Transition delay in boundary-layer flows via reactive control

by

Nicolò Fabbiane

May 2016
Technical Reports
Royal Institute of Technology
Department of Mechanics
SE-100 44 Stockholm, Sweden
Akademisk avhandling som med tillstånd av Kungliga Tekniska Högskolan i Stockholm framlägges till offentlig granskning för avläggande av teknologi doktorsexamen måndagen den 13 juni 2016 kl 10:30 i sal Kollegiesalen, Kungliga Tekniska Högskolan, Brinellvägen 8, Stockholm.

TRITA-MEK Technical report 2016:10
ISSN 0348-467X
ISRN KTH/MEK/TR-16/10-SE

Cover: uncontrolled (top) and controlled (bottom) transition to turbulence. The color-scale reports the fluctuation of streamwise wall-shear stress with respect to the laminar solution; the saturated white areas indicate the turbulent regime. Red circles and blue squares show the position of sensors and actuators, respectively. The flow is directed from right to left.

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“It. Could. Work!”

Dr. Frederick Frankenstein, Young Frankenstein (1974)
Transition delay in boundary-layer flows via reactive control

Nicolò Fabbiane
Linné FLOW Centre, KTH Mechanics, Royal Institute of Technology
SE-100 44 Stockholm, Sweden

Abstract
Transition delay in boundary-layer flows is achieved via reactive control of flow instabilities, i.e. Tollmien-Schlichting (TS) waves. Adaptive and model-based control techniques are investigated by means of direct numerical simulations (DNS) and experiments. The action of actuators localised in the wall region is prescribed based on localised measurement of the disturbance field; in particular, plasma actuators and surface hot-wire sensors are considered.

Performances and limitations of this control approach are evaluated both for two-dimensional (2D) and three-dimensional (3D) disturbance scenarios. The focus is on the robustness properties of the investigated control techniques; it is highlighted that static model-based control, such as the linear-quadratic-Gaussian (LQG) regulator, is very sensitive to model-inaccuracies. The reason for this behaviour is found in the feed-forward nature of the adopted sensor/actuator scheme; hence, a second, downstream sensor is introduced and actively used to recover robustness via an adaptive filtered-x least-mean-squares (fxLMS) algorithm.

Furthermore, the model of the flow required by the control algorithm is reduced to a time delay. This technique, called delayed-x least-mean-squares (dxLMS) algorithm, allows taking a step towards a self-tuning controller; by introducing a third sensor it is possible to compute on-line the suitable time-delay model with no previous knowledge of the controlled system. This self-tuning approach is successfully tested by in-flight experiments on a motor-glider.

Lastly, the transition delay capabilities of the investigated control configuration are confirmed in a complex disturbance environment. The flow is perturbed with random localised disturbances inside the boundary layer and the laminar-to-turbulence transition is delayed via a multi-input-multi-output (MIMO) version of the fxLMS algorithm. A positive theoretical net-energy-saving is observed for disturbance amplitudes up to 2% of the free-stream velocity at the actuation location, reaching values around 1000 times the input power for the lower disturbance amplitudes that have been investigated.

Key words: flow control, drag reduction, net energy saving, adaptive control, model-based control, optimal control, flat-plate boundary layer, laminar-to-turbulent transition, plasma actuator, direct numerical simulation, in-flight experiments.
Fördröjning av laminärt-turbulent omslag i gränsskiktströmning genom reaktiv kontroll

Nicolò Fabbiane
Linné FLOW Centre, KTH Mekanik, Kungliga Tekniska Högskolan
SE-100 44 Stockholm, Sverige

Sammanfattning

I den här avhandlingen har reglertekniska metoder tillämpats för att försenar omslaget från ett laminärt till ett turbulent gränsskikt genom att dämpa tillväxten av små instabiliteter, så kallade Tollmien-Schlichting vågor. Adaptiva och modellbaserade metoder för reglering av strömning har undersökts med hjälp av numeriska beräkningar av Navier-Stokes ekvationer, vindtunnel-experiment och även genom direkt tillämpning på flygplan. Plasmaaktuatorer och varmträdsgivare vidhäftade på ytan av plattan eller vingen har använts i experimenten och modellerats i beräkningarna.

Prestanda och begränsningar av den valda kontrollstrategin har utvärderats för både tvådimensionella och tredimensionella gränsskiktinstabiliteter. Fokus har varit på metodernas robusthet, där vi visar att statiska metoder som linjär-kvadratiska regulatorer (LQG) är mycket känsliga för avvikelser från den nominella modellen. Detta beror främst på att regulatorer agerar i förkompenseringsläge ("feed-forward") på grund av strömningens karaktär och placeringen av givare och aktuatorer. För att minska känsligheten mot avvikelser och därmed öka robustheten har en givare införts nedströms och en adaptiv fxLMS algoritm (filtered-x least-mean-squares) har tillämpats. Vidare har modelleringen av fxLMS-algoritmen förenklats genom att ersätta överföringsfunktionen mellan aktuatorer och givare med en lämplig tidsfördröjning. Denna metod som kallas för dxLMS (delayed-x least-mean-squares) kräver att ytterligare en givare införs långt uppströms för att kunna uppskatta hastigheten på de propagerande instabilitetsvägorna. Denna teknik har tillämpats framgångsrikt för reglering av gränsskiktet på vingen av ett segelflygplan.

Slutligen har de reglertekniska metoderna testas för komplexa slumpmässiga tredimensionella störningar som genererats uppströms lokalt i gränsskiktet. Vi visar att en signifikant försening av laminärt-turbulentomslag äger rum med hjälp av en fxLMS algoritm. En analys av energibudgeten visar att för ideala aktuatorer och givare kan den sparade energiåtgången på grund av minskad väggfriktion vara upp till 1000 gånger större än den energi som används för reglering.

Nyckelord: strömningsstyrning, friktionsreduktion, netto energibesparing, adaptiv styrning, modellbaserad styrning, optimal kontroll, gränsskikt över en plan platta, laminärt till turbulent omslag, plasma aktuator, DNS, flyg prov.
Preface

This thesis deals with transition delay in boundary-layer flows by reactive-control techniques. A brief introduction on the basic concepts and methods is presented in the first part. The second part contains five articles. The papers are adjusted to comply with the present thesis format for consistency, but their contents have not been altered as compared with their original counterparts.


**Paper 3.** B. Simon, N. Fabbiane, T. Nemitz, S. Bagheri, D.S. Henningson & S. Grundmann. *In-flight active-wave-cancelation via delayed-x-LMS control algorithm in a laminar boundary layer.* Under review for publication in Exp. Fluids.


May 2016, Stockholm

Nicolò Fabbiane
Division of work between authors
The main advisor for the project is Prof. Dan S. Henningson (DH). Dr. Shervin Bagheri (SB) acts as co-advisor.

**Paper 1.** The code has been developed by Nicolò Fabbiane (NF). The paper has been written by NF and Onofrio Semeraro with feedback from SB and DH.

**Paper 2.** The experimental set-up has been designed by Bernhard Simon (BS). The model-based control has been implemented by NF, while the adaptive control by Felix Fischer. The simulations have been performed by NF using the control-code developed by NF. The paper has been written by NF and BS with feedback from Sven Grundmann (SG), SB and DH.

**Paper 3.** The experimental set-up has been designed by BS. The experiment has been set-up by BS and Timotheus Nemitz (TN). In-flight experiments were performed by BS, TN and NF. The simulations have been performed by NF using the control-code developed by NF. The paper has been written by BS and NF with feedback from SG, SB and DH.

**Paper 4.** The simulations have been performed by Reza Dadfar (RD) using the control-code developed by NF. The paper has been written by RD with feedback from NF, SB and DH.

**Paper 5.** Simulations and post-processing have been conducted by NF using the control-code developed by NF. The paper has been written by NF with feedback from SB and DH.

**Other publications**
The following paper, although related, is not included in this thesis.


**Conferences**
Part of the work in this thesis has been presented or accepted to be presented at the following international conferences. The presenting author is underlined.


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Part II - Papers

Summary of the papers

Paper 1. Adaptive and model-based control theory applied to convectively-unstable flows

Paper 2. On the role of adaptivity for robust laminar-flow control

Paper 3. In-flight active-wave-cancelation via delayed-x-LMS control algorithm in a laminar boundary layer

Paper 4. Centralised versus decentralised active control of boundary layer instabilities

Paper 5. Energy efficiency and performance limitations of linear adaptive control for transition delay
Part I

Overview and summary
Chapter 1

Introduction

One of the main goals pursued by fluid dynamics in the last century is the reduction of aerodynamic drag. This became a pressing objective with the advent of the aeroplane and air transportation: the need to fly faster, cheaper and, in recent times, greener motivated – and still motivates – the research on this topic for many years. The problem has been approached from very different directions: from a better wing design to reduce the drag induced by the lift\(^1\), to a better airfoil design to keep the boundary-layer laminar as long as possible.

When a body moves through a fluid, the relative velocity between body and fluid is zero at the body surface, while far from the surface – in the free-stream – the velocity is dictated by the body geometry and motion. The link between these two regions is called *boundary-layer*. If the body moves with low speed in still air, and its motion is regular, the boundary layer that is generated is regular as well; this condition is called *laminar* flow. Since the equations that govern the flow are non-linear and very sensitive to perturbations, the flow may abruptly switch to a chaotic behaviour – the *turbulent* regime – even when small disturbances occur. The turbulent regime of the boundary layer shows a higher friction drag with respect to the laminar state, hence the effort to keep the boundary-layer laminar as long as possible.

The present work moves in the reactive control framework: the laminar-to-turbulence transition is prevented by removing those disturbances from the flow that would eventually lead to transition. The disturbances are cancelled by actuators that base their action on measurements of the disturbance in the flow.

1.1. A route to turbulence

There are several paths that lead to the turbulent regime in boundary layer flows, depending on the disturbance level in the free-stream (Saric *et al.* 2002). Weak disturbances follow a path that can be initially described by linearised equation of the flow, the Orr-Sommerfeld-Squire (OSS) equations. Linear flow instabilities – Tollmien-Schlichting (TS) waves – are triggered by free-stream disturbances; their initial growth, even if exponential, is weak. However, as the perturbation

\(^1\)The lift is the component of the aerodynamic force perpendicular to the aeroplane velocity that usually balances the weight of the aircraft.
Figure 1.1: TS-wave driven transition. The skin-friction spectra are reported for different streamwise locations; $\beta$ and $\omega$ are the spanwise wave-number and the temporal angular frequency, respectively. The flow is seeded close to the wall at $Re_X \approx 0.4 \times 10^6$ with uniform random noise. Red and blue isosurfaces indicate positive and negative values of $u' = \pm 4 \times 10^{-3} U$, i.e. the streamwise perturbation velocity with respect to the laminar solution.
1.1. A route to turbulence

When the disturbance amplitude crosses a critical level, non-linear interactions arise. At this point, the disturbances rapidly grow and transition to turbulence occurs (Kachanov 1994). By increasing the disturbance amplitude, the linear asymptotic behaviour of the perturbation is not able to describe the process any more and the short-time behaviour becomes more and more important. In fact, when a perturbation is introduced in the flow, it can experience a large transient growth, even if the flow is stable, i.e. no linear instability occurs. This is possible because OSS-modes are not mutually orthogonal and their combination may result in an amplification of the disturbance (Henningson et al. 1993). For strong free-stream disturbances the linear mechanism may be by-passed. In this case, the transition to turbulence can not be predicted via the linear theory. Non-linear dynamics theory has recently helped to have a better insight into the transition process in the presence of large amplitude disturbances (Duguet et al. 2012); the transition to turbulence appears to occur via specific structures that cluster in limited regions of the phase space, called edge states. The recent work by Kreilos et al. (2015) is an example of how transition can be predicted by dynamical-system analysis applied to boundary layers.

This work focuses on the weak disturbance scenario, where the perturbation can be initially described by the linearised Navier-Stokes (LNS) equations. A canonical boundary layer case is investigated (Blasius 1908). Consider a uniform and constant flow with velocity $U$ that encounters a semi-infinite flat plate aligned with the flow: since the free-stream flow is constant in time, a two-dimensional (2D), steady, laminar boundary-layer develops starting from the tip of the plate – the leading-edge – and extends all along the plate if the flow is not perturbed. The flow is linearly stable up to $Re_{X,c} = UX/\nu \approx 91190$, where $X$ is the streamwise distance from the leading-edge and $\nu$ is the kinematic viscosity (Schmid & Henningson 2001); downstream of this position, unstable solutions to the OS-equations always occur. This means that if the flow is perturbed at $Re_X < Re_{X,c}$ a small perturbation will decay while being convected and it will be amplified by the flow only when it reaches the critical position $Re_{X,c}$; only then, the perturbation will start to grow exponentially and eventually evolve into turbulence.

Figure 1.1 illustrates the transition scenario described in the previous paragraph. The receptivity process of the free-stream disturbances is by-passed and the flow is directly perturbed inside the boundary layer region. The flow is seeded with uniform white-noise at $Re_X \approx 400000$, see Figure 1.1a. Immediately downstream this point (Figure 1.1b), most of the seeded spatial and temporal frequencies decay and only few of them are amplified by the flow. The latter are organised in a very specific part of the spectrum: close to the zero spanwise wave-number and frequency $f = \omega/2\pi = 0.015 U/\delta_*^s$, where $\delta_*^s$ is the displacement thickness at the seeding point. The non-linear interaction between wave-packets generates the the second peak close to the zero-frequency axis in Figure 1.1c. Farther downstream these structures reach the same magnitude as the TS-wave that generated them (Figure 1.1d). They are responsible for
the streaky structures visible in the physical flow. These structures interact with the TS-waves (Figure 1.1e), break down, and eventually lead to a fully turbulent flow (Figure 1.1f).

1.2. The control problem

By preventing or obstructing the transition process described in the previous section, transition-delay and, thus, drag-reduction can be achieved. TS-waves are present in a limited frequency band in space and time, unlike the structures that define the turbulent boundary-layer. It is hence more convenient to act on TS-waves. This approach has two main advantages: (i) the temporal frequencies that describe the perturbation behaviour are low and bounded and (ii) the energy requirement to cancel the disturbance is low because of the small amplitude of the perturbation. In view of a small power consumption to perform the control action, a large benefit is achieved in terms of transition delay and consequent drag reduction; in some sense, the non-linear breakdown to turbulence is used to design a highly energy-efficient control scheme.

Wall-mounted sensors are used to detect the upcoming disturbances. Once detected, the perturbation is cancelled via a localised forcing provided by an actuator. The choice and position of these devices is the zeroth-step in the control design process, as it determines how the control algorithm will interact with the system and deeply influences the design of the control itself (Belson et al. 2013). In this work, the reference sensor $y$ – i.e. the sensor responsible for detecting the disturbance – is positioned upstream of the actuator $u$ that performs the control action, Figure 1.2. The perturbation is introduced in the flow by the disturbance source $d$. The interaction between the perturbation and the wave generated by the actuator leads to an attenuation of the disturbance amplitude, which is detected by the error sensor $z$.

The compensator is the core of the control action, as it is the system responsible for computing the actuator forcing, based on the measurement signals. It can be designed via very different strategies; a unique classification is always limiting and for sure challenging. In this work, the classification is approached according to Wiener (1961), recently reprised and extended by the insightful review by Brunton & Noack (2015). Every control strategy is based on a model of the system that it aims to control. The model can be given a-priori based on the Navier-Stokes (NS) equations; this approach is classified as white-box. In fluid-dynamics, this is usually coupled with optimal control theory (Bewley & Liu 1998). The similarities with the canonical stability theory enabled these techniques to rapidly spread in the numerical community (e.g. Barbagallo et al. 2009; Bagheri & Henningson 2011; Sharma et al. 2011; Semeraro et al. 2013b). A different example of white-box control can be found in the numerical and experimental work by Li & Gaster (2006), where opposition control is performed based on LNS equations. White-box modelling compares with the black-box approach, where the model is identified starting from experimental
data. An example can be found in the work by Juillet et al. (2014), where model-identification and optimal control are used to control natural perturbations in a channel flow.

This classification holds also for the control technique. Most of the investigations along the black-box control approach are led by the experimental community; adaptive control has been largely investigated via experiments starting from Sturzebecher & Nitsche (2003) and later by many others (e.g. Kurz et al. 2013; Kotsonis et al. 2015). Between the two extremes, several intermediate grey-box approaches are possible, as discussed later in the outlook section of the thesis (§4).

Thesis structure. This work focuses on the comparison between the white-box and the black-box approaches. The two design philosophies are analysed and compared; in particular, the model-based linear-quadratic-Gaussian (LQG) regulator (§2.2) and the adaptive filtered-x least-mean-squares (fxLMS) algorithm (§2.3) are investigated. Their performance and limitations in attenuating 2D TS wave-packets are investigated in §2.4; the robustness properties of the adaptive approach are used to design a self-tuning compensator (§2.5), where no previous knowledge of the flow is needed to perform the control action. Lastly, a 3D disturbance and control scenario is introduced (§3.1) and the transition-delay performance is investigated. An energy budget is performed: the energy saved by the drag-reduction due to transition-delay is compared to the energy required to perform the control action (§3.3).
Chapter 2

Control of boundary-layer instabilities

In this chapter, model-based and adaptive control are presented and tested for two-dimensional (2D) disturbances in a laminar zero-pressure-gradient boundary layer flow. Performance and robustness of the two design approaches are investigated. Moreover, a self-tuning compensator is introduced in the last section of the current chapter.

The plant is the system that we aim to control. In this chapter, we focus on a 2D zero-pressure-gradient boundary layer flow. In the first instance, 2D disturbances are considered. This permits us to reduce the number of sensors in the flow and to introduce the control techniques that are discussed in this work in a simpler way. A three-dimensional (3D) disturbance scenario will be discussed later in \S 3, where the transition-delay capabilities of the proposed control set-up are discussed.

A model that describes the plant is needed. The incompressible Navier-Stokes equations govern this type of flow:

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + \lambda \mathbf{u},
\]

\[
\nabla \cdot \mathbf{u} = 0,
\]

\[
\mathbf{u}(\mathbf{x}, t)\big|_{\partial \Omega} = \mathbf{u}_0(\mathbf{x}),
\]

\[
\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}).
\]

Velocity and pressure at position \(\mathbf{x} = (X, Y)\) and time \(t\) are represented by \(\mathbf{u}(\mathbf{x}, t)\) and \(p(\mathbf{x}, t)\) respectively. The Reynolds number is defined as \(Re = U_\infty \delta_0^* / \nu\), where \(U_\infty\) is the free-stream velocity, \(\nu\) the viscosity and \(\delta_0^*\) the displacement thickness in the beginning of the domain. On the boundaries \(\partial \Omega\) of the computational domain \(\Omega\) (see Figure 2.1), the following conditions are imposed (2.3): no-slip condition at the wall and asymptotic velocity in the upper boundary. A fringe technique is used to simulate inflow and outflow condition in the beginning and at the end of the domain (Nordström et al. 1999). The flow is considered periodic along the streamwise direction and a volume forcing \(\lambda(\mathbf{x})\mathbf{u}(\mathbf{x}, t)\) in the last part of the domain enforces periodicity (grey region in Figure 2.1). More details can be found in Chevalier et al. (2007), where the pseudo-spectral DNS code used in this work is described.
2.1. A linear model of the flow

As we are interested in the dynamics of small disturbances, the following decomposition is introduced:

\[ u(x, t) = U(x) + \epsilon u'(x, t), \quad (2.5) \]
\[ p(x, t) = P(x) + \epsilon p'(x, t). \quad (2.6) \]

where \( \epsilon \ll 1 \), \( \{ U(x), P(x) \} \) is the steady solution of the Navier-Stokes equations and \( \{ u'(x), p'(x) \} \) the perturbation. Applying this decomposition into (2.1–2.4) and neglecting the terms of order \( \epsilon^2 \) and higher, the following set of linear equation is obtained:

\[ \frac{\partial u'}{\partial t} = -(U \cdot \nabla) u' - (u' \cdot \nabla) U - \nabla p' + \frac{1}{Re} \nabla^2 u' + \lambda u' + f, \quad (2.7) \]
\[ 0 = \nabla \cdot u', \quad (2.8) \]
\[ u'|_{\partial \Omega} = 0, \quad (2.9) \]
\[ u'(0) = 0. \quad (2.10) \]

The term \( f(x, t) \) is used to model the forcing on the flow; spatial and time dependencies are decoupled as follows:

\[ f(x, t) = b_d(x) d(t) + b_u(x) u(t). \quad (2.11) \]

Sensors are placed in the flow in order to measure the perturbation field. The measures \( y(t) \) and \( z(t) \) are defined by:

\[ y(t) = \int_{\Omega} \mathbf{c}_y(x) \cdot u'(x, t) \, d\Omega + n(t), \quad (2.12) \]
\[ z(t) = \int_{\Omega} \mathbf{c}_z(x) \cdot u'(x, t) \, d\Omega, \quad (2.13) \]

where the kernels \( \mathbf{c}_y(x) \) and \( \mathbf{c}_z(x) \) define the sensors.

In this study, a Fourier-Chebishev expansion over \( N_X \times N_Y = 768 \times 101 \) terms is considered. The computational domain extends over \( 30\delta_0^* \) in the wall normal direction and \( 10000\delta_0^* \) in the streamwise direction. The fringe region occupies the last \( 200\delta_0^* \) of the domain along the streamwise direction. The displacement thickness based Reynolds number is set to \( Re = 1000 \) at
the beginning of the domain and a time step $\Delta t = 0.4$ is used for the time integration. The disturbance sources are modelled as synthetic vortices (Bagheri et al. 2009), while the actuators are modelled as plasma actuators according to the experimental results by Kriegseis et al. (2013). The actuator is positioned at $X_u = 400$. The sensors mimics surface hot-wires by measuring the fluctuation in the shear stress given by the disturbance; the reference sensor $y$ is placed at $X_y = 300$ and the error sensor $z$ at $X_z = 500$.

Via a Galerkin projection, it is possible to transform the partial differential equation (PDE) (2.7) into an ordinary differential equation (ODE) in time (Quarteroni 2009). The Linear Time-Invariant (LTI) system that results reads:

\begin{align*}
\dot{q}(t) &= A \, q(t) + B_d \, d(t) + B_u \, u(t), \\
y(t) &= C_y \, q(t) + n(t), \\
z(t) &= C_z \, q(t),
\end{align*}

where $q \in \mathbb{C}^{N \times 1}$ is the state vector, $A \in \mathbb{C}^{N \times N}$ is the linearised Navier-Stokes operator and $N = N_X N_Y$ is the number of degrees of freedom. The matrices $B_d, B_u \in \mathbb{C}^{N \times 1}$ allow the two inputs $d(t)$ and $u(t)$ to force the system. The output matrices $C_y, C_z \in \mathbb{C}^{1 \times N}$ filter the state $q(t)$ in order to provide the output signals $y(t)$ and $z(t)$. The stochastic signal $n(t)$ represents the measurement noise that affects the output and it is usually modelled by white-noise.

Figure 2.2 reports the impulse responses for the flow case being described. The left column depicts the response of the flow to an impulse of the disturbance source: a wave packet is generated and travels downstream while growing (Figure 2.2a). The friction trace of the wave-packet is recorded by the reference sensor $y$ (Figure 2.2c) and later by the error sensor $z$ (Figure 2.2e). This behaviour is typical of convectively unstable flows, also known as noise amplifiers. This system is asymptotically stable from an input/output point of view but the perturbation grows exponentially while travelling downstream.

The convective nature of the system also leads to an important consideration about the control setup. Figure 2.2b reports the impulse response for the control input $u$: the wave-packet travels downstream of the actuator without being detected by the reference sensor (Figure 2.2d). Therefore, when the reference sensor $y$ is positioned upstream the actuator, the setup results in a feed-forward control scheme, as in the current setup. Different relative positions of the actuator have been investigated by Belson et al. (2013); the feedback configuration – i.e. reference sensor downstream of the actuator – shows better robustness but lower performance with respect to the current feed-forward configuration. However, the feedback configuration requires the reference sensor to be very close to the actuator. Since plasma actuators are considered, this configuration is unrealistic, because of the eventual electrical interference between the sensor and the actuator.
2.1. A linear model of the flow

![Diagram](image)

Figure 2.2: The colored areas report the friction footprint of the wavepacket generated by the disturbance source (a) and the plasma-actuator (b); red indicates positive fluctuations, while blue negative. The black lines in (c-f) report the impulse response from each input to each output for the considered flow-case. The green dashed lines report the impulse response of the reduced order model.

2.1.1. Finite Impulse Response (FIR) representation

Some control techniques relax the knowledge of the plant to its input/output (I/O) relations only. The forced response of an LTI system to a generic input signal $u(t)$ can be written as:

$$z(t) = C_z e^{A t} q_0 + \int_0^t C_z e^{A \tau} B_u u(t - \tau) \, d\tau.$$  \hspace{1cm} (2.17)

If the system is stable, for large enough $t$, the first term goes to zero and the system response depends only on the forcing $u(t)$:

$$z(t) = \int_0^t C_z e^{A \tau} B_u u(t - \tau) \, d\tau = \int_0^t P_{zu}(\tau) u(t - \tau) \, d\tau$$  \hspace{1cm} (2.18)
where $P_{zu}$ is the convolution kernel. The kernel is able to describe completely the I/O relation between the input $u(t)$ and the output $z(t)$.

The time-discrete counterpart of (2.18) is of particular interest when it comes to control techniques. The time-discrete output signal $z(n) = z(n\Delta t)$ is computed as a linear combination of the time-discrete history of the input signal $u(n) = u(n\Delta t)$:

$$z(n) = \sum_{j=0}^{n} P_{zu}(i) u(n-i). \quad (2.19)$$

Since the system is stable, the convolution kernel goes to zero as the shifting index $i$ grows: this allows us to truncate the sum at an appropriate time $N_{zu}\Delta t$. Hence, the signal $z(n)$ can be obtained by the finite sum:

$$z(n) \approx \sum_{j=0}^{N_{zu}} P_{zu}(i) u(n-i). \quad (2.20)$$

The expression (2.20) is called Finite Impulse Response (FIR) filter.

The I/O relation $u \rightarrow z$ can thus be described by a finite number of coefficients $P_{zu}(i)$. These coefficients can be computed starting from a linear model of the flow, as the one provided by (2.14–2.16), or identified based on experimental data by dedicated algorithms, e.g. least-mean-squares (LMS). For more information, we refer to Paper 1.

### 2.1.2. Reduced Order Model (ROM)

Other control techniques require the direct knowledge of the system matrices $A$, $B$ and $C$. An example is the linear quadratic Gaussian (LQG) regulator that will be introduced in §2.2.1: this control technique requires the solution of a Riccati equation, whose computational cost is proportional to $N^3$, where $N = N_XN_Y$ is the number of degrees of freedom of the system describing the plant (2.14). Due to the high computational cost, handling large systems may lead to a very expensive design process and, eventually, to the impossibility of computing the control gains. Hence, system-reduction techniques applied to the Navier-Stokes linear operator are widely used to obtain smaller – and more manageable – systems that can reproduce the I/O behaviour of the flow (Rowley 2005; Kim & Bewley 2007; Bagheri et al. 2009c; Ilak et al. 2010). The reduced order system reads:

$$\dot{q}_r(t) = A_r q_r(t) + B_{r,d} d(t) + B_{r,u} u(t), \quad (2.21)$$

$$y(t) = C_{r,y} q_r(t) + n(t), \quad (2.22)$$

$$z(t) = C_{r,z} q_r(t), \quad (2.23)$$

where $A_r \in \mathbb{R}^{N_r \times N_r}$ is the ROM state matrix, $q_r(t) \in \mathbb{R}^{N_r \times 1}$ is the state vector, $B_{r,d}, B_{r,u}, C_{r,y}, C_{r,z} \in \mathbb{R}^{N_r \times 1}$ are the I/O matrices and $N_r \ll N$.

In this study, the Eigensystem Realization Algorithm (ERA) is used to provide a reduced-order model (ROM) (Juang & Pappa 1985). This algorithm
builds a realisation of an LTI system that mimics the I/O behaviour of the original system starting from its impulse responses from each input to each output, see Figure 2.2. This method is equivalent to a projection of the full system \( \{A, B, C\} \) on the set of its \( N_r \) most energetic balanced proper-orthogonal-decomposition (BPOD) modes (Moore 1981; Bagheri et al. 2009). Note that also in this approach only the I/O behaviour of the system is known. Since the transformation between the original system and the reduced order one remains unknown, the original state \( q \) can not be reconstructed starting from the reduced state \( q_r \). From a control point of view this is not necessary: the control algorithm needs only a limited knowledge of the system and, in particular, only a reliable model of the transfer functions between its inputs and outputs.

The model-reduction procedure implies information loss, that eventually leads to an error: this error can be estimated *a priori* and related to the size of the ROM (Moore 1981). This estimation can be used to choose the ROM size in order to bound the error by a given tolerance: for the current problem, the ROM size \( N_r = 112 \ll N \) is chosen in order to limit the relative model-reduction error below \( 10^{-7} \).

### 2.2. Model based control

The compensator is the system that interacts with the plant via its control inputs and outputs in order to pilot it at the desired state. In this brief review, we will focus on a linear compensator, i.e. a compensator that can be represented by a linear dynamical system. If the system that describes the compensator is time-invariant, the compensator is called static: the compensator is designed *a-priori*, usually based on a model of the system, and then it is connected to the plant. If the response of the compensator, instead, can be modified on-line, the compensator is called adaptive.

This section investigates compensators that are based on a model of the plant; the model can be either numerical (Bewley & Liu 1998; Bagheri & Henningson 2011; Semeraro et al. 2013, e.g.) or experimentally identified (Juillet et al. 2014) and it is used to compute the response of the actuator. Typical examples are Model Predictive Control (MPC) and the linear Quadratic Gaussian (LQG) regulator, discussed hereafter.

#### 2.2.1. Linear Quadratic Gaussian (LQG) regulator

The LQG regulator design is based on a complete model of the plant, Figure 2.3. It results in an LTI system that mimics the plant in order to compute the control signal \( u(t) \), given the measurement signal \( y(t) \) as an input. The compensator reads:

\[
\dot{q}_r(t) = (A_r + LC_{r,y}) \dot{q}_r(t) + B_{r,u} u(t) - Ly(t),
\]

\[
u(t) = K \dot{q}_r(t),
\]
2. Control of boundary-layer instabilities

Figure 2.3: Compensator schemes for static (LQG) and adaptive (fxLMS) strategies. An adaptive scheme may also use the error signal $z(t)$ to adapt to the current flow conditions. The grey lines indicate the I/O relations required to be modelled by each strategy.

where $\hat{q}_r(t) \in \mathbb{R}^{N_r \times 1}$ is the compensator state vector. The subscript $r$ refers to the Reduced Order Model (ROM) of the flow discussed in §2.1.2. The compensator is composed of two parts: the observer (2.24) and the controller (2.25). The former filters the measurement signal $y(t)$ by the estimation gain matrix $L \in \mathbb{R}^{N_r \times 1}$ and reconstructs an estimation $\hat{q}_r(t)$ of the state of the controlled system $q_r(t)$. The latter computes the control signal filtering the estimated state $\hat{q}_r(t)$ and the control-gain matrix $K \in \mathbb{R}^{1 \times N_r}$.

2.2.1.1. Observer: Kalman filter

The observer is designed to minimise the covariance of the difference between the plant state $q_r$ and the estimated state $\hat{q}$ when the system is excited by an unknown white-noise signal $d(t)$. To do this, the observer uses the measurement $y(t)$ affected by an error $n(t)$, also modelled as white noise, and the control signal $u(t)$. The minimization procedure leads to:

$$
L = -Y C_{r,y}^H R_n^{-1},
$$

(2.26)

where $Y \in \mathbb{R}^{N_r \times N_r}$ is the solution of the following Riccati equation:

$$
A_r Y + Y A_r^H - Y C_{r,y}^H R_n^{-1} C_{r,y} Y + B_{r,d} R_d B_{r,d}^H = 0.
$$

(2.27)

The parameters $R_d$ and $R_n$ are the expected variances of the disturbance signal $d(t)$ and measurement noise signal $n(t)$.

2.2.1.2. Controller: Linear Quadratic Regulator (LQR)

LQR design relies on the knowledge of the state $q_r$, or its estimation $\hat{q}_r$. The procedure is based on the minimization of a quadratic cost-function $\mathcal{N}$ based on the error-sensor measurements $z(t)$ and on the control signal $u(t)$:

$$
\mathcal{N} = \int_0^\infty z(t) w_z z(t) + u(t) w_u u(t) \ dt.
$$

(2.28)
The ratio between the control-strength parameter \( w_u \) and the performance parameter \( w_z \) allows the design of a controller capable of attenuating the disturbances in the system, while limiting the control effort. The minimisation procedure leads to the control law in (2.25) where the control-gain matrix is defined as:

\[
K = -w_u^{-1} B_{r,u}^H X.
\]  

(2.29)

The matrix \( X \in \mathbb{R}^{N_r \times N_r} \) is the solution of the Riccati equation:

\[
A_r^H X + X A_r - X B_{r,u} w_u^{-1} B_{r,u}^H X + C_{r,z}^H w_z C_{r,z} = 0.
\]  

(2.30)

Note that the controller design is completely independent of the observer design and vice-versa. This is commonly known as the separation principle (Glad & Ljung 2000).

2.3. Adaptive control

In an adaptive control method the compensator adjusts on-line its response in order to optimise its performance: the compensator adjustment is achieved by monitoring its performance and, based on that, the correction is computed. A typical example of this kind of compensator is the filtered-x least-mean-squares (fxLMS) algorithm, investigated by Sturzebecher & Nitsche (2003) and Kurz et al. (2013) to attenuate 2D disturbances in a boundary-layer flow.

2.3.1. Filtered-x Least-Mean-Squares (fxLMS) algorithm

The fxLMS algorithm relies on a minimisation procedure that is performed on-line. This allows the algorithm to use the actual measurements from the flow, giving this method the adaptive features that characterise it.

The compensator is a linear system. As seen in §2.1.1 for the plant, a linear system can be represented both in state-space form (like the LQG regulator in the previous section) or by a Finite Impulse Response (FIR) filter. This control technique uses the latter representation and the control signal is given by:

\[
u(n) = \sum_{i=1}^{N_K} K(i) y(n - i)
\]  

(2.31)

where \( u(n) = u(n \Delta t) \) and \( y(n) = y(n \Delta t) \) are the time-discrete representations of the time-continuous signals \( u(t) \) and \( y(t) \) and \( \Delta t \) is the sampling time step. The \( N_K \) coefficients \( K(i) \) constitute the kernel of the filter and they are related to the impulse response of the compensator. These coefficients are updated at each time step in order to satisfy the minimisation problem,

\[
\min_{K(i)} z^2(n),
\]  

(2.32)

via a steepest-descend algorithm. The resulting updating law is:

\[
K(i|n + 1) = K(i|n) + \mu z(n) \sum_{j=1}^{N_{zu}} P_{zu}(j) y(n - i - j).
\]  

(2.33)
2. Control of boundary-layer instabilities

Figure 2.4: TS amplitude $A_e(X)$. The lines report the performances of the two compensators at design condition. The shaded regions indicate the performance variation when the asymptotic velocity is changed in a $\pm 5\%$ range with respect to the design condition.

Note that the knowledge of the plant is limited to the time-discrete kernel $P_{zu}(i)$ that describes the I/O relation $u \rightarrow z$. This transfer function is commonly called secondary path (Sturzebecher & Nitsche 2003).

2.4. Controlled system

When the disturbance source is fed with uniform white-noise, it creates a train of wave packets that travels downstream while growing in amplitude. A time-averaged measure of the disturbance amplitude $A_e$ is defined:

$$A_e^2(X) = \int_0^{L_Y} \left\langle \left( \frac{u'}{U} \right)^2 \right\rangle_t\, dY,$$

(2.34)

where $L_Y$ is the wall-normal size of the computational box. The solid line in Figure 2.4 reports this quantity for the uncontrolled case; the amplitude of the perturbation grows exponentially throughout the domain and is increased by a factor of 25 at the end of the domain.

An LQG regulator is designed; the relative weight $w_u/w_z$ for the LQR controller is set to 0.1, as the disturbance-noise ratio $R_n/R_d$ for the Kalman filter. The dashed line in Figure 2.4 shows the disturbance amplitude for the LQG controlled case: the perturbation is attenuated downstream of the actuator and continues to decay downstream of the error sensor. Hence, the disturbance is attenuated by a factor of 35 with respect to the uncontrolled case in the end of the domain.

The adaptive fXLMS algorithm is capable of comparable performances, as shown by the dot-dashed line in Figure 2.4. The disturbance amplitude is effectively reduced downstream of the actuator but not completely cancelled as
2.4. Controlled system

Figure 2.5: Control kernels $K(i)$. LQG and fxLMS solutions to the control problem are reported for different free-stream conditions.

in the LQG case. The wave-packets, in fact, start growing again downstream of the error sensor, resulting in a lower attenuation at the end of the domain.

The main strength of the adaptive algorithm is revealed when the external conditions are changed with respect to the design point. The coloured areas in Figure 2.4 indicate the performance variation if the free-stream velocity varies between $0.95U$ and $1.05U$, where $U$ refers to the design condition. The LQG regulator loses almost all its performance, while the fxLMS performance is only marginally effected by the change in the free-stream condition. The adaptive algorithm acts on the control law in order to adjust to the changed condition and to compensate for the modified phase-shift and amplification of the perturbation. This can also be seen in the control kernels reported in Figure 2.5: the peak position and magnitude is modified for different flow condition to maintain the compensator performance. Further details about LQG and fxLMS sensitivity to changes in the free-stream condition can be found in Paper 1 and Paper 2.

2.4.1. Wind-tunnel experiments

Wind-tunnel experiments are conducted to investigate the robustness of model-based and adaptive control (Paper 2). The control of a zero-pressure-gradient boundary-layer over a flat-plate is investigated in the open-loop wind tunnel at TU Darmstadt. In the current setup, two-dimensional TS wave-packets are generated by 16 wall-mounted loudspeakers and a DBD plasma actuator is used to perform the control action. The control-sensors are flush-mounted surface hot-wires. Additionally, a boundary-layer hot-wire probe is mounted on a traverse system to measure the perturbation amplitude at different streamwise locations.

An LQG compensator is designed based on DNS simulations matching the experimental set-up and the measured performance matches the simulated behaviour of the control. Figure 2.6 reports the control performance for a 200 Hz TS-wave, i.e. a non-dimensional frequency $F = 2\pi \nu/U^2 f \approx 100$ (Schmid & Henningson 2001): the amplitude is attenuated downstream of the control region as predicted by the DNS simulation; Figure 2.6b-c shows the streamwise...
perturbation velocity as a function of the wall-normal position. The reported values are normalised by the free-stream velocity $U_\infty = 12$ m/s. The TS-wave is attenuated along all the wall-normal direction, and not only in the wall-region. This confirms the choice of the surface hot-wires sensors for estimating the performance of the actuator; in fact, a reduction of the perturbation amplitude in the wall region corresponds to an attenuation of the perturbation also farther away from the wall.

When the free-stream condition is changed, the LQG compensator shows in the experiments the same robustness issues that are highlighted by the simulation: the control rapidly loses performance as the free-stream velocity differs from the design-condition. The adaptive fxLMS algorithm is implemented to improve the robustness of the control strategy, showing the same properties observed in simulations.

### 2.5. A self-tuning compensator

In the previous section, it was shown how the fxLMS algorithm is able to compensate model inaccuracies and provide an effective control action. This property can be used for two different purposes: (i) enhancing the robustness of the control strategy §2.4, (ii) simplifying the model of the flow. An example of the latter is the delayed-x least-mean-squares (dxLMS) algorithm, where the secondary path $P_{zu}$ is approximated by a time delay.

This is possible because of the convective nature of the TS-wave instability. The wave-packet, in fact, travels with a specific group speed $c_g$ (Schmid & Henningson 2001). Therefore, a third sensor $p$ is introduced upstream of the reference sensor $y$ at $X_p = 250$ (Figure 2.7a); this sensor is used to evaluate the group speed at the beginning of the control region via the correlation $R_{py}$.
2.5. A self-tuning compensator

Figure 2.7: dxLMS control algorithm. A third sensor \( p \) is introduced in order to estimate the wave-packet group speed via the correlation \( R_{py} \). The grey line indicates the I/O relation required to be modelled by the control strategy, i.e. the secondary path \( P_{zu} \).

between the two measurement signals:

\[
R_{py}(\tau) = \frac{1}{T} \int_{-T/2}^{+T/2} p(t - \tau) y(t) \, dt,
\]

where \( T \) is the signal length. The maximum of the envelope of the correlation identifies the time \( \bar{\tau}_{py} \) that it takes for the wave-packet to travel from the sensor \( p \) to the sensor \( y \) (Figure 2.7b). Hence, an approximation of the group-speed is computed as:

\[
\bar{c}_g = \frac{X_y - X_p}{\bar{\tau}_{py}}.
\]

where \( X_y \) and \( X_p \) are the streamwise position of phase and reference sensors.

The time delay to be used in the secondary-path modelling is given by:

\[
\bar{\tau}_{uz} = \frac{X_z - X_u}{\bar{c}_g} = \frac{X_z - X_u}{X_y - X_p} \bar{\tau}_{py}.
\]

This estimation of the time-delay is based on the reasonable assumption that the group speed is constant throughout the control region (Paper 3; Li & Gaster 2006). The resulting time-delay model of the secondary path is reported in Figure 2.8a; It can be seen that the identified time-delay predictably falls in the middle of the trace of the wave-packet generated by the actuator.

Better insight into this approximation is given by the Bode diagram of the real and modelled secondary path. The two secondary paths are very different in magnitude (Figure 2.8b): the time-delay has a flat gain – i.e. amplifies each frequency by the same factor – while the real secondary path amplifies only the frequencies associated to an unstable TS-wave. The LMS algorithm, however, is not sensitive to this type of inaccuracies.

The stability limitation of the LMS algorithm is given by phase-errors only; this algorithm is, in fact, able to compensate a phase error up to \( \pm \pi/2 \) with
2. Control of boundary-layer instabilities

Figure 2.8: Secondary path. The solid black line in (a) depicts the real secondary path of the plant, while the dotted green line shows the time-delay approximation used by the dxLMS algorithm. Magnitude and argument of the real secondary path and the delay model are reported in (b-c). The phase error is shown in (d) for the time delay approximation with respect to the real secondary path. The dashed lines indicate the stability limit for the LMS algorithm.

respect to the real secondary path of the system (Snyder & Hansen 1994). For this reason, it is important that the phase is well approximated by the time-delay model. Figure 2.8c shows that the time delay gives a good approximation of the phase for all the frequencies that are amplified by the flow. This can be better seen in Figure 2.8d where the phase error is reported; the error stays inside the algorithm stability limits for all the amplified frequencies.

The performance given by the approximated secondary path is very close to the one given by the full model (Figure 2.9). The two techniques converge to almost the same kernel solution; the dxLMS with delay-identification has the
2.5. A self-tuning compensator

Figure 2.9: dxLMS performance. The lines in (a) report the performances of dxLMS approach compared to LQG and fxLMS. The control kernels that result from the three different compensator-design approaches are shown in (b).

so-called self-tuning property, i.e. “under certain condition [...] the controller converges eventually to the one that could be designed if the process model was known a priori” (Åström et al. 1977). Further information about this approach can be found in Paper 3.

2.5.1. In-flight experiments

A set-up similar to the one used in wind-tunnel experiments (§2.4.1) is used for in-flight investigations in collaboration with TU Darmstadt. A motor-glider Grub G109b is used for the experimental campaign; the right wing of the glider is equipped with a natural laminar flow (NLF) airfoil wind glove, where the experimental equipment is mounted. Sensors and actuators are installed on a flush-mounted plexiglass plate on the pressure side of the wing. The flat airfoil shape in this region creates an almost linear pressure gradient which is adjustable between moderate positive and negative values, depending on the angle of attack. The flight condition is monitored by an environmental data acquisition set-up on the left wing; flight velocity, angle of attack and side-slip angle are measured by a Prandtl tube and a wind vane mounted on a boom protruding upstream into the flow. All the measurements are collected with the engine turned off and the propeller feathered, i.e. when the aircraft is in glider-mode.
2. Control of boundary-layer instabilities

Figure 2.10: Control performance for fxLMS and dxLMS algorithm at an angle of attack $\alpha = 3^\circ$. The spectra show the signal reduction at position of the error sensor $z$.

The self-tuning dxLMS compensator introduced in §2.5 is tested and compared with the fxLMS algorithm. The two control approaches are equivalent, if the phase error is inside the stability region for the LMS algorithm. Figure 2.10 shows the control performance as measured by the error sensor $z$: the two compensators behave similarly and disturbance-attenuation is achieved in the TS-region of the signal spectrum. The difference between the two graphs is caused by the change in altitude during the different measurement runs; hence, a direct comparison is not possible since the change in density and viscosity slightly changes the flow condition and, therefore, the TS-wave amplification and phase.
In the previous chapter the assumption of a purely 2D flow has been made to simplify the study. This permitted to easily highlight advantages and disadvantages of the investigated control techniques. However, in real environments this assumption is far from reasonable. For this reason, it is necessary to address the control problem allowing a disturbance to develop in three dimensions.

The numerical and experimental work by Li & Gaster (2006) falls into this framework: the control of 3D disturbances via opposition control by using a linear model of the flow based on the Navier-Stokes equations. In the work by Semeraro et al. (2013), a LQG regulator is designed to control a single 3D wavepacket via localised sensors and actuators. All the sensors and actuators are connected to each other by the compensator. This leads to a prohibitive increase of the compensator complexity if a large spanwise portion of the flow is meant to be controlled. In the recent work presented in Paper 4, the possibility to limit the number of interconnections between sensors and actuators is investigated by dividing them in equal sets along the spanwise direction, each commanded by one compensator. This structure, called control units, is thus replicated along the spanwise direction in order to fill the entire domain.

The study presented in this chapter is a further development of this idea (Paper 5). However, the modularity of the control action is not based on an a-priori division in control units but rather on considerations about the control kernel. A distributed 3D disturbance field is generated using a spanwise row of independent random forcings $d$ (Figure 3.1), generating a complex, 3D, random pattern of disturbances. The control action is performed by a row of equispaced actuators forcing the flow in the proximity of the wall. Similar to the 2D case, their action $u_l(t)$ is computed based on the measurements $y_m(t)$ by a row of sensors upstream the actuators; for this set-up the number of sensors is equal to the number of actuators and they are positioned aligned with the flow direction (Figure 3.2).

This control approach is able to delay the laminar-to-turbulence transition and, consequently, provide drag-reduction with respect to the uncontrolled case (Paper 5). Moreover, the saved power due to drag reduction is compared to the power that is needed to perform the control action (§3.3). In order to compute the latter, ideal as well as real actuator models are considered.
3. Transition delay

Figure 3.1: 3D control set-up. Random 3D disturbances are generated by a row of localised independent forcings $d$. The measurements from the sensors $y$ and $z$ are used to compute the actuation signal for the actuators $u$.

3.1. A 3D compensator

A linear control law is assumed:

$$u_l(n) = \sum_m \sum_i K_{ml}(i) y_m(n-i) \quad \forall l,$$

where $K_{ml}(i) \in \mathbb{R}^{M \times M}$ is the convolution kernel of the compensator. As a consequence, the number of transfer functions between the $M$ sensors $y_m$ and the $M$ actuators $u_l$ is $M^2$. This imposes a computational constraint when $M$ is large, which is the case when covering a large spanwise region with the controller. However, since the flow is spanwise homogeneous, the same transfer function $K_m$ from all the sensors $y_{m+l}$ to one arbitrary actuator $u_l$ is replicated for each actuator $u_m$, as shown in Figure 3.2. This assumption reduces the number of transfer functions to be designed from $M^2$ to $M$. Hence, the Finite Impulse Response (FIR) filter representation of the control law reads:

$$u_l(n) = \sum_m \sum_i K_m(i) y_{m+l}(n-i) \quad \forall l$$

where one kernel dimension is suppressed and, as a consequence, $K_m(i) \in \mathbb{R}^{M \times 1}$.

3.1.1. Multi-input multi-output (MIMO) fxlMS

A multi-input multi-output (MIMO) version of the fxlMS algorithm introduced in §2.3.1 is used to dynamically design the compensator. The algorithm
3.1. A 3D compensator

minimises the sum of the squared measurement signals $z_l(n)$:

$$\min_{K_m} \left( \sum_l z_l^2(n) \right).$$  \hspace{1cm} (3.3)

The kernel is updated via a steepest descent algorithm at each time step:

$$K_m(i|n + 1) = K_m(i|n) - \mu \lambda_m(i|n).$$  \hspace{1cm} (3.4)

where the descend direction $\lambda_m(j|n)$ is given by:

$$\lambda_m(i|n) = \frac{\partial \left( \sum_l z_l^2(n) \right)}{\partial K_m(i)} = 2 \sum_l z_l(n) \frac{\partial z_l(n)}{\partial K_m(i)}.$$  \hspace{1cm} (3.5)

The error sensor signal is given by the superposition of the contributions given by the disturbance sources $d_l$ and the actuators $u_l$:

$$z_l(n) = \sum_r \sum_j P_{zd,r}(j) d_{r+l}(n-j) + \sum_r \sum_j P_{zu,r}(j) u_{r+l}(n-j) =$$

$$= \cdots + \sum_r \sum_j P_{zu,r}(j) \sum_m \sum_i K_m(i) y_{m+r+l}(n-j-i) =$$

$$= \cdots + \sum_m \sum_i K_m(i) \sum_r \sum_j P_{zu,r}(j) y_{r+m+l}(n-j-i) =$$

$$= \cdots + \sum_m \sum_i K_m(i) f_{m+l}(n-i),$$  \hspace{1cm} (3.6)

where the same spanwise homogeneity assumption has been made for the plant kernels $P_{zd,r}(j)$ and $P_{zu,r}(j)$ that represent the transfer functions $d_r \rightarrow z_l$ and $u_r \rightarrow z_l$ respectively. Therefore, the descend direction is given by:

$$\lambda_m(i|n) = 2 \sum_l z_l(n) \frac{\partial z_l(n)}{\partial K_m(i)} = 2 \sum_l z_l(n) f_{m+l}(n-i).$$  \hspace{1cm} (3.7)
Figure 3.3: Disturbance attenuation and transition delay. The shaded gray area report the skin friction fluctuations \( \tau'_w = \tau_w - \tau_{w,lam} \) with respect to the laminar solution. The green surfaces indicate the \( \lambda_2 \)-criterion with a threshold of \(-2 \times 10^{-3}\). The disturbance sources are fed with white-noise signals that range between \( \pm 2 \times 10^{-3} \), resulting in a perturbation amplitude \( A(100) = 0.11\% \). The fringe region is not shown.

This expression – except for the sum over the index \( l \) – is similar to the expression of \( \lambda(i|n) \) in the 2D case in (2.33).

3.2. Performance and limitations

Non-linear simulations are performed in a 3D extension of the domain used in the previous section. The domain extents are \([0, 2000) \times [0, 45] \times [-125, 125]\) in the streamwise, wall-normal and spanwise directions. The fringe forcing used to enforce the streamwise periodicity takes place in the last 400 spatial units along the streamwise direction (Chevalier et al. 2007). The flow is approximated by 1536 \times 384 Fourier modes along the streamwise and spanwise direction and by 151 Chebyshev polynomials in the wall-normal direction. The time step \( \Delta t \) is set to 0.4 as in the previous 2D study. Turbulence will appear in the end of the domain. Since the focus is on laminar-to-turbulence transition and not on turbulence itself, a deconvolution model is used for large-eddy simulations (LES) to avoid increasing the spatial resolution (Stolz et al. 2001).

The perturbation is introduced at \( X = 65 \) via the following volume forcing:

\[
f_d = \sum_l b_{d,l}(x) d_l(t). \tag{3.8}
\]

The 25 forcing shapes \( b_{d,l} \) are equally-spaced along the spanwise direction and are modelled as synthetic vortices and each disturbance signal \( d_l(t) \) is a uniformly-distributed white-noise signal that ranges between \( \pm a_d \) (Fabbiane
3.2. Performance and limitations

Figure 3.4: Perturbation attenuation and transition delay. The amplitude of the perturbation field as defined in (3.9) is reported as a function of the streamwise coordinate $X$ for the controlled and uncontrolled cases in Figure 3.3.

Reference and error sensors $y_l$ and $z_l$ are positioned at $X_y = 300$ and $X_z = 500$ with the same spanwise spacing $\Delta Z = 10$ as the disturbance sources. As in the previous section, the sensors measure the shear-stress fluctuation of the perturbation field averaged, in this case, over the spanwise extension of the sensor $\Delta Z$. The actuators are positioned at $X_u = 400$ and are spaced along $Z$ as the sensors. They are modelled by a modulated volume forcing similar to the disturbance sources; the forcing shapes are computed in order to model a plasma actuator, based on the experimental data by (Kriegseis et al. 2013).

The disturbance sources are fed with white-noise signals with amplitude $a_d = 2 \times 10^{-3}$. A snapshot of the resulting flow is shown in Figure 3.3a: the grey shaded area reports the skin friction fluctuation with respect to the laminar solution, while isosurfaces of $\lambda_2 = -2 \times 10^{-3}$ are depicted in green (Jeong & Hussain 1995). The random forcing triggers a pattern of random wave-packets that travel downstream in the domain while growing in amplitude, as it can be seen by their friction footprint. At $X = 900$ the flow departs from the laminar solution and the turbulent structures identified by the $\lambda_2$-criterion appear. This is the TS-wave driven transition to turbulence, cfr. §1.1.

Figure 3.3b reports the same disturbance scenario when the control is active. The perturbation is almost completely cancelled downstream of the actuators, as it can be seen by the friction fluctuations. The same transition scenario is present but moved downstream with respect to the uncontrolled case. The control is effective in delaying the laminar-to-turbulence transition.

A measurement of the perturbation field amplitude is introduced to quantitatively assess the control action:

$$A^2(X) = \max_Y \left\langle \left( \frac{u - U}{U} \right)^2 \right\rangle_{Z,t}, \quad (3.9)$$

where $U(X, Y, Z)$ is the streamwise component of the mean flow. Figure 3.4 reports the amplitude values for the controlled and uncontrolled cases illustrated in Figure 3.3. The two curves depart from each other directly downstream of
the actuators; the uncontrolled case saturates first around the uncontrolled transition location, while the controlled case shows a lower disturbance amplitude up to its transition location. The perturbation field is hence attenuated by the control action.

The kernel that results from the MIMO fxLMS dynamic design is shown in Figure 3.5: the solid thick line depicts $K_0(i)$, i.e. the connection between sensors and actuators at the same spanwise position. The kernel is similar to the 2D solution in the previous chapter and it has a compact support along the spanwise direction; hence, each sensor drives a limited number of actuators, that does not depend by the distance between sensors and actuators (Fabbiani et al. 2015a).

In order to better evaluate the effect of the control on the friction drag, the spanwise averaged friction coefficient is introduced as:

$$c_f(X) = \frac{\langle \tau_w \rangle_Z}{\frac{1}{2} \rho U^2},$$

(3.10)

where $\langle \cdot \rangle_Z$ is the spanwise average operator. The friction coefficient for the current flow case is shown in Figure 3.6a for both the controlled and uncontrolled cases. In the controlled case the transition in simply delayed without introducing any additional friction with respect to the laminar solution. Hence, the spatial delay of the transition is directly related to the drag-save resulting from applying the control (Paper 5).

The disturbance amplitude is then increased up to the point where no residual effect of the control is visible. Figure 3.6b shows the transition location as a function of the disturbance amplitude; the perturbation field amplitude at $X = 100$ is used as a measure of the amplitude of the disturbance. The transition point is identified as the point where the friction coefficient crosses the average between the laminar and the turbulent value of the friction coefficient, dot-dashed line in Figure 3.6a. The control is effective up to a seeded perturbation amplitude $A(100) = 0.40\%$, that rises up to approximately 2% at the actuator location.
3.3. Energy budget

A control strategy is successful when its benefit is larger than its cost. Hence, the power consumption of the control $P_c$ is estimated via two different actuator models: the ideal actuator and the plasma actuator. The ideal actuator is based on the fluid-dynamical power that the volume forcing exchanges with the flow; this ideal actuator, however, does not take into consideration the energy spent in order to create the volume forcing given by the actuator. A model of the plasma-actuator power consumption is obtained by combining the two experimental works by Kriegseis et al. (2011, 2013). Two different operation modes are considered: (i) dual where two plasma actuators are considered, one responsible for the positive forcing and one for the negative one, and (ii) hybrid where an offset forcing is used in order to avoid negative forcing by the actuator. Further details about this procedure can be found in Paper 5.

The power gain is defined as:

$$\Gamma = \frac{P_s}{P_c}, \quad (3.11)$$

where $P_s = U \Delta D$ is the power saved by the drag-reduction $\Delta D$ that results from applying the control. Figure 3.7 shows $\Gamma$ for the different actuator models as a function of the disturbance amplitude. In the ideal case the control strategy is capable of power gains up to $10^3$. This means that the power saved is 1000 times the power that is spent to perform the control action. For increasing amplitudes, the gain decreases until the break-even point $\Gamma = 1$ is reached for

Figure 3.6: Transition delay. (a) reports the $Z$-averaged friction coefficient for the flow case in Figure 3.3, $A(100) \approx 0.12\%$. (b) reports the transition location for increasing disturbance amplitude $A(100)$. The transition positions are computed based on a time-averaged flow over 1200 time units.
Figure 3.7: Energy budget. The reported quantities are computed based on a
time-averaged flow over 1200 time units. The solid black line indicates $\Gamma = 1$,
i.e. the break-even point for the control strategy.

$A(100) = 0.46\%$. At this point the control is on the limit of its effective range
and it is not effective in delaying the transition any more, cfr. Figure 3.6b.

Unfortunately, the power-gain $\Gamma$ barely reaches the break-even point when
a plasma actuator model is considered; a reason for this can be found in the
lower efficiency of the plasma actuator (Jolibois & Moreau 2009; Cattafesta
& Sheplak 2010). The results by Kriegseis et al. (2011) are based on research
studies on plasma actuators at the time and no new data on the state-of-the-art
is available in the literature. These results highlight that the energy efficiency
of the plasma actuator is critical to its use for this type of control. However,
this is not a rejection of this type of actuator; this preliminary estimation of
the energy consumption by a real actuator has to be considered as an invitation
for the scientific community to improve the design of plasma actuators.
Linear reactive control of boundary-layer instabilities has been addressed. The model-based control – more precisely LQG regulator – is able to attenuate the perturbation amplitude by using the knowledge of the flow status close to the wall. Because of the relative position of sensors and actuators, the control law results in a feed-forward wave-cancellation. For this technique, an accurate model of the perturbation behaviour is crucial to performance; it is in fact shown that, when model inaccuracies occur, the compensator performance rapidly degrades (Paper 2).

Robustness to model inaccuracies is recovered via adaptive control (Paper 1; Paper 2). The perturbation attenuation is monitored on-line by a second sensor downstream of the actuator and the control law is modified in order to maximize the performance. This approach gives the compensator a feed-back not directly on its action but on the validity of its control law. A recovery in robustness to model inaccuracies is shown by the fxLMS adaptive algorithm.

A self-tuning approach is proposed based on this algorithm. The modelling requirement by fxLMS are reduced to a time-delay that is computed by measuring the phase speed upstream the actuator via a third sensor. The performance of the compensator is unchanged by this model approximation both in DNS and in in-flight experiments (Paper 3).

Transition delay is achieved in a low-amplitude 3D disturbance scenario (Paper 5). The consequent drag-reduction is computed and an energy budget is performed: the power saved thanks to the delay of the laminar-to-turbulence transition is three orders of magnitude larger then the ideal power spent to perform the required forcing to control the flow.

For increasing disturbance levels, non-linear interactions arise in the flow. Since the compensator assumes a linear behaviour of the perturbation field, it gradually reduces its performance because its perturbation model fails with respect to the actual flow conditions. No transition-delay and energy-saving are achieved for perturbation amplitudes greater than 2% of the free-stream velocity. If the fxLMS algorithm is used, the adaptivity nature of the algorithm is able to marginally compensate the non-linearities in the flow.
4. Conclusions and outlook

(a) Plasma actuator (Kriegseis et al. 2013)

Figure 4.1: Actuator impulse response. Isosurfaces of positive and negative perturbation velocity are depicted in red and blue. The generated wave-packets are reported at $t = 400$ and $t = 800$; even if the two volume forcings are different, the resulting wave-packets are almost identical.

(b) Synthetic vortex (Semeraro et al. 2013)

Actuator

The net-energy-saving of the control set-up depends on the actuator and its efficiency. The theoretical results are obtained for an ideal actuator model (Paper 5); this actuator presents a unitary efficiency, i.e. the supplied energy is transferred to the flow with no losses. Real actuators behave differently: the supplied energy is not entirely transferred to the flow and part of it is lost in the forcing process. For plasma actuators this energy loss is considerable (Jolibois & Moreau 2009) and may invalidate the control performance. This confirms the overall energy-saving based on the plasma actuator model by Kriegseis et al. (2011); this estimation confirms the need to improve the design of these actuators in order to improve their energy-efficiency.

It is also possible to consider different types of actuators (Cattafesta & Sheplak 2010). In this case, the control takes place by wave-packet superposition: the actuator generates a wave-packet with opposite phase to the one that is detected by the reference sensors and, therefore, the original disturbance is cancelled. In the light of this, every actuator that is able to produce a TS wave-packet is suitable for the presented control approach. Figure 4.1 shows, as an example, the impulse response for two different volume forcings used for flow control by Semeraro et al. (2013) and Paper 5, respectively; the final wave-packet is almost identical. This leads to two conclusions: (i) it is the effect of an actuator that is important to model, and not the actuator itself (Bagheri 2010) and (ii) many different actuators may be equally suitable for reactive control.
Disturbance

In this work, the disturbances are introduced directly inside the boundary layer (Bagheri et al. 2009a; Semeraro et al. 2011, 2013). The receptivity process of free-stream disturbances is then modelled by assuming it will result in TS wave-packets. This assumption is valid for various disturbance scenarios (Saric et al. 2002): free-stream disturbances interacting with surface roughness (Goldstein 1985) or sound waves interacting with the leading-edge (Giannetti & Luchini 2006; Shahriari et al. 2016). However, TS wave-packets are not the only perturbations that populate the boundary layer: in presence of high free-stream turbulence, streaks are induced inside the boundary layer (Matsubara & Alfredsson 2001) and structures different from TS-waves may also appear in a linear framework (Dadfar et al. 2014).

This suggests further investigation of the presented control approach when the disturbance is introduced outside of the boundary-layer. Since the leading-edge has a crucial role in the receptivity process, a full flat-plate/wing geometry has to be taken into consideration for these investigations, e.g. the DNS set-up in Paper 3. This may lead to computationally challenging simulations, in particular if a 3D computational box is considered in order to evaluate the transition-delay. An example of the required simulation set-up can be found in Hosseini et al. (2013), where boundary-layer stabilisation by roughness elements is studied under a free-stream turbulence forcing.

Control algorithm

The two design approaches presented in this work – although effective for the investigated control problem – cover only a part of the possible control techniques used in flow control. Figure 4.2 is a complete schematic of the most popular control techniques used in fluid-dynamics (Brunton & Noack 2015). The proposed model-based approach combines a linear reduced-order-model obtained via BPOD and $H_2$ optimal control (§2.2.1), while the adaptive fxlMS algorithm can be seen both as opposition and extremum-seeking control (§2.3.1).

These two design approaches result in a linear compensator: the control signal is linearly dependent on the time history of the reference signal through the control kernel, that is computed based on a linear model of the system. The compensator uses the model to predict the control effect on the system. When non-linearities arise, the model fails, its prediction is not reliable and, therefore, the compensator fails at controlling the flow. Adaptive control can be used to partially recover the control performance; the fxlMS algorithm, for example, shows the ability to compensate for small non-linearities. However, since its adaptation path is also based on a linear model of the flow, the adaptation fails when the flow behaviour is too non-linear.

The performance of the control is model dependent. Enhancing the model complexity and/or robustness may result in an extension of the compensator performance-envelope: non-linear models (Noack et al. 2003), system identification (Hervé et al. 2012) and dynamic observers (Guzmán Iñigo et al. 2014) are
Conclusions and outlook

Typically very high dimensional, sometimes exceeding the capacity of computer memory. For example, a high Reynolds number three-dimensional unsteady flow will exhibit important spatial structures that span many orders of magnitude in scale. The Reynolds number can be estimated from the ratio between the largest-scale structures to the smallest structures in the flow. Thus, for a generic geometry, the state dimension will scale with $Re^{3/4}$, along with the memory cost $77-79$. The computational cost will scale with $Re^{3}$ because of the addition of multiple temporal scales, which generally scale with $Re^{3/4}=4$. For a channel flow, the scaling may even be worse with Reynolds number, as $Re^{3/4}$ in space and $Re^{4}$ in space and time. If a spatial discretization is required with 1000 elements in each direction, then a three-dimensional simulation will contain $10^{11}$ states for every flow variable (velocity, pressure, etc.).

The highest-order fully resolved simulation to date is a wall-bounded turbulent channel flow with $Re_s=5200$ (Reynolds number based on the friction velocity), containing $2.4\times10^{11}$ states. This simulation is about 3.5 times larger than the previous record holder and it uses slightly over 3/4 of a million processors in parallel. Even with Moore’s law, it will take nearly 40 yr for this type of computation to become a lightweight “laptop” computation and decades longer before being useful for in-time control, since the parallel code takes 7 real seconds per simulated time-step, as benchmarked in Ref. However, impressive and useful for design and optimization, it is unclear that this level of resolution is even necessary for many control applications.

3.2.2 Modal Representation (Gray-Box).

Instead of resolving every detail of the flow field at all scales, it is often possible to represent most of the relevant flow features in terms of a much lower dimensional state. This state represents the amplitudes of modes, or coherent structures that are likely to be found in the flow of interest. Galerkin models based on modal expansions constitute one class of gray-box models, which resolve the coherent structures of the white-box models while accounting for small-scale fluctuations with subscale closures.

The POD is one of the earliest and most successful modal representations used in fluids, resulting in dominant spatially coherent structures. POD benefits from a physical interpretation where modes are ordered hierarchically in terms of the energy content that they capture in the flow. There are numerous methods to compute POD, and the snapshot POD is efficient when a limited number of well-resolved full-state measurements are available from simulations or experiments. Snapshot POD is an example of this. In this direction, the past decade saw a growing interest in data-driven techniques (Fleming & Purshouse 2002), in particular in model free-approaches as machine-learning control. These techniques learn about the system behaviour by observing it and build input-output maps that are used to perform the control action. They are very powerful when no reliable model of the system is available a-priori, or when it comes to turbulence control – i.e. chaotic systems – since they are able to follow the chaotic-attractor dynamics modified by the control itself. However, their validity envelope depends on the extent of their training; uncertainties of the environmental condition and/or the presence of unmodelled disturbances may still present a robustness problem.
Acknowledgements

Today I send my PhD thesis for print, and I turn thirty.

As every n-th birthday, it leads to an balance about the past and a view into the future. Considering the PhD defence coming in four weeks, I must say this is one of the most important birthdays in my life.

I remember when I understood that science would have been part of my life; I was in middle school and Mr. Ballarini – my maths teacher – was giving us a written test. For me, it was like playing and I wanted to continue that game. The years passed and, in high school, Ms. Maffezzoli explained to me that the game I loved could also be used to predict complicated physical phenomena: skidding cars, flying bullets, bouncing objects and more. The maths became more and more complicated and, because of that, more and more fun.

The decision to study engineering came naturally and my passion for aviation brought me to aeronautics. I wanted to design airplanes structures. As always, never be sure about anything in your life: the first fluid dynamics course by Dr. Quartapelle made me realize that structures are overrated and fluids are where the fun is. Hence, I took my Master in fluid dynamics. When the time came for a thesis, most of my friends were looking at industry: for me it was different, I wanted to take that last chance of discovering something new and not simply designing something discovered by someone else. I asked Maurizio¹ for a thesis topic and we wrote a DNS code.

It was still not enough and I wanted to continue doing what a very good friend of mine describes as “sempre meglio che lavorare”².

This took me here, to KTH. I thank Prof. Dan Henningson for giving me the opportunity to show him he was right about hiring me. I hope I succeeded. I would also like to thank Dr. Shervin Bagheri – my co-advisor – for his guidance throughout my doctoral studies and Dr. Ardeshir Hanifi for his contagious laugh and his indisputable style. A special thank goes to Onofrio, now Dr.

¹Prof. Maurizio Quadrio.
²Transl.: “[it is and will] always [be] better than working”.

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Semeraro; he was – and still is – a good mentor and a great friend to me. I am very thankful for that.

Since I moved to Stockholm in the Swedish spring of 2012, I met a lot of people who accompanied me during these last few years. I first want to mention Cristina, my sister-in-Sweden; she listened to me complaining about everything, mostly girls I dated, girls I would have liked to date, and girls who did not want to date me. Remember I did the same for you. I thank my friends Michele, Paola, Marta, Karin and Chiara for enjoying Stockholm and its beauty with me. I also thank all my colleagues in particular Elektra\(^3\), Jacopo\(^3\), Alexandra, Mattias, Clio, Nima, Taras, Armin, Iman, Luca and Ellinor. With them I spent endless nights in photography museums without looking at a single picture, smoked pipe till late looking at the city skyline, travelled by boat to different countries, learned how to properly dance Ukrainian and Persian style. They made the office not only a nice place to work but also a nice place to be.

This project gave me the opportunity to travel around the world. In particular, I will always remember my visit at TU Darmstadt; I thank Prof. Sven Grundmann and Bernhard Simon for the fruitful collaboration and for showing me in the real world what I only dared to simulate.

My deepest gratitude goes to all my friends I left in Italy when I moved to Sweden: they never denied me their support, despite the geographical distance. Above all, I would like to thank Alberto, Agnese and their little daughter Alice for making me feel like I never left, every time I visit them.

Last, but not least, I would like to thank my grandparents, who can not read English but will nevertheless understand how grateful I am to them, and my parents: my mother for her unconstrained love and support, and my father for making me look at the stars in the sky with curiosity.

Thank you.

Nicolò

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\(^3\)Thank you also for proofreading this manuscript.
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Part II
Papers
Summary of the papers

Paper 1

*Adaptive and model-based control theory applied to convectively-unstable flows*

A review of the control methodologies aimed at delaying the laminar-to-turbulent transition in convectively unstable flows is presented. A simple one-dimensional system – the Kuramoto-Sivashinsky (KS) equation – able to replicate the stability of this type of flows is introduced to illustrate the different techniques via applied-control examples.

The compensator design is investigated as a coupling of a controller and an estimator. The former is responsible for computing the control signal assuming a complete knowledge of the system state. Optimal control techniques are reviewed: Linear Quadratic Regulator (LQR) and Model Predictive Control (MPC) are examined, in particular when saturation constraints are applied to the actuator. The estimator provides to the controller an estimation of the system state based on limited measurements in the flow. The conventional Kalman filter is introduced as well as system identification techniques borrowed from signal-processing theory.

In the end, the complete compensator is analysed. The difference between static (LQG) and adaptive (fxLMS) compensators is investigated, highlighting a strong sensitivity of the static controller to inaccuracies of the model used in the design process.

Scripts to generate all the presented data and figures are available in MATLAB format at http://www.mech.kth.se/~nicolo/ks/.

Paper 2

*On the role of adaptivity for robust laminar-flow control*

The control problem is addressed in an experimental set-up in order to investigate the necessity of adaptivity in real flow applications. An fxLMS adaptive compensator is compared with a model-based LQG regulator when attenuating 2D TS-waves in a zero-pressure-gradient boundary layer flow.

The experiments are conducted in the open-circuit wind tunnel at TU Darmstadt, Germany. A 2D disturbance is generated by a disturbance source and is downstream detected by a surface-mounted hot-wire sensor. Based on
these measurements, the compensator prescribes a suitable forcing to a dielectric-barrier-discharge (DBD) plasma actuator in order to cancel the upcoming wave. A second hot-wire sensor is placed farther downstream to monitor the compensator performance. DNS of the experimental set-up are carried out and, based on these, the LQG regulator is designed.

The model-based regulator is found to be less effective than the fxLMS compensator because of unavoidable modelling inaccuracies. Moreover, the performance of the LQG regulator degrades as the flow response departs from the design model. In particular, free-stream velocity variation is investigated: the static compensator turns out not to be able to prescribe the correct phase information to the actuator. Otherwise, the adaptive compensator is able to autonomously adjust to the modified flow conditions and effectively perform the control action for a broader interval of velocity variations.

**Paper 3**

*In-flight active-wave-cancelation via delayed-x-LMS control algorithm in a laminar boundary layer*

The adaptive control techniques investigated in Paper 2 are pushed towards a black-box approach. In particular, the stability properties of the fxLMS algorithm are used to further simplify the model of the flow and design a self-tuning compensator.

The secondary path – i.e. the transfer function between actuator and error sensor – is modelled by a time-delay only. This control technique, that is known as delayed-x LMS (dxLMS) algorithm, is successfully tested in cancelling 2D TS-waves in-flight; the experiment is set on a motor-glider Grub G-109 at TU Darmstadt. The set-up is similar to that in Paper 2: the disturbance is artificially generated by a row of 12 loudspeakers and it is detected downstream by the reference sensor, a surface hot-wire. Hence, the control action is performed by a wall-mounted DBD plasma-actuator. A second surface hot-wire is mounted farther downstream, to provide the performance information needed by the adaptive algorithm.

The time-delay for the secondary path modelling is computed via a measurement of the perturbation group speed. A third sensor is positioned upstream of the reference sensor for this purpose. This allows the design of a self-tuning controller that needs no external information about the flow; the resulting “black-box” approach is to be considered a big step forward towards a real application of this control technique.

**Paper 4**

*Centralised versus decentralised active control of boundary layer instabilities*

The control of 3D disturbances in a zero-pressure-gradient boundary-layer flow is addressed via model-based optimal control. In particular, this work focuses
on the possibility to divide and replicate the control law along the homogeneous span-wise direction in order to reduce the complexity of the controller.

DNS are performed to investigate the control performance. Evenly localised objects are distributed in the spanwise direction in the wall region (18 disturbances sources, 18 actuators, 18 estimation sensors and 18 objective sensors) and span-wise subsets of these objects are identified by signal-energy based techniques. LQG compensators are designed based on these subsets and replicated along the span-wise direction to fill the computational domain. Hence, the performance loss due to the missing connections are evaluated in order to identify a “minimal” control unit, i.e. a minimal subset of sensors and actuators able to perform an effective control action.

**Paper 5**

*Energy efficiency and performance limitations of linear adaptive control for transition delay*

Transition-delay and energy-saving capabilities of reactive flow control are investigated. A MIMO fxLMS algorithm is introduced in order to control a 3D disturbance scenario in a 2D zero-pressure-gradient boundary-layer flow.

Random disturbances are introduced in the flow via localised volume forcing inside the boundary layer. The compensator performance is evaluated for increasing disturbance level; the investigated control setup results able to delay the laminar-to-turbulence transition up to the point where transition occurs in the control region.

The energy efficiency is also evaluated: ideal as well as real actuators models are considered in order to compute the power needed to perform the control action. When ideal actuators are considered, a net-energy-saving up to 1000 times the power spent for the control is recorded and the balance remains positive as long as transition delay is achieved. However, when plasma actuators are considered, the break-even point is barely reached because of their low efficiency. This result should motivate further studies on the design and optimisation of these type of actuators.