Optimal Transmit Strategy for MISO Channels with Joint Sum and Per-antenna Power Constraints

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***Abstract*—In this paper, we study an optimal transmit strategy for multiple-input single-output (MISO) Gaussian channels with joint sum and per-antenna power constraints. We study in detail the interesting case where the sum of the per-antenna power constraints is larger than sum power constraint. A closed-form characterization of an optimal beamforming strategy is derived. It is shown that we can always find an optimal beamforming transmit strategy that allocates the maximal sum power with phases matched to the complex channel coefficients. The main result is a simple recursive algorithm to compute the optimal power allocation. Whenever the optimal power allocation of the corresponding problem with sum power constraint only exceeds per-antenna power constraints, it is optimal to allocate maximal per-antenna power to those antennas to satisfy the per-antenna power constraints. The remaining power is divided amongst the other antennas whose optimal allocation follows from a reduced joint sum and per-antenna power constraints problem of smaller channel coefficient dimension and reduced sum power constraint. Finally, the theoretical results are illustrated by numerical examples.**

***Index Terms*—Sum power constraint, per-antenna power constraint, MISO, beamforming, transmit strategy, transmission rate.**

I. Introduction

For the last two decades, there has been a huge interest in vector-valued transmit strategies in wireless communications. The optimization problem of finding the optimal transmit strategy for a Gaussian channel has been extensively studied subject to either sum power constraint or per-antenna power constraints, but, to the best of our knowledge, a combination of both constraints surprisingly has not been considered yet. While a sum power constraint limits the total power of the transmitter, a per-antenna power constraint limits the used power on each transmitter chain of each antenna. Both constraints have reasonable physical motivations. For instance, the former constraint may be imposed by regulations or to limit the energy consumption, while the latter may be imposed by hardware limitations of each RF chain. Thus, it is reasonable to consider both constraints simultaneously.

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Under a sum power constraint, when the channel state is known at both transmitter and receiver, the maximum transmission rate is obtained by performing singular value decomposition and applying water-filling on the channel eigenvalues [1]–[3]. In contrast, the per-antenna power constraints problem results in a different power allocation mechanism because the power can not be arbitrarily allocated among the transmit antennas. The per-antenna power constraints problem has received considerable attention recently [4]–[13]. Particularly, the problem of finding the capacity of point-to-point channels with per-antenna power constraints is well studied in [4]–[7]. In [4], the closed-form solution of the capacity and the optimal signaling scheme for MISO channels has been established for two separate cases assuming a constant channel which is known by both the transmitter and receiver, and also assuming Rayleigh fading where the channel coefficient is known at the receiver only. In addition, the optimal transmission schemes for point-to-point multiple-input multiple-output (MIMO) channels with per-antenna power constraints are studied in [5] and [6]. In these works, the authors derived necessary and sufficient conditions for the optimal MIMO transmission schemes and developed an iterative algorithm that converges to the optimal solution. The ergodic capacity of the MISO channel with per-antenna power constraints is considered in [4] and [7]. In [7], the authors characterize the ergodic capacity of the fading MISO channel subject to long-term average per-antenna power constraints with perfect channel state information at all nodes. Then, they consider an application to the fading two-user cognitive interference channel.

The optimization problem with per-antenna power constraints for multi-user channels is studied in [8]–[15]. In [8], the problem of transmitter optimization for the multi-antenna downlink is considered. That work mainly focuses on the minimum-power beamforming design and the capacity-achieving transmitter design. It is shown that the solution to the per-antenna power constraints problem arises from a new interpretation of the uplink-downlink duality. In [9], the authors focus on the discussion of linear signal processing strategies dealing with two optimization problems: maximizing the sum rate subject to per-antenna power constraints and maximizing the minimum user rate under per-antenna power constraints. Also, an iterative algorithm is proposed for solving the problem of maximizing the weighted rate sum for multi-user systems with per-antenna power constraints. In [10], the optimal zero-forcing beamforming in multiple antenna broadcast channels (BC) with per-antenna power constraints is considered. The results show that an optimization problem subjects to per-antenna power constraints for the broadcast channel may improve the rate considerably when the number of transmit antennas is larger than the number of receive antennas. The problem of linear zero-forcing precoder design is investigated in [11]. The authors proved that under the assumption of a sum power constraint, precoders based on the pseudo-inverses are optimal among the generalized inverses. However, this is not necessarily true under the assumption of per-antenna power constraints.

The optimal power allocation problems for the MIMO broadcast channels with per-antenna power constraints are studied in [12] and [13]. In [12], the optimal power allocation to maximize the weighted rate sum assuming a zero-forcing precoder and a per-antenna power constraint are determined. In their work, the precoding vectors for the sum power constraint are adapted and then the power allocation is optimized to maximize the sum rate under per-antenna power constraints. In [13], the author focuses on the block diagonalization based downlink precoding for a fully cooperative multi-cell system with per-base-station power constraints. To meet the per-base-station power constraints, a suboptimal heuristic method is proposed which combines the block diagonalization precoder design with an optimized power allocation scheme. In particular, the proposed solution in [13] can be reduced to the optimal zero-forcing precoder design for weighted rate sum maximization with per-antenna power constraints if single-antenna base-stations and a mobile-station are used. Optimization problems whose classical formulations have been extended by adding unconventional constraints have been considered as well. For instance, in [14], the authors focus on designing linear multi-user MIMO transceivers subject to the different quality of service constraints per user and per-antenna power constraints. In [15], the transmitter optimization problem for a MISO channel subject to general linear constraints is considered. Algorithms that solve this problem with both the optimal dirty-paper coding and simple sub-optimal linear zero-forcing beamforming are provided. The general linear constraints in their work include the sum power constraint, per-antenna power constraints and "forbidden interference direction" constraints.

Combinations of several power constraints have been considered in a range of other scenarios, for instance in the context cognitive radio channels [16], [17] or wiretap channels [18]. These works build on known results considering sum power, individual power or per-antenna power constraints, and extend them with additional power constraints to limit the received power at a third node. In particular for the cognitive radio channel, Zhang *et al.* studied in [16] the weighted rate sum maximization problem in which the secondary users have not only the sum power constraint but also interference constraints. The sum power constraint and interference constraints are also considered in [17], where the authors used the idea of antenna selection to jointly satisfy interference constraints at primary users while improving the rates of secondary users. In addition, the optimization problem with joint power and interference constraints has also received much attention in green radio setups [19]–[21]. The key difference of green radio is to focus more on the optimization of the energy efficiency instead of the transmission rate. In [19], authors designed an effective multi-user MIMO transmission strategy to maximize the system energy efficiency defined as the ratio of the rate sum to the total power consumption. The energy efficiency optimization problem for MIMO broadcast channels subject to a sum power constraint, interference constraints, and a minimum throughput constraint was studied in [20]. The solutions for the optimization problems in [19]–[21] are based on the duality between multiple access and broadcast channel as well as dirty paper coding.

In practical systems, the joint sum and per-antenna power constraints setting applies either to systems with multiple antennas or to distributed systems with separated energy sources. A sum power constraint can be, for instance, motivated by radiation limits or green aspects to limit the energy consumption. On the other hand, a per-antenna power constraint can be motivated to limit the power in the RF chain of each antenna. This also allows operating the power amplifier in the RF chain at a more energy efficient operating point. Since both aspects can be relevant in practical scenarios, it is reasonable to include them both in a classical MISO point-to-point setup. However, the joint sum and per-antenna power constraints are also relevant for future wireless systems in which base-stations are connected via high-speed links so that they can cooperate in the downlink transmission or in the uplink where mobile users cooperate in the transmission and each user has a limited power budget. Since the sum power constraint is not active if the allowed sum power is larger than the sum of the per-antenna power constraints, the problem is only interesting if the sum power constraint is smaller than the sum of the individual power constraints as illustrated in Fig. 1. Thus, the main purpose of this paper is to characterize the optimal transmit strategy for the point-to-point MISO channel with joint sum and per-antenna power constraints with the assumption of perfect channel state information at the transmitter. The solution is developed from the two original problems considering a sum power constraint or per-antenna power constraints only. A special case with two transmit antennas has been considered in [22], where we have shown that if the sum power constraint only optimal power violates a per-antenna power constraint then the optimal power allocation of the considered joint power constraints is at the intersection of the sum power constraint and the per-antenna power constraint.

The organization of this paper is as follows. In the next section, we introduce the system model and the power constraints including the sum power constraint, the per-antenna power constraint and the joint sum and per-antenna power constraint. In Section III we briefly recapitulate the known results for the problems of sum power constraint and per-antenna power constraints only. Then the properties of the optimal transmit strategy and power allocation for the joint sum and per-antenna constraints are discussed. The algorithm to find the optimal transmit strategy for joint sum and per-antenna power constraints is given in Section IV. Then, the results and numerical examples are discussed in the next section. Finally, we provide some remarks and conclusions.

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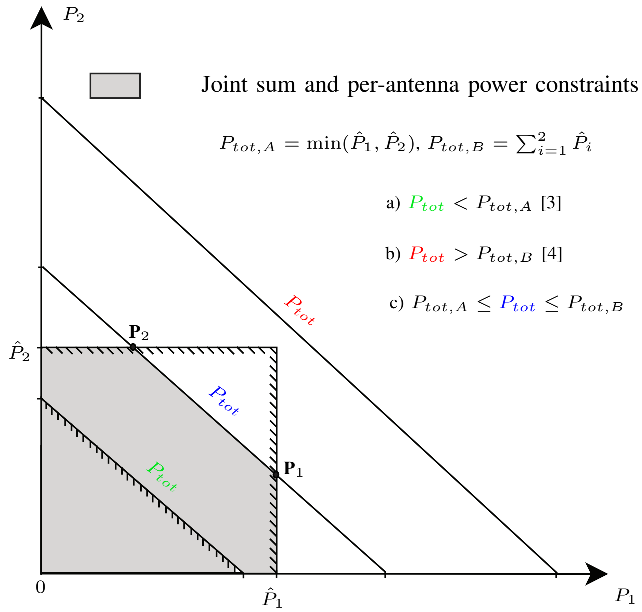

Fig. 1: Feasible power allocation region with joint sum and per-antenna power constraints with a) per-antenna power constraints are inactive, b) sum power constraint is inactive, c) sum and power antenna power constraints are all active.

Notation

We use bold lower-case letters for vectors, bold capital letters for matrices. The superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ stand for transpose, conjugate, and conjugate transpose; the superscripts $(\cdot)^{(1)}$, $(\cdot)^{(2)}$, and $(\cdot)^{(3)}$ denote the corresponding optimal values of optimization problems according to the sum power constraint, the per-antenna power constraints, and the joint sum and per-antenna power constraints. We use $\succcurlyeq$ for positive semi-definite relation, $\mathrm{tr}(\cdot)$ for trace, $\mathrm{rank}(\cdot)$ for rank, and $\mathrm{diag}\{\cdot\}$ for diagonal matrix. The expectation operator of a random variable is given by $\mathbb{E}[\cdot]$. $\mathbb{N}$, $\mathbb{R}_+$, and $\mathbb{C}$ are the sets of non-negative integers, non-negative real, and complex numbers.

II. System Model and Power Constraints

A. System Model

We consider a point-to-point MISO channel with N_t transmit antennas and one receive antenna. Further, we assume that channel state information (CSI) is available at both transmitter and receiver. The channel input-output relation of this transmission model can be written as

$$y = \mathbf{x}^T\mathbf{h} + z \tag{1}$$

where $\mathbf{x} = [x_1, ..., x_{N_t}]^T \in \mathbb{C}^{N_t \times 1}$ is the complex transmit signal vector, $\mathbf{h} = [h_1, ..., h_{N_t}]^T = [h_i]^T_{i\in\{1,...,N_t\}} \in \mathbb{C}^{N_t\times 1}$ is the channel coefficient vector and z is zero-mean scalar additive white complex Gaussian noise with power σ^2. Without loss of generality, we assume that $|h_i| > 0, \forall i \in \{1, ..., N_t\}$, because otherwise we can consider a corresponding MISO channel with a reduced number of antennas. In the following, we focus on achievable rates using Gaussian distributed input. Let $\mathbf{Q} = \mathbb{E}\left[\mathbf{x}\mathbf{x}^H\right]$ be the transmit covariance matrix of the Gaussian input $\mathbf{x}$, then the achievable transmission rate is

$$R = f(\mathbf{Q}) = \log\left(1 + \frac{1}{\sigma^2}\mathbf{h}^H\mathbf{Q}\mathbf{h}\right). \tag{2}$$

There are two questions which we are going to answer in the upcoming sections. First, we show that Gaussian distributed input is capacity-achieving for the channel (1) with joint sum and per-antenna power constraints. Second, we identify the optimal transmit strategy $\mathbf{Q}$ such that the transmission rate in (2) is maximized.

B. Power Constraints

In this part, we formally introduce the sum power, the per-antenna power and the joint sum and per-antenna power constraints problems.

1) Sum Power Constraint: If we consider a sum power constraint [1]–[3], [23]–[26], the total transmit power from all antennas is limited by P_{tot}. This power can be allocated arbitrarily among the transmit antennas, and the input covariance matrix has to satisfy the condition $\mathrm{tr}(\mathbf{Q}) \leq P_{tot}$. Let $\mathcal{S}_1$ denote the set of all power allocations which satisfy the sum power constraint, then $\mathcal{S}_1$ can be represented as

$$\mathcal{S}_1 := \{\mathbf{Q} \succcurlyeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}\}.$$

2) Per-antenna Power Constraints: In the per-antenna power constraints case [4], [7]–[10], [27]–[29], each individual transmit antenna has its own average power limitation $\hat{P}_i$, $\forall i \in \{1, ..., N_t\}$. In fact, there is no flexibility in allocating the transmit power over all transmit antennas. However, the antennas can fully cooperate with each other for the transmission. Thus, for the per-antenna power constraints, the input covariance matrix $\mathbf{Q}$ has diagonal elements satisfy $q_{ii} = \mathbf{e}_i^T\mathbf{Q}\mathbf{e}_i \leq \hat{P}_i$ with $\mathbf{e}_i$ is the i^{th} Cartesian unit vector. Let $\mathcal{S}_2$ denote the set of all power allocations which satisfy the per-antenna power constraints, then $\mathcal{S}_2$ can be represented as

$$\mathcal{S}_2 := \{\mathbf{Q} \succcurlyeq 0 : \mathbf{e}_i^T\mathbf{Q}\mathbf{e}_i \leq \hat{P}_i, i = 1, ..., N_t\}.$$

3) Joint Sum and Per-antenna Power Constraints: In this case, we combine the sum power and per-antenna power constraints. This means that each transmit antenna has a maximum individual transmit power budget of $\hat{P}_i$, $\forall i \in \{1, ..., N_t\}$ as in the per-antenna power constraints problem. Additionally, the sum power condition P_{tot} has to be satisfied as well. Let $\mathcal{S}_3$ denote the set of all power allocations which satisfy the joint sum and per-antenna power constraints, then $\mathcal{S}_3$ can be represented as

$$\begin{aligned}\mathcal{S}_3 &= \mathcal{S}_1 \cap \mathcal{S}_2 \\ &= \{\mathbf{Q} \succcurlyeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_i^T\mathbf{Q}\mathbf{e}_i \leq \hat{P}_i, i = 1, ..., N_t\}.\end{aligned}$$

In Fig. 1, the power constraint domains are illustrated with individual power constraints for two antennas and increasing sum power. Let $P_{tot,A} = \min(\hat{P}_1, \hat{P}_2)$ and $P_{tot,B} = \sum_{i=1}^{2} \hat{P}_i$, then we have three different cases of power domains as follows:

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- Case 1 - Sum power constraint domain: This domain occurs when the per-antenna power constraints are always inactive, i.e., $P_{tot} < P_{tot,A}$ [3].
- Case 2 - Per-antenna power constraints domain: This domain occurs when the sum power constraint is inactive, i.e., $P_{tot} > P_{tot,B}$ [4].
- Case 3 - Joint sum and per-antenna power constraints domain: This domain (gray area in Fig. 1) is considered when the power relations satisfy $P_{tot,A} \leq P_{tot} \leq P_{tot,B}$, i.e., both sum and power antenna power constraints can be active.

III. Problem Formulations and Solutions

In this section, we derive our main result on the optimal transmit strategy that achieves the capacity of the Gaussian MISO channel with joint sum and per-antenna power constraints. We first review the known results of the optimization problems with a sum power constraint only and per-antenna constraints only. After that, the optimization problem with joint sum and per-antenna power constraints will be studied.

A. Review of Known Results

1) Optimization Problem 1 (OP1) - Maximum Transmission Rate with Sum Power Constraint: This problem aims to find the maximum transmission rate in (2) under the set of sum power constraint S_1. The optimization problem for our given MISO channel, in this case, can be written as

$$\begin{aligned} &\text{maximize} && \log\left(1+\frac{1}{\sigma^2}\mathbf{h}^H\mathbf{Q}\mathbf{h}\right) \\ &\text{subject to} && \mathbf{Q}\in\mathcal{S}_1. \end{aligned} \tag{3}$$

The transmit strategy for the MISO channel is to send the information only in the direction of the channel vector $\mathbf{h}$ [1], [2]. The optimal solution is to perform beamforming using full power P_{tot} in the direction of the channel, i.e., $\mathbf{Q}^{(1)} = P_{tot}\mathbf{u}_1\mathbf{u}_1^H$ with $\mathbf{u}_1 = \mathbf{h}/\|\mathbf{h}\|$. The MISO channel capacity with a sum power constraint P_{tot} is

$$R^{(1)} = \log\left(1+\frac{P_{tot}}{\sigma^2}\sum_{i=1}^{N_t}|h_i|^2\right) = \log\left(1+\frac{P_{tot}}{\sigma^2}\|\mathbf{h}\|^2\right). \tag{4}$$

2) Optimization Problem 2 (OP2) - Maximum Transmission Rate with Per-antenna Power Constraints: In [4], Vu established the closed-form expression of the capacity and optimal transmit strategy for the single-user MISO channel with per-antenna power constraints. The capacity for this case can be found by solving the optimization problem

$$\begin{aligned} &\text{maximize} && \log\left(1+\frac{1}{\sigma^2}\mathbf{h}^H\mathbf{Q}\mathbf{h}\right) \\ &\text{subject to} && \mathbf{Q}\in\mathcal{S}_2. \end{aligned} \tag{5}$$

The problem in (5) can be solved by relaxing the semi-definite constraint, reducing the problem to be solvable in closed-form, and then showing that the optimal solution to the relaxed problem is also the optimal solution to the original problem [4]. In the per-antenna power constraints case, there is no power allocation among the antennas. Therefore, the transmit power from the i^{th} antenna is fixed to be $\hat{P}_i$. The optimal covariance matrix $\mathbf{Q}^{(2)}$ has rank one with $\mathbf{Q}^{(2)} = (\sum_{i=1}^{N_t}\hat{P}_i)\mathbf{v}_1\mathbf{v}_1^H$ where the beamforming vector $\mathbf{v}_1$ has the elements given as

$$v_k = \frac{\sqrt{\hat{P}_k}}{\sqrt{\sum_{i=1}^{N_t}\hat{P}_i}}\frac{h_k^*}{|h_k|}, k=1,...,N_t. \tag{6}$$

The capacity with per-antenna power constraints is then given as

$$\begin{aligned} R^{(2)} &= \log\left(1+\frac{1}{\sigma^2}\sum_{i=1}^{N_t}\hat{P}_i|\mathbf{h}^H\mathbf{v}_1|^2\right) \\ &= \log\left[1+\frac{1}{\sigma^2}\left(\sum_{i=1}^{N_t}|h_i|\sqrt{\hat{P}_i}\right)^2\right]. \end{aligned} \tag{7}$$

B. Optimization Problem 3 (OP3) - Maximum Transmission Rate with Joint Sum and Per-antenna Power Constraints

In the following proposition, we show that Gaussian distributed input is optimal for OP3.

Proposition 1. *Gaussian distributed input* $\boldsymbol{x}$ *with covariance* $\boldsymbol{Q} = \mathbb{E}\left[\boldsymbol{x}\boldsymbol{x}^H\right]$ *is capacity-achieving for the Gaussian MISO channel* (1) *with joint average sum and per-antenna power constraints, i.e.,* $\boldsymbol{Q}\in\mathcal{S}_3$.

Proof: The proof can be found in Appendix A. The achievability and converse proofs of this proposition can be derived from [1] and [30]–[33]. ■

Next, we are going to characterize the optimal transmit strategy, i.e., the optimal $\mathbf{Q}\in\mathcal{S}_3$.

1) Problem Formulation: The optimization problem to find the MISO channel capacity with joint sum and per-antenna power constraints is a convex optimization problem [34] given as follows

$$\begin{aligned} &\text{maximize} && \log\left(1+\frac{1}{\sigma^2}\mathbf{h}^H\mathbf{Q}\mathbf{h}\right) \\ &\text{subject to} && \mathbf{Q}\in\mathcal{S}_3. \end{aligned} \tag{8}$$

The objective function of problem (8) is concave while both constraints $\mathrm{tr}(\mathbf{Q}) \leq P_{tot}$ and $\mathbf{e}_i^T\mathbf{Q}\mathbf{e}_i \leq P_i\ \forall i\in\{1,...,N_t\}$ are linear in $\mathbf{Q}$. Furthermore, since $\log\left(1+\frac{1}{\sigma^2}\mathbf{h}^H\mathbf{Q}\mathbf{h}\right)$ is an increasing function in $\mathbf{h}^H\mathbf{Q}\mathbf{h}$, we can express the optimization problem in (8) as

$$\max_{\mathbf{Q}\in\mathcal{S}_3}\log\left(1+\frac{1}{\sigma^2}\mathbf{h}^H\mathbf{Q}\mathbf{h}\right) = \log\left(1+\frac{1}{\sigma^2}\max_{\mathbf{Q}\in\mathcal{S}_3}\mathbf{h}^H\mathbf{Q}\mathbf{h}\right). \tag{9}$$

Thus, we can equivalently focus on the following convex optimization problem

$$\begin{aligned} &\text{maximize} && \mathbf{h}^H\mathbf{Q}\mathbf{h} \\ &\text{subject to} && \mathbf{Q}\in\mathcal{S}_3. \end{aligned} \tag{10}$$

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2) Properties of the Optimal Transmit Strategy: The results in the following propositions will show that the optimal transmit strategy for joint sum and per-antenna power constraints is beamforming. The optimal transmission method is to transmit with the maximal sum power while the per-antenna power constraints have to be satisfied, i.e., at the optimum full transmit power is used. The phase is chosen to match the phase of the channel coefficient.

Proposition 2. *For OP3 with $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$ and a given channel $\boldsymbol{h} \in \mathbb{C}^{N_t \times 1}$ with $h_i \neq 0$, $\forall i \in \{1, ..., N_t\}$, beamforming is the optimal transmit strategy.*

Proof: The proof can be found in Appendix B. The key idea is to use Lagrange multiplier and slackness conditions of the necessary Karush-Kuhn-Tucker (KKT) conditions to show that the rank of $\mathbf{Q}^{(3)}$ has to be one at the optimum. ■

In the following, let $\mathbf{q}$ denote a beamforming vector of a rank one transmit strategy $\mathbf{Q}$, i.e., $\mathbf{Q} = \mathbf{q}\mathbf{q}^H$.

Proposition 3. *For OP3 with $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$ and a given channel $\boldsymbol{h} \in \mathbb{C}^{N_t \times 1}$ with $h_i \neq 0$, $\forall i \in \{1, ..., N_t\}$, the maximum transmission rate $R^{(3)}$ is achieved when the optimal transmit strategy $\boldsymbol{Q}^{(3)}$ uses full sum power P_{tot}, i.e., $\mathrm{tr}(\boldsymbol{Q}^{(3)}) = P_{tot}$.*

Proof: The proof can be found in Appendix C. The proof of Proposition 3 follows from the monotonicity of the rate function in terms of $\mathbf{Q}$. ■

Next, we focus on characterizing properties of the optimal beamforming vector $\mathbf{q}^{(3)}$.

Lemma 1. *Let $\boldsymbol{q}^{(3)}$ be the optimal beamforming vector corresponding to the optimal covariance matrix $\boldsymbol{Q}^{(3)}$. Then*

$$\boldsymbol{q}^{(3)} \in \mathbb{Q} := \left\{ \boldsymbol{q} : \boldsymbol{q} = \left[\frac{\sqrt{P_1} h_1^*}{|h_1|}, ..., \frac{\sqrt{P_{N_t}} h_{N_t}^*}{|h_{N_t}|} \right]^T, \boldsymbol{q}\boldsymbol{q}^H \in \mathcal{S}_3 \right\}.$$

Proof: Consider optimization problem (10) with the optimization domain $\mathcal{S}_3$, we have

$$\begin{aligned}
\max_{\mathbf{Q} \in \mathcal{S}_3} \mathbf{h}^H \mathbf{Q} \mathbf{h} &\overset{(a)}{=} \max_{\mathbf{q}:\mathbf{q}\mathbf{q}^H \in \mathcal{S}_3} |\mathbf{h}^H \mathbf{q}|^2 \\
&\overset{(b)}{=} \max_{\mathbf{q}:\mathbf{q}\mathbf{q}^H \in \mathcal{S}_3} |\sum_{i=1}^{N_t} |h_i| \sqrt{P_i} e^{j(\varphi_i - \varphi_{h_i})}|^2 \\
&\leq \max_{\mathbf{q}:\mathbf{q}\mathbf{q}^H \in \mathcal{S}_3} (\sum_{i=1}^{N_t} |h_i| \sqrt{P_i})^2 \\
&= \max_{\mathbf{q} \in \mathbb{Q}} (\sum_{i=1}^{N_t} |h_i| \sqrt{P_i})^2 \\
&\overset{(c)}{\leq} \max_{\mathbf{q}:\mathbf{q}\mathbf{q}^H \in \mathcal{S}_3} |\mathbf{h}^H \mathbf{q}|^2
\end{aligned} \tag{11}$$

where
(a) follows from Propositions 1 and 2,
(b) follows from the definition $h_i = |h_i| e^{j\varphi_{h_i}}$, $q_i = \sqrt{P_i} e^{j\varphi_i}$ with $\varphi_{h_i}, \varphi_i \in [0, 2\pi]$, and
(c) follows from the fact that $\mathbb{Q} \subseteq \{\mathbf{q} : \mathbf{q}\mathbf{q}^H \in \mathcal{S}_3\}$.

From (11) it follows that $\max_{\mathbf{Q} \in \mathcal{S}_3} \mathbf{h}^H \mathbf{Q} \mathbf{h} = \max_{\mathbf{q} \in \mathbb{Q}} |\mathbf{h}^H \mathbf{q}|^2$, i.e., the optimal beamforming vector $\mathbf{q}^{(3)}$ is in $\mathbb{Q}$. ■

3) Optimal Power Allocation for OP3: In the joint sum and per-antenna power constraints problem, Proposition 3 states that the capacity achieving transmit strategy always allocates full sum power P_{tot}. However, the optimal power allocation solution of OP1 may result in violating certain per-antenna power constraints.

In the following theorem, we will show how to allocate the powers for the MISO channel for the general case with an arbitrary number of transmit antennas. We will show that if there exists any antenna for which the optimal power allocation of OP1 exceeds the per-antenna power constraints of OP3, then for those the optimal power allocation is equal to the per-antenna power constraints and (10) reduces to a new optimization problem with a smaller total transmit power and a reduced number of channel coefficients.

Theorem 1. *Let $\mathcal{I} \subseteq \{1, \ldots, N_t\}$ and $\mathcal{P}_V := \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$, if $\mathcal{P}_V = \emptyset$ then $P_i^{(3)} = P_i^{(1)}$ $\forall i \in \mathcal{I}$, else $P_i^{(3)} = \hat{P}_i$ $\forall i \in \mathcal{P}_V$ and the remaining optimal powers can be computed by solving a reduced optimization problem*

$$\arg \max_{\boldsymbol{q}' \in \mathbb{Q}'} |\boldsymbol{h}'^H \boldsymbol{q}'|^2 \tag{12}$$

where $\boldsymbol{h}' = [h_i]_{i \in \mathcal{P}_V^c}^T \in \mathbb{C}^{|\mathcal{P}_V^c| \times 1}$, $\mathbb{Q}' := \{\boldsymbol{q}' : \sum_{i \in \mathcal{P}_V^c} |q_i|^2 \leq P_{tot} - \sum_{i \in \mathcal{P}_V} \hat{P}_i, |q_i|^2 \leq \hat{P}_i, i \in \mathcal{P}_V^c\}$ and $\mathcal{P}_V^c = \mathcal{I} \setminus \mathcal{P}_V$.

Proof: The proof can be found in Appendix D. Theorem 1 is proved in two steps. In the first step, we prove that $P_i^{(3)} = \hat{P}_i$ $\forall i \in \mathcal{P}_V$ by pointing out that the per-antenna power constraint is not active if the optimal power of OP1 solution on the i^{th} antenna does not exceed $\hat{P}_i$. After that, it is shown that the remaining problem can be reformulated as a reduced optimization problem using the properties in propositions and lemmas above. ■

It can be seen from Theorem 1 that if there exists an optimal power allocation of the OP1 solution which violates the per-antenna power constraint, then it is optimal to allocate power equal to the per-antenna power constraint for the corresponding antenna. When more power constraints are active, we have less freedom to allocate the power. In addition, Theorem 1 also shows a recursive process which leads to an efficient optimization algorithm. After satisfying the per-antenna power constraint on the power violated antenna, a reduced optimization problem with the smaller size of channel coefficient and total transmit power is formulated. The remaining optimal power allocation can be computed by solving that reduced optimization problem. The recursion finishes when all power constraints are satisfied. The number of iterations equals the times that the set of indices of optimal powers of the OP1 solution violating the per-antenna power constraints of OP3 is not empty. The following corollary states how the set of powers which violated constraints can be computed.

Corollary 1. *Let $\mathcal{P}_V^{(3)} := \{i \in \{1, ..., N_t\} : P_i^{(3)} = \hat{P}_i < P_i^{(1)}\}$ and K be the number of total iterations, then the set of violated power constraints is the union of such a set at each*

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iteration,

$$\mathcal{P}_V^{(3)} = \bigcup_{k=1}^{K} \mathcal{P}_V(k), \tag{13}$$

where $\mathcal{P}_V(k) = \{i \in \mathcal{I}(k) : P_i^{(1)} > \hat{P}_i\}$, $\mathcal{I}(k+1) = \mathcal{I}(k) \setminus \mathcal{P}_V(k)$ *and* $\mathcal{I}(1) = \mathcal{I}$ *with* $\mathcal{I} \subseteq \{1, ..., N_t\}$.

Proof: The proof can be found in Appendix E for completeness. ∎

Remark 1. *For a MISO channel with* N_t *transmit antennas and* $P_{tot} \leq \sum_{i=1}^{N_t} \hat{P}_i$, *the maximum number of violated per-antenna power constraints is* $N_t - 1$, *which also corresponds to the maximal number of iterations, i.e.,* $K \leq N_t - 1$.

4) Intersection Point: We discuss optima in the interesting joint sum and per-antenna power constraints domain, i.e., we have $P_{tot,A} \leq P_{tot} \leq P_{tot,B}$. In this domain, we can identify an intersection point where the trajectory of the sum power constraint only optimal power allocation intersects a per-antenna power constraint when increasing the allowed sum power for both setups. This point plays an important role since the power allocation behavior crossing this point changes and therewith the growth of the maximal achievable rate.

Proposition 4. *Let* $\bar{P}_{tot}$ *denote the sum transmit power at the intersection point. Then*

$$\bar{P}_{tot} = \frac{\sum_{i \in \mathcal{I}} |h_i|^2}{\sum_{j \in \mathcal{P}_V} |h_j|^2} \sum_{j \in \mathcal{P}_V} \hat{P}_j, \tag{14}$$

where $\mathcal{I} \subseteq \{1, \ldots, N_t\}$ *and* $\mathcal{P}_V := \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$.

Proof: The proof of the Proposistion 4 can be found in Appendix F. The proof idea of this proposition follows from the property that at the intersection point $P_j^{(3)} = \hat{P}_j$ for $j \in \mathcal{P}_V$ and $R^{(3)} = R^{(1)}$. ∎

In the next section, we propose an algorithm for optimal transmit strategy of MISO channels with joint sum and per-antenna power constraints derived from the analysis above.

IV. Algorithm for Optimal Transmit Strategy

We use OP1, Theorem 1, and Lemma 1 from the previous sections to provide an algorithm to compute the optimum power allocation and optimal transmit strategy for a MISO system with a given channel $\mathbf{h} = [h_1, ..., h_{N_t}]^T \in \mathbb{C}^{N_t \times 1}$, and joint sum and per-antenna power constraints where the per-antenna power constraints are denoted as $\hat{\mathbf{P}} = [\hat{P}_i, ..., \hat{P}_{N_t}]$ and the sum power constraint is denoted as $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$.

In Algorithm 1, we start with computing the optimal power allocation $\mathbf{P}^{(1)}$ of optimization problem OP1 with sum power constraint P_{tot} only. Since $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$, we know from the Proposition 3 that the optimal power allocation of OP3 always allocates full sum power. Therefore, when all powers satisfy the constraints, the optimal transmit strategy of OP1 and the optimal transmit strategy of OP3 are the same. In this situation, the optimal transmission rate is $R^{(3)} = R^{(1)}$. Otherwise, we have $R^{(3)} < R^{(1)}$. From Theorem 1, it follows that for any optimal transmit powers $P_i^{(1)}$ of OP1 which violates the maximum per-antenna power constraint, the optimal transmit power $P_i^{(3)}$ is set equal to $\hat{P}_i$. The number of antennas violating the per-antenna power constraints is $|\mathcal{P}_V|$ with $\mathcal{P}_V = \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$. In the next step, we need to find the optimal power allocation for the remaining antennas while their total power budget has reduced to $P_{tot} - \sum_{i \in \mathcal{P}_V} \hat{P}_i$. To do this, we simply repeat the computation of optimum power allocation of OP1 with a new total power $P_{tot} - \sum_{i \in \mathcal{P}_V} \hat{P}_i$ and a reduced channel defined as a new $\mathbf{h} \leftarrow [h_i]_{i \in \mathcal{I}}^T$ with $\mathcal{I} \leftarrow \mathcal{I} \setminus \mathcal{P}_V$. The algorithm stops when there is an OP1 solution with no per-antenna power constraint violated.

The optimal beamforming vector $\mathbf{q}^{(3)}$ and optimal transmit strategy $\mathbf{Q}^{(3)}$ of OP3 are then calculated using Lemma 1. The details of the algorithm are shown in Algorithm 1.

Algorithm 1: *OpTS*($\mathbf{h}, P_{tot}, \hat{\mathbf{p}}$): finding optimal transmit strategy for $P_{tot} < \sum_{i=1}^{N_t} \hat{P}_i$

Input : $\mathbf{h}, P_{tot}, \hat{\mathbf{p}}$
Output: $\mathbf{Q}^{(3)}$

1 Set of indices $\mathcal{I} := \{1, ..., N_t\}$.
2 Compute optimum power allocation $\mathbf{P}^{(1)}$ with the elements $P_i^{(1)}$, $i \in \mathcal{I}$ of OP1($\mathbf{h}$ and P_{tot}).
3 Denote $\mathcal{P}_V = \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$ as a set of indices of powers violating the per-antenna power constraints.
4 **if** $\mathcal{P}_V = \emptyset$ **then**
5     $P_i^{(3)} \leftarrow P_i^{(1)}$ for all $i \in \mathcal{I}$.
6   **Go to** 16.
7 **else**
8   **for** $i \in \mathcal{P}_V$ **do**
9         $P_i^{(3)} \leftarrow \hat{P}_i$.
10   **end for**
      `// Formulate the reduced problem`
11    $\mathcal{I} \leftarrow \mathcal{I} \setminus \mathcal{P}_V$,
12    $P_{tot} \leftarrow P_{tot} - \sum_{k \in \mathcal{P}_V} \hat{P}_k$,
13    $\mathbf{h} \leftarrow [h_i]_{i \in \mathcal{I}}^T$.
14 **end if**
15 **Return to** 2.
16 Compute optimal beamforming vector $\mathbf{q}^{(3)}$ from Lemma 1 using $[P_i^{(3)}]_{i=1}^{N_t}$, $[h_i]_{i=1}^{N_t}$.
17 Compute optimal transmit transmit strategy $\mathbf{Q}^{(3)} = \mathbf{q}^{(3)} \mathbf{q}^{(3)H}$.

V. Numerical Examples

In this section, we give some numerical examples to illustrate the theoretical results. We first show the power allocation behavior and the feasible power domains when fixing the per-antenna power constraints. After that, numerical examples to show the trends of the transmission rate in different power constraint domains with different transmit antenna configurations are discussed. The unit of power and transmission rate using in all examples in this paper are Watt and bps/Hz.

a) Power constraint domains: For the first numerical example, we consider a MISO 2×1 system with complex channel $h = [1.0984 + 0.7015i, -0.2779 - 2.0518i]^T$, noise

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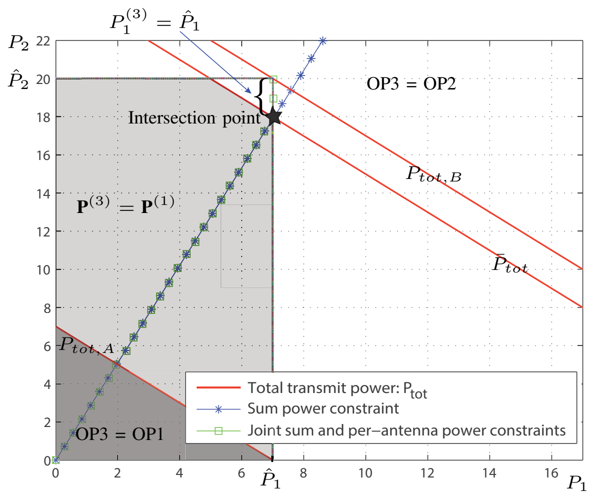

Fig. 2: Power allocation behavior with $\hat{P}_1 = 7$, $\hat{P}_2 = 20$.

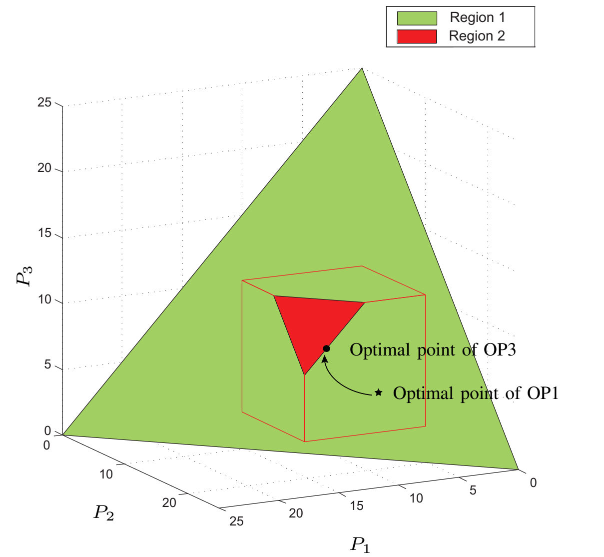

Fig. 3: MISO 3×1 optimal power allocation with joint sum and per-antenna power constraints where 'Region 1' is sum power constraint only region, and 'Region 2' is joint sum and per-antenna power constraints region when using full sum transmit power P_{tot}.

variance $\sigma^2 = 1$ and two per-antenna power constraints $\hat{P}_1 = 7$, $\hat{P}_2 = 20$ as shown in Fig. 2. Therewith, we have $P_{tot,A} = \min(\hat{P}_1, \hat{P}_2) = 7$ and $P_{tot,B} = \hat{P}_1 + \hat{P}_2 = 27$. In our simulation, we start to increase the total transmit power P_{tot} from 0 to 30 gradually. For any total transmit power $P_{tot} < P_{tot,A}$, the optimal solution of OP3 is the same as the optimal solution of OP1. Similarly, when $P_{tot} > P_{tot,B}$, the optimal solution of OP3 is the same as the optimal solution of OP2.

For the case of two transmit antennas, the intersection point can be identified at $\bar{P}_{tot} = ((|h_1|^2 + |h_2|^2)/|h_1|^2)\hat{P}_1$ if $[P_1^{(1)} = \hat{P}_1, P_2^{(1)} \le \hat{P}_2]$ or $\bar{P}_{tot} = ((|h_1|^2 + |h_2|^2)/|h_2|^2)\hat{P}_2$ if $[P_1^{(1)} \le \hat{P}_1, P_2^{(1)} = \hat{P}_2]$. Regarding the optimal power allocation behavior in this domain, we obtain from Fig. 2 that if $P_{tot,A} \le P_{tot} < \bar{P}_{tot}$, then the optimal power allocation satisfies $\mathbf{P}^{(3)} = \mathbf{P}^{(1)}$. Otherwise, the optimal power $P_1^{(1)}$ of the sum power constraint only problem violates the per-antenna power constraint $\hat{P}_1$ and $P_1^{(3)}$ is set equal to $\hat{P}_1$.

The plot in Fig. 3 shows the power allocation behavior in the case of three antennas. In this example, we consider a MISO 3×1 system with channel $h = [0.7 + 0.3i,\ 0.6 - 0.8i,\ -0.4+0.5i]^T$ and noise variance $\sigma^2 = 1$. The maximum transmit power on each antenna is set as $\hat{P}_1 = \hat{P}_2 = \hat{P}_3 = 10$ and total sum transmit power is $P_{tot} = 25$. The optimal power allocation region of OP3 is a polytope defined by $\{\mathbf{p} : P_1 \le 10, P_2 \le 10, P_3 \le 10, P_1 + P_2 + P_3 \le 25\}$. The optimum point of OP1 is found at the transmission rate $R^{(1)} = 5.0123$ with $P_1^{(1)} = 7.3$, $P_2^{(1)} = 12.6$, and $P_3^{(1)} = 5.1$. In this case, $P_2^{(1)} > 10$ violates the per-antenna power constraint $\hat{P}_2$ so that following Theorem 1, to satisfy the power constraint of OP3, $P_2^{(3)}$ is set as $P_2^{(3)} = \hat{P}_2$. Therewith, the optimal power allocation of OP3 is determined as $P_1^{(3)} = 8.8$, $P_2^{(3)} = 10$, and $P_3^{(3)} = 6.2$ at rate $R^{(3)} = 5.0017$. This point is allocated on the boundary of the joint sum and per-antenna power constraints region.

b) Optimal transmission rate examples: In this part, we illustrate the trend of the transmission rate in different optimization domains and with various number of antennas. The examples in Fig. 4 consider 2 to 5 antennas respectively. The setup of channel coefficient $\mathbf{h}_n$ and per-antenna power constraints $\hat{\mathbf{P}}_n$ corresponding to the number of antennas $N_t = 2, ..., 5$ are configured by taking the first N_t elements of $\mathbf{h} = [0.9572 + 0.8003i, 0.4854 + 0.1419i, 0.6759+0.5236i, 0.5231+0.2563i, 0.2254+0.6225]^T$ and $\hat{\mathbf{P}} = [\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4, \hat{P}_5] = [7, 10, 8, 5, 7]$. For instance, when we use two antennas, i.e., $N_t = 2$, then $\mathbf{h}_2 = [0.9572 + 0.8003i, 0.4854 + 0.1419i]^T$ and $\hat{\mathbf{P}}_2 = [\hat{P}_1, \hat{P}_2] = [7, 10]$. The noise variance is $\sigma^2 = 1$. The total transmit power increases

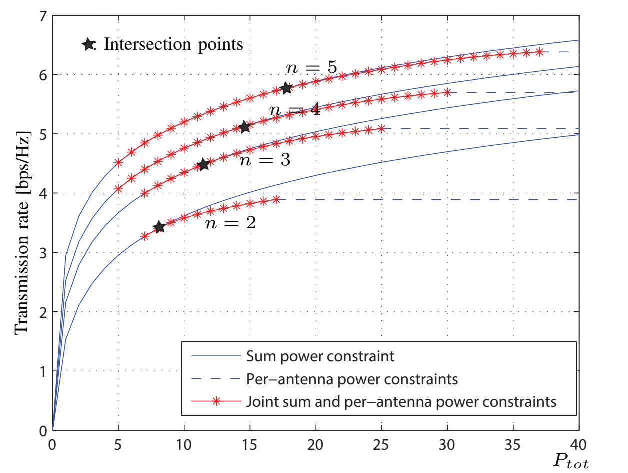

Fig. 4: Transmission rate in different power constraint domains and different transmit antenna configurations.

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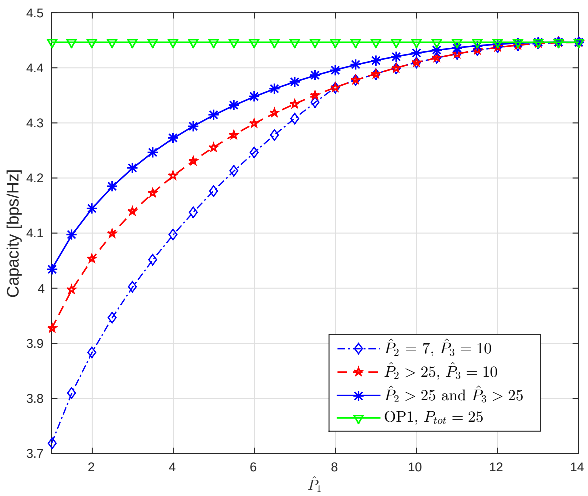

Fig. 5: The impact of choice of power constraints on the optimal power allocation and the capacity of 3×1 MISO channel with $P_{tot} = 25$. The marker symbols correspond to the following power constraint settings: sum power constraint (—▽—), additional per-antenna power constraints on P_1 (—∗—), P_1 and P_3 (−⋆−), and P_1, P_2 and P_3(−·◇·−).

from 0 to 40.

Since for cases 1 and 2 the optimal transmit strategies follow directly from OP1 and OP2, the case $P_{tot,A} \leq P_{tot} \leq P_{tot,B}$ is the most interesting. In Fig. 4, the optimal transmission rates are considered with respect to $\min(\hat{P}_i, \forall i = \{1, ..., N_t\}) \leq P_{tot,N_t} \leq \sum_{i=1}^{N_t} \hat{P}_i$ for $N_t = 2, ..., 5$. We observe that in these ranges of total transmit power, the optimal transmission rates of OP3 have similar trend as the optimal transmission rates of OP1, but the growth is slightly smaller.

The intersection points are identified when $P_{tot,N_t} = \bar{P}_{tot,N_t}$ where $\bar{P}_{tot,N_t} = (\sum_{i=1}^{N_t} |h_i|^2 / \sum_{j \in \mathcal{P}_V} |h_j|^2) \sum_{j \in \mathcal{P}_V} \hat{P}_j$ and j is summed over the indices of transmit antennas that violate the per-antenna power constraints. For instance, considering $N_t = 4$ transmit antennas, for which two antennas, antennas 2 and 3, violate per-antenna power constraints, then $\bar{P}_{tot,4} = (\sum_{i=1}^{4} |h_i|^2 / (|h_2|^2 + |h_3|^2))(\hat{P}_2 + \hat{P}_3)$. We see that $\bar{P}_{tot,N_t}$ changes with N_t. In this example, the intersection points are found as $\bar{P}_{tot,2} = 8.2$, $\bar{P}_{tot,3} = 11.5$, $\bar{P}_{tot,4} = 14.7$ and $\bar{P}_{tot,5} = 17.8$.

In Fig. 4, it is clear to see that when we keep a maximum sum transmit power while increasing the number of transmit antennas, it happens that $P_i^{(1)}$ violating $\hat{P}_i$ for a few antennas might not violates $\hat{P}_i$ for a larger number of antennas since we have more alternatives to allocate the power. Therefore, the gap between the optimal transmission rate with joint sum and per-antenna power constraints $R^{(3)}$ and the optimal transmission rate with sum power constraint $R^{(1)}$ is decreased.

c) Choices of power constraints: In this numerical example, we focus to clarify the impact of the choices of the power constraints on the optimal power allocation and the optimal transmission rate of the channel. The optimal transmission rate is shown with joint sum and per-antenna power constraints switching of a 3×1 MISO channel with $\mathbf{h} = [-1.2507 - 0.5078i, -0.9480 - 0.3206i, -0.7411 + 0.0125i]^T$. The total transmit power is set as $P_{tot} = 25$. The curves in Fig. 5 are plotted by adjusting per-antenna power constraint on antenna 1 from 0 to 14 and setting per-antenna power constraint configurations on antenna 2 and 3 as follows: (i) $\hat{P}_2 = 7, \hat{P}_3 = 10$, (ii) $\hat{P}_2 > 25, \hat{P}_3 = 10$, (iii) $\hat{P}_2 > 25, \hat{P}_3 > 25$. For those settings, it turns out that at the optimum, both per-antenna power constraints on antennas 2 and 3 are active in case (i), and only per-antenna power constraint on the antenna 3 is active in case (ii). For the last case, per-antenna powers on the antennas 2 and 3 are not restricted. In Fig. 5, the OP1 solution is also shown, which corresponds to the case when all per-antenna power constraints are not active. The OP1 optimal power allocation is $P_1^{(1)} = 13.51, P_2^{(1)} = 7.42, P_3^{(1)} = 4.07$. We can see from the figure that the optimal transmission rate decreases if more per-antenna power constraints are added. In particular, we can see from Fig. 5 that the capacity when all per-antenna power constraints are included is always smaller or equal than the others. For instance, when $\hat{P}_1 = 6$, the capacity of the case (i) reaches $R = 4.25$. This value is smaller than the capacity of case (ii) with $R = 4.3$; and both are smaller than the capacity of case (iii) with $R = 4.35$. This happens because of the fact that adding constraints limits the optimization domain, i.e., we have less freedom to allocate the power.

VI. Conclusions

In this paper, we derived the optimal power allocation for Gaussian MISO channels under joint sum and per-antenna power constraints. We further presented an iterative algorithm for this. The setup of the joint sum and per-antenna power constraints is relevant in practical systems where we have to limit the power in each RF chain for each antenna and the total radiated power across all transmit antennas due to regulation or other reasons. It is shown that beamforming is the optimal transmit strategy and the capacity is achieved when maximum sum power is used in the optimal transmit strategy. In the joint sum and per-antenna power constraints problem, the optimal powers are set equal to the per-antenna power constraints if their optimal values in sum power constraint only problem violate those per-antenna power constraints. The remaining powers can be found by solving a reduced optimization problem. Thus, we show that the optimal transmit strategy for the joint sum and per-antenna power constraints problem can be derived from the sum power constraint only and the per-antenna power constraints only problems.

Appendix

A. Proof of Proposition 1

1) Proof of Achievability: We use $\mathcal{C}(n, \mathbf{Q}, R, \epsilon)$ to denote a codebook with codewords $\mathbf{x}^n(m)$ for messages $m \in \{1, ..., M^{(n)}\}$ with $M^{(n)} = 2^{nR}$. This codebook is generated by selecting codewords of length n i.i.d. Gaussian with zero-mean and covariance $\mathbf{Q} - \rho \mathbf{I}$, where $\mathbf{Q} \in \mathcal{S}_3 := \{\mathbf{Q} \succcurlyeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, i = 1, ..., N_t\}$.

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Following [1, Theorem 8.6.5] and the proof of achievability steps in [30]–[32] with $\mathbf{Q} \in \mathcal{S}_3$, we know that the channel capacity $R^{(3)}$ for the joint sum and per antenna power constrained channel must satisfy

$$R^{(3)} \geq \sup_{\mathbf{Q}\in\mathcal{S}_3} R. \tag{15}$$

2) Proof of Converse: Following [1], [30]–[33], with a given codebook $\mathcal{C}(n, \mathbf{Q}, R, \epsilon)$, for a defined compact set $\mathcal{S}_3$ we obtain that $\frac{1}{n}\sum_{i=1}^{n}\mathbf{Q}_i \in \mathcal{S}_3$ since $\frac{1}{n}\sum_{i=1}^{n}\mathbf{Q}_i \succcurlyeq 0$, $\text{tr}(\frac{1}{n}\sum_{i=1}^{n}\mathbf{Q}_i) \leq P_{tot}$ and $\mathbf{e}_k^T\frac{1}{n}\sum_{i=1}^{n}\mathbf{Q}_i\mathbf{e}_k \leq \hat{P}_k, k = 1, ..., N_t$ hold. This implies that there exists a subsequence $(n_l)_{l\in\mathbb{N}}$ of the codeword length such that $\frac{1}{n_l}\sum_{i=1}^{n_l}\mathbf{Q}_i \to \mathbf{Q}$ as $n_l \to \infty$ with $\mathbf{Q} \in \mathcal{S}_3$. Then,

$$\begin{aligned} R &\leq \limsup_{n_l\to\infty}\{\log(1 + \frac{1}{\sigma^2}\mathbf{h}^H(\frac{1}{n_l}\sum_{i=1}^{n_l}\mathbf{Q}_i)\mathbf{h}) + \epsilon_{n_l}\} \\ &= \log(1 + \frac{1}{\sigma^2}\mathbf{h}^H\mathbf{Q}\mathbf{h}), \end{aligned} \tag{16}$$

which proves the converse. ■

B. Proof of Proposition 2

We denote $\mathbf{P} = \text{diag}\{\hat{P}_i\}$ as diagonal matrix of the per-antenna power constraints, P_{tot} as the total transmit power, $\mathbf{D} = \text{diag}\{\nu_i\}$ as diagonal matrix of Lagrangian multiplier for the per-antenna power constraints, μ as Lagrangian multiplier for the sum power constraint, and $\mathbf{K} \succcurlyeq 0$ as Lagrangian multiplier for the positive semi-definite constraint. Then the Lagrangian for problem (10) is given by

$$\mathcal{L} = \mathbf{h}^H\mathbf{Q}\mathbf{h} - \text{tr}[\mathbf{D}(\mathbf{Q}-\mathbf{P})] - \mu(\text{tr}(\mathbf{Q}) - P_{tot}) + \text{tr}(\mathbf{K}\mathbf{Q}). \tag{17}$$

Taking the first derivative and set it equal to zero, we have

$$\frac{\partial\mathcal{L}}{\partial\mathbf{Q}} = \mathbf{h}\mathbf{h}^H - \mathbf{D} - \mu\mathbf{I} + \mathbf{K} \stackrel{!}{=} 0 \tag{18}$$

or equivalently

$$\mathbf{h}\mathbf{h}^H = \mathbf{W} - \mathbf{K}, \tag{19}$$

where $\mathbf{W} = \mathbf{D} + \mu\mathbf{I}$.

Using the slackness condition $\mathbf{K}\mathbf{Q} = \mathbf{0}$, we obtain

$$\mathbf{h}\mathbf{h}^H\mathbf{Q} = \mathbf{W}\mathbf{Q}. \tag{20}$$

Since $\text{rank}(\mathbf{W}) = \text{rank}(\mathbf{D} + \mu\mathbf{I})$, which has full rank, at the optimum, we have

$$\text{rank}(\mathbf{Q}^{(3)}) \leq \text{rank}(\mathbf{h}\mathbf{h}^H) = 1. \tag{21}$$

Obviously, since $h_i \neq 0$, $\forall i \in \{1, ..., N_t\}$, $\text{rank}(\mathbf{Q}^{(3)}) = 0$ is not optimal. Therefore, the optimal rank of $\mathbf{Q}^{(3)}$ is one, i.e. beamforming is the optimal transmit strategy. ■

C. Proof of Proposition 3

Given function $f : \mathbf{Q} \to \mathbb{R}_+$ as defined in (2). Following [35] and [36], we obtain that $f(\mathbf{Q})$ is monotonic in terms of $\mathbf{Q}$. This implies that for any positive semi definite Hermitian matrices $\mathbf{Q}_1 \succcurlyeq \mathbf{Q}_2$, we have $f(\mathbf{Q}_1) \geq f(\mathbf{Q}_2)$.

Then, for OP3, if we suppose that $\mathbf{Q}^{(3)}$ is the optimal transmit strategy, the maximum transmission rate $R^{(3)} = f(\mathbf{Q}^{(3)})$ is achievable when $\mathbf{Q}^{(3)}$ allocates full sum power P_{tot}. ■

D. Proof of Theorem 1

Given function $f : \mathbf{Q} \to \mathbb{R}_+$ as defined in (2). For the proof of Theorem 1, we need following lemmas

Lemma 2. *Let $\mathcal{A} \subseteq \mathcal{I}$, $\mathcal{B} := \{i \in \mathcal{I} \setminus \mathcal{A} : P_i^{(\mathcal{S}(\mathcal{A}))} > \hat{P}_i\}$, and $\mathcal{A}' = \mathcal{A} \cup \mathcal{B}$. If $\mathcal{B} \neq \emptyset$ then $P_i^{(\mathcal{S}(\mathcal{A}'))} = \hat{P}_i$ $\forall i \in \mathcal{B}$, where $\mathcal{S}(\mathcal{A})$ and $\mathcal{S}(\mathcal{A}')$ are two given optimization domains defined as $\mathcal{S}(\mathcal{A}) := \{\boldsymbol{Q} \succcurlyeq 0 : \text{tr}(\boldsymbol{Q}) \leq P_{tot}, \boldsymbol{e}_j^T\boldsymbol{Q}\boldsymbol{e}_j \leq \hat{P}_j, j \in \mathcal{A}\}$.*

Proof (by contradiction): Since $\mathcal{S}(\mathcal{A}') \subseteq \mathcal{S}(\mathcal{A})$ we have

$$\max_{\mathbf{Q}\in\mathcal{S}(\mathcal{A})} f(\mathbf{Q}) \geq \max_{\mathbf{Q}\in\mathcal{S}(\mathcal{A}')} f(\mathbf{Q}). \tag{22}$$

For every $\mathcal{B}' \subseteq \mathcal{B}$, $\mathcal{B}' \neq \emptyset$ we have

$$\max_{\mathbf{Q}\in\mathcal{S}(\mathcal{A})} f(\mathbf{Q}) > \max_{\mathbf{Q}\in\mathcal{S}(\mathcal{A}\cup\mathcal{B}')} f(\mathbf{Q}). \tag{23}$$

If $\mathcal{B} \neq \emptyset$, suppose there exists $i \in \mathcal{B}$ for which $P_i^{(\mathcal{S}(\mathcal{A}'))} \neq \hat{P}_i$ is optimal. Since $P_i^{(\mathcal{S}(\mathcal{A}'))} \leq \hat{P}_i$ has to be satisfied for the opimization problem with domain $\mathcal{S}(\mathcal{A}')$, this implies that $P_i^{(\mathcal{S}(\mathcal{A}'))} < \hat{P}_i$ and therefore the per-antenna power constraint is not active, i.e., $\max_{\mathbf{Q}\in\mathcal{S}(\mathcal{A})} f(\mathbf{Q}) = \max_{\mathbf{Q}\in\mathcal{S}(\mathcal{A}\cup\{i\})} f(\mathbf{Q})$. However, this contradicts with (23) with $\mathcal{B}' = \{i\}$. Thus, it follows that $P_i^{(\mathcal{S}(\mathcal{A}'))} = \hat{P}_i$ $\forall i \in \mathcal{B}$. This proves Lemma 2. ■

Lemma 3. *Let $\boldsymbol{a} = [a_1, ..., a_n]^T \in \mathbb{C}^{n\times1}$, $\boldsymbol{x} = [x_1, ..., x_n]^T \in \mathbb{D} \subseteq \mathbb{C}^{n\times1}$ where $\mathbb{D}$ has the property that if $\boldsymbol{x} \in \mathbb{D}$, then $\boldsymbol{D}\boldsymbol{x} \in \mathbb{D}$ with arbitrary $\boldsymbol{D} = \text{diag}\{e^{j\varphi_k}\} \in \mathbb{C}^{n\times n}$ and $\varphi_k \in (0, 2\pi]$ $\forall k = 1, ..., n$. For $\boldsymbol{a}^T\boldsymbol{x} \geq 0$ and $c \geq 0$, we have*

$$\arg\max_{\boldsymbol{x}\in\mathbb{D}} |\boldsymbol{a}^T\boldsymbol{x} + c|^2 = \arg\max_{\boldsymbol{x}\in\mathbb{D}} |\boldsymbol{a}^T\boldsymbol{x}|^2. \tag{24}$$

Proof: We have

$$\begin{aligned} \arg\max_{\mathbf{x}\in\mathbb{D}} |\mathbf{a}^T\mathbf{x} + c|^2 &= \arg\max_{\mathbf{x}\in\mathbb{D}} |\mathbf{a}^T\mathbf{x}| + |c| \\ &= \arg\max_{\mathbf{x}\in\mathbb{D}} |\mathbf{a}^T\mathbf{x}| \\ &= \arg\max_{\mathbf{x}\in\mathbb{D}} |\mathbf{a}^T\mathbf{x}|^2. \end{aligned} \tag{25}$$

This proves Lemma 3. ■

Now, we prove Theorem 1. Since $\mathcal{S}_3 \subseteq \mathcal{S}_1$ where $\mathcal{S}_1 := \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}\}$, we have $\max_{\mathbf{Q}\in\mathcal{S}_3}\mathbf{h}^H\mathbf{Q}\mathbf{h} \leq \max_{\mathbf{Q}\in\mathcal{S}_1}\mathbf{h}^H\mathbf{Q}\mathbf{h}$. The equality occurs when $P_i^{(1)} = P_i^{(3)}$ $\forall i \in \mathcal{I}$, i.e., $P_i^{(1)} \leq \hat{P}_i$, $\forall i \in \mathcal{I}$. Otherwise there exists at least one power $P_i^{(1)}$ in the optimal power allocation of the OP1 solution that violates the per-antenna power constraints, i.e., $P_i^{(1)} > \hat{P}_i$ for some $i \in \mathcal{I}$, where the set of indices is defined as $\mathcal{P}_V := \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$.

Next, we will use Lemma 2 with $\mathcal{A} = \mathcal{I}\setminus\mathcal{P}_V$ and $\mathcal{B} = \mathcal{P}_V$. First note that with this definition of $\mathcal{A}$ and $\mathcal{B}$ we have

$$\max_{\mathbf{Q}\in\mathcal{S}_1} f(\mathbf{Q}) = \max_{\mathbf{Q}\in\mathcal{S}(\mathcal{A})} f(\mathbf{Q}) \tag{26}$$

and

$$\max_{\mathbf{Q}\in\mathcal{S}(A')} f(\mathbf{Q}) = \max_{\mathbf{Q}\in\mathcal{S}_3} f(\mathbf{Q}). \tag{27}$$

Then it follows from Lemma 2 that $P_i^{(3)} = \hat{P}_i$ $\forall i \in \mathcal{P}_V$.

Finally, we show that, in joint sum and per-antenna power constraints problem, the remaining optimal powers can be computed by solving a reduced optimization problem. We have

$$\begin{aligned}\arg\max_{\mathbf{Q}\in\mathcal{S}_3} f(\mathbf{Q}) &= \arg\max_{\mathbf{q}:\mathbf{q}\mathbf{q}^H\in\mathcal{S}_3} |\mathbf{h}^H\mathbf{q}|^2 \\ &\overset{(d)}{=} \arg\max_{\mathbf{q}\in\mathbb{Q}} |\sum_{i\in\mathcal{I}} |h_i|\sqrt{P_i^{(3)}}e^{j(\varphi_i-\varphi_{h_i})}|^2 \\ &\overset{(e)}{=} \arg\max_{\mathbf{q}\in\mathbb{Q}} (\sum_{i\in\mathcal{I}} |h_i|\sqrt{P_i^{(3)}})^2 \\ &\overset{(f)}{=} \arg\max_{\mathbf{q}\in\mathbb{Q}} (\sum_{i\in\mathcal{P}_V^c} |h_i|\sqrt{P_i^{(3)}} + \sum_{i\in\mathcal{P}_V} |h_i|\sqrt{\hat{P}_i})^2 \\ &\overset{(g)}{=} \arg\max_{\mathbf{q}'\in\mathbb{Q}'} |\mathbf{h}'^H\mathbf{q}' + \sum_{i\in\mathcal{P}_V} |h_i|\sqrt{\hat{P}_i}|^2 \\ &\overset{(h)}{=} \arg\max_{\mathbf{q}'\in\mathbb{Q}'} |\mathbf{h}'^H\mathbf{q}'|^2 \end{aligned} \tag{28}$$

where
(d) follows from the definition $h_i = |h_i|e^{j\varphi_{h_i}}$, $q_i = \sqrt{P_i^{(3)}}e^{j\varphi_i}$, $\forall i \in \mathcal{I}$ with $\varphi_{h_i}, \varphi_i \in [0, 2\pi]$,
(e) follows from Lemma 1,
(f) follows from $P_i^{(3)} = \hat{P}_i \ \forall i \in \mathcal{P}_V$,
(g) follows from substituting variables with $\mathbf{q}' = [\sqrt{P_i^{(3)}}e^{j\varphi_i}]_{i\in\mathcal{P}_V^c}$ and changing the optimization domain to $\mathbb{Q}'$,
(h) follows from Lemma 3 with $\mathbf{a} \leftarrow \mathbf{h}'$ and $\mathbf{x} \leftarrow \mathbf{q}'$, and $\mathbb{Q}' := \{\mathbf{q}' : \sum_{i\in\mathcal{P}_V^c} |q_i|^2 \le P_{tot} - \sum_{i\in\mathcal{P}_V} \hat{P}_i, |q_i|^2 \le \hat{P}_i, i \in \mathcal{P}_V^c\}$ satisfies the condition of a set $\mathbb{D}$.

Thus, we have shown that if $\mathcal{P}_V \neq \emptyset$ then $P_i^{(3)} = \hat{P}_i$ $\forall i \in \mathcal{P}_V$ and the remaining optimal powers can be allocated by solving $\arg\max_{\mathbf{q}'\in\mathbb{Q}'} |\mathbf{h}'^H\mathbf{q}'|^2$ which is equivalent to a reduced $\mathcal{S}_3$ problem. ■

E. Proof of Corollary 1

Let $\mathcal{P}_V(k) = \{i \in \mathcal{I}(k) : P_i^{(1)} > \hat{P}_i\}$ be the set of indices of optimal powers of the OP1 solution violating the per-antenna power constraints of OP3 at the k^{th} iteration. Consider optimization problem (10) with the optimization domain $\mathcal{S}_3$ and the set of all indices is $\mathcal{I} \subseteq \{1, ..., N_t\}$, the set of indices of optimal power allocations of the OP1 solution violating the per-antenna power constraints is given by

$$\mathcal{P}_V(1) = \{i \in \mathcal{I}(1) : P_i^{(1)} > \hat{P}_i\} \tag{29}$$

where $\mathcal{I}(1) = \mathcal{I}$. From Theorem 1, we know that if $\mathcal{P}_V(1) \neq \emptyset$, then $P_i^{(3)} = \hat{P}_i$ $\forall i \in \mathcal{P}_V(1)$. The reduced problem (12) with the set of all indices $\mathcal{I}(2) = \mathcal{I}(1) \setminus \mathcal{P}_V(1) = \mathcal{I} \setminus \mathcal{P}_V(1)$ is considered instead of (10) with the set of all indices $\mathcal{I}$. To find the remaining optimal power allocation, we must solve (12) in the next iteration. The set of indices of violated power allocations in this iteration is given by

$$\mathcal{P}_V(2) = \{i \in \mathcal{I}(2) : P_i^{(1)} > \hat{P}_i\}. \tag{30}$$

A new reduced problem can be formed if $\mathcal{P}_V(2) \neq \emptyset$.

The number of optimal powers of the OP1 solution violating the per-antenna power constraints of the OP3 in first two iterations is $|\mathcal{P}_V(1) \cup \mathcal{P}_V(2)|$.

Similarly, for the k^{th} iteration, the set of indices of violated power allocations is given by

$$\mathcal{P}_V(k) = \{i \in \mathcal{I}(k) : P_i^{(1)} > \hat{P}_i\} \tag{31}$$

where $\mathcal{I}(k) = \mathcal{I}(k-1) \setminus \mathcal{P}_V(k) = \mathcal{I} \setminus \{\cup_{i=1}^{k-1}\mathcal{P}_V(i)\}$. The number of optimal powers of the OP1 solution violating the per-antenna power constraints of the OP3 solution in first k iterations is $|\mathcal{P}_V(1) \cup \mathcal{P}_V(2) \cup ... \cup \mathcal{P}_V(k)|$.

Thus, assume that K is the number of iteration that the optimization problems have to reduce, it means that for any $l \ge K$, $\mathcal{P}_V(l) = \emptyset$, then the set of indices of total violated powers in joint sum and per-antenna power constraints is calculated as (13). ■

F. Proof of Proposition 4

At the intersecetion point, we have:

$$\bar{P}_{tot} = \sum_{k\in\mathcal{I}\setminus\mathcal{P}_V} P_k^{(3)} + \sum_{j\in\mathcal{P}_V} \hat{P}_j, \tag{32}$$

and it always holds

$$\sum_{i\in\mathcal{I}} |h_i|^2 = \sum_{k\in\mathcal{I}\setminus\mathcal{P}_V} |h_k|^2 + \sum_{j\in\mathcal{P}_V} |h_j|^2. \tag{33}$$

Furthermore, at the intersection point, we also have $R^{(1)} = R^{(3)}$. This implies

$$\bar{P}_{tot} \sum_{i\in\mathcal{I}} |h_i|^2 = \left(\sum_{k\in\mathcal{I}\setminus\mathcal{P}_V} |h_k|\sqrt{P_k^{(3)}} + \sum_{j\in\mathcal{P}_V} |h_j|\sqrt{\hat{P}_j} \right)^2. \tag{34}$$

Using (32) and (33), we can express (34) as the following equivalent equation

$$\frac{\sum_{k\in\mathcal{I}\setminus\mathcal{P}_V} P_k^{(3)}}{\sum_{j\in\mathcal{P}_V} \hat{P}_j} = \frac{\sum_{k\in\mathcal{I}\setminus\mathcal{P}_V} |h_k|^2}{\sum_{j\in\mathcal{P}_V} |h_j|^2}. \tag{35}$$

This can be reformulated as in (14) by using (32) and (33) once again. This proves Proposition 4. ■

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