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On the transition from stable positive glow corona to streamers

Lipeng Liu¹ and Marley Becerra¹

¹ KTH Royal Institute of Technology, School of Electrical Engineering, SE-100 44 Stockholm, Sweden

E-mail: lipeng@kth.se and marley@kth.se

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Abstract

A two-dimensional numerical simulation of the transition from stable positive glow corona to streamers in coaxial cylindrical configuration is presented. The hydrodynamic model with several convection-dominated continuity equations together with Poisson equation are solved with consideration of the ionization layer. The transition from a stable positive glow corona produced under a DC voltage to streamers is investigated under a sudden change of the applied voltage. The critical rate of rise of voltage required for the transition from positive glow to streamer corona is evaluated with a voltage ramp. By introducing either physical or numerical instabilities into the model, streamers with filamentary structures are observed, which produce a sudden increase of the discharge current by more than two orders of magnitude. It is also found that the surface electric field of the corona-generating conductor deviates from the onset electric field, casting doubts about the validity of Kaptzov's approximation to evaluate the transition from stable glow to streamers.

1. Introduction

Corona discharges in air at atmospheric pressure occur in two different modes: homogeneous glow discharges (glow corona) and filamentary streamer discharges (streamer corona) [1]. On one hand, corona discharges are undesirable in electrical power components (transmission lines, transformers, generators, etc.) since it can cause power energy loss, audible noise and insulation damage [2]. On the other, applications of corona discharges at atmospheric pressure are emerging in areas such as the ozone production, surface treatment, pollution control, thin-film deposition, etc [3].

Particularly, atmospheric pressure glow discharges play an important role in industry, for instance, in the generation of ozone. Generally, the generated glow corona should be as homogeneous as possible to obtain a high collision rate between electrons and the background gas molecules, increasing the ozone yield and decreasing power consumption [4]. It means that the transition from glow corona to filamentary streamer discharges has to be avoided.

Glow discharges also occur in nature during thunderstorms. Since thunderclouds are mostly negatively charged, high electric fields are often induced on grounded tall objects, producing positive glow corona discharges. It has been suggested that the transition from glow corona to streamers is vital when investigating the initiation of upward lightning [5-10]. For the above-mentioned reasons, it is of great interest to evaluate the conditions required for the transition from stable glow corona to streamers.

Positive glow corona was first observed by Michael Faraday in 1838 [11] and first investigated experimentally in details by Trichel in 1939 [12]. Since 1970s, numerical simulation has gradually become one of the main tools to study the mechanism of both glow and streamer discharges [13-17]. Generally, gas discharges can be numerically modelled in two different ways. The first one follows a kinetic or particle description such as Monte Carlo simulation or Boltzmann transport solution to take into account the non-equilibrium plasma phenomena. However, kinetic models are highly time-consuming. The other alternative method is to consider a fluid approximation of the plasma, which is computationally more efficient and therefore is widely used in the literature [3, 18]. The fluid

model of gas discharges is defined by several continuity equations coupled with Poisson equation. This system of partial differential equations is complicated to solve since the continuity equations are convection-dominated and strongly coupled with the fast-changing electric field. At first, the hydrodynamic model was solved only in one dimension (1D or 1.5D) considering several simplifications [13-17]. The first detailed two dimensional (2D) simulation of streamer discharge was performed by Dhali and Williams in 1987 [19], where the flux-corrected transport - finite differential method (FD-FCT) was used. After that, other numerical algorithms such as the finite element method (FEM) and the finite volume method (FVM) have also been used to solve the stiff system of differential equations describing glow and streamer corona discharges [20-23].

A detailed 1D simulation of positive glow corona was first reported by Morrow in 1997 [24]. In order to achieve stable glow corona in a spherical coaxial configuration with a 2 cm long air gap, Morrow extended FD-FCT to a non-uniform mesh [25]. Nevertheless, it took several days to finish such a 1D simulation for the several microseconds required to reach a stable glow [26]. It was observed that streamer-like ionizing waves developed from a stable glow corona produced on the surface of the inner spherical conductor with a diameter of 2 mm when the applied voltage was raised at a rate larger than 1 kV/ μ s [24]. 1D simulation is not enough to evaluate the glow to streamer transition for at least two reasons: 1) streamers have filamentary structures which are impossible to describe in 1D; 2) photo-ionization calculation is a multi-dimensional convolution which cannot be computed accurately in 1D. Although 3D simulation is necessary to describe filamentary streamers, the space charge distortion of the electric field distribution prior to the transition and the streamer initiation phase is rather uniform and can be considered to be two dimensional as a first approximation.

The basic mechanism of positive glow corona formation in air can be summarized as follows: 1) with the presence of high positive DC applied voltage to the inner conductor (anode), the air in the close proximity of anode is ionized and onset streamers are produced; 2) negative charges are absorbed by the anode and positive ions move to the outer conductor (cathode); 3) as positive ions move away from anode, the electric field around anode will increase high enough to ionize the nearby air again; 4) the above processes repeat and result in a stable positive glow corona discharges in the gap.

The ionization layer where intensive ionization (impact ionization, photo-ionization, etc.) takes place in front of the anode is usually difficult to solve under the long simulation times required to reach stable glow corona. One general strategy to avoid the complexity of stable glow corona calculations is to neglect the ionization layer by assuming the surface electric field to be constant and equal to the onset electric field [6, 9, 27, 28]. Several studies of glow

corona to streamer transition can be found in the literature where this assumption, also known as Kaptzov's approximation, is used. For example, Bazelyan, *et al.* derived a criterion for the transition based on the corona current [6]. However, there are doubts about the validity of Kaptzov's approximation under fast-changing applied voltages [29-31].

The aim of this paper is to perform a 2D simulation of the transition from stable positive glow corona to streamers in a cylindrical coaxial configuration based on the hydrodynamic model. In order to handle the difficulties associated to the convection-dominated continuity equations, the position-state separation method (POSS) [32] is used. POSS is a method that does not require numerical flux correction and its time complexity when dealing with the convection of charge particles increases linearly with the number of unknowns. These features make POSS suitable to solve the 2D hydrodynamic model of corona discharges for long simulation times required to reach steady conditions.

In order to numerically investigate the transition from glow corona to streamers, the generation of glow corona under DC voltage is first simulated. Once the stable glow corona under DC voltage is reached, the simulation is continued under ramp voltages rising with a given rate (dV/dt). Due to the strong uniformity of the space charge generated by glow corona in a coaxial cylindrical configuration, the transition to filamentary streamers cannot be produced unless additional instabilities either physical or numerical are considered. In this paper, three different types of instability are taken into account. These instabilities are compared with each other to verify if they change the critical condition at which streamer filaments are formed from a stable glow corona. The sensitivity of the critical dV/dt to other parameters (photo-ionization, detachment and positive ions mobility) is also discussed. At last, the comparison of positive glow to streamer transition for electrodes with different curvatures is performed.

The outline of the rest of this paper is as follows. The 2D numerical model, the method and the configuration are described in Section 2. Section 3 and 4 are devoted to present simulation results and discussions, respectively. Conclusions are given in Section 5.

2. Two-dimensional model and configuration

2.1. Numerical model and method

As an extension of the 1D model presented in [24], the set of continuity equations for electrons, positive ions, negative ions and metastable molecules in 2D Cartesian coordinates is given by

$$\begin{aligned} \frac{\partial N_e}{\partial t} = & S + (\alpha - \eta)N_e|\mathbf{W}_e| - \beta N_e N_p \\ & + k_d O_2^* O_2^- - \nabla \cdot (N_e \mathbf{W}_e) \end{aligned} \quad (1)$$

$$\frac{\partial N_p}{\partial t} = S + \alpha N_e |\mathbf{W}_e| - \beta N_e N_p - \beta N_p O_2^- - \nabla \cdot (N_p \mathbf{W}_p) \quad (2)$$

$$\frac{\partial O_2^-}{\partial t} = \eta N_e |\mathbf{W}_e| - k_d O_2^* O_2^- - \beta N_p O_2^- - \nabla \cdot (O_2^- \mathbf{W}_n) \quad (3)$$

$$\frac{\partial O_2^*}{\partial t} = \alpha_m N_e |\mathbf{W}_e| - k_d O_2^* O_2^- - k_q O_2^* O_2 \quad (4)$$

where t is the time, N_e, N_p, O_2^-, O_2 and O_2^* are the number densities of electrons, positive ions, negative ions, oxygen molecules and metastable ($^1\Delta_g$) oxygen molecules, respectively. $\mathbf{W}_e, \mathbf{W}_p, \mathbf{W}_n$ are the drift velocities for electrons, positive ions and negative ions. The gas medium is air at atmospheric pressure and the symbols $\alpha, \eta, \beta, \alpha_m$ denote the ionization, attachment, electron-ion (ion-ion) recombination coefficients and the rate of creation of metastable oxygen molecules due to electron impact, respectively. k_d, k_q are the detachment rate coefficient and quenching rate constant, respectively. The diffusion movement of charged particles is neglected. S is the photo-ionization source term, which is calculated following Penny and Hummert [33]. Instead of neglecting the photo-ionization radiation towards the central electrode as in [24], the radiation from all the directions is here included. Thus, the photo-ionization term at the center of the i^{th} element is integrated from all the other elements as:

$$S_i = \frac{P \Delta x_i}{A_i} \int_{j=1, j \neq i}^M |\alpha N_e \mathbf{W}_e|_j \Psi(i, j) A_j dx_j \quad (5)$$

where Δx_i is the characteristic length of the i^{th} mesh element and M is the total number of the cells discretized from the computation region; $|\alpha N_e \mathbf{W}_e|_j$ is the number of ionizing events per unit volume at the j^{th} element and A_i is the volume of the i^{th} element in the mesh. Here P is the pressure and $\Psi(i, j)$ is an experimentally determined function of the distance between i^{th} element and j^{th} element [33].

The continuity equations are solved using the position-state separation method (POSS) [32]. Equations (1-4) are coupled with Poisson's equation given by

$$\nabla^2 \varphi = -\frac{e}{\varepsilon} (N_p - O_2^- - N_e) \quad (6)$$

where ε is the permittivity of air, e the electron charge and φ the electric potential. Poisson's equation is solved by FEM combined with incomplete Cholesky conjugate gradient (ICCG) method [34]. Once the potential distribution is obtained, the electric field is calculated according to

$$\mathbf{E} = -\nabla \varphi \quad (7)$$

The discharge current per unit length including the contributions of electrons, positive ions and negative ions is given by

$$I = \frac{e}{V} \int (N_p \mathbf{W}_p - O_2^- \mathbf{W}_n - N_e \mathbf{W}_e) \cdot \mathbf{E}_L ds \quad (8)$$

where V is the applied voltage and E_L the Laplacian electric field. The values of the parameters used in this model are given in the appendix. The validation of the model has been performed by comparison with the previous 1D simulation results by Morrow [24] and experimental data by Waters [1] presented in [29].

2.2. Numerical configuration

2.2.1. Computation region and boundary condition

For sake of comparison with the 1D study reported in [24], a concentric cylindrical configuration with inner and outer radius of 0.1 cm and 2.1 cm is modeled. Due to symmetry, only half of the region is considered as shown in figure 1. For solving Poisson's equation, the Dirichlet condition is applied to boundaries 1 and 2, where the potential is fixed to the applied voltage (V) and zero, respectively. The Neumann condition is applied to boundary 3. Following [35], the boundary condition for continuity equations is described below. At the anode (boundary 1), the positive ions fluxes are fixed to zero and Neumann boundary condition is applied to negative ions and electrons. Similarly, at the cathode (boundary 2), the fluxes of negative ions and electrons are fixed to zero while Neumann boundary condition is applied to positive ions.

Figure 1. The computation region of the numerical model, where r_1, r_2 denote the radius of inner and outer conductor, respectively.

In order to accurately compute the photo-ionization term S in (1), the image of the computation region is taken

into account. Moreover, the current integration using equation (8) on the computation region is doubled as to consider the entire discharge current. Observe that the blocking effect of the inner conductor on the light beams emitted is neglected in the considered geometry. This is justified by the fact that the amount of photo-ionization term decreases about two orders of magnitude at a distance 0.2 cm from the radiation source at atmospheric pressure [33].

2.2.2. Mesh discretization

An extremely fine mesh is used around the inner conductor to resolve early streamer ionizing fronts. The computation region is firstly divided into N_r parts exponentially in radial direction and N_ϕ parts equally in angular direction using a mapped mesh, which results in $N_r \times N_\phi$ quadrilateral cells. Then the mesh is converted into triangular mesh by dividing each quadrilateral into two triangles. In this paper, 80 000 triangular elements are used in total, corresponding to $N_r = N_\phi = 200$. The size of the elements around the inner conductor is in the order of 10 μm . Half of the mesh in the computation region is plotted in figure 2(a).

In order to save computation time, the calculation of photoionization is performed following a multigrid approach in a coarser mesh with 3 200 triangular elements ($N_r = N_\phi = 40$) and then projected into the fine mesh. A quarter of the mesh used for photoionization is shown in figure 2(c). Numerical experiments have shown that this simulation strategy is efficient without losing accuracy.

2.2.3. Initial condition and applied voltage

A constant voltage of 30 kV is applied to the inner conductor from time zero. A few electron-positive ion pairs are placed around the inner conductor as seed to trigger the discharge, as it is usually done in the electrical discharge simulations [19, 26, 36, 37]. The seed of the pairs follows a Gaussian distribution expressed as below:

$$N_e|_{t=0,x,y} = N_p|_{t=0,x,y} = N_0 \exp\left(-\left(\frac{\sqrt{x^2 + y^2} - r_1 - 0.01}{0.02}\right)^2\right) \quad (9)$$

where r_1 is the radius of inner conductor and x, y are coordinates (unit in cm). The maximum density $N_0 = 10^4 \text{ cm}^{-3}$ is assumed as a reasonable value of nature background ionization caused by radioactivity and cosmic rays [38]. Numerical experiments show that the value of N_0 with a range from 10^2 to 10^{12} cm^{-3} has little effect on the obtained results under the very long simulation times considered here.

Figure 2. (a) Overall view of a half of the fine mesh; (b) View of a quarter of the coarse mesh for photo-ionization calculation.

Once stable glow corona is formed under a DC voltage, for instance at time t_s , the applied voltage V is raised to investigate its transition to streamers. The applied voltage is raised with a constant rate of rise dV/dt , namely 4, 8, 10 kV/ μs corresponding to A1-A3 as illustrated in figure 3. The case A0 represents no rise of the applied voltage.

Figure 3. Voltage waveforms considered for the evaluation of the glow to streamer transition.

2.2.4. Physical and numerical instabilities

Additional instabilities either physical or numerical are needed to produce filamentary streamers since the space charge generated by glow corona is quite uniform in coaxial cylindrical configuration. Unfortunately, very little information related to such instabilities in stable glow corona is found in the literature. In this paper, the influence of three different types of instabilities on the glow to streamer transition is considered.

The first type of instability is introduced by the ‘plasma spots’ model which is often used to create disturbances in the modelling of streamer discharges to study the features of branching [39]. There are some other reasons that may cause localized instabilities, for example, the microscopic inhomogeneities (electron density fluctuations [40], pressure instabilities [39] *et al.*) and macroscopic impurity perturbations (dust particles or other objects) [41]. For sake of simplicity, these factors are not taken into account.

Plasma spots usually have a Gaussian distribution of charge density with a radius of several micrometers [39]. In this paper, the plasma spots are arbitrarily placed at the point (0.1, 0.1) close to the inner conductor during each simulation step and are expressed as

$$\begin{aligned} \Delta N_e|_{x,y} &= \Delta N_p|_{x,y} \\ &= N_t \exp \left(- \left(\frac{\sqrt{(x-0.1)^2 + (y-0.1)^2}}{0.01} \right)^2 \right) \end{aligned} \quad (10)$$

where N_t is the maximum density of plasma spots and it is set to 10^6 cm^{-3} . This value is comparable to the corresponding local electron density and is much smaller than the density of positive ions. The simulation time step used during the transition is $4 \times 10^{-11} \text{ s}$, leading to a maximum charge density production rate of $2.5 \times 10^{18} \text{ cm}^{-3} \text{ s}^{-1}$. This is about four orders of magnitude smaller than that selected in [39]. However, such charge density production rate is still very high, which is impossible to be produced by radioactivity and cosmic rays [38].

Since the hydrodynamic approximation is used in the model, a second type of physical instability generally present in fluid dynamics is also taken into consideration. Such instability is produced by micrometric irregularities in the geometry, and it is considered when simulating the creation of downstream vortices of a stable fluid passing a body at certain velocities. Such phenomenon is known as vortex shedding. In order to accelerate vortex shedding in numerical simulation, such a perturbation is usually created by adding extra force at the boundary layer [42]. A similar approach can be used for the evaluation of the transition to streamers by considering protrusions of micrometric size on the inner cylindrical conductor. Such an instability is here modelled by setting a very small perturbation (+0.1%) of the electric potential at a local point $(\frac{r_1}{\sqrt{2}}, \frac{r_1}{\sqrt{2}})$ on the surface of inner conductor.

Numerical instabilities can also be introduced by poor space discretization or by artificial disturbances or oscillations caused by the numerical algorithm used [41]. Unstructured meshes can also produce numerical disturbances in numerical models. Such numerical instabilities should be avoided as much as possible or at least they should be separated from actual physical perturbations. Interestingly, the use of unstructured meshes have been used as numerical strategy in the simulation of vortex shedding [43]. For example, if a very high-resolution algorithm with a structured mesh is used, vortex shedding is never reproduced in simulation. However, the simulation with an unstructured mesh does not require additional disturbances to produce vortex shedding. In this paper, this third type of numerical instability is introduced by using an unstructured mesh.

3. Numerical results

3.1. Stable glow corona discharges under DC voltage

3.1.1. The current pulses of glow corona

During the first several hundreds of nanoseconds after the DC voltage is suddenly applied, onset streamers are formed. Since the applied voltage is not large enough to drive streamers to the cathode, onset streamers will stop propagating. Once onset streamers stop propagating after several tens of microseconds, a uniform glow starts to form. For a detailed discussion of the formation of stable glow corona, the reader is referred to [24].

Figure 4 illustrates the simulated current per unit length and its (electronic and ionic) components under a stable positive glow corona. Observe that the current curves are only shown after $54 \mu\text{s}$ when the current starts to oscillate steadily such that a streamer-free glow corona discharge is formed. As it can be seen from figure 4, the total current i oscillates roughly from 15 to 35 mA/m with a period of about $2 \mu\text{s}$. Most of total current is due to the positive ionic component i_p with the electronic component i_e only contributing during the sharp rise of each current pulse. The current component due to negative ions i_n is much smaller.

3.1.2. The role of different electron production processes

In order to understand the mechanism of the glow to streamer transition, it is worth to investigate the production of electrons in the glow discharge. Figure 5 compares the rate of detachment, photo-ionization and impact ionization processes producing electrons at the instant when the current reaches its minimum point and maximum peak (which corresponds to point A and B marked in figure 4). The figure shows that impact ionization is the dominant electron generation mechanism in the close proximity of the inner conductor. In this paper, the region where impact ionization dominates over the other processes is defined as the ionization layer. As it can be seen in figure 4, the

ionization layer is roughly 0.1 cm thick for the simulated configuration, which is almost equal to the radius of the inner conductor.

Figure 4. Simulated discharge current during the formation of a stable glow corona. The labels i , i_e , i_p and i_n correspond to the total current and its components due to electrons and positive and negative ions.

Figure 5. Comparison of rate of detachment, photo-ionization and impact ionization as a function of radial distance (units in cm) at two instants marked as A and B ((minimum and maximum current peaks)) in figure 4.

Photo-ionization dominates the production of electrons outside the ionization layer. Thus, it plays an important role as a feedback mechanism providing free electrons for subsequent avalanches to develop in the ionization layer. However, the strength of photo-ionization does not have significant influence either on the magnitude or the

oscillation frequency of the glow corona current [24]. Similar analysis applies to the effect of detachment.

3.1.3. The formation of coaxial shells of positive charge

Figure 6 illustrates the 2D distribution of positive ions at the instant when the simulated current reaches its minimum and maximum level, which corresponds to the point A and B in figure 4. As it is shown in figure 6, once positive ions move away from inner conductor, the next ionization wave is stimulated forming a sequence of charged coaxial cylindrical shells.

Figure 6. Positive ions distribution near the inner conductor when one current pulse reaches its minimum value (a) and maximum value (b). The 1D radial plot of (a) and (b) is shown in (c) for sake of clarity.

The new electrons produced during a period of the glow corona discharge are rapidly absorbed by the anode while negative ions are absorbed at a much slower rate. This gives the chances for metastables to detach electrons from negative ions. The positive ions are left in the gap and propagate towards cathode with an average velocity of 5×10^4 cm/s. It takes about 2 μ s for positive ions to move out of the ionization layer (0.1 cm). This time corresponds to the period of the pulsating corona current. Thus, the oscillation frequency of positive glow corona mainly depends on the mobility of positive ions and the electric field close to the anode.

3.2. Transition from glow to streamer under rising voltages

3.2.1. Initial condition for the transition

In order to evaluate the transition from stable glow corona to streamers, the change from a DC to a rising voltage is set to start when one of the steady current pulses reaches its minimum value (e.g. at $t_s = 61\mu\text{s}$ in figure 4). At this time, the evaluation of the transition under different voltage ramps is performed with a new simulation time scale t' set to 0. In this section, the simulations are performed considering plasma spots only. The evaluation of other instabilities will be later discussed in Section 4.1.1.

3.2.2. Streamer-less glow corona under rising voltages

The spatial distribution of electric field, positive ions and electrons in the close proximity of the inner conductor under different ramp voltages are shown in figure 7. For sake of comparison, the plots are shown when the applied voltage in the cases A1 to A3 reaches 41 kV. The corresponding corona current is plotted in figure 8. As reference, case A0 with instability but without rising of the applied voltage is also shown. As it can be seen, the plasma spot instability (case A0) causes only a minor disturbance on the electron and ion densities without distortion of the electric field. The corresponding corona current also remains unaffected with an average value of about 20 mA/m.

If the applied voltage is raised slowly, for instance case A1 ($dV/dt = 4\text{ kV}/\mu\text{s}$), the magnitude and frequency of the current pulses increase. Even though there is a local increase in the electron and positive ion density triggered by the instability, no significant change in the electric field is observed. For this reason, no self-sustained, filamentary streamer structures are seen at 41kV. Simulation experiments confirmed that a streamer-less glow corona could be maintained even at higher voltages since the new produced positive ions have enough time to drift out of the ionization layer without accumulating before the next pulse.

3.2.3. Glow to streamer transition

As it can be seen from cases A2 and A3 in figure 7, the homogeneity of the ionization layer and the electric field distribution around the surface of the inner conductor is broken when the applied voltage is raised with a rate equal to or larger than $8\text{ kV}/\mu\text{s}$. As the value of dV/dt increases, the inhomogeneity created by the instability and its corresponding corona current increase. After some time, very localized 'streamer' filamentary structures can be seen clearly in figure 7 (case A3), leading to a sudden current increase by more than two orders of magnitude ($> 6\text{ A/m}$) as shown in figure 8.

When the applied voltage increases fast enough (A3), the new produced positive ions produced by the instability have no time to 'run away', accumulating around the surface of inner conductors. Such inhomogeneity will increase locally the electric field which in turn will make

the inhomogeneity even worse due to increased ionization. In this way, streamers are incepted, resulting in a significant increment of the corona current within a very short time (several hundreds of nanoseconds) as observed in figure 8. It also can be seen that the steeper the applied voltage is, the earlier the streamers are incepted. Since the discharge current per unit length provides a direct indication of the formation of the streamer filaments, it is selected as the main parameter to investigate the glow to streamer transition.

In order to demonstrate the effect of the plasma spot instability, the simulations are also performed at $dV/dt = 10\text{ kV}/\mu\text{s}$ without introducing the instability. As it can be seen in figure 7 (case A3*), only a symmetric, cylindrical ionization front forms when the simulation is performed without instability. Interestingly, the discharge currents with and without instabilities (A3 and A3*) are similar within $1\mu\text{s}$ after the rising of the applied voltage (figure 7). During this period, the current increases as the result of the formation of coaxial ionization waves of glow corona under the rising applied voltage. After this, a sudden increase of the current takes place for case A3 due to the efficient accumulation of space triggered by the instability. In this case, there is a significant intensification of the existing electric field outside the ionization layer, leading to the formation of a well-defined filamentary streamer. While for case A3*, no sudden increase of current is found even when the applied voltage rises to 43 kV. This suggests that it is not suitable to evaluate the transition from glow corona to streamers in 1D since filamentary streamers cannot be produced.

Since the 2D simulations in this paper do not consider the entire three-dimensional nature of streamer filaments, the absolute values of the local physical properties (e.g. electric field and electron density) of the filamentary streamers seen in figure 7 (case A3) are underestimated. For example, the peak electric field in the front of the streamers head in figure 7 (case A3) is about 50 kV/cm which is at least two times smaller than that of a normal streamer propagating in a weak field ($\sim 100\text{ kV/cm}$) [44]. Furthermore, the peak electron density obtained in this paper is roughly $3 \times 10^{12}\text{ cm}^{-3}$, which is one orders of magnitude smaller than the electron density of a normal streamer head ($10^{13} \sim 10^{14}\text{ cm}^{-3}$) [44].

Figure 7. The distribution of electric field and the density of positive ions and electrons in the close proximity of the inner conductor at different instants. Case A0 is simulated as a reference where plasma spot is added without rising of the voltage. Cases A1-A3 correspond to different dV/dt as labeled in figure 3. Case A3* is simulated without introducing any instability.

Figure 8. Corona discharge current simulated during the transition for different voltage ramps

3.2.4. On the discharge current during transition

Aleksandrov, *et al.* proposed in [8] a criterion to assess the glow corona to streamer transition based on the discharge current. This criterion is widely used in their future studies [5-8, 10, 45, 46]. Thus, a critical discharge current required for the transition was derived based on a 1D model assuming Kaptzov's approximation. For a corona between coaxial cylinders [6], it is expressed as

$$i_c = 2\pi\epsilon\mu E_p^2 \quad (11)$$

where μ is mobility of positive ions and E_p is the onset electric field which can be calculated by Peek's empirical formula [47] given by

$$E_p = 30\delta \left(1 + \frac{0.3}{\sqrt{\delta r_1}} \right) \quad (12)$$

where δ is the relative air density and is here set to be 1. The units for inner conductor radius r_1 and E_p are cm and kV/cm, respectively.

The critical current corresponding to the simulation of this paper where $\mu \approx 2.3 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ and $E_p \approx 58 \text{ kV/cm}$ would be $i_c \approx 430 \text{ mA/m}$. Figure 9 shows the total discharge current of case A3 and its different components. It is clear that the transition is driven by a growing electronic current, which becomes eventually larger than the current component due to positive ions. Since Aleksandrov and coauthors did not take into account electrons in their calculations, the critical value calculated by equation (11) cannot consider the actual transition to streamer structures caused by the sudden increase in the electronic current component. Moreover, our simulation results show that the ionic component of the current remains below 100 mA/m during the glow to streamer transition. This value is significantly smaller than the critical (ionic) current i_c calculated by equation (11). Furthermore, observe that streamers can develop and propagate within a very short time compared with the drift time of positive ions. For this reason, electrons in the ionization layer have to also be taken into account when

evaluating the glow to streamer transition. Due to the reasons above described, equation (11) is not correct to assess when streamers are initiated in the presence of glow corona.

Figure 9. Changes of the different components of the discharge current during the transition from glow to streamers.

3.2.5. The surface electric field during transition

The average value of surface electric field of the inner conductor as a function of time is shown in figure 10. As reference line, the corona onset electric field $E_p \approx 58 \text{ kV/cm}$ is also plotted. Observe that the average surface electric value of the stable DC glow corona is about 50 kV/cm , which is about 15% smaller than the onset electric field calculated by Peek's formula. Moreover, the difference between the actual surface electric field and the onset field E_p augments as the voltage rate of rise increases. It is found that the average surface electric field of the inner conductor can decrease close to the critical breakdown electric field, around 24 kV/cm or even below once the streamers are incepted (for the case with $dV/dt = 10 \text{ kV}/\mu\text{s}$). This means that the Kaptzov's approximation does not hold when the voltage in the configuration increases and therefore it is doubtful that it could provide any means to assess the glow to streamer transition.

Figure 10. The average value of surface electric field under different applied voltage as a function of time. The Peek's onset electric field obtained with equation (12) is also shown.

4. Discussion

4.1. Sensitivity of the critical dV/dt for the transition

4.1.1. The instabilities

The analysis of the transition of glow corona to streamer reported in the previous section considers instabilities due to plasma spots only. Note that the maximum density of the spots N_t should be selected properly. If the density of plasma spots is very low, no effect will occur. On the other hand, a very large concentration of plasma could itself disturb sufficiently the electric field distribution, such that the transition is predicted even without rising of the applied voltage. Thus, the principle of selecting N_t is to choose it comparable to the local electron density. This means that fluctuations of space charge leading to the transition should have densities similar to the local electron density in the glow corona.

All the above simulations are repeated considering the other two sources of instabilities discussed in Section 2.2.4: the protrusion on the anode and the unstructured mesh. It is interesting to find that the critical dV/dt required for the transition under the configuration in this paper is quiet stable and independent on the source of instability introduced. This can be seen in figure 11 where no significant differences are found in the discharge current for the different instabilities considered.

Figure 11. Simulated corona currents under different sources of instabilities under $dV/dt = 10 \text{ kV}/\mu\text{s}$.

4.1.2. Photo-ionization and detachment

Even though the strength of photo-ionization and detachment processes have little effect either on the magnitude or the oscillation frequency of the glow corona current, as discussed in Section 3.1, it is worth to check their effect on the transition to streamers. For this, figure 12 compares the discharge current obtained by considering the photoionization term (S in equation (1)) or the detachment rate k_d multiplied by a factor of either 0.1 or 10. As it can be seen, significant differences are found when changing the photo-ionization term. In the case of a higher rate of photoionization, the current increases immediately at time t' zero due to the suddenly increase of feedback electrons. After this pulse, the current decreases to less than 100 mA/m. Even when the applied voltage reaches 41 kV, the current does not increase enough as to produce filamentary

streamers. It is interesting that reduced photo-ionization leads to an earlier transition, compared even with the intensified photoionization. This counterintuitive result is consequence of the fact that the transition to streamers is triggered by an instability that causes an electrostatic disturbance comparable with the local electric fields produced by the discharge itself. Therefore, the increased electron density produced by intensified photo-ionization suppresses the effect of the instability in the discharge, making it difficult for streamers to be incepted. While for the case of low intensity photo-ionization a stronger disturbance triggered by the instability is obtained, making the transition easier. Unsurprisingly detachment has no significant influence on the current during the transition since it is negligible compared with photoionization.

Figure 12. The corona currents under different strength of photo-ionization and detachment when $dV/dt = 10 \text{ kV}/\mu\text{s}$ with plasma spots instability.

4.1.3. Positive ions mobility

As it has been mentioned in Section 3.1.3, the frequency of glow corona current depends on the mobility of positive ions (μ). If the mobility μ is increased, the accumulation of positive ions is less efficient since they drift out of the ionization layer faster. This would lead to a larger oscillation frequency of the current, delaying the transition to streamers as it can be seen in figure 13. If the mobility is doubled (2μ) a larger dV/dt would be required for the glow-to-streamer transition ($dV/dt = 12 \text{ kV}/\mu\text{s}$ in figure 13).

Figure 13. Corona currents calculated under different positive ions mobility and dV/dt .

4.2. The effect of the radius of the inner conductor

4.2.1. Stable glow corona under DC

In order to further understand the effect of the radius of the inner conductor on the glow to streamer transition, two other configurations are modeled. Inner conductor radii (r_1) of 0.05 and 0.20 cm are considered. The radius of the outer cylinder and the applied DC voltage is maintained as in the previous sections.

Figure 14 compares the stable glow current simulated for the three different values for r_1 . For sake of comparison, the calculated currents are shifted in time such that at least one pulse in each case starts at the same time. As expected, the average value and oscillation frequency of the calculated current decreases as r_1 increases.

Figure 14. Stable glow corona currents simulated under different radii of inner conductor.

4.2.2. Transition to streamers

As shown in figure 15, the sudden increase of the current occurs earlier for the configuration with larger inner radius r_1 for a dV/dt of 10 kV/ μ s, leading to an earlier transition to streamers. This is opposite to the initiation of streamers from glow-free electrodes, where it takes place earlier for sharp electrodes. Thus, our results show that glow corona plays a significant role in the streamer inception from corona generating objects. The more glow corona is produced, the harder it is for streamers to be incepted.

Figure 15. Corona currents simulated under different radii of inner conductor with a rise rate of the applied voltage $dV/dt = 10$ kV/ μ s.

5. Conclusions

This paper presents a two-dimensional numerical simulation of the transition from stable positive glow corona to streamers. The conclusions related to such transition are summarized as:

- The electric field of a glow corona generating surface deviates from the onset electric field under fast changing background electric field. In this situation, Kaptzov's approximation does not hold and therefore the ionization layer has to be considered.
- It is not suitable to evaluate the transition from glow to streamers using 1D simulation. Furthermore, extra physical or numerical instabilities need to be introduced in the simulations in order to reproduce filamentary streamers.
- During the transition from glow corona to streamers, the discharge current increases significantly by at least two orders of magnitude in several hundreds of nanoseconds. This increase is mostly caused by the sudden increase of the electronic component of the current. In two-dimensional simulations, this is produced by the very localized increase in the density of positive ions and electrons.
- In general, the production of electrons by detachment has no influence on the glow corona and its transition to streamers. Photo-ionization instead plays a major role since it affects the conditions for the transition from glow to streamers.
- The more glow corona space charge is generated from a surface, the higher critical rate of rise of the applied voltage is required for the transition to streamers. For this reason it is easier for streamers to be incepted from blunt corona generating rods than for sharp ones.

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Appendix: transport parameters and reaction rates in air

Due to some typing errors regarding the transport parameters and reaction rates in [24], this appendix summarizes all the parameters used in the simulation. The numerical model assumes atmospheric pressure (760 Torr) with neutral gas density N equal to $2.45 \times 10^{19} \text{ cm}^{-3}$. The electron-ion and ion-ion recombination coefficients (β_{ei}, β_{ii}) are set to $2.0 \times 10^{-7} \text{ cm}^3/\text{s}$. The detachment rate coefficient $k_d = 2 \times 10^{-10} \text{ cm}^3/\text{s}$ and the quenching rate constant $k_q = 2.22 \times 10^{-18} \text{ cm}^3/\text{s}$. $|E|$ is the modulus of the electric field, units in V/cm.

The ionization coefficient is estimated as:

$$\begin{aligned} \frac{\alpha}{N} &= 2.0 \times 10^{-16} \exp\left(\frac{-7.248 \times 10^{-15}}{\frac{|E|}{N}}\right) \text{ cm}^2 \\ &\quad \text{if } \frac{|E|}{N} > 1.5 \times 10^{-15} \text{ V cm}^2 \\ &= 6.619 \times 10^{-17} \exp\left(\frac{-5.593 \times 10^{-15}}{\frac{|E|}{N}}\right) \text{ cm}^2 \\ &\quad \text{if } \frac{|E|}{N} \leq 1.5 \times 10^{-15} \text{ V cm}^2 \end{aligned} \quad (\text{A1})$$

The attachment coefficient η is calculated by taking into account the two-body attachment and three-body attachment, i.e.,

$$\eta = \eta_2 + \eta_3 \quad (\text{A2})$$

where

$$\begin{aligned} \frac{\eta_2}{N} &= 8.889 \times 10^{-5} \frac{|E|}{N} + 2.567 \times 10^{-19} \text{ cm}^2 \\ &\quad \text{if } \frac{|E|}{N} > 1.05 \times 10^{-15} \text{ V cm}^2 \\ &= 6.089 \times 10^{-4} \frac{|E|}{N} - 2.893 \times 10^{-19} \text{ cm}^2 \\ &\quad \text{if } \frac{|E|}{N} \leq 1.05 \times 10^{-15} \text{ V cm}^2 \end{aligned} \quad (\text{A3})$$

and

$$\frac{\eta_3}{N^2} = 4.7778 \times 10^{-59} \left(\frac{|E|}{N}\right)^{-1.2749} \text{ cm}^5 \quad (\text{A4})$$

The drift velocities $\mathbf{W}_e, \mathbf{W}_p, \mathbf{W}_n$ are for electrons, positive ions and negative ions are

$$\mathbf{W}_e = -3.1 \times 10^5 \frac{\mathbf{E}}{P} \text{ cm/s} \quad (\text{A5})$$

$$\mathbf{W}_p = \mathbf{W}_n = 1800 \frac{\mathbf{E}}{P} \text{ cm/s} \quad (\text{A6})$$

where the P units in Torr.

The $\Psi(i, j)$ function is obtained by fitting the experimental data from Penny and Hummert [33] as:

$$\begin{aligned} \Psi(i, j) &= 0, \quad \text{if } 0 \leq \Delta_{i,j} P \leq 1 \\ &= 5.7236 \times 10^{-5}, \quad \text{if } 1 < \Delta_{i,j} P \leq 7 \\ &= 5.483 \times 10^{-3} \exp(-0.7351 \Delta_{i,j} P) + 3.675 \\ &\quad \times 10^{-5} \exp(-0.05333 \Delta_{i,j} P) \\ &\quad \text{if } 7 < \Delta_{i,j} P \leq 36 \\ &= 9.832 \times 10^{-6} \exp(-0.02337 \Delta_{i,j} P) + 1.517 \\ &\quad \times 10^{-6} \exp(-0.008621 \Delta_{i,j} P) \\ &\quad \text{if } \Delta_{i,j} P > 36 \end{aligned} \quad (\text{A7})$$

where $\Psi(i, j)$ units in $(\text{cm Torr})^{-1}$ and $\Delta_{i,j}$ is the distance between the centers of the i^{th} and j^{th} elements. The unit of $\Delta_{i,j} P$ is cm Torr. In order to eliminate numerical errors, $\Psi(i, j)$ is set to be zero where $N_e < 10^{-3} \text{ cm}^{-3}$.

The rate of creation of metastable oxygen molecules due to electron impact is expressed as

$$\begin{aligned} \frac{\alpha_m}{N} &= 5.16524 \times 10^{-18} \text{ cm}^2 \\ &\quad \text{if } \frac{|E|}{N} > 6 \times 10^{-19} \text{ V cm}^2 \\ &= 1.09025 \times 10^{-17} - 9.56211 \times 10^{-21} \left(\frac{|E|}{N}\right) \text{ cm}^2 \\ &\quad \text{if } 2 \times 10^{-19} < \frac{|E|}{N} \leq 6 \times 10^{-19} \text{ V cm}^2 \\ &= -4.83689 \times 10^{-18} + 1.59660 \times 10^{-19} \left(\frac{|E|}{N}\right) \\ &\quad - 4.5072 \times 10^{-22} \left(\frac{|E|}{N}\right)^2 \text{ cm}^2 \\ &\quad \text{if } 0.45 \times 10^{-19} < \frac{|E|}{N} \leq 2 \times 10^{-19} \text{ V cm}^2 \\ &= 2.303 \times 10^{-18} - 6.5209 \times 10^{-20} \left(\frac{|E|}{N}\right) + 1.009 \\ &\quad \times 10^{-21} \left(\frac{|E|}{N}\right)^2 \text{ cm}^2 \\ &\quad \text{if } 0.2 \times 10^{-19} < \frac{|E|}{N} \leq 0.45 \times 10^{-19} \text{ V cm}^2 \\ &= -4.69072 \times 10^{-19} + 3.16824 \times 10^{-19} \left(\frac{|E|}{N}\right) \\ &\quad - 1.13913 \times 10^{-20} \left(\frac{|E|}{N}\right)^2 \text{ cm}^2 \\ &\quad \text{if } 0.02 \times 10^{-19} < \frac{|E|}{N} \leq 0.2 \times 10^{-19} \text{ V cm}^2 \\ &= 0 \\ &\quad \text{if } \frac{|E|}{N} \leq 0.02 \times 10^{-19} \text{ V cm}^2 \end{aligned} \quad (\text{A8})$$

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