Analysis of Traffic Load Effects on Railway Bridges

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Abstract

The work presented in this thesis studies the load and load effects of traffic loads on railway bridges. The increased knowledge of the traffic loads, simulated using field measurements of actual trains, are employed in a reliability analysis in an attempt at upgrading existing railway bridges.

The study utilises data from a weigh-in-motion site which records, for each train, the train speed, the loads from each axle and the axle spacings. This data of actual train configurations and axle loads are portrayed as moving forces and then used in computer simulations of trains crossing two dimensional simply supported bridges at constant speed. Only single track short to medium span bridges are considered in the thesis. The studied load effect is the moment at mid-span. From the computer simulations the moment history at mid-span is obtained.

The load effects are analysed by two methods, the first is the classical extreme value theory where the load effect is modelled by the family of distributions called the generalised extreme value distribution (GEV). The other method adopts the peaks-over-threshold method (POT) where the limiting family of distributions for the heights to peaks-over-threshold is the Generalised Pareto Distribution (GPD). The two models are generally found to be a good representation of the data.

The load effects modelled by either the GEV or the GPD are then incorporated into a reliability analysis in order to study the possibility of raising allowable axle loads on existing Swedish railway bridges. The results of the reliability analysis show that they are sensitive to the estimation of the shape parameter of the GEV or the GPD.

While the study is limited to the case of the ultimate limit state where the effects of fatigue are not accounted for, the findings show that for the studied cases an increase in allowable axle load to 25 tonnes would be acceptable even for bridges built to the standards of 1940 and designed to Load Model A of that standard. Even an increase to both 27.5 and 30 tonnes appears to be possible for certain cases. It is also observed that the short span bridges of approximately four metres are the most susceptible to a proposed increase in permissible axle load.

**Keywords:** bridge, rail, traffic load, load effect, dynamic amplification factor, extreme value theory, peaks-over-threshold, reliability theory, axle loads, field data.
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I thank my supervisor Professor Håkan Sundquist for his almost boyish enthusiasm, for his encouragement and also for his patience with me. Thanks are also due to my co-supervisor Dr. Raid Karoumi partly for the use of his computer program and also for his advice and guidance. I also thank both of them for the proof-reading of this thesis and their helpful comments and suggestions.

The research shown here is part of a larger research program, within the Structural Design and Bridges Division, designed to investigate the traffic loads and traffic load effects on bridges. The group has been interesting and stimulating to work within and has provided many an interesting discussion. Thanks go to the members of this group who apart from my supervisors also include Dr. Abraham Getachew who has become a good friend.

Thanks are also expressed to Mr. Lennart Askling and Mr. Lennart Andersson of the Swedish National Rail Administration. The first for his help with supplying documents and the second for his assistance with the field data.

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A warm thankyou to all the staff at the Department of Civil and Architectural Engineering and the former members of staff at the Department of Structural Engineering for creating a pleasant working environment and providing much laughter during coffee breaks. A special mention as regards the last matter goes to Dr. Gunnar Tibert and Dr. Anders Ansell.

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I dedicate this thesis to my mum and dad, Maureen and Ken James who have provided me with a secure upbringing from which it was possible to build.

Stockholm, April 2003
Gerard James
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<td>a stochastic variable to describe the geometric properties</td>
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</tr>
<tr>
<td>C</td>
<td>a stochastic variable to describe the uncertainty of the resistance model</td>
<td>56</td>
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<tr>
<td>Cov</td>
<td>coefficient of variation</td>
<td>57</td>
</tr>
<tr>
<td>Cov_A</td>
<td>coefficient of variation of geometrical properties</td>
<td>57</td>
</tr>
<tr>
<td>Cov_C</td>
<td>coefficient of variation of model uncertainty</td>
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</tr>
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<td>Cov_F</td>
<td>coefficient of variation of material strength</td>
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<td>Cov_G</td>
<td>coefficient of variation of the self-weight</td>
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<td>Cov_P</td>
<td>coefficient of variation of the axle load</td>
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<td>Cov_R</td>
<td>coefficient of variation of resistance</td>
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<td>Cov_R</td>
<td>coefficient of variation of resistance</td>
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<td>Cov_S</td>
<td>coefficient of variation of load effect</td>
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<td>Cov_\phi</td>
<td>coefficient of variation of the dynamic coefficient</td>
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<td>C_X</td>
<td>covariance matrix</td>
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<td>D(X)</td>
<td>standard deviation of a s.v. ( X )</td>
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<tr>
<td>E</td>
<td>expected value</td>
<td>47</td>
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<td>G</td>
<td>a stochastic variable to describe the strength of material</td>
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<tr>
<td>\hat{F}</td>
<td>fitted theoretical distribution function</td>
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<td>F_R(x)</td>
<td>the cumulative density function for the material strength</td>
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<td>\tilde{F}</td>
<td>empirical distribution function</td>
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<td>G</td>
<td>a stochastic variable describing the load effect of the self-weight</td>
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<td>G_d</td>
<td>dimensioning value of the load effect due to self-weight</td>
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</table>
$G_k$ characteristic value of the load effect due to self-weight, page 24

$G(z)$ the cumulative distribution function used in extreme value theory, page 69

$K$ the speed parameter, page 10

$L$ span of a bridge in metres, page 10

$L_{ec}$ effective length in metres, page 29

$L_{\Phi}$ the determinant length in metres, page 34

$L_X$ limit state surface, page 41

$M$ the safety margin, page 41

$M_n$ s.v. representing the maximum of $n$ sets of i.i.d. variables, page 68

$M_{UIC71\Phi}$ moment due to the application of the characteristic code load of load model UIC71 or SW2 including the dynamic effect, page 34

$P_y$ stochastic variable for the maximum yearly axle load, page 89

$Q$ a stochastic variable describing the traffic load effect, page 41

$Q_d$ dimensioning value of the load effect due to traffic live load, page 24

$Q_k$ characteristic value of the load effect due to traffic live load, page 24

$Q_n$ nominal traffic load effect from the UIC71 or the SW/2 load model including the dynamic effects, page 34

$R$ a s.v. describing the load bearing capacity, page 41

$R_{all}$ allowable strength of material, applies to allowable stress code format, page 24

$R_d$ dimensioning value of the load bearing capacity, page 24

$R_k$ characteristic value of the load bearing capacity, page 24

$S$ a stochastic variable for the load effect, page 41

$S_K$ the characteristic value of load effect, page 45

$S_{r_{50}}$ 50 year return load, page 73

$V$ train speed in km/h, page 12

$V()$ variance or variance-covariance, page 73

$\text{Var}$ variance, page 47

$W$ a stochastic variable describing the load effect from wind, page 41
Roman Lower Case

a  a scale parameter of the extreme value distributions, page 69
a  stochastic variable for axle spacing within a bogie, page 89
b  a location parameter of the extreme value distributions, page 69
f  first natural bending frequency of a bridge in Hz, page 10
f_{cek}  characteristic concrete compressive strength, page 25
f_{pt}  allowable stress, page 26
f_R(x)  the probability density function for the material strength, page 43
f_S(x)  the probability density function for the load effect, page 43
f_{uk}  ultimate tensile strength, page 26
f_{yk}  lower yield point, page 26
g(x)  limit state function, page 41
h  height of fill material in metres, page 29
h  height to peak above threshold, or excursion height, page 75
k_R  a factor associated with the characteristic material strength, page 45
k_S  a factor associated with the characteristic load effect, page 45
m  mean number of events of a Poisson point process in a certain time period, page 80
n_o  first natural frequency of a bridge using Eurocodes, page 166
p_f  probability of failure, page 43
u  threshold value, page 74
v  train speed, page 10
x^*  the design point in the \textit{x}-space, page 49
y^*  the design point in the \textit{y}-space, page 51

Greek Upper Case

\Phi  the dynamic amplification factor, page 35
\Phi  cdf of the standardised normal probability distribution, page 45
\Phi^{-1}  the inverse of the cdf of the standardised normal distribution, page 45
\( \Phi_2 \) Dynamic amplification factor using simple method, page 34

\( \Phi_3 \) Dynamic amplification factor for normally maintained track from Eurocodes, page 165

\( \Phi_{td} \) the dynamic amplification factor due solely to the track defects, page 116

**Greek Lower Case**

\( \alpha \) a shape parameter of the extreme value distributions Type II and III, page 69

\( \alpha \) sensitivity factor or directional cosines, page 47

\( \alpha_R \) sensitivity factor for the material strength, page 49

\( \alpha_S \) sensitivity factor for the load effect, page 49

\( \beta \) safety index also reliability index, page 47

\( \beta_C \) Cornell safety index, page 47

\( \beta_{HL} \) Hasofer and Lind reliability index, page 50

\( \epsilon \) dynamic coefficient in %, page 29

\( \eta_c \) factor for systematic difference between strengths in structures and those from tests, page 24

\( \gamma_g \) safety factor for the load effect of self-weight, page 24

\( \gamma_{ind} \) the inferred overall safety factor for the allowable stress code format, page 25

\( \gamma_m \) safety factor for material strengths, page 24

\( \gamma_n \) safety factor for the safety class of structure, page 24

\( \gamma_q \) safety factor for the load effect of imposed traffic load, page 24

\( \kappa \) factor to describe the increase in permissible axle loads, page 63

\( \mu \) location parameter, page 70

\( \hat{\mu}_{0.95} \) estimate of the location parameter that yields the 95% quantile of the 50 year return load, page 73

\( \mu_A \) mean of the geometric property, page 57

\( \mu_C \) mean of the model uncertainty, page 57

\( \mu_F \) mean of the material strength, page 57

\( \mu_G \) mean of the self-weight, page 62

xiv
\(\hat{\mu}\) estimate of the location parameter, page 72

\(\mu_{\text{inc}}\) the location parameter of the GEV when allowing for an increase in permissible axle loads, page 63

\(\mu_M\) mean value of the safety margin, page 47

\(\mu_Q\) mean of the traffic load effect, page 62

\(\mu_Q'\) normal tail approximation of the mean of the traffic load, page 62

\(\mu_R\) mean value of resistance, page 45

\(\mu_S\) mean value of load effect, page 45

\(\mu_{\phi}\) mean value of the dynamic coefficient, page 90

\(\mu_X'\) normal tail approximation of the mean of \(X\), page 52

\(\nu\) ratio of characteristic load effect of self-weight to nominal traffic load effect, page 61

\(\varphi\) pdf of the standard normal distribution, page 52

\(\psi\) load combination factors, page 55

\(\rho\) correlation coefficient, page 11

\(\rho_u\) the probability that an observation will exceed the threshold limit \(u\), page 78

\(\sigma\) scale parameter, page 70

\(\hat{\sigma}_{0.95}\) estimate of the scale parameter that yields the 95\% quantile of the 50 year return load, page 73

\(\sigma_G\) standard deviation of the self-weight, page 62

\(\sigma_Q\) standard deviation of the traffic load effect, page 62

\(\sigma_Q'\) normal tail approximation of the standard deviation of the traffic load, page 62

\(\hat{\sigma}\) estimate of the scale parameter, page 72

\(\sigma_{\text{inc}}\) the scale parameter of the GEV when allowing for an increase in permissible axle loads, page 63

\(\sigma_M\) standard deviation of the safety margin, page 47

\(\sigma_R\) standard deviation of the material strength, page 45

\(\sigma_{\phi}\) standard deviation of the dynamic coefficient, page 10
\( \sigma'_X \) normal tail approximation of the standard deviation of \( X \), page 52

\( \varphi \) the dynamic coefficient or dynamic effect, page 8

\( \varphi' \) dynamic component from a geometrically perfect track, page 8

\( \varphi'' \) dynamic component due to track irregularities, page 8

\( \xi \) damping ratio, page 110

\( \xi \) shape parameter, page 70

\( \hat{\xi}_{0.95} \) estimate of the shape parameter that yields the 95\% quantile of the 50 year return load, page 73

\( \hat{\xi} \) estimate of the shape parameter, page 72

\( \xi_{inc} \) the shape parameter of the GEV when allowing for an increase in permissible axle loads, page 63

**Abbreviations**

AAR Association of American Railroads, page 16

AREA American Railway Engineering Association, page 16

cdf cumulative distribution function, page 45

DB Deutsche Bahn, page 13

ERRI European Rail Research Institute, page 9

EVI extreme value index, same as shape parameter \( \xi \), page 70

EVT extreme value theory, page 65

GEV Generalised Extreme Value, page 70

GPD Generalised Pareto Distribution, page 74

i.i.d. independent identically distributed, page 68

JCSS Joint Committee on Structural Safety, page 39

LRFD Load and Resistance Factor Design, page 23

MLE Maximum Likelihood Estimates, page 72

ML Maximum Likelihood, page 72

MSE mean square error, page 81

NKB The Nordic Committee on Building Regulations, page 39
<table>
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<th>Abbreviation</th>
<th>Full Form</th>
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<tr>
<td>ORE</td>
<td>Office for Research and Experiments of the International Union of Railways, page 9</td>
</tr>
<tr>
<td>Pr</td>
<td>probability of an event occurring, page 68</td>
</tr>
<tr>
<td>PWM</td>
<td>Probability Weighted Moments, page 72</td>
</tr>
<tr>
<td>SLS</td>
<td>serviceability limit state, page 25</td>
</tr>
<tr>
<td>s.v.</td>
<td>stochastic variable/variables, page 56</td>
</tr>
<tr>
<td>UIC</td>
<td>Union International de Chemin de Fer (International Union of Railways), page 8</td>
</tr>
<tr>
<td>ULS</td>
<td>ultimate limit state, page 25</td>
</tr>
<tr>
<td>WAFO</td>
<td>A Matlab Toolbox For Analysis of Random Waves and Loads, page 72</td>
</tr>
<tr>
<td>WIM</td>
<td>weigh-in-motion, page 53</td>
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Chapter 1

Introduction

1.1 Background

In Europe, the railway’s market share of the transport sector has been in steady decline since the beginning of the 1970’s. According to (Nelldal et al., 1999), in 1970 the European railways had 31 % of the transport market, measured in tonne-km, while by 1995 this figure had decreased to just 15 %. During this same period road haulage had increased its share from 54 % to 77 %. The transport sector as a whole during this period increased by almost 75 %. The situation in Sweden over this period is not quite as dismal, with the equivalent market share falling from 43 % to 32 %. This situation in Europe can be compared to that of the United States where the railways managed to almost retain their share of the market, experiencing only a small decline from 51 % to 49 %, while at the same time the transport market in the US increased by approximately 86 % from 1970 to 1995.

The environments for the European and the American rail operators are very different. In the US the permissible axle load is between 30 and 35.7 tonnes while in Europe the standard is only 22.5 tonnes, the US situation allowing each wagon to be loaded to a much higher level. If one considers the self-weight of a wagon to be approximately 5.5 tonnes per axle, then this increase from 22.5 to 35.7 tonne represents a 76 % increase in the amount of goods that can potentially be transported per wagon. Even a moderate increase to a 25 tonne allowable axle load represents a 15 % increase in the weight of goods per wagon. This advantageous situation for the American rail operators can also be compared with the situation for road haulage companies from Europe and the US, which is reversed, Europe generally allowing heavier road vehicles than the US.

The advantages of a higher permissible axle load are not solely confined to that of increased net-to-tare ratios. Capacity of a line is often a critical factor and therefore the more goods than can be transported per train the better, thus reducing the need for expensive investments such as increasing the number of tracks or the number of extra sidings, the need for better signaling systems, etc. Also, energy costs are reduced as less energy goes to transporting the self-weight of the wagon per net...
tonne transported goods. Overhead costs are also reduced when measured per tonne transported goods, examples are increased efficiency at terminals, marshalling yards and during loading and unloaded. The effect of higher axle loads are however not all beneficial, track deterioration and the fatigue rate of bridges are increased with greater axle loads. The optimal allowable axle load is route specific (Abbott, 1999) with many influential factors, such as bridge capacities, type of freight traffic, line capacity, operating cost and marshalling yard. The advantages and disadvantages of an increased allowable axle load are discussed in (Zarembski, 1998; Abbott, 1999; James, 2001).

However, it would be misleading to suggest that the difference between the European and the American situation depended solely on the permissible axle loads. The European railways do not have an integrated railway system and problems such as differing electrification systems, railway gauge, infrastructure charges, safety systems and language to name but a few. These create barriers that prevent the free flow of freight traffic between nations within Europe. The longer hauls i.e. international freight transports should, in principle, be an area where the railways are more competitive than road transport, but unfortunately this is by no means the case. The infrastructure owners of the European railways are generally nationally owned and subsidised. The American infrastructure is, on the contrary, privately owned and despite this heavy burden, their rail freight companies managed to stay profitable. Another major difference in the prerequisites for the two continents is that in the States the freight transport has high priority over passenger transport, whereas in Europe the situation is reversed. This low priority of freight in Europe, creates long waiting times in sidings while higher speed passenger trains are allowed to pass the slower moving freight trains.

Not withstanding these other obviously daunting problems for the European rail freight industry, the increase in permissible axle loads is seen as a concrete measure with which to increase its competitiveness, at least for the transport of bulk goods where this increase can be utilised. Extra demands are therefore placed upon existing infrastructure when attempting to increase allowable axle loads. Key parts of the infrastructure are often the bridges and a renewed assessment of their load bearing capacity is required.

The dimensioning philosophy, adopted when writing new bridge standards and consequently when designing new structures, is conservative in nature. Bridges have an expected design life that often exceeds 100 years and there are many unknowns, especially where the anticipated future development of traffic loads are concerned. Also, it has been shown that an increased in the load bearing capacity at the design stage of a bridge is relatively inexpensive. An increase in the basic investment costs of approximately 3–5% resulting in an approximate increased load bearing capacity of 20%, (Fila, 1996). This conservative philosophy must however be abandoned when considering existing bridges as replacement cost or strengthening cost may be extremely expensive, especially when traffic disturbances are taken into consideration. From a national economics point of view, the possible extra reserves, incorporated at the design stage, should be exploited.
In the north of Sweden, there is a heavy-haul route (Malmbanan) used for the
transportation of iron-ore. During the latter half of the 1990’s there was an extensive
study, undertaken by the Swedish National Rail Administration (Banverket), into
increasing the allowable axle load to 30 tonnes. A report (Eriksson and Voldsund,
1997) showed that this was viable from a national economic point of view. The
positive findings of this project spurred the work into increase allowable axle loads
on the mainline network.

At about the same time a project termed \textit{STAX 25} was started, which involved
increasing the allowable axle loads to 25 tonne on the main Swedish railway network.
The article (Paulsson et al., 1998) provides an interesting account of the problems
involved in increasing the allowable axle loads on bridges, for the Swedish situation,
and how Banverket have approached the problem. As an example of these problems,
it is described how, even along the same route, bridges may have been designed to
different traffic loads.

In Sweden classification of railway bridges are done using a Banverket publication
(Banverket, 2000). The load models in this publication are partly in the form of
equivalent load models and partly in the form of freight wagons. The equivalent load
models are not intended to resemble actual trains while the latter mentioned freight
wagons do. These load models are placed at the position to cause a maximum for
the load effect under consideration. The effects of dynamics are accounted for by
increasing the loads by a dynamic factor. The equivalent load models are based on
statistical evaluation of actual traffic loads, see e.g. (Specialists’ Committee D 192,
1994a; Specialists’ Committee D 192, 1993) for the case of Load Model 71. The
wagon loads are however more of a deterministic nature, with deterministic axle
loads and axle weights. The uncertainties involved in the load models are treated
by the use of partial safety factors on the loading.

In this thesis another approach is adopted, whereby results from field measurements
of actual axle loads, axle spacings and train configurations are used to investigate
the actual traffic load effects on railway bridges. The thesis proceeds further to
formulate a model of the traffic load effects based an extreme value theory that can
then be incorporated into a reliability design.

1.2 Aim and Scope

The aim of this thesis is to develop a method by which field data, containing in-
formation about real trains, may be used to model traffic load effects. The study
investigates ‘actual’ traffic loads on single track railway bridges and applies statisti-
cal models by which the traffic load effects on railway bridges may be described.
It is also intended to investigate the appropriateness of these statistical models in
describing the traffic load effects on railway bridges. A further aim is to incorporate
the resulting traffic load models into a reliability design, which can then be used
to investigate the possibility of increasing allowable axle loads on Swedish railway
bridges. When an increase in allowable axle load has been considered its has been
assumed that the purpose is to raise the allowable axle loads of normal currently operating freight wagons with usual configurations of bogies and vehicle lengths i.e. two and four axle wagons. The study does not intend to cover the introduction of new wagon types with many and closely spaced axles and axle configurations.

The study has been limited to the case of simply supported single track bridges of relatively short span, where the mid-span moment was chosen as the load effect of interest. The spans of the bridges studied, range from 4–30 m. These types of bridges are chosen partly for simplicity and partly because they are the objects chosen for the European studies that led to the justification of the UIC71 load model (Specialists’ Committee D 192, 1993; Specialists’ Committee D 192, 1994a; Specialists’ Committee D 192, 1994b). Also spans between inflection points may be approximated as simply supported spans. Further, it is often these typical simple structures that are used in code calibration processes.

Both the bridges and the trains have been simplified to two dimensional models. The dynamic effects have been considered by adopting the moving force model, where constant forces traverse a beam at constant speed and therefore model the dynamic effects of moving trains on a perfect track. Allowance for the dynamic effects due to track irregularities have been subsequently accounted for in the analysis. As a result of the two dimensional modelling of both the trains and the bridges the effects of lateral motion of the train perpendicular to the direction of the bridge and hence the resulting increase in load effects have been neglected. Only one first natural bending frequency has been considered per span, i.e. one value of bending stiffness and one value of mass per metre for each span, although 11 modes have been included in the analysis. Only one value of damping has been used throughout the simulations.

The action of rail, sleepers and eventual ballast in redistributing loading from axles has not been taken into account, which is a conservative approach.

In the reliability calculations only the ultimate limit state has been considered. The loads considered in the analysis are the self-weight of the bridge and the traffic loads. No account has been made of the increase in the fatigue rate which would occur as a result of an increase in allowable axle loads. Also railway bridges in Sweden have been designed to many combinations of traffic load models and design codes. The study has only considered four of these cases. The reliability calculations are approximate and should be seen as representative values rather than precise values of the probability of failure. Precise calculations would require specific information which is related to a certain bridge and therefore falls outside the scope of this thesis.

1.3 Layout of the Thesis

Chapter 2 of the thesis starts with a review of the European recommendations used for the calculation of the dynamic amplification factor and the investigations that led up to these recommendations and even the studies done subsequent to them. This chapter also shows the results of a similar study done in the United States and
1.3. LAYOUT OF THE THESIS

compares the findings of these studies.

The next chapter reviews some of the Swedish railway traffic load models and some of the design codes used during the twentieth century. A comparison is also performed of the resulting mid-span moment of simply supported beams under these applied loads. Both the static and the dynamic mid-span moments are compared and even the dynamic amplification factors are compared. The last part of this chapter defines the combination of traffic load models and material properties such as material safety factors and characteristic quantiles assumed from the different codes.

Chapter 4 begins with an introduction into the theory of reliability. The first sections, i.e. sections 4.1–4.7, are of an introductory nature and attempt to concisely explain the basics of reliability theory and can therefore be bypassed by the reader familiar with these concepts. However, the iteration procedure used to locate the design point and the normal tail approximations, both of which are used in this thesis are described in sections 4.6.5–4.6.6. The remaining sections of this chapter explain more specifically the underlying assumptions of the reliability analysis used in this thesis. The basic variables considered in the analysis are presented together with their associated properties and assumed distributions. The manner in which the mean values of the resistance of the existing structure are taken from the varying code formats are presented. Also described is the manner in which an increase in allowable axle load is treated in the analysis. The properties and the assumed distributions of the traffic load models are treated separately in Chapter 5.

The traffic load effects are modelled using extreme value theory and this is the main subject of Chapter 5. Two representations are possible under this theory the first, described as the classical approach, using the family of distributions known collectively as the Generalised Extreme Value distribution. The alternative method under the extreme value theory is the peaks-over-threshold method where the family of distributions known as the Generalised Pareto Distribution are used for modelling the load effect. In this chapter the theory behind these two families of distributions are briefly discussed and the distributions themselves are presented. The methods used to estimate the parameters of the fitted distributions are presented together with the method of choosing a suitable threshold level in the peaks-over-threshold analysis. The chapter also discusses the analytical tools used to judge the validity of the theoretical models against the data and to assist the choice of the threshold level.

Chapter 6 is a study done on short span bridges and for four-axled wagons to investigate the sensitivity of some selected parameters on the outcome of a reliability analysis. The chapter is mostly independent of the main part of the thesis although it does utilise theories described in the other chapters of the thesis. The chapter studies the effect of the different underlying assumptions in order to identify the factors relevant to a reliability analysis.

Chapter 7 first describes the method used for the collection of field data, including train speed, static load per axle and the identification of wagon and locomotive types. The chapter attempts to study the accuracy of the measuring device in establishing
the static load per axle. In this chapter the method of modelling the bridges and
the trains is described together with the assumptions for damping. Also described
is the method used to make allowances for the dynamic effects due to track defects.

In Chapter 8 the results of the entire thesis are presented. The first part of the
chapter contains results of the measured static loading from axles and the results of
the dynamic effect due to track defects. Also presented are examples of histograms
of the resulting maximum moment distribution per train and per sets of 50 trains.
Results of the distribution fitting to both the family of generalised extreme value
distributions and the generalised pareto distributions are presented together with
obtained parameter estimates and subsequent return loads and an evaluation of their
variances. The chapter also presents the results of the findings of the reliability
analysis done for the considered design codes and load models. Results are also
compared between the two models of the classical approach to extreme value theory
and the approach using the peaks-over-threshold method as applied to this particular
problem.

Finally, the findings of the thesis are discussed and conclusion are made in Chapter 9.
Also, possible improvements to the models are made and areas for further research
are discussed.

In the appendices, the bridge data used is detailed in Appendix A, while in Ap-
pendix B the calculation methods for the dynamic amplification factor from the
Eurocodes are shown. In Appendix C several results in the form of diagrams and
tables are shown that are deemed to be superfluous for the main text but may be of
interest to future researchers or readers wishing to clarify some of the conclusions
of the main text.

1.4 Notation

The notation used in the different sections of the thesis are in some cases conflicting.
The author has attempted to retain the accepted notation within the differing sub-
ject areas, however, this unfortunately means that the same symbol may sometimes
be used to represent different quantities. Also when quoting equations from other
literature, that is mostly used in isolation within this thesis, the author has mostly
chosen to use the notation of the original literature in order to easily orientate a
reader familiar with this said literature or when referral is necessary. This again may
create confusion but the intention is that the notation is clear within the context of
the text.
Chapter 2

Dynamic Amplification Factor

2.1 Introduction

This chapter details the historical background behind the European rules for the dynamic effects of trains on railway bridges. The amount of literature on the dynamic factor is extensive and a full review is not intended here. As a matter of interest some early Swedish work on the subject was carried out at this department e.g. (Hilleborg, 1952; Hilleborg, 1951; Hilleborg, 1948).

2.2 General

The dynamic amplification factor for railway bridges results from a complex interaction between the properties of the bridge, vehicle and track. The technical report (Specialists’ Committee D 214, 1999b) provides an interesting summary of the key parameters affecting the dynamics of a railway bridge. These are split into four categories:

- Train characteristics,
- Structure characteristics,
- Track irregularities, and
- Others.

The train characteristics that affect the dynamics are:

- Variation in the magnitude of the axle loads,
- Axle spacings,
- Spacing of regularly occurring loads,
• The number of regularly occurring loads, and
• Train speed.

It is also noted that the characteristics of the vehicle suspension and sprung and
unsprung masses play a lesser role.

The structure characteristics are listed in the report as:

• Span or the influence length (for simply supported beams these are equivalent),
• Natural frequency (which is itself a function of span, stiffness, mass and sup-
  port conditions),
• Damping, and
• Mass per metre of the bridge.

The effects from the track irregularities are governed by the following:

• The profile of the irregularity (shape and size),
• The presence of regularly spaced defects, e.g. poorly compacted ballast under
  several sleepers or alternatively the presence of regularly spaced stiff compo-
  nents such as cross-beams, and
• The size of the unsprung axle masses, an increase in unsprung mass causing
  an increased effective axle force.

Under the category others, out-of-round wheels and suspension defects are included.

2.3 Historical Study of the European Recommen-
dations

The European code of practice (CEN, 1995) for traffic loads on railway bridges
details two methods by which the dynamic amplification factor may be estimated.
The first can be described as a simplified method, cf. (B.1) and (B.2) for well and
normally maintained track, respectively and is intended to be used together with one
of the characteristic load models, e.g. Load Model 71, see Figure 3.4. This dynamic
factor is in some manner of speaking 'fictitious' although when used together with
the characteristic load models is designed to describe an envelope that will contain
the dynamic bending moments of real trains that travel at their associated realistic
operating speed. The other method described in (CEN, 1995) is derived from the
Union International de Chemin de Fer (UIC) leaflet (UIC, 1979) and is designed to
be used in conjunction with real trains under certain limiting conditions of bridge
frequency, train speed and risk for resonance.
2.3. HISTORICAL STUDY OF THE EUROPEAN RECOMMENDATIONS

This second method is a more refined method and is detailed here in Appendix B. The approach of the UIC leaflet, i.e. the refined method, splits the dynamic coefficient, \( \varphi \), into two components. The first component \( \varphi' \) is the dynamic effect due to the vibration of a beam traversed by forces travelling at speed on a perfect track. The other component, \( \varphi'' \), is the dynamic effect due to the influence of track irregularities. The method is actually the one used to justify the simplified method described earlier. In Appendix 103 of the UIC 776-1R leaflet (UIC, 1979) six characteristic trains varying from special vehicles, high-speed passenger trains and freight trains are used to obtain maximum mid-span moments for varying lengths of simply supported spans. These moments are then multiplied by the factor \( (1 + \varphi) \), see Appendix B, related to the maximum speed of the vehicle in question. In this manner the type of train is associated with a feasible speed and hence a feasible dynamic factor. The characteristic trains are deterministic with deterministic axle loads, axle spacings and speed (maximum allowable for the train type). The static moments and shear forces multiplied by the dynamic factors, as described above, are then compared with the static moments and shear forces obtained by applying the UIC 71 load model multiplied by the appropriate dynamic factor \( \Phi \) using the simplified method. This is done to ensure that the dynamic load effects of the UIC 71 load model incorporate the values obtained by considering these characteristic trains. The check is done for the cases of good and normal track standard.

This UIC leaflet is in turn based on two studies done by the Office for Research and Experiments of the International Union of Railways, abbreviated ORE, now changed to the European Rail Research Institute (ERRI). The first of these studies are detailed in Question D23—Determination of dynamic forces in bridges which contains a total of 17 reports, the findings of which are summarised in the final report (Specialists' Committee D 23, 1970c), while the other is a single report (Specialists Committee D 128 RP4, 1976).

The D23 reports are a combination of both theoretical studies and of field measurements on actual railway bridges. The testing was carried out during the 50’s and the 60’s on a total of 43 bridges of varying types and materials. Within the study the dynamic response of these different types of bridges were measured during the passage of different types of locomotives. One of these reports (Specialists’ Committee D 23, 1964a) provides details of measured dynamic effects for reinforced concrete slab and beam bridges. Tests were carried out on seven bridges using steam and diesel locomotives. Only the locomotives were used in this study and not dynamic effects from service trains. It should be remembered that at the time the axle loads of the locomotives were greater than those of the wagons. The locomotive traffic loading was therefore the dimensioning case and that of most interest. The report (Specialists’ Committee D 23, 1964b) provides a similar account of the tests done on prestressed concrete bridges.

One of the reports (Specialists’ Committee D 23, 1970b) was a theoretical study of the factors affecting the dynamics of railway bridges. Of particular interest to the author was the aforementioned summary and a statistical evaluation (Specialists’ Committee D 23, 1970a) of the test results from the field measurements. It should be noted that the test trains used are referred to in the reports as not normal trains.
but "heavy test trains producing a high dynamic effect". By this I believe they mean heavy locomotives that are designed to run at high speeds.

The definition of the relative dynamic effect $\varphi$ referred to in the ORE reports is:

$$
\varphi = \left( \frac{\varepsilon_v - \varepsilon_0}{\varepsilon_0} \right) 
$$

(2.1)

where $\varepsilon_v$ is the maximum strain measured from a passage of a locomotive at speed $v$ and $\varepsilon_0$ is the maximum strain measured during the passage of the locomotive at a speed $v \leq 10$ km/h.

In the statistical report (Specialists’ Committee D 23, 1970a) analysis of variance is used to identify the parameters that form the major contribution to the dynamic effect. This report used the information from all the diesel and electric test trains and all the bridges using the maximum dynamic coefficient, $\varphi$, for each locomotive. The major factor was found to be a factor they designated as $K$ which is often referred to as the velocity or speed parameter cf. (Frýba, 1996; Specialists Committee D 128 RP4, 1976). It was finally however decided to use a factor $K/(1 - K)$ for the calculation of a linear regression. Only the runs using the diesel and electric trains are used in this regression process and it is these basic regression curves that form the basis behind the dynamic effect used in the appendix of the Eurocodes (CEN, 1995). The factor $K$ is given by the following:

$$
K = \frac{v^2}{2fL} 
$$

(2.2)

where $f$ is the first natural bending frequency of the loaded bridge in Hz, i.e. with the traffic load present, $v$ is the speed of the train in m/s and $L$ is the span of the bridge in m. The choice of the definition of $f$ is somewhat strange as the value of $K$ will vary with the position of the load and it is therefore often to use the unloaded natural frequency of the bridge rather than the loaded bridge, this is done for example in report 16 (Specialists’ Committee D 23, 1970b). However, this has significance later when the value of $K$ is simplified to the deflection from a combination of the dead and the live load.

The following is the regression curve using the least square method for the results of the investigated steel bridges:

$$
\varphi = 0.532 \frac{K}{1 - K} + 0.0032 
$$

(2.3)

$$
\sigma_\varphi = 0.0262 \left( 1 + 14 \frac{K}{1 - K} \right) 
$$

(2.4)

where $\varphi$ is the mean value and $\sigma_\varphi$ is its standard deviation. This applies for values of $K \leq 0.45$.

A similar expression was found for the prestressed and reinforced concrete bridges, as follows:
2.3. HISTORICAL STUDY OF THE EUROPEAN RECOMMENDATIONS

\begin{align}
\phi &= 0.533 \frac{K}{1-K} - 0.0009 \quad (2.5) \\
\sigma_\phi &= 0.014 \left( 1 + 21 \frac{K}{1-K} \right) \quad (2.6)
\end{align}

with the same notation as above but with the restriction that \( K \leq 0.2 \).

The two above equations (2.3) and (2.5) are the best fits to the data collected from the previously described tests. They have however the problem of the restrictions of the value of \( K \). Extrapolation past these regions to higher values of \( K \) proved to result in what was regarded as values of \( \phi \) that were too large when compared with results of theoretical models, cf. (Specialists’ Committee D 23, 1970c). The reason for extrapolation was to include speed values up to 240 km/h and to obtain a generally applicable equation. The values of \( \phi \) according to the theoretical models, namely showed a levelling off at higher values of \( K \). In report 17, in order to incorporate this levelling off of the dynamic effect an extra \( K^2 \) term was included into the original regression formula and the following was obtained by means of a regression analysis for \( K \leq 0.45 \), i.e. presumably using the best fit to the test data:

\begin{align}
\phi &= 0.615 \frac{K}{1-K + K^2} \quad (2.7) \\
\sigma_\phi &= 0.024 \left( 1 + 18 \frac{K}{1-K + K^2} \right) \quad (2.8)
\end{align}

This expression was subsequently ‘simplified’ and at the same time made more conservative to become:

\begin{align}
\phi &= 0.65 \frac{K}{1-K + K^2} \quad (2.9) \\
\sigma_\phi &= 0.025 \left( 1 + 18 \frac{K}{1-K + K^2} \right) \quad (2.10)
\end{align}

this final equation results in a higher degree of correlation between the mathematical model of (2.9) and the results from the field studies than the previous models of (2.3) and (2.5), \( \rho = 0.672 \). This value can be compared with the earlier correlation coefficient for steel structures of 0.577, the correlation for concrete was not calculated in the report. This expression is valid for steel bridges with \( K \leq 0.45 \) and concrete bridges with \( K \leq 0.2 \) and for unballasted track. However the report then states that since this formula agrees with both the experimental and the calculated results it can be used for all bridges and all \( K \) values. It also states that the presence of ballast will provide greater safety as the dynamic effects are reduced.

Cross-girders and rail-bearers with spans less than 6.5 m were treated with the simple formula:

\begin{align}
\phi &= 0.0033V \quad (2.11) \\
\sigma_\phi &= 0.066 (1 + 0.01V) \quad (2.12)
\end{align}
where \( V \) is the speed in km/h. This equation applies for speeds less than 200 km/h and was evaluated using results from unballasted track.

According to the summary of the reports the dynamic effects are reproducible given that all the parameters are the same. However, it also states that because of the complexity and the great number of parameters involved, the net effect of all these parameters is that the dynamics of the railway bridges can be regarded as stochastic.

Also of interest is that from conversations with Mr. Ian Bucknell of Railtrack who sat on the D214 committee and has also had conversations with the members of the D23 committee, the author has learnt that during the use of the regression analysis of the D23 report, the committee members had no theoretical basis from which to build their regression. This come at a later stage and the expression for \( \varphi' \) given in (B.5) can be proven theoretically.

The expression (B.5) may well have provided a better coefficient of correlation than their final equation (2.9). However, not withstanding this, the regression curves do provide information of the mean and the standard deviation of the actual measured results including the effects from track defects that are inherent in the measurements.

The ORE report (Specialists Committee D 128 RP4, 1976) studies the parameters relating to the dynamic loading of railway bridges paying special attention to the effects of track irregularities. The effects of track irregularities are especially dominant for small span bridges whereas for larger spans the effect of \( \varphi' \) is more governing.

Two types of track irregularities are studied in this report one is referred to as a rail gap. The other track defect studied in this report is that of a gap or poorly compacted ballast under a sleeper referred to in the report as a rail dip. This track imperfection, in the UIC leaflet, is stated as being a dip of 2 mm per 1 m or 6 mm per 3 m.

The first part of this report is a theoretical parametric study using computer simulations. Part I of the report studies simple spans between 5 and 40 m, for each span and each train type a total of four combinations of high/low natural frequency and high/low damping are studied. Three train types are studied all with varied train speeds typical for their respective types. The trains studied are a freight train (one locomotive and two freight wagons), a passenger train (one locomotive and two passenger cars) and a high-speed diesel train. The track imperfection studied in this first part of the study is a track dip of 2 mm per 1 m.

The results of this part of the study find that, for a perfect track, only 5.6% of the simulations exceed the theoretical curve of the UIC leaflet (UIC, 1979), \( \varphi' = k/(1 - k + k^4) \), and all of these are found for the cases of low damping.

The effect of the track irregularities are isolated by subtracting the results of the simulations without track defects from those with defects. The dip of 2 mm per 1 m is studied in this first part. For the high frequency bridges the UIC formula for \( \varphi'' \), cf. (B.9) is well above those calculated except for the case of the 5 m span. This, however, is not the case for the low frequency bridges although the UIC formula does account for the majority of the cases studied cf. Fig 7 and 8 of the said report.
Tables 3 of the report show values obtained for these simulations of the mean \( \varphi'' \), standard deviation \( \sigma_{\varphi''} \) and the 95\% quantile \( \varphi'' + 1.65\sigma_{\varphi''} \), for each span.

In Part II of the theoretical study the number of typical trains is increased to seven and the case of zero damping is included following results from damping measurements done by Deutsche Bahn (DB) on actual bridges which showed that the damping was lower than expected. There is no reference to any article here so it is not possible to ascertain what type of bridges (concrete/steel, ballasted/unballasted) were measured or in which manner. The track defects introduced in this part are 1 mm per 1 m and 3 mm per 3 m i.e. only half the defects of the UIC leaflet on which the formula for \( \varphi'' \) were based. The consequent calculated values of \( \varphi'' \) are therefore compared to half the value obtained from the UIC formula.

Again this part of the study confirmed the use of the UIC formula for \( \varphi' \), i.e. a perfect track. For the cases with high damping all the calculated values were below the UIC formula. For low damping there are a number of times the results exceed the UIC formula and this number increases with decreasing damping.

The effects of the track irregularities are isolated using the same procedure as in Part I. In Table 4 of this ORE report values of \( \varphi'' + 1.65\sigma \) are compared to \( 0.5\varphi'' \) of the UIC formula. Unfortunately, neither the values of \( \varphi'' \) nor the standard deviation of \( \varphi'' \) are shown for this part of the report making their individual components impossible to determine. Again the importance of damping is highlighted as all the calculated values obtained that exceed half the UIC value are for low or zero damping, at least for the case of the 5 m bridge.

The next part of this report is a theoretical study on the parameters affecting the dynamic factor done by the former Czech and Slovak Rail ČSD. The parameters are introduced as dimensionless quantities. The train model is relatively complex including unsprung and sprung masses. The bridge is modelled as a simply supported beam and the deck qualities as an elastic layer. The study is confined to a 10 m prestressed concrete bridge although many parameters are included in the study e.g. the speed parameter, axle spacing, track irregularities, the bridge damping, frequency parameters of the sprung and unsprung masses.

The study finds that the dynamic increment for both the deflection and the bending moment, increase generally with the speed parameter, \( K \). The dynamic increment for the bending moment was found to be generally greater than that of the deflection and showed resonant peaks corresponding to natural frequencies of the selected mechanical model. The study also finds that for this short span bridge the effects of track defects are negligible, for high and very high speeds. In this study, contrary to the earlier part of the report, the dynamic increment is found to depend only marginally on the damping.

The study also finds that the influence of the different properties of the track irregularities are small, i.e. depth and length of defect, an isolated defect or regularly recurring. However all these were calculated for a high speed of 300 km/h and the earlier part of the report finds that the track defects have little effect for a combination of small spans and high speeds.
In the ERRI reports D 214 RP 1-9 investigate the validity of the UIC leaflet (UIC, 1979) with regard to the dynamic effect for train speeds in excess of 200 km/h and near resonance. The reports concentrate on the effects of high speed trains and especially effects near bridge resonance. The investigation is extensive although almost entirely theoretical based. The final report (Specialists' Committee D 214, 1999b) is a summary of the findings of this investigation including a proposed UIC leaflet on the subject. Whilst the emphasis of the investigation, namely railway bridges for speeds greater than 200 km/h, is hardly applicable to the assumed extreme loading case of this thesis i.e. freight trains with possible overloading, it does provide an interesting overview of the European design rules for the dynamic effect and the assumptions on which they are based.

The final report (Specialists' Committee D 214, 1999b) states that underestimation/overestimation of the effects of track irregularities, $\varphi''$, is compensated by overestimation/underestimation of the dynamic impact component $\varphi'$ so that the total dynamic effect is in agreement with predicted values, at least away from resonance.

It is observed in this ERRI investigation that bridge deck accelerations on high speed lines can be excessive and cause amongst other phenomena, ballast instability, unacceptably low wheel/rail contact forces. However, this appears to be a problem associated with high speed lines and was therefore assumed not to be a design criteria when attempting to increase allowable axle loads on freight wagons. This is also true of the problem noted in the investigation concerning the effects of resonance. According to this report resonance effects occur as the train speeds coincide with the natural frequency of the bridge, the speed parameter designated $K$ approaches unity. For the spans studied in this thesis this would require speeds much greater that those of modern freight trains and even those of the foreseeable future. However, the investigation does mention the case of repeated loading that coincides with natural frequency of the bridge or a multiple thereof. This repeated loading is caused from the axles or boogies of the same or similar freight wagons joined together to form a train. This situation is common and is judged to be a possible area of interest as the speeds are within the range of current freight train operating speeds, at least if one considers the case of the repeated loading coinciding with twice the natural period of the bridge. This situation has not been investigated explicitly in this thesis, but the simulations using actual wheel spacings of actual trains together with measured speeds was assumed to cover this eventuality.

One of the reports (Specialists' Committee D 214, 1998) is entirely devoted to the estimation of the damping coefficient to be used in railway bridge dynamic calculations. It is also of interest to possible further work in that there are several existing railway bridges listed in one of the appendices together with the measured, span, bridge type, first frequency, measured damping coefficient and in some cases the structural stiffness $EI$. The report finds that the measured damping varies substantially even for bridges of the same type and span. It is noted in the report that the damping measurements are highly dependent on the estimation method used and the amplitude of vibrations during measurement. The latter observation being well known see e.g. (Frýba, 1996). The report states that modern railway bridges
have generally lower damping ratios and that the measured damping ratios were
generally lower than indicated by previous measurements. It is stated that this last
phenomena is believed to be as a result from better measuring instruments in com-
bination with better estimation techniques. The report also notes that the damping
coefficient is by no means a single value for a single bridge and that it will vary
depending on the train type that causes the excitation of the bridge. It recommends
that several measurements should be made on a bridge with different train types
and means and standard variations be used to obtain bounds for the damping value.

One of the reports (Specialists’ Committee D 214, 1999a) of the D214 committee
studies the effects of track irregularities for high-speed lines. The study uses a com-
plicated model of the track, pads, ballast and sleepers. Even the passenger trains
are well modelled as unprung, bogie and car masses including primary and sec-
ondary suspensions. The track defects used in the report are the same as those used
previously in the D128 report (Specialists Committee D 128 RP4, 1976) and are
intended to represent gaps under sleepers due to poorly compacted ballast. Three
spans are studied of 5, 10 and 20 m. The track defects for the 10 and 20 m spans
are intended to represent normally maintained track standards and are therefore
compared with $\varphi''$, see (B.10a-B.10b) while the 5 m bridge has a defect that repre-
sents a well maintained track and the results for this span are therefore compared
to $0.5\varphi''$, see earlier for details of the defects. The report shows that when the
dynamic effect is measured using the deflection as a criteria then it is always lower
or comparable to that obtained using the UIC 776-1R leaflet (UIC, 1979) for the
spans considered, even close to resonance. For the short span 5 m bridge with a high
natural frequency, the value of $\varphi''$ is calculated to be 0.23. This is compared with
the value of 0.4 obtained from the UIC 776-1R method for a well maintained track,
i.e. $0.5\varphi''$.

The size and shape of the defect and comparison with modern railway maintenance
standards is not discussed in the report as the main objective was to see if the UIC
leaflet adequately predicted the dynamic effects due to track irregularities close to
resonance and the study was therefore only comparative in nature.

Also included in report 5 (Specialists’ Committee D 214, 1999a) is a summary of the
assumptions and assumed range of parameters that were used in the previous studies
conducted by ERRI and which form the research behind the recommendations of
the UIC leaflet 776-1R. Some interesting conclusions of this report are that the
calculated dynamic effects excluding the effects of track defects may be multiplied by
the appropriate factor $(1 + \varphi'')$ or $(1 + 0.5\varphi'')$ to obtain the dynamic effects including
the effects of track irregularities. It is also concluded that the expression for $\varphi''$ as
calculated according to the UIC leaflet may overestimate the effects of the assumed
track defects. However, caution is advised as only a limited number of calculations
were conducted. But it is concluded that with modern track maintenance and for
the assessment of older bridges that this may well be a source with which to reduce
the dynamic effect.

The report also notes that the dynamic effect due to track defects, $\varphi''$, decreases
with span and increases with increased natural frequency. While the opposite is true
for the dynamic effect, $\varphi'$, due to the passage of the trains on a perfect track. In general, both $\varphi'$ and $\varphi''$ increase with speed.

The dynamic effect based on the bridge deck accelerations is, however, more pronounced especially if wheel lift-off occurs. When the unsprung masses regain contact with the rail, high frequency deck accelerations are imposed. However, for the spans and defects considered this occurred at speeds greater than 260 km/h. This effect can therefore be ignored as it lies outside the range of current freight speeds.

2.4 Studies done in the US

During approximately the same period as the D 23 studies done in Europe, the Association of American Railroads, AAR, undertook a series of field measurements of the dynamic amplification factor on American railway bridges which was subsequently reported by the American Railway Engineering Association, AREA.

The article (Byers, 1970) uses the field measurements of the AREA to make suggestions for the distributions to be used in a probabilistic code. The articles provides a useful summary of the AREA survey describing types of bridges speeds and impact factors. A normal distribution was found to be the best fit to the field data and in contrast to (Tobias, 1994) found the log-normal distribution to poorly represent the data. This paper was based on the results using diesel locomotives. It is somewhat unclear from this article and that of (Tobias, 1994) whether the tests were carried out using service trains or just locomotives. In this article it is stated that the AREA test program involved 37 girder spans with some 1800 test runs using trains with diesel locomotives. Of the bridges in the span range 10–20 m, eight were open deck and seven, ballast deck. The ballast deck bridges were of concrete, timber and steel plate. However, the article does not describe the actual bridges in this 10–20 m range.

For 10–20 m unballasted bridges and speeds in excess of 40 mph (64 km/h) a normal distribution was fitted to the data with a mean value of 24% and a standard deviation of 15%. There is a very interesting table in the said article that lists for the two categories of bridges (ballast or open deck) the mean impact factor, the standard deviation, the number of tests for each train speed category.

The article also stated that, not surprisingly, ballasted bridges had a lower impact factor than comparable open deck bridges, this phenomena is also noted in a comparable study done in Europe (Specialists’ Committee D 23, 1970c). The article also provides a good description of the problems of axle loads and dynamics on bridges. For example it comments on varying sources such as high tolerances in the fabrication of goods wagons and locomotives for springs and components, wear on wheels constantly changing the dynamics, track irregularities both on the bridge and on the approach to the bridge as a major factor to the resulting impact factor. Also mentioned is the effect of run-in and run-out i.e. the taking up of slack due to braking and acceleration of the train between wagons. This causes pitching of the
wagons and hence a redistribution of the load to adjoining wagons and boogies via the couplings. I am not sure how relevant this is for European wagons as they do not have the same coupling system as the US.

The article (Sanders and Munse, 1969) discusses the distribution of axle loads to bridge floor systems. This is not the subject under discussion, however, it does refer to the AREA tests and it would appear that the tests solely used information related to locomotives in service trains and not the entire train. It also states that locomotive axle weights could vary as much as 20% compared to the specifications of the manufacturer. It should be remembered, however, that this is based on old manufacturing procedures where a vast majority of the locomotives were steam locomotives.

Articles (Tobias et al., 1996; Tobias and Foutch, 1997) are both very interesting, they present work carried out in the US on statistical analysis of measured axle loads using the same technique, although in a simpler form, as that used for the data collection of this thesis, see Chapter 7. The article (Tobias et al., 1996) mostly presents studies of the axle loads. It presents information obtained from field studies of five instrumented bridges all steel riveted.

The other article (Tobias and Foutch, 1997) has a description of the distributions used for the loading and a description of some Monte-Carlo simulations that they have used for the assessment of the remaining service life of steel railway bridges. It uses reliability theory for this purpose. The original dimensioning load for these bridges, built after the war, were the steam locomotives and not the trailing weights from the wagons which is a comparable situation to the Swedish traffic loads for this period. The article also refers to the study on the impact factor done by the AAR. The article finds that the best distribution to describe the impact factor is log-normal. Fig. 5 of this article shows that for many of these steel bridges the mean measured DAF was approximately 1.1. These values can be compared with a summary (Hermansen, 1998) done on the ORE report (Specialists’ Committee D 23, 1964b) which states that for prestressed concrete bridges the mean DAF was approximately 1.1 and maximum noted was 1.3.

The Ph.D. thesis (Tobias, 1994) studies the fatigue evaluation of riveted steel railway bridges. The thesis presents Monte-Carlo traffic models for evaluating the load effects in riveted railway bridges for American conditions. It uses results of weigh-in-motion studies, locomotive and wagon types and train frequencies and configurations. In the thesis, the dynamic effect is found to closely follow a log-normal distribution. In Figure 2.1, which is redrawn from (Tobias, 1994), the fitted distributions are shown for the 9.1–18.3 m ballasted decks for different speed ranges. The properties of the fitted log-normal distributions are shown in the thesis for both open deck and ballasted bridges, for differing ranges of span and for the ranges of train speed shown in Figure 2.1.
2.5 Comparison of Results from the Described Studies

In the following section, a comparison has been done between the dynamic effect obtained from the original D23 reports, the dynamic effect obtained using the refined method and the dynamic effects taken from (Tobias, 1994) for both ballasted and unballasted girder span bridges. See Appendix B for details of the Eurocode calculations.

One problem encountered was the definition of the natural frequency. In the Eurocode, the natural frequency designated \( n_0 \), is the unloaded natural frequency of the bridge and according to these formulae have an upper and lower bound for each bridge span. The definition of \( K \) is, apart from this frequency definition, the same as in the D23 committee reports, compare (B.6) and (2.2). It was chosen to relate the dynamic effects by a choice of span \( L \) and speed \( v \). In one of the D23 committee report (Specialists’ Committee D 23, 1970c) an estimation of the loaded natural frequency can be obtained using the following:

\[
f \approx \frac{5.6}{\sqrt{\delta}} \tag{2.13}
\]

where \( \delta \) is the deflection due to the traffic plus the dead load, measured in cm which yields \( f \) in Hz. In (Specialists’ Committee D 23, 1970c) a limiting value from a
2.5. COMPARISON OF RESULTS FROM THE DESCRIBED STUDIES

Figure 2.2: Dynamic effect according to the UIC leaflet method, the D23 reports (2.3) and (2.9) and finally the findings from USA for $L = 10\, m$. Mean values and 95% upper confidence limits are detailed.

Table 2.1: Listings of the bridge properties to Figures 2.2–2.3.

<table>
<thead>
<tr>
<th>Span $L$</th>
<th>Loaded frequency $f$</th>
<th>Unloaded frequency range $n_o$</th>
<th>$\Phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5.0</td>
<td>$8.0 \leq n_o \leq 16.9$</td>
<td>1.31</td>
</tr>
<tr>
<td>18</td>
<td>3.7</td>
<td>$4.0 \leq n_o \leq 10.1$</td>
<td>1.168</td>
</tr>
</tbody>
</table>

serviceability state requirement is used such that the deflection should be limited to $L/800$. Using this restriction as the limit case for $\delta$ and changing to metres yields:

$$f \approx \frac{5.6}{\sqrt{L/8}} \quad (2.14)$$

where $f$ is obtained in Hz provided $L$ is inserted in metres. This value of $f$ has then continuously been used when calculating the dynamic effects from the equations of the D23 committee reports, i.e. (2.3–2.10).

In Figures 2.2 and 2.3 a comparison has been made between the dynamic factor as calculated in accordance with the refined method of the UIC leaflet (good track maintenance), the original regression curves of the D23 final report and the Ph.D. thesis, based on the measurements done by the AAR. The figures show the cases of a 10 m and a 18 m bridge, respectively. In the case of the UIC calculated effect both the upper and the lower frequencies have been shown, so that the maximum of these two values should be used in design. The mean value of the dynamic factor plus the 95% upper confidence limit has been shown for the dynamic effect calculated using the final equation of the D23 report, see (2.9–2.10). Also shown is the 95%
upper confidence limit calculated using (2.3–2.4) for steel bridges. The dynamic factors calculated from the American studies are also shown with the mean and the 95\% upper confidence limit drawn for both the ballasted and the unballasted decks. These are easily identified as they appear as stairs, the original data being categorised into speed ranges.

As can be seen from the Figure 2.2, the 95\% upper confidence limit of the original D23 regression curves almost coincide with both the upper limit according to the UIC 776-1R method and the 95\% upper confidence limit of the unballasted bridges of the US study, at least for train speeds less than approximately 70\,km/h. For ballasted bridges, the American study shows a maximum dynamic effect that is only 0.25 and 0.11 for the 95\% upper confidence limit and the mean, respectively, which can be compared with the value of between 0.4–0.46 when using the method of the UIC leaflet. From the US study only the 95\% upper confidence limit for unballasted decks produce the dynamic effects predicted by the UIC leaflet. Below speeds of 120\,km/h, the 95\% upper confidence limits of the D23 findings and the 95\% upper confidence limits of the US study on unballasted bridges are in good agreement. This is not surprising as the regression curves of the D23 report were based on the unballasted bridges stating the ballasted case to be more beneficial. The findings of the US study for ballasted bridges show a maximum value of the dynamic coefficient of 0.11 and 0.25 for the mean and the 95\% upper confidence limit respectively, for the studied speed ranges.

The case of the 18\,m span is shown in Figure 2.3. Again the 95\% upper confidence limits of both the US unballasted bridges and the D23 regression curves are in good
agreement. For speeds less than 120 km/h, the dynamic effect from the UIC leaflet are in close agreement with the 95% upper confidence limit of the ballasted bridges of the US study. The mean value for the US ballasted bridges remains much smaller than the UIC leaflet recommendations, especially under typical operating speeds for freight trains of approximately 70–110 km/h.

This small comparison seems to suggest that the dynamic effect as proposed by the UIC leaflet are in the order of the 95% upper confidence limit at least for the smaller span bridges and for normal freight operating speeds. Indeed in both the UIC leaflet and in (Weber, 1998) it is stated that the given formula for $\varphi'$ is described as representing the 95% upper confidence limit, while the formulae for $\varphi''$ are described as an upper bound that may be exceeded by as much as 30% for special cases of high train speeds and special vehicles. The wording for the properties of $\varphi''$ are difficult to interpret and since there is no comment in the UIC leaflet that gives either the mean values or the associated standard deviations it is impossible to statistically describe the dynamic effects. Either of these quantities would be sufficient to approximate a normal or a log-normal distribution.

For the larger of the two spans studied, the UIC leaflet recommendations produce dynamic effects that appear to underestimate the 95% upper confidence limit for the unballasted bridges but remain above the mean value of the AAR findings for unballasted decks. For this span the UIC calculated dynamic effect is more in line with the AAR 95% upper confidence limit for the ballasted bridges. The AAR test show that the dynamic effect reached a maximum at speeds of approximately 100–120 km/h for both the ballasted and the unballasted decks, at least within the speed ranges considered.

The experimental results of the UIC findings and the AAR tests are in good agreement for the 95% upper confidence limits of the unballasted bridges. It may therefore be possible to use the AAR test results for the European case and assume that the dynamic properties of the US ballasted bridges are similar to the European. This would therefore provide the required means and standard deviations for a statistical description of the dynamic effects and also enable a distinction between ballasted and unballasted bridges. The means and standard deviations of the individual components $\varphi'$ and $\varphi''$, however, remain elusive. The only report that separates these parameters is (Specialists Committee D 128 RP4, 1976) and then only the means and the standard deviations are shown in part I of the report, where only three trains are considered. Also, these values contain a mixture of simulations from the freight, passenger and high-speed train.

### 2.6 Summary and Comments

The European studies in the D23 report are based on field measurements of the dynamic response of actual railway bridges under the action of travelling locomotives. However, the subsequent reports from ERRI have widely been of a theoretical nature with computer simulations forming the basis for the findings. The use of the
field data appears to have been abandoned in preference to computer simulations and theoretical studies.

The track imperfections on which the UIC leaflet 776-1R are based have been used throughout these ERRI studies and used as a means of comparison. How these imperfections relate to actual modern track standards and maintenance does not appear to be discussed, with the exception of the D 214 final report which states that this may be a source of saving when assessing existing railway bridges.

The dynamic effects of the UIC leaflet are approximately the 95% quantiles although it is stated that these values may be exceeded in exceptional cases. The method appears to generally overestimate the dynamic effect especially for ballasted bridges.

The European equations from the D23 report (2.9–2.10) may be suitable for use in a reliability method to describe the dynamic effect for unballasted bridges and for speeds less than approximately 100 km/h, after which they appear to heavily overestimate the values. The American studies are better, in that a distinction is made between ballasted and unballasted bridges. They could therefore be implemented in a reliability application to describe the dynamic factor.

Modern railway bridges have lower damping and natural frequencies than were anticipated at the time of the implementation of the UIC leaflet. This does not have a great significance for this study although it may be of interest to note for studies done on modern railway bridges.
Chapter 3

Earlier Swedish Building Codes

3.1 General

In order to study the possibility of upgrading older bridges, knowledge of the earlier building standards and their format are essential. In this chapter some of the earlier Swedish standards will be presented at least to the extent used in this thesis. Prior to the introduction of the building codes (Statens Betong Kommitté, 1989a; Statens Betong Kommitté, 1989b; Statens Planverk, 1987) in the 1980’s, the Swedish building code format was the *allowable stress* format. In the allowable stress format the loads are treated as being deterministic and there is no increase of the loads to allow for the uncertainties inherent in them. However, it is recognised in this code format that there are uncertainties involved both for the material strengths and for the loading. In order to account for these uncertainties the actual material strengths are decreased by divided by a factor greater than unity, thus yielding the allowable stress. However this factor or overall safety factor is not easily available within the code format. Ascertaining the overall safety factor is more difficult than in the partial safety factor format and has therefore been extracted using assumptions which will be described in the relevant section.

After 1980 the code format changed to the *partial safety factor* format also known as the *Load and Resistance Factor Design* abbreviated LRFD. In the post 1980 codes it is easy to see the partial safety factors used on both the loadings and the material strengths as they are clearly stated within the codes.

This chapter is split into two main parts, the first describing the material properties and the appropriate safety factor, either taken directly from the codes or assumed. The second part describes some of the static traffic load models on railway bridges used in Sweden over the last century and the accompanying dynamic amplification factor or dynamic effect as appropriate.
3.2 Reinforced Concrete

3.2.1 1980’s Swedish Codes

The code format from the post 1980’s has been the partial safety factor format. Between 1980 and the current day there has been several versions of the Swedish codes for reinforced concrete. However, it has generally remained unchanged through these different versions, at least as regards values for partial safety factors and material strengths of concrete and reinforcement bars. When the simple case of only the dead weight and the traffic live load is considered then, in the ultimate limit state, the dimensioning criterion is expressed as

\[ R_d \geq G_d + Q_d \]  \hspace{1cm} (3.1)

where \( R \), \( G \) and \( Q \) are the bearing capacity, the load effect of the self-weight and the load effect of the traffic live load respectively. The index \( d \) indicating the dimensioning values of the aforementioned variables.

The above equation (3.1) can be rewritten to include the relevant partial safety factors:

\[ \frac{R_k}{\eta_c \gamma_m \gamma_n} \geq G_k \gamma_g + Q_k \gamma_q \]  \hspace{1cm} (3.2)

where \( \eta_c \) takes into consideration the systematic difference between the strength of a test specimen with that of the strength in the actual construction (Boverket, 1995; Statens Betong Kommitté, 1989a). The partial safety factors \( \gamma_m, \gamma_n, \gamma_g, \gamma_q \) are the safety factor for the material, the structure’s safety class, the self-weight and the imposed load respectively. The index \( k \) is used to indicate the characteristic values.

3.2.2 Swedish Concrete Code 1949 and 1968

During the late 1940’s and the late 1960’s the building codes for structural concrete were (Kommunikationsdepartementet, 1951) and (Statens Betong Kommitté, 1968), respectively. As mention earlier the code format of this period was the allowable stress format which can be expressed, when only the self-weight and the traffic load are considered, by the following:

\[ R_{all} \geq G_d + Q_d \]  \hspace{1cm} (3.3)

where \( R_{all} \) is the allowable strength of the material. \( G_d \) and \( Q_d \) are the dimensioning values as described above. Their is no distinction between characteristic values and dimensioning values in this code format. The safety factors are not stated in the codes and have therefore been induced. If one attempts to include the safety factors that are implicit in the expression for \( R_{all} \) we obtain

\[ \frac{R_k}{\gamma_{\text{inf}}} = R_{all} \]  \hspace{1cm} (3.4)
where $\gamma_{\text{inf}}$ in this case is the overall safety factor, $R_k$ is the unstated characteristic strength. The index $\text{inf}$ is used to indicate that the safety factor has been inferred.

The case of the concrete compressive strength $K_{400}$ has been considered. The notation $K_{400}$ is from the code (Statens Betong Kommitté, 1968) and is assumed to be equivalent to the concrete defined in the later codes (Statens Betong Kommitté, 1989a; Statens Betong Kommitté, 1989b; Boverket, 1995) as $K_{40}$. The later codes provide details of the characteristic compressive strength, $f_{ck}$ and for this concrete is equal to 28.5 MPa\(^1\). The allowable compressive strength in bending for the $K_{400}$ concrete, according to both the 1968 and the 1949 concrete codes (Statens Betong Kommitté, 1968; Kommunikationsdepartementet, 1951), is 12.5 MPa for what is referred to as the *normal* load case, which is equivalent to the serviceability limit state, SLS. This value should be increased by 20\% for the *exceptional* load case, equivalent to the ultimate limit state, ULS. Re-arranging (3.4) and inserting the above values produces an overall safety factor in bending for concrete given by:

$$
\gamma_{\text{inf}} = \frac{28.5}{(12.5 \cdot 1.2)} = 1.9
$$

Similar assumptions are employed to infer the overall safety factor for the reinforcement steel. The reinforcement steels chosen are $K_{s40}$ and $K_{s60}$ according to the notation used in (Boverket, 1995; Statens Betong Kommitté, 1989a), which are common reinforcement steel qualities in bridge building. These steels are assumed to be equivalent to the steel qualities of (Kommunikationsdepartementet, 1951; Statens Betong Kommitté, 1968), which also use the same notation. According to (Boverket, 1995) the characteristic yield strengths, $f_{yk}$ in tension are approximately 400 and 600 MPa respectively for $K_{s40}$ and $K_{s60}$ steel\(^2\). According to the concrete code of 1968 (Statens Betong Kommitté, 1968) the highest allowable tensile stresses are 220 MPa and 330 MPa for the two steel qualities, respectively. The allowable tensile stress varies depending on the bar diameter, however, using the highest values produce the lowest assumed safety factors which will in turn produce conservative results of the reliability analysis to be performed at a later stage. For the case of the 1949 concrete code (Kommunikationsdepartementet, 1951) only the steel quality $K_{s40}$ is listed and this is given the same value as above i.e. 220 MPa. Again these values should be increased by 20\% for the case of *exceptional* loading. As before, inserting these values into (3.4) and re-arranging yields:

$$
\gamma_{\text{inf}} = \frac{400}{(220 \cdot 1.2)} = \frac{600}{(330 \cdot 1.2)} = 1.52
$$

The characteristic strength of concrete changed from the 10\% quantile to the 5\% in 1986 according to (Banverket, 2000). However there is also an equation in this handbook on adjusting the characteristic concrete strength due to the general increase in strength of concrete with age. For bridges built before 1986 it is suggested that

\(^1\)According to (Boverket, 1995) the characteristic concrete compressive strength is defined as 0.85 of the 5\% quantile of the cylindrical compression strength. The factor 0.85 is intended to take into account long term effects.

\(^2\)According to (Boverket, 1995) the values represent the 5\% quantile of the upper yield point or the 0.2-percent offset method.
Table 3.1: Steel properties according to (Kommunikationsdepartementet, 1948). Allowable stresses are for exceptional loads i.e. equivalent to ULS.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Lower yield strength $f_{yk}$ [MPa]</th>
<th>Ultimate strength $f_{uk}$ [MPa]</th>
<th>Allowable stress $f_{pt}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>St 38</td>
<td>220</td>
<td>$\geq 390$</td>
<td>135</td>
</tr>
<tr>
<td>St 44</td>
<td>260</td>
<td>$460 \leq f_{uk} \leq 500$</td>
<td>160</td>
</tr>
</tbody>
</table>

the characteristic strengths be adjusted by an increase of 15% for the hardening of the concrete with age but a decrease of 2 MPa to account for the change from the 10 to 5% quantile. This would increase the characteristic strengths of the concrete classes considered in this thesis. The effect then of changing from the two quantile values has been ignored in this thesis as the concrete hardening with age appears to more than compensate for this change. This therefore ignores the increase in strength that the concrete undergoes and this thesis assumes, conservatively, that the 5% quantile represents the characteristic values for the concrete strength.

3.3 Structural Steel

3.3.1 1980’s Swedish Codes

The format for the Swedish steel codes of approximately 1980 was the partial safety factor format. The formulation in the ultimate limit state was similar to that of the concrete code, see (3.2). The characteristic strength is defined in (Boverket, 1997) as representing the 1% quantile of the upper yield point. The material partial safety factors are tabulated in Table 3.2.

3.3.2 1938 Swedish Code for Structural Steel

Again the format of this code (Kommunikationsdepartementet, 1948) is the allowable stress format. Structural steel grades often used for railway bridges of the time are St 37 and St 44, using this code’s notation. Within the text of the code the lower yield point i.e. the limit of proportionality is used to define the material strength and the minimum value for the steel grades are listed in Table 3.1 as $f_{yk}$. It is also stated in the code that a 5% deviation from this value is allowed for test results. Another method of classifying the grade of the steel through testing according to (Kommunikationsdepartementet, 1948) is to see if the ultimate tensile strength, $f_{uk}$, lie within the limits shown in Table 3.1. The allowable stress, denoted $f_{pt}$, for a slenderness ratio of zero is listed in Table 3.1. This slenderness ratio was chosen as it produces the safety factor without the any inherent safety factor against buckling. It therefore produces the smallest value and will lead to the most conservative calculation in the forthcoming reliability analysis.

The inferred overall safety factors were then assumed to be given by (3.4) by inserting
Table 3.2: Summary of safety factors, the approximate code year and the values of the quantiles which the material strengths are supposed to represent according to the appropriate code.

<table>
<thead>
<tr>
<th>Material</th>
<th>Year</th>
<th>( \gamma_{\text{inf}} )</th>
<th>( \eta )</th>
<th>( \gamma_{\text{m}} )</th>
<th>( \gamma_{\text{n}} )</th>
<th>Quantile %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete (compression)</td>
<td>1980-</td>
<td>–</td>
<td>1.5</td>
<td>–</td>
<td>1.2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1968</td>
<td>1.9</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1949</td>
<td>1.9</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>5</td>
</tr>
<tr>
<td>Reinforcement Steel (tension)</td>
<td>1980-</td>
<td>–</td>
<td>1.15</td>
<td>–</td>
<td>1.2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1968</td>
<td>1.52</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1949</td>
<td>1.52</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>5</td>
</tr>
<tr>
<td>Structural Steel (tension)</td>
<td>1980-</td>
<td>–</td>
<td>1.0</td>
<td>–</td>
<td>1.2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1938</td>
<td>1.63</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1</td>
</tr>
</tbody>
</table>

\(^a\) The values given for \( \gamma_{\text{n}} \) apply to bridges.

\(^b\) There is a double conversion here from the 1% quantile of the lower yield point (from Swedish testing standards) to the 5% quantile of the upper yield point cf. (Statens Betong Kommitté, 1989a; Boverket, 1995). The latter being the definition of the characteristic strength.

The values of \( f_{\text{pt}} \) and \( f_{\text{yk}} \) which yield

\[
\gamma_{\text{inf}} = \frac{220}{135} \approx \frac{260}{160} \approx 1.63
\]

(3.7)

### 3.4 Summary of Material Properties

Table 3.4 is a summary of the material properties assumed throughout this thesis. The year refers to the approximate year of the code, the safety factors are the stated or the inferred safety factors used on the material properties. Also listed are the values of the quantiles which the material strengths are supposed to represent according to the appropriate code.

### 3.5 Traffic Load Models

#### 3.5.1 Swedish Load Model 1901

According to (Paulsson et al., 1998) many of the older bridges in Sweden were built between 1900 and 1920 to the load model from 1901. The traffic load model is shown in Figure 3.1 and the equivalent uniformly distributed load is shown in Table 3.3 together with the dynamic coefficient for a train speed of approximately 90 km/h. The article states that the load Model A was intended for lines with heavier freight traffic and Model B for normal lines. It is also noted that there are bridges designed to an even lighter load called Model C and that some of the routes in Sweden contain lines with a mixture of bridges designed to these different specifications.
Table 3.3: Uniformly distributed load and the dynamic coefficient for a train speed of 90 km/h from 1901. Reproduced from (Paulsson et al., 1998).

<table>
<thead>
<tr>
<th>Theoretical span (m)</th>
<th>Load Model A (kN/m)</th>
<th>Load Model B (kN/m)</th>
<th>Dynamic coefficient (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>308</td>
<td>277</td>
<td>75</td>
</tr>
<tr>
<td>2.3</td>
<td>174</td>
<td>157</td>
<td>65</td>
</tr>
<tr>
<td>3.3</td>
<td>158</td>
<td>143</td>
<td>55</td>
</tr>
<tr>
<td>4.3</td>
<td>158</td>
<td>143</td>
<td>48</td>
</tr>
<tr>
<td>5.3</td>
<td>147</td>
<td>133</td>
<td>40</td>
</tr>
<tr>
<td>6.4</td>
<td>143.5</td>
<td>129</td>
<td>33</td>
</tr>
<tr>
<td>7.4</td>
<td>136.5</td>
<td>123</td>
<td>29</td>
</tr>
<tr>
<td>8.5</td>
<td>128</td>
<td>115</td>
<td>26</td>
</tr>
<tr>
<td>9.5</td>
<td>120</td>
<td>108</td>
<td>24</td>
</tr>
<tr>
<td>10.5</td>
<td>113.5</td>
<td>104</td>
<td>22</td>
</tr>
<tr>
<td>11.5</td>
<td>107</td>
<td>101</td>
<td>20</td>
</tr>
<tr>
<td>12.5</td>
<td>103</td>
<td>94.5</td>
<td>19</td>
</tr>
<tr>
<td>14.5</td>
<td>100</td>
<td>91</td>
<td>18</td>
</tr>
<tr>
<td>15.6</td>
<td>96.5</td>
<td>90</td>
<td>17</td>
</tr>
<tr>
<td>16.6</td>
<td>96.3</td>
<td>88.5</td>
<td>17</td>
</tr>
<tr>
<td>18.6</td>
<td>93.5</td>
<td>85.5</td>
<td>16</td>
</tr>
<tr>
<td>19.6</td>
<td>92</td>
<td>84.5</td>
<td>16</td>
</tr>
<tr>
<td>20.6</td>
<td>90</td>
<td>83</td>
<td>15</td>
</tr>
<tr>
<td>22.7</td>
<td>89</td>
<td>81</td>
<td>14</td>
</tr>
</tbody>
</table>

The article provides a concise and informative summary of the problems facing the Swedish National Railway Administration in its work to increase allowable axle loads on exiting railway bridges. The article also states that in 1919 the train load model was modified and that the dynamic coefficient was increased.

In this thesis Load Model A is studied whenever reference is made to this year, together with the dynamic coefficient of Table 3.3.

### 3.5.2 Swedish Load Model 1920

The revised load models can be found in (Simonsson et al., 1929) although they are very similar to the models shown in Figure 3.1 except for a minor alterations in the axle spacings. The dynamic coefficient changed substantially and is detailed in (Simonsson et al., 1929) as being

\[
\epsilon = \frac{1000}{13 + 0.7L_e} \tag{3.8}
\]

which applies for an intended maximum train speed of 100 km/h and where the effective length \(L_e\) is inserted in metres, the dynamic coefficient \(\epsilon\) is obtained in %.
3.5. TRAFFIC LOAD MODELS

Load Model A. Locomotive with 20 tonnes axle weight

<table>
<thead>
<tr>
<th></th>
<th>Locomotive 95 t</th>
<th>Tender 54 t</th>
<th>Locomotive 95 t</th>
<th>Tender 54 t</th>
<th>Ore wagons 60 t each</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (Tonnes)</td>
<td>95</td>
<td>54</td>
<td>95</td>
<td>54</td>
<td>60</td>
</tr>
<tr>
<td>Maximum Moment</td>
<td>11.6</td>
<td>6.6</td>
<td>11.6</td>
<td>6.6</td>
<td>7.2</td>
</tr>
<tr>
<td>Shear Force</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Distribution</td>
<td>2.7 1.4 1.4 1.4</td>
<td>2.7 1.4 1.4 1.4</td>
<td>2.7 1.4 1.4 1.4</td>
<td>2.7 1.4 1.4 1.4</td>
<td>2.7 1.4 1.4 1.4</td>
</tr>
</tbody>
</table>

Load Model B. Locomotive with 18 tonnes axle weight

<table>
<thead>
<tr>
<th></th>
<th>Locomotive 85 t</th>
<th>Tender 48 t</th>
<th>Locomotive 85 t</th>
<th>Tender 48 t</th>
<th>Loaded wagons 28 t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (Tonnes)</td>
<td>85</td>
<td>48</td>
<td>85</td>
<td>48</td>
<td>28</td>
</tr>
<tr>
<td>Maximum Moment</td>
<td>11.8</td>
<td>6.4</td>
<td>11.8</td>
<td>6.4</td>
<td>6.5</td>
</tr>
<tr>
<td>Shear Force</td>
<td>2.8</td>
<td>2.8</td>
<td>2.8</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>Distribution</td>
<td>2.6 1.4 1.4 1.4</td>
<td>2.6 1.4 1.4 1.4</td>
<td>2.6 1.4 1.4 1.4</td>
<td>2.6 1.4 1.4 1.4</td>
<td>2.6 1.4 1.4 1.4</td>
</tr>
</tbody>
</table>

Load Model C. Locomotive with 14 tonnes axle weight

<table>
<thead>
<tr>
<th></th>
<th>Locomotive 63 t</th>
<th>Tender 36 t</th>
<th>Locomotive 63 t</th>
<th>Tender 36 t</th>
<th>Loaded wagons 28 t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (Tonnes)</td>
<td>63</td>
<td>36</td>
<td>63</td>
<td>36</td>
<td>28</td>
</tr>
<tr>
<td>Maximum Moment</td>
<td>10.6</td>
<td>6.3</td>
<td>10.6</td>
<td>6.3</td>
<td>6.5</td>
</tr>
<tr>
<td>Shear Force</td>
<td>1.4</td>
<td>2.4</td>
<td>1.4</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Distribution</td>
<td>1.3 1.3 1.3 1.3</td>
<td>1.8 1.8 1.8 1.8</td>
<td>1.8 1.8 1.8 1.8</td>
<td>1.8 1.8 1.8 1.8</td>
<td>1.8 1.8 1.8 1.8</td>
</tr>
</tbody>
</table>

Figure 3.1: The traffic load models from the Swedish codes of 1901. All measurements are in metres and weights are in tonnes. Redrawn from (Paulsson et al., 1998).

This book also states that the dynamic coefficient can be decreased for concrete and reinforced concrete bridges when a fill material is present whose height exceeds 1.0 m, measured from underside of the rail. The revised dynamic coefficient is stated as being

\[ \epsilon_h = \frac{6 - h}{5} \epsilon \]  

(3.9)

where \( \epsilon \) is obtained from (3.8) and \( h \) is the height of the fill in metres.

The book also states that the superstructure of the bridge may or may not contain ballast. However, it also comments that the case without ballast is the more common as the relatively large extra weight of the ballast increases the cost of the bridge. It is therefore assumed in this thesis that the majority of the bridges built around this period do not contain ballast.

3.5.3 Swedish Load Model 1940’s

The traffic load models used in Sweden during the 1940’s are detailed in ( Kommunikationsdepartementet, 1948) and reproduced in Figure 3.2. There are also tables in this document which give details of the maximum static moment, the maximum shear force and the equivalent uniformly distributed load for the case of simply supported beams, which correspond to the loading models A and B. Part of the table for the case of load model A is reproduced in Table 3.4

The dynamic coefficient, \( \epsilon \) in %, to be used is also specified in this document and is
CHAPTER 3. EARLIER SWEDISH BUILDING CODES

Load Model A

<table>
<thead>
<tr>
<th>Load</th>
<th>Locomotive</th>
<th>Tender</th>
<th>Locomotive</th>
<th>Tender</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>2</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>3</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>4</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>5</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>7</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>8</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>9</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>10</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>12</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>15</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>20</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>25</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
<td>2.0 2.6</td>
<td>1.4 1.6</td>
</tr>
</tbody>
</table>

Loaded Wagons 8.5 t/m
Unloaded Wagons 1.0 t/m

Load Model B

<table>
<thead>
<tr>
<th>Load</th>
<th>Locomotive</th>
<th>Tender</th>
<th>Locomotive</th>
<th>Tender</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>13.5 t</td>
<td>1.4 1.6</td>
<td>13.5 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>2</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>3</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>4</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>5</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>7</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>8</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>9</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>10</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>12</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>15</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>20</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
<tr>
<td>25</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
<td>16.2 t</td>
<td>1.4 1.6</td>
</tr>
</tbody>
</table>

Loaded Wagons 5.0 t/m
Unloaded Wagons 1.0 t/m

Figure 3.2: The traffic load models from the Swedish codes of 1938. All measurements are in metres and weights are in tonnes. Redrawn from (Kommunikationsdepartementet, 1948).

Table 3.4: Maximum moments, without the dynamic coefficient, for a simply supported beam exposed to Load Model A from the Swedish code of 1938. Reproduced in part from (Kommunikationsdepartementet, 1948).

<table>
<thead>
<tr>
<th>Span L (m)</th>
<th>Max moment (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>169</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>450</td>
</tr>
<tr>
<td>6</td>
<td>619</td>
</tr>
<tr>
<td>7</td>
<td>816</td>
</tr>
<tr>
<td>8</td>
<td>1014</td>
</tr>
<tr>
<td>9</td>
<td>1230</td>
</tr>
<tr>
<td>10</td>
<td>1468</td>
</tr>
<tr>
<td>12</td>
<td>1943</td>
</tr>
<tr>
<td>15</td>
<td>2784</td>
</tr>
<tr>
<td>16</td>
<td>4597</td>
</tr>
<tr>
<td>18</td>
<td>6922</td>
</tr>
</tbody>
</table>

Table 3.5: Maximum moments, without the dynamic coefficient, for a simply supported beam exposed to Load Model B from the Swedish code of 1938. Reproduced in part from (Kommunikationsdepartementet, 1948).

given by the following formula:

\[ \epsilon = \frac{1765}{20 + L_e} \]  \hspace{1cm} (3.10)

where \( L_e \) is the theoretical length i.e. distance between zero points of the load effect to be studied. This formula (3.10) applies to a maximum speed of 100 km/h and a more general formula is also given in the document by:

\[ \epsilon = \frac{370 + 15.5(v - 10)}{20 + L_e} \]  \hspace{1cm} (3.11)
3.5. TRAFFIC LOAD MODELS

The traffic load model for the 1960 Swedish code is detailed in (Kommunikationsdepartementet, 1961) and reproduced in Figure 3.3. The Load Model F is intended for the heavy haul lines of the time. The traffic load could be reduced to 85% of the load effect from Load Model F for other normal gauge lines or be designed to Load Group 1 (see Figure 3.3) whichever gave the greatest effect.

The dynamic coefficient, in %, can be calculated according to (Kommunikationsdepartementet, 1961) from the following formula

\[
\epsilon = \frac{2200 + 11L_e}{25 + L_e}
\]  

where \( L_e \) is the effective length in metres. There is an accompanying table to decide the effective lengths for different types of constructions and different parts of a construction, for a simply supported beam the effective length is the span length of the beam. There is also another table to be used in conjunction with the dynamic coefficient to take into account the type of track and its joints. This table is reproduced in Table 3.5. However, in the main text it is stated that for the cases 1, 2 and 3a the dynamic coefficient should not be less than 20%.

For this study the case of a welded track with a ballast bed of between 0.4–0.6 m has been considered, i.e. case 3a of Table 3.5. This will yield results on the conservative side as it assumes the lowest dynamic coefficients were used at the time of construction, if one disregards the case of high ballast or fill in excess of 0.6 m.

Figure 3.3: The traffic load models from the Swedish codes of 1960. All measurements are in metres and weights are in tonnes. Redrawn from (Kommunikationsdepartementet, 1961).

where \( v \) is the maximum speed of the line in km/h and \( L_e \) is the theoretical distance, in metres, between zero points as before. No distinction is made between a bridge built with or without ballast in this load model, as regards the dynamic amplification factor.

In this thesis the Load Model A has been used together with the dynamic effect given by (3.10) whenever reference is made to the year 1940.

3.5.4 Swedish Load Model 1960

The traffic load model for the 1960 Swedish code is detailed in (Kommunikationsdepartementet, 1961) and reproduced in Figure 3.3. The Load Model F is intended for the heavy haul lines of the time. The traffic load could be reduced to 85% of the load effect from Load Model F for other normal gauge lines or be designed to Load Group 1 (see Figure 3.3) whichever gave the greatest effect.

The dynamic coefficient, in %, can be calculated according to (Kommunikationsdepartementet, 1961) from the following formula

\[
\epsilon = \frac{2200 + 11L_e}{25 + L_e}
\]  

where \( L_e \) is the effective length in metres. There is an accompanying table to decide the effective lengths for different types of constructions and different parts of a construction, for a simply supported beam the effective length is the span length of the beam. There is also another table to be used in conjunction with the dynamic coefficient to take into account the type of track and its joints. This table is reproduced in Table 3.5. However, in the main text it is stated that for the cases 1, 2 and 3a the dynamic coefficient should not be less than 20%.

For this study the case of a welded track with a ballast bed of between 0.4–0.6 m has been considered, i.e. case 3a of Table 3.5. This will yield results on the conservative side as it assumes the lowest dynamic coefficients were used at the time of construction, if one disregards the case of high ballast or fill in excess of 0.6 m.
Table 3.5: Dynamic coefficient for different types of track and track joints. Reproduced from (Kommunikationsdepartementet, 1961).

<table>
<thead>
<tr>
<th>Type of track</th>
<th>Type of rail joint</th>
<th>Bolted</th>
<th>Welded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Rail placed with direct contact to the superstructure</td>
<td></td>
<td>1.1$\epsilon$</td>
<td>1.0$\epsilon$</td>
</tr>
<tr>
<td>2 Rail placed on wooden sleepers which are themselves in direct contact with the superstructure</td>
<td></td>
<td>0.9$\epsilon$</td>
<td>0.8$\epsilon$</td>
</tr>
<tr>
<td>3 Continuous ballast bed or fill with mean height $h$, measured from the underside of the rail</td>
<td></td>
<td>0.8$\epsilon$</td>
<td>0.8$\epsilon$</td>
</tr>
<tr>
<td>a. $h = 0.4 - 0.6$ m</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b. $h \geq 6$ m</td>
<td>Dynamic coefficient is linearly proportional between the values in 3a and 3b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. $0.6 &lt; h &lt; 6$ m</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.4: The characteristic vertical loads referred to as UIC 71 in (Banverket, 1997) and Load Model 71 in (CEN, 1995).

Whenever reference is made to 1960 in the following chapters of the thesis Load Model F has been used without any reduction. In this context, the dynamic factor used has been calculated according to case 3a of Table 3.5 and using (3.12).

### 3.5.5 Swedish Load Model 1971

The load models that were used for the design of railway bridges in Sweden after approximately 1980 and until recently, are shown in Figures 3.4 and 3.5. These models represent the static vertical characteristic loads and in the Swedish code of practice a bridge should be designed for these two load models, whichever gives rise to the worst case for the parameter under design. The loads should be placed at the most unfavourable position for the structural component and load effect in question.

The UIC leaflet (UIC, 1974) provides a table in one of its appendices of the maximum moment in a simply supported beam, together with its position, when loaded with
3.5. TRAFFIC LOAD MODELS

Figure 3.5: The characteristic vertical loads for heavy rail traffic, referred to as SW/2 in both (Banverket, 1997) and (CEN, 1995).

Figure 3.6: A typical wagon used for transportation of a heavy load, a Uaai-z reproduced from (StatensJärnväg, 2000).

Load Model 71. There is, however, an allowance in the codes of practice (Banverket, 1997; CEN, 1995) for the point loads to be distributed when a minimum height of ballast is present. The code moments used in this thesis, therefore, assume a ballasted track and adopt a simplified method where the point loads are replaced by an equivalent UDL of 156 kN/m over a length of 6.4 m. These moments are lower than those of (UIC, 1974) and will therefore yield higher moment ratios, i.e. results on the conservative side.

The UIC71/Load Model 71 load is meant to represent normal traffic on international lines and has been used, in Sweden, in conjunction with the maximum allowable axle weight of 22.5 tonnes and a maximum allowable weight per metre of 8 tonnes for the best class of line.

The SW/2 load comes from the German Schwerlast meaning heavy load. This type of load is typically for the transportation of large loads such as turbines, large industrial machines. The loads are often known exactly and the speeds of the trains are often limited to 30 km/h. In Sweden, the Swedish National Rail Administration, (Banverket), individually check the capacity of the bridges along a proposed route before permission is granted for the transportation of such loads. A typical wagon for such a load is shown in Figure 3.6. A comparison of Figures 3.5 and 3.6 reveals the similarities between the actual load and the load model, at least in format if not numerically.

In order to obtain the design loads these characteristic static vertical traffic loads are
Table 3.6: Partial safety factors $\gamma_q$ for traffic loads.

<table>
<thead>
<tr>
<th></th>
<th>Swedish Code</th>
<th>Eurocode</th>
</tr>
</thead>
<tbody>
<tr>
<td>UIC71</td>
<td>1.4</td>
<td>1.2</td>
</tr>
<tr>
<td>SW/2 Load Model71</td>
<td>1.45</td>
<td>1.2</td>
</tr>
</tbody>
</table>

multiplied by a partial safety factor for variable loads and by a dynamic amplification factor. The respective partial safety factors for the two load models are tabulated in Table 3.6 for both the Swedish and the European code of practice.

According to the simplified rules of (CEN, 1995; Banverket, 1997) the dynamic amplification factor for carefully maintained track is given by the following equation:

$$\Phi_2 = \frac{1.44}{(\sqrt{L_\Phi} - 0.2)} + 0.82$$  \hspace{1cm} (3.13)

where $L_\Phi$ is the determinant length in metres. In the Swedish code of practice there is a limitation that $1.05 \leq \Phi_2 \leq 1.67$ while in the Eurocodes the equivalent limitation is $1.00 \leq \Phi_2 \leq 1.67$. The definition of the effective lengths vary somewhat between the Swedish code and the European code (Kindberg and Rydberg, 1997), although for the studied case of simply supported beams they are the same.

The dimensioning load effect of the load models illustrated in Figures 3.4 and 3.5 multiplied by the dynamic amplification factor of (3.13) have been used in future analysis as a denominator in order to produce a normalised load effect. It should be stressed that the load effect obtained in this manner is purely from the characteristic load and the dynamic effect. The Swedish code of practice has been used for this purpose, taking into consideration the partial safety factors when determining which of the load models will be the dimensioning one but not using them. The moment from this load model has been used throughout this thesis and has been given the notation $M_{\text{UIC71}\Phi}$. Since the load effect considered has been the moment at mid-span then this traffic load effect has also been referred to as the nominal load effect, $Q_n$ in the reliability chapters.

### 3.5.6 Comparison of Traffic Load Models 1901–1971

A comparison of the dynamic amplification factors for the four Swedish codes of practices studied, are shown in Figure 3.7. It is interesting to note that the dynamic factor increased substantially between 1901 and 1940 and remained at this level during the 1960’s before being reduced back closer to the original 1901 code rules. These changes are interesting when it is borne-in-mind that the vehicle and track technology during this period has undergone many changes i.e. from being bolted to continuously welded rail, from steam locomotives inducing extra dynamics from hammer blows to the modern day suspensions (at least for some vehicles) and from rails fastened immediately to the superstructure (or via wooden sleepers) to essentially ballasted bridges. These positive effects (i.e. generally act to decrease the
Figure 3.7: Comparison of the dynamic amplification factor, $\Phi$, used in some of the Swedish railway bridge codes during the last century.

Table 3.7: Dynamic mid-span moment for the studied years as a fraction of the 1971 dynamic mid-span moment.

<table>
<thead>
<tr>
<th>Code Year</th>
<th>Span $L$ in metres</th>
<th>4</th>
<th>8</th>
<th>10</th>
<th>13</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>1.1211</td>
<td>1.2075</td>
<td>1.2487</td>
<td>1.2874</td>
<td>1.1042</td>
<td>1.1309</td>
<td></td>
</tr>
<tr>
<td>1938</td>
<td>0.9917</td>
<td>0.9545</td>
<td>0.9451</td>
<td>0.9209</td>
<td>0.7426</td>
<td>0.7216</td>
<td></td>
</tr>
<tr>
<td>1901</td>
<td>0.9384</td>
<td>0.8027</td>
<td>0.7523</td>
<td>0.7177</td>
<td>0.6074</td>
<td>0.5962</td>
<td></td>
</tr>
</tbody>
</table>

dynamic effect) have, however, been accompanied by a general increase in allowable vehicle speed, at least for passenger trains, which generally tend to increase the dynamic effect.

A comparison of the mid-span moment due to the characteristic loads of the various Swedish codes can be seen in Figure 3.8 for the cases excluding and including the dynamic effect.
Figure 3.8: Comparison of the characteristic mid-span moment vs. span from some of the load models of the Swedish railway bridge codes during the last century.
### 3.6 Combinations of Material Properties and Load Models

In the following chapters of the thesis the years 1901, 1940, 1960 and 1980 are used to present the results. The definitions of the use of these years are presented here.

Table 3.8 shows the combination of load models and material codes used when defining the above mentioned years. For example, when referring to the year 1980 in the forthcoming text, this means that the dimensioning load was assumed to be the dimensioning case of the UIC 71 or the SW2 load model including its associated dynamic factor $\Phi_2$. The material code was that from 1980, see Table 3.2 for the partial safety factors for this year. Similar definitions are included for each year definition. All traffic load models include the allowance for dynamic effects.

As regards the material properties of the constructional steel, only the code (Kommunikationsdepartementet, 1948) was used in order to imply the overall safety factor of 1.63, see Table 3.2, which was used throughout for the allowable stress code formats, i.e. prior to approximately 1980. This assumed safety factor was then used in combination with the different load models prior to 1980 to produce the different results. This is obviously incorrect and safety factors should be obtained or inferred for these code years. However, as regards concrete the inferred safety factors did not change from the 1949 code to the 1968 code and it may be reasonable to assume that the safety factors for steel constructions did not change significantly during this period.

<table>
<thead>
<tr>
<th>Material</th>
<th>Definition Year</th>
<th>Traffic Load Model</th>
<th>Material Code Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>1980</td>
<td>UIC 71 or SW/2 (cf. section 3.5.5)</td>
<td>1980</td>
</tr>
<tr>
<td></td>
<td>1960</td>
<td>Load Model F (cf. section 3.5.4)</td>
<td>1968</td>
</tr>
<tr>
<td></td>
<td>1940</td>
<td>Load Model A (cf. section 3.5.3)</td>
<td>1949</td>
</tr>
<tr>
<td>Structural Steel</td>
<td>1980</td>
<td>UIC 71 or SW/2 (cf. section 3.5.5)</td>
<td>1980</td>
</tr>
<tr>
<td></td>
<td>1960</td>
<td>Load Model F (cf. section 3.5.4)</td>
<td>1938</td>
</tr>
<tr>
<td></td>
<td>1940</td>
<td>Load Model A (cf. section 3.5.3)</td>
<td>1938</td>
</tr>
<tr>
<td></td>
<td>1901</td>
<td>Load Model A (cf. section 3.5.1)</td>
<td>1938</td>
</tr>
</tbody>
</table>
Chapter 4

Reliability Theory and Modelling

4.1 General

In this chapter, reliability theory and the methods of assessing the safety or relative safety of a structure are discussed. The first part of the chapter, sections 4.2–4.7, is a short introduction into the subject of reliability theory and illustrates the iteration procedures used to estimate the design point and hence the safety index. These parts of the chapter are taken from various textbooks, e.g. (Thoft-Christensen and Baker, 1982; Melchers, 1999; Madsen et al., 1986), and can therefore be skipped by the reader familiar with the concept of reliability theory. These early sections are only intended as an introduction to the basic concepts, an interested reader is referred to the aforementioned textbooks. Section 4.8 and onwards deal with the specific situation as it relates to the work done in this thesis.

The textbooks mentioned provide a sound theoretical background on this subject, however the publications by The Nordic Committee on Building Regulations (NKB) (NKB, 1987) and that by The Joint Committee on Structural Safety (JCSS) (JCSS, 2001) are of more practical help for the engineer.

4.2 Introduction

Structures such as bridges, our places of work and apartment buildings need to be safe, we do not expect a bridge to collapse when we drive over it or a building to fall down while sitting at home with our families. However buildings can never be built to be totally safe, there must always be a risk of failure. Building a totally safe building would mean using an infinite amount of resources in order to withstand an infinitely large load, this would be undesirable. The risk of failure of a building should be very low, one does not expect to be exposed to the same degree of risk sitting at work in the office as a mountaineer faces climbing the north face of Mount Everest.

Loads on buildings vary with time and are rarely known in advance, they can be
described by stochastic processes. Even the geometry and material strengths of a building vary and can therefore be described as stochastic variables. If hundreds of identical buildings were built beside one another the thickness e.g. of the floor slab will vary between the buildings and so would the material strengths of any equivalent sections within the buildings. The built structure is therefore only one realisation of the unlimited number of possible outcomes.

Safety and reliability theory is a method based on mathematical statistics whereby the material strengths, the loading process with time and model uncertainties can be described by stochastic processes and variables and statements can be made about the safety or the mean time to failure of a structure.

The origins of safety and reliability theory come from the aero- and manufacturing industries and from electronic components where it was of interest to establish the failure rate of certain components or system of components. The field of expertise was further used in the space industry where a high level of reliability was essential and also in computers and telecommunications.

The structural engineer, as opposed to the electronic engineer, does not have the luxury of being able to build hundreds of exactly similar structures and test them to failure. Indeed each structure is unique, however the theory of safety and reliability provides a rational treatment for the assessment of the safety of a structure.

The codes for design of structures are deterministic in there nature. The load on a structure has a certain upper value known as the characteristic value. The material strength parameter has a certain lower value, these parameters are multiplied by factors of safety in the load case and divided by factors of safety in the material parameter case. These safety factors allow for the uncertainty involved in these characteristic values and for the uncertainty of the models used, i.e. testing the cube strength to ascertain the compressive strength of the concrete in the structure. The newer codes, however, do allow for an assessment of the uncertainties using more probabilistic methods.

Mathematical statistics is an integral part of structural reliability. The following chapter assumes that the reader is familiar with basic statistics, however, the theory of extremes and the methods of parameter estimations have been given a chapter of their own, see Chapter 5.

The aim of this chapter is to present the main parts of structural reliability used in this work. Although most of what is written is collected from the following references it is included here partly for the sake of completeness, partly because it is essential for understanding the way in which simulation results are handled in later chapters and also to recognise the fact that for many civil engineers this subject is often overlooked in a basic education. For a more detailed understanding of the subject, however, the reader is advised to read (Thoft-Christensen and Baker, 1982) which provides a good introduction into the subject as does (Schneider, 1997) and (NKB, 1996a; NKB, 1996b; NKB, 1987). For a more detailed explanation of the theory see (Madsen et al., 1986). A more intricate coverage of the theory is provided in (Ditlevsen and Madsen, 1996). However, in this authors personal
4.3 Fundamentals

The basis for this theory is that the variables influencing the strength and loading of a structure can be described by stochastic variables, shortened to s.v. or by a stochastic process. The variables affecting the strength are often denoted by the stochastic variable \( R \) where \( R \) is a function of several other s.v. such as the modulus of elasticity \( E \), the area of the cross-section \( A \) or the flexural strength in bending \( f_b \). Generally, the notation used throughout this chapter is that the stochastic variables are shown as capitals while numerical values or outcomes such as sample values are denoted with small letters, compare \( X \) and \( x \). This may be new to some readers to think of say the cross-sectional area of a floor slab as being a variable but one must imagine that although a drawing specifies a certain dimension. If we were to build several hundred of these floor slabs to the same drawing specifications the resulting floor slabs will of course have a variation in thickness.

A structures reaction to a load is referred to as a load effect, e.g. the moment that arises from a traffic load at a section in a bridge or the deflection created in a structure from a load. The load effect is often denoted by \( S \) and can arise from a variety of loads such as those from wind loading \( W \), traffic loading \( Q \), self-weight \( G \), etc.

In the ultimate limit state, the safety margin denoted \( M \), can in many cases be defined by the following:

\[
M = R - S \tag{4.1}
\]

and for the structure to be safe \( M \geq 0 \). Equation (4.1) appears fairly innocuous, but we should remember that \( M, R \) and \( S \) are all s.v. so that the equation has an infinite number of outcomes. Using the examples of material strength parameters and load effects from above, (4.1) becomes

\[
M(E, A, f_b, ..., W, Q, G, ...) = R(E, A, f_b, ...) - S(W, Q, G, ...) \tag{4.2}
\]

In general terms the s.v. of \( E, A, f_b, W, Q, G, ... \) can be described as \( X_1, X_2, ..., X_n \) so that (4.2) becomes

\[
M(X_1, X_2, ..., X_n) = R(X_1, X_2, ..., X_n) - S(X_1, X_2, ..., X_n) \tag{4.3}
\]

All the s.v. \( X_1, X_2, ..., X_n \), known as the basic variables, will not necessarily appear in both \( S \) and \( R \). However some will e.g. the thickness of a beam will act as a strength parameter but also in the form of the self-weight as a load parameter.

The limit state function or failure function, denoted \( g(x_i) \) is a function that splits the \( x \)-space into a safe set \( S \) and a failure set \( F \). The surface that divides these two regions is the limit state surface or failure surface \( L_X \). Expressed mathematically
yields

\[ g(x_i) > 0, \quad x_i \in S \]  
\[ g(x_i) = 0, \quad x_i \in L_X \]  
\[ g(x_i) < 0, \quad x_i \in F \]

(4.4a)
(4.4b)
(4.4c)

The safety margin, \( M \), introduced earlier is the s.v. obtained from replacing the parameters \( x_i \) with the s.v. \( X_i \) in the limit state function such that

\[ M = g(X_i) \]  

(4.5)

The above form of the safety margin, as expressed in (4.1) is typical of the ultimate limit state. The safety margin does not always follow this form and the function \( g \) may take any form provided that \( M \leq 0 \) corresponds to a failure state and \( M > 0 \) to a safe state. In the serviceability limit state a serviceability requirement for the deflection of a beam of span \( l \) may be

\[ \frac{S}{R} \leq \frac{l}{300} \]  

(4.6)

which expressed in the limit state form will yield a safety margin such that

\[ M = \frac{l}{300} - \frac{S}{R} \]  

(4.7)

A load can be defined as a permanent or a variable load. The self-weight of a structure being described as a permanent load while traffic and wind loads are variable loads as they are time dependent.

Structural reliability theory is broken up into three levels with increasing order of complexity.

- **Level 1.** This level regards the different variables as a single value usually the mean or some characteristic value e.g. the 5% quantile. This is the level used for most design work under the most current codes of practice. At this level it is not possible to make statements about the probability of failure of a structure.

- **Level 2.** At this level the stochastic variables are represented by two properties: the mean value and the variance and where necessary sometimes even the covariance. This method provides an approximation of the probability of failure.

- **Level 3.** For a level 3 analysis it is assumed that information is known that fully describes the entire distributions for all the different stochastic variables involved in the analysis. Given that these assumptions are correct it is possible to make statements regarding the precise value for the probability of failure.
4.4 The Fundamental Reliability Case

The basic reliability problem is that of the two dimensional case where the resistance \( R \) and the load effect \( S \) are each described by one stochastic variable. The probability distribution function for the resistance and the load effect are given by \( f_R(x) \) and \( f_S(x) \), respectively. For the sake of simplicity let us consider the case of a single member or a particular section, so that failure is defined when the load effect \( S \) is greater than or equal to the resistance \( R \). The probability of failure of this particular member or section will therefore be:

\[
p_f = \text{Prob}(R \leq S)
\]

or in general terms

\[
p_f = \text{Prob}[g(R, S) \leq 0]
\]

where \( g(r, s) \) is the limit state function. This probability can be defined in terms of the joint distribution function \( f_{R,S}(r, s) \) by the following double integral

\[
p_f = \int_{F} \int_{F} f_{R,S}(r, s) dr ds
\]

where \( F \) indicates the failure domain.

In the event that the variables \( R \) and \( S \) are independent then (4.10) can be simplified, using the relationship \( f_{R,S}(r, s) = f_R(r)f_S(s) \). The two distributions of \( S \) and \( R \) can then be treated independently. The probability of failure is the probability that the load \( S \) will adopt a value between \( x \) and \( x + dx \) while at the same time the strength \( R \) takes on a value less than or equal to \( x \) and then summed for the entire range of possible \( x \) values, in the limit as \( dx \to 0 \). This is shown schematically in Figure 4.1 for a simple 2-dimensional case, where both \( R \) and \( S \) are normally distributed and uncorrelated. However in general terms, the probability that \( S \) will adopt a value between \( x \) and \( x + dx \) is given by the following

\[
\text{Prob}(x \leq S \leq x + dx) = f_S(x)dx
\]

where \( f_S(x) \) is the probability density function for the load effect. The probability that the strength of the material will take a value less than or equal to \( x \) is given by

\[
\text{Prob}(R \leq x) = F_R(x)
\]

where \( F_R(x) \) is the cumulative distribution function for the material strength. The probability of failure will thus be given by the joint probability of both events occurring

\[
p_f = \sum_{S} \sum_{R} P(x \leq S \leq x + dx) \cdot P(R \leq x)
\]

\[
= \int_{-\infty}^{+\infty} F_R(x)f_S(x)dx
\]

While (4.13) is the formal expression for the probability of failure, closed-form solutions do not always exist for the integral (Thoft-Christensen and Baker, 1982), except in some special cases e.g. normal distributions as will be discussed in the next section. The integral of (4.13) is often referred to as the convolution integral.
Figure 4.1: Probability of failure where $S$ is the load effect, $R$ is the resistance and the two variables are independent. The hatched area is the probability that $R \leq x$ and the shaded area is the probability that $S$ will lie in the range $x$ to $x + dx$ in the limit as $dx \to 0$.

4.5 Characteristic Values

Most current codes of practice for civil engineering incorporate the use of safety factors and characteristic values. The characteristic values are typically high or low quantiles for the load effect and the resistance respectively. Typical characteristic values for the resistance of, for example, the yield stress of a steel reinforcement bar is the 5% quantile. This means that 95% of reinforcement bars will have a yield stress greater than this value. There is a 5% probability, for any steel bar, that its yield stress will be less than this characteristic value. A typical characteristic value

$$f_R$$

Figure 4.2: Typical characteristic value for the resistance variable $R$. A 5% one-sided tail is detailed.

for the material strength is the 5% quantile according to (Svensk Byggtjänst, 1994). An illustration of a typical distribution for the strength of a material is shown in
4.5. CHARACTERISTIC VALUES

Figure 4.2. From this figure it is possible to see that the characteristic value is given by

\[ R_K = \mu_R (1 + k_R Cov_R) \] (4.14)

where \( \mu_R \) is the mean value of \( R \), \( Cov_R \) is the coefficient of variation of the resistance variable and \( k_R \) is a factor depending on the type of distribution assumed for \( R \) and the quantile specified as being the characteristic value. The basic form of (4.14) has its origins from the normal distribution although choices of \( k_R \) can be made to express quantiles from other distributions. Equation (4.14) may also be expressed as

\[ R_K = \mu_R + k_R \sigma_R \] (4.15)

For a normal distribution the value of \( k_R \) representing the 0.05 quantile will be given by

\[ 0.05 = \Phi \left( \frac{R_K - \mu_R}{\sigma_R} \right) \] (4.16a)

\[ 0.05 = \Phi(k_R) \] (4.16b)

\[ k_R = \Phi^{-1}(0.05) \] (4.16c)

\[ k_R \approx -1.645 \] (4.16d)

where \( \Phi \) and \( \Phi^{-1} \) are the cumulative distribution function (cdf) and the inverse cdf for the standardised normal probability distribution, tables of which can be found in most statistic textbooks or mathematical handbooks, see e.g. (Råde and Westergren, 1998).

Another commonly used distribution for the strength of a material is the log-normal distribution, the characteristic value of which can be approximated by

\[ R_K = \mu_R \exp(k_R Cov_R) \] (4.17)

for values of \( Cov_R < 0.3 \), according to (Melchers, 1999) while (NKB, 1987) prefers \( Cov_R < 0.25 \). Where \( k_R \) can again be taken from the standard normal distribution, i.e. a value of \( k_R = -1.645 \) corresponds to the 5% quantile.

The characteristic value for the load is given in an analogous manner by

\[ S_K = \mu_S (1 + k_S Cov_S) \] (4.18)

and can be seen graphically in Figure 4.3. Typical characteristic values are 50% quantile for permanent loads such as self-weight (Svensk Byggtjänst, 1994; CEN, 1996; NKB, 1987) which is the mean value. For variable loads, however, the 98% quantile of the yearly maximum is the characteristic value, again according to (Svensk Byggtjänst, 1994; CEN, 1996; NKB, 1987). The variable loads, such as traffic loads or wind loading, are in fact time dependent and a proper description of these loads would only be obtained by considering the loading as a stochastic process. This would, however, be a complicated model and a way to circumnavigate this problem is to assume that the stochastic process is stationary, i.e. it is time invariant. The loading distribution is often related to a specified reference period.
This reference period is usually, but not always, a one year period and hence any reliability index or probability of failure is thus related to this time period e.g. the yearly probability of failure. Another usual practice is to take the maximum within each reference period and form a distribution from these maximum values. Using a reference period of one year, the distribution for the maxima for this reference period and the characteristic value of the 98% quantile, the characteristic value will hence represent a 2% probability that this value will be exceeded within any one year. Different methods of statistically modelling of the load will be discussed in Chapter 5.

Load combinations factors are also included in this level of structural reliability. As one can understand from the previous section the characteristic values for variable loads are chosen as high percentiles with only a small chance that this value will be exceeded within any one year. The likelihood that two such variable loads will be at such high values at the same time will be even smaller. The load combination factors allow for this by reducing one or more of the loads by a suitable factor representing the chance of these loads occurring simultaneously. This particular aspect of structural reliability is not used in this work. However, it is described here briefly for the sake of completeness. The interested reader is referred to the aforementioned textbooks on structural reliability or (Ferry-Borges and Castenheta, 1971; Östlund, 1993).

4.6 Second Moment Reliability Methods

4.6.1 General

In second moment reliability methods the distributions of the basic variables that constitute $R$ and $S$ are described solely by two properties namely their expected values, first moment, and their variances or covariances, second moment, hence the name. In the event that both the strength $R$ and the load effect $S$ can be described
by normal distributions then the second moment representation fully describes the
distributions and it is possible to calculate the probability of failure.

The second moment reliability methods can be split into two parts: one dealing with
procedures for a linear failure surface and the other for the non-linear case. The
following subsection 4.6.2 deals with the linear failure function while the remainder
of this section deals with the non-linear case.

4.6.2 Cornell Safety Index

If the safety margin is given by, recalling (4.1)

\[ M = R - S \]  \hspace{1cm} (4.19)

and failure is defined by

\[ M \leq 0 \]  \hspace{1cm} (4.20)

Then the mean value of \( M \) is given by

\[ \mu_M = E(M) = E(R) - E(S) = \mu_R - \mu_S \]  \hspace{1cm} (4.21)

and the variance, if \( R \) and \( S \) are independent variables, is given by

\[ \sigma^2_M = \text{Var}(M) = \text{Var}(R) + \text{Var}(S) = \sigma^2_R + \sigma^2_S \]  \hspace{1cm} (4.22)

Equations (4.21) and (4.22) come from the general properties of subtracting two
distributions, see a standard statistic text book e.g. (Johnson, 1994; Blom, 1989).

Figure 4.4 shows the basic two-dimensional case where \( R \) and \( S \) are normally dis-
tributed, although this method is not confined to a normal distribution. With
reference to Figure 4.4, the safety index \( \beta \) is the number of standard deviations the
mean value of \( M \) is from the failure surface, which in this simplified case is the point
\( m = 0 \). The design point is the point where the design load effect \( s^* \) and the design
material strength \( r^* \) are equal and produce the required value of safety index \( \beta \). The
Cornell safety index, \( \beta_C \), is given by

\[ \beta_C = \frac{E(M)}{D(M)} = \frac{\mu_M}{\sigma_M} \]  \hspace{1cm} (4.23)

In the case where the variables are normally distributed it is possible to relate the
safety index to the probability of failure using the standard normal distribution
found in most statistical textbooks.

\[ p_f = \Phi(-\beta_C) \]  \hspace{1cm} (4.24)

where \( \Phi \) denotes the cumulative density function of the standardised normal distri-
bution. Another interesting parameter can be seen in Figure 4.4, namely that of the
sensitivity factors, denoted \( \alpha \), given by

\[ \alpha_S = \frac{\sigma_S}{\sqrt{\sigma^2_R + \sigma^2_S}} \]  \hspace{1cm} (4.25)

\[ \alpha_R = \frac{\sigma_R}{\sqrt{\sigma^2_R + \sigma^2_S}} \]  \hspace{1cm} (4.26)
Figure 4.4: The upper figure shows the load effect, $S$ and the material strength, $R$ both normally distributed with the design point $s^* = r^*$. The lower figure shows the resulting safety margin $M$ as defined by (4.19).
It can also be shown from (4.25) and (4.26) that the following relationship applies
\[ \alpha_R^2 + \alpha_S^2 = 1 \] (4.27)

In the event that the failure surface is a hyperplane then the limit state function can be described by the following
\[ g(x_i) = a_0 + \sum_{i=1}^{n} a_i x_i \] (4.28)

and the corresponding safety margin is given by
\[ M = a_0 + \sum_{i=1}^{n} a_i X_i = a_0 + a^T X \] (4.29)

where the bold font indicates matrix notation. The Cornell reliability index of (4.23) can hence be written as
\[ \beta_c = \frac{a_0 + a^T E(X)}{\sqrt{a^T C_X a}} \] (4.30)

where \( E(X) \) is a vector of expected values of \( X \) and \( C_X \) is the covariance matrix of \( X \). According to (Madsen et al., 1986) it can be shown that (4.30) is invariant under any linear transformation of the basic variables.

### 4.6.3 The Non-Linear Limit State Surface

As mentioned in the previous section, if the limit state surface is not a hyperplane it is not possible to calculate the expected value and variance of the safety margin, \( M \), solely from the expected values and variance of the basic variables, \( X \).

One method to solve this problem is to linearise the safety margin by approximating it as a tangent hyperplane using the Taylor series expansion about a point. The choice at which the series is expanded affects the resultant safety index. The choice of safety margin is also arbitrary and affects the resulting safety index. To avoid arbitrariness of the safety index, definition of both the limit state function and the chosen point of expansion of the Taylor series is necessary. This requirement is a major drawback for the use of the safety index as a means of indicating the safeness of a structure if even for the same structure, section and loading different safety indexes can be obtained. A method to avoid this arbitrariness was to use a transformation of the basic variables suggested by Hasofer and Lind and to use the design point \( x^* \) as the expansion point.

### 4.6.4 Hasofer and Lind Safety Index

In section 4.6.2 it was shown, for the basic case, that the safety index is a measure of the number of standard deviations the failure surface is from the mean value of the
safety margin. A first step, proposed by Hasofer and Lind, to obtain an invariant reliability index was to transform the basic variables $X$ so that they each had an expected value of zero and in the uncorrelated case a standard deviation of unity.

The transformation for independent variables in $X$ is given by

$$Y_i = \frac{X_i - E(X_i)}{\sigma_{X_i}}$$  \hspace{1cm} (4.31)

where the transformed variable $Y_i$ will now have $\mu_i = 0$ and $\sigma_{Y_i} = 1$.

In the general case where the basic variables are dependent then the sought transformation will yield a transformed variable $Y$ such that

$$E(Y) = 0$$  \hspace{1cm} (4.32)

and the covariance matrix

$$C_Y = \text{Cov} \left[ Y, Y^T \right] = I$$  \hspace{1cm} (4.33)

where $I$ is the identity matrix. The general transformation into the $y$-space can therefore be written as

$$Y = A(X - E[X])$$  \hspace{1cm} (4.34)

and the covariance matrix from (4.33) becomes

$$\text{Cov} \left[ Y, Y^T \right] = AC_X A^T = I$$  \hspace{1cm} (4.35)

The transformation matrix $A$ can be obtained using eigenvalues and eigenvectors.

The limit state surface $L_x$ in the $x$-space must also be correspondingly transformed to a limit state surface $L_y$ in the $y$-space. The transformation is hence

$$y = A(x - E[X])$$  \hspace{1cm} (4.36)

The distance from the origin in the $y$-space to a point on the limit state surface $L_y$ is thus the number of standard deviations from the mean value in the $x$-space to corresponding point on the limit state surface $L_x$. The distance to the limit state surface is thus

$$\beta(y) = (y^T y)^{1/2}$$  \hspace{1cm} (4.37)

where $y$ is a point on the limit state surface $L_y$. Alternatively, it may be preferable to express this in the $x$-space

$$\beta(x) = \left\{ (x - E[X])^T C_X^{-1} (x - E[X]) \right\}^{1/2}$$  \hspace{1cm} (4.38)

The Hasofer and Lind safety or reliability index, denoted $\beta_{HL}$ is the shortest distance from the origin to the limit state surface.

$$\beta_{HL} = \min \left( \sum_{i=1}^{n} y_i^2 \right)^{1/2} = \min(y^T y)^{1/2}$$  \hspace{1cm} (4.39)
provided the point $y$ is a point on the limit state surface. The solution to the
minimisation problem of (4.39) yields a point on the limit state surface $y$ called the
design point and to maintain the convention from earlier sections will be denoted
$y^*$. The design point was mentioned earlier as the point of maximum likelihood
within the failure domain.

The design point was used as the point of expansion for the Taylor series. If only
the first term of the Taylor expansion is used, i.e. linearisation of the failure surface,
then this is known as a first order second method shortened FOSM.

### 4.6.5 Iteration Method to Locate the Design Point

This iteration method adopts a linearisation of the limit state surface at the design
point and is taken from (Madsen et al., 1986). For a full description of the theory
behind this algorithm refer to (Madsen et al., 1986). Similar algorithms are detailed
in (Melchers, 1999; Thoft-Christensen and Baker, 1982).

The iteration procedure will be denoted $y^{(1)}$, $y^{(2)}$, ..., $y^{(m)}$, ..., where each $y$ is an
approximation of the design point. The design point in the $y$-space, is given by the
following

$$
y^* = \beta_{HL} \alpha^* \tag{4.40}
$$

where $\alpha^*$ are the sensitivity factors or directional cosines at the design point. See
subsection 4.6.2 and Figure 4.4 for a simple two-dimensional representation of the
sensitivity factors.

The sensitivity factors at the iteration point $y^{(m)}$ are denoted $\alpha^{(m)}$ and are given by

$$
\alpha^{(m)} = -\frac{\nabla g(y^{(m)})}{|\nabla g(y^{(m)})|} \tag{4.41}
$$

where the suffix $m$ represents the $m$th iteration and where the denominator is the
Euclidean norm and $\nabla g(y^{(m)})$ denotes the gradient vector i.e.

$$
\nabla g(y^{(m)}) = \left[ \frac{\partial g}{\partial y_1}(y^{(m)}) \ldots \frac{\partial g}{\partial y_n}(y^{(m)}) \right] \tag{4.42}
$$

where $n$ is the number of basic variables.

The iteration algorithm is then

$$
y^{(m+1)} = \left( y^{(m)^T} \alpha^{(m)} \right) \alpha^{(m)} + \frac{g(y^{(m)})}{|\nabla g(y^{(m)})|} \alpha^{(m)} \tag{4.43}
$$

and if this converges to a point $y^*$, then

$$
y^* = \beta \alpha^* \tag{4.44}
$$

$$
g(y^*) = 0 \tag{4.45}
$$

The steps of the algorithm can be summarised as
1. Transform the basic variables into the \(y\)-space using (4.31) if independent or (4.32)–(4.35) if correlated

2. Transform the limit state function using (4.36)

3. Perform the partial differentiation of the limit state function according to (4.42)

4. choose a starting point \(y^{(1)}\)

5. calculate the numerical value \(g(y^{(1)})\) from (4.36)

6. calculate the numerical value of the gradient vector from (4.42) and its Euclidean norm

7. calculate the sensitivity factors from (4.41)

8. calculate the new value of the design point from (4.43)

9. check \(|y^{(2)} - y^{(1)}| \leq\) tolerance if not go back to step 5 with \(y^{(2)}\) as the new start point.

10. when satisfied calculate \(\beta\) from (4.37)

The above algorithm was programmed in Matlab by the author of this thesis and checked with worked examples from (Thoft-Christensen and Baker, 1982), see (James, 2001).

4.6.6 Normal Tail Approximation

This transformation is chosen such that the values of the original cdf, \(F_X\), and the original pdf, \(f_X\) of the s.v. \(X\) are exactly the same as the values of the equivalent normally distributed variable at the design point. Expressed mathematically yields

\[
F_X(x^*) = \Phi \left( \frac{x^* - \mu'_X}{\sigma'_X} \right) \quad (4.46)
\]

\[
f_X(x^*) = \frac{1}{\sigma'_X} \varphi \left( \frac{x^* - \mu'_X}{\sigma'_X} \right) \quad (4.47)
\]

where \(\Phi\) and \(\varphi\) are the cdf and the pdf of the standard normal distribution, \(x^*\) is the value of \(X\) at the design point, and \(\mu'_X\) and \(\sigma'_X\) are the mean and the standard deviation of the unknown approximate normal distribution. Expressions for \(\mu'_X\) and \(\sigma'_X\) can be obtained by rearranging (4.46) and (4.47) which yield

\[
\sigma'_X = \frac{\varphi \left\{ \Phi^{-1} [F_X(x^*)] \right\}}{f_X(x^*)} \quad (4.48)
\]

\[
\mu'_X = x^* - \{ \Phi^{-1} [F_X(x^*)] \} \sigma'_X \quad (4.49)
\]

Equations (4.48) and (4.49) can be incorporated into the iteration procedure of subsection 4.6.5. The values of \(\mu'_X\) and \(\sigma'_X\) must be calculated at each iteration step.
4.7 Code Calibration

4.7.1 Basic Principles

There was great activity in the subject of code calibration with the introduction of the new code format that changed from the old allowable stress method to the limit states design, also called partial safety factor method. The limit states design is the first move towards a probabilistic code format and is the method used in many of the most recent code formats including the Eurocodes. This format is also referred to as the Load and Resistance Factor Design, abbreviated LRFD, which is the term used for the code format of, amongst others, the United States.

In (CEN, 1995) it is stated that there are two methods to determine the numerical values of the partial safety factors. This therefore implies that there are two calibration methods. The first is stated as being a method based on a long and successful history of building tradition, this is described as being the leading method used for the edition of Eurocodes of the time. The other method is described in (CEN, 1995) as being a method based on statistical evaluation of experimental data and field observations.

The basic concept behind the historical method of code calibration is to retain the same safety levels from the old code format to the new proposed format. The idea is that the old code represents societies and the engineering communities concept of an acceptable degree of safety as the structures built to these specifications have proved themselves over time. It is not possible to make statements about actual safety levels in the level II reliability method and only comparative safety can be calculated, i.e. one method of design is safer than the other.

The calibration procedure is described in (Nowak, 1995) as containing the following steps. A representative structure, is chosen as the object for calibration. A statistical database is developed for the load and resistance parameters. Typical sources for these are material tests and weigh-in-motion (WIM) statistics. Load and resistance models are developed where basic variables of these models are treated as stochastic variables described by distribution functions. A reliability procedure is then established where the reliability of the structure is measured in terms of the safety index $\beta$. A target safety index is selected based on the dimensioning of a range of the selected calibration object according to some code format. The set of partial safety factors are then chosen for the new format that maintains the selected target safety index.

The report (Östlund, 1991) provides detailed information about the calibration of concrete structures showing distribution types, means and standard deviations for material parameters, geometry, loads and load effects. The material parameters used for the calibration are based on Swedish material data from quality control tests made on both the concrete and the reinforcement. The report shows detailed examples of the calibration procedure including the choice of target reliability safety index, the effects of varying the coefficient of variation of certain variables and the
Table 4.1: Values of the safety index for a yearly reference period, from (NKB, 1987).

<table>
<thead>
<tr>
<th>Safety class</th>
<th>Safety Index $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>3.71</td>
</tr>
<tr>
<td>Normal</td>
<td>4.26</td>
</tr>
<tr>
<td>High</td>
<td>4.75</td>
</tr>
</tbody>
</table>

effect of assuming different distribution types.

In (Ellingwood, 1996) it is pointed out that the reliability index has decreased over the last century reflecting the engineering communities increased confidence in material quality assurance, construction techniques and design methods. The fact that structures have survived for 50 years or more is seen as proof that this increased confidence is justified.

### 4.7.2 Target Reliability Index

The Joint Committee on Structural Safety (JCSS) have made a first attempt at producing a document of rules and regulations of designing new and existing structures using a probabilistic approach (JCSS, 2001) the document provides recommendations as to the target safety index to be used for the design of new structures. For the ultimate limit state and a reference period of one year, the target reliability indices for the highest safety class labelled as "Large consequences of failure" to which a bridge belongs is 3.7, 4.4 and 4.7 depending on whether the relative cost of the safety measure is, respectively, Large, Normal or Small. The document also states that for existing structures where the cost of increasing the safety level is relatively high in comparison to the case of a new structure, lower safety requirements should be accepted. However the document does not at this stage go on to make recommendations as to the reliability indices for existing structures. It is also noted in this document that the target reliability index should be related to the type of failure i.e. brittle failure or from a ductile failure with or without structural reserves.

The recommendations for the safety index from the JCSS document are in line with those of The Nordic Committee on Building Regulations abbreviated NKB (NKB, 1987) which do not make any distinction between the relative cost of the safety measure but only distinguish between the structures safety class, see Table 4.1

The target reliability index as given in (CEN, 1996) for the ultimate limit state is 3.8 for the design working life and 4.7 for the reference period of one year.

The work done in (Carlsson, 2002) studies, amongst others, the effect on the safety index of adopting the different guidelines of the JCSS and the NKB for short span road bridges.

In civil engineering practice there are rarely situations where all the variables and their distributions are known so that a full probabilistic analysis can be performed. The target safety index $\beta$ provides, therefore, only a conceptual safety value and
Table 4.2: Final set of partial safety factors for railway bridges chosen for the original Eurocode 1.

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Self Weight</td>
<td>$\gamma_{G1}$</td>
<td>1.35</td>
</tr>
<tr>
<td>Ballast</td>
<td>$\gamma_{G2}$</td>
<td>1.8</td>
</tr>
<tr>
<td>Prestress</td>
<td>$\gamma_{pr}$</td>
<td>1</td>
</tr>
<tr>
<td>Traffic LM 71</td>
<td>$\gamma_Q$</td>
<td>1.45</td>
</tr>
</tbody>
</table>

can not be related to the actual probability of failure.

4.7.3 Code Calibration of Railway Bridges

The method used to choose the load factors for railway bridges for the use in Eurocodes is described in (Snidjer and Rolf, 1996). The article is an account of the findings of the UIC subcommittee 'Bridges' to establish safety factors for railway loading. It is noted in this article that a code calibration based on a probabilistic study is not possible due to the lack of data. The method chosen by the committee is to compare the safety factors inherent in the design codes of five selected countries and compare them with those obtained using, what was at the time, the proposed Eurocode. Six steel bridges and three concrete bridges are the subject of this study. All the bridges are simply supported and half the number of steel bridges are without ballast. Different sets of partial safety factors are studied and the final set used in part 3 of Eurocode 1 are presented. One of the recommendations of the article is that research into a probabilistic approach should be undertaken to justify the proposed safety factors. The article therefore implies that level II reliability calculations were not used at the time of the proposed Eurocodes to calibrate the new code.

However the calibration of the partial safety factors of the traffic loads due to trains have subsequently been investigated in the reports (Specialists’ Committee D 221, 1999a; Specialists’ Committee D 221, 1999b). These reports by ERRI provide background information and worked examples which study the safety factors for the load model LM2000 and especially the combination factors $\psi$ for vertical rail traffic actions. The first of the two reports is a general background information on the vertical loads on railway bridges and the factors affecting them. It also provides important information on the current code regulations and the previous reports that led to them. The second report (Specialists’ Committee D 221, 1999b) is of extra interest where the maximum moment per train at the mid-span of simply supported beams are investigated. For simplicity the moment distributions of several bridge spans are grouped together although these are clearly dissimilar. Another point to note is that the distribution used to fit the arbitrary-point-in-time moment distribution is a Weibull distribution for minima. This is strictly incorrect to use when attempting to describe a right tail i.e. a maxima phenomena, see (Castillo, 1987), although the report states that the distribution provides a reasonable fit to the data and also has the same properties of the mean and coefficient of variance of the data. However, despite these criticisms the reports are very informative and provide infor-
mation about the types of distributions to use, including means and coefficients of variance, when performing a probabilistic analysis. The reports address many of the problems of modelling the vertical train loads on railway bridges including dynamic factor, overloading, future traffic conditions and model uncertainties. Even permanent loads are discussed in these reports and suggestions are made for the mean and coefficient of variation values to be used for both self-weight and ballast weights. The reports also discuss other vertical loads such as accidental, wind, centrifugal and thermal loads although these are not included in the subject of this thesis and shall therefore not be discussed.

A further criticism of the report (Specialists’ Committee D 221, 1999b) is again the choice of the distribution to describe the relative moment ratio symbolised as \( \lambda \) which stems from the original ERRI reports (Specialists’ Committee D 192, 1993; Specialists’ Committee D 192, 1994a; Specialists’ Committee D 192, 1994b). These original reports use 124 characteristic trains to describe the present and future traffic conditions on an international line. These 124 trains are mainly deterministic in nature with specific axle spacings and determinate axle loads. Only the mixed freight trains of this traffic load model and the frequency with which the trains are chosen are in any way stochastic. The frequency of the trains will however tend towards the predetermined train frequency of the simulation model as the number of simulations increase. The number of mixed freight trains of the 124 characteristic trains is 50 and the number of these trains per day is 60 out of a total of 190 trains per day of this model. The outcome of this model will therefore produce discrete moment ratios for the majority of the trains. This is especially true of the trains likely to cause the high moment ratios i.e. the special heavy loaded trains and the block trains. It is therefore questionable to fit a distribution function to what is essentially discrete values, especially as the behaviour of the right tail is important when extrapolating beyond the range of the data.

4.8 Resistance Model

The load effect studied in this thesis is the bending moment at mid-span of simply-supported beams consisting of either reinforced concrete or structural steel. In this section the stochastic model for the resistance shall be described. The model mainly follows that described in the NKB recommendations (NKB, 1987).

The reliability analysis adopts representative values for the materials in question and not specific detailed ultimate limit state criteria which would only be possible for specific objects. This approach is necessary to maintain the generality of the analysis.

The stochastic variable, abbreviated s.v., for the material strength is often described as consisting of three components as shown below

\[
R = CAF
\]

(4.50)

where \( C \), \( A \) and \( F \) are s.v. representing the uncertainty of the resistance model,
the geometric properties and the strength of the material, respectively. The model uncertainty describes the uncertainties of the engineering community in the static resistance model of the material in question, e.g. part of the model uncertainty, for a reinforced concrete beam in bending, may arise from the simplified rectangular stress block assumption. Differing degrees of uncertainties for differing failure modes are reflected in the properties assigned to $C$ for that failure mode. Examples of the geometric properties that affect the resistance of a section are the dimensions of a beam, the height to the reinforcement etc. The strength of the material is also a factor that affects the resistance of the section or beam, e.g. the yield strength of the steel or the compression strength of the concrete. These resistance variables are all modelled as log-normally distributed according to both the JCSS and NKB (JCSS, 2001; NKB, 1987). Under this distribution assumption, the mean of the resistance model is given by the product of the three variables as shown below

$$\mu_R = \mu_C \mu_A \mu_F$$  \hspace{1cm} (4.51)

where $\mu$ is used as a notation for the mean values, the indexing reflecting the respective basic variable. The coefficient of variation for the resistance is given by the square root of the sum of the squares of the three basic variables, i.e. by

$$Cov_R = \sqrt{Cov_C^2 + Cov_A^2 + Cov_F^2}$$  \hspace{1cm} (4.52)

where $Cov$ is used to indicate the coefficient of variation, the index again indicating the different variables. Different values of the coefficient of variation are given for the different variables depending on the guidelines used, the failure mode and the material in question.

Two sources have been used for the values of coefficient of variation for bending in concrete and steel structures. The first is the aforementioned NKB guidelines and the other is from a study by Prof. Östlund done in Sweden on the required partial safety factors for a special heavy haul route, Malmbanan, where iron ore is transported (Östlund, 1997). This latter reference can also be found in (Nilsson et al., 1999) although the author has noted that a few minor errors have been created during the transferal into this publication from the original material. The guidelines of the NKB publication are conveniently summarised in (Carlsson, 2002), which incidently also summarises guidelines from (JCSS, 2001) and an earlier publication of the NKB recommendations. The properties used throughout this work are summarised in Table 4.3 of this section.

In the work (Östlund, 1997), the characteristics of the concrete compressive strength for a class of concrete, K40, is cited when assessing the coefficient of $Cov_F$. Östlund states the mean of the compression strength of this concrete (derived from cube compression tests) as being approximately 50 MPa and a standard deviation of 5 MPa which yields a $Cov_F = 0.1$. It is not stated where these values come from although they are approximately the same as values obtained by (Degerman, 1981). No justification of the use of a coefficient of variation of 0.06 is given for the steel, however this figure is also in line with the findings (Degerman, 1981) and follow the guidelines of the JCSS according to (Carlsson, 2002).


Table 4.3: Properties of the log-normal basic resistance variables.

<table>
<thead>
<tr>
<th>Material</th>
<th>Basic Variable</th>
<th>Coefficient of Variation $Cov$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>NKB 55</td>
</tr>
<tr>
<td>Concrete</td>
<td>Model Uncertainty $C$</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Geometric Property $A$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Material Strength $F$</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td><strong>Resistance $R$</strong></td>
<td><strong>0.117</strong></td>
</tr>
<tr>
<td>Reinforcement and Structural Steel</td>
<td>Model Uncertainty $C$</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Geometric Property $A$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Material Strength $F$</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td><strong>Resistance $R$</strong></td>
<td><strong>0.117</strong></td>
</tr>
</tbody>
</table>

The Ph.D. thesis (Degerman, 1981) compiles a précis of cube tests done, in Sweden, on factory produced concrete with different strengths between 1965–1974. For this concrete class some 63885 tests were performed and the mean and standard deviation are found to be 50.2 MPa and 5.26 MPa, respectively. As regards reinforcement steel, tests results from more than 30 thousand tensile tests are used to establish the coefficient of variation of the material strength using the lower yield strength as the means of measurement. Tests from Swedish steelworks and tests carried out by The Certification Authority for Steel in Building Structures, Svensk Byggstålkontroll, are summarised in the thesis. The results from the steelworks are shown in the form of mean values and standard deviations which give a coefficient of variation that ranges from 0.07–0.05 for steel grades of Ks 40–Ks 60, respectively. The coefficient of variation is seen to vary with nominal bar diameter. A value of $Cov_F$ of 0.06 seems therefore reasonable. It is unclear from the thesis when these tensile tests were performed although it is stated in the thesis that the tests from The Certification Authority for Steel in Building Structures are from 1972–74. This value of $Cov$ appears to be relevant even to structural steel as JCSS use the value of 0.07, while (Östlund, 1997) and the NKB publication both use a value of 0.06.

It should be noted that no knowledge of the variation in strength of the earlier steel and concrete grades has been obtained and therefore a key assumption is that the above coefficient of variations are even assumed to apply to earlier constructions.

Since all the basic variables of the resistance are defined as being log-normally distributed, it follows that the resistance itself is also log-normal.

This yields an expression for the characteristic material strength as being

$$R_K = \mu_C \exp(k_CCov_C) \mu_A \exp(k_ACov_A) \mu_F \exp(k_FCov_F)$$

$$= \mu_R \exp(k_C Cov_C) \exp(k_A Cov_A) \exp(k_F Cov_F)$$ (4.53)

where the values of $k$ are given by the inverse of the standard normal distribution which represents the quantile, cf. section 4.5. The characteristic value of the geometric basic variable, $A$ is usually taken as the mean value, which gives $k_A = 0$ this
Table 4.4: Values of $k$ for typical quantile values from standard normal tables.

<table>
<thead>
<tr>
<th>Quantile (%)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability $p$</td>
<td>0.01</td>
<td>0.05</td>
<td>0.10</td>
<td>0.5</td>
</tr>
<tr>
<td>$k$ from $\Phi^{-1}(p)$</td>
<td>-2.33</td>
<td>-1.65</td>
<td>-1.28</td>
<td>0.00</td>
</tr>
</tbody>
</table>

simplifies (4.53) to become

$$R_K = \mu_R \exp(k_C Cov_C) \exp(k_F Cov_F) \quad (4.54)$$

In the aforementioned work by Östlund it is stated that the calculation models are conservatively chosen and can be assumed to yield the 5% quantile of the strength. The author has not seen this in other literature, however, Prof. Östlund is highly merited in the field of structural reliability and code calibration. Using the suggested 5% quantile, from Table 4.4 or alternatively from inverse of standard normal tables, a value of $k_C = -1.65$ is obtained. As regards the material strengths the values of $k_F$ depend on the quantile used to represent the characteristic value of the material strength. These quantile values for the respective materials can be collected from Table 3.2.

As an example the case of concrete is considered together with the values suggested by Östlund. The characteristic compression strength represents the 5% quantile, cf. Table 3.2, this gives $k_F = -1.65$. The values of the coefficient of variation for concrete in bending, from Table 4.3, are $Cov_F = 0.15$, $Cov_A = 0.0$ and $Cov_F = 0.1$.

With the value of $k_C = -1.65$ from above, the value of the characteristic resistance set in relation to the mean value of resistance becomes, after insertion of the values into (4.54)

$$R_K = \mu_R \exp(-1.65 \cdot 0.15) \exp(0.0 \cdot 0.0) \exp(-1.65 \cdot 1.0)$$

$$= 0.662 \mu_R \quad (4.55)$$

In the work (Carlsson, 2002) no inclusion of the model uncertainty is made in his equivalent expression to (4.54). It is not stated if it is assumed that the calculation model yields the mean value of the resistance, but this must be the conclusion as the exponential term is not included. The characteristic resistance according to (Carlsson, 2002) is given by

$$R_K = \mu_C \mu_A \mu_F \exp(k_F Cov_F)$$

$$= \mu_R \exp(k_F Cov_F) \quad (4.56)$$

Using the same example as above, for concrete in bending, one obtains

$$R_K = \mu_R \exp(-1.65 \cdot 0.1)$$

$$= 0.85 \mu_R \quad (4.57)$$

The values of the characteristic resistance expressed in terms of the mean value of resistance are tabulated in Table 4.5. This has been done for both (4.54) and (4.56). The values have been calculated using coefficient of variation values taken from Table 4.3 and quantile values from Table 3.2.
Table 4.5: Values of $R_K/\mu_R$.

<table>
<thead>
<tr>
<th></th>
<th>Calculated using (4.54)</th>
<th>Calculated using (4.56)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NKB55</td>
<td>0.768</td>
<td>0.848</td>
</tr>
<tr>
<td>Östlund</td>
<td>0.662</td>
<td>0.906</td>
</tr>
<tr>
<td>Steel</td>
<td>0.737</td>
<td>0.870</td>
</tr>
</tbody>
</table>

4.9 Level I Reliability

In this section, the current and the past Swedish codes will be expressed in terms of a level I reliability analysis. The reason for this is to establish the mean value of resistance that can then be used in a level II reliability analysis. The assumptions is therefore that the bridge was built correctly to the building standard of the time and therefore produce a corresponding mean value of resistance. As will be seen all the values are put in relation to the characteristic load effect from the dimensioning load of the UIC 71 or the SW/2 load model including the dynamic amplification factor.

4.9.1 Codes with the Partial Safety Factor Format

As detailed in section 3.2, the partial safety factor format is used in the Swedish codes after approximately 1980. Recalling from earlier, cf. (3.1–3.2), the dimensioning criterion in the ultimate limit state when the imposed traffic load and the self-weight are considered, after rearranging, can be written as

$$R_k \geq \gamma_m \gamma_n (G_k \gamma_g + Q_k \gamma_q)$$  \hspace{1cm} (4.58)

Inserting the expression for the characteristic resistance given in (4.54) yields

$$\mu_R \exp(k_C \text{Cov}_C) \exp(k_F \text{Cov}_F) \geq \gamma_m \gamma_n (G_k \gamma_g + Q_k \gamma_q)$$  \hspace{1cm} (4.59)

$$\mu_R \geq \exp(k_C \text{Cov}_C) \exp(k_F \text{Cov}_F) \left( \frac{\gamma_m \gamma_n}{G_k \gamma_g + Q_k \gamma_q} \right)$$  \hspace{1cm} (4.60)

If we now recall the load effect caused by the traffic load model from the dimensioning case of the UIC 71 load model or the SW/2 load model including the dynamic amplification factor $\Phi_2$ for carefully maintained track defined as $Q_n$, cf. section 3.5.5. Dividing (4.60) throughout by $Q_n$ yields

$$\frac{\mu_R}{Q_n} \geq \frac{\gamma_m \gamma_n}{G_k \gamma_g + Q_k \gamma_q}$$  \hspace{1cm} (4.61)

Recognising that for this code format the characteristic traffic load effect is the same as the defined nominal traffic load effect, since the UIC 71 and the SW/2 load models
applied at this time, i.e. $Q_k = Q_n$. Also introducing a factor $\nu$ which is defined as

$$\nu = G_k/Q_n$$

(4.62)

Simplifying (4.61) yields

$$\frac{\mu_R}{Q_n} \geq \frac{\eta c \gamma_m \gamma_n}{\exp(k_C Cov_C) \exp(k_F Cov_F)} (\nu \gamma_g + \gamma_q)$$

(4.63)

It can be recognised that setting $k_C = 0$ in the above equation yields an equivalent expression to the one that would be obtained if one regards $R_K$ as being defined according to (4.56).

Appropriate values of $\eta c \gamma_m$, and $\gamma_n$ can be obtained from Table 3.2, while values, applicable to the various materials, of $\exp(k_C Cov_C) \exp(k_F Cov_F)$ can be collected from Table 4.5. The relevant partial safety factors for the traffic load, $\gamma_q$ are tabulated in Table 3.6 and the value of $\gamma_g = 1.05$ for the self-weight. For selected values of $\nu$ the mean value of resistance put in relation to the the nominal load effect, $\mu_R/Q_n$ can thus be calculated. This value is then used at a later stage in a level II reliability method to calculate the safety index.

### 4.9.2 Codes with the Allowable Stress Format

The method described here is the equivalent to the above, however applied to the allowable stress code format, i.e. prior to 1980’s. Recalling from section 3.2.2, the dimensioning criterion in this code format is given by (3.3). Combining this equation with (3.4) yields the following

$$\frac{R_k}{\gamma_{inf}} \geq G_d + Q_d$$

(4.64)

Since no distinction is made in this code format between characteristic and dimensioning load effects these are interchangeable, i.e. $G_k = G_d$ and $Q_k = Q_d$. As before, inserting the definition of the characteristic resistance as defined in (4.54) yields

$$\mu_R \exp(k_C Cov_C) \exp(k_F Cov_F) \geq \gamma_{inf} (G_k + Q_k)$$

(4.65)

As in the previous section dividing throughout by $Q_n$ while at the same time using the definition of $\nu$ from (4.62) gives

$$\frac{\mu_R}{Q_n} \geq \frac{\gamma_{inf}}{\exp(k_C Cov_C) \exp(k_F Cov_F)} \left( \nu + \frac{Q_k}{Q_n} \right)$$

(4.66)

the inferred values of $\gamma_{inf}$ can be obtained from Table 3.2 for the material and year under study. Corresponding values of $\exp(k_C Cov_C) \exp(k_F Cov_F)$ can be obtained from Table 4.5. Values of $Q_k/Q_n$ can be collected from Table 3.7.

The obtained values of $\mu_R/Q_n$ are then used later in a level II reliability analysis.
4.10 Level II Reliability Analysis

In this section a level II formulation of the ultimate limit state shall be undertaken. This reliability analysis is used to calculate the safety index, $\beta$, which is then compared to the target safety index.

The basic assumptions of this method is that the strength of the material is a s.v. denoted $R$, that can be described by a log-normal distribution. The self-weight of the bridge, denoted $G$ is a s.v. which is normally distributed and the traffic load effect, $Q$, is described by either the first and second moments which is based on the assumption of a normal distribution or by a normal tail approximation of the actual distribution cf. section 4.6.6. The failure function then becomes:

$$g(r, g, q) = r - g - q$$  \hspace{1cm} (4.67)

The basic variables are then transformed into the standard normal space. The transformation of the s.v. $R$ into the standard normal space $Y_1$ was done using the approximation

$$R = \mu_R \exp(Y_1 \text{Cov}_R)$$  \hspace{1cm} (4.68)

while the variables $G$ and $Q$ are transformed using the transformations

$$G = \mu_G + Y_2 \sigma_G$$  \hspace{1cm} (4.69)

$$Q = \mu'_Q + Y_2 \sigma'_Q$$  \hspace{1cm} (4.70)

where $\mu'_Q$ and $\sigma'_Q$ are purely the mean, $\mu_Q$, and standard deviation, $\sigma_Q$, of the traffic load effect in the case of first and second moment approximation. Whereas $\mu'_Q$ and $\sigma'_Q$ are the normal tail approximations (see subsection 4.6.6) in the case of non-normal distributions.

Substitution of (4.68)–(4.70) into (4.67) yields the normalised coordinate system for the limit state function as

$$g(y) = \mu_R \exp(y_1 \text{Cov}_R) - (\mu_G + \sigma_G y_2) - (\mu'_Q + \sigma'_Q y_3)$$  \hspace{1cm} (4.71)

recognising that the coefficient of variation, $\text{Cov}$, is given by $\text{Cov} = \sigma/\mu$, and dividing throughout by the nominal traffic load effect defined earlier in section 4.9.1, $Q_n$, (4.71) becomes

$$g(y) = \frac{\mu_R}{Q_n} \exp(y_1 \text{Cov}_R) - \frac{\mu_G}{Q_n} (1 + \text{Cov}_G y_2) - \frac{\mu'_Q}{Q_n} (1 + \text{Cov}'_Q y_3)$$  \hspace{1cm} (4.72)

Also by substitution of $\nu$ from (4.62) into (4.72) yields

$$g(y) = \frac{\mu_R}{Q_n} \exp(y_1 \text{Cov}_R) - \nu(1 + \text{Cov}_G y_2) - \frac{\mu'_Q}{Q_n} (1 + \text{Cov}'_Q y_3)$$  \hspace{1cm} (4.73)
Differentiation of the above limit state function yields the gradient vector which at the $m$-th iteration

$$\nabla g(y^{(m)}) = \left[ \frac{\mu R}{Q_n} \text{Cov}_R \exp(y_i^{(m)} \text{Cov}_R) \right] - \nu \text{Cov}_G - \frac{\mu' Q}{Q_n} \text{Cov}_Q \right]$$  (4.74)

The above equations (4.73) and (4.74) can hence be used in the iteration procedure described in subsection 4.6.5 using the normal tail approximation of section 4.6.6 or not, for the normalised load variable, depending on the type of distribution assumed for the traffic load, i.e. normal or non-normal.

### 4.10.1 Allowance for Increased Allowable Axle Load

In the reliability method a future increase in allowable axle load was accounted for by adjusting the parameters of the distribution used to describe the traffic load effect. The increase was assumed to be linear, shifting the mean value of the distribution by a factor equivalent to the percentage increase in allowable axle load. The increase in variance of the traffic load effect will therefore be the square of the percentage increase. For the GEV and Gumbel distribution this is equivalent to increasing the location and the scale parameters by the percentage increase, the scale parameter being unchanged. If the allowable axle load is increased by a factored $\kappa$ then the corresponding adjustment to the GEV and the Gumbel parameters will be given by

$$\sigma_{\text{inc}} = \kappa \sigma$$  (4.75)
$$\mu_{\text{inc}} = \kappa \mu$$  (4.76)
$$\xi_{\text{inc}} = \xi$$  (4.77)

the Gumbel distribution being the special case of $\xi = 0$. 
Chapter 5

Statistical Models and Tools

5.1 General

In this chapter the statistical tools and mathematical models used in this thesis are introduced together with some of their basic concepts and underlying assumptions. The chapter is broken into three main parts, the first of which describes some of the general statistical tools, terminology and definitions used in the thesis. The second and the third parts both deal with extreme value theory. The second part introduces what is often referred to as the classical approach to extreme value theory, describing the three extreme value distributions and their generalised form of the Generalised Extreme Value distribution. It also discusses how they can be applied to the case of modelling the traffic load effect. The final part describes an alternative method to the classical approach, namely the Peaks-Over-Threshold model and the associated Generalised Pareto Distribution.

Extreme value theory, abbreviated EVT has been used in many disciplines of civil engineering to describe the loading distribution of variable loads and even the strength of materials. Extreme value theory has been used in oceanography to describe maximum sea levels (Shim et al., 1993; de Haan, 1993), in wind engineering (Gross et al., 1993), in mining to estimate the occurrence of large diamond and precious stone deposits (Caers et al., 1998) and to predict extreme traffic load effects on road bridges (Cremona and Carracilla, 1998; O’Connor et al., 1998). It has even been used in financial applications to describe and predict high insurance claims and in stock market applications (Embretchts et al., 1997; Emmer et al., 1998; Rootzén and Tajvidi, 1997).

As an engineer one is often interested in establishing the maximum loading on a structure in a given reference period of time or more importantly the likelihood of a certain load level being exceeded in a given reference period. This reference period is often, but not always, chosen to be one year, or it may be the structures anticipated service life. We often refer to a 50-year return load, in many instances this may even be as much as a 100-year or even 1000-year. This use of the phrase means the loading level that is likely to be exceeded on average once every 50 years (a 50-
year return load) which is true provided the loading can be described as stationary and that the time between events, i.e. the exceedance of the return load, can be approximated to a Poisson process. Because of the long design lives of bridges and other civil engineering structures we are often forced to extrapolate long past the available data in order to make predictions about events with a very low risk of occurrence. This extrapolation from a mathematical point of view is not strictly justifiable (Caers and Maes, 1998). In the preface to (Coles, 2001) the validity of extrapolation is discussed at length and it is admitted in the book that it is easy to criticise extrapolation of this kind. However, it is also noted that extrapolation is demanded and that the extreme value theory provides the only real model with which to do this.

In order to extrapolate in this manner, a sound mathematical model must form the foundation on which to build. This is provided by the models of the extreme value theory, at least asymptotically, as will be discussed in sections 5.3–5.4.

There are several text books on the subject of extreme value theory and their applications. One of the pioneering works can be found in (Gumbel, 1958), while from an engineering applications point of view the book (Castillo, 1987) is very useful in explaining the fundamentals, while the books (Embrechts et al., 1997; Leadbetter et al., 1997) provide a mathematically more rigorous account of the subject together with the underlying assumptions. More recent works on the subject can be found in (Reiss and Thomas, 2001; Coles, 2001). The book (Coles, 2001) is, in my opinion, very informative and well structured, providing the background to the distributions without getting too involved in the pure mathematics. It also shows several examples of the use of the distributions and how they can be used for inference purposes.

However, before discussing extreme value theory some of the diagnostic tools together with some general definitions will be discussed.

## 5.2 General Tools and Definitions

### 5.2.1 Goodness of Fit Tests

There were three types of goodness of fit tests done on the peaks-over-threshold and the fitted distributions. These were the Kolmogorov-Smirnov test, the Anderson-Darling test and the \( \chi^2 \) goodness of fit test. The R-squared value of the data compared with the assumed distribution was also calculated for the empirical and theoretical cumulative distribution functions (cdf) evaluated at the data values of \( x \).

The \( \chi^2 \) goodness of fit test has the disadvantage that it is only valid for sample sizes of 120 or more. This limits the range of its use especially as one moves towards the right tail and the number of data in this area becomes limited.
All the above test are standard statistical tests, which can be found in textbook literature such as (Johnson, 1994; Blom, 1989; Press et al., 1988).

5.2.2 Empirical Distribution Function

If we have a sample \(x_1 \ldots x_n\) which are identical independent observations from an unknown distribution function \(F\). Usually one does not know the actual distribution \(F\) and this must be estimated. This is usually done by means of the empirical distribution function, \(\hat{F}\). First, by ordering the sample into ascending values such that

\[x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(i)} \ldots \leq x_{(n)}\] \hspace{1cm} (5.1)

Note that the use of the brackets in the index indicates the ordered sample. Also common in the ordered sample notation is the use of \(x_{i:n}\), denoting the \(i\)-th largest sample in sample of size \(n\). The empirical distribution function used in this thesis is defined as

\[\hat{F}(x) = \frac{i - 0.5}{n} \text{ for } x_{(i)} \leq x < x_{(i+1)}\] \hspace{1cm} (5.2)

There are several estimations of the empirical distribution function to be found in literature. Another commonly used estimation is \(i/(n + 1)\). In (Schneider, 1997; Castillo, 1987) the advantages and disadvantages of the different estimations are discussed. Castillo recommends the above estimation (5.2) for use in extreme value analysis, however for large samples the choice of estimation has little effect.

From the sample \(x_1 \ldots x_n\) it is then possible to fit an assumed distribution to this sample so that the estimated distribution function of \(F\) is given by \(\hat{F}\), this is usually done under the assumption that the sample comes from a certain distribution or family of distributions. There are then several diagnostic tools available to see if the theoretical model represented by \(\hat{F}\) is a good fit to \(F\), estimated by \(\hat{F}\). Two such tools, used throughout this thesis, are the probability plot and the quantile plot, also known as the QQ plot, which are described in the following subsections 5.2.3–5.2.4. The quantile plot and the probability plot essentially show the same information but use different scales. However, what appears to be a good fit on the one plot is not necessarily so on the other.

5.2.3 Probability Plot

The probability plot is used to compare the empirical cdf, \(\hat{F}\), with the fitted mathematical model’s cdf, estimated by \(\hat{F}\), see the above subsection. The plotting positions of the ordered independent observations given by (5.1) are

\[\left(\frac{i - 0.5}{n}, \hat{F}(x_{(i)})\right) \text{ for } i = 1, 2, \ldots, n\] \hspace{1cm} (5.3)

The resulting plot should produce a straight line at 45 degrees if the fitted mathematical model is a good representation of the sample.
An alternative to this type of plot is to use probability papers, such as the Gumbel or Normal probability papers. These involve a transformation of the data such that a plot of the data under the transformation create a straight line on the plotting paper, see e.g. (Castillo, 1987; Blom, 1989; Johnson, 1994).

5.2.4 Quantile–Quantile Plot

The quantile–quantile plot is a general tool to check if two samples come from the same distribution and is often abbreviated as a QQ-plot, (Embrechts et al., 1997; Emmer et al., 1998). In such a plot the empirical quantiles can be plotted against theoretical quantiles for an assumed distribution, \( \hat{F} \). Agreement is established if the plotting points lie on a 45-degree line, (Emmer et al., 1998; Coles, 2001). A quantile plot can be drawn by using the ordered sample of (5.1).

The plotting positions are then given by

\[
x(i), \hat{F}^{-1}\left(\frac{i - 0.5}{n}\right) \quad \text{for} \quad i = 1, 2, \ldots, n
\]

(5.4)

5.3 Classical Extreme Value Theory

This next subsection provides a brief introduction into what is often described as the classical approach to extreme value analysis. The section is not intended to provide a full description of the theory and the interested reader is referred to (Leadbetter et al., 1997; Embrechts et al., 1997; Rydén and Rychlik, 2001; Brodtkorb et al., 2000; Coles, 2001). The approach considers distribution of \( M_n \) such that

\[
M_n = \max(X_1, X_2, \ldots, X_n)
\]

(5.5)

where \( X_1, X_2, \ldots, X_n \) are independent identically distributed (abbreviated i.i.d.) random variables that come from the distribution function \( F \). The random variables are often related to a time period e.g. the maximum daily sea level or the maximum daily wind speed. The distribution of \( M_n \) will then also be related to a time period depending on the value of \( n \). In the above examples if \( n \) is chosen as 365 then the distribution of \( M_n \) will be the yearly maximum.

The distribution of \( M_n \) may be calculated from the original distribution \( F \) by

\[
\Pr(M_n \leq z) = \Pr(X_1 \leq z, \ldots, X_n \leq z) = \Pr(X_1 \leq z) \cdot \Pr(X_2 \leq z) \cdot \ldots \cdot \Pr(X_n \leq z) = [F(z)]^n
\]

(5.6)

where \( \Pr \) denotes the probability of an event.

However small discrepancies in the estimation of \( F \) can lead to large errors for the distribution of \( M_n \), (Coles, 2001).
5.3. CLASSICAL EXTREME VALUE THEORY

(a) Maximum load effect, $M_n$, in time $\Delta t$.  (b) Distribution of $M^*_n$ tends to the GEV as $n \to \infty$.

Figure 5.1: Illustration of the extreme value method. As $\Delta t$ or the number of measuring points in $\Delta t$ tend to infinity, then the distribution of $Z$ tends to the Generalised Extreme Value distribution.

As $n$ increases $M_n \to \infty$ therefore a normalisation is required such that $M^*_n = (M_n - b_n)/a_n$. The asymptotic distribution of $M^*_n$, i.e. as $n \to \infty$, can be shown to belong to one of three families of distribution known collectively as the extreme value distributions, cf. (Embrechts et al., 1997; Castillo, 1987; Coles, 2001).

$$\Pr \left[ \frac{(M_n - b_n)}{a_n} \leq z \right] \to G(z) \quad \text{as} \quad n \to \infty \quad (5.7)$$

and provided $G$ is non-degenerate then $G(z)$ denotes one of the extreme value distribution functions. The cdf’s of the three families of distributions are shown below. Figure 5.3 illustrates the classical approach to the extreme value theory.

The cdf of the Gumbel or extreme value Type I family of distributions is given by

$$G(z) = \exp \left\{ - \exp \left[ - \left( \frac{z - b}{a} \right) \right] \right\} ; \quad -\infty < z < \infty \quad (5.8)$$

provided $a > 0$.

The cdf of the Fréchet or extreme value Type II family of distributions is given by

$$G(z) = \begin{cases} 
\exp \left[ - \left( \frac{z - b}{a} \right)^{-\alpha} \right] & \text{if } z > b \\
0 & \text{otherwise}
\end{cases} \quad (5.9)$$

provided $a > 0$ and $\alpha > 0$.

The cdf of the Weibull or extreme value Type III family of distributions is given by

$$G(z) = \begin{cases} 
\exp \left[ - \left( \frac{b - z}{a} \right)^{\alpha} \right] & \text{if } z < b \\
1 & \text{otherwise}
\end{cases} \quad (5.10)$$

provided $a > 0$ and $\alpha > 0$. 

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The parameters \( a \) and \( b \) in all three distributions are known as the *scale* and *location* parameters, respectively. For the Types II and III the extra parameter \( \alpha \) is referred to as the *shape* parameter. The above functions described by (5.8)–(5.10) are all for maximum extreme values, similar expressions can be developed for minimum values, see e.g. (Castillo, 1987).

### 5.3.1 Generalised Extreme Value Distribution

The above three family of distributions can be joined together to form a single distribution family called the *Generalised Extreme Value* distribution commonly called the GEV distribution, see e.g. (Embrechts et al., 1997; Brodtkorb et al., 2000; Coles, 2001; Rydén and Rychlik, 2001). This form is also referred to as the *von Mises* form, (Castillo, 1987; Embrechts et al., 1997). The cdf of the GEV is given by

\[
G(z; \xi, \sigma, \mu) = \begin{cases} 
\exp \left\{ - \left( 1 + \frac{\xi(z - \mu)}{\sigma} \right)^{-\frac{1}{\xi}} \right\} & \text{if } \xi \neq 0, \\
\exp \left\{ - \exp \left( \frac{-z}{\sigma} \right) \right\} & \text{if } \xi = 0,
\end{cases}
\]

where \( \sigma, \mu \) and \( \xi \) are the scale, location and shape parameters respectively. The formula is valid for values of \( z \) that fulfill the condition \([1 + \xi(z - \mu)/\sigma] > 0\) and \( \sigma > 0 \) and \( \xi \) and \( \mu \) are arbitrary. The case of \( \xi = 0 \), \( \xi > 0 \) and \( \xi < 0 \) correspond to the Gumbel, Fréchet and the Weibull distribution respectively. The shape parameter \( \xi \) is also know as the extreme value index often abbreviated EVI, (Beirlant et al., 1996b; Smith, 1987; Dekkers and de Haan, 1989; Dekkers et al., 1989; Grimshaw, 1993). The value of the shape parameter is vital in determining the tail behaviour of the distribution (Caers and Maes, 1998). When \( \xi < 0 \) the distribution has a finite upper limit given by \( \mu - \sigma/\xi \). When \( \xi > 0 \) the distribution has a heavy tail i.e. a slow approach towards infinity. The three cases are illustrated in Figure 5.2, where the scale and location parameters are the same throughout with values of \( \sigma = 1 \) and \( \mu = 11 \), respectively. The upper two subplots in Figure 5.2 have no upper limit while the final subplot has an upper bound of 13.

The significance of the tail behaviour is most apparent when extrapolation past the available data is attempted. If the tail behaviour is of the Weibull type then for extremely high quantiles only the upper bound or close to the upper bound values will be achieved.

The GEV distribution has the advantage that the parameters of this distribution can be estimated without first deciding whether the shape parameter, \( \xi \), is equal to zero, positive, or negative, which is the effect of restraining the estimates to a Type I, II or III extreme value distribution.
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Figure 5.2: Illustration of the tail behaviour due to value of the shape parameter $\xi$. A Gumbel tail, $\xi = 0$, the distribution has no upper limit. A Fréchet tail or heavy tail, $\xi > 0$, the distribution has no upper limit and a slow approach to infinity. A Weibull tail, $\xi < 0$, the distribution has an upper limit.

5.3.2 Return Loads

The return load or return level, as mentioned earlier, is defined as the load that has a probability $p$ of being exceeded in the time period relating to the distribution, i.e. on average the load will occur with a period of $1/p$. If the distribution $G(z)$ is the distribution of the yearly maxima, then the load with a probability $p$ of being exceeded, will on average be exceeded once every $1/p$-years. The return load can be found by inverting (5.11) which can be shown to yield the result

$$z_p = \begin{cases} \mu - \frac{\sigma}{\xi} \left\{1 - \left[-\log(1-p)\right]^{-\xi}\right\} ; & \text{if } \xi \neq 0 \\ \mu - \sigma \log \left[-\log(1-p)\right] ; & \text{if } \xi = 0 \end{cases}$$

(5.12)

such that $G(z_p) = 1 - p$. Similar expressions can also be found for the inverse of the three extreme value distributions Type I–III of (5.8)–(5.10).

The characteristic load is often defined, for variable loads, as the load with an expected return period of 50 or 100 years, (CEN, 1995; NKB, 1987). The characteristic load is therefore equivalent to a return load with a defined return period.
5.3.3 Parameter Estimates

There are various methods to estimate the parameters of the generalised extreme value distributions and the three extreme value distributions. The book (Castillo, 1987) deals extensively with the different methods for calculating the parameters of the extreme value distributions, Types I–III. However, this thesis has mostly concentrated on the estimation of the parameters of the GEV as this does not require a predetermined decision as to which type of tail behaviour is to be expected. The book (Coles, 2001) provides details of downloading computer programs for the estimation of the parameters of the GEV together with many other useful functions to be used in conjunction with extreme value analysis. The functions mentioned in the book are all S-PLUS applications and readers familiar with this computer application are referred to the aforementioned book. In this thesis the Matlab® toolbox, from the Department of Mathematics at Lund University, was used called WAFO. The toolbox together with an accompanying manual (Brodtkorb et al., 2000) are free on the internet, however the toolbox requires Matlab in order to run. The possible methods for the parameter estimates of the GEV within the WAFO toolbox are the Probability Weighted Moments and the Maximum Likelihood Estimates, abbreviated PWM and MLE or ML respectively. According to the help files for the function to estimate the parameters of the GEV, the maximum likelihood method generally yield lower variances i.e. provide better estimates, however the PWM method is valid for a wider range of parameter values.

The function used from WAFO is called \texttt{wgevfit} for the parameter estimates. Besides providing the parameter estimates ($\hat{\xi}, \hat{\sigma}, \hat{\mu}$) the function also produces an asymptotic variance-covariance matrix of the estimates. It can be shown, according to (Brodtkorb et al., 2000; Coles, 2001), that the estimates are the unbiased estimates of ($\xi, \sigma, \mu$) and that the distribution of the estimates are multivariate normal with the above mentioned variance-covariance matrix, at least asymptotically. This fact will be useful later when building confidence intervals, either of the estimates or of the return or characteristic load.

5.3.4 Characteristic Load Estimation

From the estimated GEV it is then possible to estimate the value of the load, in our case moment, that returns with an expected period $r$. This can be chosen so that the return period coincides with that used for the definition of the characteristic design load. The units of the return period $r$ must be related to the extreme value distribution i.e. in our case one day, as the daily maximum moment was used for the parameter estimates. The return period will then be $365 \cdot 50$ and $365 \cdot 100$ for the 50 and 100 years respectively. Assume we are interested in the 50 year design
5.3. CLASSICAL EXTREME VALUE THEORY

load denoted $S_{r50}$ then the daily probability of this value being exceeded will be

$$\Pr(X > S_{r50}) = 1/r$$ (5.13)

$p = \Pr(X < S_{r50}) = 1 - 1/r$ (5.14)

$$S_{r50} = G^{-1}(p; \hat{\xi}, \hat{\sigma}, \hat{\mu})$$ (5.15)

where $G^{-1}(p; \hat{\xi}, \hat{\sigma}, \hat{\mu})$ denotes the inverse of the GEV using the parameter estimates obtained by either the PWM or the ML estimate. This inverse can be calculated from (5.12).

5.3.5 Uncertainty of the Characteristic Load

The value of the characteristic load obtained in the manner described in the previous section will obviously contain uncertainties associated with the parameter estimates of the GEV. However it is possible to estimate the variation of the characteristic load $S_{r50}$. As mentioned previously when estimating the parameters of the GEV distribution, the WAFO toolbox also returns a variance-covariance matrix $V(\hat{\xi}, \hat{\sigma}, \hat{\mu})$ of the parameter estimates as well as the parameters themselves. These parameters can, according to (Leadbetter et al., 1997), be shown to be dependent and normally distributed, at least asymptotically. Random values of $\hat{\xi}$, $\hat{\sigma}$ and $\hat{\mu}$ can therefore be simulated whose values retain this dependency. According to (Englund, 2000) the Choleski method can be used whereby a matrix $A$ is found such that $V = AA^T$. A $d$-dimensional independently standard normally distributed variables $N(0,1) Z_1, Z_2, Z_3, \ldots Z_d$ can then be produced. A column vector, $Z = (Z_1, Z_2, Z_3, \ldots Z_d)^T$ can then be used such that

$$X = m + AZ$$ (5.16)

where $m$ are the mean values, in our case $d$ is three dimensional and $m = (\hat{\xi}, \hat{\sigma}, \hat{\mu})^T$. The matrix $A$ can easily be found with the help of the Matlab function sqrtm. These randomly produced parameters of the GEV distribution are then used in (5.15) to produce the estimated distribution of $S_{r50}$. From the estimated distribution of $S_{r50}$ it then a simple task to estimate the 95% confidence limit of $S_{r50}$. The values of $S_{r50}$ are structured into ascending order having index from say 1 to $n$, and the 95% quantile value is estimated using the $0.95n$-th value in this ordered sample. Typically one thousand simulations were used to estimate the distribution of $S_{r50}$. The parameter estimates corresponding to the mean value and the 95% quantile of the return load were recorded to be used later in the reliability analysis. The original shape, scale and location parameter estimates were denoted $\hat{\xi}$, $\hat{\sigma}$ and $\hat{\mu}$, respectively. The parameter estimates relating to the 95% quantile of the 50 year return load were denoted $\hat{\xi}_{0.95}$, $\hat{\sigma}_{0.95}$ and $\hat{\mu}_{0.95}$.

5.3.6 The Gumbel Model

The method of estimating the parameters and the characteristic loads was also applied to the special case of a Gumbel extreme value distribution, $\xi = 0$. The WAFO
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Figure 5.3: Illustration of the principle of the Peaks-Over-Threshold method with a Generalised Pareto Distribution fitted to the heights of the peaks, $h$ over threshold $u$.

The function to estimate the parameters of the Gumbel distribution is called \texttt{wgumbfit}. The maximum likelihood method is used in this function and, similarly to that described above, the function produces both the estimates of the distribution $(\hat{a}, \hat{b})$ and also a variance-covariance matrix $V(\hat{a}, \hat{b})$. The estimates are again asymptotically unbiased multivariate normally distributed and hence the same procedure as above can be used to build confidence intervals for the characteristic loads under the assumption of the load effects belong to the Gumbel family of distributions.

### 5.4 Peaks-Over-Threshold

The Peaks-Over-Threshold method, often abbreviated as the POT method, is an alternative method to the traditional extreme value distributions described in the previous section. The POT method has been used in many fields to identify extremal events such as loads, wave heights, floods, wind velocities, insurance claims, etc cf. (Pandey et al., 2001; Naess and Clausen, 2001; Embrechts et al., 1997; Emmer et al., 1998). The theory of this method is that regardless of the behaviour of the main part of the distribution, the tail of a distribution above a high enough threshold, $u$, can be shown to tend to a Generalised Pareto Distribution, abbreviated GPD. The main advantage of this method compared to the taking maxima of blocks of data is that it avoids losing possibly interesting information. In a block of data there may be several high values within a given block, however only one, the maximum, is registered. The information provided by these other high values are lost, this is not the case with the peaks over a high threshold as all significant peaks are registered. The principle of the POT method is illustrated in Figure 5.3.

The theory behind the POT method will not be discussed and the interested reader is referred to (Leadbetter et al., 1997; Brodtkorb et al., 2000; Rydén and Rychlik, 2001; Coles, 2001).

As before in the previous section, consider an sequence of samples $X_1, X_2, \ldots, X_n$ which are independent identically distributed from the distribution function $F$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{peaks_over_threshold}
\caption{Graphical representation of the principle of the Peaks-Over-Threshold method with a Generalised Pareto Distribution fitted to the heights of the peaks, $h$ over threshold $u$.}
\end{figure}
Then as in (5.5) let

\[ M_n = \max (X_1, X_2, \ldots, X_n) \]

We know from earlier that for large \( n \) then

\[ \Pr(M_n \leq z) \approx G(z) \]

such that \( G(z) \) is given by (5.11), under the condition that \( G \) is non-degenerate. Then provided the threshold level \( u \) is large enough the distribution of the excursion heights \( (X - u) \) under the condition that \( X > u \) is approximately given by

\[
G(h) = \begin{cases} 
1 - \left( 1 + \frac{\xi h}{\tilde{\sigma}} \right)^{-1/\xi} & \text{if } \xi \neq 0, \\
1 - \exp \left( -\frac{h}{\tilde{\sigma}} \right) & \text{if } \xi = 0,
\end{cases}
\]

for \( 0 < h < \infty \) if \( \xi \geq 0 \) and for \( 0 < h < -\tilde{\sigma}/\xi \) if \( \xi < 0 \). Where \( \tilde{\sigma} \) is a positive scale parameter, \( \xi \) is the shape parameter and \( h \) is the height of the peak above the threshold value. The shape parameter corresponds to the shape parameter of the associated GEV i.e. \( G(z) \) given by (5.11). The scale parameter, however is somewhat different and according to (Brodtkorb et al., 2000; Coles, 2001) can be shown to be

\[
\tilde{\sigma} = \sigma + \xi(u - \mu)
\]

(5.18)

where \((\xi, \sigma, \mu)\) are the parameters of the corresponding GEV.

As with the GEV the shape parameter of the GPD is dominating in determining the behaviour of the tail. If \( \xi \geq 0 \) the tail is unbounded while on the contrary if \( \xi < 0 \) the tail has an upper bound at \( u - \tilde{\sigma}/\xi \). Figure 5.4 shows the significance of the shape parameter in determining the behaviour of the tail under the GPD model assumption.

The notation \( \tilde{\sigma} \) of the scale parameter was used to distinguish that of the GPD from that of the GEV. The notation will be dropped in the forthcoming passages and even the scale parameter of the GPD will referred to simply as \( \sigma \). The reason for this is mostly that distinguishing between the parameter \( \sigma \) and the parameter estimate \( \hat{\sigma} \) becomes very difficult otherwise.

### 5.4.1 Mean Exceedance Plot

The \textit{Mean Exceedence Plot}, also known as the \textit{Mean Residual Life Plot} can be used to determine at what threshold level the exceedances start to behave as predicted by the GPD model. The basis of the plot comes from a useful property of the GPD, namely that for \( \xi < 1 \) the mean exceedance over a threshold level \( u \) is a linear function of the threshold level \( u \) (Brodtkorb et al., 2000; Coles, 2001). Mathematically expressed

\[
E(X - u|X > u) = \frac{\sigma + \xi u}{1 - \xi}
\]

(5.19)
where $E(X - u | X > u)$ expected value of the height of the exceedance and is quite simply estimated as the mean of the exceedance heights $h$. This implies that a plot of the mean of $h$ against $u$ will produce linearity if the GPD model is applicable. Confidence intervals for the mean can also be represented in this plot by adopting the approximate normal properties of a sample mean.

An example of a mean exceedance plot is shown in section 5.5 together with how it can be used in the selection of a suitable threshold.

### 5.4.2 Generalised Pareto Quantile Plot

The Generalised Pareto Quantile plot is similar, in principle, to the mean exceedance plot of the previous subsection, in that linearity indicates the validity of the GPD model. The main difference is that this plot is valid for all real values of the shape parameter $\xi$ as opposed to the restriction of $\xi < 1$. However, the disadvantage with this plot is that the computational time involved is much greater than the mean exceedance plot, especially for large samples. The algorithms for the plotting points of the generalised pareto quantile plot can be found in (Caers and Maes, 1998; Beirlant et al., 1996a; Beirlant et al., 1996b). They are not shown here, as for the majority of this work the mean residual life plot was sufficient, having values of $\xi < 1$. Also, the plotting positions are somewhat complicated and have been judge to be less interesting for most applications. However, those readers with shape parameters outside the range of validity of the mean residual plot are referred to these articles. The articles (Beirlant et al., 1996a; Beirlant et al., 1996b) also show how extreme value index, $\xi$, can be estimated from the plots.
5.4.3 Parameter Estimates of the GPD

Within the WAFO toolbox there is a function \texttt{wgpdfit} that can be used to estimate the parameters of the GPD that best fit the data. There are four estimation methods provided in the function, Pickands estimator, Probability Weighted Moments method, Moment method and the Maximum Likelihood method. Some of the estimation methods are only applicable within certain ranges of the shape parameter. The Pickands estimator is valid for all real values of the shape parameter and is thus the most robust, however, it generally, produces the greatest variance i.e. provides the greatest discrepancies from the data. It also has the disadvantage of being the most processor time consuming. Another large disadvantage of this estimator is that it does not possess a property of asymptotic normality as do the other estimators, which are asymptotically normal for certain values of the shape parameter. From all but the Pickand estimator, therefore the asymptotical normal properties of the estimates of the shape and scale parameter are thus produced with this function in the form of a variance-covariance matrix.

The ML estimator is valid for $(\xi > -1)$, however the asymptotically normal properties of the ML estimator is only valid for $(\xi > -1/2)$. The moment and the PWM estimators are valid for shape parameters of $(\xi < 1/4)$ and $(\xi < 1/2)$ respectively.

The methods of estimating the shape parameter or EVI have been discussed in many articles. The original estimator proposed by Hill is only applicable for the special case of a pareto type tail (no upper bound) with the EVI $\xi > 0$, see e.g. (de Haan, 1993; Caers et al., 1998). This estimator has then been made more general to apply to all real values of the EVI in the form of the generalised Hill estimator.

The article (de Haan, 1993) gives a description of both the Hill and the generalised Hill estimator, the latter estimator has hence be known as the de Haan estimator cf. (Naess and Clausen, 2001). It also describes in full the theory behind the estimators and the asymptotically normal behaviour including their covariance matrix. The article (Naess and Clausen, 2001) provides an easy to follow description of the estimation method by de Haan and the moment method. The article (Yun, 2000) provides a summary of several estimators of the EVI including by the Hill estimator, the Pickands estimator and the Weissman estimator together with their respective asymptotic properties.

5.4.4 Return Loads under the POT Model

As for the case of the GEV it is often of interest to extrapolate the return loads. However, this time the GPD model will be used as the basis for this extrapolation. Under the assumption that the GPD model is applicable and that the parameters of the GPD are $\xi$ and $\sigma$, then the probability of the exceedances of a variable $X$ over a suitable high threshold $u$, can be written as

$$
\Pr(X > x | X > u) = \left[ 1 + \xi \left( \frac{x - u}{\sigma} \right) \right]^{-1/\xi}
$$

(5.20)
on the condition that \( x > u \) and that \( \xi \neq 0 \). If we denote the probability \( \Pr(X > u) = \rho_u \), then
\[
\Pr(X > x) = \rho_u \left[ 1 + \xi \left( \frac{x - u}{\sigma} \right) \right]^{-1/\xi} \tag{5.21}
\]
Note the use of the subscript \( u \) in \( \rho_u \), as this value will be dependent on the choice of threshold. The level \( x_m \) which on average will return once every \( m \)-observations will then be obtained from
\[
\rho_u \left[ 1 + \xi \left( \frac{x - u}{\sigma} \right) \right]^{-1/\xi} = \frac{1}{m} \tag{5.22}
\]
which after rearranging yields
\[
x_m = u + \frac{\sigma}{\xi} \left[ (m\rho_u)^\xi - 1 \right] \text{ for } \xi \neq 0 \tag{5.23}
\]
This is valid under the condition that \( m \) is sufficiently large to ensure that \( x > u \). The above equation (5.23) is valid for a non-zero values of \( \xi \). For \( \xi = 0 \) then a similar procedure as above, but applied to the appropriate equation in (5.17) leads to the result
\[
x_m = u + \sigma \log (m\rho_u) \tag{5.24}
\]
The above equations (5.23–5.24) are not usually very helpful as they are not related to a time period and one is normally interested in the say \( N \)-year return period. In order to introduce the concept of time the number of exceedances must be related to a time period, this however can be achieved in the following manner. Suppose that we are interested in the \( N \)-year return period and we have made \( n \) observations during a certain time. It is then possible to estimate the equivalent number of observations that would be required over a period of one year, which we denote \( n_y \). The value of \( m \) to be inserted into (5.23) or (5.24) is given by
\[
m = Nn_y \tag{5.25}
\]
The probability \( \rho_u \) that an observation will exceed the threshold limit \( u \) is also required in order to solve (5.23) or (5.24). The ML estimator of \( \rho_u \) is given by
\[
\hat{\rho}_u = \frac{k}{n} \tag{5.26}
\]
where \( k \) is the number of observations that exceed the threshold and \( n \) is the number of observations. According to (Coles, 2001) the number of exceedances of \( u \) is binomially distributed Bin\((n, \rho_u)\). Inserting (5.25–5.26) into (5.23) and (5.24) produces the following
\[
\hat{z}_N = \begin{cases} 
  u + \frac{\sigma}{\xi} \left[ (Nn_y\rho_u)^\xi - 1 \right] & \text{for } \xi \neq 0 \\
  u + \sigma \log (Nn_y\rho_u) & \text{for } \xi = 0 
\end{cases} \tag{5.27}
\]
An estimate of the \( N \)-year return level can be made by inserting the parameter estimates \((\hat{\xi}, \hat{\sigma})\) from the WAFO function \texttt{wgpdfit} or equivalent together with the estimate for \( \rho_u \) obtained from (5.26).
5.4.5 Uncertainties of Return Loads

If the asymptotically normal properties of the estimates apply then the variance-covariance matrix can be used to simulate values of the shape and the scale parameters of the estimated GPD. The shape and scale parameters are thus 2-dimensional normally distributed variables with mean values of $\hat{\xi}$ and $\hat{\sigma}$ respectively and their mutual dependency is described by the variance-covariance matrix. These properties of the estimates can then be used to simulate new values of $\xi$ and $\sigma$ retaining this dependency to produce simulated values of the sought after return load.

In actual fact the variance-covariance matrix should also include the uncertainties involved in the estimate of $\rho_u$, however according to (Coles, 2001) these are usually small in comparison with the variances of the estimators of the GPD parameters.

A problem that was noted when applying this simulation technique was that when the mean i.e. estimated value of the scale parameter was low then it was possible to simulate negative values of this parameter. However, by definition the scale parameter is always real and positive and hence these negative simulated values should not be possible and therefore when used produced errors. In order to counteract this problem the negative values of $\sigma$ together with their simulated value of $\xi$ where pairwise replaced with new random values. This step was repeated until all negative values of $\sigma$ had been replaced. Generally this was not a problem and at worst in the order of a 100 values had to be replaced from a simulation of 10000. A check was done after the simulations to verify that the simulated values of $\xi$ and $\sigma$ retained their original mean values and variance-covariance matrix.

The method of producing the two dimensional normally dependent variables was the same as described earlier in section 5.3.5.

5.4.6 Assumption Gumbel Tail

If one assumes that the data belongs to a Gumbel tail of attraction, then the distribution becomes the special case of the GPD such that the shape parameter $\xi = 0$. This is the same as the exponential distribution. Under this assumption the cdf for the GPD is the second case of (5.17), i.e.

$$F(h; \xi, \sigma) = 1 - \exp\left(-\frac{h}{\sigma}\right) \quad \text{if } \xi = 0$$

(5.28)

which is the same as the expression for the exponential distribution. The estimate for the parameter of the exponential distribution $\sigma$ is given by the mean of $h$. According to (Rydén and Rychlik, 2001), the estimator is approximately normally distributed, is unbiased and a standard deviation given below. Thus summarising the estimator
and its properties:

\[ \hat{\sigma} = \frac{1}{n} \sum_{i=1}^{i=n} h_i \]  
(5.29)

\[ E(\hat{\sigma}) = \sigma \]  
(5.30)

\[ D(\hat{\sigma}) = \frac{\sigma}{\sqrt{n}} \]  
(5.31)

where \( n \) is equal to the length of the vector \( h \), the exceedance heights.

The above relationship implies that confidence intervals can be constructed for the estimate of the exponential distributions parameter, \( \sigma \) and for the return loads.

### 5.4.7 Relationship between the GEV and the GPD

According to (Brodtkorb et al., 2000) there is a relationship between the GPD and the GEV that can be useful. It was stated in section 5.4.4 that the number of exceedances is a random variable that is binomially distributed. Provided the threshold value is high then the number of exceedances can be approximated by a Poisson distribution, due to the Poisson distribution approximating a Binomial distribution under small probability conditions and assuming the events are independent. The manual (Brodtkorb et al., 2000) states that ”the maximum of a Poisson distributed number of independent GPD variables has a GEV distribution”. This is shown in the manual by considering the summation of probabilities. Assume \( k \) to be a Poisson distributed random variable with mean \( m \) and the maximum of \( k \) independent GPD variables such that \( M_k = \max(X_1, X_2, \ldots, X_k) \) therefore:

\[ \Pr(M_k \leq x) = \sum_{n=0}^{\infty} \Pr(k = n) \cdot \Pr(X_1 \leq x, X_2 \leq x, \ldots, X_n \leq x) \]  
(5.32)

The first part of the right hand side is simply the Poisson distribution and hence

\[ \Pr(M_k \leq x) = \sum_{n=0}^{\infty} \exp(-m) \frac{m^n}{n!} \cdot \left[ 1 - \left( 1 + \frac{x}{\sigma} \right)^{-1/\xi} \right]^n \]  
(5.33)

which according to (Brodtkorb et al., 2000) can be shown to simplify to

\[ \Pr(M_k \leq x) = \exp \left\{ - \left[ 1 + \xi \left( \frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \right\} \]  
(5.34)

The above equation (5.34) can be recognised by comparison with (5.11) as being a GEV distribution. Where, again according to (Brodtkorb et al., 2000), the scale and location parameters of this GEV are given by the following equations:

\[ \sigma = \hat{\sigma} m_u^\xi \]  
(5.35)

\[ \mu = u + \frac{(\sigma - \hat{\sigma})}{\xi} \]  
(5.36)
where $\hat{\sigma}$ is the scale parameter from the GPD, $m_u$ is the mean number of exceedances in the relevant time period from the Poisson distribution and $\xi$ is the shape parameter which assuming the model to be corresponds to both the GEV and the GPD. It can be seen that (5.36) is exactly the same as (5.18) from earlier.

The estimation of the parameters of the GPD are used to estimate $\hat{\sigma}$ and $\xi$, thus the only remaining unknown is the mean number of exceedances in the Poisson distribution $m_u$. This can be estimated as the mean number exceedances above the threshold $u$ from the observed data, with an adjustment for the desired time period. The relationship is used to estimate a GEV from the estimated GPD model. The estimated GEV can then be included in a reliability analysis in accordance with the theories detailed in Chapter 4.

### 5.4.8 Optimal Choice of Threshold

The estimated value of the EVI and consequently any extrapolated high percentiles are dependent on the choice of threshold value. The subject of an optimum threshold choice has been the subject of many articles (Cremona, 2001; Caers and Maes, 1998; Pickands III, 1993; Pandey et al., 2001) to name but a few. There is a trade of between bias and variance (Caers et al., 1998; Caers and Maes, 1998). A high threshold value reduces the bias as this satisfies the convergence towards the extreme value theory but however increases the variance for the estimators of the parameters of the GPD, there will be fewer data from which to estimate parameters. A low threshold value results in the opposite i.e. a high bias but a low variance of the estimators, there is more data with which to estimate the parameters.

In (Cremona, 2001) the Kolmogorov-Smirnov goodness-of-fit statistic is used as a means of measuring the quality of the assumed statistical model (in this case Rice’s formula) to the data. This method has also been used in (Getachew, 2003) for the assessment of the girder distribution factor for road bridges. In the article (Beirlant et al., 1996b) the asymptotic properties of the Hill estimator is used so that the minimisation of the Asymptotic Mean Square Error (AMSE) is used as a criterion to select the threshold. However, the article was difficult to follow and appears to consider the case of the Fréchet tail behaviour, i.e. $\xi > 0$. In a paper (Pandey et al., 2001) the bias and the root mean square error (RMSE) are used to assess different methods for calculating the parameters of the GPD. In this theoretical study, samples were generated from assumed distributions (the Gumbel distribution was used) it was therefore possible to calculate the correct answer and hence the bias.

In the article (Caers and Maes, 1998), a minimisation of the Mean Square Error is used as the criterion for the threshold selection. A semi parametric bootstrap method is used to estimate the Mean Square Error (MSE). This method was judged to be attractive to the author of this thesis and is therefore described below.

The idea of the non-parametric bootstrap method is to create 'new' sets of data by
simulation from the original data. If the original data was a vector size $n$ then new data is created by simulation from the original set using selection with replacement, whereby each sample within the original data set has a $1/n$ chance of being selected. The new data must also be the same size as the original data. This is then repeated say $B$ times so that $B$ sets of bootstrap samples are created. From these new sets of bootstrap samples it is then possible to make estimates of bias and variance and therefore the MSE.

Another variation is to use a parametric bootstrap, although this is not as preferable as the first. In this method a parametric fitting of the data is made, e.g. the GPD is fitted to the data such that the actual unknown parameters $\xi$ and $\sigma$ are estimated by $\hat{\xi}$ and $\hat{\sigma}$. From this estimated distribution or function new bootstrap samples are created by random selection from this estimated function or distribution.

In (Caers and Maes, 1998) a solution is proposed such that the bootstrap samples fall somewhere between these two categories and hence the name semi-parametric bootstrap. It is described in the article that one of the problems with the bootstrap method is that as one comes closer to the tail the empirical distribution $\hat{F}$ provides a poor representation of the tail, for this reason the bootstrap re-sampling is replaced by the parametric bootstrap, when one of the observations falls above the threshold level. This semi-parametric bootstrap denoted $\hat{F}_s$ is according to (Caers and Maes, 1998) given by

$$\hat{F}_s(x|u) = \begin{cases} 
1 - \hat{F}(u) \left\{ 1 - \left[ 1 + \frac{\xi(x - u)}{\sigma} \right]^{-1/\xi} \right\} + \hat{F}(u) & \text{for } x > u \\
\hat{F}(x) & \text{for } x \leq u
\end{cases}$$

(5.37)

The term inside the curly brackets can be recognised as the cdf of the GPD given that $x > u$. In the article this method is tested for various parent distributions and the MSE of a number of quantities are studied and successfully compared with theoretical solutions.

The author of this current thesis has unsuccessfully attempted to use this method to select the optimal threshold for my application and even to the examples shown in the described article. My interpretation of the article is that at each chosen level of threshold $u$, the parameters of the GPD are estimated using the original data that exceed the threshold. Bootstrap samples are created but every observation that exceeds the chosen threshold is replaced by an observation from the estimated GPD of the original data i.e. the parametric part of the bootstrap. Once the bootstrap samples have been created using (5.37) then for every new sample a quantity can be estimated e.g. the shape and scale parameters, or the return load. This in turn means that only the observations exceeding the threshold are used in the estimation of say the shape parameter. The MSE of the shape parameter from these $B$ bootstrap samples may then be evaluated and the minimum MSE yields the optimal threshold. However, in effect, the only difference between this method and a pure parametric method is that the number of observations falling above the threshold level will vary within the bootstrap samples for a given threshold level. Given enough number of observations above the threshold level on which to estimate the parameters, even
if this is a poor description of the data, then the 'new' observations should yield data that results in parameter estimates of the GPD that are close to the original parameter estimates. Then provided $B$ is chosen large enough, the evaluation of the MSE will be small and are therefore biased towards low thresholds. This is precisely the results obtained by my attempt to use this method and it is believed to be the reason why this method was unsuccessful. However, it may well be that I have misinterpreted the article. Another point to make is that the parameter estimation method chosen by myself was the method of moments as this was computationally more efficient than the parameter estimation method used in (Caers and Maes, 1998). This may cause different results to be obtained. Generally, unlike the results described in the aforementioned article, the bootstrap estimates of the bias were minimal in comparison to those of the estimates of variance. Also the number of bootstrap samples simulated were greater than those of the article and perhaps the reason for the smaller amount of bias.

5.5 Threshold Choice

In this section the method used for the selection of the threshold value will be illustrated by using the results of simulations of trains crossing a 2D bridge at constant speed, see Chapter 7. The case of a 20 m span bridge is used as an example.

There are several factors that need to be considered when choosing the threshold value:

- The mean exceedance plot should be a linear function of the threshold value, provided $\xi < 1$,
- The shape parameter should remain constant for increasing threshold levels if the GPD model applies,
- The resulting exceedance heights should be well represented by a GPD model, and
- The threshold value should be high enough so that the tail is well represented and that bias towards the middle of the distribution is avoided.

The mean exceedance plot was used for the first criterion, attempting to locate signs of linearity, while a plot of the estimated shape parameter versus the threshold level was used in assessing the second criterion. The first and the second criteria are actually a consequence of one another. Figure 5.5 shows the mean exceedance plot for the 20 m span case. As one can see from this plot, linearity occurs at approximately $u = 0.42$. A plot of the estimated shape parameter, or EVI, against the threshold level can be seen in Figure 5.6. From this figure it can be seen that $\hat{\xi}$ remains relatively constant over a range of threshold level from approximately 0.42–0.49. The confidence intervals of the return loads also show a stability in this range.
Figure 5.5: Mean exceedance versus threshold level $u$, for a 20 m span. The plot shows a linear behaviour after $u = 0.42$ indicating compliance with the GPD model. The chosen threshold was 0.458.

Figure 5.6: Confidence interval of the return load and the estimated value of the shape parameter versus the threshold value for a 20 m span.
Figure 5.7: Goodness-of-fit statistics, the $R^2$, the Kolmogorov-Smirnov, the Anderson-Darling and the $\chi^2$ goodness-of-fit tests versus the threshold level for a 20 m span. The chosen threshold for this span was $u = 0.458$.

Also for varying values of threshold, goodness-of-fit statistics were also evaluated and used in the decision process. This meant that for every value of threshold the parameters of the GPD were estimated and goodness-of-fit statistics were calculated for each level, comparing the data with the estimated theoretical model under the GPD assumption. Figure 5.7 shows these plots for the 20 m span. The uppermost sub-figure shows the $R^2$ value versus the threshold. A value of $R^2 = 1$ represents a perfect fit, likewise the Kolmogorov-Smirnov test indicates a good fit as the significance level $Q_{KS}$ approaches 1. For the Anderson-Darling test, at the 5% significance level, the value of 2.492 is suggested in (Johnson, 1994), i.e. the test value should fall below this level if there is no significant difference at this probability level. In the third sub-figure the Anderson-Darling test value falls below this value for all the threshold values $u > 0.41$. In the case of the $\chi^2$ goodness-of-fit test the test value, shown continuous in the sub-figure, should fall below the $\chi^2$ distribution value, for the correct degree of freedom, at the required significance level (in this case 5% was used). This value is shown dashed in the sub-figure. The test value falls below the $\chi^2$ value shortly after $u = 0.38$.

Another measure of the goodness-of-fit used in this process was the mean square error, abbreviated MSE. This is a measure of the variation of the data from that predicted by the fitted theoretical model and is given by

$$MSE = \frac{\sum_{i=1}^{n} \left[ \hat{F}(h_{(i)}) - \bar{F}(h_{(i)}) \right]^2}{n - 1}$$

(5.38)
Figure 5.8: Mean Square Error goodness-of-fit, see (5.38), versus the threshold level. Small values of this quantity denote the better fit.

where $n$ is the number of exceedances over the threshold, $h_{(i)}$ are the ordered height to peaks over the threshold level, and as before $\hat{F}$ and $\tilde{F}$ are the empirical and the estimated cdf’s, which are calculated at the observed values of the excursion heights. Small values of $MSE$ indicate a good fit. Figure 5.8 shows the $MSE$ versus the threshold level and as can be seen from this figure a threshold of between 0.40 and 0.47 may be justified.

From the above, a value of anywhere in the range of $0.42 < u < 0.46$ would therefore seem a reasonable choice. The final choice was $u = 0.458$ which was quite long into the tail, thus hopefully avoiding bias, but still had a large number of data points (695) on which to make the parameter estimates.

Also once a threshold had been chosen a final check was carried out by the use of probability and QQ plots. The QQ plots are not shown here but are detailed in Appendix C and discussed in the results, Chapter 8. A similar procedure was carried out for all six spans.
Chapter 6

Reliability Investigation of 25 t on a Four Axle Wagon

The main purpose of this section is to study the sensitivity of a reliability analysis on certain parameters, namely the assumed number of axles per year, the assumed properties of the dynamic factor and the assumed properties of the loading from the axles. In order to achieve this, a small span bridge of 4 metres has been chosen for the study whereby it is loaded with a four-axled freight wagon. The study is also confined to concrete bridges designed to the 1980 standard and an increased to a proposed allowable axle load of 25 tonnes has been investigated. As in the rest of the thesis, the study purely considers the ultimate limit state analysis and does not consider the increased fatigue effects and the consequent shortening of the bridge’s life as a result of an increase in permissible axle load. This section can be seen as independent of the main thesis and was done to identify the key parameters of the analysis.

A typical four-axled freight wagon is shown in Figure 6.1 and its bogie and axle configuration is shown in Figure 6.2. A six metre bridge is drawn to scale under the axles of the coal wagon. As can be seen from this figure the other axle loads do not fit onto the bridge when the bogie of the one wagon is placed at the point to cause the greatest mid-span moment. It is also possible to see from this figure that critical dimensions for these short span simply supported spans are the axle spacings within a bogie and the spacing of the last axle to the end of the buffers. The axle spacings of freight wagons operating in Sweden have been taken from (StatensJärnväg, 2000), and typical dimensions were 1800–2000 mm within a bogie and 1620–2000 mm for the distance from the centre of the last axle to the end of the buffers. The vast majority of these four-axled wagons had an axle spacing within the bogie of 1800 and a distance from the centre of the last axle to the end of the buffer of 1620 mm, for this reason these two dimensions have been assumed in this chapter. Some special wagons had spacings within a bogie of only 1500 mm, but as mentioned before the idea behind this thesis is to investigate raising allowable axle loads on normal freight wagons. Although the study is limited to the case of the four metre span bridge it can seen from Figure 6.2 that the formulation is applicable to spans of less than 6.4 m, i.e. four times 1.6 metres.
Figure 6.1: A faoos wagon used to transport mass goods. Picture courtesy of Green Cargo’s internet home page.

Figure 6.2: Dimensions of a faoos goods wagon in mm. A six metres span bridge is shown at the position to cause maximum moment.
6.1 Level II Analysis

Consider the case of two point loads from a single bogie set, placed on a simply-supported bridge of span less than 6.4 metres to cause the maximum moment at mid-span. The situation is shown in Figure 6.3, where \( a \) is the distance between axles within the bogie, and \( P_y \) is the yearly maximum axle load in a bogie. Both \( a \) and \( P_y \) are stochastic variables while the span length \( L \) is taken as a deterministic value. It is assumed here that the axle loads within a bogie are the same.

The traffic load effect \( Q \), also a stochastic variable, under consideration is the maximum moment at mid-span and is given by:

\[
Q = \frac{P_y}{2} (L - a) \tag{6.1}
\]

If the dynamic amplification factor is considered then the combined effect of both the traffic load and the dynamic amplification factor becomes:

\[
Q (1 + \varphi) = \frac{P_y}{2} (L - a) (1 + \varphi) \tag{6.2}
\]

The load effect, in this case the moment at mid-span can be written:

\[
S = G + Q (1 + \varphi) \tag{6.3a}
\]

\[
S = G + \frac{P_y}{2} (L - a) (1 + \varphi) \tag{6.3b}
\]

where \( G \) is the load effect due to the self-weight of the bridge. The limit state function can then be written

\[
g(R, G, P_y, a, \varphi) = R - G - \frac{P_y}{2} (L - a) (1 + \varphi) \tag{6.4}
\]

the material strength \( R \) is assumed to be log-normally distributed with mean \( \mu_R \) and coefficient of variation \( Cov_R \). The axle spacing within a bogie is assumed to be normally distributed with mean \( \mu_a \) and standard deviation \( \sigma_a \), the values chosen.
throughout this chapter were 1.8 m and 0.18 m respectively for these quantities. The axle load $P_y$ is assumed to be Gumbel distributed with an equivalent normal tail approximation for each iteration value of mean $\mu_P$ and standard deviation $\sigma_P$. The dynamic factor $\varphi$ is assumed to be normally distributed with mean $\mu_\varphi$ and standard deviation $\sigma_\varphi$.

Transformation of the individual s.v. into the standard normal $y$-space yields the following:

\[
R = \mu_R \exp (Y_1 \text{Cov}_R) \tag{6.5}
\]
\[
G = \mu_G (1 + Y_2 \text{Cov}_G) \tag{6.6}
\]
\[
P_y = \mu'_P + Y_3 \sigma'_P \tag{6.7}
\]
\[
a = \mu_a + Y_4 \sigma_a \tag{6.8}
\]
\[
\varphi = \mu_\varphi + Y_5 \sigma_\varphi \tag{6.9}
\]

Substitution of (6.5–6.9) into (6.4) yields the limit state function in the standard normal plane.

\[
g(y) = \mu_R \exp (y_1 \text{Cov}_R) - \mu_G (1 + y_2 \text{Cov}_G) - \frac{(\mu'_P + y_3 \sigma'_P)}{2} [L - (\mu_a + y_4 \sigma_a)] [1 + (\mu_\varphi + y_5 \sigma_\varphi)] \tag{6.10}
\]

Also defining a value $\nu$ such that

\[
\nu = \frac{\mu_G}{Q_n} \tag{6.11}
\]

where $Q_n$ is defined as the load effect obtained by applying the code’s traffic load model including the dynamic amplification factor. Further dividing the entire equation (6.10) by $Q_n$ and at the same time substituting the definition of $\nu$ from (6.11) yields

\[
g(y) = \frac{\mu_R}{Q_n} \exp (y_1 \text{Cov}_R) - (1 + y_2 \text{Cov}_G) \nu - \frac{(\mu'_P + y_3 \sigma'_P)}{2Q_n} [L - (\mu_a + y_4 \sigma_a)] [1 + (\mu_\varphi + y_5 \sigma_\varphi)] \tag{6.12}
\]

\[
\nabla g(y) = \begin{pmatrix}
\frac{\mu_R}{Q_n} \exp (y_1 \text{Cov}_R) \\
-\nu \text{Cov}_G \\
-\frac{\sigma'_P}{2Q_n} [L - (\mu_a + y_4 \sigma_a)] [1 + (\mu_\varphi + y_5 \sigma_\varphi)] \\
\frac{(\mu'_P + y_3 \sigma'_P) \sigma_a}{2Q_n} [1 + (\mu_\varphi + y_5 \sigma_\varphi)] \\
-\frac{(\mu'_P + y_3 \sigma'_P)}{2Q_n} [L - (\mu_a + y_4 \sigma_a)] \sigma_\varphi
\end{pmatrix}^T \tag{6.13}
\]

The above equations (6.12–6.13) can then be used in the iteration procedure described in sections 4.6.5 and 4.6.6 to obtain values of the safety index $\beta$ for an
assumed value of $\mu_R/Q_n$. The assumed value of $\mu_R/Q_n$ is subsequently adjusted until the required safety index of 4.75 is obtained. This was done automatically by using the Matlab function \texttt{fzero}, see (The MathWorks, 1999). The routine for calculating $\beta$ was therefore adjusted by the subtraction of 4.75 in order to obtain a value of zero.

To summarise, the sought after quantity is the value of the mean value of resistance, put in relation to the nominal traffic load effect, that will yield the desired target safety index of 4.75. This value of $\mu_R/Q_n$ will then be used in a reliability level I analysis in order to study the required partial safety factor for the traffic load effect as will be seen in the next section.

6.2 Level I Analysis

The level I analysis used in this chapter is the same as that described in section 4.9.1 as the study was limited to the 1980 code format. Therefore recalling (4.63) and rearranging to express the equation in terms of $\gamma_q$ yields

$$\gamma_q = \frac{\mu_R}{Q_n} \cdot \frac{\exp(k_C Cov_C) \exp(k_F Cov_F)}{\eta c \gamma_m \gamma_n} - \nu \gamma_g$$

(6.14)

where the values of $\mu_R/Q_n$ are obtained from the above section and for assumed values of $\nu$. In this manner the required value of $\gamma_q$ to obtain the required safety index can be investigated. The values of $\eta_c, \gamma_m$ and $\gamma_n$ can be obtained from Table 3.2 and values of the resistance model can be obtained from section 4.8.

6.2.1 The Bogie Load per Axle $P$

The properties of the distribution of the yearly maximum axle load $P_y$ was obtained using simulation. The basic axle load $P$ was assumed to be normally distributed with a mean value of 25 tonnes and a coefficient of variation $Cov_P$. The simulations assumed that the number of fully loaded bogies that travelled the line per year was one million. The maximum axle load per year was noted and 500 years of simulations were used to calculate the properties of the Gumbel distribution. The Gumbel distribution was assumed as it is the extreme distribution when the basic variable is normally distributed. In this manner it was possible to vary the properties of the assumed single axle distribution, i.e. by varying $Cov_P$, in order to investigates its effect on the outcome of the required partial safety factor. The one million bogies approximately represent 1400 wagons per day which are intended to be loaded to the 25 t limit.
6.2.2 The Dynamic Coefficient $\varphi$

The dynamic coefficient used in this reliability study was derived from the more rigorous method of one of the appendices of (CEN, 1995) and discussed earlier in Chapter 2. In this study it is of interest to establish the mean value of the dynamic factor, whereas the dynamic factor obtained from section B.2 approximately represents the 95% quantile cf. (Weber, 1998; UIC, 1979). In the Eurocodes there is an equation for the dynamic factor that is to be used in conjunction with ‘real trains’ when calculating the effects of fatigue. The Eurocode (CEN, 1995) states that the following equation can be used for this purpose

\[
1 + \varphi = 1 + 0.5 (\varphi' + 0.5 \varphi'') \tag{6.15}
\]

\[
\varphi = 0.5 (\varphi' + 0.5 \varphi'') \tag{6.16}
\]

where $\varphi'$ and $\varphi''$ can be calculated in the manner described in section B.2. The above equation is for track with good quality maintenance. Since this applies to fatigue calculations, it is reasonable to assume that this value represents a mean value of the dynamic factor rather than the high quantile value. This equation (6.16) has, therefore, been consistently used in this chapter to calculate the mean value of the dynamic coefficient.

With reference to section B.2 it can be seen that the dynamic coefficient using this method is a function of the effective length, the speed of the train and the first bending frequency of the element under design. The given upper and lower limits of the natural frequency of a bridge imply that for every span and train speed, there exits an upper and a lower limit of the dynamic coefficient $\varphi$. In this context the greater of the two values was always chosen for the mean value to be used in the reliability analysis.

The speed considered for these freight trains was 30 m/s which is approximately equivalent to 110 km/h. This value is rather high, however, it will produce conservative values of $\varphi$, as $\varphi$ increases with speed, see Figure 6.4, at least over the range of velocities considered here.

The mean value of the dynamic coefficient, $\mu_\varphi$, used in (6.9–6.13) was therefore calculated using (6.16). The coefficient of variation $Cov_\varphi$ was assumed to vary from 0.2–0.5.

It should be stressed here that although the mean value of the dynamic coefficient is used in the level II reliability method, this will not be the value at the design point, as this will be determined by the values of the directional cosines $\alpha$ and as this variable is coupled to the load effect will obtain a value higher than the mean, see e.g. section 4.6.

An alternative to the above described method of calculating the mean of the dynamic coefficient, although not used here, is to calculate backwards from the quantile value to the mean. The values of the dynamic effect $\varphi$ as given in (UIC, 1979; CEN, 1995) approximately represents the 95% quantile. Therefore, for an assumed value of the
Figure 6.4: Dynamic coefficient $\varphi$ calculated using the equations for fatigue analysis of ‘real trains’ for different spans, (CEN, 1995).

coefficient of variation, $Cov_\varphi$ the assumed mean value of $\varphi$ can be calculated using

$$\mu_\varphi = \frac{\varphi_{0.95}}{1 + kCov_\varphi}$$

(6.17)

where $k = 1.645$ (see standard normal tables) assuming a normal distribution and the 95\% quantile. The value $\varphi_{0.95}$ is taken as $\varphi$, where the latter is obtained from the method described in (UIC, 1979; CEN, 1995) and Appendix B.

6.3 Results of the Sensitivity Study

6.3.1 The Number of Axle Passages per Year

In this section, the sensitivity of the required safety factor for traffic loads, $\gamma_q$, to the number of fully loaded bogies per year has been studied. The number of bogies considered has been varied from 200 thousand up to one million. The basic bogies in this section have then been restricted to a mean value of 25 t and a coefficient of variation of 0.05 or 0.1. The assumption that the axle loads vary according to a normal distribution has been used throughout this section. The speed considered was 108 km/h. Figure 6.5 shows the case for a four metre span simply supported bridge and a coefficient of variation of the dynamic amplification factor of 0.5. The variation, in this case, of the required safety factor is relatively small approximately 1.4\%. A check was done on the same values but with a decrease of $Cov_\varphi$ to 0.2. Exactly the same percentage difference was noted. Another run was also done using
a coefficient of variation of $\text{Cov}_P = 0.1$ for the basic axle load $P$. The percentage difference between the results of $n = 0.2 \cdot 10^6$ and $n = 1.0 \cdot 10^6$ increased slightly to 2\%, see Figure 6.6. However this section shows that the required safety margin, $\gamma_q$ is relatively insensitive to an increase in the number of axle considered to cross a bridge in any one year. At least for the large number of axle that can be anticipated to traffic a main railway.

Table 6.1 shows the results of the yearly maximum bogie load $P_y$ for various values of anticipated number of bogies per year $n$ and for different assumed properties of the underlying normal distribution. It is possible to see from this table that the major factor that effects the value of the mean maximum yearly bogie load is the assumption of the properties of the underlying normal distribution and not the number of bogies per year. It is also worth noting that while the mean value of $\mu_{P_y}$ increases with increased $n$ the standard deviation $\sigma_{P_y}$ decreases and therefore the design point for the value of $P_Y$ remains relatively unchanged and therefore unaffected by the increase in the number of bogies per year. It is therefore reasonable to assume from this section that an increase in $n$ greater than one million is not warranted.
Figure 6.6: The required value of the partial safety factor $\gamma_q$ to maintain the target safety index versus the ratio of self-weight to traffic load effect, $\nu$. A 4 metre span of concrete is detailed. The number of bogies has little effect on the required partial safety factor for the considered number of axles. ($\mu_P = 25 \text{ t}, \sigma_P = 2.5 \text{ t}, Cov_P = 0.1$).

Table 6.1: Variation of the properties of the maximum bogie load for a differing number of assumed bogies per year.

<table>
<thead>
<tr>
<th>$n$ $(10^6)$</th>
<th>$\mu_P$ (kN)</th>
<th>$\sigma_P$ (kN)</th>
<th>Estimated Gumbel Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{a}$ (kN)</td>
<td>$\hat{b}$ (kN)</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>306.76</td>
<td>3.6</td>
<td>2.7185</td>
</tr>
<tr>
<td>0.4</td>
<td>308.64</td>
<td>3.4</td>
<td>2.6937</td>
</tr>
<tr>
<td>0.6</td>
<td>309.75</td>
<td>3.1</td>
<td>2.5251</td>
</tr>
<tr>
<td>0.8</td>
<td>310.24</td>
<td>3.2</td>
<td>2.5313</td>
</tr>
<tr>
<td>1.0</td>
<td>310.87</td>
<td>3.0</td>
<td>2.4359</td>
</tr>
</tbody>
</table>

Basic properties of $P$

- Normally distributed $\mu_P = 250 \text{ kN}, Cov_P = 0.05$
- Normally distributed $\mu_P = 250 \text{ kN}, Cov_P = 0.1$
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\[ \frac{1}{\gamma^2} = 1.00 \]
\[ \frac{1}{\gamma^2} = 1.35 \]
\[ \frac{1}{\gamma^2} = 1.70 \]
\[ \frac{1}{\gamma^2} = 2.05 \]
\[ \frac{1}{\gamma^2} = 2.40 \]
\[ \frac{1}{\gamma^2} = 2.75 \]
\[ \frac{1}{\gamma^2} = 3.10 \]
\[ \frac{1}{\gamma^2} = 3.45 \]
\[ \frac{1}{\gamma^2} = 3.80 \]
\[ \frac{1}{\gamma^2} = 4.15 \]
\[ \frac{1}{\gamma^2} = 4.50 \]
\[ \frac{1}{\gamma^2} = 4.85 \]
\[ \frac{1}{\gamma^2} = 5.20 \]
\[ \frac{1}{\gamma^2} = 5.55 \]

Figure 6.7: The required value of the partial safety factor \( \gamma \) to maintain the target safety index versus the ratio of self-weight to traffic load effect, \( \nu \). A 4 metre span of concrete is detailed. \( (\mu_P = 25 \text{ t} \sigma = 2.5 \text{t}) \). The mean value of the dynamic factor has a significant effect on the required safety factor.

6.3.2 Sensitivity to \( \sigma_P \)

The sensitivity of the reliability analysis to the assumed value of the standard deviation of the normal distribution that describes the variation of the axle load can be seen by comparing the results of Figure 6.5 and Figure 6.6. The underlying assumptions that give rise to the figures are exactly the same except that in Figure 6.5 a standard deviation of 1.25 tonnes was used whereas the corresponding value for Figure 6.6 was 2.5 tonnes. Comparison of a few equivalent points within the figures show that the difference in the required safety factor is approximately 19\% for this change from a coefficient of variation of 5 to 10\%. The result of the reliability analysis therefore appears to be sensitive to the choice of the properties of the parent distribution assumed for the original bogie loads.

6.3.3 Sensitivity to \( \mu_\varphi \) and \( Cov_\varphi \)

In this section the results of the sensitivity of the reliability analysis is shown for different assumed values of the mean of the dynamic amplification factor, \( (1 + \mu_\varphi) \). The method of studying this was to vary the speed of the trains as this is related to the dynamic amplification factor via equation (6.15) and Appendix B.2.
6.3. RESULTS OF THE SENSITIVITY STUDY

Figure 6.8: The required value of the partial safety factor $\gamma_q$ to maintain the target safety index versus the ratio of self-weight to traffic load effect, $\nu$. A 4 metre span of concrete is detailed. $\mu_\varphi = 1.111$. ($\mu_P = 25\, \text{t}$, $\sigma_P = 2.5\, \text{t}$).

Figure 6.7 shows the required value of $\gamma_q$ to maintain the required safety target of 4.75 for a four metre span. The figure shows different curves for different train speeds and hence different values of the dynamic amplification factor, $(1 + \mu_\varphi)$. The assumption for the axle loads was that the mean value was 25 t with a standard deviation of 2.5 t. It can be seen from this figure that the value of the dynamic factor has a significant effect on the outcome of the reliability analysis. The shown curves represent train speeds of 36 to 108 km/h which are raised in steps of 18 km/h.

In this part the sensitivity of the result of the required $\gamma_q$ to the coefficient of variation for the dynamic coefficient $\text{Cov}_\varphi$ was studied.

A comparison of Figures 6.8 and 6.9 shows that the value obtained for $\gamma_q$ is dependant on both the value chosen for the coefficient of variation for the dynamic coefficient, $\text{Cov}_\varphi$, and the speed of the train, $v$, which ultimately translates to the mean value of the dynamic factor. A comparison of the results obtained is shown in Table 6.2. It can be seen from this table that the results are most sensitive to the initial choice of the speed of the train and therefore ultimately the choice of the mean value of the dynamic factor $(1 + \mu_\varphi)$. The table shows that for a 13.5% increase in this mean value the corresponding values of required $\gamma_q$ are increased by 14.5 and 20.2% for values of $\text{Cov}_\varphi$ of 0.1 and 0.5 respectively. The results are however not as sensitive to the choice of the value of $\text{Cov}_\varphi$. Compare the percentage increase of 1.7 and 6.7 for an increase of $\text{Cov}_\varphi$ from 0.1 to 0.5 for speeds of 36 and 108 km/h, respectively.
CHAPTER 6. RELIABILITY INVESTIGATION OF 25 T ON A FOUR AXLE WAGON

Figure 6.9: The figure shows the sensitivity of the reliability analysis to the choice of coefficient of variation of the dynamic coefficient. The required value of \( \gamma_q \) to maintain the target safety value for a 4 metre span of concrete, \( \mu_\phi = 1.264 \). (\( \mu_P = 25 \text{ t}, \sigma_P = 2.5 \text{ t} \))

Table 6.2: A table showing the percentage difference in required value of \( \gamma_q \) for differing values of \( \mu_\phi \) and \( \text{Cov}_{\phi} \). The mean value of \( \gamma_q \) over the range of values of \( \nu \) is used for the comparison. The table shows the required value of \( \gamma_q \) is more sensitive to changes in the mean value of the dynamic factor than the coefficient of variation of the dynamic coefficient.

<table>
<thead>
<tr>
<th></th>
<th>( v = 36 \text{ km/h} )</th>
<th>( v = 108 \text{ km/h} )</th>
<th>Diff. Horizontally %</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1 + \mu_\phi))</td>
<td>1.1107</td>
<td>1.2638</td>
<td>13.5</td>
</tr>
<tr>
<td>\text{Cov}_{\phi}</td>
<td>mean value of ( \gamma_q )</td>
<td>mean value of ( \gamma_q )</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1.1901</td>
<td>1.4299</td>
<td>20.2</td>
</tr>
<tr>
<td>0.1</td>
<td>1.1713</td>
<td>1.3383</td>
<td>14.5</td>
</tr>
<tr>
<td>Diff. Vertically %</td>
<td>1.7</td>
<td>6.7</td>
<td></td>
</tr>
</tbody>
</table>
6.3.4 Summary of Two Axle Study

To summarise this section the outcome of the results for the required partial safety factor $\gamma_q$ is sensitive to the values chosen for the mean value of the dynamic coefficient, $\mu_\phi$ and the initial properties of the variation of the axle loads within a bogie, $P$. The study showed that results were relatively insensitive to the number of bogies assumed to pass a stretch of line per year, at least for the large numbers that can be expected on a main line route. Also the results show, somewhat surprisingly, that the reliability analysis was relatively insensitive to the assumed coefficient of variation of the dynamic coefficient. However, the study was limited and it is difficult to say if these results are generally applicable, although under the presented assumptions they do provide a good indication as to the significant parameters.

Another interesting point that came from this study and is somewhat obvious for these small spans, is that the axle spacings within a bogie is an important parameter together with the perhaps not so obvious parameter, the spacing from the last axle to the end of the buffers. This last spacing seems, in the majority of the studied four-axled freight wagons, to be 1620 mm. However, on a special heavy-haul route in the north of Sweden, there are freight wagons for transporting iron-ore that have much smaller distances between the last axle and the end of the buffer. This allows for a higher concentration of axle loads over a smaller area which can be concentrated towards the centre of the bridge and thus increasing the mid-span moments, which from the bridge’s point of view should be avoided.
Chapter 7

Traffic Load Simulations

7.1 Train Data Collection

7.1.1 General

Data was collected from two sites in Sweden that provided information about the loads from each axle, the spacing between axles and the train speed from moving trains as they passed through the measuring site. One of the sites was in the north of Sweden at Notviken and the other more in the middle of the country at Sannahed, see Figure 7.1.

The traffic situation at the two sites are very different. The Notviken site is on a heavy-haul route where the traffic consists almost entirely of iron-ore transports, where the wagons are now weighed before they are allowed to traffic the route, although this was not the case of all the wagons at the time of the data collection.

The traffic at Sannahed is of a mixed freight and passenger type and deemed to be more representative of normal traffic conditions, for this reason the main efforts of this thesis has been directed towards this site. The site is on a straight stretch of line where the trains are able to achieve reasonable speeds, at least when considering freight traffic. Speeds of 150 km/h were noted on this line and the average speed of the over 7400 trains analysed was 102 km/h. The allowable axle load at the site was 22.5 tonnes during the analysed measuring period of January–April 2001.

Information about the measuring methods are presented in the next section and is a similar method to that used for measuring wheel weights presented in (Tobias et al., 1996).

It was not possible to receive direct untreated data from the sites and the information available was in the form of text files. The text files contained information on the loading of each axle, the speed of the train and notations for the locomotive and wagon types. From these text files the axle loads were read into a Matlab matrix using a computer program written by the author. The axle spacings were reproduced by first reading in the locomotives and wagon notations and then via a
library of wagon types, their axle spacings and length over buffers, it was possible to recreate the axle spacings of the trains. This reading in of the data and the consequent building of the trains was again done automatically using a computer program written as a Matlab code.

Once the axle loads together with the relevant axle spacings and tags indicating the start and finish of a train had been read from the text files they were converted and stored as Matlab data files. The Matlab data files were then used to simulate the trains moving over hypothetical simply-supported bridges of differing span, described later in Section 7.2 of this chapter.

One noted problem with this method was that in certain cases the measuring system had not recognised a wagon or locomotive type. This was registered by the measuring system as e.g. 1 AXLES UNDEF or 4 AXLES UNDEF implying one respectively four axles were undefined. This phenomena raises question about the validity of the adjoining wagon not perhaps in the case of a two or four undefined axles as these could be a whole wagon but definitely in the case of a single undefined axle.

7.1.2 The WILD System

The system used for the collection of field data is a commercial system produced and marketed by Salient Systems, Inc. The product is designed for the detection of wheel flats, where WILD is an abbreviation for Wheel Impact Load Detector. The two sites used for the recording field data are not fully blown Weigh-in-Motion, WIM, sites although there are two such sites situated on the heavy-haul route described earlier. It was decided not to use the data from these WIM sites as the iron-ore wagons are now individually weighed and therefore not representative even of block
7.1. TRAIN DATA COLLECTION

Two gauges are placed on each side of each rail.

The WILD system is described briefly here but for a full description refer to (Salient Systems, Inc., 1996). The system consists of strain gauges welded to the neutral axis of the rail. Each gauge consist of two 350 ohm variable resistors that are positioned with a 45 degree orientation to the rails neutral axis, see Figure 7.2. Between two adjacent sleepers two gauges are welded on each side of a rail and on both rails making a total of eight gauges between adjacent sleepers. The gauges on each rail are welded to form a Wheatstone bridge the total resistance of which is 700 ohms (±5%) per leg. The length of each gauge is approximately 20 mm and they are welded to the rail using 100 welding points. Unfortunately the manufacturer of the strain gauges is unknown to the author. A crib is defined as the area between two adjacent gauges and there are ten cribs to each measuring station.

The sampling frequency of the system is 30 kHz and the front end processor has both high and low pass filters. The low filter is designed to produce a clear signal of the load from the axles while the high pass filter is designed to produce a good representation of the peak loading from the dynamic action of wheel irregularities. The peaks indicate wheel flats, when present, by comparing the peak and average reading of the loading from the axle.

The average reading is thus the most interesting from this studies point of view as it provides information about the loading from the axles hopefully with the dynamics due to the wheel motion and irregularities removed.
CHAPTER 7. TRAFFIC LOAD SIMULATIONS

Strain Gauges

Figure 7.3: A typical WILD site showing the wiring and the strain gauges welded onto the rails. Copied from Salient’s homepage with their permission.

7.1.3 Measuring Errors

Both the measuring stations are somewhat old, Notviken was installed in 1990 although it has received several upgrades since then and it was difficult to receive details of the accuracy of the measurements. An assessment of measuring errors was therefore attempted using known information about nominal axle weights of two types of locomotive, namely the DM3 and the Rc. According to (Tobias et al., 1996) this method has been used extensively by the Association of American Railroads and has proven to be quite accurate. However, assessment as to the accuracy of the axle spacings was not possible to determine as we did not have access to the raw data.

7.1.4 Rc and DM3 Locomotive Axle Weights

The Rc locomotive has four axles of known weight of 19.5 tonnes or 195 kN. Figure 7.4 shows a histogram of the axle load for the second of the four axles as measured at Notviken over a six-month period. The histogram shows a shift of the mean towards the right compared with the nominal value by a value of 23 kN. This would suggest a bias in the measuring device that overestimates the axle weights. The highest (426 kN) and lowest (96 kN) noted values were approximately twice and half the nominal values respectively. The latter measurement may well be from a freight wagon but with a axle spacing similar to that of an RC locomotive and hence mistaken for one by the measuring system. The coefficient of variation for this measured axle was approximately 7%.

Figure 7.5 shows a similar plot for the 14th axle of the DM3 locomotive. The
company that operate this line, MTAB, only have 16 DM3’s in their ownership, ideally the records should therefore only show 16 values. The spread of the axle weights was less in this case with a coefficient of variation of only 4.7 %. The chosen axle was not a driving axle but only a guide. Unfortunately it was not possible to get any accurate information as to the individual axle weights although the overall weight is stated as being 273 tonnes and the maximum axle weight as 20 tonnes, see (Diehl and Nilson, 2000). The first three and the last three axles in the measurement series all indicate lower axle loads than the middle eight axles. If one assumes all these eight axles to be 20 tonnes, this appears from the measured data to be a reasonable assumption, then this leaves $273 - 8 \cdot 20 = 113$ tonne to be spread over the remaining six axles. If one assumes further that these axles bear the same weight then the remaining six axles are approximately 19 tonnes, this assumption also seems reasonable judging from the collected data. The value of 19 tonnes complies with the median value in Figure 7.5. The maximum and minimum recorded axle weights were 252 and 176 kN respectively. This represents a 30 % increase and 7 % decrease on the nominal axle load using the 190 kN true value assumption. These values are much closer to the nominal values compared with the Rc locomotive case.

Figures 7.6 and 7.7 show scatter plots for the speed of the locomotive against the static axle weight as measured for the second axle on a Rc locomotive. The correlation coefficient $\rho$ was quite high at 0.491 indicating a correlation between the speed of the train and the static axle weight. This, of course, does not exist in reality and indicates the inability of the measuring system to filter what is presumably the
dynamic effects of the moving trains. One can also observe, especially from Figure 7.7, that there is a larger spread of the recorded axle weights above a speed of approximately 70 km/h.

Figure 7.8 shows a scatter plot for the speed of the locomotive against the axle weight as measured for the 14th axle on a DM3 locomotive. The correlation coefficient \( \rho \) in this case was only 0.104. However, one can also observe from this diagram that the speed of the train rarely exceeds 70 km/h. It would appear from the observations from Figures 7.6–7.8 that the measurement of the load from the axles becomes more unreliable with increased train speed. Indeed this is possibly the reason for the much smaller spread in results for the axle weight of the DM3 compared with the Rc locomotive, the DM3 having generally much lower speeds.

### 7.1.5 Axle Weights of Unloaded Iron-ore Wagons

It was suggested that problems are experienced when measuring the axle loads of locomotives, due to various problems involving acceleration of the driving axles amongst others. For this reason a study was also undertaken of the results of the unloaded iron-ore wagons denoted Faao. These wagons are four axle wagons consisting of two boogies. The nominal axle weight of these wagons due to self-weight is 5.55 tonnes or 54.5 kN. Figure 7.9 shows a histogram of the measured axle weights of the Faao wagon that were under 70 kN. The mean of these axle weights is 54.24 kN which is almost exactly the nominal load of 54.5 kN, thus indicating
7.1. TRAIN DATA COLLECTION

Figure 7.6: Scatter diagram to show the correlation between speed and axle weight of a Rc locomotive.

Figure 7.7: Scatter diagram to show the correlation between speed and axle weight of a Rc locomotive.
Almost negligible bias for these measurements. The coefficient of variation for these axles was approximately 5%.

A similar study was undertaken on the unloaded wagons registered from the measuring station at Sannahed. This time the four-axled wagon denoted Zacs was chosen. A total of 8711 Zacs bogies were registered by the station and of these some 579 were deemed to be empty. Figure 7.10 shows a normal probability plot for all the axles less than 68 kN which was the upper limit set for the definition of an empty wagon. The mean, standard deviation and coefficient of variation for the loads from
these axles were 58.6 kN, 4.3 kN and 7.3% respectively. This can be compared with the given empty weight of these wagons of 58.75 kN.

From this evaluation of the errors from the measuring station it may be concluded that the axle measurements of the locomotives are somewhat biased and tend to overestimate the actual loading from the axle. They are also dependent on the speed of the locomotive, which should not be the case. The studies done on the empty freight wagons, however, show negligible bias and a coefficient of variation between 5 and 7%. The assumption that the driving axles of the locomotives may act to distort the measurements of their axles may be well founded. An assumption from this study is that the measurements of non-driving axles have negligible bias and a coefficient of variation similar to the study of the empty wagons.

### 7.2 Moving Force Model

The model used to calculate the dynamic response of the bridges and the subsequent dynamic mid-span moment was the so-called moving force model. The program used was developed by Dr. Raid Karoumi, a colleague at the department, and was written as a sequence of Matlab program functions. The program is presented in the Ph.D. thesis (Karoumi, 1998) although this particular application only uses a small part of its capabilities, see (Karoumi, 1998) for an explanation of the program. The principle of modal superposition is utilised in this program to calculate the dynamic response of the beam. The damping is modelled as viscous damping.
The bridges were modelled as simply supported two dimensional beams using FEM beam elements. The beam was assumed to be of constant cross-section and stiffness. The chosen length of the beam elements was 0.2 m, which according to (Freda, 2001) provided adequate accuracy when used in a very similar application of trains crossing two dimensional bridges of small to medium span.

The beam is assumed to be at rest before the approach of each train, however as each successive axle arrives at the start of the beam the motion of the beam is that calculated for the appropriate time step. In this model no allowance was made for the effects of track defects and does not model interaction problems between the vehicle and the bridge. The dynamics of the bridge is due solely to the variation of the position of the forces on the bridge. Only the dynamic effects of a train crossing a bridge with a geometrically perfect track can be modelled by this method.

The trains are modelled as a succession of constant forces (axle loads) travelling at a constant speed. The collected field data was used as the source for the size of the forces, the spacing between the loads and the speed of the loads, which together comprise sets of complete train. In this simple model no allowance was made for the suspension systems of the locomotives or the wagons. Only single track bridges are considered and therefore only one train, or part thereof, is present on the bridge at any one time.

The dynamic and static internal forces are calculated, via the stiffness matrix of the beam, from the dynamic and static deflections for each time step. The data collected in these simulations were the mid-span dynamic and static moments.

This model was deemed sufficient as the purpose was not to investigate the dynamic amplification factor on railway bridges but to allow for these effects when assessing the variation of the traffic load effect. A complicated model was judged to increase the computer simulation time many fold especially for the large number of trains and axles considered in this analysis.

### 7.2.1 Number of Modes and Damping Effects

A comparison of Figures 7.11 and 7.12 show that their was very little difference in user time to go from including the first eight and the first eleven natural frequencies. However, there was an increased accuracy, although slight, and for this reason the greater number of frequencies were included. Work presented in (Freda, 2001) shows that there is a minimal increase in accuracy above this number of modes.

At first no damping was included in the modelling, however this produced results with resonance, or possibly numerical instabilities, which were markedly reduced when even small amounts of damping were included. Figures 7.13–7.15 show the mid-span moment response to the passage of an empty iron-ore train travelling at 102.1 km/h for the cases of the damping ratio, $\xi$, equal to 0, 0.02 and 0.05 respectively. The thicker and thinner lines represent the static and dynamic response respectively.
7.2. MOVING FORCE MODEL

Figure 7.11: Dynamic and Static Moment history diagram for the mid-span of a 10 metre bridge. The first eight modes are included in the analysis.

Figure 7.12: Dynamic and Static Moment history diagram for the mid-span of a 10 metre bridge. The first 11 modes are included in the analysis.
Figure 7.13: Static and dynamic moment at mid-span vs. time for a ten metre bridge. The damping ratio used was $\xi = 0.0$. The bridge is traversed by an empty iron-ore train travelling at 102 km/h.

Figure 7.14: Static and dynamic moment at mid-span vs. time for a ten metre bridge. The same train is used for simulation as in Figure 7.13 but with a damping ratio, $\xi = 0.02$. 
The final damping ratio chosen and used constantly throughout the simulations was 0.02. This was seen as a compromise between typical values for steel and concrete bridges of 0.013 and 0.028, respectively which are taken from (Frýba, 1996).

### 7.2.2 The Chosen Time Increment

Figures 7.16–7.17 show the same train but with a decreasing choice of time increment. The top halves of the figures show the passage of the entire train while the lower halves shows the locomotive and the first few wagons. The time increment used is detailed in the figures along with the calculated maximum dynamic moment. The time increment used for the figures was approximately 7 ms and 1.7 ms, respectively. The calculated maximum dynamic moment changed only slightly between figures from 925.3 to 929.9 kNm respectively. This represents an increase in accuracy of 0.4% while increasing computer time more than four fold. An intermediate stage with a time increment at 3.4 ms was also calculated although it was so similar to the 1.7 ms plot that it was deemed superfluous to show here.

The trace of the dynamic moment distribution for all three results are shown in Figure 7.18. The figure shows the area around the passage of the locomotive which gave rise to the largest discrepancies between results. A comparison was also made for the static moments, however these were so similar it was deemed meaningless to show them in a diagram. From the evidence of this small study it was deemed sufficiently accurate to use a time increment not exceeding 7 ms.
CHAPTER 7. TRAFFIC LOAD SIMULATIONS

Figure 7.16: Mid-span moment time history for train 466 from Notviken. The maximum dynamic moment at mid-span was 925.3 kNm. The time increment is approximately 7 ms.

Figure 7.17: Mid-span moment time history for train 466 from Notviken. The maximum dynamic moment at mid-span was 929.9 kNm. The time increment is approximately 1.7 ms.
7.3 Turning Points

The results from the simulations of trains passing a two dimensional bridge produce a substantial amount of data. A method used to limit this amount but at the same time retain much of the information is to store or convert the signal in the form of turning points. The method of obtaining these turning points was to use a Matlab routine from the Wafo toolbox (Brodtkorb et al., 2000) called dat2tp which converts the original data, in the form of a moment history diagram, to selected points of key interest, namely the turning points. The turning points are the points where the slope of the moment history diagram changes from positive to negative or vice versa. The routine also allows for a tolerance level to be set such that cycles of smaller amplitude than this tolerance level are not included and their turning points are hence neglected. Figure 7.19 illustrates the use of the turning points and the tolerance level. In this figure the continuous line is the original moment history diagram. The x-axis only show the index of the moment vector, however, this is related to a chosen time interval. The crosses in the diagram show all the turning points where the tolerance level is set to zero. The squares, however, show the turning points where a tolerance level of 0.016 was chosen. Hence, only those turning points that are related to cycle amplitudes greater than this tolerance are included. The index of the time increment can be retained in this function and therefore a reasonable good approximation of the key points of interest of the signal.
7.4 Allowance for Track Defects

The moving force model does not allow for the dynamic coefficient due to the track defects. Unfortunately the author has been unable to find any literature that suggests how to incorporate this phenomena into a reliability analysis. The dynamic coefficient due to the track defects, $\varphi''$, was therefore taken into account by adjusting the results of the moving force model. This dynamic coefficient was estimated using the expression in (UIC, 1979) which can also be found in (CEN, 1995) and is reproduced in Appendix B, see this appendix for the calculation of $\varphi''$. It is assumed that the track quality is of a high standard which allows the lower value to be used. The dynamic factor due to allowance for the effect of track defects will therefore be given by:

$$\Phi_{td} = 1 + 0.5\varphi''$$  \hspace{1cm} (7.1)

The speed of each train was recorded by the WILD system and the first bending frequency was either taken from the 2D bridge model used in the moving force model or the upper frequency of Appendix B, see (B.9), which yields the higher of the two possible values of $\varphi''$. 

![Figure 7.19: Illustration of the use of turning points to condense the information of the moment distribution. The crosses denote all the turning points i.e. using a tolerance of zero. The squares only show the turning points associated with a cycle amplitude of, at least, 0.016. This reduces the data from over 900 points to just 19.](image)
Chapter 8

Results

8.1 General

This chapter shows the results of the simulations, the distribution fitting and the reliability analysis. Descriptions of the methodology and assumptions are given in earlier chapters. However, this introduction briefly recalls some of the main assumptions in order to clarify the presentation of the results.

The track defects were not taken into consideration in the simulations of the trains crossing hypothetical bridges and only the dynamic effects due to constant moving forces were considered. The obtained moments were therefore adjusted to allow for the effects of track defects. This was done using the UIC recommendations, see section 7.4, and was carried out using two different first bending frequency values for each bridge. The first being the same as the bending frequency used during simulation and the other being the upper frequency of the UIC leaflet (UIC, 1979). These three variations will therefore be presented as no defect, defect using actual frequency and defect using UIC upper frequency in the following tables and figures.

The results are also presented for different load models, those using extreme value theory and those using a Peaks-Over-Threshold method, see Chapter 5. Within the EVT model, the GEV distribution is often used as the distribution for modelling the load effect. The special case of the GEV, the Gumbel distribution, is also used for modelling the load effect and these distinctions should hopefully be clear within the the context of the text. In the case of the POT method, the GPD is used to model the load effect and to calculate return loads. However, in order to perform a reliability analysis using the POT method, the GPD has been converted to a GEV using (5.34–5.36). Although this method ultimately uses the GEV in the reliability analysis it will be referred to as the POT or the GPD method in order to distinguish it from the method using classical extreme value theory.

In the chapter discussing reliability theory and modelling, two assumptions were made as to the modelling of the resistance of the material. The first being that the resistance model is conservatively chosen and represents the 5% quantile of the real value of resistance, while the other assumes that the resistance model represents the
mean value of resistance. These two different assumptions lead respectively to (4.54) and (4.56) which are used to estimate the mean value of resistance. As an example, the simplified stress block in reinforced concrete beams is used to model the concrete resistance, now is this model conservatively chosen and if so which quantile does it represent? These are referred to in the following sections as the 5% quantile and the mean value assumption for the resistance model.

All the results presented in this chapter are for a 3.5 month period and for the mixed freight and passenger line at Sannahed close to a marshalling yard called Hallsberg, see section 7.1.1.

The load effects studied in this thesis are the mid-span moments of simply supported bridges. In the presentation, all the moments including the return loads have been divided by the characteristic moment including the dynamic factor $M_{uic71\Phi}$ but excluding the partial safety for traffic load, $\gamma_q$, see section 3.5.5.

The self-weight ratio $\nu$ is the load effect of the self-weight of the bridge from the original design expressed as a ratio of the characteristic load effect of the UIC 71 or SW/2 load model including the dynamic amplification factor, see section 4.9.

In the following sections the results are often calculated showing two values for each span, e.g. in the reliability analysis and for the return loads. These two results are derived from the range of parameter estimates for the GEV and GPD models. The parameter estimates used for the different calculations are the original estimates obtained directly from the data using one of the parameter estimation methods i.e. $\hat{\xi}$, $\hat{\sigma}$, $\hat{\mu}$. The other set of parameters are those that produced the 95% quantile of the 50 year return load using the covariance-variance matrices from the original parameter estimates, see section 5.3.5 and section 5.4.5. These parameter estimates, or the calculations deriving from them, have been referred to as the 95% quantile value, although strictly speaking these are not 95% quantiles of the value itself but of that defined above. The parameter estimates in these cases are denoted $\hat{\xi}_{0.95}$, $\hat{\sigma}_{0.95}$, $\hat{\mu}_{0.95}$.

## 8.2 General Results

In this section general results are detailed such as axle loads, values of the dynamic factor due to track defects, etc, which can be of assistance in judging results detailed later and also of interest for future work.

### 8.2.1 Axle Loads

Over 300 thousand axle loads were registered in the 3.5 months of records analysed in this thesis and collected from the measuring station at Sannahed, see section 7.1.1 for a description of the site and its traffic.

Figure 8.1 shows the distribution of the measured axle loads with three main peaks
8.2. GENERAL RESULTS

Figure 8.1: A histogram of the observed axle loads collected at Sannahed over a period of 3.5 months. Median values of the peaks are indicated in the diagram. The allowable axle load was 22.5 tonnes. Three clear peaks are apparent. In ascending order these are presumably, empty wagons, half-full wagons and finally a mixture of locomotives and fully loaded wagons.

Figure 8.2: A normal distribution plot for the axles that exceed 150 kN. The straight line indicates that a normal distribution is a good representation of the data. The vertical line at 150 kN is the truncation point. Also shown are the estimated mean and standard deviation of the approximated normal distribution.
Table 8.1: Properties of the measured axle loads from Sannahed, including mean, standard deviation and number in each category. The categories approximately represent axle loads from empty wagons, half-full wagons and a mixture of locomotive and fully loaded wagons.

<table>
<thead>
<tr>
<th>Axle Load $P$ (kN)</th>
<th>Number in Interval</th>
<th>Mean Value (kN)</th>
<th>Standard Deviation (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq P &lt; 100$</td>
<td>59137</td>
<td>72.1</td>
<td>12.7</td>
</tr>
<tr>
<td>$100 \leq P &lt; 150$</td>
<td>98918</td>
<td>122.2</td>
<td>9.9</td>
</tr>
<tr>
<td>$P \geq 150$</td>
<td>154490</td>
<td>199.1</td>
<td>18.7</td>
</tr>
</tbody>
</table>

Table 8.2: Dynamic amplification factor $(1 + 0.5\varphi'')$ using the actual frequency used in the simulations. The standard deviations are small and the values are near deterministic in the majority of cases.

<table>
<thead>
<tr>
<th>Span $L$ (m)</th>
<th>4</th>
<th>8</th>
<th>10</th>
<th>13</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median of $(1 + 0.5\varphi'')$</td>
<td>1.72</td>
<td>1.46</td>
<td>1.3</td>
<td>1.16</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Std. dev. of $(1 + 0.5\varphi'')$</td>
<td>0.086</td>
<td>0.056</td>
<td>0.036</td>
<td>0.019</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

clearly visible from the histogram. The maximum observed value was 306 kN which, if correct, represents an overloading of 36%. Approximations were made of the three peaks splitting them into ranges between 0–100, 100–150 and over 150 kN. These three peaks were well represented by truncated normal distributions as can be seen from the normal probability plot of Figure 8.2 for the example of the axles in excess of 150 kN. The number of axles falling in the aforementioned groups are shown in Table 8.1 together with the values of the means and standard deviations. The highest peak includes axles from both locomotives and loaded wagons. The weights of the locomotives should be deterministic while the loaded weights of the freight wagons will obviously be stochastic.

### 8.2.2 Dynamic Amplification Factor

As described in section 7.4 the allowance for the dynamic effects due to track defects was made after the simulations. Table 8.2 shows the properties of the calculated dynamic factor using the same bridge frequencies as used in the simulations. As can be seen from this table the dynamic effect $(1 + 0.5\varphi'')$ calculated in this manner produces almost deterministic values. This is due to a combination of the speeds of the trains that were registered at this site and the limiting upper value of $\varphi''$ for train speeds greater than 79.2 km/h (22 m/s), when calculated in accordance with Appendix B. Only approximately 1600 of the 7400 measured trains had speeds less than this limiting upper value, i.e. the vast majority of the dynamic factors due to the track defect were calculated to be this upper limit and therefore resulted in a near deterministic value for this quantity. Similar deterministic results were obtained using the upper frequency of the UIC recommendations, although different numerical values were obtained than those shown in Table 8.2.
8.3 Distribution Fitting and Return Loads

8.3.1 General

In this section the results of the model fitting and the resulting return loads are presented. See chapter 5 for a discussion on parameter estimates and return loads and the methods used to estimate their associated uncertainties.

The 50 year return loads, denoted $S_{50}$, were estimated under the assumptions of the GEV, the Gumbel and the POT models using the respective estimated parameters. Returns loads related to other return periods were also calculated.

8.3.2 GEV Method

Figure 8.3 shows the results of the maximum moment ratio per train for a 10 m simply supported bridge when no allowance for the dynamics of track defects has been considered. As can be seen from this figure there are several peaks in this histogram representing different types of traffic such as passenger, mixed freight and block trains. There were over 7400 successful notations of trains during this 3.5 months measuring period and approximately 100 thousand wagons, coaches and locomotives.
CHAPTER 8. RESULTS

Figure 8.4: Histogram of the maximum moment ratio per 50 train for the case of a 10 m simply supported bridge from the simulations using measured data from Sannahed. Allowance for track defects are included using actual frequencies. The estimated GEV and Gumbel distributions are shown in the figure. Maximum values are noted at approximately 78% of the characteristic load effect.

Figure 8.4 shows the histogram of the maximum moment ratio from 50 consecutive trains together with the estimated pdf’s of the GEV and the Gumbel distributions for the case where the dynamic effects due to track defects is taken into account. A comparison between Figure 8.3 and Figure 8.4 show a shift in the final peak from approximately 0.5 to 0.65 respectively. This shift is due to the allowance for track defect dynamics and is approximately its median value of 1.3 for a 10 m span, see Table 8.2. The number of 50 when splitting the trains into sets was chosen somewhat arbitrarily although it represents almost the daily number of trains recorded on this stretch of line and generally provided good fitting to the GEV model. The figure contains 148 data points, which is obtained from the original over 7400 trains split into sets of 50 consecutive trains noting the maximum moment ratio for each set. As can be seen from this figure the ML parameter estimates for the two distributions gave very similar results. The parameter estimate for the GEV gave the shape parameter, $\xi$, very close to zero with a value of 0.0403 and is therefore the reason why the two models differ only slightly.

Figure 8.5 shows the results of the simulations for the case of a 8 m bridge, adopting the dynamic effects from track defects using the actual frequency. Again the histogram of the maximum moment per train shows several peaks. The sub-figures (b)–(d) of this figure all show how the theoretical estimated model suit the data. Both the probability plot and the QQ plot show good agreement between the data and the two models, both are straight lines close to the straight line passing through (0,0) and (1,1), this line is shown dashed in the sub-figures. For this span, the Gum-
8.3. DISTRIBUTION FITTING AND RETURN LOADS

(a) Max. moment ratio per train

(b) Max. moment ratio per 50 trains

(c) Probability Plot

(d) QQ Plot

Figure 8.5: Results for an eight metre span bridge. The results include the dynamic effects from track defects using the actual frequency. Both the probability plot and the quantile plot (QQ plot) show good agreement between the data and the GEV model. The GEV and the Gumbel models produce similar results as the estimated shape parameter of the GEV was close to zero, i.e. the special case of the Gumbel distribution.

In the analysis when no allowance was made for the track defects, the maximum value for the 4 m bridge was omitted in this analysis as it was abnormally high compared with the other results for this bridge span and was believed to be an erroneous measurement. While this axle showed no signs of being strange, a closely adjoining axle was noted as having a wheel defect by the measuring station. This may then have an affect on this measured axle. This single point produced a moment
Table 8.3: Estimated properties of the 50 year return load and the parameter estimates of the GEV and the Gumbel models for different spans. No allowance for the dynamic effects due to track defects. Results are based on the maximum of sets of 50 consecutive trains.

<table>
<thead>
<tr>
<th>$L$</th>
<th>Distribution</th>
<th>Estimated 50 year return loads</th>
<th>Parameter estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>4</td>
<td>GEV</td>
<td>0.6087</td>
<td>0.0027</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.7792</td>
<td>0.0162</td>
</tr>
<tr>
<td>8</td>
<td>GEV</td>
<td>0.5699</td>
<td>0.0165</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.6717</td>
<td>0.0127</td>
</tr>
<tr>
<td>10</td>
<td>GEV</td>
<td>0.6571</td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.6880</td>
<td>0.0126</td>
</tr>
<tr>
<td>13</td>
<td>GEV</td>
<td>0.6267</td>
<td>0.0036</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.7094</td>
<td>0.0133</td>
</tr>
<tr>
<td>20</td>
<td>GEV</td>
<td>0.6913</td>
<td>0.0125</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.7485</td>
<td>0.0198</td>
</tr>
<tr>
<td>30</td>
<td>GEV</td>
<td>0.6125</td>
<td>0.0059</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.7466</td>
<td>0.0200</td>
</tr>
</tbody>
</table>

*Sets of 70 trains were used for this span.*

Note that the result for the 13 m bridge are based on the maximum moment of 70 trains as the estimates based on the 50 train maxima gave poor curve fitting to the extreme value distributions. In the case of the 4 m bridge the curve fitting to the distributions was poor. This was mainly due to the three highest values which can clearly be seen in the QQ plot for this span as the three points in the right-hand part of the plot, see Figure C.1. It can be seen from this plot how these three points depart from that of the estimated theoretical GEV model. An inspection of all the sub-figures of Figure C.1 show that in general the GEV and Gumbel models are well suited to the data, with perhaps the exception of the 10 m span and the previously mentioned 4 m span.

Table 8.3 shows the 50 year return loads as a fraction of the characteristic moment. The variation of the 50 year return load are simulated from the variance-covariance matrix of the parameter estimates, see section 5.3.5. As we see from the table the means range from between 60–75% of the characteristic value of the 1980 model including the dynamic factor, $M_{\text{ic71}}$. However, the values shown in Table 8.3 do not include any allowance for the dynamic effect due to track defects. According
to the UIC formula, these will be small to negligible for the 20 and 30 m spans but become increasingly important with decreasing span. The table also shows that there was very little variation in the value of the 50 year return load. The restriction to the Gumbel family of distributions produced consistently higher return loads. This is a result of the GEV approximation yielding consistently Weibull estimations i.e. $\xi < 0$. This implies an upper limit to the distributions under the GEV assumption while the under the Gumbel assumption no such upper limit exists and hence the higher values.

The results of the parameter estimates and the return loads for the dynamic effects due to track irregularities calculated using the actual bridge frequency of the UIC recommendations (UIC, 1979; CEN, 1995) are shown in Table 8.4. Again the simulated moments from the 4 m span bridge provided a poor fit to the GEV model and it is questionable if this model should be used for these small spans. Again the parameter estimates are based on the maximum moment of 50 train passages.

In the case of the 10 m span there were several maxima that had low values, this was true for all of the GEV analyses. This created a situation where the majority of the results were in excess of 0.6 while a few gave results around 0.45, these values apply to the case of track defect effects calculated using actual frequencies. These low values were clearly a part of the main distribution and did not belong to the tail area. They also created problems when attempting to fit the extreme values distributions to them. To avoid this problem the low values were treated as not belonging to the

Table 8.4: Estimated properties of the 50 year return load and the parameter estimates of the GEV and the Gumbel models for different spans. Estimated properties for different spans based on the maximum of 50 trains.

Dynamic effect from track defects calculated using **actual bridge frequencies**.

<table>
<thead>
<tr>
<th>$L$</th>
<th>Distrib.</th>
<th>Estimated 50 year return loads</th>
<th>Parameter estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>4</td>
<td>GEV</td>
<td>1.1742</td>
<td>0.0118</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>1.3066</td>
<td>0.0274</td>
</tr>
<tr>
<td>8</td>
<td>GEV</td>
<td>0.8924</td>
<td>0.0093</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.9704</td>
<td>0.0188</td>
</tr>
<tr>
<td>10</td>
<td>GEV</td>
<td>0.8474</td>
<td>0.0101</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.9027</td>
<td>0.0175</td>
</tr>
<tr>
<td>13</td>
<td>GEV</td>
<td>0.7187</td>
<td>0.0038</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.8506</td>
<td>0.0161</td>
</tr>
<tr>
<td>20</td>
<td>GEV</td>
<td>0.7460</td>
<td>0.0862</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.7356</td>
<td>0.0184</td>
</tr>
<tr>
<td>30</td>
<td>GEV</td>
<td>0.6169</td>
<td>0.0059</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.7524</td>
<td>0.0198</td>
</tr>
</tbody>
</table>
CHAPTER 8. RESULTS

extreme value distribution and therefore ignored in the extreme value distribution fitting. It was also attempted to increase the number of consecutive trains from which to take the maximum. However, a few of these values persisted in turning up in the analysis, while at the same time the overall number of points on which to estimate the parameters of the GEV reduced. For this reason the above approach was adopted.

Note the choice of first natural bending frequency has a very substantial effect on the calculation of $\varphi''$. The 4–13 m bridges chosen for this study were all above the UIC frequency limits shown e.g. in (CEN, 1995; UIC, 1979). These high values produce very high track irregularity effects. Strictly speaking the use of the UIC formula is not justified outside the range of the UIC upper and lower frequency limits.

The results of the parameter estimates and the return loads for the dynamic effects due to track irregularities calculated using the upper bridge frequency of the UIC recommendations (UIC, 1979; CEN, 1995) are shown in Table C.1 of Appendix C. In general the results have a similar format to those shown previously and are therefore not shown here, although there is a shift of scale due to the different values obtained for the dynamic effects of track irregularities. For the 13 m case the GEV was the better fit although there were indications from the generalised pareto plot that the distribution belonged to the Gumbel domain of attraction. The estimations were therefore carried out for the maximum of a larger group of trains, i.e. increasing from the maximum moment of 50 to 80 trains. The results of the distribution parameter estimates combined with the return loads are indicated by the suffix 80 in the case of the 13 m span, see the mentioned table.

The choice of the number of trains that are incorporated in a group of which the maximum moment is taken has an effect on the estimation of the return load. This can clearly be seen from the two values obtained from the case of the 13 m span. Ideally i.e. asymptotically, this should not have an effect on the obtained result, but in practice the number of observations from which the extreme is taken is limited and not infinite. The distribution will therefore only be an extreme value distribution in the limiting case as $n$ tends to infinity. The choice of the number of trains will therefore have an effect on the outcome of the parameter estimates and consequently the extrapolation to obtain high quantiles, i.e. in this case the 50 year return load. For a discussion on the domain of attraction see (Castillo, 1987; Caers and Maes, 1998).

8.3.3 GPD Method

In the POT method only the dynamic effects using the actual bridge bending frequencies used in the modelling, have been analysed. This method of accounting for the defects was chosen as it provides a consistent analysis method even if it is not more correct than the other two methods. Also the intention was to compare the results using the GEV and the GPD method and therefore only one analysis was necessary.
8.4. COMPARISON BETWEEN GEV AND POT RESULTS

Table 8.5: Results using of the return loads and parameter estimates of the GPD using the peak-over-threshold approach. Dynamic effects due to track defects using the actual frequencies used in the simulations.

<table>
<thead>
<tr>
<th>Span</th>
<th>Threshold</th>
<th>Prop. 50-year return load</th>
<th>Simulated 50 year return load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean(\mu(S))</td>
<td>Std. Dev.(\sigma(S))</td>
</tr>
<tr>
<td>4</td>
<td>0.8770</td>
<td>1.0340</td>
<td>0.0091</td>
</tr>
<tr>
<td>8</td>
<td>0.6980</td>
<td>0.9038</td>
<td>0.0408</td>
</tr>
<tr>
<td>10</td>
<td>0.6570</td>
<td>0.9745</td>
<td>0.1030</td>
</tr>
<tr>
<td>13</td>
<td>0.6110</td>
<td>0.7082</td>
<td>0.0108</td>
</tr>
<tr>
<td>20</td>
<td>0.4580</td>
<td>0.7233</td>
<td>0.0453</td>
</tr>
<tr>
<td>30</td>
<td>0.4400</td>
<td>0.5875</td>
<td>0.0104</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Span</th>
<th>Parameter Estimates of the GPD</th>
<th>Estimation Method</th>
<th>KS test (Q_{KS})</th>
<th>Number (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>(-0.2392) (\xi_{0.50}) (\xi_{0.95})</td>
<td>mom (^a)</td>
<td>0.7975</td>
<td>2083</td>
</tr>
<tr>
<td>8</td>
<td>0.0011 (\xi_{0.50}) (\xi_{0.95})</td>
<td>mom</td>
<td>0.9728</td>
<td>645</td>
</tr>
<tr>
<td>10</td>
<td>0.0823 (\xi_{0.50}) (\xi_{0.95})</td>
<td>mom</td>
<td>0.8561</td>
<td>471</td>
</tr>
<tr>
<td>13</td>
<td>(-0.1473) (\xi_{0.50}) (\xi_{0.95})</td>
<td>mom</td>
<td>0.8760</td>
<td>902</td>
</tr>
<tr>
<td>20</td>
<td>(-0.0406) (\xi_{0.50}) (\xi_{0.95})</td>
<td>mom</td>
<td>0.9733</td>
<td>695</td>
</tr>
<tr>
<td>30</td>
<td>(-0.1712) (\xi_{0.50}) (\xi_{0.95})</td>
<td>ml (^b)</td>
<td>0.2157</td>
<td>1260</td>
</tr>
</tbody>
</table>

\(^a\)mom is an abbreviation for Method of Moments

An example of the method used to choose the threshold level has been shown in section 5.5 and is therefore not repeated here.

The fitting of the data to the GPD model was generally good, see Figure C.2 in Appendix C. The exceptions were the 4 m and the 30 m spans where the last data points showed a higher rate of occurrence than predicted by the estimated GPD models. It is possible to argue that these few points are most likely erroneous measurements, however it may also be argued that these represent the true behaviour in the tail area.

Table 8.5 show the results of the estimated 50-year return loads together with the parameter estimates for the GPD. Also shown in the table are the 95\% quantile estimates for these quantities.

8.4 Comparison between GEV and POT results

In this section, a comparison has been made between the results of the return loads and the estimate of the all important shape parameter, \(\xi\), for the two model types
of the GEV and the GPD. For this purpose, the results for the dynamic effects using the simulations with the actual bridge frequencies have been compared. Tables 8.4 and 8.5, of the GEV and the GPD models, respectively. A large number of both the GEV and the GPD models yield a shape parameter close to zero, i.e. a Gumbel tail. The exceptions are the 13 and 30 m spans for the GEV case and the 4, 13 and 30 m spans for the GPD case. Both the GPD and the GEV models yield evidence of a Weibull tail for the 13 and the 30 m spans, while they produce conflicting results for the 4 m span. For the 4 m span, the GEV and the GPD models suggest a near Gumbel tail and a distinctly Weibull tail respectively. Generally the 50-year return loads produce comparable results between the two models. The Gumbel tail assumption of the GEV model produces, in all but the case of a 10 m span, conservative results as regards the 50-year return load when compared to the GEV and the GPD models. From this fact together with the results suggesting a near Gumbel tail from the GEV and the GPD parameter estimates, it may be argued that the Gumbel model will provide an adequate model for the traffic load effects for mid-span moments of railway bridges. The equivalent to the Gumbel model in the peaks-over-threshold approach being the exponential distribution, see (5.17) for the case of $\xi = 0$.

In Figure 8.6 the results of the return load versus the return period are shown for two spans. For both the GEV and the GPD two sets of parameters are used to produce these curves. These are the mean parameter estimates and the parameter estimates that yielded the 95% quantile of the 50-year return load. For the mean value of the parameter estimates the two models produce similar return loads, however there is a larger difference between the results using the estimates that produced the 95% quantile values of the 50 year return load.
8.5 Results of Reliability Analysis

In this part, the results of the reliability analysis using the traffic load model of the GEV and the GPD are presented. The assumptions of the reliability model together with the underlying theory are presented in Chapter 4. For the majority of the cases considered here, the resistance model is assumed to be chosen conservatively and hence represents the 5% quantile, unless otherwise stated, see section 4.8 and the discussion in conjunction with (4.54). All the calculations have been carried out with the results of the traffic load effects using the bridge bending frequencies used in the simulations in order to calculate the dynamic effects due to track defects.

All the figures shown in this section show the results of the calculated safety index $\beta$ against the self-weight to nominal load effect ratio, $\nu$, see (4.62). The figures show the safety index for all the studied spans and for the three materials studied, i.e. concrete, reinforcement steel and constructional steel. The calculated safety index is the daily safety index and can be compared to a target daily safety index of 5.88, which is shown in all the plots of this nature. This target daily safety index is equivalent to the yearly target safety index of 4.75 recommended in (CEN, 1996; NKB, 1987; JCSS, 2001). Calculated safety indices greater than the target safety index indicate results on the safe side.

The intended increase of the allowable axle load is indicated in the caption to each figure. Generally two lines have been drawn for each year, one of these is calculated using the original parameter estimates and produces the higher safety indices. The other is calculated using the parameters that yielded the 95% quantile of the 50 year return load and therefore produce the lower safety indices. Sometimes these values coincided and are therefore difficult to distinguish.

8.5.1 GEV Model

Figures 8.7–8.9 show the results of the reliability analysis under the assumption of the GEV traffic load model and an intended increase to a 25 tonne axle load.

The first two figures relate to reinforced concrete and as can be observed, the results show that bridges built to the 1980 and 1960 codes of practice will tolerate an increase to a 25 tonne allowable axle load. The only exception is the 4 m span, which even for some of the bridges built to the 1980 standards lack the reserves to tolerate an increase. It is believed that this is because these short span bridges are adversely affected by heavily overloaded single axles and bogies. The highest moment ratio calculated for this span was due to a two axle-bogie with each axle load producing a load of 294 kN (approx. 29.4 tonnes), which can be compared with the allowable of 22.5 tonnes.

In the JCSS publication a yearly target safety index of 3.7 is recommended for structures where the consequence of failure is large but the relative cost of increasing the safety level is high. This definition is, in most cases, applicable to existing
Figure 8.7: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for concrete assuming an increase to 25 tonne axle using the assumption of a GEV traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
8.5. RESULTS OF RELIABILITY ANALYSIS

Figure 8.8: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for re-bars assuming an increase to 25 tonne axle using the assumption of a GEV traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
CHAPTER 8. RESULTS

Figure 8.9: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for steel bridges assuming an increase to 25 tonne axle using the assumption of a GEV traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
railway bridges especially if the alternative is strengthening or replacement of major elements of the bridge. The equivalent daily target safety index to this yearly value of 3.7 is approximately 5.0. Using this definition even the bridges designed to the standards of 1940 would be capable of withstanding the increase to 25 tonnes. However, caution should be advised here, as the JCSS use a different recommendations for the properties of the material strength, model uncertainties, etc. when calculating the reliability index. This will result in different beta values than the ones calculated here. However, the target safety indices of the NKB (NKB, 1987) are intended for the design of new structures and a lower safety level, similar to the JCSS recommendations, may well be acceptable for existing structures.

It can also be observed from these figures that, for the majority of cases, it was the concrete and not the reinforcement steel that was the determining factor when deciding the load bearing capacity of the bridges. The only exception was the 20 m span bridge using the 95\% quantiles of the parameter estimates. Generally, the calculated values of $\beta$ were unaffected by the choice of the mean contra the 95\% quantiles of the parameter estimates, for all three materials. However, for the 20 m span there was a large variation between the outcomes using the different parameter estimates. The reason for this variation, in this case, is possibly caused from the shape parameter changing from negative to positive, i.e. from a Weibull to a Fréchet type tail.

In the case of constructional steel, see Figure 8.9, the results show that for all the studied spans and all the studied codes of practice, an increase to 25 tonnes should be allowable within the required safety margins. This was even noted for the structures built at the beginning of the 1900’s. However, it should be stressed that first, the load model used for this year assumes the heavier loading of Load Model A see Figure 3.1 and secondly that the assumed manufacturing tolerances, coefficient of variation of material strength and other parameters used in the reliability design are the same for these older steel structures as for more modern steel structures constructed and manufactured using latter day techniques, which is of course an over simplification.

### 8.5.2 GPD Model

In this section the results of the reliability analysis, under the assumption of a GPD model, are detailed.

Figure 8.10 shows the result for the case of the concrete in a reinforced concrete structure assuming an increase to an allowable axle load of 25 tonnes. Comparison between this figure and Figure 8.7 which is the equivalent figure under the GEV model assumption show that the results are comparable with the exceptions where one of the models had a heavy tail, i.e. for the GEV model with a span of 20 m and for the GPD a span of 10 m. The results for the GPD model for the other materials are very similar to that already shown under the GEV model assumption and for that reason they are not shown here. The interested reader is referred to Appendix C, Figures C.3 and C.4.
Figure 8.10: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for concrete assuming an increase to 25 tonne axle using the assumption of a GPD traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
A more comprehensive comparison between the results of the two models is presented in the section 8.5.3.

### 8.5.3 Comparison of the GPD and the GEV Reliability Results

This section compares the results of the reliability analysis using the GEV and the GPD models for the load effect. Figures 8.11–8.12 show the results of the reliability analysis for the case of an assumed increase to 30 tonnes and for steel bridges. In the figures the GPD and the GEV models are shown on the left and the right hand side respectively. Also, as sub-captions to each sub-figure both the spans and the estimated parameters of the shape parameter are shown. Both the original estimation, $\hat{\xi}$, and the one producing the 95% quantile of the 50 year return load, $\hat{\xi}_{0.95}$, are given.

From Figure 8.11 it is possible to see that the results for the 4 m span bridge are essentially the same. For the 8 m span bridges the mean value estimates, i.e. the upper of the two sets of curves, produce approximately the same results, although the safety indices of the GPD are generally lower than those of the GEV. However, the GPD model produces a greater variation when comparing the values using the 95% quantile estimate values and again these values are lower than the comparable GEV results. The next span of 10 m produced the largest discrepancies between the two methods of all the spans considered, the GPD model producing much lower safety indices than the equivalent values under the GEV model assumption.

The quantile-quantile plots for this ten metre span and the other spans, under the GPD model, can be seen in Figure C.2. It can be observed from the QQplot of the ten metre span bridge, sub-figure (d), that the empirical quantiles imply a more Weibull tail than that predicted by the theoretical model, with the data in the right tail suggesting an upper limit at approximately 0.12 above the threshold of 0.657. While the theoretical model predicts that values above this level can be achieved. It is possible that another choice of threshold would yield a better description of the right tail behaviour, however this has not been investigated here. It can also be seen from this figure, but for the case of the four metre span, that the theoretical model badly represents the data especially in the last few data points. The theoretical model underestimating the quantile values when compared with the data.

It can be observed from Figure 8.11 and its sub-captions, that the discrepancies occur when one of the models predict a Fréchet type tail behaviour, i.e. $\hat{\xi} > 0$. This is the case for $\hat{\xi}_{0.95}$ of the eight metre span bridge and for both the parameter estimates of the ten metre span bridge under the GPD model assumption. This latter span yields the largest differences in results and also has the largest discrepancy between the estimates of the shape parameter. The GPD model predicts a Fréchet type tail behaviour with values of $\hat{\xi} > 0$, the value of $\hat{\xi}_{0.95}$ is large when compared with the parameter estimates for the other spans. This tail behaviour, as remarked earlier, approaches infinity slowly and therefore produces higher quantile values for a given
Figure 8.11: Comparison of the results for the GPD and the GEV models. The reliability index $\beta$ versus self-weight ratio $\nu$ see (4.62). Results for steel bridges of spans 4–10 m, assuming an increase to 30 tonne. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
8.5. RESULTS OF RELIABILITY ANALYSIS

Figure 8.12: Comparison of the results for the GPD and the GEV models. The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for steel bridges of spans 13–30 m, assuming an increase to 30 tonne. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.

(a) $L = 30 \text{ m } \hat{\xi} = -0.17$, $\hat{\xi}_{0.95} = -0.14$ GPD

(b) $L = 30 \text{ m } \hat{\xi} = -0.15$, $\hat{\xi}_{0.95} = -0.15$ GEV

(c) $L = 20 \text{ m } \hat{\xi} = -0.04$, $\hat{\xi}_{0.95} = 0.013$ GPD

(d) $L = 20 \text{ m } \hat{\xi} = -0.01$, $\hat{\xi}_{0.95} = 0.09$ GEV

(e) $L = 13 \text{ m } \hat{\xi} = -0.15$, $\hat{\xi}_{0.95} = -0.08$ GPD

(f) $L = 13 \text{ m } \hat{\xi} = -0.20$, $\hat{\xi}_{0.95} = -0.18$ GEV
probability than for a Gumbel or Weibull tail and therefore lower safety indices. The GEV model for the ten metre span bridge, on the contrary, predicts a Weibull type tail i.e. an upper limit to the traffic load effect. This significance of the shape parameter can also be observed for the eight metre span bridge, where again $\hat{\xi}_{0.95} > 0$ and produces significantly lower safety indices.

Figure 8.12 shows the comparison between the two models but for the larger of the studied spans. The GPD and the GEV models for the 13 and the 30 m span bridges provide very similar results, while for the 20 m span bridge the results are not too dissimilar, the major difference for this last span coming from the 95% quantile values. Both, the models produce an original estimate of the shape parameter that is slightly smaller than zero, while the 95% values both produce shape parameters that have slightly Fréchet tail behaviour. This is the first span where the GEV model predicts lower safety indices than the GPD model, for all other spans the GPD model yielding lower safety indices.

Figure 8.13 shows the comparison between the two models for the worst case, i.e. the ten metre span bridge. For the comparison the results using the 1901 standards and a 30 tonne axle load are shown in the figure for both the GPD and the GEV models. The figure shows the importance of the estimated shape parameter, a positive shape parameter producing lower safety indices than the negative estimate. Also the greater the value of the shape parameter the lower the resulting safety
8.6 Increases of Allowable Axle Loads

8.6.1 Concrete Bridges

The cases discussed here will generally be taken from the results of the GPD model, as these produced the most conservative results. However, the GEV model will be used for the 20 m span bridge as for this case the GEV model was the most conservative. Reference to the GEV cases should be apparent within the context of the text.

From Figure 8.10 it is possible to see that, for the majority of the spans and the building codes and design loads considered, the concrete would tolerate an increase to an allowable axle load of 25 tonnes. The exceptions to this rule are some of the 1940 span bridges and the four metre span. The bridges built to the 1940 standard with spans of eight metres or more, yield results close to or slightly less than the desired daily target safety level of 5.9 (equivalent yearly of 4.7). However, all of these do retain the target safety level of 5.0 (equivalent yearly of 3.7). For the four metre span bridge all the standards show values less than 5.9 but greater than the 5.0 level, with the exception of the 1940 standard which hovers around this last value. Bearing in mind that for this span the theoretical models generally underestimated the moment levels it is questionable whether, according to this analysis, they should tolerate this increase to 25 tonnes.

The results assuming an increase to 27.5 tonnes under the GPD model are detailed in Figure 8.14. From this figure it is possible to see that the bridges built to the 1980 and the 1960 standards should tolerate this increase except again in the case of the four metre span. The 1940 code bridges do not attain the 5.9 safety index but do achieve the 5.0 level. For the four metre span bridges all the standards fall below the 5.9 level, while both the 1980 and the 1960 standards achieve the 5.0 level. However, for the four metre span bridges built to the 1940 standards, the increase in allowable axle load will not be tolerated according to this analysis and even the bridges built to the 1960 standard are close the lower limit of acceptability.

Figure 8.15 show the results for the GPD model assuming an increase to 30 tonnes. The results are similar to that of the 27.5 tonnes, although the four metre span bridges built to the 1960 standards now fall below the required daily safety level of 5.0. Figure C.7 shows the equivalent figure but for the GEV model. As explained earlier it was the 20 m span bridges of this model that produced safety indices lower than the equivalent GPD model. From sub-figure (b) of this figure it is possible to see that even for the GEV model the above observations are true even if the obtained safety indices are lower.
Figure 8.14: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for concrete assuming an increase to 27.5 tonne axle using the assumption of a GPD traffic load effect. The mean and the 95% quantile values is shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
8.6. INCREASES OF ALLOWABLE AXLE LOADS

Figure 8.15: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for concrete assuming an increase to 30 tonne axle using the assumption of a GPD traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
8.6.2 Steel Bridges

Figures 8.9 and C.4 show the results of the reliability analysis for steel bridges and a proposed allowable axle load of 25 tonnes for the GEV and the GPD model respectively. The GPD model shows that an increase would be acceptable for the majority of cases and spans. The exception to this is the span of 10 m, where the bridges built to the 1901 code lack reserves. This is even true in some cases for the lower target safety factor of 5.0, when the 95% quantile values are used for the parameter estimates of the GPD. For this span, in certain cases even the bridges built to the 1980 and 1940 codes don’t obtain the 5.9 target safety index. For all of these cases the shape parameter was positive implying a heavy tail. For the GEV model, Figure 8.9, the results also show that generally the increase would be permissible. The exception, according to this model is the 20 m span bridge, where some of the cases of bridges built to the 1901, 1940 and 1980 fail to reach the 5.9 level but are greater than the lower 5.0 level. Again these cases occur when using the parameter estimates that produced the 95% quantile of the 50 year return load and again these coincided with positive values of the shape parameter. It is interesting to note that the bridges built to the 1960 code appear to be safer than those built to the later 1980 code.

Figures 8.11 and 8.12 show the results of the reliability analysis for both the GPD and the GEV models and for a raise in allowable axle load to 30 tonnes. For the majority of the spans and the standards studied the bridges will tolerate an increase to a 30 tonne allowable axle load. The exceptions are the four metre span bridges and most of the 95% quantile values of the ten metre span bridges. The case of the ten metre span bridge shows the vulnerability of the analysis to a higher positive value of the shape parameter.

8.7 The Effect of Some Studied Parameters

8.7.1 Increase in Allowable Axle Load

In this short subsection the effect of raising the allowable axle load has been isolated and studied. Figure 8.16 shows the effect of raising the allowable axle load to 25, 27.5 and 30 tonnes for the case of a ten metre concrete span bridge using the assumption of the GPD load model. The studied standard in this case is the 1940 standard. As can be seen from the figure the increase causes a parallel displacement of the curves in the vertical direction, an increase in axle load decreasing the safety index.

8.7.2 Assumption of the Model Uncertainty

In this section the assumption behind the resistance model uncertainty is changed to see its impact on the reliability analysis. All the previous calculations have assumed
Figure 8.16: The figure shows the effect of increasing the allowable axle load using the GPD model. Shown are the results for 1940, a span of 10 m, a concrete material and for the three cases of 25, 27.5 and 30 tonnes. The increase causes an almost parallel displacement in the vertical axis, representing a decrease in safety.

that the engineering models used to describe the resistance of a section have been chosen conservatively and represents the 5% quantile, see section 4.8. Another alternative is that the resistance model represents the mean value, in which case the assumed mean value of resistance will be lower by approximately 20% in the case of the concrete, compare (4.55) and (4.57). In Figure 8.17 a comparison is made between the results of the reliability analysis under these two assumptions. The figure shows how the change of assumptions causes a significant drop in the resulting safety index. This is a reasonably obvious result, however, it should be borne in mind when comparing models from older design codes which may have been lacking in some areas or where known problems of design have subsequently been discovered.

Figure 8.18 shows the same comparison but for the case of a 13 m steel bridge with an allowable axle load of 30 tonnes. The change in mean value of resistance from the one assumption to the other represents a 14% lower value. This change of assumption produces a lower safety index, with the respective curves undergoing a parallel displacement in the vertical plane. This relatively small change in the assumed value of the mean of the resistance produces considerably lower safety indices.
Figure 8.17: The figure shows the effect of changing the underlying assumption of the resistance model. The case of the resistance model used in design representing the mean and the 5% quantile are detailed. Shown are the results for a concrete span bridge of 8 m and for the cases of 27.5 tonnes axle load under the assumption of a GPD model. The mean value assumption yields a lower assumed mean value of resistance of the section and hence a lower safety index.

Figure 8.18: The figure shows the effect of changing the underlying assumption of the resistance model. The case of the resistance model used in design representing the mean and the 5% quantile are detailed. Shown are the results for a 13 m steel bridge and for the cases of 30 tonnes axle load under the assumption of a GPD model. The mean value assumption yields a lower assumed mean value of resistance of the section and hence a lower safety index.
Chapter 9

Discussions and Conclusions

9.1 General

The work in this thesis presented methods by which field data, such as measured axle loads, axle spacings, train configurations and speeds, may be analysed to create a model of the traffic load effects on railway bridges.

The traffic load effect considered in this thesis was the mid-span moment of simply supported bridges. Only short to medium span bridges have been studied in the thesis with spans ranging from 4–30 metres.

The model used to simulate the load effect from the collected field data was the so-called moving force model. In this model, the bridge was portrayed as a two dimensional beam of constant cross-section and bending stiffness. Each train was idealised as a succession of point loads travelling at constant speed. Accordingly, only the dynamic effect of trains travelling across a bridge with a geometrically perfect track can be simulated in this model. The effects of track defects was allowed for afterwards by multiplying the dynamic moment history by a factor, which was dependent on the train speed and the bridge span. Also the forces were modelled as having direct contact with the bridge, therefore neglecting the distribution of axle loads via rail, sleepers and possible ballast.

The model used to describe the traffic load effects from the results of the moving force model was based on Extreme Value Theory and adopts the use of the family of distributions known collectively as the Generalised Extreme Value (GEV) or alternatively the Generalised Pareto Distribution (GPD). This model allows for the extrapolation past the available data. While the extrapolation past the existing data can not be justified mathematically the extreme value theory provides the only consistent model for this extrapolation. The uncertainties involved in the parameter estimates were incorporated and, at least to some degree, allowed for in the method. Since this model builds on the theory of extreme values it is only suitable for use in the ultimate limit state analysis.

Further, the model of the load effect was incorporated into a reliability analysis,
making it possible to investigate upgrading of existing railway bridges to greater permissible axle loads. Only the ultimate limit state was considered in this analysis. In order to perform a reliability analysis, information was gathered concerning the codes to which the existing bridges were designed. Only a limited number of material codes and a limited number of code traffic load models have been considered. In some cases quite coarse assumptions were made, especially as regards the inferred safety factors of the code formats using permissible stresses. Also due to lack of information, the material codes of one period are used in conjunction with load models from another period, which is obviously incorrect but at least provides an indication as to the safety of the structures.

To the knowledge of the author, no previous work has adopted the use of measured axle loads and train configurations when attempting to describe a model for the traffic load effects. The majority of the earlier work appears to assume deterministic axle loads and in the majority of the cases even deterministic train configurations.

9.2 Field Data

The measuring errors in the field data of the axle loads were difficult to quantify. The study of the empty freight wagons gave results that implied negligible bias and a coefficient of variation of 5–7%. However, the studies of the locomotives showed the readings to contain bias, generally overestimating the loading from the axles of the locomotives. They also showed a dependency on the speed of the locomotive, which of course is false. This may be due to the proposed idea that the driving axles of the locomotives act to distort the readings. However, while feasible, this can not be confirmed with the material at hand. The other possibility, at least for the case of the RC locomotives, is that freight wagons with axle spacings close to those of the locomotives are mistakenly identified as RC locomotives and therefore included in their statistics. However, this does not account for the dependency of the axle loads on the speed of the vehicle and some form of bias at least for the locomotives can be expected. If one assumes that the hypothesis that the driving axles of the locomotives yield erroneous measurements is true, then it implies that these biased measurements were also included into the analysis, as all successfully registered axles were included. Hence, the field measurements would tend to overestimate the actual axle loads. The highest measured axle at Sannahed was noted on a two axled freight wagon and was 306 kN, approximately 30 t, this appears to very high when compared with the allowable axle load of 22.5 t. The highest measured axle of an RC locomotive, from the Notviken site, was registered at approximately 42 t despite these axle possessing almost deterministic values at approximately half this value. Although the measurements from Notviken were not used in the final analysis, the measuring method was similar to the one at Sannahed and it does raise questions about the accuracy of the measuring stations. However, the study does seem to suggest that the measurements produce sometimes erroneously high axle loads. Since the extreme value analysis concentrates on the right tail of the distributions and therefore on the high values this may well mean that it yields
conservative results.

Another problem with the data received from the measuring station was that if the wagon type was not contained within the library of wagons for the system then the system has no way of recognising them and therefore are eliminated from the analysis. Indeed since the purpose of this thesis was to investigate an increase of the allowable axle loads for normal freight wagons, special transports such as those shown in Figure 3.5, were purposely excluded from my library of wagons despite being included in the weighing stations in order that they would not be recognised. This was done to exclude these special cases from the analysis which would otherwise have distorted the results for normal rail traffic. Therefore, the problem was that possibly normal freight wagons are not recognised or even attributed to other wagon types and therefore dangerous axle configurations may be missed. The more desirable information from the stations would be the raw data or at least data which contains the axle spacings, static loads and train speed. However for various practical limitations this type of data acquisition was not possible.

9.3 Moving Force Model and the Dynamic Factor

For the short span bridges, \( L \leq 8 \text{ m} \), the computer simulations gave very little information on the overall dynamic factor as the dynamic effects for short high frequency span bridges are almost entirely composed of the dynamic effect from track defects. However, it did, at a minimum, yield the 'static' moment distribution, as the contribution of \( \varphi' \) being relatively small. This was not so computationally expensive when compared with merely calculating the static moment distribution for these 7400 trains consisting of more than 300 thousand axles.

One major disadvantage of the proposed method is that the computer simulation time for the moving forces across the two dimensional bridges is exhaustive. For the 7400 trains this involved three weeks of continuous simulation per span on a 500 MHz Pentium III processor, with 524 MB RAM. This is unsatisfactory when attempting to use this method systematically for the reclassification of bridges. However, the computer program used was written to incorporate many other types of phenomena such as track irregularities and vehicles with suspensions, track wheel interaction etc. It is therefore possible that developing the program to solely deal with the moving force scenario and using a compiled version of the program may improve its performance and make this approach more viable. Also, the continuing development of faster computer hardware will help to make this type of approach less time consuming.

The allowance of the dynamic effect due to the track imperfections was very unsatisfactory and produced close to deterministic results. The hope when using the approach was that the variation in speed would also produce a variation in the factor \((1 + 0.5\varphi'')\). The author had however overlooked the upper limit of this value at speeds in excess of 79 km/h, see (B.9). It is believed that the allowance for this factor is conservative. Therefore, this part of the analysis needs to be reviewed.
CHAPTER 9. DISCUSSIONS AND CONCLUSIONS

and improved. The only problem being that the literature study into the dynamic amplification factor show that isolation of this term has proven to be difficult.

The dynamic amplification factor is difficult to describe as a purely random variable. Certain unfavourable combinations of train type, bridge type and damping etc. caused repeatedly high values. However, it appears from the literature study of this phenomena that even when considering the same bridge, the properties that affect this quantity are by no means constant and the resulting actual dynamic amplification factor will therefore vary, and seemingly quite substantially, around a theoretical value. In Europe, the studies of the dynamic coefficient have concentrated on theoretical studies with relatively few field measurements. Also, from the results it has been difficult to isolate the dynamic effects for different types of trains, i.e. freight, passenger etc. Another aspect is that even when the dynamic coefficient has been measured it has been done using locomotives and not the freight wagons. There is an apparent need to undertake field measurements of the dynamic amplification factor. It would be advantageous if this could be separated into differing categories, such as train types (freight or passenger), bridge types (concrete and steel) and especially to distinguish between ballasted and unballasted bridges. Of special interest when raising allowable axle loads is the dynamic factors associated from freight wagons, as it is these wagons that produce the extreme loading cases. Field studies of the dynamic amplification have begun in Sweden, see e.g. (Ansell, 2000; Enckell-El et al., 2003).

9.4 Extreme Value Description of the Load Effect

The method produces some unsatisfactory results, where the reliability analysis is sensitive to the assumed tail behaviour of the load effect. Logically there is no apparent reason why the GPD method should yield differing tail behaviour for bridges with similar spans. This, however, was observed, e.g. for the spans of 10 and 13 m. The first of the spans producing a heavy Fréchet type of tail, while the 13 m span produced a tail with an upper end point, i.e. a Weibull tail. Although a further investigation of the 10 m span suggested that the tail was at least less heavy than the original parameter estimates suggested, it does highlight the problem of the sensitivity of the analysis to the tail behaviour.

The Fréchet type tail in the GPD model, \( \xi > 0 \), yields much lower safety indices and the analysis appears to be very sensitive to variations of \( \xi \) above zero. This is not surprising as the tail of this type of distribution approaches infinity relatively slowly. However, this does illustrate a weakness in this type of analysis as the choice of threshold affects the estimated value of \( \xi \). This theoretically should not be the case in the GPD model as an increase in threshold should produce the same value of shape parameter. In practice this does not appear to be the case, which may imply that the model is unsuitable for this data, although the paper (Caers and Maes, 1998) also highlights this problem and uses the GPD model for analysis.

A Weibull type tail was often found to be the result of fitting of the GEV or the
9.5. RELIABILITY ANALYSIS

By fitting the GPD to the data, which produces an upper limit of the load effect. However, the results were sometimes contradictory between the two models. It may be prudent to choose a Gumbel tail instead of a Weibull tail in these instances as the Gumbel tail does not have an upper bound. Also from the evidence of the study done on the axle loads exceeding 150 kN, it seems reasonable to model the axle loads as normally distributed and since these have a dominating effect on the load effect then a normal distribution may be a reasonable assumption for the load effect itself. The asymptotic distribution of a normally distributed parent distribution is a Gumbel distribution (Leadbetter et al., 1997) and therefore a reasonable portrayal of the tail behaviour. However, this does have the restriction of limiting the estimation into one type of family of extreme distributions while the beauty of the GEV formulation is that no prior knowledge of the family is necessary and it can be decided from the data. Also, using the asymptotic variance-covariance matrix of the parameter estimates allows for the variation of the parameters including the shape parameter $\xi$. Should the parameter estimate of $\xi$ lie close to zero, then one can expect the variation to include a Gumbel tail.

Despite the above described difficulties, the fitting of the data to both the GEV and the GPD models seems to suggest their suitability to the representation of the traffic load effects. The GEV method has the advantage of not requiring a threshold choice, however, the choice of the number of trains in a set, from which the maximum is taken, is somewhat analogous. The disadvantage with the GEV model is that within each set there may exist several interesting high values that are discarded as only the maximum per set is included in the analysis. This loss of informative data is avoided in the POT model as all maxima above the threshold level are included in the analysis.

The method for allowing for the effect of an increase in the allowable axle load was assumed to be linear although the variance was assumed to be the square of the increase. This does not allow for any systematic change in behaviour when loading the wagons. The linear increase was based on the gross loaded weights, it may on hindsight have been better to use the net weights, when assessing the change of variance.

9.5 Reliability Analysis

The results of the reliability analysis using the load effects from the field data, show that for concrete structures an increase in allowable axle load to 25 t would be permitted for bridges of eight metres and up to the maximum 30 m span studied. This applies to the bridges design to the what is referred to as the 1960 and the 1980 standards, see section 3.6, for this definition. For these spans, even the concrete bridges design to the 1940 standard would tolerate an increase if the lower yearly target safety index of 3.7 is found acceptable for these existing structures. This was even found true for the majority of the cases for a proposed increase to 27.5 tonnes.

The concrete bridges appear, from this study, to even tolerate an increase to 30 tonnes
for spans ranging from 13–30 m and built to the 1960 and 1980 standards. For the majority of the studied cases and for span bridges greater or equal to eight metres, the safety level was found acceptable if the yearly target safety index of 3.7 is adopted.

It should be remembered that only the ultimate limit state has been investigated and only the load effect of the mid-span moment of simply supported beams and that these results should be seen as an indicator as to potential rather than precise answers. However, this type of analysis would be valid even for other types of load effects and should, if so desired, be easily extended to investigate other effects.

The small span bridge of four metres was, according to this study the most sensitive to a proposed increase in allowable axle load. The reasons for this is believed to be a combination of the adverse effects of overloading of isolated bogies and axles and of the choice of modelling the dynamic effect due to track defects. The dynamic effect due to the track defect is believed to be erroneously high, due to the high choice of natural frequency of the bridge. However, this overestimation of the dynamic effect from the track defect may well compensate for what is most likely an underestimation of the effects from $\varphi'$ which will occur as a result of the chosen high natural frequency. Another point to note for these small span bridges is that the load distribution effects of any ballast present, or of the distribution effect of even the rail and the sleepers has been ignored. For these small span bridges, the effects of load distribution must be quite substantial especially for ballasted track. It was also this short span bridge that produced the worst fitting to the extreme value models. The data, especially the last point, showing higher values than that predicted by the GPD model, the values being 1.13 and 1.03 respectively, cf. Figure C.2. This implies that the model is on the unsafe side with observed values larger than the predicted values.

The steel bridges studied, in the majority of cases, appear to permit a proposed increase to 25 tonnes, even for the bridges built to the 1901 codes, at least in the studied ultimate limit state. The study also indicates that an increase to both 27.5 and 30 tonnes appears to be possible for many of the studied spans. However, there was limited information when inferring the safety factors for most of these codes and since the analysis is sensitive to small changes in the underlying assumptions for these steel structures, further investigation is warranted. Also, for the older bridges fatigue considerations rather than ultimate limit state analysis are presumably of upmost importance.

The sensitivity of the reliability results to the type of tail behaviour, Weibull or Fréchet, was particulary noticeable for the case of steel. It is believed that the Weibull type behaviour of the load effect coupled with the small variations associated with the resistance model give rise to two near deterministic variables that even if close together only have a small risk of ‘overlapping’ one another. This yields high safety levels which are decreased dramatically when the load effect is modelled with a slowly varying tail behaviour, compare sub-figures (a) and (b) of Figure 8.11. This would also imply that the results are probably sensitive to the chosen mean value of resistance. Some form of model uncertainty on the load side may therefore be appropriate for these cases.
The overall safety factor of 1.63, which was assumed for steel constructions designed according to the allowable stress code format appears to be high and this figure together with the assumed properties of steel needs to be studied further before too many conclusions can be made from the reliability studies. In the case of steel structures, the reliability analysis is very sensitive to an increase in the assumed value of the mean of the resistance value as can be seen from the small increase of 14% associated with the allowance for the resistance model being equivalent to a 5% quantile or the mean value, cf. Figure 8.18. Also, for steel constructions, the only material data regarding the variability of the material strengths have been based on presumably post 1960 steel testing. The variation of steel quality and manufacturing tolerances, in these earlier constructions may well posses a different coefficient of variation which will affect the results. A larger variation will affect the results adversely while the contrary will be true for smaller variations. The most likely results being a wider variation, as the manufacturing techniques and tolerances were presumably of a lower standard than those of the latter day.

This lack of knowledge of the material data from the earlier constructions is also true of the concrete structures and the coefficient of variation of the yield strength of rebars and the strength of the concrete has been based on these latter day qualities. However, with regard to the concrete, no allowance for the strengthening with age has been taken into account which is a conservative approach. It should be stressed that all the results assume that the structural integrity of the bridges have been maintained, i.e. no adverse corrosion, carbonisation deterioration of the structure etc.

The results are very sensitive to changes in the underlying assumptions and a number of uncertainties may be associated with this type of modelling of the load effect. It may be advisable to include some form of model uncertainty for the load effect in the reliability calculations. This is especially so if a more refined reliability analysis is to be undertaken where the dynamic effect from track irregularities are more clearly quantified together with the material properties.

As regards uncertainties, on the one hand there are the problems of choosing a threshold and the uncertainties involved in the theoretical model suiting the data. The latter is in some way quantified in this analysis. However, the dynamic effect from track defects has been chosen conservatively, for the smaller spans that showed problems in allowing an increasing in allowable axle load. Another conservative assumption, as mentioned previously, was to neglect the effects of load spreading via the track and possible ballast. The results of the analysis show consistently small variances for the load effect and this seems to be almost deterministic even when regarding the distribution of the daily maxima. When compared to road vehicles and therefore road bridges the uncertainties associated with the lateral position of the vehicle are negligible for railway bridges. For railway bridges, the point of application of the loading is clearly defined as one knows precisely the position of the rails. Also, when determining the moments for which the bridges where originally designed, wherever the code format gave a distinction between a ballasted or unballasted track, the assumption was an ballasted track. This yields a lower assumed moment for which the bridge was designed and therefore a conservative
approach. However, the choice of the larger of the load models specified in the respective codes, was non-conservative.

The increase in fatigue rate as a result of an increase in the allowable axle load has not been considered in this analysis. However, within the WAFO toolbox there is a function that enables the stress cycles to be counted using the rainflow counting method, cf. (Frýba, 1996; Brodtkorb et al., 2000). There are also other functions within this toolbox for calculating the damage effects from hysteresis loops. This would however require more specific information on typical bridges so that correct stress levels could be obtained when performing a fatigue analysis. This could be incorporated into a traditional or a reliability analysis to check the fatigue criteria.

Only the main beams have been studied in this thesis, how local details are affected has not been considered. Also the analysis has shown that the short span bridges are the most susceptible to the effects of overloaded axles. This implies that stringers and cross-bearers may also be highly strained, especially when there are small capabilities of load distribution to surrounding elements, which is especially true for unballasted bridges.

With the exception of the four metre span bridge, the traffic load effects from real traffic yield much lower values than those given by the UIC 71 code loading despite using large dynamic factors when allowing for track irregularities, at least for the shorter span bridges. This was believed to be a result of the actual axle spacings being more beneficial that those of the load models. This raises the question of wagon design where, from a bridge’s point of view, a concentration of axles should be avoided. Examples of such concentrations being triple axle boogies, bogies with short distances between axles and wagons with a short distance from boogie/axle centres to end of buffer. For locomotives this is not as important as the loads from the axles are well defined. Also the question of overloading is of great importance when attempting to increase the permissible axle loads. It may be possible to introduce a system where handlers prepared to weigh their loaded wagons can be privileged by allowing a higher axle load for their freight.

9.6 Further Research

Field measurements of the dynamic factor appear to be the exception rather than the rule. Many of the measurements date back to the 50’s and 60’s where the measuring techniques may well be questioned. In the series done in Europe on built-in beams, the error of the measuring instruments were of the same order of the measurements themselves, (Specialists’ Committee D 23, 1970a). Field measurements of the dynamic factor especially related to freight trains could be of invaluable interest in the work to upgrade existing railway bridges. For short span high frequency bridges the effect of the dynamic effect due to track irregularities form a large part of the overall load effect. It would be an interesting investigation to study the effects of track irregularities specifically for freight trains with typical freight operating speeds.
A reliability study for the shorter span bridges, could be limited to the situation of two four-axle freight wagons where the dimensioning case is the rear of one wagon and the front of the other. One of the sets of bogie axles could thus be modelled using an extreme value distribution, similar to that done in Chapter 6, while the other set of bogie axles could assume a point-in-time distribution e.g. a normal distribution with an appropriate coefficient of variation. Differing means and standard deviations could be chosen to represent an increase in allowable axle load. This would however only yield information in the ultimate limit state.

Modelling of the material properties prior to 1960 needs to be reviewed, so that the reliability calculations for these early bridges should be seen more as a guidance than a definitive answer. Further research is required in the collection of data from which the material properties of this older steel and concrete can be described in the form of mean and standard deviations.

Only a limited number of combinations of load models and structural codes have been investigated, in some cases quite coarse assumptions have been made to infer the safety factors inherent in the allowable stress code formats. This work needs to be reviewed and a full study of the older codes and their safety factors needs to be assessed. Also the code load models used in this thesis need to be complimented with the other load model cases that have not been considered. As an example, when discussing the 1901 bridges only the heavier model of Load Model A has been considered, see Figure 3.1, while bridges of the age may well have been built to one of the other load models shown in the figure. However, once this information is gathered it should be a simple exercise to incorporate the new values into the analysis. Also the distinction between ballasted and unballasted bridges should be introduced.

There is another method, other than the one described here, for the treatment of the uncertainties associated with extrapolation under the GPD and the GEV load models. It is discussed in (Coles, 2001) and adopts a method called the profile likelihood which uses a logarithmic scale so that the greater the extrapolation the greater the uncertainties of say the return load. This would be an interesting method to pursue and to investigate the effects on the inference. The problem of clustering of results has not been tackled in this thesis and the effects of the use of a de-clustering algorithms on the peaks-over-threshold method need also to be pursued. Also the choice of the optimal threshold value remains unresolved and requires further investigation for these applications.

No variation of the first natural frequency per span was investigated and only one value of damping was used throughout the simulations. The study should therefore be complimented with a study of how a variation in these values affect the final results of the reliability calculations.

The shear criteria has not been checked for these bridges and while it is quite possible that the moment ratios are also indicative of the safety indices in shear this should be investigated. The design criteria, safety factors and material properties, such as model uncertainty and coefficient of variation, in shear may differ substantially from...
those assumed in this study.

The measuring stations provide a wealth of information as to the ‘real’ traffic loads on Swedish railway bridges. The analysis of this field data could be developed further to form a statistical description of typical railway traffic. For example, from the results of the measured axle loads, detailed in section 8.2.1, it should be possible to formulate a statistical representation of the axle loads and use them in a reliability method, similar to the method described in Chapter 6. No distinction is made in these results between locomotives and freight wagons, although it is not a difficult task to compliment these findings by isolating the statistics for the freight wagons.


Kommunikationsdepartementet (1948). Normalbestämmelser för järnkonstruktioner till bygnadsverk.


Specialists’ Committee D 192 (1993). Loading diagram to be taken into consideration for the calculation of rail-carrying structures on lines used by international services—Theoretical basis for verifying the present UIC 71 loading. Technical report, European Rail Research Institute (ERRI), Utrecht. RP 1.

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Specialists’ Committee D 192 (1994b). Loading diagram to be taken into consideration for the calculation of rail-carrying structures on lines used by international services—comparison of the effects of current and future rail traffic on international lines with the effects from uic 71 loading on a probabilistic basis. Technical report, European Rail Research Institute (ERRI), Utrecht. RP 3.


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International Union of Railways UIC, Office for Research and Experiment ORE, Utrecht.


Appendix A

Standard Trough Bridge Data

This appendix gives details of the section properties of the bridges used in the simulations using the moving force model of section 7.2. The bridges chosen are standard trough bridges that were used to bridge small openings in Sweden. The section properties are taken from drawings that were used for standard bridges from the period circa 1950–1970.

A.1 Spans of 2–4.5 Metres

Concrete quality Std K 400 reinforcement Ks 40 and Ks 60. The building codes specified in the drawings are (Kommunikationsdepartementet, 1961; Statens Betong Kommitté, 1968) and VV bronorm 1969.

The troughs were filled with 600 mm of ballast so that the underside of the rail was level with the top of the edge beams. In the calculation of the section properties, the sloping part that constitutes the wall of the inner trough, has been neglected so that the wall is assumed to be homogeneously 250 mm thick. The extra mass effect due to the ballast has been taken into account when calculating the natural bending frequencies of the bridges.

Table A.1 shows a summary of the properties of the bridges used in the computer

![Figure A.1: Section of a typical trough bridge with a free opening of 2 to 4 metres. Redrawn from drawing B2447-2.](image-url)
Table A.1: Summary of the bridge properties.

<table>
<thead>
<tr>
<th>Span [m]</th>
<th>(d) [mm]</th>
<th>(I) [m(^4)]</th>
<th>(E) [GPa]</th>
<th>(A) [m(^2)]</th>
<th>(m_{\text{eff}}) [kg/m]</th>
<th>(f_1) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>300</td>
<td>0.1142</td>
<td>30</td>
<td>1.65</td>
<td>9075</td>
<td>60.3</td>
</tr>
<tr>
<td>8.0</td>
<td>-</td>
<td>0.3946</td>
<td>30</td>
<td>2.46</td>
<td>11538</td>
<td>24.9</td>
</tr>
<tr>
<td>10.0</td>
<td>-</td>
<td>0.3946</td>
<td>30</td>
<td>2.46</td>
<td>11538</td>
<td>15.9</td>
</tr>
<tr>
<td>13.0</td>
<td>-</td>
<td>0.4579</td>
<td>30</td>
<td>2.80</td>
<td>11536</td>
<td>10.1</td>
</tr>
<tr>
<td>20.0</td>
<td>-</td>
<td>0.3322</td>
<td>30</td>
<td>3.54</td>
<td>9205</td>
<td>4.1</td>
</tr>
<tr>
<td>30.0</td>
<td>-</td>
<td>1.4000</td>
<td>30</td>
<td>4.00</td>
<td>10400</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Figure A.2: Section of a standard trough bridge with a free opening of 8 to 11 metres.
Redrawn from drawing B2447-32

simulations, where \(d\) is the depth of the bottom slab, \(E\) is the elastic modulus of the concrete, \(I\) is the moment of inertia, \(A\) the cross-sectional area, \(m_{\text{eff}}\) is the effective mass per metre and finally \(f_1\) is the first bending frequency.

A.2 Bridges of Spans 8-11 metres

Figure A.2 shows the dimensions of a standard simply supported trough bridge for spans between 8 and 11 metres. The thickness of the slab in the original drawings varies with a greater dimension in the middle than at the supports, this however has been ignored. The slab is approximated as homogeneous and the dimension used is the smallest, i.e. that at the supports. When calculated the section properties the slanting walls have been approximated as vertical with a thickness of 500 mm. The basic dimensions of the bridges are unchanged from span to span however the number of reinforcement bars differ.
Appendix B

Dynamic Factors from Eurocodes

B.1 Dynamic Amplification Factor using simplified Method of the Eurocodes

The dynamic amplification factor according to the Eurocodes (CEN, 1995) is given by the following for carefully maintained track

\[ \Phi_2 = \frac{1.44}{\sqrt{L_\Phi} - 0.2} + 0.82 \]  
(B.1)

where the limits \(1.00 \leq \Phi_2 \leq 1.67\) apply.

For normally maintained track, standard maintenance the following is used

\[ \Phi_3 = \frac{2.16}{\sqrt{L_\Phi} - 0.2} + 0.73 \]  
(B.2)

where the limits \(1.00 \leq \Phi_3 \leq 2.00\) apply.

B.2 Dynamic Annex E and F of Eurocode

According to this method the dynamic amplification factor is given by either:

\[ 1 + \varphi = 1 + \varphi' + \varphi'' \]  
(B.3)

or:

\[ 1 + \varphi = 1 + \varphi' + 0.5\varphi'' \]  
(B.4)

where (B.4) should be used unless otherwise specified, as this equation represents a well maintained track. The factors \(\varphi'\) and \(\varphi''\) are, according to (UIC, 1979) factors associated with dynamics of the bridge from a track in perfect condition and the effects due to track irregularities respectively.
The factor $\varphi'$ is given by:

$$\varphi' = \frac{K}{1 - K + K^4} \quad (B.5)$$

where the factor $K$ is given by

$$K = \frac{v}{2L_\Phi n_o} \quad (B.6)$$

where $v$ is the velocity of the train, $L_\Phi$ the determinant length in metres and $n_o$ is the natural frequency of the bridge. The determinant length, in the case of a simply supported beam, is the span of the bridge and the limits of the natural frequency is given by (B.7–B.8)

If the natural frequency, $n_o$, of a proposed railway bridge is unknown then using the formulae from (CEN, 1995) an upper and lower limit can be calculated. The upper limit is hence given by:

$$n_o = 94.76L_\Phi^{-0.748} \quad (B.7)$$

and the lower limit by

$$\begin{align*}
  n_o &= 80/L_\Phi \quad \text{for } 4 \text{ m} \leq L_\Phi \leq 20 \text{ m} \quad (B.8a) \\
  n_o &= 23.58L_\Phi^{-0.592} \quad \text{for } 20 \text{ m} < L_\Phi \leq 100 \text{ m} \quad (B.8b)
\end{align*}$$

The dynamic factor due to track irregularities, $\varphi''$, is given by:

$$\varphi'' = \frac{\alpha}{100} \left\{ 56 \exp \left[ - \left( \frac{L_\Phi}{10} \right)^2 \right] + 50 \left( \frac{L_\Phi n_o}{80} - 1 \right) \exp \left[ - \left( \frac{L_\Phi}{10} \right)^2 \right] \right\} \quad (B.9)$$

where $\alpha$ is a speed coefficient given by:

$$\begin{align*}
  \alpha &= v/22 \quad \text{for } v \leq 22 \text{ m/s} \quad (B.10a) \\
  \alpha &= 1 \quad \text{for } v > 22 \text{ m/s} \quad (B.10b)
\end{align*}$$
Appendix C

Miscellaneous Results

Table C.1: Estimated properties of the 50 year return load and the parameter estimates of the GEV and the Gumbel models for different spans. Estimated properties for different spans based on the maximum of 50 trains. Dynamic effect from track defects calculated using upper UIC bridge frequencies.

<table>
<thead>
<tr>
<th>$L$</th>
<th>Distrib.</th>
<th>Estimated 50 year return loads</th>
<th>Parameter estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>4</td>
<td>GEV</td>
<td>0.9471</td>
<td>0.0086</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>1.0684</td>
<td>0.0225</td>
</tr>
<tr>
<td>8</td>
<td>GEV</td>
<td>0.8268</td>
<td>0.0083</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.9013</td>
<td>0.0174</td>
</tr>
<tr>
<td>10</td>
<td>GEV</td>
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<td>0.0104</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
<td>0.9200</td>
<td>0.0179</td>
</tr>
<tr>
<td>13</td>
<td>GEV</td>
<td>0.7816</td>
<td>0.0042</td>
</tr>
<tr>
<td></td>
<td>Gumbel</td>
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Table C.2: The parameter estimates of the GEV and the Gumbel distributions that gave rise to the 95 % quantile of the return load.

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Figure C.1: QQ plots for the GEV model estimates. Dynamic effects from track defects use the actual frequency used in the simulations.
Figure C.2: QQ plots for the heights over threshold for GPD model estimates. Dynamic effects from track defects use the actual frequency used in the simulations.
Figure C.3: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for re-bars assuming an increase to 25 tonne axle using the assumption of a GPD traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
APPENDIX C. MISCELLANEOUS RESULTS

Figure C.4: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for steel bridges assuming an increase to 25 tonne axle using the assumption of a GPD traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
Figure C.5: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for steel bridges assuming an increase to 27.5 tonne axle using the assumption of a GPD traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
Figure C.6: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for re-bars bridges assuming an increase to 27.5 tonne axle using the assumption of a GEV traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
Figure C.7: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for concrete assuming an increase to 30 tonne axle using the assumption of a GEV traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
Figure C.8: The reliability index $\beta$ versus $\nu$, the ratio of load effect from self-weight to UIC71/SW2 load effect, see (4.62). Results for re-bars bridges assuming an increase to 30 tonne axle using the assumption of a GPD traffic load effect. The mean and the 95% quantile values are shown for each code year. The target daily reliability index is shown at approx. 5.9, values above this level are acceptable.
Table C.3: The estimated properties of the GEV calculated from the properties of the GPD and used in the reliability analysis, see section 5.4.7. Two values are shown one for the original parameter estimates and one for the parameter estimates that produced the 95% quantile of the 50 year return load.

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