higher influence since the stiffness and the damping are dependent on the temperature. Periodically changing forces, which generate vibrations in the machine can be caused by internal (i.e. cutting process, unbalanced rotation elements) and external sources (i.e. vibration transferred through the floor) [29].

Although dynamic errors are often traceable and characterizable, their prediction and the compensation are the most difficult of all of the error sources. Considering the mass and stiffness, the goal should be to increase stiffness and reduce mass of the moving components.

2.3.4. Static error

In case of static errors, the non-rigid body behaviour have to be considered. Location errors and component errors change due to internal or external forces. The weight of the workpiece and the moving carriages can have a significant influence on the machine’s accuracy due to the finite stiffness of the structural loop [30]. These effects can be observed during static machining or force measurement. Besides cutting force and gravity accelerating parts can also provide static loads. These errors compared to kinematic errors can be more significant. For example, if guideways bend due to the weight of the moving slide, the slide will show a vertical straightness and a pitch error motion, which is called “quasi-rigid behaviour” [26].
2.4. Connecting terms and definitions

2.4.1. Machine coordinate frame

A right-hand rectangular machine coordinate system with X, Y, Z principal axes and the corresponding A, B, C rotary axes about each of these axes can be seen in the Figure 7.

![Figure 7. A right-hand rectangular machine coordinate system [15]](image)

The machine’s global coordinate system is independent from the movement of the components, thus capable to describe the relative positions of the elements of the kinematic structure. It is usually attached to the base structure of the machine.

The local frames are rigidly attached to the components and follow their movements. The local frames should be defined to support the modeling and measurement process. The arrangement of the global coordinate system and the local frames have to be considered separately in case of different kinematic structure. Furthermore note that a frame is a coordinate system, where in addition to the orientation we give a position vector which locates its origin relative to some other embedding ‘universe coordinate system’ [31].

2.4.2. Kinematic structure – structural loop

Kinematics teaches the knowledge and mathematical descriptions of the movement of bodies and particles in space [32]. The kinematic structure of a machine includes the functional component such as the drives and guides, and also the whole supporting frame.

During a modeling process several assumption required for a more effective description. In case of a machine tool the first most common simplification is the rigid-body assumption. If the movements of the components are several magnitudes bigger, than the deformations in their shape, the rigidity is acceptable. The next step in this abstraction is the connections between the rigid machine elements. Joints are defining
the constraints in the relative motions of the parts. The arrangement of the joints has to serve the wanted movements of the components.

The movements of each part is generally described in a Cartesian, cylindrical or spherical three-dimensional coordinate system and with transformations it is possible to switch from one system to another [33]. The kinematic chain of a serial structure is only closed during the manufacturing process by the connection to the workpiece [34].

The illustration of the kinematic structures requires a certain level of abstraction. The kinematic chain shows all of the axes, the workpiece, the tool and even the bed. The most important function is to symbolize the flow of the movement in a kinematic structure. However most machine tools and measuring machines have a serial structure, several combinations are possible depending on the number of the axes. Along the kinematic chain, nominal position and orientation of the tool centre point (TCP) can be calculated at one end of the chain [26]. In further chapter examples are presented about the illustration. A notation presented by ISO 10791-7 (Figure 8) describes the order of the connected parts (tool, bed and workpiece) in the kinematic chain [35].

![Diagram](image)

*Figure 8. Serial kinematics abstracted to kinematic chain (t: tool; b: bed; w: workpiece) [35]*

### 2.4.3. Homogenous transformation matrices

Multi-axis machines contain a sequence of links connected by joints enabling rotational or translational motion between each elements. Since the assumptions of rigid-body kinematics, all of the links and joints can be modelled with a homogeneous coordinate
transformation. A homogeneous transformation matrix (HTM) describes the relative position of two coordinate systems which in our case are rigidly connected to the components of the machine tool. In real systems, some small imperfections exist in machine tool structural elements. HTMs are able to include those imperfections by applying [36] three rotational and three translational components.

A HTM is a 4x4 matrix constructed to represent one frame to another with translational and rotational transformations. Thus we can state that HTMs are operators in matrix form which realize the mapping from one frame to another. In the Equation number 1 the decomposition of the HTM (with null perspective transformation) can be seen. \( \mathbf{O}_1, \mathbf{O}_2 \) and \( \mathbf{O}_3 \) vectors are the orientation cosines, which defines the orientation of a matrix in respect to the mapped one. The \( [p_x \quad p_y \quad p_z]^{T} \) vector is the position vector which point from the origin of the mapped frame to the origin of the result frame.

\[
^{R}T_N = \begin{bmatrix}
1 & 0 & 0 & p_x \\
0 & 1 & 0 & p_y \\
0 & 0 & 1 & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
O_{1x} & O_{2x} & O_{3x} & 0 \\
O_{1y} & O_{2y} & O_{3y} & 0 \\
O_{1z} & O_{2z} & O_{3z} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
= \begin{bmatrix}
O_{1x} & O_{2x} & O_{3x} & p_x \\
O_{1y} & O_{2y} & O_{3y} & p_y \\
O_{1z} & O_{2z} & O_{3z} & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

To demonstrate the usage of HTM’s consider the following example. Equation number 2 describes a change in the representation of the position vector \( \mathbf{p} \) from \( \mathbf{B} \) to \( \mathbf{A} \). \( \mathbf{R} \) is the rotational matrix, which defines the orientation of \( \mathbf{B} \) to the orientation of \( \mathbf{A} \) and \( \mathbf{r} \) vector is the translational vector, which locates the origin of \( \mathbf{B} \) to the origin of \( \mathbf{A} \).

\[
^{A}P = \begin{bmatrix}
A \mathbf{R} & A \mathbf{r}
\end{bmatrix}
\begin{bmatrix}
^B \mathbf{p}
\end{bmatrix}
\]

In a Cartesian three-dimensional coordinate system a rigid body has 6 degrees of freedom. We can differentiate 3 translational movements in the 3 perpendicular axes and 3 rotational about the perpendicular axes. Equation 3, 4, 5 shows the translational
movements in x, y and z direction (with the amount of x, y and z) and Equation 6, 7, 8 show the rotational transformations about x (A), y (B) and z (C) (with the amount of $\theta_x$, $\theta_y$ and $\theta_z$). These equations will be used for further discussion.

\[
^RT_x = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}
\]

\[
^RT_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}
\]

\[
^RT_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}
\]

\[
^RT_A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x & 0 \\ 0 & \sin \theta_x & \cos \theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}
\]

\[
^RT_B = \begin{bmatrix} \cos \theta_y & 0 & \sin \theta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta_y & 0 & \cos \theta_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}
\]

\[
^RT_C = \begin{bmatrix} \cos \theta_z & -\sin \theta_z & 0 & 0 \\ \sin \theta_z & \cos \theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}
\]
Series of coordinate transformation matrices defines the relative position of each axes in a modeling process. The machine structure decomposition starts at the tip and ends at the base reference coordinate system \((n=1)\). If \(N\) rigid bodies are connected in series then the tip will be the \(N\)th axis and the relative HTMs of the axes between the tip and the base will be the sequential product of all the HTMs. Starting from the tip the successive multiplication of the HTMs to the reference described by the Equation 9 [8].

\[
R_T^N = \prod_{n=1}^{N} n^{-1}T_n = T_n^{-1}T_2^{-1}T_3^{-1} \ldots N^{-1}T_N
\]  

### 2.4.4. Functional point

The functional point (FP) regarding the ISO 230-1 definition is the cutting tool centre point or the point associated with a component on the machine tool where cutting tool would contact the part for the purposes of material removal. However the functional point can move within the machine tool working volume ISO 230 recommend to apply tests of geometrical characteristics that can locate the relative position between a moving tool and the hypothetical centre of a moving workpiece [15].

For a deeper understand of the role of functional point we have to consider their relation to the measurement point (MP). In an optimal case the measurement point can be positioned anywhere around the works volume. For the most accurate description of the effects of errors the measurement point should coincide with the functional point \((MP \approx FP)\). In reality the measurement capabilities are limited with several constraints, moreover measuring every possible functional point position would require huge effort. However, if the position of the measurement point in the measured axis’ coordinate system is known then the linear errors can be transformed and this can lead to a better estimation of the errors at any functional point in the work volume [37].

Linear errors are usually more sensitive to the deviation between the measurement and the functional point. They are affected by the angular error motions and this effect can vary depending on the point of the trajectory and the deviation between measurement and the functional point. The Abbé and Bryan Principles [38] describe this problem. The Abbé Offset is the offset distance between points and the angular error motions are related by the differences in magnitude of the positioning system.
Furthermore, according to the detailed researches of Laspas [5] it can be stated, that the error motion of the individual machine tool components and the effect of the component errors due to their configuration in the structural loop results in the actual relative position between the tool and the workpiece—the functional point. The error motion of components needs to be known at the trajectory of the functional point to enable prediction of its contribution to workpiece geometrical accuracy.

2.4.5. Stiffness of machines

The stiffness of the machines generally understood as the ability of a structure to resist deformation. The structural stiffness gives the capacity of a structure to resist deformations induced by applied loads. This capacity can be described by its stiffness matrix. The most important performance criteria of machining operations, productivity and accuracy are defined by the static and dynamic stiffness of the machining system [24]. The static and dynamic stiffness are defined differently and the stiffness values for the same structure regarding the two definitions, will differ. However, as the two values are calculated on the basis of the same structure, i.e. using the same stiffness matrix, a relation can be defined between them. This thesis considers the static behaviour of machines, neglecting the dynamic stiffness and the vibration of machine tools, which can result in the relative displacement of the tool and the table as well.

The static stiffness of a structure is defined with a unit load applied on the structure. “Point stiffness is the inverse of the displacement in the load direction on a node where a unit load is applied” [39]. Thus the point stiffness values at different positions and directions of the applied unit load are different. The stiffness of machine tools should be properly designed for reducing the unwanted relative displacement of the tool and the table in different directions. The stiffness cannot be easily calculated from the material properties and geometry of the machine. Finite element models and different measurement methodologies are commonly used to define static stiffness.

The characteristics how a machine react for an applied load can be compared to the strength of elastic materials. This comparison can be seen in the

Figure 9 and Figure 10. The general shape of a force–deflection diagram in case of an elastic material and a measured spring characteristic of a vertical turret lathe are introduced. Three different points on the force–deflection curve can be taken as representative of the strength of a material. In a somehow different way each shows a maximal level of load what the material can resist. The first two points pursuit to present an elastic limit of the material. In these points permanent deformations were
first observed. Since machine tools are built up from materials with similar behaviour these points and characteristics can be found on machine tools as well. However, the measurement of the failure point extremely difficult since it requires the damage of the machine tool, the linear proportional limit can be observed easier.

![Figure 9. Representation of the strength of a material: general shape of a force–deflection curve](image9.png)

![Figure 10. Force–deflection curve or spring characteristic of a vertical turret lathe [40]](image10.png)
2.5. Parameters of machine tool geometric errors

Donmez [10] classified the component errors into four groups according to their characteristics and measurement instrumentation and implementation: linear displacement errors, angular errors, straightness-parallelism orthogonality errors (or so called location errors) and spindle thermal drift.

The linear displacement errors and angular errors usually called together component errors. The spindle static and dynamic errors can be significant error sources in machine tools. Investigation showed that the spindle drift characteristics are more complex than the other error components in the machine tool [10]. In this thesis the static and dynamic errors due to the spindle imperfections are neglected. The focus will be on the component and location errors.

2.5.1. Component errors of a linear axis

A linear axis of machine is designed to travel precisely along a reference straight line and stop at the predefined position. From practical point of view there is deviation between the actual path and the reference straight line at the guideways due to inaccurate lead and misalignment of the ballscrew or errors in the feedback system of the control system. The effects of the reversal errors have to be considered as well. The component errors of the linear axis are linear positioning (in the direction of the designed movement), two straightness errors, and three angular errors (yaw, pitch and roll). The component errors can be seen in Figure 11.

Figure 11. Component errors of a linear axis (x).

\( E_{XX} \) : positioning error; \( E_{YX} \) : straightness error in Y direction; \( E_{ZX} \) : straightness error in Y direction;
\( E_{AX} \) : angular error, roll; \( E_{BX} \) : angular error, yaw; \( E_{CX} \) : angular error, pitch
2.5.2. Component errors of a rotational axis

Angular errors are rotational errors. The component errors in the case of rotational axis are the two radial errors, the axial error, the two tilt errors and the radial positioning error. These can be seen in Figure 12. The angular errors usually caused by misalignment in the assemblies.

![Figure 12. Representation of the six deviation of a rotary kinematic component](image)

2.5.3. Location errors

However in literature several different approaches exist the most common definition for location errors are the following. The linear axis location errors represent orientations of its reference straight line in the reference coordinate system [18]. In case of a rotary axis location errors, they are defined analogously, representing the position and orientation of the axis average line of a rotary axis, i.e. the straight line representing the mean location and orientation of its axis of rotation [16]. Important to understand, that location errors operate with positions and orientations, which is the “average” trajectory of the moving component. To calculate this average the connecting standards offers several methods.

Typical location errors are the straightness, parallelism and orthogonality. Slide straightness errors are the nonlinear translational movement in the two orthogonal directions other than its axis of motion. Parallelism and orthogonality location errors are described by the angular orientation of the machine axes with respect to each other [10].
Due to the error motions of the horizontal and vertical straightness, roll, pitch and yaw, the trajectory of the linear moving component of the machine will not be the nominal line. The shape of the trajectory depends on the location of the trajectory in the work envelope as well as on the magnitudes and directions of the error motions, these dependencies can be seen in the Figure 13 [15], where three squareness (location) errors are represented.

Figure 13. Machine tool with error motions along the linear Y-axis. (after [15])

a, b: actual trajectories of Y-axis linear motion (blue is nominal and red is actual), EA0Z squareness error of Z to Y, EB0Z squareness error of Z to X, EC0X squareness error of Y to X, EAY: angular error of Y in A-axis direction (pitch), EBY angular error of Y in B-axis direction (roll), ECY angular error of Y in C-axis direction (yaw), EXY straightness deviation of Y in X-axis direction, EYY positioning error of Y-axis, EZY straightness deviation of Y in Z-axis direction
3. The bottom-up approach

In the bottom-up methodology the sub-systems are investigated as individual systems and the whole top-level is built up from these fragments. The output of this approach is a prediction of the effects of the geometric errors in the workspace of a machine tool. The implemented direct measurement method is the laser interferometer measurement. The aim of the computational model is to compose these direct measurement data and use them as inputs and calculate the deviations from a given toolpath in the workspace with geometric errors.

In this modeling method two machines were investigated. The specifications of the three-axis and a five-axis machine tool can be find in Error! Reference source not found. and 2.

Table 1. Specifications of the investigated three-axis machine tool

<table>
<thead>
<tr>
<th>AFM R-1000 “Baca”</th>
<th>Maximum offset (effective travel range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X axis (longitudinal)</td>
<td>1000 [mm]</td>
</tr>
<tr>
<td>Y axis (transverse)</td>
<td>510 [mm]</td>
</tr>
<tr>
<td>Z axis (vertical)</td>
<td>561 [mm]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fast feed rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>X axis (longitudinal)</td>
</tr>
<tr>
<td>Y axis (transverse)</td>
</tr>
<tr>
<td>Z axis (vertical)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Resolutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>X,Y,Z axis</td>
</tr>
</tbody>
</table>

Table 2. Specifications of the investigated five-axis machine tool

<table>
<thead>
<tr>
<th>Hermle C501</th>
<th>Maximum offset (effective travel range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X axis (longitudinal)</td>
<td>1000 [mm]</td>
</tr>
<tr>
<td>Y axis (transverse)</td>
<td>1100 [mm]</td>
</tr>
<tr>
<td>Z axis (vertical)</td>
<td>750 [mm]</td>
</tr>
<tr>
<td>A axis</td>
<td>230 [°]</td>
</tr>
<tr>
<td>C axis</td>
<td>360 [°]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fast feed rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>X axis (longitudinal)</td>
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<td>Y axis (transverse)</td>
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<td>Z axis (vertical)</td>
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<tr>
<td>A axis</td>
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<td>C axis</td>
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<tr>
<th>Resolutions</th>
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<tbody>
<tr>
<td>X,Y,Z axis</td>
</tr>
<tr>
<td>A axis</td>
</tr>
<tr>
<td>C axis</td>
</tr>
</tbody>
</table>
3.1. The laser interferometer measurement process

Selecting a robust test method for characterizing quasi-static positioning performance (e.g., linear positioning, straightness, and angular deviation) is essential for building up an empirical modeling method. Several tests are well defined by existing machine tool standards and have been thoroughly tested and implemented by industry [37]. A direct measurement method is required to measure the individual geometric error components. The computational model populated from these data to predict the displacement between the tip of the machine tool and the clamped workpiece.

The laser interferometer measurement as a commonly used direct measurement method is originally dedicated to measure the individual errors of axes. Since a laser interferometer equipped with different type of optics, it provides a number of different measurement options, including: linear positioning accuracy and repeatability of an axis, angular pitch and yaw, straightness of an axis, squareness between axes, flatness of a surface. Most of the equipment enables to measure in low uncertainty and high precision compared to other methods. Each setup and components can be seen in Figure 14. The most important disadvantages of the method are its time consuming procedure and the need for qualified operators.

A laser interferometer system has a modular architecture allowing to select components to meet specific measurement needs. The most important parts of such system are the laser head, the optics (including the reflectors and the beam splitter for different type of measurements), the stage, the tripod, the mounting kits and the mirrors. The setup is the following. One retro-reflector is rigidly attached to a beam-splitter, to form a fixed length reference arm. The other retro-reflector moves relatively to the beam-splitter and forms the variable length measurement arm. The laser beam emerging from the laser head has a constant frequency which is stable with a nominal wavelength. When this beam reaches the polarising beam-splitter it is split into two beams: a reflected beam and a transmitted beam. The two beams travel to their retro-reflectors and are then reflected back through the beam-splitter to form an interference beam at the detector. If the difference in path length changes, the detector sees a signal varying. These variations used to compute the change in the difference between the two path lengths. It should be noted that the wavelength of the laser beam will depend on the refractive index of the air through which it is passing. The wavelength value used to compute the measured values may need to be compensated for changes in these environmental parameters [41].
Linear positioning measurement

Angular measurement (Yaw, Pitch + Roll)

Straightness measurement

Figure 14. The measurement setups [41] and pictures from the implemented measurements

Table 3. XL-80 Laser measurement system characteristics [41]

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal wavelength</td>
<td>633 [nm]</td>
</tr>
<tr>
<td>Frequency accuracy</td>
<td>±0.1 [ppm]</td>
</tr>
<tr>
<td>Linear meas. resolution and axial range</td>
<td>0.001 [µm]; 0 - 80 [m]</td>
</tr>
<tr>
<td>Angular meas. resolution and axial range</td>
<td>0.1 [µm/m]; 0-15 [m]</td>
</tr>
<tr>
<td>Straightness meas. resolution and axial range</td>
<td>0.01 [µm]; 0.1 - 4 [m]</td>
</tr>
<tr>
<td>Squareness meas. resolution and axial range</td>
<td>0.01 [µm/m]; ± 3/M[m/m]</td>
</tr>
</tbody>
</table>
3.1.1. The setup parameters and the review of the measurement

The laser interferometer measurements were executed on the AFM R-1000 three-axis machine. Following the guidelines of ISO 230-1 the measuring intervals shall be no longer than the 1/10 of the length of the axis and shall be between 25 mm and 250 mm [15]. Between two measurement points before capturing the measurement data the machine tool had a 3 second pause to eliminate transients from the sensor process interferometer or the move of the axes. All of the measurements were repeated five times in each direction of the axis. Bidirectional measurements are suitable to investigate the hysteresis of the machine tool. To choose the error (positive and negative) directions correctly is essential during the test. Besides the sign convention great care must be taken in a Cartesian coordinate system for the right hand rule have to be carefully followed. All of the measurements were taken in cold condition, however it is suggested to execute warm up cycles.

Renishaw’s XL-80 laser interferometer system was used to execute the measurements. The system included an environmental compensation unit called XC-80 which increase the accuracy by measuring changes in air temperature, air pressure and relative humidity. The sensor readings are used to compensate the laser readings only in linear measurement mode. Regarding the Renishaw’s XL-80 laser interferometer systems specifications (Table 3) different type of optics were used for linear positioning, angular and straightness measurement. During the measurement of a linear axis should be ensured an alignment of the laser beam with the axis of motion as close to the centre line of the table as possible [42]. Renishaw’s system contains a software which analyse the captured data and plotted the captured results. Since this system was not able to measure the roll angular error of the machine, it was measured by a precision levels (can be seen in the middle of Figure 14).

3.1.2. The measurement results

All of the measurement data was analysed and stored by the Renishaw’ systems software. The presented results for each axis were calculated from the average of the 5 repetition in each direction. The measurement functional point was individually defined for each axis, including the rod and the length of the kits which were used to mount the optics.

In case of the X axis the length of the measurement interval was 20 mm in the range of 0 mm to 980 mm. The cover of the machine was removed for all the measurements. The reflector was rigidly attached to the table and the interferometer or
the prism (in case of the straightness measurements) was mounted to the spindle. The linear errors of X axis are shown in the Figure 15 and the angular errors in the Figure 16. Judging from the results, the pitch error of X is the biggest error of the axis (and in the whole machine). This can be caused by the buckle of the axis, which is the longest in the machine and carries the table and mounted on the Y axis. This magnitude of the error has a huge effect on the result. The pitch error of X axis were repeated to verify the results.

Figure 15. Linear errors of X axis

Figure 16. Angular errors of X axis
Measurement of the Y axis was taken with 20 mm intervals between two target points and the measurement range was from -30 mm to -490 mm (according to the machine coordinate system). The setups for all of the component errors were the same as for the X axis. The linear errors and the angular of Y axis are shown in the Figure 17.

![Figure 17. Linear (right) and angular (left) errors of Y axis](image)

Z axis carries the spindle. The measurement of this axis was taken with 20 mm intervals and the measurement range was from -30 mm to -450 mm (according to the machine coordinate system). The setup for all of the component errors were required a turning mirror to divert to the vertical direction the laser beam. The linear errors and the angular of Z axis are shown in the Figure 18.

![Figure 18. Linear (right) and angular (left) errors of Z axis. Since the spindle is attached to the Z axis the measurement of the roll error is unnecessary.](image)
3.2. Geometric error model development for machine tools

We can separate two major ways, which have been proposed to describe the quasi-static errors for different configurations of machine tools [36]. The first is the so called workspace approach, which contains the resultant errors without analysing the sources. All of the errors are modelled as a distortion of the workspace. The model parameters are dependent on the machine state and require several measurement data. The second is the element approach, which describes the error propagation of each errors. With additional information from the kinematic structure of the machine, the effects on the volumetric error can be defined. Thus the resultant error can be evaluated as the outcome of the errors from the all positioning elements. A third approach can be also differentiated, which is the combination of the previous two. This is the synthetic approach, which bridges the gap between the two approaches. It describes how the elemental errors are amplified or suppressed during their propagation through the kinematic system of the machine to influence the three-dimensional accuracy in its workspace [43].

In this thesis the elemental approach was implemented through the geometric error model development. The first step is to continue the work of Laspas [5]. His thesis contains the description of the methodology that includes and explains aspects necessary for the development of the machine geometric error model. The thesis presented an evaluation of machine tool’s accuracy. The main focus was on geometric and kinematic error modeling for predicting the toolpath accuracy in order to assess machined part’s accuracy.

The goal is to improve his model and apply it to a three-axis machine with a new kinematic structure and extend it to be applicable to a five-axis machine tool with two rotary axes. Furthermore to make the model more generally applicable for multi-axis machines in an arbitrary serial configuration. Finally the validation of the model is to be done, to prove the implementation of the methodology.

3.2.1. Modeling methodology

In order to describe the proposed model the most significant aspects should be emphasized. The first aspect is the kinematic structure and the kinematic model of the machine with the local and global coordinate frames characterise the relative orientation and position of the axes of motion. The local errors of the system and the component errors of each axis gives the considerable error parameters. The functional
point as it was introduced previously is one of the most sensitive part of the model and also has an important role to connect the model with the input parameters from the measurements. The core of the model is the mathematical relations which was implemented in a MATLAB program. The model will be presented through the following steps.

- Kinematic structure and model of the machine
- Global and local coordinate frames
- Local and component errors
- The functional point
- Connection with the input measurement data
- The composition of the HTMs

Since the generalization is also desired the boarders which describes the utility of the model will be emphasized.

3.2.2. The presentation of the model

3.2.2.1. Kinematic structure and model of the machine

The illustration of the kinematic structures requires a certain level of abstraction. The kinematic chain shows all of the axes, the workpiece, the tool and even the bed. regarding the notation presented by ISO 10791-7 (see Figure 8), the investigated three-axis machines kinematic chain is [t (C) Z b Y X w] and the five-axis machines [t (C) Z Y X b A C w], where the C in the brackets notes the spindle.

The schematic representation of the kinematic chain can be seen in Figure 19. Along the kinematic chain, nominal position and orientation of the tool centre point (TCP) can be calculated at one end of the chain [26]. Homogeneous transformation matrices are used to express the relative spatial position and the coordinate transformation of the rigid body frames.
3.2.2.2. Global and local coordinate frames

One of the most significant steps in establishing a model for machine tools is the assignment of the global coordinate system and the local coordinate frames. A machine’s global coordinate system is independent from the movement of the components, thus used to describe the relative positions of the elements of the kinematic structure. It is usually attached to the base structure of the machine. The local frames are rigidly attached to the components and follow their movements. All of the coordinate frames should be defined to support the modeling and measurement process.

According to ISO 230-1 [15] the reference coordinate frame should be chosen to support the measurement (or the modeling) process, but can be defined arbitrary. According to ISO 10303-105 [44] a first and second pair frame shall be defined. The first pair frame is the reference frame of the axis where the direction and the dimension of motion of the axis is described. The second pair frame, the local coordinate frame follows each movement of the axis.

The identification of the coordinate frames for the three-axis machine tool can be seen in the Figure 20. The local coordinate frames were defined to the 0 position for each axis, which are the wing positions. The global coordinate frame defined to meet the corresponding 0 coordinate for each axes. Thus, for instance the right wing position of the X axis determines the 0 coordinate of X axis for the reference coordinate system.
Figure 20. The assignment of the coordinate frames for the three-axis machine tool. The red coordinate system corresponds to the reference coordinate frame. The blacks correspond to the local coordinate frames of the X, Y and Z axis.

The assignment of the coordinate frames for the five-axis machine tool were defined on the table and can be seen in the Figure 21. The reference coordinate frame and the local coordinate frames of the X, Y, Z and A axis are in the same position, where the C axis line shifted in Y direction intersects the axis line of A. This position is important in the model building point of view. The easiest way to describe the rotational effect of a rotary axis when one of the local coordinate frame’s axis is attached to the reference coordinate frame. The effect of the second rotary table still not trivial. The local coordinate system of the C axis defined on the surface of the table. The relative position of the Z axis of this local frame to the reference frame remains the same after any transformation of the rotary axes. But the X and Y coordinate line rotates relatively to the reference frame as C axis rotates. HTMs have to be used to describe this relation.
3.2.2.3. Local and component errors

The geometric and kinematic imperfections can be described in six degree of freedom in the form of HTMs. To do so, all of the component and local error have to be clarified. Table 4 shows the previously introduced component and local errors. For the three-axis machine the component errors of the translational axes and the first three local errors have to be considered. In case of a five-axis machine with two rotary axes all of the errors in the table are necessary.

The naming convention follows ISO 230-1 [15], where $E$ stands for the word error, the first index means the axis which corresponds to the direction of the motion and the second shows the axis of the motion.

<table>
<thead>
<tr>
<th>TRANS. AXES</th>
<th>Linear Errors</th>
<th>Rotational/Angular Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>X AXIS</td>
<td>$E_{XX}$</td>
<td>$E_{AX}$</td>
</tr>
<tr>
<td></td>
<td>$E_{XY}$</td>
<td>$E_{BY}$</td>
</tr>
<tr>
<td></td>
<td>$E_{XZ}$</td>
<td>$E_{BY}$</td>
</tr>
<tr>
<td></td>
<td>$E_{YZ}$</td>
<td>$E_{BY}$</td>
</tr>
<tr>
<td>Y AXIS</td>
<td>$E_{YY}$</td>
<td>$E_{AX}$</td>
</tr>
<tr>
<td></td>
<td>$E_{XY}$</td>
<td>$E_{BY}$</td>
</tr>
<tr>
<td></td>
<td>$E_{YZ}$</td>
<td>$E_{BY}$</td>
</tr>
<tr>
<td>Z AXIS</td>
<td>$E_{ZZ}$</td>
<td>$E_{AX}$</td>
</tr>
<tr>
<td></td>
<td>$E_{XZ}$</td>
<td>$E_{BY}$</td>
</tr>
<tr>
<td></td>
<td>$E_{YZ}$</td>
<td>$E_{BY}$</td>
</tr>
<tr>
<td>ROT. AXES</td>
<td>Axial</td>
<td>Angular pos.</td>
</tr>
<tr>
<td>A AXIS</td>
<td>$E_{AX}$</td>
<td>$E_{AA}$</td>
</tr>
<tr>
<td></td>
<td>$E_{ZA}$</td>
<td>$E_{BA}$</td>
</tr>
<tr>
<td></td>
<td>$E_{YA}$</td>
<td>$E_{CA}$</td>
</tr>
<tr>
<td>C AXIS</td>
<td>$E_{ZC}$</td>
<td>$E_{AC}$</td>
</tr>
<tr>
<td>LOCATION ERRORS</td>
<td>$E_{A0Y}$ $E_{C0X}$ $E_{B0X}$ $E_{B0A}$ $E_{C0A}$ $E_{Y0C}$ $E_{A0C}$ $E_{B0C}$</td>
<td>$E_{BC}$</td>
</tr>
</tbody>
</table>
The local errors presented (and noted) according to the ISO 230-1 [15]. For a five-axes machine tool (including the spindle) 23 local errors can be differentiated. Since all zero positions of linear and rotary axes can be set to zero when checking the geometric accuracy 5 local errors can be neglected. In this thesis the local errors connected to the spindle are simplified, which means 4 local errors less to be considered. With choosing the Z axis as the primary and in order Y, X, A and C are lower level axes, the final 8 errors are reached, which can be found in the Table 5.

In the notation, $E$ stands for the word error, the first letter in the index signs the direction of the deviation, the second in this case is always zero and the third is the investigated axis.

<table>
<thead>
<tr>
<th>Table 5. The local errors of a three- and five axis machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>X axis</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>-</td>
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<tr>
<td>-</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>$E_{B0X}$</td>
</tr>
<tr>
<td>$E_{C0X}$</td>
</tr>
</tbody>
</table>

$E_{C0A}$ squareness error of A to Y; $E_{B0A}$ squareness error of A to Z; $E_{A0Y}$ squareness error of Y to Z; $E_{C0X}$ squareness error of X to Y; $E_{B0X}$ squareness error of X to Z; $E_{Y0A}$ Y offset error from A to C; $E_{B0C}$ parallelism error of C to Z in the reference ZX plane; $E_{A0C}$ parallelism error of C to Z in the reference ZY plane;

These errors can be integrated to the model in HTM form. The transformational meaning of the local errors with respect to the order of the axes are the following.

The HTM of the location error of Z axis, which is the reference axis:

$$S_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (10)

The HTM of the location error of Y axis comes from the squareness error of Y respect to Z.
The HTM of the location error of X axis comes from the squareness error of X axis to Y and Z.

\[ S_Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -E_{A0Y} & 0 \\ 0 & E_{A0Y} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11) \]

The HTM of the location error of A axis comes from the squareness error of A to Z and Y.

\[ S_X = \begin{bmatrix} 1 & -E_{C0X} & E_{B0X} & 0 \\ E_{C0X} & 1 & 0 & 0 \\ -E_{B0X} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12) \]

The HTM of the location error of C axis comes from the Y offset error of C to B \((O_C)\), the parallelism error of C to Z in the reference ZX plane \((P_{CZX})\) and the parallelism error of C to Z in the reference ZY plane.

\[ O_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & E_{Y0A} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (14) \]

\[ P_{CZX} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -E_{B0C} & 0 \\ 0 & E_{B0C} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15) \]

\[ P_{CZY} = \begin{bmatrix} 1 & 0 & E_{A0C} & 0 \\ 0 & 1 & 0 & 0 \\ -E_{A0C} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16) \]

\[ S_A = O_C \cdot P_{CZX} \cdot P_{CZY} = \begin{bmatrix} 1 & 0 & E_{A0C} & 0 \\ 0 & 1 & -E_{B0C} & E_{Y0A} \\ -E_{A0C} & E_{B0C} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17) \]
3.2.2.4. The functional point

The error motion of components needs to be known at the trajectory of the functional point to be able to predict its effect to workpiece geometrical accuracy. Thus the measurement of the displacement errors need to be carried at the toolpath where the manufacturing will be executed. A measurement point which coincides with the functional point avoids the effect of other component errors. The uncertainty of the predicted accuracy relates with the distance between the functional point and the measurement point. If the measurement point’s position is well documented the effects of the angular errors can be calculated [5].

Linear errors are usually more sensitive to the deviation between the measurement and the functional point. According to the work of Bryan [38] linear errors are affected by the angular error motions and this effect can vary depending on the point of the trajectory and the deviation between measurement and the functional point. This effect was deeply investigated in Laspas [5].

In case of a rotary axes similar effects can be observed due the deviation between measurement and the functional point. An example present this effect in Figure 22. The level effect of the tilt error of C axis in Y direction can be seen on measured radial error of C axis in X direction due the offset between the functional point and the measurement point. Admittedly if the measured points are well documented, so the distances between the functional points and the measurement points are known, the effects of the tilt errors can be corrected from the radial errors. In this research measurements were taken on the three-axis machine, where separated measurement points were defined for each axis of motion.
3.2.2.5. The composition of HTMs

Considering a rotating rigid body, in Figure 12 the possible error components are shown. These errors can be described as the function of the rotating angle ($\theta$). A general HTM of the component errors of a rotating axes are calculated by the substituting Equation number 3 - 8 to Equation 9. The sequential multiplication result in the general HTM (Equation number 18, where operator S=sin and C=cos).

$$\text{ERR} = \begin{bmatrix} C\varepsilon_Y \cdot C\varepsilon_Z & -C\varepsilon_Y \cdot S\varepsilon_Z & S\varepsilon_Y & \delta_X \\ S\varepsilon_X \cdot S\varepsilon_Y \cdot C\varepsilon_Z + C\varepsilon_X \cdot S\varepsilon_Z & C\varepsilon_X \cdot C\varepsilon_Z - S\varepsilon_X \cdot S\varepsilon_Y \cdot S\varepsilon_Z & -S\varepsilon_X \cdot C\varepsilon_Y & \delta_Y \\ -C\varepsilon_X \cdot S\varepsilon_Y \cdot C\varepsilon_Z + S\varepsilon_X \cdot S\varepsilon_Z & S\varepsilon_X \cdot C\varepsilon_Z + C\varepsilon_X \cdot S\varepsilon_Y \cdot S\varepsilon_Z & C\varepsilon_X \cdot C\varepsilon_Y & \delta_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (18)

In the following steps the small angle approximation ($\sin \varepsilon = \varepsilon$ and $\cos \varepsilon = 1$) will be used several times, but it has to be noted that in the nanometer precision range second order effects can be significant, thus this simplification cannot be used.

In case of rotation about different axis in the Cartesian coordinate system different second order terms can be neglected and substitutions have to be executed in Equation number 18.
If the line of the rotation is C axis, from Equation 18 after the small angle approximation and the substitution of $\theta_Z = \varepsilon_Z$ the result will be Equation number 19.

$$\text{ERR}_C = \begin{bmatrix}
\cos \theta_Z & -\sin \theta_Z & \varepsilon_Y & \delta_X \\
\varepsilon_X \cdot \varepsilon_Y \cdot \cos \theta_Z + \sin \theta_Z & \cos \theta_Z - \varepsilon_X \cdot \varepsilon_Y \cdot \sin \theta_Z & -\varepsilon_X & \delta_Y \\
-\varepsilon_Y \cdot \cos \theta_Z + \varepsilon_X \cdot \sin \theta_Z & \varepsilon_X \cdot \cos \theta_Z + \varepsilon_Y \cdot \sin \theta_Z & 1 & \delta_Z \\
0 & 0 & 0 & 1
\end{bmatrix}$$ (19)

After neglecting the second order term $\varepsilon_X \cdot \varepsilon_Y = 0$, the simplified form, which expresses the component error propagation of the C rotary axis will be Equation 20.

$$\text{ERR}_C = \begin{bmatrix}
\cos \theta_Z & -\sin \theta_Z & \varepsilon_Y & \delta_X \\
\varepsilon_X \cdot \varepsilon_Y \cdot \cos \theta_Z + \sin \theta_Z & \cos \theta_Z & -\varepsilon_X & \delta_Y \\
-\varepsilon_Y \cdot \cos \theta_Z + \varepsilon_X \cdot \sin \theta_Z & \varepsilon_X \cdot \cos \theta_Z + \varepsilon_Y \cdot \sin \theta_Z & 1 & \delta_Z \\
0 & 0 & 0 & 1
\end{bmatrix}$$ (20)

If the line of the rotation is B axis, from Equation 18 after the small angle approximation and the substitution of $\theta_Y = \varepsilon_Y$ the result will be Equation number 21.

$$\text{ERR}_B = \begin{bmatrix}
\cos \theta_Y & -\cos \theta_Y \cdot \varepsilon_Z & \sin \theta_Y & \delta_X \\
\varepsilon_X \cdot \sin \theta_Y + \varepsilon_Z & 1 - \varepsilon_X \cdot \sin \theta_Y & -\varepsilon_X \cdot \cos \theta_Y & \delta_Y \\
-\sin \theta_Y + \varepsilon_X \cdot \varepsilon_Z & \varepsilon_X + \sin \theta_Y \cdot \varepsilon_Z & \cos \theta_Y & \delta_Z \\
0 & 0 & 0 & 1
\end{bmatrix}$$ (21)

In this case after neglecting the second order term $\varepsilon_X \cdot \varepsilon_Z = 0$, the simplified form, which expresses the component error propagation of the C rotary axis will be Equation 22.

$$\text{ERR}_B = \begin{bmatrix}
\cos \theta_Y & -\cos \theta_Y \cdot \varepsilon_Z & \sin \theta_Y & \delta_X \\
\varepsilon_X \cdot \sin \theta_Y + \varepsilon_Z & 1 - \varepsilon_X \cdot \sin \theta_Y & -\varepsilon_X \cdot \cos \theta_Y & \delta_Y \\
-\sin \theta_Y + \varepsilon_X \cdot \varepsilon_Z & \varepsilon_X + \sin \theta_Y \cdot \varepsilon_Z & \cos \theta_Y & \delta_Z \\
0 & 0 & 0 & 1
\end{bmatrix}$$ (22)

Finally if the line of the rotation is A axis, from Equation 18 after the small angle approximation and the substitution of $\theta_X = \varepsilon_X$ the result will be Equation number 23.

$$\text{ERR}_A = \begin{bmatrix}
1 & -\varepsilon_Z & \varepsilon_Y & \delta_X \\
\sin \theta_X \cdot \varepsilon_Y + \cos \theta_X \cdot \varepsilon_Z & \cos \theta_X - \sin \theta_X \cdot \varepsilon_Y & -\sin \theta_X & \delta_Y \\
-\cos \theta_X \cdot \varepsilon_Y + \sin \theta_X \cdot \varepsilon_Z & \sin \theta_X + \cos \theta_X \cdot \varepsilon_Y & \cos \theta_X & \delta_Z \\
0 & 0 & 0 & 1
\end{bmatrix}$$ (23)

In this case after neglecting the second order term $\varepsilon_Y \cdot \varepsilon_Z = 0$, the simplified form, which expresses the component error propagation of the C rotary axis will be Equation 24.
\[
ERR_A = \begin{bmatrix}
1 & -\varepsilon_Z & \varepsilon_Y & \delta_X \\
\sin \theta_X \cdot \varepsilon_Y + \cos \theta_X \cdot \varepsilon_Z & \cos \theta_X & -\sin \theta_X & -\delta_Y \\
-\cos \theta_X \cdot \varepsilon_Y + \sin \theta_X \cdot \varepsilon_Z & \sin \theta_X & \cos \theta_X & \delta_Z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (24)

Considering linear motion description an ideal motion can be expressed by substituting Equation number 3, 4 and 5 to Equation 9.

\[
\mathbf{r}_{T_L} = \begin{bmatrix}
1 & 0 & 0 & X \\
0 & 1 & 0 & Y \\
0 & 0 & 1 & Z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (25)

To construct the general HTM of the component errors (\(E_L\)) in case of a linear carriage Equation number 18 can be used. After the small angle approximation and the simplification of the second order terms Equation 26 will be the result.

\[
\mathbf{E}_L = \begin{bmatrix}
1 & -E_C & E_B & E_X \\
E_C & 1 & -E_A & E_Y \\
-E_B & E_A & 1 & E_Z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (26)

### 3.2.3. The function of the model

In the bottom-up methodology the output of the model is a prediction of the effects of the geometric errors in the workspace of a machine tool. The goal is to calculate the deviations from a given toolpath in the workspace with geometric errors. This was realised by a MATLAB program, which read the input data and executed all of the calculations before finally plotting the resultant errors for a given toolpath. Besides the measurement data and the predefined location of the coordinate frames the specification of the kinematic structure will facilitate the input parameters about the investigated machine. The reference axis can be chosen arbitrary, but it determines the form of the position independent location errors.

After the statistical analysis of the measurement data the component errors and the documented measurement positions and functional points are still separated for each axis. The actual position of one axis (\(H_a\)) in the local coordinate frame of the axis defined by the multiplication of four matrices, where to the variable \(a\) the notation of the proper axis shall be substituted (\(X, Y, Z, A, B, C\)).

\[
H_a = S_a \cdot R_{T_a} \cdot E_a \cdot P_{aFP}
\] (27)
Where $S_a$ is the matrix of the local errors, which was presented for all the axes with the Equation number 10, 11, 12, 13 and 17. The values of the matrix are independent from the position where the actual position is calculated. $^R T_a$ is the nominal position of the axis, which is defined by the given toolpath. $^a E$ is the matrix of the summarized (and simplified) component errors, which was described for linear and rotary axes in the chapter 3.2.2.5. The component errors are position-dependent. In the model the values of the variables of the component error matrices are determined by the interpolation of the measurement data. $P_{a,FP}$ is the difference between the functional point or point of interest and the measured one [5].

The sum of the errors for one axis can be calculated (Equation 28, where $I$ is a 4x4 unit matrix). Finally Equation 29 shows the aggregated errors from each axis in a given point of the predefined toolpath, assuming a five-axis machine with an A and C rotary axes.

$$E_{a,Sum} = H_a - (P_{a,FP} + ^R T_a - I)$$

(28)

$$E_{tot} = E_{X,Sum} + E_{Y,Sum} + E_{Z,Sum} + E_{A,Sum} + E_{C,Sum}$$

(29)

Figure 23, including the main steps of the three-axis model and the expanded five-axis model. All of the input parameters can be defined in an .xlsx input data sheet which makes the process easier to control and more transparent. The data sheet can be find in the appendix.

It must be emphasised that in case of the five-axis machine, one point of the workspace can be reached from several composition of the axes. That required a totally new approach in the definition of the toolpath. An own method was developed to handle this problem, which focused on the movement of the rotary axes. This method determined the position of each axis after a defined toolpath.
Figure 23. The model construction in case of a three (blue) - and a five (green) - axis machine (inputs are orange)
3.2.4. The validation

In a validation process the adequate operation of the achieved computational model of the local and component errors can be investigated. During the validation each input parameters effect on the program considered ‘ceteris paribus’. That means the adjustment just one parameter between two states, while all the others are held on constant zero. In this case the operation of the program can be validated, since the effect of each error parameter can be calculated by hand (or practically with another MATLAB code).

In the first case all of the error parameters were set to 0. In this case the resultant error had to be 0. In the next steps the displacement of one arbitrary chosen point (P) in the workspace is investigated. In the validation P' indicates the value which was calculated by hand. P'' indicates the output values from the model. The proposed computational model has to give the same result.

\[ P = \begin{pmatrix} -50 \\ -150 \\ -50 \end{pmatrix} \rightarrow P' = \begin{pmatrix} X' \\ Y' \\ Z' + \text{tool length} \end{pmatrix} = P'' = \begin{pmatrix} X'' \\ Y'' \\ Z'' + \text{tool length} \end{pmatrix} \]

If \( P' = P'' \) that means the program is valid for the given parameter. The positioning and straightness errors validation process is trivial, since these displacements are linear. More consideration is required in the case of rotational errors.

![Figure 24](image1.png)  
(a)  

![Figure 25](image2.png)  
(b)

Figure 24. a The offset between the measurement point and the functional point

Figure 25. b The deviation from the nominal point due to the rotational error
The lever effect of the angular errors need to be acknowledged. In Figure 24 the offset between the measurement point and the functional point can be seen. Depending on the axis of rotation the proper component of the offset has to be considered. Since $\theta \ll 1$ the deviation from the nominal point can be calculated due to the error with the small angle approximation as $E = L \cdot \theta$ (see Figure 25). It is very important to consider which direction of the rotation is examined and with respect to the right hand rule how the coordinate system can be specified. To determine the rotational errors, 100 [arcsec] was used to get bigger errors for a more punctual supervision. This is at least one magnitude bigger than in real measurements.
Certain parts are taken out from this version of the thesis, because of the confidential and unpublished content.
References


5. Laspas, T., Modeling and measurement of geometric error of machine tools, in School of Industrial Engineering and Management, Department of Production Engineering. 2014, KTH Royal Institute of Technology: Stockholm.


42. Standardization, I.O.f., *ISO 10791-2, Test conditions for machining centers - Part 2: Geometric tests for machines with vertical spindle or universal heads with vertical primary rotary axes*.


## Appendix

### Appendix 1: The input datasheet

**Datasheet (Punched card) for 5-axes machining tool**

<table>
<thead>
<tr>
<th>Machine axes effective travel range</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Tool length</td>
<td></td>
</tr>
<tr>
<td>2.0 Type of Toolpath (D/1)</td>
<td></td>
</tr>
<tr>
<td>Rectangular(1), Spindle(2), Linear(3), Circle(4), Point(5), Generated by axis positions (6)</td>
<td></td>
</tr>
<tr>
<td>Step size (mm)</td>
<td>0.1</td>
</tr>
<tr>
<td>3.0 Linear data in (mm)</td>
<td></td>
</tr>
<tr>
<td>X1: 400</td>
<td></td>
</tr>
<tr>
<td>X2: 400</td>
<td></td>
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<tr>
<td>X3: 400</td>
<td></td>
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<tr>
<td>X4: 400</td>
<td></td>
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<tr>
<td>X5: 400</td>
<td></td>
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<tr>
<td>Y1: 400</td>
<td></td>
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<td>Y2: 400</td>
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<td>Y3: 400</td>
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<td>Z1: 400</td>
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<td>Z2: 400</td>
<td></td>
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<tr>
<td>Z3: 400</td>
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<tr>
<td>Z4: 400</td>
<td></td>
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<tr>
<td>4.1 Rectangular, data in (mm)</td>
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<tr>
<td>X1: 400</td>
<td></td>
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<tr>
<td>X2: 400</td>
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<tr>
<td>X3: 400</td>
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<td>Y5: 400</td>
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<td>Z1: 400</td>
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<td>Z2: 400</td>
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<tr>
<td>Z3: 400</td>
<td></td>
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<tr>
<td>Z4: 400</td>
<td></td>
</tr>
<tr>
<td>4.2 Circular data in (mm)</td>
<td></td>
</tr>
<tr>
<td>Center X: 400</td>
<td></td>
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<tr>
<td>Center Y: 400</td>
<td></td>
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<tr>
<td>Center Z: 400</td>
<td></td>
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<tr>
<td>Radius: 400</td>
<td></td>
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<tr>
<td>Rot. X*: 400</td>
<td></td>
</tr>
<tr>
<td>Rot. Y*: 400</td>
<td></td>
</tr>
<tr>
<td>5.0 Interpolation (D/1)</td>
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<tr>
<td>Piecewise linear interpolation</td>
<td></td>
</tr>
<tr>
<td>Piecewise Cubic interpolation</td>
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<tr>
<td>Polynomial interpolation - Minimax, Newton and Lagrange forms</td>
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</tr>
<tr>
<td>6.0 Measurements</td>
<td></td>
</tr>
<tr>
<td>6.1 X-axis</td>
<td></td>
</tr>
<tr>
<td>Freq.: 400</td>
<td></td>
</tr>
<tr>
<td>Freq.: 200</td>
<td></td>
</tr>
<tr>
<td>6.2 Y-axis</td>
<td></td>
</tr>
<tr>
<td>Freq.: 400</td>
<td></td>
</tr>
<tr>
<td>Freq.: 200</td>
<td></td>
</tr>
</tbody>
</table>

**Interpolation** (between the measurement points)

**Component errors** (for all of the axes, filled with measurement data)

**Location errors** (according to ISO 230-1)

**Kinematic structure** (distance between the local coordinate systems)

---

**Machine axes effective travel range**

**The length of the tool**

**Toolpath type selection** (different forms can be defined in the work space)

**Toolpath position** (the definition of the given form)
Appendix 2: The effect of the change in the feed rate

The effect of the change in the feed rate
(On the figures position 2 measurement results can be seen, data in [mm])

Counter-clockwise direction
The deviations in the measurement result in case of two different feed rate
- FR=500
- FR=1000

F = 740.65 [N]
F = 363.16 [N]
F = 111.50 [N]
F = 28.45 [N]

With kinematic errors (the unloaded case included)

The differences in the effect of the loads (without kinematic errors)
- F = 740.65 [N]
- F = 363.16 [N]
- F = 111.50 [N]

The relative difference in percentage between the two feed rates

- F = 740.65 [N]
- F = 363.16 [N]
- F = 111.50 [N]