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Jointly modelling individual's daily activity-travel time use and mode share by a nested multivariate Tobit model system

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Abstract

Understanding mechanisms underlie the individual's daily time allocations is very important to understand the variability of individual's time-space constraints and to forecast his/her daily activity participation. At most of previous studies, activity time allocation was viewed as allocating a continuous quantity (daily time budget) into multiple discrete alternatives (i.e. various activities and trips to engage with). However, few researches considered the influence of travel time that needs to be spent on reaching the activity location. Moreover, travel time itself is influenced by individuals' mode choice. This can lead to an over- or under-estimation of particular activity time location. In order to explicitly include the individual's travel time and mode choice considerations in activity time allocation modelling, in this study, a nested multivariate Tobit model is proposed. This proposed model can handle: 1. Corner solution problem (i.e. the present of substantial amount of zero observations); 2. Time allocation trade-offs among different types of activities (which tends to be ignored in previous studies); 3. Travel is treated as a derived demand of activity participation (i.e. travel time and mode share are automatically censored, and are not estimated, if corresponding activity duration is censored). The model is applied on a combined dataset of Swedish national travel survey (NTS) and SMHI (Swedish Meteorological and Hydrological Institute) weather record. Individuals' work and non-work activity durations, travel time and mode shares are jointly modelled as dependent variables. The influences of time-location characteristics, individual and household socio demographics and weather characteristics on each dependent variable are examined. The estimation results show a strong work and non-work activity time trade-offs due to the individual's time-space constraints. Evidences on a potential positive utility of travel time added on non-work activity time allocation in the Swedish case, are also found. Meanwhile, the results also show a consistent mode choice preference for a given individual. The estimated nested multivariate Tobit model provides a superior prediction, in terms of the deviation of the predicted value against the actual value conditional on the correct prediction regarding censored and non-censored, compared to mutually independent Tobit models. However, the nested multivariate Tobit model does not necessarily have a better prediction for model components regarding non-work related activities.

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1. Introduction

Individuals every day make choices intentionally or unintentionally of investing what amount of time in which type of activity. On the one hand, such choices depend partially on the available resources they access to, such as car availability and other travel resources. On the other hand, it also depends partially on the constraints that the individuals have on the given day, such as available time in a day, compulsory and non-compulsory commitments, required time to commute and travel between locations, stores' opening hours, etc. Such constraints and resources affect our daily activity participations and travel related decisions (Hägerstrand, 1970). In general, activity time allocation problem is a subset problem of activity scheduling problem which includes activity time allocation and activity sequencing. Modelling activity time allocation can be viewed as allocating a continuous quantity (daily time budget) into multiple discrete choices (i.e. activity types to engage).

Whilst there have been a lot of studies modelling individuals' daily activity time allocation problem, most of these studies ignored the relationships between the individual's activity participation decisions and his/her activity duration and travel time, mode choice. Many researches pointed out the negative utility of travel time in the activity scheduling problem (Supernak, 1992 and Joh et al., 2001). Given that an individual's ability to travel and engage in the activity is highly influenced by the amount of the activity duration available and that, in many circumstances, an individual has to do trade-offs between his/her travel distances and activity durations in selecting his/her activity locations (Susilo and Dijst, 2010), it is therefore important to consider the influence of travel time on the activity time allocation problem in one integrated model structure. Assuming an individual would like to invest his/her time on a certain type of activity *i*, the corresponding travel time for this kind of activity is zero if this activity is not conducted, otherwise the corresponding travel time is non-zero and this amount of travel time would potentially affect individual's activity time allocation. If this activity is to be conducted, the individual would also need to decide which travel mode he/she wants to take, whilst considering the whole trip chaining pattern, which indeed affects the travel time of this particular trip. Besides, the interaction among activity time allocation, travel time and mode choice also varies between different activity types (work or non-work). The previous studies (e.g. Kang and Scott, 2010; Susilo and Kitamura, 2005; Susilo and Axhausen, 2014) have shown that the time allocation for work activities are more predictable and less dependent on an individual's daily space-time constraints than that for non-work activities.

In order to model the relationship among the individual's activity-travel indicators (i.e. his/her activity time allocation with his/her travel time spent and selected travel modes), for different types of activities on the given day, an integrated model framework should be able to address at least the following three methodological issues. First, there are activities which many individuals do not participate in the given day. For example, for an individual who is not working on the given day, his/her work activity duration is zero. Thus the proposed model structure should have zero as a boundary, and not allow negative values as a solution, which is called corner solution and censoring. Secondly, since an individual has a limited available time to allocate to different types of activities, more time allocated in one type of activity would conceptually lead to a fewer amount of time allocated in other types of activities. Such correlations among different activity types as well as the correlations among activity duration, travel time and mode choices, should be considered in the proposed model. Thirdly, travel indeed is a derived demand of activity participation, thus individuals make mode choice and have travel time only when this particular type of activity is conducted. Thus, the proposed model structure should model the corresponding travel time and mode choice conditional on observing a non-zero activity duration, which can be handled by the sample selection method.

Generally there are two basic categories of model systems that serve the purpose of modelling the activity time allocation. The first category is the utility maximization-based Kuhn–Tucker (KT) demand systems (Kim et al, 2002). This type of model formulates an individual's utility of activity time allocation as the sum of the utilities of the duration of each activity type, while the utility of the duration of each activity type is then parameterized as a function of several variables of interest, such as individuals' social demographic, land use characteristics, etc. Multiple Discrete-Continuous Extreme Value (MDCEV) model (Bhat, 2005) is the most widely used model of this type, not only in modelling individuals' activity time use but also in modelling household car ownership (Bhat et al, 2009) and household mode choice (Rajagopalan and Srinivasan, 2008). MDCEV model is based on utility maximization theory. By applying Kuhn–Tucker condition, MDCEV assumes that an individual would optimally allocate his/her activity time budget in a way that the marginal utilities of the activities that this individual invests time in are equal to each other, while the marginal utilities of the activities that this individual does not invest time in are always less than the marginal utilities of the activities that he/she invests time in (and this is the reason why he/she does not invest time in some particular type of activity). MDCEV has two advantages. First the model is based on the utility maximization, thus is believed to have behavioural interpretations and to be more attached to economic theory (on the other hand it is also criticized due to its functional and parametric assumptions). Second, the likelihood function of MDCEV has a closed form, thus is computationally efficient and the model can incorporate a fairly large number of activity types. However, MDCEV model does not consider other time quantities other than activity duration such as travel time. Even if travel time can be considered in MDCEV model framework, the assumption that individual would optimally allocate travel time is conceptually inconvincible and the fact that travel is a derived demand of activity participation is not well taken in the model. Besides, another restriction of MDCEV model is that total daily activity time budget of each individual is assumed to be fixed, which is likely to change if certain attribute changes, for example, weather. In prediction, MECEV model requires total daily activity time budget of a given individual being known, which is usually unavailable in prediction scheme.

The second category is the multivariate discrete–continuous framework (examples are: Srinivasan and Bhat, 2006; Fang, 2008). The models in this category are based on limited dependent variable models, usually a combination of Tobit models (Tobin, 1958), dealing with the censored cases if the continuous quantity is large and can be partitioned arbitrary such as the time allocation model, or ordered Probit model, dealing with the ordered cases if the continuous quantity is small and can only be partitioned in integer such as the trip generation model (Ferdous et al., 2010). In this study, only the Tobit model case is discussed since the dependent variables (time and share) in this paper can be partitioned arbitrary. Unlike models derived from the utility maximization-based Kuhn–Tucker demand systems, models from the multivariate discrete–continuous frameworks model each dependent variable separately but not independently. The models have a SUR (seemingly unrelated regression) form in the sense that the error term of each dependent variable is not independent with each other but jointly distributed. Models of this framework, though are less attached to economic theory, have advantages in terms of estimation and data fitting (Fang, 2008).

Multivariate Tobit model (e.g. Nelson and Olson, 1978; Maddala, 1983) is one such model from the multivariate discrete–continuous frameworks, which adopts an error covariance structure. An example of bivariate Tobit model is shown as:

$$\begin{aligned} &\mathrm{y}^*_{\mathrm{AD}} = f_1\left(X_{\mathrm{AD}}, X\right) + \varepsilon_{\mathrm{AD}}, \quad \mathrm{y}_{\mathrm{AD}} = \max(0, \mathrm{y}^*_{\mathrm{AD}}) \\ &\mathrm{y}^*_{\mathrm{TT}} = f_2\left(X_{\mathrm{TT}}, X\right) + \varepsilon_{\mathrm{TT}}, \quad \mathrm{y}_{\mathrm{TT}} = \max(0, \mathrm{y}^*_{\mathrm{TT}}) \\ &[\varepsilon_{\mathrm{AD}}, \varepsilon_{\mathrm{TT}}] \sim N(0, \Sigma) \end{aligned} \tag{1}$$

In Eq.(1), $\mathrm{y}^*_{\mathrm{AD}}$ and $\mathrm{y}^*_{\mathrm{TT}}$ are latent variables associated with observed activity duration variable y_{AD} and observed travel time variable y_{TT}. $\varepsilon_{\mathrm{AD}}$ and $\varepsilon_{\mathrm{TT}}$ are jointly normal distributed. X_{AD} and X_{TT} denote the attributes that only influence $\mathrm{y}^*_{\mathrm{AD}}$ and $\mathrm{y}^*_{\mathrm{TT}}$ while X denotes the common attributes that influence both $\mathrm{y}^*_{\mathrm{AD}}$ and $\mathrm{y}^*_{\mathrm{TT}}$. $\varepsilon_{\mathrm{AD}}$ and $\varepsilon_{\mathrm{TT}}$ are likely to share common unobserved attributes due to imperfect specification of X_{AD}, X_{TT} and X. The correlation between travel time and activity duration is then partially explained by the common attributes X and partially explained by the covariance between $\varepsilon_{\mathrm{AD}}$ and $\varepsilon_{\mathrm{TT}}$. However, despite the applications of multivariate Tobit model in economics (e.g. Carlos and Cox, 2001; Hang, 2009), it has rarely been applied in transportation field. The multivariate Tobit model is very flexible to consider the sequential nature of activity and travel by adding sample selection into the model. The sample selection treats the activity time model component and model share model component as selection models to the follow-up model component of travel time. Thus the parameters in the model component of travel time are then conditional on the outcome of the model component of activity time allocation and mode share. Lee (1992) proposed such an adjustment in a bivariate case, namely Nested Tobit model. The nested Tobit model provided a solution that two dependent variables are sequentially censored. For example, the travel time is incidentally censored at 0 when the activity duration is censored at 0.

Beside these two general approaches, the instrumental variable (IV) techniques are computationally easy and serve the same purpose. IV technique can be applied in the Tobit model in order to consider the interactions among activity time, travel time and mode share. For example, the predicted activity duration ($\widehat{y_{\mathrm{AD}}}$) can be an instrumental variable in the Tobit model of travel time (y_{TT}):

$$\begin{aligned} &\mathrm{y}_{\mathrm{AD}}^{*} = f_1\left(X_{\mathrm{AD}}, X\right) + \varepsilon_{\mathrm{AD}}, \qquad &\mathrm{y}_{\mathrm{AD}} = \max(0, \mathrm{y}_{\mathrm{AD}}^{*}) \\ &\mathrm{y}_{\mathrm{TT}}^{*} = f_2\left(\widehat{\mathrm{y}_{\mathrm{AD}}}, X_{\mathrm{TT}}, X\right) + \varepsilon_{\mathrm{TT}}, \quad &\mathrm{y}_{\mathrm{TT}} = \max(0, \mathrm{y}_{\mathrm{TT}}^{*}) \end{aligned} \tag{2}$$

Where $\varepsilon_{\mathrm{AD}}$ and $\varepsilon_{\mathrm{TT}}$ are independent with each other. The notations of symbols are the same as in Eq.(1). IV technique is another way of dealing with correlated unobserved attributes that affect both activity duration and travel time. In other words, the effects of X_{AD} serve the same role as the covariance between $\varepsilon_{\mathrm{AD}}$ and $\varepsilon_{\mathrm{TT}}$ in SUR form. Lee (2007, 2009) used Tobit model with IV to examine the determinants of activity time allocation and the relationship between activity duration and travel time. However, the limitation of IV technique is also well-known. The requirement of IV is in itself a contradiction. First, the instrumental variable $\widehat{y_{\mathrm{AD}}}$ must not be significantly correlated with $\varepsilon_{\mathrm{TT}}$. Second, $\widehat{y_{\mathrm{AD}}}$ must be significantly correlated with $\mathrm{y}_{\mathrm{AD}}^{*}$ to control for endogeneity. However, not only finding suitable X_{AD} with which to model y_{AD}^{*} can be difficult (this is because the attributes in X_{AD} tend to move into X in order to keep $\widehat{y_{\mathrm{AD}}}$ not significantly correlated with $\varepsilon_{\mathrm{TT}}$), but also modelling y_{AD}^{*} as a function of X_{AD} and X which is uncorrelated with $\varepsilon_{\mathrm{TT}}$ will necessarily leave a substantial amount of variance in y_{AD}^{*} unexplained, therefore making $\widehat{y_{\mathrm{AD}}}$ less correlated with y_{AD}^{*} (Mokhtarian and Cao, 2007). Such limitation may potentially yield inconsistent estimation in IV model.

Thus this paper provides a sample selection version of multivariate Tobit model, namely Nested multivariate Tobit model, to model activity duration, travel time and mode share. Such a model covers all the issues discussed above, which are: the substantial amount of censored observations, the correlations (and trade-offs) among dependent variables, and the sequential nature of activity participation and travel. This paper is structured in the following manner: Section 2 describes modelling framework, Section 3 presents an application of this model using Swedish National Transport Survey Data. Section 4 discusses the estimation results. Section 5 provides a prediction case, and Section 6 offers the summary section.

2. Modelling framework

In this paper, individuals' daily activity-travel patterns are of interest. Six individual's daily activity-travel indicators are jointly modelled as dependent variables in this paper: (1) out-of-home work activity duration, y_{wd}, (2) out-of-home non-work activity duration, y_{nd}, (3) slow-mode share (i.e. percentage of walking and cycling trips) in out-of-home work trips, y_{wm}, (4) slow mode share in out-of-home non-work trips, y_{nm}, (5) out-of-home work related travel time, y_{wt}, and (6) out-of-home non-work related travel time, y_{nt}. Mode choice at trip level is aggregated into mode share at daily level which is used as the dependent variable in this paper, since the activity time allocation problem is formulated at daily level. The purpose of including mode share as a dependent variable is to model travel time in a more meaningful manner. Mode share model component is used as a selection model to the travel time model component, so that the travel time model component is dependent on the outcome of mode share model component, presented in the following paragraphs.

Let four latent variables $y_{wd}^{*}, y_{nd}^{*}, y_{wm}^{*}, y_{nm}^{*}$ denote the uncensored (unobserved) activity durations and mode shares. The model system that describes the relationship among an individual's activity durations, travel time, and mode shares of work and non-work activities can be written as following. Note that the sub index of individual is omitted:

$$y_i^{*} = X_i\beta_i + \varepsilon_i \quad i \in \left(wd, nd, wm, nm\right) \tag{3}$$

$$\begin{cases} \text{if } y_{wd}^* > 0,\ \ y_{wd} = y_{wd}^* \text{ and } y_{wm} = \begin{cases} 100 \text{ if } y_{wm}^* \ge 100 \\ y_{wm}^* \quad \text{if } y_{wm}^* \in (0,100) \\ 0 \quad \text{if } y_{wm}^* \le 0 \end{cases} \text{and } y_{wt} = \begin{cases} X_{wt}\beta_{wt}^{y_{wm}=0} + \varepsilon_{wt} & \text{if } y_{wm} = 0 \\ X_{wt}\beta_{wt}^{y_{wm}>0} + \varepsilon_{wt} & \text{if } y_{wm} > 0 \end{cases} \\ \text{if } y_{wd}^* \le 0 \quad y_{wd} = 0 \text{ and } y_{wm} = 0 \text{ and } y_{wt} = 0 \end{cases}$$

$$\begin{cases} \text{if } y_{nd}^* > 0,\ \ y_{nd} = y_{nd}^* \text{ and } y_{nm} = \begin{cases} 100 \text{ if } y_{nm}^* \ge 100 \\ y_{nm}^* \quad \text{if } y_{nm}^* \in (0,100) \\ 0 \quad \text{if } y_{nm}^* \le 0 \end{cases} \text{and } y_{nt} = \begin{cases} X_{nt}\beta_{nt}^{y_{nm}=0} + \varepsilon_{nt} & \text{if } y_{nm} = 0 \\ X_{nt}\beta_{nt}^{y_{nm}>0} + \varepsilon_{nt} & \text{if } y_{nm} > 0 \end{cases} \\ \text{if } y_{nd}^* \le 0 \quad y_{nd} = 0 \text{ and } y_{nm} = 0 \text{ and } y_{nt} = 0 \end{cases} \tag{4}$$

In these equations,

y_i: the observed dependent variables, $i \in (\mathrm{wd, nd, wt, nt, wm, nm})$.

y_i^*: the corresponding unobserved latent variable associated with y_i

X_i: the set of exogenous variables associated with y_i

β_i: the vector of coefficients associated with y_i^*

$\beta_{\mathrm{wt}}^{y_{\mathrm{wm}}=0}$, $\beta_{\mathrm{wt}}^{y_{\mathrm{wm}}>0}$, $\beta_{\mathrm{nt}}^{y_{\mathrm{nm}}=0}$, $\beta_{\mathrm{nt}}^{y_{\mathrm{nm}}>0}$: the vector of coefficients associated with y_{wt} and y_{nt}. Each set of coefficients is only applied when the condition shown in superscript holds.

ε_i: the joint normal distributed error term.

The vector of error terms [$\varepsilon_{wd}\ \varepsilon_{nd}\ \varepsilon_{wt}\ \varepsilon_{nt}\ \varepsilon_{wm}\ \varepsilon_{nm}$] follows multivariate normal distribution with zero mean and covariance matrix $\sum$ but remains independent across observations.

In Eq.(4), latent variables y_{wd}^* and y_{nd}^* are left side censored at 0 while latent variables y_{wm}^* and y_{nm}^* are both left side censored at 0 and right side censored at 100, since mode share ranges from 0% to 100%. The model has a two-level structure, the censoring pattern in first level differs depending on whether y_{wd}^* or y_{nd}^* is censored, and the censoring pattern in second level differs depending on whether y_{wm}^* or y_{nm}^* is censored conditional on observing y_{wd}^* or y_{nd}^* is not censored. Besides, the coefficients of work/non-work related travel time are dependent on the outcomes of the corresponding mode share. For example, two sets of coefficients, $\beta_{wt}^{y_{wm}=0}$ and $\beta_{wt}^{y_{wm}>0}$, are estimated depending on the y_{wm}. The nested multivariate Tobit model is one type of sample selection model in the sense that, 1. Mode shares in work/non-work trips are incidentally censored when work/non-work activity durations are censored, showing that travel is a derived demand of activity participation. 2. The coefficients of work/non-work related travel time is dependent on the outcomes of mode share variables since the travel time is significantly influenced by the travel modes chosen by an individual. The interaction among activity participations, travel time and mode choices and the trade-offs between work and non-work activity-travel engagements are reflected in the covariance matrix as well as the effects of common attributes in X. The difference between a nested multivariate Tobit model and a multivariate Tobit model is similar as the difference between a multinomial logit model (MNL) and a nested logit model (NL), but the former deals with continuous response.

Obviously, each individual may have different activity-travel patterns, or censoring patterns. For example, one may work at the office on the given day ($y_{wd} > 0$ and $y_{wt} > 0$), driving during all his work trips ($y_{wm} = 0$) but may not conduct any out-of-home non-work activities ($y_{nd} = 0, y_{nt} = 0$ and $y_{nm} = 0$), while the other may not work on the given day ($y_{wd} = 0, y_{wt} = 0$ and $y_{wm} = 0$) but spend some time walking and eating outside ($y_{nd} > 0, y_{nt} > 0$ and $y_{nm} > 0$). For $n+m$ dimensional multivariate Tobit model, there are $2^n \times 3^m$ censoring patterns, where n is the number of dimensions which is censored only at one side (in this study, $n \in \{\text{work duration, non-work duration}\}$) and m is the number of dimensions which is censored at both left and right sides (in this study, $m \in \{\text{slow mode share in work trips, slow mode share in non-work trips}\}$). However, for $n+m$ dimensional nested multivariate Tobit model, the number of censoring pattern is less than $2^n \times 3^m$ due to the sample selection.

Full information maximum likelihood can be established for estimating the above model. The likelihood function is very similar as that of the multivariate Tobit model, which is the product of all individual likelihood values. In general, the probability of latent variables associated with individual i who belongs to censoring pattern c, unconditional on observed values, can be expressed as the probability density function of multivariate normal distribution:

$$L_{i,c} = \phi(Y^*, Y \mid X\beta, \Sigma) \tag{5}$$

Where $L_{i,c}$ is the likelihood function of individual i who belongs to the censoring pattern c, and ϕ denotes the multivariate normal density function, $Y*$ is the vector of latent variables $[y^*_{wd}, y^*_{nd}, y^*_{wm}, y^*_{nm}]$, Y is the vector of observed variables $[y_{wt}, y_{nt}]$ and $X\beta$ denotes the mean vector of the multivariate normal distribution and $\sum$ denotes the covariance matrix of the error terms.

However, the latent variables are not observed, the likelihood function conditional on the observed values Y is needed and it varies according to the specific censoring pattern of each individual.

If no latent variables are censored, all latent variables become observed and the likelihood function L_i for individual i is just Eq. (5) but substituting Y^* with Y. If one or more latent variables are censored, the corresponding censored latent variables then need to be integrated out from Eq. (5). Due to the sequential nature shown in Eq. (4), the censoring pattern differs depending on whether the latent variables in the first level $[y^*_{wd}, y^*_{nd}]$ are censored.

If no latent variables in the first level are censored, the likelihood function L_i would differ depending on whether the latent variables in the second level are censored. For example, the likelihood function L_i when observing $y^*_{wd} > 0$, $y^*_{nd} > 0, y^*_{wm} \leq 0, y^*_{nm} \geq 100$ is:

$$L_i = \int_{-\infty}^{0} \int_{100}^{+\infty} \phi(Y^*_c, Y_{uc} \mid X\beta, \Sigma) dy^*_{nm} dy^*_{wm} \tag{6}$$

Where Y^*_c denotes the vector of the censored variables in Y^*, $[y^*_{wm}, y^*_{nm}]$. Y_{uc} denotes the vector of the remaining uncensored variables $[y_{wd}, y_{nd}, y_{wt}, y_{nt}]$. The vector $\beta^{y_{wm}=0}_{wt}$ is applied instead of $\beta^{y_{wm}>0}_{wt}$ in β since $y_{wm} = 0$.

If one of the latent variables $[y^*_{wd}, y^*_{nd}]$ in the first level are censored, for example, $y^*_{wd} \leq 0$ and $y^*_{nd} > 0$, the corresponding second level variables (y^*_{wt} and y^*_{wm}) then disappear in the likelihood function (integrated out from -∞ to +∞), since the observed values of y_{wt} and y_{wm} become deterministic (zero). Conditional on observing $y^*_{nd} > 0$, the censoring in second level, $y^*_{nm} \leq 0$ or $y^*_{nm} \geq 100$, leads to an additional integral of y^*_{wm} on the likelihood function. For example, the likelihood function L_i when observing $y^*_{wd} \leq 0$, $y^*_{nd} > 0$, $y^*_{nm} \geq 100$ is:

$$L_i = \int_{-\infty}^{0} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{100}^{+\infty} \phi(Y^*_c, Y_{uc} \mid X\beta, \Sigma) dy^*_{nm} dy^*_{wm} dy^*_{wt} dy^*_{wd} \tag{7}$$

Where Y^*_c is $[y^*_{wd}, y^*_{wt}, y^*_{wm}, y^*_{nm}]$. Y_{uc} is $[y_{nd}, y_{nt}]$. Similarly, the vector $\beta^{y_{nm}>0}_{nt}$ is applied in β.

If both latent variables $[y^*_{wd}, y^*_{nd}]$ in the first level are censored, all dependent variables in the second level are 0 and then need to be integrated out from -∞ to +∞ in the likelihood function:

$$L_i = \int_{-\infty}^{0} \int_{-\infty}^{0} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \phi(Y^* \mid X\beta, \Sigma) dy^*_{wt} dy^*_{nt} dy^*_{wm} dy^*_{nm} dy^*_{wd} dy^*_{nd} \tag{8}$$

The final likelihood function is the production of all observations in all censoring patterns:

$$L = \prod_{c=1}^{C} \prod_{i=1}^{N_c} L_{i,c} \tag{9}$$

Where C is the total number of censoring patterns, C=15 in this paper, N_c is the total number of observations in c^{th} censoring pattern.

The sample selection (nested structure) actually reduces the number of dimensions needed to be integrated in the individual likelihood function, compared with the model without sample selection (multivariate Tobit model). This is because if the latent variables in the first level are censored at 0, the corresponding latent variables in the second level are integrated out from -∞ to +∞, thus disappear in the individual likelihood function. For a two-level structure as shown in this paper, the dimensions needed to be integrated are no more than two. Thus it is preferred to partition the censored dimensions from the uncensored components in the individual likelihood function. The theoretical foundations of such separation can be found in Pudney (1991). Let the likelihood function of individual i who belongs to a censoring pattern c be expressed as the following:

$$L_{i,c} = \int_{\Omega_{Y_c^*}} \phi(Y_c^*, Y_{uc} \mid X\beta, \Sigma) \tag{10}$$

Where $\Omega_{Y_c^*}$ is the integration domain of censoring pattern c. Conditional on the uncensored part, the joint distribution can be rewritten as:

$$\begin{aligned} \phi\left(Y_c^*, Y_{uc}|X\beta,\Sigma\right) &= \phi\left(Y_c^*|Y_{uc}, X\beta_c, \Sigma^c\right)\cdot\phi\left(Y_{uc}|X\beta_{uc},\Sigma^{uc}\right) \\ &= \phi\left(v_{uc}|0,\Sigma^{uc}\right)\cdot\phi\left(v_c^*|\mu^*,\Sigma^*\right) \end{aligned} \tag{11}$$

Where
$v_{uc} = Y_{uc} - X\beta_{uc}$
$v_c^* = Y_c^* - X\beta_c$
$\Sigma^{uc} = \mathrm{E}(v_{uc}v_{uc}{}')$
$\Sigma^* = \Sigma^c - \Sigma^{c,uc}(\Sigma^{uc})^{-1}\Sigma^{uc,c}$
$\Sigma^c = \mathrm{E}(v_c v_c{}')$
$\Sigma^{c,uc} = \Sigma^{uc,c'} = \mathrm{E}(v_c v_{uc}')$
$\mu^* = \Sigma^{c,uc}(\Sigma^{uc})^{-1}v_{uc}$
Substituting Eq. (11) in Eq. (10):

$$L_{i,c} = \phi\left(v_{uc}|0,\Sigma^{uc}\right)\cdot\int_{\Omega_{v_c^*}}\phi\left(v_c^*|\mu^*,\Sigma^*\right) \tag{12}$$

Since $\Omega_{v_c^*}$ has only two dimensions, the integral in Eq. (12) can be easily evaluated by any program that embeds a bivariate normal cumulative density function. Thus, no simulation machinery is required.

3. Model application and data source

The proposed model framework is applied on a combined dataset of Swedish national travel survey (NTS) dataset and SMHI (Swedish Meteorological and Hydrological Institute) weather record (SMHI, 2013). The NTS data is a person-trip dataset which records all trips that respondents took on the observed day. In NTS, a trip is defined that a certain errand is achieved at the trip destination. Changing travel mode is not counted as an errand. If a trip contains multiple travel modes, the one with the longest travel distance is taken as the travel mode of this trip. The NTS dataset is composed of datasets of four periods and each covers respectively from 1998 to 2001, 2003 to 2004, 2005 to 2006 and 2011. The information includes all travel pattern characteristics (e.g. main travel mode, travel purpose, start and end point, departure and arrival time etc.), as well as individual and household characteristics (Algers, 2001). The weather data contains weather information from 1961 to 2011 in the whole of Sweden at municipality level, including average, minimum and maximum temperatures (only average temperature is used in this study), precipitation amount (mm), visibility (visible distance measured in km), wind speed (km/h), relative humidity, snow depth and air pressure. The weather data was collected at three hour intervals every day. Weather information was assigned to each trip by matching the weather data from the weather station nearest to the departure point of the trip and selecting the weather variable with the measured time closest to the departure time. A detailed discussion of the pros and cons of combining these two datasets can be found in Liu et al., (2014).

The exogenous variables considered, include the time-location characteristics, individual social demographics, household social demographics and weather characteristics. The dependent variables and all exogenous variables that are included in the model are listed in Table 1 below. It is worth noting that "work activity" also includes school activities for children and teenagers.

Table 1. Variables in the Nested multivariate Tobit model.

Variable Category	Description
Dependent variables	
Work activity duration (C)	Daily out-of-home time expenditure for work activities (min)
Non-work activity duration (C)	Daily out-of-home time expenditure for non-work activities (min)
Work related travel time (C)	Total travel time spent on work trips on the given day (min)
Non-work related travel time (C)	Total travel time spent on non-work trips on the given day (min)
Slow-mode share in work trips (%) (C)	Percentage of walk and cycling trips in all daily work trips
Slow-mode share in non-work trips (%) (C)	Percentage of walk and cycling trips in all daily non-work trips
Exogenous variables	
Time-location characteristics	
Weekday (D)	The activities and trips were conducted from Monday to Friday
Municipality level (D)	Rural: municipality residences were less than 20,000 (reference)
	Sub urban: municipality residences were from 20,000 to 100,000
	Urban: municipality residences were more than 100,000
Individual social-demographics	
Male (D)	The respondent is male
Age (D)	Age under 19 years old (reference)
	Age from 19 to 30 years old
	Age from 31 to 50 years old
	Age from 51 to 65 years old
	Age over 65 years old
Household social-demographics	
Household type (D)	Family has no child (reference)
	Family has youngest child at 0-6 years old
	Family has youngest child at 7-18 years old
Household size (C)	Number of household members
Car ownership (C)	Number of cars in household
Household Income (C)	Log(income) (in SEK)
Weather characteristics	
Monthly temperature variation (C)	Monthly mean temperature in the corresponding municipality
Daily temperature variation (C)	Cold day: Z score interval less than 0.
	Warm day: Z score interval greater than 0.
Relative humidity (C)	Relative humidity measure
Wind speed square (C)	Square of wind speed $(m/s)^2$
Precipitation (C)	Precipitation amount (mm)
Visibility (D)	Poor visibility: visibility up to1 km
	Good visibility: visibility over 1 km (reference)
Snow covered (D)	Ground without snow covered (reference)
	Ground with snow covered

Note: C in the parenthesis denotes it is a continuous variable while D in the parenthesis denotes it is a dummy variable.

In Table 1, observed temperature values were separated into measures of monthly variation and daily variation, in order to differentiate the impact of variation of "normal"/"as expected" weather conditions between municipalities with the impact of variation of "un-usual"/"unpredictable" weather conditions from a local perspective (Liu et al. 2013). The observation of "Monthly temperature variation" is calculated as:

$$T_{m,l} = \frac{\sum_{n=y-11}^{y-1} \sum_{d \in m} t_{n,m,d,l}}{N_m} \tag{13}$$

Where $T_{m,l}$ is the measure of temperature variation in month m at municipality l, $t_{n,m,d,l}$ is the temperature record of year n, month m, day d, at location l. N_m is the total number of days in corresponding month m during ten years previous to the analysed year. Thus "monthly temperature variation" is measured as the monthly mean temperature during the ten years previous to the analysed year. Correspondingly, "daily temperature variation" is calculated as:

$$D_{y,m,d,l} = \frac{t_{y,m,d,l} - T_{m,l}}{\sqrt{\left(\sum_{n=y-11}^{y-1} \sum_{d \in m} \left(t_{n,m,d,l} - T_{m,l}\right)^2\right) \Big/ \left(N_m - 1\right)}} \tag{14}$$

Where $D_{y,m,d,l}$ is the measure of daily temperature variation at day d, month m, year y at location l. The denominator is the sample standard deviation of $t_{y,m,d,l}$ with respect to its monthly mean. Thus, $D_{y,m,d,l}$ is the Z-score representing the deviation of temperature with respect to its historical monthly mean value.

In this study, only individuals whose trips were all conducted in daytime (from 6:00 to 21:00) are included since trips at night may differ significantly from trips in daytime. For example, shops are closed, and temperature drops significantly at night. The datasets were checked and observations with missing values were eliminated. The full dataset was separated into two parts. 75% of random sampled observations were used for model estimation (estimation sample) while the rest 25% were used for prediction purpose (prediction sample) After data screening, 9862 observations were used as the estimation sample while 3271 observations were used as the prediction sample. The descriptive statistics of the estimation sample and prediction sample are presented in Table 2.

Table 2. Descriptive statistics of the dependent variables in the estimation sample and prediction sample.

Dependent variables	Total number (%) of non-censored cases		Mean with censored cases		Mean without censored cases	
	Estimation sample	Prediction sample	Estimation sample	Prediction sample	Estimation sample	Prediction sample
Work activity duration	3484(35%)	1178(36%)	135.7(min)	140.6(min)	384.2(min)	390.4(min)
Non-work activity duration	8004(80%)	2623(80%)	112.8(min)	112.8(min)	138.9(min)	140.6(min)
Work related travel time	3484(35%)	1178(36%)	15.6(min)	15.6(min)	44.2(min)	43.3(min)
Non-work related travel time	8004(81%)	2623(81%)	53.9(min)	56.3(min)	66.4(min)	70.2(min)
Slow-mode share in work trips	663(6.7%)	207(6.3%)	9.6%	8.6%	44.4 %	42.7%
Using slow modes in all work trips	550(5.6%)	193(5.9%)	/	/	/	/
Slow-mode share in non-work trips	2071(21%)	666(20%)	21.2%	21.0%	40.8%	40.9%
Using slow modes in all non-work trips	1225(12.4%)	414(12.7%)	/	/	/	/
Number of observations	9862	3271				

Note: The non-censoring cases for slow mode share denote that slow mode shares of those cases are neither 0% nor 100%. "Using slow modes in all work/non-work trips" refers to the cases with slow mode share equal to 100%

The estimation sample and prediction sample are generally comparable. Around 35% of the observed individuals conducted work activities in the given day with around 385 minutes of working duration while 80% of the observed individuals conducted non-work activities in the given day with around 140 minutes of non-work duration. This suggests that a substantial amount of individuals may have both work and non-work activities in the given day. The total travel time spent for non-work trips (68 min) is slightly longer than that of work trips (44 min). As expected, the slow mode share in non-work trips (21%) is much higher than that in work trips (9%).

4. Estimation results

The parameters need to be estimated in the model include 176 (22×8) coefficients associated with six dependent variables, and 21 elements in cholesky factorization of the variance-covariance matrix $\sum$. The estimation results are shown in Table 3. The salient results are summarized as follow.

Table 3. Estimation results of nested multivariate Tobit model

Dependent variable / Exogenous variable	Work activity duration		Non-work activity duration		Slow-mode share in work trips(%)		Slow-mode share in non-work trips(%)	
	Estimates	T-values	Estimates	T-values	Estimates	T-values	Estimates	T-values
Intercept	-609.6	-12.18	98.17	6.93	-251.6	-5.99	41.65	4.74
Time-location characteristics								
Weekday	606.7	29.74	-44.10	-13.51	201.2	8.11	9.37	4.62
Rural area	Ref		Ref		Ref		Ref	
Sub-urban area	53.33	4.69	-8.32	-2.81	17.01	2.98	3.72	1.55
Urban area	103.7	8.07	-7.90	-1.84	40.34	5.65	11.88	4.47
Individual social-demographics								
Male	-9.60	-1.13	5.00	1.50	-7.92	-2.03	-9.31	-5.12
Age <19	Ref		Ref		Ref		Ref	
Age 19-30	-78.76	-4.22	-7.15	-1.04	-78.25	-7.48	-25.14	-5.81
Age 31-50	-84.44	-6.46	7.40	1.46	-79.72	-10.91	-32.88	-10.46
Age 51-65	-179.4	-9.55	21.64	3.30	-118.9	-8.96	-34.85	-8.25
Age >65	-901.3	-18.11	39.42	5.63	-541.6	-6.81	-28.65	-6.56
Household social-demographics								
No child	Ref		Ref		Ref		Ref	
Youngest child 0-6 years old	-68.21	-3.95	31.33	5.01	-31.68	-3.35	3.66	0.89
Youngest child 7-18 years old	-28.80	-1.83	11.37	1.94	-29.18	-3.42	-1.11	-0.67
Household size	0.89	0.21	-0.49	-0.27	1.87	0.86	-0.78	-0.67
Number of cars in household	18.16	3.24	3.45	1.73	-34.13	-11.60	-22.10	-16.17
Log(household income)	10.09	6.37	0.93	1.44	2.87	3.70	0.26	0.67
Weather characteristics								
Monthly temperature variation	-1.87	-2.58	0.61	2.48	-0.73	-2.13	-0.57	-3.79
Temperature Z score increase in <0 interval	-93.28	-3.46	-2.82	-0.27	-31.24	-2.43	47.98	2.49
Temperature Z score increase in >0 interval	-61.48	-3.24	12.85	2.21	14.75	1.57	8.96	2.55
Relative humidity	3.20	8.92	0.03	0.25	1.00	4.99	0.03	0.05
Wind speed square	-1.80	-0.87	0.06	0.09	0.73	0.75	0.32	0.75
Precipitation amount	-6.72	-3.14	0.70	0.92	-2.18	-2.04	-1.24	-2.61
Visibility <1km	-59.31	-3.49	-15.54	-2.59	-16.63	-2.06	-5.54	-1.48
Ground with snow cover	-103.1	-5.43	19.09	2.95	-23.34	-2.60	-13.38	-3.33

Exogenous variable \ Dependent variable	Work related travel time, when Slow-mode share in work trips=0		Work related travel time, when Slow-mode share in work trips>0		Non-work related travel time, when Slow-mode share in work trips=0		Non-work related travel time, when Slow-mode share in work trips>0	
	Estimates	T-values	Estimates	T-values	Estimates	T-values	Estimates	T-values
Intercept	73.55	5.58	26.10	0.64	19.36	1.80	54.99	4.52
Time-location characteristics								
Weekday	0.22	0.02	-5.40	-0.21	-13.50	-5.76	-27.06	-9.76
Rural area	Ref		Ref		Ref		Ref	
Sub-urban area	0.63	0.29	3.82	1.13	-6.11	-2.23	-4.74	-1.40
Urban area	-1.36	-0.40	5.67	0.98	-9.50	-3.01	-10.61	-2.86
Individual social-demographics								
Male	1.08	0.73	-0.64	-0.27	8.89	4.06	6.29	2.51
Age <19	Ref		Ref		Ref		Ref	
Age 19-30	16.86	2.53	10.48	0.86	-0.16	-0.03	25.81	4.39
Age 31-50	6.17	0.90	16.30	1.67	12.27	3.04	37.94	8.59
Age 51-65	7.48	0.73	20.68	1.29	19.39	3.69	40.99	7.09
Age >65	29.09	0.98	30.10	0.35	14.28	2.56	47.69	8.03
Household social-demographics								
No child	Ref		Ref		Ref		Ref	
Youngest child 0-6 years old	-9.97	-2.36	-1.52	-0.22	-6.73	-1.36	-12.06	-2.18
Youngest child 7-18 years old	-5.30	-1.65	-0.66	-0.10	0.02	0.005	-5.51	-1.03
Household size	2.07	2.53	1.44	1.69	-1.28	-0.88	1.25	0.78
Number of cars in household	-5.40	-2.26	0.08	0.02	8.88	5.89	18.62	10.11
Log(household income)	-1.21	-3.42	-0.46	-0.90	-0.16	-0.32	1.26	2.23
Weather characteristics								
Monthly temperature variation	0.24	1.79	0.17	0.89	0.86	4.73	1.08	5.17
Temperature Z score increase in <0 interval	0.42	0.09	1.28	0.18	-3.08	-0.44	-18.11	-2.00
Temperature Z score increase in >0 interval	-3.63	-1.02	0.81	0.14	2.09	0.48	13.74	2.86
Relative humidity	-0.09	-0.85	-0.04	-0.27	-0.10	-1.30	0.04	0.43
Wind speed square	-0.34	-0.92	0.74	1.50	-0.88	-1.72	-0.27	-0.46
Precipitation amount	-0.20	-0.46	-0.73	-1.42	1.26	2.34	0.002	0.003
Visibility <1km	9.35	2.79	3.76	0.94	-4.43	-1.02	0.69	0.13
Ground with snow cover	6.75	1.75	0.71	0.15	31.81	6.73	20.64	3.69
Final Log-likelihood	-170669							
Adjusted R^2	0.160							
Adjusted R^2 of single Tobit models	0.104							

Note: estimates in grey grid are not significant at 10% level. The adjusted R^2 of single Tobit models are the adjusted R^2 of models that model each dependent variable independently using single Tobit model.

4.1. Variable effects

The adjusted R^2 of nested multivariate Tobit model is 0.16 which is significantly larger than that of single Tobit models, 0.104, showing a great model improvement. It is also worth noting that since the travel time and mode share are incidentally censored depending on the outcome of activity duration, the coefficients on travel time/mode share should be interpreted as the impacts on travel time/mode share only for those individuals whose corresponding activity duration is non-zero. For example, the coefficient of "weekday" on work activity duration is significantly positive (606.7), as expected, showing that people work more on weekday than on weekend. Correspondingly the coefficients of "weekday" on work related travel time are not significant. However, it does not mean that work related travel time on weekday is not significantly different from that on weekend. Instead, it indicates that the work related travel time of those who are working on weekday is not significantly longer than that of those who are working on weekend. For convenience, the following interpretation would not emphasize this particularity in interpretation.

As expected, people living in sub-urban and urban areas tend to have longer work activity duration and shorter non-work activity duration than those living in rural areas. People are also more likely to walk and cycle in more urbanized areas, presumably due to better accessibility in urbanized areas or congestion.

There is no significant gender difference observed in terms of work/non-work activity time allocations. Male tends to have smaller slow mode share in both work and non-work trips than female. Elders (age >65) have the longest non-work activity duration in all age groups and correspondingly have the longest non-work related travel time, while teenagers (age<19) walk and cycle more often in their daily trips than other age groups.

The presence of young children corresponds to shorter work activity duration and longer non-work activity duration for an individual. Interestingly, those who have young children also use motorised modes more often on a workday. However for those who conduct non-work activities on the given day, the presence of young children has no significant influence on their slow mode usages in non-work trips. The presence of young children also corresponds to a decrease of work related travel time especially with motorised modes. This is because picking up/dropping off children is counted as non-work activities thus a work trip splits into a children-related trip and a follow-up work trip. Having more cars in household corresponds to an increase in work activity duration and a decrease in non-work activity duration. People from higher income household also have longer work activity duration, but no evidence is found that they use motorised modes more often than their counterparts.

Comparing to daily (short term) temperature variation, monthly (long term) temperature variation shows a more significant effect on individuals' work/non-work activity-travel engagements. People spend less time on work activities but more time on non-work activities in warmer months. Presumably this is due to a summer season mood in Sweden where people are more relax towards working load than in other seasons. Slow mode shares in work and non-work trips decrease in warm months which highlights Swedish habit of staying in their summer house and have longer commute and other trips, compared to other seasons. This echoes the previous study's results on mode choice which is also using Swedish data (Liu et al, 2013). This phenomenon is also shown by the increasing of work and non-work related travel time in warmer months, especially for non-work related travel time. However, a warmer day, compared to its monthly mean, encourages the use of slow modes in non-work trips. This highlights the fact that short term and long term temperature variations have different influences on individuals' mode choices.

A high relative humidity corresponds to a rise in work activity duration and slow modes share in work trips. This is mainly due to the seasonal effect as summer averagely has a low relative humidity compared to winter in Sweden. Wind speed does not show any significant impacts on work/non-work activity-travel engagements. Precipitation corresponds to less walking and cycling in both work and non-work trips. Bad visibility (fog) corresponds to a decrease of work and non-work activity durations, especially for work activities. The shifts in morning departure time and prolonged motorised travel time in bad visibility conditions may contribute to commuters being late for work.

As expected, snow covered ground shows a negative effect on work activity duration and a positive effect on non-work activity duration which is likely due to the extra snow-related activities. The slow mode shares in work and non-work trips also decrease in a situation with snow covered ground. Also both work and non-work related travel time increase in snow covered ground situation, especially non-work related travel time associated with motorised modes. However, the increase of work related travel time may be mainly due to the bad road condition while the increase of non-work related travel time may be mainly due to the extra travel for snow-related activities.

4.2. Correlation and standard deviation

The estimated correlations and standard deviation are presented in Table 4. The estimated standard deviations (diagonal elements) indicate a higher variance for work activity duration than for non-work activity duration, but a lower variance for work related travel time than for non-work related travel time. Further, there is a strong negative correlation (-0.49) between work and non-work activity durations, showing the trade-offs due to the remaining unobserved attributes that reflect individual's time-space constraints. Besides, non-work related travel time is also positively correlated with non-work activity duration (0.18). It implies the further an individual travels to a non-work location, the more the individual would like to get utility (longer visit duration) at the destination location (Susilo and Dijst, 2009). This indicates a potential positive utility of travel time added on non-work activity time allocation in the Swedish case. However, work related travel time is not significantly correlated with work activity duration, indicating that the length of commute time is generally irrelevant to the duration of working. Besides, work related travel time also negatively correlates with slow mode share in work trips (-0.33), indicating the further an individual commutes, the less likely he/she would cycle or walk, and vice versa. Similar trend also appears in the relationship between non-work related travel time and slow mode share in the non-work trips, and the negative correlation tends to be even stronger (-0.74). Slow mode share in work trips is highly correlated with slow mode share in non-work trips (0.92). This reflects strongly consistent mode choice preference in both work and non-work trips.

Table 4. Correlation and standard deviation derived from estimated covariance matrix

Dependent variables / ***Dependent variables***	Work activity duration	Non-work activity duration	Slow-mode share in work trips(%)	Slow-mode share in non-work trips(%)	Work related travel time	Non-work related travel time
Work activity duration	330.3 (57.08)					
Non-work activity duration	-0.49 (-1.85)	139.9 (135.6)				
Slow-mode share in work trips(%)	0.73 (1.25)	-0.007 (-0.004)	137.2 (63.3)			
Slow-mode share in non-work trips(%)	0.66 (1.54)	-0.08 (-0.16)	0.92 (1.84)	75.0 (107.8)		
Work related travel time	-0.14 (-1.38)	-0.06 (-0.54)	-0.33 (-1.68)	-0.03 (-0.21)	35.0 (23.2)	
Non-work related travel time	-0.53 (-1.32)	0.18 (2.56)	-0.64 (-2.14)	-0.74 (-2.79)	0.02 (1.54)	84.0 (162.3)

Note: t-value in parenthesis. Estimates in grey grid are not significant at 10% level. The diagonal elements in "***Correlation and standard deviation***" are standard deviation of each dependent variable and off-diagonal elements in lower triangle are correlation coefficients. The corresponding t-values of standard deviations and correlations are calculated by "delta" method (Greene, 2003).

4.3. Marginal effects

The estimated coefficients shown in Table 4 only represent the variable effects on a subsample depending on the sample selection. As discussed in section 4.1, the insignificant coefficients of "weekday" on work related travel time are only applied on those who are working on weekday (work duration is non-zero). However, the policy makers are often interested in the variable effects on the entire sample/population. For example, work related travel time in weekday is longer than that in weekend simply because of fewer individuals are working in weekend. Besides, in order to compare the variable effects of the Nested multivariate Tobit model and mutually independent Tobit model, the marginal effects of the two models are both calculated. The marginal effects provide the changes of a dependent variable due to the unit change of an explanatory variable, for the whole sample. Due to the sample selection and covariance matrix, it is difficult to derive the analytical form for marginal effects of the nested multivariate Tobit model. The marginal effects are then calculated by simulating the covariance structure and applying sample selection (Eq. (4)) on the simulated outcomes. 5000 draws are simulated for each observation which is sufficient to cover the entire distribution of covariance structure. The mean of simulated outcomes then represents the expected outcome of a given dependent variable for each observation. In general, three kinds of marginal effects are calculated for continuous variable, ordinal variable and dummy variable respectively. For a continuous variable, the value of the variable is increased by 10% for the whole sample and the marginal effects are calculated as the difference between the expected outcomes of the before- and after-change sample for each observation. The same draws are used in before- and after-change samples to eliminate the simulation error. And the final marginal effect takes the mean of all observations. For an ordinal variable, the marginal effects are calculated such that the value of the variable is increased by 1. For a dummy variable, the dummy variable is changed to 1 for a subsample where the dummy variable takes a value of 0 and to 0 for the subsample where the dummy variable takes a value of 1. Then the marginal effects of the two subsamples are calculated respectively and added up after reversing the sign of the marginal effect of second subsample. It is worth noting that the marginal effects are not comparable across the three types of variables due to the differences in the variable scale. The marginal effects by variable category for each dependent variable are presented in Table 5 for nested multivariate Tobit model and mutually independent Tobit model respectively.

In general, the marginal effects in Table 5 are the changes in unit of a dependent variable of interest. For instance, the first number 217.8 can be interpreted as the work duration in weekday is averagely 217.8 min less than that in weekend in the nested multivariate Tobit model. The number 0.43 for the effects of "household size" in the first column means that the work duration would increase 0.43min if the household sizes for all observations are increased by 1. Similarly, the number 6.02 for the effect of "log(household income)" indicates that the work duration would increase by 6.02 min due to 10% increase of log of household income for all observations.

The marginal effects shown in Table 5 indicate several policy implications. Living in urban area would marginally increase individual's work activity duration by 52min compared to rural area, but the difference between living in urban and rural area in non-work activity duration is small, only 6min. Slow mode shares increase more in work trips, 9.5%, than in non-work trips, 3.7%, for travellers living in urban area compared to rural area. This indicates that urbanization plays more important roles in influencing the mode choice for work trips than that for non-work trips. Besides, work related travel time increases marginally by 3.92min in urban area, while non-work related travel time decreases marginally by 4.08min in urban area. This indicates that car/bus work trips suffer congestion in urban area, even with more commuters choosing to walk and cycle to work due to closer commute distance in urban area. However, this seems to be not the case for car/bus non-work trips. Travellers having young children in the family would marginally have 32min less work activity duration but 25min more non-work activity duration than those who do not have young children in the family. This is presumably due to more children related activities such as leaving work early for picking up children at kindergarten. Moreover, the slow mode share for travellers who have young children is marginally 6.6% less in their work trips but 3.3% more in their non-work trips, compared to that for travellers those who do not have young children. This indicates the potential of a high usage of motorised modes for picking-up/dropping-off children while a high walk and cycling share for children-related leisure activities. Among the effects of weather characteristics, the effect of "ground with snow cover" plays the most important roles. Slow mode shares in work and non-work trips decrease 5.0% and 3.4% respectively in a situation with snowy ground, implying more traffic in this situation. Longer non-work activity duration and non-work related travel time also indicate more snow related activity participations which may require long trips. On the contrary, precipitation and wind speed play the least important roles among all weather variables.

Besides, there are differences in the marginal effects between the nested multivariate Tobit model and mutually independent Tobit models. Combined with better data fit of nested multivariate Tobit model, such differences point to the inconsistent marginal effects from mutually independent Tobit models. In general, mutually independent Tobit models tend to underestimate the variable effects on work activity duration but overestimate the variable effects on non-work activity duration. Since work and non-work activity durations are first level dependent variables, these over/underestimations are mainly due to the ignorance of covariance structure. For the second level dependent variables, slow mode share and travel time, the differences in the corresponding marginal effects are both due to the ignorance of covariance structure and the sample selection. Mutually independent Tobit models also tend to underestimate the variable effects on slow mode share in work trips but overestimate the variable effects on slow mode share in non-work trips. Sometimes, corresponding marginal effects of the two models may not necessarily have the same sign, pointing to the misleading interpretations from the results of mutually independent Tobit models. For instance, the marginal effect of "age>65" on slow mode share in non-work trips is 10.1 in the mutually independent Tobit model while that in the nested multivariate Tobit model is -7.1. Overall, ignoring the covariance structure and sample selection can lead to inconsistent variable effects that lead to misleading policy implications.

Table 5. The marginal effects of nested multivariate Tobit model and mutually independent Tobit model by variable category

Dependent variable / Exogenous variable	Work activity duration		Non-work activity duration		Slow-mode share in work trips(%)	
	NMT[a]	MIT	NMT	MIT	NMT	MIT
Time-location characteristics						
Weekday	217.8	161.0	-35.19	-43.86	27.39	16.45
Sub-urban area	25.78	19.61	-6.46	-7.91	3.84	1.71
Urban area	51.69	39.62	-6.11	-10.38	9.54	4.41
Individual social-demographics						
Male	-4.64	0.97	3.89	3.64	-1.73	-1.88
Age 19-30	-36.40	-27.21	-5.51	8.32	-14.11	-10.77
Age 31-50	-40.00	-32.91	5.77	22,43	-16.33	-16.46
Age 51-65	-80.85	-67.38	17.08	33,06	-21.63	-16.28
Age >65	-228.2	-162.2	31.60	50,39	-33.44	-22.34
Household social-demographics						
Youngest child 0-6 years old	-32.00	-32.31	24.96	23.65	-6.61	-5.98
Youngest child 7-18 years old	-13.81	-0.43	8.89	4.00	-6.19	-1.72
Household size	0.43	1.69	-0.38	-2.13	0.41	0.77
Number of cars in household	8.66	6.72	2.70	4.48	-6.54	-5.44
Log(household income)	6.02	0.33	0.89	1.71	0.79	-0.04
Weather characteristics						
Monthly temperature variation	-0.53	-0.73	0.30	0.38	-0.09	-0.09
Temperature Z score increase in <0 interval	0.38	0.35	0.02	-0.01	0.06	0.05
Temperature Z score increase in >0 interval	-0.56	-0.57	0.23	0.11	0.06	0.003
Relative humidity	12.33	7.81	0.16	0.92	1.82	0.74
Wind speed square	-0.30	-1.17	0.02	0.05	0.05	-0.02
Precipitation amount	-0.30	-0.05	0.05	0.04	-0.04	0.004
Visibility <1km	-27.58	-21.34	-11.86	-11.60	-3.58	0.11
Ground with snow cover	-46.43	-36.56	15.15	20.84	-4.98	-4.23

Dependent variable / Exogenous variable	Slow-mode share in non-work trips(%)		Work related travel time		Non-work related travel time	
	NMT[a]	MIT	NMT	MIT	NMT	MIT
Time-location characteristics						
Weekday	0.70	-9.47	22.37	20.18	-16.69	-23.63
Sub-urban area	0.76	1.74	2.63	2.25	-3.79	-4.29
Urban area	3.68	5.22	3.92	4.69	-4.08	-5.02
Individual social-demographics						
Male	-2.90	-5.64	-0.002	0.41	3.02	0.63
Age 19-30	-8.10	-4.58	5.44	0.46	-3.05	16.61
Age 31-50	-10.18	-4.18	3.63	-1.67	5.80	24.21
Age 51-65	-9.75	-2.27	1.01	-5.71	10.73	33.56
Age >65	-7.08	10.14	-22.45	-19.67	14.57	47.23
Household social-demographics						
Youngest child 0-6 years old	3.25	6.36	-4.75	-5.36	-2.76	6.92
Youngest child 7-18 years old	0.33	-2.99	-1.90	-0.31	-1.09	2.02
Household size	-0.30	-0.81	0.85	0.63	-0.48	-1.30
Number of cars in household	-6.56	-11.00	-1.41	0.59	1.70	2.59
Log(household income)	0.18	0.35	-0.14	-0.21	0.70	0.70
Weather characteristics						
Monthly temperature variation	-0.10	-0.11	0.02	-0.08	0.37	0.36
Temperature Z score increase in <0 interval	-0.01	-0.09	0.02	0.02	0.05	-0.04
Temperature Z score increase in >0 interval	0.002	0.07	-0.07	-0.08	0.14	0.03
Relative humidity	0.02	-0.64	0.48	0.94	-0.20	-1.08
Wind speed square	0.04	0.03	-0.01	-0.12	-0.12	0.01
Precipitation amount	-0.03	-0.02	-0.04	-0.02	0.02	0.03
Visibility <1km	-2.81	-5.08	1.20	-2.52	-5.56	-1.94
Ground with snow cover	-3.37	1.08	-1.77	-3.53	19.01	18.97

[a] NMT denotes the nested multivariate Tobit model while MIT denotes mutually independent Tobit model

5. Model prediction

The final object is to test whether the nested multivariate Tobit model is superior to mutually independent Tobit models in terms of prediction. The prediction procedure of nested multivariate Tobit model is very straightforward. Since the vector of error terms [ε_{wd} ε_{nd} ε_{wt} ε_{nt} ε_{wm} ε_{nm}] have zero means, the expected values of the latent variables are just variable effects, as: $\widehat{Y^*}=X\hat{\beta}$. And the predicted vector $\hat{Y}$ can be calculated through Eq.(4) from $\widehat{Y^*}$. Two measures are used to evaluate the performance of the model. The first measure is the "hit rate", which calculates the percentage of correct predictions regarding censored and non-censored. The "hit rate" shows the ability of model to correctly predict whether an individual participates in certain type of activity and takes non-motorised modes. The second measure is the symmetric mean absolute percentage error (SMAPE) (Flores, 1986), which measures the deviation of prediction from the actual value, conditional on the correct prediction regarding censored and non-censored. Thus SMAPE only measures those cases which are correctly predicted as censored or non-censored. The SMAPE is calculated as:

$$\text{SMAPE} = \frac{\sum_{n=1}^{N} \frac{|P_n - A_n|}{P_n + A_n}}{N} \times 100\% \tag{15}$$

Where P_n is the predicted value of observation n and A_n is the actual value of observation n. SAMPE ranges from 0 to 100, and a small SMAPE indicates good fit. "Hit rate" and SMAPE of both nested multivariate Tobit model and single Tobit models are presented in Table 5.

Table 6. Prediction performance

Model		Work activity duration	Non-work activity duration	Slow-mode share in work trips(%)	Slow-mode share in non-work trips(%)	Work related travel time	Non-work related travel time	Total
Nested multivariate Tobit model	Hit rate	76.4%	80.2%	85.5%	57.3%	76.4%	76.9%	75.5%
	SMAPE	29.4%	40.1%	45.2%	60.1%	30.6%	39.9%	35.9%
Single Tobit models	Hit rate	77.2%	80.3%	88.1%	68.1%	75.3%	81.7%	78.5%
	SMAPE	49.8%	41.4%	52.6%	57.0%	47.4%	36.6%	41.6%

Seen from Table 5, both nested multivariate Tobit model and mutually independent Tobit models show reasonable fit in terms of hit rate. Separated univariate Tobit model even shows slightly better prediction, around 3 percentage points. Moreover, close to 80% of the observations can be correctly predicted as censored or non-censored, except slow mode share in non-work trips, for which the hit rate of nested multivariate Tobit model is around 60%. In terms of SMAPE, nested multivariate Tobit model provides much better prediction than mutually independent Tobit models, lowering SMAPE by 6 percentage points, showing the necessity of considering the covariance structure and sample selection. The nested multivariate Tobit model shows better predictions of work related activity-travel variables, but not necessarily better for non-work related activity-travel variables. This echoes the previous studies (e.g. Kang and Scott, 2010; Susilo and Axhausen, 2014) which shows that the time allocation for work activities are more predictable and less dependent on an individual's daily space-time constraints than that for non-work activities.

6. Summary

This paper formulates a sample selection version of multivariate Tobit model, namely nested multivariate Tobit model. The model was proposed and constructed since it is considered to be able to offer advantages to deal with three issues: 1 substantial amount of censoring observations, 2 correlations among activity duration, travel time and mode share due to common unobserved factors, 3 the sequential nature of activity participation and travel. Estimating nested multivariate Tobit model is considered as fairly straightforward and can be achieved by splitting censored dimension from uncensored components in the multivariate normal distribution. For a two-level nested model proposed in this paper, the number of censored dimensions is no more than two, thus no simulation machinery is required. The proposed model is then applied on a combined dataset of Swedish national travel survey (NTS) dataset and SMHI (Swedish Meteorological and Hydrological Institute) weather record. The influences of time-location characteristics, individual and household socio demographics and weather characteristics on individuals' work and non-work activity-travel engagements were examined.

The estimation results demonstrate that the nested multivariate Tobit model can be used to predict and capture the detailed interaction between individuals' activity-travel engagements on the given day. The estimates of unrestricted covariance matrix show clear trade-offs due to the remaining unobserved attributes that reflects individual's time-space constraints. It also implies the further an individual travels to a non-work location, the more the individual would like to get utility (longer visit duration) at the destination location. After all, ignoring the covariance structure and sample selection can lead to inconsistent estimation of marginal effects, thus lead to misleading policy implications. The prediction performance of nested multivariate Tobit model is superior to that of mutually independent single Tobit models in terms of "SMAPE" measure. Meanwhile mutually independent single Tobit models perform slightly better in terms of "Hit rate" but both hold a reasonable fit, over 75%. The nested multivariate Tobit model does not necessarily perform better for model components regarding non-work related activities. This may be because either there were some important attributes missing for predicting non-work activities or more detailed classification in "non-work" is needed, which means more equations are estimated.

However, one should also be aware of the limitation of this study. First, as this study uses the data derived from the national travel survey, the detailed origin-destination locations are quite rough, only at municipality level. Thus, more detailed geo-coded data is required for exploring built environment effects on the activity time allocation problem. Besides, the number of trips for work/non-work activities is currently not included in the analysis which means the inference on trip demand is not available. However, one can extend the current model structure by adding number of trips as an ordered Probit model component in the first level of the current model structure. Correspondingly, if equations of activity types and equations of number of trips are added in the model system, simulation techniques such as GHK simulator (Hajivassiliou et al. 1996) can be applied in order to approximate the multiple integral in multivariate normal distribution. Alternatively, composite marginal likelihood approach (e.g. Bhat et al, 2010) is also an appealing approach that can avoid the use of simulation, and can be a plausible direction for future research. It needs to be noted here that this model is only applied for trips with purposes, not for trip for the sake of the trip itself.

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