Multiuser Diversity Orthogonal Frequency Division Multiple Access Systems

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Abstract

Multiuser diversity can be used to significantly increase system throughput in wireless communication systems. The idea is to schedule users when they experience good channel conditions and let them wait when the channels are weak. In this thesis, several aspects on multiuser diversity OFDMA systems are investigated. An adaptive reduced feedback scheme for multiuser diversity OFDMA is proposed. It significantly reduces the total feedback overhead while maintaining a multiuser diversity gain. The scheme uses clusters of sub-carriers as feedback units and only feeds back information about the fading peaks. Furthermore, an opportunistic beamforming scheme for clustered OFDM is presented and evaluated. A key aspect of the opportunistic beamforming scheme is that it increases the frequency fading of users with relatively flat channels, which increases the likelihood of being scheduled. Scheduling is an important aspect of multiuser diversity. A modified proportional fair scheduler is proposed in this thesis. It incorporates user individual target bit-rates and delays and a tunable fairness level. These features make the scheduler more attractive for future mixed service wireless systems. The use of the feedback information in the opportunistic beamforming process is discussed and evaluated. This extra information can help to increase the performance of unfairly treated users in the system. Several aspects of the proposed system are evaluated by means of simulation, using the 3GPP spatial channel model. In the simulations, the clustered beamforming performs better than three comparison systems. The modified proportional fair scheduler manages to divide the resources according to the user targets, while at the same time exploiting the multiuser diversity as well as the standard proportional fair algorithm. The thesis also includes results on coded packet error rate estimation from a channel realization by means of a two dimensional table. This can be useful in large network simulations as well as in designing adaptive modulation schemes.
Acknowledgements

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Patrick Svedman
Stockholm, December 2004
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Chapter 1

Introduction

1.1 Background

The last decade in the wireless world has meant a slow but steady move from pure voice services, like NMT and early GSM, to more mixed voice and data services, like SMS, WAP, WLAN and 3G. People are beginning to use their mobile devices more like their internet-connected stationary computers. Most mobile devices differ from stationary computers in significant ways, for example the screen size and the input interface. Still, we can expect an increasing demand for and usage of internet browsing, email, file downloading, multiplayer gaming, streaming media, video telephony, instant messaging, business applications etc. with mobile devices. This vision holds both opportunities and challenges for telecom operators and vendors. There is an opportunity to increase revenue by billing as many customers as possible for new services they are willing to pay for. One major challenge is to design future communication systems that can support many users while still offering a high enough user-perceived service quality. The “high enough” user-perceived service quality level depends on the price of the service. Generally, there is a tradeoff between the offered service quality and the number of users that the system can accommodate. Hence, the business model of the operator affects the desirable behavior of the communication system. An operator that aims at guaranteeing high service quality can support few, but highly billable, customers in the system. An operator that aims at offering medium service quality levels to more users, can instead bill more customers less.

The traditional design paradigm in the telecommunications indus-
try has been to guarantee the users a certain Quality-of-Service (QoS). Enough resources were given to each user to provide some margin for communication disturbances. The goal was an “anytime anywhere” service level. However, if the communication resources were already occupied, a new user would not be admitted to the system. The internet world has a slightly different perspective. New users are basically always admitted, but if the network or a particular service is heavily loaded some users might experience a degraded service level. The offered service level is “best effort”.

The last few years, “best effort” approaches have become more popular in wireless communication systems and research. This is mainly due to the emergence of the concept of multiuser diversity.

1.2 Multiuser Diversity

Diversity in communications is similar to the everyday life concept of diversity: “Do not put all the eggs in the same basket!”. If you drop the basket, all eggs are lost. Diversity in communications means that the same message is sent over several random channels. If one channel turns out to be bad, there is still a chance of successful reception through the other channels. The different channels can for instance be different frequencies (frequency diversity), different transmit and/or receive antennas (antenna diversity) or different time-instants (time diversity) [Pro01]. Multiuser diversity means that there are several users that want to communicate, each with different and time-varying communication channel quality [TV05]. The multiuser diversity gain can be obtained by letting the user with the best instantaneous channel quality communicate. A fundamental property of multiuser diversity is that the more users that are available, the higher expected channel quality of the best user.

Diversity is needed because the wireless channel quality is random. This randomness is also called fading. The major loss in signal power is usually due to the distance and objects between the transmitter and the receiver (path loss and shadow fading). This kind of fading changes relatively slowly. Fast fading however, arises from the constructive or destructive superposition of time-delayed copies of the signal, arriving via different propagation paths (multipaths). As the paths change, due to movement, the fading can change rapidly. Furthermore, if the time-delays of the different propagation paths differ significantly (time-dispersive channel), the fading may differ between different frequencies in the received
1.2 Multiuser Diversity

signal. This is called frequency-selective fading. The channel gains as a function of time of two flat fading users are illustrated in Figure 1.1. The user that moves fast is far away from the basestation and has consequently a higher path loss.

![Figure 1.1](image)

**Figure 1.1:** As an example, this figure shows how the channel gains of two users vary with time due to fast fading. The fast moving user is farther away from the base-station, which results in the lower average gain.

The multiuser diversity effect was first noticed in an information theoretic context. Knopp and Humblet studied the uplink of a single-cell, with users experiencing time-varying flat fading channels [KH95a]. It turns out that the sum capacity is maximized if, for each time-instant, the user with the best channel gain is allowed to transmit. The optimal solution also includes a power-control law which uses more transmit power for strong channels than for weak channels. This is an opposite strategy to conventional power control, which uses transmit power to compensate for weak channels. Related information theoretic results for fading multiuser
channels can be found in [KH95b, Tse97, TH98, HT98, LG01a, LG01b].

To exploit multiuser diversity, channel access has to be given to users with instantaneously good channel conditions. The scheduling scheme that achieves capacity gives channel access to the user with the best channel conditions. This maximum throughput scheme is potentially very unfair to users with weak slow fading condition, e.g. users that are far away. A more fair scheduler is the proportional fair (PF) scheduler [VTL02]; this scheduler is used in the downlink of the US standard for data delivery, IS-856 [Qua01]. The proportional fair scheduler tries to schedule a user when its instantaneous channel quality is high in relation to its average channel quality. The original PF scheduler is designed for flat fading users that are treated equally. Proportional fair scheduling is more suitable for data traffic than for voice. The reason is that users cannot be guaranteed any data-rate or delay. A more general PF scheduler is proposed and evaluated in this thesis, that can take individual user QoS needs into account when giving channel access to users. This can enable mixed services in the multiuser diversity system, even if QoS guarantees cannot be given.

In multiuser diversity schemes, the scheduler must know the instantaneous channel quality of the users. In a time division duplex (TDD) system, the same frequency band is used for both uplink and downlink communication during different time-slots. The downlink channel quality can be estimated from the uplink quality and vice versa, provided that the time variations are not too fast in relation to the duplex time. In a frequency division duplex (FDD) system, where the uplink and downlink communication are in different frequency bands, this is more difficult. Instead, the instantaneous downlink quality, for instance, can be fed back to the scheduler via the uplink. This signalling overhead for FDD multiuser diversity systems is addressed in this thesis.

### 1.3 Multiple Antennas

Multiple transmit and receive antennas can be used to provide diversity in space (antenna or spatial diversity). The receiver receives several different copies of the transmitted signal through different spatial channels. The different signal copies can be combined at the receiver to increase the communication reliability or to increase the data rate. The spatial channels can also be used to transmit different signals, so-called spatial multiplexing, which could increase the data rate significantly. An overview
of multiple-input multiple-output (MIMO) schemes is given in [GSS+03].

The tradeoff between spatial diversity and spatial multiplexing is studied in [ZT03].

An important difference between MIMO schemes is the degree of channel state information (CSI) at the transmitter. Several spatial diversity schemes like space-time coding require no CSI at the transmitter. This thesis addresses the downlink of a wireless communication system, which means that the base-station is the transmitter and the users are the receivers. With channel state information at the transmitter, it is possible to use the multiple transmit antennas to form a beam that directs the transmitted energy towards the desired user, so-called beamforming. If the channels to users in adjacent cells are known, it is often possible to form the beams in a way that reduces the interference, see for instance [Ben04]. MIMO schemes with partial CSI at the transmitter are for instance weighted orthogonal space-time block codes [Jön03] and beamforming with covariance feedback [VM01, JVG01, MBO04].

Multiuser diversity schemes rely on the fast fading of the users. If the users are moving slowly their fading will likely be slow, which can lead to a low multiuser diversity gain or long idle periods for the users. This problem can be reduced if opportunistic beamforming is applied. The traditional use of multiple antennas is to provide diversity to combat fading. Opportunistic beamforming is an opposite philosophy in that it aims at increasing the fading rate of the users [VTL02]. The opportunistic beamforming is not based on CSI at the transmitter, but random and is changed regularly. The random beamforming will give many users low signal strengths, but if there are many users in the system the beam will likely “hit” a user. In addition, compared to single antenna transmission, the interference level will fluctuate faster with opportunistic beamforming, called opportunistic nulling [VTL02]. Combined with a multiuser diversity scheduler, opportunistic beamforming enables multiuser diversity gain and fairness in addition to beamforming gain, even if users are moving slowly.

It is worthwhile to note that MIMO schemes that use explicit CSI at the transmitter usually require high quality channel estimates to work well [ZO03, YG04, Rup02]. For opportunistic beamforming, only the channel quality has to be estimated, which is less complex. Also, the overall channel quality can be represented with fewer bits than the full MIMO channel estimate. On the other hand is it feasible to schedule users in a round-robin fashion in CSI-at-the-transmitter schemes, enabling feedback of only one channel estimate at a time. For opportunistic schemes,
all users have to feed back all the time. Opportunistic schemes are more feasible in data systems where the requirements on QoS are lower. Furthermore, multiuser diversity schemes are more suitable in systems with many users, for obvious reasons.

1.4 OFDM

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation technique that is a promising candidate for future wireless broadband systems. It has been implemented in several wireless systems, such as Digital Audio Broadcast (DAB) [DAB01] and wireless LAN [IEE99]. It is also used in wired communication systems such as ADSL [ADS02].

OFDM divides the available bandwidth into orthogonal low-rate sub-carriers. Usually, the Discrete Fourier transform (DFT) is used to modulate the information onto the sub-carriers [WE71]. OFDM has several attractive features. For instance, the need for advanced equalization in multipath environments is avoided. OFDM is also attractive in a multiuser perspective. Since the bandwidth is divided into orthogonal sub-carriers, it is possible to schedule different users simultaneously on different frequency bands [WCLM99]. This enables an extension of the multiuser diversity concept to not only include temporal fading, but also fading in the frequency domain. Frequency division multiple access with OFDM (OFDMA), i.e. several simultaneous users on different frequencies, is more suitable in the downlink. For uplink OFDMA to work, the received signals from the users have to be synchronized in time and frequency, which is practically difficult [vdBBB+99].

However, there are several practical problems with OFDM. To maintain the orthogonality between the sub-carriers, carrier frequency synchronization has to be accurate and the guard period between consecutive OFDM symbols has to longer that the channel impulse response. Furthermore, OFDM signals suffer from a high peak-to-average power ratio (PAR), which puts high requirements on the power amplifiers and the dynamic ranges of digital-to-analog and analog-to-digital converters (DAC and ADC).

1.5 Outline and Contributions

In this section, an outline of the thesis is presented along with an overview of its contributions. The thesis addresses several issues related to mul-
tiuser diversity for the downlink of FDD OFDMA systems, like feedback, opportunistic beamforming and scheduling.

1.5.1 Chapter 2, Multiuser Diversity OFDM Systems

This chapter discusses several aspects of OFDM systems that exploit multiuser diversity. A review of previous work on multiuser diversity OFDM is followed by a presentation of the system model used in the following chapters. The practical problem of feedback reduction is addressed and an adaptive reduced feedback scheme is proposed and evaluated.

1.5.2 Chapter 3, Opportunistic Beamforming for OFDM

Opportunistic beamforming was originally designed for a flat fading single-carrier scenario [VTL02]. In this chapter, an extension to a multi-carrier scenario is proposed. The extension is compatible with the reduced feedback scheme proposed in the previous chapter. The work in Chapters 2 and 3 was published in


1.5.3 Chapter 4, Multiuser Diversity Scheduling for OFDM

In this chapter, first a review of multiuser OFDM resource allocation is presented. The single-carrier PF scheduler of [VTL02] is extended to incorporate user individual QoS requirements as well as a tunable fairness level. The proposed scheduler is presented in an OFDM context. Furthermore, a method to couple the opportunistic beamformer of Chapter 3 with the scheduler is presented. The work in this chapter was published in

1.5.4 Chapter 5, Numerical Results for Chapters 2-4

The system-level performance of feedback, beamforming and scheduling schemes are connected. The impact of different beamforming schemes on the system throughput depends on what kind of scheduling that is applied. The performance of different scheduling methods depends on what kind of CSI that is available, which is connected to the used feedback scheme. Therefore we present combined numerical simulation results on the different proposed schemes in Chapters 2-4. Comparisons to several alternatives are made. The results show that the clustered beamforming scheme performs better than the alternatives. The modified proportional fair scheduler manages to differentiate the QoS for most users according to their target QoS.

1.5.5 Chapter 6, Table-based Performance Evaluation

The work in this chapter is not directly related to the work in Chapters 2-5. This chapter addresses the problem of estimating the Packet Error Rate (PER) of a coded communication system from a particular channel realization. This issue is especially relevant in the simulation of large communication networks. In particular, the connection between the channel realization of a coded and interleaved OFDM system is studied. The work in this chapter was published in


A similar approach was presented in [LRZ03], which was published after the submission of [SBO04].

1.6 Notation

Plain letters, e.g. $a$ and $A$, are used for scalars. Boldface letters, e.g. and $\mathbf{A}$, are used for vectors and matrices respectively. Uppercase letters
are also used for “frequency domain" variables, so in some cases boldface uppercase letters are used for frequency domain vectors. Calligraphic uppercase letters, e.g. $A$ are used to denote sets.

- $\Pr(\text{event})$: The probability of event
- $\Pr(\text{event}_1|\text{event}_2)$: The probability of event$_1$, given event$_2$
- $E[X]$: The expected value of the random variable $X$
- $\text{var}(X)$: The variance of the random variable $X$
- $A \in \mathbb{C}^{M \times N}$: The elements of the matrix $A$, with $M$ rows and $N$ columns, are complex.
- $A \in \mathbb{R}^{M \times N}$: The elements of the matrix $A$, with $M$ rows and $N$ columns, are real.
- $||a||$: The Euclidean vector norm of $a$
- $a^T \ A^T$: The transpose of the vector $a$ and matrix $A$
- $a^* \ A^*$: The Hermitian transpose of the vector $a$ and matrix $A$
- $A^K$: The matrix $A$ multiplied $K$ times with itself
- $A^{1/2}$: The matrix square-root of $A$
- $\delta(t)$: The time-continuous Dirac delta-function
- $\delta(k)$: The time-discrete Kronecker delta-function
- $\text{sinc}(t)$: The sinc-function $\frac{\sin(\pi t)}{\pi t}$
- $x(t) \ast g(t)$: The convolution of $x(t)$ and $g(t)$
- $\mathcal{F}\{x(t)\}$: The Fourier transform of $x(t)$
- $\arg(a)$: The phase of the complex number $a$
- $\arg \max_{a} f(k)$: The $k$ that maximizes $f(k)$.
- $a \in [b, c)$: $b \leq a < c$
- $k \in \{0, \ldots, N - 1\}$: $k$ can take on the integer values between 0 and $N - 1$
- $\{a : f(a) = b\}$: The set of $a$ such that $f(a) = b$.
- $j$: The imaginary unit, $j^2 = -1$

### 1.7 Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog-to-Digital Converter</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Density Function</td>
</tr>
<tr>
<td>CL-BF</td>
<td>Clustered Beamforming</td>
</tr>
<tr>
<td>C/N</td>
<td>Sub-carrier-to-Noise power ratio</td>
</tr>
<tr>
<td>Conv-BF</td>
<td>Conventional Beamforming</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>Acronym</td>
<td>Definition</td>
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<tr>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>DAC</td>
<td>Digital-to-Analog Converter</td>
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<tr>
<td>DD</td>
<td>Delay Diversity</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DSL</td>
<td>Digital Subscriber Line</td>
</tr>
<tr>
<td>EQ-BF</td>
<td>Equal Beamforming</td>
</tr>
<tr>
<td>FDD</td>
<td>Frequency Division Duplex</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>H/2</td>
<td>HIPERLAN/2</td>
</tr>
<tr>
<td>HDR</td>
<td>High Data Rate</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>ICI</td>
<td>Inter Cell Interference</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter Symbol Interference</td>
</tr>
<tr>
<td>LAN</td>
<td>Local Area Network</td>
</tr>
<tr>
<td>M-PF</td>
<td>Modified Proportional Fair</td>
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<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>MISO</td>
<td>Multiple-Input Single-Output</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
</tr>
<tr>
<td>PER</td>
<td>Packet Error Rate</td>
</tr>
<tr>
<td>PF</td>
<td>Proportional Fair</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RR</td>
<td>Round Robin</td>
</tr>
<tr>
<td>SDMA</td>
<td>Spatial Division Multiple Access</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal to Interference plus Noise power Ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise power Ratio</td>
</tr>
<tr>
<td>TDD</td>
<td>Time Division Duplex</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunication System</td>
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Chapter 2
Multiuser Diversity OFDM Systems

2.1 Summary
This chapter contains:

- A multi-cell downlink OFDM baseband model.
- A reduced feedback scheme for multiuser diversity OFDM. By clustering adjacent sub-carriers and letting the users feed back only the strongest clusters, the fading can be exploited with significantly reduced feedback.
- A scheme for adaptive feedback rate per user as a function of the number of users. A problem with the reduced feedback scheme is that many sub-carriers can be left unscheduled when there is little feedback and few users. By letting each user feed back more when there are few active users, this can be avoided.

2.2 Why Multiuser Diversity and OFDM?
The original idea of multiuser diversity, also described in Section 1.2, is based on scheduling of flat fading users in time [KH95a, VTL02]. In time-dispersive multipath channels however, the users experience frequency-selective fading. Equalization is a way to combat this kind of fading [Pro01].
The basic principle is to amplify the weaker frequencies to effectively create a frequency flat channel. In a single-carrier system with frequency-selective fading and equalization at the receivers, energy is transmitted also on frequency bands that are in deep fades.

Instead, if OFDM is used to cope with the time-dispersive channels, the frequency-selective fading can be exploited. OFDM divides the communication channel into orthogonal sub-channels, called sub-carriers, each using a small part of the available frequency band. The frequency-selective fading will result in different sub-carrier gains. This enables the exploitation of multiuser diversity in the frequency domain, i.e. users can be scheduled also on their frequency fading peaks [SOAS03, WOS+03]. The effective frequency-domain channel will consist of orthogonal parts from different users. With a good resource allocation algorithm, transmission of energy on badly fading parts of the spectrum can be avoided.

Multiuser diversity systems use adaptive modulation instead of power control to achieve the target error rates. By scheduling users with good instantaneous channel conditions, higher order modulation can be used and high system throughput can be achieved [VTL02]. Multiuser diversity systems are more feasible for data systems than for voice systems, since no guarantees on QoS can be given. If the fading of a user is unfavourable, there is a risk for long delays. However, by applying OFDM, the users can expect a more even data flow than for a single carrier TDMA multiuser diversity system. This is because several user can be scheduled simultaneously on different frequencies.

In an FDD multiuser diversity system, the users feed back channel information to the base-station for each scheduling decision. If OFDMA is used, the amount of feedback required from the users increases significantly. Since users can be scheduled on different frequency sub-carriers, users must feedback measurement information about several sub-carriers. A related problem with fast scheduling schemes is the required feedforward overhead. Since the scheduling changes, the scheduler needs to inform the users about the new scheduling decisions. If the scheduling rate is high and all sub-carriers are assigned to different users, the feedforward overhead becomes significant. These issues are addressed in Section 2.4. Note that there are efforts to estimate the downlink channel from the uplink also in FDD systems [JMB01].

In the following chapters, the downlink of an FDD multiuser OFDMA system will be studied. The use of OFDMA in the multiuser downlink is more feasible than in the multiuser uplink (also called the multichannel). OFDMA in the uplink requires that the transmissions from
the users are synchronized so that all signal contributions are received simultaneously [vdBBB’99].

2.3 Multi-cell Downlink OFDM Baseband Model

In this section, the multiuser downlink baseband communication model is presented for single-input-single-output (SISO) channels and multiple-input-single-output (MISO) channels. The aim of this section is to motivate the simulation model in Chapter 5. Consider the downlink of an FDD OFDMA system with $I$ base-stations that are synchronized in time, i.e., the OFDM symbols are transmitted simultaneously from all base-stations. This synchronism is advantageous in OFDMA multiuser diversity systems, because it provides the maximum intercell interference (ICI) during the training periods if all sub-carriers are used during the training. The scheduling decisions in interfering cells can result in some sub-carrier not being used, which can give a lower, but not higher, interference level during the data transmission. For the same reason, it is advantageous if all cells use the same transmit power per sub-carrier during both training and data transmission. Adaptive power allocation also introduces extra signalling in the downlink if a different modulation mode for data transmission is decided for a user than the mode fed back based on training. This is avoided if the same transmit power is used during training and data transmission. In [SWCO04b], adaptive power allocation was considered in a single-cell study, but this thesis will assume a fixed transmission power per sub-carrier. Other assumptions are:

- The channel fading is constant during one OFDM symbol.
- The frequency error between all transmitters and receivers is zero.
- All transmitters and receivers use the same baseband sampling frequency.

2.3.1 SISO

Assume that the base-stations transmit with one antenna and the users receive with one antenna (single-input-single-output or SISO). Omitting the time index, the received symbol on sub-carrier $n$ of a user in cell 0,
synchronized to base-station 0 is

\[ y_n = \sqrt{P} e_n^0 G_n^0 + \sqrt{P} \sum_{i=1}^{I-1} e_n^i \tilde{G}_n^i + w_n \]  \hspace{1cm} (2.1)

where

- \( P \) is the transmit power per sub-carrier in all cells
- \( c_n^i \) is the complex-valued symbol transmitted on sub-carrier \( n \) by base-station \( i \),
- \( G_n^i \) is the complex-valued sampled frequency response on subcarrier \( n \) between base-station \( i \) and the user,
- \( \tilde{G}_n^i = e^{j \phi_n} G_n^i \) is the rotated sampled frequency response on subcarrier \( n \) between base-station \( i \) and the user,
- \( w_n \) is AWGN with variance \( \sigma_w^2 \).

See Appendix 2.A for more details on how the base-band model is derived. Here, it is assumed that all \( I \) base-stations transmit symbols on subcarrier \( n \). In case they do not, the corresponding base-stations can be removed from the ICI sum. In the simulations in Chapter 5, the sum of the interference signals will be assumed to be Gaussian distributed. Since frequency synchronization between all transmitters and receivers is assumed, the inter-carrier interference in the inter-cell interference signals is zero.

Assuming that \( E[|c_n^i|^2] = 1 \) and that the channel realizations are given, the instantaneous received SINR on sub-carrier \( n \) can be written as

\[
\text{SINR}_n = \frac{|G_n^0|^2}{\sum_{i=1}^{I-1} |G_n^i|^2 + \sigma_w^2/P}. \hspace{1cm} (2.2)
\]

### 2.3.2 MISO

Now, assume that all base-stations use \( M \) antennas to transmit one symbol and that all users have one receive antenna (multiple-input-single-output or MISO). Base-station \( i \) applies on sub-carrier \( n \) the beamforming vector \( b_n^i \) in order to distribute the symbol among the antennas. A user in cell 0 receives on sub-carrier \( n \)

\[ y_n = \sqrt{P} G_n^0 b_n^0 c_n^0 + \sqrt{P} \sum_{i=1}^{I-1} \tilde{G}_n^i b_n^i c_n^i + w_n \]  \hspace{1cm} (2.3)
where $T$ denotes transpose and

- $b_n^* \in \mathbb{C}^{M \times 1}$ is the beamforming vector on sub-carrier $n$, normalized so that $\|b_n^*\| = 1$ and
- $G_n^T = [G_{n,0}^* \cdots G_{n,M}^*] \in \mathbb{C}^{1 \times M}$ is the frequency domain channel vector from base-station $i$ to the user.

In this setup, the multiple antenna transmission is transparent to the user. The beamforming and MISO-channel can be combined into a complex scalar effective channel, $H_n^* = G_n^* b_n^*$. The received signal can then be written as

$$y_n = \sqrt{P} e_n^* H_n^0 + \sqrt{P} \sum_{i=1}^{I-1} c_n^i H_n^i + w_n. \tag{2.4}$$

Assuming that $E[|c_n|^2] = 1$ and that the channel realizations are given, the received SINR on sub-carrier $n$ can be written as

$$\text{SINR}_n = \frac{|H_n^0|^2}{\sum_{i=1}^{I-1} |H_n^i|^2 + \sigma_n^2 / P}. \tag{2.5}$$

which is similar to the SISO SINR, but with the SISO channels replaced by the effective channels, $H_n^i$. In the simulations in Chapter 5, the sum of the interference signals will be assumed to be Gaussian distributed.

### 2.4 Clustered OFDM for Multiuser Diversity

#### 2.4.1 Feedback Reduction

One of the main problems with FDD multiuser diversity OFDM systems is the large amount of feedback required from the users. Since different users can be scheduled on different frequency sub-carriers, users must feed back measurement information about each sub-carrier. We propose to reduce the feedback by:

- grouping adjacent sub-carriers into clusters and
- only feeding back information about the strongest clusters.

The quantization of the feedback is not treated in this thesis. In [Joh03, GA03, FEM03], it was shown that a significant multiuser diversity gain can be obtained, even if the feedback is heavily quantized. Clustered
OFDM and multiuser diversity were also studied in [SOAS03] and [WOS+03], where the reduced feedback aspect was not mentioned. A different feedback scheme for opportunistic OFDM was presented in [SNA04], where one bit per user and sub-carrier is allowed.

The amount of feedback can be very large if there are many users and sub-carriers in the system. In this section, we propose a way to reduce the amount of feedback in an FDD OFDM system exploiting multiuser diversity. The correlation between adjacent OFDM sub-carriers and the fact that the information about the strongest sub-carriers is most valuable to the scheduler is used.

Consider a system with $K$ active users using OFDM with $N$ sub-carriers. Let $M_{\text{mod}}$ be the number of modulation and coding modes that the system supports. To exploit multiuser diversity, the scheduler requires channel state information about the different users. The full channel state information consists of $K \times N$ complex numbers, and even more if multiple antennas are used. Since the multiuser diversity scheduler only requires supportable rates, $K N \log_2(M_{\text{mod}})$ bits have to be fed back. This feedback information can create a very large overhead. To reduce the amount of overhead without sacrificing too much in performance, the following measures can be taken. Since the correlation between sub-carriers generally is high, due to the limited delay spread in the channel, the $N$ sub-carriers can be divided into $Q$ clusters of $R$ adjacent sub-carriers [CDS96], which can be used as feedback units. The feedback information is a measure of the channel quality in the cluster, for instance, the minimum or average sub-carrier supportable rate within the cluster. In a well-designed system, the cluster-size, $R$, is chosen so that the sub-carriers within one cluster are highly correlated. This will be analyzed in more detail in Section 2.4.3 and Appendix 2.C. Clustering is illustrated in Figure 2.1.

![Figure 2.1: Illustration of clustering. The 64 sub-carriers are grouped in 16 clusters of 4 adjacent sub-carriers each ($N = 64$, $Q = 16$ and $R = 4$).]
Furthermore, a scheduler exploiting multiuser diversity usually does not schedule users on their weaker clusters. Hence, the amount of feedback information can be further reduced by letting each user feed back information only about its $S$ strongest clusters. This, however, introduces the need to also feed back the indices of $S$ clusters, which gives a total feedback rate of $SK \log_2(QM_{mod})$ bits per transmission block. The feedback reduction is illustrated in Figure 2.2. The total feedback rate per scheduling block as a function of the cluster-size, $R$, and the number of fed back clusters, $S$ is illustrated in Figure 2.3.

### 2.4.2 Adaptive Feedback Rate

When only a few users are active and each user feeds back a small number of clusters ($K$ and $S$ small), the spectral usage can be low. By spectral
usage we mean the fraction of the downlink bandwidth that is used. This is because the scheduler cannot reliably assign users and modulation levels to clusters that no user has fed back information about. A second problem with a low spectral usage is that a low average number of users compete for the same cluster, resulting in lower modulation levels, as shown in [WSC04]. It is not feasible to transfer transmit power from unused clusters to used clusters since it makes the interference in adjacent cells unpredictable. However, a low spectral usage can be avoided by introducing an adaptive feedback rate. If we assume a high correlation between the activity in the uplink and the downlink, it is feasible with a higher feedback rate per user when there are few active users in the downlink. Let the random variable $U_K \in S, \ldots, Q$ denote the number of clusters that all $K$ users feed back for one transmission block. The probability that less than $u$ of the $Q$ clusters are assigned by the scheduler, $P_{fb}(S, K) = \Pr(U_K \leq u)$, is a function of the number of users, $K$, and how many clusters they each feed back, $S$ (See Appendix 2.B). For a fixed probability, $P_{fb}(S, K)$, and $u$, the adaptive feedback rate $S = f(K)$

**Figure 2.3:** This figure shows the total feedback in bits per scheduling block for a cell with 32 users, 512 sub-carriers and 8 adaptive modulation modes as a function of the cluster-size, $R$, and the number of fed back clusters, $S$. This can be compared to the feedback without reduction, which would be almost 50 kbit per scheduling block.
can be computed so that the spectral usage, $E[U_K/Q]$, remains fairly constant. Information about at least one cluster should always be fed back, no matter how many users are active. To illustrate, Figure 2.4 compares the reduced feedback scheme with fixed $S$ with the adaptive reduced feedback scheme, where $S = f(K)$.

Since the number of active users in the downlink of a cell is a slowly changing parameter, the adaptive feedback scheme introduces negligible signalling overhead in the downlink. However, the uplink signalling frame structure complexity is increased due to the adaptive feedback rate per user.

### 2.4.3 Reduced Feedback - Reduced Performance?

The cluster-size is an important design parameter. A larger cluster-size implies that fewer clusters have to be fed back to maintain a specific spectral usage. However, it is important that the cluster bandwidth is lower than the coherence bandwidth of the channel. Otherwise, the reported supportable rate does not represent all the sub-carriers within the cluster. This will lead to less efficiency in the adaptive modulation.

In this section, a simple simulation example is given to illustrate the effect of the clusters-size. 32 users in one cell have channels generated according to a HIPERLAN/2 (H/2) channel model [MAS+98]. In this example, no inter cell interference (ICI) is considered. The base-station has one transmit antenna and the users have one receive antenna. The impulse responses are of the form

$$g(\tau) = \sum_{l=0}^{L-1} \beta_l \delta(\tau - lT_g)$$  \hspace{1cm} (2.6)

where $T_g = 10$ ns. The sample period of H/2 is $5T_g$. The channel taps $\beta_l$ are independent, complex Gaussian variables with an exponentially decaying power delay profile

$$E[|\beta_l|^2] = Ae^{-\gamma T_g}$$  \hspace{1cm} (2.7)

where $\gamma$ is the expected root mean square (RMS) delay spread and $A$ is chosen so that $E\left[\sum_{l=0}^{L-1} |\beta_l|^2\right] = 1$. It is assumed that all users have the same power delay profile, but independent channels. H/2 uses 64 sub-carriers and a cyclic prefix of 800 ns.
The downlink throughput during one OFDM symbol estimated as

$$\text{Throughput} = \frac{R}{2} \sum_{q \in \mathcal{Q}} \log_2(1 + \text{SNR}_q/\Gamma)$$

(2.8)

where $R$ is the number of sub-carriers per cluster, $\mathcal{Q}$ is the set of clusters that have been assigned to users, $\text{SNR}_q$ is the minimum instantaneous received SNR in cluster $q$ for the assigned user, and $\Gamma$ is the gap corresponding to a symbol error rate of $10^{-4}$ for QAM [CDEGDF95]. Note that $\mathcal{Q}$ does not necessarily contain all available clusters. It is assumed that all users estimate and feed back their SNR perfectly (or equivalently supportable rate). For each cluster in the downlink, the user with the highest reported SNR is scheduled. This maximizes the throughput in the downlink. Also note that the use of (2.8) assumes that the modulation can be perfectly adapted to the reported SNRs.

Figure 2.5 shows the downlink throughput in bits per OFDM symbol for different cluster-sizes, $R$, as a function of the mean RMS delay spread of the channels, $\gamma$. Each user feeds back the minimum sub-carrier supportable rate within the 4 strongest clusters ($S = 4$). The throughput of the full feedback system is also shown, in which the users feed back the SNR’s of all sub-carriers. The full feedback scheme performs best, since the scheduler can select the user with the highest SNR on each sub-carrier. For the reduced feedback scheme, the throughput depends on the channel delay spread. For relatively flat channels (low delay spread), large clusters ($R = 8$ and $R = 16$) give the highest throughput. The reason for this is twofold. Firstly, the feedback value for each cluster is highly representative for all sub-carriers within the cluster, which gives high accuracy in the adaptive modulation. Secondly, users with strong channels will likely experience high SNR’s across all sub-carriers, due to the low delay spread. The feedback of four large clusters enables the scheduler to allocate a large portion of the OFDM symbol to the strongest user. For large clusters, the probability that some clusters are left unassigned (since no user fed them back) is very low. For low delay spread and small clusters ($R = 1$), the strongest users will be assigned only a small part of the OFDM symbol. Also, the expected spectral usage for $R = 1$ is 87% whereas it is 100% for $R \geq 4$. For increasing delay spread, the frequency selectivity of the channels increases. This is exploited by the schemes using smaller clusters, since the users can be scheduled only on their frequency fading peaks. Large clusters are not suitable for larger delay spreads since there can be significant channel variations within the cluster. Since the SNR of the weakest sub-carrier within the cluster is
fed back in this example, the base-station will use a too low modulation level for most sub-carriers within the large clusters.

From this example, one can see that a suitable cluster-size depends on the average channel delay spread of the users. Since it is not possible to use different cluster-sizes for different users, a compromise that gives good performance for most expected channels is a suitable choice. For this example and delay spread in the range of Figure 2.5, a cluster-size of 2 or 4 sub-carriers seems to be appropriate. For the channel model used in this example, it is possible to derive the probability that a sub-carrier within a cluster deviates significantly from the average within the cluster. It is a function of the covariance matrix of the channel as well as the cluster-size. The details are given in Appendix 2.C.

2.4.4 Feedforward Overhead

As discussed in the previous sections, the overhead of measurement information from the users back to the base-station can be significant. However, the feedforward overhead can also be significant in an adaptive resource allocation system, especially if the reallocation rate is high. The scheduler has to inform the users about the scheduling decisions. If there are $K$ users in the cell, competing for the $Q$ clusters, the feedforward overhead is $Q \log_2(K)$ bits per scheduling block. For each cluster, the base-station can start the block by sending an identification number of the user ($\log_2(K)$ bits) that was allocated to the cluster. Unfortunately, the base-station has to use the lowest modulation order to send the user identification, in order to guarantee that all users competing for the particular cluster can receive the user identification correctly. The modulation order that will be used in the cluster during the rest of the scheduling block does not have to be sent, since the selected user knows which supportable rate it fed back. If the resource allocation algorithm is allowed to change the power allocation and adaptive modulation level, also the selected modulation level has to be fed forward.
Figure 2.4: In (a)-(d), the adaptive feedback rate scheme is compared to the fixed feedback scheme as a function of the number of users, $K$. In this example, the number of sub-carriers $N = 512$, the cluster-size $R = 4$ and the number of modulation levels $M_{\text{mod}} = 8$. For the fixed feedback scheme, each user feeds back the modulation level and index of the 8 strongest clusters ($S = 8$). For the adaptive feedback rate, $S$ is chosen so that the probability that less than 75% of the clusters are fed back is $\approx 10\%$. In (a), this probability is shown. The abrupt changes in the probability for the adaptive scheme for large $K$ is due to the integer granularity of $S$. In (b), the resulting $S$ for the adaptive scheme is plotted together with the fixed $S$ for the fixed scheme. For few users ($K < 25$), each user feeds back more information than in the fixed scheme, but less when there are many users ($K > 25$). From (c), it is clear that the adaptive feedback scheme results in a relatively constant expected spectral usage. The total feedback rate in bits per scheduling block is depicted in (d). Since, the feedback rate per user is fixed for the fixed scheme, the total feedback rate increases linearly with the number of users.
Figure 2.5: In this figure, the total cell throughput in bits per OFDM symbol is plotted as a function of the RMS delay spread of the channel. In the full feedback scheme, each user feeds back information about all subcarriers (no clustering or reduced feedback). For the reduced feedback scheme, large cluster-sizes, $R$, are suitable for relatively flat channels (low delay spread), whereas small cluster-sizes are more suitable for more frequency-selective channels (large delay spread). In this example, the number of users is 32. In the reduced feedback scheme, each user feeds back the information about the 4 strongest clusters.
Appendix 2.A  Baseband Model

In this section, the baseband model for multiuser multicell downlink OFDM communication is studied. Carrier frequency modulation methods of the baseband OFDM signal do not differ from the traditional [Pro01]. In this thesis, perfect frequency synchronization between transmitters and receivers is assumed. An FDD system is considered, where all base-stations transmit on the same frequency band, but a different band than the users use for uplink communication. Therefore, there is no interference between users in the downlink, but only from other base-stations. Hence, it is sufficient to study the communication link to one user.

Assume that $I$ base-stations are deployed in the area of the user. The channels from all base-stations to the user are assumed to be time-dispersive, with impulse responses $g^i(t)$, where $i$ is the base-station index. The time dispersion of each channel impulse response, $\Delta^i$, is at most $\Delta_{CP}$, which is the length of the OFDM cyclic prefix [PR80]. The time-delays between the base-stations and the user, $\tau^i$, are included in the impulse responses, so that $g^i(t) = 0$ for all $t < \tau^i$ and for all $t \geq \Delta^i + \tau^i$.

Base-station $i$ transmits baseband OFDM symbols

$$x^i(t) = \sum_{l=-\infty}^{\infty} x^i_l(t - lT_s)$$

where $T_s = NT + \Delta_{CP}$ is the OFDM symbol period, $N$ is the number of sub-carriers, and $x^i_l(t)$ is the $l$th OFDM symbol:

$$x^i_l(t) = \left\{ \begin{array}{ll} \sum_{k=0}^{N-1} c^i_k e^{j2\pi kt/NT} & \text{when } t \in [-\Delta_{CP}, NT) \\ 0 & \text{otherwise} \end{array} \right.$$  \hspace{1cm} (2.10)

Assume that the studied user communicates with base-station 0. The user is assumed to be perfectly synchronized to this base-station. For convenience, let $\tau^0 = 0$, which means that $\tau^i$ can be seen as the difference in physical delay to the user between base-station $i$ and 0. The user receives the signal

$$r(t) = x^0(t) * g^0(t) + \sum_{i=1}^{I} x^i(t) * g^i(t) + z(t) = r^0(t) + r_{ICI}(t) + z(t)$$  \hspace{1cm} (2.11)

where $*$ denotes convolution and $z(t)$ is AWGN. The received signal consists of the desired signal, $r^0(t)$, inter-cell interference, $r_{ICI}(t)$, and
Figure 2.6: The received signal is the sum of the received signals from all base-stations \( r^i(t) \) (plus noise). Since the user is synchronized to base-station 0, the delay \( \tau^0 \) can be set to zero. The delays from the other base-stations \( \tau^i \) are then the additional delay compared to base-station 0.

The timing relation between the different terms in the sum of received signal is illustrated in Figure 2.6.

AWGN, \( z(t) \). We can study the reception of one symbol without loss of generality. The received signal \( r(t) \) is sampled at \( t = mT \) for \( m = 0, \ldots, N - 1 \), which means that the cyclic-prefix samples are thrown away. The decision variables are obtained after DFT of the samples.

\[
y_n = \frac{1}{N} \sum_{m=0}^{N-1} r(mT)e^{-j2\pi mn/N} = y_n^0 + y_n^{\text{ICI}} + w_n \tag{2.12}
\]

for \( n = 0 \ldots N - 1 \). The desired signal part is successfully demodulated.
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\[ y_n^0 = \frac{1}{N} \sum_{m=0}^{N-1} \int_0^{\Delta^0} g^0(\tau) x^0(mT - \tau) d\tau \ e^{-j2\pi mn/N} \]

\[ = \frac{1}{N} \sum_{m=0}^{N-1} \int_0^{\Delta^0} g^0(\tau) \sum_{k=0}^{N-1} c_k^0 e^{j2\pi k(mT - \tau)/NT} d\tau \ e^{-j2\pi mn/N} \]

\[ = \frac{1}{N} \sum_{k=0}^{N-1} c_k^0 \int_0^{\Delta^0} g^0(\tau) e^{-j2\pi k\tau/NT} d\tau \sum_{m=0}^{N-1} e^{-j2\pi m(n-k)/N} \]

\[ = \sum_{k=0}^{N-1} c_k^0 G^0\left(\frac{k}{NT}\right) \delta(n - k) \]

\[ = c_n^0 G^0\left(\frac{n}{NT}\right) = c_n^0 G_n^0 \tag{2.13} \]

where \( G^0(f) \) is the Fourier transform of \( g^0(t) \). Hence, the desired signal part in the demodulated signal is the desired symbols \( c_n^0 \) multiplied with the frequency response of the channel, \( G^0(f) \), sampled at \( f = n/NT \).

The inter-cell interference term, \( y_{n IC1}^n \), contains contributions from all interfering base-stations. Due to the different propagation times to the user, the sampling and demodulation will not be correctly synchronized. Let \( \mathcal{I}_{n IS1} \) denote the set of interfering cells for which \( \Delta^1 + \tau^1 < \Delta_{CP} \). Sampling the signal from any cell in this set at \( t = mT \) for \( m = 0, \ldots, N-1 \) gives samples from only one OFDM symbol, hence no ISI. Let \( \mathcal{I}_{IS1} \) denote the other cells. Hence, \( y_n^1 = y_{n IC1}^n + y_{n IS1}^n \). Sampling and demodu-
lution of the signals from the cells in $I_{\text{No ISI}}$ gives

$$
y_{n}^{\text{No ISI}} = \sum_{i \in I_{\text{No ISI}}} \frac{1}{N} \sum_{m=0}^{N-1} \int_{\tau^{i}}^{\tau^{i}+\Delta^{i}} g^{i}(\tau)x^{i}(mT - \tau) d\tau \ e^{-j2\pi mn/N}
$$

$$
= \sum_{i \in I_{\text{No ISI}}} \frac{1}{N} \sum_{m=0}^{N-1} \int_{0}^{\tau^{i}+\Delta^{i}} g^{i}(\tau) \sum_{k=0}^{N-1} c_{k} e^{j2\pi k(mT-\tau)/NT} d\tau \ e^{-j2\pi mn/N}
$$

$$
= \sum_{i \in I_{\text{No ISI}}} \frac{1}{N} \sum_{k=0}^{N-1} c_{k} \int_{0}^{\tau^{i}+\Delta^{i}} g^{i}(\tau) e^{-j2\pi k(mT-\tau)/NT} d\tau \ \sum_{m=0}^{N-1} e^{-j2\pi m(n-k)/N}
$$

$$
= \sum_{i \in I_{\text{No ISI}}} \frac{1}{N} \sum_{k=0}^{N-1} c_{k} \int_{0}^{\Delta^{i}} g^{i}(\tau + \tau^{i}) e^{-j2\pi k(mT-(\tau^{i}+\tau^{i}))/NT} d\tau^{i} \ \sum_{m=0}^{N-1} e^{-j2\pi m(n-k)/N}
$$

$$
= \sum_{i \in I_{\text{No ISI}}} \sum_{k=0}^{N-1} c_{k}^i \tilde{G}^{i}(\frac{N}{NT}) \delta(n-k)
$$

$$
= \sum_{i \in I_{\text{No ISI}}} c_{n}^i \tilde{G}^{i}(\frac{N}{NT}) = \sum_{i \in I_{\text{No ISI}}} \tilde{c}_{n}^{i} \tilde{c}_{n}^{i} \quad (2.14)
$$

where $\tilde{G}^{i}(f) = e^{-j2\pi f\tau^{i}} \mathcal{F}[g^{i}(t + \tau^{i})]$, which is the rotated frequency response of the non-delayed channel, $g^{i}(t + \tau^{i})$, from the $i$th base-station.

The channels from the base-stations that belong to the set $I_{\text{ISI}}$ have a time-delay $\tau^{i}$ and an impulse response length $\Delta^{i}$ such that $\tau^{i} + \Delta^{i} \geq \Delta_{CP}$. Some of the receiver samples at $mT$, $m \in \{0, \ldots, N-1\}$, contain contributions from two different OFDM symbols, which makes the $y_{n}^{\text{ISI}}$ term more complicated. Demodulation by DFT of a sum of interfering non-synchronized signals is closely related to the phenomenon of cross-talk in DSL, and has been investigated thoroughly in this context [GK02].

According to [SCM97], $y_{n}^{\text{ISI}}$ can be written as

$$
y_{n}^{\text{ISI}} = \sum_{i \in I_{\text{ISI}}} c_{n}^i \frac{N - c_{n}^i}{N} \tilde{G}^{i}(\frac{n}{NT}) + \tilde{w}_{n}^{i} \quad (2.15)
$$
where $\epsilon_i$ is the number of samples within the DFT window that contain contributions from two symbols, and $\tilde{w}_i^n$ is an extra noise term that is introduced since the orthogonality of the system is disturbed. It can be assumed that $\sum_{i \in \mathcal{I}_1} \tilde{w}_i^n = \tilde{w}_n$ is approximately Gaussian, which can be motivated by the general central limit theorem for sums of unequally distributed random variables [Cra61]. For simplicity, assume that $\epsilon_i \ll N$ for all $i$, resulting in

$$y_1^\text{n} = \sum_{i \in \mathcal{I}_1} c_i^j \hat{G}^i \left( \frac{n}{NT} \right)$$  \hfill (2.16)

It is reasonable to assume that the ICI signals with most delay are also among the weakest, which helps to motivate the previous approximations. To reduce the effects of asynchronous interference signals, windowing of the OFDM symbols can be applied [WE71]. This technique removes or reduces the discontinuities between consecutive OFDM symbols.

The sampling and demodulation of the receiver noise $z(t)$ is as for standard OFDM. As in [Wil03], assume that the receiver has a filter $f(t)$ that band-limits the signal, with frequency response

$$F(f) = \begin{cases} 1 & f \in [-\frac{K}{T}, \frac{K}{T}] \\ 0 & \text{otherwise} \end{cases}$$  \hfill (2.17)

for some large integer $K$. Then consider the filtered noise $z_f(t) = z(t) * f(t)$, where $f(t) = \frac{2K}{T} \text{sinc} \left( \frac{2Kt}{T} \right)$. Furthermore, assume that $K$ is large enough so that the fact that the frequencies of the OFDM signals outside the filter were cut off can be ignored. To avoid aliasing in the sampled received signal, the receiver sampling frequency has to be at least $\frac{2K}{T}$.
The autocorrelation of the noise becomes

\[
E[z_f(t)z_f(s)] = \frac{4K^2}{T^2} E[ \int \int z(t - \tau) \text{sinc}(\frac{2\pi K}{T}) z(s - \sigma)^* \text{sinc}(\frac{2\sigma K}{T}) d\tau d\sigma ]
\]

\[
= \frac{2N_0 K^2}{T^2} \int \int \delta(t - \tau - s + \sigma) \text{sinc}(\frac{2\tau K}{T}) \text{sinc}(\frac{2\sigma K}{T}) d\tau d\sigma
\]

\[
= \frac{2N_0 K^2}{T^2} \int \text{sinc}(\frac{2\tau K}{T}) \text{sinc}(\frac{2\tau K}{T}) d\tau
\]

\[
= \frac{N_0}{2} \int F(f) e^{j2\pi f(s-t)} df
\]

\[
= \frac{N_0 K}{T} \text{sinc} \left( \frac{(s-t)2K}{T} \right)
\]

(2.18)

where \(N_0/2\) is the power spectral density of \(z(t)\). Sampling the filtered noise and demodulating by DFT gives the frequency domain noise \(w_n\) with autocorrelation

\[
E[w_n w_{n+m}] = E \left[ \frac{1}{N} \sum_{n=0}^{N-1} z_f(nT)e^{-j2\pi n/N} \frac{1}{N} \sum_{k=0}^{N-1} z_f(kT)^* e^{j2\pi km/N} \right]
\]

\[
= \frac{N_0 K}{N^2 T} \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} \text{sinc}(2K(n-k)) e^{j2\pi (km-ln)/N}
\]

\[
= \frac{N_0 K}{N^2 T} \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} \delta(n-k) e^{j2\pi (km-ln)/N}
\]

\[
= \frac{N_0 K}{N^2 T} \sum_{n=0}^{N-1} e^{j2\pi n(m-l)/N} = \frac{N_0 K}{N T} \delta(m-l)
\]

(2.19)

Hence, \(w_n\) is additive white Gaussian noise with variance \(\frac{N_0 K}{N T}\).

Combining (2.12) - (2.19) gives

\[
y_n = c_n^0 G_n^0 + \sum_{i=1}^{I} c_i \tilde{G}_n^i + w_n
\]

(2.20)
Appendix 2.B  Adaptive Feedback Rate Derivation

In this appendix, the probability that less than \( u \in \{S, \ldots, Q\} \) different clusters are fed back from \( K \) users is derived \((K \geq 1)\). This result can be used to find a proper adaptive feedback rate as a function of the number of users \( K \).

It is assumed that the users independently feed back \( S \) out of the \( Q \) clusters. Let the random variable \( U_k \) denote the number of different clusters, out of the \( Q \) clusters, that have been fed back when \( k \) users have made their pick. The probability that less than \( S \) clusters are picked is zero, as is the probability that more than \( Q \) are picked. The non-zero probabilities can be stacked in a vector and computed recursively from

\[
P_k = \begin{pmatrix} \Pr(U_k = S) \\ \Pr(U_k = S + 1) \\ \vdots \\ \Pr(U_k = Q) \end{pmatrix} = A \begin{pmatrix} \Pr(U_{k-1} = S) \\ \Pr(U_{k-1} = S + 1) \\ \vdots \\ \Pr(U_{k-1} = Q) \end{pmatrix} = AP_{k-1}
\]

(2.21)

where the lower triangular matrix \( A \) denotes the probability transitions when one more user picks clusters.

\[
A = \begin{pmatrix} \\
\Pr(U_k = S | U_{k-1} = S) \\
\Pr(U_k = S + 1 | U_{k-1} = S) & \ddots & 0 \\
\vdots \\
\Pr(U_k = Q | U_{k-1} = S) & \cdots & \Pr(U_k = Q | U_{k-1} = Q) 
\end{pmatrix}
\]

(2.22)

The elements of \( A \) are computed as

\[
\Pr(U_k = S + t | U_{k-1} = S + r) = \\
\begin{cases} \\
\frac{Q - S - r}{S - (t - r)} & \text{if } r \leq t \text{ and } t - r \leq S \\
\frac{Q}{S} & \text{if } r > t \\
0 & \text{otherwise}
\end{cases}
\]

(2.23)
Since the first user \((k = 1)\) picks \(S\) different clusters,

\[
\mathbf{P}_1 = \begin{pmatrix}
\Pr(U_1 = S) \\
\Pr(U_1 = S + 1) \\
\vdots \\
\Pr(U_1 = Q)
\end{pmatrix} = \begin{pmatrix}
1 \\
0 \\
\vdots \\
0
\end{pmatrix}
\] (2.24)

and

\[
\mathbf{P}_K = \mathbf{A}^{K-1} \begin{pmatrix}
1 \\
0 \\
\vdots \\
0
\end{pmatrix}.
\] (2.25)

Finally, the probability that \(u\) or fewer different clusters are fed back by \(K\) users can be computed as

\[
\Pr(U_K \leq u) = \sum_{i=0}^{u-S} \mathbf{P}_K(i)
\] (2.26)

where \(\mathbf{P}_K(i)\) denotes the \(i^{th}\) element of \(\mathbf{P}_K\). To find the feedback rate \(S\) so that \(\Pr(U_K \leq u)\) remains fairly constant for all \(K\), a full search of all integer \(S \in \{1, \ldots, Q - 1\}\) can be done.

For a fixed \(S\), the expected spectral usage can be computed as

\[
E[U_K/Q] = \frac{1}{Q} \sum_{u=S}^{Q} u \Pr(U_K = u) = \frac{1}{Q} \sum_{u=S}^{Q} u \mathbf{P}_K(u - S).
\] (2.27)

**Appendix 2.C On Sub-carrier Variations Within a Cluster**

**2.C.1 Correlation Between Sub-carriers**

This section assumes the channel model (2.6)-(2.7) with a sample spaced exponentially decaying power delay profile.

The correlation between the \(k^{th}\) and \(n^{th}\) sub-carriers is

\[
\rho_{kn} = \frac{E[G_k G_n^*]}{\sqrt{\text{var}(G_k) \text{var}(G_n)}}.
\] (2.28)
If the channel impulse response is $T_g$ spaced,

$$g(\tau) = \sum_{l=0}^{L-1} \beta_l \delta(\tau - lT_g)$$  \hspace{1cm} (2.29)

where the taps are independent and with variances $\sigma_l^2$, the numerator can be found as

$$E[G_kG_n^*] = \sum_{l=0}^{L-1} \sigma_l^2 e^{-j2\pi l(k-n)T_g/NT}.$$ \hspace{1cm} (2.30)

The variance of $G_k$ is $\sum_{l=0}^{L-1} \sigma_l^2$. Hence, the correlation between sub-carrier $k$ and $n$ is

$$\rho_{kn} = \frac{\sum_{l=0}^{L-1} \sigma_l^2 e^{-j2\pi l(k-n)T_g/NT}}{\sum_{l=0}^{L-1} \sigma_l^2}.$$ \hspace{1cm} (2.31)

If the power delay profile of the taps is exponentially decaying, as in (2.7), $\sigma_l^2 = Ae^{-LT_g/2\gamma}$, the correlation can be written as

$$\rho_{kn} = \frac{1 - e^{-T/2\gamma}}{1 - e^{-LT_g/2\gamma}} \frac{1 - e^{-LT_g/2\gamma} - e^{-LT_g/2\gamma - j2\pi L(k-n)T_g/NT}}{1 - e^{-LT_g/2\gamma}}.$$ \hspace{1cm} (2.32)

where the normalization $\sum_{l=0}^{L-1} \sigma_l^2 = 1$ is assumed. See [ESvdB+98] for a similar derivation. As an illustration, the magnitude of the correlation, $|\rho_{kn}|$, between sub-carriers as a function of the distance between the sub-carriers, $n-k$, is shown in Figure 2.7 for three different delay spreads $\gamma$.

### 2.C.2 Probability of a Weak Sub-carrier Within a Cluster

To help evaluate the appropriate cluster-size $R$, the probability that the gain of the $i$th sub-carrier in a cluster is less than $\alpha$ times the mean gain of the cluster is derived (a similar derivation can be found in [CH92]).

For convenience, consider the cluster with lowest index.

$$\Pr \left[ |G_r|^2 < \frac{\alpha}{R} \sum_{r=0}^{R-1} |G_r|^2 \right]$$ \hspace{1cm} (2.33)
2.C On Sub-carrier Variations Within a Cluster

Figure 2.7: The correlation magnitude $|\rho_{kn}|$ between the sub-carriers $k$ and $n$ as a function of $n - k$ (assuming $n \geq k$) for H/2 and the channel model (2.6)-(2.7). In the figure, the RMS delay spread, $\gamma$, is 10, 40, and 120 ns. The impulse response is truncated at 78 samples ($L = 78$), which is slightly shorter than the H/2 cyclic prefix.

which can be written in matrix form as

$$\text{Pr} \left[ \mathbf{G}^* \left( \mathbf{A} - \frac{\alpha}{\mathbf{I}_R} \right) \mathbf{G} < 0 \right]$$

where $\mathbf{G} = [G_0 \cdots G_{R-1}]^T$, $\mathbf{A}_i$ is an all zero $R \times R$ matrix except for the $(i, i)^{th}$ element, which is one, $\mathbf{I}_R$ is the $R \times R$ identity matrix and $^T$ and $^*$ denote transpose and conjugate transpose respectively. Equation (2.34) can be rewritten as

$$\text{Pr} [\mathbf{G}^* \mathbf{I}_{\Delta} \mathbf{G} < 0] = \text{Pr} \left[ \mathbf{u}^* \mathbf{R}_G^{1/2} \mathbf{I}_{\Delta} \mathbf{R}_G^{1/2} \mathbf{u} < 0 \right]$$

where $\mathbf{u}$ is an $R \times 1$ vector of i.i.d. zero-mean, complex, unit variance Gaussian random variables and $\mathbf{R}_G$ is the covariance matrix of $\mathbf{G}$, which
for example can be obtained from (2.32). Taking the singular value decomposition of $R_G^{1/2}I_\Delta R_G^{1/2}$ we find

$$\Pr[\tilde{u}^*A\tilde{u} < 0] = \Pr\left[\sum_{r=0}^{R-1} \lambda_r |\tilde{u}_r|^2 < 0\right] = (2.36)$$

$$\Pr\left[\sum_{r \in \mathcal{R}} \lambda_r |\tilde{u}_r|^2 < 0\right] = \Pr\left[\sum_{r \in \mathcal{R}} X_r < 0\right] = \Pr[X < 0] = (2.37)$$

where $\tilde{u} = [\tilde{u}_0 \cdots \tilde{u}_{R-1}]^T$ contains i.i.d zero-mean, complex, unit variance Gaussian random variables and $\Lambda$ is a diagonal matrix of the singular values of $R_G I_\Delta$. $\mathcal{R}$ is the set of $r$ for which the singular values $\lambda_r$ are non-zero. The $X_r$ are exponentially distributed with characteristic functions

$$\Phi_{X_r}(f) = \frac{1}{1 + j2\pi f \lambda_r} \quad (2.39)$$

leading to the characteristic function of $X$ as

$$\Phi_X(f) = \prod_{r \in \mathcal{R}} \frac{1}{1 + j2\pi f \lambda_r}. \quad (2.40)$$

Assuming no duplicate singular values, the product can be partial fraction expanded as

$$\Phi_X(f) = \sum_{r \in \mathcal{R}} \frac{B_r}{1 + j2\pi f \lambda_r} + Rem(f). \quad (2.41)$$

Now, ignoring the remainder $Rem(f)$, which generally is zero

$$\Pr[X < 0] = \int_{-\infty}^{0} \int_{-\infty}^{\infty} \Phi_X(f) e^{2\pi jfx} df dx = (2.42)$$

$$= \int_{-\infty}^{0} \int_{-\infty}^{\infty} \sum_{r \in \mathcal{R}} \frac{B_r e^{2\pi jfx}}{1 + j2\pi f \lambda_r} df dx \quad (2.43)$$

Moving the summation outside the integrals and solving the integral over $f$ yields

$$\Pr[X < 0] = \sum_{r \in \mathcal{R}^+} B_r \int_{-\infty}^{0} \frac{1}{\lambda_r} e^{-x/\lambda_r} u(x) dx \quad (2.44)$$

$$= \sum_{r \in \mathcal{R}^-} B_r \int_{-\infty}^{0} \frac{1}{\lambda_r} e^{-x/\lambda_r} u(-x) dx \quad (2.45)$$
where $\mathcal{R}^+$ and $\mathcal{R}^-$ are the sets of $r$ for which $\lambda_r$ are positive and negative respectively and $u(x)$ is the step function. The integrals over the positive $\lambda_r$ are all zero, which finally gives us the wanted probability.

$$
\Pr[X < 0] = \sum_{r \in \mathcal{R}^-} B_r \int_{-\infty}^{0} \frac{1}{\lambda_r} e^{-x/\lambda_r} \, dx = \sum_{r \in \mathcal{R}^-} B_r. \tag{2.46}
$$

This result agrees well with numerical simulations.
Chapter 3

Opportunistic Beamforming for OFDM

3.1 Summary
This chapter contains:

- An introduction to opportunistic beamforming.
- An opportunistic beamforming design for clustered OFDM. By applying different beamforming weights in different clusters, but the same within a cluster, the frequency fading is increased in a favorable way.
- Discussions on the frame structure of opportunistic beamforming systems and opportunistic beamforming versus sectorized systems.

3.2 Introduction
The concept of multiuser diversity relies on the fading of the users. If the channel quality of each user varies with time, it is possible to schedule users that are near their fading peaks. In real systems, however, some users may be stationary or moving very slowly. For such users, the time between fading peaks may be long, which can result in long delays.

Opportunistic beamforming was introduced in [VTL02] to cope with this problem. A flat fading single-carrier scenario was considered where
only one user at a time could be scheduled. Figure 3.1 roughly describes how opportunistic beamforming works.

1. First, a random beam is formed and a training signal is transmitted. The SINR is estimated by each user in the cell using this training signal. This scheme assumes that all nearby base-stations change their beams and transmit their training signals simultaneously. Also, the training signals of the different cells have to be orthogonal, so that the users can distinguish the own base-station from the interference.

2. The estimated SINR, or equivalently the supportable rate, is fed back by each user to the base-station.

3. Based on the fed back information, the scheduler selects one user for communication and sends the user identification followed by the data using the supportable modulation level.

4. After the block of data has been transmitted, the process starts over again with the base-station forming a new random beam and transmitting a training signal.

It is not necessarily the user with the strongest received signal power that will have the highest received SINR. The opportunistic beamforming in the adjacent cells can give high fluctuations in the interference level, giving a very low interference level for some users. This opportunistic nulling can act as an interference suppression mechanism. It is clear that an opportunistic system works better if there are many active users in the system to select from.

The opportunistic beamforming structure of [VTL02] is shown in Figure 3.2. The same signal is transmitted over the two transmit antennas but with different power and phase. The random power distribution coefficient \( \alpha \) is uniformly distributed between 0 and 1 and changed between each transmission block, as described above. The random phase coefficients \( \theta_1 \) and \( \theta_2 \) are uniformly distributed between 0 and \( 2\pi \) and changed in the same way. The flat fading complex values channel coefficients of user \( k \), \( g_{1k} \) and \( g_{2k} \) vary with time, but slower than the change rate of the opportunistic beamformer. The highest received signal power at user \( k \) is obtained if the beamforming weights happen to be in the beamforming
3.3 Opportunistic Beamforming for Clustered OFDM

configuration:

\[
\alpha = \frac{|g_{1k}|^2}{|g_{1k}|^2 + |g_{2k}|^2}, \\
\theta_1 = -\arg(g_{1k}), \\
\theta_2 = -\arg(g_{2k}) \quad (3.1)
\]

This can be extended to more transmit antennas. The beamforming configuration maximizes the received SNR to

\[
\text{SNR}_{BF} = \frac{|g_{1k}|^2 + |g_{2k}|^2}{\sigma^2} \quad (3.2)
\]

where \(\sigma^2\) is the receiver AWGN power. It is shown in [VTL02] that as number of users grow, the likelihood that one of the users will be in the beamforming configuration goes to 1. The SNR in (3.2) can be compared to the SNR if only transmit antenna 1 is used,

\[
\text{SNR}_{SISO} = \frac{|g_{1k}|^2}{\sigma^2} \quad (3.3)
\]

or if for example Alamouti space-time block-coding [Ala98] is used,

\[
\text{SNR}_{Alamouti} = \frac{|g_{1k}|^2 + |g_{2k}|^2}{2\sigma^2} \quad (3.4)
\]

The increased received SNR for the beamforming configuration compared to the single transmit antenna SNR is called the beamforming gain.

Opportunistic beamforming can be extended directly to OFDM, as outlined in [VTL02]. Since such a system consists of parallel flat fading channels, opportunistic beamforming as described above can be applied to each subcarrier.

3.3 Opportunistic Beamforming for Clustered OFDM

In an OFDMA multiuser diversity system, it is favorable if the users fade, both in time and frequency. There is not much multiuser diversity gain to be exploited in a system with only stationary and flat fading users. The opportunistic beamforming scheme proposed in this section is a way
to increase the fading rates of the users in both time and frequency. The scheme is based on the clustering described in Chapter 2.

We assume a base-station with \( M \) transmit antennas and \( K \) users with one receive antenna each. Assume that the downlink OFDM symbol can be shared among several users. To reduce the amount of feedback from the users to the base-station scheduler, the feedback scheme proposed in Section 2.4.1 is used.

The antenna configuration used in this chapter is similar to that used in [VTL02, LLRS03, SWCO04b, SWO04]. Assuming synchronous base-stations and omitting the time index (c.f. Section 2.3.2), the \( k^{th} \) user that belongs to base-station 0 receives, on the \( n^{th} \) sub-carrier,

\[
y_{n,k} = \mathbf{G}_{n,k}^\text{T} \mathbf{c}_n^0 \sqrt{P \mathbf{c}_n^0} + \sqrt{P} \sum_{i=1}^{I} \mathbf{G}_{n,k}^\text{T} \mathbf{b}_n^i \mathbf{c}_n^i + w_{n,k} \tag{3.5}
\]

where \( \mathbf{c}_n^i \) is the unit-energy transmitted symbol from base-station \( i \), \( w_{n,k} \) is additive white Gaussian noise, \( \mathbf{G}_{n,k}^i \in \mathbb{C}^{M \times 1} \) is the baseband frequency response vector from the \( M \) antennas of the \( i^{th} \) base-station to the \( k^{th} \) user on the \( n^{th} \) sub-carrier, \( \mathbf{b}_n^i \in \mathbb{C}^{M \times 1} \) is the transmitter beamforming vector for the \( n^{th} \) sub-carrier and \( i^{th} \) base-station, and \( P \) is the transmit power per sub-carrier. For the interfering channels, \( \mathbf{G} \) means that the frequency response is rotated, which does not change the received SINR. In [LLRS03], different designs for \( \mathbf{b}_n^i \) are discussed. In this thesis, the elements of \( \mathbf{b}_n^i \) are i.i.d. complex variables with uniformly distributed phase over \([0, 2\pi]\) and magnitude over \([0, 1]\) and normalized so that \( ||\mathbf{b}_n^i||^2 = 1 \).

Let \( H_{n,k}^i = \mathbf{G}_{n,k}^\text{T} \mathbf{b}_n^i \) denote the complex-valued effective baseband frequency response from base-station \( i \) to user \( k \) on sub-carrier \( n \), so that

\[
y_{n,k} = H_{n,k}^0 \sqrt{P \mathbf{c}_n^0} + \sqrt{P} \sum_{i=1}^{I} H_{n,k}^i \mathbf{c}_n^i + w_{n,k} \tag{3.6}
\]

Hence, the users effectively experience SISO channels on all sub-carriers. To increase the frequency fading rate, we propose a clustered beamforming design for \( \mathbf{b}_n^i \). For clustered beamforming (CL-BF), the \( \mathbf{b}_n^i \) are identical within one cluster, but independent between the clusters. Hence, the sub-carrier correlation within the clusters is maintained and the sub-carrier correlation between the clusters is reduced. The transmitter structure of CL-BF is illustrated in Figure 3.3. By having different
beamforming weights in different clusters (CL-BF), we can increase the frequency fading rate. This is an extension of the idea in [VTL02] where different beamforming weights were used in different transmission blocks to increase the temporal fading rate. Figure 3.4 illustrates the effect of clustered opportunistic beamforming on two channels.

As a comparison, the equal beamforming design (EQ-BF) of $b^i_n$ will also be used. For equal beamforming, $b^i_n = b^i \forall n$. Hence, only one IDFT has to be performed at the transmitter, compared to $M$ for CL-BF, but no extra frequency fading is induced. The beamforming weights are constant during one transmission block, which consists of a number of OFDM symbols, but change from block to block.

A second alternative to CL-BF is delay diversity (DD). In a delay diversity system, the same signal is transmitted on all antennas but time-delayed on a subset of the antennas. This results in an artificially longer channel impulse response and more diversity also in the frequency domain. The advantage of such an approach is its low complexity. The first drawback of a delay diversity solution is that the length of the cyclic prefix has to be increased to incorporate the longest length of the artificial impulse response of any user. The second drawback is that it increases the frequency variability not only between the clusters, but also within the clusters. This leads to less accuracy in the feed-back rates.

To illustrate the effect of clustered opportunistic beamforming and delay diversity, the effective gains across the sub-carriers of a user are shown in Figure 3.5. The used simulation parameters are described in Section 5.2. The performance of the described schemes are compared in Chapter 5.

### 3.4 Opportunistic Beamforming Frame Structure

In a multiuser diversity system, all users have to track their signal-to-interference-plus-noise ratio (SINR). SINR tracking and prediction is an easier problem than full channel estimation [FSES04]. Before each scheduling decision all users feed back their supportable rates based on the estimated or predicted SINR. However, in a system with opportunistic beamforming, tracking and prediction of the SINR needs to be coordinated with the change of beamforming weights. This brings the need for a training period with the new beamforming weights before the feedback and scheduling. Since there is a processing and feedback delay between
the training and the scheduling decision, a frame structure as in Figure 3.6 is feasible. The training with the new beamforming weights, $b(l+1)$, can be done within the previous transmission block, which uses the previous beamforming weights, $b(l)$.

Since the users estimate their SINR during the training period, it is important that both the signal and the interference power does not change between the training and the data transmission. This can be guaranteed by synchronizing the base-stations and not allowing shifting of transmit power from unused to used sub-carriers.

### 3.5 Opportunistic Beamforming vs Sectors

One can argue that the multiple transmitters at the base-station are better used by dividing the cell into sectors, using directive antennas. This enables the base-stations to transmit to several users simultaneously. The cell throughput of a single-carrier sectorized system scales as

$$\text{Throughput} \sim \sum_{i=1}^{N_{\text{sect}}} \log(1 + \text{SINR}_i)$$

(3.7)

where $\text{SINR}_i$ is the SINR of the scheduled user in sector $i$ and $N_{\text{sect}}$ is the number of sectors. For the opportunistic beamforming setup in this thesis, the number of sectors is equal to one. The average SINR of the scheduled user in the opportunistic system can be expected to be higher than the average $\text{SINR}_i$ of the scheduled users in the sectored cell. This is due to the increased multiuser diversity (more users), beamforming gain and opportunistic nulling. Still, the increase in SINR gives a logarithmic increase in throughput, whereas the simultaneous users give a linear increase in throughput. However, the gain from sectorizing is diminishing with the number of sectors. In a highly sectorized cell or if there is significant local scattering around the base-station, the inter-sector interference becomes significant and users on the border between two sectors will experience low SINRs. This leads to the combination of sectorized cells and opportunistic beamforming. By employing several directive transmit antennas in each sector and using opportunistic beamforming, both the multiplexing gain from transmitting to several users simultaneously and the benefits of opportunistic beamforming to multiuser diversity systems can possibly be achieved. However, this is left for future research.
3.5 Opportunistic Beamforming vs Sectors

Figure 3.1: This figure illustrates the single-carrier opportunistic beamforming principle. In 1., the base-station forms a random beam and transmits a training signal using this beam. Each user estimates the SINR from this training signal. The SINR or the corresponding supportable rate is fed back to the base-station in 2., which will be the basis for the scheduling decision. In 3., the base-station transmits a data block to the scheduled user based on the supportable rate. After the finished transmission, the base-station forms a new beam and transmits the training signal again in 4. and so on.
Figure 3.2: The single-carrier opportunistic beamforming of [VTL02]. The same signal is transmitted over the two antennas with different phase and powers.

Figure 3.3: Transmitter structure for clustered opportunistic beamforming (CL-BF) with two transmit antennas ($M = 2$) and three subcarriers per cluster ($R = 3$).
3.5 Opportunistic Beamforming vs Sectors

Figure 3.4: In these three plots, the effect of clustered beamforming on a slowly moving user is illustrated. The two uppermost plots show the magnitudes of the channels from the two transmit antennas to a user. The lower plot shows the effective channel after clustered opportunistic beamforming is applied to the both channels. It can be seen that the fading rate in both time and frequency is increased, which is advantageous in a multiuser diversity system. The number of sub-carriers is 128, the cluster-size is 4 sub-carriers and the beamforming pattern is changed every 16th OFDM symbol.
Figure 3.5: Example of effective gains of a user for different transmitter schemes. The lowest plot shows the gains for a SISO channel. By adding one transmit antenna and delay diversity in the middle plot, more frequency variability is induced. In the uppermost plot, the effective gains for clustered beamforming is shown. The cluster-size, $R$, is 4 sub-carriers.

Figure 3.6: Frame structure for opportunistic beamforming. During the $l^{th}$ transmission block, the beamforming weights, $b(l)$, are used. The scheduling and adaptive modulation in the next transmission block $(l+1)$ is based on measurements of training symbols (Tr) with the new beamforming weights, $b(l+1)$. Due to the feedback delay, the training (Tr) with the new beamforming weights, $b(l+1)$, is done inside the previous transmission block $l$.  

\begin{tabular}{c|c|c|c}
\hline
\text{Data} & \text{Tr} & \text{Data} \\
\hline
$b(l)$ & $b(l+1)$ & $b(l)$ & $b(l+1)$ \\
\hline
\end{tabular}
Chapter 4

Multiuser Diversity
Scheduling for OFDM

4.1 Summary
This chapter contains:

- An overview of previous multiuser diversity scheduling, especially for OFDM.
- A modified Proportional Fair scheduler (M-PF). The scheduler can handle individual bit-rate and delay requirements and incorporates a variable fairness level.
- A method to use scheduling information in the opportunistic beamforming.

4.2 Introduction
To exploit multiuser diversity, a proper scheduler has to be used. To maximize the cell throughput, the user with the highest supportable rate should always be selected [KH95a]. An alternative to this potentially very unfair scheduler is the proportional fair (PF) scheduler [Qua01]. It offers a compromise between fairness and multiuser diversity exploitation. The PF scheduler is presented in Section 4.3. In [BH02], delay sensitive users are given priority in the single-carrier PF algorithm when
their delays reach a certain threshold. A similar approach was considered in [KH02] where unfairly treated users are successively given higher priority. In [ALS+03], the single-carrier PF scheduler is extended to OFDM. We will use the recommended scheme in [ALS+03] as a comparison, calling it PF. In this chapter, we propose a modified PF (M-PF) scheduler for OFDM. It differs from previous PF schedulers in that it can accommodate different quality-of-service (QoS) classes and that it has a tunable fairness level.

Many results on sub-carrier, bit and power allocation schemes for multiuser OFDM are available, for instance [WCLM99, RC00, ZL02, ZL03, ECV03]. Optimization problems of different forms are solved to satisfy the minimum rate requirements of the different users. However, these schemes only consider resource allocation for one transmission block. Hence, resources may be wasted on a demanding user that is in a deep fade on most sub-carriers. The philosophy of the PF scheduler is different in that it can let a user wait several transmission blocks if it is fading unfavorably. Hence, a particular QoS can not be guaranteed, but rather a best effort QoS. The M-PF scheduler proposed here tries to satisfy the different rate requirements of the different users, while at the same time exploiting multiuser diversity. This can lead to many users not reaching their target rates. On the other hand, the system can accommodate more users that are near their target rates, increasing the multiuser diversity effect.

Recently, resource allocation schemes that directly aim at user satisfaction instead of QoS have emerged, e.g. [SL03]. Also, the business model of the operator influences the desirable behavior of the scheduler [BLZZ03]. A good scheduler in a system with few high-paying users is not necessarily good in a system with many moderately-paying users. Recent studies on user satisfaction indicate that the satisfaction of an already “well-served” user increases only marginally by increasing the service level even further [JMK+01][EL03]. However, if the service level is decreased below some level, the satisfaction level drops significantly. The PF scheduler proposed in this chapter aims at the scenario with many moderately-paying users, since it does not give hard QoS guarantees. Rather, it tries to offer moderate QoS levels to as many users as possible. Simulation results show that most users within a QoS class are equally distributed in terms of bit-rates and delay. If the QoS class is well defined for a particular application and most users are near their target QoS, the average users satisfaction level will be high.
4.3 Proportional Fair

The proportional fair (PF) scheduling algorithm was designed for the downlink of the single carrier CDMA/TDMA IS-856 system, also known as 1xEV or HDR [Qua01, BGP+00]. In this system, voice and data traffic is employed on different carriers. The data traffic part of the system exploits multiuser diversity by letting all active users feed back their estimated instantaneous supportable rates and giving channel access according to the PF algorithm. The system has 12 different adaptive coding and modulation levels and supports 60 active users per sector. Multiuser diversity and the proportional fair algorithm is also being considered for the evolution of UMTS [PDF+01].

For each transmission block \( l \), the scheduler receives the instantaneous supportable bit-rate from all active users, \( C_k(l) \). Furthermore, the scheduler keeps track of the average bit-rate of each user, \( F_k(l) \), in a historical time-window, \( t_c \). For each transmission block \( l \), the user \( k^*(l) \) is scheduled according to

\[
k^*(l) = \arg \max_k \frac{C_k(l)}{F_k(l)}.
\]  

(4.1)

This is basically the maximum throughput scheduling, but weighted by the inverse of the historical bit-rate. If a user has not been scheduled for long time, it will be favoured. If a user has been given channel access recently and therefore has a high historical throughput, it will need a very high channel quality, \( C_k(l) \), to be scheduled. This gives a compromise between multiuser diversity and fairness. The time-update of the historical bit-rate, \( F_k(l) \), can be done in several different ways. A low complexity update equation that also gives low memory requirements is

\[
F_k(l+1) = \begin{cases} 
(1 - \frac{1}{t_c})F_k(l) + \frac{1}{t_c}R_k(l) & \text{if } k^*(l) = k \\
(1 - \frac{1}{t_c})F_k(l) & \text{if } k^*(l) \neq k 
\end{cases}
\]  

(4.2)

which is an exponentially weighted low-pass filter [VTL02].

4.4 Modified Proportional Fair

In future communication systems, users will likely have different requirements, for instance real-time data, voice or background downloading of data. This is not taken into account by the PF algorithm presented in Section 4.3, which treats all users equally. In this section, we modify
the PF algorithm to take user requirements in terms of bit-rates and delays into account. Furthermore, the fairness level of PF is fixed. In this section, a tunable fairness parameter is incorporated into the scheduling metric. The modified proportional fair (M-PF) algorithm is presented in the OFDMA context of this thesis, but it can also be applied to a single-carrier system.

We assume that each user, $k$, belongs to a QoS class with parameters $R_k$ and $T_k$, which is the target rate in bits/s and averaging window time in seconds, respectively. The averaging window time reflects the target delay of the QoS-class. Long delay windows allow longer periods of not being scheduled, whereas a short delay window will help schedule the user more often.

The channel state information at the transmitter are the $SK$ rate values $C_{s,k}$ and the $SK$ corresponding indices, $I_{s,k}$, where $s \in \{1, \ldots, S\}$, $k \in \{1, \ldots, K\}$, $I_{s,k} \in \{1, \ldots, Q\}$, $S$ is the number of fed-back clusters and $K$ is the number of users. $C_{s,k}$ is the minimum supportable sub-carrier bit-rate in cluster $I_{s,k}$. For each user, $k$, the scheduler keeps track of the total throughput, $A_k$, in the historical time window, $T_k$; the rate is $F_k = A_k / T_k$. If a user has a higher rate than the target rate, $R_k$, the user will be disadvantaged in the scheduling process. If a user has a lower rate than the target rate, it will be favored in the scheduling. If a user is not scheduled and has a long time window, $T_k$, its average rate, $A_k / T_k$, will drop slowly with time. If the user instead has a short delay window, its average rate will drop quickly if it is not scheduled.

To find the scheduling during the next transmission block, the scheduler computes the following measure for each user $k$:

$$B_k = \frac{A_k}{T_k R_k} + \epsilon$$

where $\epsilon$ is a small positive regularization term. $B_k$ is a measure of how well the user has met its rate requirements in the historical time window, $T_k$:

- $B_k > 1$: User $k$ has exceeded its target rate.
- $B_k = 1$: User $k$ has just met its target rate.
- $B_k < 1$: User $k$ has undershot its target rate.

For each cluster, $q$, the scheduler chooses a user according to the criterion

$$\text{user}_q = \arg \max_k \left\{ \frac{C_{s,k}}{B_k^q} \ : \ I_{s,k} = q \right\}.$$
In other words, the scheduler chooses the user for cluster \( q \) that has feedback the highest requested bit-rate, \( C_{s,k} \), but weighted with the fairness measure, \( B_k^c \). Users that have not met their target rates (low \( B_k \)) will be favored and users that have overshot their target rates (high \( B_k \)) will be disadvantaged.

The classical PF algorithm of Section 4.3 uses an exponentially weighted low-pass filter to update the average bit-rates in the time window. Here, the actual average throughputs, \( A_k \), in the specified windows, \( T_k \), are stored, which increases the memory requirements of the scheduler. However, the actual average rate, \( F_k = A_k/T_k \) can now be compared to the target rate. Note that the low-pass update of \( A_k \) would also be possible.

The cluster assignment order within one OFDM symbol matters since the throughput, \( A_k \), is updated after each cluster assignment. Here, the strongest cluster of all users is assigned first and the weakest cluster is assigned last. This does not necessarily maximize the throughput of the current OFDM symbol. The optimal assignment order is left for future research.

Should the user with the highest supportable rate within the cluster be scheduled? Or should the user that is most below its target rate be scheduled? This is a tradeoff between cell throughput and fairness. The fairness parameter \( \kappa \in [0, \infty) \) can be used to tune the fairness of the scheduler. Setting \( \kappa = 0 \) gives the maximum throughput scheduling criterion

\[
\lim_{\kappa \to 0} \arg \max_k \left\{ \frac{C_{s,k}}{B_k^c} : I_{s,k} = q \right\} = \arg \max_k \left\{ C_{s,k} : I_{s,k} = q \right\} = \arg \max_k \left\{ C_{s,k} : I_{s,k} = q \right\} \quad (4.5)
\]

whereas a high \( \kappa \)-value gives a scheduler that only tries to meet the target rates without exploiting the multiuser diversity. Since \( C_{s,k} \) is bounded,

\[
\lim_{\kappa \to \infty} \arg \max_k \left\{ \frac{C_{s,k}}{B_k^c} : I_{s,k} = q \right\} = \arg \min_k \left\{ B_k : I_{s,k} = q \right\} . \quad (4.6)
\]

This gives the operator the possibility to easily set the fairness level that is suitable for a particular system. The classical PF algorithm can be obtained by setting \( \kappa = 1, R_k = R, T_k = T, \epsilon = 0 \) and letting the throughputs, \( A_k \), be updated through an exponentially weighted low-pass
filter as in Section 4.3,

\[
\text{user}_q = \arg\max_k \left\{ \frac{C_{s,k}}{B_k^t} : I_{s,k} = q \right\} \\
= \arg\max_k \left\{ \frac{C_{s,k}TR}{A_k} : I_{s,k} = q \right\} \\
= \arg\max_k \left\{ \frac{C_{s,k}}{F_k} : I_{s,k} = q \right\}.
\]  \tag{4.7}

4.5 Semi-random Opportunistic Beamforming

For slowly moving users, the temporal channel correlation between transmission blocks is high. If clustered beamforming is used, this can be exploited by the scheduler to help users that are well below their target rates. By keeping the beamforming weights for the user’s strongest cluster during the next transmission block the probability that the cluster is strong also during the next transmission block is increased.

1. A user, \( k \), is considered to be well below its target rate if \( B_k < \nu \), where \( \nu < 1 \) is a threshold value.

2. For each user, \( k \), that is well below its target rate, keep the beamforming weights on the strongest cluster of the user during the next transmission block.

Other ways to utilize the partial channel state information in the opportunistic beamforming could for instance be

- To keep beamforming weights on the strongest cluster of the users with lowest speed or fading rate. This would assume speed knowledge or estimates at the transmitter.

- To let a fixed portion of the sub-carriers keep their beamforming weights from block to block. This would allow locking a successful beam to a stationary user for a long time-period.

- To keep beams only in the clusters that support the highest modulation level. This could possibly help boost the cell throughput.
Chapter 5

Numerical Results for Chapters 2-4

5.1 Summary

This chapter contains:

- A description of the parameters and assumptions in the multi-cell simulations.
- Simulation results for the schemes proposed in Chapters 2-4.

5.2 Simulation Environment and Assumptions

To evaluate the performance of the proposed schemes, the downlink of an FDD system with seven cells has been simulated. Results are collected from one cell which is surrounded by six interfering cells. All base-stations are assumed to be synchronous and use the same frequency band for the downlink. All base-stations use the same beamforming and scheduling schemes. The users are assumed to perfectly estimate the SINR on all sub-carriers and the feedback delay is assumed to be zero. The adaptive feedback rate is such that the probability that less than 80% of the sub-carriers can be assigned is 0.2 (see Section 2.4).

If we assume that the sum of the interference is Gaussian, the downlink channel consists of parallel approximately Gaussian sub-channels,
with contributions from different users. In the simulations, the supportable number of bits per OFDM symbol for the $n^{th}$ sub-carrier and the $k^{th}$ user in cell 0 is estimated as

$$T_{n,k} = \frac{1}{2} \log_2 \left( 1 + \frac{\text{SINR}}{\Gamma} \right) = \frac{1}{2} \log_2 \left( 1 + \frac{|H_{0,n,k}^0|^2}{\Gamma \left( \sum_{i=1}^{I} |H_{i,n,k}^i|^2 + \sigma_z^2/P \right)} \right)$$

(5.1)

where the notation is as in (3.6) and $\Gamma$ is the gap corresponding to a symbol error rate of $10^{-4}$ for QAM [CDEGDF95]. This means that it is assumed that the adaptive modulation can handle also non-integer number of bits per symbol. It is assumed that the scheduled users achieve the rates they estimate from (5.1).

The system parameters are described in Table 5.1. The 3GPP spa-

<table>
<thead>
<tr>
<th>Sampling frequency</th>
<th>4 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sub-carriers $N$</td>
<td>128</td>
</tr>
<tr>
<td>Cyclic prefix length</td>
<td>12 $\mu$s</td>
</tr>
<tr>
<td>Total OFDM symbol period $T_s$</td>
<td>32+12=44 $\mu$s</td>
</tr>
<tr>
<td>Sub-carrier spacing</td>
<td>31.25 kHz</td>
</tr>
<tr>
<td>Total bandwidth</td>
<td>4 MHz</td>
</tr>
<tr>
<td>Total transmit power $P_{tot}$</td>
<td>500 W</td>
</tr>
<tr>
<td>Average SNR at cell boundary</td>
<td>10 dB</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>1900 MHz</td>
</tr>
<tr>
<td>Number of transmit antennas $N_t$</td>
<td>2</td>
</tr>
<tr>
<td>Antenna separation</td>
<td>$4\lambda$</td>
</tr>
<tr>
<td>Maximum user speed</td>
<td>100 km/h</td>
</tr>
<tr>
<td>Maximum relative Doppler</td>
<td>0.77%</td>
</tr>
<tr>
<td>Cell radius</td>
<td>1 km</td>
</tr>
<tr>
<td>Distance between base-stations</td>
<td>2 km</td>
</tr>
<tr>
<td>Number of interfering base-stations</td>
<td>6</td>
</tr>
<tr>
<td>Cluster-size $R$</td>
<td>2 sub-carriers</td>
</tr>
<tr>
<td>Fairness parameter $\kappa$</td>
<td>0.5</td>
</tr>
<tr>
<td>Weight-keeping threshold $\nu$</td>
<td>0.8</td>
</tr>
<tr>
<td>Transmission block $N_{tb}$</td>
<td>16 OFDM symbols</td>
</tr>
<tr>
<td>Simulation time</td>
<td>$1536T_s \approx 68$ ms</td>
</tr>
</tbody>
</table>

**Table 5.1:** Simulation parameters. In some Figures, $\kappa$ and $N_t$ are varied.

tial channel model for MIMO simulations for urban environments is used [3GP03]. It is modified for omni-directional antennas by chang-
ing the antenna gain pattern to be uniform. The users are uniformly distributed over the cells and their speeds are uniformly distributed between 0 and 100 km/h. Temporal correlation over the simulated OFDM symbols is added using Jakes model [DBC93]. In one simulation, flat channels are used. The same channel model is used for this case, but the taps of each impulse response are summed to one tap.

Three user-classes are considered: one class with high bit-rate requirements and short delay-window, corresponding to a user using a real-time application, one class with medium bit-rate and delay requirements, and one class with a low bit-rate and long delay-window, corresponding to a user downloading data in the background. Half of the users belong to Class 3 and one quarter of the users belong to Class 1 and 2 each. In practice, each user can have an individual rate and delay requirement.

- Class 1: \( R = 256 \text{kbps} \) and \( T = 20N_{tb}T_s = 0.9 \text{ms} \)
- Class 2: \( R = 128 \text{kbps} \) and \( T = 40N_{tb}T_s = 1.8 \text{ms} \)
- Class 3: \( R = 64 \text{kbps} \) and \( T = 60N_{tb}T_s = 2.6 \text{ms} \)

The opportunistic beamforming schemes with PF schedulers are compared to a conventional beamforming scheme with round-robin scheduling (Conv. BF RR). In the conventional beamforming scheme, the beamforming is computed independently on all OFDM sub-carriers. This scheme requires channel state information at the transmitter, which is also assumed. The base-stations have no channel knowledge of the users in the adjacent cells, so interference nulling is not possible. The transmitter computes the beamforming weights for each sub-carrier by singular-value decomposing the channel vector as described in [And00]. The effective channel, \( H_{n,k} \), is equal to the singular value of the channel vector, \( G_{n,k}^{\text{sv}} \). By following this procedure, the beamforming vector on each sub-carrier will be in the beamforming configuration mentioned in Section 3.2. Waterfilling of the transmit power across the sub-carriers is done, giving the stronger sub-carriers more power [CT91]. This may result in some sub-carriers not being used. For the conventional beamforming scheme, round-robin scheduling is used, with fixed allocated time-slots for each user. Users with high-rate requirements will be assigned correspondingly more time-slots. One time-slot here is equal to one transmission block. One problem with the conventional beamforming scheme is that the inter-cell interference may change between channel estimation and data transmission, due to transmit power waterfilling in adjacent cells.
This would degrade the performance of this scheme. In the simulations, however, we neglect this effect and assume that the interference level during the data transmission is known when the modulation level is selected. However, the waterfilling of power does not take the ICI into account.

The modified proportional fair scheduler (M-PF) proposed in Chapter 4.4 is also compared to the OFDM proportional fair extension recommended in [ALS+03], which will be called standard proportional fair (PF). In this scheme as in single-carrier PF, the users have no particular QoS requirements, but compete for the channel with equal priority. The user-specific historic throughput is updated after the allocation of all sub-carriers using 4.2. The sub-carrier assignment order is as for M-PF.

To evaluate the performance of clustered beamforming (CL-BF), it is compared to equal beamforming weights across the sub-carriers (EQ-BF), delay diversity with delay half the length of the cyclic prefix (see Section 3.3) and single antenna transmission (SISO).

Delay is an important QoS measure. In the simulations, we have defined the maximum scheduling delay a user experiences as the maximum time between the reception of two consecutive packets. We have defined a packet to be 512 bits. A user which is rarely scheduled will have a high maximum delay. Maximum delay is also partly a function of the rate of the user. Users with very low bit-rates will experience longer delays, even if they are scheduled each OFDM symbol.

5.3 Simulation Results

The cell throughput is the sum of the bit-rates of all users in one cell. In Figure 5.1, the cell throughput for different beamforming and scheduling schemes is compared as a function of the number of users in each cell. The schemes based on clustered beamforming perform best. The low cell throughput of the conventional beamforming scheme is due to the unpredictable inter-cell interference and to the round-robin scheduling. In an environment free of ICI, the conventional beamforming scheme outperforms the opportunistic systems except when many users are active [SWO04]. The cell throughput of the delay diversity scheme decreases due to the increased cyclic prefix overhead. Multiuser diversity schemes usually perform worse for few users. By adapting the feedback rate, as described in Section 2.4, we keep the spectral usage high even for few users. The location of the users strongly affects the performance. This is the reason why the conventional beamforming curve is not com-
5.3 Simulation Results

Figure 5.1: Cell throughput as a function of the number of users in each cell is plotted for two transmit antennas. Conventional beamforming with round-robin scheduling, clustered beamforming with M-PF and PF scheduling, equal beamforming on all clusters, single transmit antenna and delay diversity with M-PF scheduling are compared.

pletely flat as can be expected. Even if several hundred random scenarios are simulated, there are small differences for different number of users. Note that the same user locations and channel realizations have been used for all schemes. For the PF scheduler, there is small performance drop for about 10 to 20 users. This can be explained by the fact that, in this scheme, the user throughputs are updated after the allocation of all clusters. When there are few users, the probability that different users compete for the same cluster is lower than when there are more users in the system. For a moderate number of users, they feed back several clusters each and the probability that they compete for the same cluster is larger than for few users. This increases the risk for overassignment of a user, which can lead to a oscillating scheduling behavior. For many user, on the other hand, they feed back few clusters each, so the effect of overassignment is smaller. For M-PF, user throughputs are updated after
Numerical Results for Chapters 2-4

each cluster assignment, resulting in a smoother scheduling behavior.

The M-PF scheduler is designed to give users the rates they require, but not much more. This is not possible in the standard PF scheduler. To evaluate how well this discrimination works, user rate CDFs are plotted in Figure 5.2 when there are 32 users per cell. The cell throughputs for CL-BF M-PF and CL-BF PF are nearly the same for 32 users (see Figure 5.1). However, the M-PF scheduler manages to differentiate the rates of the different QoS-classes as can be seen in Figure 5.2. Since users are not treated differently by the PF algorithm, the CDF for PF All is for all users. The rates of the users within the QoS classes are fairly concentrated. This indicates that the system is fair. If the rate requirements of the classes are properly set and the system is not overloaded, most users will also be satisfied.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5_2.pdf}
\caption{The figure shows the CDF of the bit-rates of the 32 users for clustered beamforming with M-PF and PF scheduling. Note that the CDF for PF is for all users. The cell throughputs of the two schemes are nearly the same for 32 users.}
\end{figure}

In Figure 5.3, CDFs of maximum delay for CL-BF, M-PF, and CL-BF PF are plotted. For all three classes and also for PF, most users
experience a maximum delay below 10 transmission blocks, which equals 7 ms. The low delay can be explained by the many time-frequency slots that are available to the scheduler (at most $Q$ users can be scheduled in the same OFDM symbol). The maximum delay of users in QoS Class 3 (background download) is larger, which is acceptable.

![Graph showing CDFs of maximum delays]

**Figure 5.3:** The figure shows the CDFs of the maximum delays of the 32 users in the cell for clustered beamforming with M-PF and PF scheduling.

Adding more transmit antennas in a beamforming system enables narrower beams. In the clustered opportunistic beamforming system studied here, more transmit antennas is advantageous as long as there are enough active users, as can be seen in Figure 5.4.

One important function of the clustered opportunistic beamforming is to induce extra frequency fading for users with flat channels. To show the gain from having different beamformers in different clusters, the cell throughputs of the different schemes are shown in Figure 5.5, where all users are given flat channels. The EQ-BF and SISO systems perform worst, since no frequency fading is induced and exploited. For the conventional beamforming scheme, performance is also severely degraded since the waterfilling can not exploit any frequency selective fading.
Figure 5.4: Cell throughput as a function of the number of users is plotted for two and four transmit antennas, $N_t$, for the CL-BF M-PF scheme.

To evaluate the effect of the fairness parameter, $\kappa$, in the M-PF scheduler, the cell throughput is plotted as a function of $\kappa$ in Figure 5.6. The cell throughput of the PF scheduler (which has $\kappa = 1$) is plotted as a comparison. As expected, the highest cell throughput is achieved with $\kappa = 0$, when the strongest user is always scheduled, without taking fairness into account. User rate and delay CDFs are plotted in Figures 5.7 and 5.8. A higher fairness level gives more equally distributed users within the QoS-classes. For fairness levels $\kappa > 1$, there are only small differences in user rates and delays. This indicates that a suitable tuning interval for $\kappa$ is between 0 and 1.

To further help the weakest users, the M-PF scheduler interacts with the beamformer, as described in Chapter 4.5. The beamforming weights of the strongest cluster of the weakest users are kept to the next transmission block. The effect of this method can be seen in Figure 5.9, which is a zoom-in on the CDF of the weakest users of QoS Class 1 with a target rate of 256 kbps. The gain of this method is limited because of the
5.3 Simulation Results

Figure 5.5: The cell throughput as a function of the number of users is plotted for two transmit antennas for flat channels. Conventional beamforming with round-robin scheduling (Conv-BF RR), clustered beamforming with modified proportional fair scheduling (CL-BF M-PF) and with standard proportional fair scheduling (CL-BF PF), equal beamforming on all clusters with modified proportional fair (EQ-BF M-PF) and single transmit antenna with modified proportional fair scheduling (SISO M-PF) are compared.

Changing inter-cell interference between transmission blocks. Even if the base-station keeps the beamforming weights on the strongest cluster, the inter-cell interference might change radically. Still a higher throughput for the weakest users can be observed if this extra information is used. The throughput for the stronger users is not affected by this technique.
Figure 5.6: Cell throughput for 32 users as a function of the fairness parameter $\kappa$ is plotted for clustered beamforming with M-PF scheduling. As a comparison, the throughput of clustered beamforming with standard PF is plotted. The standard PF has no fairness parameter.
5.3 Simulation Results

Figure 5.7: User rate CDFs for QoS class 1 for the clustered beamforming with M-PF scheme are plotted for various fairness levels $\kappa$. There are 32 users in the cell.
Figure 5.8: Maximum delay CDFs for QoS class 3 for the clustered beamforming with M-PF scheme are plotted for various fairness levels $\kappa$. A delay of 10 transmission blocks is about 7 ms. There are 32 users in the cell.
5.3 Simulation Results

Figure 5.9: The CDFs for the 60% of the users in class 1 with the lowest rates. Clustered beamforming with M-PF **with** beamforming weight keeping is compared to clustered beamforming with M-PF **without** beamforming weight keeping. Beamforming weight keeping means keeping of the beamforming weights for the reported strongest cluster of the weakest users during consecutive transmission blocks. The threshold for weight keeping is here 0.8*target rate, which is 205 kbps.
Chapter 6

Table-based Performance Evaluation

6.1 Summary
This chapter contains:

- Two packet error probability estimation methods by table lookup.
- An evaluation of the methods, both for SISO and MIMO schemes, using a HIPERLAN/2 system.

6.2 Introduction
In the development and evaluation of different algorithms, Monte Carlo simulations are an essential tool to assess the resulting performance. However, this can be a time-consuming task, for example in system-level evaluations of communications systems. A common strategy in such situations is to use lookup tables to approximately estimate the link performance with a reasonable simulation time. To simulate the full implementation of the physical layer may take hours, whereas the table lookup takes less than a second. These lookup tables, in turn, are typically based on detailed link level simulations. Such strategies have previously been described, for example, in [MdRMU94, OAJ+97, LRZ03]. Typical parameters of interest are the throughput or Packet Error Rate for each link.
There have also been approaches to analytically obtain approximate expressions for the coded performance of OFDM systems, for example [SWB97]. This is usually only feasible in special cases, like for low-order modulation.

Here, we consider the evaluation of wireless LAN systems like HIPER-LAN/2 [HIP00], which uses OFDM. A simple approach in system simulations to avoid extensive link-level simulations is to just map the empirical mean sub-carrier to noise power (C/N) level at the receiver to a PER level. Because of the properties of the HIPERLAN/2 physical layer and since the channels typically exhibit considerable frequency variations across the 20MHz frequency band that the system uses, the performance will not only depend on the mean C/N across the sub-carriers, but on the full statistics of the frequency selective channel, deteriorating the performance of methods based on only mean C/N.

Here, we propose two different two-dimensional tables to better estimate the PER.

- A lookup table with the empirical first and second order moments, across the frequencies, as input parameters.
- A lookup table with the empirical first order moment and the number of sub-carriers below a certain C/N-threshold as input parameters.

The 2-dimensional parameterization of wideband channels used here may also be applied in several other situations, as described in Section 6.6, e.g. to improve the accuracy of adaptive modulation.

6.3 System Model and Assumptions

6.3.1 HIPERLAN/2

In HIPERLAN/2 [HIP00], which is an OFDM based system, 7 different combinations of modulation and channel coding are available for adaptive modulation. However, the same modulation is used for all data-carrying sub-carriers and the channel coding is performed across the frequencies, combined with interleaving to avoid burst errors. HIPERLAN/2 communicates over 52 sub-carriers, of which 48 carry data.

For each packet (Protocol Data Unit or PDU), the most interesting fact in a system evaluation, is if it is correctly received or if at least one bit has been corrupted. Therefore, the PER is defined as the fraction of
6.3 System Model and Assumptions

the PDUs that are corrupted with at least one bit at the receiver. Each PDU consists of 54 bytes of data.

6.3.2 Two Extensions to MIMO

Now, consider a multiple-input-multiple-output (MIMO) extension of HIPERLAN/2, e.g. so called eigen-beamforming [And00] or Alamouti space-time block coding (STBC) [Ala98]. The added MIMO processing at the transmitter and receiver together with the MIMO-channel can often be represented by an equivalent scalar channel (see Figure 6.1).

First, consider eigen-beamforming on the $k$:th sub-carrier. The MIMO-channel $H_k$ can be singular value decomposed as $H_k = U_k S_k V_k^*$, where $^*$ denotes conjugate transpose, $U_k$ and $V_k$ are unitary matrices with the left and right singular vectors of $H_k$ respectively as columns and $S_k$ is diagonal with the singular values of $H_k$ as elements. Denote the largest singular value $s_k$ and the corresponding singular vectors $u_k$ and $v_k$. Multiplying the symbol to be transmitted on sub-carrier $k$, $x_k$, with $v_k$ and the received vector with $u_k^*$ gives the received symbol:

$$y_k = u_k^* H_k v_k x_k + u_k^* n_k = s_k x_k + \tilde{n}_k$$  \hspace{1cm} (6.1)

where $n_k$ and $\tilde{n}_k$ is additive white Gaussian noise. Hence, for eigen-beamforming as described above, the equivalent scalar channel $\alpha_k$ equals $s_k$.

If Alamouti STBC is used at the transmitter (which assumes two transmit antennas) and maximum-ratio-combining is used at the receiver, the equivalent scalar channel gain $\alpha_k$ is related to the MIMO channel as

$$|\alpha_k|^2 = \sum_{m,n} |H_k(m,n)|^2$$  \hspace{1cm} (6.2)

where $H_k(m,n)$ is the scalar channel gain between antenna transmit antenna $m$ and receive antenna $n$ on sub-carrier $k$.

6.3.3 Channel Assumptions and Simulator Description

In our simulations, the SISO channel between two antennas is modeled as frequency-selective Rayleigh fading. The power delay profile is either exponentially decaying with various expected RMS delay-spreads, or according to the HIPERLAN/2 channel models A or B, which correspond
Figure 6.1: MIMO processing and channel can often be transformed into a scalar equivalent. The transformation is done for each sub-carrier $k$.

to typical office environments and typical large open space environments respectively [MAS+98]. The expected total channel power is equal in all cases. The channel impulse response is always shorter than the cyclic prefix of the OFDM symbol. The MIMO channel is modeled as independent SISO channels.

In the SISO setup of the link-level simulator used in this chapter, the transmitter model contains scrambling, convolutional channel coding, interleaving, symbol mapping and OFDM modulation according to the HIPERLAN/2 physical layer specification. In addition to the corresponding transmitter functions, the receiver implements zero-forcing equalization and uses hard decision decoding [Öhm03].

For the MIMO setups studied in this chapter, eigen-beamforming and Alamouti STBC as described above, the MIMO transmitter processing is done between the symbol mapping and the OFDM modulation. Hence, the OFDM modulation needs to be done for each transmit antenna. In the receiver, the MIMO processing is done between the OFDM demodulation and the equalization. It is assumed that the channel is constant during one packet. Furthermore, both the receiver and the transmitter are assumed to have perfect estimates of the channel. The noise is assumed to be additive white and Gaussian. No interference is considered in this chapter, but an extension to a noise and interference scenario should be possible by replacing the sub-carrier to noise power ratios with sub-carrier to noise and interference power ratios.
6.4 Performance Evaluation

For each given channel realization, the packet error probability will depend on the full channel response. However, because of the channel coding and interleaving across frequencies, it is reasonable to assume that the most significant parameters affecting the packet error probability is the empirical mean C/N level and the empirical C/N variability across the sub-carriers. Three different mappings from the full channel response to a PER are considered in this work.

- The traditional 1-dimensional mapping from the empirical mean C/N across the sub-carriers to a PER.

- 2-dimensional mapping from the empirical mean and variance of C/N across the sub-carriers to a PER. The variance is normalized by the squared mean C/N in order to make most realizations fall into a rectangular table.

- 2-dimensional mapping from the empirical mean C/N across the sub-carriers and the empirical number of sub-carriers with a C/N below a certain threshold value. The threshold value should be chosen according to the used transmission mode. An appropriate threshold value was found by trial and error.

The PER estimation qualities of the methods mentioned above are benchmarked to the link-level simulator, whose PER is considered to be true.

6.4.1 Lookup Table Generation

Based on link-level simulations, using a full implementation of the HIPER-LAN/2 physical layer (see section 6.3), the average PER is estimated as a function of the parameters of interest. A large number of different channel realizations with widely varying delay spread are used to determine the table in order to be able to predict the performance of a large scope of different channels. The following steps describe how to generate a lookup table.

1. Define the table in terms of boundaries and grid-resolution. Two integers per table bin will be needed, ‘number of packets’ and ‘number of erroneous packets’.
2. Generate a channel realization. Vary the expected delay spread between iterations in order to cover as much of the table as possible.

3. Use the channel realization in the link simulator to transmit and receive one packet. Also vary the noise level between iterations.

4. Increase the 'number of packets' variable in the bin corresponding to the channel realization and noise level.

5. If the packet was received erroneously, also increase the 'number of erroneous packets' variable.

6. Goto 2. until enough packets have been transmitted and received to collect sufficient statistics for all bins in the table.

7. Generate the lookup table by dividing the 'number of erroneous packets' with the 'number of packets' for each bin.

As an example, Figure 6.2 shows the table for transmission mode 3 (QPSK modulation with rate 1/2 convolutional coding). Clearly, the frequency variability of the channel affects the resulting PER level.

6.4.2 PER Estimation using the Lookup Table

Once the lookup table has been generated, it can be used to estimate the PER of a particular channel realization and noise level. Note that the table is valid only for the transmission mode that was used during the generation of the table. Linear interpolation is used in the table. For channel realizations outside the table, special measures have to be taken. We have extended the table with the boundary values of the table, i.e. the PER is estimated as the PER of the point in the table that is closest to the realization. If the span of the table is chosen well, the outlier realizations will be few and the mismatch of little importance.

When estimating the PER of a MIMO link, the MIMO channel realizations have to be converted into their scalar SISO equivalents, as described in section 6.3. The equivalent scalar channel realization is then used for table lookup.

6.5 Validation

In the validation of the lookup tables, the channels were generated according to simulation channels A and B for HIPERLAN/2 [MAS`98].
Figure 6.2: A lookup table using mean C/N and variance of C/N across the frequencies, for transmission mode 3.

The same channel realizations have been used for all methods, including the full link-level simulation. Note that the x-axis in the plots is the expected value of C/N. This means that the actual channel realizations used for a particular expected C/N value typically deviate from the expected C/N value. The PER in the plots was computed as the average PER for the different channel realizations as estimated by the different methods. In this section only transmission mode 3 (QPSK modulation with rate 1/2 convolutional coding) is considered, but it has been verified that the methods perform equally well for other modes.

The ability of the lookup table methods to estimate the PER for different levels of expected C/N in a SISO setting using channel model B is illustrated in Figure 6.3. The figure shows the result from the full implementation of the physical layer as a benchmark and the corresponding results based on the lookup table techniques described above. Both 2-dimensional tables provide better estimates of the true PER than the 1-dimensional table, although the difference is small.

The same lookup tables were used to estimate the PER of a MIMO-
Figure 6.3: Performance of transmission mode 3 for channel model B (SISO). 2-d table 1 uses the variance of C/N whereas 2-d table 2 uses the number of sub-carriers below a threshold value.

extended HIPERLAN/2 link, using the technique described in section 6.4.2. In Figure 6.4 and Figure 6.5, 1- and 2-dimensional table lookup is compared to the full implementation of the MIMO link. The MIMO channel was constructed from independent channel realizations of simulation channel A. In Figure 6.4, 2 transmit antennas and 2 receive antennas were used together with beamforming. In Figure 6.5, 2 transmit antennas and 4 receive antennas were used together with Alamouti STBC. Clearly, the 2-dimensional tables predict the PER more accurately. The big difference can by explained by the fact that, in the MIMO case, the equivalent frequency responses generally are much more flat than the SISO frequency responses, due to multiple antenna diversity. This property is not captured in the 1-dimensional table. Note that the lookup tables used in the MIMO settings were generated using the SISO setting. If the tables had been generated in the same setting as they were used, the difference between the 1-dimensional and 2-dimensional tables...
6.6 Summary and Applications

This chapter has presented and evaluated two table lookup methods to quickly estimate the resulting PER from a channel realization without simulating the full link-level. The extension of this method to a MIMO scenario is also presented and evaluated. It was shown that the proposed 2-dimensional mappings more accurately estimate the PER from the channel realizations, especially when the link settings differ from when

Figure 6.4: Performance of transmission mode 3 for channel model A, using 2x2 beamforming. 2-d table 1 uses the variance of C/N whereas 2-d table 2 uses the number of sub-carriers below a threshold value.

would probably have been smaller. However, this is the strength of the 2-dimensional tables. The time-consuming task of table-generation has to be done only once, which makes it easy to flexibly configure the system level simulation model in terms of number of antennas and used MIMO technique.
the lookup table was generated.

The PER estimation strategy described above is primarily useful for system level simulations, but also on the link level, especially when comparing and designing different transmit and receive strategies for MIMO systems. The schemes presented herein for link performance estimation may also be applied to adaptive modulation schemes that use the same transmission mode across the sub-carriers, e.g. HIPERLAN/2 and IEEE 802.11a [HIP00][IEE99]. Currently, the transmission mode is normally determined by the empirical mean C/N. Taking the frequency variability into account, as described above, may substantially improve the accuracy of the modulation adaption [Öhm03].

Also, the general principle of designing multi-parameter tables for performance evaluation is applicable to other kinds of systems. In this evaluation, differences in coded symbol SNR was due to different C/N
in the channel frequency response. The table-based methods presented here can be applied to other systems where the coded symbols are sent over different channels, for instance fast fading single-carrier systems or spatial multiplexing schemes where symbols are transmitted over different spatial channels with different SNR.
Chapter 7

Conclusions

In this thesis, several aspects of multiuser diversity OFDM systems were discussed. A solution to the practical problem of feedback overhead was proposed and analyzed. It involves clustering of sub-carriers and feedback of only a small part of the spectrum. To overcome the low spectral usage when few users are active and the reduced feedback scheme is used, an extension to an adaptive reduced feedback scheme was derived. The idea is to let each user feed back more when there are few users in the system and less when the system is loaded.

Based on the clustering, which was a core element in the reduced feedback scheme, an opportunistic beamforming scheme was proposed. It uses random beamforming in each cluster, which increases the frequency selective fading, thereby increasing the multiuser diversity effect. Important to note is that the opportunistic beamforming scheme does not increase the fading between sub-carriers within the clusters, which enables high accuracy in the reduced feedback scheme. By changing all beamforming weights each scheduling block, the temporal fading rate of slowly moving users is increased, increasing both fairness and system throughput.

The scheduling algorithm is central in multiuser diversity schemes. In this thesis, a modification of the proportional fair scheduling method was proposed and evaluated. The proportional fair algorithm tries to combine multiuser diversity and fairness. The proposed modified proportional fair scheduler incorporates two important aspects into the standard proportional fair scheduler. Firstly, the modified scheduler allows users to have individual target bit-rates and delays. This is necessary in order
to meet the diverse user requirements on future wireless voice and data traffic. Secondly, a fairness parameter was introduced into the scheduler, allowing the operator to tune the behavior of the scheduling algorithm in the wide range from a pure multiuser diversity scheduler to a “completely fair” scheduler. The proposed scheduling metric can also be used in single-carrier multiuser diversity systems.

Opportunistic beamforming originally requires no channel state information, but forms the beams completely randomly. In an FDD multiuser diversity system however, there is always some feedback from the users to assist the scheduling. In this thesis, it was proposed that this information could be used by the opportunistic beamforming to help the unfairly treated users in the system by letting the “good beams” for these users remain for a period of time.

The last chapter of the thesis deals with a slightly different topic. In many coded and interleaved packet-based communication systems, the connection between the channel realization and the resulting packet error probability is difficult to analyze. In this thesis a table-based method is proposed and investigated. The table is a mapping from a parameterization of the channel realization to a packet error rate. As parameters, the average SNR was used together with either the variance of the SNRs across the coded block or the number of symbols in the block under a particular SNR-threshold. This technique can be useful in for instance large wireless network simulations as well as in the design of adaptive coding and modulation schemes. The method was evaluated in a HIPERLAN/2 scenario.

7.1 Ideas for Future Work

Several practical aspects and evolutions of multiuser diversity remain open. Listed below are a few topics for future research.

- In this thesis we used the multiple transmit antennas to send only one symbol stream (per sub-carrier). To increase throughput, several symbol streams can be used. Different beams with different symbols can be sent to different users, creating a form of opportunistic SDMA [SH03, VTL02].

- The “cross-layer” interaction between scheduler and beamformer was only briefly touched upon in this thesis. It allows for smarter control of the “random” beams.
7.1 Ideas for Future Work

- The cluster or sub-carrier assignment order under the proportional fair constraint that maximizes the total throughput is not applied in this thesis. A low complexity solution to this problem is left for future research.

- The proportional fair algorithm does not take data queue lengths into account. The PF algorithm has shown instability in the case of finite data queues [And04]. This encourages the design of a fair multiuser diversity scheduling algorithms for finite data queues.

- In multiuser diversity systems, data blocks are usually short and consecutively received blocks may have been transmitted with completely different coding and modulation modes. This rises the question of how to decode such a data stream in the best way.
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